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OPTIMAL ROUTING OF BATTLE GROUP
VERTREP ASSETS

by

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Optimal Routing Of Battle Group VERTREP Assets

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ABSTRACT

During battle group operations ships regularly require the transfer of material and personnel. The VERTREP of personnel and high priority cargoes is accomplished by logistics helicopter. This study describes an implicit enumeration algorithm to schedule the delivery route for a single helicopter. The algorithm employs a depth first search technique to solve the multiple constraint, multiple time window routing problem. Several fathoming techniques are demonstrated and computational results for eleven ship battle groups are presented.

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I. INTRODUCTION

"I don't know what logistics is but I want some!" ADM Ernest King, CNO 1942

A. BACKGROUND

Over forty years later, this battle cry can once again be heard echoing through the Navy halls of the Pentagon. Navy planners have sounded the call for an increased awareness of the implications of logistics on warfighting capabilities. Additionally, in an era of ever decreasing defense budgets, planners can no longer afford to throw money and resources at a logistics problem in an attempt to solve it. Instead, logistic efforts must concentrate on cost effective execution. In particular, efficient methods to maintain the logistic sustainability of the naval forces must be developed.

Logistic sustainability is the ability of the logistic support infra-structure to adequately provide the necessary materials for a force to conduct an assigned mission within a theater of operation for an extended period of time. Whether the force contains hundreds of ships supporting the Okinawa invasion in 1945, or a few ships escorting convoys in the Persian Gulf in 1988, the logistic support of these forces enables them to remain on station thousands of miles from their home ports. The logistic sustainability of naval forces is a key component in the successful completion of their mission. It is also the foundation for the use of naval forces as instruments of power projection which supports the United States Maritime Strategy of 'taking the fight to the enemy'.

Two essential elements in maintaining a high level of logistic sustainability are the availability and rapid delivery of material. The logistic material needed by the battle group (BG) is carried aboard the combat logistics force (CLF) vessel, or station ship. The transfer of material from the station ship to a ship requiring resupply (customer ship) is referred to as underway replenishment (UNREP). Typical methods of UNREP are connected replenishment (CONREP) and vertical replenishment (VERTREP).

During a CONREP, the customer and station ships must be alongside each other with lines and hoses fastened between them. These lines and hoses are used to transfer a wide variety of cargoes. In fact, any items within the capacity of the transfer rigs can be transported to the customer ship. However, CONREP of cargoes is relatively slow and manpower intensive. Moreover the proximity of the ships, which is often less than 150 yards, makes CONREP a dangerous evolution that requires expert seamanship as well as competent crews. This proximity also requires either the customer ship to leave its defensive screening station or the station ship to approach the fringes of the screen, neither of which are desirable alternatives since they both increase the vulnerability of the BG. Nevertheless, CONREP is the more versatile method of underway replenishment. In particular, CONREP is the only method for replenishing liquids such as fuel and water.

Instead of lines and hoses, VERTREP transfer of cargoes and passengers is conducted by one of the two CH-46 helicopters carried aboard most station ships. While helicopters have a smaller carrying capacity, their speed allows them to make many trips in a short period of time, enabling them to move a large volume of material in a short time. With VERTREP, both the customer and station ship can remain in their positions within the formation, thereby preserving the integrity of the BG screen. Thus, VERTREP provides an excellent alternative to CONREP, especially in a hostile environment where rapid transfers and high combat readiness are critical.

To efficiently perform a VERTREP, the helicopter must be scheduled so as to minimize the flying time to deliver all the necessary cargoes in a timely manner. Depenuing on the situation, the problem of scheduling the helicopter can be relatively easy or rather hard. In the next section, different types of VERTREP are discussed.

B. VERTICAL REPLENISHMENT

There are three basic types of VERTREP: Intra-theater lift, alongside VERTREP, and logistics helicopter (LOG HELO). A description of each is detailed below.

First, the intra-theater lift involves the resupply of the BG from an advanced logistic site (ALS). In this case, the helicopter is used mainly to carry cargoes from the ALS to the station ship. Since the helicopter is simply flying back and forth between the ALS and the station ship, the scheduling is simple.

The alongside VERTREP is used to transfer large amounts of material, e.g., ammunition, often to augment a CONREP. As with CONREP, customer ships need to rendezvous within a few thousand yards of the station ship to speed replenishment and allow resupply of multiple customer ships at one time. During a peacetime operation, the alongside VERTREP again does not pose any problem in terms of scheduling the helicopter. However, during combat, cargoes are often assigned priority by combat value and the scheduling of helicopters becomes more complicated. For more information, the reader is referred to Pilnick (1989).

Finally, the LOG HELO is typically used to deliver personnel and smaller high priority cargoes. During a LOG HELO operation, the helicopter takes off from the station ship with the cargoes and flies to various customer ships who some cases, the cargoes to be delivered are available on another ship. Hence, the helicopter may also have to visit ships other than the customer ships in order to pick up cargoes.

Unlike the first two types of VERTREP, the scheduling of a LOG HELO operation can be complicated. In fact, the routing of the helicopter to visit various ships in the BG can be viewed as a generalization of the traveling salesman problem which is a hard problem in combinatorial optimization. Moreover, the fact that the BG is constantly moving forward during the entire LOG HELO operation makes the travel time between ships asymmetric. For example, if ship A is at the front of the formation and ship B is in the back, then the time to fly from A to B is less than the one from B to A. So, one would not expect a person without the aid of a computer to produce an 'optimal' routing for the helicopter. Here, an optimal route means a route(s) which allows the helicopters to complete the entire LOG HELO operation in the least amount of time.

It is clear that an optimal delivery route for the helicopter would directly decrease the usage of fuel, personnel time, and the helicopter itself. Indirectly, an optimal delivery route extends the service life of the helicopter, thereby lengthening the time between overhauls and increasing helicopter availability which, in effect, improves the logistic flexibility of the BG. Therefore, it is the focus of this thesis to develop an algorithm to produce a more efficient route for the helicopter to deliver cargoes to ships in the BG.

The outline of this thesis is as follows. Chapter 2 lists the problem assumptions, formulates a mathematical model of the LOG HELO problem, and reviews alternative solution methods. Chapter 3 describes the implicit enumeration procedure for the problem. Chapter 4 presents results from the computer experimentation and Chapter 5 conclude the thesis with the discussion of further extensions and the work necessary for fleet implementation.

II. PROBLEM FORMULATION

A. PROBLEM STATEMENT

The task of routing the helicopters to deliver cargoes to various ships in a BG is generally the responsibility of the material control officer (MATCONOFF) or the BG logistics coordinator (BGLC). Each day, the BGLC is given a list of cargoes and passengers, some with both the pick up and delivery points (ships) and others with only the delivery points (i.e., the cargo is already on the station ship). Then, the BGLC must construct a route or routes for the helicopter to pick up and deliver all the cargoes and passengers on the list. Any cargoes left undelivered at the end of the day are put on the list to be delivered the next day. As for passengers, they are usually required to be at their destinations by certain times on a given day. So, the BGLC must ensure that these requirements with passengers are satisfied. Figure 1 depicts a schematic picture of a helicopter route.

At the present, the BGLC constructs the routes manually using various rules of thumb such as visiting ships in a clockwise or counter clockwise fashion. However, more experienced BGLC's may have more complicated rules or insights which often lead to very good routes. Below, the problem of

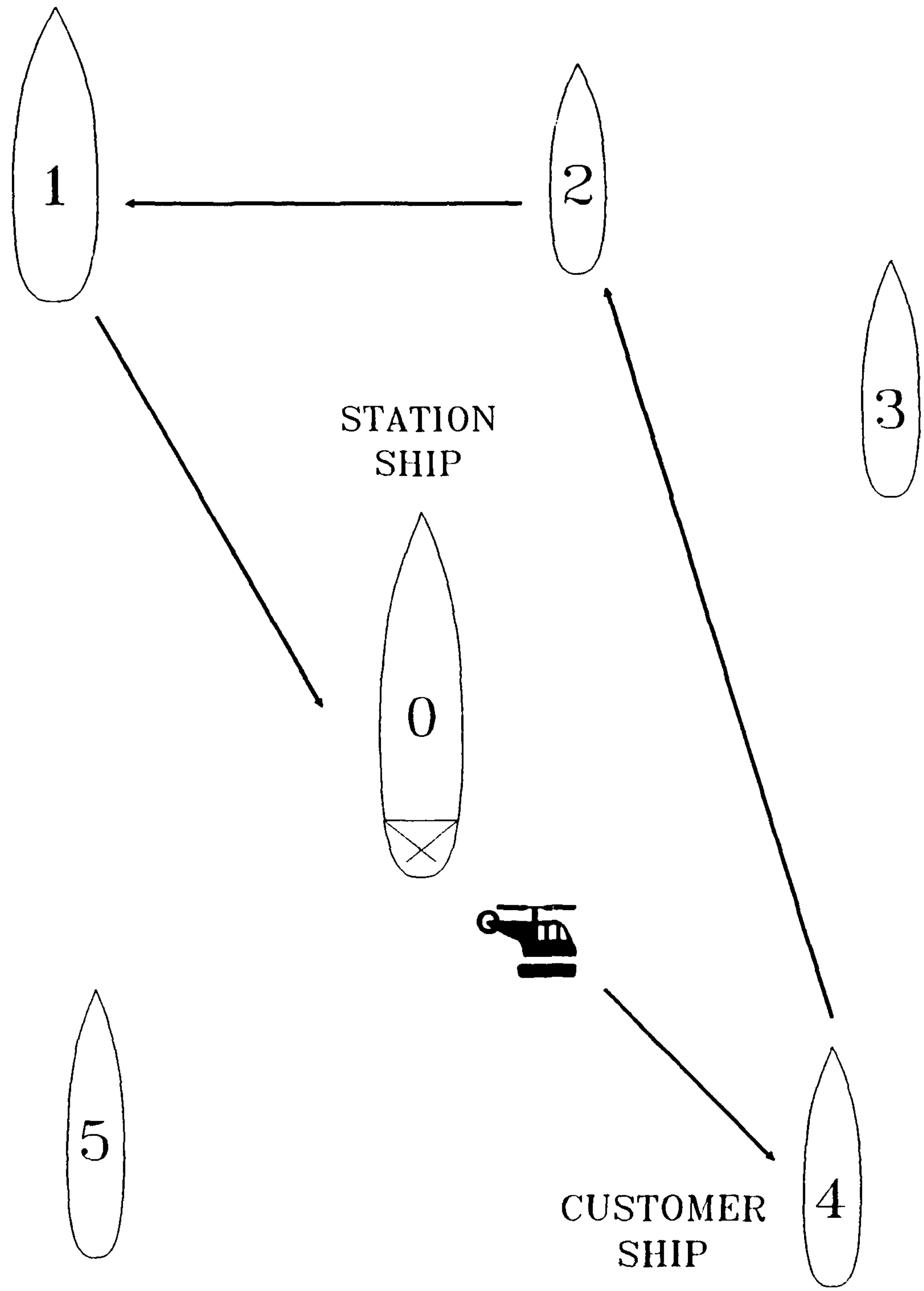


Figure 1. A Logistics Helicopter Route

constructing routes for the LOG HELO operation is formulated as a mathematical programming model. As with any model, there are some aspects of the problem which must be excluded in order to make the resulting formulation tractable.

B. MODEL ASSUMPTIONS

1. Objective Function

The model below assumes that the BG is performing a peacetime mission and the objective is to gain an efficient use of the helicopter. During a crisis, the objective of a BG is to maintain the highest state of combat readiness. This in turn implies that the BGLC should schedule the LOG HELO operation to support this objective. Although a mathematical programming model can be formulated (see Pilnick (1989) for discussion of similar problems), it requires knowing combat values for all cargo types. Since the assignment of combat values to cargoes is beyond the scope of this thesis, the peace time operation of LOG HELO is assumed. Although there are generally two helicopters on a station ship, the model schedules only one helicopter as the second is usually assigned to other duties or grounded for maintenance. Further, the helicopter is assumed to depart from the station ship carrying all cargoes and passengers for delivery, and does not pick up any cargoes on the route.

The objective is to minimize the flying time of this helicopter during a LOG HELO operation. Note that, when the helicopter can carry all cargoes to be delivered on a given day in one trip, minimizing flying time means finding the quickest route to visit all delivery points once. This problem is known as the traveling salesman problem (TSP). When cargoes must be split into multiple trips, the problem consists of two stages: one to partition the cargoes into trips and the other to find the quickest route for each trip. If the optimal partition is known, the problem reduces to solving several traveling salesman problems, one for each trip. In general, the optimal partition is not known and to solve the problem optimally would require partitioning and resolving the traveling salesman problems. On the other hand, one can produce a good solution by simply maximizing the amount of cargoes carried by the helicopters and at the same time minimizing the delivery time for the onboard cargoes.

2. Travel Times

In this thesis, the travel time from ship i to ship j is assumed to consist of three components: the pick up time at ship i (if any), the flying time from i to j, and the unloading time at ship j. The pick up and unloading times (VERTREP time) are assumed to be known prior to the scheduling of the helicopter and are treated as constants in the model. The flying time is given by the following equation (see Praprost, 1989; and Hardgrave, 1989):

$$T_{i,j}=\frac{F\times(Y_j-Y_i)+\sqrt{[F\times(Y_j-Y_i)]^2+(H^2-F^2)\times[(X_j-X_i)^2+(Y_j-Y_i)^2]}}{(H^2-F^2)} \quad (1)$$

where (x_i, y_i) and (x_j, y_j) denote the coordinates of ship i and j respectively, F is the speed of the formation, and H is the helicopter speed.

The above calculations assume that weather is perfect and ships remain in their assigned position for the entire LOG HELO operation. With regard to the latter assumption, ships may need to reduce speed or change course, hence falling behind their assigned position during UNREP. However, the effect of falling behind (fallback) during a VERTREP is negligible for three reasons: the large flight envelope of the CH-46 helicopter reduces the need to deviate from formation course or speed; the time spent on a VERTREP is short, usually less than ten minutes; and because of the relatively fast speed of the helicopter, even a 5 NM fallback would change flight times by less than three minutes.

As for the weather assumption, the formulation for T_{ij} could be modified to include the variation in time as a function of weather. However, such modification would, again, be beyond the scope of this thesis.

3. Delivery Time Windows

During a mission, ships have a daily schedule of events such as gunnery exercise and engineering drills. Some of these events might endanger the helicopter and some would preclude VERTREP operations. It is therefore assumed that the periods in which each ship is available for VERTREP are known apriori.

4. Cargo Consolidation

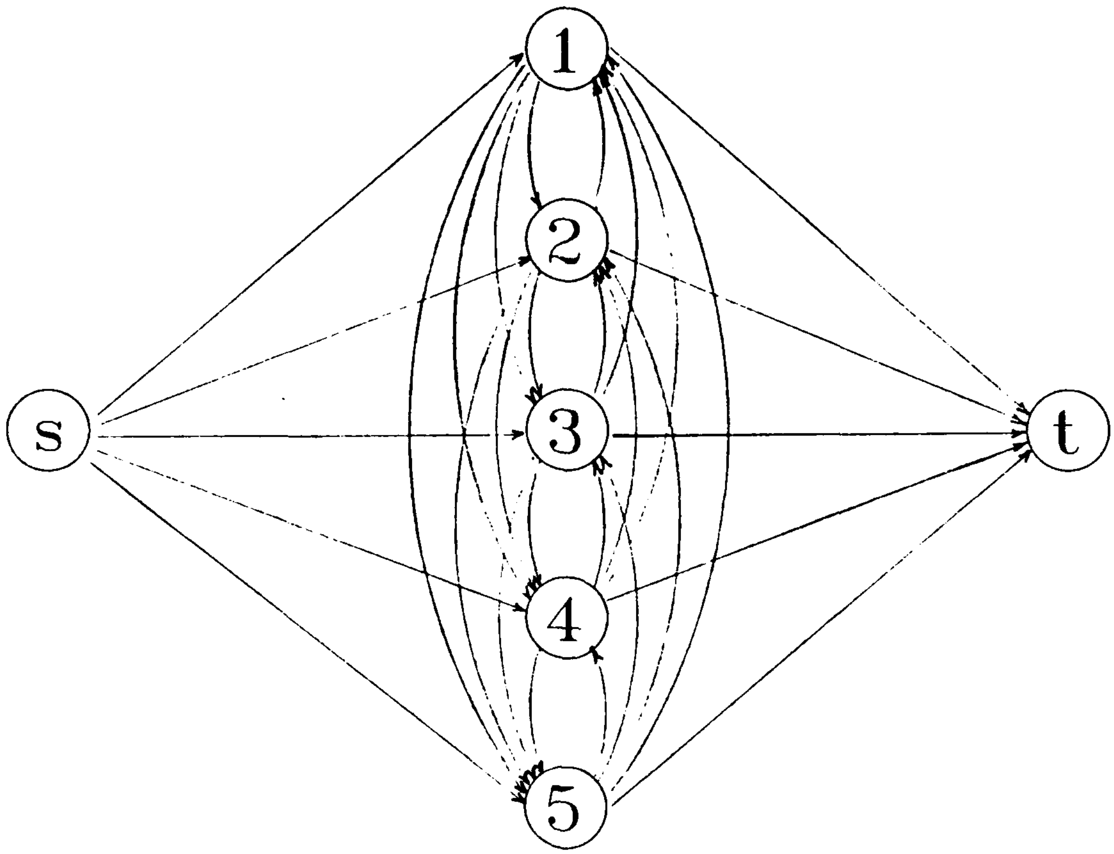
Given a list of cargoes, the problem of loading the maximum number of cargoes onto the helicopter is difficult. Without considering the weight capacity of the helicopter, the problem is a three dimensional knapsack problem which is a hard combinatorial problem. To make the problem more tractable, the following assumptions are made:

(i) All cargoes destined for the same ship are consolidated into a single and inseparable piece of cargo.

(ii) Similarly, all cargoes and passengers destined for the same ship must fly on the same flight and be delivered at the same time.

C. FORMULATION

Under the cargo consolidation assumption, selecting ships to be visited on a route implicitly determines the cargoes to be delivered. This relationship reduces the problem to finding a sequence of ships to be visited which can be represented as a network. Figure 2 presents the network of helicopter movements for the five ship BG depicted in Figure 1. Nodes numbered 1 to 5 represent the customer ships and



s,1,2,3,4,5,t	$\in$ N, the set of all nodes
s	source node (departure from station ship 0)
t	sink node (return to ship 0)
1,2,3,4,5	transshipment nodes
i ⟶ j	arc pair (i,j) $\in$ A, the set of all arcs

Figure 2. A Network of Helicopter Movements

node s and t represent the start and finish of the flight. Implicitly, both s and t can also be viewed as nodes representing the station ship at two different times: start and finish. The directed arc, say from node i to j, indicates the flight from ship i to ship j. A sequence of arcs from s to t would then represent a route for the helicopter. For example, (s,1), (1,3), (3,5), and (5,t) means that the helicopter delivers cargoes to ships in the following order: 1, 3, and 5. This route then implies that cargoes destined for ship 2 and 4 are to be delivered by future flights. Let G(N,A) denote the network for possible helicopter movements where N={s,t,1,2,...,NS}, NS = number of (customer) ships in the BG, and A is the set of arcs in the network. Then, the logistic helicopter routing problem can be stated as follows:

Indices:

i,j,k	nodes in the network, i.e., i = s, t, 1, 2,..., NS
h	passenger section, h = 1, 2, 3 (CH-46 can be considered to have three passenger sections)
q	delivery time window

Data:

W_j	total weight of all passengers and cargoes destined for ship j
V_j	total volume of all cargoes destined for ship j
P_j	number of passengers destined to ship j
PS	seats in a passenger section (six per section for a CH-46)

PV	volume of each passenger section on helicopter (approximately 240 ft^3 for a CH-46)
T_{ij}	flight time from ship i to ship j
VT_j	VERTREP or transfer time at j
Q^j	number of time windows for ship j
C_q^j	start of the q^{th} delivery time window for ship j
F_q^j	finish of the q^{th} delivery time window for ship j
B	the node-arc incidence matrix for G (N,A)
b	A column vector with NS+2 components, all of which are zero except for two. The component corresponding to node s is equal to -1, and the one for node t is equal to 1.
WC	weight capacity of the helicopter (4000 lbs for CH-46)
VC	volume capacity of the helicopter (720 ft^3 for CH-46)
TC	maximum allowable flight time (Normally 2 hours per flight, more if refueling at customer ships is considered.)

Decision Variables:

X_{ij}	A binary variable to indicate whether the arc (i,j) is included in the optimal route.
Z_h	A binary variable to indicate whether passenger section h is used.
Y_q^j	A binary variable to indicate the time window in which the helicopter is to make delivery at ship j.
D_j	A continuous variable representing the departure time from ship j. By convention, D_s = the start time of the flight, and D_t = the completion time.

Model:

Primary Objective: $MAX \sum_{(i,j) \in A} X_{ij}$

Secondary Objective: $MIN \; D_t$

Constraints:

$$\sum_{(i,j) \in A} W_j \times X_{ij} \leq WC \qquad (2)$$

$$\sum_{(i,j) \in A} P_j \times X_{ij} - PS \times \sum_h Z_h \leq 0 \qquad (3)$$

$$PV \times \sum_h Z_h + \sum_{(i,j) \in A} V_j \times X_{ij} \leq VC \qquad (4)$$

$$\sum_{(i,j) \in A} (T_{ij} + VT_j) \times X_{ij} \leq TC \qquad (5)$$

$$\sum_{(i,j) \in A} \max \{ (D_i + T_{ij} + VT_j - D_j) , (\sum_q C_q^j \times Y_q^j + VT_j - D_j) \} \times X_{i,j} \leq 0 \qquad j = 1, 2, \ldots, NS, t \qquad (6)$$

$$D_j - \sum_q F_q^j \times Y_q^j \leq 0 \qquad j = 1, 2, \ldots, NS \qquad (7)$$

$$\sum_q Y_q^j = 1 \qquad j = 1, 2, \ldots, NS \qquad (8)$$

$$B x = b \qquad (9)$$

In the above formulation, the primary objective is to ensure that the helicopter visits the maximum number of customer ships hence delivering the maximum number of cargoes per flight. The secondary objective is to guarantee that the most efficient route is used to deliver the cargoes. Constraints 2 to 5 ensure that the capacities; weight, volume, and fuel, of the helicopter are not exceeded. Constraints 6, 7, and 8 force the helicopter to deliver within one of the time windows for each ship and eliminate subtours (Desrochers et al, 1988). As stated, constraint 6 is a nonlinear constraint; however, it can be replaced with the following linear constraints:

$$\sum_{(i,j) \in A} (D_i + T_{ij} + VT_j - D_j) \times X_{i,j} \leq 0 \qquad j = 1, 2, \ldots, NS, t$$

$$\sum_{(i,j) \in A} ((\sum_q C_q^j \times Y_q^j) + VT_j - D_j) \times X_{i,j} \leq 0 \qquad j = 1, 2, \ldots, NS, t$$

D. PRIOR WORK

A few UNREP models are available. BFORM (Battle Force Operation Replenishment Model, Johns Hopkins Applied Physics Laboratory) and RASM (Replenishment at Sea Model, Center for Naval Analyses) are UNREP simulation models designed to study CLF ship designs. Both models use myopic demand based scheduling algorithms for CONREP and overlook VERTREP (Harris, 1989). TACREP (Tactical Replenishment Model), currently under development as a follow-on to RASM, significantly improves the CLF scheduling algorithm, but to date does not consider

VERTREP either. Only the HELPS model (Renwick, 1975) actually models helicopters, but it simulates amphibious operations and would be more appropriate for intra-theater lift. None of these models provide an appropriate framework from which to build.

Deo and Pang (1984) have classified hundreds of shortest path and vehicle routing algorithms which fall into two categories: problems with time windows, or problems with resource constraints, bu_ none considered both as is the case with the LOG HELO problem. Desrosiers et al (1984) solve the routing problem with time windows by partitioning the problem into separate shortest path problems each with time windows which are then solved using a dynamic label setting technique. This label setting technique is improved upon by Desrochers and Soumis (1988) by altering the order in which the nodes are explored to exploit the time window structure. However, neither of these techniques consider resource constraints. Beasley and Christofides (1988) solve the multiple resource constrained problem using Lagrangian relaxation to provide a lower bound on the solution, then employ branch and bound techniques to reach the final solution. Aneja et al (1983) solve the same problem using only branch and bound. These resource constrained problems do not consider time windows. All these algorithms reviewed stressed techniques which take advantage of the specific structure of their network problem.

As the LOG HELO problem is more general, these algorithms are not directly implementable.

III. IMPLICIT ENUMERATION

Implicit enumeration (IE), or branch and bound algorithms, are an improvement over total enumeration algorithms. The IE process examines a subset of all possible paths or routes for the helicopter. This technique can be extremely effective in problems with six or more customer ships. For example, the program generates less than 0.2% of all possible paths for most ten ship problems.

A. IMPLICIT ENUMERATION ALGORITHM

The basic algorithm is divided into two steps, the first step is designed to generate all possible paths. This is done using a depth first search (DFS) branching rule. Using the DFS rule, a new path is generated by adding a ship to the end of the last path. In the second step, each path generated is tested for its potential to provide an optimal answer. Any path with no potential is eliminated from further consideration, and the path is said to be 'fathomed'. Fathoming pares a branch when further exploration of that branch will yield no path with a better solution.

Although the algorithm may generate thousands of paths, only two need to be retained at any time: the current path, and the incumbent path. As each path is generated, it replaces the previous one and is designated as the current path. Of all the paths previously generated, the incumbent path is the one which yields the best values for the objective functions. Any current path which is not fathomed, is compared to the incumbent path. When all paths have been generated and compared, the incumbent path is optimal. The algorithm is presented below.

IMPLICIT ENUMERATION ALGORITHM

STEP 0) Initialize the Algorithm:

0.1 Declare the station ship as visited, and the customer ships as unvisited.

0.2 Declare the current path as containing only the station ship

0.3 Declare the number of ships in the incumbent path as 0

STEP 1) Generate a New Current Path, using DFS: from the last ship i, in the current path-

1.1 If no ships have been scheduled to be visited from ship i, schedule each unvisited ship, j, for branching from ship i, in the order of the minimum travel time from i to each j. (dotted lines in Fig. 3)

1.2 If any scheduled ships are unvisited from i, branch to one of them, i+1. (solid lines Fig. 3), Otherwise go to 1.3

1.2.1 Remove i+1 from the schedule for ship i

1.2.2 Label i+1 as visited

1.2.3 Assign i+1 as the last ship in the new current path

1.2.4 Proceed to step 2.

1.3 If no scheduled ships exist from ship i, backtrack: (dashed lines in Fig. 3)

1.3.1 Label ship i as unvisited

1.3.2 Move to ship i-1

1.3.2.1 if no such ship exists, stop;

1.3.2.2 otherwise, go to 1.2

STEP 2) Evaluate the Current Path

2.1 If possible, fathom the path (see fathoming techniques below) and go to 1.3

2.2 Compare current path versus the incumbent path

2.2.1 If the number of ships on the current path, (CPL), exceeds the number of ships in the incumbent path (IPL); replace the incumbent

2.2.2 If the CPL = IPL , and the current total flight time is less than the incumbent total flight time; replace the incumbent

2.2.3 Otherwise, go to STEP 1.

Figure 3 demonstrates the DFS branching for the current path (0,4,2,1,0) shown in Figure 1. In step 0, the station ship is declared as visited (square) and the customer ships 1-5 are initialized as unvisited (circles). Step 1.1 schedules ships 1-5 (dotted lines) to be visited from 0. Then in step 1.2, ship 4 is visited (solid line) and the current path becomes (0,4). The path is not fathomed in step 2 and the algorithm returns to step 1. Step 1.1 schedules ships 1, 3, and 5 from ship 4 and step 1.2 branches to ship 2. The new current path is (0,4,2). This process continues until the current path (0,4,2,1) is fathomed in step 2. Then step 1.3

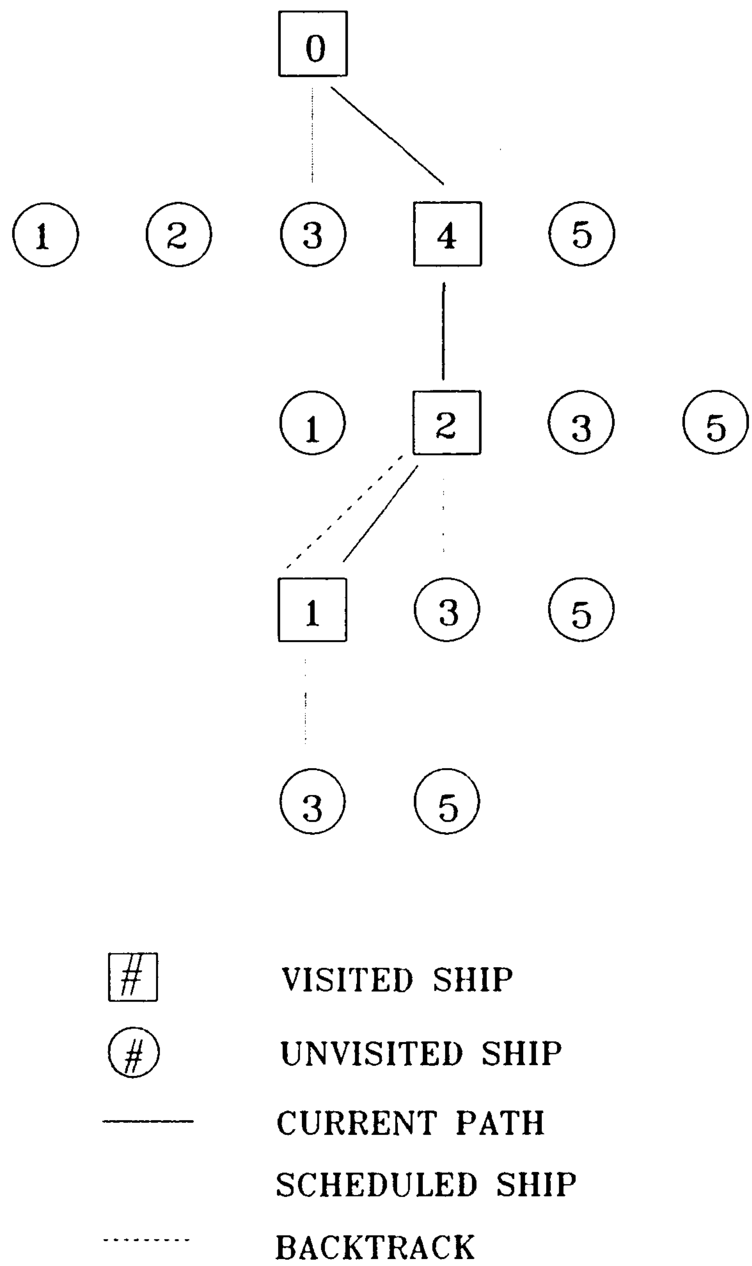


Figure 3. Example of Path Development using DFS

backtracks to (0,4,2), and step 1.2 branches to either ship 3 or 5 to form the next path.

The DFS branching rule is well suited for use with IE. DFS quickly begins exploring routes with a large numbers of ships, thereby maximizing the primary objective function. By branching to the nearest ship, the DFS rule attempts to find the optimal, or a near optimal answer as quickly as possible. Obtaining a near optimal answer early allows the algorithm to fathom more paths.

B. EXAMPLE PROBLEM

The following example LOG HELO scheduling problem is used to illustrate the fathoming techniques below. The BG from Figure 1 is updated in Figure 4 to show a current path (0,4,2,1,0) marked by the solid lines, and the incumbent path (0,5,4,3,2,0) marked by the dashed lines. The VERTREP and travel times between the ships are consolidated for simplicity and are displayed next to the respective arcs.

The characteristics of the incumbent path and current path are summarized in Table 1. The incumbent path length (IPL) is 4, which is simply the number of ships in the path. The current path length (CPL) is 3. The cumulative weights and volumes are sums of those variables for the ships on the path. The cumulative flight time is the sum of the travel times between the ships in the path, with VERTREP times included. Total flight time is this cumulative time with the added

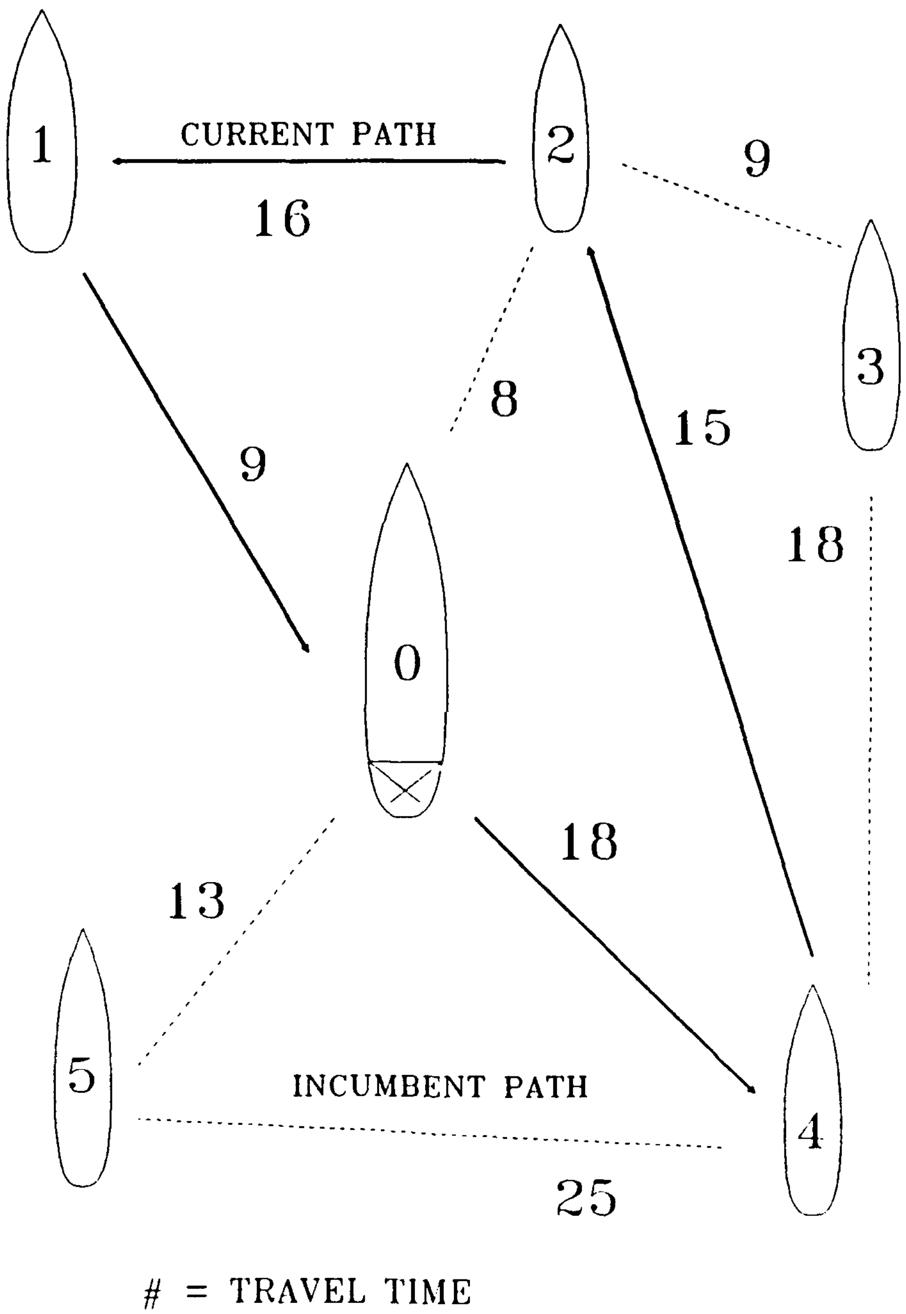


Figure 4. Current and Incumbent Paths for Example

travel time to return to ship 0. The weight, volume, and time capacities of the helicopter are 4000 lbs, 720 ft^3, and 10 hours (assumes refueling possible).

Table 1. Path Characteristics of Example Problem

CHARACTERISTICS	INCUMBENT PATH	CURRENT PATH
Path	0-5-4-3-2	0-4-2-1
Path Length	4	3
Cumulative Weight	3050	3250
Cumulative Volume	400	450
Cumulative Flight Time	65	49
Total Flight Time	73	58

C. FATHOMING

Four fathoming techniques are presented below. The first three are designed to produce paths which maximize the number of customer ships visited (the primary objective). The last technique minimizes the total flight time (the secondary objective) among those paths which optimize the primary objective.

1. Feasibility Constraints

As a current path is generated, the program accumulates statistics on the total flight time, number of passengers carried, as well as the cumulative weight and volume of the cargoes to be delivered on that path. For the example, the weights of the cargoes to be delivered are shown in Table 2. As each path is generated, the cumulative weight the helicopter must carry is determined, (see Table 1), and compared with the helicopter weight constraint. Note that the path (0,4,2,1) does not exceed the 4000 lbs limit, and the path would not be fathomed. If the cumulative weight had exceeded the weight capacity, and the program would backtrack to (0,4,2) and look for another scheduled ship.

Table 2. Consolidated Cargo List

SHIP	WEIGHT (lbs)	VOLUME (ft^3)
0	0	0
1	2000	200
2	500	150
3	800	50
4	750	100
5	1000	100

The flight time must be checked to guarantee time window feasibility. If the current path does not schedule the VERTREP within a current time window, a delay is added to the flight time which indicates that the helicopter waits until the next available time window. If no such time window exists before the time constraint is exceeded, the path is fathomed.

2. Minimum Additional Cargo (MAC)

This technique tests whether another ship can be added to the current path. The test simply adds to the current cumulative values the minimum weight, volume, and travel time for the ships/cargoes yet to be visited. If the addition of these minimum values make the current path infeasible, the path is fathomed.

3. Maximum Path Length (MPL)

This method determines the upper bound on the number of ships that can be visited prior to developing any paths. Any path containing more ships than the MPL is fathomed since no ship can be added without exceeding a constraint.

The MPL is simply the minimum of the maximum number of ships which each constraint would allow the helicopter to visit. Table 3 demonstrates how to calculate the maximum number of ships the weight constraint allows the helicopter to visit. First, the weights of the cargoes to be delivered to

Table 3. Maximum Path Length (MPL) Calculation

Ordered Cargo Weight	Associated Ship	Path Generated	Cumulative Weight	MPL
500	2	0,2	500	1
750	4	0,2,4	1250	2
800	3	0,2,4,3	2050	3
1000	5	0,2,4,3,5	3050	4
2000	1	0,2,4,3,5,1	5050	*

* Adding ship 1 exceeds helicopter capacity: MPL = 4

each ship, W_t, are ordered from smallest to largest. A path is then developed which visits the ships in that order (0,2,4,3,5,1) and the cumulative weight to be carried on the helicopter is simply the sum of these weights. When visiting another ship will cause the cumulative weight to exceed the weight capacity, the maximum path is reached since all ships not already on the path have even greater weights. For this example, the maximum path length is 4, since adding ship 1 to the route exceeds the helicopter lift capacity. Using the data from Tables 2 and 4, the ship maximums based on the volume and time constraints are both 5. Thus the MPL is 4 and any path with more than 4 ships would be fathomed. When a path contains exactly 4 ships, the path would be fathomed only when the flight time of the path exceeds that of the incumbent path.

Table 4. Travel Times* for Example Formation

From \ To	0	1	2	3	4	5
0		17	9	19	18	13
1	9		12	19	22	14
2	8	16		14	22	16
3	13	19	9		18	17
4	13	23	15	18		20
5	14	22	17	23	25	

* includes VERTREP times

4. Minimum Flight Time (MFT)

Given that the incumbent path length (IPL), equals the maximum path length (MPL), it is possible to fathom a path based on the secondary objective function, time. When the IPL and MPL are equal, they represent the maximum number of ships that can be visited. If the MFT for the current path exceeds the flight time for the incumbent path, the current path can be fathomed.

One method for calculating the MFT for the current path can be illustrated by referring back to Figure 4. In this figure, ships 3, and 5 are unvisited. The minimum travel time (MTT) to visit ship 5 (see Table 5), is the minimum between $T_{1,5}$ (14), and $T_{3,5}$ (17), or 14 minutes, since these are the only ships from which ship 5 can possibly be visited.

Table 5: Minimum Travel Time Calculation

TRANSIT TIMES FROM SHIP	TO SHIP		
	3	5	0
1	19	14	
3		17	13
5	23		14
MINIMUM TIME	19	14	13

Similarly, the minimum travel time to customer ship 3 and station ship 0 are 19 and 13, respectively. The times for the unvisited ships are added in the increasing order of the minimum time to the travel time of the current path until the number of ships in the path equals the MPL (see Table 6). In this case, the MFT equals 76 which exceeds the total flight time of the incumbent path (73 minutes) and the current path can be fathomed.

Table 6: Minimum Flight Time Calculation

PATH	SHIPS IN PATH	CUMULATIVE TRAVEL TIME	TOTAL FLIGHT TIME	COMMENTS
INCUMBENT	4	65	+ 8 = 73	IPL = MPL
CURRENT	3	49		IPL - CPL = 1
+ MTT(1)*	4	+ 14		ADD 1 SHIP
+ MTT(0)*	4	= 63	+ 13 = 76**	

* From Table V.
** MFT for current path greater than incumbent, fathom current path

D. INITIAL INCUMBENT PATH

Nine methods are used to determine the initial incumbent path. The first four simply branch to the ship with the next closest bearing in the following fashion: (1) clockwise from ship 0, (2) counter-clockwise from ship N, (3) clockwise from ship (N+1)/2 rounding up, and (4) from ship (N-1)/2 rounding down. Similarly, the fifth method branches to the next closest ship. Methods six and seven branch to the next closest ship not eliminated by the MPL rule for weight and volume, respectively. Finally, the eighth and ninth methods simply visit ships by next minimum weight or volume. Each method provides a single path. The nine paths are compared and the one which has the best values for the objective functions is assigned as the initial incumbent path.

An IE algorithm has been developed from the model in Chapter 2 utilizing the depth first search branching technique, the four fathoming rules, and the initial incumbent path methods discussed in this chapter. Several problem sets were developed to rigorously test accuracy and effectiveness of the algorithm. The problem set generation and the analysis of test results are reviewed in Chapter 4.

IV. RESULTS

A. PROBLEM GENERATION

A FORTRAN implementation of the algorithm described in Chapter 3 was tested on an IBM 3033AP. For actual fleet use, the program should run in less than 15 minutes on an microcomputer with an 8088 processor. Therefore, a run time goal of 20 CPU seconds on the IBM 3033 was set as a criteria for effective performance.

Seven problem sets were designed to find the poorest performance aspects of the algorithm under a variety of conditions. To examine the algorithm effectiveness, each of the first four problem sets contained different types of cargo. The last three sets were used to investigate the effects of different types of time windows. The random generation of data for each problem set is discussed below:

1. Battle Group Formation

All seven problems were generated with a single station ship and ten customer ships. Ship positions were determined as follows:

a. Station Ship:
   Bearing: Uniformly distributed U(0-359) degrees
   Range: U(0-10) NM

b. Customer Ships:
   Bearing: 2 or 3 per 90 degree sector U(0-90)
   Range: U(5-30) NM

A formation with ten customer ships was chosen as the upper limit of a BG to be serviced by a single logistics support ship. Most battle groups contain only five to eight combatants. In multiple carrier battle forces (BF), battle groups continue to maintain individual integrity with respect to the CLF station ship. All cargoes destined for ships outside the LOG HELO's BG would be coordinated through and delivered to the station ship for that BG, thus BF operations add only one ship to the BG problem.

2. Cargo List

Four separate cargo types were utilized to test the effects of passengers and cargoes with different volume versus weight correlations. Each cargo list was designed so that the mean total volume is 800 ft^3, and the mean total weight is 4400 lbs, thereby ensuring optimal routes of 8, 9, or 10 ships to test all possible outcomes. The list of cargoes and passengers for delivery was created by generating random demands for individual ships as shown below:

a. Problem Sets 1, 5, 6, and 7; Uncorrelated Cargoes, No Passengers:

volume U(20-140) ft^3,
weight U(340-500)lbs

b. Problem Set 2; Negatively Correlated Cargoes, No Passengers:

volume U(20-140) ft^3,
weight (500 - volume $\pm$ 20) lbs

c. Problem Set 3; Positive Correlated Cargoes, No Passengers:

volume U(20-140) ft^3,
weight (340 + volume $\pm$ 20) lbs

d. Problem Set 4; Passengers and Uncorrelated Cargoes:

Cargoes: volume U(20-80) ft^3,
weight U(140-300) lbs

Passengers: Each ship had a 30% chance of being the destination of 1 to 4 passengers

3. Time Windows

To more completely test the algorithm effectiveness the flight time constraint was relaxed to ten hours (refueling assumed) forcing the algorithm to schedule more ships and increase the run time. Five randomly chosen ships in each trial had this ten hour period restricted to test the algorithm effectiveness in dealing with common fleet restrictions on VERTREP operations. The first four problem sets included a single one hour unavailability period (UP) for each of the five ships. Unavailability periods occur when ship operations preclude VERTREP. These periods had randomly chosen start times: U(0-9) hours. In problem set 5 the UP start times occurred only in the first three hours. Sets 6 and 7 used narrow delivery time windows (NTW's). Such delivery time windows are required when passengers or cargoes must be delivered to the customer ship at a specific time.

NTW's of one hour were used, with start times spread out over three hours for set 6 and nine hours for set 7.

The characteristics of each problem set are displayed in Table 7. Twenty trials were generated for each of sets.

Table 7: Type of Data Generated for Each Problem Set

PROBLEM SET	1	2	3	4	5	6	7
NEGATIVE CORRELATION		X					
NO CORRELATION	X				X	X	X
POSITIVE CORRELATION			X				
WITH PASSENGERS				X			
UP, 9 HRS	X	X	X	X			
UP, 3 HRS					X		
NTW, 3 HRS						X	
NTW, 9 HRS							X

Several different comparisons were conducted with the trial results. The model was validated using the first four problem sets. Problem sets 1, 2, and 3 were used to evaluate the effectiveness of the initial incumbent path methods. The fathoming rules were compared using problem set 4. Sets 1, 2, and 3 were also used to analyze the effects of cargo correlation, while passenger effects were examined using sets 1 and 4. The last test evaluated the effects of various time windows using problem sets 1, 5, 6, and 7. The problem sets used for each test are displayed in Table 8, which is followed by a discussion of the results of these tests.

Table 8: Tests For Which Problem Sets Were Used

PROBLEM SET	1	2	3	4	5	6	7
VALIDATION	X	X	X	X			
INITIAL INCUMBENT	X	X	X				
FATHOMING				X			
CARGO CORRELATION	X	X	X				
TIME WINDOWS	X				X	X	X
PASSENGERS	X			X			

B. VALIDATION

The IE algorithm was validated with problem sets 1 through 4 using a second program which totally enumerated all possible paths. In all eighty trials, both the primary and secondary objective function values were identical for the two programs, although alternate optimal solutions were sometimes discovered.

C. INITIAL INCUMBENT PATH ANALYSIS

Table 9 summarizes sixty trials conducted with and without initial incumbent paths (IIP) using problem sets 1, 2, and 3. Although IIP only reduced the mean run time by 7%, IIP never significantly increased run times, and occasionally greatly reduced them, therefore IIP was used in the rest of the testing.

Table 9: Run Time Reduction Using Initial Incumbent Paths for Problem Sets 1, 2, and 3.

RUN TIME REDUCTION	PERCENT OF TRIALS
25% or more	10
10-25%	23
0-10%	57
0-4% increase	10

NOTES: Maximum reduction : 82%
Mean Reduction : 7%
Maximum increase : 4%

The results of the individual IIP methods are displayed in Table 10. In general, none of the methods out performed any other. With a combined success rate of 98%, methods 6 through 9 nearly guaranteed a path with an optimal solution for the primary objective function (ie. the optimal path length, or OPL). However, they performed poorly with regard to the second objective of optimizing the flight time. Conversely, methods 1 through 5 were less likely to generate a path with the correct OPL (only 88% combined); but when correct, the paths provided good flight times. Each of the first five methods produced a path which was within 10% of the optimal flight time in at least 16% of the trials. Utilizing the best of the nine methods on each trial, the length of the initial incumbent path was equal to the OPL in all but one out of the sixty trials and the flight time of the IIP was within 10% of the optimal time in 65% of the trials. Since the IIP procedure requires little run time, all nine methods should be retained, and augmented with additional methods like 1 through 5 to increase the IIP effectiveness in reducing average program run time.

Table 10: Effectiveness of Initial Incumbent Path Methods for Problem Sets 1, 2, and 3 (60 trials)

# of TRIAL SOLUTIONS	IIP METHODS									
	1	2	3	4	5	6	7	8	9	BEST*
WITH OPL	50	53	46	53	49	51	59	59	59	59
with OPTIMAL FLIGHT TIME	0	0	2	0	3	1	1	0	0	6
within 5% of OPTIMAL FLIGHT TIME	6	4	5	3	10	7	9	0	0	22
within 10% of OPTIMAL FLIGHT TIME	10	14	14	11	15	12	12	0	0	39
within 25% of OPTIMAL FLIGHT TIME	27	34	30	29	30	23	23	0	0	54

* method with closest value to optimal answer for each trial

D. FATHOMING TECHNIQUE ANALYSIS

The twenty trials of problem set 4 were conducted to test the effectiveness of the three fathoming rules individually and in combination as shown in Tables 11 and 12. The first column of both tables show that the maximum path length (MPL) and minimum additional cargo (MAC) techniques do not significantly reduce the problem run time. However, the minimum flight time (MFT) method fathomed an average of over 97% of the possible paths. While the benefits of MPL and MAC are minimal when combined with MFT, they do not increase run time, thus all three techniques are used for the rest of the testing.

Table 11: A Comparison of the Mean Run Times (in CPU sec) for Problem Set 4 with Differing Fathoming Techniques

TECHNIQUE	ALL TRIALS (20)	TSP ONLY (6)	non-TSP, MPL = OPL (11)	non-TSP, MPL ≠ OPL (3)
TOTAL ENUMERATION	780	780	780	780
CONSTRAINTS ONLY	436	912	247	174
MPL	380	764	227	173
MAC	521	1159	267	176
MPL,MAC	358	710	217	174
MFT	32.7	4.3	8.0	180
MPL,MAC,MFT	32.6	4.3	8.0	179

To further illustrate the effects of the fathoming techniques, Tables 11 and 12 also divide the trials into three categories. The trials in column 2 had optimal path lengths of ten ships, these are the travelling salesman problems (TSP's) mentioned in Chapter 2. The third column contains non-TSP problems for which the maximum path length (MPL) equals the optimal path length (OPL). Non-TSP problems with MPL's greater than the OPL are displayed in column 4. TSP problems are solved more quickly than non-TSP problems, but as long as the MPL matches the OPL the times are quite acceptable. However, when the MPL exceeds the OPL, the minimum flight time (MFT) technique can not be applied, and the algorithm run times exceed 120 seconds.

Recall that the MFT rule only begins fathoming paths once the incumbent path length (IPL) equals the maximum path length. Since the IPL is always less than or equal to the OPL, when the OPL is less than the MPL, the minimum flight time rule is useless, and run times may be unacceptable.

Table 12: A Comparison of the Mean Number of Paths Generated (in 1000's) for Problem Set 4 for Different Fathoming Techniques

TECHNIQUE	ALL TRIALS (20)	TSP ONLY (6)	NON-TSP, MPL = OPL (11)	NON-TSP, MPL = OPL (3)
TOTAL ENUMERATION	9900	9900	9900	9900
CONSTRAINTS ONLY	4800	9900	2900	1600
MPL	4600	9900	2500	1600
MAC	4500	9900	2400	1600
MPL,MAC	4400	9900	2300	1600
MFT	258	12	27	1600
MPL,MAC,MFT	257	12	26	1600

Trials in which the MPL exceeds the OPL can occur only when the number of ships in the optimal solution is N-2 or fewer. In these cases less than 27% of all possible paths are generated. For example, an eight ship problem with an MPL of

7 and an OPL of 6, generates less than 18,000 paths and would run in roughly five seconds. Thus only problem sets with 9 or 10 customer ships are of concern. The uncommon circumstances which cause the MPL to overestimate the OPL further reduce the likelihood and significance of this case and are discussed in the results of the cargo correlation and passenger testing.

E. CARGO CORRELATION ANALYSIS

Tables 13 and 14 present the run time and paths generated by trials using the uncorrelated, negatively, and positively correlated problem sets (1, 2, and 3 respectively). In general, the algorithm is highly effective for all cargo types with a median run time of eight seconds for all 60 trials. In the uncorrelated and positively correlated cases, the maximum path length (MPL) was correct for all 40 trials, with only one run time exceeding fifteen seconds. However, in the negatively correlated data set, three of the trials exceeded twenty seconds and two trials stand out in particular. In trial D, the MPL equalled the OPL, but the length of the IIP was less than both; and in Trial T, the MPL and OPL differed. As the MPL is calculated by checking the ordered weights and volumes, negatively correlated cargoes can sometimes cause the MPL to overestimate as in trial T, or create a poor IIP as in trial D. However, it is believed that typical helicopter loads will be positively correlated, thus the maximum path length will rarely overestimate for actual cargo lists.

Table 13: A Comparison of Run Times (in CPU sec) by Cargo Correlation using Problem Sets 1, 2, and 3

TRIAL	NEGATIVE CORRELATION Problem Set 2	NO CORRELATION Problem Set 1	POSITIVE CORRELATION Problem Set 3
A	7.1	4.0	7.8
B	6.7	3.5	7.2
C	9.1	2.1	5.8
D	48.9*	2.4	5.3
E	4.4	4.9	23.4
F	2.6	4.7	4.7
G	11.7	3.5	6.1
H	7.9	4.3	5.5
I	15.7	7.1	5.9
J	7.4	0.7	2.7
K	13.2	2.6	0.6
L	3.4	4.8	14.2
M	22.6	11.1	10.5
N	5.4	10.5	3.3
O	18.6	2.0	3.2
P	7.6	3.8	2.1
Q	8.5	8.8	3.0
R	6.6	1.1	7.9
S	8.7	0.9	6.5
T	450.8**	5.6	10.4
TOTAL	666.9	88.4	139.1
MEAN	33.3	4.4	7.0
TOTAL Less D,T	167.2		
TOTAL Less D,T	9.3		

* - no IIP, ** - MPL ≠ OPL

Table 14: A Comparison of the Number of Paths Generated (in 1000's) by Cargo Correlation using Problem Sets 1, 2, and 3

TRIAL	NEGATIVE CORRELATION Problem Set 2	NO CORRELATION Problem Set 1	POSITIVE CORRELATION Problem Set 3
A	23.0	10.3	24.2
B	20.8	10.2	23.0
C	29.3	4.8	17.3
D	201.1*	5.9	15.2
E	13.2	12.8	84.0
F	6.5	13.9	13.2
G	39.2	9.6	8.6
H	23.0	12.6	7.4
I	55.2	21.4	18.6
J	21.7	1.2	7.0
K	43.9	6.7	0.9
L	9.0	13.7	48.3
M	80.4	36.1	35.2
N	15.9	37.2	9.0
O	70.8	4.6	8.0
P	23.0	10.6	4.8
Q	26.5	26.2	6.9
R	18.9	2.5	22.9
S	30.6	1.7	21.0
T	3572.2**	16.7	35.6
Total	4324.2	258.7	411.1
Mean	216.2	12.9	20.6
Total Less D,T	550.9		
Mean Less D,T	30.6		

* - no IIP found, ** - MPL ≠ OPL

F. PASSENGER ANALYSIS

As previously mentioned, overestimation of the maximum path length (MPL) also occurred in trials with passengers from problem set 4. Tables 15 and 16 compare trials with and without passengers (problem set 1). In only three of the twenty trials were the MPL's greater than the OPL's. Without those three trials (C, E, and M), the run times are quite acceptable with a mean of 6.8 seconds.

The percentage of passenger problems which overestimate the maximum path length can be reduced to an insignificant level by modifying the program. The algorithm does not model the priority delivery of passengers in actual LOG HELO scheduling. A modification to allow the user to designate passengers or cargoes (ie. ships) for guaranteed delivery actually improves the MPL calculation. The weight and volume of passengers and cargoes destined to those ships can be subtracted from the helicopter capacity. The revised MPL is calculated based on this reduced capacity and the remaining ships with no guaranteed deliveries. Removing passengers from the MPL calculation, eliminates the source of error, and the MPL will be more accurate. Note in some cases, if too many passengers/cargoes are guaranteed delivery, no feasible route may exist.

Table 15: A Comparison of Run Times (in CPU sec) with and without Passengers using Problem Sets 1 and 4

TRIAL	NO PASSENGERS Problem Set 1	WITH PASSENGERS Problem Set 4
A	4.0	4.0
B	3.5	9.6
C	2.1	251.3*
D	2.4	4.8
E	4.9	124.6*
F	4.7	4.3
G	3.5	3.6
H	4.3	13.6
I	7.1	4.0
J	0.7	1.6
K	2.6	3.0
L	4.8	9.0
M	11.1	162.1*
N	10.5	2.4
O	2.0	9.8
P	3.8	4.3
Q	8.8	21.4
R	1.1	14.5
S	0.9	0.9
T	5.6	5.4
TOTAL	88.4	654.2
MEAN	4.4	32.7
TOTAL Less C,E, & M		116.2
MEAN Less C,E, & M		6.8

* MPL incorrect

Table 16: A Comparison of the Number of Paths Generated (in 1000's) with and without Passengers using Problem Sets 1 and 4

TRIAL	NO PASSENGERS Problem Set 1	WITH PASSENGERS Problem Set 4
A	10.3	10.4
B	10.2	32.4
C	4.8	2236.7*
D	5.9	13.8
E	12.8	1102.0*
F	13.9	12.2
G	9.6	9.6
H	12.6	47.8
I	21.4	10.4
J	1.2	3.7
K	6.7	8.9
L	13.7	24.9
M	36.1	1450.7*
N	37.2	6.1
O	4.6	31.1
P	10.6	13.0
Q	26.2	76.4
R	2.5	44.5
S	1.7	1.7
T	16.7	16.2
TOTAL	258.7	5152.5
MEAN	12.9	257.6
TOTAL Less C, E, & M		363.1
MEAN Less C, E, & M		21.4

* - MPL incorrect

G. TIME WINDOW ANALYSIS

Problem sets 1, 5, 6, and 7 were used to compare the effects of various time windows and the trial results are shown in Tables 17 and 18. With nearly identical mean run times, the spread of start times for the unavailability periods (UP's) used in sets 1 and 5 had little or no effect. The algorithm successfully solved all unavailability period trials in 12 seconds or less. However, narrow time windows did cause some difficulties. In the three hour case, the mean run time nearly doubled to 8.7, though only one trial exceeded 20 seconds. In the nine hour case, the mean run time was 20.8 seconds as seven trials exceeded the goal.

When ships have only a single hour for a delivery, the flight times of various routes begin to converge. This reduces fathoming by the minimum flight time technique and increases the run time. However, proper use of the algorithm can prevent time windows from severely increasing run times. From analysis of the trials in the first four problem sets, the total flight time was always less than four hours. However, in problem set 7, many trials had flight times in excess of five hours, indicating that the helicopter was delayed more than an hour while waiting for time windows to make deliveries. To correct this, the program should not schedule ships if their time windows do not begin in the first four hours, but should save these ships for the next flight.

Table 17: A Comparison of Run Times (in CPU sec) by Time Windows using Problem Sets 1, 5, 6, and 7

TRIAL	UP's 3 Hrs Problem Set 1	UP's 3 Hrs Problem Set 5	NTW's 3 Hrs Problem Set 6	NTW's 9 Hrs Problem Set 7
A	4.0	3.9	1.3	5.1
B	3.5	6.3	21.6	42.8
C	2.1	2.2	3.2	10.1
D	2.4	4.7	15.[illegible]	12.6
E	4.9	4.2	7.5	4.8
F	4.7	4.1	9.2	16.2
G	3.5	3.5	9.4	9.1
H	4.3	7.5	10.2	18.9
I	7.1	4.7	13.1	29.8
J	0.7	0.7	2.2	14.3
K	2.6	7.4	14.7	13.6
L	4.8	3.4	5.1	10.5
M	11.1	11.5	2.6	30.5
N	10.5	6.5	13.3	36.8
O	2.0	1.6	4.3	4.5
P	3.8	6.2	12.0	27.8
Q	8.8	5.8	9.9	28.4
R	1.1	0.8	6.3	10.4
S	0.9	0.9	9.0	18.9
T	5.6	3.1	4.5	60.9
TOTAL	88.4	89.0	174.6	405.6
MEAN	4.4	4.5	8.7	20.3

Table 18: A Comparison of the Number of Paths Generated (in 1000's) by Time Windows using Problem Sets 1, 5, 6, and 7

TRIAL	UP's 9 Hrs Problem Set 1	UP's 3 Hrs Problem Set 5	NTW's 3 Hrs Problem Set 6	NTW's 9 Hrs Problem Set 7
A	10.3	10.4	2.9	15.8
B	10.2	19.8	80.1	184.7
C	4.8	5.1	8.4	35.9
D	5.9	13.0	52.5	48.1
E	12.8	10.7	26.7	15.2
F	13.9	12.9	32.6	64.0
G	9.6	9.6	29.8	33.9
H	12.6	24.2	34.8	73.0
I	21.4	14.6	50.9	120.6
J	1.2	1.2	5.4	56.4
K	6.7	22.6	51.3	60.4
L	13.7	8.8	15.5	41.8
M	36.1	27.3	7.3	144.3
N	37.2	18.8	57.1	149.7
O	4.6	3.7	15.4	13.8
P	10.6	18.2	39.1	115.3
Q	26.2	16.4	45.4	123.9
R	2.5	1.6	19.8	35.8
S	1.7	1.7	30.7	69.8
T	16.7	8.6	13.6	254.4
TOTAL	258.7	249.2	619.3	1686.6
MEAN	12.9	12.5	31.0	84.3

In general, the algorithm is highly efficient. The problem sets constituted the most difficult scenarios the algorithm must solve. For the first four problem sets, the run time goal was exceeded only three times out of 75 trials with correct MPL's. The mean run time for these trials was 6.9 CPU seconds, with a mean of 21,100 paths generated. This implies that the program can quickly solve problems with eight or fewer customers, where the worst case (an MPL greater than OPL) generates fewer than 18,000 paths.

Only overestimation of the MPL on 9 or 10 ship problems can not be solved quickly. However, these problems are an extremely small fraction of the problems which the algorithm must solve. Therefore, adding a simple heuristic which provides a route close to the optimal answer within the run time goal would be sufficient to complete the algorithm.

V. CONCLUSIONS

An implicit enumeration algorithm for scheduling a single logistics helicopter route has been described. This algorithm employs a depth first search procedure to optimize two objective functions simultaneously. Analysis of the results in Chapter 4 reveals that the algorithm is highly effective for scheduling the helicopters. However, some improvements to the algorithm may enhance solution times, and several extensions are necessary before the algorithm can be employed.

- Identify the optimal path length for negatively correlated cargo lists to permit fathoming by minimum flight times.
- Develop additional initial incumbent path methods to include visiting ships in the order of time windows.
- Explore alternate depth first search branching rules which may be less effected by time windows.
- Modify algorithm to provide user with a choice of routes.
- Revise travel time calculation to include weather effects and refueling.
- Allow helicopters to carry external cargoes.
- Allow helicopters to pick up cargoes from customer ships.

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