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GENERALIZED SIMPLIFIED ANALYTIC SOLUTION OF OPTIMUM CYCLE PARAMETERS FOR LAND AND AIRCRAFT GAS TURBINE ENGINES AND ITS APPLICATIONS

by

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GENERALIZED SIMPLIFIED ANALYTIC SOLUTION OF OPTIMUM CYCLE PARAMETERS FOR LAND AND AIRCRAFT GAS TURBINE ENGINES AND ITS APPLICATIONS

Cui Jiya, Beijing University of Aeronautics and Astronautics

Abstract: Based on constant specific heat, a simplified solution is proposed for the conventional and optimum cycle parameters of land and aircraft gas turbine performance and its four significant pressure ratios: for optimum stress or work, for optimum core turbine outlet pressure and reheat, for optimum thermal efficiency, and for optimum specific fuel consumption, hitherto not systematically analyzed (in terms of pressure ratio). The sea level static and strato aircraft illustrative results are compared with two precise variable specific heat calculations with either isentropic or polytropic compressor and turbine efficiency kept constant. It is not unexpected to see the simplified method deviating in absolute performance values, yet it is interesting that this much simpler and quicker method serves in fairly well predicting trends in performance variation with change in pressure ratio and the four optimum pressure ratios as well as trends in component and aircraft conditional effects on these optimum pressure ratios and their performance, being thus appropriate for use in preliminary estimates in design. The proposed solution may serve as a start for precise solutions as well. Moreover, it will probably be helpful in analyzing complex cycles and also in searching for new cycles.

power units or mobile engines utilizing shaft power, with specific thrust value given by the square root of twice the shaft work, and the variation of thermal efficiency precisely equivalent to the variation of the reciprocal of specific fuel consumption in terms of shaft power.

The paper deals only with cycle analysis. The practical choice of compressor pressure ratios will be a compromise between the various optima, with due consideration given to off-design performance and a number of particular engineering requirements.

Key technical terms: gas turbine, optimal pressure ratio, simplified solution.

I. Foreword

Nearly half a century has elapsed since the advent of gas turbines. In cycle analysis, there are equations for deriving power or thrust from the specific heat, thermal efficiency, and various optimal (optimal reheating or increase in stress) pressure ratios of turbine exit pressures. However, there are no equations for the optimal pressure ratios of the fuel consumption rate with more complex relations. By using the parameter section simplified method system, the paper derives the simplified solution of the optimal pressure ratios; in addition, examples are cited in comparing the results of calculations and the precise solutions of the variable specific heat in order to have a correct evaluation of the extent of approximation and applications.

II. Simplified Solutions of Constant Specific Heat

Fig. 1 shows an actual cycle by noting the aircraft situations.

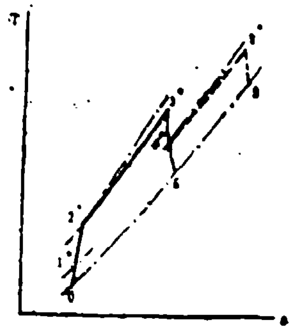

Fig. 1. An actual gas turbine cycle
Remark: 01* - impact pressure
1*2* - compressors 2*3* - combustion
3*4* - turbine 4*8 - thrust increase
or reheat 4*5 or 8*9 - exhaust nozzle
or power turbine

In the simplified solution, the constant specific heat c_p=1.0046kJ/(kg·K), for air and fuel gas; isentropic index k=1.4; the same isentropic efficiency η_T^* is used for turbine of exhaust nozzle, and the small amount of fuel (oil) g_r in the fuel gas flow is neglected.

The relation between the impact pressure ratio of intake gas and the efficiency η_{jk} of the gas intake manifold is given by the equation:

$$\pi_v = \frac{p_1^*}{p_0} = \left(1 + \eta_{jk} \frac{k-1}{2} M^2 \right)^{\frac{k}{k-1}} = \tau^{\frac{k}{k-1}} \qquad (1)$$

In the equation, τ is the isentropic temperature increase ratio; in the stationary situation, π_V=1=τ.

The relation between total effective work W′ of the cycle and $x = \pi_k^{*\frac{k-1}{k}}$ is:

$$W' = c_p\left[\eta_T^* T_3^* \left(1 - \frac{1}{\tau\tau_{,}x}\right) - \frac{T_1^*}{\eta_k^*}(x - 1)\right] \qquad (2)$$

In the equation, $\tau_r = \sigma_r^{\frac{k-1}{k}}$; however, σ_r is the total pressure coefficient of the combustion chamber. Eq. (2) is suitable for use in turbine blades and shaft (engine) of the output shaft power (not including further generation of thrust and the consideration of intake gas drag) and ground-based gas turbines.

After subtracting the work done by impact pressure from the turbojet (engine) with direct output of thrust, we obtain the effective work done:

$$W = W' - \frac{V^2}{2} \tag{3}$$

In the stationary situation, W=W'.

The unit thrust of a turbojet engine is (after subtracting the intake gas drag) given by the following equation:

$$\frac{R}{m} = c_s - V = \sqrt{2W'} - V \tag{4}$$

with a given aircraft speed V, the derivative of the above equation is zero; in other words, $\frac{d[\ ]}{dx} = 0$, thus obtaining Eq. (5) for the optimal pressure ratio $(\pi_c^*)_{R.W'}$ of work of thrust:

$$x_{R.W'} = \left(\frac{T_3^* \eta_c^* \eta_T^*}{T_1^* \tau \tau_r} \right)^{\frac{1}{2}} \tag{5}$$

It is easy to see that the above equation can be generally used with all gas turbines; these cases are quite common.

The thermal efficiency of the cycle is:

$$\eta_t = \frac{W}{c_p\left[T_3^* - T_1^* \left(1 + \frac{x-1}{\eta_c^*} \right) \right] / \xi} = \frac{F - \frac{U}{x} - Sx}{E - Sx} \cdot \xi \tag{6}$$

In the equation, ξ is the combustion chamber efficiency; the various parameter sections are:

$$F = \eta_T^* T_3^* + \frac{T_1^*}{\eta_c^*} - \frac{V^2}{2c_p}$$

$$U = \frac{\eta_T^* T_3^*}{\tau \tau_1}$$

$$E = T_3^* - T_1^* \left(1 - \frac{1}{\eta_c^*} \right)$$

$$S = \frac{T_1^*}{\eta_c^*}$$

From $\frac{d\eta_t}{dx} = 0$, Eq. (7) is derived for the pressure ratio (π_c^*), of the optimal thermal efficiency:

$$x_e = \frac{US - \sqrt{U^2S^2 - (F - E)UES}}{(F - E)S} \tag{7}$$

See reference [1] for similar derivations. The above equation can be applied also to various gas turbine types.

A detailed discussion is required for specific fuel consumption. Specific fuel consumption for a turbojet engine is generally calculated in terms of thrust: the unit thrust equation (4) can be written as

$$\frac{R}{\dot{m}} = \left(\sqrt{D - \frac{U}{x} - Sx} - V_1 \right) \sqrt{2c_p}$$

In the equation,

$$D = \eta_T^* T_3^* + \frac{T_1^*}{\eta_c^*}$$

$$V_1 = \frac{V}{\sqrt{2c_p}}$$

However, the fuel to air ratio is given by the equation:

$$g_f = \frac{c_p}{\xi H_r} (E - Sx)$$

In the equation, H_r is the heating value of the fuel (oil), then the specific fuel consumption of thrust is

$$C_R = \frac{3600 g_f}{\frac{R}{\dot{m}}} = \frac{3600\sqrt{c_p/2}}{\xi H_f} \cdot \frac{E - Sx}{\sqrt{D - \frac{U}{x} - Sx - V_1}} \tag{8}$$

Taking the derivative of x as zero, the x_{CR} of $(\pi_k^*)_{C_R}$ satisfies the following equation:

$$\sum_{i=0}^{6} A_i x^i = 0 \tag{9}$$

In the equation, the parameter sections are as follows:

$$A_0 = \frac{1}{4}U^2E^2,\ A_1 = -\frac{3}{2}U^2ES,\ A_2 = \frac{9}{4}U^2S^2 + UE\left(D - \frac{E}{2}\right)S,\ A_3 = (E - 3D + V_1^2)\ US^2,\ A_4 = \frac{3}{2}US^3 + \left[\left(D - \frac{E}{2}\right)^2 - DV_1^2\right]S^2,\ A_5 = \left(\frac{E}{2} - D + V_1^2\right)S^3,\ A_6 = \frac{1}{4}S^4.$$

In the stationary situation on the ground, V_1=0. The above equation can be simplified into the following expression:

$$S^2x^3 + (E - 2D)Sx^2 + 3USx - UE = 0 \tag{9a}$$

With trial computation on a computer, the first root begins from x_η; then x_{C_R} of the optimal pressure ratio of the thrust specific fuel consumption is derived.

The relation between the thrust specific fuel consumption and efficiency is:

$$C_R = \frac{3600 g_f}{\frac{R}{\dot{m}}} = \frac{3600V}{H_f\left(\frac{W}{g_f H_f} \cdot \frac{\frac{R}{\dot{m}}V}{W}\right)} = \frac{3600V}{H_f(\eta_t\eta_p)} = \frac{3600V}{H_f\eta_0} \tag{8a}$$

It can be seen that C_R manifests the reciprocal of the product (the total efficiency η_0 of the thermal efficiency η_t and the propelling efficiency η_p with the influence of aircraft speed V). In a stationary situation on the ground where η_p is zero, the above equation is converted into:

$$C_R = \frac{3600 g_r}{c_5} = \frac{3600(c_5/2)}{H_r \frac{c_5^2/2}{g_r H_r}} = \frac{3600(c_5/2)}{H_r \eta_t} \tag{8b}$$

In other words, this is converted into the reciprocal, manifesting the thermal efficiency; in addition, with modification of the effect of the jet gas velocity c_5.

The specific fuel consumption values of the power for turbine blades, turbine shaft, and ground-based turbine are as follows:

$$C_e = \frac{3600 g_r}{W'} = \frac{3600}{H_r\left(\frac{c_5^2 - V^2}{2g_r H_r} + \frac{V^2}{2g_r H_r}\right)} = \frac{3600}{H_r\left(\eta_t + \frac{V^2}{2g_r H_r}\right)} \tag{10}$$

or

$$\eta_t = \frac{3600/H_r}{C_e} - \frac{V^2}{2g_r H_r}$$

Hence, C_c is the reciprocal directly exhibiting thermal efficiency in a stationary situation; however, during aircraft flight, the influence of aircraft speed V and fuel gas ratio g_r is included.

We have to point out that the specific fuel consumption of a military aircraft relates to range or effective loading, thus affecting tactical or strategic performance. The optimum pressure ratio of specific fuel consumption is called "the most economical pressure ratio" (this conventional term does not represent an overall viewpoint); however, for civilian aircraft and ground-based gas turbines, it is feasible to call it the most economical pressure ratio.

For the exit pressure p_4^* (of the turbine) is the optimal value; actually, there are simultaneous optimal effective power, thrust and specific fuel consumption of reheat or thrust increase; according to the equilibrium relation of power, we have the following expression:

$$\eta_T^* T_3^* \left[1 - \left(\frac{p_4^*/p_0}{p_3^*/p_0} \right)^{\frac{k-1}{k}} \right] = \frac{T_1^*}{\eta_c^*} (x - 1) ,$$

it is not difficult to derive the following formula:

$$\frac{p_4^*}{p_0} = \left[\left(1 + \frac{T_1^*}{\eta_c^* \eta_T^* T_3^*} \right) - \frac{T_1^*}{\eta_c^* \eta_T^* T_3^*} x \right]^{\tau_T, x} \quad (11)$$

In the equation, for the derivative of x at 0, the given p_4^* is the optimal value; that is, Eq. (12) is the optimal pressure ratio $(x_4^*)_{,4}^*$ of reheat or increase in thrust:

$$x_{,4}^* = \frac{1}{2} \left(1 + \frac{\eta_c^* \eta_T^* T_3^*}{T_1^*} \right) \quad (12)$$

The above equation appears in reference [2]. Obviously, the equation can be used for various types of gas turbines.

III. Comparison Between Calculated Examples and Precise Solutions

With two calculated examples of ground-stationary and high-altitude aircraft, we can obtain the variation in the performance of a gas turbine with change in gas compressor pressure ratio; the filled circle as symbol represents four types of the optimal pressure ratio, as shown in Figs. 2 and 3.

The corresponding precise solutions of the variable specific heat are also calculated in parallel; in addition, starting with the optimal pressure ratio of the simplified solution, a stepped acceleration method is used to solve for the optimal pressure ratio in order to save computation time. One of the precise solutions is to maintain constant the isentropic efficiency of

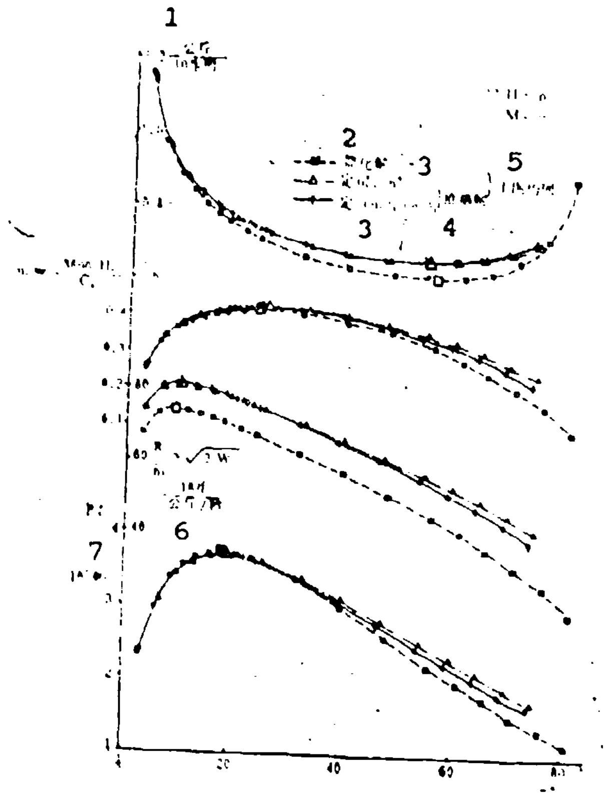

Fig. 2. Effect of gas compressor pressure ratio in a stationary situation on the ground

Remark: T_3^*=1358K, σ_r=0.96, ζ=0.97, H_r=43,124kJ/kg

Simplified solutions: η_k^*=0.825, η_T^*=0.925 --□--

Precise solutions: η_k^*=0.825, η_T^*=0.925—·△—

$(\eta_p)_k$=0.88, $(\eta_p)_T$=0.91—+—

Revised ordinate:

$$\eta_t = \frac{3600/H_r}{C_e}, \quad \frac{R}{\phi} = \sqrt{2W}$$

KEY: 1 - kg/10N-inch [letter illegible]
2 - Simplified solution 3 - Determine
4 - Precise solutions 5 - Same
in the following figure 6 - 10N/(kg/s)
7 - Poise

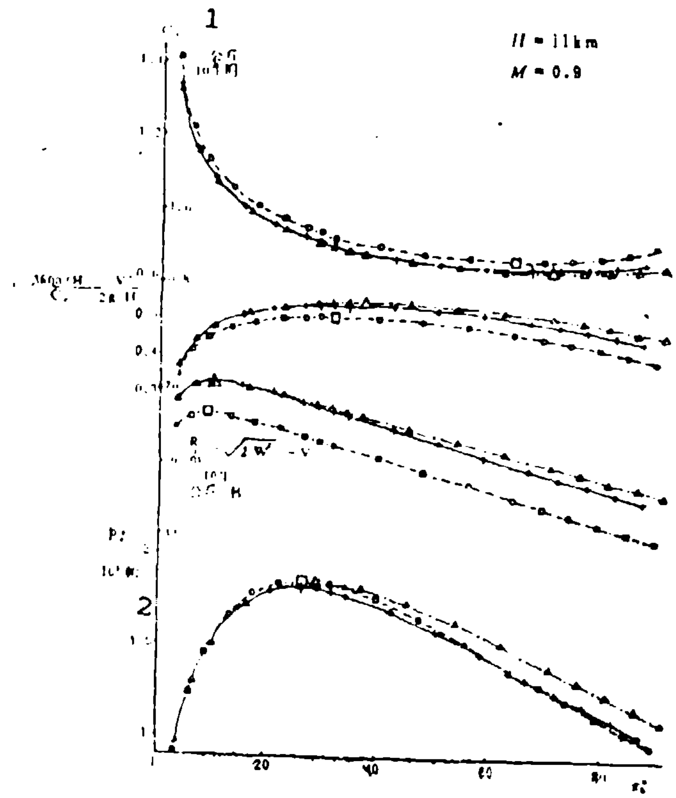

Fig. 3. Effect of pressure ratio in a gas compressor for the situation of high-altitude aircraft
REMARK: (The symbols used as the same as in Fig. 2, η_{jk}=0.99)
Revised ordinate: $\eta_t = \frac{3600/H_r}{C_t} - \frac{V^2}{2 g_r H_r}$

KEY: 1 - kg/10N-h [letter illegible] 2 - Poise

the gas compressor and turbine with consideration of the fuel oil flow g_r and its effect on the physical properties of the combustion gas; the entropy value is taken from reference [3], and the gas constant is taken from [4]. In addition, the following cooling air coefficients are taken as follows: for the combustion chamber, ν=0.91; for turbine, ν_T=0.9456; mechanical efficiency, $\eta_{[illegible]}$=0.99; and efficiency of exhaust nozzle, η_{pk}=0.9725. In the second of the precise solutions, consideration to the variation with pressure ratio for the efficiency level of the rotating components is added:

$$\eta_k^* = \frac{x_k^{*\frac{k_k - 1}{k_k}} - 1}{x_k^{*\frac{k_k - 1}{k_k(\eta_p)k}} - 1}$$

$$\eta_k^* = \frac{1 - \dfrac{1}{x_T^{*\frac{k_T - 1}{k_T}(\eta_p)_T}}}{1 - \dfrac{1}{x_T^{*\frac{k_T - 1}{k_T}}}}$$

Take the polyvariable efficiency $(\eta_p)_K$=0.88 and $(\eta_p)^r$=0.91 (as the constant) of the intermediate constant pressure ratio and constant isentropic efficiency η_K^*=0.825 and η_T^*=0.925; in other words, the calculations are conducted by assuming the stage loss as constant. The results of the two precise solutions are also compared with plotting on the diagram.

It can be seen that the sequence of the various optimal pressure ratios of three solutions is the same. The difference between two precise solutions is generally smaller; however, the various performance parameters in the simplified solution are as expected, of course, with certain errors. Even so, it should be noted that the general trend of variation with change in pressure ratio of the various performance parameters as well as the various optimal pressure ratios are quite close to precise solutions.

As previously analyzed, Fig. 2 is generally applied to various types of gas turbines; however, Fig. 3 is only generally applied to aircraft gas turbines.

IV. Effect of Component Parameters and Flight Conditions

For further testing, the approximation property of the simplified solutions, by using the flight situation in Fig. 3, is the criterion in separately changing the efficiencies of gas compressor and turbine, temperature of the combustion gas before entering the turbine, as well as the flight altitude and speed, by using the three solution methods in calculating the variation of various optimal pressure ratios and other properties, as shown in Figs. 4 to 6. Although these variations are mostly well-known, however in the past there were fewer data on the optimal parameters of the total turbine exit pressure. In the figure, the relatively secondary optimal data of thermal efficiency are neglected in these figures. The abscissa in Fig. 4 calibrates the isentropic efficiency and the determination of the polyvariable efficiencies; these are converted by randomly selecting $\pi_K^*=19.05$ ($k_K=1.383$) and $\pi_T^*=5.46$ ($k_T=1.317$). Therefore, by comparing the determined polyvariable efficiencies (the second of the precise solutions), only a general qualitative reference can be made.

It can be seen that when these two rotating components have their efficiencies increased, either by reducing power consumption or by increasing power output, almost entirely similarly to raise the optimal pressure ratios, increase of the total pressure, and improvement of the specific fuel consumption of the corresponding combustion gases exiting from the turbine, only the corresponding thrusts increase slowly; in the individual cases, the corresponding thrusts may even be reduced owing to higher pressure ratios.

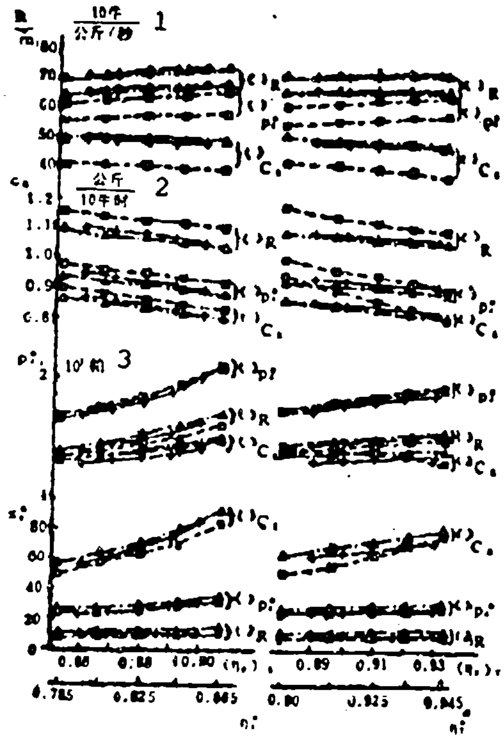

Fig. 4. Effect of gas compressor and turbine efficiencies on optimal pressure ratios and other properties
Remark: revised abscissa: $(\eta_p)_k$ in the left diagram; in the right diagram, the second dot of the lower abscissa is 0.915.
KEY: 1 - 10N/(kg/s)   2 - kg/10N-h   3 - Poise

With an increase in temperature T_3^* of the combustion gases upstream of the turbine, or cooling of the intake combustion gas T_0 due to higher H, the flight altitude (up to the isothermal layer), these cases increase the temperature range of the thermodynamic cycle, thus affecting the general phase image, in other words: an increase of the various optimal pressure ratios, increase of the ratio p_4^*/p_0 of the corresponding thrusts and the combustion gas pressure at the turbine exit (attention should be given to higher H, although p_4^* is reduced, but more slowly than p_0); the corresponding specific fuel consumption values improve with higher H. However, with an increase in T_3^*, there is only an

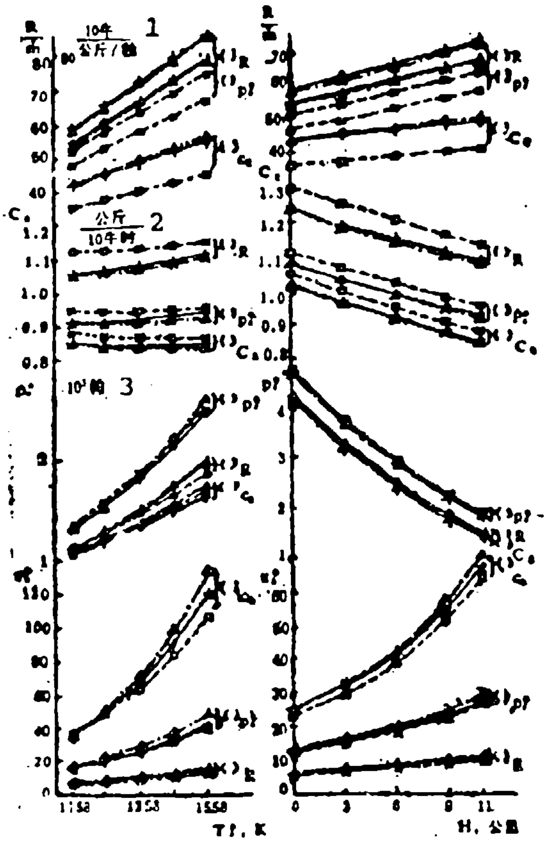

Fig. 5. Effect of combustion gas temperature upstream of turbine blades and flight altitude on various optimal pressure ratios and performance
KEY: 1 - 10N/kg/s 2 - kg/10N-h 3 - Poise 4 - kg

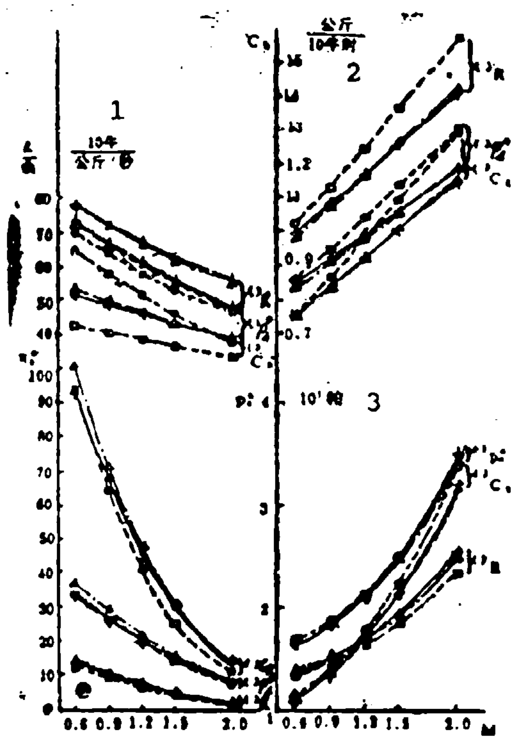

Fig. 6. Effect of flight speed on various optimal pressure ratios and performance
Remark: It is assumed that when $M \leq 0.9$, $\eta_{jk}=0.99$; when linearly reduced to M=2, $\eta_{jk2}=0.94$)
KEY: 1 - 10N/kg-s
2 - kg/10N-h 3 - Poise

improvement of $(C_R)_{C_R}$ of the optimal specific fuel consumption (in the second of the precise solutions, this is an improvement first and a deviation later; possibly, when the pressure ratio is high, η_k^* is too low); however, $(C_R)_R$ and $(C_R)_{p_4^*}$ are basically deteriorating, possibly related to the fact that the pressure ratio is not high enough.

When the flight speed M (Mach number) is increased (here no analysis is made as to whether or not the thrust is sufficient for supersonic flight), an increase in the impact pressure ratio results; however, the temperature range between the highest and the lowest temperature in the cycle remains unchanged; therefore, the various optimal pressure ratios of the gas compressor are reduced. The total pressure exiting the various corresponding turbines is increased because of higher impact pressure; an increase of the intake gas drag reduces the thrust of the various units and increases the specific fuel consumption values.

From the above figures, these two types of precise solutions are basically very similar to each other, in addition to the high pressure ratio. In the simplified solution, this still involves greater general errors. However, besides the high pressure ratios or high Mach numbers the various optimal pressure ratios are still correspondingly close to the precise solution. In some cases this is between two precise solutions; in addition, the general trend of variation of various performance parameters is quite close.

V. Conclusions

The article concretely illustrates that the ground stationary properties of aircraft gas turbines are closely related to the power of ground gas turbines.

By testing with calculated examples, the simple and rapid analytical solutions of the simplified solutions of fixed specific heat, a preliminary evaluation can be made that the performance of gas turbines exhibit a trend of variation with change in pressure ratios as well as the various optimal pressure ratios. This case can be used as a beginning of the precise solution; this finding can also be used in an initial estimation of the general trends by component parameters and flight conditions on the various optimal pressure ratios and their performance. The functions of analyzing the complex cycles and the initial exploration of new cycles are also attractive.

From the angle of cycle analysis, the article illustrates the function of simplified solutions. As for the selection of pressure ratios of an actual gas compressor, in addition to paying attention to the requirements of design situations and a trade-off among various optimal pressure ratios, comprehensive studies should be conducted with a combination of the concrete situation of an engineering project in addition to testing in practical cases.

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