

FINAL REPORT ON

NONLINEAR COMPUTERIZED METHODOLOGY

A. Angle of Arrival Estimation

B. Data Modeling and Identification

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OBJECTIVE: Multiple convolution of signals (sonar or radar) has been analyzed as a nonlinear preprocessing technique for source localization from arrays of receivers, for time-delay estimation in general, and for spectral estimation of nonstationary signals. The method has proved successful by computer simulation for many troublesome cases as a supplement to MUSIC (and its adaptations) and as a simple alternative (or representation of) the Wigner-Ville distribution.

Also, a new two-dimensional (image) processing method is being investigated by a transformed two-term Volterra series algorithm. The method appears promising for such applications as underwater green-laser data processing and other cases involving spatially coherent light and related nonlinear terms.



ISSUES

The project goal originally was for the development of nonlinear array processing for better source localization. In regards to traditional high-resolution techniques such as <u>MUSIC</u>, it was found that difficulties are common due to low numbers of receivers, multiple sources in close proximity, partially coherent signals, spatially and/or time correlated noises and low signal-to-noise ratio. A theoretical development was made to show that <u>multiple-autoconvolution</u> of received signals significantly enhances the resolution of the MUSIC algorithm and its various adaptations in many troublesome cases. Theoretically, each convolution would double the resolution for time-delay or direction-of-arrival estimation at the sacrifice of halving the search window. Of course, finite-data signals and noise distortion cause other errors. Fortunately, the convolution amplifies coherency in the signal (preserving the phase) and attenuates the noise. In reality, a single convolution followed by simple windowing is most successful. This is particularly true for nonstationary signals and noise. (I.e., signal phase and noise statistics are time variant.)

It is interesting and potentially very useful that the time-windowed, autoconvolved signal followed by a frequency-windowed Fourier transform results essentially in the <u>Wigner-Ville distribution</u> (WVD). A preliminary analysis indicates that the simple operation of autoconvolution can enhance spectral estimation of nonstationary signals in a simpler but analogous way to the WVD.

TECHNICAL APPROACH

(a) Delay Estimation

Consider two signals $S_1(t)$ and $S_1(t-D)$ which are measured with additive zero-mean noise $V_1(t)$ and $W_1(t)$ (independent of signal $S_1(\cdot)$ and delay D) so that

> $X_1(t) = S_1(t) + V_1(t)$ $Y_1(t) = S_1(t-D) + W_1(t)$



Traditionally, crosscorrelation of $x_1(t)$ and $y_1(t)$ (first-order method, FOM) is used to estimate time delay D. While windowing techniques may enhance the estimate D somewhat it is indicated here that simple autocorrelation is also successful.

Mth-order signals may be generated from x_1 , y_1 by convolution such that

$$X_{2} = X_{1} * X_{1}, \qquad Y_{2} = Y_{1} * Y_{1}$$

$$\vdots \qquad \vdots$$

$$X_{M} = X_{M-1} * X_{1}, \qquad Y_{M} = Y_{M-1} * Y_{1}$$

If x_1 is a sequence of N points, x_m would have 2^{m-1} (N-1) + 1 points.

Suppose

$$X_{1}(t) = ae^{-j\omega t}$$
, $Y_{1}(t) = ae^{-jw(t-D)}$

Then,

$$X_{M}(t) = a^{M}e^{-j\omega t}$$
, $Y_{M}(t) = a^{M}e^{-j\omega(t-MD)}$

or more generally

$$Y_1(t) = X_1(t-D) \rightarrow Y_M(t) = X_M(t-MD)$$
.

Strictly speaking, of course this holds only for continuous signals of infinite duration.

Crosscorrelation between x_M and y_M yields a peak at MD. If the FOM resolves the time delay for $D < 2\pi/\omega$, SOM is limited to $2D < 2\pi/\omega$, but with theoretically doubled resolution. MSE improvement in the noise free case is limited to M^2 with M convolutions.

(b) Power Spectrum Estimation

Consider

$$X(t) = a(t) \exp \phi(t)$$
,

where $\phi(t)$ is real and a(t) is positive and real. The Fourier transform at t may be estimated by the following, where H(u) is a positive, real function:

$$F(t,\omega) = \left|\int H(u) X (t + u) e^{-j\omega u} du\right|^2.$$

The WVD is a time-frequency distribution which provides an unbiased spectrum estimate by

W(t,
$$\omega$$
) = $\int H_1(u) \widetilde{X} (t - u/2) X (t + u/2) e^{-j\omega u} du$,

where the overbar denotes conjugate. Traditionally, time smoothing also is used to improve the estimate so that

$$W(t,\omega) = \int H_2(\tau) \left[\int H_1(u) \ \widetilde{X} \ (t + \tau - u/2) \ X \ (t + \tau + u/2) \ e^{-j\omega u} \ du \right] d\tau ,$$

or

$$W(t,\omega) = \int H_1(u) \left[\int H_2(\tau) \ \overline{X} \ (t + \tau - u/2) \ X \ (t + \tau u/2) \ d\tau \right] e^{-j\omega u} \ du \ .$$

Similarly, a windowed, short-duration convolved signal yields

$$Y(t) = \int H_2(\tau) X (t/2 - \tau) X (t/2 + \tau) d\tau ,$$

and a short-time windowed Fourier transform yields the spectrum of Y(t) from X(t/2)

$$C(t,\omega) = \int H_1(u) \left[\int H_2(\tau) X \left((t + u)/2 - \tau \right) X \left((t + u)/2 + \tau \right) d\tau \right] e^{-j\omega u} du .$$

Similarly, the power spectrum related to X at time t is

$$C_{s}(2t,\omega) = \left| \iint H_{1}(u) H_{2}(\tau) X (t + u/2 - \tau) X (t + u/2 + \tau) e^{-j\omega u} d\tau du \right|.$$

If X(t) is a pure wave of frequency $\omega_{o},$ these expressions simplify and are equivalent with

$$W_s(t,\omega) = \delta(\omega_0) \int \int H_2(\tau) H_1(u) du d\tau$$

and

$$C_{s}(2t,\omega) \approx \delta(\omega_{0}) \int \int H_{1}(u) H_{2}(\tau) d\tau du'$$

Hence, convolution over $H_2(\tau)$ followed by Fourier transform over $H_1(u)$ is analogous to WVD over $H_1(u)$ and smoothed in time over a window $H_2(\tau)$.

TECHNICAL ACCOMPLISHMENTS

Table A compares the mean error (ME) and MSE for FOM, SOM (in windowed 199 points) and SOM (59 points) with three levels of signal-to-noise ratio when the signal is a single sinusoid. Table B considers the same comparisons for an exponentially damped sinusoid. In both cases, the noise is Gaussian or uniform, 100 real data points are measured, sampling rate is 18 points/period, and Monte Carlo simulations are performed over 300 runs. Superiority of appropriately windowed SOM is apparent.

Figure 1 compares the spectrum estimate for backscatter sonar data by short-time Fourier transform, by autoconvolution, and by smoothed WVD. Figures 2 and 3 show how autoconvolution enhances periodicities for the data used in Figure 1.

SIGNIFICANCE

The following have been found for various restricted classes of problems:

- Multiple convolution can be utilized to enhance signal coherence and attenuate noise distortion such as delay or direction of arrival estimation.
- 2. Single autoconvolution seems to enhance direction of arrival resolution by MUSIC in many troublesome cases which include combinations of multiple sources in close proximity, partially coherent sources, small numbers of receivers, low signal-to-noise ratio, spatially and/or time-correlated noises, and small number of samples.
- The autoconvolution method, for successful application discussed here, is quite sensitive to windowing forms. But the window can be most simple.
- 4. For spectral estimation of nonstationary processes, the windowed (time and frequency) single autoconvolution offers similar resolution to WVD. In fact, the two methods are practically equivalent, but autoconvolution seems simpler.

S/N	Exact	FOM (100)		SOM (199)		SOM (59)	
in Delay dB	Delay	ME	MSE	ME	MSE	ME	MSE
No Noise	5	0	0	0	0	0	0
	5 33	0.33	0.11	0.17	0.027	0.17	0.027
	5.5	-0.5	0.25	0	0	0	0
-3 dB	5	-0.42	1.92	-0.23	0.848	-0.028	0.341
	5.33	-0.08	1.82	-0.02	0.852	-0.039	0.017
	5.5	-0.03	1.63	-0.011	0.833	-0.017	0.396
-9 dB	5	0.133	5.65	-1.15	5.69	0.172	2.35
	5.33	0.01	5.62	1.41	7.12	-0.265	2.74
	5.5	-0.13	5.69	-1.65	8.27	0.317	2.82

Table A. Periodic signals

.

Table B. Nonperiodic signals

S/N in dB	Exact Delay	FOM (100)		SOM (59)		Short SOM (20)	
		ME	MSE	ME	MSE	ME	MSE
No Noise	5	0	0	0	0	0	0
	5.33	0.33	0.11	0.17	0.027	0.17	0.027
	5.5	-0.5	0.25	0	0	0	0
-3 dB at Sample #1	5	0.333	7.69	0.537	4.47	0.122	2.44
	5.33	-0.36	7.54	0.666	5.15	0.111	2.65
	5.5	0.221	6.23	0.705	5.16	0.001	3 80
-9 dB at Sample #1	5	1.69	19.69	1.14	7.35	-0.32	5.48
	5.33	1.47	18.82	1.46	8.41	-0.78	6.36
	5.5	1.19	17.62	-1.38	8.34	-0.89	6.89

Figure 1: Frequency spectrum of backscatter sonar data, 256 points processed Thick line: short time Fourier transform Thin line: autoconvolution method Dotted line: smoothed WVD

Figure 2: Backscatter sonar data, real part, 256 samples.

Figure 3: Backscatter sonar data, real part of the 32-point autoconvolved signal.

Bilinear Volterra analysis can be useful for two-dimensional (image) data processing.
 In conjunction with singular-value decomposition, the method presents a good base for future research.

PUBLICATIONS AND PRESENTATIONS

Details of the autoconvolution supplement for angle of arrival estimation and its performance comparison with MUSIC alone is available in reports (OSU-EE-ONR Reports 90-1,2) previously submitted to ONR. Several recent papers and books have evolved wholly or in part from this project [1]-[11]. New papers and reports are in progress particularly related to real applications, nonstationary spectral estimation, and two-dimensional (image) processing. The submitted report OSU-EE-ONR Report 90-3 provides substantial foundation for the latter.

The applications of these algorithms to Navy data analysis was discussed with staff members at NOSC in May and at China Lake in July. Both presentations received careful scrutiny but enthusiastic interest.

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LIST OF PARTICIPANTS

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- I.K. Rhee, PhD (6-90) "A Nonlinear Approach to the Direction-Finding Problem in Array Processing" (OSU 33-OWR-90-2)
- T.H. Kwon, PhD (6-91) "Restoration of Quadratically Distorted Images"
- F.J. Bugnon (Former Post Doct) "The Second-Order Method: A Direction Finding Enhancement" (OSU-EE-OWR-90-1)

Appendix

Examples of simulated resolution of the direction of arrival problem for two sources are presented here.

SIMULATION CONDITIONS

Standard 1 Sampling rate 1MHz. Linear array of 3 sensors. Distance between sensors : D = 600m (half wave lenght) Propagation speed c - 300,000 km/s. Independent Gaussian noise of amplitude SN - 0.4 (i.e. variance of 0.32) Independent sources s_1 and s_2 of amplitude S1 and S2. s, :S1 -√2 Frequency : 2 pure complex waves at 250

±62.5 KHz

Impinging angle : 6° .

s. : S2 - 2

Frequency : 1 pure complex wave at 250 KHz. Impinging angle : 0° .

RESOLUTION OF THE FIRST ORDER.

As the two sources get closer, the peaks get smaller and wider untill the point where the sources are not distiguishible. Then, unable to separate the sources, the peak grows higher and thinner between the actual source placements.

The resolution limit, at which two maxima are distinguishible is at about 4.5° .

RESOLUTION OF THE FIRST ORDER



(Standard 1)

RESOLUTION OF THE SECOND ORDER

For the second order, the resolution limit is under 1.3° , that is four times better than for the first order.

The relative strengths of the sources, as well as the bias drift are opposite compared with the first order (previous page). It suggests a way to reduce the bias by comparing the results of both methods... provided that the first order is able to distinguish sources!

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RESOLUTION OF THE SECOND ORDER

(Standard 1)



COMPARED RESOLUTIONS

The comparison is conducted almost at the resolution limit of the second order.

The superiority of the second order is clear.

The third and fourth order are not as good as the second order. The second order shows the highest peak but is at the verge of detecting the second source, as the log plot shows (next page). The fourth order is also limit, but at a too low level.

- Though the second order can be thought of as an artificial way of doubling the array length (aperture), it preforms much better than the actual doubling of the array length, and the array must be quadrupled in length to obtain similar results.



RESOLUTION WITH CORRELATED SOURCES.

The following figures show how resolution is affected by a higher correlation between the two sources, for a constant signal power. The second order method always shows better results. Note that the correlation also produces an increased bias according to the fourth figure.

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