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"Awaiting-Parts Time" in a Multi-Indenture, Readiness-Based Sparing Model

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ABSTRACT

This research memorandum presents a means of calculating the expected length of "awaiting-parts time" of weapon replaceable assemblies, which are needed to calculate aircraft readiness in a multi-indenture, readiness-based sparing model. The method presented is appropriate for any other level of indenture. Although full-scale application of the model is not feasible at this time, the model aids in comparing and evaluating existing models.

CONTENTS

INTRODUCTION AND SUMMARY

In traditional demand-based sparing (DBS), past replacement rates are used as future, projected demand rates. After a safety factor is added, these demands form the basis for a collection of aircraft spare parts that are placed aboard a carrier and used to repair aircraft and failed components. These parts are called an Aviation Consolidated Allowance List (AVCAL).

Department of Defense (DOD) policy states that spares provisioning must be based on weapon system readiness objectives [1 and 2]. In [3], DOD amplified its position that the methodology used to determine requirements must relate requirements of secondary items and repair parts to the readiness goals of end items and weapon systems.

The Navy outlined its response to these concepts in [4]. In particular, based on the successful results of the F-14A readiness-based AVCAL test conducted aboard USS *Enterprise* in 1986, the Navy is implementing readiness-based sparing (RBS) methodology in determining AVCAL requirements.

In RBS, as in DBS, past demands are used to project future demands. In RBS, however, tradeoff among parts is performed while producing AVCAL requirements in order to maximize readiness without exceeding a cost limit or to minimize cost without causing readiness to drop below a certain level.

For the RBS development project, the Center for Naval Analyses (CNA) was tasked to examine alternative ways RBS can be used to determine spare-part requirements for multiple type-model-series (TMS) aircraft sites, and how RBS can be used in the simultaneous sparing of aircraft components and the subparts needed for their repair–the multi-indenture, readiness-based sparing (MIRBS) problem.

This research memorandum focuses on the modeling of MIRBS. Another memorandum [5] focuses on the problem of gathering correct and appropriate data.

Although full-scale application of the model is not feasible at this time, its importance lies in that it facilitates comparison and evaluation of existing models. RBS methodology cannot be used immediately in simultaneous sparing of components and their subparts because existing models use wrong assumptions and cannot be used even as approximating methods. The model developed herein is too computer intensive for a full-scale application. In addition, as detailed in

[5], the quality and type of data currently collected are not satisfactory for such an application.

The next section of this paper provides background for the model developed in the subsequent section. Then, approximations to the model are examined and conclusions given.

MODELING BACKGROUND

In studying MIRBS, the hierarchical structure of aircraft parts is an important consideration. In such a structure, an aircraft is composed of weapon replaceable assemblies (WRAs) (in this study all organizational-level removables are regarded as WRAs). Each WRA is composed of shop replaceable assemblies (SRAs), and each SRA is made up of sub-SRAs, and so on until the last sub-SRA and piece part. In such an indenture structure:

- A single part may consistute a whole WRA.
- Identical parts can be found in distinct parents.
- Identical parts can be found in distinct indenture levels.

Although there are different methods of calculating aircraft readiness, they all depend on aircraft downtime. Aircraft downtime is a function of WRAs' turnaround times (TATs). TAT is composed of "awaiting-maintenance time," "in-process time" (which is mainly the time actual work is performed), and "awaiting-parts time" (AWPT). AWPT *is the time until the required* SRAs become available. AWPT is the link between the two indenture levels. (While the models presented herein are equally applicable to any indenture level, for the sake of simplicity this research memorandum focuses on a WRA and the SRAs composing it.) To calculate aircraft readiness resulting from an AVCAL, the expected length of the AWPT of each WRA must be obtained.

Two factors are necessary in calculating expected AWPTs:

- Which SRAs fail when their parent WRA fails?
- How long does it take for replacement of those failed SRAs to become available?

The following assumptions relate to these factors:

- No redundancy is built in, so all failed parts need replacement (series systems).
- WRAs fail independently (of each other).

- SRAs fail independently (of each other).
- Total (airwing) demand for each SRA has a Poisson distribution.
- Ordering of spares is done on a one-for-one basis.
- Repair/resupply times of different types of SRAs are independent (of each other).
- Exactly one SRA fails each time its parent WRA fails (to be revisited).

Under these assumptions, the conditional probability that SRA_i failed given that its parent WRA failed is approximated by the ratio

$$\frac{\text{failure rate of } SRA_i \text{ in this WRA application}}{\sum_{\text{all } j\text{s in this WRA}} \text{failure rate of } SRA_j \text{ in this WRA application}},$$

and the conditional expected AWPT given that SRA_i and its parent WRA failed is obtained by using Little's formula [6 and 7], and equals the expected number of backorders of SRA_i divided by the failure rate of SRA_i in all its airwing applications.

Although most of the assumptions are reasonable approximations to real life, the last one is shown to be unrealistic. In table 1, data show that during the 1985 deployments of USS *Constellation* and USS *Coral Sea*, only between 15 and 25 percent of the Intermediate Maintenance Activity (IMA) WRA repairs required exactly one SRA. It is interesting to note that between 50 and 69 percent of the WRAs required no SRAs at all. Among those 16 to 26 percent of WRAs requiring two or more SRAs, some required hundreds of SRAs.

It is relatively easy to modify a model that assumes exactly one SRA failure for each parent WRA failure into one that accounts for failed WRAs that do not require any SRAs to be repaired. In this case, the conditional probability that SRA_i failed given that its parent WRA failed is approximated by the ratio

$$\frac{\text{failure rate of } SRA_i \text{ in parent WRA application}}{\text{failure rate of parent WRA (including failures needing no SRAs)}},$$

and the conditional expected AWPT remains as before for the case when an SRA failed and equals zero when no SRA has failed.

Table 1. Number of SRAs required to repair a WRA[a]

	Number of removals	Percent needing no SRAs	Percent needing one SRA	Percent needing two or more SRAs
All IMA repairs	29,604	68.4	15.3	16.3
Aircraft repairs at IMA	20,852	61.5	18.7	19.8
Sample[b]	566	50.5	23.5	26.0

a. Data: 1985 deployments of *Constellation* and *Coral Sea.*
b. Sample = IMU, radar receiver, receiver transmitter, GM launcher. All have SRAs.

It is extremely simple to calculate the expected AWPT for both cases (exactly one failed SRA, and one or zero failed SRA). Simply sum over all the SRAs composing a WRA the product of the probability for the SRA to fail and the conditional expected AWPT for this SRA. (Because in the case of no failed SRAs the conditional expected AWPT equals zero, it suffices to modify the probabilities.)

Multiple failures present a much more complicated problem. A conditional expected AWPT calculation that uses the expected number of backorders is valid if and only if at most one SRA fails. In the multiple-failure case, the distribution of the time until the last type of SRA is obtained is needed (similar to the maximum in order statistics). To illustrate the concept, consider the following very simple example.

There are no spares stocked. n distinct SRAs have failed. Repair/resupply time of each SRA has an independent exponential distribution with parameter μ. The probability that the time until the last SRA becomes available is less than t, $F(t)$, is the probability that all SRAs become available before t. Under the independence assumption, this is equal to the product of the probabilities that each SRA becomes available before t, that is:

$$F(t) = (1 - e^{-\mu t})^n = \sum_{k=0}^{n}(-e^{-\mu t})^k \binom{n}{k} = 1 - \sum_{k=1}^{n}(-1)^{k+1}(e^{-\mu t})^k \binom{n}{k} \quad ,$$

and the expected AWPT, E, equals

$$
\begin{aligned}
E &= \int_{t=0}^{\infty}[1-F(t)]dt = \sum_{k=1}^{n}(-1)^{k+1}\binom{n}{k}\int_{t=0}^{\infty}(e^{-\mu k})^{t}dt \\
&= \frac{1}{\mu}\sum_{k=1}^{n}(-1)^{k+1}\binom{n}{k}\frac{1}{k} = \frac{1}{\mu}\sum_{m=1}^{n}\frac{1}{m} \; .
\end{aligned}
$$

The average repair/resupply time for the single-failure case is $\frac{1}{\mu}$. To give a feeling for E (in terms of $\frac{1}{\mu}$) as a function of n (the number of failed SRAs), several values of E and n are presented in table 2.

Table 2. Sample of expected AWPT (E) and number of SRAs (n)

n	E
1	$1.0/\mu$
5	$2.3/\mu$
10	$2.9/\mu$
15	$3.3/\mu$
20	$3.6/\mu$
25	$3.8/\mu$
30	$4.0/\mu$
35	$4.1/\mu$
40	$4.3/\mu$
45	$4.4/\mu$
50	$4.5/\mu$

It can be seen that E increases sharply at the beginning and then stabilizes.

In the next section, the multiple-failure case with multiple applications of a type of SRA in a parent WRA and with different parameters (μ) for different types of SRA is discussed.

MODEL DEVELOPMENT

This section shows how to develop a means to calculate expected AWPTs when there are multiple SRA failures and multiple SRAs per parent WRA. The model presented recognizes that in MIRBS problems SRAs compete for repair/resupply resources with SRAs of the same type from all airwing applications; that is, given that SRAs of type i are required for WRA_1, the waiting time to get them depends on the total number in the repair/resupply pipeline, not on the number required by WRA_1. Repair/resupply of the different types of SRAs belonging to WRA_1 is performed simultaneously.

The following situation is considered.

The aircraft whose readiness is calculated, for example A/C_1, is composed of, for example, v WRAs, all of which have to be ready (that is, in working condition) for A/C_1 to be ready. When a WRA, say WRA_1, has been removed after aircraft inspection or repair, it is either replaced with a whole WRA_1, or repaired. While repairing WRA_1, occasionally SRAs need to be replaced (at least 30 percent of the time). A removed SRA is either replaced with a whole SRA or repaired. Each type of SRA can be used, in different quantities, in different WRAs of A/C_1. It can also be used in OTHER APPLICATIONS, such as other aircraft of the same type as A/C_1, or different types of aircraft. Each removed SRA goes into a repair/resupply pipeline that is characterized by the type of SRA. (See figure 1.) Under the assumption that WRAs fail independently, $WRA_2, \ldots, WRA_v$ need not be explicitly considered when calculating AWPT of WRA_1; our universe can be divided into: WRA_1 applications, and ALL OTHER APPLICATIONS ($= WRA_2 \cup \ldots \cup WRA_v \cup$ OTHER APPLICATIONS).

The following notation is used:

E = Expected length of AWPT of WRA_1

$E_{I,J}$ = Expected length of AWPT of WRA_1 given that at least one SRA type i, $\forall i \epsilon I$, is needed, but no SRA type j, $\forall j \epsilon J$, is needed. $I \cap J = \phi, I \cup J = \{1, \ldots, n\}$

$P(I,J)$ = Probability {at least one SRA type i, $\forall i \epsilon I$, is needed but no SRA type j, $\forall i \epsilon J$, is needed}

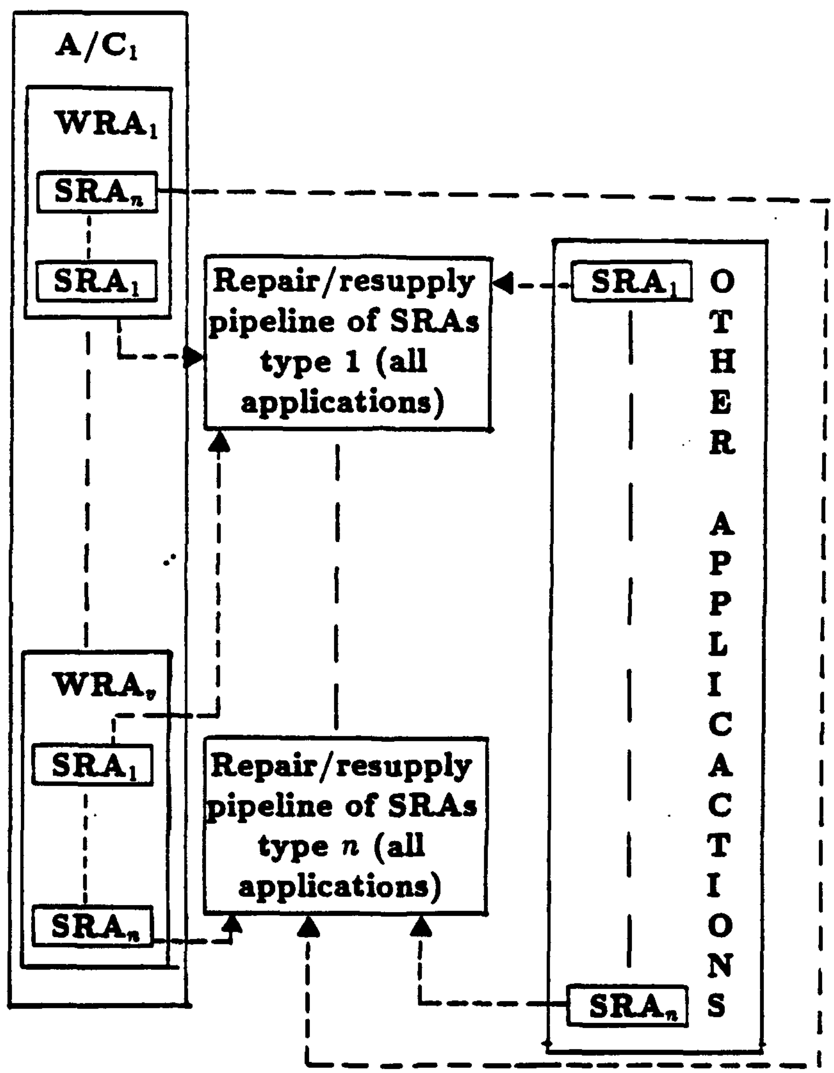

Figure 1. Flow of SRAs into repair/resupply pipelines

λ = Rate at which a WRA_1 used in A/C_1 is removed and repaired aboard ship

λ_i = Removal rate of an SRA_i used in WRA_1; $i = 1, \ldots, n$

γ_i = Total removal rate of SRA_i (all applications); $i = 1, \ldots, n$

τ_i = Mean repair/resupply time of SRA_i; $i = 1, \ldots, n$

q_i = Quantity of SRA_i per WRA_1; $i = 1, \ldots, n$

s_i = Number of spare SRAs type i initially stocked (AVCAL); $i = 1, \ldots, n$

K_i = Total number of SRAs in the repair/resupply pipeline for SRAs type i, including *all* the SRAs type i needed for WRA_1; $i = 1, \ldots, n$

$P_i(k)$ = Probability $\{K_i = k\}$; $i = 1, 2, \ldots, n$; $k = 0, 1, \ldots$

T_i = Total *time* waited for repair/resupply of *all* SRAs type i in the pipeline; $i = 1, \ldots, n$.

The derivation logic and assumptions follow those outlined in the previous section. Now, by the law of total probability, we have:

$$E = \sum_{\substack{\forall I, J \ni I \neq \phi \\ I \cap J = \phi \\ I \cup J = \{1, \ldots, n\}}} E_{I,J} \cdot P(I, J) \ . \tag{1}$$

The probability that SRA_i failed in a given WRA failure is estimated by

$$\frac{\lambda_i}{\lambda} ,$$

so

$$\left(1 - \frac{\lambda_i}{\lambda}\right)^{q_i}$$

estimates the probability that no SRA type i has failed in a given WRA failure.

Thus, $P(I,J)$ is estimated by:

$$P(I,J) = \prod_{i\in I}\left[1-\left(1-\frac{\lambda_i}{\lambda}\right)^{q_i}\right]\prod_{j\in J}\left(1-\frac{\lambda_j}{\lambda}\right)^{q_j} \quad , \tag{2}$$

and

$$\begin{aligned} E_{I,J} = \int_{t=0}^{\infty} [1 - \text{Probability}\{AWPT \text{ of } WRA_1 \leq t \mid \text{at least} \\ \text{one SRA type i}, \forall i \in I, \text{is needed, but no SRA type j}, \\ \forall j \in J, \text{is needed}\}]dt \quad . \end{aligned} \tag{3}$$

By assuming independence among SRAs of different types, equation 3 can be rewritten as:

$$E_{I,J} = \int_{t=0}^{\infty}\left[1-\prod_{i\in I}\text{Probability}\{T_i \leq t\}\right]dt \quad . \tag{4}$$

Again, using the law of total probability:

$$\text{Probability}\{T_i \leq t\} = \sum_{k=0}^{\infty}\text{Probability}\{T_i \leq t \mid K_i = k\} \cdot P_i(k) \quad . \tag{5}$$

Let us assume that total demand (all applications) for SRA_i is distributed Poisson (γ_i). Then, each repair/resupply pipeline can be modeled by a $M/G/\infty/\infty/\infty$ queueing system. (In standard queueing theory [6and 7], this notation means that interarrival times are exponential random variables, the service times' distribution is not specified, and there are infinite numbers of servers and customers/parts, and no capacity constraints are imposed on the system.) For such a system, the number of customers, that is SRAs, in the system is distributed Poisson $(\gamma_i\tau_i)$, or:

$$P_i(k) = e^{-(\gamma_i\tau_i)}(\gamma_i\tau_i)^k/k!; k = 0, 1, \ldots \tag{6}$$

And, the output of the system (that is, repaired/resupplied SRAs) is distributed Poisson (γ_i), just as the input. It should be noted that these results are independent of the service (that is, repair/resupply) time distribution.

Now, since the output is Poisson, times between outputs are exponentially distributed, so:

$$\begin{aligned}&\text{Probability}\{T_i \le t \mid K_i = k\} = \\ &\quad \text{Probability}\{\text{sum of } k \text{ exponential random variables} \le t\} \quad .\end{aligned} \tag{7}$$

Equation 7 holds only for $k > S_i$. For $k \le S_i$ there is no waiting at all, and then Probability $\{T_i \le t\} = 1$ for all $t \ge 0$, (since Probability $\{T_i = 0\} = 1$).

Thus, using equations 6 and 7, equation 5 can be rewritten as:

$$\begin{aligned}
&\text{Probability}\{T_i \le t\} \\
&= \sum_{k=0}^{S_i} 1 \cdot e^{-(\gamma_i\tau_i)}(\gamma_i\tau_i)^k/k! \\
&+ \sum_{k=S_i+1}^{\infty} \left[\int_{u=0}^{t} \frac{\gamma_i^{k-S_i}}{\Gamma(k-S_i)} u^{k-S_i-1} e^{-\gamma_i u} du\right] e^{-(\gamma_i\tau_i)}(\gamma_i\tau_i)^k/k! \\
&= \sum_{k=0}^{S_i} e^{-(\gamma_i\tau_i)}(\gamma_i\tau_i)^k/k! + \sum_{k=S_i+1}^{\infty} e^{-(\gamma_i\tau_i)}(\gamma_i\tau_i)^k/k! \frac{\gamma_i^{k-S_i}}{\Gamma(k-S_i)} \\
&\quad \left[\frac{(k-S_i-1)!}{\gamma_i^{k-S_i}} - e^{-\gamma_i t} \sum_{m=0}^{k-S_i-1} \frac{(k-S_i-1)!t^m}{m!\gamma_i^{(k-S_i-1-m+1)}}\right] \cdot \\
&= 1 - \sum_{k=S_i+1}^{\infty} e^{-(\gamma_i\tau_i)}(\gamma_i\tau_i)^k/k! \sum_{m=0}^{k-S_i-1} e^{-\gamma_i t}(\gamma_i t)^m/m! \quad .
\end{aligned} \tag{8}$$

At this point, let us return our attention to equation 4 and let us denote Probability $\{T_i \leq t\}$ by $F_i(t)$ and $1 - F_i(t)$ by $\overline{F}_i(t)$. Then,

$$
\begin{aligned}
E_{I,J} &= \int_{t=0}^{\infty} \left[1 - \prod_{i \in I} F_i(t)\right] dt \qquad (9) \\
&= \int_{t=0}^{\infty} \left[\prod_{p=1}^{|I|} (-1)^{p-1} \sum_{\substack{\forall (i_1, \ldots, i_p) \\ \ni i_j \in I; j = 1, \ldots, p \\ \text{and} \\ i_{j+1} > i_j; j = 1, \ldots, p-1}} \prod_{j=1}^{p} \overline{F}_{i_j}(t) \right] dt \\
&= \sum_{p=1}^{|I|} (-1)^{p-1} \sum_{\substack{\forall (i_1, \ldots, i_p) \\ \ni i_j \in I; j = 1, \ldots, p \\ \text{and} \\ i_{j+1} > i_j; j = 1, \ldots, p-1}} \int_{t=0}^{\infty} \left[\prod_{j=1}^{p} \overline{F}_{i_j}(t) \right] dt \ .
\end{aligned}
$$

Using equation 8 for $F_i(t)$, we solve the integral of equation 9.

$$\int_{t=0}^{\infty}\left[\prod_{j=1}^{p}\overline{F}_{i_j}(t)\right]dt \tag{10}$$

$$= \int_{t=0}^{\infty}\left[\prod_{j=1}^{p}\sum_{k_j=S_{i_j}+1}^{\infty} e^{-(\gamma_{i_j}\tau_{i_j})}(\gamma_{i_j}\tau_{i_j})^{k_j}/k_j! \sum_{m_j=0}^{k_j-S_{i_j}-1} e^{-\gamma_{i_j}t}(\gamma_{i_j}t)^{m_j}/m_j!\right]dt$$

$$= \sum_{k_1=S_{i_1}+1}^{\infty} e^{-(\gamma_{i_1}\tau_{i_1})}(\gamma_{i_1}\tau_{i_1})^{k_1}/k_1!\ldots \sum_{k_p=S_{i_p}+1}^{\infty} e^{-(\gamma_{i_p}\tau_{i_p})}(\gamma_{i_p}\tau_{i_p})^{k_p}/k_p!$$

$$\sum_{m_1=0}^{k_1-S_1-1}\frac{(\gamma_{i_1})^{m_1}}{m_1!}\ldots\sum_{m_p=0}^{k_p-S_p-1}\frac{(\gamma_{i_p})^{m_p}}{m_p!}\int_{t=0}^{\infty} e^{-\left(\sum_{j=1}^{p}\gamma_{i_j}\right)t}\, t^{\sum_{j=1}^{p} m_j}dt$$

$$= \sum_{k_1=S_{i_1}-1}^{\infty} e^{-(\gamma_{i_1}\tau_{i_1})}(\gamma_{i_1}\tau_{i_1})^{k_1}/k_1!\ldots \sum_{k_p=S_{i_p}+1}^{\infty} e^{-(\gamma_{i_p}\tau_{i_p})}(\gamma_{i_p}\tau_{i_p})^{k_p}/k_p!$$

$$\sum_{m_1=0}^{k_1-S_1-1}\frac{(\gamma_{i_1})^{m_1}}{m_1!}\ldots\sum_{m_p=0}^{k_p-S_p-1}\frac{(\gamma_{i_p})^{m_p}}{m_p!}\left(\sum_{j=1}^{p}\gamma_{i_j}\right)^{-\left(\sum_{j=1}^{p} m_j\right)-1}\left(\sum_{j=1}^{p} m_j\right)!$$

$$= \left(\sum_{j=1}^{p}\gamma_{i_j}\right)^{-1}\sum_{k_1=S_{i_1}+1}^{\infty} e^{-(\gamma_{i_1}\tau_{i_1})}(\gamma_{i_1}\tau_{i_1})^{k_1}/k_1!\ldots$$

$$\sum_{k_p=S_{i_p}+1}^{\infty} e^{-(\gamma_{i_p}\tau_{i_p})}(\gamma_{i_p}\tau_{i_p})^{k_p}/k_p! \sum_{m_1=0}^{k_1-S_1-1}\ldots$$

$$\sum_{m_p=0}^{k_p=S_p-1}\left[\prod_{j=1}^{p}\left(\frac{\gamma_{i_j}}{\sum_{u=1}^{p}\gamma_{i_u}}\right)^{m_j}\right]\frac{(\sum_{j=1}^{p} m_j)!}{\prod_{j=1}^{p}(m_j!)}\ .$$

Upon inversion of the order of summations, equation 10 can be rewritten as:

$$\int_{t=0}^{\infty}\left[\prod_{j=1}^{p}\overline{F}_{i_j}(t)\right]dt = \tag{11}$$

$$\left(\sum_{j=1}^{p}\gamma_{i_j}\right)^{-1}\sum_{m_1=0}^{\infty}\cdots\sum_{m_p=0}^{\infty}\left[\prod_{j=1}^{p}\left(\frac{\gamma_{i_j}}{\sum_{u=1}^{p}\gamma_{i_u}}\right)^{m_j}\right]\cdot$$

$$\frac{\left(\sum_{j=1}^{p} m_j\right)!}{\prod_{j=1}^{p}(m_j!)}\prod_{j=1}^{p}\sum_{k_j=S_{i_j}+1+m_j}^{\infty} e^{-(\gamma_{i_j}\tau_{i_j})}(\gamma_{i_j}\tau_{i_j})^{k_j}/k_j!\ .$$

Equation 11 has an interesting probabilistic interpretation; it equals:

$$\left(\sum_{j=1}^{p}\gamma_{i_j}\right)^{-1}\sum_{m_1=0}^{\infty}\cdots\sum_{m_p=0}^{\infty}\text{Probability}\{M_1=m_1,\ldots,M_p=m_p\}$$
$$\prod_{j=1}^{p}\text{Probability}\{K_{i_j}-s_{i_j}>M_j \mid M_j=m_j\}\quad, \tag{12}$$

with

$$(M_1,\ldots,M_p)\sim\text{Multinomial}\left(\frac{\gamma_{i_1}}{\sum_{j=1}^{p}\gamma_{i_j}},\ldots,\frac{\gamma_{i_p}}{\sum_{j=1}^{p}\gamma_{i_j}}\right)$$

and $K_{i_j}\sim$ Poisson $(\gamma_{i_j}\tau_{i_j}); j=1,\ldots,p$.

This is a generalization of $E=\frac{1}{\gamma_1}\cdot$ Expected number of backorders, which is obtained by setting p equal to 1.

This section is closed by synthesizing equations 1, 2, 9, and 11 to yield:

$$E = \sum_{\substack{\forall I,J\ni I\neq\phi\\ I\cap J=\phi\\ I\cup J=\{1,\ldots,n\}}}\left\{\sum_{p=1}^{|I|}(-1)^{p-1}\sum_{\substack{\forall(i_1,\ldots,i_p)\\ \ni i_j\in I;j=1,\ldots,p\\ i_{j+1}>i_j;j=1,\ldots,p-1}}\right. \tag{13}$$
$$\left(\sum_{j=1}^{p}\gamma_{i_j}\right)^{-1}\sum_{m_1=0}^{\infty}\cdots\sum_{m_p=0}^{\infty}\left[\prod_{j=1}^{p}\left(\frac{\gamma_{i_j}}{\sum_{u=1}^{p}\gamma_{i_u}}\right)^{m_j}\right]\frac{\left(\sum_{j=1}^{p}m_j\right)!}{\prod_{j=1}^{p}(m_j!)}$$
$$\left.\left[\prod_{j=1}^{p}\sum_{k_j=S_{i_j}+m_j+1}^{\infty}e^{-(\gamma_{i_j}\tau_{i_j})}(\gamma_{i_j}\tau_{i_j})^{k_j}/k_j!\right]\right\}$$
$$\prod_{i\in I}\left[1-\left(1-\frac{\lambda_i}{\lambda}\right)^{q_i}\right]\prod_{j\in J}\left(1-\frac{\lambda_j}{\lambda}\right)^{q_j}\quad.$$

MODEL APPLICATION

As mentioned previously, the theory developed in this paper is applicable to any level of indenture. However, the computational complexity (see equation 13) and the quality of available data (see [5]) limited application down to only the SRA level.

As mentioned in the Modeling Background section, expected AWPTs are required to calculate aircraft readiness. The method developed in the previous section was incorporated into the multi-indenture version of CNA's Multi-Item, Multi-Echelon (MIME) model. In MIME, an additional WRA or SRA is stocked if it increases most aircraft readiness per dollar, up to a prespecified budget limitation.

To implement the method, the infinite sums—of equation 13—were approximated by finite sums. This was accomplished in the following way.

The number of spares stocked initially aboard ship plus an integer, say m, was substituted for infinity. m was varied from 1 to 200, and expected AWPTs for several representative real-life data cases were observed. After m exceeded six, expected AWPT changed less than one hundredth of a day. This magnitude of change was judged acceptable. The intuitive reasoning for such a good approximation is the lack of concentration of probability masses on the righthand tail of the Poisson distribution for these real-life data.

Because of the combinatorial nature of the MIRBS problem, each subproblem must be solved many times, and each subproblem is complicated and computationally intensive. Therefore, computer central processing unit (CPU) times are in the magnitude of hours, which is unacceptable for real-life-sized applications.

To overcome the obstacle of computer resource scarcity, approximating or bounding models were sought. The first candidate was the Availability Centered Inventory Model (ACIM) [8]. Although ACIM belongs to the class of models that assumes exactly one SRA failure per WRA failure, which happens in only 15 to 24 percent of the cases considered in table 1, its very short CPU time requirements warranted a try.

The second candidate was a CNA-modified ACIM that allows zero SRA requirements when a WRA fails, the ACIMO model.

The subroutines calculating the expected AWPTs were altered to print out the values at each iteration of the AVCAL requirements determination. ACIM

AWPT and ACIMO AWPT were compared to the expected AWPT calculated by the method developed in the previous section, NEW AWPT. The comparison was performed for a radio receiver WRA with five SRAs. This WRA had (relatively) reliable data, and five is the largest number of SRAs for which NEW AWPT produces exact results. A sample of the outcome is presented in table 3.

Table 3. AWPTs for a radio receiver with five SRAs

Number of spare SRAs in stock	NEW AWPT (days)	ACIM AWPT (days)	ACIMO AWPT (days)
0	4.6	3.0	2.7
1	0.6	1.1	1.0
2	0.5	0.8	0.8
3	0.4	0.4	0.4
4	0.3	0.6	0.6

In using the table, it should not be inferred that increasing the number of spares vertically from three to four would result in an increase from 0.4 to 0.6 in AWPT, because the three spares are not necessarily a subset of the four spares in the next line.

A horizontal comparison makes it clear that ACIM AWPT and ACIMO AWPT cannot be used as bounds for NEW AWPT, because while ACIM AWPT is greater than or equal to ACIMO AWPT in all cases, no such relationship was established to hold in regard to NEW AWPT.

As approximating methods, the average percentage of

$$\frac{|\text{ ACIM AWPT} - \text{NEW AWPT }|}{\text{NEW AWPT}}$$

is 56 percent, and the average percentage of

$$\frac{|\text{ ACIMO AWPT} - \text{NEW AWPT }|}{\text{NEW AWPT}}$$

is 54 percent, These numbers are too high for an acceptable error by an approximating method.

It should be mentioned that the Aviation Readiness Requirements Oriented to Weapon Replaceable Assemblies (ARROWs) model—being intalled at the Navy Aviation Supply Office (ASO)—uses the expected number of backorders in its formulae for calculating expected AWPT. As explained in the Modeling Background section, this approach is valid only under the assumption that at most one SRA fails with a WRA.

Having seen that models making this assumption give wrong answers quickly, one more attempt to approximate the NEW model was made.

By assuming independence of random variables $M_1, \ldots, M_p$ in formula 12, the multinomial distribution term can be factorized and the summations performed individually for each of the $m_1, \ldots, m_p$ indices, which simplified calculations greatly.

Regarding the accuracy of this approximation: in the worst case, when no spares are stocked, AWPT is, on the average, 5 percent (or about a quarter of a day) off; when the number of spares stocked is approximately the same as the average number of spares in the repair/resupply pipeline, the difference in AWPTs is less than a thousandth of a day, quite a good approximation—surprisingly good, especially in view of the fact that the independence assumption is totally wrong from the probabilistic point of view.

Regarding calculation times, CPU times were reduced by a factor of 30, but this is still much slower than ACIM and too slow for large-scale application.

CONCLUSIONS

Models assuming at most one SRA failure per parent failure compute expected AWPT very quickly, but use the wrong assumption. A correct model was developed but it is very computer intensive. The fast models cannot be used as approximating or bounding methods. A good approximating method was developed, but it is still too computer intensive for full-scale application. New computing approaches, such as parallel computing, raise hope for application, but data quality problems must be resolved first.

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