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This report presents a summary of 30 papers published in archival literature, dealing with the issues of reduced order modeling, dynamics and control of space structures. Field/boundary element methods for control of nonlinear dynamic response of continuum models of space structures are discussed. Explicit expressions for tangent stiffnesses of truss and frame-type lattice structures are presented. Weak formulations of multi-body dynamics problems with constraint are presented.			
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In this final report, a descriptive summary of the research accomplished under AFOSR Grant 84-0020 is presented. Technical details of the research, mathematical formulations, their computational implementations and their verifications, are presented in the archival papers listed at the end of this report.

In [1], a singular-solution approach is used to derive n discrete coupled ordinary differential equations governing the transient dynamic responses of an initially stressed flat plate that is assumed to be a continuum model of a large space structure. Non-proportional damping is assumed. A reduced-order model of N ($\leq n$) coupled ordinary differential equations are derived in terms of the amplitudes of pseudo-modes of the nominally undamped system. Optimal control is implemented using m ($\leq N$) control-force actuators, in addition to a possible number p ($\leq m$) passive viscous dampers. Algorithms for efficient solution of Riccati equations are implemented. The problem of control spillover is discussed. Several example problems involving suppression of vibration of "free-free" and "simply-supported" plates are presented and discussed.

In Ref. [2], the problem of active control of transient dynamic response of large space structures, using the fully coupled nodal equations of motion, taking account of non-proportional passive damping, is treated. Equivalent continuum plate-models of LSS are considered. A reduced-order modeling of the structure using a boundary element method is presented. Efficient algorithms for solving the matrix algebraic Riccati equation are discussed. Several examples are presented to illustrate the controllability of the transient dynamic response of the structure, using an arbitrary number of control-force actuators.

In the work of Kondoh and Atluri [3], the influences of local (individual member) buckling and minor variations in member properties on the global response of truss-type structures are studied. A simple and effective way of forming the tangent stiffness matrix of the structure and a modified arc length method are devised to trace the nonlinear response of the structure beyond limit points, etc. Several examples are presented to indicate: (i) the broad range of validity of the simple procedure for evaluating the tangent stiffness, (ii) the effect of buckling of individual members on global instability and post-buckling response and (iii) the interactive effects of member buckling and global imperfections.

In a follow-up paper by Kondoh and Atluri [4], a simplified, economical and highly accurate method for large deformation, post-buckling, analyses of frame-type structures is presented. Using the concepts of polar-decomposition of deformation, an explicit expression for the tangent stiffness matrix of a member/element of the frame, that accounts for the nonlinear bending-stretching coupling, is derived in closed form, at any point in the load-deformation path. The ranges of validity of the present simplified approach are discussed. Several example problems to demonstrate the feasibility of the present approach, over ranges of deformation that are well beyond those likely to occur in practical large frame structures, are included.

In Ref. [5] a simple (exact) expression for the tangent-stiffness matrix of a space truss undergoing arbitrarily large deformation, as well as member buckling, is given. An arc-length method is used to solve the tangent-stiffness equations in the post-buckling range of the structural deformation. Several examples to illustrate the viability of the present approaches in analyzing large space structures, simply, efficiently, and accurately, are given.

In Ref. [6] by O'Donoghue and Atluri, the topic of vibration control of large space structures which, in this instance are modeled by equivalent continua, is addressed here. A "singular (or fundamental) solution" approach is utilized and the control algorithm is based on a fully coupled nodal system of equations which permits the effects of non-proportional damping to be monitored. Additionally, the structure will be allowed to undergo large deformations which will necessitate the implementation of a nonlinear control algorithm. In the scheme proposed here, the calculation of

the feedback control forces will be based on the feedback gain matrix obtained from a solution of the linear optimal control problem. Examples will be presented to illustrate the controllability of the vibrations in both the linear and nonlinear cases.

In a follow-up paper, O'Donoghue and Atluri [7] investigate the problem of active control of the transient dynamic response of large space structures, modeled as equivalent continua, is investigated here. The effects of initial stresses, in the form of in-plane stress resultants in an equivalent plane model, on the controllability of transverse dynamic response, are studied. A singular solution approach is used to derive a fully coupled set of nodal equations of motion which also include nonproportional passive damping. One approach considers a direct attack on this system of nodal equations. An alternative scheme implements a reduced-order model of coupled ordinary differential equations which are obtained in terms of the amplitudes of the pseudomodes of the nominally undamped system. Optimal control techniques are employed to develop a feedback control law. Algorithms for the efficient solution of the Riccati equation are implemented. Several examples are presented which involve the suppression of vibration of the transient dynamic response of the structure using an arbitrary number of control force actuators.

With the application of the shape-control of space-antennae in mind, Zhang and Atluri, in Ref. [8], derive integral equations for the representation of in-plane as well as transverse displacements of shallow shells undergoing small, quasi-static or dynamic deformations. A combined boundary/interior element method for static stress, free-vibration, and transient response analyses of shallow shells, based on these integral equations, is derived. Numerical results are presented to demonstrate not only the computational economy but also the superior accuracy realizable from the present approach, in contrast to the popular Galerkin finite element approach.

In Ref. [9], simplified procedures for finite-deformation analyses of space frames, using one beam element to model each member of the frame, are presented. Each element can undergo three-dimensional, arbitrarily large, rigid motions as well as moderately large non-rigid rotations. Each element can withstand three moments and three forces. The nonlinear bending-stretching coupling in each element is accounted for. By obtaining exact solutions to the appropriate governing differential equations, an explicit expression for the tangent-stiffness matrix of each element, valid at any stage during a wide range of finite deformations, is derived. An arc length method is used to incrementally compute the large deformation behavior of space frames. Several examples which illustrate the efficiency and simplicity of the developed procedures are presented. While the finitely deformed frame is assumed to remain elastic in the present paper, a plastic hinge method, wherein a hinge is assumed to form at an arbitrary location in the element, is presented in a companion paper.

In Ref. [10], nonlinear integral equations are derived for the representation of displacements of shallow shells undergoing moderately large, quasi-static or dynamic deformations. A combined interior/boundary element method based on these integral equations, and its implementation for the shell problem are discussed in detail. A "tangent stiffness" iterative procedure is used to obtain the nonlinear solution. Numerical results are presented to demonstrate the efficiency and the accuracy by the present approach.

In a follow-up paper [11], Atluri, Zhang and O'Donoghue discussed the following topics: (i) some of the recent advances in formulating finite deformation (large rotations as well as stretches) plate and shell theories, and attendant mixed finite element formulations based on symmetric variational statements; (ii) finite element/boundary element formulations based on unsymmetric variational statements, Petrov-Galerkin methods, and the use of fundamental solutions in infinite space, for the highest-order differential operator of the problem, as test functions in solving nonlinear plate and shell problems; and (iii) algorithms for solving the problems of control of nonlinear dynamic motion of plates and shells.

In a significant development Ref. [12], simple and economical procedures for large-deformation elasto-plastic analysis of frames, whose members can be characterized as beams, are presented. An assumed stress approach is employed to derive the tangent stiffness of the beam, subjected in general to non-conservative type distributed loading. The beam is assumed to undergo arbitrarily large rigid rotations but small axial stretch and relative (non-rigid) point-wise rotations. It is shown that if a plastic-hinge method (with allowance being made for the formation of the hinge at an arbitrary location or locations along the beam) is employed, the tangent stiffness matrix may be derived in an explicit fashion, without numerical integration. Several examples are given to illustrate the relative economy and efficiency of the method in solving large-deformation elasto-plastic problems. The method is of considerable utility in analyzing off-shore structures and large structures that are likely to be deployed in outerspace.

The problem of large deflections of thin flat plates is rederived in Ref. [13], using a novel integral equation approach. These plate deformations are governed by the von Karman plate theory. The numerical solution that is implemented combines both boundary and interior elements in the discretization of the continuum. The formulation also illustrates the adaptability of the boundary element technique to nonlinear problems. Included in the examples here are static, dynamic and buckling applications.

In a fundamentally new development in Ref. [14], the problem of transient dynamics of highly-flexible three-dimensional space-beams, undergoing large rotations and stretches is treated. The case of conservative force loading, which may also lead to configuration-dependent moments on the beam, is treated. Based on the present governing equations, a general mixed variational principle for the static problem is presented. Furthermore, using the three parameters associated with a conformal rotation vector representation of finite rotations, a well-defined Hamilton functional is established for the dynamic problem of a flexible beam undergoing finite rotations and stretches. This is shown to lead to a symmetric tangent stiffness matrix at all times. In the present total Lagrangean description of motion, the mass-matrix of a finite element depends linearly on the linear accelerations, but nonlinearly on the rotational parameters and attendant angular accelerations; the stiffness matrix depends nonlinearly on the deformation; and an "apparent" damping matrix depends nonlinearly on the rotations and attendant velocities. A Newmark time-integration scheme is used to integrate the semi-discrete finite element equations in time. An example of transient dynamic response of highly flexible beam-like structures in free-flight is presented to illustrate the validity of the theoretical methodology developed in this paper.

In Ref. [15], the non-linear field-boundary-element technique is applied to the analysis of snap-through phenomena in thin shallow shells. The equilibrium path is traced by using the arc-length method and the solution strategy is discussed in detail. The results show that, as compared to the approaches based on the popular symmetric-variational Galerkin finite element formulation, the current approach based on an unsymmetric variational Petrov-Galerkin field-boundary-element formulation gives a faster convergence while using fewer degrees of freedom. The illustrative numerical examples deal with post-buckling responses of several shallow shells with different geometries.

In a very significant development, Ref. [16], deals with elasto-plastic large deformation analysis of space-frames. It is based on a complementary energy approach. A methodology is presented wherein: (i) each member of the frame, modeled as an initially straight space-beam, is sought to be represented by a single finite element; (ii) each member can undergo arbitrarily large rigid rotations, but only moderately large relative rotations; (iii) a plastic-hinge method, with arbitrary locations of the hinges along the beam, is used to account for plasticity; (iv) the non linear bending-stretching coupling is accounted for in each member; (v) the applied loading may be non-conservative and (vi) an explicit expression for the tangent stiffness matrix of each element is given under conditions (i) to (v). Several examples, with both quasi-static and dynamic loading, are given to illustrate the accuracy and efficiency of the approaches presented. This methodology is of great relevance in

analyzing large-space-structures.

Ref. [17] deals with nonlinearities that arise in the study of dynamics and control of highly flexible large-space-structures. Broadly speaking, these nonlinearities have various origins: (i) geometrical: due to large deformations and large rotations of these structures and their members; (ii) inertia: depending on the coordinate systems used in characterizing the overall dynamic motion as well as elastic deformations; (iii) damping: due to nonlinear hysteresis in flexible joints; viscoelastic coatings, etc., and (iv) material: due to the nonlinear behavior of the structural material. The geometrical and material nonlinearities affect the "tangent stiffness operator" of the structure; the inertia nonlinearities affect the "tangent inertia operator".

To study the nonlinear transient dynamic response and control of flexible space-structures, one may think of: (i) semi-discrete approximation methods, and (ii) space-time methods. In the former class of methods, an appropriate spatial discretization is employed through weak-formulations (finite-element and field/boundary element) in space, and thus a set of coupled nonlinear ordinary differential equations (O.D.E.) is derived. These O.D.E.'s are solved often through temporal integration techniques of the finite difference-type. The semi-discrete methods are not ideally suited for travelling-wave type propagating disturbances. The second category of methods, viz., the space-time methods, wherein weak formulations in both space and time are employed, are somewhat better suited for wave-propagation type problems. In this reference, attention is primarily focused on semi-discrete methods, while some results recently obtained on space-time methods are deferred to a later publication.

Depending on the scale of the response that is required to be studied, a large-space-structure may either be modeled as an equivalent continuum, or as a lattice structure with the details of each member being accounted for. The spatial discretization in either case is required to be of the least-order possible so that the control algorithms may be meaningfully implemented. The reduced-order-modelling of the "tangent stiffness" operator of either a continuum model, or a lattice-model of a space-structure is treated in some detail in this reference, for structures undergoing large dynamic deformations.

The control of dynamic motion of space-structures is currently envisaged to be through either active processes, passive processes or some combinations thereof. One of the concepts of active control that is considered in detail here, and by other authors, is the use of piezo-ceramic actuators that are bonded to the truss and frame members of the space-structure in various locations. The controlling shear stress transmitted by the actuator to the truss by frame member depends on the axial force, transverse shear forces, and bending and twisting moments, in the member itself, as well as the excitation voltage applied to the piezo-actuator. This problem of mechanical coupling between the structural member, and actuators, is discussed in some detail in this reference.

The problem of control of nonlinear dynamic motion is addressed in this reference. The problem is posed in the form of determining the feed-back gain matrix and the attendant control force vector, such that the response as predicted by a semi-discrete system of coupled nonlinear ordinary differential equations, subject to a set of arbitrary initial conditions, is damped out in a pre-set scheme.

In the first part of the reference, continuum models of space-structures are analyzed. These include models of the space-beam type as well as the shallow shell type. In the case of space-beams, the problem of nonlinear dynamic response, when the beam undergoes large overall rigid as well as elastic motion, is discussed. The beam is assumed to undergo large rotations as well as stretches. A simple finite element algorithm to predict the response is presented. When a shallow-shell type continuum model is used, a field-boundary element approach based on nonlinear integral equations is presented as a means to create a reduced-order dynamic model of the semi-discrete type. A simple algorithm to control the response predicted by these nonlinear semi-discrete equations is discussed.

In the second part of the reference, detailed models of the lattice-type space-structures are discussed. Each member of the structural lattice is assumed to be either a "truss member", or a "frame member". The "truss member" is assumed to carry only an axial load, and has three

displacement degrees of freedom at each node. The "frame member" is assumed to carry an axial force, transverse shear forces, bending moments and a twisting moment; and is assumed to have three displacement and three rotational degrees of freedom at each node. Explicit expressions for the tangent stiffness matrices of both "truss" type and "frame" type members, which undergo arbitrarily large displacements, arbitrarily large overall rigid rotations and moderate local (relative) rotations, are derived. In all cases, each member (truss or frame type) is modelled by a single finite element, in the entire range of large deformations. Several examples are presented to illustrate the efficiency and on-board computational feasibility of these reduced-order models for lattice structures. In each instance, remarks on needs for future research are made.

Currently, there is a renewed interest in the study of multi-body-dynamics and its application in many fields of engineering. The mathematical model of a rigid body is useful whenever the overall motion, involving large rigid rotation, is of interest. The nonlinear dynamic equations of motion, in their explicit form, appear quite complex due to the expression for the absolute accelerations. In Ref. [18], weak formulations of linear and angular momentum balance laws of a rigid body undergoing large overall motion are stated a priori. Holonomic as well as nonholonomic constraints, that may exist on the motion of the rigid body, are introduced into this weak form in a fundamentally novel fashion here. Comments are made on the incremental form (and consistent linearization) of the weak formulation (with constraints), and the time-finite-element solutions thereof.

Large deformation and post-buckling analyses of structures have been studied by many researchers as an important subject in structural mechanics in the past decade or so. In all their studies, an incremental approach, either of the total Lagrangean type or the updated Lagrangean type, is employed. As the incremental approach is often based on the so-called tangent stiffness matrix, which reflects all the non-linear geometrical and mechanical effects, the majority of non-linear analyses of typical engineering structures, and especially truss- and frame-type large space structures, will be vastly simplified if an explicit expression (i.e., without involving assumed basis functions for displacements/stresses, and without involving element-wise numerical integrations) for the tangent stiffness matrix of an element can be derived. Toward this end, in Ref. [19], the authors have recently proposed a method for explicitly deriving the tangent stiffness matrix of the truss- and the frame-type structures, and have demonstrated that this procedure is not only inexpensive but also highly accurate in a wide variety of the problems involving very large deformations and highly non-linear pre- and post-buckling responses.

A novel theory and its computational implementation are presented in Ref. [20] for the analysis of strongly nonlinear dynamic response of highly-flexible space-beams that undergo large overall motions as well as elastic motions with arbitrarily large rotations and stretches. The case of conservative force loading, which may also lead to configuration-dependent moments on the beam, is treated. A symmetric tangent stiffness matrix is derived at all times even if the distributed external moments exist. An example of transient dynamic response of the beam is presented to illustrate the validity of the theoretical methodology developed herein.

The prediction of transient response of structures, in the form of traveling waves, is very important for controlling the dynamic behavior of structures. It is well known that the standard semi-discrete form of the finite element method is not suitable for predicting the wave propagation, due to the inherent dispersion involved. In Ref. [21], an application of space-time finite element method to the wave propagation problem is discussed. The main concerns in such problems consist of developing a consistent and stable scheme and also of capturing a shock wave, without wiggling. We discuss, at first, a weak form of the wave propagation problem, taking into account the jump condition associated with velocity and stress. A mixed finite element formulation plays an important role in evaluating the velocity explicitly. The application of present formulation to the linear wave equation shows that the present numerical results at the discontinuity give the mean values of jump. In the case of flexural wave propagations in Timoshenko beam, the present

method captures the wave front easily rather than the semi-discrete method.

The emphasis in Ref. [22], is on direct formulation of weak solutions, consistent linearizations, and appropriate tangent matrices for multiple rigid body systems. Multi-body dynamics can be formulated in two different ways. In the first, the connectivity among the bodies can be taken into account by appropriate constraint equations via the Lagrangian multiplier technique. The use of this technique, obviously increases the number of the unknowns of the dynamic system; hence, for large problems, a second technique is sometimes preferred in order to minimize the number of the degrees of freedom. This second formulation is only valid for systems with holonomic constraints and tree configurations. In this case one orders the tree. On the basis of this order each rigid body is a master of its follower and a slave of its predecessor. Then the absolute motion of the slave is resolved into the entrained motion with the master and into the relative motion with respect to it. Obviously this approach is more involved than the Lagrangian multipliers technique, but it is sometimes preferred not only because it involves fewer degrees of freedom, but also because the coordinates of the relative motion are very often closer to the physics of the system.

As to which technique is the best in a given situation, it clearly depends on the specific nature of the problem we are dealing with and on a general basis we can anticipate that the best performances can be obtained by a third formulation which is a convenient mixture of the previous two. In the first paper, the dynamics of a single rigid body with constraints has been formulated, and it is quite easy to extend it to the multiple-rigid body system. In this paper we focus the attention on the analytical developments involved in the second formulation.

In a significant paper, Ref. [23], the problem of transient dynamics of highly flexible three-dimensional space-curved beams, undergoing large rotations and stretches, is treated. The case of conservative force loading, which may also lead to configuration-dependent moments on the beam, is considered. Using the three parameters associated with a conformal rotation vector representation of finite rotations, a well-defined Hamilton functional is established for the flexible beam undergoing finite rotations and stretches. This is shown to lead to a symmetric tangent stiffness matrix at all times. In the present total Lagrangian description of motion, the mass-matrix of a finite element depends linearly on the linear accelerations, but nonlinearly on the rotation parameters and attendant accelerations; the stiffness matrix depends nonlinearly on the deformation; and an 'apparent' damping matrix depends nonlinearly on the rotations and attendant velocities. A Newmark time-integration scheme is used to integrate the semi-discrete finite element equations in time. Several examples of transient dynamic response of highly flexible beam-like structures, including those in free flight, are presented to illustrate the validity of the theoretical methodology developed in this paper.

A follow-up paper, Ref. [24], deals with finite rotations, and finite strains of three-dimensional space-curved elastic beams, under the action of conservative as well as nonconservative type external distributed forces and moments. The plausible deformation hypothesis of "plane sections remaining plane" is invoked. Exact expressions for the curvature, twist, and transverse shear strains are given; as is a consistent set of boundary conditions. General mixed variational principles, corresponding to the stationarity of a functional with respect to the displacement vector, rotation tensor, stress-resultants, stress-couples, and their conjugate strain-measures, are stated for the case when conservative-type external moments act on the beam. The momentum-balance conditions arising out of these functionals, either coincide exactly with, or are equivalent to, those from the "static method". The incremental variational functionals, governing both the Total and Updated Lagrangian incremental finite element formulations, are given. An example of the case of the buckling of a beam subject to axial compression and non-conservative type axial twisting couple, is presented and discussed.

Constraint equations arise in the dynamics of mechanical systems whenever there is the need to restrict kinematically possible motions of the system. In practical applications, constraint equations can be used to simulate complex, connected systems. If the simulation must be carried out

numerically, it is useful to look for a formulation that leads straightforwardly to a numerical approximation. In Ref. [25], we extended the methodology of our previous work to incorporate the dynamics of holonomically and nonholonomically constrained systems. The constraint equations are cast in a variational form, which may be included easily, in the time finite element framework. The development of the weak constraint equations and their associated "tangent" operators is presented. We also show that this approach to constraint equations may be employed to develop time finite elements using a quaternion parametrization of finite rotation. Familiarity with the notation and methodology of our previously presented work is assumed.

Weak formulations in Analytical Dynamics are developed, paralleling the variational methods in elastostatics, and including a fundamental yet novel approach for treating constraints (both holonomic and nonholonomic). In Ref. [26], a general three-field approach is presented, in which the momentum balance conditions, the compatibility conditions between displacement and velocity, the constitutive relations and the displacement and momentum boundary conditions are all enforced in weak form. A primal, or kinematic formulation is developed from the general form by enforcing the compatibility conditions and displacement boundary conditions *a priori*. The conditional stability of the kinematic formulation is the counterpart of the locking phenomenon in elastostatics and may be avoided, either by reduced order integration, or by utilizing a mixed formulation. Toward this end, a two-field mixed formulation is presented, which follows from the general form, when the constitutive relations are satisfied *a priori*. A general set of the constraint equations is introduced into the kinematic and mixed formulations, using a specific choice of multipliers, which results in modified variational principles. Several simple examples concerning rigid body dynamics are presented.

Ref. [27] presents general variational formulations for dynamical problems, which are easily implemented numerically. The development presents the relationship between the very general weak formulation arising from linear and angular momentum balance considerations, and well known variational principles. Two and three field mixed forms are developed from the general weak form. The variational principles governing large rotational motions are linearized and implemented in a time finite element framework, with appropriate expressions for the relevant "tangent" operators being derived. In order to demonstrate the validity of the various formulations, the special case of free rigid body motion is considered. The primal formulation is shown to have unstable numerical behavior, while the mixed formulation exhibits physically stable behavior. The formulations presented in this paper form the basis for continuing investigations into constrained dynamical systems and multi-rigid-body systems, which will be reported in subsequent papers.

The deformation of a beam-column, the upper and lower surfaces of which are bonded in segments with piezo-ceramic liners, is studied in Ref. [28] for the purpose of obtaining appropriate expressions for the force transferred to the structural member by the piezo-actuator. This concept may be employed for the control of large dynamic deformations for a lattice-type flexible space-structure. The present model, which is based upon a static analysis, accounts for the effects of transverse shear and axial forces in addition to a bending moment on the beam in formulating the governing equilibrium equations. The present model provides more complete expressions for the force transmitted to the structural member than a model reported earlier in literature, in which the shear and axial forces are neglected.

In a very significant development [29], a scheme for active control of nonlinear vibration of space-structures, wherein each member is modeled as a beam-column, is presented. The expressions for shear stresses transmitted to the structural member by the distributed segmented piezo-electric actuators, which are bonded on the surfaces of the member, are derived in the general case in which the structural member is subjected to moments, transverse shear forces and an axial force. Based on the weak form of the governing equations, and a complementary energy approach based on assumed stress fields, the viability of active control on nonlinear dynamic response of lattice-type space structures, using piezo actuators, is studied. Four examples are given to demonstrate

the feasibility of the approaches presented in this paper.

Finally, Ref. [30] deals with the effect of non-linearly flexible hysteretic joints on the static and dynamic response of space frames. It is shown that a complementary energy approach based on a weak form of the compatibility condition as a whole of a frame member, and of the joint equilibrium conditions for the frame, is best suited for the analysis of flexibly jointed frames. The present methodology represents an extension of the authors' earlier work on rigidly connected frames. In their present case also, an explicit expression for the tangent stiffness matrix is given when (i) each frame member, along with the flexible connections at its ends, is represented by a single finite element, (ii) each member can undergo arbitrarily large rigid rotations and only moderate relative rotations and (iii) the non-linear bending-stretching coupling is accounted for in each member. Several examples, with both quasi-static and dynamic loading, are included, to illustrate the accuracy and efficiency of the developed methodology.

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