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13. ABSTRACT *(Maximum 200 words)*

The objectives of this project were to develop and understand probability and statistical models which adequately describe the reliability of composite materials. Prerequisite to the study of fibrous composites was the study of the failure characteristics of brittle fibers (e.g. carbon fibers) that are used in such composites. Thus, new approaches to the reliability of structures with respect to their strength to withstand an environmental stress were considered.

More specifically, it was shown that the probability distribution for the tensile strength of brittle fibers had the IFR property and that it carried over to composites containing such fibers. A new probability distribution for the strength of brittle materials was developed and its characteristics were studied. Some general stress-strength models were also developed and applied to the failure of systems subject to cyclic loading. Involved in the failure of composite materials was the development of new graphical and inferential statistical methods for estimating mixtures of failure distributions or mixtures of hazard functions. The related problem of competing risks models was also investigated. Techniques for statistical testing that the strengths of materials made by different manufacturers (or at different times by a specific manufacturer) are the same were developed using statistical process control ideas and sequential design and analysis methods. Finally, smooth nonparametric quantile function estimators were studied. All of the results should prove useful in the assessment of materials reliability.

14. SUBJECT TERMS
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STATISTICAL MODELS AND INFERENCE PROCEDURES FOR STRUCTURAL AND MATERIALS RELIABILITY

FINAL REPORT

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SUMMARY

The main objectives of this research project were to develop and fully understand probability and statistical models which adequately describe the failure of modern composite materials. Prerequisite to the study of fibrous composites was the study of the failure characteristics of brittle fibers, such as carbon fibers, that are used in such composites. Thus, new approaches to the reliability of structures with respect to their strength to withstand some kind of environmental stress were considered.

The following specific results were obtained: It was shown that the probability distribution for the tensile strength of brittle fibers had the increasing failure rate (IFR) property and that it carried over to the composites containing such fibers. A theory was developed explaining this phenomenon. A new probability distribution for the strength of brittle materials was developed and its characteristics were studied. Some general stress-strength models were also developed and applied to the problem of fatigue failure of gun barrels, or to the failure of any system subject to cyclic loading. These more realistic models lead to more managable statistical estimation problems. Involved in the failure of composite materials was the development of statistical methods for estimating mixtures of failure distributions or mixtures of hazard functions. This study lead to new graphical and inferential methods for determining the presence of different types of flaws in materials causing failure under tensile load. The related problem of competing risks models was also investigated in this context. Techniques for statistical testing that the strengths of materials made by different manufacturers or made at different times by a specific manufacturer are the same were developed. These were obtained using statistical process control ideas and sequential design and analysis methods. Finally, smooth nonparametric quantile function estimators were studied with respect to optimal selection of the smoothing parameter. All of these results should prove to be useful in the assessment of the reliability of materials.

REPORT OF RESEARCH INVESTIGATIONS AND FINDINGS

A. STATEMENT OF THE PROBLEMS STUDIED

The main area of investigation in this research project concerned the development and understanding of probabilistic and statistical models which adequately describe the failure of materials. In particular, the probabilistic and statistical modeling of strength of brittle fibers and composite materials made from such fibers was one major area of investigation. The probability distributions governing the failure of brittle fibers, and how the properties of such distributions carried over into the composites containing such fibers were studied extensively. Specifically, it was noticed that the probability distribution for the tensile strength of brittle fibers had the property of increasing failure rate (IFR). Also, it was found that fibrous composites containing such fibers exhibited the same behavior. A theory was developed and studied with which to explain this phenomenon.

Involved in the failure of composite materials was the development of statistical methods for mixtures of failure distributions. This study lead to new graphical and inferential results for determining the presence of different types of flaws in materials causing failure under tensile load, based on data from strength tests. Therefore, the difficult general problem of estimation and testing for arbitrary mixtures of probability distributions was studied. A related problem is that of competing risks models which was also studied briefly.

General "stress-strength" models were also considered and their properties were investigated. One definition of the reliability of a physical system, or specimen of material, which is subjected to some kind of environmental "stress" is the probability that its "strength" exceeds the applied stress. The "strength" of the system, or specimen, depends on the material properties and other factors. The system or specimen fails if, at some application of the stress (or loading), the applied stress exceeds the intrinsic strength. During testing it is of interest to be able to draw inferences about the reliability of such a system or specimen. The testing of modern composite or ceramic materials provide examples of such reliability estimation problems. Previous stress-strength models either required parametric assumptions and observed values of strength for estimation or were not conducive to convenient estimation due to their form. Hence, more realistic models leading to more manageable estimation problems were studied.

Another problem investigated was that of testing that the strengths of materials made by different manufacturers or at different times by the same manufacturer are the same. This problem was studied from a statistical process control point of view and also from a sequential analysis approach (a "regenerative sampling plan").

The smooth nonparametric estimation of quantiles of strength (or lifetime) distributions was also considered. In particular, small percentiles of strength distributions are important in design considerations, for example, the determination of the A-basis and B-basis values for engineering purposes. Kernel-type quantile estimators were studied with respect to their optimality properties as compared with the usual nonparametric estimators such as the empirical percentiles, based on either right-censored or uncensored data.

B. SUMMARY OF MOST IMPORTANT RESULTS

Probability models for the strength of brittle materials, including fibrous composites, were developed and their properties were investigated. A different approach to the problem of estimating the strength distribution of brittle fibers than ones previously in the literature was proposed in paper number 1 in Section C of this report. The procedure assumed a Poisson flaw model, but rather than presuming a particular functional form, it was assumed that the probability distribution of strength had an increasing failure rate (IFR). The maximum likelihood estimate of the fiber strength distribution was obtained within the class of all IFR distributions, yielding a continuous distribution estimate which provided a good fit to fiber tensile strength data, typically fitting much better than the usual Weibull model (measures by root mean squared error). The method was illustrated with data for two different types of carbon fibers supplied by personnel at the U.S. Army Materials Technology Laboratory in Watertown, Massachusetts.

In paper 2, a general model for the failure of fibrous composite materials was described. It was shown to contain some of the well-known models in the literature. The composite material was viewed as a coherent system of independent identically distributed component strengths. Under the assumptions that the applied load was redistributed "homotonically" to the unfailed components upon the failure of a component and that the distributions of component strengths were increasing failure rate on average (IFRA), it was shown that the system, or composite, strength distribution was also IFRA. Empirical evidence indicated that the stronger IFR property held. A simple explanation was proposed for the IFR character of the strength distribution of a fibrous composite by assuming an "equalization of load principle." An example was given using carbon reinforced composite data to illustrate the IFR fit as compared with that of the empirical, normal, and the Weibull distributions.

Further developing the above idea in paper number 8, TP_2 orderings of distributions and of survival functions were studied. It was shown that the first passage time of a Markov process with TP_2-ordered transition distributions has an increasing failure rate. Conditions were also given for which mixtures of IFR distributions are also IFR. A formula was obtained for the failure rate when the strength distribution was a function of both load and strength. That formula, in conjunction with the TP_2-ordering on the strength survival functions and log-concavity of the strength and load variables, lead to an increasing failure rate for the strength distribution. Two models based on this theory were presented which explain the IFR character of the strength distribution of fibrous composite materials observed in test data.

From a parametric viewpoint, a new probability distribution which described the tensile strength of a single brittle fiber was developed in paper

4 and further studied in paper number 9. The new distribution was based on the Griffith theory of brittle fracture and the statistical theory of extreme values. This new "brittle fracture" (BF) distribution tended to fit experimental tensile strength data for carbon fibers better than the usual Weibull distribution and also had a sound physical basis. The methods of maximum likelihood and linear least squares for estimating the parameters of this new distribution were used and investigated by Monte Carlo simulation. Some examples with carbon fiber strength data from the USAMTL and comparisons with corresponding Weibull fits to the same data were discussed.

In the area of general "stress-strength" models, an approach in paper 3 assumed cumulative damage under Cramér's "proportionate growth" model. (See also papers 6 and 12.) This approach was applied to a model for critical crack size estimation in the fatigue failure of gun barrels. A consistent discrete estimate of the distribution of critical crack size was obtained using the theory of mixtures of probability distributions. Estimates of mean barrel life and other parameters are also available from the estimated critical crack size distribution.

The analysis of mixtures of distributions and mixtures of hazard functions was considered in papers 5 and 15. In 5, a strategy for analyzing mixed and pooled exponential distributions was studied. It was shown that a linear plot of the mean residual life on the failure rate characterized the mixture of two exponentials. This plot was used to estimate the two components in the mixing distribution with the two largest mixing proportions. The EM algorithm was used with these as initial values to obtain the maximum likelihood estimators. Gradient plots were used to see if a higher order fit was needed, and a heuristic was given on how to use the gradient plots to identify components in the higher order fit when this was the case. Graphs of an assignment function were then used to determine if the data were from a mixed model or simply pooled. In paper 15, a competing risk type of model, called the mixed hazards model, was studied. That model differed from competing risks in the usual sense in that the cause of failure could not be identified. Questions related to the identifiability of the model and to the uniqueness and support size of the maximum likelihood estimate of the mixing distribution were considered. A dual maximization problem resulted in a bound for the discrepancy of the log-likelihood from its maximum value.

The problem of inference for quantiles, or percentiles, was further considered in papers 11, 13 and 17. The Nadaraya-type kernel quantile estimator for complete samples and right-censored samples in 13 and 17, respectively, were studied with respect to the optimal bandwidth selection problem. It was shown that the optimal bandwidth in both cases was of the order $n^{-1/(2k-1)}$, where k was the order of the kernel, but with different coefficients which depend on the kernel, the distribution, and the true quantile. For second-order kernels ($k=2$), the optimal bandwidth (in a probability sense) was of the order $n^{-1/3}$, which differs markedly from the optimal bandwidth in kernel density estimators. In paper 11, the problem of detecting a shift of a percentile (usually downward) of a strength distribution in a process control setting was considered. Shewhart-type charts for lower percentiles were developed based on samll samples. Lower and upper control limits for "percentile charts" for Weibull (or extreme value) and lognormal (or normal) distributions were presented which were obtained by Monte Carlo simulations. Average run lengths of the Weibull percentile control charts were discussed, and an application to the breaking stress of carbon fibers was presented to illustrate the method.

In paper 7, the problem of choosing the "better" of two populations based on sequential sampling was investigated. Observations were assumed to be taken from one of the two populations. A randomized selection rule chooses the populations according to some predetermined odds. The odds for choosing a population the next time may be dependent on all the observations taken so far from both populations. In 7, the odds were made for the randomized selection rule in such a way that the number of observations from the "better" population relative to the number of observations from the other population grew at prescribed rates, typically of exponential order or faster. The rules included randomized play-the-winner rules and randomized analogues of Robbins' $\bar{R}$ rules. Some Monte Carlo investigations were conducted to study these randomized selection rules for moderate sample sizes.

Many other problems related to those mentioned above were investigated successfully. The resulting publications and reports are in the complete listing in Section C of this report.

C. LIST OF ALL PUBLICATIONS AND TECHNICAL REPORTS

1. S. D. Durham, J. D. Lynch and W. J. Padgett (1988). Inference for strength distribution of brittle fibers under increasing failure rate. **Journal of Composite Materials 22**, 1131-1140.

2. S. D. Durham, J. D. Lynch and W. J. Padgett (1989). A theoretical justification for increasing average failure rate strength distribution in fibrous composites. **Naval Research Logistics Journal 36**, 655-661.

3. S. D. Durham and W. J. Padgett (1989). A model for critical crack size estimation in fatigue failure of gun barrels. **Naval Research Logistics Journal 36**, 139-146.

4. C. Black, S. Durham, J. Lynch and W. Padgett (1989). A new probability distribution for the strength of brittle fibers. **Fiber-Tex 1989**, The Third Conference on Advanced Engineering Fibers and Textile Structures for Composites, NASA Conf. Publ. 3082, pp. 363-374.

5. J. Grego, H. Hsi and J. D. Lynch (1990). A strategy for analyzing mixed and pooled exponentials. **Applied Stochastic Models and Data Analysis 6**, 59-70.

6. S. D. Durham and W. J. Padgett (1990). Estimation for a probabilistic stress-strength model. **IEEE Transactions on Reliability R-39**, 199-203.

7. S. D. Durham and K. F. Yu (1990). Randomized play-the-leader rules for sequential sampling from two populations. **Probability in the Engineering and Informational Sciences 4**, 355-367.

8. S. D. Durham, J. D. Lynch and W. J. Padgett (1990). TP_2-orderings and the IFR property with Applications. **Probability in the Engineering and Informational Sciences** 4, 73-88.

9. C. M. Black, S. D. Durham and W. J. Padgett (1990). Parameter estimation for a new distribution for strength of brittle fibers: A simulation study. **Communications in Statistics, Simulation and Computation** 19, 806-825.

10. C. Baldwin, S. Durham, J. Lynch and W. Padgett (1990). A hypercube application in large scale composite materials modeling. **1990 Proceedings** of the Fifth Distributed Memory Computing Conference, pp. 490-497.

11. W. J. Padgett and J. D. Spurrier (1990). Shewhart-type charts for percentiles of strength distributions. **Journal of Quality Technology** 22, 283-288.

12. W. J. Padgett and S. D. Durham (To appear). A probabilistic stress-strength model and its application to fatigue failure in gun barrels. **Journal of Statistical Planning and Inference.**

13. Y. L. Lio and W. J. Padgett (To appear). On the asymptotically optimal bandwidth for Nadaraya's quantile estimator. **Statistics and Probability Letters.**

14. S. Durham and N. Flournoy (To appear). Convergence results for an *adaptive ordinal urn design.* Second World Congress of the Bernoulli Society for Mathematical Statistics and Probability, **1991 Proceedings.**

15. Hsiu-Li Hsi, B. G. Lindsay and J. D. Lynch (Submitted). On mixtures of hazards: Nonparametric maximum likelihood in certain competing risk failure models.

16. W. J. Padgett (Submitted). A smooth nonparametric quantile estimator for IFR distributions.

17. Y. L. Lio and W. J. Padgett (Submitted). Asymptotically optimal bandwidth for a smooth nonparametric quantile estimator under censoring.

18. S. Durham, J. Smolka and M. Valtorta (Submitted). Conditions for the statistical consistency of Dempster's rule on diagnostic trees. (Summary to appear in **Proceedings of the Second Annual Symposium on Statistics and Artificial Intelligence.**)

19. S. Durham and N. Flournoy (Submitted). An adaptive ordinal urn model for quantile estimation.

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Sneh Gulati, Graduate Student, University of South Carolina, Columbia, SC, 8/16/89-6/30/90: Ph.D. Candidate (Expected completion date-August, 1991).

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REPORT OF INVENTIONS

There no inventions by the investigators during this project.