SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-TR- 1 0 1 1 5
6a. NAME OF PERFORMING ORGANIZATION: School of Engineering and Applied Science, University of Los Angeles	6b. OFFICE SYMBOL (If applicable)
6c. ADDRESS (City, State and ZIP Code): Hilgard Avenue, Los Angeles, California 90024	7a. NAME OF MONITORING ORGANIZATION: AFOSR/Directorate Symbol
	7b. ADDRESS (City, State and ZIP Code): Building 410, Bolling AFB DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR/Directorate Symbol	8b. OFFICE SYMBOL (If applicable): NA
8c. ADDRESS (City, State and ZIP Code): Building 410, Bolling AFB DC 20332 - 6448	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: AFOSR - 89 - 0096
	10. SOURCE OF FUNDING NOS.

PROGRAM ELEMENT NO	PROJECT NO.	TASK NO.	WORK UNIT NO.
61102F	2302	B1	

11. TITLE (Include Security Classification): Development of a Micromechanic Theory of Contitutive Relations of Polycrystalline Solids (U)

12. PERSONAL AUTHOR(S): Tung-Hua Lin, Xiao-Qiang Wu, Wei Zhong

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr, Mo., Day)	15. PAGE COUNT
Annual	FROM 11/1/89 TO 10/31/90	November 30, 1990	

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB GR	Micromechanic, Plasticity, Polycrystals

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

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20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☒ DTIC USERS ☒	21. ABSTRACT SECURITY CLASSIFICATION: (U)

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE NUMBER (Include Area Code)	22c. OFFICE SYMBOL
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DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

(202) 767-0463

SECURITY CLASSIFICATION OF THIS PAGE

AD-A232 697

has 12 slip system. Plastic and creep property of a crystal depends on the slip in all the 12 slip systems. It is very difficult to obtain the effects of all the slip systems from single crystal tests (Bassani, 1990). Instead of waiting for more informative single crystal tests data, we attempt to utilize the available polycrystal tests. Presently the polycrystal creep property is derived from polycrystal test from its axial and combined tension and shear tests (Lin, 1984 Ribeiro, 1978, Wu, 1981), but the non-radial loading data were not used. The non-radial loading has a large effect on the polycrystal creep property. Hence this inclusion of the non-radial loading data in the formulation of the polycrystal creep response will improve greatly the calculated strain-time relations. This approach will enhance significantly the accuracy of creep analysis of many structures.

Mechanical equation of state has been widely used in creep analyses. It is here shown that this equation of state is valid for radial loadings.

ANNUAL PROGRESS REPORT
DEVELOPMENT OF A MICROMECHANIC THEORY OF CONSTITUTIVE RELATIONS OF POLYCRYSTALLINE SOLIDS

Grant No. AFOSR-89-0096

Nov. 1, 1989 through Oct. 31, 1990

Department of Civil Engineering
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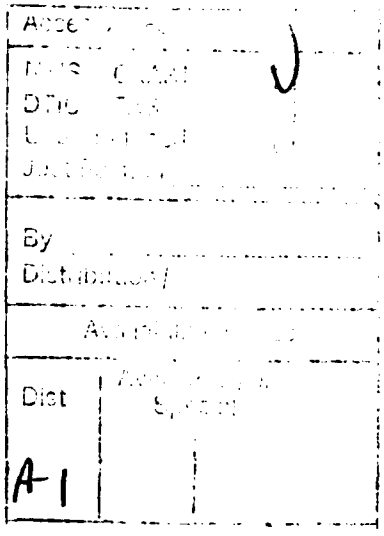

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Derivation of Stress-Strain-Time Relationship of a Polycrystal from its Radial and Non-Radial Loading Tests

by

T.H. Lin, X.Q. Wu and W. Zhong

Abstract: To analyze structures loaded at elevated temperatures, the stress-strain-time relation of the material is required. This relation given by "time-hardening" and "strain-hardening" theories (Johnson and Henderson, 1962) commonly used often have large discrepancies with experimental results and hence give large errors in the analysis. Lin et al., 1977, suggested to calculate this polycrystal creep properties from the creep data of single crystals. However, the plastic and creep property of crystals varies with their size. The size of a single crystal in tests is much larger than those in a polycrystal. There will be significant error to use single crystal data in polycrystal calculations. A f.c.c. crystal has 12 slip system. Plastic and creep property of a crystal depends on the slip in all the 12 slip systems. It is very difficult to obtain the effects of all the slip systems from single crystal tests (Bassani, 1990). Instead of waiting for more informative single crystal tests data, we attempt to utilize the available polycrystal tests. Presently the polycrystal creep property is derived from polycrystal test from its axial and combined tension and shear tests (Lin, 1984 Ribeiro, 1978, Wu, 1981), but the non-radial loading data were not used. The non-radial loading has a large effect on the polycrystal creep property. Hence this inclusion of the non-radial loading data in the formulation of the polycrystal creep response will improve greatly the calculated strain-time relations. This approach will enhance significantly the accuracy of creep analysis of many structures.

Mechanical equation of state has been widely used in creep analyses. It is here shown that this equation of state is valid for radial loadings.

INTRODUCTION

Structures subject to loads at elevated temperatures have been greatly increased during the last few decades. At these temperatures, creep strain becomes significant and induces a redistribution of stress in redundant structures. These redistributed stresses govern the creep rates and hence the life of the structure. To analyze this stress redistribution in the structure at different time instants, a realistic stress-strain-time relation of the material is needed. The stress-strain-time reaction given by "time hardening" and "strain-hardening" (Johnson and Hutchinson, 1962) commonly used for structural analysis often have large discrepancies with experimental results. These creep theories do not consider the physical mechanism of deformation. This seems to be one main source of error of these theories. The present study based on the physical mechanism of deformation is expected to give a creep theory more representative of actual metals. As Dorn and Mote, 1963, have indicated, among the different mechanisms of plastic deformation, slip is the main mechanism for face-centered cubic metals at low and intermediate temperatures. Conrad, 1961 has also stated that at these temperatures, the deformation of a f.c.c. polycrystalline metal occurs essentially by deformation of the grains. Grain boundary sliding is only significant when the temperature approaches one half the melting temperature of the metal. Present study is mainly concerned with such metals and alloys at low and intermediate temperatures, hence slip is considered to be the sole source of creep deformation.

Single crystal test at room temperature (Taylor and Elam, 1925, Taylor, 1928), have shown that under stress, slip occurs along certain crystal directions on certain crystal planes. In a face-centered-cubic crystal, there are four such planes, on each of which there are three slip directions giving twelve slip systems. These planes correspond to dislocation glide planes and these directions correspond to Burgers vectors of dislocations. From dislocation theory, (Read, 1953), the force to move a segment of dislocation line is directly proportional to the shear stress on the slip plane along the Burgers vector. This is shown in single crystal tests that slip depends on the resolved shear stress along the slip direction on the slip plane and is independent of the normal pressure on the

plane, Taylor, 1938. The quantitative relations between plastic strain and dislocation movement have been elegantly given by Mura, 1967, Kröner, 1958 and others. Tests on aluminum single crystals by Johnson et al., 1953, 1955, show that deformation at elevated temperatures occurred by slip in primarily the twelve slip systems that are operative at room temperature. There also may be some slip on (311), (211) or (100) planes but their contribution is small and requires higher resolved shear stresses than that for slip in the twelve slip systems. Hence in the present study, each crystal is considered to have time dependent slip only in these twelve systems, and the rate of slip is taken to depend on the resolved shear stress, Johnson et al., 1953, 1955.

Recent Works of Self-Consistent Theory of Polycrystal Creep:

The main difference of a polycrystal from a single crystal is the presence of grain boundaries. The grain boundary has been estimated to be only a few atoms thick, (Dorn, 1963, Barrett, 1952). Hence, in the calculation of slip field of a polycrystal, the grain boundary can be regarded as a surface of zero thickness across which crystal orientation changes from one to another. The anisotropy of elastic constants of single crystals varies from one metal to another. This anisotropy is small for aluminum, Barrett 1952. The present study concerns mainly with aluminum and its alloys, hence this anisotropy is neglected. When an aggregate of randomly oriented crystals of homogeneous and isotropic elastic constants is uniformly loaded, the stress in uniform throughout before creep occurs. However, crystals of different orientations have different resolved shear stresses. Assume that a crystal deforms by creep only when the resolved shear stress in some slip system exceeds certain critical value (Schwope et al., 1953). Creep occurs in the aggregate when the resolved shear stress in the most favorably oriented crystal reaches the critical value. Consider a fine-grained metal. The size of one crystal is much less than that of the aggregate. The slid crystal can be considered as being embedded in a infinite medium. At the beginning, only the most favorably oriented crystals slide and the distance between two slid crystals is large and the stress field within and around each slid crystal is essentially the same as that of only one slid crystal in a infinite medium. The stress field caused by slip in a crystal in a fine-grained aggregate decreases rapidly

with the distance from the slid crystal (Lin et al., 1961). This is similar to the inclusion problem in elasticity. Eshelby's results on ellipsoidal inclusions, (Eshelby, 1957), give uniform slip strain accompanying uniform resolved shear stress in the slid crystal. This agrees with the dependency of slip rate on the resolved shear stress, (Johnson et al., 1953, 1955). Hence, these Eshelby's results may be used to calculate the stresses in the slid crystal at this loading.

The sum of loads carried by all crystals cut by a section must balance the external load on the aggregate. The stress relieved by slip of one crystal must be carried by other crystals. Creep or plastic strain in one group of crystals will increase the average stress taken by other groups of crystals. This interaction effect between groups of crystals was first taken into consideration by Kröner, 1968. Budiansky and Wu, 1962, rederiving Kröner's scheme of incorporating this average interaction effect by a different physical reasoning and using Eshelby's results for ellipsoidal inclusions, obtained numerical results for the time-independent plastic stress-strain curves of strain-hardening f.c.c. polycrystals in tension and shear. Theories considering this average interaction effect are called self-consistent theories. Hill, 1965, had shown that Budiansky and Wu's self-consistent model has certain limitations, since the pronounced directional weaknesses in the constraint of an already-yielded aggregate, when loaded far beyond the elastic range, are disregarded. However, Hutchinson, 1970, has compared the predictions of the more general models of Hill with those of the simple model and has found that the two predictions are roughly similar in the range of small plastic strains.

Assuming the rate of slip in a slip system to be governed by the resolved shear stress in that system, Rice, 1970, has shown that it is possible to derive the polycrystal macroscopic creep strain rate from a potential function of stress. Phillips, 1969, has experimentally obtained loading surfaces of polycrystal aluminum under combined loading at elevated temperatures. Brown, 1970a, b, has extended the application of the Budiansky-Wu's self-consistent model of plastic deformation to creep strains. Assuming the slip rate $\dot{\gamma}$ in a slip system to be a power function of the resolved shear stress τ in that system,

$$\dot{\gamma} = c\tau^n \tag{1}$$

where c and n are constants and n varies from 3 to 8, Brown has calculated, by this self-consistent model, the creep strain rates under a given path of non-radial loading. He has compared the calculated results with experimental strain-rate vs time curves and has found that the experimental curves gives much larger creep strain rate than the calculated results following each change in loading. Hutchinson, 1975, has shown a more direct method of estimating the steady creep characteristics of polycrystals composed of f.c.c. crystals whose slip rate in a slip system is related to its resolved shear stress as given in Eq. (1). These studies have contributed much to the understanding of the relation between the creep properties of single crystals and those of polycrystals.

When a polycrystal is loaded at an elevated temperature, many or all crystals may slide; then the distance between two adjacent slid crystals may become very small. Hence the application of Eshelby's results considering the average interaction effect may cause significant error. The assumption of creep rate of a single crystal as a function of the resolved shear stress only, neglects the transient creep. This neglect of transient creep and the use of the self-consistent theory seem to be the man cause of the discrepancy between Brown's calculated and experimental results (Brown, 1970a, b).

Method of Calculating Stress Field Caused by a Given Creep Strain Distribution:

Ductile polycrystalline metals undergo considerable creep deformation before cracks occur. Hence, a rigorous theory of polycrystal creep should satisfy, throughout the aggregate, the conditions of continuity of displacement, condition of equilibrium and the stress-strain-time relation of the component crystal. A method satisfying all these three conditions to calculate the stress fields caused by given creep strains is here shown. This method was originally developed by Lin et al. 1966, 1970, 1975, 1978, for calculating plastic stress-strain relation of polycrystals from that of a single crystal and is here modified for creep calculations.

Referring to a set of rectangular coordinates (x_{ij}) where i = 1, 2, 3, the strain components e_{ij} consists of elastic and creep parts

$$e_{ij} = e_{ij}^{E} + e_{ij}^{c} \tag{2}$$

where the superscript E and c denote elastic and creep respectively. The stress-strain relationship for this isotropic medium is represented by

$$\begin{aligned} \tau_{ij} &= \delta_{ij}\,\lambda e_{kk}^{E} + 2\mu\, e_{ij}^{E} \\ &= \delta_{ij}\,\lambda(e_{kk} - e_{kk}^{c}) + 2\mu(e_{ij} - e_{ij}^{c}) \end{aligned} \tag{3}$$

where δ_{ij} is the Kronecker delta, λ and μ are Lame's constants and the repeated indices denote summation. The condition of equilibrium within a body is given as

$$\tau_{ij,j} + F_i = 0 \tag{4}$$

where subscript j after comma denotes partial differentiation with respect to x_j-axis, and F_i denotes the i-component of body force. Substituting (3) in (4), we obtain

$$\delta_{ij}\,\lambda e_{kk,j} + 2\mu\, e_{ij,j} - (\delta_{ij}\,\lambda e_{kk,j}^{c} + 2\mu\, e_{ij,j}^{c}) + F_i = 0 \tag{5}$$

It is seen that the parenthesis term is equivalent to F_i in causing strain field e_{ij} and hence is called the equivalent body force and is denoted by $\overline{F}_i$. Creep strain caused by slip produces no volumetric strain hence $\overline{F}_i$ reduces to

$$\overline{F}_i = -2\mu\, e_{ij,j}^{c} \tag{6}$$

The strain field e_{ij} caused by $\overline{F}_i$ may be obtained by solving the differential Equation (5) with $F_i = 0$. The stress field caused by this $\overline{F}_i$, denoted by τ_{ij}^{s} may be obtained from the strain field.

$$\tau_{ij}^{s} = \delta_{ij}\,\lambda\, e_{kk} + 2\mu\, e_{ij} \tag{7}$$

Consider a point $\underset{\sim}{x}'$ in the aggregate sliding in the nth slip system. Let $\underset{\sim}{\alpha}(\underset{\sim}{x}', n)$ be the unit vector normal to the sliding plane at $\underset{\sim}{x}'$ with $\alpha_i\left(\underset{\sim}{x}', n\right)$ as its component along $\underset{\sim}{x}_i$-axis and $\underset{\sim}{\beta}\left(\underset{\sim}{x}', n\right)$ be the unit vector along the sliding direction at $\underset{\sim}{x}'$ with $\beta_j\left(\underset{\sim}{x}', n\right)$ as its component along x_j-axis. Denoting the magnitude of the creep strain caused by slip in this slip system at $\underset{\sim}{x}'$ by $e^c\left(\underset{\sim}{x}', n\right)$, the creep strain components due to slip in this slip system, referring to x_i-coordinates may be written as

$$e_{ij}^c\left(\underset{\sim}{x}'\right) = n_{ij}\, e^c\left(\underset{\sim}{x}', n\right) \tag{8}$$

where $n_{ij} = \alpha_i\left(\underset{\sim}{x}', n\right)\beta_j\left(\underset{\sim}{x}', n\right) + \alpha_j\left(\underset{\sim}{x}', n\right)\beta_i\left(\underset{\sim}{x}', n\right)$. The equivalent body force is then

$$\overline{F}_k\left(\underset{\sim}{x}'\right) = -2\mu \frac{\partial e^c\left(\underset{\sim}{x}', n\right)}{\partial x_l'} n_{kl} \tag{9}$$

After creep occurs in a number of crystals, the stress in the polycrystal becomes highly heterogeneous. To find the stress field caused by the heterogeneous slip, the following imaginary process is considered. Imagine that the load is removed and the creep strain remains. This creep strain causes a residual stress field τ_{ij}^R. Noting (7), we have

$$\tau_{ij}^R = \delta_{ij}\,\lambda(e_{kk} - e_{kk}^c) + 2\mu(e_{ij} - e_{ij}^c) = \tau_{ij}^s - 2\mu\, e_{ij}^c \tag{10}$$

Then imagine that the polycrystal is reloaded. No additional creep strain occurs during this imaginary process of unloading and reloading. This reloading is purely elastic and hence causes a uniform applied stress τ_{ij}^A in the aggregate. The resulting stress in the aggregate is the sum of the heterogeneous residual stress τ_{ij}^R and the applied homogeneous stress τ_{ij}^A.

$$\tau_{ij} = \tau_{ij}^A + \tau_{ij}^R = \tau_{ij}^A + \tau_{ij}^s - 2\mu\, e_{ij}^c \tag{11}$$

Consider a fine-grained metal. Each component crystal is much smaller than the aggregate. To calculate the stress field caused by a slip in a crystal at the interior of the aggregate, the equivalent body force due to creep strain gradients can be considered to act in an infinite medium. The displacement field is an infinite isotropic elastic medium caused by body force $F_k\left(\underset{\sim}{x}'\right)$ acting in volume v' has been given by Kelvin (Love, 1927). From this displacement field, the stress field is obtained. From Kelvin's solution, the stress field $\tau_{ij}^s\left(\underset{\sim}{x}\right)$ caused by equivalent body force $\overline{F}\left(\underset{\sim}{x}'\right)$ acting in v' is given as

$$\begin{aligned}\tau_{ij}^s(x) &= \int_{v'} \phi_{ijk}\left(\underset{\sim}{x}, \underset{\sim}{x}'\right) \overline{F}_k\left(\underset{\sim}{x}'\right) dv' \\ &= 2\mu \int_{v'} \phi_{ijk}\left(\underset{\sim}{x}, \underset{\sim}{x}'\right) \left[\frac{\partial e^c\left(\underset{\sim}{x}', \cdot\right)}{\partial x_l'} n_{kl}\left(\underset{\sim}{x}'\right) \right] dv' \end{aligned} \tag{12}$$

where

$$\begin{aligned}\phi_{ijk}\left(\underset{\sim}{x}, \underset{\sim}{x}'\right) &= \frac{-3}{8\pi(1-\sigma)} \frac{(x_i - x_i')(x_j - x_j')(x_k - x_k')}{r^5} \\ &+ \frac{1-2\sigma}{8\pi(1-\sigma)} \left[\frac{\delta_{ij}(x_k - x_k') - \delta_{ik}(x_j - x_j') - \delta_{jk}(x_i - x_i')}{r^3} \right] \end{aligned} \tag{13}$$

$r^2 = (x_i - x_i')(x_i - x_i')$ and σ is the Poisson's ratio.

From creep strain field e_{ij}^c, the equivalent body force $\overline{F}_i$ is readily obtained. The stress field τ_{ij}^s caused by $\overline{F}_i$ can be found by Equation (12), then the stress field is computed by Equation (11).

Present Polycrystal Model:

A large three dimensional region embedded in an infinite isotropic elastic medium is considered to be entirely filled with innumerous identical cubic blocks, each of which is composed of 64 f.c.c. crystals (Fig. 1) having different orientations. The average incremental stress $\Delta\bar{\tau}_{ij}$ vs

average incremental creep strain $\Delta\overline{e}^c_{ij}$ of the interior center basic block is taken to represent the macroscopic incremental stress ΔT_{ij} vs creep strain ΔE^c_{ij} of the polycrystal (Lin et al., 1974, 1975). The size of the total region may be considered to be infinite as compared to the size of one block. The creep strain distribution of the blocks in the interior region are taken to be the same. Hence, to calculate the stress field in the center block, the values of $\Delta e^c\left(\underset{\sim}{x}', n\right)$ and $\frac{\partial \Delta e^c\left(\underset{\sim}{x}', n\right)}{\partial x_l'}$ at (x_1', x_2', x_3') are

taken to be the same as those at any point $(x_1' - m_1 a, x_2' - m_2 a, x_3' - m_3 a)$, where m_1, m_2, m_3, are any integer and "a" is the linear dimension of the basic cube-shaped block. Then the integral in (12) over the three dimensional infinite region reduces to an integral over one basic block with $\phi_{ijk}(x, x')$ expressed as $\phi_{ijk}(x_1, x_2, x_3, x_1' - m_1 a, x_2' - m_2 a, x_3' - m_3 a)$ with m_1, m_2, m_3, summed over all positive and negative integers.

Crystal Orientations:

Polycrystals considered are composed of crystals of random orientations. Let specimen axes be denoted by x_1, x_2 and x_3. Creep tensile strain under tensile loading along x_1-axis should be about the same as that along x_2 or x_3-axis. Under a tensile loading, the creep shear strain components E^c_{12}, E^c_{23} and E^c_{31} approach zero. Under shear loading T_{12}, T_{23} or T_{31}, E^c_{11}. E^c_{22} and E^c_{33} are zero and the macroscopic shear stress-strain-time relations should be about the same. This is here referred to as the initial isotropy of polycrystal creep.

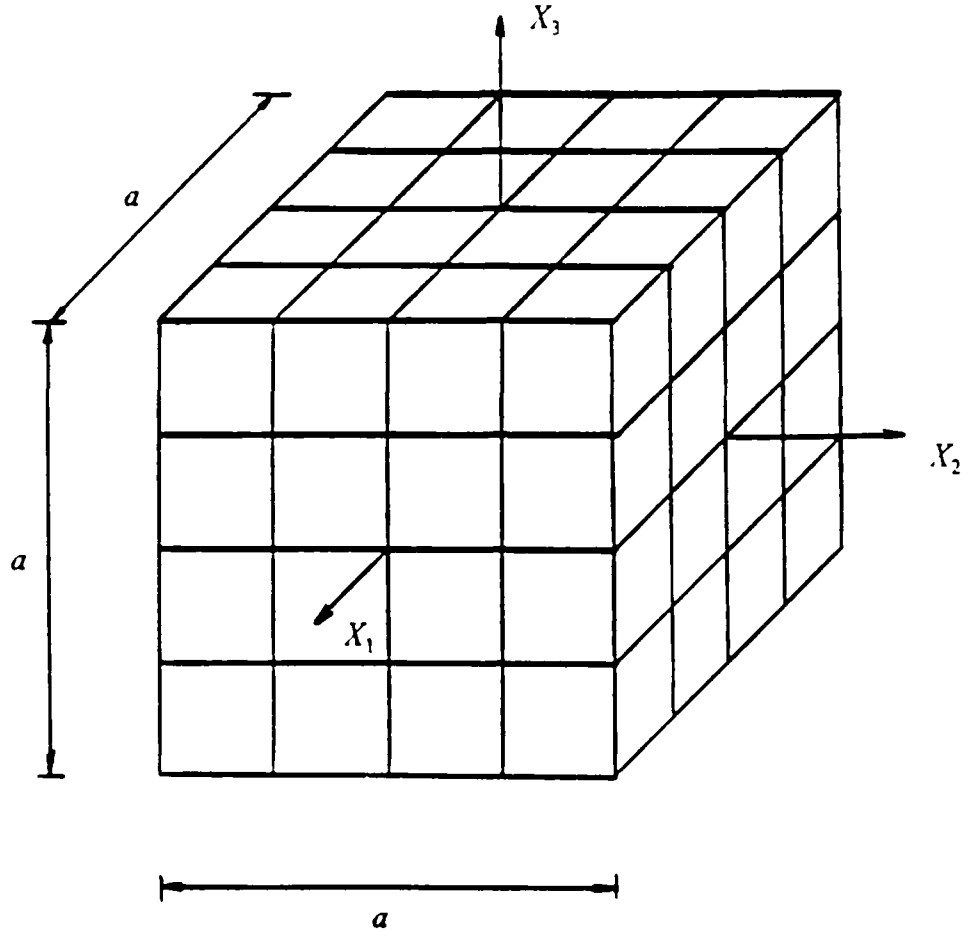


Figure 1: The Basic Block of 64-Crystals

In order to satisfy this property of initial creep isotropy, a 64-crystal model has been developed (Lin et al., 1974, Lin and Ribeiro, 1981). The 64-crystal in a basic cubic block are divided into eight groups, each of which has 8-crystals and occupies one octant of the block. The orientations and arrangement of crystals of one group are chosen so as to give mirror images of other groups with respect to three coordinate planes of the specimen axes. This gives three planes of symmetry an hence satisfies the property of orthotropy.

The actual orientations of crystals in a metal can be quite random or with preferred orientations. To simulate the one with random orientations, we orient one of the 8-crystals in the first octant such that this resolved shear stress in the most favorable slip system under T_{11} equals to that under T_{12} at

the loading T_{12} equals .577 T_{11}. This would give Von Mises criterion for initial yielding. A different orientation of this crystal can yield Tresca's criterion of yielding. Another crystal of the eight is oriented to give a mirror image to the first with respect to the plane making 45° with both x_2 and x_3 axes. These give two crystals associated with T_{11}. Similarly there are two such crystals associated with T_{22} and with T_{33}. These take six crystals. The remaining two crystals have their crystal axes coinciding with the specimen axes x_1, x_2, x_3. These 8-crystals are positioned in the first octant in such a way as shown in Fig. 1, as to give no preference to loadings T_{11}, T_{22} or T_{33}. By this way, the polycrystal has the property of cubic orthotropy i.e. E^c_{11} vs T_{11} under T_{11}, E^c_{22} vs T_{22} under T_{22} and E^c_{33} vs T_{33} under T_{33} are the same. This property holds for actual metals of random orientations with arbitrary orientations of specimen axes. The present 64-crystal model gives this initial creep isotropy only when the crystal orientations are fixed to the specimen axes. The orientations of these 8-crystals uses in our calculation and listed in Table 1.

Table I

Orientation of the First Slip Systems of the Crystals in the First Octant with Respect to Specimen Axes

Crystal Number	Slip Axes			
1	α	=	(0.577350 ,	0.577350 , 0.577350)
	β	=	(0.000000 ,	-0.707107 , 0.707107)
	γ	=	(0.816497 ,	-0.408248 , -0.408248)
2	α	=	(0.156435 ,	0.966105 , 0.205352)
	β	=	(0.987688 ,	-0.153016 , -0.032525)
	γ	=	(0.000000 ,	0.207912 , -0.978148)
3	α	=	(0.205352 ,	0.156435 , 0.966105)
	β	=	(-0.032525 ,	0.987688 , -0.153016)
	γ	=	(-0.978148 ,	0.000000 , 0.207912)
4	α	=	(0.205352 ,	0.966105 , 0.156435)
	β	=	(-0.032525 ,	-0.153016 , 0.987688)
	γ	=	(-0.978148 ,	0.207912 , 0.000000)
5	α	=	(0.966105 ,	0.205352 , 0.156435)
	β	=	(-0.153016 ,	-0.032525 , 0.987688)
	γ	=	(0.207912 ,	-0.978148 , 0.000000)
6	α	=	(0.966105 ,	0.156435 , 0.205352)
	β	=	(-0.153016 ,	0.987688 , -0.032525)
	γ	=	(0.207912 ,	0.000000 , -0.978148)
7	α	=	(0.156435 ,	0.205352 , 0.966105)
	β	=	(0.987688 ,	-0.032525 , -0.153016)
	γ	=	(0.000000 ,	-0.987148 , 0.207912)
8	α	=	(0.577350 ,	0.577350 , 0.577350)
	β	=	(0.000000 ,	-0.707107 , 0.707107)
	γ	=	(0 816497 ,	0.408248 , -0.408248)

Averaging Microscopic Strains and Stresses:

Refer to Fig. 1 that shows the basic block of 64-Crystals, let S_j or $S_j(x_j)$ denote the intersection of the plane $x_j = C$ $(-\frac{a}{2} \le C \le \frac{a}{2})$ with the block. The macroscopic displacements on the surfaces of the block are defined as

$$U_i|_{S_j\left(\pm\frac{a}{2}\right)} = \frac{1}{A}\int_{S_j\left(\pm\frac{a}{2}\right)} u_i dS \tag{14}$$

where $A = (a)^2$ is the area of each face of the block. The macroscopic strain of the block is defined as

$$E_{ij} = \frac{1}{2}\left[\frac{U_i|_{S_j\left(\frac{a}{2}\right)} - U_i|_{S_j\left(-\frac{a}{2}\right)}}{a} + \frac{U_j|_{S_i\left(\frac{a}{2}\right)} - U_j|_{S_i\left(-\frac{a}{2}\right)}}{a}\right] \tag{15}$$

Substituting (14) into 15, we have

$$E_{ij} = \frac{1}{2}\frac{1}{aA}\left[\int_{S_j}\int_{-\frac{a}{2}}^{\frac{a}{2}} u_{i,j}dx_j dS + \int_{S_i}\int_{-\frac{a}{2}}^{\frac{a}{2}} u_{j,i}dx_i dS\right]$$

(*no summation on i and j*)

$$= \frac{1}{aA}\int_V \frac{1}{2}(u_{i,j} + u_{j,i})dV = \frac{1}{V}\int_V e_{ij}dV = \bar{e}_{ij} \tag{16}$$

where *a* bar on the top denotes the volume average over the block. In the present study, the summation convention for product terms is used only for the coordinate indices unless otherwise stated. Eq. 16 indicates that the macroscopic strain of the block equals the average internal strain of this block. The above derivation also gives

$$E_{ij} = E_{ji} \tag{17}$$

Let $e_{ij}(p)$ denote the average strain of the pth grid in the block, then

$$\bar{e}_{ij} = \frac{1}{64}\sum_{p=1}^{64} e_{ij}(p) \tag{18}$$

thus

$$E_{ij} = \frac{1}{64}\sum_{p=1}^{64} e_{ij}(p) \tag{19}$$

Let the macroscopic stress of the block be defined on the surfaces $S_j\left(\frac{a}{2}\right)$ by

$$T_{ij} = \frac{1}{A}\int_{S_j\left(\frac{a}{2}\right)} \tau_{ij} dS \tag{20}$$

Since all the blocks are assumed to deform identically, the stress at any point on the surface $S_j\left(\frac{a}{2}\right)$ will be the same as that at the same point on the surface $S_j\left(-\frac{a}{2}\right)$. Consider the equilibrium of a body in the block cut by the planes $S_j\left(\frac{a}{2}\right)$ and $S_j(x_j)$, the resultant forces of the tractions on the four planes except $S_j\left(\frac{a}{2}\right)$ and $S_j(x_j)$ are zero. Therefore, the resultant forces of the tractions on $S_j\left(\frac{a}{2}\right)$ should be balanced by that on $S_j(x_j)$. Then the macroscopic stress in (20) can be evaluated on any surface $S_j(x_j)$. Hence

$$T_{ij} = \frac{1}{A}\int_{S_j} \tau_{ij} dS = \frac{1}{a}\int_{-\frac{a}{2}}^{\frac{a}{2}} \tau_{ij} dS\, dx_j = \frac{1}{V}\int_V \tau_{ij} dV = \bar{\sigma}_{ij} \tag{21}$$

(no summation on j)

From (21), we also have

$$T_{ij} = T_{ji} \tag{22}$$

By the same reasoning s (18)

$$T_{ij} = \bar{\sigma}_{ij} = \frac{1}{64}\sum_{p=1}^{64} \tau_{ij}(p) \tag{23}$$

i.e., the macroscopic stress of the block is the average of the nominal stresses of the 64-crystals.

The macroscopic elastic strains are defined as

$$E_{ij}^{E} = M_{ijkl} T_{kl} \tag{24}$$

where M_{ijkl} are the macroscopic compliance coefficients of the block which, for a homogeneous elastic medium, are the same as that of the medium. From (21), we have

$$E_{ij}^{E} = M_{ijkl}\bar{\sigma}_{kl} = \bar{e}_{ij}^{E} = \frac{1}{64}\sum_{p=1}^{64} e_{ij}^{E}(p) \tag{25}$$

The macroscopic creep strains are defined as

$$E_{ij}^{c} = E_{ij} - E_{ij}^{E} \tag{26}$$

Therefore,

$$E_{ij}^{c} = \bar{e}_{ij} - \bar{e}_{ij}^{E} = \bar{e}_{ij}^{c} = \frac{1}{64}\sum_{p=1}^{64} e_{ij}^{c}(p) \tag{27}$$

Letting $\Delta\tau_{ij} = \tau_{ij} - \bar{\tau}_{ij}$ $\Delta e_{ij} = e_{ij} - \bar{e}_{ij}$, the work done by the stress on the strain in the block is given by

$$\begin{aligned}\int_V \tau_{ij} e_{ij} dV &= \int_V (\bar{\tau}_{ij} + \Delta\tau_{ij})(\bar{e}_{ij} + \Delta e_{ij}) dV \\ &= \int_V \bar{\tau}_{ij}\bar{e}_{ij} dV + \int_V \Delta\tau_{ij}\Delta e_{ij} dV \\ &= V T_{ij} E_{ij} + \int_S \Delta\tau_{ij} n_j \Delta u_i dV \end{aligned} \tag{28}$$

Since the deformation states on $S_j\left(\frac{a}{2}\right)$ and that on $S_j\left(-\frac{a}{2}\right)$ are the same, we have

$$\Delta\tau_{ij} n_j \big|_{S_j\left(\frac{a}{2}\right)} = -\Delta\tau_{ij} n_j \big|_{S_j\left(-\frac{a}{2}\right)} \qquad \textit{(no summation on } j\textit{)} \tag{29}$$

and

$$u_i|_{S_j\left(\frac{a}{2}\right)} - U_i|_{S_j\left(\frac{a}{2}\right)} = u_i|_{S_j\left(-\frac{a}{2}\right)} - U_i|_{S_j\left(-\frac{a}{2}\right)} \tag{30}$$

i.e.,

$$\Delta u_i|_{S_j\left(\frac{a}{2}\right)} = \Delta u_i|_{S_j\left(-\frac{a}{2}\right)} \tag{31}$$

therefore,

$$\int_S \Delta\tau_{ij} n_j \Delta u_i dV = 0 \tag{32}$$

hence

$$\overline{\tau_{ij} e_{ij}} = \bar{\tau}_{ij}\bar{e}_{ij} = T_{ij}E_{ij} \tag{33}$$

This satisfies the well known virtual work equation for polycrystal aggregates, which Bishop and Hill, 1954 proved, based on a "non-correlation" hypothesis for a "unit" cube containing a sufficient number of crystals such that it is "macroscopically homogeneous". Hill, 1967, showed that this equation is valid for an aggregate as long as either the tractions or the displacements on the boundaries of the aggregate are uniform, and it is valid approximately as long as those boundary conditions are "macroscopically uniform". Havner, 1971 used these uniform boundary conditions in his discrete model, which is practical for complex loading conditions only if his "unit" cube contains a sufficient number of grains. In the present model, neither the tractions nor the displacements on the boundaries of the basic block need to be uniform. The above derivation also holds for the case that the block contains a finite number of single crystals that are of arbitrary shapes, sizes and orientations as long as all blocks are assumed to deform identically. This model removes the assumption of non-correlation shown by Bishop and Hill, 1951.

Calculation of Residual Stresses Caused by Creep:

To simplify the numerical calculation, creep strain in each crystal is assumed to be uniform and this average stress of each crystal is assumed to represent the stress over the entire crystal. With this assumption, creep strain gradient in each crystal vanishes. Across the crystal boundary creep strain drops from the uniform value to zero. This gives no equivalent body force but gives an equivalent uniform surface force of the plane boundary surface of each crystal. Eq. 12 reduces to

$$\tau^s_{ij}(x) = 2\mu \oint, \phi_{ijk}\left(\underset{\sim}{x}, \underset{\sim}{x}'\right) n_{kl}\left(\underset{\sim}{x}'\right) e^c\left(\underset{\sim}{x}', n\right) dS'_l \tag{34}$$

where dS'_l is the projected differential area of the boundary of the volume v' normal to x_1-axis.

$$dS'_l = \upsilon_l dS'$$

where υ_l is direction cosine of the normal to the boundary of the differential area dS'.

Considering the creep strain distributions in all cubic blocks to be the same as that of the center block as indicated previously, the stress field at point $\underset{\sim}{x}$ caused by the equivalent surface forces due to slip in the nth slip system of the qth crystal is then

$$\tau^s_{ijqn}\left(\underset{\sim}{x}\right) = 2\mu \oint \sum_{m_1} \sum_{m_2} \sum_{m_3} \phi_{ijk}(x, x', m_1, m_2, m_3) e^c(q, n) n_{kl}(q) dS'_l \tag{35}$$

Since $e^c(q, n)$ is assumed to be uniform in each crystal. This above can be written as

$$\tau^s_{ijqn}(x) = 2\mu\, e^c(q, n) n_{kl}(q) \sum_{m_1} \sum_{m_2} \sum_{m_3} \oint \phi_{ijk}(x, x', m_1, m_2, m_3) dS'_l \tag{36}$$

then

$$\tau^R_{ijqn}(x) = \tau^s_{ijqn}(x) - 2\mu e^c_{ij}(x) \tag{37}$$

The residual stress field is hence determined. The average residual stress of the crystal "p" due to a uniform creep strain due to slip in the nth slip system of the qth crystal $e^c(q,n)$ can be calculated and written as

$$\tau^R_{ijqn}(p) = a_{ijqn}(p)e^c(q,n) \tag{38}$$

By tensor transformation, the residual resolved shear stress in the mth slip system of the pth crystal is readily obtained and is written as

$$\tau^R_{mpnq} = a_{mpnq}e^c_{nq} \tag{39}$$

τ^R_{mpnq} is the average residual resolved shear stress in the mth slip system in the pth crystal due to a unit uniform resolved shear creep strain in the nth slip system in the qth crystal and is called the residual stress influence coefficient. These influence coefficients are the same as those used in plasticity calculations, since both creep and plastic strains are inelastic strains (Lin, 1968).

These coefficients satisfy the reciprocal relations and give a symmetrical matrix in our calculations. The above provides a more rigorous method to calculate the stress field from given creep strain distributions than other proposed methods in the literature.

Single Crystal Characteristics:

A number of theories of creep in single crystal metals have been reviewed by Johnson et al., 1953, 1955. Stowell, 1958 has indicated that all the then existing theories of transient creep which is an important part of total creep, are inadequate to account for the actual behavior of polycrystalline metals at elevated temperatures. Single crystal data under multi-axial creep under non-radial loading are very meagre and difficult to measure. Since we have developed a method to calculate the stress-strain-time relation of a polycrystal from that of the component crystal, we here calculate the component crystal characteristics from the given available test data of the polycrystal under radial and non-radial loadings. The size of single crystals used for single crystal tests is much larger than

that of the component crystal in a polycrystal and it is known that the creep property varies with the crystal size. To calculate the creep characteristics of the component crystal from the stress-strain-time relation of a polycrystal, has the advantage of automatically taking care of the size effect of the crystals.

Most formula proposed for creep in metals under constant temperature have assumed the creep rate $\dot{\gamma}^c$ on a slip system depending on its resolved shear stress τ and the summation of creep slip $\Sigma\gamma^c$ all of slip systems.

$$\dot{\gamma}^c = \dot{\gamma}^c(\tau, \Sigma\gamma^c) \tag{40}$$

One form of the above is given as

$$\dot{\gamma}^c = c\tau^{\alpha}\left(1 + \alpha e^{-\beta\frac{\Sigma\gamma^c}{Q\tau^d}}\right) \tag{41}$$

where c, d, α, Q, and β are material constants.

Plasticity studies have shown that active hardening is different from latent hardening, (Taylor, 1938). The rate of hardening also varies among this different latent slip systems. (Wu et al., 1990, Bassani et al., 1990, Bassani, 1990). Hence the use of the sum of slip $\Sigma\gamma^c$ in describing this slip rate is not adequate. Hence Eq. 40 is changed to

$$\dot{\gamma}_m^c = \dot{\gamma}_m^c(\tau_m, \gamma_n^c) \qquad n = 1, \ldots 12 \tag{42}$$

Now the creep rate in the mth slip system is a function of the resolved shear stress in this system τ_m and resolved shear creep strains in all the slip systems. A number of attempts have been made to use Eq. 41 to simulate experimental creep data covering radial and non-radial loadings (Ribeiro 1978, Wu, 1981). However, the agreements between the calculated and the experimental results are not satisfactory especially for the case of non-radial loading. The form given in (42) is much less restrictive and hence should yield better agreements with experimental results.

The present formulation divides the creep rate in a slip system on the transient and steady parts

$$\dot{\gamma}_m = \dot{\gamma}_m^s + \dot{\gamma}_m^t \tag{43}$$

$$\dot{\gamma}_m^s = c\left(\frac{\tau_m}{\tau_o}\right)^n \tag{44}$$

$$\dot{\gamma}_m^t = P\left(\frac{\tau_m}{\tau_o}\right)^n \; e^{-\frac{\gamma_m^t}{Q\left(\frac{t_m}{t_o}\right)^n}} \tag{45}$$

where τ_o is taken to be 10,000 p.s.i. in the range of loadings (1) to (3) considered, the material constants have found to be

$$\begin{aligned} n &= 4.3 \\ c &= c_o = 0.80 \times 10^{-4} \\ Q &= Q_o = 0.75 \times 10^{-4} \\ P &= 60.0 \times 10^{-4} \end{aligned} \tag{46}$$

The first step is to use Eq. 45 to calculate the strain-time curves for loadings given by the paper "Short-Time Biaxial Creep of an Aluminum Alloy with Abrupt Changes of Temperature and State of Stress" by Blass and Findley, 1971. Aluminum Alloy 2618-T61 was tested at 200°C. The test data considered are

Time		0 to 2 hrs.	2 hr to 4 hrs.	
(1) Tensile Loading	T_{11} =	20,000 p.s.i.	-	A1*
(2) Shear Loading	T_{12} =	11,500 p.s.i.	-	B1
(3) Radial Loading	T_{11} =	20,000 p.s.i.	-	C1
	T_{12} =	11,500 p.s.i.	-	

* Test number of the short-time creep test.

(4) Non-Radial Loading	T_{12}	=	11,500 p.s.i.	$T_{11} = 11{,}500$ $T_{12} = 20{,}000$	B2

It was found that for case (4), Eq. 45 greatly under represent the creep strain after the start of the non-radial loading. Due to symmetrical arrangement of the 64-crystal model, only 16-crystals have different active slip systems under T_{11} and T_{12}. Numerical calculations indicate that 14 of the 16 crystals have active slip systems changed. It is assumed that new slip systems slide more easily to give large creep rate after the non-radial loading. To represent this higher creep rate, Q and c in Eq. 45 are increased by ΔQ and Δc respectively. These ΔQ and Δc can vary from one slip system to another and from one crystal to another. To simplify the calculation, ΔQ and Δc are assumed to be constant and occur in all the 14-crystals with new active slip systems.

These ΔQ and Δc, after few trials, were taken as

$$\Delta Q = Q_1 F(\Sigma \mid \gamma_n^c \mid, \Delta\tau_m). \qquad Q_1 = 4.00 \tag{47}$$

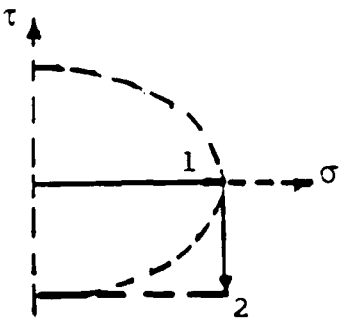
and

$$\Delta c = c_1 F(\Sigma \mid \gamma_n^c \mid, \Delta\tau_m) \qquad c_1 = 3.00 \tag{48}$$

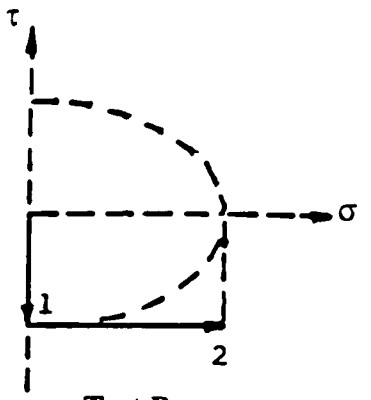
where $$F(\Sigma \mid \gamma_n^c \mid, \Delta\tau_m) = \tanh\left(\frac{1}{4}(\Sigma \mid \gamma_n^c \mid) x (\Delta\tau_m)\right) \text{ and only applies when} \tag{49}$$

$\Delta\tau_m$ in increasing,

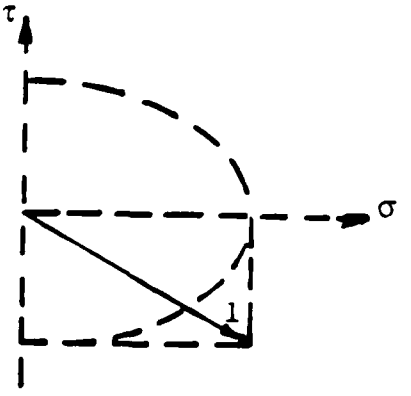
Using these equations the calculated axial and shear strain-time curves are compared with the experimental data as shown in Figs. 3 to 6. The above creep slip rate in different slip systems as given in Eqs. 45, 47 to 49 is expected to yield much more realistic polycrystal creep characteristics for this material than the "time-hardening" or "strain-hardening" theories. If the structure is loaded to give much higher stresses, creep test data covering this high stress range in radial and non-radial loadings should be included. Polycrystal creep test data seems to be much easier to obtain. This procedure seems to be able to improve the accuracy of creep analyses of structures.



Test A
200°C



Test B
200°C



Test C
200°C

Figure 2: Loading Paths of Tests

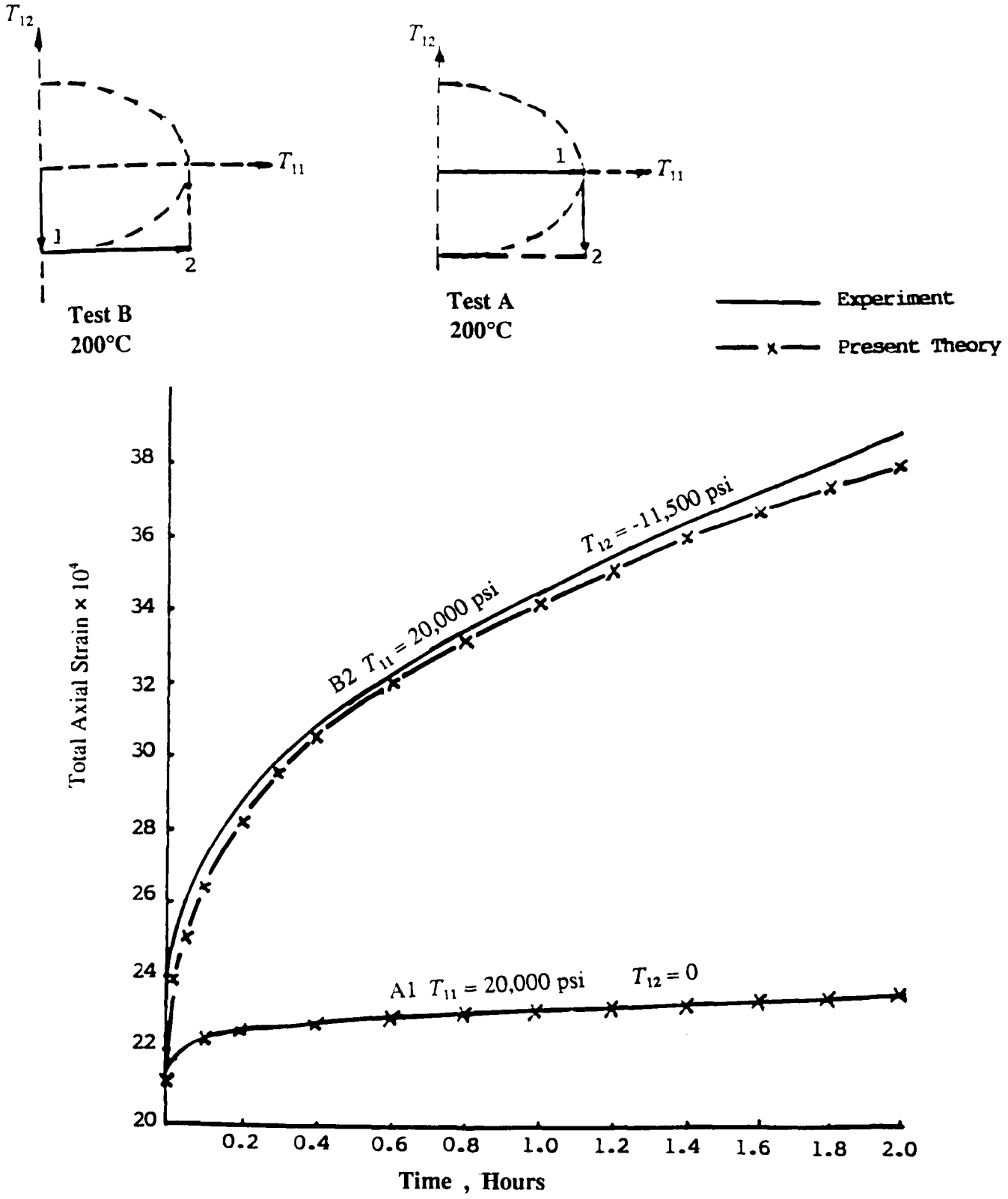


Figure 3: Calculated and Experimental Curves under Axial Loading (Al) and Non-Radial Loading (B2)

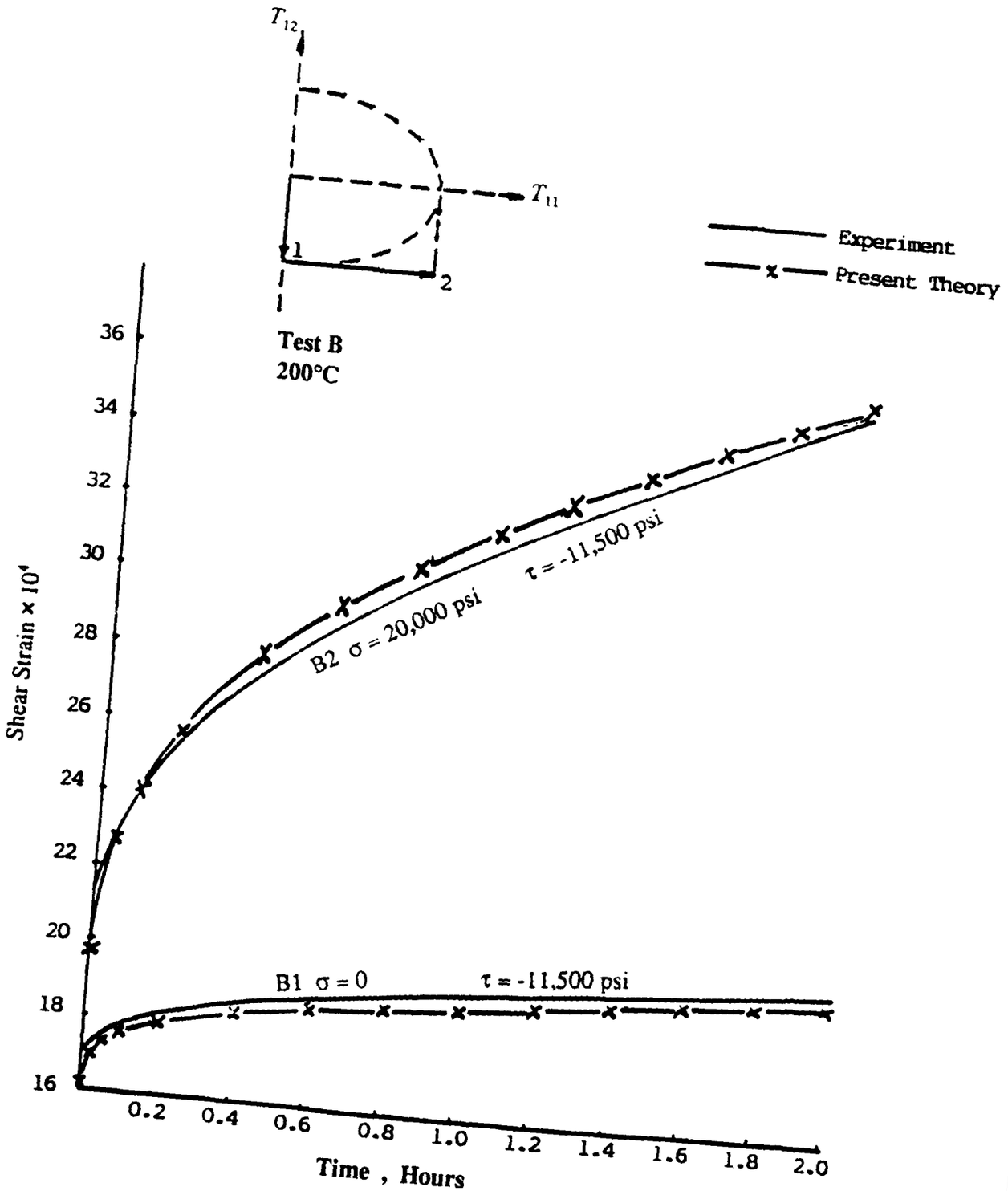


Figure 4: Calculated and Experimental Curves under a Non-Radial Loading (BI) and (B2)

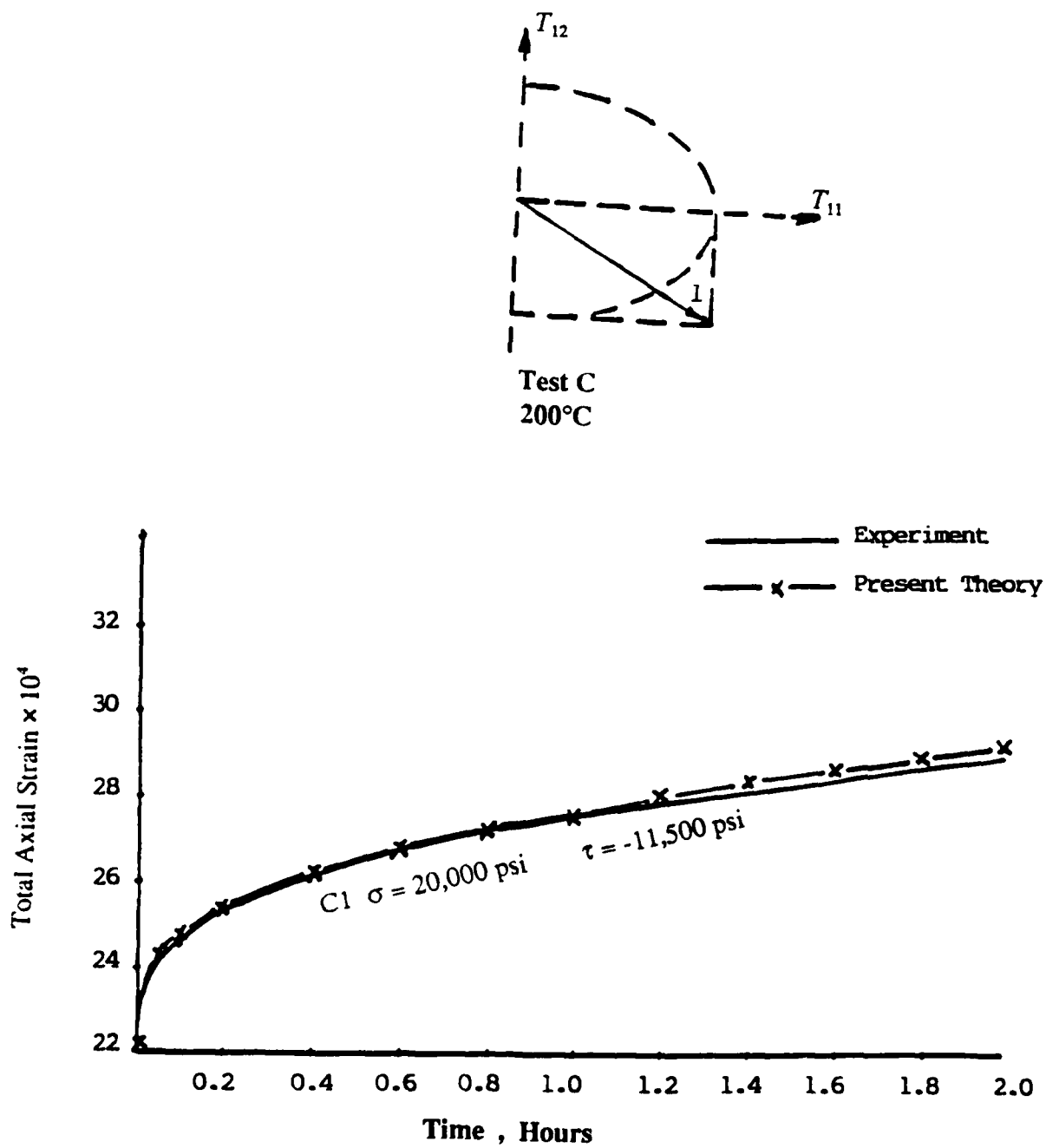


Figure 5: Calculated and Experimental Curves under a Radial Loading (Cl) Axial Strain

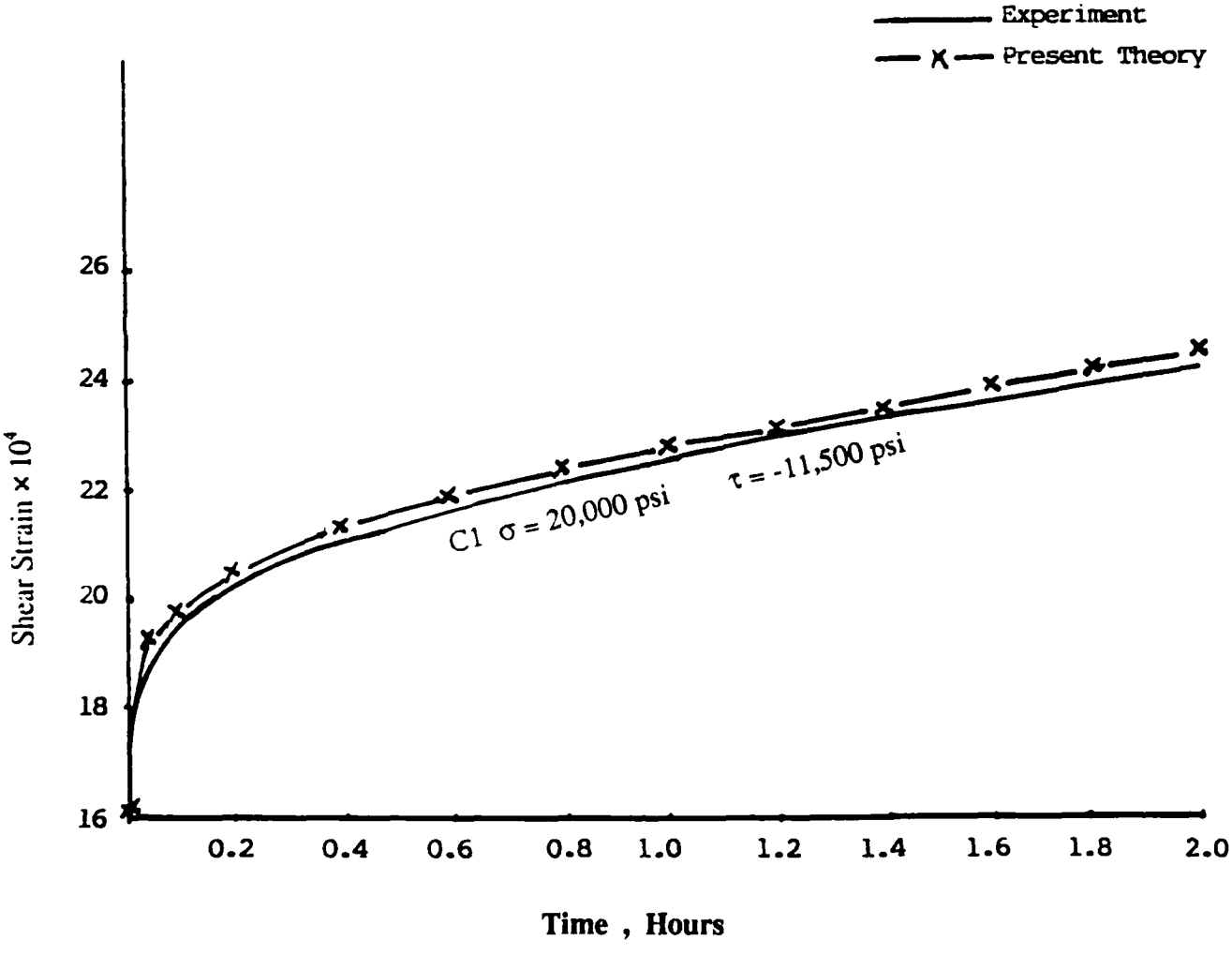


Figure 6: Calculated and Experimental Curves under a Radial Loading (Cl) Shear Strain

Mechanical Equation of State:

Most structures are subject to stresses varying with time. Creep tests have generally been done under constant load or constant stress. In applying these data to structures in which stress varies with time, certain assumptions have to be made. One commonly used assumption for creep analysis of structures is the existence of a mechanical equation between creep rate, stress, temperature, and current creep strain. This Mechanical Equation of State is not derived from the physics of the metals and therefore may not be valid for some metals. Under constant temperature, this equation may be written for a polycrystal,

$$\dot{E}^c_{11} = F(E^c_{11}, T_{11}) \tag{50}$$

where T_{11} is the macroscopic stress and F denotes a function. Here the stress is assumed to depend only on the current creep strain and its rate, and not on the strain rate during earlier stages of deformation. The creep behavior of the component crystal is generally assumed of the following form

$$\dot{\gamma}^c = F(\gamma^c, \tau) \tag{51}$$

Here τ represents the resolved shear stress, γ and $\dot{\gamma}$ denote the creep strain and creep strain rate, respectively.

Table II

Maximum Resolved Shear stress of Each Crystal at Intersection of Creep and Relaxation Test

Crystal Number	Max. Resolved Stress-Creep (psi)	Max. Resolved Stress-Relax (psi)	Resolved Stress-Diff. (psi)	Percent Error
1	11186.0273	11188.6055	2.58	.023
2	13394.6367	13377.6797	16.96	.127
3	13394.6367	13377.6797	16.96	.127
4	11186.0273	11188.6055	2.58	.023
5	11319.9180	11340.3203	20.40	.180
6	11319.9180	11340.3203	20.40	.180
7	11319.9180	11340.3203	20.40	.180
8	11319.9180	11340.3203	20.40	.180
9	11319.9180	11340.3203	20.40	.180
10	11319.9180	11340.3203	20.40	.180
11	11319.9180	11340.3203	20.40	.180
12	11319.9180	11340.3203	20.40	.180
13	11186.0273	11188.6055	2.58	.023
14	13394.6367	13377.6797	16.96	.127
15	13394.6367	13377.6797	16.96	.127
16	11186.0273	11188.6055	2.58	.023
17	12671.5273	12671.4492	0.078	.0006
18	12671.5273	12671.4492	0.078	.0006
19	12671.5273	12671.4492	0.078	.0006
20	12671.5273	12671.4492	0.078	.0006
21	13394.6367	13377.6797	16.96	.127
22	11186.0273	11188.6055	2.58	.023
23	11186.0273	11188.6055	2.58	.023
24	13394.6367	13377.6797	16.96	.127
25	13394.6367	13377.6797	16.96	.127
26	11186.0273	11188.6055	2.58	.023
27	11186.0273	11188.6055	2.58	.023
28	13394.6367	13377.6797	16.96	.127
29	12671.5273	12671.4492	0.078	.0006
30	12671.5273	12671.4492	0.078	.0006
31	12671.5273	12671.4492	0.078	.0006
32	12671.5273	12671.4492	0.078	.0006

Table II (Con't)

Maximum Resolved Shear stress of Each Crystal at Intersection of Creep and Relaxation Test

Crystal Number	Max. Resolved Stress-Creep (psi)	Max. Resolved Stress-Relax (psi)	Resoived Stress-Diff. (psi)	Percent Error
33	12671.5273	12671.4492	0.078	.0006
34	12671.5273	12671.4492	0.078	.0006
35	12671.5273	12671.4492	0.078	.0006
36	12671.5273	12671.4492	0.078	.0006
37	13394.6367	13377.6797	16.96	.127
38	11186.0273	11188.6055	2.58	.023
39	11186.0273	11188.6055	2.58	.023
40	13394.6367	13377.6797	16.96	.127
41	13394.6367	13377.6797	16.96	.127
42	11186.0273	11188.6055	2.58	.023
43	11186.0273	11188.6055	2.58	.023
44	13394.6367	13377.6797	16.96	.127
45	12671.5273	12671.4492	0.078	.0006
46	12671.5273	12671.4492	0.078	.0006
47	12671.5273	12671.4492	0.078	.0006
48	12671.5273	12671.4492	0.078	.0006
49	11186.0273	11188.6055	2.58	.023
50	13394.6367	13377.6797	16.96	.127
51	13394.6367	13377.6797	16.96	.127
52	11186.0273	11188.6055	2.58	.023
53	11319.9180	11340.3203	20.40	.180
54	11319.9180	11340.3203	20.40	.180
55	11319.9180	11340.3203	20.40	.180
56	11319.9180	11340.3203	20.40	.180
57	11319.9180	11340.3203	20.40	.180
58	11319.9180	11340.3203	20.40	.180
59	11319.9180	11340.3203	20.40	.180
60	11319.9180	11340.3203	20.40	.180
61	11186.0273	11188.6055	2.58	.023
62	13394.6367	13377.6797	16.96	.127
63	13394.6367	13377.6797	16.96	.127
64	11186.0273	11188.6055	2.58	.023

Using a component crystal with the stress-strain-time relation of the form (51), and following the polycrystal model shown in Fig. 1. The strain-time curve at constant uniaxial loading $T_{11} = 28.00$ ksi and the stress-strain-time curves of the relaxation test at initial loading 30.00 ksi had been calculated by the present theory are shown in Fig. 6.

The comparison of the maximum resolved shear stress for each crystal at the intersection point of both tests are presented in Table III. It can be seen that the error is very small. This proves that the calculated component crystal constitutive equations satisfy the mechanical equation of state by numerical calculation.

Conclusions:

1. The theory developed to calculate the polycrystal creep characteristic from that of the component crystals satisfies both equilibrium and compatibility conditions as well as the component crystal stress-strain-time relations.

2. The size effect of crystals is automatically taken care of in this proposed approach.

3. A more accurate representation of a polycrystal creep characteristics. Single crystal stress-strain-time relationship is taken to depend on the stress and the amounts of slip in different slip systems in the crystal. The slip rate in one slip system depends on the resolved shear stress in the slip system and the amount of slip in the active and all the eleven other slip systems. An expression of this dependency was found to give calculated creep strain-time curves agreeing well with the experimental curves of the test data obtained by Blass and Findley, 1971 on an aluminum alloy tested at 200°C under both radial and non-radial loadings. This expression is to be used n calculating the stress-strain-time relation of the material for other radial and non-radial loadings. It is believed that this procedure will give much more realistic creep responses than the "time-hardening" and "strain-hardening" theories. This should improve greatly the creep analyses of structures.

4. Single crystal stress-strain-time relations proposed, generally satisfies the relation of mechanical equation of state. Using such a relation in calculating the polycrystal creep response, it was found that this relation is also satisfied for polycrystals. One creep loading and one relaxation loading of a polycrystal were calculated by the present theory. It was found that at the intersection of the creep rate vs creep strain curves of the creep and relaxation loadings, the resolved shear stresses in all the crystals are the same for the two loadings. This shows this mechanical equation of state is sound for radial loadings (not for (non-radial loadings).

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