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J. R. Wait

Antenna Performance Influenced by the Finite Extent and Conductivity of Ground Planes: A Collection of Reprints by J. R. Wait, et al.

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Project 91260 "High-Frequency (HF) Antenna Element Modeling" is a MITRE sponsored research project to develop computer programs and accurate models for predicting the element pattern, radiation efficiency, and input impedance of HF monopole elements on ground planes of various forms that rest on or are in close proximity to earth. A unique aspect of this project has been the use of a technical advisor group composed of outside interested investigators to assist MITRE personnel in the development and evaluation of analytical models.

As a member of this group, Dr. James R. Wait has assembled a collection of twenty-two reprints, authored by himself and his colleagues, on antenna performance influenced by the finite extent and conductivity of ground planes primarily in the presence of earth. THe objective of this collection is to provide a focus and filter of relevant papers from among the more than 750 refereed papers by Wait and his associates; this paper presents this collection. Also included are Dr. Wait's discussions of U.S. Navy Electronics Laboratory reports related to reprint 1.11 (Wait and Walter, 1963) and an annotated listing of selected related publications.

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ABSTRACT

Project 91260 "High-Frequency (HF) Antenna Element Modeling" is a MITRE sponsored research project to develop computer programs and accurate models for predicting the element pattern, radiation efficiency, and input impedance of HF monopole elements on ground planes of various forms that rest on or are in close proximity to earth. A unique aspect of this project has been the use of a technical advisor group composed of outside interested investigators to assist MITRE personnel in the development and evaluation of analytical models.

As a member of this group, Dr. James R. Wait has assembled a collection of twenty-two reprints, authored by himself and his colleagues, on antenna performance influenced by the finite extent and conductivity of ground planes primarily in the presence of earth. The objective of this collection is to provide a focus and filter of relevant papers from among the more than 750 refereed papers by Wait and his associates; this paper presents this collection. Also included are Dr. Wait's discussions of U.S. Navy Electronics Laboratory reports related to reprint 1.11 (Wait and Walters, 1963) and an annotated listing of selected related publications. A biography of Wait and a complete listing of his refereed papers through April 1990 are given in appendices.

AUTHOR'S PREFACE

The performance of an antenna is dependent on the environment. In some cases the effect is minor but very often we need to worry about the structure upon which the antenna is mounted. It becomes part of the radiating structure! For example, if the system consists of a vertical wire fed from a perfectly conducting plane sheet of infinite extent, simple image theory would suffice to predict the radiation pattern and the input impedance. But the ground plane may be imperfectly conducting and it would always be of limited extent. Over the years, I have had an abiding interest in this problem, particularly in connection with the design of very low frequency (VLF), low frequency (LF), medium frequency (MF), and HF antennas in situations where the influence of the finitely conducting earth is significant. In such cases, radial-wire or mesh ground systems are often employed to mitigate the adverse effect of poorly conducting earth in the low angle radiation and to improve the overall power efficiency.

My publications, including those coauthored with my colleagues dealing with the topic, are scattered over many journals, reports and symposia proceedings. Some of these are no longer accessible. For this reason and because I have received numerous requests to supply copies, I have felt it is desirable to assemble some of the relevant material in a reprint collection. I have taken the opportunity to correct typographical errors in of the papers. Also I have added some extracts from related reports, particularly those from the U.S. Navy"s Ocean System Center in San Diego where I often visited in the 1960s. Finally, I have included a listing of related papers, published in the open literature, along with specific comments.

I wish to thank Melvin M. Weiner of The MITRE Corporation in Bedford, MA for getting me to assemble this collection. I am also grateful for his initial support. By way of introduction to this subject, the reader should consult Melvin Weiner's monograph, "Monopole Elements on Circular Ground Planes," (coauthored by S. Cruze, C-C Li, and W. Wilson) published by Artech House, 1987. Their analyses deal with the case where the monopole and circular disc are located in free space.

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ACKNOWLEDGMENT

M. M. Weiner, Principal Investigator of Project 91260, wrote the abstract, edited the manuscript, and motivated the author to assemble the enclosed reprints.

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SECTION 1

Reprints

Note: The reprint number in parentheses corresponds to that in the listing of refereed papers of appendix B. The reprints in this section are assembled in the order of increasing reprint number.

1.1 Wait, J. R., and W. J. Surtees, May 1954, "Impedance of a Top-Loaded Antenna of Arbitrary Length Over a Circular Grounded Screen," *Journal of Applied Physics*, Vol. 25, pp. 553-556

(Reprint No. 43)

Reprinted from JOURNAL OF APPLIED PHYSICS, Vol. 25, No. 5, 553-555, May, 1954

Impedance of a Top-Loaded Antenna of Arbitrary Length over a Circular Grounded Screen*

JAMES R. WAIT, Radio Physics Laboratory, Defence Research Board, Ottawa, Canada,

AND

WALTER J. SURTEES, University of Toronto, Toronto, Canada

(Received September 22, 1953)

The problem of a vertical monopole situated over a circular perfectly conducting screen lying on a finitely conducting ground is considered. An approximate method employed originally by Abbott to calculate the self-impedance is discussed. Using this formula explicit expressions are derived for the self-impedance of a thin, top-loaded vertical monopole. Sinusoidal current distribution is assumed.

INTRODUCTION

USUALLY the most important factor contributing to the inefficiency of low-frequency antennas is to be found in the ground system. The rules for ground system design are usually empirical and based on results of experiments on existing installations. Recently, the general problem has been investigated analytically by Abbott[1] who has developed a design procedure to select the optimum number of radial conductors specified given the values of the electrical constants of the ground.

An important related problem is the actual change of input impedance of the antenna for different sizes of ground systems. This analysis has been carried out for a vertical antenna situated centrally over a perfectly conducting disk by Leitner and Spence[2] and more recently by Storer.[3] They, however, only considered the case when the surrounding medium was free space. A more appropriate situation is when the disk is lying on the surface of a homogeneous conducting half-space which corresponds to the ground. It is not surprising that the solution of this problem is in general very difficult.

FORMULATION OF PROBLEM

With reference to a cylindrical polar coordinate system (ρ,ϕ,z) the antenna of height h is coincident with the positive z axis. The circular screen of radius a lies in the plane $z=0$ which is also the surface of the ground. The conductivity and dielectric constant of the ground are denoted by σ and ϵ, respectively, and the dielectric constant of the air by ϵ_0. The permeability of the whole space is μ which is taken to be the same as free space.

Due to the obvious polar symmetry in this problem, Maxwell's equations take the following form for time dependence according to $\exp(i\omega t)$:

$$\rho E_z = -\frac{\eta}{\gamma}\frac{\partial}{\partial\rho}(\rho H_\phi) \qquad (1)$$

$$E_\rho = -\frac{\eta}{\gamma}\frac{\partial H_\phi}{\partial z} \qquad (2)$$

$$\frac{\partial E_z}{\partial\rho} - \frac{\partial E_\rho}{\partial z} = i\mu\omega H_\phi, \qquad (3)$$

where γ and η are the intrinsic propagation constant and characteristic impedance of the medium, respectively, and defined by

$$\gamma = [i\mu\omega(\sigma+i\omega\epsilon)]^{\frac{1}{2}}$$

and

$$\eta = [i\mu\omega/(\sigma+i\omega\epsilon)]^{\frac{1}{2}}.$$

A subscript zero is affixed to these quantities in order that they should pertain to the air. That is,

$$\gamma_0 = i(\epsilon_0\mu)^{\frac{1}{2}}\omega = i2\pi/\lambda,$$

where λ is the free space wavelength in meters and

$$\eta_0 = (\mu/\epsilon_0)^{\frac{1}{2}} \simeq 377 \text{ ohms}.$$

Following the method of Storer,[3] the difference between the impedance Z of the antenna over a finite screen and the impedance Z_0 for an infinite screen is denoted by ΔZ and is given by

$$\Delta Z = Z - Z_0 = -\frac{\eta_0}{\gamma_0^2 I_0^2}\int_0^h \left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho H_\phi^s\right) I(z)dz. \qquad (4a)$$

In this equation H_ϕ^s is the change of the magnetic field H_ϕ for a finite screen from the magnetic field H_ϕ^∞ for an infinite screen. The current on the antenna is $I(z)$ and the base current is I_0. It then follows that

$$\Delta Z = -\frac{1}{I_0^2}\int_a^\infty H_\phi^\infty(\rho,0)E_\rho(\rho,0)2\pi\rho d\rho, \qquad (4b)$$

where

$$H_\phi^\infty(\rho,0) = -\frac{1}{2\pi}\frac{\partial}{\partial\rho}\int_0^h \frac{\exp[-\gamma_0(z^2+\rho^2)^{\frac{1}{2}}]}{(z^2+\rho^2)^{\frac{1}{2}}} I(z)dz. \qquad (4c)$$

THE APPROXIMATE SOLUTION

The magnetic field in the ground outside the screen is a solution of the wave equation

$$(\Delta-\gamma^2)H_\phi(\rho,z) = 0, \qquad (5)$$

where $\Delta = \frac{\partial}{\partial\rho}\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho$

* Paper presented at Joint International Scientific Radio Union–Institute of Radio Engineers meeting in Ottawa, October 6th, 1953.

[1] F. R. Abbott, Proc. Inst. Radio Engrs. 40, 846 (1952).

[2] A. Leitner and R. D. Spence, J. Appl. Phys. 21, 1001 (1950).

[3] J. E. Storer, J. Appl. Phys. 22, 1058 (1951).

and therefore

$$H_\phi(\rho,z)=\int_0^\infty J_1(\lambda\rho)e^{uz}p(\lambda)\lambda d\lambda,\quad z\leq 0, \qquad (6)$$

where $u=(\lambda^2+\gamma^2)^{\frac{1}{2}}$ and the electric field is given by

$$E_\rho=-\frac{\eta}{\gamma}\int_0^\infty uJ_1(\lambda\rho)e^{uz}p(\lambda)\lambda d\lambda,\quad z\leq 0. \qquad (7)$$

The binomial expansion of u is of the form

$$u=\gamma\left(1+\frac{\lambda^2}{2\gamma^2}+\cdots\right),$$

so that

$$E_\rho=-\frac{\eta}{\gamma}\left\{\int_0^\infty \gamma J_1(\lambda\rho)e^{uz}p(\lambda)\lambda d\lambda +\int_0^\infty (1/2\gamma)J_1(\lambda\rho)e^{uz}p(\lambda)\lambda^3 d\lambda+\cdots\right\}. \qquad (8)$$

Now the differential equation for J is given by

$$\left(\frac{\partial}{\partial\rho}\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho-\lambda^2\right)J_1(\lambda\rho)=0$$

and so it readily follows that, for $z=0$

$$E_\rho=-\eta H_\phi-\frac{\eta}{2\gamma^2}\frac{\partial}{\partial\rho}\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho H_\phi)$$

$$\pm\text{terms containing }\gamma^{-4}. \qquad (9)$$

The second term is negligible if the propagation constant in the ground is sufficiently large and if ρH_ϕ is not varying too rapidly with ρ. That is ρH_ϕ should not change appreciably in a distance equal to $|\gamma^{-1}|$ in the radial direction for points outside the screen. This approximation is always well justified under practical conditions at low radio-frequencies.

Employing this result for E_ρ the equation for the self-impedance is now given by

$$\Delta Z=\frac{\eta}{I_0^2}\int_{\rho=a}^\infty H_\phi^m(\rho,0)H_\phi(\rho,0)2\pi\rho d\rho. \qquad (10)$$

To apply this formula to an actual situation it can be assumed that $H_\phi(\rho,0)$ is not very different from $H_\phi^m(\rho,0)$ in the region of the ground plane where the losses are significant, and therefore

$$\Delta Z\simeq\frac{\eta}{I_0^2}\int_{\rho=a}^\infty [H_\phi^m(\rho,0)]^2 2\pi\rho d\rho. \qquad (11)$$

An equivalent statement of this approximation is that the electrical radial current density at the surface of the ground is not appreciably affected by the finite conductivity of the soil. The approximate expression for ΔZ in Eq. (11) was first given in this form by Abbott[4] who quoted it without proof but pointed out that it was a plausible result. Later Monteath[5] also arrived at this same formula from an extension of the compensation theorem well known in circuit theory.

A better approximation to Eq. (10) might be to set

$$H_\phi(\rho,0)=H_\phi^m(\rho,0)F(p), \qquad (12)$$

where $F(p)$ is Sommerfeld's "surface wave attenuation factor" which is a function of the numerical distance p which for a good conducting ground ($\sigma\gg\epsilon\omega$) is given by

$$p=(\pi\rho/\lambda)(\epsilon\omega/\sigma).$$

When $p\ll 1$ it can be shown or seen from tabulated numerical values[6] that $F(p)$ is very close to unity. If $p>1$ the magnitude of $F(p)$ drops appreciably below unity. However, since H_ϕ^m varies essentially as $e^{-\gamma_0\rho}/\rho$ and since $F(p)$ is a slowly varying function the contribution from the integrand in Eq. (11) for the cases where $p>1$ are negligible. At low radio-frequencies (<1000 kc) and for moderate ground conductivities ($\sigma>10^{-3}$ mhos per meter) the approximate formula given by Eq. (11) is sufficiently close to the formula in Eq. (10) to justify its use.

Up to this point no restriction has been placed on the current distribution on the antenna. For thin antennas it can usually be assumed that

$$I(z)=I_m\sin(\alpha-\beta z) \qquad (13)$$

for

$$0\geq z\geq h,$$

where $\beta=\gamma_0/i=2\pi/\lambda$. The maximum current amplitude is I_m, and the conditions at the upper end of the antenna specify α. For unterminated ends $I(h)=0$ so that $\alpha=\beta h$ and then

$$I(z)=I_0\sin\beta(h-z)/\sin\beta h.$$

In some instances at low frequencies it is customary to load the antenna near the top so that effectively $I(h)\neq 0$. It is convenient then to set

$$\alpha=\beta(h+h'),$$

where h' specifies the degree of top loading.

Employing the value of $I(z)$ given by Eq. (13) and inserting it into Eq. (4c) the integration with respect to h can be carried out to give

$$H_\phi^m(\rho,0)=-\frac{iI_m}{2\pi}\left[\frac{e^{-i\beta r}}{\rho}\cos(\beta h-\alpha)-\frac{e^{-i\beta\rho}}{\rho}\cos\alpha -\frac{ih}{\rho}\frac{e^{-i\beta r}}{r}\sin(\alpha-\beta h)\right]. \qquad (14)$$

---

[4] F. R. Abbott, Standard Radial Ground Systems for l.f. and m.f. Monopole Transmitting Antennas. U. S. Navy Electronics Laboratory (San Diego), Report No. 219, Oct. 1950.

[5] G. D. Monteath, Proc. Inst. Elect. Engrs (London) 98, Pt. IV, 23 (1951).

[6] K. A. Norton, Proc. Inst. Radio Engrs. 24, 1367 (1936).

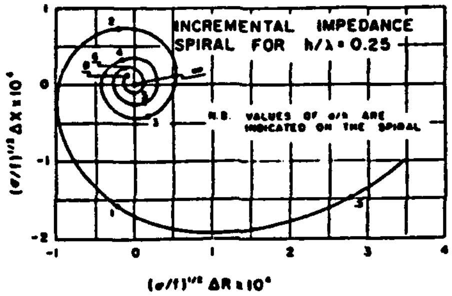

FIG. 1. The incremental self-impedance ΔZ for a quarter-wave monopole over a circular grounded screen.

where $r=(\rho^2+h^2)^{1/2}$. Using this value for $H_\phi{}^a$ the integration indicated in Eq. (11) can now be carried out to yield

$$\Delta Z=-\frac{\eta}{2\pi\sin^2\alpha}\{\cos^2(\beta h-\alpha)I_6+\cos^2\alpha I_6 -h^2\sin^2(\beta h-\alpha)I_1+2ih\cos\alpha\sin(\beta h-\alpha)I_3 -2\cos(\beta h-\alpha)\cos\alpha I_4-2ih\sin(\beta h-\alpha) \times\cos(\beta h-\alpha)I_2\}, \quad (15)$$

where $I_1, I_2, I_3\cdots I_6$ are integrals which can be expressed in terms of the exponential integral

$$Ei(-um)=-\int_u^\infty \frac{e^{-mx}}{x}dx$$

as follows:

$$I_1=\int_{\rho=a}^\infty \frac{e^{-2i\beta r}}{r^2\rho}d\rho=\frac{1}{h^2}E_i(-2i\beta r_0) -\frac{1}{2h^2}(e^{-2i\beta h}Ei[-2i\beta(r_0-h)] +e^{2i\beta h}Ei[-2i\beta(r_0+h)])$$

$$I_2=\int_{\rho=a}^\infty \frac{e^{-2i\beta r}}{r\rho}d\rho=\frac{1}{2h}\{Ei[-2i\beta(r_0+h)]e^{2i\beta h} -Ei[-2i\beta(r_0-h)]e^{-2i\beta h}\}$$

$$I_3=\int_{\rho=a}^\infty \frac{e^{-i\beta(\rho+r)}}{r\rho}d\rho=\frac{1}{h}(e^{i\beta h}Ei[-i\beta(g+1)h] -e^{-i\beta h}Ei[-i\beta(g-1)h])$$

$$I_4=\int_{\rho=a}^\infty \frac{e^{-i\beta(\rho+r)}}{\rho}d\rho=Ei(-i\beta hg) -Ei[-i\beta(g+1)h]e^{i\beta h}-Ei[-i\beta(g-1)h]e^{-i\beta h}$$

$$I_5=\int_{\rho=a}^\infty \frac{e^{-2i\beta r}}{\rho}d\rho=-\tfrac{1}{2}\{Ei[-2i\beta(r_0+h)]e^{2i\beta h} +Ei[-2i\beta(r_0-h)]e^{-2i\beta h}\}$$

$$I_6=\int_{\rho=a}^\infty \frac{e^{-2i\beta\rho}}{\rho}d\rho=-Ei(-2i\beta a),$$

where

$$r_0=(a^2+h^2)^{1/2} \quad\text{and}\quad g=[a+(a^2+h^2)^{1/2}]\frac{1}{h}.$$

The exponential integrals occurring in the above expressions are all of imaginary argument so therefore they can be expressed in terms of the sine and cosine integrals, $Si(x)$ and $Ci(x)$, respectively, as follows

$$Ei(-ix)=Ci(x)+i\left[\frac{\pi}{2}-Si(x)\right].$$

When the antenna is not terminated such that $h'=0$ or $\alpha=\beta h$ the expression for ΔZ simplifies to

$$\Delta Z=\frac{\eta}{4\pi\sin^2\beta h}\Big[Ei[-2i\beta(r_0+h)]e^{2i\beta h} +Ei[-2i\beta(r_0-h)]e^{-2i\beta h}+2\cos^2\beta h\,Ei(-2i\beta a) +4\cos\beta h\{Ei(-i\beta hg)-Ei[-i\beta h(g+1)]e^{i\beta h} -Ei[-i\beta h(g-1)]e^{-i\beta h}\}\Big]. \quad (16)$$

It has been found that the results for ΔZ can be put in a very convenient form by writing

$$\Delta Z=\Delta R+i\Delta X$$

and then plotting ΔR as a function of ΔX for values of the ratio a/h. The curves take the form of spirals which are somewhat similar to the "impedance spirals" of Storer[3] for the antenna and disk situated in free space.

As a specific example the impedance spiral for a quarter wave vertical monopole (i.e. $h=\lambda/4$) without top loading is shown in Fig. 1. Since the displacement currents in the ground can be neglected at low radiofrequencies (i.e. $\omega\ll\sigma/\epsilon$) the curves can be conveniently normalized by multiplying ΔR and ΔX by $(\sigma/f)^{1/2}$. It is interesting to note that ΔR approaches infinity as a/h approaches zero. This is not surprising since the ground loss would be infinite if the lower end of the antenna were in direct contact with a homogeneous semi-infinite conductor.

Further calculations from Eqs. (15) and (16) have been carried out in this laboratory by Mr. D. A. Trumpler. In addition the effect of the ground screen on the radiation field has also been studied. These results are the subject of subsequent papers.[6,7]

ACKNOWLEDGMENT

The authors would like to thank Mr. H. Page, Mr. G. D. Monteath, and Dr. F. R. Abbott for their interesting and helpful comments in connection with this work.

[6] W. J. Surtees, Defence Research Board Report on Grant No. 67, Department of Electrical Engineering, University of Toronto (October, 1952).

[7] J. R. Wait and W. A. Pope, Paper No. 42.2, Institute of Radio Engineers Convention Record (March, 1954).

1.2 Wait, J. R., and W. A. Pope, 1954, "The Characteristics of a Vertical Antenna with a Radial Conductor Ground System," *Applied Scientific Research*, Vol. 4, Sec. B, pp. 177-195, (The Hague)

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THE CHARACTERISTICS OF A VERTICAL ANTENNA WITH A RADIAL CONDUCTOR GROUND SYSTEM *)

by JAMES R. WAIT and W. A. POPE

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Summary

Employing an approximate method the input impedance of a ground based vertical radiator is calculated. The ground system consists of a number of radial conductors buried just below the surface of the soil. The integrals involved in the solution are evaluated, in part, by graphical methods. The final results are plotted in a convenient form to illustrate the dependence of the impedance on number and length of radial conductors for a specified frequency, antenna height, and ground conductivity. It is finally shown that under usual conditions the radiated fields are modified by only a few percent due to the presence of the ground system.

§ 1. *Introduction*. Antenna systems for low radio frequency are designed, usually, to work in conjunction with a radial wire ground system buried just below the surface of the earth. The purpose of this wire grid is to provide a low-loss return path for the antenna base current and consequently to improve the efficiency of the transmission.

The rules for ground system design are usually empirical and based on the results of experiments on existing installations. The first systematic study of this problem was carried out by Brown [1] [2]) and his associates who were mainly concerned with the operation of half-wave antennas for the broadcast band. Sometime later Abbott [3]) developed a procedure to select the optimum number of radial conductors, given the values of the electrical constants of the ground. An important related problem is the actual change of input impedance of the antenna for different sizes and types of ground systems. This analysis has been carried out by Leitner and

*) Work carried out under Project No. D 48-95-55-07. Paper presented at the U.S.A. National Convention of the Institute of Radio Engineers in New York, March 1954.

Spence[4]) and more recently by Storer[5]) for a vertical antenna situated over a perfectly conducting disc. However, they only considered the case where the surrounding medium was free space.

The purpose of this paper is to consider, in some detail, the characteristics of a vertical antenna of any length situated over a circular ground screen composed of N radial wires of equal spacing situated at the interface between the air and a semi-infinite homogeneous ground. An approximate method to calculate the input impedance will be employed similar to that described by Monteath[6]) who developed an extension to the compensation theorem of electric-circuit theory. The current distribution on the antenna is considered to be sinusoidal.

With reference to a cylindrical polar coordinate system (ϱ, φ, z) the antenna of height h is coincident with the positive z-axis as indicated in fig. 1. The ground screen is of radius a and lies in the plane

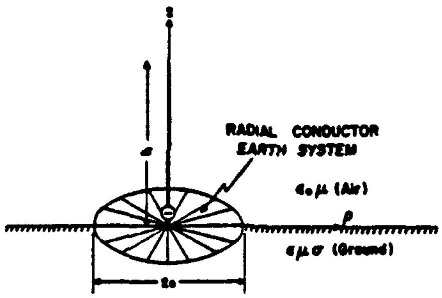

Fig. 1. The schematic representation of a vertical antenna situated over a radial conductor ground system.

$z = 0$ which is also the surface of the ground. The conductivity and dielectric constant of the ground are denoted by σ and ε respectively and the dielectric constant of the air by ε_0. The permeability of the whole space is taken as $\bar{\mu}$ which is taken to be that of free space. The intrinsic propagation constant γ and characteristic impedance η of the earth medium are defined by

$$\gamma = [i\bar{\mu}\omega(\sigma + i\omega\varepsilon)]^{\frac{1}{2}}$$

and

$$\eta = [i\bar{\mu}\omega/(\sigma + i\omega\varepsilon)]^{\frac{1}{2}},$$

where ω is the angular frequency. The propagation constant γ_0 and characteristic impedance η_0 of the air are then defined by

$$\gamma_0 = i(\varepsilon_0\bar{\mu})^{\frac{1}{2}}\,\omega = i2\pi/\lambda = i\beta$$

and

$$\eta_0 = (\bar{\mu}/\varepsilon_0)^{\frac{1}{2}} \simeq 377\ \Omega,$$

where λ is the wavelength in air.

The self-impedance at the terminals of the antenna is now denoted by Z_T and can be broken into two parts by setting, $Z_T = Z_0 + \Delta Z_T$ where Z_0 is the self-impedance of the same antenna if the ground plane were perfectly conducting and infinite in extent. On the other hand ΔZ_T is the difference between the self-impedance of the antenna over the imperfect and the perfect ground plane. It is called the self-impedance increment and can be written in terms of a real and imaginary part as follows:

$$\Delta Z_T = \Delta R_T + i\Delta X_T \tag{1}$$

where ΔR_T and ΔX_T represent the resistance and reactance increment. If the current at the terminals of the antenna is I_0, the power required to maintain this current is $I_0^2 R_T$. If the ground were perfectly conducting the input power would be $I_0^2 R_0$ where R_0 is the real part of Z_0. The additional power required to maintain the same current I_0 at the terminals is $I_0^2\,\Delta R_T$ *).

It is seen, therefore, that the quantity ΔR_T represents an important parameter of a radio frequency antenna.

§ 2. *The impedance calculation.* It is shown in appendix I that the impedance increment is given by

$$\Delta Z_T = -\frac{1}{I_0^2}\int_0^\infty H_\varphi^\infty(\varrho, o)\, E_\varrho(\varrho, o)\, 2\pi\varrho\, d\varrho, \tag{2}$$

where $H_\varphi^\infty(\varrho, o)$ is the magnetic field of the antenna tangential to a perfectly conducting ground plane and $E_\varrho(\varrho, o)$ is the tangential electric field on the imperfect ground. If the current on the antenna is $I(z)$, it follows that

$$H_\varphi^\infty = -\frac{1}{2\pi}\frac{\partial}{\partial\varrho}\int_0^h \frac{e^{-i\beta(z^2+\varrho^2)^{1/2}}}{(z^2+\varrho^2)^{1/2}}\, I(z)\, dz. \tag{3}$$

*) It is understood that I_0 is the root mean square value of the current if the power is to be expressed in watts.

For thin antennas it can be usually assumed that the current distribution is sinusoidal, that is

$$I(z) = I_0 \sin(\alpha - \beta z)/\sin \alpha \tag{4}$$

if the antenna is fed at the base (i.e. a monopole). The quantity α is determined by the height of the antenna and the top-loading, that is

$$\alpha = \beta(h + h').$$

The quantity h' specifies the amount of top loading and is usually obtained from experiment. For a thin antenna without top-loading (i.e. unterminated case) $\alpha = \beta h$. The integrals of the type indicated in (3) can be expressed in closed form for a sinusoidal current distribution. The result is given by

$$H_\varphi^\infty(\varrho, o) = -\frac{iI_0}{2\pi \sin \alpha}\left[\frac{e^{-i\beta r}}{\varrho}\cos(\beta h - \alpha) - \right.$$

$$\left. - \frac{e^{-i\beta\varrho}}{\varrho}\cos\alpha - \frac{ih}{\varrho}\frac{e^{-i\beta r}}{r}\sin(\alpha - \beta h)\right], \tag{5}$$

where $r = (\varrho^2 + h^2)^{1/2}$.

Since the electric field $E_\varrho(\varrho, o)$ is an unknown quantity, it is necessary to make several simplifications at this stage. Since $|\gamma| \gg \beta$, an approximate boundary condition (see appendix II) is employed, expressed by

$$E_\varrho(\varrho, o) \simeq -\eta_c H_\varphi(\varrho, o), \tag{6}$$

where $H_\varphi(\varrho, o)$ is the tangential magnetic field for the imperfect ground system and η_c is the surface impedance of the air-ground interface. If ϱ is greater than a, the radius of the ground screen, η_c can be replaced by η. If ϱ is less than a, η_c is the intrinsic impedance η_s of the ground screen which is in parallel with the impedance η of the ground. In a previous investigation [7]) [8]) of this problem it was assumed that η_s was zero so the ground system was equivalent to a perfectly conducting disc of radius a. The limits of the integration in (2) are then from $\varrho = a$ to $\varrho = \infty$. A more general case is when η_s is comparable in magnitude to η, in which it follows that

$$\eta_c = \frac{\eta\eta_s}{\eta + \eta_s} \quad \text{for} \quad 0 < \varrho < a, \tag{7}$$

where

$$\eta_e = \frac{i\eta_0 d}{\lambda} \ln \frac{d}{2\pi c}$$

and where d is the spacing between the radial conductors and c is the radius of the wire. The expression for η_e has been derived [9]) for a wire grid in free space where it was necessary to assume that $|\gamma_0 d| \ll 1$. Since the grid is lying on the ground plane, this restriction must be replaced by $|\gamma_e d| \ll 1$ where γ_e is the effective propagation constant for propagation along a thin wire in the interface and is given by [10])

$$\gamma_e = \left(\frac{\gamma_0^2 + \gamma^2}{2}\right)^{1/2}.$$

If there are N radial conductors, it can be seen that d can be replaced by $2\pi\varrho/N$ since N is usually of the order of 100. It is assumed also that $H_\varphi(\varrho, 0)$ is not very different from $H_\varphi^\infty(\varrho, 0)$ in the region of the ground plane where the losses are significant. This approximation has also been discussed previously [7]) [8]) and it certainly appears to be valid if $|\gamma| \gg \beta$.

The impedance increment ΔZ_T is then written in the following form:

$$\Delta Z_T = \Delta Z + \Delta Z_s, \tag{8}$$

where

$$I_0^2 \Delta Z \simeq \eta \int_a^\infty [H_\varphi^\infty(\varrho, 0)]^2 \, 2\pi\varrho \, d\varrho \tag{9}$$

and

$$I_0^2 \Delta Z_s \simeq \int_0^a \frac{\eta_e \eta}{\eta + \eta_e} [H_\varphi^\infty(\varrho, 0)]^2 \, 2\pi\varrho \, d\varrho. \tag{10}$$

The first expression ΔZ corresponds to the self-impedance of the monopole over a perfectly conducting discoid, whereas the second expression ΔZ_s accounts for the finite surface impedance of the radial conductor system.

It is instructive to consider ΔZ_s first in some detail, since the integrations can be carried out and the result expressed in terms of the exponential integral defined by

$$\mathrm{Ei}(-i\beta a) = -\int_a^\infty \frac{e^{-i\beta\varrho}}{\varrho} d\varrho. \tag{11}$$

The procedure was outlined previously [7]) [8]) where an expression for ΔZ was given for any value of the height h and top loading h'. The special case where the antenna is unterminated ($h' = 0$) is given by

$$\Delta Z = \frac{\eta}{4\pi \sin^2 \beta h} \{\mathrm{Ei}\,[-2i\beta(r_0 + h)]\, e^{i2\beta h} + \\ + \mathrm{Ei}\,[-2i\beta(r_0 - h)]\, e^{-2i\beta h} + 2\cos^2 \beta h\, \mathrm{Ei}\,(-2i\beta a) \\ + 4\cos\beta h\,[\mathrm{Ei}\,(-i\beta h g) - \mathrm{Ei}\,[-i\beta h(g+1)]\, e^{i\beta h} \\ - \mathrm{Ei}\,[-i\beta h(g-1)]\, e^{-i\beta h}]\}, \qquad (12)$$

where

$$g = \frac{1}{h}\,[a + (a^2 + h^2)^{1/2}] \text{ and } r_0 = (a^2 + h^2)^{1/2}.$$

This equation may be put in a suitable form for computation by employing the relation

$$\mathrm{Ei}\,(-i\beta a) = \mathrm{Ci}\,(\beta a) + i\left[\frac{\pi}{2} - \mathrm{Si}\,(\beta a)\right], \qquad (13)$$

where Ci (βa) and Si (βa) are the cosine and sine integrals respectively as defined and tabulated by Jahnke and Emde [11]). The results of the calculations are presented in a most general form by plotting $4\pi\Delta Z/\eta$ as a function of a/h for various values of h/λ as shown in figs. 2 and 3. It can be seen that the magnitude of ΔZ increases without limit as a approaches zero. This formulation is not actually valid in this limiting case since one terminal of the generator would then be connected directly to the earth medium (or to a disc of vanishing radius) and would lead to an infinitely resistive path for the antenna base current.

A slightly more convenient way to illustrate these calculations is to plot $(\sigma/f)^{1/2}\,\Delta R$, where ΔR is the real part of ΔZ, as a function of a/h as shown in fig. 4 where f is the frequency in Hz. This is only permissible if displacement currents in the ground are negligible (note if $\varepsilon\omega \ll \sigma$, $\eta \simeq (i\mu\omega/\sigma)^{1/2}$. It is interesting to note that ΔR actually assumes negative values under certain conditions. In this case, the input power to obtain a given current I_0 at the antenna terminals is actually less for an imperfect ground than for a perfect ground. This fact can be reconciled by showing that the radiated power is actually reduced if $\Delta R < 0$. A physical explanation for the oscillating nature of the curve is that a wave is reflected from the discontinuity in the surface impedance at $\varrho = a$. As the radius a

increases, the phase lag of the reflected wave will continually increase. This viewpoint is substantiated when it is noted that the period of the oscillations is nearly equal to twice the diameter of the ground screen.

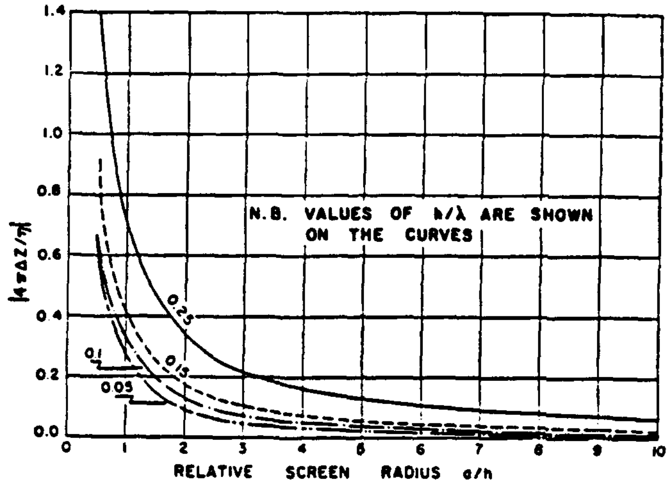

Fig. 2. The incremental self-impedance for a vertical antenna situated over an idealized perfectly conducting discoid lying on a homogeneous ground.

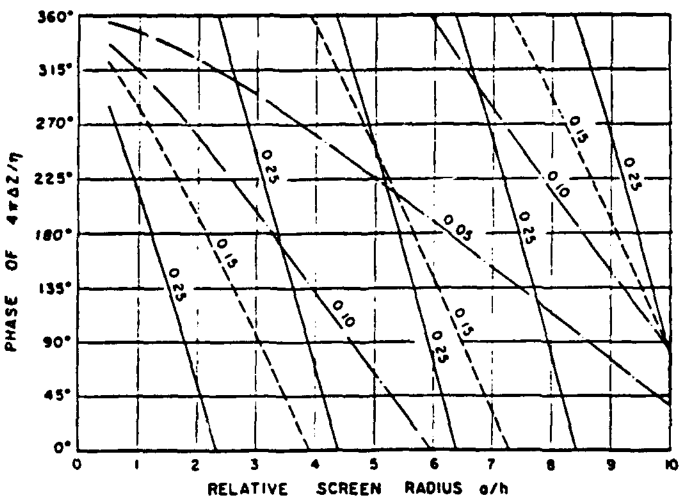

Fig. 3. The phase of the incremental self-impedance corresponding to the conditions of fig. 2.

When the ground screen of finite surface impedance is considered the integrations indicated by (10) must be evaluated. This appears to be a formidable task for the case when the antenna is of arbitrary length. However, if the antenna is a quarter-wave monopole, without top-loading, the integrations can be carried out fairly readily by graphical means. In this case

$$H_\varphi^\infty(\varrho, 0) = -\frac{iI_0}{2\pi\varrho} e^{-i\beta(\varrho^2+\lambda^2/4^2)^{1/2}}, \tag{14}$$

which is a special case of (5) with $h' = 0$ and $h = \lambda/4$, and hence the integral in (10) can then be expressed in the following form:

$$\Delta R_s = \int_0^A \frac{pq \cos\left[\pi(1-4R) + \frac{3\pi}{4} - \tan^{-1}\frac{p+q}{p}\right] \mathrm{d}P}{\sqrt{2\pi}\,[p^2 + (p+q)^2]^{1/2} P} \tag{15}$$

and, similarly, ΔX_s with cos (—) replaced by sin (—). The real dimensionless quantities p, q, A and R are defined by

$$p = 120\,\pi\delta/\sqrt{2} \text{ with } \delta = (\varepsilon_0\omega/\sigma)^{1/2},$$

$$q = \frac{240\,\pi^2 P}{N} \ln \frac{P}{NC} \text{ with } C = c/\lambda,$$

$$A = a/\lambda \text{ and } R = \sqrt{P^2 + (\tfrac{1}{4})^2}.$$

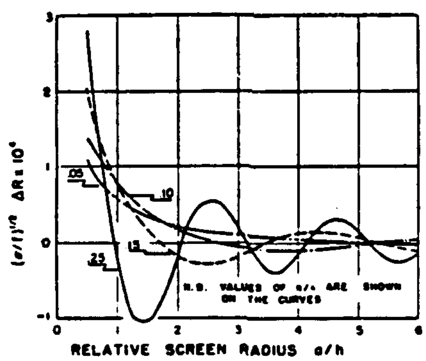

Fig. 4. The real part of the incremental self-impedance corresponding to the conditions of fig. 2.

It has been assumed in writing (15) in this form that displacement currents in the ground are negligible, that is $(\varepsilon\omega/\sigma) \ll 1$. Using the results of the numerical integration for ΔR_s, values of ΔR_T ($= \Delta R + + \Delta R_s$) are plotted as a function of A in figs. 5, 6 and 7 for various values of δ and N with $C = 0.1 \times 10^{-5}$, and in fig. 8 for various values of C for $N = 100$ and $\delta = 0.1$. The value of δ can be readily obtained from fig. 9 when the ground conductivity σ and the frequency in kHz are specified.

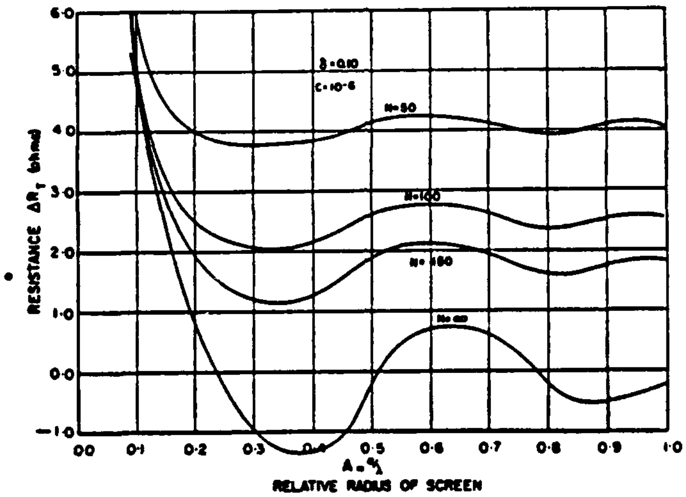

Fig. 5. The incremental self-resistance of a vertical quarter-wave monopole on a radial conductor ground system for a wire radius equal to 10^{-6} of a free-space wavelength.

It is immediately evident that the oscillations in the curves for ΔR_T have been damped if N is finite. This can be expected since there is a smaller change of the surface impedance at $\rho = a$ if the ground conductivity is reasonably high ($\delta < 0.1$). The limiting case where $N = \infty$ corresponds to the perfectly conducting disc discussed previously. It is quite apparent from these curves that a ground screen radius greater than about ½ of a wavelength is wasteful. On the other hand it would be quite feasible to choose a large number of radials to reduce the resistance increment to a low value. Although, in practice, it is usual to employ No. 8 wire ($C = 0.5 \times 10^{-5}$ at 1 MHz); some improvement could be obtained by using larger wire diameters. From a theoretical standpoint, however, it would appear that, for a given total weight of wire, it is preferable to use a conductor of smaller diameter, say No. 22 wire ($C = 0.1 \times 10^{-5}$ at 1 MHz) and to employ 150 or more radial conductors.

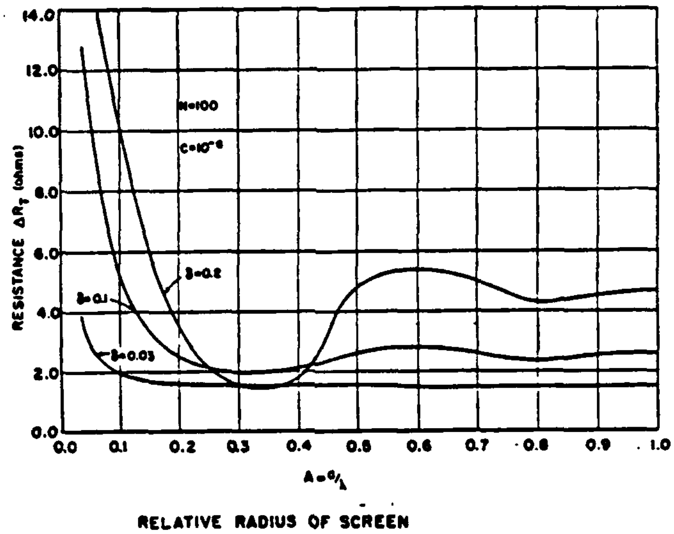

Fig. 6. The incremental self-resistance of a vertical quarter-wave monopole on a radial conductor ground system for a wire radius equal to 10^{-6} of a free-space wavelength.

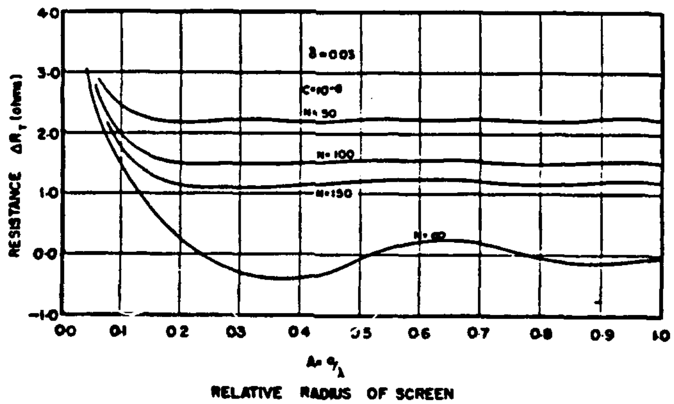

Fig. 7. The incremental self-resistance of a vertical quarter-wave monopole on a radial conductor ground system for a wire radius equal to 10^{-6} of a free-space wavelength.

§ 3. *The earth currents.* It is interesting to examine how the antenna base current is shared by the radial conducters and the ground itself. If the current flowing in the ground is denoted by I_e and the total current in the radial wires by I_w, then the ratio I_e/I_w is equal

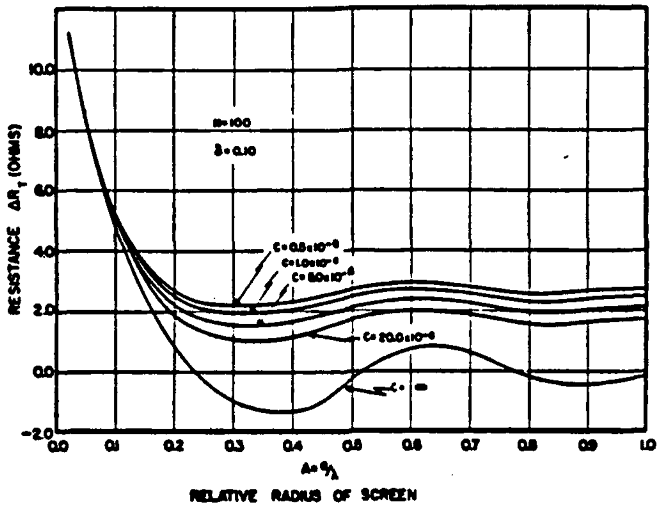

Fig. 8. The incremental self-resistance as a function of the wire radius with a fixed number of radial conductors and ground conductivity.

to the ratio of the surface impedance of the grating of the wires composing the earth system to the surface impedance of the ground. Therefore it follows that

$$\frac{I_e}{I_w} = \frac{\eta_e}{\eta} \simeq \frac{iq}{(1 + i)\, p}, \tag{16}$$

where p and q have been defined previously. Since the total current is given by $I_t = I_e + I_w$, it follows that

$$\left|\frac{I_w}{I_t}\right| = \left|\frac{1}{1 + \eta_e/\eta}\right| = \frac{\sqrt{2}\, p}{[p^2 + (p + q)^2]^{1/2}}. \tag{17}$$

It has been assumed here that displacement currents in the ground are negligible (i.e. $\varepsilon\omega\, \sigma \ll 1$).

Equation (17) is not a function of the height of the antenna and therefore it would apply also to top-loaded antennas as long as the circular symmetry is essentially retained. Employing this equation, curves are plotted in figs. 10 and 11 to show the dependence of the current in the radial wires on the various parameters. The abcissae are the lengths of the radial wires in wavelengths measured from the base of the antenna. It is noted that if the radial wires are increased beyond a certain length, nearly all the current flows in the ground. When the ratio $|I_w/I_t|$ is equal to $\frac{1}{2}$, the current in the ground is equal to the total current carried by the radial wires.

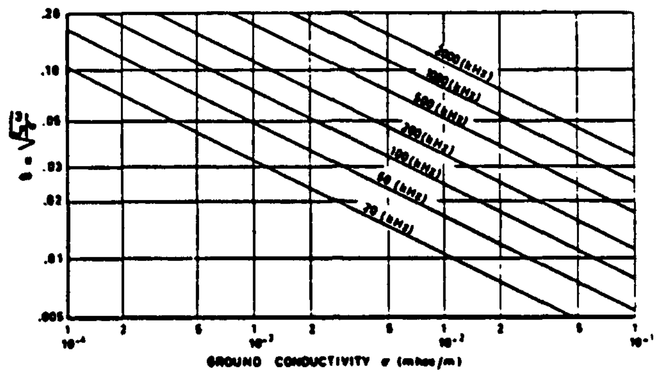

Fig. 9. Ground conductivity.

§ 4. *The radiated power.* While the main subject of this paper has been to evaluate the input impedance of the antenna, it is also of some interest to know if the presence of the ground screen appreciably changes the radiated power. A simple and approximate analysis is now carried out which indicates that this change is small.

The power dissipated in an elemental area of the ground is equal to the real part of $\eta|H_\varphi^2|$. It is then evident that the change of power lost ΔP_L in the ground due to the presence of the ground screen is given by

$$\Delta P_L = Re \int_0^a \Delta\eta |H_\varphi|^2 \, 2\pi\varrho \, d\varrho, \tag{18}$$

where $\Delta\eta$ is the difference between the surface impedance η_c of the radial conductor earth system and the surface impedance η of the ground. However, the change of input power at the antenna terminals is given by

$$\Delta P = Re \int_0^a \Delta\eta H_\varphi^2 \, 2\pi\varrho \, d\varrho. \tag{19}$$

Now, since there is conservation of power, the change of the total radiated power ΔP_r beyond the edge of the earth system is equal to $\Delta P - \Delta P_L$ or

$$\Delta P_r = Re \int_0^a \Delta\eta \, [H_\varphi^2 - |H_\varphi^2|] \, 2\pi\varrho \, d\varrho. \tag{20}$$

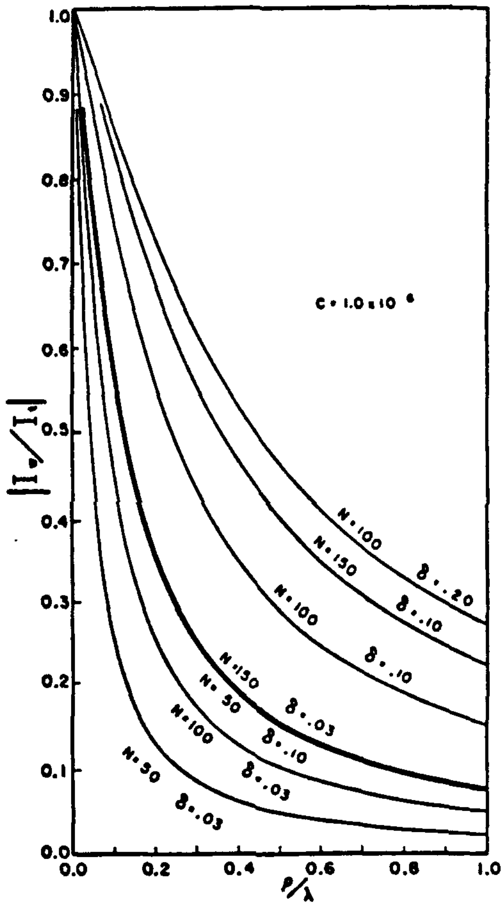

Fig. 10. The ratio of the current carried by the radial conductors to the total earth current as a function of the distance from the base of the antenna.

For a good ground screen, $\Delta\eta \simeq -\eta$ and, if the antenna is a quarter wave monopole ($h = \lambda/4$), the integration can be carried out in closed form to yield

$$\frac{\Delta P_r}{I_0^2} = \text{Real part of } \frac{\eta}{4\pi}\Bigg\{\Bigg[2\ln\frac{4a}{\lambda} + 0.5773 + \ln\frac{\pi}{2} - \text{Ci}\,(\sqrt{\pi^2 + (2\beta a)^2} - \pi) + \text{Ci}\,(2\pi) - \text{Ci}\,(\sqrt{\pi^2 + (2\beta a)^2} + \pi)\Bigg] + i\Bigg[\text{Si}\,(\sqrt{\pi^2 + (2\beta a)^2} - \pi) - \text{Si}\,(2\pi) + \text{Si}\,(\sqrt{\pi^2 + (2\beta a)^2} + \pi)\Bigg]\Bigg\}. \tag{21}$$

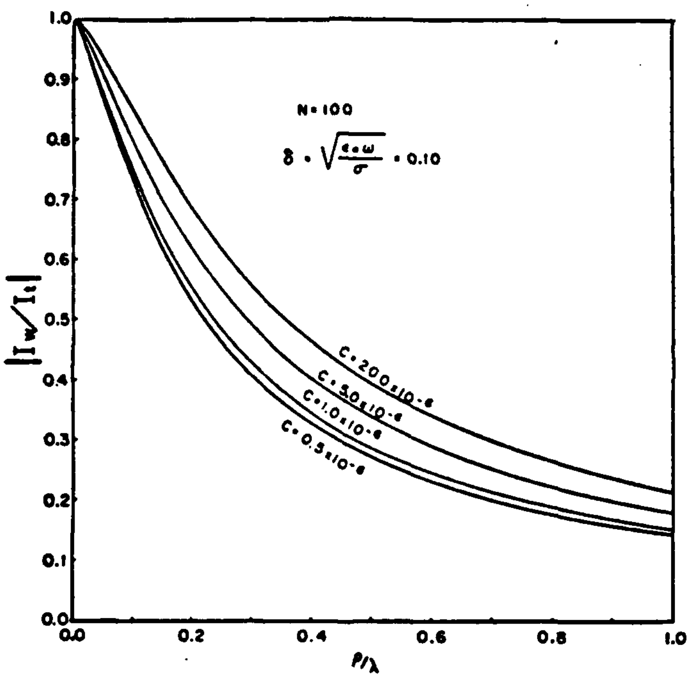

Fig. 11. The ratio of the current carried by the radial conductors to the total earth current as a function of the distance from the base of the antenna.

If the ground screen radius a is small compared with a wavelength, the change of power radiated is given approximately by

$$\frac{\Delta P_r}{I_0^2} \simeq \text{Re}\; i\eta\beta^2 a^2\, 2 \simeq -\eta\,\frac{1}{2\sqrt{2}}(\beta a)^2. \tag{22}$$

This is usually a small quantity with the total radiated power so it is of minor significance at low radio frequencies.

§ 5. ***The radiation pattern.*** The effect of the ground screen on the radiation pattern is also of some interest. For convenience, it is assumed here that the ground screen is equivalent to a thin, perfectly conducting, circular disc laid on a homogeneous ground. The magnetic field $H_\varphi(\varrho, z)$ in the air can be written as the sum of the field $H_\varphi^\infty(\varrho, z)$ for an infinite screen and a secondary field $H_\varphi^s(\varrho, z)$. In appendix I it is shown that

$$H_\varphi^s = \frac{\gamma_0}{\eta_0} \int_{\varrho'=a}^{\infty} \int_{\lambda=0}^{\infty} J_1(\lambda\varrho)\, J_1(\lambda\varrho')\, e^{-\mu_0 z} \mu_0^{-1} E_\varrho(\varrho', 0)\, \varrho' \, d\varrho' \, \lambda \, d\lambda, \tag{23}$$

where

$$\mu_0 = (\lambda^2 + \gamma_0^2)^{1/2}.$$

The change of the magnetic field ΔH_φ due to the presence of the screen is now given by

$$\Delta H_\varphi = H_\varphi^s - |H_\varphi^s|_{a=0} =$$

$$= -\frac{\gamma_0}{\eta_0} \int_{\varrho'=0}^{a} \int_{\lambda=0}^{\infty} E_\varrho(\varrho', 0)\, \varrho' \, d\varrho' \, J_1(\lambda\varrho')\, J_1(\lambda\varrho)\, e^{-\mu_0 z} \mu_0^{-1} \lambda \, d\lambda. \tag{24}$$

The integration with respect to λ can be carried out by the saddle point method of integration since $\beta\varrho \gg 1$ in the radiation zone. The result is

$$\Delta H_\varphi = \frac{\beta e^{-i\beta \bar{R}}}{\eta_0 \bar{R}} \int_0^a E_\varrho(\varrho', 0)\, J_1(\beta\varrho' \cos\theta)\, \varrho' \, d\varrho' \tag{25}$$

where

$$\bar{R} = \sqrt{\varrho^2 + z^2},\ \beta = 2\pi/\lambda = -i\gamma_0, \text{ and } \theta = \tan^{-1} z/\varrho.$$

The approximate boundary condition, $E_\varrho(\varrho', 0) \simeq -\eta\, H_\varphi(\varrho', 0)$, can now be employed so that

$$\frac{\Delta H_\varphi}{H_\varphi} = \frac{-\beta\eta}{\eta_0} \frac{e^{-i\beta\bar{R}}}{\bar{R}} \int_0^a \frac{H_\varphi(\varrho', 0)}{H_\varphi(\varrho, z)} J_1(\beta\varrho' \cos\theta)\, \varrho' \, d\varrho'. \tag{26}$$

An approximate expression for ΔH_φ is now obtained by replacing

$H_\varphi(\varrho, z)$ on the right hand side of (26) by $H_\varphi^\infty(\varrho, z)$. If the antenna is a quarter wave monopole,

$$H_\varphi^\infty(\varrho', 0) = \frac{-iI}{2\pi\varrho'} e^{-i\beta\sqrt{\varrho'^2+(\lambda/4)^2}}, \tag{27}$$

and for $\beta\varrho \gg 1$

$$H_\varphi^\infty(\varrho, z) \simeq \frac{-iI}{2\pi\varrho} e^{-i\beta R} \cos\left(\frac{\pi}{2}\sin\theta\right), \tag{28}$$

so that

$$\frac{\Delta H_\varphi}{H_\varphi} = -\frac{\cos\theta}{\cos\left(\frac{\pi}{2}\sin\theta\right)} \frac{\eta}{\eta_0} \int_0^{2\pi a/\lambda} e^{-i\sqrt{p^2+\frac{\pi^2}{4}}} J_1(p\cos\theta)\, dp. \tag{29}$$

The right hand of this equation is of the order of $|\eta/\eta_0|$ which is small compared with unity. For small screens where $a \ll \lambda$ the relation simplifies to

$$\frac{\Delta H_\varphi}{H_\varphi} \simeq -\left|\frac{\eta}{\eta_0}\right| e^{-i\frac{\pi}{4}} \left(\frac{\pi a \cos\theta}{\lambda}\right)^2 \frac{1}{\cos\left(\frac{\pi}{2}\sin\theta\right)}, \tag{30}$$

which is of second order magnitude.

§ 6. *Conclusion.* The results of this analysis, while not exhaustive, are sufficiently developed to be useful in the design of vertical antennae with radial conductor ground systems. The work has shown that the input impedance of the type of antenna discussed is dependent mainly on the number and the length of the radial ground conductors and on the conductivity of the ground in which the wires are buried. The dependence of antenna impedance on ground wire size is shown to be very slight. It may be concluded, from this discussion, that a sensible design criterion for an optimum ground system is attained by a suitable choice of number and length of ground wire radials so that they will always carry an appreciable fraction of the total earth current.

APPENDIX I

Formulation of the input impedance. An expression is here formulated for the input impedance at the terminals of an antenna situated over a circular screen. The total flux F of the vector $\mathbf{E} \times \mathbf{H}$

over a surface surrounding the antenna is given by

$$F = \int_S \mathbf{E} \times \mathbf{H}.\mathbf{n}\, ds \tag{31}$$

where $\mathbf{n}$ is the unit outward vector normal to S. As is customary in other problems of this type, S is chosen to be a slender cylindrical surface of vanishing radius ϱ concentric with the antenna so that

$$F = -\lim_{\varrho\to 0} 2\pi\varrho \int_0^h E_z H_\varphi \, dz. \tag{32}$$

It then follows that

$$Z = \lim_{\varrho\to 0}\left[-\frac{1}{I_0^2}\int_0^h E_z I(z)\, dz\right]. \tag{33}$$

It is now convenient to let $E_z = E_z^\infty + E_z'$, where E_z^∞ is the corresponding value of the electric field for a perfectly conducting ground plane and E_z' is the change of the field to account for the finite conductivity in the soil and the ground system. The impedance increment ΔZ_T is then given by

$$\Delta Z_T = \left[-\frac{1}{I_0^2}\int_0^h E_z' I(z)\, dz\right]_{\lim\,\varrho\to 0} =$$

$$= \left[-\frac{\eta_0}{\gamma_0 I_0^2}\int_0^h \left(\frac{1}{\varrho}\frac{\partial}{\partial\varrho}\varrho H_\varphi'\right) I(z)\, dz\right]_{\lim\,\varrho\to 0}. \tag{34}$$

Since H_φ' is a solution of the wave equation,

$$H_\varphi'(\varrho, z) = \int_0^\infty J_1(\lambda\varrho)\, e^{-\mu_0 z} f(\lambda)\, \lambda d\lambda. \tag{35}$$

for $z > 0$, where $\mu_0 = (\lambda^2 + \gamma_0^2)^{1/2}$. From Maxwells equations it is seen that

$$E_\varrho(\varrho, 0) = \frac{\eta_0}{\gamma_0}\int_0^\infty J_1(\lambda\varrho) f(\lambda)\, \mu_0 \lambda\, d\lambda, \tag{36}$$

and by applying the Fourier-Bessel theorem it follows that

$$f(\lambda) = \frac{\gamma_0}{\eta_0\mu_0}\int_0^\infty J_1(\lambda\varrho')\, E_\varrho(\varrho', 0)\, \varrho'\, d\varrho'. \tag{37}$$

This equation for $f(\lambda)$ can then be substituted back into (35) to obtain an expression for $H_\varphi'(\varrho, z)$ in terms of $E_\varrho(\varrho, 0)$. It is also noted that $J_1(\lambda\varrho)$ can be replaced by $\lambda\varrho/2$ as ϱ tends to zero so that

$$\lim_{\varrho\to 0} H_\varphi^s(\varrho, z) = \frac{\gamma_0}{2\eta_0} \int_0^\infty\int_0^\infty \varrho J_1(\lambda\varrho')\, e^{-\mu_0 z} \mu_0^{-1} \lambda^2\, d\lambda\, E_\varrho(\varrho', 0)\, \varrho'\, d\varrho', \quad (38)$$

and introducing Sommerfeld's Integral

$$\int_0^\infty e^{-\mu_0 z} \mu_0^{-1} \lambda J_0(\lambda\varrho)\, d\lambda = (z^2 + \varrho^2)^{-1/2}\, e^{-\gamma_0(z^2+\varrho^2)^{1/2}}, \quad (39)$$

the integration in (38) with respect to λ can now be carried out to give

$$\lim_{\varrho\to 0} H_\varphi^s(\varrho, z) = \frac{-\gamma_0\varrho}{2\eta_0} \int_0^\infty \frac{\partial}{\partial\varrho'} \frac{e^{-\gamma_0(z^2+\varrho'^2)^{1/2}}}{(z^2 + \varrho'^2)^{1/2}}\, E_\varrho(\varrho', 0)\, \varrho'\, d\varrho'. \quad (40)$$

Inserting this expression into (34) leads directly to (2) for the input impedance increment.

Appendix II

The approximate boundary condition. The magnetic field in the ground outside the screen is a solution of the wave equation

$$(\Delta - \gamma^2)\, H_\varphi(\varrho, z) = 0$$

and therefore

$$H_\varphi(\varrho, z) = \int_0^\infty J_1(\lambda\varrho)\, e^{\mu z}\, p(\lambda)\, \lambda d\lambda \quad (41)$$

for $z < 0$ and where $\mu = (\lambda^2 + \gamma^2)^{1/2}$. The electric field is given by

$$E_\varrho = -\frac{\eta}{\gamma} \int_0^\infty \mu\, J_1(\lambda\varrho)\, e^{\mu z}\, p(\lambda)\, \lambda d\lambda. \quad (42)$$

The binomial expansion of μ is of the form

$$\mu = \gamma\left(1 + \frac{\lambda^2}{2\gamma^2} + \ldots.\right)$$

so that

$$E_\varrho \simeq -\frac{\eta}{\gamma}\left\{ \int_0^\infty \gamma J_1(\lambda\varrho)\, e^{\mu z}\, p(\lambda)\, \lambda\, d\lambda + \int_0^\infty \frac{1}{2\gamma} J_1(\lambda\varrho)\, e^{\mu z}\, p(\lambda)\, \lambda^3\, d\lambda + \ldots\right\}. \quad (43)$$

Now the differential equation for J_1 is given by

$$\left(\frac{\partial}{\partial\varrho} \frac{1}{\varrho} \frac{\partial}{\partial\varrho} \varrho - \lambda^2\right) J_1(\lambda\varrho) = 0, \quad (44)$$

so it readily follows that

$$E_\varrho = -\eta H_\varphi - \frac{\eta}{2\gamma^2}\frac{\partial}{\partial\varrho}\frac{1}{\varrho}\frac{\partial}{\partial\varrho}\varrho H_\varphi \pm \text{terms in } \gamma^{-4}. \qquad (45)$$

The second and remaining terms are negligible if the propagation constant of the ground is sufficiently large and if H_φ is not varying too rapidly with ϱ. That is, ϱH_φ should not change appreciably in a distance equal to $|\gamma^{-1}|$ in the radial direction.

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Note added in proof. Further calculations for short top-loaded antennas have been carried out. The results are available in Radio Physics Lab. Report No, 19-0-7 April, 1954 by J. R. Wait and W. A. Pope. Good agreement has been obtained with experiment for a 250 foot mast at 100 kHz.

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INPUT RESISTANCE OF L.F. UNIPOLE AERIALS

With Radial Wire Earth Systems

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SUMMARY.—The input resistance of a low-frequency unipole aerial is calculated. The earth system consists of a number of radial conductors buried just below the surface of the soil. The integrals involved in the solution are evaluated, in part, by graphical methods. The final results are plotted in a convenient form to illustrate the dependence of the input resistance on number and length of radial conductors for a specified frequency and earth conductivity. The curves should be useful in the design of earth systems for low-frequency transmitting aerials. It is pointed out that increasing the radius of the earth system beyond a certain limit gives only a slight improvement in radiation efficiency.

LIST OF SYMBOLS

(m.k.s. units are employed throughout)

- (ρ, ϕ, z) = cylindrical polar co-ordinates
- h = height of an ideally top-loaded aerial or the equivalent height of an unloaded aerial
- a = length of radial wires
- σ = conductivity of ground
- ϵ = permittivity of ground
- ϵ_0 = permittivity of air ($= 8{\cdot}85 \times 10^{-12}$)
- μ_0 = permeability of free space ($= 4\pi \times 10^{-7}$)
- γ = intrinsic propagation constant of ground
- η = characteristic impedance of ground
- γ_0 = propagation constant of air
- η_0 = characteristic impedance of air ($= 120\pi$)
- β = wave number in air ($= -j\gamma_0$)
- ω = angular frequency
- λ = wavelength in air
- Z_t = self-impedance at terminals of aerial
- Z_0 = self-impedance of the aerial for a perfectly-conducting earth plane
- ΔZ_t = self-impedance increment ($= Z_t - Z_0$)
- ΔR_t = real part of ΔZ_t
- ΔX_t = imaginary part of ΔZ_t
- I_0 = current at terminals of aerial ($= \sqrt{2} \times$ r.m.s. current)
- $H_\phi^\infty(\rho, 0)$ = the tangential magnetic field of the aerial on a perfectly-conducting earth plane of infinite extent
- $E_\rho(\rho, 0)$ = the tangential electric field of the aerial on the imperfect ground
- $I(z)$ = current along the aerial
- $H_\phi(\rho, 0)$ = the tangential magnetic field of the aerial on the imperfect ground
- η_e = the surface impedance of the air-ground interface
- η_s = the surface impedance of the radial wire grid
- d = the spacing between the radial wires
- c = radius of the wires of the grid
- γ_e = the effective propagation constant of a wire in the air-ground interface
- N = number of radial wires in the earth system
- ΔZ = self-impedance increment for an ideal circular ground screen of radius a
- ΔZ_e = correction to ΔZ to account for the losses within the ground screen
- b = limit of integration for equations (12) and (13)
- $Ei(-x)$ = $-\int_x^\infty (e^{-t}/t)\,dt$
- $O(y)$ = a quantity whose order of magnitude is equal to y
- Γ and θ = magnitude and phase of the complex number $\Gamma e^{j\theta}$ defined in equation [15(b)]
- P = variable of integration ($= \rho/\lambda$)
- H_1 = height of aerial in wavelengths ($= h/\lambda$)
- F and ψ = magnitude and phase of the complex number $Fe^{j\psi}$ defined in equation (15a)
- p, q, δ, A, B, C_1 = dimensionless quantities defined in the text following equation [15(a)]
- R_0 = the input resistance of the aerial for a perfectly-conducting earth plane
- C = Euler's number ($= 0{\cdot}5772\ldots$)
- h_0 = actual height of the unloaded aerial
- $Si(x)$ = $\int_0^x \frac{\sin t}{t}\,dt$
- $Ci(x)$ = $-\int_x^\infty \frac{\cos t}{t}\,dt$
- δ = $\left(\frac{\epsilon_0\omega}{\sigma}\right)^{\frac{1}{2}}$ = ground conductivity parameter

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Introduction

AERIAL systems for low-frequencies are designed, usually, to work in conjunction with a radial-wire earth system buried just below the surface of the earth. The purpose of this wire grid is to provide a low-loss path for the aerial base current and consequently to improve the radiation efficiency.

The rules for earth-system design are usually empirical and based on the results of experiments on existing installations. The first systematic study of this problem was carried out by Brown[1,2] and his associates who were mainly concerned with the operation of half-wave aerials for the broadcast band. Sometime later Abbott[3] developed a procedure for selecting the optimum number of radial conductors, given the values of the electrical constants of the ground. An important related problem is the actual change of input impedance of the aerial with different sizes and types of earth systems. This analysis has been carried out by Leitner and Spence[4] and more recently by Storer[5], for a vertical aerial situated over a perfectly-conducting disc. However, they only considered the case where the surrounding medium was free space.

In a previous paper[6] the electrical characteristics of a vertical aerial with a radial conductor system was studied. Employing an approximate method the input impedance was calculated. To illustrate the nature of the problem only a quarter-wave unipole was considered in detail since it was the case most easily computed. Curves were plotted showing the dependence of the input base resistance on number and length of radial conductors for a specified frequency and ground conductivity. It is the purpose of this paper to extend the solution and calculations for shorter aerials with top-loading.

With reference to a cylindrical polar coordinate system (ρ, ϕ, z) the aerial of height h is coincident with the positive z axis as indicated in Fig. 1. The earth screen is of radius a and lies in the plane $z = 0$ which is also the surface of the ground. The conductivity and permittivity of the ground are denoted by σ and ϵ respectively, and the permittivity of the air is denoted by ϵ_0. The permeability of the whole space is taken as μ_0 which is taken to be that of free space. The intrinsic propagation constant γ and characteristic impedance η of the earth medium are defined by

$$\gamma = [j\mu_0\omega(\sigma + j\omega\epsilon)]^{1/2}$$

and $\eta = [j\mu_0\omega/(\sigma + j\omega\epsilon)]^{1/2}$

where ω is the angular frequency. The propagation constant γ_0 and the characteristic impedance η_0 of the air are then defined by

$$\gamma_0 = j(\epsilon_0\mu_0)^{1/2}\omega = j2\pi/\lambda = j\beta$$

and $\eta_0 = (\mu_0/\epsilon_0)^{1/2} \approx 377$ ohms

where λ is the wavelength in air.

The self-impedance at the terminals of the aerial is now denoted by Z_t which can be broken into two parts by setting, $Z_t = Z_0 + \Delta Z_t$ where Z_0 is the self-impedance of the same aerial if the earth plane were perfectly conducting and infinite in extent. Thus ΔZ_t is the difference between the self-impedance of the aerial over the imperfect and the perfect earth plane. It is called the self-impedance increment and can be written in terms of a real and imaginary part as follows

$$\Delta Z_t = \Delta R_t + j\Delta X_t \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where ΔR_t and ΔX_t represent the resistance and reactance increments. If the current amplitude at the terminals of the aerial is I_0, the power required to maintain this current is $I_0^2R_t/2$ watts. If the ground were perfectly conducting, the input power would be $I_0^2R_0/2$ where R_0 is the real part of Z_0. The additional power required to maintain the same r.m.s. current $I_0/\sqrt{2}$ at the terminals is $I_0^2\Delta R_t/2$.

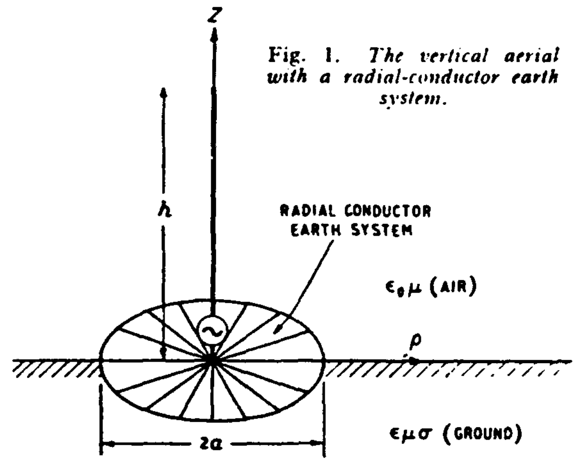


Fig. 1. *The vertical aerial with a radial-conductor earth system.*

General Theory

It was shown in the previous paper[6] that the self-impedance increment ΔZ_t could be written in the following form:

$$\Delta Z_t = -\frac{1}{I_0^2}\int_0^\infty H_\phi^z(\rho, 0)\, E_\rho(\rho, 0)\, 2\pi\rho\, d\rho \quad \ldots \quad (2)$$

where $H_\phi^z(\rho, 0)$ is the magnetic field of the aerial tangential to a perfectly conducting earth plane of infinite extent and $E_\rho(\rho, 0)$ is the tangential electric field on the imperfect earth.* This formula also follows immediately from the work of Monteath[7]. If the current on the aerial is $I(z)$ amps it follows that

$$H_\phi^z(\rho, 0) = \frac{1}{2\pi}\frac{\delta}{\delta\rho}\int_0^h \frac{\exp.[-j\beta(z^2+\rho^2)^{1/2}]}{(z^2+\rho^2)^{1/2}} I(z)\, dz \quad \ldots \quad (3)$$

The electric field $E_\rho(\rho, 0)$ is essentially an unknown quantity. However, since $\gamma \gg \gamma_0$ an approximate boundary condition can be employed expressed by,

$$E_\rho(\rho, 0) \approx -\eta_c H_\phi(\rho, 0) \quad \ldots \quad \ldots \quad (4)$$

where $H_\phi(\rho, 0)$ is the tangential magnetic field for the imperfect earth and η_c is the surface impedance of the air-ground interface.

If ρ is greater than a (the radius of the earth screen) η_c can be replaced by η. If ρ is less than a, η_c is the intrinsic impedance η_s of the earth screen in parallel with the impedance η of the earth. In a previous investigation[8,9] of this problem it was assumed that η_s was zero, so that the earth system was equivalent to a perfectly-

*N.B. $H_\phi^z(\rho, 0) = [H_\phi^z(\rho, z)]_{z \to 0}$

conducting disc of radius a. The limits of the integration in equation (2) are then from $\rho = a$ to $\rho = \infty$.

A more general case is when η_s is comparable in magnitude to η, in which if follows that

$$\eta_c = \frac{\eta\eta_s}{\eta + \eta_s} \text{ for } 0 < \rho < a$$

$$\text{where } \eta_s = j\eta_0 \frac{d}{\lambda} \log_e \frac{d}{2\pi c}$$

and where d is the spacing between the radial conductors and c is the radius of the wire. The expression for η_s has been derived[10] for a wire grid in free space where it was necessary to assume that $|\gamma_0 d| \ll 1$. Since the grid is lying on the earth plane, this restriction must be replaced by $|\gamma_e d| \ll 1$ where γ_e is the effective propagation constant for propagation along a thin wire in the interface and is given by[11]

$$\gamma_e = \left(\frac{\gamma_0^2 + \gamma^2}{2}\right)^{1/2}$$

If there are N radial conductors it can be seen that d can be replaced by $2\pi\rho/N$ since N is usually of the order of 100. It is assumed also that $H_\phi(\rho, 0)$ is not very different from $H_\phi^\infty(\rho, 0)$ in the region of the ground plane where the losses are significant. This approximation has also been discussed previously[8,9] and it certainly appears to be valid if $|\gamma| \gg |\gamma_0|$.

The impedance increment ΔZ_t is then written in the following form

$$\Delta Z_t = \Delta Z + \Delta Z_s \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

$$\text{where } I_0^2 \Delta Z \approx \eta \int_a^\infty [H_\phi^\infty(\rho, 0)]^2 \, 2\pi\rho \, d\rho \quad \ldots \quad (6)$$

$$\text{and } I_0^2 \Delta Z_s \approx \int_0^a \frac{\eta\eta_s}{\eta + \eta_s} [H_\phi^\infty(\rho, 0)]^2 \, 2\pi\rho \, d\rho. \quad (7)$$

The first term ΔZ corresponds to the self-impedance of the unipole over a perfectly conducting discoid, whereas the second term ΔZ_s accounts for the finite surface impedance of the radial-conductor earth system.

Assuming a sinusoidal current distribution for $I(z)$ the magnetic field $H_\phi^\infty(\rho, 0)$ can be expressed in closed form. The integration indicated by equation (6) can then be carried out and the result expressed in terms of sine and cosine integrals[8]. Curves of the function ΔZ for unterminated aerials have been computed from this formula[8]. The integrations indicated in equation (7), however, cannot be carried out analytically. It is necessary to resort to a numerical procedure for this case.

Impedance Calculation

From equation (3) it follows that

$$H_\phi^\infty(\rho, 0) = \frac{\rho}{2\pi} \int_{z=0}^{h} \left[\frac{\exp.[-j\beta(\rho^2 + z^2)^{1/2}]}{(\rho^2 + z^2)^{3/2}} + j\beta \frac{\exp.[-j\beta(\rho^2 + z^2)^{1/2}]}{\rho^2 + z^2}\right] I(z)\, dz. \quad \ldots \quad (8)$$

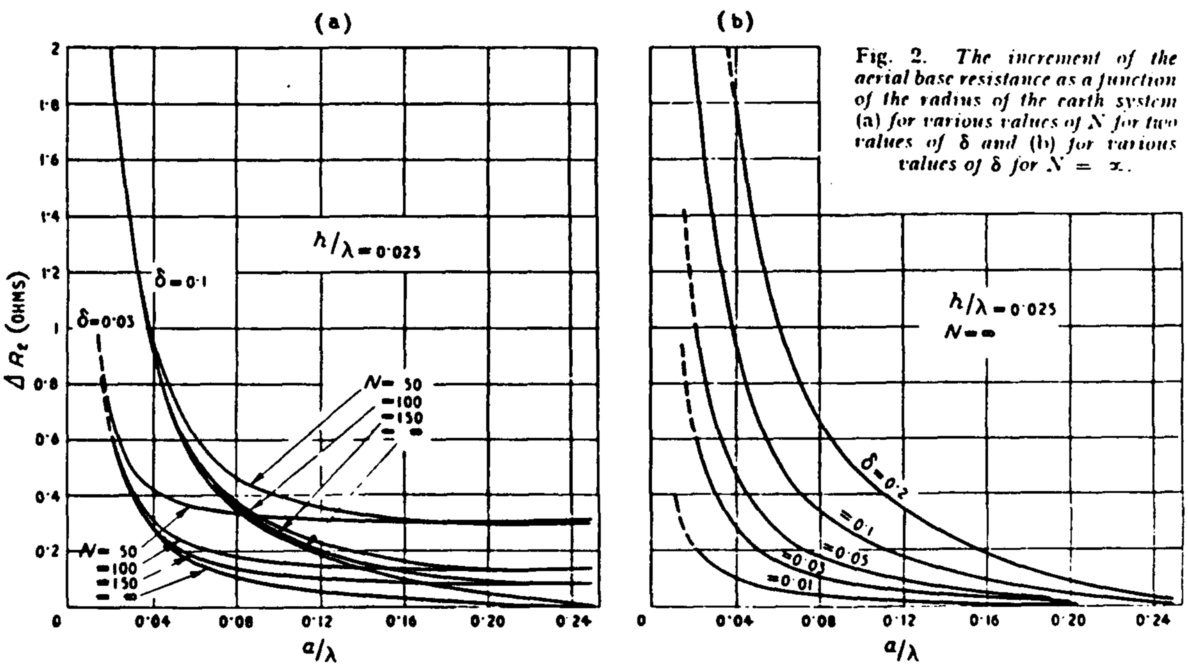


Fig. 2. *The increment of the aerial base resistance as a function of the radius of the earth system (a) for various values of N for two values of δ and (b) for various values of δ for $N = \infty$.*

For top-loaded aerials that are short compared with a wavelength a reasonable approximation is $I(z) \approx I_0$, over the length of the aerial. Also, exp. $[-j\beta(\rho^2+z^2)^{1/2}]$ is given to sufficient accuracy by exp. $(-j\beta\rho)$ so that

$$H_\phi^\infty(\rho,0) \approx \frac{e^{-j\beta\rho}\rho I_0}{2\pi}\int_0^h [(\rho^2+z^2)^{-3/2} + j\beta(\rho^2+z^2)^{-1}]\,dz \approx \frac{e^{-j\beta\rho}\rho I_0}{2\pi}\left[\frac{h}{\rho(\rho^2+h^2)^{1/2}} + \frac{j\beta}{\rho}\tan^{-1}\frac{h}{\rho}\right] \quad \text{.. .. ..} \quad (9)$$

This can be expanded in a power series in (h/ρ) as follows:

$$H_\phi^\infty(\rho,0) \approx \frac{e^{-j\beta\rho}I_0}{2\pi\rho}\left[\left(\frac{h}{\rho}\right)(1+j\beta\rho) - \left(\frac{h}{\rho}\right)^3\left(\frac{1}{2}+\frac{j}{3}\beta\rho\right)+\ldots\right] \quad \text{.. .. ..} \quad (10)$$

and correspondingly,

$$\left[H_\phi^\infty(\rho,0)\right]^2 \approx \frac{I_0^2 e^{-2j\beta\rho}}{4\pi^2\rho^2}\left[\left(\frac{h}{\rho}\right)^2(1+j\beta\rho)^2 - \left(\frac{h}{\rho}\right)^4(1+j\beta\rho)(1+2j\beta\rho/3)+\ldots\right] \quad \text{..} \quad (11)$$

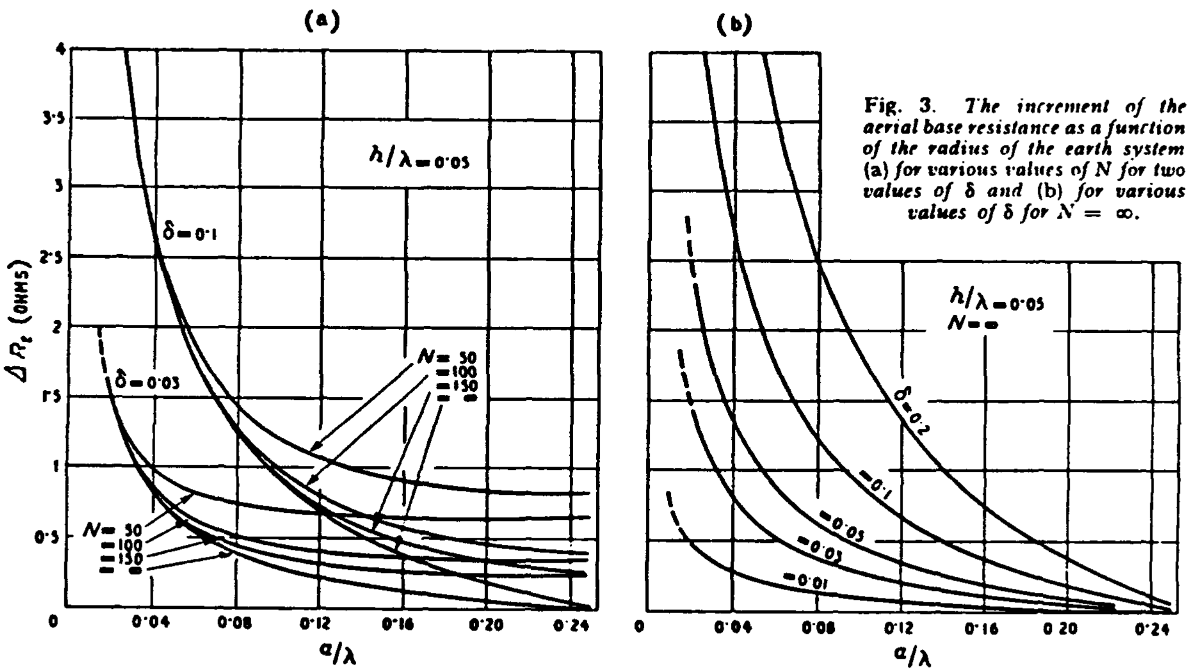


Fig. 3. *The increment of the aerial base resistance as a function of the radius of the earth system* (a) *for various values of N for two values of δ and* (b) *for various values of δ for N = ∞.*

The self-impedance ΔZ is then given by a series of integrals as follows:

$$\Delta Z = \frac{\eta}{I_0^2}\int_a^b [H_\phi^\infty(\rho,0)]^2\, 2\pi\rho\, d\rho + \frac{\eta h^2}{2\pi}\int_b^\infty e^{-2j\beta\rho}(1+2j\beta\rho-\beta^2\rho^2)\,\rho^{-3}\,d\rho$$

$$-\frac{\eta h^4}{2\pi}\int_b^\infty e^{-2j\beta\rho}(1+5j\beta\rho/3-2\beta^2\rho^2/3)\,\rho^{-5}\,d\rho + \ldots\ldots\ldots \quad \text{.. .. ..} \quad (12)$$

The integration of ρ has been broken conveniently into two ranges, from a to b and b to ∞. The distance b is chosen sufficiently large so that the series of integrals converges rapidly. The integral with limits a to b is integrated by graphical means. Using the exponential integral defined by,

$$Ei(-2j\beta b) = -\int_b^\infty e^{-2j\beta\rho}\,\rho^{-1}\,d\rho$$

the impedance ΔZ is written

$$\Delta Z = \frac{\eta}{I_0^2}\int_a^b [H_\phi^\infty(\rho,0)]^2\, 2\pi\rho\, d\rho + \frac{\eta h^2\beta^2}{2\pi}\left[\frac{1}{\beta b}\left(\frac{1}{2\beta b}+j\right)e^{-2j\beta b} - Ei\left(-2j\beta b\right)\right]$$

$$-\frac{\eta h^2\beta^2}{2\pi}\left(\frac{h^2}{b^2}\right)\left\{\left[\left(\frac{1}{4\beta^2b^2}+\frac{1}{18}\right)+j\left(\frac{7}{18\beta b}-\frac{\beta b}{9}\right)\right]e^{-2j\beta b} + \frac{2}{9}\beta^2b^2\,Ei\left(-2j\beta b\right)\right\} + \frac{\eta h^2\beta^2}{2\pi}\times O\left(\frac{h^4}{b^4}\right)$$

$$\text{.. .. .. ..} \quad (13)$$

When $h^2/b^2 \ll 1$, the terms containing higher powers of h^2/b^2 can be neglected.

The increment of the input resistance ΔR_t for purposes of calculation is now written

$\Delta R_t =$ Real part of $(\Delta Z_s + \Delta Z)$

$$= \frac{\sqrt{2}}{2\pi}\int_0^A \frac{pq\,F^2(H_1,P)}{[p^2+(p+q)^2]^{1/2}P}\cos\left(2\psi - 4\pi P + \frac{3\pi}{4} - \tan^{-1}\frac{p+q}{p}\right)dP$$

$$+ \frac{\sqrt{2}}{2\pi}\int_A^B F^2(H_1,P)\,P^{-1}\cos(2\psi - 4\pi P + \pi/4)\,dP + \sqrt{2}\,\pi\,p\,H_1^2\,V\cos(\theta + \pi/4) \quad \ldots \quad (14)$$

where $$F(H_1,P)\,e^{j\psi} = \frac{H_1/P}{[1+(H_1/P)^2]^{1/2}} + j\,2\pi P\tan^{-1}(H_1/P), \quad \ldots \quad (15a)$$

$$p = 120\pi\delta/\sqrt{2}, \qquad q = \frac{240\pi^2 P}{N}\log_e\frac{P}{NC_1}, \qquad \delta = (\epsilon_0\omega/\sigma)^{1/2},$$

$$H_1 = h/\lambda, \qquad P = \rho/\lambda, \qquad A = a/\lambda, \qquad B_1 = b/\lambda \text{ and } C_1 = c/\lambda,$$

and $$V e^{j\theta} = 2\left[\frac{j}{\beta b}\left(1 - \frac{j}{2\beta b}\right)e^{-j2\beta b} - Ei\left(-2j\beta b\right)\right] \quad \ldots \quad (15b)$$

Summarizing, this formula for ΔR_t should be accurate to within a few per cent under the restrictions that $H_1 < 0{\cdot}1$ (electrically short aerial), $\epsilon\omega/\sigma \ll 1$ (negligible displacement current in soil), and $(B_1/H_1)^2 = (b/h)^2 = 25$.

Presentation of Results

It hardly needs to be mentioned that the major part of this work has to do with the evaluation of the integrals in equation (14). A graphical procedure was adopted employing a conventional area planimeter. The resulting values of the integrals, so obtained, are believed to be accurate to within 1%.

The computed values of ΔR_t for fixed aerial heights are plotted as a function of a/λ for various values of δ and N in Figs. 2 to 4. The wire radius to wavelength ratio, c/λ, is taken to be 10^{-6} which corresponds to No. 8 B. & S. wire at 183 kc/s. The curves in Figs. 2(a), 3(a) and 4(a) are for two fixed values of the ground conductivity parameter δ, whereas the curves in 2(b), 3(b) and 4(b), for a perfect ground screen $N = \infty$, show a wider range of δ. With the results plotted in this form, values of ΔR_t for intermediate values of N and δ can be estimated quickly by interpolation. For purposes

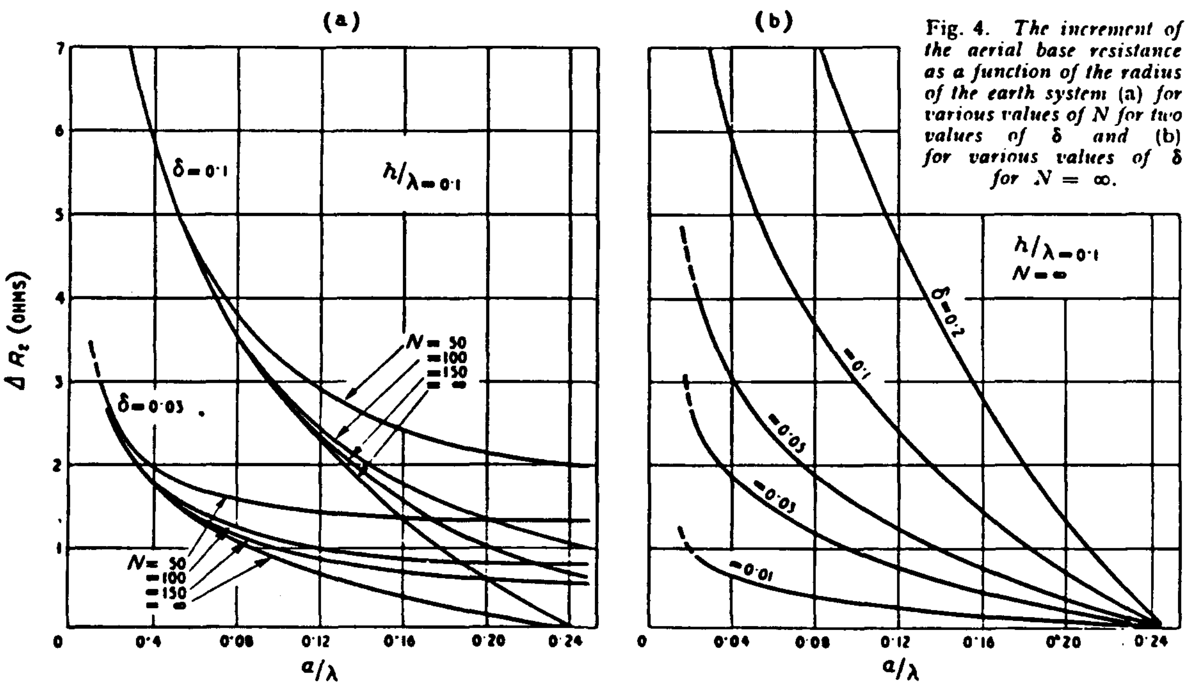


Fig. 4. *The increment of the aerial base resistance as a function of the radius of the earth system (a) for various values of N for two values of δ and (b) for various values of δ for N = ∞.*

of comparison, the values of ΔR_t for a quarter-wave unipole ($h = \lambda/4$) are shown plotted in Fig. 5(a) and 5(b). The data for these curves are taken from a previous paper[6].

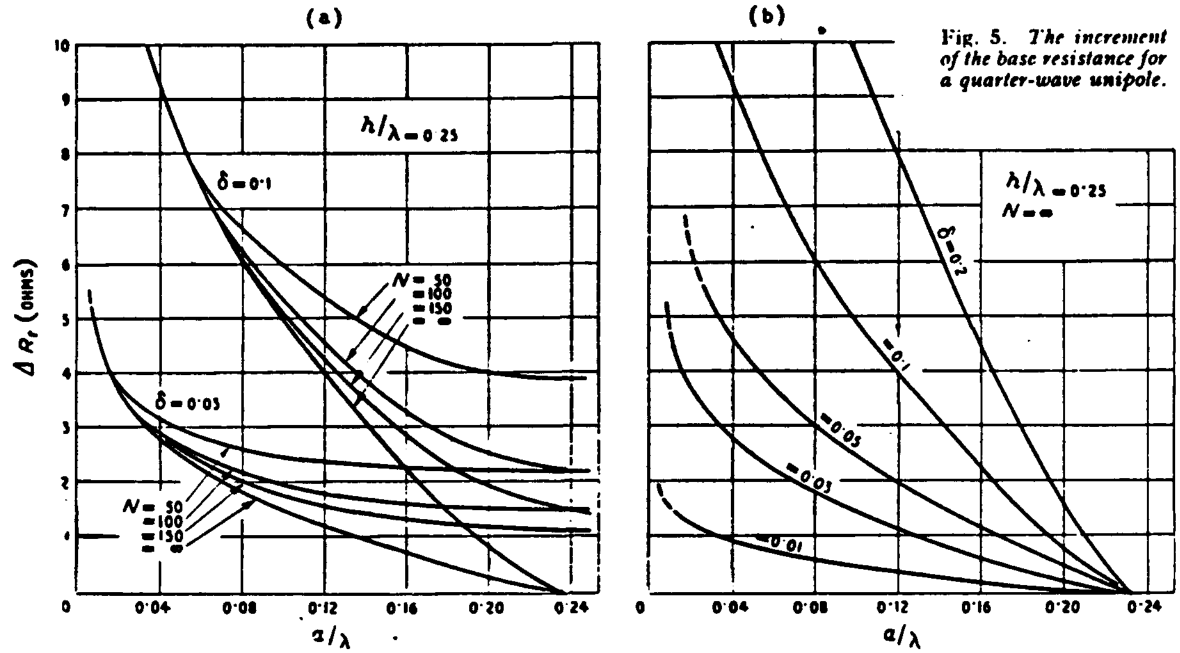


Fig. 5. *The increment of the base resistance for a quarter-wave unipole.*

It is apparent immediately that the increase of ΔR_t with diminishing earth-screen radius is much more rapid for the short aerials than for the quarter-wave unipole. This behaviour is connected with the fact that the induction and static fields of short aerials are more significant than those for a higher aerial, such as a quarter-wave unipole.

The appropriate value of δ to use in connection with these curves is obtained conveniently from Fig. 6, when the ground conductivity and the frequency are specified.

The effect of changes in wire radius is slight. This fact is illustrated in Fig. 7 where ΔR_t is shown plotted as a function of a/λ for various values of the wire radius/wavelength ratio for a quarter-wave unipole.

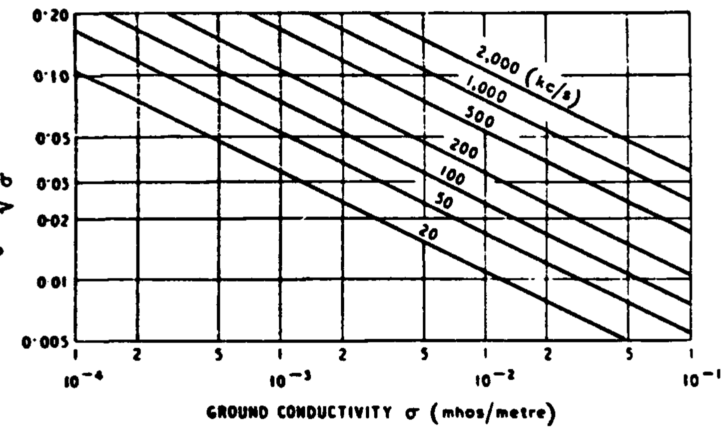


Fig. 6. *The parameter δ as a function of frequency in kc/s and ground conductivity.*

For earth screens which are small compared with the wavelength ΔR_t varies in a linear manner with δ. That is, it varies directly as the square root of the frequency and inversely as the square root of the ground conductivity. However, for larger values of a/λ as is illustrated in Fig. 8, the values of ΔR_t become somewhat less sensitive to changes in δ.

Up to this point, the discussion has been limited mainly to vertical aerials with ideal top-loading. It is usually justified in these cases to assume that the current distribution is constant from the base of the aerial to its upper end. It has been shown also by Brown[1,2], Smeby[12], and others[13] that the radiation resistance R_0 can be computed by considering only the current on the vertical portion of the aerial. The contributions for the currents flowing on the loading 'umbrella' or 'cone' are usually negligible if the top-loading has been adjusted for maximum radiation resistance. This radiation resistance is given by [14]

$$R_0 \approx 160\pi^2 (h/\lambda)^2$$

It is possible now to apply the above results to aerials without top-loading by defining an 'equivalent' or 'effective' height h. This value is obtained by assuming that the loaded and unloaded unipoles are electrically equivalent if their radiation resistances are equal. For thin unloaded unipoles the current distribution is approximately sinusoidal and if its actual height is denoted by h_0, the radiation resistance, R_0, is given by

$$\sin^2 \beta h_0 \, R_0 = 30\,(C + \log 2\beta h_0 - \mathrm{Ci}\, 2\beta h_0) + 15\,(\mathrm{Si}\, 4\beta h_0 - 2\,\mathrm{Si}\, 2\beta h_0) \sin 2\beta h_0 + 15\,(C + \log \beta h_0 - 2\,\mathrm{Ci}\, 2\beta h_0 + \mathrm{Ci}\, 4\beta h_0) \cos 2\beta h_0 \quad (16)$$

where $C = 0{\cdot}5772$. Si and Ci are the sine and cosine integral functions. R_0 is just one-half of the self-resistance of a thin, centre-driven, aerial of length $2h_0$ situated in free space[14]. When h_0/λ is small compared with unity, $R_0 \approx 40\,\pi^2\,(h_0/\lambda)^2$.

Using these formulae to calculate the equivalent height h of the unloaded aerials it follows that for $h/\lambda = 0{\cdot}025$, $0{\cdot}050$ and $0{\cdot}10$ $h_0/\lambda = 0{\cdot}050$, $0{\cdot}095$, and $0{\cdot}175$ respectively. In other words, it is probable that the curves in Figs. 2, 3, and 4 for loaded aerials also apply to unloaded aerials of heights $0{\cdot}050\lambda$, $0{\cdot}095\lambda$ and $0{\cdot}175\lambda$ respectively. The accuracy of this procedure can be checked by comparing the results with more exact previous calculations for an unloaded unipole situated over a circular, perfectly conducting, disc laid on the ground. Employing the data in Figs. 2, 3 and 4 the function ΔR_t for the loaded unipole and $N = \infty$ is plotted in Fig. 9 as a function of h/λ for selected values of a/λ. The corresponding curves, for the unloaded unipole are obtained by the above mentioned approximate procedure and are also shown in Fig. 9. The encircled points on the dotted curves are obtained from calculations carried out from equation (16) of reference 6. The points indicated by × for $h/\lambda = 0{\cdot}25$ can be obtained either from equation (16) of reference

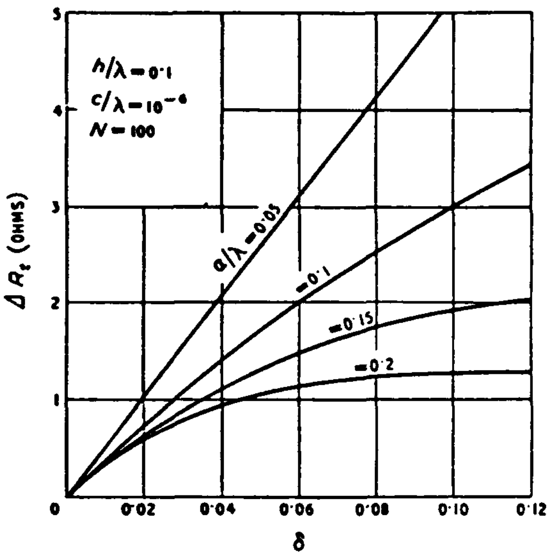


Fig. 8. *An illustration to show how ΔR_t varies with ground conductivity and frequency.*

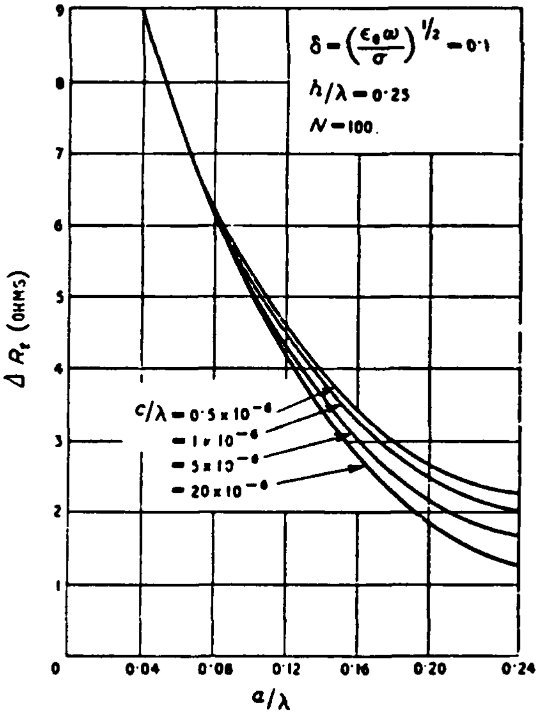
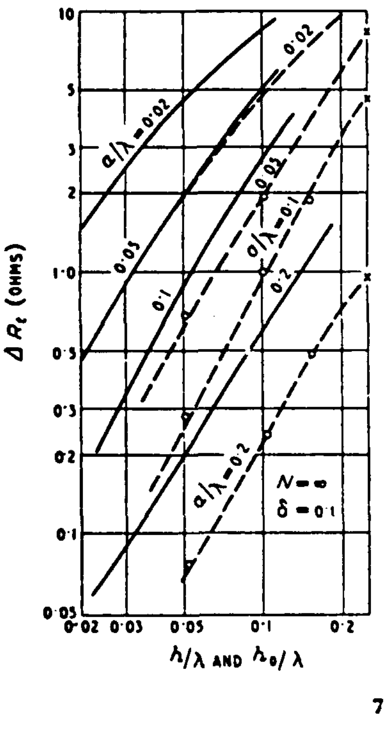


Fig. 7. *The effect of changing wire radius on the increment of input resistance ΔR_t.*

Fig. 9. *The solid curves represent ΔR_t for an ideally loaded unipole of height h, whereas the dashed curves correspond to the estimated values for an unterminated unipole of height h_0. The indicated points are plotted from more exact formulae.*



6 or directly from Fig. 5(b). The good agreement between the two methods of calculation for ΔR_t is reassuring.

It is also instructive to plot ΔR_t as a function

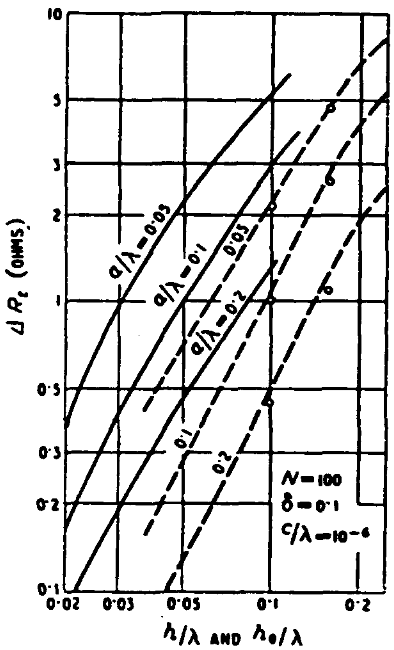


Fig. 10. *The curves for the loaded unipole and the estimated curves for the unloaded unipole are shown by the solid and dashed lines respectively. The encircled points represent values computed by Monteath.*

of h/λ for both loaded and unloaded unipoles for a finite value of N. The results are shown in Fig. 10 for $N = 100$, $\delta = 0{\cdot}1$, $c/\lambda = 10^{-6}$ employing data from Figs. 2, 3, 4 and 5. The encircled points correspond to results, communicated to us privately by Mr. G. D. Monteath of the B.B.C. for unloaded unipoles with heights of $0{\cdot}10\lambda$ and $0{\cdot}167\lambda$. Again the agreement is very satisfactory.

Both the curves in Figs. 9 and 10 illustrate the near linear log-log relationship between ΔR_t and h/λ. This behaviour is also prevalent for other values of N and δ and provides a convenient means of interpolating and extrapolating for values of h/λ other than those shown in Figs. 2 to 6.

Conclusion

No attempt has been made in this paper to consider the economic factors but rather the emphasis has been on showing the manner in which the impedance varies with the number and length of radial wires, aerial height, and ground conductivity. The curves should be useful in the design of earth systems for low-frequency transmitting aerials. It would appear that many earth systems are probably more extensive than necessary since the benefits gained by employing large radius screens are lost if there are not a sufficient number of radial conductors. This is particularly so if the ground conductivity is relatively high.

The calculations for the base input-resistance are derived on the assumption of uniform current distribution along the vertical portion of the aerial. It has been indicated, however, that the results are also applicable to aerials with non-uniform current distribution if the quantity h is regarded as an effective height.

Acknowledgments

The need for the curves presented in this paper was pointed out by Dr. T. W. Straker. Further advice and suggestions were also given by Dr. F. R. Abbott, Dr. W. J. Surtees, Mr. R. S. Thain and Mr. J. S. Belrose. Appreciation must also be expressed to Mr. G. D. Monteath of the British Broadcasting Corporation who communicated to us unpublished reports of his research.

APPENDIX

Measurements on a 250-ft umbrella top-loaded unipole show good agreement with the theory. The following observed values were supplied to us by Mr. R. S. Thain of this laboratory:

Length of Radials, a = 800 ft.
Number of Radials, N = 120
Ground Conductivity, $\sigma = 2{\cdot}0 \times 10^{-3}$ mhos/metre
Frequency, 97 kc/s
Radiation Resistance as calculated from field strength measurements, R_e = 0·50 ohm
Input resistance measured on a bridge, R_t = 0·75 ohm
Observed resistance increment, ΔR_t = 0·25 ohm
Theoretical value of resistance increment (for $h/\lambda = 0{\cdot}025$, $a/\lambda = 0{\cdot}08$, $\delta = 0{\cdot}07$), ΔR_t = 0·23 ohm.

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Effect of the Ground Screen on the Field Radiated from a Monopole*

J. R. WAIT†

SOME INTEREST has been shown recently in the radiation characteristics of an L.F. vertical antenna with a radial wire ground system. The investigations[1-3] have been primarily concerned with the input impedance at the terminals of the antenna. This was shown to be mainly a function of the number and length of radials and the ground conductivity. It was assumed in most of this work that the ground wave field for a given current on the antenna was not appreciably affected by changes in size of the ground screen. Under this assumption, the radiation efficiency of the antenna is determined mainly by the input resistance.

In an earlier paper,[2] an approximate method was given which was suitable for estimating the dependence of the ground wave field on the size of the ground screen. It is the purpose of this note to show that quantitative results can be obtained which support our earlier contention that the ground screen has only a small effect on the ground wave field intensity for a specified current on the antenna.

The ground screen is assumed to be a perfectly conducting disc of radius a lying on a homogeneous flat ground of conductivity σ. Choosing a cylindrical coordinate system (ρ, ϕ, z) the antenna is considered to be coincident with the positive z axis and the surface of the ground is defined by $z=0$. Denoting $H_\phi(\rho, z)$ as the magnetic field of the antenna in the absence of any ground screen and $\Delta H_\phi(\rho, z)$ as the change due to the presence of the ground screen, it follows from an earlier paper,[2] that

$$\frac{\Delta H_\phi(\rho, z)}{H_\phi(\rho, z)} \simeq -\frac{\beta\eta}{\eta_0} \frac{e^{-i\beta[\rho^2+z^2]^{1/2}}}{[\rho^2+z^2]^{1/2}} \cdot \int_{\rho'=0}^{a} \frac{H_\phi^{*}(\rho', 0)}{H_\phi^{*}(\rho, z)} J_1(\beta\rho' \cos\theta)\rho' d\rho' \quad (1)$$

where $\beta = 2\pi/\lambda$, λ = free space wavelength, $\eta = (\mu\omega/\sigma)^{1/2} e^{i\pi/4}$ (surface impedance of the ground), $\mu = 4\pi \times 10^{-7}$, ω = angular frequency, $\eta_0 = 120\pi$ (intrinsic impedance of free space), $\theta = \tan^{-1} z/\rho$, and where J_1 is the Bessel function of the first type. In the above, H_ϕ^{*} refers to the field of the antenna over a flat perfectly conducting ground. This expression for the fractional change of the magnetic field is approximate and neglects terms of higher order in (η/η_0). It also assumes that the attenuation of the ground wave has a dependence with distance which is independent of the size of the ground screen. $\Delta H_\phi/H_\phi$ can be regarded as the fractional change of the effective height of the antenna due to the presence of the ground screen.

As an example a quarter-wave monopole antenna is considered with an assumed sinusoidal current distribution. In this case

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† National Bureau of Standards, Boulder, Colo.

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$$H_\phi^{m}(\rho', 0) = \frac{-iI}{2\pi\rho'} e^{-i\beta[\rho'^2+(\lambda/4)^2]} \qquad (2)$$

and since $\rho \gg \lambda$;

$$H_\phi^{m}(\rho, z) \simeq \frac{-iI}{2\pi\rho} e^{-i\beta[\rho^2+z^2]^{1/2}} \cos\left(\frac{\pi}{2}\sin\theta\right). \qquad (3)$$

It then follows that the fractional change of the field is given by

$$\frac{\Delta H_\phi(\rho, z)}{H_\phi(\rho, z)} = \frac{-\beta\eta}{\eta_0} \frac{\cos\theta}{\cos\left(\frac{\pi}{2}\sin\theta\right)} \cdot \int_0^{a} e^{-i\beta[\rho'^2+(\lambda/4)^2]^{1/2}} J_1(\beta\rho'\cos\theta)d\rho'. \qquad (4)$$

The integral, apparently, cannot be evaluated in closed form. It is not too difficult, however, to evaluate it by a graphical method. This has been done for the case $z=0$ which corresponds to the ground wave field. Letting

$$\frac{\Delta H_\phi(\rho, 0)}{H_\phi(\rho, 0)} = \delta[X_1 + iX_2] \qquad (5)$$

with $\delta = |\eta/\eta_0|$, the integrals to consider are

$$X_1 = -\int_0^{2\pi a/\lambda} \cos\left[(\rho^2 + \pi^2/4)^{1/2} - \pi/4\right] J_1(\rho)d\rho \qquad (6)$$

and

$$X_2 = \int_0^{2\pi a/\lambda} \sin\left[(\rho^2 + \pi^2/4)^{1/2} - \pi/4\right] J_1(\rho)d\rho. \qquad (7)$$

Some numerical values of X_1 and X_2 are given in the following Table I.

TABLE I

$2\pi a/\lambda$	X_1	X_2
0.0	0.000	0.000
0.5	−0.042	0.040
1.0	−0.130	0.181
1.5	−0.211	0.417
2.0	−0.209	0.700
2.5	−0.102	0.947
3.0	0.042	1.093
3.5	0.155	1.131
4.0	0.171	1.133
4.5	0.113	1.178
5.0	0.050	1.300
5.5	0.050	1.468
6.0	0.119	1.612
6.5	0.205	1.674

The ratio of the field with the ground screen to the field $H_\phi(\rho, z)$ without the ground screen is then given by

$$1 + \frac{\Delta H_\phi(\rho, z)}{H_\phi(\rho, z)} = Ae^{i\Phi} \qquad (8)$$

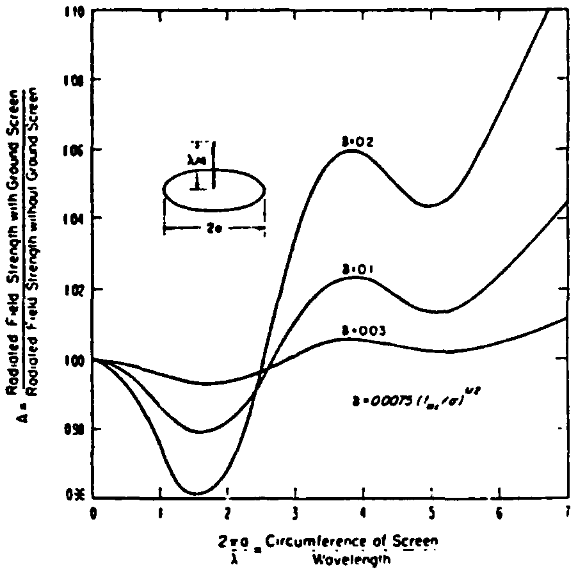

Fig. 1—The ratio of the ground wave field strengths of the antenna with and without the ground screen for a specified antenna current.

where the amplitude ratio A is given by

$$A = \left[(1 + \delta X_1)^2 + (\delta X_2)^2\right]^{1/2} \qquad (9)$$

and the phase factor is Φ given by

$$\Phi = \tan^{-1} \frac{\delta X_2}{1 + \delta X_1}. \qquad (10)$$

Since δ has already been considered small, these are given adequately by

$$A \simeq 1 + \delta X_1$$

and

$$\Phi \simeq \delta X_2 \text{ (radians)}.$$

The factor δ can be obtained conveniently from the following relation

$$\delta = 0.0075\,(f_{mc}/\sigma)^{1/2}$$

where f_{mc} is the frequency in mc and σ is the ground conductivity in mhos/meter. (Since displacement currents in the ground have been neglected throughout, the formulas are valid only when $\delta^2 \ll 1$.) Using (9), the amplitude ratio A is shown plotted in Fig. 1 as a function of $2\pi a/\lambda$, the circumference of the ground screen in wavelengths, for typical values of δ.

It is interesting to note that for small screens the ground wave field strength is actually slightly less than it would be in the absence of the screen. As the screen becomes larger, the amplitude ratio increases somewhat, but is still only a few per cent greater than unity for a less than a wavelength. Thinking in terms of the reciprocal situation where the monopole is regarded as a receiving antenna and the transmitter is located at some distant point on the surface of the ground, the increase of A above unity is characteristic of a "recovery" effect. Such a phenomenon occurs in ground wave propagation from land to sea.[4]

Another interesting feature of the curves in Fig. 1 is that the minimum values of A occur approximately where the input resistance of the antenna is a minimum. It can be generally concluded, however, that the dependence of the L.F. ground wave field strength on the size of the ground screen is of minor significance compared to the dependence of the input resistance on the size of the screen. It should be mentioned that Page and Monteath[5] have used a similar method to calculate the radiation pattern of a vertical antenna over an irregular ground plane.

ACKNOWLEDGMENT

I would like to thank W. A. Pope who carefully carried out the graphical integration of X_1 and X_2.

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(Reprint No. 72)

AN INVESTIGATION OF SLOT RADIATORS IN RECTANGULAR METAL PLATES

By D. G. FROOD, B.A., M.A., and J. R. WAIT, M.Sc., Ph.D.

(*The paper was first received 16th May, and in revised form 2nd August, 1955.*)

SUMMARY

The radiation from slots cut in conducting surfaces of limited extent is discussed. Equatorial plane patterns of an axial half-wave slot in a rectangular metal plate are measured in the X band. The experimental results compare favourably with the calculated patterns on the assumption that the plate can be represented by a thin elliptic cylinder or ribbon of infinite length. It is observed that, if the length of the plate is equal to or greater than its width, the pattern is within a few per cent of the corresponding theoretical pattern for a plate of infinite length.

The admittance of the slot in the plate was also measured and compared with the computed conductance. The agreement is seen to be quite good.

LIST OF PRINCIPAL SYMBOLS

(ρ, ϕ, z) = Radial, azimuthal and axial co-ordinates of a cylindrical co-ordinate system.
V = Voltage across the centre of the slot.
η_0 = Intrinsic impedance of free space (=120 ohms).
k = $2\pi/\lambda$.
r = Distance from observer to centre of slot.
r_1 = Distance from observer to lower end of slot.
r_2 = Distance from observer to upper end of slot.
ω = Angular frequency.
θ = Polar angle = arc tan z/ρ.
$S(\theta)$ = H-plane pattern of slot on infinite sheet.
Y = Self-admittance at centre of slot.
Z = Self-impedance of slot ($=1/Y$).
$(\hat{\rho}, \Phi, z)$ = Cylindrical co-ordinates centred at one edge of sheet for principal E-plane.
$(\hat{\rho}', \Phi', z)$ = Cylindrical co-ordinates centred at second edge of sheet for principal E-plane.
$F(s)$ = Fresnel-type integral with upper limit s defined in eqn. (6).
S_1, S_2 = Limits for the integral $F(s)$.
H_0 = Axial magnetic field of incident plane wave.
K = Factor of proportionality depending on the geometry of the slot.
v = Voltage induced at the centre of the slot by the incident wave.
$g(kd)$ = Shunt conductance of the slot in the waveguide divided by the characteristic admittance of the waveguide (= function of kd).
a, b = Inner dimensions of the broad and narrow faces of the waveguide.
λ_g = Effective wavelength in guide.

(1) INTRODUCTION

Slot antennae are becoming very extensively employed in microwave radiating systems. Their history of development has been rapid. It is difficult to say exactly when they were invented, but certainly the contributions from Watson[1] and Booker[2] and their collaborators were of major importance.

It is surprising that little attention has been paid by previous workers to the effect of cutting the slot in metal surfaces of finite extent. While the developed design procedures assumed that the exterior surface was infinite in extent, the need for a practical antenna system limits the physical size of the metal surface on which the slots are cut. Stevenson,[3] who developed an elegant theory for the radiation of resonant slots in a rectangular waveguide, assumed for convenience that the exterior region was equivalent to a half space. He admits that the assumed infinite dimension of the face of the waveguide for the exterior problem is a severe limitation to the validity of the expressions for the conductance of the slot.

It is the purpose of the paper to investigate in some detail the significance of the finite extent of the metal surface or sheet on which the slot is cut.

(2) THEORETICAL DISCUSSION

The radiation pattern of a thin slot cut in an infinite plane sheet of perfect conductivity and vanishing thickness can be obtained by an application of an electromagnetic Babinet's principle.[2] By using this technique, the well-known results[4] for the thin-wire antenna can be transformed immediately to the complementary problem. For a half-wave slot oriented in the z-direction of a (ρ, ϕ, z) co-ordinate system and centre-fed by a voltage V, the fields are given by

$$\left.\begin{aligned} H_z &= \frac{-jV}{2\pi\eta_0}\left(\frac{\varepsilon^{-jkr_1}}{r_1} + \frac{\varepsilon^{-jkr_2}}{r_2}\right)\varepsilon^{j\omega t} \\ H_\rho &= \frac{jV}{2\pi\eta_0\rho}\left[(z + \lambda/4)\frac{\varepsilon^{-jkr_1}}{r_1} + (z - \lambda/4)\frac{\varepsilon^{-jkr_2}}{r_2}\right]\varepsilon^{j\omega t} \\ E_\phi &= \frac{-jV}{2\pi\rho}(\varepsilon^{-jkr_1} + \varepsilon^{-jkr_2})\varepsilon^{j\omega t} \end{aligned}\right\} \quad . \quad (1)$$

where r_2, r_1 and r are the distances of any point P to the upper, centre and lower end, respectively, of the slot and where $\eta_0 = 120\pi$ ohms and $k = 2\pi$/(free-space wavelength). Eqns. (1) refer to the slot cut in an infinite sheet. When the point P is sufficiently far away from the slot the equations simplify somewhat to

$$\left.\begin{aligned} E_\phi &\simeq j(2\pi r)^{-1}S(\theta)\exp(-jkr + j\omega t) \\ H_z &\simeq \left(\frac{1}{\eta_0}\right)E_\phi \sin\theta \\ H_\rho &\simeq -\left(\frac{1}{\eta_0}\right)E_\phi \cos\theta \end{aligned}\right\} \quad . \quad . \quad (2)$$

where θ = arc tan (z/ρ) and where terms containing r^{-2}, r^{-3}, etc., have been neglected. The factor $S(\theta)$ can be defined as the radiation pattern of the slot and is given by

$$S(\theta) = V\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Following the method of Carter[5] for the thin-wire antenna,

Written contributions on papers published without being read at meetings are invited for consideration with a view to publication.

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the self-admittance Y at the centre of the slot can be obtained from

$$Y = \frac{-2}{V}\int_{-\lambda/4}^{\lambda/4} H_z(0, 0, z)\sin kz\,dz \quad . \quad . \quad . \quad (4)$$

The integrations can be carried out to yield the result

$$Y = 2\cdot06 + j0\cdot97 \text{ millimhos}$$

which corresponds to the case when the slot is allowed to radiate on both sides of the sheet. The centre impedance of the slot is then

$$Z = 1/Y = 365 - j212 \text{ ohms}$$

It should be noted that Begobich[6] recently gave the value $362\cdot5 + j210\cdot5$ ohms, which indicates reasonable agreement with the above, except for the sign of the reactance. Apparently he made an error in his derivation. The result derived by the authors can be checked by comparing it with Booker's statement of Babinet's principle, and connecting the impedance, Z, of the slot with the impedance, Z^w, of the complementary wire antenna, such that

$$ZZ^w = \eta_0^2/4 = 3\,600\pi^2$$

Using Carter's[5] value $73\cdot2 + j42\cdot5$ for Z^w leads back to the authors' expression for Z. Begovich erroneously employed the complex conjugate of Z^w.

When the slot radiates only on one side of the sheet the conductance G would be one-half the real part of Y given above, and hence $G = 1\cdot03$ millimhos. The corresponding susceptance is one-half the imaginary part of Y plus the susceptance of the feed system.

It should be noted that the pattern of the slot on the infinite sheet is essentially omni-directional in the azimuthal plane. It is of interest to consider the effect of truncating the sheet. For example, the slot is cut in the centre of a rectangular metal plate of width $2d$, and oriented parallel to the other sides of length L,

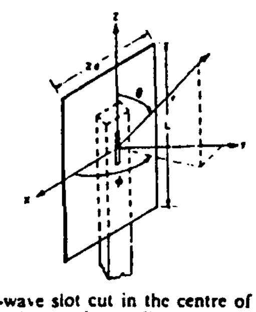

Fig. 1.—Thin half-wave slot cut in the centre of a rectangular metal sheet and co-ordinate system.

as illustrated in Fig. 1. The slot is fed by a waveguide such that radiation takes place only on one side of the plate.

The ideal E-plane pattern in the equatorial plane, corresponding to a value of d very much greater than the wavelength, would be as shown in Fig. 2, where the plane of the sheet corresponds to $\Phi = 0°$. In other words, the sheet is assumed to be sufficiently wide that diffraction around and by the edges is negligible. Of course, if the slot were allowed to radiate on both sides of the sheet the pattern would be a complete circle. The corresponding H-plane pattern for the half-wave slot is simply the function $S(\theta)$ shown in Fig. 3, where $\theta = 0°$ is in the plane of the sheet.

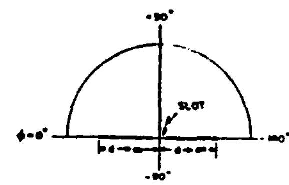

Fig. 2.—E-plane pattern of the slot for an infinitely large sheet.

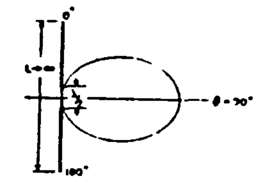

Fig. 3.—H-plane pattern of the slot for an infinitely large sheet.

It would now be expected, in view of the shape of the H-plane pattern shown in Fig. 3, that the effect of the finite value of L would not be pronounced. This is later borne out by experiment. However, the finite value of d can be expected to lead to a considerable impairment in the pattern, since the field in the broadside direction of the slot is very significant. It is possible to treat the problem of the slot in the finite plate by an approximate theoretical procedure, if the edges separated by the distance $2d$ are considered to diffract the primary field of the slot as if they were semi-infinite half-planes. The interaction between the edges is neglected, and on physical grounds this would seem justified if $2d$ is somewhat greater than the free-space wavelength. Again, this supposition is borne out by experiment and comparison with a more rigorous treatment, such as that which represents the plate by a thin elliptic cylinder.[7,8]

To facilitate the discussion it is desirable to consider the slot as a receiving element. The voltage induced in the thin slot from the wave incident on the plate is then proportional to the tangential magnetic field along the slot. This statement follows from Schelkunoff's equivalence principle.[4] From Fig. 4 a plane

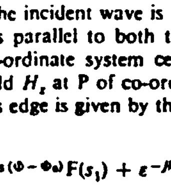

Fig. 4.—End view of the sheet showing a plane wave incident from below.

wave is shown to be incident from below the thin conducting plate of width $2d$. The incident wave is polarized such that the magnetic vector H_0 is parallel to both the slot and the edges. Choosing a polar co-ordinate system centred at the left-hand edge of Fig. 4, the field H_z at P of co-ordinates (ρ, Φ, z) due to diffraction around this edge is given by the classical Sommerfeld formula[9]

$$H_z = H_0\left[\varepsilon^{-jk\rho\cos(\Phi-\Phi_0)}F(s_1) + \varepsilon^{-jk\rho\cos(\Phi+\Phi_0)}F(s_2)\right] \quad . \quad (5)$$

where Φ_0 is the angle the incident waves makes with the plane of the sheet, where

$$\left.\begin{aligned} F(s) &= \varepsilon^{j\pi/4}\pi^{-1/2}\int_{-\infty}^{s}\varepsilon^{-jx^2}dx \\ \text{and}\qquad s_1 &= (2k\hat{\rho})^{1/2}\sin\left(\frac{\Phi-\Phi_0}{2}\right) \\ s_2 &= -(2k\hat{\rho})^{1/2}\sin\left(\frac{\Phi+\Phi_0}{2}\right) \end{aligned}\right\} \quad . \ . \ . \quad (6)$$

Similarly the field H_z' at P due to diffraction around the other or right-hand edge is identical in form to eqn. (5) if ρ, Φ and Φ_0 are replaced by $\hat{\rho}'$, Φ' and Φ_0, and H_0 is replaced by $H_0\varepsilon^{-j2kd\cos\Phi_0}$. The voltage v induced in the centrally located slot on the sheet is then given by

$$v = K(H_z + H_z') \text{ for } r' = r = d \text{ and } \Phi = \Phi' = 0$$

where K is a constant which depends on the dimensions of the slot. It then follows that

$$|v| = K\left|F[-(2kd)^{1/2}\sin(\Phi_0/2)] + F[-(2kd)^{1/2}\cos(\Phi_0/2)]\right| \quad . \ . \ . \ . \quad (7)$$

which is applicable in the range $\Phi_0 = 0°–180°$.

When the incident wave is incident on the upper side of the sheet, steps must be taken to combine the fields, as determined by the two knife-edge problems, in the proper manner. It is convenient to choose a slightly different co-ordinate system as shown in Fig. 5, where the angular co-ordinates Φ and Φ_0 are now measured from the bottom of the sheet.

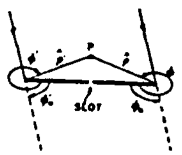

Fig. 5.—End view of the sheet showing a plane wave incident from above.

The magnetic field H_z at P, regarding the sheet as a semi-infinite half-plane with the edge at (0, 0, z), is given by eqn. (5), and a similar expression is obtained for H_z'. The voltage v induced in the slot is then given approximately by

$$v = K[H_z + H_z' - H_0\varepsilon^{jkd\cos\Phi_0}] \quad . \ . \ . \quad (8)$$

The term $H_0\varepsilon^{jkd\cos\Phi_0}$ is the primary field of the incident wave at the slot and it must be subtracted from $H_z + H_z'$, since the primary field is included both in H_z and H_z'.

The slot voltage for the wave incident on the upper side of the sheet is then given by

$$|v| = K\left|F[(2kd)^{1/2}\sin(\Phi_0/2)] + F[(2kd)^{1/2}\cos(\Phi_0/2)] - 1\right| \quad . \ . \ . \ . \quad (9)$$

for Φ_0 in the range 0°–180°.

As a numerical example, the *E*-plane pattern for a thin slot cut in the axial direction on a metal sheet of width $2d$ is computed using eqns. (7) and (9) for $kd = 141$. The pattern plotted in Fig. 6 is normalized in the broadside direction to 0 dB. It is interesting to note that the field in the *direction tangential* to the sheet is 6 dB below the maximum value. This is a characteristic which holds consistently for all sheet widths greater than a few wavelengths. It is worth mentioning that Booker has given a value of 3 dB rather than 6 dB for the reduction of the field along the sheet. His argument is based on physical grounds,

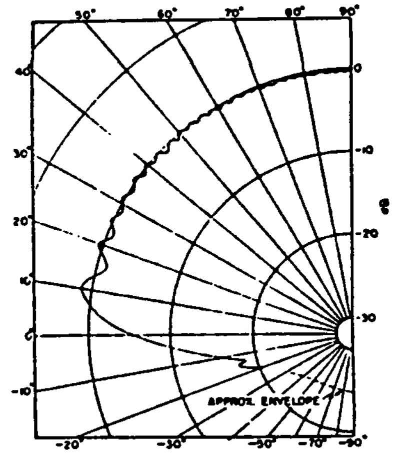

Fig. 6.—Principal *E*-plane pattern of a slot in a sheet of width $2d$ symmetrical about 90°. The plane of the sheet corresponds to 0°.
$kd = 141$
$\theta = 90°$
$V_s = 90°$
N.B.—Curve is calculated.

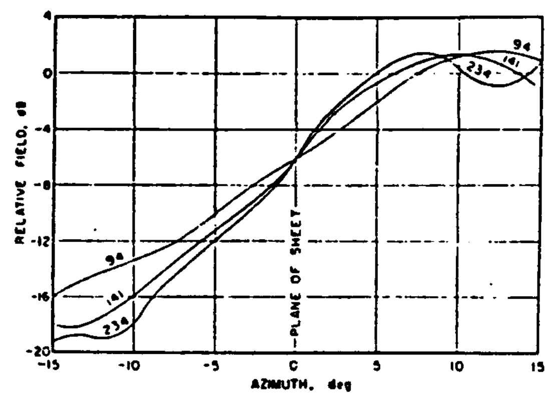

Fig. 7.—Behaviour of the *E*-plane pattern for directions 15° above and below the plane of the sheet.
Value of kd is indicated on the curves.
N.B.—Curves are calculated.

which apparently does not account properly for the energy diffracted around the edges. The calculations were also carried out for $kd = 94$ and 234, and these are shown plotted in Fig. 7 in the interesting transition region for angles within 15° above and below the plane of the sheet. It is noted that the field at the

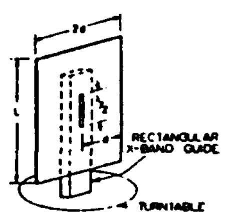

Fig. 8.—Schematic of the waveguide-fed slot mounted on the turntable for the pattern measurements.

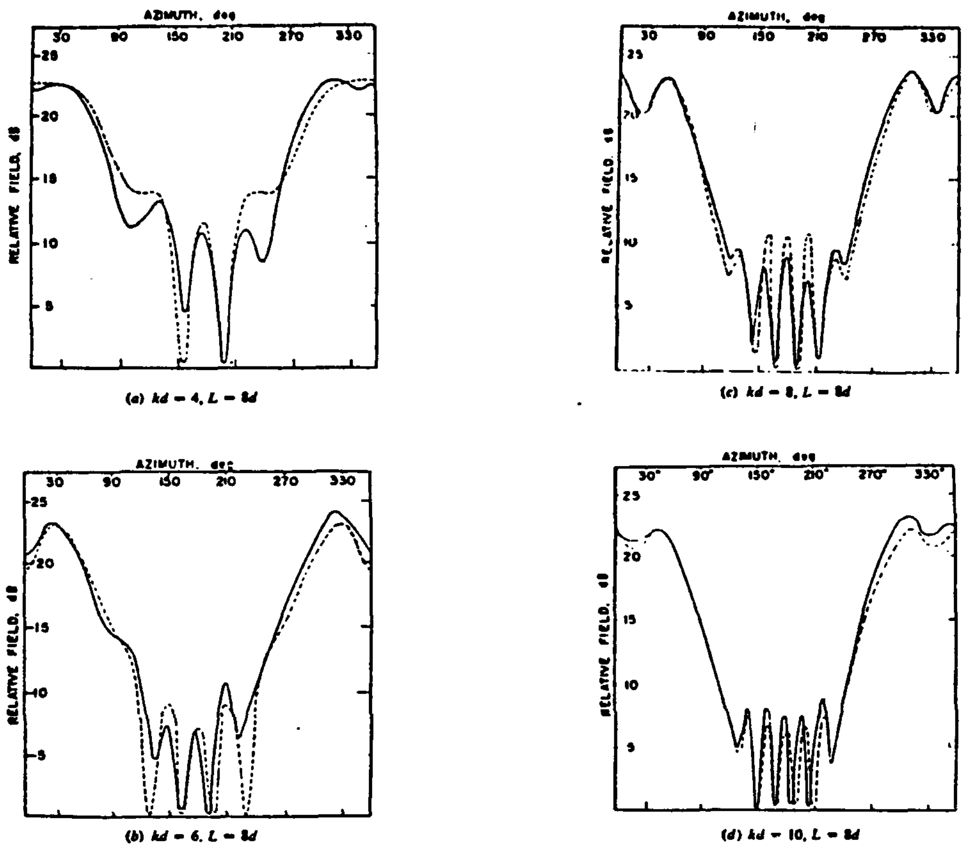

Fig. 9.—Principal *E*-plane patterns of a thin half-wave slot in the centre of a rectangular sheet of length *L* and width $2d$.

—— Experimental.
---- Theoretical.
$\theta = 90°$
$\psi_0 = 90°$
Plate 4·7 ft wide at X band.

rear of the sheet diminishes very slowly with increasing values of $2d$. In fact, it is easy to show from the asymptotic behaviour of the integral F(s), that the amplitudes of the envelopes of the back lobes are approximately inversely proportional to $\sqrt{2d}$.

The above approximate method for calculating patterns of slots in metal sheets of finite width cannot be expected to yield reliable results when $2d$ is of the order of a wavelength or less. A more rigorous approach is to represent the sheet by a thin elliptic cylinder of vanishing minor axis. Computations based on this model have been carried out previously[8] for values of kd ranging from 2 to 8. It was shown that the double knife-edge approximation is accurate to within a few decibels if kd is greater than about 6.

(3) EXPERIMENTAL PATTERNS

In this Section some experimental patterns will be compared with the patterns computed both from the double knife-edge technique and from the elliptic-cylinder method.

The experimental work was carried out on an antenna range at a wavelength of 3·2 cm. A narrow half-wave slot was cut in the broad face of an X-band waveguide illustrated in Fig. 8, with the centre of the slot approximately three-quarters of a guide wavelength from the short-circuited end. The slot was parallel to the axis of the guide and was offset approximately 0·1 in from the centre of the broad face of the guide. A number of thin rectangular aluminium plates were prepared with kd varying from 4 to 141 with length/width ratios from $\frac{1}{2}$ to 20. These plates were designed to be easily mounted on the broad face of the guide. The waveguide assembly was then mounted on a suitable turntable and illuminated by a transmitting dish antenna which was located at a distance of 100 ft. The radiation was incident normally to the axis of the guide and was horizontally polarized. The output from the waveguide was detected and amplified and its varying values were plotted on an ink recorder which had a logarithmic scale.

Measured patterns are shown in Figs. 9(a)–9(h) for kd = 4, 6, 8, 10, 12, 16, 28 and 141, for the case where the slot is centrally located in the plate. Theoretical data are also shown for comparison in some of the curves. The theoretical curve for the case $kd = 141$ is shown in Fig. 6. The vertical scale is arbitrary, and for the sake of convenience the experimental and calculated curves are matched in the direction of the maximum field. The elliptic-cylinder method of calculation was employed for kd values of up to 10 using eqn. (8) or Reference 8 with $\theta = 90°$,

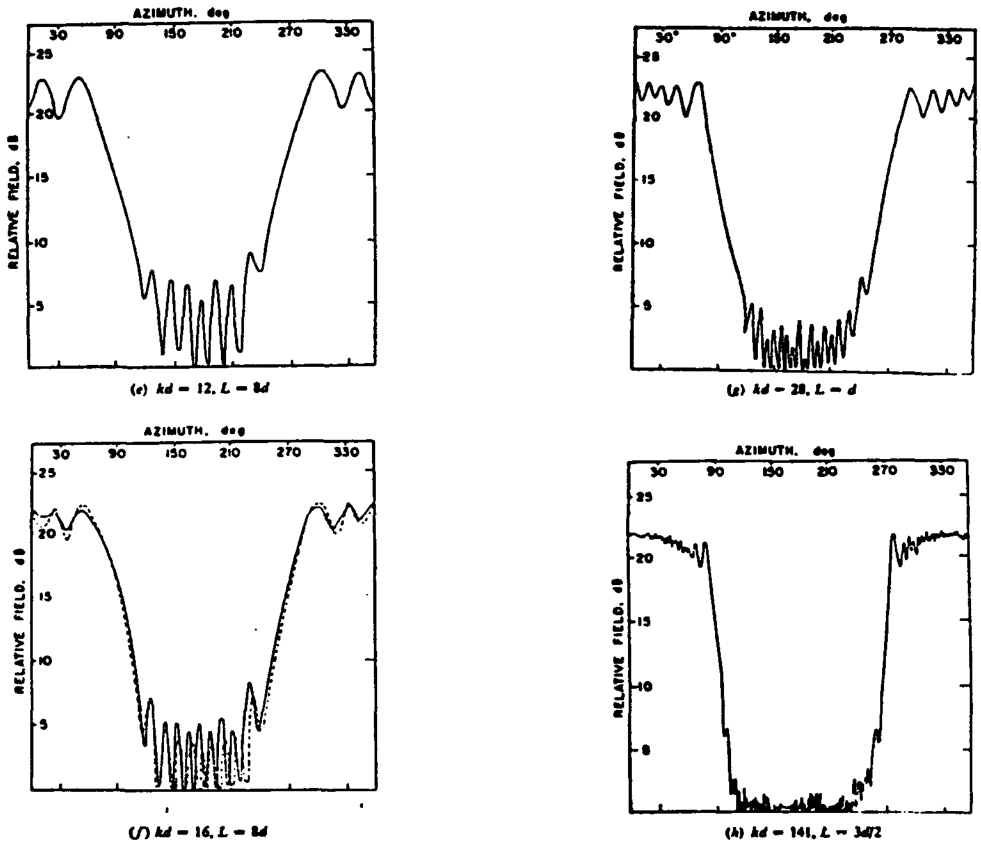

Fig. 9—*continued*

and v_0, the angular co-ordinate of the slot, equal to 90°. For larger values of *kd* the double knife-edge technique was used to obtain the theoretical curves. It should be noted that the azimuth scale of these figures are chosen so that the broadside direction from the slot corresponds to 0°.

The agreement between experiment and theory improves for the larger sheets. The probable reason for the discrepancy for small sheets is that the diffraction by the waveguide behind the sheet is becoming significant. For $kd = 4$ the plate is only about 50% wider than the broad face of the guide, and it is therefore not surprising that the pattern differs from that calculated on the basis of a thin elliptic cylinder. It is also interesting to observe that the experimental pattern for $kd = 4$ is somewhat asymmetrical; this is due, no doubt, to the fact that while the slot is centrally located in the plate, the guide is displaced slightly. The asymmetry is also seen to occur for $kd = 6$, but to a lesser extent.

To illustrate the effect of the length of the plate, patterns were recorded for $L = d$, $4d$ and $20d$, keeping *kd* constant at 6. The similarity between these curves, shown in Fig. 10, is striking, and substantiates the earlier supposition that the azimuthal patterns are determined mainly by the lateral dimension (i.e. $2d$) of the plate for an axial slot. In fact, it can be seen from the curves in Fig. 10 that the plate can be regarded as infinite in length so long as *L* is greater than about $4d$. The main effect of finite length seems to be an increase in the level of the back lobes.

In all the above-mentioned experiments the slot was situated in the centre of the plate. If the slot was displaced toward one edge by an amount $d/2$, the angular elliptic co-ordinate

Fig. 10.—Principal *E*-plane pattern for the sheet of various lengths with constant width.
Effect of length $kd = 6$, $\theta = 90°$, $v_0 = 90°$, $L = d, 4d, 20d$

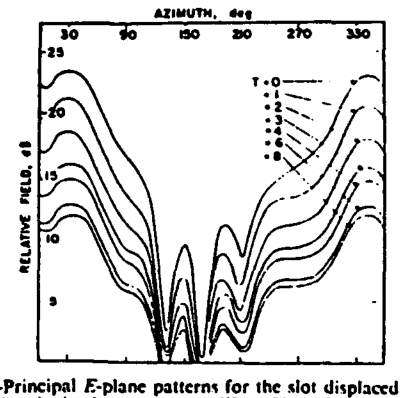

Fig. 11.—Principal E-plane patterns for the slot displaced toward the right-hand edge by an amount $d/2$. (The values of T correspond to the number of layers of tape covering the slot.)

$kd = 4$, $\theta = 90°$, $V_0 = 60°$.

becomes $v_0 = 60°$. *The pattern then became quite asymmetrical* as shown in Fig. 11. This type of asymmetry was also present in the theoretical computations of the offset slot or the thin elliptic cylinder.[8] The other curves in Fig. 11 correspond to the patterns for the slot covered with layers of plastic electrical tape of thickness 7·5 mils. The number of layers is indicated by the value T. It is interesting that the azimuthal patterns are essentially unchanged in shape, which is in accord with theory for a thin slot. It is not possible to draw any further conclusions from this set of curves, since it was not feasible at the time to measure the change of the voltage standing-wave ratio in the guide for the different thicknesses of the dielectric covering. Furthermore, the dielectric properties of the plastic tape are not known.

(4) MEASUREMENT OF SLOT ADMITTANCE

Another important characteristic of a slot cut in a metal surface is its admittance. In the earlier part of the paper it was shown that the admittance at the centre of a thin half-wave slot cut in an infinitely thin conducting sheet of infinite extent, was $2{\cdot}06 + j0{\cdot}97$ millimhos. If the slot is fed by a waveguide, which is located on one side of the sheet so that it radiates only into one of the half-spaces, the conductance at the centre of the slot is 1·03 millimhos and the susceptance, as mentioned earlier, is dependent on the nature of the evanescent structure of the field within the guide.

When the sheet is of finite size the conductance is no longer 1·03 millimhos. The variation of G with the width of the sheet was investigated theoretically by using a model of a thin axial half-wave slot at the centre of the broad face of a thin elliptic cylinder.[8] G was obtained explicitly by computing the power radiated from the slot for a specified voltage at the centre of the slot. It was shown that G was an oscillating function of the width $2d$, and it approached 1·03 millimhos as $2d$ approached infinity.

It is now worth while to examine the admittance using an experimental procedure. The slot of width 1/16 in and length 5/8 in was cut parallel to the narrow face of the X-band waveguide. Means were then taken to mount a series of plates of various widths flush with the narrow face of the waveguide in a similar manner to that employed for the pattern measurements of the slots. The slot was cut in the narrow face rather than the broad face, so that the effect of plates of small width could be examined also.

As is customary in waveguide measurements, the longitudinal slot is represented by an equivalent shunt admittance across the equivalent transmission line of the waveguide. Employing a slotted line in conjunction with an adjustable short-circuit termination in the guide beyond the slot, the conductance and susceptance are determined using a standard technique.[10] These values are normalized by dividing by the characteristic admittance of the guide, which is real. The normalized conductance denoted by g, and the normalized susceptance denoted by b_n, are plotted in Fig. 12 as a function of kd.

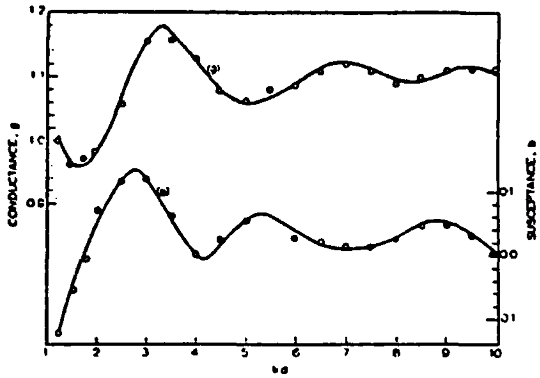

Fig. 12.—Measured normalized conductance and susceptance of the slot as a function of the width of the sheet.

Slot width = 1/16 in
Slot length = 5/8 in
$\lambda = 3{\cdot}20$ cm, $\lambda_g = 4{\cdot}48$ cm

By adapting Stevenson's theory[3] for longitudinal slots near resonance, cut in the narrow face of the waveguide, it is easy to show that the normalized conductance $g(kd)$, as a function of kd, in terms of the actual conductance G at the centre of the slot, is given by

$$g(kd) = \frac{480}{73\pi}\frac{a}{b}\frac{\lambda_g}{\lambda}\cos^2\frac{\pi\lambda}{2\lambda_g}\frac{1{\cdot}03}{G} \quad . \quad . \quad . \quad (10)$$

Stevenson's formula corresponds to eqn. (10) when G is replaced by 1·03; this corresponds to the case of an infinite plate (i.e. $kd \to \infty$). Employing the theoretical results for G, mentioned above, the value of $g(kd) - g(\infty)$ given by eqn. (10) is plotted in Fig. 13 using $a = 0{\cdot}90$ in, $b = 0{\cdot}40$ in, $\lambda = 3{\cdot}2$ cm, $\lambda_g = 4{\cdot}48$ cm, along with the corresponding experimental curve. There is reasonable agreement between the experimental and the

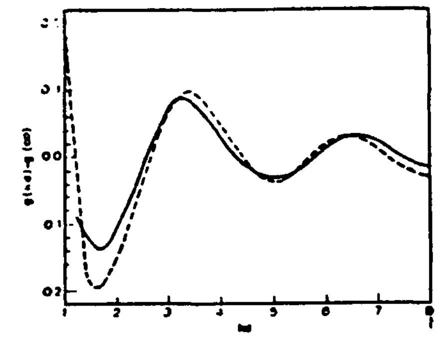

Fig. 13.—Comparison between the experimental and theoretical normalized conductance.

[N.B.—The theoretical value of $g(\infty)$ is 1·20.]
—— Experimental.
---- Theoretical.

computed curves. The disagreement for the smaller values of *kd* can probably be accounted for by the fact that the theory does not account for the diffraction by the waveguide behind the plate. It should also be mentioned that the theoretical value of *g*, which is found using Stevenson's procedure, assumes that the susceptance is much smaller than the conductance (i.e. the slot is near resonance). It can be seen from the experimental results that, although b_s is small, it is not negligible and would, no doubt, also be a source of discrepancy.

(5) CONCLUSIONS

It has been demonstrated that the radiation characteristics of an axial half-wave slot in a rectangular metal plate are mainly a function of the width, rather than the length, of the plate. The measured pattern and radiation conductance of the slot agreed quite closely with theory over a wide range of plate widths. The experimental phase of this project is continuing, and particular attention will be paid to the effect of the finite width and depth of the slot. It is also hoped to examine the effect of curvature of the metal plate in which the slot is cut.

(6) ACKNOWLEDGMENTS

We wish to thank Mr. R. G. Sinclair who ably assisted with the pattern measurements and Messrs. R. E. Walpole and W. A. Pope who carried out most of the computations. Valuable advice was also received from Dr. A. W. Adey in connection with the admittance measurements.

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The Radiation Patterns and Conductances of Slots Cut on Rectangular Metal Plates*

Radiation Patterns

It is easy to predict the form of the principal E-plane radiation pattern obtained from a radiating slot cut on a long metal plate. Consider, for example, the plate of width W, shown in Fig. 1. Let S represent

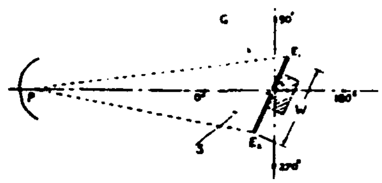

Fig. 1

the slot (whose axis is perpendicular to the plane of the paper) and let G represent the waveguide feeding power to the slot. Very far away from the waveguide and plate is a receiving antenna P.

When the slot is pointing toward P, *i.e.*, in the 0° direction, a large field strength results, but if the slot points away from P (in the 180° direction) the field strength is small, its actual value being determined by the fraction of the total power radiated which is diffracted behind the sheet to P. In the intermediate directions 90° and 270° the field strength at P would lie between the values obtained for the 0° and 180° directions. Thus, in the absence of further complications, the radiation pattern of the metal sheet antenna would resemble the dashed line of Fig. 2(a). Actually one important feature has been overlooked in this development.

It will be appreciated that the edges E_1 and E_2 of the metal sheet form a discontinuity to the field traveling outward from the slot and along the sheet. Radiating line sources will, therefore, be induced near the edges E_1 and E_2 of the sheet and these will come in and out of phase with each other as the antenna is rotated. Thus, instead of the radiation pattern looking like the dashed line of Fig. 2(a), it will resemble the full line of the same figure. The number of interference fringes in a complete rotation of the plate will be greater for wide sheets than for narrow ones since a smaller incremental rotation is required to form a given path difference for a wide sheet. The amplitude of the fringes will be smaller for wide sheets than for narrow ones, however, since the strength of the induced line sources decreases as the sheet width is increased.

From what has been said, the radiation patterns for narrow, moderately wide, wide, and infinitely wide sheets will be like the full lines of Figs. 2(a), 2(b), 2(c), and 2(d), respectively.

* Received by the IRE, June 20, 1956.

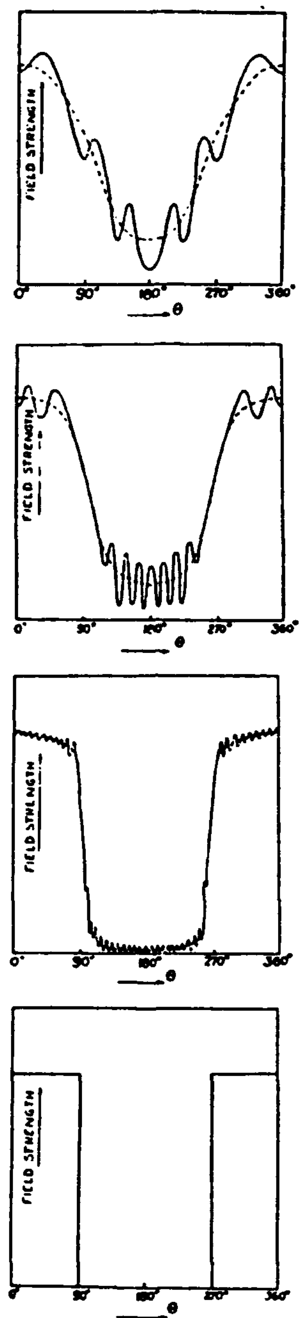

Fig. 2—Azimuth or E-plane radiation patterns for plate width, w, of about 2, 4, 15, and ∞ in wavelength.

Experimentally it was found that the radiation pattern of a plate of given width (W) was independent of its length (L) if:

$$L/W \geq 1.$$

For sufficiently long plates the calculated and measured radiation patterns agreed very closely, but for narrow plates there was some asymmetry in the patterns due to diffraction effects caused by the waveguide behind the plate. For widths greater than two or three free space wavelengths, this effect was negligible, however.

Conductance

Applying simple physical arguments, such as were used to predict the radiation patterns, it can be shown that the conductance (g) of a resonant slot in a flat metal plate is a damped oscillating function of plate width (W) with a period of about one free space wavelength, λ_0, (Fig. 3).

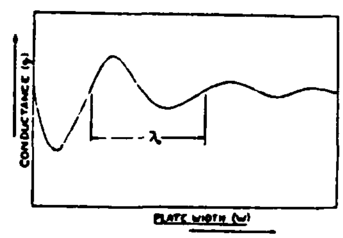

Fig. 3

It was found that the measured and calculated slot conductances, as a function of plate width, agreed very closely except that the measured conductances were always about 10 per cent lower than theoretical ones. This discrepancy may have been due to the fact that theory requires the slot length (l) and slot width (w) to be such that

$$\log_{10}\left(\frac{l}{w}\right) \gg 1.$$

Clearly, this condition is very hard to fulfill in practice.

After the preparation of this paper, further research was done on the method of measuring slot properties. It was found that the technique used above (the quarterwave method) is not, in general, as good as a new method which is independent of the vswr and any reflections from the probe in the slotted line. The essentials of this work will be published shortly. The full mathematical and experimental details of the flat plate antenna discussed here have been given.[1,2]

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Pattern of a Flush-Mounted Microwave Antenna

James R. Wait

The numerical results for the far zone radiation from an axial slot on a circular cylinder of perfect conductivity and infinite length are discussed. It is shown that the results for large diameter cylinders can be expressed in a universal form that is suitable for pattern calculations for arrays of slots on a gently curved surface. The work is compared with a related diffraction problem considered by Fock.

1. Introduction

The radiation from flush-mounted microwave antennas has aroused considerable interest in recent years [1–6].[1] From a theoretical standpoint the problem would seem to be simple enough; the surface on which the antenna is mounted is considered to be adequately represented by a sphere or cylinder whose radius of curvature is matched to that of the local curvature of the actual surface. Unfortunately, the formal solution of the problem, in terms of the classical harmonic series involving integral or half-integral order Bessel functions, always converges very poorly when the radius of curvature is large compared to the wavelength. There are two well-known alternative representations [6, 7], however, which can be used to advantage in certain cases. In the illuminated region of the antenna, geometrical optics is most satisfactory, while deep in the shadow the rigorous harmonic series can be converted into the rapidly converging residue series.

The calculation of the radiation field of a flush-mounted antenna in the tangent plane (the classical light-shadow boundary) is not readily treated by either geometrical optics or the residue series. In the former case the field is indeterminant, and in the latter case the convergence is extremely poor and would actually diverge in the illuminated region. Despite the fact that the harmonic series is cumbersome, it is valid in this transition zone between the illuminated and shadow regions of space. Therefore, it is desirable to attempt to adapt the harmonic-series representation to surfaces of large radius of curvature. This is the purpose of the present paper.

2. Theoretical Basis

A thin axial slot, cut on a circular cylinder of infinite length and perfect conductivity is considered because it gives rise to a plane-polarized radiation field. The cylinder is taken to be of radius a and coaxial with a cylindrical coordinate system (ρ,ϕ,z). The slot that extends from z_1 to z_2 at $\phi=0$, has a voltage distribution $V(z)$ throughout its length. The radiation pattern is best expressed in terms of spherical coordinates (r,ϕ,θ) where $\theta=0$ corresponds to the axis of the cylinder. It has been shown [9] that the electric field, which has only a ϕ component, is given by

$$E_\phi=\frac{e^{-ikr}}{r}S(\theta)M(x,\phi), \tag{1}$$

where

$$S(\theta)=\frac{k\sin\theta}{2\pi}\int_{z_1}^{z_2}V(z)e^{ikz\cos\theta}dz, \tag{2}$$

$$M(x,\phi)=\frac{1}{\pi x}\sum_{m=0}^{\infty}\frac{\epsilon_m e^{im\pi/2}\cos m\phi}{H_m^{(2)\prime}(x)}, \tag{3}$$

$x=ka\sin\theta$, $k=2\pi$/free space wavelength, $\epsilon_0=1, \epsilon_m=2 (m\neq 0)$, and $H_m^{(2)\prime}(x)$ is the derivative of the Hankel function appropriate for a time factor $\exp(i\omega t)$. Equation (1) is valid for $kr\sin\theta\gg 1$.

The function $S(\theta)$ is the space factor of the slot, or an array of collinear slots if they were cut in an infinite plane conducting sheet. The function $M(x,\phi)$ is called the "cylinder space factor" as it fully describes the effect of the finite diameter of the cylinder on the radiation pattern in both the θ and ϕ directions. For purposes of computation, it is written

$$M(x,\phi)=|M(x,\phi)|e^{i\alpha(x,\phi)}. \tag{4}$$

The phase function $\alpha(x,\phi)$ is a rapidly varying function of ϕ for the larger x values. For this reason, it is desirable to express α in the illuminated region $(0<|\phi|<\pi/2)$ as a geometrical-optical term plus a correction factor $\Delta(x,\phi)$, as follows [9]

$$\alpha(x,\phi)=\alpha(x,0)-x(1-\cos\phi)+\Delta(x,\phi). \tag{5a}$$

In the shadow region $(\pi/2<|\phi|<\pi)$, it is desirable to express α as a physical-optical term plus a correction, which is also designated $\Delta(x,\phi)$, in the following manner:

$$\alpha(x,\phi)=\alpha(x,\pi/2)+(\pi/2-\phi)x+\Delta(x,\phi). \tag{5b}$$

The amplitude $|M(x,\phi)|$ and the phase correction $\Delta(x,\phi)$ of the cylinder space factor are shown plotted in figures 1 and 2 for ϕ from 0 to 180° for $x=10$, 12, 15, 18, and 21. To prevent troublesome overlapping, the ordinates are shifted for each curve by a constant amount.

[1] Figures in brackets indicate the literature references at the end of this paper.

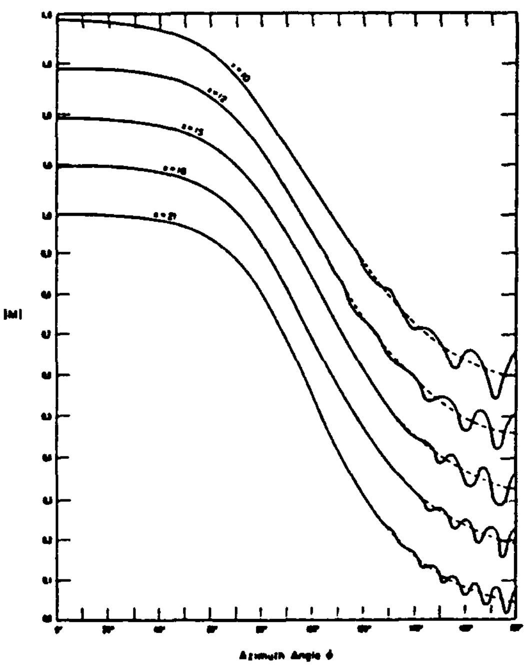

FIGURE 1. *Amplitude of cylinder space factor for narrow axial slot.*

N. B. vertical scale is shifted for each curve.

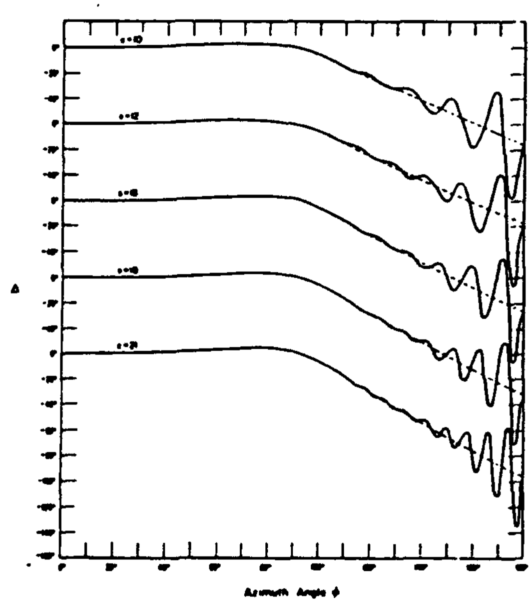

FIGURE 2. *Phase of cylinder space factor for narrow axial slot.*

N. B. vertical scale is shifted for each curve.

For $|\phi|<90°$ but not near 90°, the value of the space factor is quite well approximated by the geometrical-optics approximation. That is

$$\Delta(x,\phi)\cong 0$$

and

$$|M(x,\phi)|\cong 1$$

Deep in the shadow, that is $|\phi|>90°$ but not near 90°, $M(x,\phi)$ has the form of a damped standing wave, and can be characterized by a function of the form

$$A(\phi-\pi/2)e^{-ix(\phi-\pi/2)}+A(3\pi/2-\phi)e^{-ix\left(\frac{3\pi}{2}-\phi\right)}$$

where

$$A(\beta)\cong \exp\left[-i\tau(\pi a/\lambda)^{1/3}\beta\right]$$

with $\tau=0.808\exp(-i\pi/3)$. This exponential form for $A(\phi)$ is the first term of a rather complicated residue series [7]. It is only valid for $ka\gg 1$. The higher order terms (higher modes) are also parametric in $(\pi a/\lambda)^{1/3}\beta$, where β is some angular distance. It is, therefore, logical to expect $M(x,\phi)$ to be parametric in $(x/2)^{1/3}(\phi-\pi/2)$ and $(x/2)^{1/3}[(3\pi/2)-\phi]$. In fact, if the ripples are ignored, the function $M(x,\phi)$ should be a function only of $(x/2)^{1/3}(\phi-\pi/2)$. The dotted curves in figures 1 and 2 are believed to be the appropriate form of $M(x,\phi)$ when the standing-wave effect, due to the interfering traveling wave from the other side of the cylinder, is removed. These "smoothed" curves are then replotted as a function of X, where $X=(x/2)^{1/3}(\phi-\pi/2)$. The results are shown in figure 3a where both the values of amplitude $|M(X)|$ and the phase correction $\Delta(X)$ for $x=10$, 12, 15, 18 and 21 fall on the same set of curves for the range of X indicated. The amplitude is also shown in figure 3b, being plotted on a log scale.

Having the space factor for the cylinder plotted in this universal form suggests that one may extrapolate the results to larger values of x. This concept, embodied in presenting the results in terms of a parameter X, is not unrelated to the notion of angular distance that has been successfully employed in the representation of field strengths of a transmitter at large ranges over a spherical earth [10]. In the latter instance, the angular distance is defined as the angle subtended at the center of the earth between the horizons of transmitting and receiving antennas. In the present situation, the angular distance is $\phi-\pi/2$ since the source antenna is on the cylinder and the observer is at infinity.

While the present problem is concerned directly with the radiation of an axial slot on a circular cylinder, there is a direct application of the results to the reciprocal case where a plane wave is incident on the cylinder (the E vector is to be perpendicular to the cylinder axis). Then it is possible to regard $M(x,\theta)$ as being proportional to the surface current excited on the cylinder. In other words, the axial slot can be regarded as a receiving antenna and the source is taken to be at infinity.

In view of the reciprocal nature of the problem, it is desirable to compare the present results with those

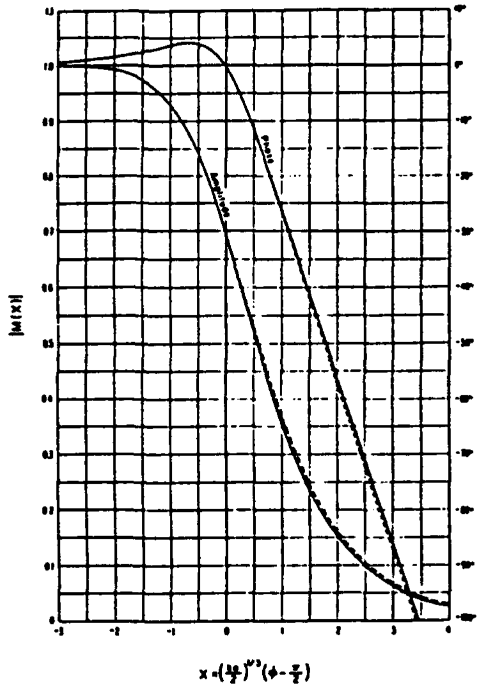

FIGURE 3a. *Amplitude and phase of space factor in parametric form.*

——From circular cylinder data; . . . from Fock.

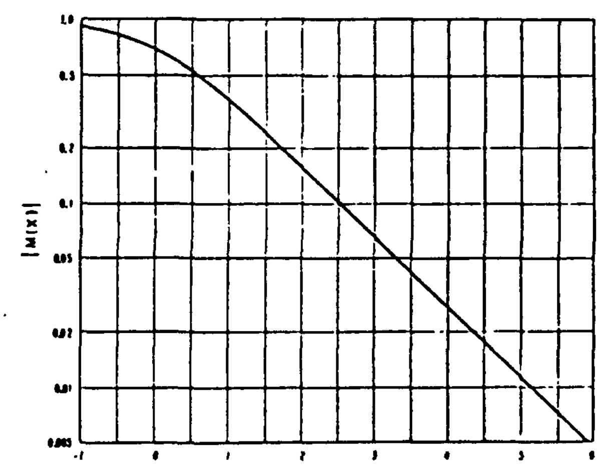

FIGURE 3b. *Amplitude of space factor in parametric form.*

From Circular - cylinder data.

of Fock [11] for the currents excited on a curved surface by an incident plane wave. Fock first considers the diffraction of plane wave by a paraboloid and, in particular, focuses his attention on the tangential magnetic field in the penumbral region (near the boundary of light and shadow). Then, after postulating that the surface currents are only dependent on the local radius of curvature, he obtains a representation for the current distribution that is only dependent on the parameter l/d where l is the distance from the light-shadow boundary and d is the width of the penumbral region. Fock then believes that his results are reasonably valid for any shaped bodies so long as the radius of curvature is always very large compared to the wavelength. As a crucial test of Fock's approximate formula, his results are plotted in figure 3a along with the cylinder computed data. The agreement is very good. There appears to be a slight discrepancy for larger positive X values which is not entirely unexpected because some of Fock's restrictions are becoming violated. Nevertheless, considerable justification is given to the validity of the extrapolation of the cylinder curves to larger values of x.

3. An Application

An interesting application of the foregoing theory is the calculation of the pattern of an end-fire array, of slots on a gently curved surface. For example consider the array of $2N+1$ parallel and axial slots on a cylindrical surface of radius a indicated in figure 4. The angle Φ defines the direction of the observer

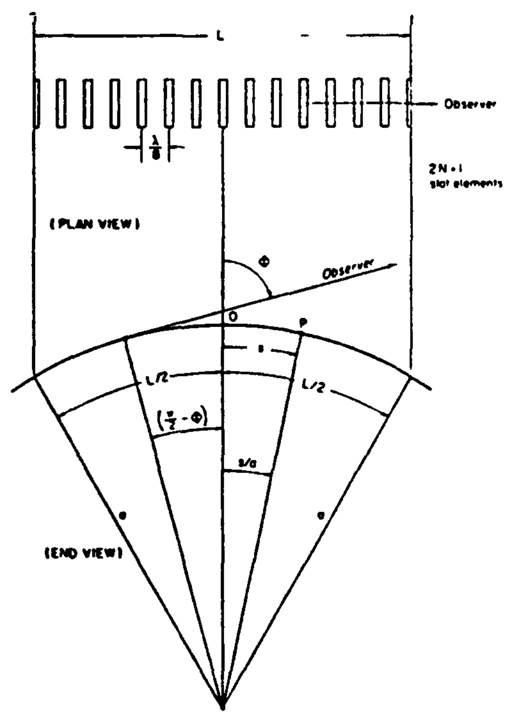

FIGURE 4. *Schematic diagram of end-fire array of slots on a curved conducting surface.*

with respect to the normal at 0, the center of the array. The array and the observer are taken to be in the principal plane (i. e., the plane of the paper). s is the distance from 0, measured along the cylinder surface, to P, the location of any one of the elements. The field due to the source at P is then proportional to

$$e^{iky}K(s)A(s)$$

where $K(s)$ is the relative strength of the element, y is the projection of s on the ray from 0 to the observer and $A(s)$ is the correction factor to geometrical optics. In terms of the function M, $A(s)$ is given by

$$A(s)=M(X) \text{ for } X\leqq 0, \tag{6a}$$

$$\cong M(X)e^{-i\frac{X^3}{3}} \text{ for } X\geqq 0, \tag{6b}$$

where

$$X=-\left[\left(\frac{\pi}{2}-\Phi\right)+\frac{s}{a}\right]\left(\frac{ka}{2}\right)^{1/3}.$$

The factor y is given by $y=c\sin[\Phi-(s/2a)]$ when c is the chord length OP and because

$$c=\left[2a^2\left(1-\cos\frac{s}{a}\right)\right]^{1/2}\cong s\left(1-\frac{s^2}{24a^2}\right), \tag{7a}$$

it follows that

$$y\cong s\left(1-\frac{s^2}{24a^2}\right)\sin\left(\Phi-\frac{s}{2a}\right) \tag{7b}$$

subject to $s<<a$.

Remembering that there are $2N+1$ elements spaced at intervals Δs, it is seen that

$$2N\times\Delta s=L$$

where L is the length of the array. The source is now written

$$K(s)=e^{-\Gamma s}$$

where Γ is the propagation constant of the exciting wave along the array. For the present purpose

$$\Gamma=+imk,$$

where m is a constant real number.

The total field of the array is now proportional to

$$\sum_{n=-N}^{N} e^{iky}e^{-ikms}A(s)$$

where $s=n\cdot\Delta s$ with $n=-N, -N+1, \ldots -1, 0, 1, 2 \ldots N-1, N$. Introducing dimensionless parameters, defined by

$$\Delta S=k\Delta s \text{ and } S=n\Delta S$$

it follows that the pattern $T(\Phi)$ of the array is given by

$$T(\Phi)=\sum_{n=-N}^{N} e^{i(Y(S)-mS)}A(X), \tag{8a}$$

where

$$Y(S)=S\left(1-\frac{S^2}{24(ka)^2}\right)\sin\left(\Phi-\frac{S}{2(ka)}\right) \tag{8b}$$

and

$$X=-\left[\left(\frac{\pi}{2}-\Phi\right)+\frac{S}{ka}\right]\left[\frac{ka}{2}\right]^{1/3}. \tag{8c}$$

When ka tends to infinity such that the surface is effectively flat, the pattern becomes

$$T(\Phi)=T_0(\Phi)=\sum_{n=-N}^{N} e^{i(Y(S)-mS)}, \tag{9a}$$

where $Y(S)=S\sin\Phi$ and since $S=n\cdot\Delta S$,

$$T_0(\Phi)=\frac{e^{-iNK}(1-e^{i(2N+1)K})}{1-e^{iK}} \tag{9b}$$

where $K=\Delta S[\sin\Phi-m]$.

According to the analysis of Hansen and Woodyard [12], the maximum gain in the end-fire direction (i. e., $\Phi=90°$) is obtained when $m\cong(N+2)/N$. Then, taking the electrical spacing between the elements as 45° or $\Delta S=\pi/4$, the array has 65 elements, $N=32$ and the total length of the array is 8 wavelengths. The pattern $T_0(\Phi)$ in this case is only defined for the region $-90°<\Phi<90°$. The main lobe and the first side lobe are shown plotted in figure 5 normalized such that the maximum field is 0 db. Using the same values of N and ΔS, the corresponding pattern $T(\Phi)$ is plotted for $ka=200$. The main lobe is seen to be somewhat changed in

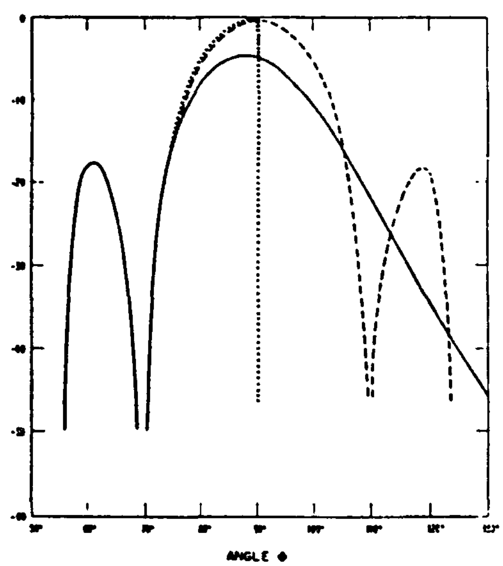

FIGURE 5. *Pattern of end-fire array of slots on a curved surface.*

...... $ka=\infty$; ——— $ka=200$; — — — $ka=200$, $A(X)=1$; array length$=8\lambda$; number of elements$=65$.

form, particularly in the directions near $\Phi=90°$ which is the tangent plane of the center of the array. The decibel level of the field in the diffraction zone ($\Phi>90°$) is seen to decrease linearly with angle. There are no lobes formed in this region. It is of interest to know how the curvature of the array itself would modify the pattern. Therefore, for sake of comparison, the pattern $T(\Phi)$ of the end-fire array of isolated elements on a circular arc ($ka=200$) is also plotted in figure 5. The working formula here is identical to equation (8a) with $A(X)$ being replaced by 1 because diffraction effects are no longer present. The pattern is almost identical to that for an end-fire array where elements are on a straight line. Lobes, in this case, are formed on both sides of the main beam although they are not quite symmetrical about it.

The pattern $T(\Phi)$ is also computed for $ka=100$, and is shown plotted in figure 6 along with the curve for $ka=\infty$ and the curved array of isolated elements.

It would seem that an end-fire array of slots on a curved surface has some rather interesting properties. The main effects of the non-flatness of the surface is to tilt the maximum of the main beam up slightly, and to extend the total width of the main beam into the shadow region. There is no evidence of lobes on this side of the beam, although at large angles (Φ approaching 180°), there is the possibility that the back lobe from the end-fire array could creep around the surface in the other direction and form lobes. This effect from a practical standpoint, however, is completely insignificant for $ka>100$.

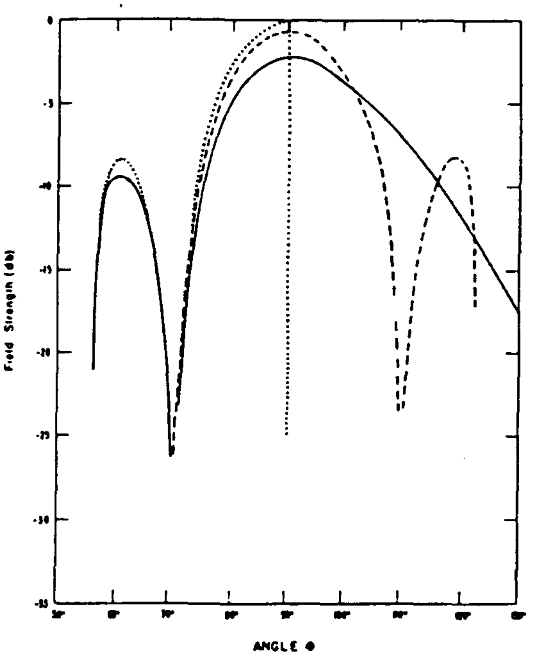

FIGURE 6. *Pattern of end-fire array of slots on a curved surface.*

...... $ka=\infty$; ——— $ka=100$; — — — $ka=100$, $A(X)=1$; array length $=8\lambda$; number of elements $=65$.

4. Concluding Remarks

It is seen that the radiation pattern of a flush-mounted antenna is influenced, to a considerable extent, by the radius of curvature of the surface on which it is mounted. In most cases, the main beam is tilted upward away from the surface, although the total width of the beam can be broadened into the shadow. Similar phenomena had been observed in the calculated patterns of slots on a conducting half-plane [13]. It would appear that any attempt to reduce the side lobes of an end-fire array by using a diffracting edge or surface will lead to a broadening of the main beam. It is also quite apparent that the resultant pattern of a flush mounted antenna is not simply obtained by multiplying the free space pattern by the pattern of a single slot on the curved surface. Such a process would produce a pattern with lobes in the diffraction or shadow region which do not exist in reality for a nonclosed surface. Furthermore, this "multiplication" technique would not predict the broadening of the beam in the tangent plane.

The author thanks William Briggs for his assistance in the calculation of the patterns in figures 5 and 6.

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(Reprint No. 121)

A Study of Earth Currents Near a VLF Monopole Antenna with a Radial Wire Ground System*

The efficiency of antennas for very low radio frequencies is determined to a large extent by the ohmic losses in the soil near the base of the antenna. It is customary to install a radial wire ground system buried just below the surface of the earth. The purpose of this wire grid is to provide a low-loss return path for the antenna base current in an effort to improve the efficiency of transmission.

The rules for ground system design in the past have been usually empirical and based on the results of experiments on existing installations. The first attempt to design an optimum system was carried out by Abbott.[1] Extensive calculations of the input resistance of a monopole with a radial wire ground system have been carried out by Wait and Pope.[2,3] It should be emphasized that in this latter work no attempt was made to evaluate the losses associated with high-voltage insulators, tuning coils, and copper losses. The attention was devoted to the ohmic losses in soil and their dependence on number and length of radial wires. Furthermore, the radial wires were assumed to be in intimate contact with the soil being located just below the air-earth interface. The working formula for the component of the input resistance, ΔR, due to ohmic losses in the soil is given by

$$\Delta R \cong \text{real part of } \frac{1}{I_0^2}\int_0^\infty [H_t^\infty]^2 Z(\rho) 2\pi\rho d\rho \quad (1)$$

where I_0 is the base current of the antenna, H_t^∞ is the tangential magnetic field of the antenna assuming a perfectly conducting ground plane, ρ is the distance measured along the ground plane from the base of the antenna, and $Z(\rho)$ is the effective surface impedance of the actual ground plane. $Z(\rho)$ was taken to be equal to the intrinsic impedance of the soil, η, in parallel with the surface impedance of the radial wire grid, Z_s. That is,

$$Z(\rho) = \frac{\eta Z_s}{\eta + Z_s} \quad \text{for } \rho < a$$
$$= \eta \quad \text{for } \rho > a \quad (2)$$

where

$$\eta = \left[\frac{i\mu\omega}{\sigma + i\epsilon\omega}\right]^{1/2} \cong \left[\frac{i\mu\omega}{\sigma}\right]^{1/2}$$

and

$$Z_s = \frac{i\mu\omega d}{2\pi} \ln \frac{d}{2\pi c}$$

where a is the length of the radial wires, μ is the permeability of the ground ($\cong 4\pi \times 10^{-7}$), ω is the angular frequency, σ is the ground conductivity in mhos/meter, ϵ is the dielectric constant of the ground, c is the radius of the radial wires, and d is the spacing between the wires. (If there are N radial wires equally spaced about the base of the antenna, $d \cong 2\pi\rho/N$.)

The validity of (2) for the composite surface impedance was not established in the above mentioned work. The expression used for Z_s, the equivalent shunt impedance for a wire grid, was taken to be the same as that for an infinite parallel wire grid in free space. A recent analysis indicates that the equivalent shunt impedance for an infinite wire grid in the plane interface of two homogeneous media is indeed almost identical to Z_s for the isolated grid if the interwire spacing, d, is always much less than a wavelength in the electrically denser medium.[4] In the present situation, this restriction is equivalent to stating that d should be somewhat less than an electrical skin depth in the soil, that is $d \ll (2/\sigma\mu\omega)^{1/2}$. In most practical cases this condition is met. A question also rises as to the applicability of the formula for surface impedance of a parallel wire grid to a radial wire grid system where the wire spacing is not constant. Furthermore, the wires are not of infinite length being terminated by rods or some other means at a finite distance from the base of the antenna. The assumption that the radial wire behaves locally as a parallel infinite wire grid would seem to be very difficult to justify on purely theoretical grounds. It could be expected, however, that, if the length of the radial wires is large compared to a skin depth in the soil, the wave reflected from the end of the radial wires would be highly attenuated.

As a check of the wire grid theory for antenna ground loss calculations, an experimental test was carried out in Cutler, Me., which is the proposed site of a high-power (1-megw) vlf transmitter for the U. S. Navy. A small test antenna was erected on the site and radial wires were installed in the same manner as in a permanent installation. The actual currents carried by the wires were measured using a small loop pickup placed in proximity to the wires. The average currents carried by the ground were also measured. Now the radio of the total current carried by the wires, I_w, to the current in the soil, I_s, is equal to the ratio of the surface impedance, η, of the soil to the surface impedance, Z_s, of the grid.[5] Therefore, such a test of the splitting of the current between the radial wires and the ground is a good check on the validity of the theory.

Before discussing the experimental results[5] calculations of I_w are presented for pertinent values of the parameters. Denoting the effective antenna height by h, the tangential magnetic field, H_t^∞, on the ground plane at distance ρ from the base is given by

$$H_t^\infty \cong \frac{I_0}{2\pi} \frac{h}{\sqrt{\rho^2 + h^2}}$$

for $\rho \ll$ wavelength. Actually, H_t^∞, is the radial current density in amperes per meter emanating from the antenna for an idealized perfectly conducting ground plane. In the case of the imperfect conducting ground with the radial wire ground system present, the total ground current, I_t, is essentially numerically equal to H_t^∞ and is composed of two parts, I_s, the total earth current and, I_w, the total wire current. We can now write, for purposes of computation,

$$X = \left|\frac{I_w}{I_t}\right| = \left|\frac{1}{1 + \eta/Z_s}\right|$$
$$= \frac{\sqrt{2}p}{[p^2 + (p + q)^2]^{1/2}}$$

where

$$p = 120\pi\delta/\sqrt{2} \qquad \delta = (\epsilon_0\omega/\sigma)^{1/2}$$

and

$$q = \frac{240\pi^2 P}{N} \ln \frac{P}{NC},$$

$C = c/\lambda$, $P = \rho/\lambda$. The amplitude of the current in one radial wire, i_w, is then given by

$$i_w = \frac{|I_w|}{N} = \frac{I_0}{2\pi N} \frac{h}{\sqrt{\rho^2 + h^2}} X.$$

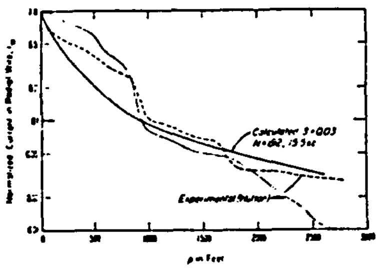

Fig. 1—Decay of current in radial wire as a function of distance ρ from base of antenna.

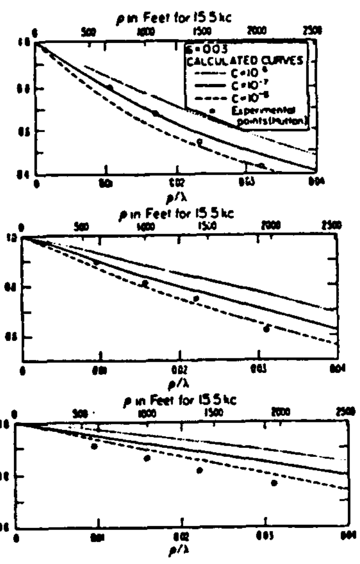

Fig. 2—Comparison of calculated and observed currents in radial wire, for N = 48, 96, and 192 reading from top to bottom.

In Fig. 1 the computed current, i_w, normalized to unity at $\rho = 0$, is shown plotted as a function of ρ in feet for $h = 100$ feet, $\lambda = 63{,}400$ feet (15.5 kc), $N = 192$, $\delta = 0.03$ ($\sigma = 1$ milli-mho/meter), $C \cong 10^{-7}$ (no. 8

* Received by the IRE, March 11, 1958.

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wire, awg at 15.5 kc). In Fig. 2, computed values of X are shown plotted as a function of ρ/λ for $N=48$, 96, and 192, respectively. Various values of C are indicated on the curves.

In the experimental setup, the antenna was a monopole with a height of 100 feet with a circular capacity hat with a radius of about 200 feet. In view of the difficulty in clearing land in the heavily wooded and rocky terrain of Maine, it was not feasible to employ many radials emanating in all directions from the antenna. To effect a compromise three radials at azimuth angles of 0°, 180°, and 270° were installed. The sector between about 60° and 100° was then selected as the region for further tests. In case 1, this sector was filled with radials at an angular separation of 7.5°, in case 2 it was 3.75°, and in case 3 it was 1.88°. For these three separations the equivalent value of N would be 48, 96, and 192 respectively.

The indicated points in Fig. 1 are measured current values in two of the central wires of the fan as a function of ρ. The ordinates are normalized to unity at $\rho=0$. In this case the base current of the antenna was kept constant at about 1.0 ampere. There is a general agreement with the calculated curve. Discrepancies could be ascribed to the varying nature of the soil conductivity along the radial wires. Measurements of soil conductivity by the four-probe method indicated a random variation of 50 per cent about 1.0 milli-mho/meter. The abnormally large measured values of i_w for the smaller values of ρ in Fig. 1 might be ascribed to the asymmetrical variation of the total earth current resulting from using a fan of radials rather than a complete array equally spaced for 360°.

A more appealing experiment which overcomes to some extent the asymmetry of the experimental setup is carried out as follows. The receiving coil is moved in a direction transverse to the radial wires at a constant height and constant distance ρ. The measured magnetic field is then essentially constant for the region between the wires and rises to rather pronounced maxima over the wires. The ratio of the maximum field to that between the wires is denoted by B which is given by the relation

$$B \cong \frac{|I_w|/2\pi s}{|I_e|/d}$$

where s is the height of the receiving or pickup coil above the wire and d is their separation. Now, since *the current in the wire lags the current in the ground by 45°*, it follows that[4]

$$\frac{I_w}{I_e} = \frac{2\pi s}{d} B e^{i\pi/4}$$

or

$$X = \left|\frac{I_w}{I_e + I_w}\right| = \left[\frac{2A^2}{1 + 2A + 2A^2}\right]^{1/2}$$

with

$$A = \frac{\sqrt{2}\pi s}{d} B.$$

Values of X calculated from the experimental data are shown plotted in Fig. 2, for $N=48$, 96, and 192 respectively for $s=4$ feet and $d=2\pi\rho/N$. The agreement between the experimental points and the calculated curve for $C=10^{-7}$ (no. 8 wire at 15.5 kc) is quite good for the two cases of $N=48$ and 96. The departure for the case $N=192$ can be attributed to contribution from the current in the wires to the measured field in the minimal position.

In general, one may say that satisfactory agreement between theory and experiment has been reached. Therefore, further justification is given to the validity of the published calculations of the ground loss component of the base resistance of a monopole with a radial conductor ground system. It should be mentioned, however, that the present study has not considered losses within the aperture or near the base of the antenna. Recent studies by Gustafson, Devaney, and Smith[4] indicate that these losses may be quite large in special antennas for high-power vlf installations.

The author wishes to thank A. D. Watt and T. E. Devaney for helpful advice, C. E. Smith and W. G. Hutton for permission to quote their experimental results, and Anabeth Murphy for assistance with the calculations.

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[4] W. E. Gustafson, T. E. Devaney, and A. N. Smith, "Ground System Studies of High Power V.L.F. Antennas," *paper no. 38, Record of the VLF Symposium*, Boulder, Colo.; January 23–25, 1957.

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7

LES DIAGRAMMES DE RAYONNEMENT D'UNE ANTENNE A FENTES PLACÉES SUR UN PLAN HORIZONTAL LIMITÉ ET IMPARFAIT

THE PATTERNS OF A SLOT-ARRAY ANTENNA ON A FINITE AND IMPERFECT GROUND PLANE

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The radiation from a slot aperture on a perfectly conducting half-plane is discussed in some detail. The extension to end-fire arrays of slots is also treated. Numerical computations are carried out for radiation patterns. It is shown that in general, the main beam is tilted up and away from the edge of the half-plane. The situation where the half-plane is lying on the surface of a homogeneous flat earth is also considered. The results have application in the design of flush mounted antennas of the end-fire type where, because of practical limitations, the ground plane is of finite extent and may be located on the surface of a «lossy» plane earth.

Introduction

The concept that highly directive antennas can be mounted flush to a metallic surface such as the fuselage of a high speed aircraft has been successfully utilized in many instances in recent years. At very short wavelengths, the surface on which the antenna is mounted or embedded can often be considered flat and infinite in extent for purposes of design. In some cases, however, the effect of the finite extent of the ground plane must be considered. An example is when the pattern is of the «end-fire» type such as a linear array of slots excited by a travelling wave. The main lobe in this case is in the end-fire direction if the travelling wave has a velocity equal to or slightly less than the speed of light. The finite extent of the ground plane tends to reduce the directivity and produces a tilt of the main beam away from the surface.

Another important example is when a travelling wave antenna is mounted on a metallic plane of finite extent which itself is lying on the surface of the earth. Antennas of this type are used to obtain a low-angle beam for aircraft landing devices.

It is the purpose of this paper to study this problem from a theoretical standpoint. The model chosen for the ground plane is an infinitesimally thin perfectly conducting half-plane. The array is considered to be made up continuous and discrete distributions of parallel magnetic sources or slots mounted on one (the upper) side of the half-plane. The following effects will be considered : the distance of the array to the edge of the half-plane, the length of the array, the velocity of the exciting travelling wave, and the electrical constants of the half-space on which the half-plane is lying.

Earlier Investigations

There have been a number of prior investigations which are closely related to the subject of this paper. It is desirable if these are mentioned and placed in their proper perspective. Radiation patterns of a magnetic line source adjacent to a perfectly isolated half-plane were computed by Wait[1] for a variety of cases. The generalization to arbitrary sources such as an aperture or finite slots has been carried out by Tai[2] and others[3, 4]. The radiation from a parallel array of line magnetic sources mounted on a half-plane has been considered by Hurd[5]. He actually presented computed patterns which illustrated the beam tilt of end-fire arrays due to the finite distance of the array from the edge of the half-plane.

The influence of a homogeneous lossy flat earth on the radiation pattern of a vertical antenna is included in the comprehensive analysis by Norton[6]. The effect of a circular ground screen on the pattern of a ground based monopole antenna has been approached by a number of investigators[7-9]. These methods are usually based on the assumption that the influence of the imperfect ground beyond the edge of the screen can be treated by a perturbation method in conjunction with an approximate (Leontovich) boundary condition. An exception is the work of Bekefi[10] who provides a variational formulation for a vertical electric dipole at the center of a circular metal disk laid on a plane earth. It appears, however, that due to an oversimplified choice of trial function for the tangential electric field over the ground, the derived results for the propagation constant of the surface wave is in serious conflict with reality. Actually the velocity of the surface wave should approach the velocity, c, of free space as the refractive index n of the ground approaches infinity, whereas Bekefi's surface wave velocity is approaching a value less than c/10. While this might not greatly impair the impedance calculation, it would have a profound effect on the derived radiation pattern. It would be worthwhile to improve Bekefi's method by choosing a more recondite trial function.

In a very recent paper Carwell and Flammer[11] have calculated the pattern of a vertical electric dipole located on a half-plane which is located on the surface of a homogeneous flat earth or half-space. Using an argument based on Green's theorem they derive an expression for the radiation field in terms of the (unknown) electric tangential field over the earth's surface outside the half-plane. It is then assumed that this electric field is unmodified by the presence of the half-plane. Such an assumption violates the edge singularity at the edge of the half-plane. It can be shown, however, (see appendix) that such an approach is probably valid at low angles and for a highly conducting earth.

A mathematically rigorous approach to the problem of a plane wave incident on a conducting half-plane which is lying on a homogeneous flat earth has been carried in an elegant fashion by Clemmow[12]. The scattered field is expressed as an angular spectrum of plane waves for the general case. When the incident wave is due to a line magnetic source on the imperfectly conducting flat earth and the receiver is on the half-plane, various reasonable approximations can be made and the final solution for the «ground-to-ground» field can be expressed in terms of a modified Fresnel integral. Clemmow's analysis is both extensive and complicated, even though the solutions themselves reduce to quite simple forms in many of the cases of practical interest. In view of this fact, Senior[13] has suggested a method which is a union of ray theory and rigorous diffraction theory. The presence of the plane earth is accounted for by the introduction of a suitable image field and the problem thereby reduced to one in which two fields are incident obliquely upon a perfectly conducting half-plane in free space. Since Senior's method is closely related to the one used in the present paper more will be said about it in the following text and also in the appendix.

This paper was presented by P. Robieux of C.S.F. to whom the authors are deeply indebted.

Single Slot on a Half-plane

With reference to a conventional cylindrical coordinate system (ρ, Φ, z), a slot of length $2l$ is located on a perfectly conducting half-plane sheet defined by $\Phi = 0, 2\pi$. The slot is parallel to and a distance ρ from the edge of the half-plane sheet[1]. The slot is assumed to be narrow and radiates only on one side of the sheet. The electric field in the far or radiation field in the equatorial plane (i.e. $z = 0$) has only Φ component and is given by

$$E_\Phi = \frac{ik}{4\pi\rho_o}\left[\int_{-l}^{+l} V(z')\,dz'\right] \exp(-ik\rho_o + i\omega t)\,Q \quad (1)$$

where $V(z')$ is the distribution of transverse voltage along the slot and where Q is a function of $k\rho$ and Φ_o the azimuthal angle. Two alternative representations are available for Q; the first is

$$Q = \frac{1}{2}\sum_{m=0}^{\infty} \epsilon_m i^{m/2} \cos\left(m\Phi_o/2\right) J_{m/2}(k\rho) \quad (2)$$

where $\epsilon_o = 1$, $\epsilon_m = 2$ $(m \neq 0)$ and $J_{m/2}(k\rho)$ is a Bessel function of order $m/2$ and argument $k\rho$. The second is

$$Q = e^{ik\rho\cos\Phi_o}\left(\frac{i}{\pi}\right)^{\frac{1}{2}} \int_{-\infty}^{p} e^{-it^2}\,dt \quad (3)$$

where $p = (2k\rho)^{\frac{1}{2}} \cos(\Phi_o/2)$. For present purposes the latter form is desirable and it can be written in terms of Fresnel integrals as follows

$$Q = e^{ik\rho\cos\Phi_o} F(u) \quad (4)$$

where

$$F(u) = \frac{1}{2}\left\{1 \pm (1+i)\left[C(|u|) - i\,S(|u|)\right]\right\} \quad (5)$$

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2}t^2\right) dt,$$

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right) dt,$$

$$u = (4k\rho/\pi)^{\frac{1}{2}} \cos(\Phi_o/2) = (8\rho/\lambda)^{\frac{1}{2}} \cos\Phi_o/2,$$

and where the upper (+) sign is to be employed when u is positive and the lower sign (-) when u is negative.

The function $F(u)$ characterizes the radiation pattern of a single axial slot radiating on one side of a half-plane conducting sheet in terms of ρ the distance of the slot to the edge and the azimuthal angle Φ. $F(u)$ is readily computed from tables of Fresnel integrals. To illustrate its properties $|F(u)|$ is plotted in figs. 2A for $u > o$ and in fig. 2B for $u < o$. The phase Ψ, defined by $\Psi = -\arg F(u)$ is plotted in figs. 2C for $u > o$ and fig. 2D for $u < o$. It can be seen that for u sufficiently positive $|F(u)|$ is approaching unity indicating that the half-plane is behaving essentially as an infinite plane conductor. At $u = o$ the value of $F(u)$ is identically 0.5. When u becomes large in the negative direction, $F(u)$ asymptotically approaches zero. The limiting behavior of the function is summarized in the following

$$F(u) \cong 1 + \frac{e^{i\frac{3\pi}{4}}}{\sqrt{2}\pi u} e^{-i\frac{\pi}{2}u^2} \quad \text{for } u \to +\infty \quad (6a)$$

$$= 0.5 \quad \text{for } u = 0 \quad (6b)$$

$$\cong \frac{e^{-i\frac{\pi}{4}} e^{-i\left(\frac{\pi}{2}\right)u^2}}{\sqrt{2}\pi u} \quad \text{for } u \to -\infty \quad (6c)$$

Slot Arrays on a Half-Plane

The extension of the foregoing to an array of parallel N axial slots at $\rho_1, \rho_2, \ldots\rho_n, \ldots\rho_N$ from the edge of the half-plane is now carried out. The situation is illustrated in fig. 3a where the observer is again assumed to be in the equatorial plane $(z = 0)$ and Ψ is the 180° complement of the azimuthal angle Φ used previously. Each slot is allowed to have an arbitrary distribution of voltage $V_n(z)$ and further they may be of arbitrary lenght $2l_n$. The excitation coefficient for each slot is defined by

$$A_n = \int_{-l_n}^{l_n} V_n(z)\,dz \quad (7)$$

where $V_n(z)$ is the transverse (complex) voltage along the nth slot with the phase reference at $z = 0$ and $\rho = \rho_n$. From the principle of superposition it then follows that the pattern of the array in the equatorial or principal plane is

$$G(\Psi) = k\sum_{n=1}^{N} e^{i\beta_n\rho_n} e^{-ik\rho_n\cos\Psi} A_n F(u_n) \quad (8)$$

where

$$u_n = \left(4k\rho_n/\pi\right)^{\frac{1}{2}} \sin\Psi/2$$

and where β_n is phase factor which specifies the relative phase of each of the slots. For example, the phase difference between ρ_n and ρ_{n+1} is $\rho_n\beta_n - \rho_{n+1}\beta_{n+1}$ radians. In what follows, A_n is always real and β_n is a constant which is replaced simply by β. In other words, the slots are excited by a travelling wave with a phase velocity ω/β. Furthermore, the slots are to be equally spaced with a separation Λ between centers. It then follows that

$$\rho_n = D + \left(n - \frac{1}{2}\right)\Lambda$$

and the length of the array is

$$L = \Lambda N$$

Consequently

$$(\beta - k\cos\Psi)\rho_n = \left(\beta - k + 2k\sin^2\Psi/2\right)\rho_n$$

$$= \left(g\pi + X^2\right)\left(\frac{a}{1-a} + \frac{n-\frac{1}{2}}{N}\right)$$

if $g = (\beta - k)L/\pi$, $X = (2kL)^{\frac{1}{2}} \sin(\Psi/2)$ and $a = D/(D+L)$[2].

The pattern can now be written conveniently in terms of the dimensionless parameters g, X and a:

$$\frac{G(\Psi)}{kL} = \frac{2}{N}\sum_{n=1,2,3\ldots}^{N} \exp\left[i\left(g\pi + X^2\right)\left(\frac{a}{1-a} + \frac{n-\frac{1}{2}}{N}\right)\right] A_n F(u_n) \quad (9)$$

(1) See fig. 1.

(2) The parameter X is plotted as a function of Ψ in fig. 3B.

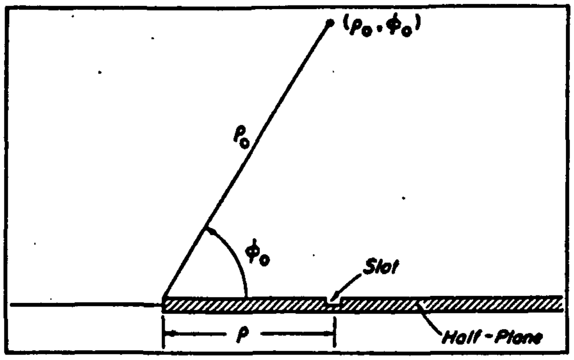


Fig 1

where

$$u_n = X\left[\left(\frac{\alpha}{1-\alpha} + \frac{n-\frac{1}{2}}{N}\right)\frac{2}{\pi}\right]^{\frac{1}{2}}$$

When the plane is infinite in extent, the pattern is identical to the above if $F(u)$ is replaced by unity. In this case, diffraction effects due to the edge of the half-plane are ignored. Furthermore, if A_n were unity corresponding to a uniform amplitude of excitation, the series can be summed to give

$$\frac{G(\Psi)}{kL} = 2\exp\left[i\left(g\pi + X^2\right)\left(\frac{\alpha}{1-\alpha} + \frac{1}{2}\right)\right]\frac{\sin\left[\left(g\pi + X^2\right)\frac{1}{2}\right]}{N\sin\left[\left(g\pi + X^2\right)\frac{1}{2N}\right]} \quad (10)$$

The pattern of a continuous distribution of axial slots is obtained by letting N become indefinitely large but keeping L fixed such that

$$L = N\,d\rho$$

where $d\rho$ is the (infinitesimal) separation between the slot elements. This leads to

$$\left.\begin{aligned} \frac{G(\Psi)}{kL} = {} & 2\,\frac{\exp\left[i\left(g\pi + X^2\right)\frac{1}{1-\alpha}\right]}{i\left(g\pi + X^2\right)}F(v_1) - \\ & - 2\,\frac{\exp\left[i\left(g\pi + X^2\right)\frac{\alpha}{1-\alpha}\right]}{i\left(g\pi + X^2\right)}F(v_2) + \\ & + \frac{X(i-1)}{(g\pi)^{\frac{1}{2}}\left(g\pi + X^2\right)}\left[G(W_1) - G(W_2)\right] \end{aligned}\right\} \quad (11)$$

where

$$G(W) = C(W) + i\,S(W) = \int_0^W \exp\left(i\frac{\pi}{2}t^2\right)dt.$$

$$v_1 = X\left(\frac{2}{\pi}\frac{1}{1-\alpha}\right)^{\frac{1}{2}}, \quad W_1 = \left(2g\frac{1}{1-\alpha}\right)^{\frac{1}{2}}$$

$$v_2 = X\left(\frac{2}{\pi}\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}, \quad W_2 = \left(2g\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2}}$$

The corresponding pattern for the infinite ground plane (ie. diffraction effects ignored) is simply

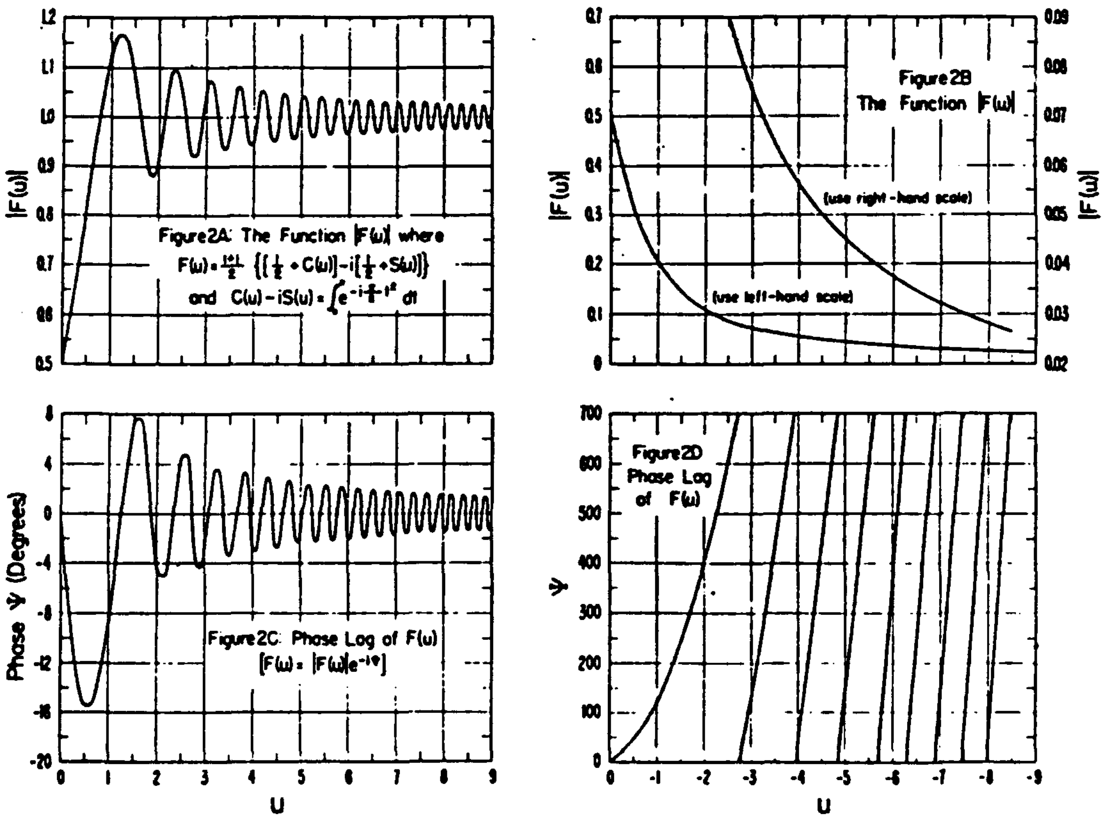


Fig 2

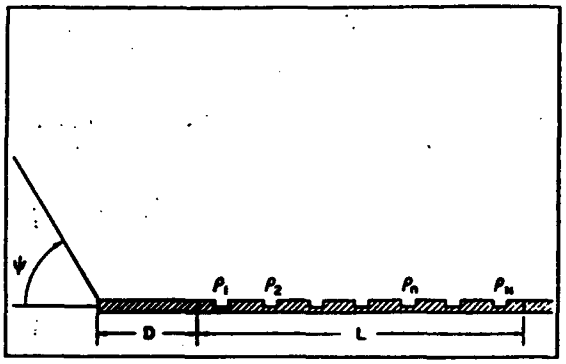


Fig 3a

$$\frac{G(\Psi)}{kL} = 2\exp\left[i\left(g\pi + X^2\right)\left(\frac{\alpha}{1-\alpha} + \frac{1}{2}\right)\right]\frac{\sin\left[\left(g\pi + X^2\right)\frac{1}{2}\right]}{\left(g\pi + X^2\right)\frac{1}{2}} \quad (12)$$

which of course is well known[14] having a magnitude equal to $2 \sin\theta/\theta$ where $\theta = (g\pi + X^2)/2$.

Employing equation (10), the pattern $|G(\Psi)/kL|$ is plotted as a function of the parameter X in fig. 4 for $g = 1$ and various values of N. This is the situation where the array consists of N discrete elements fed with a constant amplitude. The condition $g = 1$ is the criterion for optimum gain of such an array as found by Hansen and Woodyard[16]. It means that the electrical length kL of the array is π radians less than βL where β is the phase constant of the wave exciting the array. The patterns in fig. 4 are shown for both positive and negative values of X. In the case of slots embedded in an infinite ground plane only positive values are physically realizable. On the other hand, the region of negative X is accessible if the elements are located in free space. It is seen that as N becomes greater than about 8 the pattern does not depend on N at least for $|X|$ less than 5. In other words, the array of discrete sources can be replaced by continuous array ($N = \infty$) if N is not too small.

Using equation (12) the influence of varying g is illustrated in fig. 5 for the continuous uniform array. For $X > 0$ this corresponds to an array of length L embedded in an infinite ground plane, or if negative values of X are to be included, the array is located in free space. Here it can be seen that the effect of increasing g beyond unity is to reduce the major (end-fire) lobe considerably.

The information in figs. 4 and 5 is, of course, well known[14] but it is desirable that it be presented here for comparison of what follows.

From the basic equation (9) for N discrete elements on a half-plane, the pattern $|G(\Psi)/kL|$ is plotted in fig. 6 for the conditions, $A_n = 1$, $N = 8$, $\alpha = 0$, $g = 0$. In the case of such an end-fire antenna on a half-plane, both positive and negative values of X are accessible. The corresponding pattern for the continuous array ($N = \infty$), with $A_n = 1$, $\alpha = 0$, $g = 0$ is also shown. For purposes of comparison the pattern of the same continuous array on a infinite sheet is also illustrated in fig. 6.

The effect of varying g, the excitation parameter, for the uniform continuous array ($A_n = 1$) on a half-plane is shown in fig. 7 for $\alpha = 0$. Contrary to the cases of an infinite plane sheet or for an isolated array, the main lobe appears to be narrowest for $g = 0$ rather than $g = 1$.

When the array is moved away from the edge of the half-plane ($\alpha > 9$) the pattern is again modified. This is illustrated in fig. 8 for the case of the uniform continuous array on the half-plane with $g = 0$ and α varying from 0 to 0.9. In each case the pattern of the array on an infinite ground plane ($\alpha = 1.0$) is shown for comparison.

The foregoing has been restricted to uniform excitation wherein $A_n = 1$, the effect of tapering the illumination is shown in fig. 9 for the case of a discrete array ($N = 8$) on both an infinite plane and a half-plane for $g = 0$ and $\alpha = 0$. The coefficients are specified by the condition that the minor lobes for the infinite plane case should be of equal amplitude. For this reason the array is called a Tchebyscheff array[14] following the work of Dolph and Riblet. Using their procedure, it is found that

$$A_1 = A_8 = 0.632\,,$$
$$A_2 = A_7 = 0.675\,,$$
$$A_3 = A_6 = 0.881\,, \text{ and}$$
$$A_4 = A_5 = 1.000.$$

It is interesting to note that the minor lobes for the half-plane case are no longer of equal amplitude and the major lobe is broadened as it was in the uniform end-fire array. A similar effect has been recently observed by Barnett and Tai[17] who calculated the pattern of a Tchebyscheff array of electric current elements in the vicinity of a half-plane.

In the preceding calculations attention has been confined entirely to the situation where the half-plane sheet is in free space. An interesting extension is when the half-plane is resting on the surface of a homogeneous ground. In this case the pattern of the array of slots for an observer in the air is obtained by introducing an image source on the under edge of the half-plane. The strength of the image array is modified by the Fresnel reflection coefficient appropriate for (vertically-polarized) plane waves incident on the homogeneous ground at a grazing angle of Ψ. The formula for the pattern for a single element at distance ρ from the edge is given by

$$P = e^{-ik\rho\cos\Psi}\left[F(u) + R(\Psi)F(-u)\right] \qquad (13)$$

where

$$u = (8\rho/\lambda)^{\frac{1}{2}} \sin(\Psi/2)$$

and

$$R(\Psi) = \frac{K\sin\Psi - \left(K - \cos^2\Psi\right)^{\frac{1}{2}}}{K\sin\Psi + \left(K - \cos^2\Psi\right)^{\frac{1}{2}}}$$

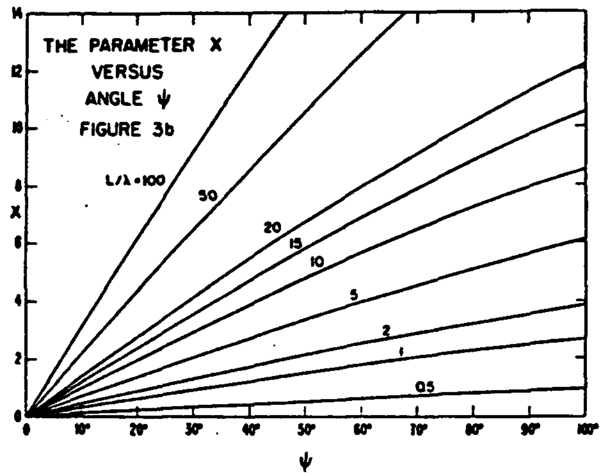
where K is the complex dielectric constant of the ground relative to free space. The above equation for P is a good approximation for values of Ψ from 0 to 180° and is again restricted to the principal or equatorial plane. The nature of the approximations are discussed in the appendix.

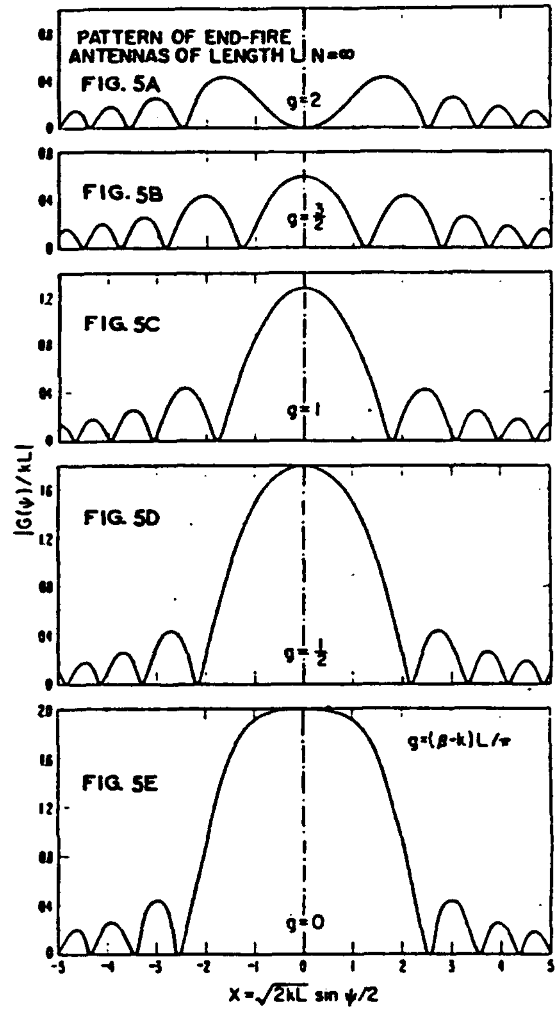
The pattern $|P|$ for a single slot on a half-plane is shown in fig. 10 for a lossless or dielectric ground for K ranging from 1 to 100 and three values of ρ/λ. The case $K = 1$, of course, corresponds to free space below as well as above the half-plane. As K increases the diffraction effects due to the edge of the half-plane apparently are de-emphasized. When K is allowed to approach infinity with $\Psi > 0$, then $|P|$ approaches unity. Physically, this corresponds to complete reflection from the ground plane ($R(\Psi) = 1$) and the pattern has the usual omni-directional characteristic of a single slot on an infinite ideally conducting ground plane. It is interesting to note that, for $1 < K < \infty$, if Ψ is reduced to zero the pattern $|P|$ is always zero. In other words, at low angles the field of the image source tends to cancel the field of the direct source. In the case of $K = 1$ the pattern $|P|$ is identically 0.5 at $\Psi = 0$ since there is no image source. On the other hand for Ψ large, the pattern $|P|$ is oscillating about unity. These ripples asymptotically approach zero as ρ/λ approaches infinity and they are due to the interference of the direct radiation from the slot and the wave scattered from the edge of the half-plane.

The picture is not changed appreciably if the ground is dissipative, (ie. lossy), whence K has a negative imaginary part. The pattern $|P|$ for a complex dielectric constant of $10 - ix$ with x = 4, 16, 64, 256(1) are shown in fig. 11 for two values of ρ/λ. Again as x becomes large the magnitude of $R(\Psi)$ approaches unity and $|P|$ approaches unity.

The prescription for calculating the pattern of an array on

(1) The corresponding loss-tangent values would be 0.4, 1.6, 6.4 and 25.6, respectively.


THE PARAMETER X
VERSUS
ANGLE ψ
FIGURE 3b
L/λ = 100
50
20
15
10
5
2
1
0.5
X
ψ


PATTERN OF END-FIRE
ANTENNAS OF LENGTH L N=∞
FIG. 5A
g=2
FIG. 5B
g=3/2
FIG. 5C
g=1
FIG. 5D
g=1/2
FIG. 5E
g=0
g=(β-k)L/π
|G(ψ)/kL|
X=√2kL sin ψ/2

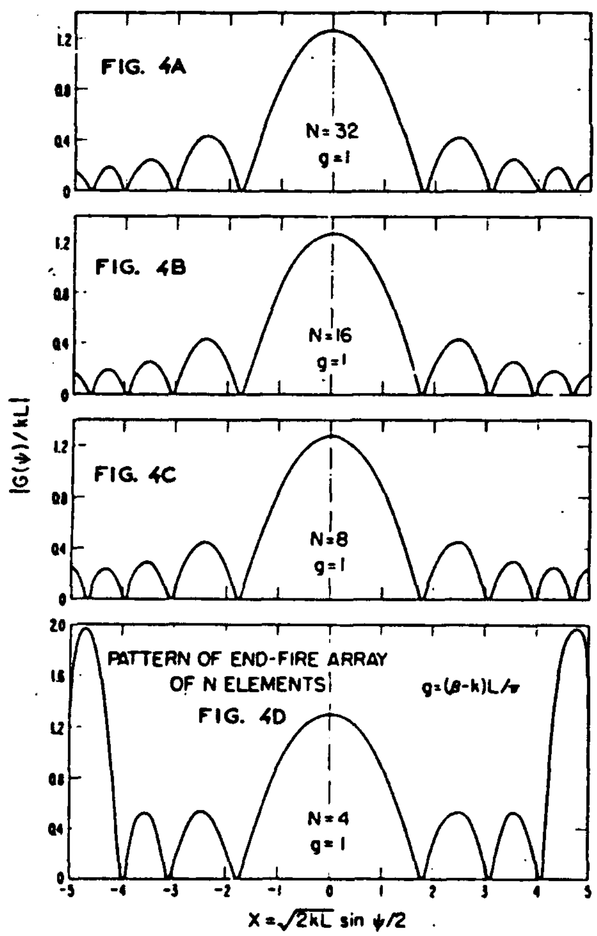
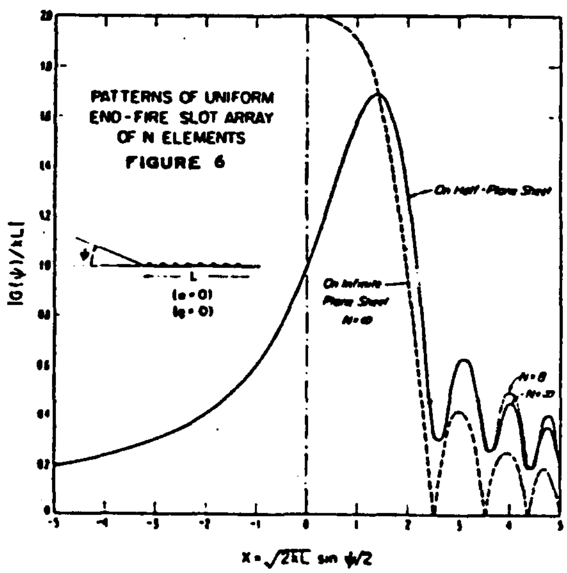


FIG. 4A
N=32
g=1
FIG. 4B
N=16
g=1
FIG. 4C
N=8
g=1
PATTERN OF END-FIRE ARRAY
OF N ELEMENTS
FIG. 4D
g=(β-k)L/π
N=4
g=1
|G(ψ)/kL|
X=√2kL sin ψ/2


PATTERNS OF UNIFORM
END-FIRE SLOT ARRAY
OF N ELEMENTS
FIGURE 6
On Half-Plane Sheet
On Infinite Plane Sheet N=∞
N=8
N=32
|G(ψ)/kL|
X=√2kL sin ψ/2

the half-plane lying on a homogeneous ground is identical to that above. For example if $G(\Psi)$ is the pattern of the array on the isolated half-plane then the desired pattern $\overline{G}(\Psi)$ is given by

$$\overline{G}(\Psi) = G(\Psi) + R(\Psi) \cdot G(-\Psi) \qquad (14)$$

To illustrate this, the pattern $|\overline{G}(\Psi)/kL|$ for the case of uniform continuous end-fire array of length L with $\alpha = 0$ and $g = 0$ is shown in fig. 12 for various real values of K and in fig. 13 for the complex value 10-ix.

Conclusion

The pattern of a flush mounted end-fire antenna is seen to be markedly influenced by the truncation of the ground plane. The general tendency is for the main beam to be tilted upward and also broadened. As the distance of the array from the edge of the half-plane is increased the pattern approaches that for an infinite ground plane. Such a behavior is in qualitative agreement with the experimental results of Elliott[10] for corrugated surface antennas and Simon and Robieux[18] for dielectric-sheet antennas. It is not possible to make a detailed comparison with their work since the experimental conditions do not correspond to the model chosen in this paper. It is hoped in the future to elaborate the model to include other forms of non-uniform excitation[20, 21] such as velocity modulation of the exciting wave and various forms of amplitude modulation.

Acknowledgement

We would like to thank Mrs. P. Murdock and Mrs. A. Murphy for their help in the preparation of the paper and Mr. John C. Ilsman for drafting the many illustrations.

Appendix

The Half-plane on the Surface of a Half-space.

The diffraction of a plane wave by a half-plane sheet of perfect conductivity located in free space was solved by Arnold Sommerfeld in 1899. The corresponding solution for the situation where the half-plane is located in the plane interface between two homogeneous media is a much more complicated problem. Clemmow[12] gave a formal solution to this problem in 1953 but, except in special cases, it was not suitable for numerical computation. As mentioned, Senior has proposed a method based on a combined use of ray theory and rigorous diffraction theory yielding a simple solution. It is the purpose of this appendix to discuss the limitations in Senior's solution and its applicability to the reciprocal radiation problem when the source is on the half-plane. A comparison with Flammer's[11] method is also made.

A half-plane located in the interface between two homogeneous media is indicated in fig. 14. A cylindrical coordinate system (ρ, Φ, z) is chosen such that the half-plane is defined by $\Phi = 0$ (or $\Phi = 2\pi$). A plane with the magnetic vector parallel to the edge of the half-plane is incident at an angle Φ_0 with the upper surface of the half-plane. The incident field for a time factor $\exp(i\omega t)$ is thus given by

$$H_z^i = H_0\, e^{ik\rho\cos(\Phi - \Phi_0)} \qquad (15)$$

where H_0 is the amplitude of the incident field.

If the whole region $(0 < \Phi < 2\pi)$ is free space, the Sommerfeld solution for the resulting field is

$$H_z(\rho, \Phi) = H_0 \left\{ e^{ik\rho\cos(\Phi - \Phi_0)} F\left[(8\rho/\lambda)^{\frac{1}{2}} \cos\frac{\Phi - \Phi_0}{2}\right] + e^{ik\rho\cos(\Phi + \Phi_0)} F\left[(8\rho/\lambda)^{\frac{1}{2}} \cos\frac{\Phi + \Phi_0}{2}\right]\right\}$$

where

$$F(u) = (1 + i)\int_{-\infty}^{u} e^{-i\frac{\pi}{2}t^2}\, dt$$

is the same function previously defined. Now if $\Phi = 0$ the observer is on the half-plane sheet at a distance ρ from the edge. It then follows that

$$H_z(\rho, 0) = 2H_0\, e^{ik\rho\cos\Phi_0} F(u) \qquad (16)$$

where

$$u = (8\rho/\lambda)^{\frac{1}{2}} \cos\Phi_0/2$$

It is thus seen that the field $H_z(\rho, 0)$ has the same azimuthal dependence on Φ_0 as the radiation field of a slot element at $(\rho, 0)$. This is, of course, a consequence of the reciprocity theorem.

The radial component of the electric field is obtained from

$$E_\rho(\rho, \Phi) = \frac{1}{i\epsilon\omega\rho}\,\frac{\delta H_z(\rho, \Phi)}{\delta\Phi} \qquad (17)$$

and in the plane $\Phi = \pi$ it follows that

$$E_\rho(\rho, \pi) = E_0\left[e_r - \sin\Phi_0\right] \qquad (18)$$

where

$$e_r = 2\sin\Phi_0\, e^{-ik\rho\cos\Phi_0} F\left[-(8\rho/\lambda)^{\frac{1}{2}}\sin\Phi_0/2\right] - e^{-i\pi/4} e^{-ik\rho}\left(\frac{2}{\pi k\rho}\right)^{\frac{1}{2}} \cos\frac{\Phi_0}{2} \qquad (19)$$

This quantity varies as $(k\rho)^{-\frac{1}{2}}$ as $k\rho$ approaches zero whereas for large $k\rho$ asymptoticylly becomes

$$e_r \cong \cos\frac{\Phi_0}{2}\,\frac{\exp[-i(k\rho - \pi/4)]}{\sqrt{\pi}\,(2k\rho)^{3/2}\sin^2\frac{\Phi_0}{2}}\,\exp\left[i\,\frac{3}{4k\rho\sin^2\frac{\Phi_0}{2}}\right] \qquad (20)$$

The quantity e_r is the reflected electric field from the edge in the plane of the half-plane as normalized to the amplitude E_0 of the incident electric field.

To illustrate the variation of the magnitude of the reflected or scattered field from the edge, $|e_r|$ is plotted in fig. 15 as a function of $k\rho$ as a function of Ψ $(= \pi - \Phi_0)$. When $k\rho > 2\pi$ (or $\rho > \lambda$), $|e_r|$ is less than 10^{-3} for angles Ψ less than about $10°$. Furthermore, the field is decaying as $\rho^{-3/2}$ indicating that little radiation takes place in this direction.

When the lower half-space $(+\pi < \Phi < 2\pi)$ is filled with homogeneous material whose (complex) dielectric constant is k relative to free space, then the solution as proposed by Senior for the magnetic field $\overline{H}(\rho, \Phi)$ in the upper half space is

$$\overline{H}_z(\rho, \Phi) = H_z(\rho, \Phi) + R(\Psi)\, H_z(\rho, 2\pi - \Phi) \qquad (21)$$

It can readily be verified that $E_\rho(\rho, 0) = 0$ which is the required boundary condition on the surface of the half-plane sheet. The tangential fields on the surface of the ground are conveniently written

$$E_\rho(\rho, \pi) = E_t + e_r[1 - R(\Psi)] \qquad (22)$$

where

$$E_t = -E_0 \sin\Psi\,[1 - R(\Psi)]$$

and

$$H_\Phi(\rho, \pi) = H_t = \eta_0 E_0 [1 + R(\Psi)] \qquad (23)$$

It can be immediately seen that E_t/H_t has the value appropriate for a vertically polarized plane wave at oblique incidence on a homogeneous ground. Consequently, the field $e_s[1 - R(\Psi)]$ does not satisfy the boundary condition but it has been shown that e_s is small except close to the edge and furthermore for a highly conducting ground, $R(\Psi)$ is near unity. It may therefore be concluded that the boundary condition is only violated for a small region of the ground surface near the edge of the ground plane.

Assuming that this procedure is valid, the tangential magnetic field on the ground plane becomes

$$\overline{H}_z(\rho, 0) = H_z(\rho, 0) + R(\Psi) H_z(\rho, 2\pi) \qquad (24)$$

$$= 2H_o e^{-ik\rho \cos\Psi}[F(u) + R(\Psi) F(-u)]$$

where

$$u = (8\rho/\lambda)^{\frac{1}{2}} \sin(\Psi/2)$$

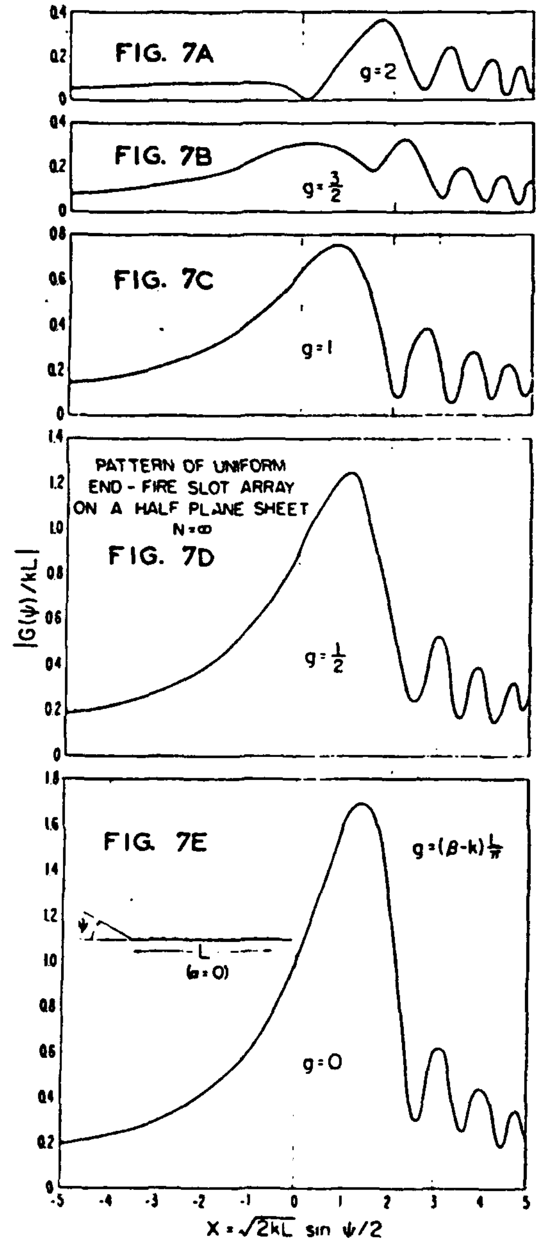


Fig. 7

The quantity $\overline{H}_z(\rho, 0)$ has the same function dependence on Φ_o as the radiation field of a single axial slot at a distance ρ from the edge of the half-plane.

It is of interest to compare the present technique with one suggested recently by Flammer for a similar problem. With reference to fig. 14 and the cartesian coordinates (x, y, z) the incident field makes an angle of incidence Ψ with the negative x axis following the derivation of Flammer, the integral equation for the tangential magnetic field on the ground plane is

$$H_z\Big]_{y=0} = 2H_o e^{-ikx\cos\Psi} + \qquad (25)$$

$$+\frac{i}{2}\int_0^\infty H_o^{(2)}[k(x - x')]\left[\frac{\delta H_z}{\delta y}\right]_{y=0} dx'$$

This can be written in terms of the (unknown) tangential electric field $E_x(x', 0)$ by using the result

$$\left[\frac{\delta H_z}{\delta y}\right]_{y=0} = i\epsilon\omega E_x(x', 0)$$

The next step in Flammer's procedure is to set

$$E_x(x', 0) = E_o[1 - R(\Psi)] \sin\Psi\, e^{-ikx'\cos\Psi} \qquad (26)$$

which neglects completely the presence of the half-plane for all negative values of x. Furthermore, $E_x(x', 0)$ does not have the correct order of singularity[17] as $x' \to 0$. After some algebraic manipulation the resulting expression for the tangential magnetic field on the upper surface of the half-plane is

$$H_z(x, 0) = 2H_o e^{-ikx\cos\Psi} T \qquad (27)$$

where

$$T = 1 - \cos\frac{\Psi}{2} + [F(u) + R(\Psi) F(-u)] \cos\frac{\Psi}{2}$$

It can be seen that for Ψ near zero (glancing incidence)

$$T \cong F(u) + R(\Psi) F(-u) \qquad (28)$$

which is in agreement with equation (24) derived using Senior's method.

The near coincidence of the two approximate formulas for small values of the grazing angle Ψ lends confidence to the approach used in the body of the paper for calculating patterns of end-fire antennas on a half-plane which are lying on a half space.

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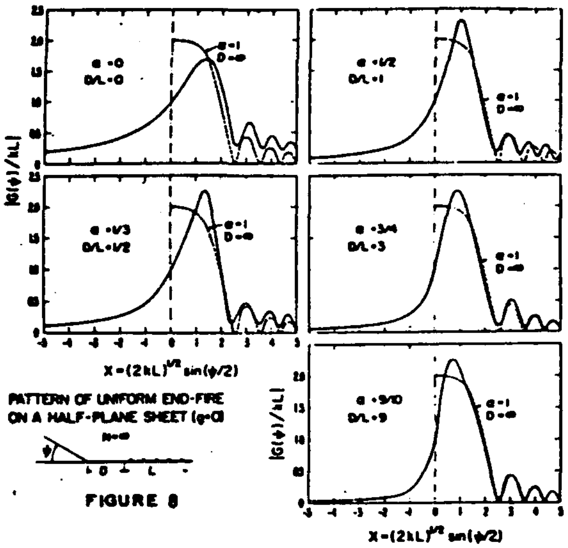
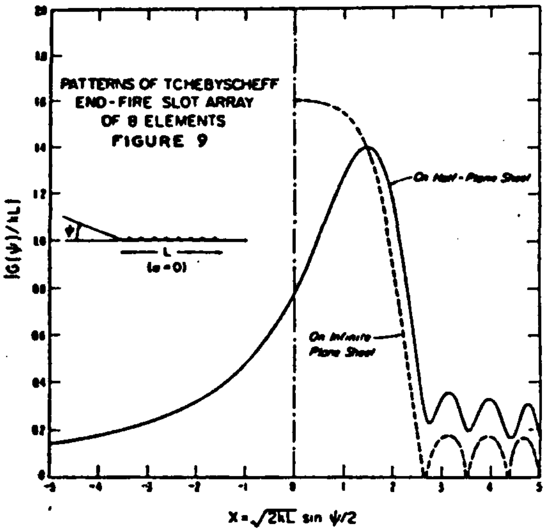


PATTERN OF UNIFORM END-FIRE ON A HALF-PLANE SHEET (α=0)

FIGURE 8



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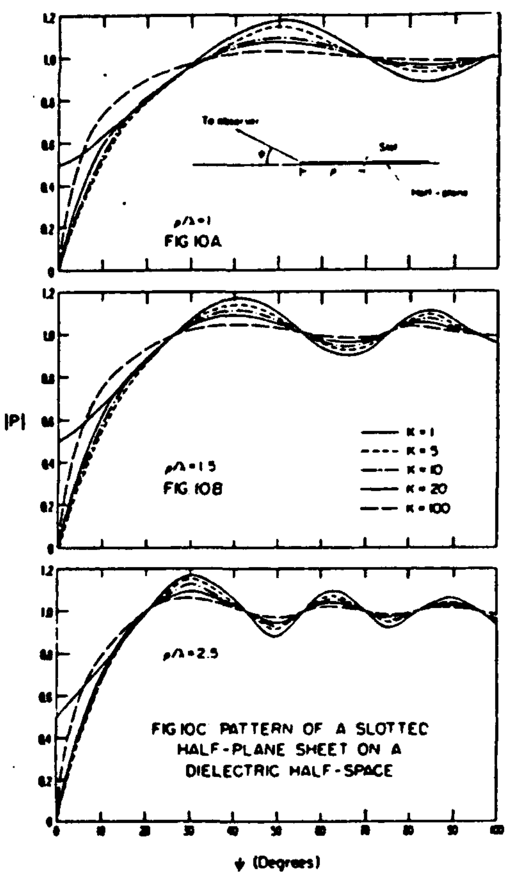

FIG 10C PATTERN OF A SLOTTED HALF-PLANE SHEET ON A DIELECTRIC HALF-SPACE

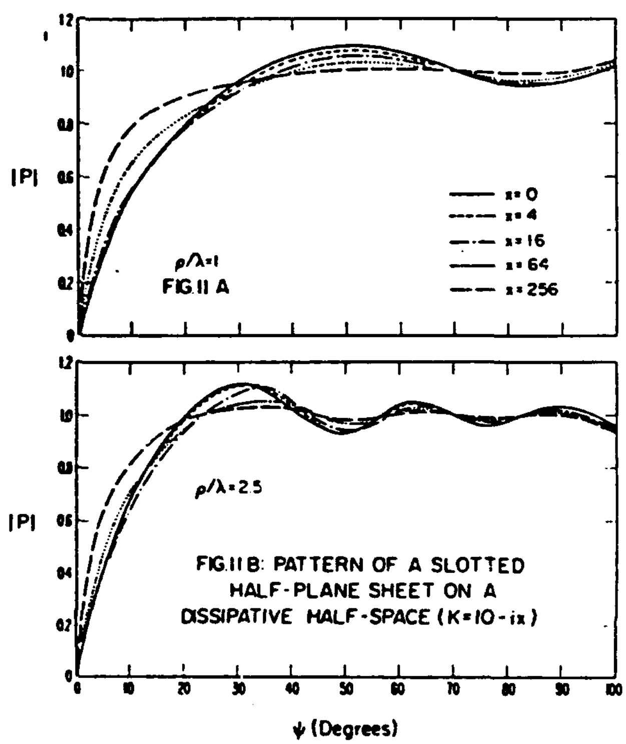

FIG.11 B: PATTERN OF A SLOTTED HALF-PLANE SHEET ON A DISSIPATIVE HALF-SPACE (K=10-ix)

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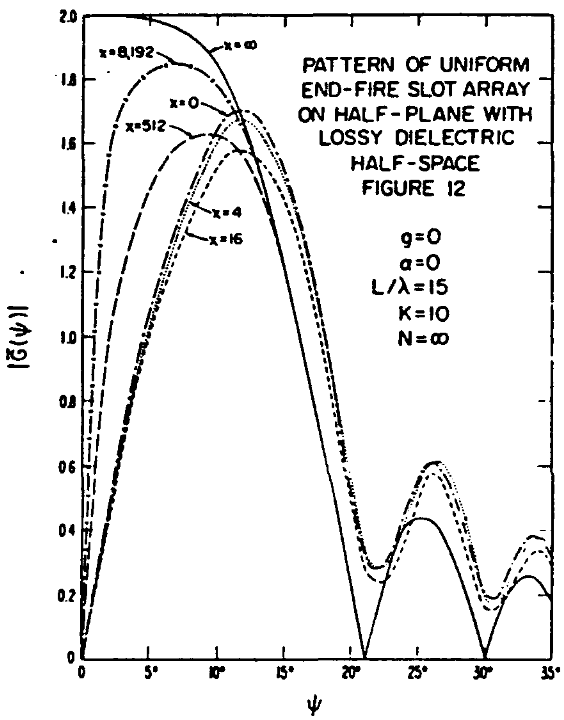
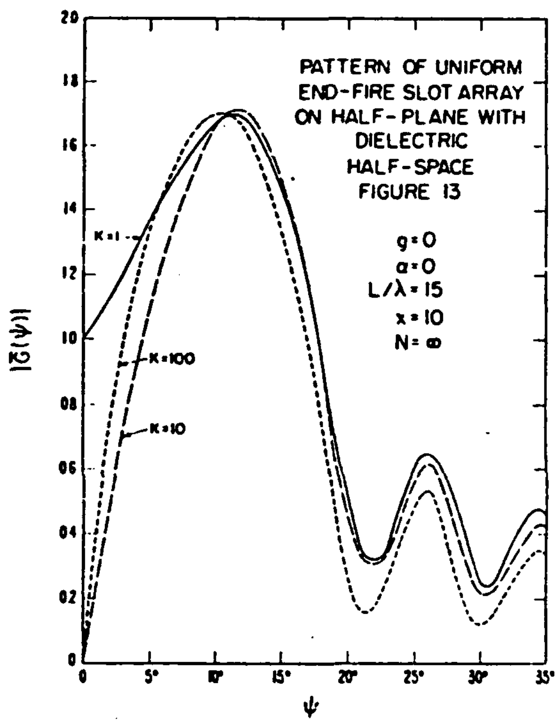


PATTERN OF UNIFORM END-FIRE SLOT ARRAY ON HALF-PLANE WITH LOSSY DIELECTRIC HALF-SPACE
FIGURE 12



PATTERN OF UNIFORM END-FIRE SLOT ARRAY ON HALF-PLANE WITH DIELECTRIC HALF-SPACE
FIGURE 13

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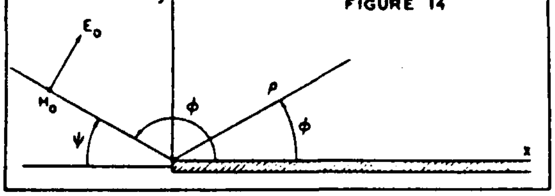
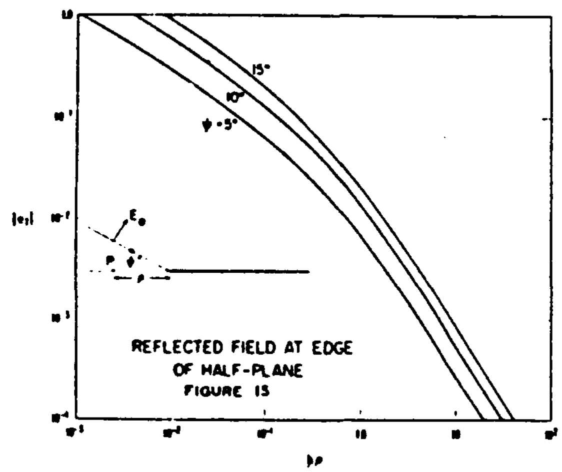
FIGURE 14





REFLECTED FIELD AT EDGE OF HALF-PLANE
FIGURE 15

1.10 Wait, J. R., 1963, "The Theory of an Antenna Over an Inhomogeneous Ground Plane," extracted from *Electromagnetic Wave Theory*, edited by E. C. Jordan, pp. 1079-1097, Pergamon Press, Oxford.

(Reprint No. 208)

extracted from : Electromagnetic Wave Theory edited by E.C. Jordan, Pergamon Press, Oxford , 1963

THE THEORY OF AN ANTENNA OVER AN INHOMOGENEOUS GROUND PLANE

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ABSTRACT

We consider an antenna over a flat ground plane which is characterized by a variable surface impedance as in a radial-wire screen. The problem is formulated in terms of the mutual impedance between two vertical dipoles, one which is raised, and the other is located on the ground plane. The ground screen is taken to be in the combined form of a circular disc and a concentric sector. An approximate solution of the problem is obtained and the results are compared with previous investigations of closely related work.

1. INTRODUCTION

The influence of the ground plane in antenna radiation is a subject which has not received the attention it deserves. In many cases it is assumed that the ground behaves as a flat perfectly conducting surface. Unfortunately, the principal characteristics of the antenna are influenced—in general, in an adverse way—by the finite ground conductivity. At broadcast frequencies, it has been common practice for many years to improve the situation by the use of a radial-wire ground system. Typically, the approach has been empirical. Apparently the first systematic study was carried out by Brown and his associates (Gihring and Brown, 1935; Brown *et al.*, 1937). More recent investigations have been focused mainly on the influence of the ground system on the impedance of the antenna (Abbott, 1952; Monteath, 1951; Wait and Surtees, 1954; Wait and Pope, 1954, 1955). It has been assumed usually that the radiated field for a given current on the antenna was not appreciably affected by the presence of the ground screen.

In an earlier paper (Wait and Pope, 1954) an approximate method was given which is suitable for estimating the dependence of the ground wave on the size of the screen. Calculations (Wait, 1956) based on this work support the contention that the ground screen has only a small effect on the radiated field for screens with a radius of the order of a wavelength or less. Very similar conclusions have been arrived at by Monteath (Page and Monteath, 1955; Monteath, 1958).

† This work was carried out while the author was on a visit to the Technical University of Denmark, Copenhagen, in the fall of 1960.

In this paper the theory is extended to ground screens which may be large in terms of wavelength. The problem now bears some similarity to previous investigations of propagation across a land-sea boundary (Bremmer, 1958; Wait, 1956, 1957). While the literature on this subject is quite extensive, the results are not sufficiently general to be applied without modification. Thus it seems worthwhile to reformulate the problem in a fairly general way.

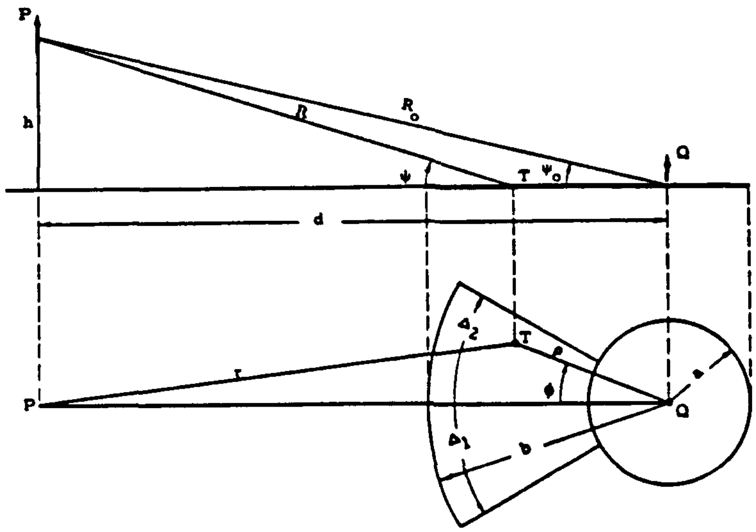

FIG. 1. Side and plan views of the dipoles and the ground plane. (Note that $\tan \psi = h/r$ and $\tan \psi_0 = h/d$.)

2. FORMULATION

The problem to be considered is illustrated in Fig. 1. Two vertical electric dipole antennas are located at P and Q which are located at height h and height zero, respectively, over a flat earth. The effective lengths of these dipoles are l_1 and l_2, respectively. The surface of the ground is characterized by a surface impedance Z everywhere except at a ground screen about Q. This ground screen is made up of a circular disc of radius a which is lying on the surface of the ground. Furthermore, a sector of radial wires emanates from this circular disc in the manner shown in Fig. 1. The tangential fields over the ground screen are also assumed to be related by an effective surface impedance Z'. Over the disc (i.e. $\rho < a$) this is denoted Z_a' and over the sector (i.e. $b > \rho > a$, $\Delta_2 > \phi > -\Delta_1$) it is denoted Z_b'.

The mutual impedance z between dipoles P and Q is written

$$z = z_0 - \Delta z \tag{1}$$

where z_0 is the mutual impedance between dipole P and Q when no ground plane was present. Thus Δz is the change of the impedance resulting from the presence of the ground plane. Having the problem formulated in this way allows the compensation theorem from network theory to be used to express Δz in terms of a surface integral over the area of the ground screen S. Thus (Monteath, 1951)

$$\Delta z = \frac{1}{I_0^2} \int\int_S (Z' - Z)\, \mathbf{H}_{pt} \,.\, \mathbf{H}'_{qt}\, \mathrm{d}S \tag{2}$$

where $\mathbf{H}_{pt}$ is the tangential magnetic field on the ground plane, *with no ground screen*, resulting from a current I_0 impressed on terminals of dipole P and where $\mathbf{H}'_{qt}$ is the tangential magnetic field on the ground plane, *with the ground screen present.*

Equation (2) is an exact formulation of the problem provided that the tangential electric and magnetic fields may be related by a surface impedance. Essentially it is a two-dimensional integral equation since the integrand contains the unknown field $\mathbf{H}'_{qt}$. It has been shown in a previous paper (Wait, 1956) that such an equation can be reduced to a one-dimensional integral equation by employing a stationary-phase principle. Such a method is particularly appropriate for studying propagation over land–sea boundaries. Here a somewhat different approach is used since the two-dimensional nature of the problem must be preserved.

3. THE APPROXIMATE SOLUTION

The mutual impedance z_0, in the absence of the screen is easily obtained from the known solution (Norton, 1957; Wait, 1957) of a dipole over a flat conducting plane of surface impedance Z. Thus

$$z_0 = \frac{i\mu_0 \omega l_1 l_2}{2\pi R_0} \mathrm{e}^{-ikR_0} \cos^2 \psi_0\, W(R_0, Z) \tag{3}$$

where

$$W(R_0, Z) = 1 - i(\pi p_0)^{1/2}\, \mathrm{e}^{-w_0}\, \mathrm{erfc}(iw_0^{1/2}), \tag{4}$$

$$w_0 = \left(1 + \frac{h\eta_0}{R_0 Z}\right)^2 p_0,$$

$$p_0 = -\frac{ikR_0}{2}\left(\frac{Z}{\eta_0}\right)^2, \quad k = 2\pi/\text{wavelength},$$

$$\eta_0 = 120\pi \quad \text{and} \quad \mu_0 = 4\pi \times 10^{-7}.$$

This result is valid subject to the approximation that $kd \gg 1$ and $|Z/\eta_0|^2 \ll 1$.

It should be noted that $W(R_0, Z)$ can be written in the form (Norton, 1957; Wait, 1957)

$$W(R_0, Z) = W_r + W_s \tag{5}$$

where

$$W_r = \frac{1 + R(\psi_0)}{2}$$

is the radiation or space field and W_s is the Norton surface wave, and $R(\psi_0)$ is a Fresnel reflection coefficient. Provided ψ_0 is not near zero, and if $|p_0| \gg 1$, W_r is the principal part of W. By definition, the antenna pattern of dipole Q in the absence of the ground screen is the function W_r. Now at near-grazing conditions where ψ approaches zero W_s becomes the principal part of W since W_r tends to zero. This is an important point to keep in mind when discussing the effects of the ground on the radiation from the antenna at low angles.

Another quantity required is the tangential magnetic field $\mathbf{H}_{pt}$ resulting from a current I_0 in dipole P. It is a vector with amplitude given by

$$H_{pt} = \frac{ikI_0 l_1}{2\pi R} e^{-ikR} \cos\psi \, W(r, Z) \tag{6}$$

where

$$W(r, Z) = 1 - i(\pi p)^{1/2} e^{-w} \operatorname{erfc}(iw^{1/2}), \tag{7}$$

$$w = p\left(1 + \frac{h\eta_0}{RZ}\right)^2,$$

$$p = -\frac{ikR}{2}\left(\frac{Z}{\eta_0}\right)^2, \quad \text{and} \quad R = (r^2 + h^2)^{1/2}.$$

This result is valid subject to $kr \gg 1$ and $|Z/\eta_0|^2 \ll 1$.

Now the tangential magnetic field $\mathbf{H}'_{qt}$ on the ground plane in the presence of the screen may be written in the form

$$H'_{qt} = \frac{ikI_0 l_2}{2\pi\rho}\left(1 + \frac{1}{ik\rho}\right) e^{-ik\rho} \, W'(\rho, Z', Z), \tag{8}$$

where W' is an unknown function of the radial distance ρ and the surface impedances Z and Z' of the screen and of the ground Equation (8) is normalized so that W' would approach unity if $Z' = Z = 0$.

In a similar manner the mutual impedance z in the presence of the screen is written

$$z = \frac{i\mu_0\omega l_1 l_2}{2\pi R_0} e^{-ikR_0} \cos^2\psi_0 \, W'(R_0, Z, Z') \tag{9}$$

where W' is an unknown function which is also normalized such that it becomes unity for $Z = Z' = 0$.

Using (3), (6), (8), and (9), it readily follows that (2) may be expressed by the equivalent form

$$
\left.\begin{aligned}
W'(R_0, Z, Z') = W(R_0, Z) \\
+ \frac{ik}{2\pi \cos\psi_0} \int\int \left(\frac{Z' - Z}{\eta_0}\right) e^{-ik\rho} \\
e^{-ik(R-R_0)} (\mathbf{i}_p \cdot \mathbf{i}_q) \left(\frac{R_0}{R}\right) \frac{\cos\psi}{\cos\psi_0} \\
\times \left(1 + \frac{1}{ik\rho}\right) W(R, Z)\, W'(\rho, Z', Z)\, d\phi\, d\rho,
\end{aligned}\right\} \quad (10)
$$

where the variables of integration are ρ and ϕ, the polar coordinates about Q. Also, $\mathbf{i}_p$ and $\mathbf{i}_q$ are unit vectors in the direction of the fields $\mathbf{H}_{pt}$ and $\mathbf{H}'_{qt}$.

In the case when h approaches zero, (10) reduces to

$$
\left.\begin{aligned}
W'(d, Z, Z') = W(d, Z) \\
+ \frac{ik}{2\pi} \int\int \left(\frac{Z' - Z}{\eta_0}\right) (\mathbf{i}_p \cdot \mathbf{i}_q) \left(\frac{d}{r}\right) \left(1 + \frac{1}{ik\rho}\right) \\
W(r, Z)\, W'(\rho, Z', Z)\, d\phi\, d\rho.
\end{aligned}\right\} \quad (11)
$$

This is a two-dimensional integral equation for the function W'. In principle one could solve (11) for W' and then using this, (10) becomes an explicit integral formula for $W'(R_0, Z, Z')$ at any height h. Because of the complexity of such an approach, it is customary to reduce (11) to a one-dimensional integral equation by employing a stationary-phase principle (Bremmer, 1958; Feinberg, 1946). Essentially this makes use of the idea that the important part of the surface integral is a narrow ellipse with foci at P and Q. In the geometry of Fig. 1 we would then find that (11) could be reduced to

$$
W'(d, Z, Z') \cong W(d, Z) - \left(\frac{ikd}{2\pi}\right)^{1/2} \int_0^b \left(\frac{Z' - Z}{\eta_0}\right) \frac{W(d-\rho, Z)\, W'(\rho, Z', Z)}{[\rho(d-\rho)]^{1/2}}\, d\rho \quad (12)
$$

provided kb is somewhat greater than unity. An interesting special case of (12) is when Z' is allowed to be constant with respect to ρ. Then, to within the stationary-phase approximation, $W'(\rho, Z', Z)$ in the integrand can be replaced by $W(\rho, Z')$. This leads to the following integral formula for the mixed-path function W',

$$
W'(d, Z, Z') \cong W(d, Z) - \left(\frac{ikd}{2\pi}\right)^{1/2} \left(\frac{Z' - Z}{\eta_0}\right) \int_0^b \frac{W(d-\rho, Z)\, W(\rho, Z')}{[\rho(d-\rho)]^{1/2}}\, d\rho. \quad (13)
$$

All quantities on the right-hand side are known, since

$$
W(\rho, Z') = 1 - i(\pi p)^{1/2} e^{-p} \operatorname{erfc}(ip^{1/2}) \quad (14)
$$

where $p = -\frac{ik\rho}{2}\left(\frac{Z'}{\eta_0}\right)^2$.

The preceding results, borrowed from the previous work (Wait, 1956) on ground-wave propagation over mixed paths, suggest that a plausible approximation to (10) is obtained if $W'(\rho, Z', Z)$ is replaced by the function $W(\rho, Z')$ appropriate for propagation over a plane of surface impedance Z'. In the limiting case where Z' was zero $W(\rho, Z')$ would be unity. Furthermore, even if Z' were finite and slowly varying, but satisfied the inequality

$$\frac{kb}{2}\left|\frac{Z'}{\eta_0}\right|^2 < 1,$$

it would be an excellent approximation to replace $W(\rho, Z')$ by unity. In most ground screens, this condition would be met.

In order to simplify the required integrations, it is now assumed that d is somewhat greater than b. Thus, in the integrand of (10) we make the following additional approximations

$$\mathbf{1}_p \cdot \mathbf{1}_q \cong -\cos\phi,$$

$$\frac{R_0}{R}\frac{\cos\psi}{\cos\psi_0} \cong 1$$

for the whole range of integration. Thus

$$W'(R, Z, Z') = W(R, Z)\,[1 + \Omega] \tag{15}$$

where

$$\Omega \cong -\frac{ik}{2\pi\cos\psi_0}\int\int_S e^{-ik\rho}\, e^{-ik(R-R_0)}\left(\frac{Z'-Z}{\eta_0}\right) \times \left(1 + \frac{1}{ik\rho}\right)\cos\phi\, d\phi\, d\rho, \tag{16}$$

where

$$R = [\rho^2 + d^2 + h^2 - 2\rho d\cos\phi]^{1/2}$$

and

$$R_0 = [d^2 + h^2)^{1/2}.$$

The quantity Ω is the fractional change of the mutual impedance between P and Q resulting from the presence of the screen S. It is now convenient to split Ω into two parts in the manner

$$\Omega = \Omega_a + \Omega_b \tag{17}$$

where Ω_a is the contribution from the semi-circular screen of radius a, and Ω_b is the contribution from the sector.

4. THE INTEGRAL Ω_a FOR THE CIRCULAR GROUND SCREEN

Over the range $0 < \rho < a$, it is permissible to retain only first-order phase terms, thus $R - R_0 \cong -\rho \cos\phi \cos\psi_0$. Therefore,

$$\Omega_a \cong \frac{-ik}{2\pi \cos\psi_0} \int_{\rho=0}^{a} \int_{\phi=-\pi}^{\pi} e^{-ik\rho}\left(1 + \frac{1}{ik\rho}\right) e^{ik\rho\cos\phi\cos\psi_0} \cos\phi \left(\frac{Z_a' - Z}{\eta_0}\right) d\phi\, d\rho. \quad (18)$$

The ϕ integration can be carried out in closed form to give

$$\Omega_a \cong \frac{k}{\cos\psi_0} \int_{\rho=0}^{a} e^{-ik\rho}\left(1 + \frac{1}{ik\rho}\right) J_1(k\rho\cos\psi_0)\left(\frac{Z_a' - Z}{\eta_0}\right) d\rho \quad (19)$$

where it has been assumed that Z_a' does not depend on ϕ. $J_1(x)$ is the Bessel function of the first type of order. An alternate derivation of (19) is given in the appendix.

In the absence of the sector portion, Ω_a represents the total effect of the ground screen. If $ka \gg 1$ the integrand in (19) can be approximated by employing only the first term of the asymptotic expansion for the Bessel function. Thus, for the major range of the integrand,

$$J_1(k\rho\cos\psi_0)\, e^{-ik\rho}\left(1 + \frac{1}{ik\rho}\right) \cong \left(\frac{1}{2\pi k\rho\cos\psi_0}\right)^{1/2} e^{-i3\pi/4}\,(1 - ie^{-i2k\rho\cos\psi_0})\, e^{ik\rho(\cos\psi_0 - 1)}. \quad (20)$$

Thus

$$\Omega_a \cong -\sqrt{\left(\frac{i}{2\pi\cos^3\psi_0}\right)} \int_0^{ak} \left(\frac{Z_a' - Z}{\eta_0}\right)(1 - ie^{-2ix\cos\psi_0}) \frac{e^{-ix(1-\cos\psi_0)}}{x^{1/2}}\, dx. \quad (21)$$

When Z_a' is essentially a constant over the range of integration, Ω_a can be expressed in terms of Fresnel integrals. Thus

$$\Omega_a \cong \sqrt{\left(\frac{i}{2\cos^3\psi_0}\right)}\left(\frac{Z - Z_a'}{\eta_0}\right) \left[\frac{1}{\sin\frac{\psi_0}{2}} \int_0^{\sqrt{\left(\frac{4ka}{\pi}\right)}\sin\frac{\psi_0}{2}} e^{-i(\pi/2)t^2}\, dt - \frac{i}{\cos\frac{\psi_0}{2}} \int_0^{\sqrt{\left(\frac{4ka}{\pi}\right)}\cos\frac{\psi_0}{2}} e^{-i(\pi/2)t^2}\, dt\right]. \quad (22)$$

As ψ_0 tends to zero, the above equation reduces to

$$\Omega_a \cong \left(\frac{i2ka}{\pi}\right)^{1/2} \left(\frac{Z - Z_a'}{\eta_0}\right) \left[1 - i\left(\frac{\pi}{4ka}\right)^{1/2} \int_0^{\left(\frac{4ka}{\pi}\right)^{1/2}} e^{-i(\pi/2)t^2}\, dt\right]. \quad (23)$$

Since $ka \gg 1$

$$\Omega_a \cong \left(\frac{i2ka}{\pi}\right)^{1/2} \left(\frac{Z - Z_a'}{\eta_0}\right) \left[1 - \left(\frac{i\pi}{8ka}\right)^{1/2}\right]. \quad (24)$$

When the second term in the square bracket is neglected the result agrees with a previous analysis (Wait, 1956) for the transmission across a land–sea boundary. It is concluded that the second term in the square brackets of (24) results from the circular shape of the screen. Usually in the land–sea boundary problems both media are semi-infinite. Further support to this contention is given below.

It is of interest to compare (22) with the solution obtained on the assumption that the ground screen may be replaced by a perfectly conducting half-plane. Assuming an incident plane wave, the voltage induced in antenna Q is calculated by a modification of Sommerfeld's results (Sommerfeld, 1899) for diffraction by a knife edge. The method suggested by Senior (1956) is a union of ray theory and rigorous diffraction theory. A direct application leads to

$$1 + \Omega_a = 2\,\frac{F(u) + R(\psi_0)\,F(-u)}{1 + R(\psi_0)} \quad (25)$$

where $R(\psi_0)$ is the Fresnel reflection coefficient given by

$$R(\psi_0) = \frac{\sin\psi_0 - (Z/\eta_0)}{\sin\psi_0 + (Z/\eta_0)} \quad (26)$$

and

$$F(u) = 1 + \sqrt{(2i)} \int_0^u e^{-i(\pi/2)t^2}\, dt \quad (27)$$

with $u = (4ka/\pi)^{1/2} \sin\psi_0/2$. The function $F(u)$ can be identified as the response in the dipole at Q resulting from an incident plane wave (at grazing angle ψ_0) on a semi-infinite conducting plane. The function $R(\psi_0)\,F(-u)$ is then the response of an (image) plane wave incident from below the half-plane. Such a solution is very plausible since the tangential magnetic field is continuous across the air–ground interface and the fields have the proper singularity at the edge of the half-plane and satisfy the boundary conditions on the half-plane itself. However, the tangential electric field is not continuous across the air–ground interface. For near-grazing angles this violation of the boundary conditions takes place within a very small distance from the edge *of the half-plane. The result given by (25) can be rewritten in the form*

$$\Omega_a = \sqrt{\left(\frac{i}{2}\right)} \left(\frac{Z}{\eta_0}\right) \frac{1}{\cos\frac{\psi_0}{2}} \frac{1}{\sin\frac{\psi_0}{2}} \int_0^{\sqrt{\left(\frac{4ka}{\pi}\right)}\sin\frac{\psi_0}{2}} e^{-i(\pi/2)t^2}\, dt \qquad (28)$$

which is almost identical to (22) when $Z_a' = 0$ and the second integral is disregarded. It is also required that ψ_0 is small. (It might also be mentioned that a similar formula for this situation has been proposed by Carswell and Flammer (1957). Their result is derived by an approximate evaluation of the one-dimensional integral equation of the same type that occurs in mixed-path theory for ground-wave propagation (Wait, 1956).)

An exact correspondence between (22) and (28) is not expected because of the differing approximations inherent in the two approaches. However, this demonstration strongly suggests that the second integral in the square bracket is the contribution from the back edge (i.e. $\phi \sim \pi$) of the circular disc. As mentioned, for $ka \gg 1$, this term can be neglected.

Another aspect of the circular ground screen also deserves mention. In the formulation of the problem it was assumed that antennas P and Q were vertical electric dipoles of infinitesimal length. The modification of the theory for antennas of finite length is not difficult although the complexity is greatly increased. For near-grazing angles, it is not difficult to show that the principal results are not changed essentially. For example, if Q is a quarter-wave monopole with an assumed sinusoidal current distribution, the changes in (16), (17), and (18) are:

$$\frac{1}{\cos\psi_0} \quad \text{is replaced by} \quad \frac{\cos\psi_0}{\cos\left(\frac{\pi}{2}\sin\psi_0\right)}$$

and

$$\left(1 + \frac{1}{ik\rho}\right) e^{-ik\rho} \quad \text{is replaced by} \quad e^{-ik\sqrt{[\rho^2 + (\lambda/4)^2]}}.$$

For large ground screens ($ka \gg 1$), the difference between the latter two factors is negligible. For screens comparable in size to the wavelength, it may be important to use the correct form of this factor. In the case of the quarter-wave monopole, Ω_a can be written in the form

$$\Omega_a = \frac{\cos\psi_0}{\cos\left(\frac{\pi}{2}\sin\psi_0\right)} \int_0^{ka} \left(\frac{Z_a' - Z}{\eta_0}\right) e^{-i\sqrt{[x^2 + (\pi/2)^2]}} J_1(x\cos\psi_0)\, dx. \qquad (29)$$

When Z_a' is a constant over the range of x and ψ_0 tends to zero, it is convenient to write

$$\Omega_a = \delta[X_1 + iX_2'] \qquad (30)$$

where

$$\delta = \left(\frac{Z - Z_a'}{\eta_0}\right) e^{-i\pi/4},$$

$$X_1 = -\int_0^{ka} \cos\left[\left(x^2 + \frac{\pi^2}{4}\right)^{1/2} - \frac{\pi}{4}\right] J_1(x)\, dx \tag{31}$$

and

$$X_2 = \int_0^{ka} \sin\left[\left(x^2 + \frac{\pi^2}{4}\right)^{1/2} - \frac{\pi}{4}\right] J_1(x)\, dx. \tag{32}$$

If the screen is perfectly conducting

$$\delta = \frac{Z}{\eta_0} e^{-i\pi/4} = \left(\frac{\epsilon_0\omega}{\sigma + i\epsilon\omega}\right)^{1/2} \left(1 - \frac{i\epsilon_0\omega}{\sigma + i\epsilon\omega}\right)^{1/2}$$

in terms of the electrical constants, σ and ϵ, of the ground. At low frequencies, where $\omega \ll \sigma/\epsilon$,

$$\delta \cong \left(\frac{\epsilon_0\omega}{\sigma}\right)^{1/2} = 0\cdot0075\,(f_{Mc}/\sigma)^{1/2}$$

which is essentially a real quantity. (In the above, f_{Mc} is the frequency in megacycles and σ is the ground conductivity in mhos per meter.)

Numerical values of X_1 and X_2 for ka in the range from 0 to 6·5 are given in Table 1.

TABLE 1

ka	X_1	X_2
0·0	0·00	0·000
0·5	−0·042	0·040
1·0	−0·130	0·181
1·5	−0·211	0·417
2·0	−0·209	0·700
2·5	−0·102	0·947
3·0	0·042	1·093
3·5	0·155	1·131
4·0	0·171	1·133
4·5	0·113	1·178
5·0	0·050	1·300
5·5	0·050	1·468
6·0	0·119	1·612
6·5	0·205	1·674

These values were obtained by graphical integration so the accuracy of the last significant figures are doubtful. For large values of ka, X_1 and X_2 can be represented by Fresnel integrals if $(x^2 + \pi^2/4)^{1/2}$ is replaced by x and $J_1(x)$ is replaced by the first term of its asymptotic expansion. Thus

$$X_1 \cong \frac{1}{\sqrt{2}} \int_0^u \cos\left(\frac{\pi}{2} t^2\right) dt \tag{33}$$

and

$$X_2 \cong (2ka/\pi)^{1/2} - \frac{1}{\sqrt{2}} \int_0^u \sin\left(\frac{\pi}{2} t^2\right) dt \tag{34}$$

where

$$u = (4ka/\pi)^{1/2} = (8a/\lambda)^{1/2}.$$

It can be seen from Table 1 and from (33) and (34) that X_2 is generally quite large compared with X_1. Consequently, the imaginary part of Ω_a is somewhat larger than its real part which indicates that the presence of the ground screen influences the phase to a greater extent than the amplitude.

5. THE INTEGRAL FOR THE SECTOR GROUND SCREEN

Attention is now turned to the sector portion of the screen. The antenna at Q is again assumed to be a dipole. If $b \ll d$ it is again permissible to retain only first-order phase terms. Therefore, it follows from (10) and (14), that

$$\Omega_b = -\frac{ik}{2\pi \cos\psi_0} \int_{\rho=a}^{b} \int_{\phi=-\Delta_1}^{\Delta_1} e^{-ik\rho}\left(1 + \frac{1}{ik\rho}\right) e^{ik\rho\cos\phi\cos\psi_0} \times$$

$$\times \cos\phi \left(\frac{Z_b' - Z}{\eta_0}\right) d\phi\, d\rho. \tag{35}$$

This result is analogous to (18) for the circular screen. To effect the ϕ integration we use the basic relation

$$e^{ix\cos\phi} = \sum_{n=0}^{\infty} \epsilon_n e^{in\pi/2} J_n(x) \cos n\phi$$

which is a generating function for Bessel functions, $J_n(x)$. Using this result, it readily follows that

$$\Omega_b = \frac{k}{\cos\psi_0} \int_{\rho=a}^{b} e^{-ik\rho}\left(1 + \frac{1}{ik\rho}\right) \Lambda(k\rho\cos\psi_0)\left(\frac{Z_b' - Z}{\eta_0}\right) d\rho \tag{36}$$

where

$$\Lambda(x) = \frac{1}{2\pi i}\sum_{n=0}^{\infty} \epsilon_n e^{in\pi/2} J_n(x) \int_{-\Delta_1}^{\Delta_2} \cos n\phi \cos\phi \, d\phi$$

$$= \frac{1}{4\pi i}\sum_{n=0}^{\infty} \epsilon_n e^{in\pi/2} J_n(x) \left[\frac{\sin(n-1)\Delta_2 + \sin(n-1)\Delta_1}{n-1} + \right.$$

$$\left. + \frac{\sin(n+1)\Delta_2 + \sin(n+1)\Delta_1}{n+1}\right]. \quad (37)$$

It is noted that if $\Delta_1 = \Delta_2 = \pi$,

$$\Lambda(x) = J_1(x) \quad (38)$$

which corresponds to a circular screen. Another special case is

$$\Delta_1 = \Delta_2 = \frac{\pi}{2}$$

which leads to

$$\Lambda(x) = \frac{J_1(x)}{2} + \frac{i}{\pi}\sum_{n=0}^{\infty} \frac{\epsilon_n J_{2n}(x)}{(4n^2-1)}. \quad (39)$$

This particular formula was quoted by Monteath (1958).

Equation (36), which generally requires a numerical integration and a summation, becomes very complex if the screen is large in terms of wavelength. This complication results from the poor convergence of the Bessel function series when $k\rho \cos\psi_0 \gg 1$. An alternate representation for Ω_b can be obtained if a modified stationary-phase evaluation of the ϕ integration is used. This approach is particularly suitable when the $kb \gg 1$. The phase factor $k(R - R_0)$ is now approximated in the following way

$$(R - R_0) = \left[R_0^2 + \rho^2 - 2\rho d\left(1 - \frac{\phi^2}{2} + \ldots\right)\right]^{1/2} - R_0$$

$$\cong R_1 - R_0 + \frac{\rho d\phi^2}{2R_1} \quad (40)$$

where $R_1 = [R_0^2 + \rho^2 - 2\rho d]^{1/2}$. Thus we have retained second-order phase variations. Consequently,

$$\Omega_b \cong -\frac{ik}{2\pi\cos\psi_0}\int_{\rho=a}^{b}\left(\frac{Z_b' - Z}{\eta_0}\right) e^{-ik\rho} e^{-ik[R_1-R_0]} \times$$

$$\times \int_{-\Delta_1}^{\Delta_2} e^{-ik\rho d\phi^2/2R_1} \, d\phi \, d\rho. \quad (41)$$

This can be written in the form

$$\Omega_b \cong -\left(\frac{ik}{2\pi}\right)^{1/2} \frac{1}{\cos^{3/2}\psi_0} \int_a^b \left(\frac{Z_b' - Z}{\eta_0}\right) \frac{e^{-ik\rho}\, e^{-ik(R_1-R_0)}}{\rho^{1/2}} F(k\rho)\, d\rho \quad (42)$$

where

$$F(k\rho) = \frac{1}{1-i} \int_{-\Delta_1\sqrt{\left(\frac{k\rho d}{\pi R_1}\right)}}^{\Delta_2\sqrt{\left(\frac{k\rho d}{\pi R_1}\right)}} e^{-i(\pi/2)t^2}\, dt$$

$$= \frac{\left\{C\left[\Delta_2\sqrt{\left(\frac{k\rho d}{\pi R_1}\right)}\right] + C\left[\Delta_1\sqrt{\left(\frac{k\rho d}{\pi R_1}\right)}\right]\right\} - i\left\{S\left[\Delta_2\sqrt{\left(\frac{k\rho d}{\pi R_1}\right)}\right] + S\left[\Delta_1\sqrt{\left(\frac{k\rho d}{\pi R_1}\right)}\right]\right\}}{1-i}, \quad (43)$$

and

$$C(u) - iS(u) = \int_0^u e^{-i(\pi/2)t^2}\, dt. \quad (44)$$

It can be seen that if

$$\Delta_2\sqrt{\left(\frac{k\rho d}{\pi R_1}\right)} \quad \text{and} \quad \Delta_1\sqrt{\left(\frac{k\rho d}{\pi R_1}\right)}$$

are greater than about 5, $F(k\rho)$ may be replaced by unity and thus the sector is behaving as a circular screen of radius b. Equation (42) is in a suitable form for numerical integration in the general case when $F(k\rho)$ cannot be replaced by unity.

In the far zone, where only first-order phase variations are employed, (42) has the form

$$\Omega_b \cong -\left(\frac{ik}{2\pi}\right)^{1/2} \frac{1}{\cos^{3/2}\psi_0} \int_a^b \frac{Z_b' - Z}{\eta_0} \frac{e^{-ik\rho(1-\cos\psi_0)}}{\rho^{1/2}} F(k\rho)\, d\rho \quad (45)$$

and

$$F(k\rho) = \frac{1}{1-i} \int_{-\Delta_1\sqrt{\left(\frac{k\rho\cos\psi_0}{\pi}\right)}}^{\Delta_2\sqrt{\left(\frac{k\rho\cos\psi_0}{\pi}\right)}} e^{-i(\pi/2)t^2}\, dt. \quad (46)$$

This is also in suitable form for numerical integration with respect to ρ (or $k\rho$).

An interesting special case of (45) is when $\psi_0 \to 0$ and Z_b' can be regarded as a constant. Then

$$\Omega_b \cong -\left(\frac{ik}{2\pi}\right)^{1/2} \frac{Z_b' - Z}{\eta_0} \int_a^b \frac{F(k\rho)}{\rho^{1/2}}\, d\rho. \quad (47)$$

After an integration by parts, it readily follows that

$$\Omega_b = -\left(\frac{2ikb}{\pi}\right)^{1/2}\left(\frac{Z_b' - Z}{\eta_0}\right)\left\{\left[F(kb) - i\sqrt{\left(\frac{2i}{\pi kb}\right)}\left(\frac{e^{-ikb\Delta_2^2/2}}{2\Delta_2} + \frac{e^{-ikb\Delta_1^2/2}}{2\Delta_1}\right)\right]\right.$$
$$\left. - \sqrt{\left(\frac{a}{b}\right)}\left[F(ka) - i\sqrt{\left(\frac{2i}{\pi ka}\right)}\left(\frac{e^{-ika\Delta_2^2/2}}{2\Delta_2} + \frac{e^{-ika\Delta_1^2/2}}{2\Delta_1}\right)\right]\right\} \quad (48)$$

where $F(kb)$ and $F(ka)$ are defined by (46). If $(kb)^{1/2}\Delta_2$ and $(kb)^{1/2}\Delta_1 \gg 1$, the first term in square brackets can be replaced by unity. Furthermore, if $(b/a)^{1/2} \gg 1$, the second square bracket term is negligible compared with unity. To within this approximation, the sector screen is behaving as a circular screen of radius b (e.g. compare (24) and (48)).

When ψ_0 is finite but small, it is possible to extend the preceding result by replacing the factor $\exp[-ik\rho(1 - \cos\psi_0)]$ in (45) by its power series expansion. Then, again assuming Z_b' is a constant, it is found that

$$\Omega_b = -\left(\frac{2ikb}{\pi}\right)^{1/2}\left(\frac{Z_b' - Z}{\eta_0}\right)\sum_{m=0}^{\infty}\frac{[-ikb(1-\cos\psi_0)]^m}{m!(2m+1)}$$
$$\times\left\{\left[F(kb) - \frac{1}{(2\pi ikb)^{1/2}}\left(\Delta_2 G_m\left(-\frac{i\Delta_2^2}{2}, kb\right) + \Delta_1 G_m\left(-\frac{i\Delta_1^2}{2}, kb\right)\right)\right]\right.$$
$$-\left(\frac{a}{b}\right)^{m+1/2}\left[F(ka) - \frac{1}{(2\pi ika)^{1/2}}\right.$$
$$\left.\left.\left(\Delta_2 G_m\left(-\frac{i\Delta_2^2}{2}, ka\right) + \Delta_1 G_m\left(-\frac{i\Delta_1^2}{2}, ka\right)\right)\right]\right\} \quad (49)$$

where

$$G_m(\alpha, x) = \frac{1}{x^m}\frac{\partial^m}{\partial\alpha^m}\frac{e^{\alpha x}}{\alpha}. \quad (50)$$

When $\psi_0 = 0$ only the $m = 0$ term of the series is finite and the result is identical to (48). When ψ_0 is finite but small enough that $(kb)^{1/2}\psi_0 < 1$, the series converges very rapidly and only a few terms are needed.

6. CONCLUDING REMARKS

The collected results presented here should be useful in making estimates of the influence of an inhomogeneous ground plane on antenna radiation. As we have seen, the subject is closely related to the question of ground-wave propagation over mixed paths such as occur at land–sea boundaries.

In the present study, the electrical characteristics of the ground are assumed to be characterized by a surface impedance which is a (complex) constant Z outside a surface S. Within S the impedance Z' is allowed to be variable. In the case of a radial-wire system emanating from Q, it is appropriate to use formulas which have been developed for the surface of a wire grid in the interface of a conducting half-space (Wait, 1958). In general these are complicated, but recently some numerical results have been obtained which should be useful in this problem. At low radio frequencies for moderately or well-conducting soils it is a satisfactory approximation to regard the surface Z' as the parallel combination of the surface impedance Z_s of the equivalent grid and the ground beneath. Thus

$$Z' \cong \frac{Z_s Z}{Z + Z_s} \tag{51}$$

where

$$Z_s \cong \frac{i\eta_0 d}{\lambda_0} \log_e \frac{d}{2\pi c}, \tag{52}$$

$$Z \cong (i\mu_0\omega/\sigma)^{1/2},$$

and d is the spacing between the radial conductors and c is the radius of the wires. Such a formula is strictly valid only if $(\sigma\mu_0\omega)^{1/2} d \ll 1$ everywhere within the ground system. If there are N radial conductors, it can be seen that d can be replaced by $2\pi\rho/N$ where N is usually of the order of 100.

It is admitted that the theory in this paper is rather involved. In order to obtain numerical results it is necessary to evaluate the integrals Ω_a and Ω_b by numerical or analytical means. The quantity $\Omega(= \Omega_a + \Omega_b)$ is then regarded as the fractional increase of the field as a result of the presence of the sector ground screen (as indicated by equation (15)). The final results should be valid when the surface impedances Z and Z' are reasonably small compared with η_0 or 120 π ohms. However, this is a condition which is also required in Sommerfeld's theory for a dipole over a conducting half-space, and is not overly restrictive.

Extensive numerical results based on the theory given in this paper have now been obtained. They will be included in a forthcoming paper co-authored with Mrs. L. C. Walters [N. B. S. Monograph No. 60, 1963].

7. APPENDIX

The various approximate formulas used in this present work start from the mutual impedance formula given by (2). As Monteath (1951) shows, this is based upon Ballentine's (1929) formulation of the electromagnetic reciprocity theorem. The author (1954) has obtained similar results directly from solutions of the wave equation. The latter method has the advantage that a perturbation procedure can readily be applied to obtain higher order corrections. It has the disadvantage that only relatively simple geometries are easily treated. A brief presentation of the alternate method is given here since it sheds some light on the nature of the approximations used in the body of the text.

We consider a vertical antenna erected over a flat ground plane. Choosing a cylindrical coordinate system (ρ, ϕ, z) the antenna extends from z_1 to z_2 on the z axis and the ground plane is $z = 0$. On the assumption that the fields do not vary in the ϕ direction it is evident that the resultant magnetic field has only a ϕ component, H_ϕ. Furthermore, in the homogeneous space $z > 0$, H_ϕ may be derived from a scalar function which satisfies

$$\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho} + k^2 + \frac{\partial^2}{\partial z^2}\right)\psi = 0 \tag{53}$$

from the relation

$$H_\phi = \frac{\partial\psi}{\partial\rho}. \tag{54}$$

Thus, in general,

$$H_\phi = H_\phi^s + H_\phi^\infty,$$

$$H_\phi^s(\rho, z) = \int_0^\infty J_1(\lambda\rho)\, e^{-u_0 z} f(\lambda)\, \lambda\, d\lambda \tag{55}$$

where H_ϕ^∞ is the field of the antenna over a perfectly conducting ground plane for $z \geqq 0$, where $u_0 = (\lambda^2 - k^2)^{1/2}$. Then from Maxwell's equations

$$\Big[\, E_\rho(\rho, z) \,\Big]_{z=0} = E_\rho(\rho, 0) = \frac{\eta_0}{ik}\int_0^\infty J_1(\lambda\rho) f(\lambda)\, u_0\, \lambda\, d\lambda. \tag{56}$$

On an application of the Fourier-Bessel theorem, it follows that

$$f(\lambda) = \frac{ik}{\eta_0 \mu_0}\int_0^\infty J_1(\lambda\rho)\, E_\rho(\rho, 0)\, \rho\, d\rho. \tag{57}$$

Equation (55) may then be written in the form

$$H_\phi^s(\rho, z) = \frac{ik}{\eta_0}\int_{\rho'=0}^\infty \int_{\lambda=0}^\infty J_1(\lambda\rho)\, J_1(\lambda\rho')\, e^{-u_0 z}\, u_0^{-1}\, E_\rho(\rho', 0)\, \rho'\, d\rho'\, \lambda\, d\lambda. \tag{58}$$

We consider that the ground is modified in some way so that the new tangential field becomes $E_\rho'(\rho, 0)$. This in turn leads to a new secondary field $H_\phi^{s'}$. Therefore, the change of the field ΔH_ϕ resulting from the modification of the ground plane is

$$\Delta H_\phi(\rho, z) = \frac{ik}{\eta_0}\int_{\rho'=0}^\infty \int_{\lambda=0}^\infty J_1(\lambda\rho)\, J_1(\lambda\rho')\, e^{-u_0 z}\, u_0^{-1}\, [E_\rho'(\rho', 0) - E_\rho(\rho', 0)]\, \rho'\, d\rho'\, \lambda\, d\lambda. \tag{59}$$

appropriate for a perfectly conducting ground plane. When the antenna is a quarter-wave monopole with a sinsuoidal current distribution

$$H_\phi^\infty(\rho', 0) = -\frac{iI}{2\pi\rho'}\, e^{-ik\sqrt{[(\rho')^2+(\lambda/4)^2]}} \tag{63}$$

and, for $k\rho \gg 1$,

$$H_\phi^\infty(\rho, z) \cong -\frac{iI}{2\pi\rho}\, e^{-ikR} \cos\left(\frac{\pi}{2}\sin\psi\right). \tag{64}$$

Thus

$$\Omega \cong \frac{\cos\psi}{\cos\left(\frac{\pi}{2}\sin\psi\right)} \int_0^{k\rho_0} \left(\frac{Z' - Z}{\eta_0}\right) e^{-i\sqrt{(x^2+\pi^2/4)}}\, J_1(x\cos\psi)\, dx \tag{65}$$

This is in agreement with (29).

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Influence of a Sector Ground Screen on the Field of a Vertical Antenna [1]

James R. Wait and Lillie C. Walters

The field of a short vertical antenna on a homogeneous ground is shown to be modified by the presence of a metallic screen. The screen is taken in the form of a circular disk and a concentric sector. The modification of the field is expressed in the form of surface integrals over the disk and the sector. Extensive numerical results for these basic integrals are given and a number of applications are illustrated.

1. Introduction

The influence of the ground on the fields of antennas has been discussed sporadically in the literature for many years. In most propagation calculations it is assumed that the transmitting antenna has a fixed dipole moment and the ground is taken to be a perfect conductor or possibly a homogeneous imperfectly conducting half space. In practice, however, some kind of ground system is used. Usually this takes the form of a metal screen or radial wire system which is on the surface of the ground or may be buried slightly beneath the surface. The design of such systems has been typically empirical. Apparently, the first analytical approach was carried out by Brown et al. [1].[2] Later works [2, 3] have dealt mainly with the influence of the ground system on the impedance. In most cases it has been assumed that the radiated field for a given current on the antenna was not appreciably affected by the presence of the ground screen. In fact, an approximate analytical method was given previously by Wait and Pope [4] which is suitable for estimating the dependence of the ground wave on the size of a circular ground screen. Calculations [5] based on this work supported the contention that a screen has only a small effect on the radiated field provided the radius of the screen is of the order of a wavelength or less. Very similar conclusions have been arrived at by British workers [6, 7].

In this paper consideration is given to ground screens which may be large in terms of a wavelength. Since the theory has been treated quite generally in a previous paper [8], attention will be focused here on the numerical calculations and the predicted performance.

2. Formulation and Description of Problem

The situation is described as follows. A vertical electric dipole is located on a flat homogeneous ground of conductivity σ and dielectric constant ϵ. The vertical electric field E at a distance R_0 and elevation angle ψ_0 is given as follows

$$E_z=\frac{i\mu_0\omega lI}{2\pi R_0}\, e^{-ikR_0} \cos^2\psi_0 W(R_0, Z) \qquad (1)$$

where

$\mu_0=4\pi\times 10^{-7}$
ω=angular frequency
l=effective height of transmitting dipole
I=current at terminals of transmitting dipole
$k=2\pi$/wavelength.

In the above, $W(R_0, Z)$ is a complex quantity which is a function of the surface impedance Z of the ground. Over a perfectly conducting ground, W would approach unity. In the case of finite ground conductivity [9]

$$W(R_0, Z)\simeq 1-i(\pi p_0)^{1/2}e^{-w_0} \operatorname{erfc}(iw_0^{1/2}) \qquad (2)$$

where

$$w_0=[1+(\eta_0/Z)\sin\psi_0]^2 p_0, \qquad (3a)$$

$$p_0=-\frac{ikR_0}{2}\left(\frac{Z}{\eta_0}\right)^2, \qquad (3b)$$

and

$$\eta_0=120\pi.$$

This result is valid for $k R_0 >> 1$ and $|Z/\eta_0|^2 << 1$. If

$$Z=\left(\frac{i\mu_0\omega}{\sigma+i\epsilon\omega}\right)^{1/2}\left[1-\frac{i\epsilon_0\omega}{\sigma+i\epsilon\omega}\cos^2\psi_0\right]^{1/2} \qquad (4)$$

the expression for W coincides exactly with the result given by Norton [10] for the same situation. It may be noted that this value of Z is exactly equal to the ratio of the tangential electric and magnetic fields for a vertically polarized plane wave incident at an angle $90°-\psi_0$ on the homogeneous flat ground.

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[1] This work was sponsored by the Electronics Research Directorate of the Air Force Cambridge Research Laboratories, Office of Aero-Space Research (USAF), Bedford, Mass., under contract PRO-61-568.

[2] Figures in brackets indicate the literature references on page 8

In applications to practical communication problems it is very convenient to split off the surface wave portion W_s by writing

$$W = W_r + W_s \quad (5)$$

where, by definition,

$$W_r = [1 + R_v(\psi_0)]/2$$

is the radiation or space wave field, and

$$R_v = \frac{\sin\psi_0 - Z/\eta_0}{\sin\psi_0 + Z/\eta_0} \quad (6)$$

is the Fresnel reflection coefficient. This decomposition of the total field into space and surface wave was first made by Norton [10] and it is a convenient procedure in radio engineering since by definition, W_r is the radiation pattern of the antenna in the presence of the ground plane. It may be the dominant term in many cases of practical interest although as ψ_0 approaches zero W_r actually vanishes. Methods for estimating the relative importance of W_s are given in the papers by Norton [10].

The central task in the present paper is to indicate how a wire mesh or a similar metal screen lying on the ground will modify the field at the receiving antenna. The surface impedance is assumed to be modified to Z' over the area of the screen, but remains the same outside the screen. The field E_z' in the presence of the screen is then written

$$E_z' = \frac{i\mu_0\omega I}{2\pi R_0} e^{-ikR_0}\cos^2\psi_0 W'(R_0, Z, Z'), \quad (7)$$

where W' is an unknown complex quantity which is now a function of Z' in addition to R_0 and Z. The quantity W' reduces to W if $Z' = Z$.

In a previous paper [8] an integral equation for W' was obtained by an application of the Lorentz reciprocity theorem. Although it would be possible to solve this equation directly using a digital computer it was indicated that a first order iteration was satisfactory. In this case it was found that

$$W'(R, Z, Z') \cong W(R, Z)\,[1 + \Omega] \quad (8)$$

where Ω is the fractional change of the field due to the presence of the screen. Within the approximations stated, the factor Ω can be regarded as the modification of the effective height of the transmitting antenna, since it influences W_r and W_s to the same extent.

Before proceeding further it is convenient to introduce a polar coordinate system (ρ, ϕ) centered at the source dipole as indicated in figure 1. Thus an element of area of the ground plane is $\rho\, d\phi\, d\rho$.

From the analysis in the previous paper by Wait [8], it was shown that

$$\Omega \cong -\frac{ik}{2\pi\cos\psi_0}\iint_S e^{-ik\rho} e^{-ik(R - R_0)}\left(\frac{Z' - Z}{\eta_0}\right)$$

$$\times\left(1 + \frac{1}{ik\rho}\right)(\cos\phi)\, d\phi\, d\rho. \quad (9)$$

where

$$R = [\rho^2 + d^2 + h^2 - 2\rho d\cos\phi]^{1/2}$$

$$R_0 = (d^2 + h^2)^{1/2},\; h = R_0\sin\psi_0$$

and

$$d = R_0\cos\psi_0.$$

The integral may be evaluated when the shape of the ground screen is specified. In the following, attention will be confined to screens which are in the form of a sector. A special case is a circular screen and this is considered first.

3. The Circular Screen

Over the range $0 < \rho < a$ it is assumed that the surface impedence is $Z' = Z_s'$. Beyond the screen (i.e., $\rho > a$), $Z' = Z$. Furthermore, it is assumed that the receiving antenna is in the far field such that

$$R - R_0 \cong -\rho\cos\phi\cos\psi_0.$$

Thus

$$\Omega_s \cong -\frac{ik}{2\pi\cos\psi_0}\int_{\rho=0}^{a}\int_{\phi=-\pi}^{\pi} e^{-ik\rho}$$

$$\times\left(1 + \frac{1}{ik\rho}\right) e^{ik\rho\cos\phi\cos\psi_0}\cos\phi\left(\frac{Z_s' - Z}{\eta_0}\right) d\phi\, d\rho. \quad (10)$$

If Z_s' does not depend on ϕ the integration with respect to ϕ may be readily carried out to give

$$\Omega_s \cong \frac{k}{\cos\psi_0}\int_{\rho=0}^{a} e^{ik\rho}\left(1 + \frac{1}{ik\rho}\right) J_1(k\rho\cos\psi_0)$$

$$\left(\frac{Z_s' - Z}{\eta_0}\right) d\rho, \quad (11)$$

where J_1 is the Bessel function of the first type of order one. When dealing with large screens the argument $k\rho\cos\psi_0$ can be regarded as a large quantity over the major portions of the integrand. Thus, J_1 may be replaced by the first term of its asymptotic expansion. Therefore,

$$\Omega_s \cong -\left(\frac{i}{2\pi\cos^3\psi_0}\right)^{1/2}\int_0^{ka}\left(\frac{Z_s' - Z}{\eta_0}\right)$$

$$\times(1 - ie^{-2ix\cos\psi_0})\,\frac{e^{-ix(1-\cos\psi_0)}}{x^{1/2}}\, dx. \quad (12)$$

When Z_s' is essentially constant over the range of integration. Ω_s can be expressed in terms of Fresnel integrals. After a change of variable it readily follow that

$$\Omega_s \simeq \frac{Z-Z_s'}{\eta_0} e^{-i\pi/4} G$$

where

$$G = \frac{i}{(2\cos^3\psi_0)^{1/2}} \left[\frac{1}{\sin(\psi_0/2)} \int_0^{(4ka/\pi)^{1/2}\sin(\psi_0/2)} \exp[-i(\pi/2)t^2]dt - \frac{i}{\cos(\psi_0/2)} \int_0^{(4ka/\pi)^{1/2}\cos(\psi_0/2)} \exp[-i(\pi/2)t^2]dt \right] \quad (13a)$$

As ψ_0 approaches zero the above equation reduces to

$$G = i\left(\frac{2ka}{\pi}\right)^{1/2} \left[1 - i\left(\frac{\pi}{4ka}\right)^{1/2} \int_0^{(4k/\pi)^{1/2}} \exp[-i(\pi/2)t^2]dt \right] \quad (13b)$$

and, if $ka >> 1$, this may be approximated by

$$G \cong i\left(\frac{2ka}{\pi}\right)^{1/2} \left[1 - \left(\frac{i\pi}{8ka}\right)^{1/2} \right] \simeq i\left(\frac{2ka}{\pi}\right)^{1/2}. \quad (14)$$

It is interesting to note that, if the integral in (12) is evaluated by a stationary phase method, the second Fresnel integral in the square bracket term of (13a) is not present. This would correspond to the approximation usually employed in the practical theories of mixed-path ground wave propagation. The value of G corresponding to this situation is denoted $G(1)$.

Numerical values of the integrals G and $G(1)$ are given in table 1. The values of ka (denoted KA) take the values 5, 10, 20, 30, and 100, while ψ_0 (denoted PSI) runs from 0° to 45°. It is immediately evident that, for small values of ψ_0 (i.e., near grazing), the integrals G and $G(1)$ are not significantly different. As will be clear from the following section the integral $G(1)$ would correspond physically to the situation where the screen is semicircular in shape (i.e., ϕ extends from $\pi/2$ to $-\pi/2$ only).

To illustrate the application of the results in table 1, values of the complex quantity $1+\Omega_s$ have been computed for several values of the surface impedances of the ground plane. For this purpose it is convenient to write

$$\frac{Z-Z_s'}{\eta_0} = \frac{1}{N} e^{i\beta} \quad (15)$$

where N and β are real. If the ground screen is a metal sheet $Z_s' << Z$ and, consequently, $Ne^{-i\beta}$ could be regarded as the complex refractive index of the ground itself. However, in general, N and β have a more general meaning as defined by (15). Taking $N=3$ and $\beta=0°$, the amplitude and phase of $1+\Omega_s$ are shown plotted in figures 2a and 2b, respectively, as a function of ψ_0 for various values of ka. It is emphasized that such curves should not be regarded as radiation patterns but rather as modifications of the effective height of the transmitting antenna due to the ground screen. It is apparent that for the low angles involved in HF communication the ground screen will increase the effective height of the transmitting antenna by a significant amount. The value of N given in this example corresponds to a dielectric constant of 3^2 or 9 which is typical of very dry ground. The effect of choosing a large value of N is shown in figures 3a and 3b where $N=10$ and $\beta=0°$. The curves are very similar in shape but the overall effectiveness of the ground screen is reduced somewhat.

The value of β, as defined by (15), determines the phase of the complex refractive index of the ground. For a very dry or nonconducting ground β is zero as indicated in figures 2a to 3b. However, when the conductivity becomes important β may be greater than zero. In fact, for a highly conducting ground where displacement currents are negligible, β may approach 45°. To illustrate the influence of finite β, the amplitude and phase of $1+\Omega_s$ are shown in figures 4a and 4b for $ka=20$, $N=10$, and various values of β between 0° and 45°. It is evident from these curves that the presence of the conduction currents tends to diminish the amplitude but it does increase the phase.

It is becoming apparent that at the lower frequencies and highly conducting ground the presence of the ground screen has a small effect on the total field (for a given strength Il of the source dipole). To illustrate this point, the amplitude and phase of $1+\Omega_s$ are shown in figures 5a and 5b for $ka=20$ and $\beta=45°$ for $N=10$ and 30. The modification of the effective height of the antenna is less than 2 db and here the radius of the screen is almost 3 wavelengths.

4. The Sector Screen

It is clear from the previous results that a large circular ground screen will, indeed, improve the low angle radiation from a ground-based vertical antenna. However, one might ask if any portions of the circular ground screen could be removed without materially affecting the performance of the system. This is certainly a valid question. In the first place it is known [5] that the impedance of the antenna is not affected by anything beyond about one-half wavelength from the antenna. Therefore, to throw some light on the question posed above, the ground screen is taken to be in the form of a sector extending from $\rho=a$ to $\rho=b$ from the base of the transmitting antenna. From

$\rho=0$ out to $\rho=a$ the screen is circular in shape. The situation is illustrated in figure 6. The surface impedance over the area of the sector is Z_b. In terms of the polar coordinate system (ρ, ϕ), the area of the sector is defined by $-\Delta_1<\phi<\Delta_2$ and $a<\rho<b$.

It is convenient to express the factor Ω as the sum of two parts in the manner

$$\Omega=\Omega_a+\Omega_b \tag{16}$$

where Ω_a is the contribution from the circular screen of radius a and Ω_b is the contribution from the sector which extends from a to b. The portion Ω_b can be written

$$\Omega_b=-\frac{ik}{2\pi\cos\psi_0}\int_{\rho=a}^{b}\int_{\phi=-\Delta_1}^{\Delta_2} e^{-ik\rho}$$

$$\left(1+\frac{1}{ik\rho}\right)e^{ik\rho\cos\phi\cos\psi_0}\times\cos\phi\left(\frac{Z_b'-Z}{\eta_0}\right)d\phi d\rho \tag{17}$$

where the receiving antenna is assumed to be in the plane $\phi=0$. The integral for Ω_b given above is sufficiently general to determine the effect of the sector as a function of elevation and azimuth angle. Actually, the integral is analogous to (10) for the circular screen where the limits of ϕ extend from $-\pi$ to π. As before, only first-order phase terms are retained so that the receiving antenna must be in the far field.[3] The extension to the near-field case has been considered previously by Wait [8]. In actual communication circuits the receiving antenna would always be in the far field.

To evaluate the integral in (17) it is convenient to use the approximation

$$\cos\phi=1-\frac{\phi^2}{2}$$

for the exponent in the integrand while, in the integrand, $\cos\phi$ is replaced by unity. This is valid since the principal contributions correspond to small values of ϕ. An interesting check on this statement is given below.

Following the procedure used in the previous section, a dimensionless function G_b is introduced by setting

$$\Omega_b=\frac{Z-Z_b'}{\eta_0}e^{-i\pi/4}G_b. \tag{18}$$

The integral for G_b may now be written in the form

$$G_b=\frac{i}{(2\pi)^{1/2}\cos^{3/2}\psi_0}\int_{ka}^{kb}\frac{e^{-ix(1-\cos\psi_0)}}{x^{1/2}}F(x)dx \tag{19}$$

where

$$F(x)=\frac{1}{1-i}\int_{-\Delta_1[(x/\pi)\cos\psi_0]^{1/2}}^{\Delta_2[(x/\pi)\cos\psi_0]^{1/2}}\exp\left[-i\frac{\pi}{2}t^2\right]dt.$$

The Fresnel integral $F(x)$ is normalized so that $\lim_{x\to\infty} F(x)=1$, provided Δ_1 and Δ_2 are both positive. In this limiting case the sector is behaving essentially as a circular screen. For example, one may note that

$$G_b\Big]_{\Delta\to\infty}=G_1(kb)-G_1(ka)$$

where G_1 is the integral described by omitting the second term of (13a).

An interesting special case of (19) is when $\psi_0\to 0$ and Z_b' can be regarded as a constant. Then

$$G_b=i\left(\frac{1}{2\pi}\right)^{1/2}\int_{ka}^{kb}\frac{F(x)}{x^{1/2}}dx. \tag{20}$$

After an integration by parts it readily follows that

$$G_b=i\left(\frac{2kb}{\pi}\right)^{1/2}\Bigg\{F(kb)$$

$$-\left(\frac{2}{\pi kb}\right)^{1/2}e^{i3\pi/4}\left[\frac{\exp(-ikb\Delta_2^2/2)}{2\Delta_2}+\frac{\exp(-ikb\Delta_1^2/2)}{2\Delta_1}\right]$$

$$-\left(\frac{a}{b}\right)^{1/2}\Bigg[F(ka)-\left(\frac{2}{\pi ka}\right)^{1/2}e^{i3\pi/4}\Bigg(\frac{\exp(-ika\Delta_2^2/2)}{2\Delta_2}$$

$$+\frac{\exp(-ika\Delta_1^2/2)}{2\Delta_1}\Bigg)\Bigg]\Bigg\}. \tag{21}$$

The integral G_b has been evaluated for a range of values of kb. To simplify the situation, the lower limit ka is fixed at 5 and $\Delta_1=\Delta_2=\Delta$. The numerical results for G_b [denoted G(B)] are given in tables 2 to 8 for Δ [DELTA] ranging from 5° to 60°. Within each table kb [KB] varies from 10 to 100 and ψ_0 [PSI] varies from 0° to 45°.

As a check on the numerical work, G_b for $\psi_0=0$ was calculated using both (19) and (21). Also, it may be noted that

$$G_b(kb)\Big]_{\Delta\to\infty}=G_1(kb)-G_1(5)$$

where the values of $G_1(x)$ are listed in table 1 and where x is to be identified with KA.

To illustrate the effect of a finite value of Δ, some typical cases are shown in figures 7a and 7b where the amplitude and phase of G_b are plotted as a function of ψ_0 for $ka=5$, $kb=40$, and various values of Δ. It appears that for these conditions the total sector angle 2Δ need not be greater than about 50° in order to be fully effective.

In order to demonstrate the effect of the sector on the total field it is convenient to consider both Z'_a and Z'_b small compared with Z. Thus

$$\frac{Z-Z_b'}{\eta_0}\simeq\frac{Z-Z_a'}{\eta_0}\simeq\frac{Z}{\eta_0}\simeq\frac{1}{N}e^{i\theta}$$

where $Ne^{-i\theta}$ is the complex refractive index of the

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[3] This far-field condition can be written

$$kR_0\left[\left(1+\frac{a^2}{R_0^2}-\frac{2ad}{R_0^2}\cos\phi\right)^{1/2}-1\right]-\frac{kad}{R_0}\cos\phi<\frac{\pi}{4}.$$

ground. Consequently, it follows from (16), that

$$1+\Omega=1+\Omega_a+\Omega_b\simeq 1+\frac{e^{i(\beta-\pi/4)}}{N}(G_a+G_b).$$

The amplitude of this quantity (expressed in db) and the phase are shown in figures 8a and 8b for $ka=5$, $kb=40$, $\beta=0$, and $N=3$. This would correspond to a relatively dry soil. It is certainly evident here that considerable improvement results from the presence of the sector. The corresponding set of curves shown in figures 9a and 9b are for a highly conducting soil characterized by $N=10$ and $\beta=45°$. The sector screen here has a negligible effect on the performance of the system. In fact, there is even a slight degradation for the very low grazing angles.

The marked improvement by using a large sector screen on a dry ground is indicated in figure 10. Here $ka=5$, $kb=200$, $N=3$, and $\beta=0$. At low angles the gain is greater than 12 db even with a total sector angle, 2Δ, of 20°.

In the preceding discussion it has been tacitly assumed that the receiving antenna is located in the vertical plane which bisects the sector. Normally, this would be the optimum location and for a fixed communication link it would be considered good practice to orient the sector toward the receiving antenna. However, there may be certain applications where the receiving antenna is located off the center line. The formulas given above are actually valid for this case since Δ_1 and Δ_2 may take any positive or negative value. However, rather than computing directly from the general formulas, it is desirable to establish some simple identities which enable the results in tables 2 to 8 to be used.

It may be readily verified that $G_b(\Delta_1, \Delta_2)$, as defined by (18), has the following property

$$G_b(\Delta_1,\Delta_2)=\frac{G_b(\Delta_1,\Delta_1)+G_b(\Delta_2,\Delta_2)}{2}=\frac{G_b(\Delta_1)+G_b(\Delta_2)}{2}.$$

Numerical values of $G_b(\Delta, \Delta)$ or $G_b(\Delta)$ for various positive values of Δ are given in tables 2 to 8 inclusive. If negative values of Δ are encountered it is useful to note that

$$G_b(\Delta)=-G_b(-\Delta).$$

With this information it is a simple matter to compute $(1+\Omega)$ as a function of the azimuth angle δ which is defined by

$$\delta=(\Delta_2-\Delta_1)/2.$$

Thus

$$1+\Omega_a+\Omega_b\cong 1+\frac{Z-Z'_a}{\eta_0}G_a e^{-i\pi/4}+\frac{Z-Z'_b}{\eta_0}e^{-i\pi/4}G(\Delta_1,\Delta_2) \qquad (22)$$

which is an obvious generalization of (18).

To illustrate the azimuthal variation of the field when using a sector it is again desirable to write

$$\frac{Z-Z'_a}{\eta_0}\cong\frac{Z-Z'_b}{\eta_0}\cong\frac{Z}{\eta_0}\cong\frac{1}{N}e^{i\beta}.$$

Then again denoting the total width of the sector by 2Δ, the amplitudes of $1+\Omega_a+\Omega_b$ are shown in figures 11 and 12 for $N=3$, $\beta=0$, $ka=5$, $\Delta=20°$, and various values of ψ_0 from 0° to 25°. In figure 11, $kb=40$ whereas in figure 12, $kb=200$. As expected, the maximum response corresponds to small values of δ. In fact, as δ increases the response decreases quite significantly for the larger sector.

5. Final Remarks

In the present study, the electrical properties of the ground are assumed to be characterized by a surface impedance which is a (complex) constant Z outside a surface S. Within S, the impedance Z' is allowed to be variable. In the case of a radial wire system emanating from Q, it is appropriate to use formulas which have been developed for the surface of a wire grid in the interface of a conducting half space [11]. In general these are complicated, but recently some numerical results have been obtained which should be useful in this problem.[4] At low radiofrequencies for moderately or well-conducting soils it is a satisfactory approximation to regard the surface Z' as the parallel combination of the surface impedance Z_s of the equivalent grid and the ground beneath. Thus

$$Z'\cong\frac{Z_s Z}{Z+Z_s} \qquad (23)$$

where

$$Z_s\cong\frac{i\eta_0 d}{\lambda_0}\log_e\frac{d}{2\pi c}, \qquad (24)$$

$$Z\cong(i\mu_0\omega/\sigma)^{1/2},$$

and d is the spacing between the radial conductors and c is the radius of the wires. Such a formula is strictly valid only if $(\sigma\mu_0\omega)^{\frac{1}{2}}d<<1$ everywhere within the ground system. If there are N radial conductors, it can be seen that d can be replaced by $2\pi\rho/N$ where N is usually of the order of 100.

[4] Available from Mrs. T. Larsen, Laboratory of Electromagnetic Theory Technical University of Denmark, Copenhagen

6. Appendix

6.1. Evaluation of the Fresnel Integral

The integrals occurring in G (13a), (13b), and $F(x)$, (19) are of the type

$$\int_0^u e^{-(i/2)\pi t^2} dt = C(u) - iS(u). \quad (25)$$

These Fresnel integrals were evaluated by the method proposed by Boersma [12]. This method is based on the τ method of Lanczos [13]. The Fresnel integral defined by Boersma is

$$f(x) = \int_0^x \frac{e^{-it}}{\sqrt{2\pi t}} dt \cdot \quad (26)$$

The definition in (25) conforms to the one used by Boersma [14] in eq (26) if

$$x = \frac{\pi u^2}{2}.$$

For values of the argument $0 \leq x \leq 4$ in (26), $f(x)$ is computed by a finite power series in x; for values of the argument $x \geq 4$, $f(x)$ is approximated by a polynomial in $1/x$. For $n=12$, the power series in x valid for $0 \leq x \leq 4$ is

$$f(x) \cong e^{-ix} \sqrt{\frac{x}{4}} \sum_0^{11} (a_n + ib_n) \left(\frac{x}{4}\right)^n \cdot \quad (27)$$

The power series in $\frac{1}{x}$, valid for $x \geq 4$ is

$$f(x) \cong \frac{1-i}{2} + e^{-ix} \sqrt{\frac{4}{x}} \sum_0^{11} (c_n + id_n) \left(\frac{4}{x}\right)^n \cdot \quad (28)$$

The numerical values of the coefficients a_n, b_n, c_n, and d_n as developed by Boersma [14, 12] are given in table 9. With these coefficients the Fresnel integrals can be computed over the range $0 \leq x \leq \infty$, in general, to eight decimal points. The subroutine used in evaluating the Fresnel integral was checked with the tables of Pearcy [15] and those of Wijngaarden and Scheen [16]. The former tables, using definition (26), are accurate to six or seven digits depending on the size of the argument while the latter, using definition (25), are accurate to five digits.

6.2. Evaluation of G_b by Gaussian Quadrature

With a procedure for evaluating the Fresnel integral, the remaining problem was to compute the integral in (19). The method used, Gaussian quadrature, is described briefly below [17].

In quadrature methods a definite integral is approximated by a weighted sum of particular values of the ordinate with the abscissas properly distributed in the limits of integration. Thus,

$$\int_a^b f(x) dx = \sum_{j=1}^n H_j f(a_j) + E_n. \quad (29)$$

The abscissas a_j are roots of the Legendre polynomials, the weights H_j are functions of these roots, and E_n is the error term which can, in general, be made arbitrarily small with increasing n. The Gaussian roots and weights are tabulated for various n for limits between -1 and 1 by Davis and Rabinowitz [18], but other limits can be used by a change of variable as follows:

$$\int_a^b f(x) dx = \frac{b-a}{2} \int_{-1}^1 g(y) dy,$$

where

$$x = \frac{b-a}{2} y + \frac{b+a}{2}. \quad (30)$$

Furthermore, in the Gaussian quadrature procedure, the integrand is approximated by a polynomial of $(2n-1)$ degree which has the same ordinates as the function for n discrete abscissas.

To obtain accuracy for G_b eq (19) was written

$$G_b = \frac{i}{\sqrt{2\pi} \cos^{3/2} \psi_0} \left[\int_5^{10} \frac{e^{-ix(1-\cos\psi_0)} F(x)}{\sqrt{x}} dx + \int_{10}^{15} \frac{e^{-ix(1-\cos\psi_0)} F(x)}{\sqrt{x}} dx \ldots + \int_{kb-5}^{kb} \frac{e^{-ix(1-\cos\psi_0)} F(x)}{\sqrt{x}} dx \right] \quad (31)$$

and Gaussian quadrature was used with $n=16$ in eq (29) for each interval of 5 for kb. This work was checked against (21) for $\psi_0 = 0$ and various values of Δ_1 and Δ_2. The answers agreed to the five digits asked for in the results.

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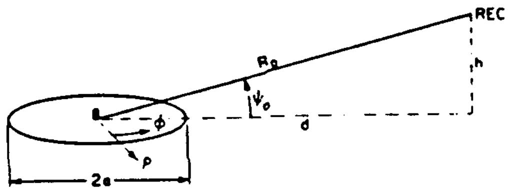

FIGURE 1. *Vertical electric dipole located over a circular metal screen which, itself, is lying on a homogeneous ground.*

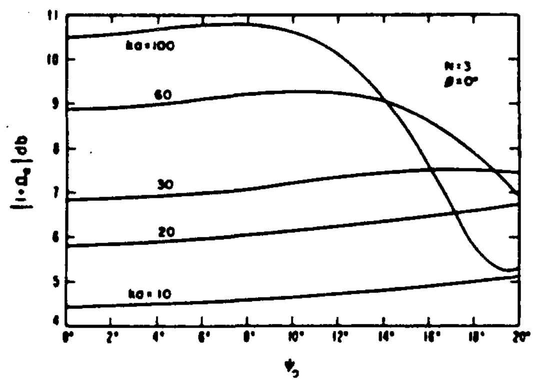

FIGURE 2a. *The amplitude of $1+\Omega_0$ as a function of ψ_0 with parameter* ka *for nonconducting ground illustrating the effect of the circular screen of radius a*

[The ordinate can be regarded as the modification of the effective height of the monopole resulting from the presence of the ground screen.]

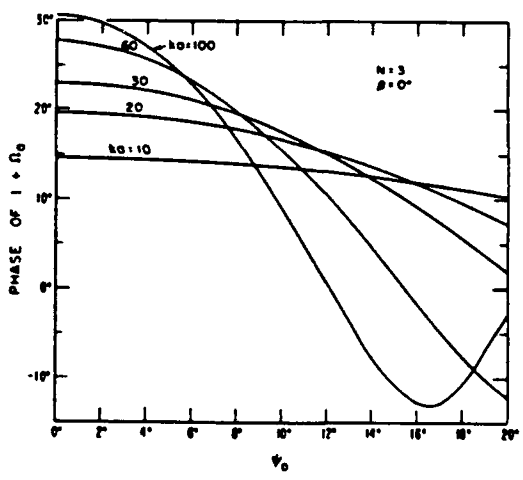

FIGURE 2b. *The phase of $1+\Omega_0$ as a function of ψ_0 with parameter* ka *for nonconducting ground.*

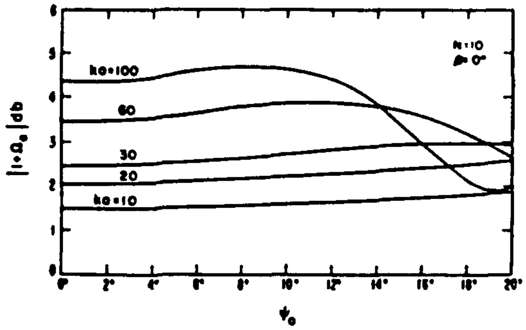

FIGURE 3a. *The amplitude of* $1+\Omega_a$ *as a function of* ψ_0 *with parameter* ka *and nonconducting ground illustrating the effect of large* N.

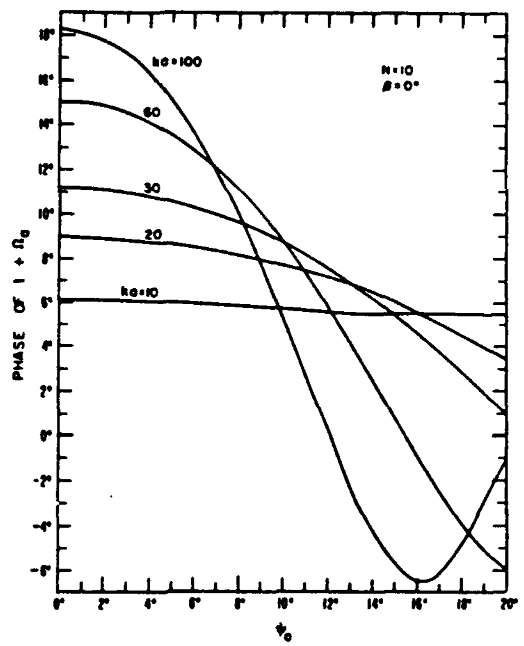

FIGURE 3b. *The phase of* $1+\Omega_a$ *as a function of* ψ_0 *with parameter* ka *and nonconducting ground illustrating the effect of large* N.

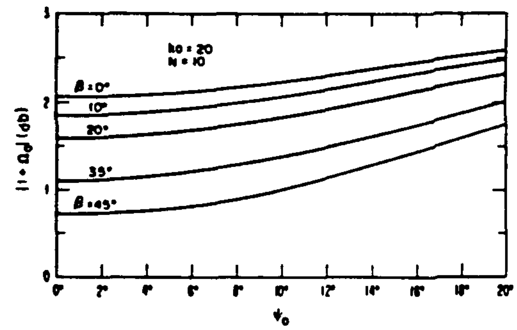

FIGURE 4a. *The amplitude of* $1+\Omega_a$ *as a function of* ψ_0 *illustrating the effect of finite* β.

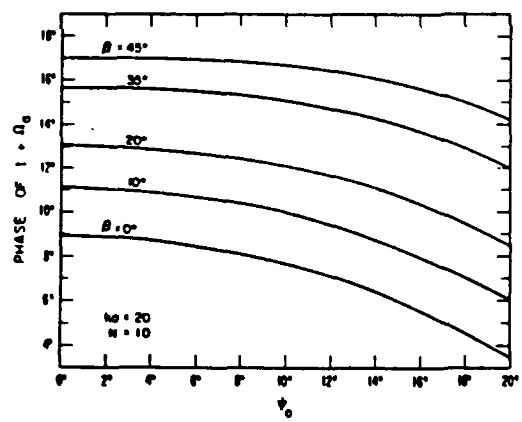

FIGURE 4b. *The phase of* $1+\Omega_a$ *as a function of* ψ_0 *illustrating the effect of finite* β.

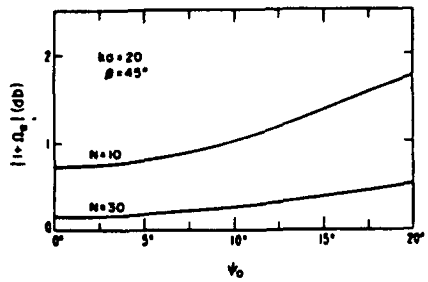

FIGURE 5a. *The amplitude of* $1+\Omega_a$ *as a function of* ψ_0 *for highly conducting ground illustrating the effect of large* N.

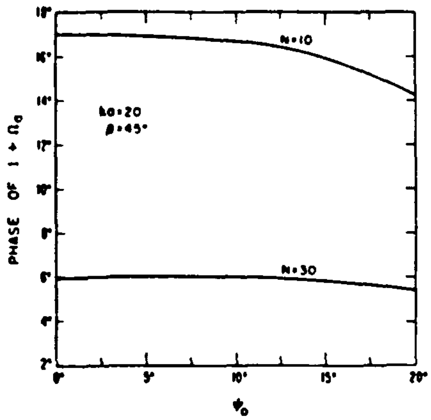

FIGURE 5b. *The phase of* $1+\Omega_a$ *as a function of* ψ_0 *for highly conducting ground illustrating the effect of large* N.

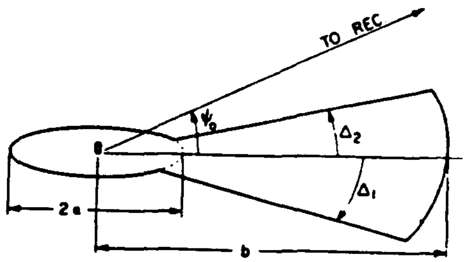

FIGURE 6. *Vertical electric dipole located over a combination circular-sector screen which, itself, is lying on a homogeneous ground.*

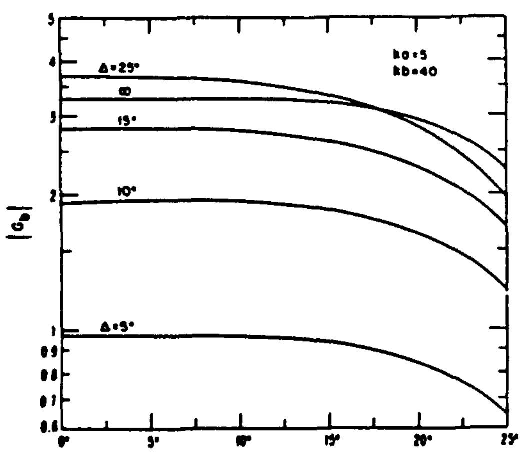

FIGURE 7a. *The amplitude of* G_b *as a function of* ψ_0 *illustrating the effect of finite* Δ. (These curves are for a circular screen of radius a and a sector which extends from a to b).

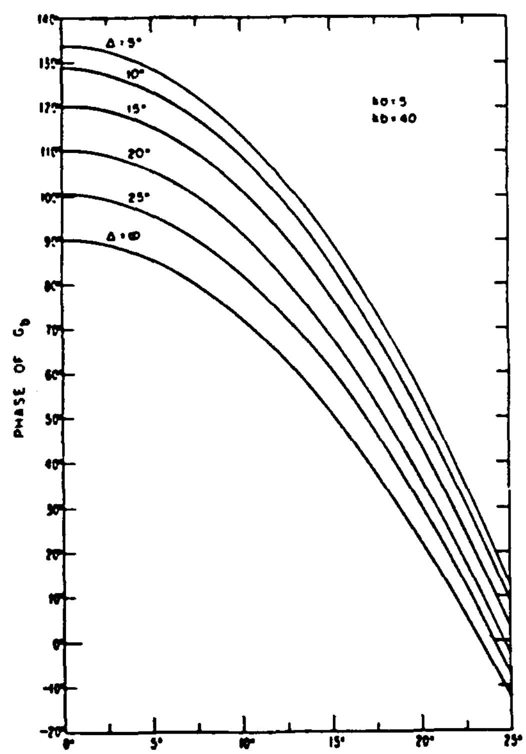

FIGURE 7b. *The phase of* G_b *as a function of* ψ_0 *illustrating the effect of finite* Δ.

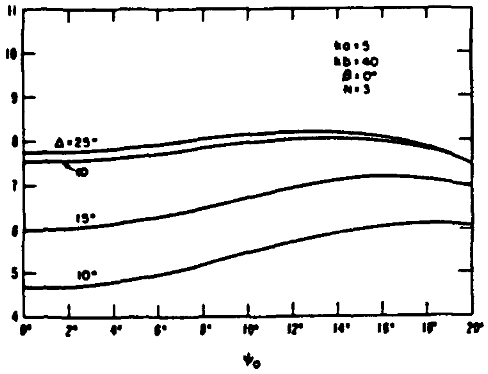

FIGURE 8a. *The amplitude of $1+\Omega_a+\Omega_b$ as a function of ψ_0 illustrating the effect of finite Δ for nonconducting ground.*

[The ordinate can be regarded as the modification of the effective height of the monopole resulting from the presence of the ground screen which is in the combined form of a circular disk and a sector.]

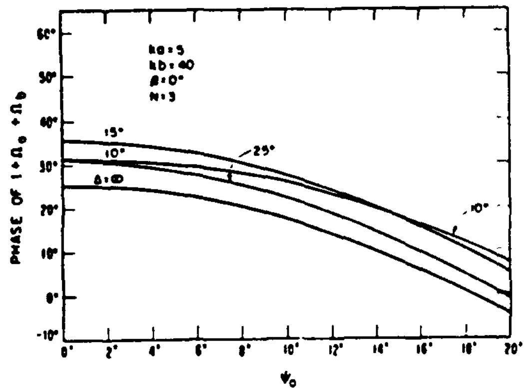

FIGURE 8b. *The phase of $1+\Omega_a+\Omega_b$ as a function of ψ_0 illustrating the effect of finite Δ for nonconducting ground.*

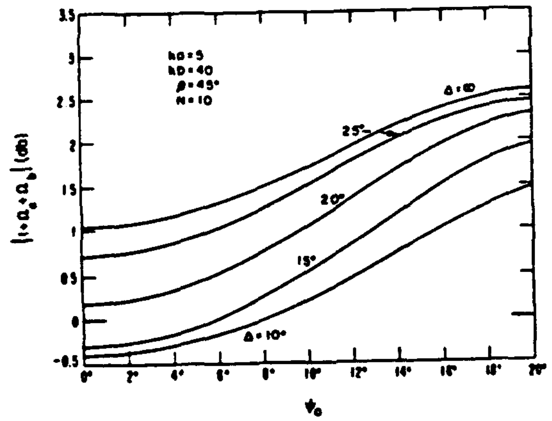

FIGURE 9a. *The amplitude of $1+\Omega_a+\Omega_b$ as a function of ψ_0 for highly conducting ground and large N illustrating the effect of finite Δ.*

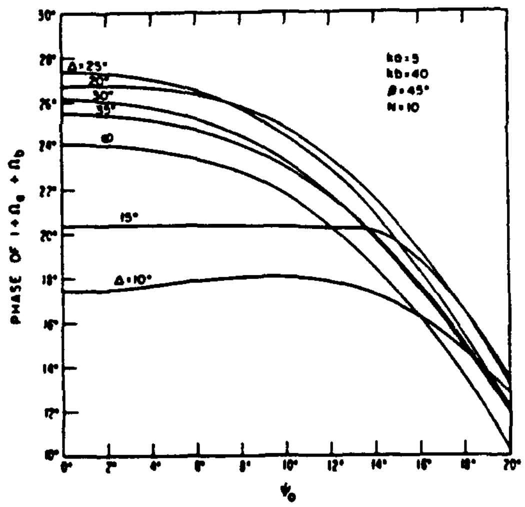

FIGURE 9b. *The phase of $1+\Omega_a+\Omega_b$ as a function of ψ_0 for highly conducting ground and large N illustrating the effect of finite Δ.*

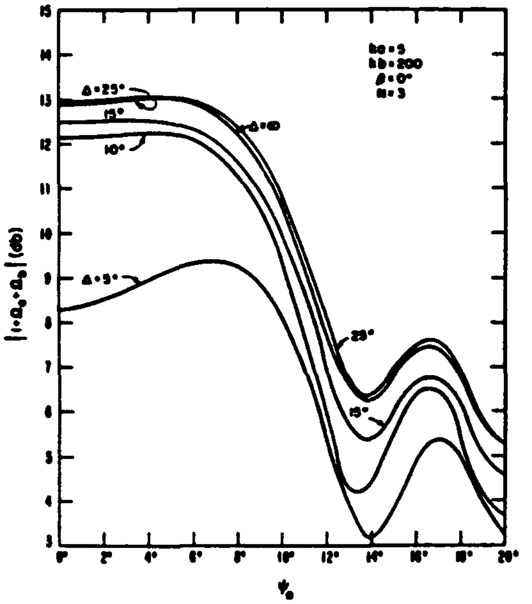

FIGURE 10. *The amplitude of* $1+\Omega_a+\Omega_b$ *as a function of* ψ_0 *illustrating the effect of finite* Δ *for nonconducting ground.*

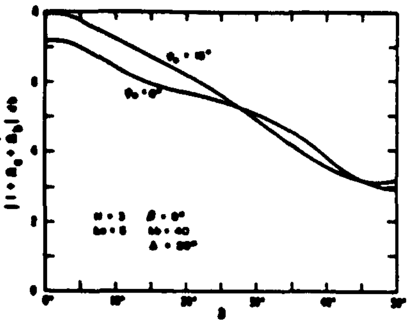

FIGURE 11. *The amplitude of* $1+\Omega_a+\Omega_b$ *as a function of the azimuthal angle* δ *for nonconducting ground illustrating the effect of finite* ψ_0 *for* kb=40.

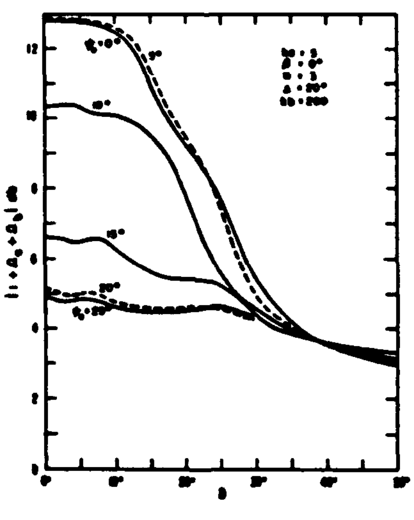

FIGURE 12. *The amplitude of* $1+\Omega_a+\Omega_b$ *as a function of the azimuthal angle* δ *for nonconducting ground illustrating the effect of finite* ψ_0 *for* kb=200.

1.12 Wait, J. R., 1967, "Pattern of a Linear Antenna Erected Over a Tapered Ground Screen," *Canadian Journal of Physics*, Vol. 45, pp. 3091-3101

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PATTERN OF A LINEAR ANTENNA ERECTED OVER A TAPERED GROUND SCREEN[1]

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A theory is presented for the radiation from a thin vertical antenna located above and at the center of a sector ground system. The formulation is carried out for a general variation of the surface impedance of the system. To facilitate discussion, certain limiting cases are considered in some detail. Of special interest is the possibility that the low-angle radiation pattern of an h.f. antenna over a dielectric-type ground will be vastly improved by using a ground screen whose surface impedance varies exponentially, in the radial direction, from the base of the antenna.

1. INTRODUCTION

On a number of occasions it has been suggested that low-angle radiation from ground-based h.f. antennas can be enhanced by the use of large ground planes (Wait 1956, 1963; Wilson 1961; Andersen 1963). To be effective, these must reduce significantly the surface impedance of the foreground out to distances from the antenna comparable with or greater than a Fresnel zone. Usually, in the analytical formulations of this problem, it has been assumed that the transmitting antenna is equivalent to a vertical electric dipole located on the screen. In this paper, we wish to indicate the generalizations of the theory required to account for an antenna which may be of both arbitrary length and arbitrary height above ground. At the same time, we shall also consider the ground screen or earth mat to be tapered in the sense that, in general, its surface impedance is not constant.

The geometry of the situation is indicated in Fig. 1 where we have used cylindrical coordinates (ρ, ϕ, z), with the surface of the ground being $z = 0$ and the linear antenna extending from $z = h_1$ to $z = h_2$ on the z axis. Without

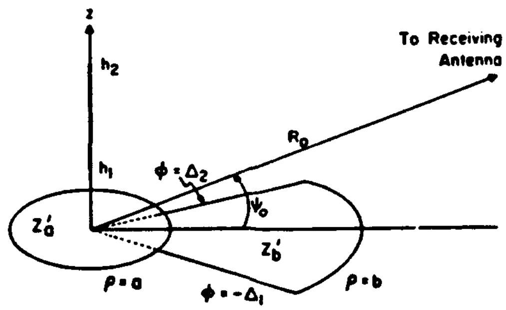


FIG. 1. A linear antenna erected over a sector ground system whose surface impedance differs from the surrounding ground plane.

[1]This work was carried out while the author was a visiting professor at Harvard University Cambridge, Massachusetts.

loss of generality, the receiving antenna is located in the plane $\phi = 0$ at a radial distance R_0 measured from the origin. The angle of radiation is denoted ψ_0 and is measured from the ground as indicated in Fig. 1. Within the circular portion of the ground screen (i.e., $\rho < a$), the surface impedance is denoted $Z_a'(\rho)$, which we regard to be a function of ρ. The outer portion of the ground screen (i.e., $a < \rho < b$ and $-\Delta_1 < \phi < \Delta_2$), the surface impedance is denoted $Z_b'(\rho, \phi)$, which may be a function of both ρ and ϕ. To facilitate subsequent discussion, we shall refer to the latter as the sector contribution. (In previous studies, both Z_a' and Z_b' were regarded as constants, in which case, the outer portion of the ground screen was truly a sector.)

The formulation of the present problem is really a straightforward application of the Lorentz reciprocity theorem to the free-space region $z > 0$, which is bounded by an inhomogeneous surface where the tangential fields satisfy impedance boundary conditions. Using some of the formalism developed for solving problems in network theory, such as Monteath's (1951) extension of the compensation theorem, we find an expression for the radiated field in terms of the specified antenna current and the tangential magnetic field on the surface $z = 0$.

2. THE AZIMUTHALLY SYMMETRIC PROBLEM

To illustrate the purely theoretical aspect of the problem, we consider first the azimuthally symmetric situation where the surface impedance at the plane $z = 0$ is $Z'(\rho)$. The resulting magnetic field then has only a ϕ component which we denote H_ϕ'. On the other hand, if, instead, the plane $z = 0$ can be characterized by a constant surface impedance Z, the corresponding field, of the *same* antenna with the same current distribution, is readily computed and we designate this H_ϕ. Now, under the rather nonrestrictive assumption that the receiving antenna is in the far field, we find the following:

$$(1)\quad H_\phi' = H_\phi + \frac{k\exp(-ikR_0)}{2R_0}(1 + R_v) \times \int_0^\infty \frac{Z'(\rho') - Z}{\eta_0} H_\phi'(\rho', 0) J_1(k\rho' \cos\psi_0)\rho' d\rho',$$

where

$$R_v = \frac{\sin\psi_0 - (Z/\eta_0)}{\sin\psi_0 + (Z/\eta_0)}$$

is a Fresnel reflection coefficient. Here, $k = (\epsilon_0\mu_0)^{\frac{1}{2}}\omega = \omega/c$, $\eta_0 = (\mu_0/\epsilon_0)^{\frac{1}{2}} = 120\pi$ ohms, and ω is the angular frequency which enters into the implied time factor $\exp(i\omega t)$. Then, of course, J_1 is the Bessel function of order 1. The derivation of (1) follows directly from the material in the Appendix of the paper referenced above. It is rather important to note that $H_\phi'(\rho', 0)$ is the tangential magnetic field over the ground plane whose surface impedance is $Z'(\rho')$.

The usefulness of (1) lies in the fact that the integrand may be approximated by assuming a value for $H_\phi'(\rho', 0)$ which need only be valid when

$Z'(\rho')$ differs appreciably from Z. Thus, for example, if the ground screen is a perfectly conducting circular plate of radius a, we replace the upper limit of the integral by a and $Z'(\rho')$ is zero. In this case, we may assume that $H_\phi'(\rho', 0)$ over the range of $\rho' = 0$ to a is the same as $H_\phi^\infty(\rho', 0)$, which is the tangential magnetic field over a perfectly conducting ground plane of infinite extent. It is clear that this assumption involves the neglect of waves reflected from the edge of the screen, but otherwise it seems perfectly reasonable.

First of all, we have the following exact expression for the radiated field in the absence of any ground system:

$$(2) \quad H_\phi(\rho, z) = \frac{ik \cos \psi_0 \exp(-ikR_0)}{4\pi R_0} \times \int_{h_1}^{h_2} I(h)[\exp(ikh \sin \psi_0) + R_v \exp(-ikh \sin \psi_0)]dh.$$

This may be interpreted as the direct radiation of the linear antenna with a specified current $I(h)$ and its image-carrying current $I(h)R_v$. For convenience in what follows, we rewrite (2) in the form:

$$(3) \quad H_\phi(\rho, z) = \frac{i \cos \psi_0}{4\pi R_0} (1 + R_v)\exp(-ikR_0)I_m F(\psi_0),$$

where

$$(4) \quad F(\psi_0) = \frac{k}{1 + R_v} \int_{h_1}^{h_2} f(h)[\exp(ikh \sin \psi_0) + R_v \exp(-ikh \sin \psi_0)]dh,$$

$$I(h) = I_m f(h).$$

Here $F(\psi_0)$ is a dimensionless pattern function and $f(h)$ is a dimensionless current distribution function, while I_m is some suitably defined reference current on the antenna.

The tangential magnetic field, under the perfect conductivity assumption, is

$$(5) \quad H_\phi^\infty(\rho', 0) = -\frac{\partial}{\partial \rho'} \int_{h_1}^{h_2} I(h) \frac{\exp(-ikr_e)}{2\pi r_e} dh,$$

where $r_e = [(\rho')^2 + h^2]^{\frac{1}{2}}$. An equivalent form of (5) is

$$(6) \quad H_\phi^\infty(\rho', 0) = \frac{ikI_m}{2\pi} \int_{h_1}^{h_2} f(h) \frac{\rho'}{r_e^2} \left(1 + \frac{1}{ikr_e}\right) \exp(-ikr_e)dh.$$

We now use (1), in combination with (3) and (6), and restrict attention to the perfectly conducting circular ground screen or plate of radius a. Thus, we find the following expression for the total radiation field:

$$(7) \quad H_\phi'(\rho, z) = \frac{iI_m \cos \psi_0}{4\pi R_0} \exp(-ikR_0)(1 + R_v)[F(\psi_0) + \Omega],$$

where

$$(8)\qquad \Omega = -\frac{k^2}{\cos\psi_0}\frac{Z}{\eta_0} \times \int_0^a \left[\int_{h_1}^{h_2} f(h)\frac{(\rho')^2}{r_q^2}\left(1+\frac{1}{ikr_q}\right)\exp(-ikr_q)\mathrm{d}h\right] J_1(k\rho'\cos\psi_0)\mathrm{d}\rho'.$$

As an interesting check on this result, we may let $a \to \infty$, whence

$$(9)\qquad \Omega|_{a\to\infty} = k\frac{1-R_v}{1+R_v}\int_{h_1}^{h_2} f(h)\exp(-ikh\sin\psi_0)\mathrm{d}h,$$

which follows from the Appendix of a previous paper (Wait 1967). Then, (7) reduces to

$$(10)\qquad H_\phi'(\rho, z)|_{a\to\infty} = \frac{ikI_m\cos\psi_0\exp(-ikR_0)}{2\pi R_0}\int_{h_1}^{h_2}\cos(kh\sin\psi_0)f(h)\,\mathrm{d}h,$$

which is the exact expression for the radiation field of the linear antenna over a perfectly conducting ground plane of infinite extent. In passing, we mention that the recovery of this exact result is a consequence of using the integral relation (1) with the identity $H_\phi'(\rho', 0) = H_\phi^\infty(\rho', 0)$. It is important to note that the latter is only an approximation when the screen is of finite size.

3. GENERAL FORMULATION

We now return to the general configuration indicated in Fig. 1. In this case, we no longer have azimuthal symmetry. This means that, in place of (1), we use

$$(11)\qquad H_\phi' = H_\phi + \frac{k\exp(-ikR_0)}{2R_0}(1+R_v) \times \int_S\int \frac{Z'(\rho',\phi)-Z}{\eta_0}H_\phi'(\rho',0)\frac{\exp(ik\rho'\cos\psi_0\cos\phi)}{2\pi i}\rho'\cos\phi\mathrm{d}\rho'\mathrm{d}\phi,$$

where the integration extends over the surface S of the ground screen, whose surface impedance is $Z'(\rho', \phi)$, being a function of both ρ' and ϕ. We note here that the observer is located at $(\rho, 0, z)$.

Actually, (11) is an approximation in that the depolarization of the scattered field is neglected. In other words, it is assumed that the magnetic field has only a ϕ component even when the ground screen is not perfectly symmetrical. Furthermore, in order to bring (11) into a tractable form, the tangential field $H_\phi'(\rho', 0)$ over the ground screen is assumed to be the same as $H_\phi^\infty(\rho', 0)$ over the range of integration indicated in (11). With these simplifications, we use (11) to show that

$$(12)\qquad H_\phi'(\rho, 0, z) \cong \frac{iI_m\cos\psi_0\exp(-ikR_0)}{4\pi R_0}(1+R_v)[F(\psi_0)+\Omega_a+\Omega_b],$$

where

$$(13)\quad \Omega_a = \frac{k^2}{\cos\psi_0}\int_0^a \frac{Z_a'(\rho') - Z}{\eta_0}\left[\int_{h_1}^{h_2} f(h)\frac{(\rho')^2}{r_e^2}\left(1 + \frac{1}{ikr_e}\right) \times \exp(-ikr_e)dh\right]J_1(k\rho'\cos\psi_0)d\rho'$$

and

$$(14)\quad \Omega_b = \frac{k^2}{\cos\psi_0}\int_a^b \frac{1}{2\pi i}\int_{-\Delta_1}^{\Delta_2}\frac{Z_b'(\rho',\phi) - Z}{\eta_0} \times\left[\int_{h_1}^{h_2} f(h)\frac{(\rho')^2}{r_e^2}\left(1+\frac{1}{ikr_e}\right)\exp(-ikr_e)dh\right] \times \exp(ik\rho'\cos\psi_0\cos\phi)\cos\phi\, d\phi\, d\rho'.$$

It is evident that Ω_a represents the contribution from the circular portion of the ground screen, whereas Ω_b represents the contribution from the sector portion of the screen.

The results given by (12), (13), and (14), while formidable in appearance, are in a form suitable for specific applications. For example, if the ground screen consists of a radial wire system, the surface impedances Z_a' and Z_b' are expressible in terms of the physical parameters, such as the number of wires, their spacing, and their radii. We shall not enter into this aspect of the problem here since it has been discussed previously (Wait 1959). Another factor is the current distribution on the antenna. Obviously, this plays a role and cannot be normalized out of the problem. However, for sufficiently thin linear antennas, the sinusoidal current assumption may be made.

4. SOME SIMPLIFICATIONS

Considerable simplification results if the length of the antenna is small compared with a wavelength. Then the quantity r_e may be replaced by a constant r_0 which is given by

$$r_0 = [(\rho')^2 + h_0^2]^{1/2} \qquad \text{where} \qquad h_0 = (h_1 + h_2)/2.$$

Then (12) is written in the simpler form:

$$(15)\qquad H_\phi' \cong \frac{iI_m\cos\psi_0\exp(-ikR_0)}{4\pi R_0}(1 + R_v)k\int_{h_1}^{h_2} f(h)dh[G_0(h_0) + \hat{\Omega}_a + \hat{\Omega}_b],$$

where

$$G_0(h_0) = \frac{\exp(ikh_0\sin\psi_0) + R_v\exp(-ikh_0\sin\psi_0)}{1 + R_v},$$

$$(16)\quad \hat{\Omega}_a = \frac{k}{\cos\psi_0}\int_{\rho'=0}^{a}\exp(-ikr_0)\frac{(\rho')^2}{r_0^2}\left(1+\frac{1}{ikr_0}\right)J_1(k\rho'\cos\psi_0)\frac{Z_a' - Z}{\eta_0}d\rho',$$

and

$$(17) \quad \hat{\Omega}_b = -\frac{ik}{2\pi\cos\psi_0}\int_{\rho'=a}^{b}\int_{\phi=-\Delta_1}^{\Delta_2}\exp(-ikr_0)\frac{(\rho')^2}{r_0^2}\left(1+\frac{1}{ikr_0}\right) \times \exp(ik\rho'\cos\phi\cos\psi_0)\cos\phi\left(\frac{Z_b'-Z}{\eta_0}\right)\mathrm{d}\phi\mathrm{d}\rho'.$$

It is also worthwhile to note that (15) can be expressed in the familiar form

$$(18) \quad H_\phi' = \frac{ikp_m}{2\pi R_0}\exp(-ikR_0)\cos\psi_0 W',$$

where

$$p_m = I_m\int_{h_1}^{h_2} f(h)\mathrm{d}h = \int_{h_1}^{h_2} I(h)\mathrm{d}h,$$

and

$$(19) \quad W' = (1+R_v)[G_0(h_0)+\hat{\Omega}_a+\hat{\Omega}_b]/2.$$

If now $kh_0 \ll 1$, the expression for the pattern factor W' reduces to that given previously (Wait 1963) where it was assumed, at the outset, that the transmitting antenna was a source dipole located at the center of the circular screen. In (16) and (17), this amounts to replacing r_0 by ρ' which is a valid approximation for finite values of h_0 provided $k\rho' \gg kh_0$.

5. SOME APPLICATIONS AND CONCLUSIONS

In order to illustrate the general applicability of the preceding results, several simple situations will be considered where the relevant formulas for the pattern functions are expressible in closed form. First of all, we consider the pattern function W'' for a dipole located on and at the center of a circular screen of radius a. From (19) and making use of (16), the appropriate form is

$$(20) \quad W'' = (1+R_v)[1+\hat{\Omega}_a]/2,$$

where

$$(21) \quad \hat{\Omega}_a = -\frac{k}{\cos\psi_0}\int_0^a F(k\rho')\exp(-ik\rho')\left(1+\frac{1}{ik\rho'}\right)J_1(k\rho'\cos\psi_0)\mathrm{d}\rho',$$

and

$$F(k\rho') = [Z - Z_a'(\rho')]/\eta_0.$$

As indicated before, $\hat{\Omega}_a$ is the fractional correction to the pattern factor which accounts for the presence of the ground screen whose surface impedance $Z_a'(\rho')$ differs from the surface impedance Z of the otherwise homogeneous half-space. When this impedance contrast $F(k\rho')$ is zero, then, of course, $\hat{\Omega}_a$ vanishes. In an earlier paper (Wait 1963) some consideration was given to the evaluation of the integral in (21) for $F(k\rho')$ = const. and a finite. Even in this relatively simple situation numerical integration was required, although, for sufficiently large values of ka, useful approximations could be made to yield results in terms of Fresnel integrals.

It is rather interesting to note that (21) may be evaluated in closed form if $F(k\rho')$ is allowed to vary exponentially such that $F(x) = F_0 \exp(-bx)$, where $x = k\rho'$. This can be regarded as a special case of a circular screen whose effectiveness is maximum very near the antenna. Then, we find without difficulty that

$$(22) \quad \hat{\Omega}_a = -\frac{F_0}{\cos\psi_0}\left[\int_0^{ka} \exp[-(b+i)x] J_1(x\cos\psi_0)\mathrm{d}x - i\int_0^{ka} \exp[-(b+i)x_s - 1] J_1(x\cos\psi_0)\mathrm{d}x\right].$$

exp[-(b+i)x]x^-1

When $\exp[-b(ka)] \ll 1$, it is evident that the upper limit of these two integrals is effectively ∞. The integrals are then of the standard types (Gradshteyn and Ryzhik 1965):

$$(23) \quad \int_0^\infty e^{-\alpha x} J_1(\beta x)\mathrm{d}x = \frac{(\alpha^2+\beta^2)^{\frac{1}{2}} - \alpha}{\beta(\alpha^2+\beta^2)^{\frac{1}{2}}}$$

and

$$(24) \quad \int_0^\infty e^{-\alpha x} x^{-1} J_1(\beta x)\mathrm{d}x = \frac{(\alpha^2+\beta^2)^{\frac{1}{2}} - \alpha}{\beta},$$

which are valid for $\mathrm{Re}\,\alpha > 0$ when β is real. Using (23) and (24), we find that, for $ka = \infty$, equation (22) is given by

$$(25) \quad \hat{\Omega}_a = -\frac{F_0}{\cos^2\psi_0}\frac{(1+S_a)[S_a - (1-ib)]}{S_a},$$

where

$$S_a = [(1-ib)^2 - \cos^2\psi_0]^{\frac{1}{2}}.$$

If b now becomes much less than $\sin^2\psi_0$, it is seen that (25) reduces to

$$(26) \quad \hat{\Omega}_a \cong F_0/(\sin\psi_0),$$

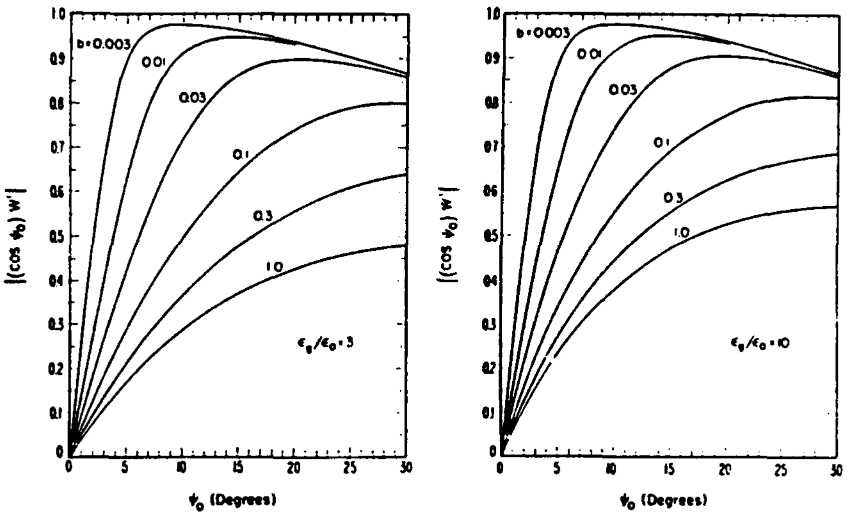
which is the expected value for radiation over an infinite ground plane of constant surface impedance Z_a'.

On the other hand, if b becomes sufficiently large, $\hat{\Omega}_a$ vanishes and the resultant pattern corresponds to that for a dipole located on a ground plane of surface impedance Z. To illustrate the behavior of the fields for this case, the pattern function $|(\cos\psi_0)W'|$ is plotted (in Fig. 2(a), (b)) as a function of ψ_0 for the case where $Z_a'(0) = 0$ and a pure dielectric ground is chosen such that

$$Z/\eta_0 = [(\epsilon_g/\epsilon_0) - \cos^2\psi_0]^{\frac{1}{2}}(\epsilon_0/\epsilon_g)$$

and $\epsilon_g/\epsilon_0 = 3$ and 10. The latter are two typical values of the relative dielectric constant of dry ground. The values of b on the curves are a measure of the extent of the circular screen. For example, the radian distance $1/b$ is the electrical length from the dipole to a point where $[Z - Z_a(\rho')]/\eta_0$ is $1/e$ times its value at $\rho' = 0$.

Note: (26) is exactly true for $Z_a' = 0$; then $F_0 = (Z/\eta_0) \sin\psi_0$



(*a*) (*b*)

FIG. 2. The pattern of a ground-based dipole antenna with a symmetrical ground system with a radially tapered surface impedance.

It is not surprising that the curves in Fig. 2(*a*), (*b*) show that the low-angle radiation is much enhanced when b is chosen to be sufficiently small. Also it is significant that the pattern does not have any lobe structure. This feature is to be contrasted with the case for the abruptly truncated ground screen (Wait 1963).

To give some insight into the sector contribution $\hat{\Omega}_b$, we note first that (17) may be written in the form:

$$(27)\quad \hat{\Omega}_b = -\frac{k}{\cos\psi_0}\int_{\rho'=a}^{b} \exp(-ikr_0)\frac{(\rho')^2}{r_0^2}\left(1+\frac{1}{ikr_0}\right) \times \Lambda(k\rho'\cos\psi_0, \Delta_1, \Delta_2)M(\rho')d\rho',$$

where

$$(28)\quad \Lambda(y, \Delta_1, \Delta_2) = \frac{1}{2\pi i}\int_{-\Delta_1}^{\Delta_2} \exp(iy\cos\phi)(\cos\phi)N(\phi)d\phi,$$

and we have assumed that

$$(29)\quad (Z_s' - Z)/\eta_0 = -M(\rho')N(\phi),$$

being a product of two functions $M(\rho')$ and $N(\phi)$. Clearly, if $N(\phi)$ = const. and $\Delta_1 = \Delta_2 = \pi$, then $\Lambda(y, \Delta_1, \Delta_2) = J_1(y) \times$ const. and the expression for $\hat{\Omega}_b$ has the required form for a circular ground system whose surface impedance varies only with ρ'.

To illustrate in a qualitative manner the influence of a ϕ dependence in Z_s', we choose

(30) $$N(\phi) = \exp(-p(\phi - \phi_m)^2),$$

where p is a scale factor and $\phi = \phi_m$ is the direction where the sector has a maximum value of $N(\phi)$. Then, because we restrict attention to values of a and b which are large compared with a wavelength, it is permissible to replace $\cos\phi$ by $1 - (\phi^2/2)$ in the exponential in the integrand of (28) but replace it by unity elsewhere. At the same time, we permit Δ_1 and Δ_2 to be sufficiently large that $\exp(-p\Delta_1)$ and $\exp(-p\Delta_2)$ are both $\ll 1$. Then, it is clear that

$$\Lambda(y, \Delta_1, \Delta_2) \cong \Lambda_1,$$

where

(31) $$\Lambda_1 = \frac{1}{2\pi i}\int_{-\infty}^{+\infty} \exp[-p(\phi - \phi_m)^2] \exp[iy(1 - \tfrac{1}{2}\phi^2)] \mathrm{d}\phi,$$

and where $y = k\rho' \cos\psi_0$. The integral may be written in the equivalent form

(32) $$\Lambda_1 = \frac{1}{2\pi i} e^{iy} \exp\left[-\left(p - \frac{p^2}{p + \frac{1}{2}iy}\right)\phi_m^2\right] \times \int_{-\infty}^{+\infty} \exp\left[-\left(\sqrt{p + \tfrac{1}{2}iy}\,\phi - \frac{p\phi_m}{\sqrt{(p + \frac{1}{2}iy)}}\right)^2\right] \mathrm{d}\phi$$

and thus

(33) $$\Lambda_1 = -\frac{e^{i\pi/4}}{(2\pi y)^{\frac{1}{2}}} e^{iy} \exp\left(-\frac{i\phi_m^2 p y}{2p + iy}\right) \frac{1}{[1 - i(2p/y)]^{1/2}}.$$

If p is now sufficiently small,

(34) $$\Lambda_1 = -\frac{e^{i\pi/4}}{(2\pi y)^{1/2}} e^{iy},$$

which is to be compared with the asymptotic approximation

(35) $$J_1(y) \simeq -\frac{e^{i\pi/4}}{(2\pi y)^{1/2}} e^{iy}(1 - i e^{-2iy}),$$

which is valid for $y \gg 1$. Thus, we recover the expected value for a circular screen except for the factor ie^{-2iy} which leads to a rapidly varying factor in the integrand for Ω_b' and is of no significance for low-angle radiation over large screens (Wait 1963).

To illustrate the influence of the azimuthal tapering of the ground screen, we show, in Fig. 3, a plot of the function $\Lambda_1(p)/\Lambda_1(0)$ which is the ratio of the right-hand side of (33) and (34). For this example, $y = 10$ and ϕ_m takes the values 0, 5, 10, and 20°. These results indicate that the sector width parameter p plays an important role. Also any asymmetry of the propagation path, relative to the direction of the sector, will degrade the radiated field.

It should be stressed that the curves in Fig. 3 apply only to the relative contribution from a small annular ring of the ground system. Without detailed, numerical evaluations, it is not possible to draw further conclusions about the

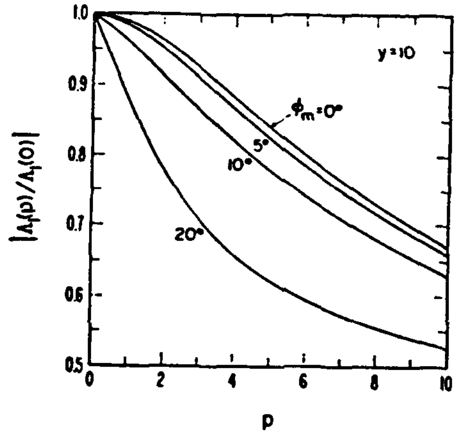


FIG. 3. The influence of azimuthal tapering of the surface impedance of the sector portion of the ground system.

relative merit of azimuthal versus radial tapering of the ground system. It is evident that the whole subject warrants further study.

Finally, we should like to indicate that a formal extension of the theory can be made to account for the departure of the field $H_\phi'(\rho', 0)$ from its assumed value $H_\phi^\infty(\rho', 0)$. First of all, we observe that the surface-wave attenuation over the screen can be considered if the function $F(k\rho')$ in (21) is defined by

$$F(k\rho') = \frac{Z - Z_s'(\rho')}{\eta_0} W(\rho'),$$

where

$$W(\rho') = H_\phi'(\rho', 0)/H_\phi^\infty(\rho', 0)$$

is the attenuation function (Wait 1963). Formally, (20) and (22) still hold if F_0 and b are defined in accordance with the substitution $F(x) = F_0 \exp(-bx)$, where $x = k\rho'$. However, more work is needed to fully develop this approach.

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On the Theory of Radiation From a Raised Electric Dipole Over an Inhomogeneous Ground Plane

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The influence of a finite ground plane on the field of a raised electric dipole is considered from an analytical standpoint. The method for evaluating the integrals is discussed briefly and some asymptotic results are given which illustrate the limiting behavior for ground screens of large diameter. It is shown that the resulting pattern may be crudely interpreted in terms of the direct radiation of the source dipole, its image in an infinite ground plane, and contributions from the edges of the ground screen.

1. Introduction

The possibility that low-angle radiation from vertical antennas can be improved by the use of extended ground systems has been discussed occasionally in the past. While the improvement is very modest for ground screens of wavelength dimensions (Wait and Pope, 1954; Page and Monteath, 1955), the increase in low-angle radiation is substantial when the radial wires or mesh are extended to many wavelengths (Wait, 1963). The use of sector-shaped screens has also been considered, both from the theoretical and the experimental standpoints (Wait and Walters, 1963; Gustafson et al., 1966; Bernard et al., 1966). In fact, the agreement between the observed radiation patterns and those based on the surface impedance model is sufficiently encouraging to pursue further this approach.

In this paper, we wish to generalize the theory to account for the finite height of the antenna located over the ground system. In the previous formulation, the dipole was located on and at the geometrical center of the sector. This extension of the analysis is considered worthwhile in view of the need to choose the optimum relation between ground-system size and antenna height. However, here we will not dwell on the engineering economics of the situation, but will direct our attention to the method of calculation. At the same time, some aspects of the asymptotic behavior of very large screens will be pointed out. The resulting formulas lead to some interesting limiting cases which provide physical insight.

In what follows, we will deal only with the far-field radiation pattern which is assumed to be vertically polarized. This is clearly a special case of the general mutual impedance formulation (Wait, 1963). The situation is illustrated in figure 1.

2. Formulation

With respect to a cylindrical coordinate system (ρ, ϕ, z), the earth's surface is $z=0$ and the dipole Q is located at $z=h$ on the axis. As indicated in figure 1, the circular portion of the ground screen of surface impedance Z_a' is bounded by $\rho=a$, while the sector of surface impedance Z_b' is bounded by $\rho=b$ and $\phi=+\Delta_1$ and $-\Delta_2$. In the absence of any ground system, the surface impedance of the earth is Z and is assumed constant. Apart from the finite value of h, the geometry is identical to that used in the quoted references (Wait, 1963; Wait and Walters, 1963; Bernard et al., 1966; Gustafson et al., 1966). However, in the general mutual impedance formulation, there is one important modification. This is easily seen by noting that the expression for the tangential magnetic

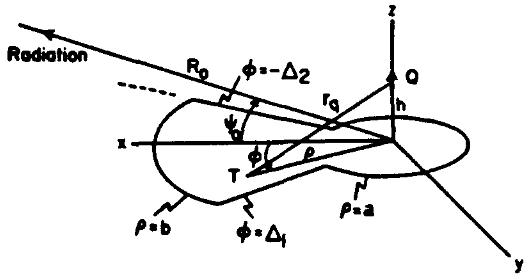

FIGURE 1. *Vertical electric dipole located over a combination circular-sector screen which, itself, is lying on a homogeneous ground.*

field at T of a source dipole at Q is of the form

$$H_\varphi = \frac{ikI_0\ell\rho}{2\pi r_q^2}\left(1+\frac{1}{ikr_q}\right)\exp(-ikr_q) \tag{1}$$

where $r_q=(\rho^2+h^2)^{1/2}$, $k=\omega/c=2\pi/(\text{wavelength})$, and where I_0 is the current and ℓ is the length of the dipole. As usual, the time factor is $\exp(i\omega t)$. The form of (1) indicates that, whenever the factor $[1+(ik\rho)^{-1}]\exp(-ik\rho)$ appears in the previous formulations, we should insert, in its place, the factor $(\rho/r_q)^2[1+(ikr_q)^{-1}]\exp(-ikr_q)$.

Keeping in mind the necessary modifications for a finite value of h, we find the expression for the radiation field to be

$$E_z \cong \frac{i\mu_0\omega\ell I_0}{2\pi R_0} e^{-ikR_0}\cos^2\psi_0 W' \tag{2}$$

where

$$W' = (1+R_v)[G_0(h)+\Omega]/2, \tag{3}$$

$$R_v = \frac{\sin\psi_0-(Z/\eta_0)}{\sin\psi_0+(Z/\eta_0)}, \tag{4}$$

$$G_0(h) = \frac{e^{+ikh\sin\psi_0}+R_v e^{-ikh\sin\psi_0}}{1+R_v}, \tag{5}$$

$$\Omega = \Omega_a + \Omega_b, \tag{6}$$

where

$$\Omega_a = \frac{k}{\cos\psi_0}\int_{\rho=0}^{a} e^{-ikr_q}\frac{\rho^2}{r_q^2}\left(1+\frac{1}{ikr_q}\right)J_1(k\rho\cos\psi_0)\left(\frac{Z_a'-Z}{\eta_0}\right)d\rho \tag{7}$$

and

$$\Omega_b = -\frac{ik}{2\pi\cos\psi_0}\int_{\rho=a}^{b}\int_{\phi=-\Delta_1}^{\Delta_2} e^{-ikr_q}\frac{\rho^2}{r_q^2}\left(1+\frac{1}{ikr_q}\right)e^{ik\rho\cos\phi\cos\psi_0}\cos\phi\left(\frac{Z_b'-Z}{\eta_0}\right)d\phi\, d\rho, \tag{8}$$

where $\eta_0=(\mu_0/\epsilon_0)^{1/2}=120\pi$. The results given by (2)–(8) reduce to the corresponding forms given previously (Wait, 1963; Wait and Walters, 1963) when h is set equal to zero. For $\Omega=0$, the pattern function reduces to the familiar form

$$2W' = (1+R_v)G_0(h) = e^{ikh\sin\psi_0}+R_v e^{-ikh\sin\psi_0}$$

where R_v is the appropriate Fresnel coefficient to account for reflection from the homogeneous flat earth of surface impedance Z. Thus, the dimensionless parameter $\Omega (= \Omega_a + \Omega_b)$ may be identified as the contribution from the combined ground system. Just as in the earlier work, the crux of the problem is the evaluation of the integral expressions for Ω_a and Ω_b.

3. The Contribution From the Circular Screen

First of all, we shall consider the integral Ω_a which is the contribution from the circular portion of the ground system. To simplify the discussion, it is assumed that the surface impedance Z_a' is constant over the area of the screen. Then, after a trivial change of variable, (7) may be rewritten in the form

$$\Omega_a = \frac{Z - Z_a'}{\eta_0} e^{-i\pi/4} G(ka) \tag{9}$$

where

$$G(ka) = -\frac{e^{i\pi/4}}{\cos\psi_0} \int_0^{ka} \frac{x^2}{x^2 + H^2} \left(1 + \frac{1}{i(x^2 + H^2)^{1/2}}\right) J_1(x \cos\psi_0) \exp\left[-i(x^2 + H^2)^{1/2}\right] dx \tag{10}$$

and where $H = kh$. When the radius of the screen becomes infinite, (10) may be evaluated in closed form (as indicated in the appendix) to give

$$G(\infty) = e^{i\pi/4} (\sin\psi_0)^{-1} e^{-iH \sin\psi_0}. \tag{11}$$

This means that, in this limiting case,

$$\Omega_a = \frac{Z - Z_a'}{\eta_0} \frac{e^{-iH \sin\psi_0}}{\sin\psi_0}. \tag{12}$$

Then, at the same time, if the ground screen is sufficiently well conducting such that $|Z_a'/Z| \ll 1$,

$$\Omega_a \cong \frac{1 - R_r}{1 + R_r} e^{-ikh \sin\psi_0}. \tag{13}$$

This combines with (3) to give

$$W'|_{ka=\infty} = \cos(kh \sin\psi_0) \tag{14}$$

which is the required pattern factor for a vertical electric dipole at height h over a perfectly conducting plane. This merely demonstrates that the general surface impedance formulation leads to consistent results when the extent of the screen is unlimited.

In order to develop a suitable asymptotic formula for large values of ka, we combine (9), (10), and (11) to get

$$\Omega_a = \frac{Z - Z_a'}{\eta_0} \left[\frac{e^{-iHS}}{S} + \frac{1}{C} \int_{ka}^{\infty} \frac{x^2}{x^2 + H^2} \left(1 + \frac{1}{i(x^2 + H^2)^{1/2}}\right) J_1(xC) \exp\left[-i(x^2 + H^2)^{1/2}\right] dx\right], \tag{15}$$

where $S = \sin\psi_0$ and $C = \cos\psi_0$. If $ka \gg 1$, and since $x > ka$ over the range of integration, we may replace the Bessel function by the first term of its asymptotic expansion.[1] Thus, (15) is expressible in the form

$$\Omega_a \cong \frac{Z - Z_a'}{\eta_0}\left[\frac{e^{-ikS}}{S} + \frac{e^{-i3\pi/4}}{(2\pi C^3)^{1/2}}\int_{ka}^{\infty} \exp\left[-i(x^2+H^2)^{1/2} + ixC\right]x^{-1/2}\quad [1 - i\exp(-2ixC)]\,dx\right]. \tag{16}$$

For further analysis, (16) is written

$$\Omega_a - \Omega_a^{(\infty)} \cong K\left[\int_{ka}^{\infty} e^{-ix(1-C)}F(x)\,dx - i\int_{ka}^{\infty} e^{-ix(1+C)}F(x)\,dx\right], \tag{17}$$

where

$$K = \frac{Z - Z_a'}{\eta_0}\,\frac{e^{-i3\pi/4}}{(2\pi C^3)^{1/2}}, \qquad \Omega_a^{\infty} = \frac{Z - Z_a'}{\eta_0}\,\frac{e^{-ikS}}{S},$$

and

$$F(x) = x^{-1/2}\exp\left[-i(x^2+H^2)^{1/2} + ix\right].$$

It is clear that the right-hand side of (17) vanishes when the ground system is of infinite extent. Thus, when ka is not infinite, the two integrals may be interpreted as the influence of the finite size of the circular ground system.

Further insight into the problem is achieved by integrating the right-hand side of (17) successively by parts. This leads easily to an expansion of the type

$$\Omega_a - \Omega_a^{(\infty)} \cong -iK\left[\frac{e^{-i(1-C)ka}}{(1-C)}\sum_{n=0}^{N}\frac{F^{(n)}(ka)}{(1-C)^n}(-i)^n - i\,\frac{e^{-i(1+C)ka}}{(1+C)}\sum_{n=0}^{N}\frac{F^{(n)}(ka)}{(1+C)^n}(-i)^n\right] + R_N, \tag{18}$$

where $F^{(n)}(ka) = d^nF(x)/dx^n|_{x=ka}$ for $n = 1, 2, 3, \ldots N$ and $F^{(0)}(ka) = F(ka)$. For sufficiently large ka, the remainder R_N can be made arbitrarily small if N is chosen judiciously.

When the leading two terms of (18) are retained and the remainder R_N is neglected we easily find that

$$\Omega_a - \Omega_a^{\infty} \cong -\left(\frac{Z - Z_a'}{\eta_0}\right)e^{-i\pi/4}\frac{e^{-i\Phi}}{(2\pi C^3)^{1/2}}\left\{\left[\frac{e^{-i(1-C)ka}}{(1-C)(ka)^{1/2}}\left(1 + \frac{i(1+ip)}{2ka(1-C)}\right)\right]\right.$$

$$\left. - i\left[\frac{e^{-i(1+C)ka}}{(1+C)(ka)^{1/2}}\left(1 + \frac{i(1+ip)}{2ka(1+C)}\right)\right]\right\}, \tag{19}$$

where

$$\Phi = [(ka)^2 + H^2]^{1/2} - ka$$

and

$$p = 2ka\left(\frac{ka}{[(ka)^2 + H^2]^{1/2}} - 1\right).$$

If H is sufficiently small, Φ and p can be replaced by zero, and the asymptotic development in (19) is equivalent to the previous formulation (Wait, 1963; Wait and Walters, 1963) where $H = 0$ at the outset.

[1] For example, $J_1(xC) \cong \left(\frac{2}{\pi xC}\right)^{1/2}\cos(xC - 3\pi/4)$ where neglected terms vary as $(xC)^{-3/2}$, $(xC)^{-5/2}$, etc.

Unfortunately, in most cases of practical interest, the asymptotic series of the type given by (18) does not converge well. The difficulty is that $(1-C)(ka)^{1/2}$ is not a large parameter for near grazing angles (i.e., $C \cong 1$) even when ka is large. This suggests that we return to (16) and express it approximately in terms of Fresnel integrals which are well tabulated. Taking a clue from the nature of the asymptotic form given by (19), we simplify the integrand of (16) by employing the approximation

$$\exp\,[-i(x^2+H^2)^{1/2} \pm ixC] \cong \exp\,(-i\Phi)\,\exp\,[-ix(1 \mp C)]$$

which, of course, is exact if either $x = ka$ or if $H = 0$. Without difficulty, it is then found that

$$\Omega_a - \Omega_a^{(\infty)} \cong -\left(\frac{Z-Z_a'}{\eta_0}\right) e^{i\pi/4} e^{-i\Phi} \frac{1}{(2\cos^3\psi_0)^{1/2}} \left\{ \frac{1}{\sin(\psi_0/2)} \int_{2(ka/\pi)^{1/2}\sin(\psi_0/2)}^{\infty} e^{-i(\pi/2)t^2} dt \right.$$

$$\left. - \frac{i}{\cos(\psi_0/2)} \int_{2(ka/\pi)^{1/2}\cos(\psi_0/2)}^{\infty} e^{-i(\pi/2)t^2} dt \right\} \qquad (20)$$

where $\Phi = [(ka)^2 + H^2]^{1/2} - ka \cong H^2/(2ka) = (kh)^2/(2ka)$.

Equation (20) is in a form suitable for numerical evaluation since tables of Fresnel integrals are available for all values of the argument which, in this case, is the parameter $2(ka/\pi)^{1/2}\sin(\psi_0/2)$ or $2(ka/\pi)^{1/2}\cos(\psi_0/2)$.[2] When the argument is sufficiently large, we may use the asymptotic approximation

$$\int_u^{\infty} e^{-i\frac{\pi}{2}t^2} dt \cong -i\,\frac{1}{\pi u}\, e^{-i(\pi/2)u^2}$$

where neglected terms contain higher inverse powers of u.

Employing this form, (20) simplifies considerably to

$$\Omega_a - \Omega_a^{(\infty)} \cong -\left(\frac{Z-Z_a'}{2\eta_0}\right) \frac{e^{-i\pi/4}e^{-i\Phi}}{(2\pi\cos^3\psi_0)^{1/2}} \frac{1}{(ka)^{1/2}} \cdot \left[\frac{e^{-i2ka\sin^2(\psi_0/2)}}{\sin^2(\psi_0/2)} - i\frac{e^{-i2ka\cos^2(\psi_0/2)}}{\cos^2(\psi_0/2)} \right] \qquad (21)$$

where

$$\Omega_a^{(\infty)} = \frac{Z-Z_a'}{\eta_0} \frac{e^{-ikh\sin\psi_0}}{\sin\psi_0} \text{ and } \Phi \cong (kh)^2/(2ka).$$

To give further insight into this result, we assume that $|Z_a'/Z| \ll 1$, corresponding to a highly conducting screen. Then from (3), we find that the corresponding pattern function W' (with no sector screen present) is

$$W' \cong \cos(kh\sin\psi_0) - \frac{(1-R_c)}{4} \frac{e^{-i(\Phi+\frac{\pi}{4})}}{(2\pi ka\cos^3\psi_0)^{1/2}} \sin\psi_0 \cdot \left\{ \frac{e^{-i2ka\sin^2(\psi_0/2)}}{\sin^2(\psi_0/2)} - i\frac{e^{-i2ka\cos^2(\psi_0/2)}}{\cos^2(\psi_0/2)} \right\}. \qquad (22)$$

Here, the first term $\cos(kh\sin\psi_0)$ is the pattern for an infinite ground plane, while the second term is the correction for the finite size of the actual circular screen. The result as given is only valid when $ka\sin^2(\psi_0/2) \gg 1$, $(ka)^2 \gg (kh)^2$, and, as indicated above, $|Z_a'/Z| \ll 1$.

The first term in the curly bracket in (22) represents the contribution from the front edge of

[2] One may note that $\int_u^{\infty} e^{-i(\pi/2)t^2} dt = [\frac{1}{2} - C(u)] - i[\frac{1}{2} - S(u)]$ where $C(u)$ and $S(u)$ are tabulated by Pearcy (1956).

the screen whereas the second term may be identified with a reflection from the back edge. Actually, for angles near grazing, the latter is relatively insignificant. Thus, if $\psi_0^2 \ll 1$, in addition to $ka\psi_0^2 \gg 1$, eq (22) may be further approximated by

$$W' \cong \cos(kh\psi_0) - (1 - R_v)\frac{e^{-i(\Phi + \frac{\pi}{4})} e^{-ika\psi_0^2/2}}{(2\pi ka)^{1/2}\psi_0}. \tag{23}$$

While this is a greatly oversimplified formula, it does indicate that the finite ground plane will only appear "infinite" if $(ka)^{1/2}\psi_0$ is sufficiently large. Any further quantitative discussions necessitate the evaluation of the Fresnel integral form given by (20).

4. The Contribution From the Sector

A few remarks will now be made about the method of handling the contribution Ω_b of the sector screen. The double integral to contend with is given by (8). The approximate method for carrying out the ϕ integration is identical to that used before (Wait, 1963; Wait and Walters, 1963). In the case where both ka and $kb \gg 1$ and when Z_b' is a constant, the result may be written

$$\Omega_b \cong \frac{Z - Z_b'}{\eta_0} e^{-i\pi/4} G_b, \tag{24}$$

where

$$G_b = \frac{i}{(2\pi)^{1/2}(\cos^3\psi_0)^{1/2}} \int_{ka}^{kb} \frac{e^{-ix(1-\cos\phi_0)}}{x^{1/2}} e^{-i\Phi(x)} F_s(x)\,dx, \tag{25}$$

where

$$\Phi(x) = (x^2 + H^2)^{1/2} - x \cong H^2/(2x),$$

and

$$F_s(x) = \frac{1}{1-i} \int_{-\Delta_1[(x/\pi)\cos\phi_0]^{1/2}}^{\Delta_2[(x/\pi)\cos\phi_0]^{1/2}} \exp[-i(\pi/2)t^2]\,dt. \tag{26}$$

The Fresnel integral $F_s(x)$, appearing here, approaches one if Δ_2 and Δ_1 are sufficiently large.

The method for evaluating the integral in (25) is identical to that used before (Wait and Walters, 1963). The only difference is the additional factor $\exp[-i\Phi(x)]$ in the integrand. In fact, if $ka \gg (kh)^2$, this exponential factor may be replaced by one over the range of integration in (25). In this case, the previous results (Wait and Walters, 1963) for the contribution Ω_b and the resulting tabulated integrals may be taken over without change. The pattern function including the effect of the circular screen may then be obtained without difficulty from (3) and (6).

5. Final Remarks

In this paper, we have indicated the necessary extensions of the theory of the "finite ground plane effect" to account for a raised antenna. While we have considered a vertical electric dipole with its free-space pattern described by $\cos\psi_0$, it is clear that a vertical antenna of finite length may, in most cases, be treated by simply replacing $\cos\psi_0$ by the appropriate free-space pattern function. Also, it should be stressed that the discussion given above refers to the far-field pattern of the composite radiating system. From the receiving standpoint, this means that the patterns so calculated are only valid for a downcoming plane-wave incident at a grazing angle ψ_0. Some of the complications which arise when the incident wave emanates from a test transmitter in the "near field" is discussed in the references quoted (Wait, 1963; Wait and Walters, 1963).

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6. Appendix. Evaluation of the Infinite Integral $G(\infty)$

The integral given by (10) for $|ka| = \infty$ may be written in the equivalent form

$$G(\infty)=\frac{e^{-i\pi/4}}{\cos\psi_0}\int_0^{\infty e^{-i\delta}} xJ_1(x\cos\psi_0)\frac{\partial}{\partial x}\left[\frac{\exp[-i(x^2+H^2)^{1/2}]}{(x^2+H^2)^{1/2}}\right]dx \tag{A1}$$

where $e^{-i\delta}=k/|k|$. If the air is allowed to have a vanishingly small conductivity, δ is an arbitrarily small positive constant. Following an integration by parts, (A1) is transformed to

$$G(\infty)=-\frac{e^{-i\pi/4}}{C}\int_0^{\infty}\frac{\exp[-i(x^2+H^2)^{1/2}]}{(x^2+H^2)^{1/2}}\frac{\partial}{\partial x}[xJ_1(Cx)]dx, \tag{A2}$$

where $C=\cos\psi_0$. We now observe that

$$\frac{\partial}{\partial x}[xJ_1(Cx)]=-\frac{\partial^2}{\partial C\partial x}J_0(Cx)=\frac{\partial}{\partial C}[CJ_1(Cx)],$$

which permits (A2) to be expressed by

$$G(\infty)=-\frac{e^{-i\pi/4}}{C}\frac{\partial}{\partial C}C\int_0^{\infty}\frac{\exp[-i(x^2+H^2)^{1/2}]}{(x^2+H^2)^{1/2}}J_1(xC)dx \tag{A3}$$

$$=-\frac{e^{-i\pi/4}}{C}\frac{\partial}{\partial C}C\left[-\frac{2i}{HC}\sin\left[\frac{H}{2}(1-S)\right]\exp\left[-i\frac{H}{2}(1+S)\right]\right] \tag{A4}$$

$$=e^{i\pi/4}S^{-1}\exp(-iHS). \tag{A5}$$

The infinite integral occurring in (A3) is well known.[3]

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Impedance characteristics of electric dipoles over a conducting half-space

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The input resistance of a Hertzian electric dipole located over a homogeneous conducting half-space is considered. It is shown that the functional dependence of the input resistance on the 'loss tangent' of the half-space is consistent with electrostatic concepts, provided the frequency is sufficiently low. On the other hand, at the higher frequencies, the results are compatible with the surface impedance formulations based on the compensation theorem. The results have possible application to remote sensing of planetary surfaces from an elevated source.

INTRODUCTION

When an antenna is located near or on the earth's surface, a substantial fraction of the input power is 'wasted' in the ground. A measure of the performance is the ratio of the radiation resistance to the loss resistance. The loss resistance is proportional to the energy dissipated in the ground. To improve the performance, the antenna may be raised to a sufficient height above the earth to ensure that the reaction of the ground is negligible. For low-frequency operation, this is not very practical because the height must be comparable or somewhat greater than the operating wavelength. Another approach is to employ a metallic ground screen at the base of the antenna [*Wait and Pope*, 1955]. For VLF transmitting antennas, this takes the form of an extensive radial wire configuration, which may extend over several square kilometers. Nevertheless, the radiation efficiencies may still be less than 50%.

Another important aspect of the situation is the inverse problem where the data on the measured input impedance are used to estimate the electrical characteristics of the ground [*Keller and Frischknecht*, 1965]. Such an approach has promise in remote-sensing applications for lunar exploration.

In this paper, we wish to review briefly the relevant electromagnetic theory of the dipole impedance problem. Various approximations and interrelations in available formulations are discussed. We hope as a result to have provided a clearer understanding of the physical bases for remote sensing of the environment for dipole antennas located in the vicinity of conducting surfaces.

We consider an elementary Hertzian electric dipole located at a height z_0 over a homogeneous dissipative half-space whose conductivity is σ and dielectric constant is ϵ. If we superimpose the solutions for vertical and horizontal dipoles in an appropriate manner, the results for an arbitrary inclination of the dipole are obtained without further analysis. Therefore, in what follows we consider only the purely vertical or purely horizontal orientations. The formulation of the problem is very similar to the one used previously [*Wait*, 1962] in a different context.

VERTICAL DIPOLE FORMAL SOLUTION

With regard to a cylindrical coordinate system (ρ, ϕ, z), the vertical dipole of length ds, carrying a current I, is located at $z = z_0$ on the z axis over a dissipative half-space which occupies the region $z < 0$. The situation is illustrated in Figure 1. *Sommerfeld* [1949] gave the exact formal solution many years ago. For this problem, the fields in the insulating upper half-space can be obtained from an electric Hertz vector which has only a z component given by

$$\Pi = \frac{I\,ds}{4\pi i\epsilon_0\omega}\left\{\frac{\exp(-\gamma_0 R)}{R} + \int_0^\infty R(\lambda)\cdot\frac{\lambda}{u_0}\exp[-u_0(z+z_0)]\,J_0(\lambda\rho)\,d\lambda\right\} \quad (1)$$

where

$$u_0 = (\lambda^2 + \gamma_0^2)^{1/2}, \qquad \gamma_0 = i(\epsilon_0\mu_0)^{1/2}\omega = ik,$$

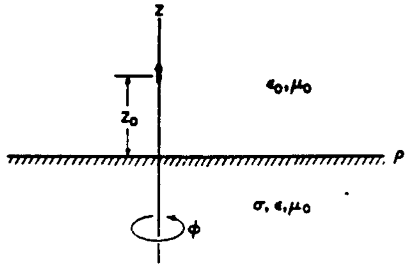

Fig. 1*a*. Vertical electric dipole (VED) over a homogeneous conducting half-space.

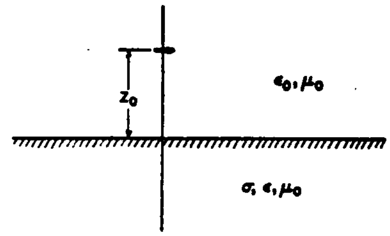

Fig. 1*b*. Horizontal electric dipole (HED) over a homogeneous half-space.

$$R(\lambda) = (N^2u_0 - u_1)(N^2u_0 + u_1)^{-1};$$

$$u_1 = (\lambda^2 + \gamma_1^2)^{1/2},$$

$$\gamma_1 = [i\mu_0\omega(\sigma_1 + i\epsilon_1\omega)]^{1/2}, \qquad N = \gamma_1/\gamma_0,$$

$$R = [\rho^2 + (z - z_0)^2]^{1/2}$$

Here we have designated the dielectric constant and the magnetic permeability of the upper half-space to be ϵ_0 and μ_0, respectively. To satisfy radiation conditions at infinity, the real parts of u_0 and u_1 are defined to be positive for λ ranging from 0 to ∞ on the real axis. The harmonic time dependence, according to the factor exp $(i\omega t)$, is understood.

To perform a complete calculation of the self-impedance of the source dipole, the current distribution on the dipole must be determined. This aspect of the problem may, however, be deferred if we confine our attention to the change of the impedance *resulting* from the presence of the lower dissipative half-space. Thus for electrically short antennas it is permissible to retain the dipole approximation. By definition, the change δZ is given by $\delta Z = Z - Z_0$, where Z is the self-impedance of the dipole in the presence of the half-space and Z_0 is the impedance of the same dipole located in free space. Thus, $Z_0 = \lim (z_0 \rightarrow \infty)\, Z$.

According to the 'emf' method, we can calculate the impedance increment δZ from the formula

$$\delta Z = \lim_{\substack{z \to z_0 \\ \rho \to 0}} \left[-\frac{\delta E_z ds}{I} \right] \qquad (2)$$

where

$$\delta E_z = (k^2 + \partial^2/\partial z^2)(\Pi - \Pi_0) \qquad (3)$$

and

$$\Pi_0 = \lim_{z_0 \to \infty} (\Pi)$$

After performing the derivative operations and proceeding to the indicated limits, we obtain

$$\delta Z = -\frac{(ds)^2}{4\pi i\epsilon_0\omega}\int_0^\infty R(\lambda)\frac{\lambda^3}{u_0}e^{-\alpha u_0}\,d\lambda \qquad (4)$$

where $\alpha = 2z_0$. It is convenient to normalize this by writing $\delta = R_0T$, where $R_0 = 20k^2(ds)^2$, the real part of Z_0, is the free-space radiation resistance of the dipole. (Here we assume that $(\mu_0/\epsilon_0)^{1/2} = 120\pi$.) Thus we may write

$$\frac{\delta Z}{R_0} = i\,\frac{3}{2}\frac{1}{k^3}\int_0^\infty \frac{N^2u_0 - u_1}{N^2u_0 + u_1}\cdot\frac{\lambda^3}{u_0}\exp(-\alpha u_0)\,d\lambda \qquad (5)$$

A simple yet revealing limiting case is to consider the static or dc limit. Then $\omega \rightarrow 0$ and, as a consequence, $u_0 = u_1 = \lambda$. Thus

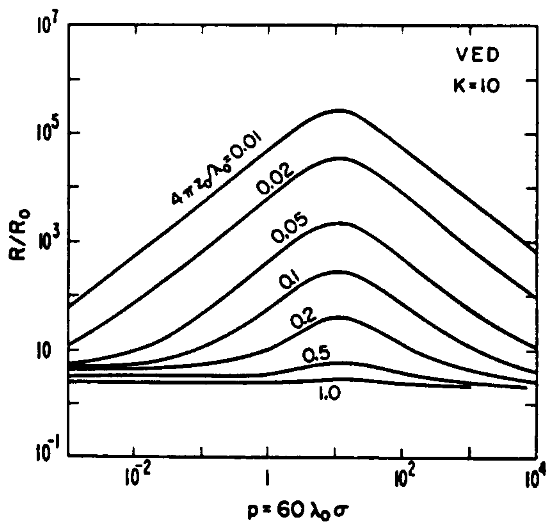

Fig. 2. The input resistance of a *vertical* electric dipole (VED) in the *presence* of a conducting half-space.

$$\frac{\delta Z}{R_0} = i\frac{3}{2}\frac{1}{k^3}\frac{N^2-1}{N^2+1}$$

$$\cdot\int_0^\infty \lambda^2 e^{-\alpha\lambda}\, d\lambda = i\frac{3}{(2kz_0)^3}\frac{N^2-1}{N^2+1} \quad (6)$$

If we write $N^2 = K - ip$ where $K = \epsilon_1/\epsilon_0$ is the relative dielectric constant and $p/K = \sigma_1/\epsilon_1\,\omega$ is the 'loss tangent' of the lower half-space, we obtain for the real part δR of the impedance increment

$$\frac{\delta R}{R_0} = \frac{3}{(2kz_0)^3}\frac{2p}{(K+1)^2+p^2} \quad (7)$$

For a fixed value of kz_0 and K, it is evident that δR has a maximum value when $p = K + 1$.

To illustrate the quasi-static behavior, the function R/R_0, based on (5), is shown plotted in Figure 2 as a function of p for $K = 10$. This is mainly a measure of the power absorbed in the half-space for a vertical electric dipole (VED) placed immediately above the surface. As indicated, for small values of the dipole height (i.e., $4\pi z_0/\lambda \ll 1$), the input resistance change δR ($= R - R_0$) greatly exceeds the free-space radiation resistance R_0 of the dipole. Also, in accordance with the simple static limit, the maximum value of R occurs, for a value of $p = K + 1 = 11$, for the example shown. However, as the parameter $4\pi z_0/\lambda$ becomes comparable with unity, the curves lose their characteristic shape and a clear-cut maximum is no longer evident. The behavior of the impedance increment in this range is discussed below.

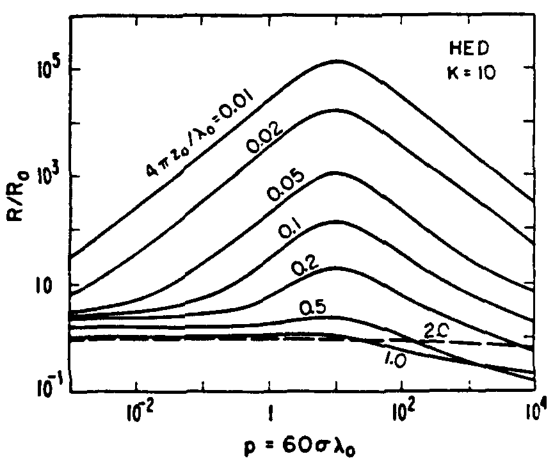

Fig. 3. The input resistance of a *horizontal* electric dipole (HED) in the *presence* of a conducting half-space.

THE HORIZONTAL ELECTRIC DIPOLE

The formulation for the horizontal electric dipole (HED) is very similar to the case of the VED discussed above. The only slight additional complication is that the Hertz vector for the problem is required to have both a vertical and a horizontal component. Then following a straightforward application of the emf method, we find that the impedance increment δZ, resulting from the presence of the conducting half-space, is

$$\frac{\delta Z}{R_0} = i\frac{3}{2k^3}$$

$$\cdot\int_0^\infty\left[\frac{(k^2-u_0^2)}{2}\frac{u_0-u_1}{u_0+u_1}+\frac{\lambda^2u_0^2(u_0-u_1)}{k^2(N^2u_0+u_1)}\right]$$

$$\exp(-\alpha u_0)\frac{\lambda}{u_0}\,d\lambda \quad (8)$$

In proceeding to the static limit (i.e., $\omega \to 0$), we note that $(u_0 - u_1)/(u_0 + u_1) \to 0$, but $(u_0 - u_1)/k^2 \to (N^2 - 1)/(2\lambda)$. Thus, in this limit

$$\frac{\delta Z}{R_0} = i\frac{3}{4k^3}\frac{N^2-1}{N^2+1}\int_0^\infty \lambda^2 e^{-\alpha\lambda}\,d\lambda$$
$$= i\frac{1}{2}\frac{3}{(2kz_0)^3}\frac{N^2-1}{N^2+1} \quad (9)$$

which is identical to (6) except for the factor 1/2.

Using numerical integration data from *Volger and Noble* [1963], the function R/R_0, for the HED, is plotted in Figure 3 as a function of p for $K = 10$ and for various values of the parameter $k\alpha$ or $4\pi z_0/\lambda_0$. (The approach of these authors is similar to that of *Wait* [1962].) The similarity of these curves and those for the VED in Figure 2 is striking. In both cases, there is a maximum in the energy absorption where $p = K + 1$, as predicted by the simple static approximation.

To illustrate the dependence on the relative dielectric constant K, the normalized input resistance R/R_0 is plotted in Figure 4 as a function of p, for a range of K values for two fixed values of the dipole height. Although the curves have the same general shape, the maxima in R are shifted to the left as K is reduced. In all cases, the general behavior is in accordance with the static approximation given by (9). A significant point is that, for a fixed value of z_0/λ_0, a single measured value of R/R_0 will, in general, lead to two possible sets of values of K and p.

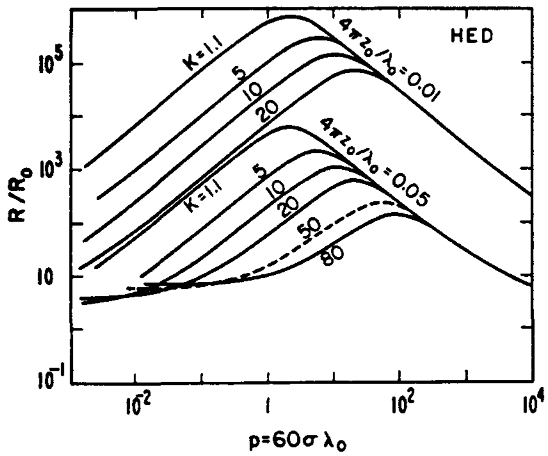

Fig. 4. The input resistance of HED showing the influence of the relative dielectric constant ($K = \epsilon/\epsilon_0$) of the half-space.

ASYMPTOTIC APPROXIMATIONS

The results we have just discussed are dominated by the electrostatic coupling of the dipoles to the dissipative or lossy dielectric half-space. Certainly, there will always be a sufficiently low frequency where the change of the input impedance due to the half-space can be computed on the basis of a static theory. The question then arises as to how this approach to the problem can be reconciled with the usual methods that involve the surface impedance description of the half-space. To shed some light on this question, we return to the integral formula given by (4) above.

First of all, we consider the limit where the half-space is perfectly conducting. Thus, if $N^2 \to \infty$, we have

$$\delta Z = \delta Z^{(\infty)} = -\frac{(ds)^2\eta_0}{4\pi ik}\int_0^\infty \lambda^3 u_0^{-1} \exp(-\alpha u_0)\, d\lambda$$

$$= -\frac{(ds)^2\eta_0}{2\pi ik\alpha^3}(1 + ik\alpha)\exp(-ik\alpha) \qquad (10)$$

Normalizing by R_0 and considering the real part Re $\delta Z^{(\infty)}$ or $\delta R^{(\infty)}$, we find that

$$\frac{\delta R^{(\infty)}}{R_0} = \frac{3}{(k\alpha)^3}[\sin(k\alpha) - (k\alpha)\cos k\alpha]$$

where, as usual $\alpha = 2z_0$. Here we confirm that $\delta R^{(\infty)}/R_0 \to 1$ as $z_0/\lambda_0 \to 0$ corresponding to the expected doubling of the input resistance (i.e., $R \to 2R_0$), for a vertical electric dipole located on a perfectly conducting surface.

Having dispensed with the perfectly conducting case, we now define the impedance increment ΔZ resulting from the *finite* conductivity of the half-space as follows:

$$\Delta Z = Z - Z^{(\infty)} = \delta Z - \delta Z^{(\infty)} \qquad (11)$$

or

$$\frac{\Delta Z}{R_0} = i\frac{3}{k^3}\int_0^\infty \frac{\lambda^3}{N^2u_0 + u_1}\,\frac{\exp(-\alpha u_0)}{u_0}\, d\lambda \qquad (12)$$

Now, for sufficiently high conductivity and/or high frequency, $u_1 = (\lambda^2 - N^2k^2)^{1/2}$ can be replaced by iNk over the significant range of the integrand. Then, (12) reduces to

$$\frac{\Delta Z}{R_0} \cong \frac{3}{Nk^3}\int_0^\infty \frac{\lambda^3 \exp(-\alpha u_0)}{[u_0 + i(k/N)]u_0}\, d\lambda \qquad (13)$$

Noting that $N = [(\sigma_1 + i\epsilon_1\omega)/i\epsilon_0\omega]^{1/2}$ has an argument between 0 and $\pi/4$ (for nonvanishing σ_1 and ϵ_1), we can express (13) in the form

$$\frac{\Delta Z}{R_0} = \frac{3}{Nk^3}\left[\frac{1}{\alpha^3}(1 + ik\alpha)e^{-ik\alpha} - \frac{ik}{N\alpha}e^{-ik\alpha} + k^3\left(1 - \frac{1}{N^2}\right)Q\right] \qquad (14)$$

where

$$Q = \int_0^\infty \frac{\lambda}{u_0}\,\frac{\exp(-\alpha u_0)}{u_0 + i(k/N)}\, d\lambda = -e^{ik(\alpha/N)}Ei[-ik(1 + N^{-1})\alpha]$$

where

$$Ei[-ix] = -\int_x^\infty \frac{e^{-iy}}{y}\, dy \qquad (15)$$

is the exponential integral as conventionally defined. Now, if $|N|$ is sufficiently large, (14) can be further approximated by

$$\Delta Z/R_0 = (1/N)f(k\alpha) \qquad (16)$$

where

$$f(x) = 3[x^{-3}(1 + ix)e^{-ix} - Ei(-ix)]$$

This result shows that Re ΔZ or ΔR varies inversely with N or $p^{1/2}$. This is in accordance with the curves in Figure 2 if p is sufficiently large. On the basis of (16), we might be led to the (erroneous) conclusion that ΔR or δR increase indefinitely with decreasing loss tangent p/K. As indicated by the curves in Figure 2, this does not happen. It is important that (16) not be used outside its range of validity; namely, $|N|$

should be large compared with unity. This is a rather important point since the compensation theorem approach, if improperly applied, may also lead to an incorrect conclusion. For example, on the basis of the compensation theorem, in its general form, the result for the change of the input impedance ΔZ of the VED resulting from the finite conductivity of the half-space is [*Wait and Pope*, 1955]

$$\Delta Z = \frac{1}{I_0^2} \int_0^\infty E_\rho H_\phi^{(\infty)} 2\pi\rho \, d\rho \qquad (17)$$

Here, I_0^2 is the current at the terminals of the electric dipole, E_ρ is the resulting tangential electric field in the radial direction ρ, whereas $H_\phi^{(\infty)}$ is the tangential magnetic field in the azimuthal direction ϕ, for an assumed perfectly conducting half-space. If (17) were applied with the proper value for E_ρ, it would lead to an exact formula for ΔZ. However, to facilitate its application, E_ρ is usually first approximated by assuming that E_ρ is proportional to H_ϕ, which in turn is replaced by $H_\phi^{(\infty)}$. In fact, it is an extremely interesting exercise to show that, on using the approximation

$$E_\rho = \eta_0(1/N)H_\phi^{(\infty)} \qquad (18)$$

(17) reduces precisely to (16). Furthermore, the detailed analysis of *Sommerfeld and Renner* [1942] gives an identical result for Re ΔZ or ΔR if their complicated 'second order' correction is ignored.

A similar argument applies to the horizontal electric dipole (HED), where the integral formula (8) and the assumption $|N| \gg 1$ reduces to the simple result

$$\frac{\Delta Z}{R_0} \cong i\frac{3}{Nk\alpha}\left(1 - \frac{i}{k\alpha}\right)e^{-ik\alpha} \qquad (19)$$

This again illustrates the inverse dependence on $|N|$, which is in accord with the curves in Figures 3 and 4, for sufficiently large values of p.

CONCLUDING REMARKS

The results given in this paper should clarify the various mechanisms responsible for energy loss from an electric dipole source located above a conducting ground. As indicated, at sufficiently low frequencies the coupling between the antenna and the environment (i.e., the half-space) is electrostatic in nature. This is related to the so-called E-field loss discussed in the context of high-power VLF transmitting antennas [*Watt*, 1967; *Wait*, 1963] operating over poorly conducting soils. On the other hand, the higher frequency approximation to the exact integral formulas leads to results that are normally associated with the compensation theorem approach. In this method, the Poynting vector or some variant of it is integrated over the whole ground plane, and in this way the ground reaction on the antenna is estimated. Actually, there is no contradiction between these seemingly different approaches if we keep in mind the nature of the approximations and the ranges of their validity. Of course, in the case of the half-space problem, it is possible to work with the exact integral formula, and the numerical data so obtained are valid for the whole range of the parameters. However, in many cases of practical interest, the environmental configuration is more complicated than a homogeneous half-space. Then, a quasi-static assumption or a surface-impedance description is useful. We have attempted to give some indication of the meaning of these limiting situations and to remove some apparent contradictions that have been alluded to from time to time.

Finally, we stress that the dipole formulations considered here are only valid for wire antennas that are small compared with both the wavelength and the height above the half-space. Relaxing either or both of these restrictions leads to a different class of problems.

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(Reprint No. 389)

SURFACE-WAVE EFFECTS WITH LARGE ANTENNA EARTH SCREENS

It is pointed out that a perfectly conducting earth screen may not be the optimum if low-angle radiation is to be maximised for a given antenna current. This is a consequence of the surface wave excited for an earth screen with finite inductive reactance.

Extended earth screens are sometimes used for improving low-angle radiation from h.f. antennas. They usually take the form of a radial wire or a metallic mesh which is placed on the earth's surface in the immediate vicinity of the antenna.[1] In the theoretical treatment for these systems, it is convenient to describe the surface of the earth screen by an effective surface impedance Z', which, in general, is different from the surface impedance Z of the ground beyond the screen. At first glance, one would think that the ideal system would be to have $Z' = 0$ corresponding to a screen of effectively perfect conductivity. It is the purpose of this letter to show that this is not the case.

The situation is illustrated in Fig. 1. A vertical electric dipole is located on a flat ground. The effective surface impedance of the ground system is a constant Z' out to a radius b from the base of the antenna. The ground beyond radius b is taken to have a constant surface impedance Z, and we assume that the system is rotationally symmetric. As indicated, ρ is the radial distance to an arbitrary point on the earth's surface from the dipole.

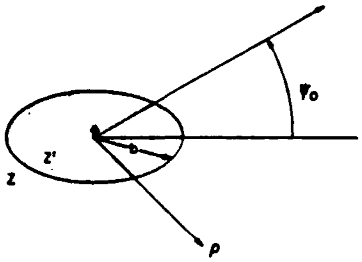

Fig. 1 *Electric dipole located on a circular earth screen*

The radiation field E, for a time factor $\exp(j\omega t)$, at an infinite distance R and elevation angle ψ_0, is expressible in the form[2]

$$E = \frac{j\mu_0 \omega I ds}{2\pi} W' \frac{e^{-jkR}}{R} \cos\psi_0 \qquad (1)$$

where $\mu_0 = 4\pi \times 10^{-7}$, $k = 2\pi/(\text{wavelength})$ and Ids is the current moment of the dipole. Here W' is a pattern function normalised so that it is unity over a perfectly conducting ground plane of infinite extent (i.e. $Z' = Z = 0$). Now, following an early derivation,[2]

$$W' = (1 + R_v)(1 + \Omega)/2 \qquad (2)$$

where $R_v = \{\sin\psi_0 - (Z/\eta_0)\}/\{\sin\psi_0 + (Z/\eta_0)\}$, $\eta_0 = 120\pi$, and Ω is a 'modification' which vanishes if the earth screen is absent (i.e. $Z' = Z$ or if $b \to 0$). In the limiting case of no earth screen,

$$W' = W = (1 + R_v)/2 = \sin\psi_0/\{\sin\psi_0 + (Z/\eta_0)\} \qquad (3)$$

Here Z/η_0 is the normalised surface impedance of the 'unmodified' ground. If the latter is homogeneous and lossless, we have

$$Z/\eta_0 = (1/K)^{\frac{1}{2}}\{1 - (\cos^2\psi_0)/K\}^{\frac{1}{2}} \qquad (4)$$

where K is the relative permittivity of the dielectric ground. The corresponding result for W is exact.

The modification of the surface impedance in a circular region of radius b, from Z to Z', results in improving the 'cut back' by a factor $1 + \Omega$. Also, it can be shown[2] that the ground-wave field of the dipole is also modified by the factor $1 + \Omega$, where this is to be evaluated as $\psi_0 \to 0$. In fact, as we will see below, $1 + \Omega$ is essentially independent of ψ_0 for small angles. Here Ω is to be obtained from[2]

$$\Omega = -\frac{kF}{\cos\psi_0}\int_0^b e^{-jk\rho} W'_e(k\rho)\left(1 + \frac{1}{jk\rho}\right) J_1(k\rho\cos\psi_0)\,d\rho \qquad (5)$$

where

$$W'_e(k\rho) = 1 - j(\pi p)^{\frac{1}{2}} e^{-p}\,\mathrm{erfc}(jp^{\frac{1}{2}}) \qquad (6)$$

$p = -\frac{jk\rho}{2}\left(\frac{Z'}{\eta_0}\right)^2$, $F = (Z - Z')/\eta_0$, and where J_1 is the Bessel function of order one while W'_e is an 'attenuation function' which accounts for the modification of the tangential magnetic field by the finite surface impedance Z' of the earth screen. In obtaining eqn. 5, we have neglected the effects of back reflections from the edge of the earth system (i.e. at $\rho = b$). Other investigations have shown this is a well justified assumption.[3,4]

Now the modified surface impedance Z' will be imagined to be composed of the parallel combination of the ground impedance Z (evaluated at grazing incidence) and the effective impedance Z_g of the wire grid.[1,3] Thus

$$Z' = ZZ_g/(Z + Z_g) \qquad (7)$$

where $Z = \eta_0 K^{-\frac{1}{2}}(1 - K^{-1})^{\frac{1}{2}}$ is the surface impedance of the ground for grazing incidence. If the wire-grid spacing is much less than a wavelength and if ohmic losses in the wires are negligible, we know that Z_g is purely inductive. Thus we set

$$Z_g = j\eta_0\delta_g$$

where δ_g is a dimensionless reactance parameter. A good approximation is $\delta_g = (d/\lambda_0)\ln\{d/(2\pi c)\} + S_c$, where d is the spacing between the wires in the mesh, λ_0 is the free-space wavelength, c is the wire radius, and S_c is a correction factor which is negligible if d/λ_0 is sufficiently small.[1]

As indicated by eqn. 5, the integrand, apart from the factor J_1, is proportional to the tangential magnetic field at a distance ρ from the dipole. If here $W'_e(k\rho)$ is replaced by unity, we have the assumption that the tangential magnetic field is the same as if the surface were perfectly conducting throughout. Clearly, if the reactance parameter δ_g were effectively zero, this would be a good approximation even for a screen

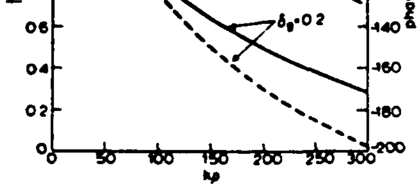

Fig. 2 *'Attenuation function' W_g for a partially inductive surface*
—— amplitude
- - - - phase

of finite extent. However, if the reactance of the screen is an appreciable fraction of the impedance of the ground, it is not justified to set $W_e = 1$ in the integrand of eqn. 5. This fact is clearly illustrated in Fig. 3, where the amplitude and phase of W_e are shown plotted as a function $k\rho$ for a lossless homogeneous earth with a relative permittivity $K = 10$. For

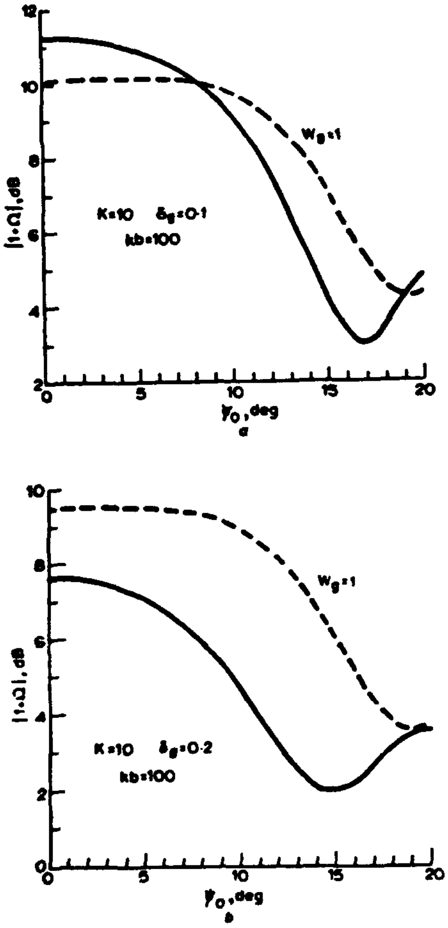

Fig. 3 Modification of the radiation field due to the presence of the earth screen
The curves for the $W_e = 1$ assumption are also shown

the value of $\delta_e = 1$, it is apparent that the inductive portion of Z' is sufficient to allow a partially trapped surface wave to propagate along the surface. This surface-wave effect for $\delta_e = 0{\cdot}2$ is only noticeable for the shorter distances. Nevertheless, we see that W_e is substantially different from unity over most of the range of $k\rho$ indicated. As pointed out by Patton,[5] a similar effect should be observed for a lossy earth, but we do not consider this case here.

The amplitude of $1 + \Omega$, for $\delta_e = 0{\cdot}1$ and $0{\cdot}2$, is shown in Figs. 3a and b, respectively, for $kb = 100$ and $K = 10$. The broken curve shows the corresponding quantity with the assumption that $W_e = 1$.

Finally, in Fig. 4, we indicate how $|1 + \Omega|$ varies with the screen reactance parameter δ_e for $K = 3$ and 10, $kb = 100$, and when the grazing angle ψ_0 tends to zero. In this case, $|1 + \Omega|$ is a measure of how effective the earth screen is in improving the earth-wave field.[2] The curves indicate that an optimum value of δ_e exists, which is of the order of $0{\cdot}1$. However, using the $W_e = 1$ assumption, the enhancement due to the surface effect would not be predicted. This fact could be kept in mind in the design of practical earth screens for h.f. antennas over poorly conducting ground.

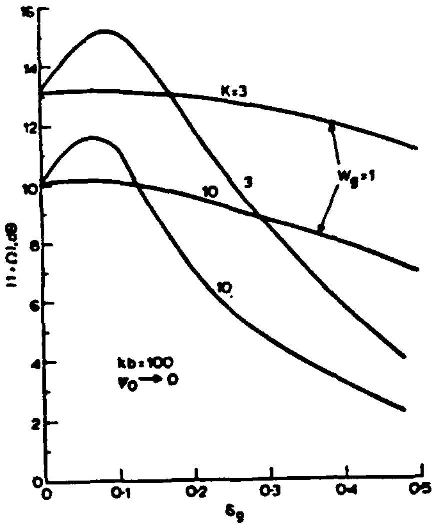

Fig. 4 Dependence on the earth-screen reactance

I wish to thank K. P. Spies for his assistance in the numerical calculations.

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30th September 1969

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FIELDS OF ELECTRIC DIPOLE ON RADIALLY INHOMOGENEOUS GROUND SURFACE

An integral-equation method is employed to calculate the fields for low-angle radiation over a plane boundary with an exponentially varying surface impedance. It is shown that the effective dipole moment of the source is increased by a complex factor which depends on both the taper and the extent of the inhomogeneous region.

There is an intriguing possibility that the low-angle radiation from h.f. antennas may be enhanced by modifying the foreground.[1-3] Physically, this can be achieved by using an extended radial-wire or a mesh ground system. In the previous theoretical treatments of the subject, it has been assumed usually that the ground system is effectively a perfect conductor over its total extent whether it is disc or sector shaped.[4] In this letter, we wish to consider some implications of tapering the ground system. In this case, the effective surface impedance is a smooth function of the radial distance from the base of the antenna. Analytically, this is a considerably more involved problem than for a constant or zero surface impedance.

For simplicity, we assume azimuthal symmetry, and we also represent the source as a ground-based Hertzian electric dipole. The situation is illustrated in Fig. 1A, where the

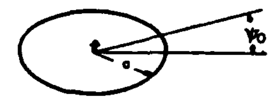


Fig. 1A *Basic geometry*

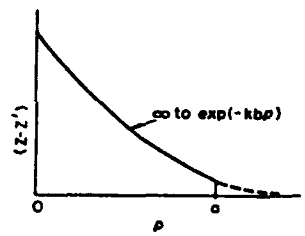


Fig. 1B *Assumed surface-impedance variation*

surface impedance of the ground beyond a specified radius a is a constant Z. Within the ground system (i.e. $0 < \rho < a$), the surface impedance $Z'(\rho)$ is taken to be a function of ρ, the radial distance from the source dipole. As indicated, ψ_0 is the angle subtended by the observer's direction and the ground plane.

In the absence of the ground system [i.e. $a = 0$ or $Z'(\rho) = Z$], the far-field radiation pattern $P(\psi_0)$ of the ground-based dipole on the homogeneous flat earth is

$$P(\psi_0) = |1 + R_v|(\cos\psi_0)/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$R_v = \frac{\sin\psi_0 - (Z/\eta_0)}{\sin\psi_0 + (Z/\eta_0)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and $\eta_0 = 120\pi$. Here, of course, R_v is the appropriate Fresnel-reflection coefficient for a normalised surface impedance Z/η_0. It was shown before[5] that the corresponding pattern function is

$$P(\psi_0) = |(1 + R_v)(1 + \Omega_a)|(\cos\psi_0)/2 \quad . \quad . \quad . \quad (3)$$

where Ω_a is the all important correction factor which accounts for the presence of the ground system. The latter is

$$\Omega_a = -\frac{k}{\cos\psi_0}\int_0^a W'(k\rho)F(k\rho)e^{-jk\rho} \times \left(1 + \frac{1}{jk\rho}\right)J_1(k\rho\cos\psi_0)d\rho \quad (4)$$

where $k = 2\pi/(\text{wavelength})$, J = Bessel function of the first kind, and $F(k\rho) = \{Z - Z'(\rho)\}/\eta_0$ is the surface-impedance contrast function, which, by definition, vanishes if $\rho > a$. In this integral representation for the perturbed radiation field, $W'(\rho)$ is the unknown attenuation function for the tangential magnetic field of the source dipole in the region $0 < \rho < a$. This function was shown to satisfy a Volterra-integral equation of the form,

$$W'(x) = W(x) + \left(\frac{jx}{2\pi}\right)^{1/2}\int_0^x F(y)\frac{W(x-y)W'(y)}{\{y(x-y)\}^{1/2}}dy \quad . \quad . \quad . \quad . \quad (5)$$

$$\text{where } W(x) = 1 - j(\pi p)^{1/2}e^{-p}\operatorname{erfc}(jp^{1/2}) \quad . \quad . \quad . \quad (6)$$

$x = k\rho$, and $p = -(jx/2)(Z/\eta_0)^2$. Here $W(x)$ is the well known attenuation function for propagation to a 'numerical distance' p over an homogeneous flat surface. In eqn. 6, erfc is the complement of the error function. Methods of the solution of eqn. 5 for the attenuation function $W'(x)$ are described elsewhere.[6]

As mentioned above, the specification of the functional form of the surface-impedance contrast function $F(x)$, where $x = k\rho$, will determine the form of the resulting attenuation function $W'(x)$. For present purposes, we will consider

$$Z - Z'(x) = Ze^{-bx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where Z corresponds to the appropriate value for a homogeneous lossless ground of relative permittivity ϵ_g/ϵ_0. The situation is illustrated in Fig. 1B. In this case, we choose

$$Z/\eta_0 = (\epsilon_0/\epsilon_g)^{1/2}(1 - \epsilon_0/\epsilon_g)^{1/2} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

which is the exact form for a vertically polarised plane wave at grazing incidence. The exponential variation indicated by eqn. 7 is an idealisation for a ground screen which is tapered smoothly from the base of the antenna. Here we do not dwell on the physical realisability of such a ground system, although we could point out that a radial-wire system will have a smoothly decreasing effective impedance as one recedes from the base of the antenna along the ground surface. For our example, the dimensionless parameter b is a measure of the rapidity of the impedance taper. For example, $b = 0$ would correspond to a perfectly conducting ground plane, while $b = \infty$ corresponds to a homogeneous earth of surface impedance Z throughout.

Once we have the numerical values for the attenuation function $W'(k\rho)$, the pattern function $P(\psi_0)$ can be obtained directly from eqn. 3, following the numerical integration of eqn 4. In this case, the upper limit a of the integration

Table 1

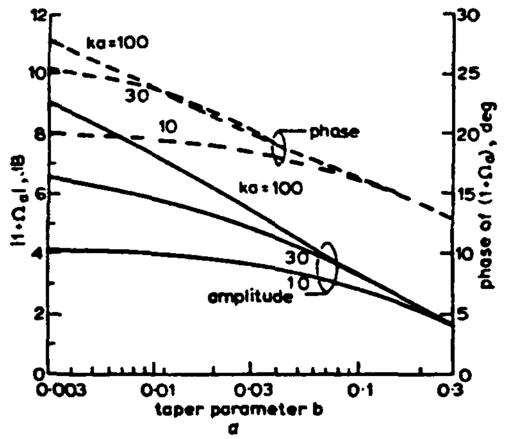
ψ_0, deg	$\lvert 1 + \Omega_a\rvert$, dB	Phase of $(1 + \Omega_a)$, deg
1	4·92	20·5
2	4·93	20·4
3	4·94	20·3
4	4·95	20·1
5	4·96	19·8
6	4·98	19·5
7	5·00	19·1
8	5·01	18·7
9	5·03	18·1
10	5·05	17·6

$b = 0.01$, $\epsilon_g/\epsilon_0 = 10$, $ka = 30$

corresponds physically to the outer extremity of the ground system

As indicated by eqns. 1 and 3, the complex factor $(1 + \Omega_a)$ is the ratio of the field with and without the tapered ground system. As it turns out, this factor is a slowly varying function of the angle ψ_0, at least for the low angles of interest to long-distance h.f. propagation. To illustrate this point, we show the amplitude and phase of $(1 + \Omega_a)$ in Table 1, for a typical selection of the relevant parameters.

The near constancy of $1 + \Omega_a$ as a function of ψ_0 justifies the use of this factor as a quantitative measure of the effectiveness of the ground system. In fact, we can assert that the effective moment of the source dipole is modified by the factor $1 + \Omega_a$.



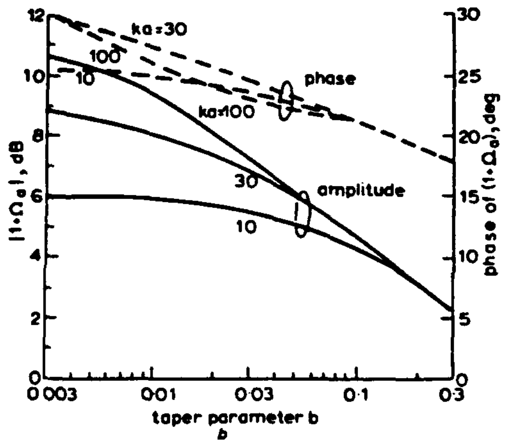


Fig. 2 *Modification of effective dipole moment resulting from assumed surface-impedance variation*

a $\epsilon_g/\epsilon_0 = 10$, $\psi_0 \leq 5°$
b $\epsilon_g/\epsilon_0 = 3$, $\psi_0 \leq 5°$

This useful concept was also borne out in an earlier study for constant-impedance ground screens.[4]

For the tapered exponential model considered here, some concrete numerical values are illustrated in Figs. 2*a* and *b* for relative permittivities of 10 and 3, respectively. In each case, the abscissa is the taper parameter *b*, while the ordinates are the amplitude and the phase of $1 + \Omega_a$. In each case, three values of the ground-system radius *a* are indicated. [Here, $ka = 2\pi a/(\text{wavelength})$].

From the curves in Figs. 2*a* and *b*, it is evident that, if the taper parameter is sufficiently small (i.e. almost constant surface impedance), the results are equivalent to a perfectly conducting circular screen of radius *a* lying on a dielectric half-space. This fact is confirmed by comparing the results here with those obtained previously[4] for such a situation. As the taper parameter *b* is increased, it is evident that the ground system is less effective in the sense that $1 + \Omega_a$ is approaching 1. Also, we see that the dependence on the finite value of *ka* is less as *b* increases. In fact, if $b > 0{\cdot}1$ the discontinuity in the surface impedance at $\rho = a$ is not significant.

The method employed here can be applied to more realistic situations which involve elevated antennas of finite length erected over inductive wire-grid systems. The incorporation of ohmic losses in the ground is also straightforward. Work on the subject continues.

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25th August 1969

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Transient Response of a Dipole over a Circular Ground Screen

Abstract—The time-domain response of a vertical electric dipole located over a circular ground screen is considered. The relevant time-harmonic solution is used as a starting point. By making a number of approximations a very simple formula is obtained for the transient response of the far field when the dipole current moment is a ramp function of time. It is shown that for early times, the pattern response appears as if the ground screen were of infinite extent, while at very long times, the response approaches that expected for a homogeneous flat ground.

Introduction

Low-angle radiation from a dipole antenna over a finitely conducting ground may be improved by the use of a metallic ground screen. The limited size of any practical system does not permit one to realize fully the ideal pattern for a perfectly conducting ground plane. Quantitative studies of this important effect have been the subject of various theoretical and experimental investigations [1]–[3]. Without exception the analyses have been carried out for time-harmonic fields. In this communication we wish to develop a transient solution. To simplify the discussion, a relatively simple model is adopted, and the source dipole moment is assumed to be a step function of time.

Formulation of the Time-Harmonic Problem

With respect to a cylindrical coordinate system (ρ,ϕ,z) the earth's surface is at $z = 0$, and the source electric dipole is located at $z = h$ on the axis. As indicated in Fig. 1, the circular ground screen is located on the surface $z = 0$ and is bounded by $\rho = a$. In the absence of the ground system the surface impedance is Z, which does not vary with ρ. On the other hand, the surface impedance of the ground screen, in parallel with the homogeneous earth, is Z_g'. This, in effect, is saying that the effective surface impedance for the entire ground plane is Z_g' for $0 < \rho < a$, and Z for $\rho > a$.

For convenience, we will discuss the problem initially for a time factor exp $(i\omega t)$. The current moment of the dipole is designated $P_0(\omega)$, and the magnetic field in the far zone has the form

$$H_\phi(\omega) = \frac{i\omega P_0(\omega)}{2\pi c R_0} \exp(-i\omega R_0/c) \cos\psi_0 W(\omega) \qquad (1)$$

where R_0 is the distance to the observer, ψ_0 the angle subtended by R_0 and the earth's surface, and c the velocity of light in a vacuum. W is a pattern function which is normalized to be unity if the dipole height h were zero, and if the entire ground plane were perfectly conducting.

An approximate formula for W is given as follows [4]:

$$W \simeq (1 + R_v)[G_v(h) + \Omega]/2 \qquad (2)$$

where

$$R_v = \frac{\sin\psi_0 - (Z/\eta_0)}{\sin\psi_0 + (Z/\eta_0)} \qquad (3)$$

$$G_v(h) = \frac{\exp[i\omega(h/c)\sin\psi_0] + R_v \exp[-i\omega(h/c)\sin\psi_0]}{1 + R_v} \qquad (4)$$

and

$$\Omega = \frac{\omega}{c\cos\psi_0}\int_{\rho=0}^{a} \exp(-i\omega r_a/c)\,\frac{\rho^2}{r_a^2}\left(1 + \frac{c}{i\omega r_a}\right) \cdot J_1\left(\frac{\omega\rho}{c}\cos\psi_0\right)\left(\frac{Z_g' - Z}{\eta_0}\right)d\rho \qquad (5)$$

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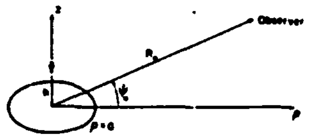

Fig. 1. Vertical electric dipole over circular ground screen.

where $\eta_0 = (\mu_0/\epsilon_0)^{1/2} = 120\pi$ and $r_q = (\rho^2 + h^2)^{1/2}$. The results given by (2)–(5) reduce to the forms given previously [1], [2], where h is set equal to zero. While (2) is approximate it does reduce to various correct limiting forms. For example, if $a \to 0$, then Ω vanishes, and

$$2W' = (1 + R_v)G_0(h)$$
$$= \exp [i\omega(h/c) \sin \psi_0] + R_v \exp [-i\omega(h/c) \sin \psi_0] \qquad (6a)$$

where R_v is the appropriate Fresnel reflection coefficient for oblique reflection from a flat earth whose surface impedance is Z. The other limiting case is when the ground screen radius becomes infinitely large. For example, if $|Z_s'| \ll Z$, we find that [4], in the limit $a \to \infty$,

$$\Omega = (Z/\eta_0) \exp [-i\omega(h/c) \sin \psi_0]/\sin \psi_0. \qquad (6b)$$

This combines with (2) and (3) to give

$$W' = \cos [\omega(h/c) \sin \psi_0] \qquad (7)$$

which is the expected pattern for a vertical electric dipole over a flat perfectly conducting ground plane.

Transient Analysis

In order to approach the transient problem, it is imagined that *at time $t = 0$ the dipole is suddenly turned on. Then the current* moment is

$$p(t) = \frac{\exp(-\alpha t) - \exp(-\beta t)}{\beta - \alpha} u(t) \qquad (8a)$$

where α and β are constants, and where $u(t) = 1$ for $t > 0$ and $u(t) = 0$ for $t < 0$. As will be shown, this double exponential form has certain mathematical advantages. The Fourier transform of the current moment is given by

$$P_0(\omega) = \int_{-\infty}^{\infty} p(t) \exp(-i\omega t)\, dt = \frac{1}{\beta-\alpha}\left(\frac{1}{\alpha+i\omega} - \frac{1}{\beta+i\omega}\right). \qquad (8b)$$

By superposition the transient response $h_\phi(t)$ of the radiation field is then obtained by inverting the transform

$$H_\phi(\omega) = \int_{-\infty}^{\infty} h_\phi(t) \exp(-i\omega t)\, dt \qquad (9)$$

or, more explicitly, from

$$h_\phi(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_\phi(\omega) \exp(i\omega t)\, d\omega. \qquad (10)$$

By utilizing the shift rule, it is easy to obtain

$$h_\phi(t) = \frac{p_0}{2\pi c R_0} \cos \psi_0 A\left(t - \frac{R_0}{c} + \frac{h \sin \psi_0}{c}\right) \qquad (11)$$

where

$$A(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} W'(\omega) \exp [-i\omega(h/c) \sin \psi_0] \cdot \frac{i\omega \exp(i\omega t)}{(\alpha + i\omega)(\beta + i\omega)}\, d\omega. \qquad (12)$$

The function $A(t)$ is designated as the pattern response. In order to see the meaning of this quantity, the whole ground plane is considered to be perfectly conducting. Thus W' in the integrand of (12) is to be replaced by (7), giving

$$A(t) = A_0(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{1}{2} \{1 + \exp [-i\omega(2h/c) \sin \psi_0]\} \cdot \frac{i\omega \exp(i\omega t)}{(\alpha + i\omega)(\beta + i\omega)}\, d\omega. \qquad (13)$$

This integration is readily carried out with the following result:

$$A_0(t) = \frac{1}{2(\beta - \alpha)} \{[\beta \exp(-\beta t) - \alpha \exp(-\alpha t)]u(t) + [\beta \exp(-\beta t') - \alpha \exp(-\alpha t')]u(t')\} \qquad (14)$$

where $t' = t - (2h/c) \sin \psi_0$. The terms multiplied by $u(t)$ and $u(t')$ are proportional to the direct signal and the signal reflected from the ground plane, respectively. If β and α are allowed to approach zero, it can be seen that

$$A_0(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{1}{2} \{1 + \exp [-i\omega(2h/c) \sin \psi_0]\} \frac{1}{i\omega} \exp(i\omega t)\, d\omega$$
$$= \tfrac{1}{2}[u(t) + u(t')]. \qquad (15)$$

From the nature of this limiting process it is clear that the contour in (15) is to be indented below any singularities which occur on the real axis. In the following, it is understood that this convention holds.

In the limiting case of a perfectly conducting ground plane the radiated field is seen to be two superimposed step functions. The corresponding source dipole moment is a ramp function, i.e., $p(t) = tu(t)$. This is a convenient type of source for an initial study of the transient response of a ground screen excited by a dipole.

Some Simplifications

In order to proceed with the general case, a number of simplifications are now made. First, it is assumed that the ground screen is perfectly conducting (i.e., $Z_s' = 0$). Thus (2) is expressible in the simpler form.

$$W' = \cos(\omega h S/c) + \delta W \qquad (16)$$

where

$$\delta W = \frac{S\Delta}{S + \Delta} \frac{\omega}{cC} \int_{-\infty}^{\infty} \frac{\rho^2}{\rho^2 + h^2}\left[1 + \frac{c}{i\omega(\rho^2 + h^2)^{1/2}}\right] \cdot \exp [-i\omega(\rho^2 + h^2)^{1/2}/c] J_1(\omega\rho C/c)\, d\rho \qquad (17)$$

with $\Delta = Z/\eta_0$, $S = \sin \psi_0$, and $C = \cos \psi_0$. For the ramp source function $p(t) = tu(t)$, the response pattern is then

$$A(t) = A_0(t) + G(t) \qquad (18)$$

where

$$G(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{\delta W}{i\omega} \exp(-i\omega h S/c) \exp(i\omega t)\, d\omega. \qquad (19)$$

To facilitate the integrations in (17), the following asymptotic forms for the Bessel function are employed:

$$(\omega/c)J(\omega\rho C/c) \simeq -\left[\frac{i\omega}{2\pi c \rho C}\right]^{1/2} \cdot [\exp(i\omega\rho C/c) - i \exp(-i\omega\rho C/c)]. \qquad (20)$$

The consequences of using this high-frequency approximation are discussed in the Appendix. Thus (17) reduces to

$$\frac{\delta W \exp(-i\omega h S/c)}{i\omega} \simeq -\frac{S\Delta}{S + \Delta} \frac{1}{(2\pi c)^{1/2}} \frac{1}{C^{1/2}} \int_0^{\infty} \frac{1}{\rho^{1/2}} \frac{\rho^2}{\rho^2 + h^2} \cdot \left[1 + \frac{c}{i\omega(\rho^2 + h^2)^{1/2}}\right] \frac{1}{(i\omega)^{1/2}} \{\exp [-i\omega(\rho/c)(1 - C)] - i \exp [-i\omega(\rho/c)(1 + C)]\} \exp [-i\omega\Phi(\rho)]\, d\rho \qquad (21)$$

Fig. 2. Time-ray interpretation of total transient signal

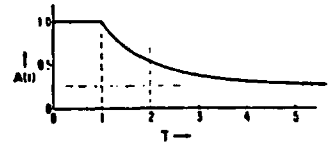

Fig. 3. Transient response of ground-based dipole with circular screen.

where

$$\Phi(\rho) = \frac{(\rho^2 + h^2)^{1/2}}{c} - \frac{\rho}{c} + \frac{hS}{c}.$$

In view of the approximations in the original derivation [4], the factor $C^{3/2}$ can be replaced by unity. Also, since we are dealing with large screens where $(\omega a/c) \gg 1$, the first square bracket term in the integrand of (21) may be replaced by unity. On the other hand, in the second square bracket, the term multiplied by $-i$ can be ignored since the phase is varying rapidly. As indicated before [4], this neglected term is related to the wave reflected from the back edge of the screen. With these simplifications, it is found that (21) reduces to

$$\frac{\delta \Pi^* \exp(-i\omega hS/c)}{i\omega} \simeq -\frac{S\Delta}{S+\Delta}\frac{1}{(2\pi c)^{1/2}}\int_a^\infty \frac{1}{\rho^{1/2}}\frac{1}{(i\omega)^{1/2}} \cdot \exp[-i\omega(\rho/c)(1-C)]\exp[-i\omega\Phi(\rho)]\,d\rho. \quad (22)$$

Using this form for the integrand in (19), the integration over ω can now be readily effected. Thus

$$G(t) = -\frac{S\Delta}{S+\Delta}\frac{1}{(2c)^{1/2}\pi}\int_a^\infty \frac{1\ u[t-(\rho/c)(1-C)-\Phi(\rho)]}{\rho^{1/2}[t-(\rho/c)(1-C)-\Phi(\rho)]^{1/2}}\,d\rho \quad (23)$$

where it is implicitly assumed that the normalized surface impedance Δ is independent of the frequency.[3] It can be seen that the integrand in (23) is zero for values of $\rho > \rho_0$, where

$$\frac{1}{c}(\rho_0^2 + h^2)^{1/2} - \frac{\rho_0 C}{c} + \frac{hS}{c} - t = 0 \quad (24)$$

which is equivalent to

$$\rho_0^2(1 - C^2) - 2\rho_0(ct - hS)C - [(ct - hS)^2 - h^2] = 0. \quad (25)$$

The relevant root is

$$\rho_0 = \{(ct - hS)C + [(ct - hS)^2 - h^2S^2]^{1/2}\}S^{-2}. \quad (26)$$

An alternative form of (23) which is suitable for numerical work is

$$G(t) = -\frac{S\Delta}{S+\Delta}\frac{1}{(2c)^{1/2}\pi}\int_a^{\rho_0}\frac{1}{\rho^{1/2}} \cdot \frac{1}{[t-(1/c)(\rho^2+h^2)^{1/2}+\rho C/c - hS/c]^{1/2}}\,d\rho\ u(\rho_0 - a). \quad (27)$$

For the raised dipole (i.e., $h > 0$), further reduction of this integral does not seem possible.

The time-ray interpretation of the results is indicated in Fig. 2. The dipole at T is excited at $t = 0$. If the points M and N are on the common wavefront, then, obviously, the ground reflected signal at N will arrive at the observer at time t seconds later, such that

$$ct = TN - TM = (\rho_0^2 + h^2)^{1/2} - \rho_0 C + hS. \quad (28)$$

The sum total of these rays, integrated from the edge of the screen out to infinity, gives the contribution $G(t)$ to the transient response of the dipole.

[3] Here $\Delta = Z/\eta_0$ is defined to be effectively a constant which is independent of frequency. This is strictly valid only for a lossless dielectric-like ground [1].

A considerable analytical simplification is obtained when the dipole is located on the screen. Then (27) reduces to

$$G(t) = \begin{cases} -\dfrac{S\Delta}{S+\Delta}\dfrac{1}{(2c)^{1/2}\pi}\displaystyle\int_a^{ct/(1-C)}\dfrac{1}{\rho^{1/2}} \cdot \dfrac{1}{[t-(\rho/c)(1-C)]^{1/2}}\,d\rho\, u\left[t - \dfrac{a}{c}(1-C)\right] & (29) \\ -\dfrac{\Delta}{S+\Delta}\dfrac{S}{[2(1-C)]^{1/2}}\dfrac{1}{\pi}\left\{\arcsin\left[1 - \dfrac{2(1-C)a}{ct}\right] + \dfrac{\pi}{2}\right\} u\left[t - \dfrac{a}{c}(1-C)\right]. & (30) \end{cases}$$

The factor $S/[2(1-C)]^{1/2}$ in (30) may be replaced by unity since it is already assumed that $1 - C \ll 1$. The resultant pattern response for this limiting situation is then given by the following remarkably simple result:

$$A(t) = u(t) - [\Delta/(S+\Delta)]f(T)u(T-1) \quad (31)$$

where

$$f(T) = (1/\pi)[\arcsin(1 - 2/T) + \pi/2] \quad (32)$$

and

$$T = (ct/a)/(1-C) \simeq (ct/a)/(S^2/2) \simeq (ct/a)/(\psi_0^2/2).$$

Thus it can be seen at the initial instant $t = 0$ the response is a unit positive step. This is followed by a linearly decreasing signal which begins at $T = 1$. It then flattens off to a constant value in accordance with the limit:

$$A(t)]_{t\to\infty} = 1 - [\Delta/(S+\Delta)] = S/(S+\Delta)$$

which is the expected value for the pattern factor for a dipole on the surface of a flat impedance surface of infinite extent. The situation is illustrated in Fig. 3 where $A(t)$ is sketched as a function of the time parameter.

Conclusion

While the present analysis is based on a highly idealized model, the results do indicate that the finite extent of a ground screen will modify significantly the pulse shape of the radiated field. This may be an important factor in timing systems where some feature of the radiated pulse shape is tagged as a point to measure the travel time. Further work on this subject should consider the frequency dependence of the ground-surface impedance and the equivalent impedance of the wire mesh which makes up the ground screen. Also, raised antennas of finite length will lead to more complicated integral evaluation but, in principle, the present results can be adapted if superposition is used.

Appendix

In performing the integration over ω in (19), the Bessel function was replaced by the first term in its asymptotic expansion. The significance of this step will be discussed briefly.

We are concerned with the evaluation of the following Fourier integral:

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\exp(-i\omega\rho/c)}{Cc}\left(1 + \frac{c}{i\omega\rho}\right)iJ_1(\omega\rho C/c)\exp(i\omega t)\,d\omega. \quad (33)$$

Noting that $iJ_1(Z) = I_1(iZ)$, the Fourier integral tables [5] can be used to show that

$$g(t) = \frac{1}{\pi C^2 \rho}\left\{\frac{c}{\rho}\left[\left(\frac{\rho C}{c}\right)^2 - \left(t - \frac{\rho}{c}\right)^2\right]^{1/2} - \left(t - \frac{\rho}{c}\right)\left[\left(\frac{\rho C}{c}\right)^2 - \left(t - \frac{\rho}{c}\right)^2\right]^{-1/2}\right\} u\left[-\left|t - \frac{\rho}{c}\right| + \frac{\rho C}{c}\right]. \quad (34)$$

For the times such that $|t - (\rho/c)(1 - C)| \ll (\rho/c)\ (1 - C)$, it can be seen that

$$g(t) \simeq \frac{u[t - (\rho/c)(1 - C)]}{\pi C^{3/2}(2c\rho)^{1/2}[t - (\rho/c)(1 - C)]^{1/2}}. \quad (35)$$

While this is an admittedly crude approximation, it is qualitatively satisfactory, for this reason, and because it facilitates the subsequent integration over ρ. In fact, the result given by (35) corresponds to the form used in the main text which is based on the limiting asymptotic form for the Bessel function $J_1(\omega\rho C/c)$ in the frequency plane.

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Integral equation approach to the radiation from a vertical antenna over an inhomogeneous ground plane

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The fields of a dipole source located over a variable impedance surface are considered. A method of obtaining solutions of the integral equation for the tangential field is outlined. Numerical results are given for the resulting radiation patterns for an exponentially tapered variation of the surface impedance. It is shown that the tapering has the advantage of reducing the lobe structure of the low-angle radiation.

INTRODUCTION

The use of extended ground screens to enhance low-angle radiation from HF antennas is now common. To be effective the ground screen should reduce the effective surface impedance of the foreground out to distances comparable with a Fresnel zone. The utility of the general analytical formulation [*Wait*, 1963] has been demonstrated on a number of recent occasions [*Bernard et al.*, 1966; *Gustafson et al.*, 1966; *Collin and Zucker*, 1969].

In this note, we wish to discuss the evaluation of the radiated fields for a situation where the surface impedance of the ground is tapered as a function of distance from the base of the antenna. For simplicity here, we assume azimuthal symmetry, and also we represent the source as a ground-based Hertzian electric dipole. This situation is illustrated in Figure 1, where the surface impedance of the ground beyond a specified radius ρ_0 is a constant Z. Within the ground system (i.e., $0 < \rho < \rho_0$), the surface impedance $Z'(\rho)$ is taken to be a function of ρ that is the radial distance from the source dipole. As indicated, ψ_0 is the angle subtended by the observer's direction and the ground plane.

Now, in the absence of the ground system (i.e., $\rho_0 = 0$ or $Z'(\rho) = Z$), the far-field radiation pattern $P(\psi_0)$ of the ground-based dipole on the homogeneous flat earth is given by

$$P(\psi_0) = |1 + R_v| (\cos \psi_0)/2 \qquad (1)$$

$$R_v = \frac{\sin \psi_0 - (Z/\eta_0)}{\sin \psi_0 + (Z/\eta_0)} \qquad (2)$$

and $\eta_0 = 120\pi$. Here, of course, R_v is the appropriate Fresnel reflection coefficient for a normalized surface impedance Z/η_0. It was shown earlier that the corresponding pattern function is given by

$$P(\psi_0) = |(1 + R_v)(1 + \Omega_v)| (\cos \psi_0)/2 \qquad (3)$$

where Ω_v is the all-important correction factor that accounts for the presence of the ground system. The ground system factor is given by

$$\Omega_v = -\frac{k}{\cos \psi_0} \int_0^{\rho_0} W'(k\rho)F(k\rho)e^{-ik\rho} \cdot \left(1 + \frac{1}{ik\rho}\right) J_1(k\rho \cos \psi_0)\, d\rho \qquad (4)$$

where $k = 2\pi/(\text{wavelength})$, where J_1 is the Bessel function of the first kind, and where $F(k\rho) = [Z - Z'(\rho)]/\eta_0$ is the surface impedance contrast function which by definition vanishes if $\rho > \rho_0$. In this integral representation for the perturbed radiation field, $W'(\rho)$ is the *unknown* attenuation function for the tangential magnetic field of the source dipole in the region $0 < \rho < \rho_0$. This function was shown to satisfy

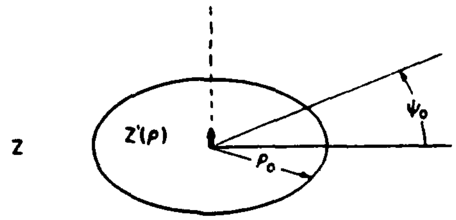

Fig. 1. Vertical dipole located centrally over a radially varying impedance surface.

a Volterra integral equation of the form

$$W'(x) = W(x) + \left(\frac{ix}{2\pi}\right)^{1/2} \int_0^x F(y) \frac{W(x-y)W'(y)}{[y(x-y)]^{1/2}}\, dy \tag{5}$$

where

$$W(x) = 1 - i(\pi p)^{1/2} e^{-p} \operatorname{erfc}(ip^{1/2}) \tag{6}$$

and $p = -(ix/2)(Z/\eta_0)^2$. Here, $W(x)$ is the well-known attenuation function for propagation to a 'numerical distance' p over an homogeneous flat surface. In (6), erfc is the complement of the error function.

SOLVING THE INTEGRAL EQUATION

We now outline a series method of solving (5) for $W'(x)$ for small values of the argument x. Since $W(x)$ has an expansion of the form

$$W(x) = \sum_{n=0}^{\infty} \alpha_n x^{n/2} \tag{7}$$

it seems plausible to assume that $W'(x)$ has a similar expansion

$$W'(x) = \sum_{n=0}^{\infty} \alpha'_n x^{n/2} \tag{8}$$

where the coefficients α'_n are to be determined. In addition, we assume the known impedance function $(Z'(x) - Z)/\eta_0$ can be expanded as

$$F(x) = -\frac{Z'(x) - Z}{\eta_0} = -\sum_{n=0}^{\infty} \zeta_n x^{n/2} \tag{9}$$

Upon substituting the appropriate expansions into the integral equation we obtain, on interchanging the order of summation and integration,

$$\sum_{n=0}^{\infty} \alpha'_n x^{n/2} = \sum_{n=0}^{\infty} \alpha_n x^{n/2} - \left(\frac{ix}{2\pi}\right)^{1/2} \sum_{n=0}^{\infty} \sum_{p=0}^{n} \sum_{j=0}^{p} \zeta_j \alpha'_{p-j} \alpha_{n-p} \cdot \int_0^x y^{(p-1)/2} (x-y)^{(n-p-1)/2}\, dy \tag{10}$$

Noting the integral in (10) is $x^{n/2}$ times the beta function $B[(p+1)/2, (n-p+1)/2]$ and using the standard relation

$$B(u, v) = \frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)} \tag{11}$$

(10) becomes

$$\alpha'_0 + \sum_{n=1}^{\infty} \alpha'_n x^{n/2} = \alpha_0 + \sum_{n=1}^{\infty} \left[\alpha_n - \left(\frac{i}{2\pi}\right)^{1/2} \delta_{n-1}\right] x^{n/2} \tag{12}$$

where

$$\delta_n = \sum_{p=0}^{n} \sum_{j=0}^{p} \zeta_j \alpha'_{p-j} \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{n-p+1}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right)} \alpha_{n-p} \tag{13}$$

Finally, we equate coefficients of like powers of $x^{1/2}$ on the two sides of (12) to get

$$\alpha'_0 = \alpha_0$$

and

$$\alpha'_n = \alpha_n - \left(\frac{i}{2\pi}\right)^{1/2} \delta_{n-1} \qquad (n = 1, 2, 3, \cdots) \tag{14}$$

which is effectively a recurrence formula for the evaluation of α'_n in terms of $\alpha'_0, \alpha'_1, \cdots, \alpha'_{n-1}$.

When these results are applied to the case where

$$\frac{Z'(x) - Z}{\eta_0} = -\frac{Z}{\eta_0} e^{-bx} \tag{15}$$

(where b is a nonnegative constant), the series for $W'(x)$ is given by

$$W'(x) = 1 - \tfrac{1}{2}\sqrt{\pi}\, b\xi x^{3/2} + \tfrac{3}{16}\sqrt{\pi}\, b^2\xi x^{5/2} + \tfrac{8}{15} b^2\xi^2 x^3 - \tfrac{5}{[illegible]}\sqrt{\pi}\, b^3\xi x^{7/2} - \tfrac{3}{8} b^3\xi^2 x^4 + \tfrac{1}{[illegible]}\sqrt{\pi}\,([illegible]\, b^4\xi - 56 b^3\xi^3) x^{9/2} + \tfrac{[illegible]}{[illegible]} b^4\xi^2 x^5 + \cdots \tag{16}$$

where $\xi = (i/2)^{1/2} Z/\eta_0$.

When x is not sufficiently small, series 15 is not useful for evaluating $W'(x)$, and we must resort to numerical methods of solving integral equation 5. The technique outlined below is an adaptation of one described by *Wagner* [1953] and yields $W'(x)$ for a prescribed set of x values, $x_1, x_2, x_3, \cdots$. The first step is to compute $W'(x_1)$ and $W'(x_2)$ using series 16; the solution then proceeds inductively, $W'(x_n)$ being computed when $W'(x_1), W'(x_2), \cdots, W'(x_{n-1})$ are known. From integral equation 5, we write

$$W'(x_n) = W(x_n) + \left(\frac{ix_n}{2\pi}\right)^{1/2} \cdot \left\{\int_0^{x_1} K(x_n, y) W'(y)\, dy + \int_{x_1}^{x_2} K(x_n, y) W'(y)\, dy + \cdots + \int_{x_{n-1}}^{x_n} K(x_n, y) W'(y)\, dy\right\} \tag{17}$$

where

$$K(x, y) = \frac{F(y)W'(x-y)}{[y(x-y)]^{1/2}} \tag{18}$$

Now assume that $W'(y)$ may be approximated in the interval (x_{j-1}, x_j) $(j = 3, 4, \cdots, n)$ by a quadratic polynomial fitted at x_{j-2}, x_{j-1}, and x_j; thus

$$\begin{aligned} W'(y) = {} & \frac{(y - x_{j-1})(y - x_j)}{(x_{j-2} - x_{j-1})(x_{j-2} - x_j)} W'(x_{j-2}) \\ & + \frac{(y - x_{j-2})(y - x_j)}{(x_{j-1} - x_{j-2})(x_{j-1} - x_j)} W'(x_{j-1}) \\ & + \frac{(y - x_{j-2})(y - x_{j-1})}{(x_j - x_{j-2})(x_j - x_{j-1})} W'(x_j) \end{aligned} \tag{19}$$

which is just the well-known Lagrange interpolation polynomial of degree 2. When the appropriate polynomials are substituted into (17), one may solve the resulting equation for $W'(x_n)$ and write

$$W'(x_n) = \frac{W(x_n) + \left(\frac{ix_n}{2\pi}\right)^{1/2} \sum_{j=1}^{n} I_j(x_n)}{1 - \left(\frac{ix_n}{2\pi}\right)^{1/2} \hat{I}(x_n)} \tag{20}$$

where

$$I_1(x_n) = \int_0^{x_1} K(x_n, y) W'(y)\, dy \tag{21}$$

$$I_2(x_n) = \int_{x_1}^{x_2} K(x_n, y) W'(y)\, dy \tag{22}$$

$$\begin{aligned} I_j(x_n) = \int_{x_{j-1}}^{x_j} & K(x_n, y)[A_j(y) W'(x_{j-2}) \\ & + B_j(y) W'(x_{j-1}) + C_j(y) W'(x_j)]\, dy \\ & (j = 3, 4, \cdots, n-1) \end{aligned} \tag{23}$$

$$\begin{aligned} I_n(x_n) = \int_{x_{n-1}}^{x_n} & K(x_n, y)[A_n(y) W'(x_{n-2}) \\ & + B_n(y) W'(x_{n-1})]\, dy \end{aligned} \tag{24}$$

$$\hat{I}(x_n) = \int_{x_{n-1}}^{x_n} K(x_n, y) C_n(y)\, dy \tag{25}$$

$$A_j(y) = \frac{(y - x_{j-1})(y - x_j)}{(x_{j-2} - x_{j-1})(x_{j-2} - x_j)} \tag{26a}$$

$$B_j(y) = \frac{(y - x_{j-2})(y - x_j)}{(x_{j-1} - x_{j-2})(x_{j-1} - x_j)} \tag{26b}$$

$$C_j(y) = \frac{(y - x_{j-2})(y - x_{j-1})}{(x_j - x_{j-2})(x_j - x_{j-1})} \tag{26c}$$

and $K(x_n, y)$ is obtained from (18). The integrals (21)–(25) were approximated by a 12-point Gaussian quadrature formula. To remove infinite singularities in the integrand, the integration variable was first changed to θ by the relation $y = x_1 \sin^2\theta$ in $I_1(x_n)$ and by $x_n - y = (x_n - x_{n-1}) \sin^2\theta$ in $I_n(x_n)$ and $\hat{I}(x_n)$. In $I_1(x_n)$ and $I_2(x_n)$, $W'(y)$ was evaluated at the Gaussian abscissas by means of series 16.

Computing time is a rapidly increasing function of the number of points at which $W'(x)$ is evaluated (about 5 minutes was required for 60 points on a CDC 3800 computer); thus it is desirable to space the points $x_1, x_2, x_3, \cdots$ as widely as the demands of accuracy will permit. One must, of course, take x_1 and x_2 to be small enough so that $W'(x_1)$ and $W'(x_2)$ may be computed from (16); some numerical experiments then indicated that adequate accuracy was obtained by taking the interval $x_j - x_{j-1}$ to be roughly $x_j/10$. Further confidence in the results was gained by applying the numerical technique outlined above to the integral equation for $W(x)$:

$$W(x) = 1 - \left(\frac{ix}{2\pi}\right)^{1/2} \frac{Z}{\eta_0} \int_0^x \frac{W(y)}{[y(x-y)]^{1/2}}\, dy \tag{27}$$

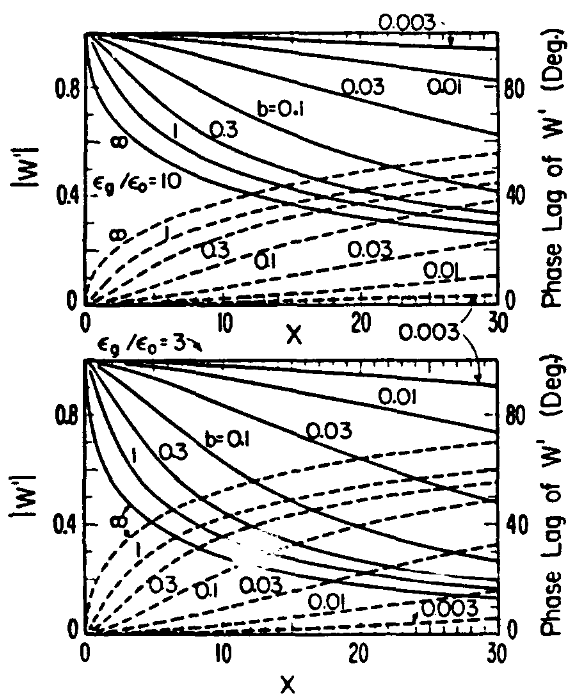

Fig. 2a, b. The attenuation function for an exponential variation of the surface impedance. The abscissa is the normalized distance $x (= 2\pi\rho/\text{wavelength})$ for the dipole.

To start the calculation, $W(0.10)$ and $W(0.11)$ were computed from (6); then a formula analogous to (20) was used to evaluate $W(x)$ at a sequence of points ending at $x = 30$. These results were compared with values obtained from (6); at no point did the magnitude of the relative error exceed 3×10^{-5}.

THE ATTENUATION FUNCTION

As mentioned above, the specification of the functional form of the surface impedance contrast function $F(x)$ will determine the form of the resulting attenuation function $W'(x)$. Using the numerical methods outlined above, some representative results are obtained for the case where

$$Z - Z'(x) = Ze^{-bx} \qquad (28)$$

and where Z corresponds to the appropriate value for a homogeneous lossless ground of relative dielectric constant ϵ_g/ϵ_0. In this case, we choose

$$Z/\eta_0 = (\epsilon_0/\epsilon_g)^{1/2}(1 - \epsilon_0/\epsilon_g)^{1/2} \qquad (29)$$

which is the exact form for a vertically polarized plane wave at grazing incidence. The exponential variation indicated by (28) is an idealization for a ground screen that is tapered smoothly from the base of the antenna. Here, we do not dwell on the physical realizability of such a ground system, although we could point out that a radial wire system will have a smoothly decreasing effective impedance as one recedes from the base of the antenna along the ground surface. For our example, the dimensionless parameter b is a measure of the rapidity of the impedance taper. For example, $b = 0$ would correspond to a perfectly conducting ground plane, whereas $b = \infty$ corresponds to a homogeneous earth of surface impedance Z *throughout*.

Specific values of the amplitude and phase lag of $W''(x)$ are shown plotted in Figure 2 for $\epsilon_g/\epsilon_0 = 10$ and for $\epsilon_g/\epsilon_0 = 3$. The abscissa here is the electrical distance x measured from the base of the antenna along the ground [i.e., $x = 2\pi\rho/(\text{wavelength})$]. Various values of b are indicated on the curves. As ex-

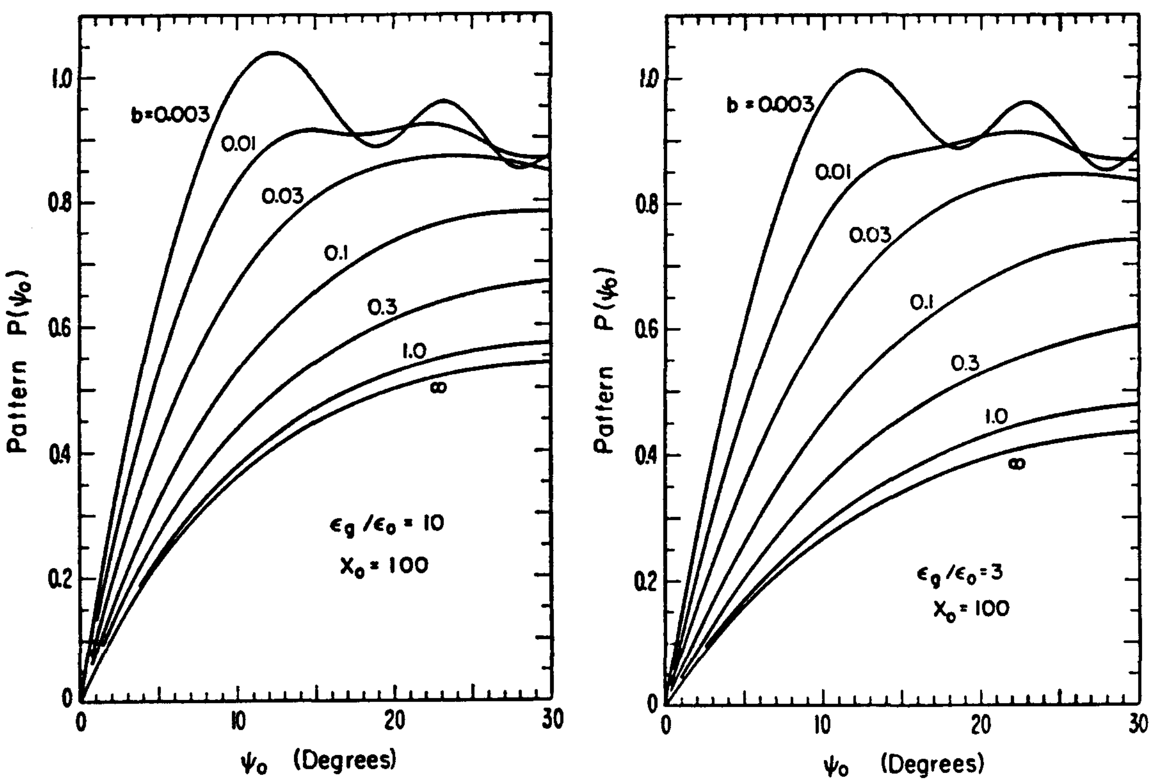

Fig. 3a, b. The far-field radiation pattern of the dipole as a function of the elevation angle ψ_0 for an exponentially tapered ground system that is truncated at $x_0 = 100$ (i.e., $2\pi\rho_0 = 100$ wavelengths). Various values of b are shown.

pected, the magnitude of the attenuation function $W'(x)$ is considerably less than unity for the lower dielectric constant of the ground. We stress here that the curves shown in Figure 2 apply only to a pure dielectric ground. This would be a reasonable representation for a real physical situation of operation at HF over relatively dry ground or thick ice layers.

RADIATION PATTERNS

Once we have the numerical values for the attenuation function $W'(k\rho)$, the pattern function $P(\psi_0)$ can be obtained directly from (3) following the numerical integration of (4). In this case, the upper limit ρ_0 of the integration corresponds physically to the outer extremity of the ground system.

By use of the numerical results for $W'(x)$ shown in Figure 2, the pattern function $P(\psi_0)$ is shown in Figure 3*a* and 3*b* for $\epsilon_g/\epsilon_0 = 10$ and 3, respectively, for $x_0 = k\rho_0 = 100$. It is not surprising that the low-angle portion of the pattern is greatly enhanced when b is decreased to small values. In fact, for the case $b = 0.003$, the ground system is very similar in effectiveness to a perfectly conducting circular disk of radius ρ_0. This fact is confirmed by comparing the results for $b = 0.003$ in Figures 3*a* and 3*b* with the curves given previously for a perfectly conducting circular disk of the same radius [*Wait and Walters*, 1963] and for the same values of the relative dielectric constant of the ground.

For smaller values of b, the curves in Figures 3*a* and 3*b* are smooth and monotonically increasing functions of the grazing angle ψ_0. This can be attributed to the fact that, for the value of x_0 (or $k\rho_0$) chosen, the surface impedance contrast function $F(x)$ becomes negligible before x reaches x_0. Thus, there is no appreciable scattering from the edge of the ground system at $\rho = \rho_0$. For some practical applications, this may be a desirable characteristic for the radiated fields.

The influence of varying the extent of the ground

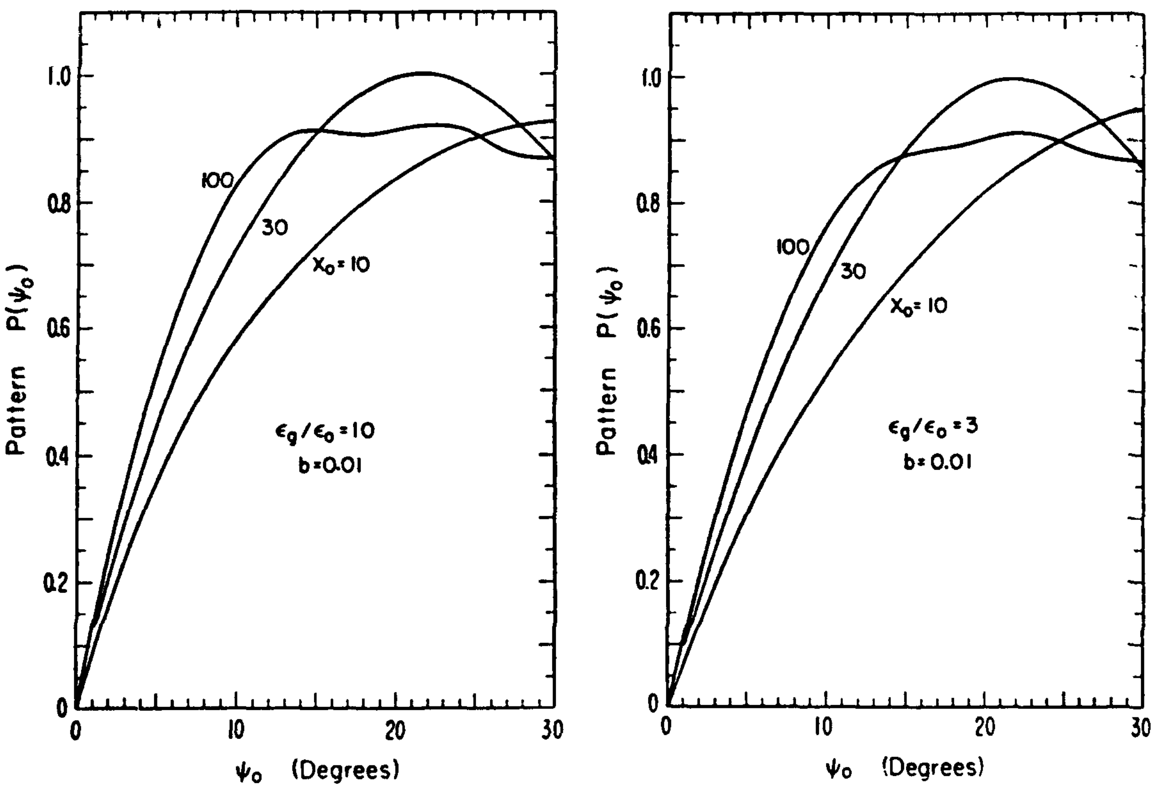

Fig. 4*a*, *b*. The far-field pattern for an exponentially tapered ground system showing the effect of different truncation distances for $b = 0.01$.

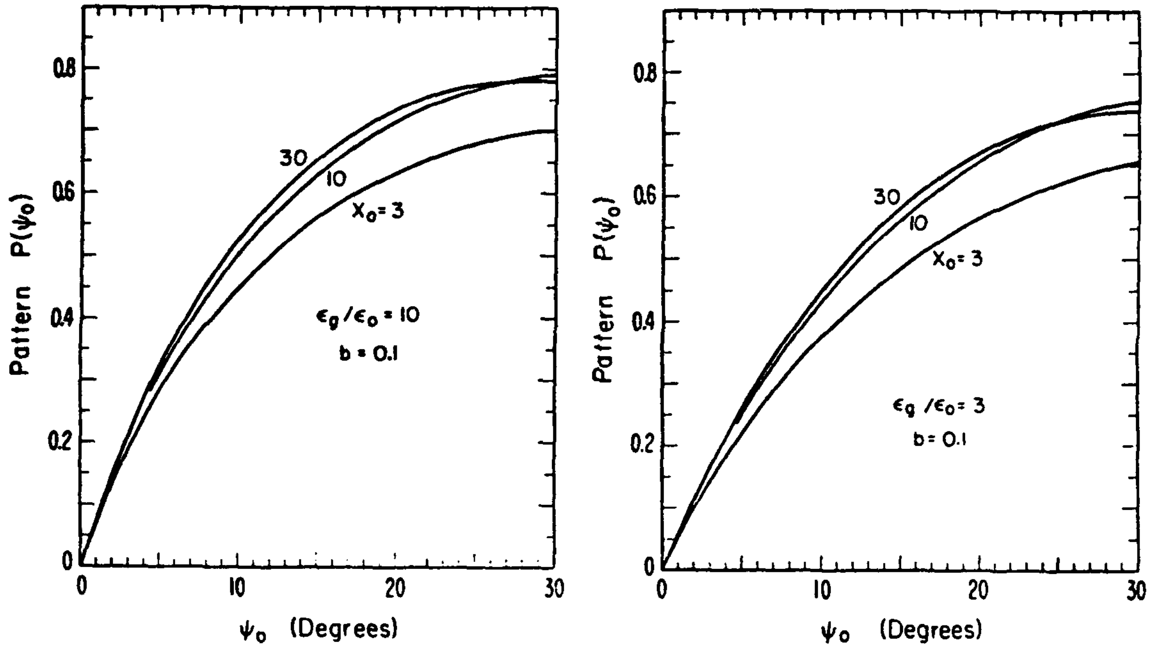

Fig. 5a, b. The far-field pattern for an exponentially tapered ground system showing the effect of different truncation distances for $b = 0.1$.

system is illustrated in Figures 4a and 4b for $\epsilon_g/\epsilon_0 = 10$ and 3, respectively, for a fixed value of b ($= 0.01$). It is not surprising that for these cases the low-angle pattern is again improved as x_0 is increased. Also, there is some evidence that the scattered wave from the edge of the ground system is producing some minor wobbles in the patterns. Similar results are shown in Figures 5a and 5b, where the value of b is increased by a factor of 10. Here, the influence on the finite value of x_0 is much smaller because the surface impedance contrast has decreased to a negligible value before the 'edge' of the ground system is reached.

In a previous study [*Wait*, 1967] of tapered ground systems, the attenuation function $W'(k\rho)$ was approximated by unity in the field integral given by (4) in the present paper. In effect, this is a physical optics or Kirchhoff type of approximation. It has the merit of leading to a relatively simple closed-form expression for the resulting pattern functions. Certainly, for a truncated ground system of low surface impedance, the approximation is very good (i.e., where $b\,k\,\rho_0 \ll 1$ in the present context). Also, if the ground system taper is sufficiently rapid, the contribution from the integral in (4), where $W'(\rho)$ differs appre-

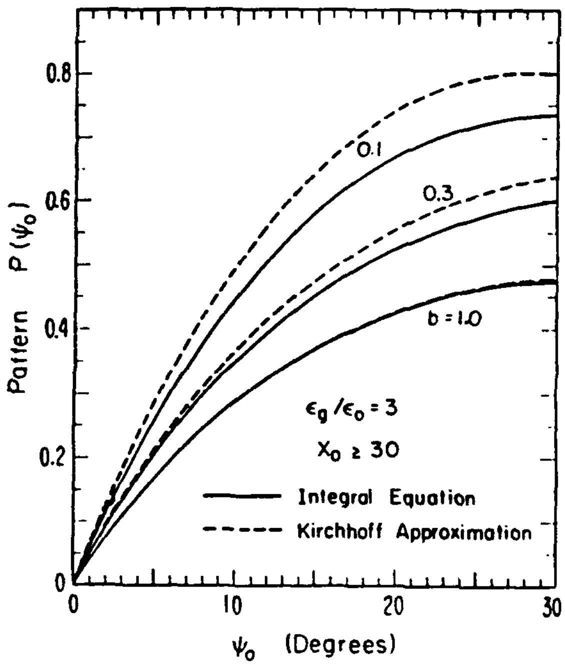

Fig. 6. The far-field pattern using the integral solution for $W'(x)$ compared with the Kirchhoff-type approximation which sets $W'(x) = 1$ in the field integral.

ciably from unity, would be expected to be negligible, even when the upper limit ρ_0 is effectively infinite. This fact is confirmed by comparing the computed pattern function $P(\psi_0)$ for the case (1) where the integral equation is used to obtain $W'(\rho)$ and (2) where $W'(\rho)$ is replaced by unity. The comparative results are shown in Figure 6 for ϵ_g/ϵ_0, where the value of x_0 is effectively infinite. As we see from the plotted curves, the Kirchhoff-type approximation is overly optimistic for small values of b when the tapered ground system is effectively infinite in extent.

CONCLUDING REMARKS

The results given in this paper apply to a rather idealized radiating system. For example, the source is a Hertzian ground-based dipole and the ground system is represented by a surface impedance boundary that is allowed to taper smoothly from the source to the extremity of the system. To consider a more realistic system, we should take into account the ohmic losses in the ground, the inductive reactance of the radial wire ground system, and the finite height and length of the source antenna. The inclusion of such factors constitutes a straightforward extension of the present formulation.

In general, we feel that the intentional tapering of the ground system can lead to major improvements in the low-angle radiation characteristics of HF antennas.

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On the Radiation from a Vertical Dipole with an Inductive Wire-Grid Ground System

Abstract—The far field of a vertical electric dipole on a sectionally homogeneous ground plane is considered. The specific model used is a dielectric-like ground which is modified by using an inductive wire grid or mesh screen in a region surrounding the dipole. Attention is focused on the modification of the radiation pattern resulting from the presence of the inductive ground screen. It is demonstrated that the low-angle radiation may be greatly enhanced by a ground screen which extends out to 15 or more wavelengths.

Introduction

The performance of HF antennas is adversely influenced by the presence of a poorly conducting ground. Unfortunately, for one reason or another, it is sometimes required to operate a communication system with at least one terminal over ice, frozen ground, or other dielectric-like material. In such cases the low-angle radiation is greatly reduced. One remedy is to raise the antenna to a height of several wavelengths. While this may suffice for certain applications, it usually is accompanied by deep nulls in the vertical radiation pattern, not to mention the increased cost of the support structure. The other remedy is to employ a wire grid or mesh ground mat beneath the antenna. Not only does this stabilize the impedance

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of the antenna but, if it is sufficiently extended, the vertical radiation pattern will approach the ideal expected for a perfectly conducting ground plane. There is now a substantial literature [1]–[5] on the subject of extended ground screens covering both theoretical and experimental investigations.

The general theory of the radiation field has been discussed rather extensively. The method used is based on the compensation theorem which is closely related to the Lorentz reciprocity relations for vector electromagnetic fields. In the actual calculations, it is usually assumed that the current distribution over the ground screen is identical to that for the same antenna located over a perfectly conducting ground screen. If the wire spacing in the ground screen or mesh is sufficiently small this may be an excellent approximation. However, in the general case, the currents excited on the ground screen will be attenuated or modified due to the nonzero value of the effective surface impedance. In the present communication, we examine this question. To facilitate the discussion, an idealized model is employed.

Analytical Model

The situation is illustrated in Fig. 1. A vertical electric dipole is located on flat ground. The effective surface impedance of the ground system is a constant Z' out to a radius b from the base of the antenna. The ground beyond radius b is taken to have a constant surface impedance Z. In the present formulation, we assume that the system is rotationally symmetric. The consequences of this assumption are discussed later on.

The radiation field E, for a time factor $\exp(i\omega t)$, at a distance R and elevation angle ψ_0, is expressible in the form

$$E = \frac{i\mu_0\omega I ds}{2\pi} W''(\cos\psi_0)\frac{\exp(-ikR)}{R} \tag{1}$$

where $\mu_0 = 4\pi \times 10^{-7}$, $k = 2\pi/(\text{wavelength})$, and Ids is the current moment of the dipole. Here, W'' is a *pattern function normalized* such that it is unity over a perfectly conducting ground plane of infinite extent (i.e., $Z' = Z = 0$). Now, following an early derivation [2], the first-order iteration of an integral equation yields

$$W'' = (1 + R_v)(1 + \Omega)/2 \tag{2}$$

where $R_v = [\sin\psi_0 - (Z/\eta_0)]/[\sin\psi_0 + (Z/\eta_0)]$, $\eta_0 = 120\pi$, and Ω is a "modification" which vanishes if the ground screen is absent (i.e., $Z' = Z$ or if $b \to 0$). In the limiting case of no ground screen,

$$W'' = W' = (1 + R_v)/2 = \sin\psi_0/[\sin\psi_0 + (Z/\eta_0)]. \tag{3}$$

Here, Z/η_0 is the normalized surface impedance of the "unmodified" ground. If the latter is homogeneous and lossless, we have

$$Z/\eta_0 = (1/K)^{1/2}[1 - (\cos^2\psi_0)/K]^{1/2} \tag{4}$$

where K is the relative dielectric constant of the dielectric ground. The corresponding result for W' is exact. If now we focus our attention on low grazing angles, we see that

$$W' \simeq \psi_0/(Z/\eta_0) \tag{5}$$

where $(Z/\eta_0) \simeq (1/K)^{1/2}[1 - (1/K)]^{1/2}$, and ψ_0 is the grazing angle in radians. If, for example, $K = 3$, we have

$$W' \simeq \psi_0(3/2^{1/2}) = \psi_0 \times 2.12$$

while if $K = 10$,

$$W' \simeq \psi_0(10/3) = \psi_0 \times 3.16.$$

At a typical angle of 3° we have $W' = 0.111$ and 0.165, for $K = 3$ and 10, respectively. This "cutback" effect, where W' varies linearly with ψ_0 at low angles, is a consequence of locating the antenna over a dielectric-type ground.

The modification of the surface impedance in a circular region of radius b, from Z to Z', results in improving the cutback by a

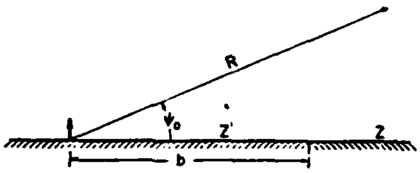

Fig. 1. Essential geometry showing side view of dipole on flat ground plane.

factor of $1 + \Omega$. Also, it can be shown [2] that the ground wave field of the dipole is also modified by the factor $1 + \Omega$, where this is to be evaluated as $\psi_0 \to 0$. In fact, as we will see below, $1 + \Omega$ is essentially independent of ψ_0 for small angles while, of course, the pattern factor W'' itself varies linearly with ψ_0.

The working formula for Ω which forms the basis of the present paper is given by [2]

$$\Omega \simeq -\frac{kF}{\cos\psi_0}\int_0^b \exp(-ik\rho)W'_g(k\rho)(1 + 1/ik\rho)J_1(k\rho\cos\psi_0)\,d\rho \tag{6}$$

where

$$W'_g(k\rho) = 1 - i(\pi p)^{1/2}\exp(-p)\,\mathrm{erfc}\,(ip^{1/2}) \tag{7}$$

where $p = -(ik\rho/2)(Z'/\eta_0)^2$ and $F = (Z - Z')/\eta_0$. Here, J_1 is the Bessel function of order one while W'_g is a "ground wave attenuation function" which accounts for the modification of the tangential magnetic field by the finite surface impedance Z' of the ground screen. Often, W'_g is replaced by unity which is justified, of course, if $Z' = 0$ and if back reflections from the edge of the ground screen are ignored [5]. One may observe that W'_g is closely related to the plasma dispersion function [6].

Actually, (6) was derived on the basis that ψ_0 is a small parameter, but this assumption is not overly restrictive and in fact ψ_0 can be used up to 30° or so with negligible error. Also, it should be mentioned that the attenuation function W'_g is usually derived on the basis that $|Z'/\eta_0| \ll 1$ but, *in fact, this is overly restrictive*. If, in fact, the value of Z' is employed which is appropriate for grazing incidence, W'_g is valid even when Z'/η_0 is not small [5]. In any case, for present purposes, we will assume that the formulas are valid. Furthermore, the modified surface impedance Z' will be imagined to be composed of the parallel combination of the ground impedance Z (evaluated at grazing incidence) and the effective impedance Z_g of the wire grid. Thus

$$Z' = ZZ_g/(Z + Z_g) \tag{8}$$

where $Z = \eta_0 K^{-1/2}(1 - K^{-1})^{1/2}$, is the surface impedance of the ground for grazing incidence. If the wire grid spacing is much less than a wavelength and if ohmic losses in the wires are negligible, we know that Z_g is purely inductive. Thus we set

$$Z_g = i\eta_0\delta$$

where δ is a dimensionless reactance parameter. A good approximation is $\delta = (d/\lambda_0)\log_e[d/(2\pi c)] + S_c$, where d is the spacing between the wires in the mesh, λ_0 is the free-space wavelength, c is the wire radius, and S_c is a correction factor which is negligible if d/λ_0 is sufficiently small [1], [5]. For present purposes, we will not dwell on the geometrical parameters of the wire grid, but simply specify the effective value of δ.

We stress that, in the present note, the ground is assumed to be a lossless dielectric. While the theory has been developed for a general dissipative ground, it simplifies the discussion if the relative dielectric constant is taken to be real. This neglect of conduction currents is valid when $\sigma/K\epsilon_0\omega \ll 1$, where σ is the conductivity of the ground.

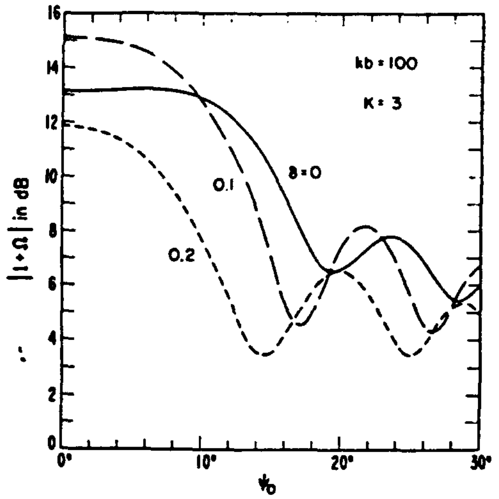

Fig. 2. Modification, in decibels, of radiation field of dipole resulting from presence of ground screen of radius b.

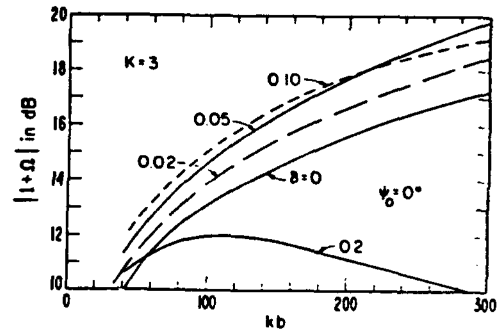

Fig. 3. Dependence on wire-mesh screen diameter for various reactances of wire mesh for $K = 3$.

Discussion of Numerical Results

A numerical evaluation of (6) was carried out for a wide range of the parameters ψ_0, kb, K, and δ. The range of integration was suitably subdivided, and a Gaussian quadrature was applied to each subinterval. The detailed tabulations of the pattern factor W'' and the all-important parameter $1 + \Omega$ are available elsewhere [7]. Here, we will present some of the results in graphical form and discuss the physical significance of the radiation patterns.

In Fig. 2 we illustrate the magnitude of $1 + \Omega$ as a function of the grazing angle ψ_0. The ordinate which is $20 \log_{10} |1 + \Omega|$ is the decibel ratio of the radiation field with and without the ground system. The radius b of the ground screen is $(100/2\pi)\lambda_0$ or about 16 wavelengths and the relative dielectric constant of the (lossless) ground is 3. Various values of δ, the normalized screen reactance, are shown. Note the dependence on the grazing angle ψ_0 and the normalized reactance δ of the wire mesh. Actually, $\delta = 0$ corresponds to a perfectly conducting screen of radius b (i.e., the wire spacing d is effectively zero). The other two curves (i.e., for $\delta = 0.1$ and 0.2) correspond to typical values of the normalized reactance. All the curves in Fig. 2 illustrate that $1 + \Omega$ is approaching a well-defined limit as ψ_0 tends to zero. The magnitude of the ordinate for these small values of ψ_0 is a measure of the ability of the ground screen to enhance both the low-angle sky wave radiation and the ground wave from the dipole. Immediately, we see that the best ground screen is not necessarily the one which is perfectly conducting. More will be said about this later.

A particularly interesting feature of Fig. 2 is the manner in which $|1 + \Omega|$ varies with ψ_0 for the different values of the reactance parameter δ. In general, we see that as ψ_0 is increased from zero, $|1 + \Omega|$ decreases to a minimum, then rises again to subsidiary maximum, and then oscillates with a sinusoidal-like ripple. As indicated before [5], for the case $\delta = 0$ the pattern can be interpreted as an interference between the direct radiation of the dipole and the diffraction from the edge of the ground screen. The situation is similar for nonzero values of δ, but now the first null in the pattern occurs at lower angles. This is a consequence of the reduced phase velocity of the "ground wave" excited by the dipole along the ground screen.

The dependence of the ground screen modification factor $|1 + \Omega|$ on the extent of the ground screen is shown in Fig. 3 for $K = 3$ and for the vanishingly small grazing angles. Various values of the reactance parameter δ are shown. The dependence on kb for the $\delta = 0$ case is fully consistent with the expected proportionality of Ω to $(kb)^{1/2}$. A similar trend is observed for other values of δ up to about 0.1. For larger values of δ, there is a marked degradation, particularly for the larger kb values.

Other calculations [7] show that for δ values of order of 0.1 the low-angle radiation is maximized. This enhancement can be attributed to the trapped surface wave character of the ground wave excited over an impedance boundary that has an appreciable inductive component. In fact, the magnitude of the ground wave "attenuation function" W_g may exceed unity if the numerical distance parameter $|p|$ is in the range from 1 to 10 while at the same time $\arg p > 0$. These conditions are met over a portion of the distance range in the present calculations.

Conclusions

The results given in this note illustrate the rather significant modification, for an inductive-type ground screen, of the radiation from HF antennas. It is believed the present calculations are the first to show the effect of a quasi-trapped surface wave excited on the ground screen. While the model consists of an azimuthal symmetric system, the results can be applied with some confidence to sector-shaped ground screens if the sector is directed toward the receiving point. Of course, the angular width of the sector should be sufficiently large to encompass at least one Fresnel zone, as indicated in some detail elsewhere [5].

The major conclusion from the present work is that a perfectly conducting ground screen may not be the best ground screen if low-angle radiation is to be maximized. We emphasize, however, that this conclusion must be tempered by the realization that the vertical radiation patterns are less desirable if the low-angle radiation is maximized.

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RADIATION FROM A VERTICAL DIPOLE WITH AN INDUCTIVE WIRE-GRID GROUND SYSTEM

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Abstract

The far field of a vertical electric dipole on a sectionally homogeneous ground plane is considered. The specific model used is a dielectric-like ground which is modified by using an inductive wire grid or mesh screen in a region surrounding the dipole. Attention is focussed on the modification of the radiation pattern resulting from the presence of the inductive ground screen. It is demonstrated that the low-angle radiation may be greatly enhanced by a ground screen which extends out to 15 or more wavelengths. However, some rather curious effects are noted, such as pronounced lobes in the vertical radiation pattern when the reactance of the mesh screen is of the order of 0.1 times the characteristic impedance of free space.

INTRODUCTION

The performance of HF antennas is adversely influenced by the presence of a poorly conducting ground. Unfortunately, for one reason or another, it is sometimes required to operate a communication system with at least one terminal over ice, frozen ground, or other dielectric-like material. In such cases, the low-angle radiation is greatly reduced. One remedy is to raise the antenna to a height of several wavelengths. While this may suffice for certain applications, it usually is accompanied by deep nulls in the vertical radiation pattern, not to mention the increased cost of the support structure. The other remedy is to employ a wire grid or mesh ground mat beneath the antenna. Not only does this stabilize the impedance of the antenna but, if it is sufficiently extended, the vertical radiation pattern will approach the ideal expected for a perfectly conducting ground plane. There is now a substantial literature [1-5] on the subject of extended ground screens covering both theoretical and experimental investigations.

The general theory of the radiation field has been discussed rather extensively. The method used is based on the compensation theorem which is closely related to the Lorentz reciprocity relations for vector electromagnetic fields. In the actual calculations, it is usually assumed that the current distribution over the ground screen is identical to that for the same antenna located over a perfectly conducting ground screen. If the wire spacing in the ground screen or mesh is sufficiently small this may be an excellent approximation. However, in the general case, the currents excited on the ground screen will be attenuated or modified due to the non-zero value of the effective surface impedance. In the present paper, we plan to examine this question. To facilitate the discussion, an idealized model is employed.

THE ANALYTICAL MODEL

The situation is illustrated in Fig. 1. A vertical electric dipole is located on a flat ground. The effective surface impedance of the ground system is a constant Z' out to a radius b from the base of the antenna. The ground beyond radius b is taken to have a constant surface impedance Z. In the present formulation, we assume that the system is rotationally symmetric. The consequences of this assumption are discussed later on.

The radiation field E, for a time factor $\exp(i\omega t)$, at a distance R and elevation angle ψ_o, is expressible in the form

$$E = \frac{i\mu_o \omega I ds}{2\pi} W'(\cos\psi_o) \frac{e^{-ikR}}{R} \qquad (1)$$

where $\mu_o = 4\pi \times 10^{-7}$, $k = 2\pi/(\text{wavelength})$, and Ids is the current moment of the dipole. Here, W' is a pattern function normalized such that it is unity over a perfectly conducting ground plane of infinite extent (i.e., $Z' = Z = 0$). Now, following an early derivation [2],

$$W' = (1 + R_v)(1 + \Omega)/2 \qquad (2)$$

where $R_v = [\sin\psi_o - (Z/\eta_o)]/[\sin\psi_o + (Z/\eta_o)]$, $\eta_o = 120\pi$, and Ω is a "modification" which vanishes if the ground screen is absent (i.e., $Z' = Z$ or if $b \to 0$). In the limiting case of no ground screen,

$$W' = W = (1 + R_v)/2 = \sin\psi_o/[\sin\psi_o + (Z/\eta_o)] \quad . \qquad (3)$$

Here, Z/η_o is the normalized surface impedance of the "unmodified" ground. If the latter is homogeneous and lossless, we have

$$Z/\eta_o = (1/K)^{\frac{1}{2}}[1 - (\cos^2\psi_o)/K]^{\frac{1}{2}} \quad . \qquad (4)$$

where K is the relative dielectric constant of the dielectric ground. The corresponding result for W is exact. If now we focus our attention on low grazing angles, we see that

$$W \approx \psi_o/(Z/\eta_o) \tag{5}$$

where $(Z/\eta_o) \approx (1/K)^{\frac{1}{2}}[1-(1/K)]^{\frac{1}{2}}$, and ψ_o is the grazing angle in radians. If, for example, K = 3, we have

$$W \approx \psi_o(3/2^{\frac{1}{2}}) = \psi_o \times 2.12$$

while, if K = 10,

$$W \approx \psi_o(10/3) = \psi_o \times 3.16 \quad .$$

At a typical angle of 3^o we have $W \approx 0.111$ and 0.165 for $K = 3$ and *10, respectively*. This "cut-back" effect, where W varies linearly with ψ_o at low angles, is a consequence of locating the antenna over a dielectric-type ground.

The modification of the surface impedance in a circular region of radius b, from Z to Z', results in improving the "cut-back" by a factor of $1+\Omega$. Also, it can be shown [2] that the ground wave field of the dipole is also modified by the factor $1+\Omega$, where this is to be evaluated as $\psi_o \to 0$. In fact, as we will see below, $1+\Omega$ is essentially independent of ψ_o for small angles while, of course, the pattern factor W' itself varies linearly with ψ_o.

The working formula for Ω which forms the basis of the present paper is given by

$$\Omega = -\frac{kF}{\cos\psi_o}\int_0^b e^{-ik\rho}\, W_g(k\rho)\left(1+\frac{1}{ik\rho}\right)J_1(k\rho\cos\psi_o)\,d\rho \quad , \tag{6}$$

where

$$W_g(k\rho) = 1 - i(\pi p)^{\frac{1}{2}}e^{-p}\,\mathrm{erfc}(i\,p^{\frac{1}{2}}) \quad , \tag{7}$$

$p = -\frac{ikp}{2}\left(\frac{Z'}{r_o}\right)^2$ and $F = (Z-Z')/r_o$. Here, J_1 is the Bessel function of order one while W_g is a "ground wave attenuation function" which accounts for the modification of the tangential magnetic field by the finite surface impedance Z' of the ground screen. Often, W_g is replaced by unity which is justified, of course, if $Z' = 0$ and if back reflections from the edge of the ground screen are ignored [5]. Some simplifications and limiting cases of (6) are discussed in the Appendix.

Actually, (6) was derived on the basis that ψ_o is a small parameter, but this assumption is not overly restrictive and, in fact, ψ_o can be used up to 30^o or so with negligible error. Also, it should be mentioned that the attenuation function W_g is usually derived on the basis that $|Z'/\eta_o| << 1$ but, in fact, this is overly restrictive. If, in fact, the value of Z' is employed which is appropriate for grazing incidence, W_g is valid even when Z'/η_o is not small [5-7]. In any case, for present purposes, we will assume that the formulas are valid. Furthermore, the modified surface impedance Z' will be imagined to be composed of the parallel combination of the ground impedance Z (evaluated at grazing incidence) and the effective impedance Z_g of the wire grid. Thus,

$$Z' = Z Z_g/(Z + Z_g) \quad , \tag{8}$$

where $Z = \eta_o K^{-\frac{1}{2}}(1 - K^{-1})^{\frac{1}{2}}$, is the surface impedance of the ground for grazing incidence. If the wire grid spacing is much less than a wavelength and if ohmic losses in the wires are negligible, we know that Z_g is purely inductive. Thus, we set

$$Z_g = i\,\eta_o\,\delta$$

where δ is a dimensionless reactance parameter. A good approximation is $\delta = (d/\lambda_o) \log_e [d/(2\pi c)] + S_c$ where d is the spacing between the wires in the mesh, λ_o is the free-space wavelength, c is the wire radius, and S_c is a correction factor which is negligible if d/λ_o is sufficiently small [1, 5]. For present purposes, we will not dwell on the geometrical parameters of the wire grid, but simply specify the effective value of δ.

We stress that, in the present paper, the ground is assumed to be a lossless dielectric. While the theory has been developed for a general dissipative ground, it simplifies the discussion if the relative dielectric constant is taken to be real. This neglect of conduction currents is valid when $\sigma/K\epsilon_o\omega << 1$, where σ is the conductivity of the ground.

DISCUSSION OF NUMERICAL RESULTS

A numerical evaluation of (6) was carried out for a wide range of the parameters ψ_o, kb, K, and δ. The range of integration was suitably subdivided and a Gaussian quadrature was applied to each subinterval. The detailed tabulations of the pattern factor W' and the all-important parameter $1+\Omega$ are available from the authors. Here, we will present some of the results in graphical form and discuss the physical significance of the radiation patterns.

In Fig. 2, we illustrate the magnitude of $1+\Omega$ as a function of the grazing angle ψ_o. The ordinate which is $20 \log_{10} |1+\Omega|$ is the decibel ratio of the radiation field with and without the ground system. For this figure, the radius b of the ground screen is $(100/2\pi)\lambda_o$ or about 16 wavelengths, and the relative dielectric constant of the (lossless) ground is 3. Various values of δ, the normalized screen reactance, are shown. Actually, $\delta = 0$ corresponds to a perfectly conducting screen of radius b (i.e., the wire spacing d *is effectively zero*). The other two curves (i.e., for $\delta = 0.1$ and 0.2) correspond to typical values of the normalized reactance. All the curves in Fig. 2 illustrate that $1+\Omega$ is approaching a well-defined limit as ψ_o tends to zero. The magnitude of the ordinate for these small values of ψ_o is a measure of the ability of the ground screen to enhance both the low-angle sky wave radiation and the ground wave from the dipole. Immediately, we see that the "best" ground screen is not necessarily the one which is perfectly conducting. More will be said about this below.

A particularly interesting feature of Fig. 2 is the manner in which $|1+\Omega|$ varies with ψ_o for the different values of the reactance parameter δ. In general, we see that as ψ_o is increased from zero, $|1+\Omega|$ decreases to a minimum then rises again to subsidiary maximum and then oscillates with a sinusoidal-like ripple. As indicated before [5], for the case $\delta = 0$, the pattern can be interpreted as an interference between the direct radiation of the dipole and the diffraction from the edge of the ground screen. The situation is similar for non-zero values of δ, but now the first null in the pattern occurs at lower angles. This is a consequence of the reduced phase velocity of the "ground wave" excited by the dipole along the ground screen.

The influence of the extent of the ground screen is illustrated in Fig. 3 for $\delta = 0$ and in Fig. 4 for $\delta = 0.10$. As expected, at low angles, the pattern modification factor $|1 + \Omega|$ is proportional to $(kb)^{\frac{1}{2}}$ for $\delta = 0$, but this is only approximately true for $\delta = 0.1$. Also, we see from Fig. 3 that, for $\delta = 0$, the factor $|1 + \Omega|$ has a relatively weak dependence on ψ_o, at least for low angles. Again, as expected, for $\delta = 0$ the first null occurs at the lowest angles for the largest value of kb. A similar but more marked trend is seen in Fig. 4. Here, again the relative slowness of the ground wave on the screen is producing a more drastic mutilation of the pattern.

The dependence on the relative dielectric constant K of the ground is illustrated in Fig. 5 for $kb = 100$ and $\delta = 0.10$. While only two values of K are shown (i.e., 3 and 10), the results are sufficient to demonstrate that $|1 + \Omega|$, at low angles, is greater for the lower value of K [i.e., ground screens are most effective for the poorer ground conditions]. Also, we note from Fig. 5 that the general form of vertical radiation pattern (i.e., the variation with ψ_o) is not critically dependent on K.

The dependence of the ground screen modification factor $|1 + \Omega|$ on the extent of the ground screen is shown in Fig. 6 for $K = 3$ and for the vanishingly small grazing angles. Various values of the reactance parameter δ are shown. The dependence on kb, for the $\delta = 0$ case, is fully consistent with the proportionality of Ω to $(kb)^{\frac{1}{2}}$, as discussed in the Appendix. A similar trend is observed for other values of δ up to about 0.1. For larger values of δ, there is a marked degradation, particularly for the larger kb values. The same effect is observed in Fig. 7 when $K = 10$.

The actual dependence of $|1 + \Omega|$ on δ is illustrated in Fig. 8. Here, we see that, for δ values of the order of 0.1, the low-angle radiation is maximized. This enhancement can be attributed to the trapped surface wave character of the ground wave excited over an impedance boundary that has an appreciable inductive component. In fact, the magnitude of the ground wave "attenuation function" W_g may exceed unity if the numerical distance parameter $|p|$ is in the range from about 1 to 10, while at the same time, $\arg p > 0$. These conditions are met over a portion of the distance range in the present calculations.

The departures of the attenuation function W_g from unity over the range of the distance ρ are illustrated in Tables Ia and Ib for a selected set of parameters. The values of W_g based on (7) are shown in the tables in complex polar form. It is evident, for the distance ranges involved, $|W_g|$ exceeds unity by a substantial amount. However, if the reactance δ is increased beyond about 0.1, it is evident that $|W_g|$ decreases significantly below unity. Thus, the behavior of W_g indicated in Tables Ia and Ib is consistent with the features of the radiation patterns discussed above.

Table Ia

Amplitude and Phase of W_g for K = 3

	δ = 0.02	0.1	0.2
kρ = 10	1.055(-3.4°)	1.224(-19.7°)	1.241(-41.5°)
20	1.078(-4.8°)	1.323(-28.5°)	1.306(-60.2°)
50	1.127(-7.7°)	1.523(-47.1°)	1.335(-99.3°)
100	1.183(-11.1°)	1.744(-69.8°)	1.179(-146.0°)
200	1.268(-15.9°)	2.004(-105.3°)	0.696(-146.8°)
300	1.337(-19.8°)	2.120(-135.2°)	0.310(-99.8°)

Table Ib

Amplitude and Phase of W_g for K = 10

	δ = 0.02	0.1	0.2
kρ = 10	1.053(-3.5°)	1.169(-20.1°)	1.075(-38.1°)
20	1.076(-4.9°)	1.237(-29.0°)	1.069(-54.4°)
50	1.123(-7.9°)	1.362(-47.4°)	0.990(-86.9°)
100	1.178(-11.3°)	1.477(-69.4°)	0.812(-123.0°)
200	1.259(-16.2°)	1.563(-102.8°)	0.495(-169.4°)
300	1.324(-20.2°)	1.546(-130.1°)	0.286(-162.7°)

CONCLUDING REMARKS

The results given in this paper illustrate the rather significant modification, for an inductive-type ground screen, of the radiation from HF antennas. It is believed the present calculations are the first to show the effect of a quasi-trapped surface wave excited on the ground screen. While the model consists of an azimuthal symmetric system, the results can be applied with some confidence to sector-shaped ground screens if the sector is directed toward the receiving point. Of course, the angular width of the sector should be sufficiently large to encompass at least one Fresnel zone, as indicated in some detail elsewhere [5].

The major conclusion, from the present work, is that a perfectly conducting ground screen may not be the best ground screen if low-angle radiation is to be maximized. We emphasize, however, that this conclusion must be tempered by the realization that the vertical radiation patterns are less desirable if the low-angle radiation is maximized.

APPENDIX: Analytical Simplifications

While the general equation (6) can be employed for numerical work, it is desirable to investigate the analytical structure of the integral form and examine certain limiting cases.

First of all, we decompose (6) in the following manner:

$$\Omega = \Omega_a + \Omega_b$$

where

$$\Omega_a = -\frac{F}{\cos\psi_o}\int_0^{ka} e^{-ix}\, W_g(x)\left(1+\frac{1}{ix}\right) J_1(x\cos\psi_o)\, dx \qquad (9a)$$

and

$$\Omega_b = -\frac{kF}{\cos\psi_o}\int_a^{b} e^{-ik\rho}\, W_g(k\rho)\left(1+\frac{1}{ik\rho}\right) J_1(k\rho\cos\psi_o)\, d\rho \qquad (9b)$$

Now we usually choose the parameter ka so that, for $0 < x < ka$, $W(x)$ can be approximated by 1 in the integral for Ω_a while, at the same time, ka is sufficiently large that $J_1(k\rho\cos\psi_o)$ can be adequately represented by its asymptotic approximation in the integral for Ω_b.

With the judicious choice of ka indicated, we find that

$$\Omega_a = -e^{-i\pi/4}\, F\, G(ka) \quad . \qquad (10)$$

where

$$G(ka) = e^{i\pi/4}\int_0^{ka} e^{-ix}\left(1+\frac{1}{ix}\right) J_1(x\cos\psi_o)\, dx \qquad (11)$$

is an integral which has been tabulated and discussed rather thoroughly [8]. Thus, we can dispense with it.

In the integral for Ω_b, we now utilize the fact that $k\rho >> 1$. Then, in the integrand,

$$1 + \frac{1}{ik\rho} \approx 1$$

and

$$J_1(k\rho\cos\psi_o)e^{-ik\rho} \approx \frac{e^{-i3\pi/4}}{(2\pi k\rho\cos\psi_o)^{\frac{1}{2}}}\left[e^{-ik\rho(1-\cos\psi_o)} - i\,e^{-ik\rho(1+\cos\psi_o)}\right].$$

Then, (9b) is approximated by

$$\Omega_b \approx \frac{e^{i\pi/4}\,kF}{(2\cos^3\psi_o)^{\frac{1}{2}}}\,\frac{1}{(\pi k)^{\frac{1}{2}}}\left[\int_a^b \frac{W(k\rho)}{\rho^{\frac{3}{2}}}\,e^{-ik(1-\cos\psi_o)\rho}\,d\rho\right.$$

$$\left. - i\int_a^b \frac{W(k\rho)}{\rho^{\frac{3}{2}}}\,e^{-ik(1+\cos\psi_o)\rho}\,d\rho\right]. \qquad (12)$$

Now, for small angles of ψ_o, we see that the first integral in square brackets above is dominant because of the highly oscillating term $\exp[-ik(1+\cos\psi_o)\rho]$ in the second integral. In fact, this second integral can be interpreted as the contribution from the back edge of the ground screen. While this contribution could be retained without difficulty, we will discard it here.

Using the definitions of $W(k\rho)$ given by (7), we now find that (6) is given by

$$\Omega_b \approx \frac{F\,e^{i\pi/4}}{(2\cos^3\psi_o)^{\frac{1}{2}}}\left[\left(\frac{k}{\pi}\right)^{\frac{1}{2}}\int_a^b \frac{e^{-ik(1-\cos\psi_o)\rho}}{\rho^{\frac{3}{2}}}\,d\rho\right.$$

$$\left. - i\,k^{\frac{1}{2}}\,\alpha^{\frac{1}{2}}\int_a^b e^{-\alpha\rho}\,e^{-ik(1-\cos\psi_o)\rho} \times \operatorname{erfc}[i(\alpha\rho)^{\frac{1}{2}}]\,d\rho\right],$$

(13)

where $\alpha = -ik(Z'/\eta_o)^2/2$. Now, the first integral in the square bracket above can be converted to a Fresnel integral and the second can be evaluated by integration by parts. Thus, after some manipulation, we find that

$$\Omega_b \sim \frac{F e^{i\pi/4}}{(2\cos^3\psi_o)^{\frac{1}{2}}}\left[\frac{1}{\sin(\psi_o/2)}\,\frac{ik(1-\cos\psi_o)}{\alpha+ik(1-\cos\psi_o)}\int_{(4ka/\pi)^{\frac{1}{2}}\sin(\psi_o/2)}^{(4kb/\pi)^{\frac{1}{2}}\sin(\psi_o/2)} \exp[-i(\pi/2)z^2]\,dz\right.$$

$$+\frac{i(ka)^{\frac{1}{2}}}{\alpha+ik(1-\cos\psi_o)}\Big(\exp[-\alpha b - ik(1-\cos\psi_o)b]\,\mathrm{erfc}\,[i(\alpha b)^{\frac{1}{2}}]$$

$$\left.-\exp[-\alpha a - ik(1-\cos\psi_o)a]\,\mathrm{erfc}\,[i(\alpha a)^{\frac{1}{2}}]\Big)\right] \quad . \tag{14}$$

If now $\psi_o \to 0$, we obtain the limiting form

$$\Omega_b \sim \frac{F e^{i3\pi/4}}{2^{\frac{1}{2}}}\left(\frac{k}{\alpha}\right)^{\frac{1}{2}}\left[e^{-\alpha b}\,\mathrm{erfc}\,[i(\alpha b)^{\frac{1}{2}}] - e^{-\alpha a}\,\mathrm{erfc}\,[i(\alpha a)^{\frac{1}{2}}]\right] \quad . \tag{15}$$

If, in addition, since $|\alpha a|$ has already been assumed small,

$$\Omega_b \sim \frac{F e^{i3\pi/4}}{2^{\frac{1}{2}}}\left(\frac{k}{\alpha}\right)^{\frac{1}{2}}\left[e^{-\alpha b}\,\mathrm{erfc}\,[i(\alpha b)^{\frac{1}{2}}] - 1\right] \quad . \tag{16}$$

In the extreme limiting case where $|\alpha b| << 1$, we obtain the simple form

$$\Omega_b \sim F(2kb/\pi)^{\frac{1}{2}} e^{i\pi/4} \quad . \tag{17}$$

which is well known [5]. It corresponds to the case where the ground wave attenuation function W_g is effectively unity over the whole extent of the ground screen. This can be seen by returning to (13) or (14) and setting $\alpha = 0$, whence

$$\Omega_b \sim \frac{F e^{i\pi/4}}{(2\cos^3\psi_o)^{\frac{1}{2}}}\,\frac{1}{\sin(\psi_o/2)}\int_{(4ka/\pi)^{\frac{1}{2}}\sin(\psi_o/2)}^{(4kb/\pi)^{\frac{1}{2}}\sin(\psi_o/2)} \exp[-i(\pi/2)z^2]\,dz \quad , \tag{18}$$

which is also well known [5]. Then, if now we let $\psi_o \to 0$,

$$\Omega_b \simeq F(2k/\pi)^{\frac{1}{2}} (b^{\frac{1}{2}} - a^{\frac{1}{2}}) e^{i\pi/4}$$

$$\simeq F(2kb/\pi)^{\frac{1}{2}} e^{i\pi/4} \qquad (19)$$

which is consistent with (17) above.

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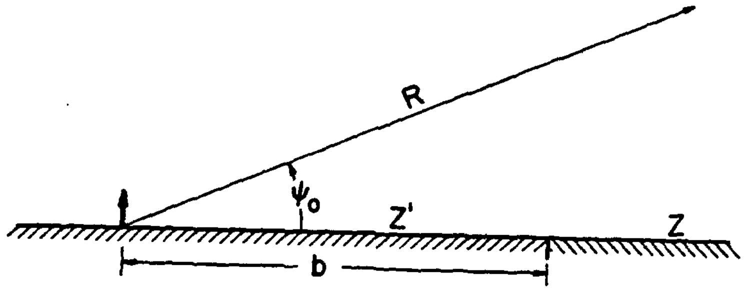

Fig. 1 The essential geometry showing a side view of the dipole on the flat ground plane.

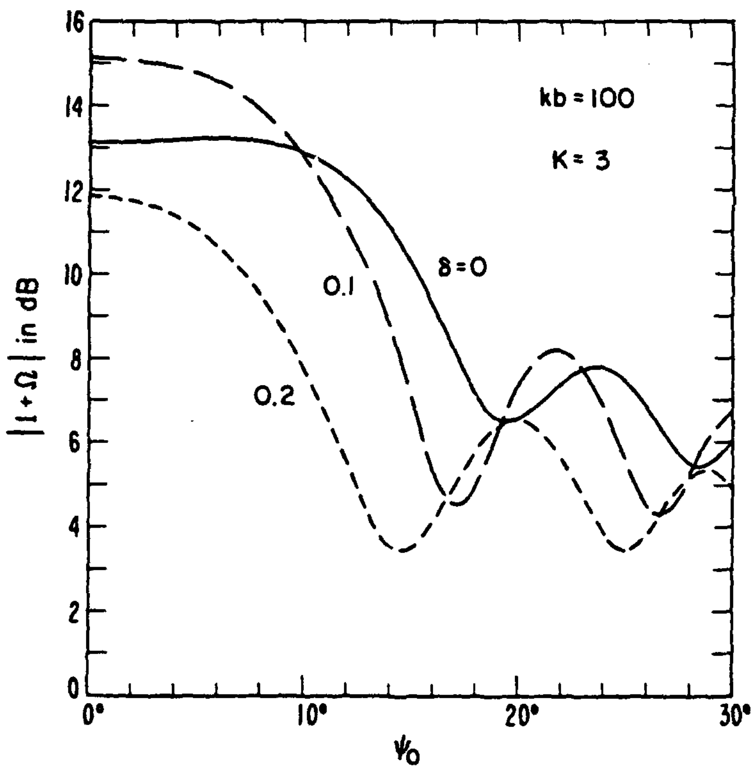

Fig. 2 The modification, in decibels, of the radiation field of the dipole resulting from the presence of the ground screen of radius b. Here, we show the dependence on the grazing angle ψ_o and the normalized reactance δ of the wire mesh.

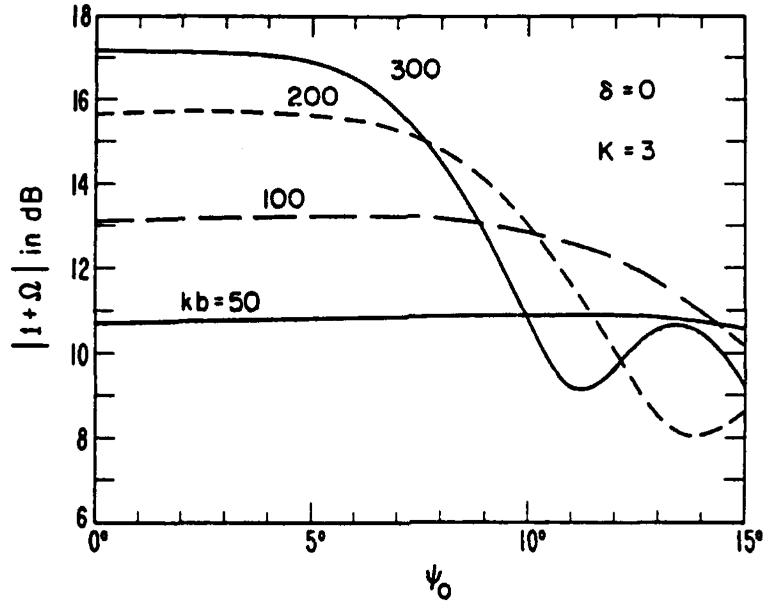

Fig. 3 The dependence on grazing angle for various wire-mesh screen diameters for effectively zero wire spacing.

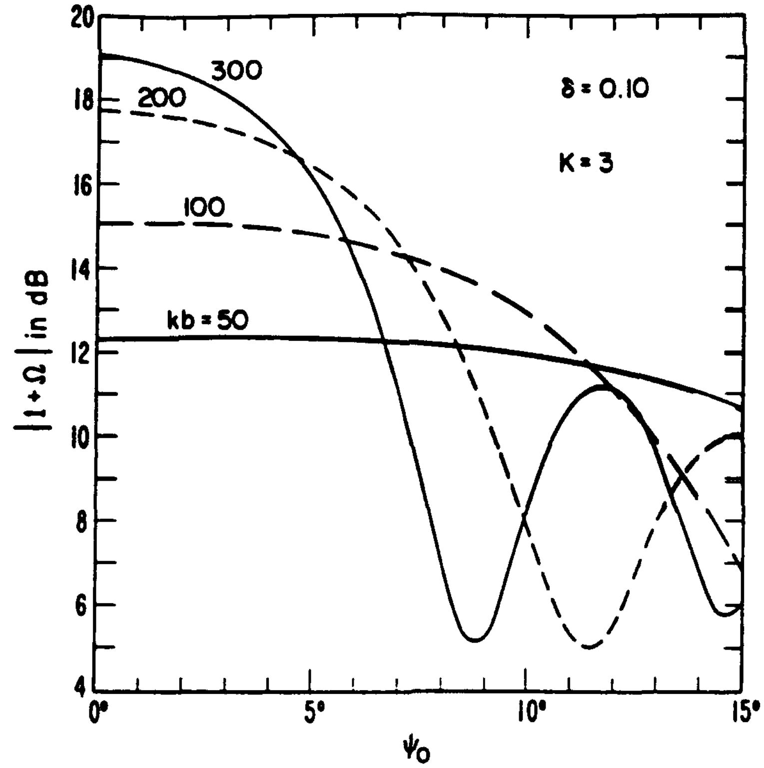

Fig. 4 The dependence on grazing angle for various wire-mesh screen diameters for non-zero wire spacing.

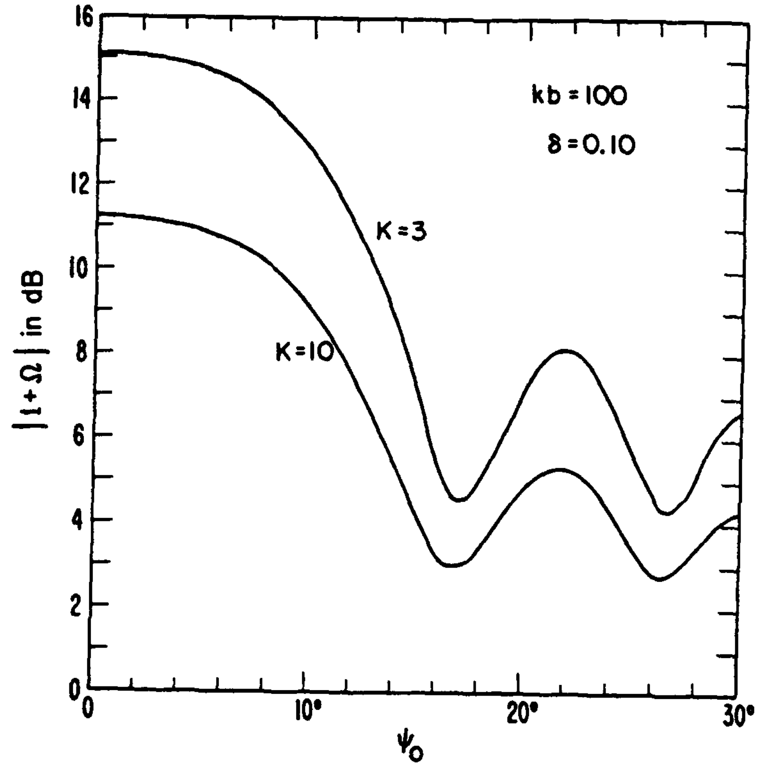

Fig. 5 The dependence on grazing angle for two dielectric constants of the ground.

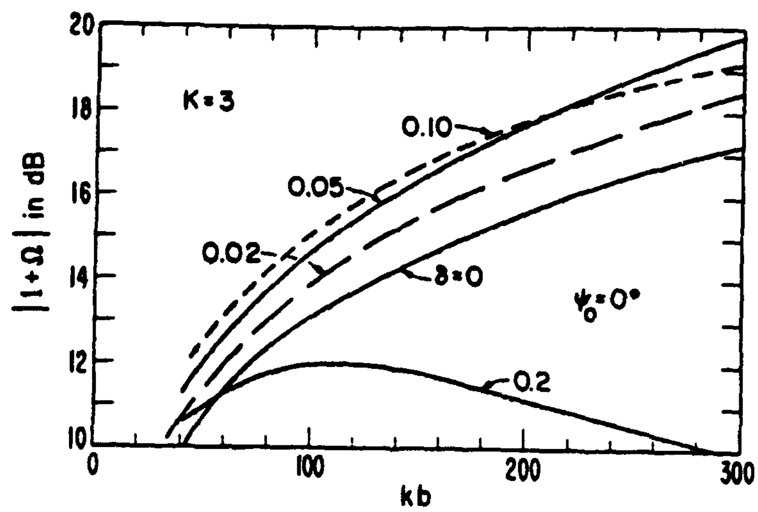

Fig. 6 The dependence on the wire-mesh screen diameter for various reactances of the wire mesh for K = 3 .

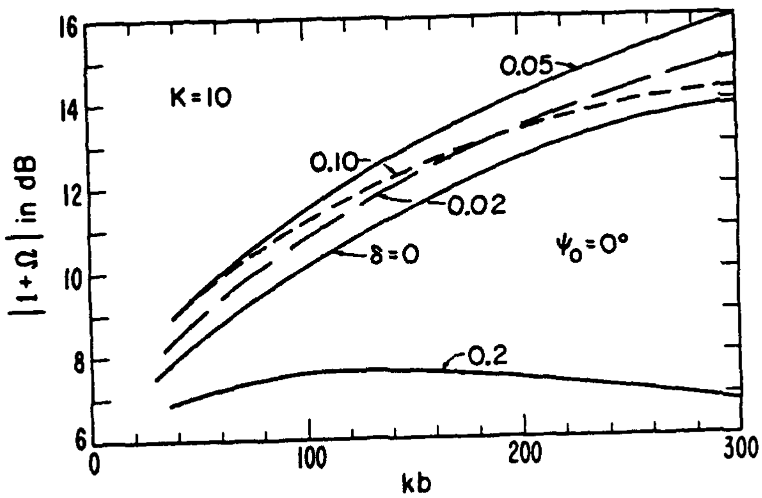

Fig. 7 The dependence on the wire-mesh screen diameter for various reactances of the wire mesh for K = 10 .

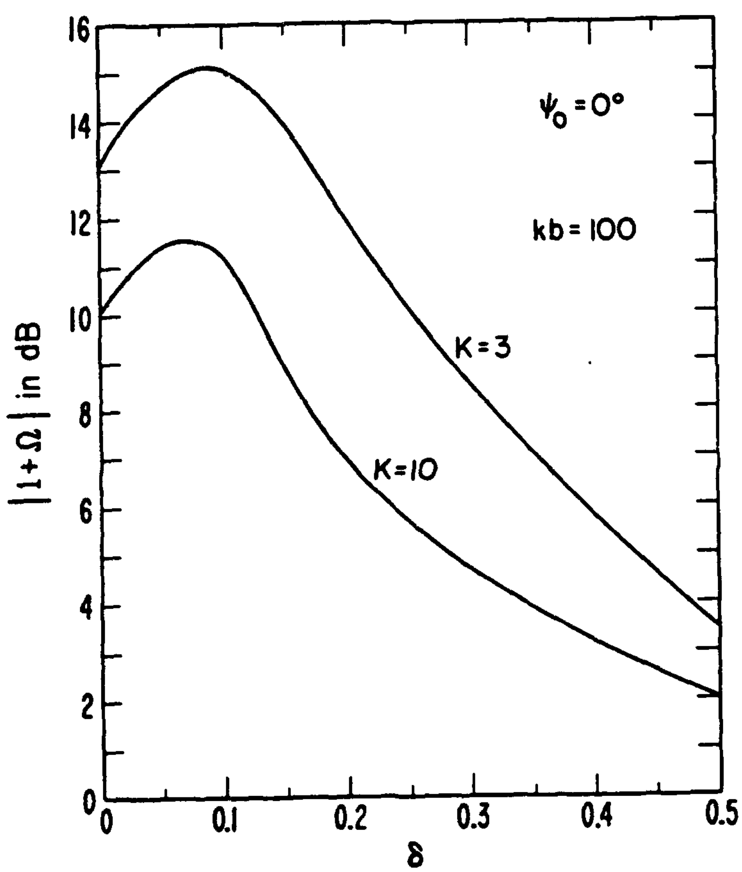

Fig. 8 The dependence on the effective reactance of the wire mesh for two different dielectric constants of the ground.

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Calculated pattern of a vertical antenna with a finite radial-wire ground system

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The appropriate form for the surface impedance of a radial-wire ground screen is utilized to obtain quantitative effects of the important parameters (number of radials, length of radials, wire radius, and ground constants) on the low-angle radiation of a vertical antenna. An approximate correction term is derived to account for currents reflected from the edge of the screen. The effect is small for large screens, but it may be important for smaller screens.

INTRODUCTION

Wire ground screens are commonly used to enhance low-angle radiation from HF antennas. The theory of impedance ground screens has been treated quite generally [*Wait*, 1963, 1967*a*; *Collin and Zucker*, 1969], and extensive numerical work has been done [*Wait and Walters*, 1963; *Horn*, 1967, 1968]. The complication of an exponentially varying impedance ground screen has also been examined [*Wait*, 1967*b*], and an integral-equation approach has been used to determine the attenuation of the ground wave over the screen [*Wait and Spies*, 1969, 1970].

Here we consider a radial-wire ground screen which has a variable impedance as a result of its variable wire spacing. Through the use of such a ground screen, the dependence of the low-angle radiation on the important parameters (number, length, and radius of radials) can be directly examined. Also, the importance of the wave reflected from the edge of the screen is evaluated by an approxmiate method. Such an evaluation is important since the possibility of such waves is ignored when using the usual impedance representation of the ground screen.

VARIABLE IMPEDANCE FORMULATION

The geometry of the transmitting antenna and circular screen is shown in Figure 1. The screen is of radius a while the source is a Hertzian dipole of current moment Il located at the origin. As indicated, the elevation angle ψ equals $\theta - \pi/2$. In addition, Z is the impedance of the ground, while $Z'(\rho)$ is the parallel impedance combination of the screen and the ground.

The far-zone radiated magnetic field with no screen present, $H_\phi^0(\psi)$, is given by the geometrical-optical formula

$$H_\phi^0(\psi) = \{[ikIl \exp(-ikr)]/4\pi r\}(1 + R_v)\cos\psi \quad (1)$$

where

$$R_v = [\sin\psi - (Z/\eta_0)]/[\sin\psi + (Z/\eta_0)]$$

$$Z/\eta_0 = \epsilon_r^{-1/2}[1 - (\cos^2\psi)/\epsilon_r]^{1/2}$$

where ϵ_r is the relative complex dielectric constant of the ground, k is the wave number, and η_0 is the intrinsic impedance of free space. In writing (1), and in what follows, $\exp(i\omega t)$ time dependence is assumed.

With the screen present, the far-zone magnetic field $H_\phi(\psi)$ is given by

$$H_\phi(\psi) = H_\phi^0(\psi)(1 + \Omega_s + \Omega_e) \quad (2)$$

where Ω_e is a correction for the wave reflected from the edge of the screen and where

$$\Omega_s = (-k/\cos\psi)\int_0^a W'(k\rho)\{[Z - Z'(\rho)]/\eta_0\} \cdot \exp(-ik\rho)(1 + 1/ik\rho)J_1(k\rho\cos\psi)\,d\rho$$

where J_1 is the first-order Bessel function and $W'(k\rho)$ is the unknown attenuation function.

The determination of $W'(k\rho)$ is a considerable task in itself, and consequently, we choose to make the simplifying assumption $W'(k\rho) = 1$. The validity of this assumption for an exponentially tapered im-

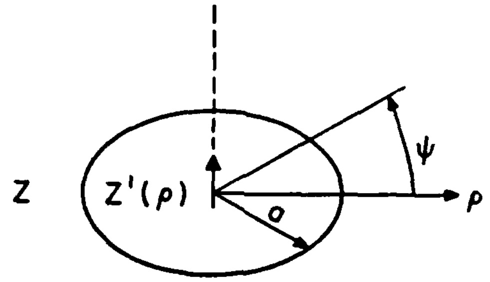

Fig. 1. Vertical electric dipole located at the center of a radial-wire ground screen.

pedance has been studied by *Wait and Spies* [1970], and they point out that the assumption is essentially a physical-optics or Kirchhoff approximation. Their numerical results indicate that this approximation yields a slightly larger value for $1 + \Omega_e$ than the more rigorous calculation in some cases, but almost no difference in others.

The parallel impedance of the screen and ground for grazing incidence is given by [*Wait*, 1959]:

$$Z'(\rho) \cong ZZ_s/(Z + Z_s) \tag{3}$$

where

$$Z = [\eta_0/(\epsilon_r^{1/2})](1 - \epsilon_r^{-1})^{1/2} \qquad \text{at } \psi = 0$$

$$Z_s = (i\omega\mu_0\rho/N) \ln (\rho/Nb)$$

where b is the radius of the wire (assumed perfectly conducting) and N is the number of radials. By substituting (3) into (2) and making a change of variable, we obtain a form suitable for computation:

$$\Omega_e = \frac{-(Z/\eta_0)}{\cos \psi} \cdot \int_0^{ka} \frac{[\exp(-ix)](1 + 1/ix) J_1(x \cos \psi)}{1 + \{ix/[(Z/\eta_0)N]\} [\ln (x/Nk_0 b)]} dx \tag{4}$$

Evaluation of (4) and numerical results are discussed later.

CORRECTION FOR REFLECTED CURRENTS

Since the edge of the ground screen results in a discontinuity in the surface impedance, a reflected wave is to be expected. Equivalently, since the current of the ends of the radial wires must be zero to satisfy the continuity equation, a reflected current is necessary to produce zero wire current at the end. Since this current is normally neglected, it is desirable to determine whether it has a significant effect on the far-zone pattern. Since a rigorous computation of the reflected current would necessitate the solution of an extremely complicated boundary-value problem, we choose to make the following simplifying assumptions: (1) the reflected current at the edge is the negative of the primary screen current (equivalent to -1 reflection coefficient), and (2) the reflected current propagates inward with the known propagation constant for a wire located at the interface. If anything, assumption (1) above will be an overestimate of the magnitude of the reflected wave.

Now, the primary surface-current density carried by the screen at $\rho = a$ is given by

$$J_\rho^{\,p}(a) = [E_\rho(a)]/[Z_s(a)] \tag{5}$$

Furthermore, the radial electric field can be approximated by

$$E_\rho(a) \cong -Z_\rho'(a) H_\phi^{\,\infty}(a) \tag{6}$$

where $H_\phi^{\,\infty}$ is the magnetic field which would exist if the ground were perfectly conducting.

Using (3), (5), and (6), the reflected current, $J_\rho^{\,r}(a)$, at the edge can be written

$$J_\rho^{\,r}(a) \cong [Z/(Z + Z_s)] H_\phi^{\,\infty}(a)$$

$$\cong \frac{ikIl(1 + 1/ika) \exp(-ika)}{2\pi a\{1 + ika/[N(Z/\eta_0)] [\ln (ka/Nkb)]\}} \tag{7}$$

Then the inward traveling current $J_\rho^{\,r}(\rho)$ is approximated by

$$J_\rho^{\,r}(\rho) \cong J_\rho^{\,r}(a)(a/\rho) \exp[-ik_w(a - \rho)] \tag{8}$$

where $k_w = k[(\epsilon_r + 1)/2]^{1/2}$ is the propagation constant for the wire in the interface [*Wait*, 1972]. The factor (a/ρ) results, of course, from the convergence of the ingoing wave.

Now that the current density is known, the far-zone reradiated magnetic field $H_\phi^{\,r}(\psi)$ can be calculated from a straightforward surface integration. Thus

$$H_\phi^{\,r}(\psi) \cong \{[-ik(1 - R_v) \sin \psi \exp(-ikr)]/4\pi r\} \cdot \int_0^a \rho J_\rho^{\,r}(\rho) \left[\int_0^{2\pi} \cos \phi \exp(ik\rho \cos \psi \cos \phi)\, d\phi \right] d\rho \tag{9}$$

where the factor $(1 - R_v) \sin \psi$ can be derived by reciprocity. The ϕ integration can be done exactly by using the integral representation of the first order Bessel function. Also, we note that

$$(1 - R_v) \sin \psi = (1 + R_v)(Z/\eta_0) \tag{10}$$

where Z is the ψ-dependent form as given by (1). Consequently, $H_\phi^e(\psi)$ is given by

$$H_\phi^e(\psi) \cong [k(1 + R_v)(Z/\eta_0) \exp(-ikr)]/zr \cdot \int_0^\infty \rho J_\rho^e(\rho) J_1(k\rho \cos\psi)\, d\rho \qquad (11)$$

The change in the far-zone pattern due to reflected current Ω_e that was introduced in (2) can now be defined by:

$$\Omega_e \cong H_\phi^e(\psi)/H_\phi^0(\psi)$$

$$\cong \frac{(1 + 1/ika)[Z(\psi)/\eta_0] \exp[-i(k + k_w)a]}{\cos\psi\,\{1 + ika/[N(Z/\eta_0)] \ln(ka/Nkb)\}} \cdot \int_0^{ka} J_1(x\cos\psi) \exp[i(k_w/k)x]\, dx \qquad (12)$$

where $Z(\psi)$ indicates the ψ-dependent form rather than the grazing incidence form that appears in the denominator.

In general, the integral appearing in (12) requires numerical evaluation. However, for large screens (i.e., $ka \gg 1$) and if k_w has a significant imaginary part $[-\mathrm{Im}(k_w a) \gg 1]$, the main contribution to the integral occurs for large x. Consequently, the Bessel function is approximated asymptotically as follows

$$J_1(z) \sim [1/(2\pi z)^{1/2}] \cdot \{\text{exp}[i(z - 3\pi/4)] + \exp[-i(z - 3\pi/4)]\} \qquad (13)$$

Consequently, Ω_e can be expressed in the form

$$\Omega_e \sim \frac{[Z(\psi)/\eta_0] \exp[-i(k + k_w)a]}{(2\pi)^{1/2}(\cos\psi)^{3/2}((1 + \{ika/[N(Z/\eta_0)]\}[\ln(ka/Nkb)]))} [I_+ + I_-] \qquad (14)$$

where

$$I_\pm = \exp(\mp i3\pi/4) \cdot \int_0^{ka} ((\{\exp[i(k_w/k \pm \cos\psi)x]\}/x^{1/2}))\, dx$$

By a change of variable, we find that

$$I_\pm = \{[\exp(\mp i3\pi/4)]/[(k_w/k \pm \cos\psi)^{1/2}]\} \cdot \int_0^{(k_w \pm k\cos\psi)a} \{[\exp(iz)]/z^{1/2}\}\, dz \qquad (15)$$

The leading term in the asymptotic ~epresentation of $I_\pm$ can be obtained either from the asymptotic expansion of the Fresnel integral for complex argument or from an integration by parts. Thus, in either case,

$$I_\pm \sim \{[-i(k/a)^{1/2}]/(k_w \pm k\cos\psi)\} \cdot \exp\{i[a(k_w \pm k\cos\psi) \mp (3\pi/4)]\} \qquad (16)$$

Finally, on substituting (16) into (14) and replacing k_w by its definition in (8), we obtain

$$\Omega_e \sim \frac{-i[Z(\psi)/\eta_0] \exp(-ika)}{(2\pi ka)^{1/2}(\cos\psi)^{3/2}((1 + \{ika/[N(Z/\eta_0)]\}[\ln(ka/Nkb)]))} \cdot \left\{\frac{\exp[i(ka\cos\psi - 3\pi/4)]}{[(\epsilon_r + 1)/2]^{1/2} + \cos\psi} + \frac{\exp[-i(ka\cos\psi - 3\pi/4)]}{[(\epsilon_r + 1)/2]^{1/2} - \cos\psi}\right\} \qquad (17)$$

We note that for large ka, Ω_e becomes quite small. This is to be expected, since the individual wire currents decrease, and the increased spacing results in an effectively small surface-current density. Equivalently, this can be thought of as the result of a large surface impedance at the edge of the screen.

NUMERICAL RESULTS

In order to estimate the importance of reflected currents, the quantities $|1 + \Omega_a|$ and $|1 + \Omega_a + \Omega_e|$ were evaluated for various screen sizes, numbers of radials, and complex dielectric constants. Ω_a was evaluated by a numerical integration of (4), and Ω_e was evaluated from the asymptotic form in (17). A comparison of the two quantities is given in Figure 2 for a moderate-size screen ($ka = 30$). The value for kb corresponds to #14 wire at a frequency of 30 MHz. The effect of reflected currents is less than -1 db for all angles, and, for larger screens, the effect is even smaller. For small screens in which the effect may be of some importance, the asymptotic form in (17) will probably not be valid, and a numerical integration of (12) is required. However, this should pose no problem since the integration interval ($0 \rightarrow ka$) will not be large.

The effect of the number of radials for a larger ground screen (i.e., $ka = 60$) is shown in Figures 3 and 4 for two representative values of ϵ_r. It is seen that significant improvement is obtained by increasing the number of radials. For both cases, the effect of Ω_e is insignificant.

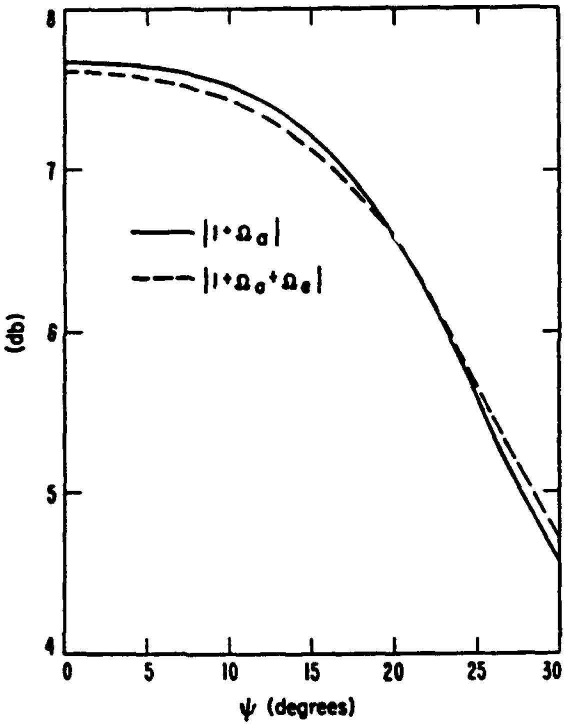

Fig. 2. The effect of edge-reflected currents: $ka = 30$; $kb = 0.512 \times 10^{-2}$, $\epsilon_r = 3 - i\ .3$, and $N = 150$.

The effect of screen size is shown in Figures 5 and 6. It is seen that little is to be gained by increasing ka and that the pattern becomes more oscillatory at larger angles for increased ka. Again, the effect of Ω_e is insignificant.

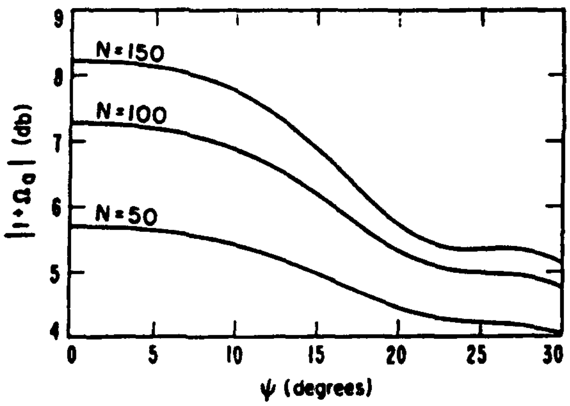

Fig. 3. The effect of the number of radials for a small dielectric constant: $\epsilon_r = 3 - i\ .3$, $ka = 60$, and $kb = 0.512 \times 10^{-2}$.

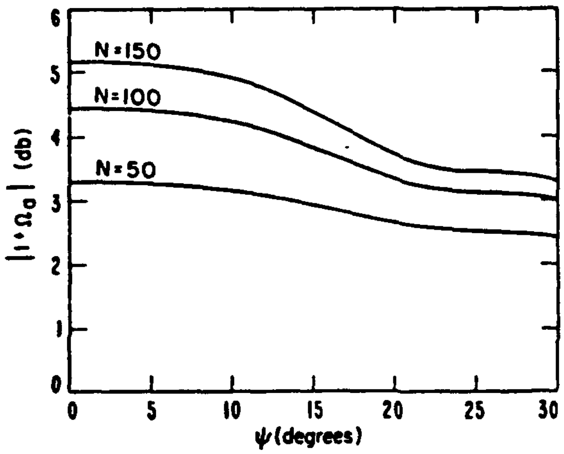

Fig. 4. The effect of the number of radials for a larger dielectric constant: $\epsilon_r = 10 - i\ 2$, $ka = 60$, and $kb = 0.512 \times 10^{-2}$.

CONCLUDING REMARKS

It was found that the effect of currents reflected from the edge of the screen is quite small when the screen is reasonably large (i.e., $ka > 30$). However, the situation is not quite so clear-cut for smaller screens, and a numerical integration is required to yield quantitative results.

For radial-wire screens, greater improvement can be obtained by increasing the number of radials than by increasing the lengths of radials.

Since these calculations are based on a simplified model in which the unknown attenuation function of the ground wave is set equal to one, a worthwhile extension would be to solve the integral equal for the

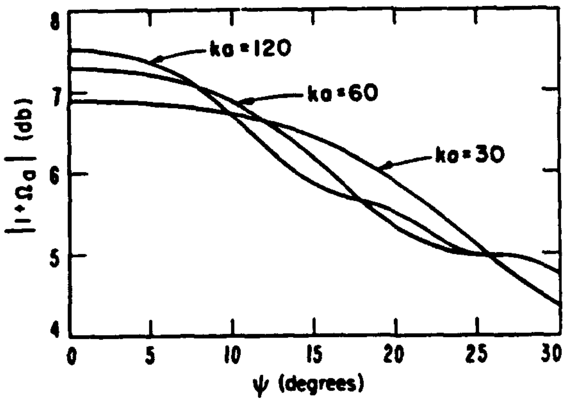

Fig. 5. The effect of screen size for a small dielectric constant: $\epsilon_r = 3 - i\ .3$, $N = 100$, and $kb = 0.512 \times 10^{-2}$.

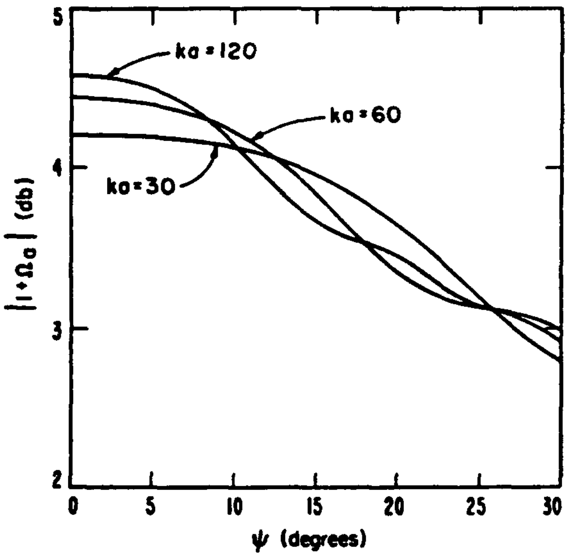

Fig. 6. The effect of screen size for a larger dielectric constant: $\epsilon_r = 10 - i\,2$, $N = 100$, $kb = 0.512 \times 10^{-3}$.

attenuation function. Such a procedure would improve the accuracy of both Ω_a and Ω_e. However, we do not feel that the conclusions would be appreciably modified.

Acknowledgments. We would like to thank Philipp Blac'smith for instigating this study, and Mrs. Loys Gappa for her help in preparing the manuscript.

Editor's Note. Readers are reminded that in shilling-fraction notation, a/bc is to be interpreted to mean $a/(bc)$.

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Effect of Edge Reflections on the Performance of Antenna Ground Screens

DAVID A. HILL AND JAMES R. WAIT

Abstract—A method for computing the effect of the wave reflected inward from the edge of circular ground screens is developed. Numerical results show that the reflected wave is unimportant to the performance of very large screens and only of small importance for moderate size screens. However, as the screen size is further decreased, the relative importance of the reflected wave increases.

I. Introduction

Wire ground screens are commonly used to improve the performance of HF antennas operating in the presence of a poorly conducting ground. The improvement in the radiation field has been discussed extensively [1]–[4], and the method of analysis was based on the compensation theorem. In such treatments, the effect of a wave reflected inward from the edge of the screen has been considered negligible. In this communication we develop a method for evaluating the importance of the inward reflected wave for circular ground screens of constant surface impedance.

II. Formulation

The geometry is illustrated in Fig. 1. A vertical Hertzian dipole of current moment Il is located at the center of a circular ground screen of radius a. The effective surface impedance of the ground system is a constant Z', and the ground beyond radius a has a constant surface impedance Z.

The far-zone magnetic field $H_\phi(\psi_0)$ can be written in the following form [1]:

$$H_\phi(\psi_0) = \frac{ikIl(1 + R_v)\cos\psi_0 \exp(-ikr)}{4\pi r}(1 + \Omega) \quad (1)$$

where $R_v = [\sin\psi_0 - (Z/\eta_0)]/[\sin\psi_0 + (Z/\eta_0)]$, k is the wavenumber, η_0 is the impedance of free space, and an $\exp(i\omega t)$ dependence is assumed. If the ground is homogeneous, the normalized surface impedance Z/η_0 is given by [4]

$$\frac{Z}{\eta_0} = \left(\frac{1}{\epsilon_r}\right)^{1/2}\left[1 - \frac{(\cos^2\psi_0)}{\epsilon_r}\right]^{1/2} \quad (2)$$

where ϵ_r is the relative complex dielectric constant. The term Ω accounts for the presence of the screen and vanishes if the screen is absent.

As indicated by (1) the far-zone pattern is modified by a factor $1 + \Omega$. It can also be shown that the ground wave is modified by $1 + \Omega$, where Ω is evaluated as ψ_0 approaches zero [1].

For convenience, Ω is now split into two parts:

$$\Omega = \Omega_a + \Omega_r \quad (3)$$

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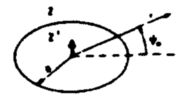

Fig. 1. Vertical electric dipole at center of circular ground screen.

where Ω_a is the usual form which neglects reflections from the edge of the screen and Ω_r is a correction term which accounts for the reflection. First of all we know from previous work [1] that Ω_a is given by

$$\Omega_a = \frac{-F}{\cos\psi_0}\int_0^a \exp(-ik\rho)W_g(k\rho)\left(1 + \frac{1}{ik\rho}\right)J_1(k\rho\cos\psi_0)k\,d\rho \quad (4)$$

where $F = (Z - Z')/\eta_0$, J_1 is the first-order Bessel function and $W_g(k\rho) = 1 - i(\pi p)^{1/2}\exp(-p)\,\mathrm{erfc}(ip^{1/2})$, where $p = -(ik\rho/2)\cdot(Z'/\eta_0)^2$. In the preceding, W_g is a ground wave attenuation function which accounts for the modification of the magnetic field by the finite surface impedance Z'. In general, a numerical evaluation of (4) is required, and the substitution $x = k\rho$ is useful.

The modified surface impedance Z' is assumed to be the parallel combination of the ground impedance Z (at $\psi_0 = 0$) and the effective impedance Z_g of the wire grid [4]. Thus

$$Z' \simeq \frac{ZZ_g}{(Z + Z_g)}. \quad (5)$$

If the wire grid spacing is much less than a wavelength and the wires are highly conducting, then Z_g is purely inductive [3], [4] and we may set $Z_g = i\eta_0\delta$, where δ is dimensionless and independent of ρ.

An estimate of Ω_r can be derived by modifying the magnetic field over the screen to include a reflected wave. Mathematically, this is equivalent to rewriting (4) with the following transformation:

$$\exp(-ik\rho)W_g(k\rho)\left(1 + \frac{1}{ik\rho}\right) \rightarrow \exp(-ika)W_g(ka)\left(1 + \frac{1}{ika}\right)R(\rho) \quad (6)$$

where

$$R(\rho) = \frac{F\exp(-i\pi/4)\exp[-ik(a - \rho)]}{2(2\pi)^{1/2}[k(a - \rho)]^{1/2}}W_g[k(a - \rho)].$$

The factor $R(\rho)$ is a reflection function which was derived from an earlier analysis involving reflection of a ground wave from a land–sea boundary [5], and we assume here that $R(\rho)$ is not affected by the curvature of the boundary. The singular nature of $R(\rho)$ at $\rho = a$ suggests that the integral be split into an integrable term involving the singularity and a remainder. Thus

$$\Omega_r \simeq \frac{-F^2\exp[-i(ka + \pi/4)]}{2(2\pi\cos\psi_0)^{1/2}}W_g(ka)\left(1 + \frac{1}{ika}\right)I_r \quad (7)$$

where

$$I_r = 2(ka)^{1/2}J_1(ka\cos\psi_0) + \int_0^{ka}(ka - x)^{-1/2}\{-J_1(ka\cos\psi_0) + W_g(ka - x)\exp[-i(ka - x)]J_1(x\cos\psi_0)\}\,dx.$$

The integrand now is well behaved, and (7) is a suitable form for numerical evaluation.

III. Numerical Results

A numerical evaluation of (4) and (7) was carried out for ranges of interest of ψ_0, ka, ϵ_r, and δ. It was found that, for large screens ($ka \geq 100$), Ω_r had an insignificant effect (<0.1 dB) on the quantity $1 + \Omega$. In Fig. 2 we illustrate results, at grazing incidence for a poorly conducting ground, for two somewhat smaller screens. Even here, the maximum difference in the curves with and without the reflected wave is only about 0.2 dB and the value of δ which yields the maximum value of $|1 + \Omega|$ at $\psi_0 = 0$ is essentially unchanged. As ka decreases, the ratio $|\Omega_r|/|\Omega_a|$ increases. How-

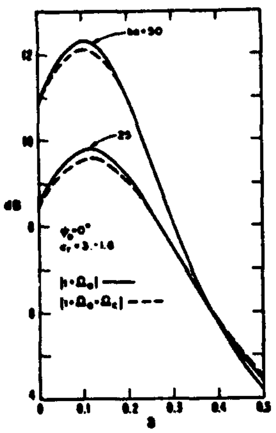

Fig. 2. Effect of reflected waves on moderate size ground screens.

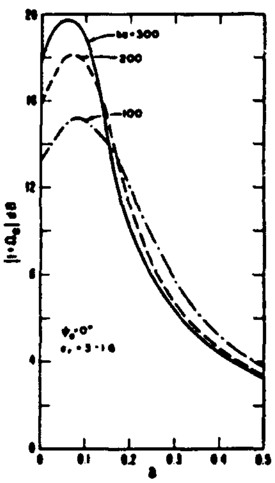

Fig. 3. Dependence on δ for various size ground screens.

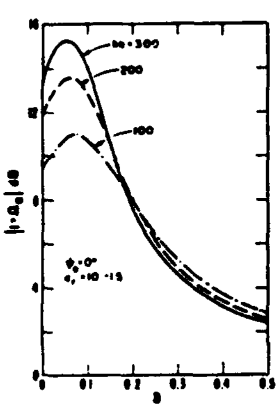

Fig. 4. Dependence on δ for various size ground screens with larger complex dielectric constant.

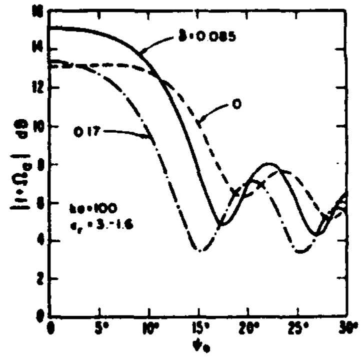

Fig. 5. Dependence on ψ_0 for various values of δ.

ever, small screens are usually of minor interest here because they do not enhance the radiation field sufficiently. Results for larger screens are shown in Figs. 3 and 4, with Ω_c being insignificant to graphical accuracy in each case. The optimum value of δ decreases slightly with increasing ka and with increasing $|\epsilon_r|$.

The optimum value of δ for $\psi_0 = 0$ yields a less desirable pattern for higher angles [4]. This is shown well in Fig. 5 where the pattern for the optimum value of δ for $\psi_0 = 0$ (as determined from Fig. 3) is compared to the pattern of the perfectly conducting disk. For increasing δ, the nulls are seen to occur for smaller values of ψ_0.

IV. Conclusions

The results confirm the validity of earlier calculations for large ground screens where waves reflected from the edge of the screen have been neglected. In fact, even for moderate sizes ($ka \simeq 25$), the reflections are relatively unimportant and do not change the general trends. However as ka decreases, the relative importance of the reflected field increases.

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Anno XXXV, N.5 - Settembre-Ottobre 1980 -Pagg. 576-583
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Low-angle radiation of an antenna over an irregular ground plane

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RIASSUNTO. - *Usando il teorema della Compensazione, si ottengono le espressioni al primo ordine dei campi di radiazione di un'antenna a monopolo posta sopra una superficie irregolare con conduttività finita. Si considera il caso a simmetria di rotazione, e si applicano i risultati al caso in cui l'antenna è posta sopra una piattaforma circolare con pareti inclinate. Si mostra che il guadagno d'antenna e l'eccitazione delle onde di terra sono maggiori che nel caso di una superficie uniformemente piatta.*

SUMMARY. - *Using the Compensation Theorem, first-order expressions are obtained for the radiation fields of a monopole antenna located over an irregular ground surface with finite conductivity. The azimuthally symmetric case is considered and the results are applied to the situation where the monopole is located on an elevated circular plateau with sloping sides. It is indicated that the antenna gain and the ground wave excitation are enhanced over that for a uniformly flat surface.*

Introduction.

There are a number of important applications where the irregular ground surface influences the desired performance of antennas. For example in many H. F. communication systems, the total transmission loss is critically dependent on the nature of the radiation at low angles from the transmitting antenna.

In this note, we present a general first-order theory for calculating the radiation field of a monopole antenna located on or over an irregular boundary. The development is based on the electromagnetic Compensation Theorem in the form developed by Monteath (1) over twenty years ago. The full scope of the Compensation Theorem has not always been

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appreciated, since Monteath only illustrated its application to relatively simple problems. Also, it has only been realized quite recently that the formulation is sufficiently general to facilitate integral equation solution of rather complicated problems (2). For the present, however, we shall invoke physical approximations with a view to obtaining relatively concise first-order expressions for the radiation pattern of a vertical monopole.

Formulation.

As illustrated in Fig. 1, we located a vertical electric dipole of effective length l_a at point A over a deterministic surface $z = f(x, y)$, where $z = 0$ is the reference surface. At some point B located in the (x, z) plane, we have a similar electric dipole of effective length l_b. The distance or range between A and B is R, and this straight line subtends an angle Θ with the positive z-axis. The general problem is to calculate the mutual impedance between the two dipoles for a prescribed surface profile $f(x, y)$ and its electrical properties.

First of all, we assume that the mutual impedance Z_{AB} between the dipoles is known for the simpler auxiliary problem of having $f(x, y) = 0$ everywhere, and taking the region $z < 0$ to be a homogeneous half-space. The corresponding mutual impedance for the actual profile $f(x, y)$ is denoted as Z'_{AB}, where the prime signifies the unknown or sought quantity. Monteath's formulation (1) of the Compensation Theorem tells us immediately that:

$$[1] \qquad Z'_{AB} - Z_{AB} = \frac{1}{I_a I_b} \iint_S (\vec{E}_b \times \vec{H}'_a - \vec{E}'_a \times \vec{H}_b) \cdot \vec{n}\, dS$$

In [1], the terminal currents in dipoles A and B are I_a and I_b, respectively, and $\vec{E}_b$ and $\vec{H}_b$ are the known fields of dipole B for the auxiliary situation, whereas $\vec{E}'_a$ and $\vec{H}'_a$ are the unknown fields for the actual profile where $z = f(x, y)$. Here, the integration of these quantities is over the surface that coincides with $z = f(x, y)$ and is closed by any surface at some large negative value of z. We assume here that it is always possible to select this lower surface so that its contribution to the surface integral in [1] is negligible. The unit vector $\vec{n}$ is thus the outward normal to surface $z = f(x, y)$, as indicated in Fig. 1. For relatively gentle slopes, we can see that the Cartesian components of $\vec{n}$ are given by:

$$\vec{n} \simeq (-\partial f/\partial x, -\partial f/\partial y, 1)$$

The electrical characteristics of the actual surface are to be described by the Leontovich or surface impedance boundary condition. This surface impedance $\eta'(x, y)$ may be a slowly varying function of the horizontal coordinates x and y. Strictly speaking, it should not vary significantly in a distance equal to $|\gamma^{-1}|$, where γ is the propagation constant of the medium for $z < f(x, y)$. Then, for convenience, we characterize the homogeneous half-space in the auxiliary problem by a constant surface impedance η.

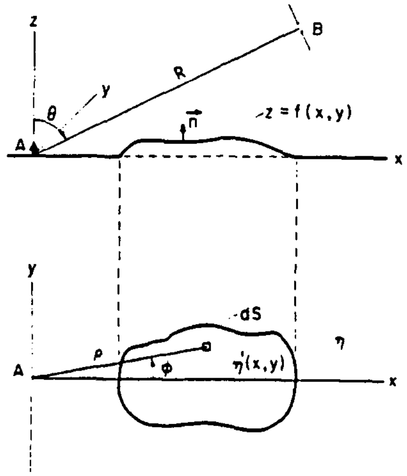

Fig. 1

An approximate solution.

Adopting the various simplifications and physical approximations described above, we now can write down the required field expressions that are to be inserted in [1]. Here, we assume that the dipole B is effectively located at an infinite distance from the surface. Thus, the relevant unprimed field components over S are:

$$[2] \qquad E_{bz} = \frac{-ik\eta_0 I_b l_b}{4\pi R} \sin\theta\, e^{-ikR}\, e^{ik\rho\cos\varphi\sin\theta} \left(e^{ikf\cos\theta} + e^{-ikf\cos\theta}\right),$$

$$[3] \qquad E_{bx} = \frac{ik\eta_0 I_b l_b}{4\pi R} \cos\theta\, e^{-ikR}\, e^{ik\rho\cos\varphi\sin\theta} \left(e^{ikf\cos\theta} - e^{-ikf\cos\theta}\right),$$

$$- \eta H_{by}$$

and:

$$[4] \qquad H_{oy} = \frac{-ikI_bl_b}{4\pi R} e^{-ikR} e^{ik\rho\cos\varphi\sin\theta} (e^{ikf\cos\theta} + e^{-ikf\cos\theta})$$

These formulas correspond to the radiation field of a Hertzian dipole oriented at right angles to the radius vector R and carrying a current equal to the real part of $I_b \exp(i\omega t)$, where ω is the angular frequency. Here, $k = \omega/c$ is the free-space wavenumber, and $\eta_0 \cong 120\pi$ ohms. Also, as indicated in Fig. 1, φ is the azimuthal angle with reference to a cylindrical coordinate system (ρ, φ, z). In addition, we should observe that in writing [2], [3], and [4], we are, in effect, assuming that the reference surface at $z = 0$ is a relatively good conductor.

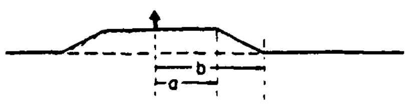

FIG. 2

Now the primed or unknown field components over S are postulated to have the form:

$$[5] \qquad H'_{ax} = \frac{-I_a l_a}{2\pi\rho^2}(1 + ik\rho) e^{-ik\rho} \sin\varphi \,.\, F(\rho, \varphi)\, G(z_a, z)$$

$$[6] \qquad H'_{ay} = \frac{I_a l_a}{2\pi\rho^2}(1 + ik\rho) e^{-ik\rho} \cos\varphi \,.\, F(\rho, \varphi)\, G(z_a, z)$$

$$[7] \qquad E'_{ax} \cong -\eta' H'_{ay} - \frac{\partial f}{\partial x} E'_{az}$$

and:

$$[8] \qquad E'_{ay} \cong \eta' H'_{ax} - \frac{\partial f}{\partial y} E'_{az}$$

where $F(\rho, \varphi)$ is an « attenuation function » that to a first approximation is replaced by unity. In writing [5], [6], [7], and [8], we assume that the field components are the same as those of a vertical Hertzian dipole located at $z = 0$, but modified by a suitable height-gain function $G(z_a, z)$ that depends only on the heights of the dipole A and the observer over the surface $z = 0$. It is important to note that these primed expressions include near field effects.

To proceed further, we observe that:

$$[9] \qquad (\vec{E}_b \times \vec{H}'_a - \vec{E}'_a \times \vec{H}_b) \cdot \vec{n} \simeq E_{bz} H'_{ay} \frac{\partial f}{\partial x} - E_{bz} H'_{ax} \frac{\partial f}{\partial y}$$

$$+ E_{bx} H'_{ay} - E'_{ax} H_{by} \frac{\partial f}{\partial x} - E'_{ax} H_{by}$$

Here, we note that the sum of the latter two terms can be replaced by $\eta' H'_{ay} H_{by}$ by virtue of [7]:

Using all the previous equations, we find without difficulty that:

$$[10] \qquad \frac{Z'_{ab} - Z_{ab}}{Z_a} = \int_0^{2\pi} d\varphi \int_0^{\infty} d\rho \, e^{ik\rho \cos\varphi \sin\theta} \Big[i \cos\theta \sin(kf\cos\theta) \cos\varphi$$

$$- \sin\theta \cos(kf\cos\theta) \frac{\partial f}{\partial \rho} - \frac{\eta' - \eta}{\eta_0} \cos\varphi \cos(kf\cos\theta) \Big] \frac{1}{2\pi\rho} (1 + ik\rho) e^{-ik\rho} \cdot F(\rho, \varphi)$$

where:

$$[11] \qquad \frac{\partial f}{\partial \rho} = \frac{\partial f}{\partial x} \cos\varphi - \frac{\partial f}{\partial y} \sin\varphi .$$

and:

$$[12] \qquad Z_a \quad \frac{ik\eta_0 I_a I_b}{2\pi R} e^{-ikR}$$

We note here that Z_a is the mutual impedance of the two dipoles if they were located on a flat perfectly conducting plane and separated by a distance R. Also, in writing [10], we have assumed that $G(z_0, z) = 1$.

Azimuthally symmetric case.

An immediate simplification of [10] occurs when η', f and $\partial f/\partial \rho$ are independent of the azimuth angle φ. Then, we can use the following known formulas from Bessel function theory:

$$[13] \qquad \frac{1}{2\pi} \int_0^{2\pi} e^{iZ\cos\varphi} \, d\varphi = J_0(Z)$$

and:

$$[14] \qquad \frac{-i}{2\pi} \int_0^{2\pi} e^{iZ\cos\varphi} \cos\varphi \, d\varphi = J_1(Z)$$

Then, [10] becomes:

$$[15]\quad \frac{Z'_{ab}-Z_{ab}}{Z_o} = -k\int_o^{\infty}\left\{\left[i\cos\theta\sin(kf\cos\theta) - \frac{\eta'-\eta}{\eta_o}\cos(kf\cos\theta)\right]J_1(k\rho\sin\theta)\right.$$

$$\left. + i\sin\theta\cos(kf\cos\theta)\frac{\partial f}{\partial\rho}J_o(k\rho\sin\theta)\right\}\left(1+\frac{1}{ik\rho}\right)e^{-ik\rho} \approx F(\rho)\,d\rho$$

where $F(\rho)$ is azimuthally symmetric attenuation function.

In the case of a uniformly flat surface $f = \partial f/\partial\rho = 0$ everywhere; thus [15] reduces to:

$$[16]\quad \frac{Z'_{ab}-Z_{ab}}{Z_o} = k\int_o^{\infty}\left(\frac{\eta'-\eta}{\eta_o}\right)\left(1+\frac{1}{ik\rho}\right)e^{-ik\rho}J_1(k\rho\sin\theta)F(\rho)\,d\rho$$

This expression or closely related forms have been used to predict the form of the radiation pattern and the enhancement of the surface wave field of antennas with large ground metallic ground screens [3], [4]. In such cases, η' is reduced to be much less than η and/or the impedance η' is made to be effectively inductive so that $F(\rho)$ may be enhanced over the ground system.

Another special case is to allow the surface to be everywhere perfectly conducting. Then, $\eta' = \eta = 0$ for all values of ρ. Consequently, [15] reduces to:

$$[17]\quad \frac{Z'_{ab}-Z_{ab}}{Z_o} = -ik\int_o^{\infty}\left\{\sin\theta\cos(kf\cos\theta)J_o(k\rho\sin\theta)\cdot\frac{\partial f}{\partial\rho}\right.$$

$$\left. + \cos\theta\sin(kf\cos\theta)J_1(k\rho\sin\theta)\right\}\left(1+\frac{1}{ik\rho}\right)e^{-ik\rho}F(\rho)\,d\rho$$

On setting $F(\rho) = 1$, which is a reasonable approximation, this result reduces to an equivalent form derived by Page and Monteath (5).

An application.

One rather revealing example is to let $\eta = 0$, corresponding to an assumed perfectly conducting sea that occupies the region $\rho > b$. Within this circular island, we have a circular plateau of radius a with a flat top. At the same time, we allow $\theta \to 90°$, corresponding to grazing incidence. For this case, we can write:

$$[18]\quad Z'_{ab} \simeq Z_o[1 + \Omega_g + \Omega_e]$$

where:

$$\Omega_e = k \int_a^b \frac{\eta'}{\eta_0}\left(1 + \frac{1}{ik\rho}\right) e^{-ik\rho} J_1(k\rho)\, F(\rho)\, d\rho \tag{19}$$

and:

$$\Omega_t = -ik \int_a^b \left(\frac{\partial f}{\partial \rho}\right)\left(1 + \frac{1}{ik\rho}\right) e^{-ik\rho} J_0(k\rho)\, F(\rho)\, d\rho \tag{20}$$

Thus Ω_e is the modification of the field resulting from the electrical property contrast of the « island », while Ω_t is the corresponding modification due to the *non-flatness*. As mentioned above, Ω_e has been discussed on numerous earlier occasions. The topographical effect represented by Ω_t is usually ignored, although, in many applications, the transmitting monopole is located on a bluff or some other elevated terrain feature. To illustrate its effect, we assume that the beach of the island has a uniform downward slope, so that $-\partial f/\partial \rho$ is a positive constant that we designate by K. Also, to simplify the discussion, we assume that the radius of the plateau is much greater than a wavelength. Thus, ka and $kb >> 1$, and, consequently, $J_0(k\rho)$, can be replaced by the first term of its asymptotic expansion over the integration range in [20]. Thus, we find that:

$$\Omega_t \simeq \frac{e^{i\pi/4}}{(2\pi)^{1/2}} kK \int_a^b \frac{1}{(k\rho)^{1/2}} (1 + i e^{-2ik\rho})\, F(\rho)\, d\rho \tag{21}$$

With suitable assumptions about the form of $F(\rho)$, this integral can be evaluated *in terms of error integrals of* complex argument (4). The essential nature of the problem, however, appears if we recognize that the *contribution from the rapidly varying* term $i \exp(-2ik\rho)$ can be neglected. Also, over the region $a < \rho < b$, the slowly varying function $F(\rho)$ can be replaced by $F(a)$. Thus, [21] reduces to the remarkably simple form:

$$\Omega_t \simeq (2/\pi)^{1/2} K e^{i\pi/4} [(kb)^{1/2} - (ka)^{1/2}] F(a) = 2K e^{i\pi/4} [(b/\lambda)^{1/2} - (a/\lambda)^{1/2}) F(a) \tag{22}$$

where λ *is the* free-space wavelength. For example, if $b/\lambda = 100$, $a/\lambda = 64$, $K = 0.1$, and assuming $F(a) \simeq 1$, we see that:

$$\Omega_t \simeq 0.4 \times e^{i\pi/4}$$

This corresponds approximately to a 4 *dB* gain. For this case, the heigth of the plateau above the sea is 3.6λ, and the sloping beach extends over a radial distance equal to approximately 36λ.

Actually, the improvement of locating the monopole on an elevated plateau applies also to the effective ground wave radiated for a given current moment (3). Thus, the main conclusion from this analysis is that the elevation of the ground plane should be considered in any design of surface wave launchers for high-frequency radio waves over the sea.

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SECTION 2

DISCUSSION OF U.S. NAVY ELECTRONICS LABORATORY REPORTS BASED ON REPRINT 1.11

by

J. R. Wait
5 March 1990

Following the publication of the Monograph by Wait and Walters(1963), a series of unclassified reports were issued by investigators at the U.S. Navy Electronics Laboratory (now known as the Naval Ocean System center) in San Diego. We will identify these as NEL 1346, 1359, 1567 and 1430 in the order we discuss them below.

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W.E. Gustafson, W.M. Chase, and N.H. Balli, Ground system effect on high frequency antenna propagation, NEL Report 1346, 29 pgs., 4 Jan 1966

G.D. Bernard, W.E. Gustafson, and W.M. Chase, HF extended ground systems : results of a numerical analysis, NEL Report, 52 pgs., 24 Feb 1966 , NEL report 1359

J.M. Horn, HF vertical-plane patterns of monopoles and elevated vertical dipoles with and without extended ground systems,142 pgs., 25 Jun 1968 , NEL Report 1567

J.M. Horn, A numerical analysis of H.F. extended-sector ground systems, 52 pgs., 14 Jan 1967, NEL Report 1430

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These four NEL reports were distributed widely including libraries and they were released to the Federal Clearinghouse now known as NTIS (National Technical Information Service) . Now such results are in the public domain. Thus we need not describe their content in great detail. But it does seem appropriate to identify some of the significant results and conclusions reached by the authors. In some cases , an alternative interpretation of the data is offered without meaning to detract from the noble efforts of the NEL investigators .

In NEL 1346 it is pointed out that the ground based monopole with extended ground screens may have a major advantage over elevated vertical antennas (with or without ground screens) if pattern lobing is to be minimized. With this motivation , the authors undertook physical model runs for circular ground sreens employing both square meshes of various sizes(½ to 4 inches) and radial wires with various angular separations (1½ , 2½ and 5 degrees). This experimental program became manageable by using frequencies in the VHF range (130, 250 and 360 MHz) which were appropriate to model frequencies lower by a factor of 12 when a relatively well conducting ground site was selected (e.g. 0.1 mhos/m). Some useful and meaningful pattern plots were illustrated by the authors to confirm the expectation of enhanced radiated fields at low take-off angles.

Dr. Gary Beɪnard, who was a consultant to NEL, presented a concise tabulation of the experimental data along with results generated by computer calculations using the formulation in Wait(1963) and Wait and Walters(1963). This comparison of experiment and theory is reproduced here from NEL Report 1346. The calculated quantity tabulated is equivalent to $20 \ln | 1 + \Omega_a |$ in dB . The source adopted is a vertical (ground based) electric dipole at the center of the circular screen. Actually the experimental data were obtained using a quarter wave monopole with a small radius (impedance stabilizing) ground plane . The general agreement between experiment and theory is reasonably good. It is also worth noting that, for low take-off angles, the angular dependence of the field is small. This is particularly the case for 130 MHz (i.e ka = 16) which is the lowest test frequency.

Further radiation pattern calculations are reported in NEL 1359 which again are based on Wait and Walters(1963). The frequencies chosen were 4, 8, 16 and 32 MHz. The full circular ground screen radius varied in octave steps from 2 to 128 wavelengths and the grid spacings were 6, 12, 24 and 48 inches, The ground conductivities chosen were 3,10, 30 and 100 milli-mhos/m with corresponding dielectric constants of 4, 10, 20 and 30 , respectively. In each case, computations were carried out for vertical (take-off) angles of 2, 5, 10, 15, 20 and 25 degrees. For all computations, the ground system was circular (i.e. full azimuthal symmetry). The copper wire size was No.10 (American Wire Guage) . The source in all cases was assumed to be a short vertical current element or (infinitesimal) electric dipole.

Some examples of the extensive calculations, from NEL 1359 are shown in Table 2 where the quantity $\hat{G}$ in dB is again the quantity $20 \ln | 1 + \Omega_a |$. Here $\hat{G}$ represents the improvement for the antenna with extended ground screen compared with the same element over the imperfect ground when the dipole moment is fixed .In this example, the ground conductivity is 10 milli-mhos/m and the dielectric constant is 10 . In the case, where the radius is a = 16 wavelengths or ka = 100, the results for 32 MHz, for the smallest mesh-spacing, are compatible with Fig.2a, in Wait and Walters(1963) carried out for a dielectric constant of 9, zero ground conductivity, and an assumed perfectly conducting ground screen.

In NEL 1567, many additional calculations are reported again for the fully circular ground screen.. For the most part, the results are based on Wait(1967)'s paper for vertical electric dipole of finite length erected over the center of the ground screen. Many graphical plots are presented in a format which allows the reader to see the relative enhancement of the low angle radiation when an extended ground screen is employed . As J.M. Horn, the diligent author,points out the advantages of such a ground screen are reduced somewhat for elevated antennas. Also, as indicated before, the economic benefit,of large ground screens over well conducting ground, is small.

In 1430, Horn deals with the sector ground screen geometry, using essentially the formulation of Wait(1963) and following the methods outlined in Wait and Walters(1963). He adopts the ground based quarter-wave monopole which would not be very different than for the infinitesimal dipole model unless the ground screen was less than several wavelengths in size. Horn includes a listing of two computer programs. One is based on the mesh ground screen where the effective surface impedance Z' is essentially a constant and the other is for various radial wire configurations where Z' varies with radius.

In the four above mentioned NEL reports, it is stated that the ground wave is neglected.Indeed,it is true that the results are presented in the context of the modification of the radiation fields - due to the presence of the ground screen. But the quantity $|1 + \Omega_a|$ tabulated , for low angles (e.g. $\psi_0 = 2^o$), is applicable to the modification of the ground wave field as compared to the case of no (or small) ground screen for a fixed current at the terminals of the transmitting dipole. It is a pity that the NEL investigators did not carry out their calculations for the grazing condition where $\psi_0 \rightarrow 0^o$ as done in Wait and Walters (1963).

While there is no reason to doubt the accuracy of the calculations in the four NEL reports, one might question the validity of the results in cases of very low ground conductivity where sparse ground screens are employed. In such cases some of the initial assumptions appear to be violated.

TABLE 1. GAIN, IN DB, OF EXTENDED GROUND SYSTEM (20-FOOT-RADIUS) OVER IMPEDANCE GROUND (2-FOOT-RADIUS)

(reproduced from NEL Report 1346)

(20 FEET = 2.5 WAVELENGTHS AT 130.5 MC/S.)

Elev. Angle (degrees)	Square Mesh Size (inches)				Radial Wire Separation (degrees)		
	1/2	1	2	4	1.25	2.5	5
130.5 Mc/s							
5	4.9	4.9	5.1	5.1	4.7	4.3	3.8
10	5.2	5.2	5.5	5.4	4.9	4.4	3.9
15	4.9	4.9	5.1	4.9	4.9	4.3	3.6
20	5.0	5.0	5.2	4.8	4.8	4.1	3.4
25	4.8	4.8	4.9	4.2	4.5	3.9	2.8
250 Mc/s							
5	4.6	4.9	4.9	3.6	4.9	3.7	2.3
10	5.0	5.0	5.0	3.2	4.9	3.4	2.0
15	4.8	4.9	4.6	2.2	4.2	2.6	1.6
20	4.6	4.5	3.9	0.5	3.8	1.9	1.1
25	2.8	2.3	1.4	-2.8	1.4	-0.4	-0.3
360 Mc/s							
5	6.7	6.8	6.7	3.5	6.4	4.5	2.5
10	6.9	6.9	6.4	3.0	6.1	3.9	2.0
15	6.9	6.9	5.7	1.3	5.5	3.2	1.6
20	5.6	5.1	3.3	-1.2	3.4	1.7	1.0
25	2.4	1.6	-1.1	-1.5	1.3	1.1	0.6

TABLE 2. GAIN, IN DB, OF EXTENDED GROUND SYSTEM (20-FOOT-RADIUS) OVER NO GROUND SYSTEM FOR QUARTER-WAVELENGTH ANTENNA, EXPERIMENTAL RESULTS, AND SHORT DIPOLE, COMPUTED RESULTS.
(20 FEET = 2.5 WAVELENGTHS AT 130.5 MC/S.)

(reproduced from NEL Report 1346)

	Quarter Wavelength Measured and Modified*			Short Dipole -- Computed				
Elev. Angle (degrees)	Square Mesh Size (inches)							
	1/2	1	2	1/4	1/2	1	2	4
	130.5 Mc/s							
5	4.9	4.9	5.1	4.2	4.2	4.2	4.2	3.9
10	5.1	5.1	5.1	4.4	4.4	4.4	4.4	4.1
15	5.5	5.4	5.4	4.9	4.9	4.9	4.8	4.4
20	5.0	5.1	5.1	5.3	5.3	5.3	5.2	4.8
25	5.0	5.0	5.0	5.7	5.7	5.6	5.5	5.0
	250 Mc/s							
5	6.4	6.8	6.6	5.7	5.7	5.7	5.5	4.6
10	6.2	6.4	6.3	6.2	6.1	6.1	5.8	4.9
15	6.7	6.9	6.4	6.6	6.6	6.5	6.1	5.1
20	6.2	6.3	5.7	6.7	6.7	6.6	6.1	5.0
25	5.2	4.9	4.1	6.2	6.2	6.0	5.5	4.3
	360 Mc/s							
5	7.3	7.7	7.1	6.7	6.7	6.6	6.1	4.8
10	7.1	6.5	6.7	7.2	7.2	7.0	6.4	5.0
15	7.3	7.6	6.3	7.5	7.5	7.7	6.6	5.0
20	6.2	6.1	4.3	7.1	7.1	6.8	6.0	2.9
25	3.6	3.2	1.1	5.3	5.3	5.0	4.2	2.8

* Extrapolated from results with 2-foot-radius impedance ground.

Table 2

Gain $\hat{G}$ in dB for large screen relative to small or no screen for ground conductivity of 10 milli-mhos/m and relative dielectric constant of 10 (reproduced from NEL Report 1359 by Gary Bernard et al.)

A. 4 Mc/s

$a = 2\lambda = 491.8$ ft

Elev. Angle (deg.)	Mesh Spacing (inches) 6	12	24	48
2	1.3	1.3	1.4	1.3
5	1.3	1.4	1.4	1.4
10	1.5	1.5	1.5	1.5
15	1.7	1.7	1.7	1.6
20	2.0	2.0	2.0	1.8
25	2.3	2.3	2.2	2.0

$a = 4\lambda = 983.5$ ft

Elev. Angle (deg.)	Mesh Spacing (inches) 6	12	24	48
2	1.9	1.9	2.0	1.9
5	2.0	2.0	2.0	1.9
10	2.3	2.3	2.3	2.2
15	2.7	2.7	2.7	2.4
20	3.1	3.1	3.0	2.7
25	3.3	3.3	3.1	2.7

$a = 8\lambda = 1967.0$ ft

2	2.9	2.9	2.9	2.7
5	3.1	3.1	3.0	2.8
10	3.7	3.6	3.5	3.2
15	4.1	4.1	3.9	3.5
20	3.9	3.8	3.6	3.1
25	2.7	2.6	2.4	2.0

$a = 16\lambda = 3934.1$ ft

2	4.3	4.3	4.2	3.8
5	4.7	4.6	4.5	4.1
10	5.4	5.3	5.1	4.5
15	4.5	4.4	4.2	3.6
20	2.5	2.4	2.3	2.0
25	3.1	3.1	2.9	2.5

$a = 32\lambda = 7868$ ft

2	6.2	6.1	5.9	5.3
5	6.8	6.7	6.4	5.8
10	6.2	6.0	5.7	5.0
15	3.3	3.2	3.1	2.7
20	2.6	2.6	2.5	2.1
25	2.4	2.4	2.2	1.9

$a = 64\lambda = 15{,}736$ ft

2	8.6	8.4	8.1	7.3
5	8.9	8.8	8.4	7.5
10	4.0	4.0	3.8	3.3
15	3.7	3.6	3.4	3.0
20	2.8	2.8	2.6	2.3
25	2.5	2.5	2.3	2.0

$a = 128\lambda = 31{,}472$ ft

Elev. Angle (deg.)	Mesh Spacing (inches) 6	12	24	48
2	11.2	11.0	10.5	9.5
5	9.6	9.5	9.0	8.0
10	4.3	4.2	4.0	3.5
15	4.0	3.9	3.7	3.3
20	3.1	3.0	2.8	2.4
25	2.6	2.5	2.4	2.1

Table 2 cont.

B 8 Mc. s

$a = 2\lambda = 245.9$ ft

Elev. Angle (deg.)	Mesh Spacing (inches) 6	12	24	48
2	2.2	2.2	2.2	2.0
5	2.2	2.2	2.2	2.0
10	2.4	2.4	2.4	2.1
15	2.6	2.6	2.6	2.2
20	3.0	2.9	2.8	2.4
25	3.3	3.3	3.1	2.6

$a = 4\lambda = 491.8$ ft

Elev. Angle (deg.)	Mesh Spacing (inches) 6	12	24	48
2	3.1	3.1	3.1	2.7
5	3.0	3.2	3.2	2.8
10	3.5	3.5	3.4	3.0
15	4.0	3.9	3.7	3.2
20	4.3	4.2	4.0	3.3
25	4.4	4.3	4.0	3.2

$a = 8\lambda = 983.5$ ft

2	4.5	4.4	4.3	3.8
5	4.7	4.6	4.4	3.9
10	5.2	5.1	4.9	4.2
15	5.5	5.4	5.1	4.2
20	5.0	4.9	4.5	3.6
25	3.5	3.3	3.0	2.3

$a = 16\lambda = 1967.0$ ft

2	6.3	6.2	5.9	5.1
5	6.6	6.5	6.2	5.3
10	7.0	6.9	6.5	5.5
15	5.8	5.6	5.2	4.2
20	3.3	3.2	3.0	2.4
25	4.1	4.0	3.6	2.9

$a = 32\lambda = 3934.1$ ft

2	8.4	8.3	7.9	6.8
5	8.8	8.7	8.2	7.1
10	7.7	7.5	7.0	5.8
15	4.4	4.3	4.0	3.4
20	3.5	3.4	3.2	2.6
25	3.3	3.2	2.9	2.3

$a = 64\lambda = 7868$ ft

2	11.0	10.7	10.2	8.8
5	11.0	10.8	10.2	8.8
10	5.3	5.2	4.9	4.0
15	4.9	4.8	4.4	3.7
20	3.8	3.6	3.3	2.7
25	3.4	3.3	3.0	2.4

$a = 128\lambda = 15{,}736$ ft

Elev. Angle (deg.)	Mesh Spacing (inches) 6	12	24	48
2	13.6	13.4	12.7	11.1
5	11.6	11.3	10.6	9.1
10	5.6	5.5	5.1	4.2
15	5.2	5.1	4.7	3.9
20	4.0	3.9	3.6	2.9
25	3.4	3.3	3.1	2.5

Table 2 cont.

C. 16 Mc/s

$a = 2\lambda = 122.9$ ft

Elev. Angle (deg.)	Mesh Spacing (inches) 6	12	24	48
2	3.2	3.2	3.1	2.4
5	3.3	3.3	3.1	2.4
10	3.4	3.4	3.2	2.5
15	3.7	3.7	3.4	2.6
20	4.0	3.9	3.6	2.7
25	4.3	4.2	3.8	1.5

$a = 4\lambda = 245.8$ ft

Elev. Angle (deg.)	Mesh Spacing (inches) 6	12	24	48
2	4.4	4.4	4.2	3.3
5	4.5	4.5	4.2	3.3
10	4.8	4.8	4.4	3.4
15	5.2	5.1	4.6	3.6
20	5.4	5.3	4.7	3.5
25	5.4	5.1	4.5	3.2

$a = 8\lambda = 491.6$ ft

2	6.1	6.0	5.6	4.4
5	6.2	6.1	5.7	4.5
10	6.6	6.5	5.9	4.6
15	6.8	6.6	5.9	4.4
20	6.0	5.7	5.0	3.5
25	4.1	3.9	3.2	2.1

$a = 16\lambda = 983.2$ ft

2	8.1	7.9	7.3	5.9
5	8.3	8.1	7.5	6.0
10	8.5	8.3	7.5	5.8
15	6.9	6.5	5.7	4.0
20	4.1	4.0	3.5	2.4
25	5.0	4.7	4.0	2.7

$a = 32\lambda = 1966.4$ ft

2	10.4	10.1	9.4	7.5
5	10.6	10.4	9.5	7.7
10	9.0	8.7	7.7	5.7
15	5.5	5.3	4.7	3.5
20	4.4	4.2	3.6	2.5
25	4.0	3.8	3.3	2.3

$a = 64\lambda = 3932.8$ ft

2	12.9	12.6	11.7	9.6
5	12.8	12.4	11.4	9.2
10	6.5	6.3	5.6	4.2
15	6.0	5.8	5.1	3.8
20	4.6	4.4	3.8	2.6
25	4.1	4.0	3.4	2.3

$a = 128\lambda = 7865.6$ ft

Elev. Angle (deg.)	Mesh Spacing (inches) 6	12	24	48
2	15.6	15.2	14.1	11.8
5	13.2	12.8	11.6	9.2
10	6.8	6.6	5.8	4.3
15	6.3	6.1	5.4	3.9
20	4.9	4.7	4.0	2.7
25	4.2	4.0	3.5	2.4

Table 2 cont.

D. 32 Mc/s

a = 2 λ = 61.5 ft

Elev. Angle (deg.)	Mesh Spacing (inches) 6	12	24	48
2	4.0	3.9	3.3	2.1
5	4.0	4.0	3.3	2.1
10	4.2	4.1	3.4	2.1
15	4.4	4.3	3.5	2.1
20	4.7	4.4	3.6	2.1
25	4.9	4.6	3.6	2.1

a = 4 λ = 122.9 ft

Elev. Angle (deg.)	Mesh Spacing (inches) 6	12	24	48
2	5.4	5.2	4.4	2.9
5	5.5	5.3	4.4	2 9
10	5.7	5.5	4.6	2.9
15	6.0	5.7	4.6	2.8
20	6.1	5.7	4.5	2.6
25	5.9	5.4	4.0	2.1

a = 8 λ = 245.9 ft

2	7.1	6.9	5.8	3.2
5	7.2	7.0	5.9	3.9
10	7.5	7.2	6.0	3.9
15	7.5	7.0	5.6	3.8
20	6.5	5.9	4.3	3.4
25	4.4	3.8	2.5	2.2

a = 16 λ = 491.8 ft

2	9.1	8.8	7.5	5.1
5	9.3	8.9	7.6	5.1
10	9.3	8.8	7.2	4.6
15	7.3	6.7	5.0	2.5
20	4.6	4.2	3.1	1.6
25	5.3	4.8	3.4	1.6

a = 32 λ = 983.5 ft

2	11.5	11.0	9.5	6.7
5	11.6	11.1	9.5	6.5
10	9.6	8.9	7.1	4.1
15	6.1	5.7	4.4	2 5
20	4 8	4.3	3.2	1 6
25	4.4	4.0	2.9	1.4

a = 64 λ = 1967.0 ft

2	14.0	13.4	11.7	8 4
5	13.7	13.0	11.1	7.7
10	7.1	6.7	5.2	3.0
15	6.6	6.1	4.8	2 7
20	5.0	4.5	3.2	1 6
25	4.5	4.1	3.0	1.5

a = 128 λ = 3934.1 ft

Elev. Angle (deg.)	Mesh Spacing (inches) 6	12	24	48
2	16.6	16.0	14.0	10 4
5	13.9	13.2	11.0	7.2
10	7.4	6 9	5.4	3.1
15	6 9	6 3	4 9	2.7
20	5 3	4 8	3.4	1.7
25	4 6	4.2	3 0	1.5

SECTION 3

ANNOTATED LISTING OF SELECTED RELATED PUBLICATIONS (1967-1989)

by

J. R. Wait

5 March 1990

3.1 Krause, L. O., November 1967, "Enhancing H. F. received fields with large planar and cylindrical ground screens," *IEEE Trans. on Antennas and Propagation*, No. 6, pp. 785-795

The author presents a series of interesting pattern calculations (using image theory) for tilted ground screens located over a conducting half-space. For such geometries, the low angle radiation may be enhanced. Cylindrical shaped screens are also considered. The author cautions on the use of the method for angles at grazing angles below 3°.

3.2 Collin, R. E. and F. J. Zucker, 1969, *Antenna Theory*, Pt. II, McGraw Hill, N.Y.

In Chap. 23 authored by J. R. Wait, eq.. 123 has a misprint; e^{jx} should be e^{-jx}. Eqs.. 23.122 is OK. Also the Fressel function J_1 here was approximated by the first term of its asymptotic expansion which can lead to 1 dB discrepancies in computed patterns but general features are correct. This point was confirmed by Lt. Joe Fortney, of RADC, Hanscom AFB, Bedford, Mass.., who recomputed the integral given by 23.123 (private communication Jan 1990).

3.3 Waldman, A., January 1970, "Elevation steering of the pattern of vertically polarized elements over a ground screen," *IEEE Trans. on Antennas and Propagation*, Vol. AP-18, pp. 105-107

Author presents calculation based on reprint 1.13 (Wait, 1967) for an elevated vertical antenna over the sector ground screen system. An approximation pattern multiplication scheme is adopted. The objective is to develop a method to steer the principal maximum in the low angle elevation pattern.

3.4 Balanis, C. A., July 1970, "Pattern distortion due to edge diffractions," *IEEE Trans. on Antennas and Propagation*, Vol. AP-18, pp. 561-563

This is a systematic treatment for an arbitrary aperture source in a finite size ground plane of perfect conductivity. The ambient medium is free space. Excellent correlation with experiment is shown.

3.5 Yu, J. S., K. J. Scott, and A. R. Spatuzzi, November 1970, "A modified elevation angle of radiation from a monopole on ground screens," *IEEE Trans. on Antennas and Propagation*, Vol. AP-18, pp. 795-799

The authors employ an approximate diffraction technique to estimate the pattern of a vertical monopole on a square wire mesh screen of finite extent. The experimental data appears to justify the procedure for the pattern calculation for the pattern calculation for the particular examples given.

3.6 Rafuse R. P., and J. Ruze, December 1975, "Low angle radiation from vertically polarized antennas over radially heterogeneous flat ground," *Radio Science*, Vol. 10, pp. 1011-1018

The authors construct an admittedly approximate analytical solution based on an assumed current distribution on both the ground screen and the external ground surface. The results seem to be compatible with the more rigorous formulations based on the compensation theorem. The latter can be used to form an integral equation to deduce the surface current distribution over the ground screen rather than assuming it [e.g. see reprint 1.18 (Wait and Spies, 1970) and Hill and Wait, 1973)]. Rafuse and Ruze introduce an "empirical normalization" of the near field surface impedance to bring their results into conformity with physical expectations. The comment in the paper that the surface current at the edge of the ground screen is discontinuous is a bit surprising. Of course, the radial electric field, indeed, is discontinuous. In an important observation, the authors confirm that the classical Fresnel zone description of the "active" area in front of the antenna does not hold. Indeed, it is much smaller as has been shown by J. Bach Anderson (1963).

3.7 Teng, C. J., and R. J. P. King, January-March 1981, "Surface Fields and Radiation Patterns of a Vertical Electric Dipole Over a Radial-Wire Ground System," *Electromagnetics*, Vol. 2, pp. 129-146

The authors use the compensation theorem to deal with a vertical electric dipole located over a radial wire ground system at the interface of the flat earth and air. The radiation patterns were found to be independent of the length of the radials beyond the point where the effective impedance of the composite impedance was within 90 percent of the underlying ground.

3.8 Park, K. S., R. J. P. King, and C. J. Teng, April-June 1982, "Radiation Patterns of an HF Vertical Dipole Near a Sloping Beach," *Electromagentics*, Vol. 2, pp. 129-146

The effective surface impedance of the beach is calculated from a two-layer stratified earth model so the formulation proceeds as if the equivalent ground plane has a constant elevation. It is found that the low angle radiation is reduced somewhat from that of no beach. The usual lobing occurs in the elevation pattern when the dipole is raised above 0.3 wavelengths.

3.9 Burke, G. J., R. J. P. King, and E. K. Miller,, September 1984, *Surface Wave Excitation Study*, Lawrence Livermore National Laboratory, Report 20214

In this report, results are shown for the terminal impedance of a vertical antenna with a radial wire ground system for up to sixteen radials. The "Numerical Electromagnetics Code", known as NEC, is employed. An interesting comparison is made with corresponding results based on the approximated compensation theorem approach. The agreement is good when the restrictions in the latter method are adhered to. But, for a dielectric-like earth, agreement is poor because of reflections from the end of the radials. As Burke et al point out, resonance effects are most noticeable when the earth system is elevated and not grounded at the end points by stakes.

3.10 Weiner, M. M., S. P. Cruze, C. C. Li, and W. J. Wilson, 1987, *Monopole Elements on Circular Ground Planes*, Artech House, Norwood, MA.

The analyses deal with the case where the monopole element and circular disk are located in free space. The models which are used and compared are the induced electromotive force (EMF) method, integral equation, method of moments, oblate spheroidal wave functions, scalar diffraction theory, geometric theory of diffraction, method of moments combined with the geometric theory of diffraction, and the method of images.

3.11 Burke, G. J., January 1988, *A Model for Insulated Wires in the Method of Moments Code (NEC)*, Lawrence Livermore National Laboratory, Report 21301

Among other things, Burke shows how the propagation constant of a buried insulated wire depends on the depth of burial. Excellent agreement is obtained with Wait's paper (Canadian Journal of Physics, Vol. 50, pp. 2402-2409, 1972) who employed an analytical method. The results are relevant to the radial wire ground system.

3.12 Nagy, L., 1989, *Input Impedance and Radiation Pattern of a Top-Loaded Monopole Having a Radial Wire Ground System,* (Summary only), Proceedings of the International Radio Scientific Union (URSI) Symposium on Electromagnetic Theory, The Royal Institute of Technology, Stockholm

The author deals mainly with the input impedance calculation using the compensation theorem along with first order perturbation. The radiation patterns, as presented, ignore the presence of the ground system which is partly justified because of the short length of the radials.

3.13 James, G. L. and G. T. Poulton, August 14-17, 1989, *Effects of Ground Screen on HF Antennas* (Summary only), Proceedings of the URSI Symposium on Electromagnetic Theory, The Royal Institute of Technology, Stockholm

Authors employ a GTD (geometrical theory of diffraction) technique to reduce radiation patterns for a vertical antenna over a circular ground screen for both radial wire and square mesh (bonded and unbonded) geometries. Approximate boundary conditions due to Kontorovich et al are exploited.

APPENDIX A

BIOGRAPHY OF JAMES R. WAIT

Dr. James R. Wait was born in Ottawa, Canada, on January 23, 1924. From 1942 to 1945 he was a radar technician in the Canadian Army. He received his B.A.Sc. and M.A.Sc. degrees in engineering physics in 1948 and 1949, respectively, and his Ph.D degree in electromagnetic theory in 1951, all from the University of Toronto, Toronto, Canada.

From 1948 to 1951, he was associated with Newmont Exploration Ltd. of New York, N.Y., and Jerome, Arizona, where his research led to several patents for electromagnetic and induced-polarization methods of geophysical prospecting. From 1952 to 1955, he was a section leader at the Defense Research Telecommunications Establishment, Ottawa, where he was primarily concerned with electromagnetic problems. Since joining the National Bureau of Standards (NBS), Boulder, Colorado, in 1955, he was appointed adjunct professor of electrical engineering at the University of Colorado, Boulder. During 1960, he was a visiting research fellow at the Laboratory of Electromagnetic Theory in the Technical University of Denmark, Copenhagen. For the academic year 1966-1967, he was a visiting professor at Harvard University, Cambridge, Massachusetts. From 1963 to 1967, he was a senior research fellow at Boulder, where he was also a consultant to the director of the Institute for Telecommunication Sciences and Aeronomy. From 1967 to 1970, he was senior scientist in the Office of the Director of the Environmental Sciences Services Administration (ESSA) Research Laboratories, Boulder. In September of 1970, he became the senior scientist in the Office of the Director of the Institute for Telecommunications Sciences (ITS) of the Office of Telecommunications. In October, 1971, he returned to his position in the Office of the Director of the Environmental Research Laboratories of the National Oceanic and Atmospheric Administration (NOAA), formerly ESSA, but remained as a consultant to the director of ITS. In addition, he was a fellow of the Cooperative Institute for Research in Environmental Sciences. In May, 1971, Dr. Wait was a visiting professor at the Catholic University of Rio de Janeiro, Brazil, and in September, 1971, he was a guest of the U.S.S.R Academy of Sciences in Moscow and Tbilisi.

Dr. Wait was awarded the Department of Commerce Gold Medal in 1959 for "highly distinguished authorship in the field of radio propagation"; the Boulder Scientist Award sponsored by the Scientific Research Society of America in 1960; the NBS Samuel Wesley Stratton Award in 1962; and the Arthur S. Flemming Award, Washington, D.C. Chamber of Commerce, and the Harry Diamond Award from the IEEE, both in 1964. He is also a fellow of the Institute of Electrical and Electronic Engineers (IEEE) and the American Association for the Advancement of Science (AAAS). In September 1972, Dr. Wait received an Outstanding Publication Award from the Office of Telecommunications in Washington, D.C. Dr. Wait received NOAA's 1973 Scientific Research and Achievement Award "in recognition of his outstanding achievement as a scientist and research leader in theoretical studies of electromagnetic wave propagation in the earth and its atmosphere". In 1975 he received a Special Achievement Award from the Office of Telecommunications "in recognition of outstanding authorship of a great variety of publications and journal articles".

He was mainly responsible for the establishment of the journal *Radio Science*, which started in 1959 as Section D of the *NBS Journal of Research*; he served three terms as editor. He has also served three terms as an associate editor of the *Journal of Geophysical Research*. For four years, he was U.S. regional editor of *Electronics Letters*. Currently, he is a member of the Editorial Board of *GeoExploration* (Sweden) and he is co-editor of the Institution of Electrical Engineers (IEE) series, *Monographs on Electromagnetic (EM) Waves* (England).

Dr. Wait has published eight books and numerous papers on subjects ranging from electromagnetics to geophysics. He was secretary (1974 to 1978) of the U.S. National Committee of the International Scientific Radio Union(URSI). He has been a U.S. delegate to URSI General Assemblies in Boulder (1957), London (1960), Tokyo (1963), Ottawa (1969), Warsaw (1972), Lima (1975) and Helsinki (1978). In 1977, Dr. Wait was elected to the U.S. National Academy of Engineering. Also, in June 1977, he was elected to be a fellow of the IEE. On July 31, 1978, Dr. Wait received the Balth van der Pol Gold Medal at the General Assembly of URSI held in Helsinki, Finland.

In August, 1980, Dr. Wait became professor of electrical engineering and geosciences at the University of Arizona in Tucson. Since then he received the IEEE Centennial Medal in 1984 and the IEEE Geoscience and Remote Sensing Society's Distinguished Achievement Award in 1985. He was appointed regents professor in the University of Arizona in 1988. Jim Wait retired from the University in September 1989 to become a private consultant in his home office, 2210 East Waverly, Tucson AZ 85719. He keeps in close touch with the campus (being a 10 minute bike ride away).

APPENDIX B

COMPLETE LISTING (TO APRIL 1990) OF REFEREED PAPERS OF JAMES R. WAIT

by

J. R. Wait
August 1990

Note: Reprints included in section 1 are denoted by an asterisk in front of the reprint number.

Reprint
Number

1 J.R. Leslie and J.R. Wait, *Detection of overheated transmission line joints by means of a bolometer*, Trans. AIEE, Vol. 68, 1-5, May 1949; also Elect. Eng., Vol. 68, November 1949.

2 *Electromagnetic radiation in the earth (A theoretical investigation)*, Dept. of Electrical Engineering, University of Toronto, April 1950.

3 *Transient electromagnetic propagation in a conducting medium*, Geophys., Vol. XVI, 213-221, April 1951.

4 *The magnetic dipole over the horizontally stratified earth*, Can. J. Phys., Vol. 29, 577-592, November 1951.

5 *A conducting sphere in a time varying magnetic field*, Geophys., Vol. XVI, 666-672, October 1951.

6 *The cylindrical ore body in the presence of a cable carrying an oscillating current*, Geophys., Vol. XVII, 378-386, April 1952.

7 *Current-carrying wire loops in a simple inhomogeneous region*, J. Appl. Phys., Vol. 23, 497-498, April 1952.

8 *The basis of a method for measuring the complex dielectric constant at millimetre wavelengths*, Radio Physics Laboratory, Project Report, 15 Sept. 1952.

9 *Mutual inductance of circuits on a two-layer earth*, Can. J. Phys., Vol. 30, 450-452, September 1952.

10 *Electromagnetic fields of current-carrying wires in a conducting medium*, Can. J. Phys., Vol. 30, 512-523, September 1952.

11 *The magnetic dipole antenna immersed in a conducting medium*, Proc. IRE, Vol. 40, 1244-1245, October 1952.

12 *A note of dipole radiation in a conducting medium*, Geophys., Vol. XVII, 978-979, October 1952.

13 *Reflection of electromagnetic waves obliquely from an inhomogeneous medium*, J. Appl. Phys., Vol. 23, 1403-1404, December 1952.

14 *The electric fields of a long current-carrying wire on a stratified earth*, J. Geophys. Res., Vol. 47, 481-485, December 1952.

15 *Receiving properties of a wire loop with a spheroidal core*, Can. J. Tech., Vol. 31, 9-14, January 1953.

16 with J.E.T. Mousseau, *Calculated field patterns for horizontal travelling wave antennas*, Radio Physics Laboratory, Project Report No. 19-0-2, 15 January 1953.

17 *Transient coupling in grounded circuits*, Geophys., Vol. 18, 138-141, January 1953.

18 *A transient magnetic dipole source in a dissipative medium*, J. Appl. Phys., Vol. 24, 341-343, March 1953.

19 with L.L. Campbell, *Effect of a large dielectric constant on ground-wave propagation*, Can. J. Phys., Vol. 31, 456-457, March 1953.

Reprint
Number

20 *On the feasibility of measuring ground conductivity from an aircraft,* Defence Research Telecommunications Establishment, Ottawa, Canada (March 9, 1953).

21 with L.L. Campbell, *The fields of an electric dipole in a semi-infinite conducting medium,* J. Geophys. Res., Vol. 58, 21-28, March 1953.

22 *Induction by a horizontally oscillating magnetic dipole over a conducting homogeneous earth,* Trans. Amer. Geophys. Union, Vol. 34, 185-188, April 1953.

23 *The potential of two current point sources in a homogeneous conducting prolate spheroid,* J. Appl. Phys., Vol. 24, 496-497, April 1953.

24 *Propagation of radio waves over a stratified ground,* Geophys., Vol. 18, 416-422, April 1953.

25 *A conducting permeable sphere in the presence of a coil carrying an oscillating current,* Can. J. Phys., Vol. 31, 670-678, May 1953.

26 with L.L. Campbell, *Transmission curves for ground wave propagation at low radio frequencies,* Report R-1, Def. Res. Tele. Est., Radio Physics Lab., April 1953.

27 *The receiving loop with a hollow prolate spheroidal core,* Can. J. Tech., Vol. 31, 132-139, June 1953.

28 *An approximate method of obtaining the transient response from the frequency response,* Can. J. Tech., Vol. 31, 127-131, June 1953.

29 with L.L. Campbell, *The fields of an oscillating magnetic dipole immersed in a semi-infinite conducting medium,* J. Geophys. Res., Vol. 58, 167-178, June 1953.

30 *Radiation from a vertical electric dipole over a stratified ground,* Trans. IRE, Vol. AP-1, 9-12, Part I, July 1953.

31 *Radiation resistance of a small circular loop in the presence of a conducting ground,* J. Appl. Phys., Vol. 24, No. 5, 646-649, May 1953.

32 *The radiation fields of a horizontal dipole in a semi-infinite dissipative medium,* J. Appl. Phys., Vol. 24, 958-959, July 1953.

33 *Electromagnetic coupling between a circular loop and a conducting sphere,* Geophys., Vol. 18, 970-971, October 1953.

34 *Radiation from a line source adjacent to a conducting half plane,* J. Appl. Phys., Vol. 24, No. 12, 1528-1529, December 1953.

35 *Induction in a conducting sheet by a small current-carrying loop,* Appl. Sci. Res., Sec. B, Vol. 3, 230-235, May 1953.

36 *Complex magnetic permeability of spherical particles,* Proc. IRE, Vol. 41, No. 11, 1665-1667, November 1953.

37 *The fields of a line source of current over a stratified conductor,* Appl. Sci. Res., Sec. B, Vol. 3, 279-292, 1953.

38 *Radiation from a ground antenna,* Can. J. Tech., Vol. 32, 1-9, January 1954.

39 with W.C.G. Fraser, *Radiation from a vertical dipole over a stratified ground, Part II,* Trans. IRE, Vol. AP-3, No. 4, October 1954.

Reprint
Number

40 *Note on the theory of radio propagation over an ice-covered sea,* Def. Res. Tele. Est., Radio Physics Lab., Project Report 18-0-7, March 31, 1954.

* 41 with W.J. Surtees, *Impedance of a top-loaded antenna of arbitrary length over a circular grounded screen,* J. Appl. Phys., Vol. 25, No. 5, 553-555, May 1954.

42 *On anomalous propagation of radio waves in earth strata,* Geophys., Vol. 19, 342-343, April 1954.

* 43 with W.A. Pope, *The characteristics of a vertical antenna with a radial conductor ground system,* Appl. Sci. Res., Sec. B, Vol. 4, 177-195, 1954.

44 with K.F. Hill and W.A. Pope, *Reflection from a mirror surface with an absorbent coating,* J. Opt. Soc. Amer., Vol. 44, No. 6, 438-441, June 1954.

45 *On the relation between telluric currents and the earth's magnetic field,* Geophys., Vol. 19, 281-289, April 1954.

46 *Mutual coupling of loops lying on the ground,* Geophys., Vol. 19, No. 2, 290-296, April 1954.

47 *On the theory of an antenna with an infinite corner reflector,* Can. J. Phys., Vol. 32, 365-371, May 1954.

48 *Reflection from a wire grid parallel to a conducting plane,* Can. J. Phys., Vol. 32, 571-579, September 1954.

49 with S. Kahana, *Radiation from a slot on a cylindrically tipped wedge,* Can. J. Phys., Vol. 32, 714-721, November 1954.

50 C. Froese and J.R. Wait, *Calculated diffraction patterns of dielectric rods at centimetric wavelengths,* Can. J. Phys., Vol. 32, 775-781, December 1954.

51 with L.L. Campbell, *Fields of dipoles in a semi-infinite conducting medium,* J. Geophys. Res., Vol. 58, 21-28, March 1953; 167-168, June 1953; Def. Res. Tele. Est., Project Report 19-0-9, September 1954.

52 with W.A. Pope, *Evaluation of errors in an eight-element adcock antenna,* Trans. IRE, Vol. AP-3, No. 4, 159-162, October 1954.

53 *Theory of electromagnetic surface waves over geological conductors,* Geofisica pura e Appl., Vol. 28, 47-56, 1954.

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