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# Final Report For Phase I SBIR Project:

Superconducting SQUID Amplifiers for Infrared Detectors And Other Applications

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David Osterman Hypres, Inc. 175 Clearbrook Road Elmsford, New York 10523 (914) 592-1190 Ext. 7818





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# 1. INTRODUCTION

Superconducting thin film electronics have properties which make them potentially superior to conventional electronics in applications where low power and high data processing rates are required. One such application is infrared focal plane arrays (IRFPAs), for which it is now widely accepted that the best strategy for achieving high performance is to locate the processing electronics as close to the focal plane array as possible. An extremely attractive version of this strategy is to physically locate the electronics directly on the focal plane array substrate. This is feasible if the infrared detectors themselves operate at low temperatures. Detectors that operate at low temperatures include superconducting detectors, now in development at HYPRES, and long wavelength cut-off semiconductor detectors, typically designed for space applications. Clearly this requires circuit elements to be small in area so the maximum real estate on the array substrate can be devoted to the detectors themselves.

In applying these considerations to superconducting circuitry it becomes evident that, because of the relatively large area that they occupy, superconducting amplifiers will be difficult to fit onto a focal plane array if one amplifier is assigned to each detector. The most well developed version of the superconducting amplifier is the SQUID (Superconducting QUantum Interference Device). In its thin film version the SQUID has a multi-turn coil for inductive coupling of the input which, for practical applications, may extend tens or hundreds of microns on a side. One solution to this space problem is to multiplex inputs into a single SQUID amplifier thereby allowing many detectors to share one amplifier. This report gives the results of a Phase I SBIR project to design and investigate a superconducting multiplexing circuit that is compatible with a SQUID amplifier.

The multiplexing circuit uses as a basic unit an element that functions as a variable mutual inductance. It has been named a parametric flux transformer or PFT for short. By means of a "control" input PFT's allow mutual inductances to be reduced to zero or increased to large values, thereby controllably preventing signals from passing or, alternatively passing them with gain.

This report describes how the mulitplexer works and a circuit schematic for a five input version is given. It is clear from this schematic how to extend the circuit to multiplexers with greater (or fewer) numbers of inputs. Calculations have been performed of the noise arising from the multiplexer itself as well as the effect of the multiplexer on the noise contribution from the SQUID amplifier. Crosstalk between multiplexer inputs has also been investigated as well as the effects of circuit parameters that are slightly different from the ideal parameters owing to variations introduced in circuit fabrication. Finally, a set of equations has been derived for optimizing the multiplexer circuit so that it will not be too noisy and will also fit into a reasonably small area. These equations have been solved for one specific instance, where the inputs are from superconducting infrared detectors, and

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solutions are derived which give all the circuit parameters.

During the time of this Phase I SBIR project, in addition to the calculations performed, properly functioning dc SQUID amplifiers were fabricated at HYPRES. A description of these amplifiers is included in this report. For the sake of giving a complete background there is also a brief description of HYPRES' superconducting infrared detectors. Sections 2-6 of this report have been incorporated into the proposal for Phase II SBIR funding of this work.

## 2. <u>EXPLANATION OF SOUID AMPLIFIERS AND HYPRES</u> SUPERCONDUCTING IR DETECTORS

## 2.1 SQUID AMPLIFIER - OPERATING PRINCIPLE

The operation of the dc SQUID amplifier may be understood with the help of Figs. 1 and 2. The components of the amplifier, shown in Fig. 1a are: 1). the 4-terminal SQUID detector and 2). an input circuit which inductively couples the voltage to be amplified into the SQUID. The dc SQUID is a nonlinear superconducting device composed of two Josephson junctions in a closed loop. A related device, the rf SQUID, has a single Josephson junction in a closed loop. The junctions of the dc SQUID must be resistively shunted to remove hysteresis. Like most superconducting devices it has a critical current I<sub>c</sub> below which it has no voltage. When current-biased at the correct level above I<sub>c</sub> its voltage output is periodic in the flux applied to it. This periodic relation is shown in Fig. 2 for an actual SQUID fabricated and tested at HYPRES. One half period of this flux-to-voltage input is converted into flux via the input circuit and the working point of the SQUID is maintained on one of the steep sections of the transfer relation.

Noise in the amplifier is determined by two factors. The first is the intrinsic noise of the SQUID. This has been shown<sup>1</sup> to be dependent on the critical current and resistive shunts of the Josephson junctions of which it is composed. The second factor is the characteristics of the input circuit in translating this intrinsic noise into an equivalent noise at the amplifier input<sup>2</sup>. This depends on parameters of the input circuit and its inductive coupling to the SQUID. The intrinsic noise of SQUIDs already fabricated at HYPRES has been measured and found to be sufficiently low for these SQUIDs to meet the requirements of the proposed amplifiers.

Like noise, the gain and dynamic range of the amplifier depend on the intrinsic properties of the SQUID as well as the input circuit. The process of optimizing the input noise, dynamic range and gain of a SQUID amplifier must therefore take both of these factors into account.

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# 2.2 SUPERCONDUCTING INFRARED DETECTORS - OPERATING PRINCIPLE

The mechanism of HYPRES' superconducting infrared detectors is well understood on both a theoretical and experimental level<sup>3,4</sup>. The detectors are superconducting Josephson tunnel junctions. Because they exhibit a temperature dependent current-voltage characteristic, they function as detectors of light when they are properly configured, both thermodynamically and electrically, as thermal detectors. The effect of incident light on the current-voltage relation is then as shown in Fig. 4. To utilize this effect for optical detection the Josephson junction is current-biased on the steep quasiparticle branch of the current-voltage curve and the detector output is a voltage. For temperatures below the superconducting transition temperature  $T_c$ , this voltage is strongly temperature dependent and is proportional to the sum of the superconducting gap energies of the upper and lower layers of the Josephson junction. The detector can therefore operate at temperatures up to  $T_c$ . For niobium junctions  $T_c = 9.2$  K and for niobium nitride junctions  $T_c = 16$  K. Because the detection principle is thermal the optical bandwidth is broad, extending to well beyond 100  $\mu$ m.

Figure 5 is a schematic of the detector test devices that have been studied extensively. Detectors are niobium  $(Nb/Al/AlO_x/Nb)$  or niobium nitride (NbN/MgO/NbN) Josephson junctions fabricated on low thermal conductivity substrates by the standard thin film process used at HYPRES for superconducting electronics. The substrate of choice has been fused quartz  $(SiO_2)$ , however, development of back-etched silicon substrates is being pursued in parallel.

# 3. TEST AND EVALUATION RESULTS FROM SOUID AMPLIFIER CHIPS.

Figure 1b shows the basic schematic of the SQUID amplifier chips tested and evaluated in the Phase I program. On the same chip as the amplifier, and connected to the amplifier input, is a Josephson junction which functions as a superconducting IR detector. Several versions of this chip were fabricated, in part to test that important characteristics of the amplifier did in fact vary as they were predicted to. Some of these versions are listed in table I along with the predicted and measured gains. The measured gain for each amplifier is given as a range of values because the gain varies over this range as the bias point is changed, as it does in general for a properly functioning SQUID.

### TABLE I

Chip Type	R <sub>series</sub>	Predicted Gain	Measured Gain	
1	2.5 Ohms	100X	80-110X	
2	1.0 Ohms	200X	220X	
3	2.5 Ohms	100X	150X	
5	2.5 Ohms	100X	100-133X	

### Comparison of designed and measured gains in SQUID amplifiers

Note: Chips 1,3,5 are same amplifier design but with different designs of superconducting IR detector at the input.

Comparing the predicted and measured gain values in Table I it is apparent that there is good agreement. There is also verification of, among other relations, the dependence of the gain on  $R_{series}$ , (see Fig. 1b) which is one of inverse proportionality. The effects of this as well as various other parameters on the functioning of the amplifiers are thus well understood, allowing engineering of the amplifier characteristics as needed. This ability is important for making the amplifiers compatible with a multiplexing circuit, as is required for the phase II program.

The input noise of the SQUID amplifiers has been measured experimentally and calculated. The dominant noise source for the amplifiers of table I is from Johnson noise in  $R_{series}$  which is at 4.2 K. For  $R_{series} = 1$  Ohm, as for amplifier chip type 2 in table I, this gives a noise floor at the input of the amplifier of 15 pV//Hz (15 x 10<sup>-12</sup> V//Hz). This value has in fact been measured.

The dynamic range of the amplifiers is determined by the ratio of the largest to smallest signal which may be measured<sup>5</sup>. At the low end this is the noise level of the amplifier, 15 pV per  $\sqrt{Hz}$  of bandwidth in a representative amplifier. At the high end the largest input voltage which will be amplified is, for amplifier 2 of table I, 0.2  $\mu$ V. Then the dynamic range is 0.2  $\mu$ V/15pV = 13333 or 82 dB in a 1 Hz bandwidth. (The dynamic range is reduced in proportion to the square root of bandwidth). The use of room-temperature feedback electronics which are commercially available will increase the dynamic range above this value.

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The calculated bandwidth of the amplifiers is determined by the L/R time constant of the input circuit which is typically 5 MHz for the amplifiers referred to in table I. The bandwidth has not yet been measured experimentally.

The probe constructed for measuring the characteristics of SQUID amplifiers is unusual and also worthy of mention. This probe is constructed especially for low noise measurements including low noise filtering. Electromagnetic pickup during noise measurements was further reduced by conducting the measurements within a shielded room on the premises of HYPRES.

### 4. <u>NEW DESIGN FOR MULTIPLEXING MANY INPUTS INTO A SINGLE SOUID</u> <u>AMPLIFIER.</u>

The basic unit of the multiplexing scheme is a new superconducting device, the "parametric flux transformer", which incorporates an older superconducting device into a novel circuit configuration. Figure 7 shows the parametric flux transformer (PFT). It makes use of a closed superconducting loop with a single Josephson junction in the loop. In this basic aspect it is like the well-known rf SQUID (not to be confused with the <u>dc</u> SQUID which has <u>two</u> Josephson junctions in the loop, as described in Section 2.1). What is different is that there are three transformer-coupled inputs to this loop denoted "signal in", "signal out" and " control". As will be explained below current injected in the "signal in" input will produce flux at the "signal out" in an amount that can be controlled by the "control input". Since the controlled quantity is the amount of flux out for a given current in, the device functions like a transformer with variable mutual inductance  $M_{eff}$ . When inserted into a circuit a small signal gain will result that is proportional to the mutual inductance. For the correct circuit parameters,  $M_{eff}$  and the small signal gain can be varied from zero to infinity, by means of the PFT "control" input.

This capability may be applied to multiplexing in a circuit such as that of Fig 8 which is designed to multiplex five inputs into one dc SQUID. In Fig 8 the output of each PFT is coupled into a single closed superconducting loop. This is the "standard" flux transformer almost invariably used to couple into a dc SQUID. By virtue of the Meissner effect in a closed superconducting loop, the total magnetic flux within the standard flux transformer is conserved. Thus any flux generated by the outputs of the PFTs is compensated by currents flowing in the standard transformer which cancel the externally imposed flux, and the current generated injects a signal into the SQUID. When this circuit is used as a multiplexer the gain of all PFT units but one is set to zero by use of the control inputs. The one PFT with non-zero gain transmits its signal to the standard flux transformer and amplifies it as well. Thus only one chosen input reaches the dc SQUID.

There is one extremely important advantage of the PFT multiplexing scheme which needs to be emphasized and which is significant when the multiplexed inputs are from infrared or other detectors. This advantage is that the multiplexer, in addition to blocking off the signal from the detectors which are not being read, also blocks off the noise. This cannot be said about all multiplexing circuits. Multiplexing schemes which read the summed voltage output of detectors linked in a series chain, only one of which is in the "on" state, pick up the noise from all detectors in the series which are in the "off" state. These multiplexers thus pass on far more noise than exists for a single detector alone. This is avoided in the PFT multiplexing circuit. There is no extra noise from detectors which are not selected for reading.

#### 5. HOW THE MULTIPLEXER WORKS

The PFT of Fig. 7 is rendered into a nonlinear device by the presence of the single Josephson junction in its loop. The Josephson junction will pass a current that is a non-linear function of the <u>time-integrated voltage</u> across the junction. This is expressed by the well-known Josephson equations (ref. 6, pg. 166):

$$\mathbf{I} = \mathbf{I}_e \sin \phi \tag{1}$$

$$V = (h/2e) d\phi/dt$$
(2)

The important parameters for solving the circuit equations are the Josephson junction critical current  $I_e$ , the total inductance of the PFT loop L and the input and output mutual inductances  $M_1$  and  $M_2$ , all labeled in Fig. 7. From ref. 6, pg 222, the solution relating the total flux in the PFT,  $\Phi_{TOTAL}$ , to the flux imposed by the input coil,  $\Phi_{INPUT}$  is:

$$\Phi_{\text{TOTAL}} = \Phi_{\text{INPUT}} - LI_c \sin \left( 2\pi \Phi_{\text{TOTAL}} / \Phi_0 \right)$$
(3)

Here  $\Phi_0$  is the fundamental flux quantum  $\Phi_0 = h/2e = 2.07 \times 10^{-15}$  Wb and the "quasistatic" approximation is used which is a valid approximation in the present case. To obtain a more useful form it is desirable to introduce the current in the "signal in" input of the PFT,  $I_{SIG}$ , the current in the PFT loop itself,  $I_{PFT}$ , and the flux generated at the "signal out" leads,  $\Phi_{OUT}$ . These are also labeled in Fig. 7 and are related to previously defined quantities by

$$I_{SIG} = \Phi_{INPUT}/M_1$$
(4)  
$$I_{PFT} = (\Phi_{TOTAL} - \Phi_{INPUT})/L$$
$$\Phi_{OUT} = M_2 I_{PFT}$$

Substituting the definitions in Eq (4) into Eq (3) finally gives the relationship between flux at the signal out,  $\Phi_{OUT}$ , and current in the signal in,  $I_{SIG}$ .

$$I_{SIG} = (-1/M_1) \left[ L\Phi_{OUT} + (\Phi_0/2\pi) \sin^{-1}(\Phi_{OUT}/M_2I_c) \right]$$
(5)

The solution of this equation depends on the value of the parameter  $\beta_1 = L I_c 2\pi/\Phi_0$ . There are three qualitatively different types of solution, shown in Fig. 10, depending on whether  $\beta_1 < 1$ ,  $\beta_1 > 1$  or  $\beta_1 = 1$ . The behavior necessary for the PFT to function properly is obtained for  $\beta_1 = 1$ . In Fig. 11 this solution is again displayed with highlights of two extremely useful features: 1) There is a point of zero slope so that an input current will not result in any output flux for very small input signals. This is the operating point of zero mutual inductance M<sub>eff</sub> and, in the small signal limit, of zero gain. 2) There is a point of infinite slope so that an input current will produce an output flux that is infinitely larger for very small input signals. This is the point of infinite mutual inductance Meff and, in the small signal limit, of infinite gain. To use the PFT on a signal arising from an IR detector, for example, the detector signal, which is assumed to be small, is injected into the "signal in" input. Then a large signal injected into the "control" input adds to the first signal. The "control" signal is adjusted so as to place the total signal on either the zero gain or infinite gain operating points. By this method, the IR detector output is either not transmitted to the "signal out" leads, or transmitted with gain; and the choice of these two options is determined by current applied to the control output.

# 6. OPERATION OF MULTIPLEXER WITH NON-IDEAL PARAMETERS

As noted above, the ideal condition for multiplexing is that the parameter  $\beta_1 = L I_c 2\pi/\Phi_0$  satisfies  $\beta_1 = 1$ . When this is the case it is possible to vary the small signal gain between zero and infinity. For real circuits there is always some spread in parameters like  $I_c$  and L, so it is natural to ask if the circuit will work with  $\beta_1$  close to but not exactly equal to one.

From Fig 10 it is evident that the case of  $\beta_1 = 5$  results in a non-single-valued relationship between input and output. This in fact is the case whenever  $\beta_1 > 1$ . There are still two operating points where the small signal gain is zero and infinite. The double-valued output for a single input however means that the circuit will at certain times jump from one branch to another, which would produce unacceptable spikes in the output. Operating with  $\beta_1 > 1$ is therefore to be avoided.

For  $\beta_1 < 1$  the relationship between input and output is single valued and, in addition, there are points where the gain is zero. This is shown in Fig 10 for  $\beta_1 = 0.5$ . What is different from the case of  $\beta_1 = 1$  is that there are no points where the small signal gain is infinite. Instead, a solution of Eq. (5) shows that the maximum gain is proportional to  $\beta_1/(1-\beta_1)$ . The maximum gain achievable is proportional to the maximum mutual inductance  $M_{eff}^{max}$  which is given in Table II.

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### TABLE II

Maximum mutual inductance  $M_{eff}^{max}$  from non-ideal parametric flux transformer with  $\beta_1 \neq 1$  in units of  $M_1 M_2 / L$ 

 β <sub>1</sub>	M <sub>eff</sub> <sup>max</sup>	
.90	9.0	
.92	11.5	
.94	15.7	
.96	24.0	
.98	49.0	
.99	99.0	

Clearly it is still possible to obtain the multiplexing function for  $\beta_1 < 1$ . It is still possible to set the gain of the individual PFTs in the multiplexer to zero, thus shutting off the signal completely. While it is not possible to turn the PFT mutual inductance  $M_{eff}$  and gain up as high as infinity, it is still possible to get the high values of  $M_{eff}$  shown in Table II. Multiplexing is still possible with the ability to vary the gain from zero to a large finite value.

In real circuits where there is some spread in parameters due to the circuit fabrication process, multiplexing is still feasible as long as  $\beta_1 < 1$ . This becomes a practical consideration because inhomogeneity can cause variations in I<sub>c</sub> in a given wafer of  $\pm 4\%$  typically. This will be reflected in a  $\pm 4\%$  variation in  $\beta_1$ . With HYPRES' standard niobium Josephson junction circuit fabrication process a capability exists to fine tune <u>downward</u> all I<sub>c</sub>'s on a wafer by elevating the temperature for a controlled length of time. It should therefore be possible in a real circuit to fabricate one or many multiplexers with  $\beta_1 < 1$  but also within 8% of  $\beta_1 = 1$  for all multiplexers and PFTs within each multiplexer. This shows explicitly that the circuit described here will operate correctly when its components are real circuit elements which exhibit non-ideal behavior.

# 7. OPTIMIZATION OF THE COMPLETE MULTIPLEXER CIRCUIT.

Having described the characteristics of the most important part of the multiplexing circuit, namely the Parametric Flux Transformer (PFT), this section will discuss the operation of the complete multiplexer circuit and derive optimization equations for it's performance.

A schematic of a complete circuit for multiplexing five inputs into a single dc SQUID is shown in Fig. 8. It should be clear from this figure how the circuit would be modified for more (or less) multiplexed inputs. For each input the signal must be introduced through a PFT into the large common flux transformer loop. The values of all inductances and critical currents must be chosen according to equations derived below.

As pointed out in Section 5, the PFT part of the mulitplexer acts as a transformer with variable mutual inductance. The complete multiplexer circuit of Fig. 8 may therefore be modeled as a number of transformers in series each with a mutual inductance to an input,  $M_{eff}$  that may be varied between  $M_{eff} = 0$  and a maximum value  $M_{eff} = M_{eff}^{max}$ . An expression for  $M_{eff}^{max}$  may be written with the aid of Fig. 9 which labels the various inductances in the complete multiplexer circuit of Fig. 8. The expression is:

$$M_{eff}^{max} = M_1 M_2 M_3 [\beta_1 / (\beta_1 - 1)] / [(L_1 + L_2 + L_6)(L_3 + L_4 + L_7)$$
(6)

Here N is the number of inputs into the multiplexer. To produce the maximum flux in the SQUID for a given current input one must maximize  $M_{eff}^{max}$  by choosing the optimum value for the various inductances in Eq (6). This procedure is equivalent to minimizing the equivalent input noise from the dc SQUID and yields the following expression:

$$M_{eff}^{max} = K_1 K_2 K_3 [\beta_1 / (\beta_1 - 1)] / [(L_0 L_2 L_{sq}) / (L_2 + L_6)] [1 / (4 \sqrt{N})]$$
(7)

(M<sub>eff</sub><sup>max</sup> optimized)

The K<sub>i</sub>'s in this equation are the mutual inductance coefficients, for example  $K_1 = M_1/(L_0L_1)$ . The conditions for optimization, i.e. the conditions for Eq (7) to be valid, are:

$$L_4 = NL_3 \tag{8}$$

and

$$L_1 = L_2 + L_6$$
 (9)

Equations 8 and 9 are impedance matching conditions where all impedances are purely inductive.

### 8. NOISE CONSIDERATIONS IN THE COMPLETE MULTIPLEXER CIRCUIT

It is important that the noise introduced by the multiplexer not be so great as to degrade the sensitivity of the entire circuit beyond some minimum level. What is this level? The answer depends on the specific application. In the following calculations the specific application that will be considered is multiplexing among a set of superconducting infrared detectors of the type described in Section 2.2. A thermal conductance of 5 x  $10^{-7}$ 

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Watts/Kelvin is assumed between the Josephson junction and a cold block. Then the current noise coming from such a detector is  $40 \times 10^{-12}$  Amps//Hz. Thus the equivalent input noise of the multiplexer should not be significantly greater than this noise level.

The equivalent input noise levels originating from two sources were calculated in the Phase I study. These are the noise of the dc SQUID used to read out the multiplexer output, and the noise introduced by the multiplexer itself.

For the first of these, the noise from the dc SQUID, the <u>intrinsic</u> SQUID noise is not directly affected by the presence of the mulitplexer circuit. It is necessary however to calculate not just the intrinsic noise at the dc SQUID, but the equivalent input noise at the input of the entire multiplexer circuit. This calculation <u>is</u> affected by the presence of the multiplexer.

From measurements of dc SQUIDs fabricated at HYPRES it is known that the intrinsic flux noise within the SQUID is typically  $\phi^n(SQUID) = 1 \times 10^{-5} \phi_0 / /Hz$ . To relate this to an equivalent current noise at the multiplexer input it is necessary to divide by  $M_{eff}^{max}$  which is the coefficient relating current at the multiplexer input to flux injected into the SQUID. Assuming an optimized multiplexer system the relevant expression is given in Eq. (7) and the equivalent input SQUID noise is:

$$I^{n} (SOUID) = \phi^{n}(SQUID) / M_{eff}^{max} =$$
(10)

$$1 \times 10^{-5} \phi_0 \{ K_1 K_2 K_3 [\beta_1 / (\beta_1 - 1)] / [(L_0 L_2 L_{sq}) / (L_2 + L_6)] [1 / (4 \sqrt{N})] \}^{-1}$$

From this equation it is evident that choosing  $L_0$  or  $L_{sq}$  large will reduce the equivalent input SQUID noise. Since  $L_0$  is the inductance from a coil associated with each input, making  $L_0$  large will require a large area coil for each input which defeats the area-saving aim of the multiplexer circuit. The primary strategy for reducing equivalent input SQUID noise is therefore to increase the SQUID inductance  $L_{sq}$ .

The intrinsic noise of the multiplexer arises from current fluctuations in the PFT loop. This loop is in fact an rf SQUID. The noise of rf SQUIDs has been calculated by numerous people and there are many publications describing the results. It is generally agreed that the intrinsic noise of an rf SQUID is due to current fluctuations within the SQUID loop whose origin is Johnson noise through the quasiparticle resistance of the single Josephson junction<sup>7</sup>. This is given by:

$$i^{n} (PFT) = \sqrt{(4k_{b} T/R_{qp})}$$
(11)

where  $R_{qp}$  is the quasiparticle resistance of the Josephson junction in the PFT. Referring this intrinsic noise back to the multiplexer input gives

$$I^{n} (PFT) = \sqrt{(4k_{b}T/R_{qp})} [(1-\beta_{l})/(\beta_{l} K_{l})][/L_{l}/L_{0}]$$
(12)

Keeping the equivalent input noise derived in Eqs. (10) and (12) below the acceptable limit of  $40 \times 10^{-12}$  A//Hz is one of the design goals for the multiplexer. In the next section this goal and others are used to derive circuit parameters.

### 9. DESIGNS FOR 5-INPUT AND 20-INPUT MULTIPLEXERS

A set of equations describing the characteristics of the multiplexer may be solved to give circuit parameters. Besides electrical circuit parameters an important quantity to solve for is the area of the circuit components. One of the reasons for multiplexing is to save space by reducing the number of amplifiers. This is especially the case for focal plane arrays where space is at a premium, and even more important for superconducting circuits where SQUID amplifiers are one of the largest circuit elements.

Both 5-input and 20-input multiplexers have been considered. The 5-input multiplexer will actually be fabricated for the Phase II follow-on to the present SBIR program. The 20-input multiplexer may be relevant to eventual very large scale (ten thousand detectors or more) focal plane arrays.

The equations to be solved are the following:

1. Equations (8) and (9) above which are inductance matching conditions.

2. For non-hysteretic operation of all PFT's in the multiplexer,  $\beta_1 = 1$ . Using the definition of  $\beta_1$ , and inserting actual inductance values this is  $(L_1 + L_2 + L_6)I_c 2\pi / \phi_0 = 1$ . Combining with Eq. (9) gives

$$4 \pi L_1 I_c / \phi_0 = 1$$
 (13)

3. The equivalent input noise due to intrinsic noise in the dc SQUID should be less than 40 x  $10^{-12}$  A//Hz. Assuming the intrinsic noise itself is 1 x  $10^{-5} \phi_0$  //Hz this gives:

$$1 \times 10^{-5} \phi_0 \{ K_1 K_2 K_3 [\beta_1 / (\beta_1 - 1)] / [(L_0 L_2 L_{sq}) / (L_2 + L_6)] [1 / (4 \sqrt{N})] \}^{-1} < 40 \times 10^{-12}$$
(14)

4. The equivalent input noise due to intrinsic noise in the multiplexer itself should be less than  $40 \times 10^{-12} \text{ A}/\text{Hz}$ . This gives:

 $\sqrt{(4k_{\rm b}T/R_{\rm ap})} [(1-\beta_{\rm l})/(\beta_{\rm l} K_{\rm l})][/L_{\rm l}/L_{\rm o}] < 40 \ X \ 10^{-12}$ (15)

Two aspects of the equations stand out. The first is that  $L_3$  and  $L_4$  only enter through the inductance matching conditon Eq. (8). The absolute magnitudes of  $L_3$  and  $L_4$  are not specified by Eq. (8), they both may be increased by an arbitrary factor as long as it is the same factor for both. A second observation is that the magnitudes of both equivalent input noise contributions go to zero as  $\beta$  goes to 1. Fabricating a circuit with  $\beta_1 = 1.0$  exactly would therefore be ideal from the standpoint of noise. In reality however it is impossible to be so precise, so a value of  $\beta_1 = 0.9$  was assumed for the calculations below. It is possible with HYPRES' present fabrication process to achieve a  $\beta_1$  in the range of 0.9 to 1.0. A more detailed discussion of this consideration is given in Section 6. Assuming  $\beta_1 = 0.9$  in the calculations will give an upper bound or worst case for the calculated noise, and the circuit parameters so derived will therefore include a margin for operation within the desired performance specifications.

Solutions for N = 5:

There are a range of solutions to Eqs. 8, 9, and 13-15. Not all of these are useful from the standpoint of saving space. One solution which does save space is  $L_0 = 10$  pH,  $L_2 = 8$  pH,  $L_3 = 5$  pH,  $L_6 = 5$  pH and and  $L_{sq} > 81$  nH. To get  $L_0 = 10$  pH and  $L_1$  13 pH a thin-film transformer can be used in which the primary and secondary are concentric square films with square holes in the center of each. This can be fit in a 18 $\mu$ m x 18 $\mu$ m area.

The dc SQUID required for this solution would have the two Josephson junctions in series with a multiturn loop of 57 turns. These turns would couple to  $L_4$  inductively,  $L_4$  being formed by a single turn loop. This is the reverse of the most common dc DQUID configuration where the Josephson junctions are in series with a one turn loop and the other side of the transformer is multiturn. This non-standard geometry should be, however, no more difficult to fabricate than the common one.

The Josephson junctions in the PFT loops of the multiplexer will be circular junctions 4  $\mu$ m in diameter and with critical current densities  $J_c = 100 \text{ A} / \text{ cm}^2$ . This is all within the normal capabilities of HYPRES' standard Josephson junction fabrication process.

Solutions for N = 20

One solution is  $L_1 = 13$  pH,  $L_2 = 8$  pH,  $L_6 = 5$ pH and  $L_0 = 10$  pH and  $L_{sq} > 330$  nH.

## 10. CROSSTALK

When a single input is selected for reading by the multiplexer it is necessary that all other inputs are rejected. Insufficient rejection of non-selected inputs will result in crosstalk. An inspection of the circuit of Fig. 8 shows that some precautions must be taken to prevent this from occurring.

To enable a reading of one of the inputs in Fig. 8 current is injected into the control line of the selected input. This current is coupled through a transformer to the PFT corresponding to the selected input. However there is also an unwanted, but unavoidable, coupling to the PFT's of all other inputs. The unwanted coupling is via a path of superconducting loops and transformers. Because of this coupling, the action of turning one input "on" also turns all inputs slightly away from the "off" state. In more precise terms, all non-selected PFT's should be at the operating point labeled "zero gain" in Fig. 11 but are displaced to a point of non-zero gain by the unwanted coupling to the selected control line. Although the non-selected inputs are still far from being completely *on*, there is some contamination from the introduction of their signals into the multiplexer output.

There are several ways to correct for this effect. The simplest is to calculate or measure the amount of flux,  $\phi$ (non-select) coupled into the non-selected PFT's and then inject an equal but opposite flux with the control lines. The calculation has been performed and the result is  $\phi$ (non-select) =  $K_2^2 / (2.3 \text{ N}) \times \phi$ (select). Here  $\phi$  (select) is the amount of flux that must be injected into a PFT to switch it from the "off" state to the "on" state.

For a 5-input multiplexer the correction current is approximately 7 % of the *on* current, for typical parameter values. For a 20-input multiplexer the correction current is 2 % of the *on* current.

### 11. <u>CONCLUSION</u>

A circuit for multiplexing 5 and 20 inputs into a single dc SQUID amplifier has been produced for this Phase I SBIR project. Equations have been derived which specify the parameters of the circuit necessary for optimization. Optimization consists of ensuring that the equivalent input noise added by the multiplexer and dc SQUID not exceed some specified level. Crosstalk between inputs due to insufficient rejection of non-selected inputs was examined and a simple means for compensating for it was devised.

The specific case of 5-input and 20-input multiplexing was considered when the inputs came from highly sensitive superconducting infrared detectors. The circuit parameters were derived subject to the condition that the dc SQUID and multiplexer does not significantly degrade the sensitivity of the overall system including the multiplexer, dc SQUID and detectors themselves. These parameters were found to accomplish one of the objectives of the circuit, namely a reduction in the amount of area occupied by the complete circuit. The 5-input multiplexer will be laid out, fabricated, and tested for the second phase of this SBIR program.

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1. a) General dc SQUID amplifier circuit. b) SQUID amplifier circuit with Josephson junction infrared detector input.

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2. Flux to voltage transfer relation of a dc SQUID fabricated at HYPRES.

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3. Photo of detail of thin-film dc SQUID fabricated at HYPRES.

Doc. # 179



V (1 mv/DIV) -

4. Current-voltage characteristic of a superconducting Josephson junction infrared detector with and without illumination.

Doc. # 179



5. Test configuration of superconducting Josephson junction infrared detectors.

Doc. # 179



6. Output of superconducting Josephson junction infrared detector as an attenuated laser is scanned over the detector.

Doc. # 179



7. A parametric flux transformer (PFT), basic unit of the multiplexing circuit.

Doc. # 179



8. A five-input multiplexer with output into a dc SQUID.

Doc. # 179



9. Detail of the five-input multiplexer with labeled inductances (L)

Doc. # 179



10. The flux at the "signal out" port of the PFT as a function of current in the "signal in" port for three values of  $\beta_1$ : a)  $\beta_1 = 0.5$  b)  $\beta_1 = 1.0$  c)  $\beta_1 = 5.0$ 



 $\mathrm{I}_{\mathrm{SIG}}~(2\pi\mathrm{M_{1}}/\Phi_{0})$ 

11. Detail from Fig. 4 for the case of  $\beta_1 = 1.0$ .

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