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13. ABSTRACT *(Maximum 200 words)*

Algorithms for signal processing and image processing have been developed and implemented on a variety of architectures including a Cray Y-MP, an Intel iPSC/2, a Connection Machine, and an Alliant FX/40. Research has also been conducted on interative least squares and related methods with applications to structural analysis.

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Parallel Algorithms for Least Squares and Related Computations

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Final Report - Table of Contents

1. Project Objectives

This represents a *comprehensive final report* on our research project, which is concerned with the design and testing of new algorithms for least squares computations, with particular emphasis on applications to signal and image processing and to computational methods in structural analysis. The primary objectives of the project were to mathematically develop, test, and analyze fast numerical algorithms for the efficient solution to computational problems on modern high performance computers.

Matrix computations, including the solution of systems of linear equations, least squares problems and algebraic eigenvalue problems, govern the performance of many applications on modern vector and parallel computers. In order to meet some of the challenges of this emerging new generation of machines, the **goals of this project** have been to develop techniques in matrix computations for efficient implementation on advanced computer architectures. Multiprocessing technology is still evolving and is predicted to be the dominant methodology in the computer industry by the next decade. The full potential of parallel computation will be realized in signal and image processing computations only when parallel architectures are combined with system interfaces, computational strategies, and numerical algorithms into integrated signal environments. Our purpose has been to address each of these topics. The research described here is expected to have impacts on science and engineering as part of a continuing development of the computational foundations of technology. As in other fields, scientific computation has emerged in these areas as a third major paradigm beside theory and experimentation.

We have developed and implemented algorithms on a variety of architectures, including: a Cray Y-MP supercomputer at our newly operational North Carolina Supercomputing Center in the Research Triangle Park, an Intel iPSC/2 distributed memory multiprocessor at the Oak Ridge National Laboratory, a Connection Machine CM/2 at the Argonne National Laboratory, and our own Alliant FX/40 vector-multiprocessor. The Alliant was purchased by AFOSR funds under the Defense Department DURIP Program in 1988.

Applications of our work to the practical real-world problems of least squares estimation methods in signal and image processing and to computational methods in structures and fluids have been made. Our main objective in **least squares computations** has been to develop fast recursive algorithms for linear prediction of one and two dimensional time series in **adaptive signal processing**: identification, estimation and control. Our schemes are amenable to implementation on parallel processing systems, especially distributed memory architectures such as the Intel iPSC/2 Hypercube. This work in developing parallel algorithms for recursive least squares has produced some especially important recent results which have led to several technical research papers involving the principal investigator and his students during the reporting period.

Very recently, the principal investigator, along with co-researchers W. Ferng, G. Golub and D. Pierce, have made some important strides in developing fast adaptive condition estimation schemes for recursive matrix modifications. Condition estimation is important is assessing the reliability and accuracy of linear predictors in signal processing and control, and has important potential applications to target tracking. Figure 4 in Section 2 illustrates applications to target tracking.

Applications of our work on **image processing** algorithms include large scale computations for full 3D image reconstruction problems. We would like to accelerate these computations by the use of parallel multilevel methods. Defense Department contractors, such as General Electric, would like to build (and are building) massively parallel computer systems for these applications. Even with 1024 processors, the computations may be too slow for the DOD applications, e.g., fast inspection of jet engine rotor blades: thus the need for a multilevel approach in a parallel environment to accelerate the computations. (For instance, the B-1 Bomber did not participate in the recent Gulf War, in part, because of defects in the engine turbine rotor blades.) One of my former Ph.D. students, Air Force Major Douglas James, is involved in research on this topic. He is currently assigned to the U.S. Air Force Academy.

We have also worked in the areas of **iterative least squares and related methods** with applications to **structural analysis**. Here we are concerned with the solution of the fundamental problems of equilibrium: for instance in elastic analysis - that of finding the stresses and strains and solving redesign problems, given a finite element model of a large complex structure and a set of external loads. To obtain the solution of this constrained minimization problem, a variety of algorithms involving the displacement method or the force method can be applied. Our work on this topic has led to new approaches involving least squares conjugate gradient iterative nullspace methods combined with substructuring. Major Douglas James has written his Ph.D. dissertation in this area, and several technical publications have been written on this aspect of our project.

In addition to research with direct applications to the areas of recursive least squares methods in signal and image processing, and to computational mechanics, the principal investigator has been involved, along with K. Gallivan and A. Sameh at the University of Illinois, with the development of a comprehensive treatise (publications 7, 9) on parallel algorithms for dense computations in linear algebra. The work has recently been published in a general reference book on parallel algorithms by SIAM.

2. Summaries of Major Accomplishments

Abstracts of the most significant research findings are given in this section of the report. Referenced publications can be found in Section 4. We include a cumulative record of research summaries of papers written under this grant.

- **Least Squares Modifications with Inverse Factorizations: Parallel implications.** The process of modifying least squares computations by updating the covariance matrix has been used in control and signal processing for some time in the context of linear sequential filtering. Here we give an new approach to the process and provide extensions to downdating. Our purpose is to develop algorithms that are amenable to implementation on modern multiprocessor architectures. In particular, the inverse Cholesky factor $\mathbf{R}^{-1}$ is considered and it is shown that the inverse $\mathbf{R}^{-1}$ can be updated (or downdated) by applying the same sequence of orthogonal (hyperbolic) plane rotations that are used to update (downdate) $\mathbf{R}$. We have attempted to provide some new insights into least squares modification processes and to suggest parallel algorithms for implementing Kalman type filters in the analysis and solution of estimation problems in signal processing. This is joint work with former Ph.D. student C. Pan. (See publication 1.)

- **Least Squares Multiple Updating Algorithms on a Hypercube.** Parallel algorithms for multiple updating methods in recursive least squares computations are investigated. Comparisons of updating algorithms by carefully implemented orthogonal Householder and Givens algorithms are made on the iPSC hypercube distributed memory multiprocessor system. Overall, the performance of updating by orthogonal Householder reflections using a row oriented storage scheme is superior to those of Givens rotations using a row oriented scheme and the greedy Givens sequence on the hypercube, for our application. In particular, the *communication complexity is independent of the number of vectors being updated.* The methods we describe can also be adapted to the parallel computation of general orthogonal factorizations involved in least squares problems. We have in mind applications to windowed recursive least squares filtering schemes for near real-time computations on distributed memory architectures. This is joint work with D. Agrawal and Computer Engineering graduate student S. Kim. (See publication 2.)

- **Recursive Least Squares Computations on a Vector-Multiprocessor.** We consider parallel implementations of algorithms for recursive least squares computations based upon the information matrix and the covariance matrix updating methods. The target architecture is a shared-memory multiprocessor, and test results on an Alliant FX/40 system with vectormultiprocessors (*purchased through an AFOSR DURIP grant*) demonstrate the parallel efficiencies of the algorithms. The results also show that the covariance method in a form *suggested by Pan and Plemmons is easily the most efficient* on the Alliant multiprocessor computer. This is an invited paper presented at the International Conference on the Mathematics of Networks and Systems, Amsterdam, June 1989. (See publication 3.)

• **Optimality Relationships for p-cyclic SOR.**

The optimality question for block **p**-cyclic SOR iterations discussed in the classic textbooks by Young and Varga, as well as in the monograph by Berman and Plemmons, is answered under natural conditions on the spectrum of the block Jacobi matrix. In particular, it is shown that repartitioning a block **p**-cyclic matrix into a block **q**-cyclic form, $\mathbf{q} < \mathbf{p}$, results in asymptotically faster SOR convergence for the same amount of work per iteration. As a consequence block **2** - cyclic SOR is shown to be optimal under these conditions. New applications of this work include **p**-cyclic iterative methods for queuing network analysis and constrained least squares computations arising in structural analysis. This is joint work with A. Hadjidimos at Purdue and D. Pierce at Boeing. (See publication 4.)

• **Substructuring Methods for Computing the Nullspace of Equilibrium Matrices.**

Equations of equilibrium arise in numerous areas of engineering. Applications to electrical networks, structures and fluid flow are elegantly described in a recent book on applied mathematics by Strang. The context in which equilibrium equations arise may be stated in the *constrained minimization form*:

$$\min\ (\mathbf{x}^T\mathbf{F}\mathbf{x} - \mathbf{x}^T\mathbf{r}) \quad \text{subject to} \quad \mathbf{E}\mathbf{x} = \mathbf{s},$$

Here **F** is generally some symmetric positive definite matrix associated with the minimization problem. For example, **F** is the element flexibility matrix in the structures application. An important approach (called the force method in structural optimization) to the solution to such problems involves dimension reduction nullspace schemes based upon computation of a basis for the nullspace for **E**. In our approach to solving such problems we emphasize the *parallel computation* of a basis for the nullspace of **E** and examine the applications to structural optimization and fluid flow. Several new block decomposition and node ordering schemes are suggested and reanalysis computations are investigated. The following figure illustrates our novel approach, which combines substructuring with nullspace decompositions.

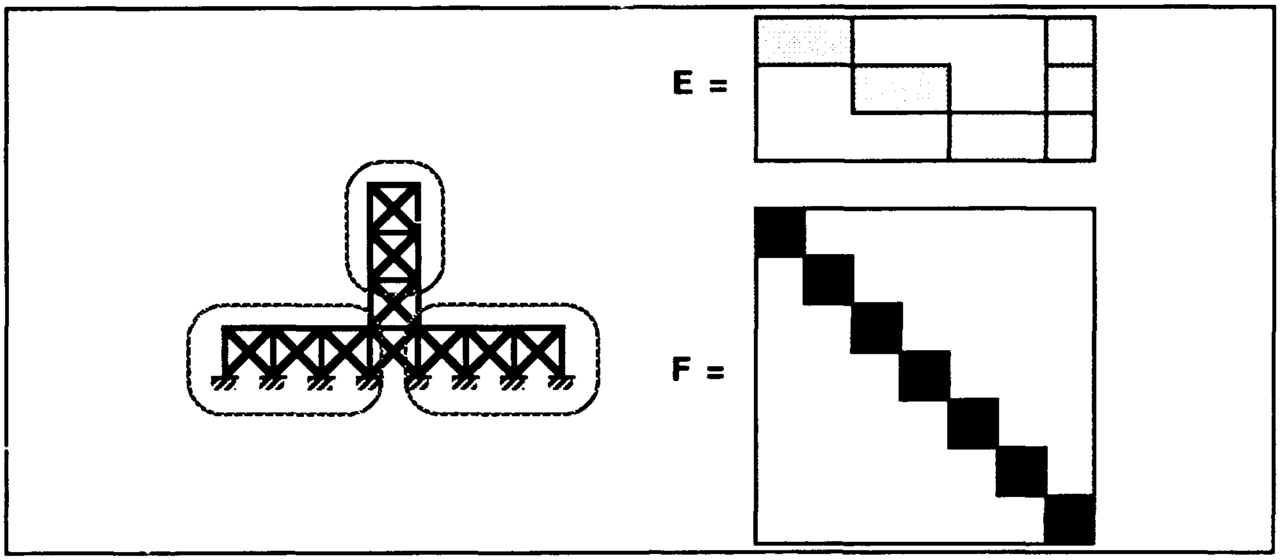

Figure 1. Truss Structure Problem Formulation.

Comparisons of these schemes are also made with those of C. Hall at the University of Pittsburgh, and others, for fluid flow computations. This is joint work with R. White. (See publication 5.)

- **An Iterative Substructuring Algorithm for Equilibrium Equations.** The topic of iterative substructuring methods, and more generally domain decomposition methods, has been extensively studied over the past few years, and the topic is well advanced with respect to first and second order *elliptic problems*. However, relatively little work has been done on more general constrained least squares problems (or equivalent formulations) involving equilibrium equations such as those arising, for example, in *realistic structural analysis applications*. The potential is good for effective use of iterative algorithms on these problems, but such methods are still far from being competitive with direct methods in industrial codes. The purpose of this paper is to investigate an order reducing, preconditioned conjugate gradient method proposed by *Barlow, Nichols and Plemmons* for solving problems of this type. The relationships between this method and nullspace methods, such as the force method for structures and the dual variable method for fluids, are examined. Convergence properties are discussed in relation to recent optimality results for Varga's theory of p-cyclic SOR. We suggest a mixed approach for solving equilibrium equations, consisting of both direct reduction in the substructures and the conjugate gradient iterative algorithm to complete the computations. Some typical problems considered in our numerical tests are indicated in the next figure.

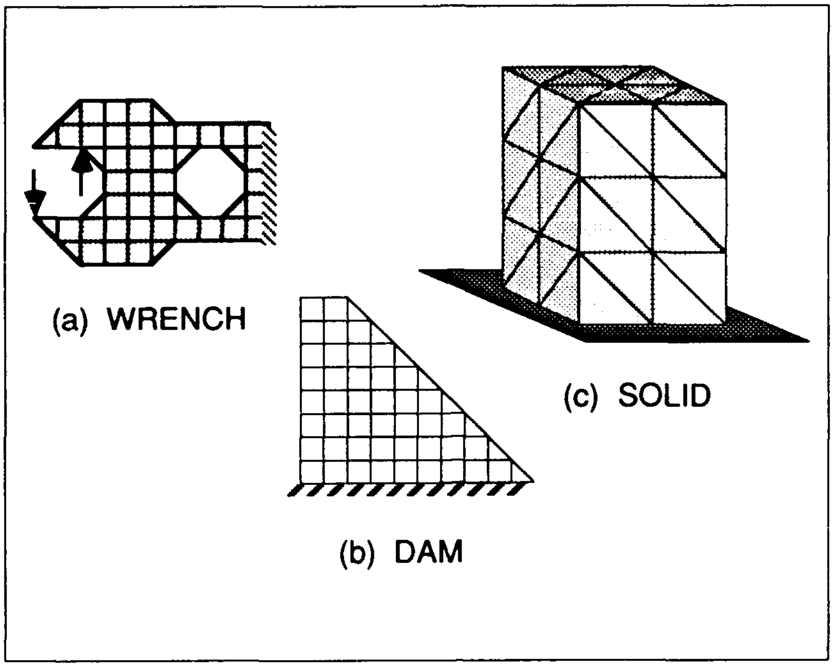

Figure 2. Some Two and Three Dimensional Problems Considered .

This is joint work with Major Douglas James who has written his Ph.D. dissertation with the principal investigator. (See publication 6.)

• **Parallel Algorithms for Dense Linear Algebra Computations**. Our purpose in this comprehensive *81 page typeset paper* is to provide an overall perspective of parallel algorithms for dense matrix computations in linear system solvers, least squares problems, eigenvalue and singular-value problems, as well as rapid elliptic PDE solvers. Numerical Linear algebra is a fundamental tool which is indispensable to scientific and engineering research, and these computations are becoming increasingly dependent upon the development and implementation of parallel algorithms on modern high-performance computers. With this in mind we have attempted in this paper to collect and describe and to put into perspective a selection of the more important parallel algorithms for numerical linear algebra. We give a *major new emphasis to certain computational primitives* whose efficient execution on parallel and vector computers is essential in order to obtain high performance algorithms. This is joint work with K. Gallivan and A. Sameh from the University of Illinois Center for Supercomputing Research and Development. (See publications 7, 9.)

• **Recursive Least Squares on a Hypercube Multiprocessor Using the Covariance Factorization**. We have developed an efficient parallel implementation of an algorithm for recursive least squares computations based upon the covariance updating method. The target architecture is a distributed-memory multiprocessor, and test results on an Intel iPSC/2 hypercube demonstrate the parallel efficiency of the algorithm. A 64-node system is measured to execute the algorithm over *48 times as fast* as a single processor for the largest problem that fits on a single node (fixed size speedup). Moreover, the computation times increase only slightly with an increase in the number of processors when the problem size per processor remains constant. Applications include robust regression in statistics and modification of the Hessian matrix in optimization, but the primary motivation for this work is the need for fast recursive least squares computations in signal processing. This is joint work with C. E. Henkel. (See publications 8, 11.)

• **Implicit Nullspace Iterative Methods for Constrained Least Squares Problems**. We propose a class of iterative algorithms for solving equality constrained least squares problems, generalizing an order-reducing algorithm first analyzed by Barlow, Nichols, and Plemmons. These algorithms, which we call implicit null space methods, are based on the classical nullspace method, except that a basis for the nullspace of the constraint matrix is not explicitly formed. The implicit methods allow great flexibility in the choice of preconditioner, and are suitable for parallel implementation on substructured problems. We offer some numerical results for both structural engineering applications and Stokes Flow. The paper is by Major Douglas James, and is part of his dissertation work under this AFOSR grant. (See publication 10).

• **Order-Reducing Conjugate Gradients vs Block AOR for Constrained Least Squares Problems**. We compare the convergence properties of two iterative algorithms for solving equality constrained least squares problems of the form

$$\min \|\mathbf{G}\mathbf{y} - \mathbf{c}\|_2 \text{ such that } \mathbf{E}\mathbf{y} = \mathbf{b}.$$

The first algorithm, due to Barlow, Nichols, and Plemmons, applies a variation of the conjugate gradient algorithm to a symmetric positive definite system which is smaller than the original problem. The second, Block Accelerated Over-relaxation, is a two parameter generalization of block SOR. Barlow, Nichols, and Plemmons have proven that their order-reducing conjugate gradient algorithm converges faster than block SOR. We extend their result to show that the algorithm is also superior to block AOR. Numerical examples arising in structural analysis confirm the analysis. The paper is by Ph.D student Major Douglas James, and is part of his dissertation work under this AFOSR grant. (See publication 12).

• **Fast Adaptive Condition Estimation**. Recursive condition number estimates of matrices are useful in many areas of scientific computing, including: recursive least squares computations, optimization, eigenanalysis, and general nonlinear problems solved by linearization techniques where matrix modification techniques are used. Our purpose in this paper is to propose a fast adaptive condition estimator, which we call *ace*, for tracking the condition number of the modified matrix over time, in terms of its triangular factors. *ace* is fast in the sense that only $O(\mathbf{n})$ operations are required for $\mathbf{n}$ parameter problems, and is adaptive over time, *i.e.*, estimates at time $\mathbf{t}$ are used to produce estimates at time $\mathbf{t} + 1$.

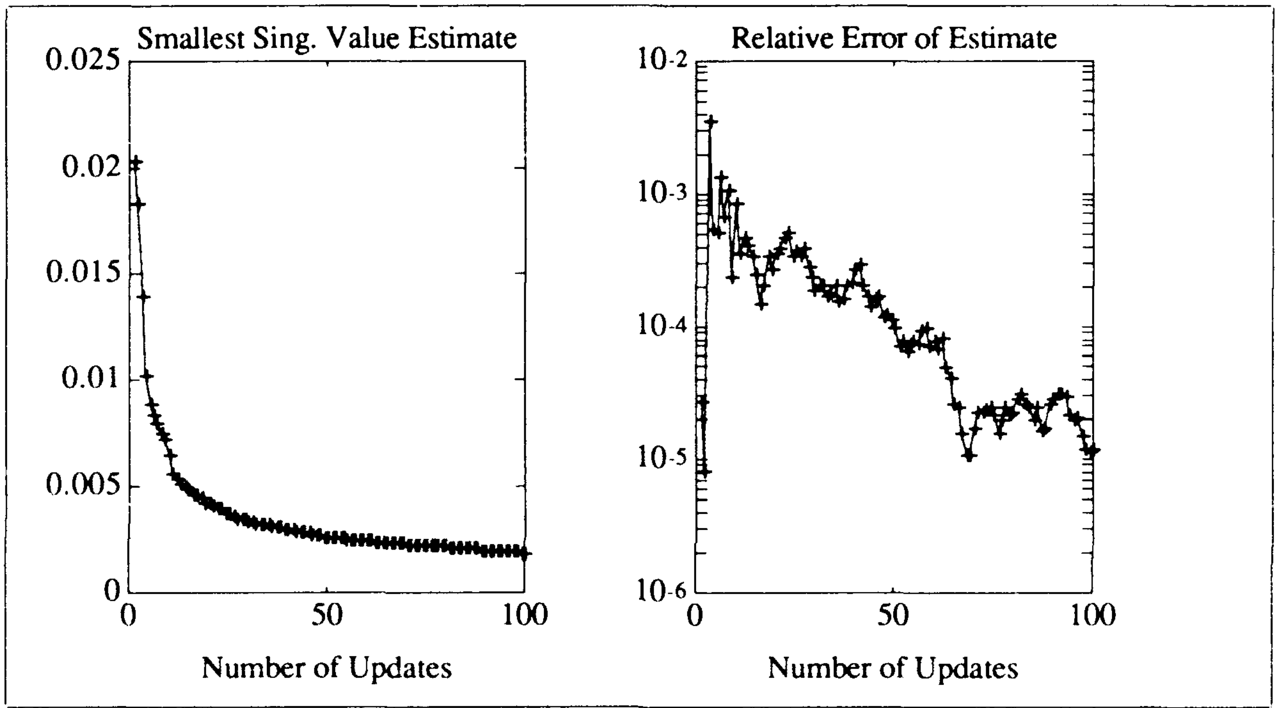

Figure 3. Performance of our Adaptive Condition Estimator on Signal Processing Data.

Traditional condition estimators for triangular factors, such as the LINPACK and LAPACK type schemes, generally require $O(\mathbf{n}^2)$ operations. *ace* is in the spirit of the popular incremental condition estimation scheme, *ice*, developed by Bischof and used in LAPACK, in the sense that the estimates are based on max-min principles. Numerical experiments are reported (see Figure 3), indicating that the scheme *ace* yields an accurate and robust, yet inexpensive, adaptive condition estimator for recursive matrix modifications. This is Joint work with D. Pierce at the Boeing Company. (See publication 13).

• **Tracking the Condition Number for RLS in Signal Processing**. We apply a fast adaptive condition estimation scheme, called *ace*, for recursive least squares (RLS) computations in signal processing. *ace* is fast in the sense that only O($\mathbf{n}$) operations are required for $\mathbf{n}$ parameter problems, and is adaptive over time, i.e., estimates at time $\mathbf{t}$ are used to produce estimates at time $t + 1$. RLS algorithms for linear prediction of time series are applied in various fields of signal processing: identification, estimation and control. However, RLS algorithms are known to suffer from numerical instability problems under finite word length conditions, due to ill-conditioning. We develop adaptive procedures, linear in the order of the problem, for accurately tracking relevant extreme singular values and associated condition numbers over time t. In this paper exponentially-weighted data windows are considered. *ace* is in the spirit of an incremental condition estimation scheme, *ice*, proposed by Bischof in conjunction with orthogonal factorization. Numerical experiments indicate that *ace* yields a very accurate, yet inexpensive, RLS condition estimator. The following figure illustrates how our Kalman filtering schemes find applications in target tracking.

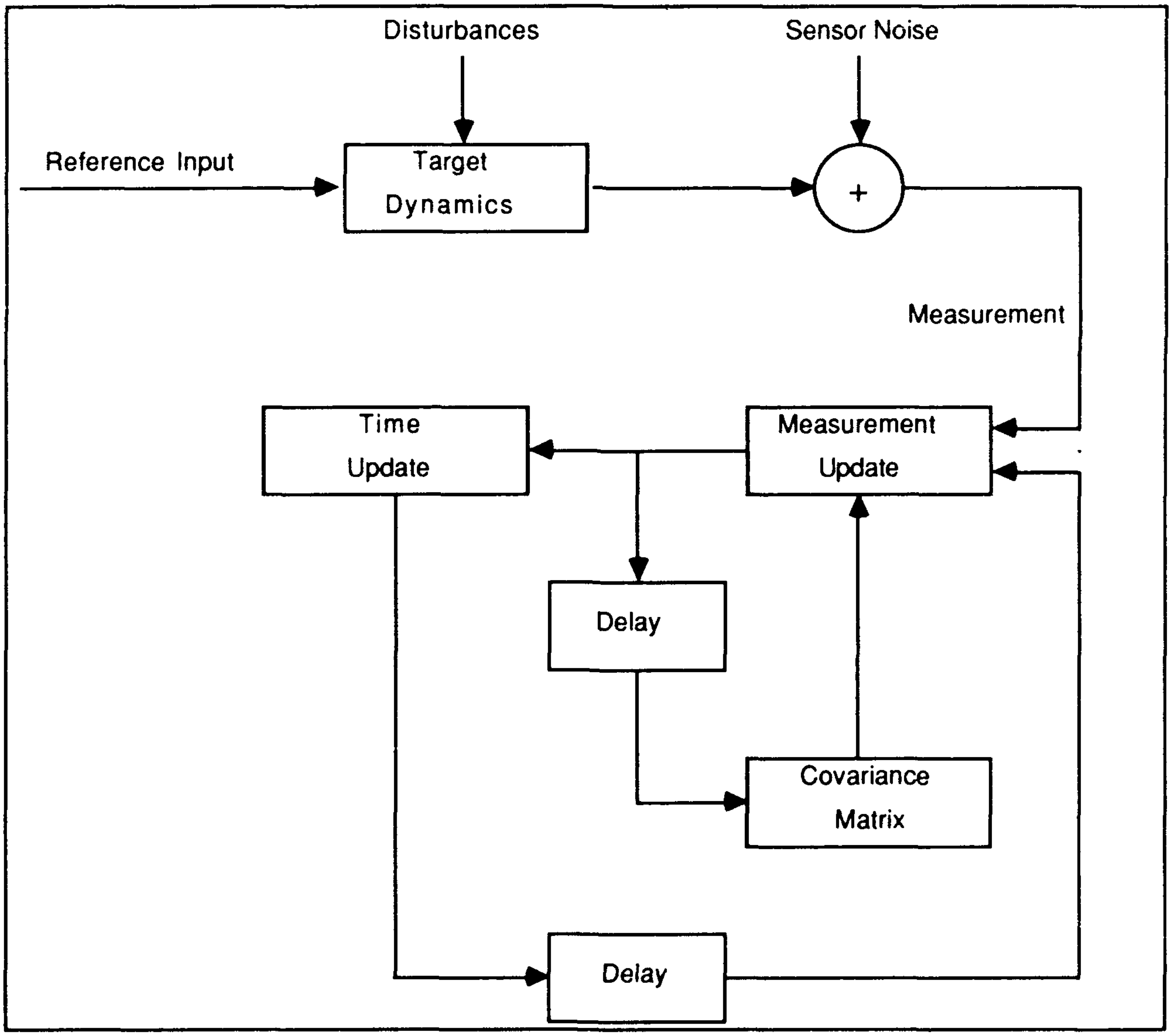

Figure 4. Tracking Filter based on updating Covariance Matrix.

As a side benefit, *ace* also provides accurate bounds on the power spectral densities in the context of adaptive filtering in signal processing. This is Joint work with D. Pierce at the Boeing Company. (See publication 14).

• **Block Cyclic SOR for Markov Chains with p-cyclic Infintesimal Generator.**
The block SOR method for the computation of the steady state distribution of finite Markov chains that possess **p**-cyclic infinitesimal generators is considered. It is shown that convergence, in a sense more general than the usual, may be obtained, even if the SOR iteration matrix violates the usual conditions for semiconvergence. Necessary and sufficient conditions for convergence in this, extended, sense are derived. They are then applied in the case where the p^{th} power of the associated Jacobi matrix of the system to be solved possesses only nonnegative eigenvalues. Exact convergence intervals and the optimal ω values are derived for this case. In addition to the "usual" optimal ω in the interval $\left(1, \frac{p}{p-1}\right)$, other ω values, that yield convergence in the extended sense, are found to achieve the same, optimal, convergence rate. Numerical tests indicate that small perturbations of ω around the optimal value affect the convergence factor much less, if these newly introduced optimal ω values are used. Numerical tests fully support the theory developed here. This is joint work with Kimon Kontovasilis and William J. Stewart. (See publication 15).

• **Adaptive Lanczos Methods for Recursive Condition Estimation.**
Estimates for the condition number of a matrix are useful in many areas of scientific computing, including: recursive least squares computations, optimization, eigenanalysis, and general nonlinear problems solved by linearization techniques where matrix modification techniques are used. The purpose of this paper is to propose an adaptive Lanczos estimator scheme, which we call *ale*, for tracking the condition number of the modified matrix over time. Applications to recursive least squares (RLS) computations using the covariance method with sliding data windows are considered. The scheme *ale* is fast for arbitrary **n** parameter problems arising in RLS methods in control and signal processing, and is adaptive over time, *i.e.*, estimates at time **t** are used to produce estimates at the next time step **t + 1**. Comparisons are made with other adaptive and non-adaptive condition estimators for recursive least squares problems. Numerical experiments are reported in our studies indicate that *ale* yields a very accurate recursive condition estimator. This is joint work with graduate assistant William Ferng and with Gene H. Golub at Stanford University. (See publication 16).

• Inverse Factorization Methods in Linear Prediction.

A new inverse factorization technique is presented for solving linear prediction problems arising in signal processing. The algorithm is similar to a scheme of Luk and Qiao in that it uses the rectangular Toeplitz structure of the data to recursively compute the prediction error and to solve the problem when the optimum filter order has been found. The novelty of the scheme presented here is the use of an inverse factorization scheme by Pan and Plemmons for solving the linear prediction problem with low computational complexity and without the need for solving triangular systems. We also provide a linear systolic array for solving these problems. Extensions of this work to two dimensional signal processing problems are being made. Here, one works with block Toeplitz matrices. This is joint work with graduate assistant James Nagy and the research overlaps the new project: AFOSR-91-0163. (See publication 17).

• Iterative Lanczos-Based Condition Estimators for Linear Systems.

For the system of linear equations $\mathbf{Ax} = \mathbf{b}$, with a fixed nonsingular matrix $\mathbf{A}$, the condition number $K(\mathbf{A})$ is important since it provides information about the sensitivity of the solution to perturbations in the data. We suggest here an iterative approach to estimating the condition number of $\mathbf{A}$, based on the Lanczos method. We call this scheme ILE: Iterative Lanczos Estimator. The results of numerical experiments on over 600 test matrices indicate that this scheme is robust and accurate. Three different condition estimators, including a generalization of the LINPACK algorithm with "look behind" strategy suggested by Cline, Conn and Van Loan, the probabilistic condition estimator suggested by Higham, and the incremental condition estimator ICE suggested by Bischof, are compared with ILE. Parallel implementations of ILE are discussed and computations on a Cray Y-MP are reported. This is joint work with graduate assistant William Ferng and the research overlaps the new project: AFOSR-91-0163. (See publication 18).

3. Graduate Students

The following graduate students are have worked under the principal investigator for this grant.

• **Douglas James**. Ph.D. August 1990. Douglas is a Major in the U.S. Air Force. He received his B.S. degree at the Air Force Academy and his M.S. degree at MIT under the direction of Professor Gilbert Strang, before enrolling at NCSU to pursue his Ph.D in mathematics under the direction of the principal investigator. His graduate study was funded by the Air Force Institute of Technology, WPAFB, OH. Major James' dissertation topic was on iterative least squares substructuring methods. Two papers from his dissertation have already been accepted for publication, and a third was given as an invited student paper at a national SIAM conference. Major James is now assigned to the Air Force Academy.

• **William Ferng**. William is pursuing his Ph.D. in mathematics with a minor in computer science under the direction of the principal investigator. He did his undergraduate work in engineering at Taiwan National University. Mr. Ferng has considerable parallel processing and supercomputing experience. His dissertation is in parallel algorithms for least squares computations. He is expected to graduate under the principle investigator's direction in 1992.

• **James Nagy**. James is pursuing his Ph.D. in mathematics with a minor in electrical and computer engineering under the direction of the principal investigator. He did his undergraduate and M.S. work at Northern Illinois University. His M.S. thesis at NIU involved a study of fast Toeplitz algorithms in sequential estimation. His dissertation topic is signal and image processing: identification, estimation and control. He is expected to graduate under the principle investigator's direction in August 1991. He has been awarded a Post Doctoral position with the Institute for Mathematics and its Applications, University of Minnesota, for the academic year 1991-92.

4. Technical Publications

1. *Least squares modifications with inverse factorizations: parallel implications*, **J. Computational and Appl. Math.**, 27(1989), pp. 109-127 (with C. Pan). Research on this paper overlaps AFOSR-83-19500.

2. *Least squares multiple updating algorithms on a hypercube*, **Inter. J. Parallel and Dist. Computing**, 8 (1990), pp. 80-88 (with D. Agrawal and S. Kim).

3. *Recursive least squares on a vector-multiprocessor*, **Proc. International Symp. MTNS-89, Sig. Proc. and Numer Meth.**, Amsterdam 1989, Birkhauser Press Boston, Inc., 3(1990), pp. 495-502.

4. *Optimality relationships for p-cyclic SOR*, **Numer. Math.**, 56 (1990), pp. 635-643 (with A. Hadjidimos and D. Pierce).

5. *Substructuring methods for computing the nullspace of equilibrium matrices*, **SIAM J. Matrix Analysis**, 11 (1990), pp. 1-22 (with R. White).

6. *An iterative substructuring algorithm for equilibrium equations*, **Numer. Math.**, 57(1990), pp. 625-633 (with D. James).

7. *Parallel algorithms for dense linear algebra computations*, **SIAM Review**, 32 (1990), pp. 54-135 (with K. Gallivan and A. Sameh).

8. *Parallel recursive least squares on a hypercube multiprocessor*, in **Numerical Linear Algebra, Digital Signal Processing and Parallel Algorithms**, NATO ASI Series F, Springer-Verlag, Ed. by G. Golub and P. Van Dooren, (1990), pp. 571-579.

9. **Parallel Algorithms for Matrix Computations**, SIAM Press, Philadelphia PA, November (1990), 197pp. (Joint with K. Gallivan, M. Heath, E. Ng, J. Ortega, B. Peyton, C. Romine and R. Voigt).

10. *Implicit nullspace iterative methods for constrained least squares problems*, Invited student paper at the Copper Mt. Conf. on Iterative Methods, Copper Mt., CO (1990), submitted to the **SIAM J. Sci. Stat. Comp.** (by Ph.D. student Major Douglas James, and part of his dissertation work supported under this AFOSR grant.)

11. *Recursive least squares on a hypercube multiprocessor using the covariance factorization*, **SIAM J. Sci. Stat. Comp.**, 12(1991), pp. 95-106 (with C. Henkel).

12. *Order-reducing conjugate gradients vs block AOR for constrained least squares problems*, to appear in **Lin. Alg. and It's Applications**, (1991) (by Ph.D student Major Douglas James, and part of his dissertation work supported under this AFOSR grant).

13. *Fast adaptive condition estimation*, to appear in **SIAM J. Matrix Anal.** (1991) (with D. Pierce).

14. *Tracking the condition number for RLS in signal processing*, to appear in **Mathematics of Control, Signals and Systems**, (1991) (with D. Pierce).

15. *Block cyclic SOR for Markov chains with p-cyclic infintesimal generator*, **Lin. Alg. Applic**. Special Issue on Iterative Methods, to appear (1991) (with K. Kontovasilis and W. J. Stewart).

16. *Adaptive Lanczos methods for recursive condition estimation*, to appear in **Numerical Algorithms**, (1991) (with William R. Ferng and Gene H. Golub).

17. *An inverse factorization algorithm for linear prediction*, preprint, (1991) (with J. Nagy). Research on this paper overlaps the new project: AFOSR-91-0163.

18. *Iterative Lanczos-based condition estimators*, preprint, (1991) (with W. Ferng). Research on this paper overlaps the new project: AFOSR-91-0163.

5. Invited Research Presentations

- *Recursive Least Squares on a Hypercube Multiprocessor*, **NATO Workshop on Num. Lin. Alg., Signal Proc. and Parallel Algorithms**, Leuven, Belgium, August 1988.
- *Least Squares Computations in Signal Processing*, **University of Illinois-Urbana**, Colloquium, October 1988.
- *Parallel Algorithms for Recursive Least Squares Computations*, **University of Pittsburgh and Carnegie-Mellon**, Colloquium, December 1988.
- *An Iterative Substructuring Algorithm for Equilibrium Equations*, **R. S. Varga Conf. on Approx. Theory and Numerical Linear Algebra**, Kent OH, March 1989.
- *Recursive Least Squares Computations*, **International Conf. on Mathematics in Networks and Systems**, Amsterdam , June 1989.
- *Modified Least Squares Methods*, **Minisymp. - SIAM Conf.**, San Diego CA, July 1989.
- *Fast RLS Methiods in Signal Processing*, **Boeing Aircraft Company**, Seattle WA, Colloquium, August 1989.
- *Least Squares and Related Computations on High Performance Architectures*, **Wake Forest University**, Colloquium, January 1990.
- *Panel Presentation on Scientific Computing*, **ACM - IEEE Supercomputing '89**, Reno NV, November 1989.
- *Department Heads Presentation: What is Scientific Computing?*, **ACM Annual Computer Science Conference**, Washington, DC, February 1990.
- *Fast Adaptive Condition Estimation in Signal Processing*, **Householder XI Symposium**, Halmstad, Sweden, June 1990.
- *Congugate Gradient Methods for Least Squares Computations*, **Symposium at the USSR Academy of Sciences**, Moscow, Russia, July 1990.
- *Software Needs in Signal Processing*, **DARPA Workshop on Science and Technology for the 90's**, Oak Ridge, TN, September 1990.
- *Comparison of Adaptive Condition Estimators in Signal Processing and Control*, **SIAM Conf. on Lin. Alg. in Signals, Systems and Control**, San Francisco, CA, November 1990.

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This final report summarizes the activities in support of the Air Force research project, AFOSR-88-0285, and identifies the important accomplishments over the period of the grant.

Our research project has been concerned with the design and testing of new algorithms for least squares computations with particular emphasis on applications to signal processing and to optimization methods in structural analysis, as well as to related problems in science and engineering. The objectives were to mathematically develop, test, and analyze fast numerical algorithms for the efficient solution to computational problems on modern high performance computers. Our recent work on fast recursive least squares computations in signal and image processing and computational methods in structural analysis have led to especially significant results. Some highlights of these results are outlined in this report.

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