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ABSTRACT

A variety of modifications to the modeled dissipation rate transport equation that have been proposed during the past two decades to account for rotational strains are examined. The models are subjected to two crucial test cases: the decay of isotropic turbulence in a rotating frame and homogeneous shear flow in a rotating frame. It is demonstrated that these modifications do not yield substantially improved predictions for these two test cases and in many instances give rise to unphysical behavior. An alternative proposal, based on the use of the tensor dissipation rate, is made for the development of improved models.

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1. Introduction

Many turbulent flows of practical importance involve combinations of rotational and irrotational mean strains. The rotational strains can arise from the effects of curvature, a system rotation, or swirl, as well as from shear. Much of the motivation for formulating second-order closure models has its origins in the desire to account for the distinct physical effects of rotational and irrotational strains, in a framework where directional information is incorporated. Reynolds stress models of the eddy viscosity type, since they only depend on the mean velocity gradients through their symmetric part (i.e., the mean rate of strain tensor), are oblivious to the presence of rotational strains. Consequently, these models cannot distinguish between the considerably different turbulence physics exhibited in plane shear, plane strain, and rotating plane shear flows – a deficiency that is now well-known [1, 2]. However, the same deficiency still remains with the modeled form of the dissipation rate transport equation that has been commonly used in the turbulence modeling literature starting with the work of Launder and co-workers [3]. This frequently used model has no explicit dependence on rotational strains; rotational strains can only have an indirect effect through the changes that they induce in the Reynolds stress tensor. Such a limited dependence is known to be detective, making it impossible to properly describe rotating isotropic turbulence and swirling jets, among other turbulent flows [4, 5]. Consequently, several independent researchers over the past two decades have tried to develop rotational modifications to the modeled dissipation rate transport equation to resolve this problem. These alterations will be the subject of the present study.

In this paper, a variety of previously proposed modifications to the modeled dissipation rate transport equation will be considered:

(1) the Pope [5] model,

(2) the Hanjalic and Launder [6] model,

(3) the Bardina model [7], and

(4) the Raj [8] model.

These models represent a good cross-section of the rotational modifications to the modeled dissipation rate transport equation that have been proposed during the last fifteen years. Two flows from homogeneous turbulence will be used to test the models: (a) the decay of isotropic turbulence in a rotating frame, and (b) homogeneous shear flow in a rotating frame. These two test cases are chosen since they represent homogeneous flows whose structure is significantly altered by rotation, without the added complication of walls or other sources of mean turbulent diffusion. In this fashion, the question of the interaction of rotational and irrotational strains can be studied in isolation, independent of the complicating features introduced by solid boundaries and mean turbulent diffusion. The performance of each of the models will be documented in detail and specific proposals will be made for the development of improved models. A new model based on the tensor dissipation – formulated in order to include more directional information – will be proposed and tested.

2. The Models and Test Cases to be Considered

We will restrict our attention to the incompressible flow of a viscous fluid with constant properties. The velocity field $\mathbf{v}$ and pressure P are decomposed into mean and fluctuating parts, respectively, as follows:

$$\mathbf{v} = \overline{\mathbf{v}} + \mathbf{u}, \quad P = \overline{P} + p. \tag{1}$$

The exact transport equation for the turbulent dissipation rate in homogeneous flows takes the form [2]

$$\dot{\varepsilon} = \mathcal{P}_\varepsilon - \Phi_\varepsilon \tag{2}$$

where

$$\mathcal{P}_\varepsilon = -2\nu \overline{\frac{\partial u_i}{\partial x_j}\frac{\partial u_k}{\partial x_j}}\frac{\partial \overline{v}_i}{\partial x_k} - 2\nu \overline{\frac{\partial u_j}{\partial x_i}\frac{\partial u_j}{\partial x_k}}\frac{\partial \overline{v}_i}{\partial x_k} - 2\nu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_i}{\partial x_j}\frac{\partial u_k}{\partial x_j}} \tag{3}$$

$$\Phi_\varepsilon = 2\nu^2 \overline{\frac{\partial^2 u_i}{\partial x_j \partial x_k}\frac{\partial^2 u_i}{\partial x_j \partial x_k}} \tag{4}$$

are, respectively, the production and destruction of dissipation terms given that $\varepsilon \equiv 2\nu \overline{\partial u_i/\partial x_j \partial u_i/\partial x_j}$ is the scalar dissipation rate and ν is the kinematic viscosity of the fluid. Here, since the turbulence is homogeneous, the overbar denotes a spatial mean, and the superposed dot denotes an ordinary time derivative.

The modeled version of the transport equation for ε that has been used widely in the turbulence modeling community – which is due to Launder and co-workers [3] – takes the form

$$\dot{\varepsilon} = C_{\varepsilon 1}\frac{\varepsilon}{K}\mathcal{P} - C_{\varepsilon 2}\frac{\varepsilon^2}{K} \tag{5}$$

where $\mathcal{P} = -\overline{u_i u_j}\partial \overline{v}_i/\partial x_j$ is the turbulence production, and $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ are dimensionless constants that typically assume the values of 1.44 and 1.92, respectively. Equation (5) is obtained from (2) by making the assumption that the production (or destruction) of dissipation is proportional to the production (or destruction) of turbulent kinetic energy, i.e.,

$$\mathcal{P}_\varepsilon \propto \mathcal{P}/\tau_0, \quad \Phi_\varepsilon \propto \varepsilon/\tau_0 \tag{6}$$

where τ_0 is the turbulent time scale that is needed for dimensional consistency. The added assumption that the turbulent time scale τ_0 is given by

$$\tau_0 = \frac{K}{\varepsilon} \tag{7}$$

then yields the modeled equation (5).

Due to the symmetry of the Reynolds stress tensor $\tau_{ij} \equiv \overline{u_i u_j}$, it follows that

$$\mathcal{P} = -\tau_{ij}\overline{S}_{ij} \tag{8}$$

where

$$\overline{S}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{v}_i}{\partial x_j} + \frac{\partial \overline{v}_j}{\partial x_i}\right) \tag{9}$$

is the mean rate of strain tensor. Hence, (5) has *no explicit dependence* on the mean rotation tensor

$$\overline{\omega}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{v}_i}{\partial x_j} - \frac{\partial \overline{v}_j}{\partial x_i}\right). \tag{10}$$

It therefore follows that the commonly used modeled dissipation rate equation can only be affected indirectly by rotational strains through the changes that they induce in the Reynolds stress tensor. As we will show later, this makes it difficult to properly describe certain rotating turbulent flows.

The model suggested by Pope [5] is based on the addition of a mean vortex stretching term and takes the form

$$\dot{\varepsilon} = C_{\varepsilon 1}\frac{\varepsilon}{K}\mathcal{P} - C_{\varepsilon 2}\frac{\varepsilon^2}{K} + C_{\varepsilon 3}\frac{K^2}{\varepsilon}\overline{S}_{ij}\overline{\omega}_{jk}\overline{\omega}_{ki} \tag{11}$$

in homogeneous turbulence where $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$, and $C_{\varepsilon 3} = 0.79$. The standard ε-equation is obtained in the limit as $C_{\varepsilon 3} \to 0$. In the modification proposed by Hanjalic and Launder [6] to "sensitize the dissipation equation to irrotational strains," the ε-equation takes the following form in homogeneous turbulence:

$$\dot{\varepsilon} = C_{\varepsilon 1}\frac{\varepsilon}{K}\mathcal{P} - C_{\varepsilon 2}\frac{\varepsilon^2}{K} - C_{\varepsilon 3}K e_{mij}e_{mkl}\frac{\partial \overline{v}_i}{\partial x_j}\frac{\partial \overline{v}_k}{\partial x_l} \tag{12}$$

where e_{ijk} is the permutation tensor and $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$, and $C_{\varepsilon 3} = 0.27$. Since,

$$\frac{\partial \overline{v}_i}{\partial x_j} = \overline{S}_{ij} + \overline{\omega}_{ij} \tag{13}$$

it is clear that this modification introduces rotational strains; in the limit as $C_{\varepsilon 3} \to 0$ the standard ε-equation is recovered. The Bardina model [7, 9] takes the form

$$\dot{\varepsilon} = C_{\varepsilon 1}^{*}\frac{\varepsilon}{K}\mathcal{P} - C_{\varepsilon 2}^{*}\frac{\varepsilon^2}{K} \tag{14}$$

in a homogeneous turbulence, where

$$C_{\varepsilon 1}^{*} = 1.50 - 0.015\frac{K}{\varepsilon}\left(\frac{1}{2}\overline{\omega}_{ij}\overline{\omega}_{ij}\right)^{\frac{1}{2}} \tag{15}$$

$$C_{\varepsilon 2}^{*} = 1.83 + 0.15\frac{K}{\varepsilon}\left(\frac{1}{2}\overline{\omega}_{ij}\overline{\omega}_{ij}\right)^{\frac{1}{2}} \tag{16}$$

are functions of the mean rotation tensor. The models (11), (12), and (14)-(16) are written for an inertial frame. In a non-inertial frame, the rotation tensor $\overline{\omega}_{ij}$ must be replaced by the absolute mean rotation tensor

$$\overline{W}_{ij} = \overline{\omega}_{ij} + e_{mji}\Omega_m \tag{17}$$

where Ω_m is the angular velocity of the non-inertial frame relative to an inertial framing (see Speziale [10]).

Raj [8] developed a model based on an analysis of the exact transport equation of the true dissipation. In an inhomogeneous turbulence, this exact transport equation contains terms that depend explicitly on the rotation rate of the reference frame. Based on an empirical argument, Raj [8] developed a simple correction term to account for the effect of rotations which was guided by his turbomachinery research. For homogeneous turbulence, the model proposed by Raj [8] takes the form

$$\dot{\varepsilon} = C_{\varepsilon 1}\frac{\varepsilon}{K}\mathcal{P} - C_{\varepsilon 2}\frac{\varepsilon^2}{K} + C_{\varepsilon 3}\Omega\varepsilon\left(\frac{3\tau_{zz}}{2K} - 1\right) \tag{18}$$

in a rotating turbulent flow, where Ω is the angular velocity of the reference frame – which, for simplicity, is aligned along the z direction – and τ_{zz} is the component of Reynolds stress tensor along the axis of rotation. Here, $C_{\varepsilon 3}$ is a constant of $\mathcal{O}(1)$. In the limited practical applications where this model has been used, $C_{\varepsilon 3}$ has assumed different values ranging from 1 to 5. For simplicity, we will set $C_{\varepsilon 3} = 5$ (the largest of these values); $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ assume the traditional values of 1.44 and 1.92, respectively. It must be said at the outset that (18) does not constitute a general model and has some internal inconsistencies in the way it was derived [11]. Nonetheless, the model does have some interesting features that are worth testing in the problems to be considered in this study (e.g., the model directly accounts for the effect of the rotationally induced anisotropy in the Reynolds stress tensor, along the axis of rotation, on the dissipation).

In order to test these models in the flows to be considered, a Reynolds stress model is needed. A simple second-order closure model is chosen – namely, the shortened form of the Launder, Reece, and Rodi model which is now referred to as the "Basic Model" by Launder and co-workers [12]. For a homogeneous turbulence in a rotating frame, this Basic Model of Launder and co-workers takes the form [13]:

$$\begin{aligned}\dot{\tau}_{ij} &= (C_2 - 1)\left(\tau_{ik}\frac{\partial\overline{v}_j}{\partial x_k} + \tau_{jk}\frac{\partial\overline{v}_i}{\partial x_k}\right) \\ &\quad + (C_2 - 2)(\tau_{ik}e_{mkj}\Omega_m + \tau_{jk}e_{mki}\Omega_m) \\ &\quad - C_1\frac{\varepsilon}{K}\left(\tau_{ij} - \frac{2}{3}K\delta_{ij}\right) - \frac{2}{3}C_2\tau_{kl}\frac{\partial\overline{v}_k}{\partial x_l}\delta_{ij} - \frac{2}{3}\varepsilon\delta_{ij}\end{aligned} \tag{19}$$

where C_1 and C_2 are dimensionless constants that assume the values of 1.8 and 0.6, respectively. This model is derived by making two major assumptions – namely that the dissipation rate tensor is isotropic, i.e.,

$$\varepsilon_{ij} = \frac{2}{3}\varepsilon\delta_{ij} \tag{20}$$

and the pressure strain correlation is of the general form

$$\Pi_{ij} = \varepsilon\mathcal{A}_{ij}(\mathbf{b}) + K\mathcal{M}_{ijkl}(\mathbf{b})\frac{\partial\overline{v}_k}{\partial x_l} \tag{21}$$

where $\mathcal{A}_{ij}$ and $\mathcal{M}_{ijkl}$ are *linear* functions of the anisotropy tensor $\mathbf{b}$.

The first problem to be considered as a test case is isotropic turbulence in a rotating frame. Here, an initially decaying isotropic turbulence wherein

$$\tau_{ij} = \frac{2}{3}K_0\delta_{ij}, \quad \varepsilon = \varepsilon_0, \tag{22}$$

at time $t = 0$, is subjected to a solid body rotation with constant angular velocity

$$\Omega_i = (0, 0, \Omega) \tag{23}$$

and vanishing mean velocity gradients

$$\frac{\partial \overline{v}_i}{\partial x_j} = 0. \tag{24}$$

The substitution of (23) - (24) into (19) – along with the use of the initial condition of isotropy (22) – then yields the equation

$$\dot{K} = -\varepsilon \tag{25}$$

where $\tau_{ij} = \frac{2}{3}K\delta_{ij}$. This is the same form as its non-rotating counterpart. Hence, the Launder, Reece, and Rodi model predicts that an initially isotropic turbulence subjected to a rotation decays isotropically – a result that is consistent with the Navier-Stokes equations [14, 15]. For rotating isotropic turbulence, the modeled ε- transport equation takes the standard form

$$\dot{\varepsilon} = -C_{\varepsilon 2}\frac{\varepsilon^2}{K} \tag{26}$$

(where $C_{\varepsilon 2} = 1.92$) for the Pope and the Raj models; consequently, these models do not distinguish the difference between isotropic turbulence in a rotating frame and in an inertial frame. The Bardina model, however, does have a non-zero rotational correction; for rotating isotropic turbulence (14) takes the form

$$\dot{\varepsilon} = -C_{\varepsilon 2}\frac{\varepsilon^2}{K} - C_{\varepsilon 3}\Omega\varepsilon \tag{27}$$

where $C_{\varepsilon 2} = 1.83$ and $C_{\varepsilon 3} = 0.15$. Likewise, so does the Hanjalic and Launder model for which we have the following ε-transport equation in rotating isotropic turbulence:

$$\dot{\varepsilon} = -C_{\varepsilon 2}\frac{\varepsilon^2}{K} - 4C_{\varepsilon 3}K\Omega^2 \tag{28}$$

where $C_{\varepsilon 2} = 1.92$ and $C_{\varepsilon 3} = 0.27$.

The second problem to be considered is homogeneous shear flow in a rotating frame. The mean velocity gradients here take the form

$$\frac{\partial \overline{v}_i}{\partial x_j} = \begin{pmatrix} 0 & S & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{29}$$

which correspond to a plane shear. They are applied in a steadily rotating frame where the axis of rotation is normal to the plane of the shear, i.e.,

$$\Omega_i = (0, 0, \Omega).$$

The substitution of (28) - (29) into (19) yields the Reynolds stress transport equation

$$\dot{\tau}_{ij} = \tau_{ik}A_{jk} + \tau_{jk}A_{ik} - 1.8\frac{\varepsilon}{K}\left(\tau_{ij} - \frac{2}{3}K\delta_{ij}\right) + \frac{2}{3}\left(\frac{3}{5}\mathcal{P} - \varepsilon\right)\delta_{ij} \tag{30}$$

where

$$A_{ij} = \begin{bmatrix} 0 & -\frac{2}{5}S + \frac{7}{5}\Omega & 0 \\ -\frac{7}{5}\Omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{31}$$

and $\mathcal{P} \equiv -\tau_{12}S$ is the turbulence production. The ε-transport equation for the Pope model is given by

$$\dot{\varepsilon} = -C_{\varepsilon 1}\frac{\varepsilon}{K}\tau_{12}S - C_{\varepsilon 2}\frac{\varepsilon^2}{K} \tag{32}$$

in rotating shear flow. This is the same as the standard model due to the fact that the mean vortex stretching term in (11) vanishes for any plane homogeneous turbulence. The Bardina model for rotating shear flow is obtained by replacing $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ in (32) with

$$C^*_{\varepsilon 1} = 1.50 - 0.015|\tfrac{1}{2}S - \Omega|K/\varepsilon \tag{33}$$

$$C^*_{\varepsilon 2} = 1.83 + 0.15|\tfrac{1}{2}S - \Omega|K/\varepsilon. \tag{34}$$

Both the Hanjalic and Launder model and the Raj model take the form

$$\dot{\varepsilon} = -C_{\varepsilon 1}\frac{\varepsilon}{K}\tau_{12}S - C_{\varepsilon 2}\frac{\varepsilon^2}{K} + R \tag{35}$$

in rotating shear flow where $C_{\varepsilon 1} = 1.44$ and $C_{\varepsilon 2} = 1.92$. For the Hanjalic and Launder model

$$R = -C_{\varepsilon 3}K(S - 2\Omega)^2 \tag{36}$$

where $C_{\varepsilon 3} = 0.27$, whereas for the Raj model

$$R = C_{\varepsilon 3}\Omega\varepsilon\left(\frac{3\tau_{zz}}{2K} - 1\right) \tag{37}$$

where $C_{\varepsilon 3} = 5$.

In rotating shear flow, the transport equations for τ_{ij} and ε are solved subject to the initial conditions of isotropy (22) at time $t = 0$.

3. Discussion of the Results

For the two homogeneous turbulence problems to be considered, the models give rise to a coupled set of first-order nonlinear ordinary differential equations which are solved by a fourth-order accurate Runge-Kutta numerical integration scheme. First, the results for isotropic turbulence in a rotating frame will be presented. For this problem, the solutions depend on Ω, K_0, and ε_0 only through the dimensionless parameter $\Omega K_0/\varepsilon_0$. In Figures 1(a) - (b) the time evolution of the dimensionless turbulent kinetic energy ($K^* \equiv K/K_0$ and $\tau \equiv \varepsilon_0 t/K_0$ is the dimensionless time) obtained for the Bardina model is shown for two moderate rotation rates: $\Omega K_0/\varepsilon_0 = 0.123$ and $\Omega K_0/\varepsilon_0 = 0.469$. These two rotation rates correspond to the test cases considered in the physical experiments of Wigeland and Nagib [15]. It is clear that the agreement between the model predictions and the experiments is good (however, it should be noted that the difference between these results and the predictions of the standard model are small). This is not surprising since the Bardina model was calibrated based on the same experimental data [7]. While the Bardina model performs well for weak rotation rates, it has substantial problems for rapidly rotating isotropic turbulence. In Figure 1(c), the time evolution of the turbulent kinetic energy predicted by the Bardina model for $\Omega K_0/\varepsilon_0 = 69.5$ is compared with the direct simulations of Speziale, Mansour, and Rogallo [14]. It is clear that the model *drastically underpredicts* the decay rate of the turbulent kinetic energy. This arises from a deficiency in the model; it is a simple matter to show that, for $\Omega K_0/\varepsilon_0 > 1.11$, the Bardina model predicts that the turbulence Reynolds number Re_t *increases* during the decay – a result that is unphysical.

The time evolution of the turbulent kinetic energy predicted by the Hanjalic and Launder model is shown in Figure 2(a) along with the predictions of the standard model and the experimental data of Wigeland and Nagib on rotating isotropic turbulence corresponding to $\Omega K_0/\varepsilon_0 = 0.123$. While the model does not do too bad a job for this case, it yields worse results than the standard model. It then breaks down entirely for appreciable values of $\Omega K_0/\varepsilon_0$. In Figure 2(b), the predictions of the Hanjalic and Launder model are shown for $\Omega K_0/\varepsilon_0 = 0.469$. The model yields extremely bad results for this case and becomes unrealizable when $\tau \doteq 4.2$ (i.e., the dissipation rate becomes negative causing us to terminate the calculations). It is the source term added by Hanjalic and Launder to the right-hand-side of the dissipation rate equation that causes the model to become unrealizable for discernible values of $\Omega K_0/\varepsilon_0$. We will not show the results for the Pope model and the Raj model since they are identical to the predictions of the standard model. Hence, like the standard model, they are unable to predict the reduction in the decay rate of the turbulent kinetic energy that arises from a system rotation of isotropic turbulence [14, 15].

Now, we will discuss the results obtained for rotating shear flow. In Figures 3(a) - (c), the time evolution of the turbulent kinetic energy predicted by the Bardina model is compared with the standard model and the large-eddy simulations of Bardina, Ferziger, and Reynolds [16] for three rotation rates: $\Omega/S = 0$, 0.25, and 0.5. (Again, $K^* = K/K_0$ whereas $t^* = St$). From Figures 3(b) - 3(c) it is clear that the Bardina model has little effect for appreciable rotation rates; like the standard model, the Bardina model drastically underpredicts the growth rate of the turbulent kinetic energy for $\Omega/S = 0.25$ and $\Omega/S = 0.5$. However, what is more serious is that the Bardina modification degrades considerably the results for pure shear flow ($\Omega/S = 0$) as shown in Figure 3(a). The growth rate of the turbulent kinetic energy predicted by the Bardina model is too high. This results from the overprediction of the equilibrium value of the shear anisotropy $(b_{12})_\infty = -0.191$; the standard model yields a value of $(b_{12})_\infty = -0.18$ which is closer to the experimental value of $(b_{12})_\infty = -0.16$.

In Figure 4, the predictions of the Hanjalic and Launder model are compared with the standard model and the large-eddy simulation of Bardina, Ferziger, and Reynolds [16] for homogeneous shear flow ($\Omega/S = 0$). It is clear that the Hanjalic and Launder model predicts much too strong a growth rate for the turbulent kinetic energy in homogeneous shear flow. This results from an underprediction of the dissipation rate. In fact, the calculation is terminated a short time after $St = 2$ due to the model becoming unrealizable through negative dissipation rates. Since the Hanjalic and Launder model yields anomalous results for pure shear flow (the most basic case) we will not bother to show the results for non-zero rotation rates.

In Figures 5(a) - (b), the results predicted by the Raj model for rotating shear flow are compared with those of the standard model as well as with the large-eddy simulations of Bardina, Ferziger, and Reynolds [16]. We do not show the results of the Raj model for pure shear flow ($\Omega/S = 0$) since they are identical to those of the standard model. From Figure 5(a), it is clear that the Raj model yields improved predictions over the standard model for the $\Omega/S = 0.25$ case. However, there are problems with the Raj model for rotation rates that are appreciably larger than $\Omega/S = 0.25$ as can be seen in Figure 5(b). For the case where $\Omega/S = 0.5$, the Raj model yields unrealizable results (i.e., the dissipation rate becomes negative for values of St appreciably larger than 7).

It is clear from these results that simple corrections to the modeled dissipation rate equation obtained by adding a source term that depends on rotational strains can give rise to realizability problems (see Lumley [1]). Since the standard modeled dissipation rate equation is realizable with respect to K and ε (i.e., as shown by Speziale [17], the standard model will always yield positive kinetic energies and dissipation rates in homogeneous turbulent flows), it is clear that these kind of ad hoc corrections are counterproductive. In order to properly describe the turbulent flows discussed in this section, directional and two-point information is needed in the modeling of the dissipation rate. Speziale and Gatski [18] have been recently working on the development of a modeled transport equation for the tensor dissipation rate

$$\varepsilon_{ij} \equiv 2\nu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}.$$

This model for the tensor dissipation takes the form

$$\begin{aligned} \dot{\varepsilon}_{ij} &= -\varepsilon_{ik}\left(\frac{\partial \bar{v}_j}{\partial x_k} + 2e_{mkj}\Omega_m\right) - \varepsilon_{jk}\left(\frac{\partial \bar{v}_i}{\partial x_k} + 2e_{mki}\Omega_m\right) \\ &\quad + \frac{2}{3}C_{\varepsilon 1}\frac{\varepsilon}{K}\mathcal{P}\delta_{ij} - C_{\varepsilon 2}\frac{\varepsilon}{K}\varepsilon_{ij} + C_{\varepsilon 3}\varepsilon\bar{S}_{ij} \\ &\quad - C_{\varepsilon 4}(\varepsilon_{ik}\overline{W}_{jk} + \varepsilon_{jk}\overline{W}_{ik}) \end{aligned} \tag{38}$$

in a rotating frame where $C_{\varepsilon 1} = 1.46$, $C_{\varepsilon 2} = 1.83$, $C_{\varepsilon 3} = 1.50$, and $C_{\varepsilon 4} = 1.0$. It was developed for use in conjunction with the SSG model for the pressure-strain correlation derived recently by Speziale, Sarkar, and Gatski [19]. In Figures 6(a) - (c), the results predicted by the tensor dissipation model (used in conjunction with the SSG model) are compared with the predictions of the Launder, Reece, and Rodi model and the large-eddy simulations [16]. It is clear that this new tensor dissipation model does a much better job in capturing the trends of the large-eddy simulations. Most notably, the substantially stronger growth rate of the $\Omega/S = 0.25$ case is, for the most part, captured; furthermore, the model does not restabilize for the $\Omega/S = 0.5$ case – a result that is consistent with linear stability theory as well as with the large-eddy simulations. These improvements are not that surprising in light of the recent findings of Durbin and Speziale [20] which suggest that the dissipation rate tensor can be *anisotropic* in equilibrium homogeneous shear flows at high turbulence Reynolds numbers.

4. Conclusions

Several modifications to the modeled dissipation rate transport equation that have been proposed to account for rotational strains are compared critically for two test flows. The following general conclusions can be drawn:

(1) The Pope [5] model yields the *same* predictions as the standard model for rotating isotropic turbulence and for rotating homogeneous shear flow. These predictions are deficient in that they fail to properly account for the reduction in the turbulence decay rate that results from a system rotation of isotropic turbulence and they fail to account accurately for the changes in the growth rate of the turbulent kinetic energy that result from a system rotation of homogeneous shear flow which can either stabilize or destabilize the flow.

(2) The Hanjalic and Launder [6] model has major problems with the violation of realizability. It yields negative dissipation rates in rotating isotropic turbulence for appreciable values of $\Omega K_0/\varepsilon_0$. Furthermore, this modification degrades significantly the predictions for homogeneous shear flow (the predicted growth rate of the turbulent kinetic energy is much too large).

(3) The Bardina model [7] is only able to accurately predict the reduction in the decay rate of the turbulent kinetic energy in rotating isotropic turbulence for weak to moderate rotation rates where this effect is small. For $\Omega K_0/\varepsilon_0 \gg 1$, this model predicts too strong a reduction in the turbulence decay rate. Furthermore, this model does not yield any improvements for rotating homogeneous shear flow and mildly degrades the predictions for pure shear flow.

(4) The Raj [8] model yields the *same* deficient predictions as the standard model in rotating isotropic turbulence. In rotating homogeneous shear flow, this model does yield some improvements for moderate rotation rates for which $\Omega/S < 0.3$. However, for stronger rotation rates the model becomes unrealizable and yields negative values for the turbulent dissipation rate.

In order to properly describe rotating turbulent flows, some directional and two-point information needs to be incorporated into models for the dissipation rate rather than the ad hoc modifications discussed above. The directional information can be incorporated through the use of the tensor dissipation. In this regard, the preliminary calculations presented in Section 3 for rotating shear flow based on a modeled tensor dissipation rate equation appear to be promising. However, the reduction in the decay rate of the turbulent kinetic energy in rotating isotropic turbulence is a two-point phenomena; the inertial waves generated by a system rotation scramble the transfer term in such a way that the phase coherence needed to cascade energy from large to small scales is disturbed. Some limited two-point information needs to be included based on an appropriate integral length scale which responds to rotational strains. This issue is currently under investigation and will be the subject of a future paper.

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References

[1] Lumley, J. L., 1978, *Computational Modeling of Turbulent Flows*, Adv. Appl. Mech. **18**, pp. 123-176.

[2] Speziale, C. G., 1990, *Analytical Methods for the Development of Reynolds Stress Closures in Turbulence*, Ann. Rev. Fluid Mech. **23**, pp. 107-157.

[3] Launder, B. E. and Spalding, D. B., 1974, *The Numerical Computation of Turbulent Flows*, Comput. Methods Appl. Mech. & Engrg. **3**, pp. 269-289.

[4] Launder, B. E., 1990, *Phenomenological Modeling: Present and Future*, Lecture Notes in Physics (J. L. Lumley, ed., Springer-Verlag, New York) **357**, pp. 439-485.

[5] Pope, S. B., 1978, *An Explanation of the Turbulent Round Jet/Plane Jet Anomaly*, AIAA J. **16**, pp. 279-281.

[6] Hanjalic, K. and Launder, B. E., 1980, *Sensitizing the Dissipation Equation to Irrotational Strains*, ASME J. Fluids Eng. **102**, pp. 34-40.

[7] Bardina, J., Ferziger, J. H., and Rogallo, R. S., 1985, *Effect of Rotation on Isotropic Turbulence: Computation and Modeling*, J. Fluid Mech. **154**, pp. 321-336.

[8] Raj, R., 1975, *Form of the Turbulence Dissipation Equation as Applied to Curved and Rotating Turbulent Flows*, Phys. Fluids **18**, pp. 1241-1244.

[9] Bardina, J., 1988, *Turbulence Modeling Based on Direct Simulation of the Navier-Stokes Equations*, Proceedings of the 1st National Fluid Dynamics Congress, Paper No. 88-3747-CP.

[10] Speziale, C. G., 1989, *Turbulence Modeling in Non-Inertial Frames of Reference*, Theoret. & Comput. Fluid Dynamics **1**, pp. 3-19.

[11] Raj, R. and Speziale, C. G., 1990, *A Note on the Dissipation Equation in Rotating Turbulent Flows*, Phys. Fluids A, to be submitted for publication.

[12] Launder, B. E., Reece, G., and Rodi, W., 1975, *Progress in the Development of a Reynolds Stress Turbulence Closure*, J. Fluid Mech. **68**, pp. 537-566.

[13] Speziale, C. G. and Mac Giolla Mhuiris, N., 1989, *On the Prediction of Equilibrium States in Homogeneous Turbulence*, J. Fluid Mech. **209**, pp. 591-615.

[14] Speziale, C. G., Mansour, N. N., and Rogallo, R. S., 1987, *The Decay of Isotropic Turbulence in a Rapidly Rotating Frame*, Proceedings of the 1987 Summer Program of the Center for Turbulence Research (P. Moin, W. C. Reynolds, and J. Kim, eds., Stanford University Press), pp. 205-211.

[15] Wigeland, R. A. and Nagib, H. M., 1978, *Grid-Generated Turbulence With and Without Rotation about the Streamwise Direction*, IIT Fluids and Heat Transfer Report R78-1, Illinois Institute of Technology.

[16] Bardina, J., Ferziger, J. H., and Reynolds, W. C., 1983, *Improved Turbulence Models Based on Large-Eddy Simulation of Homogeneous, Incompressible Turbulent Flows*, Stanford University Technical Report TF-19.

[17] Speziale, C. G., 1990, *Discussion of Turbulence Modeling: Present and Future*, Lecture Notes in Physics (J. L. Lumley, ed., Springer-Verlag, New York) **357**, pp. 490-512.

[18] Speziale, C. G. and Gatski, T. B., 1990, *A Model for the Tensor Dissipation Rate of Turbulence*, Phys. Fluids A, to be submitted for publication.

[19] Speziale, C. G., Sarkar, S., and Gatski, T. B., 1989, *Modeling the Pressure-Strain Correlation of Turbulence – An Invariant Dynamical Systems Approach*, J. Fluid Mech., in press.

[20] Durbin, P. A. and Speziale, C. G., 1990, *Local Anisotropy in Strained Turbulence at High Reynolds Numbers*, Phys. Fluids A, submitted for publication.

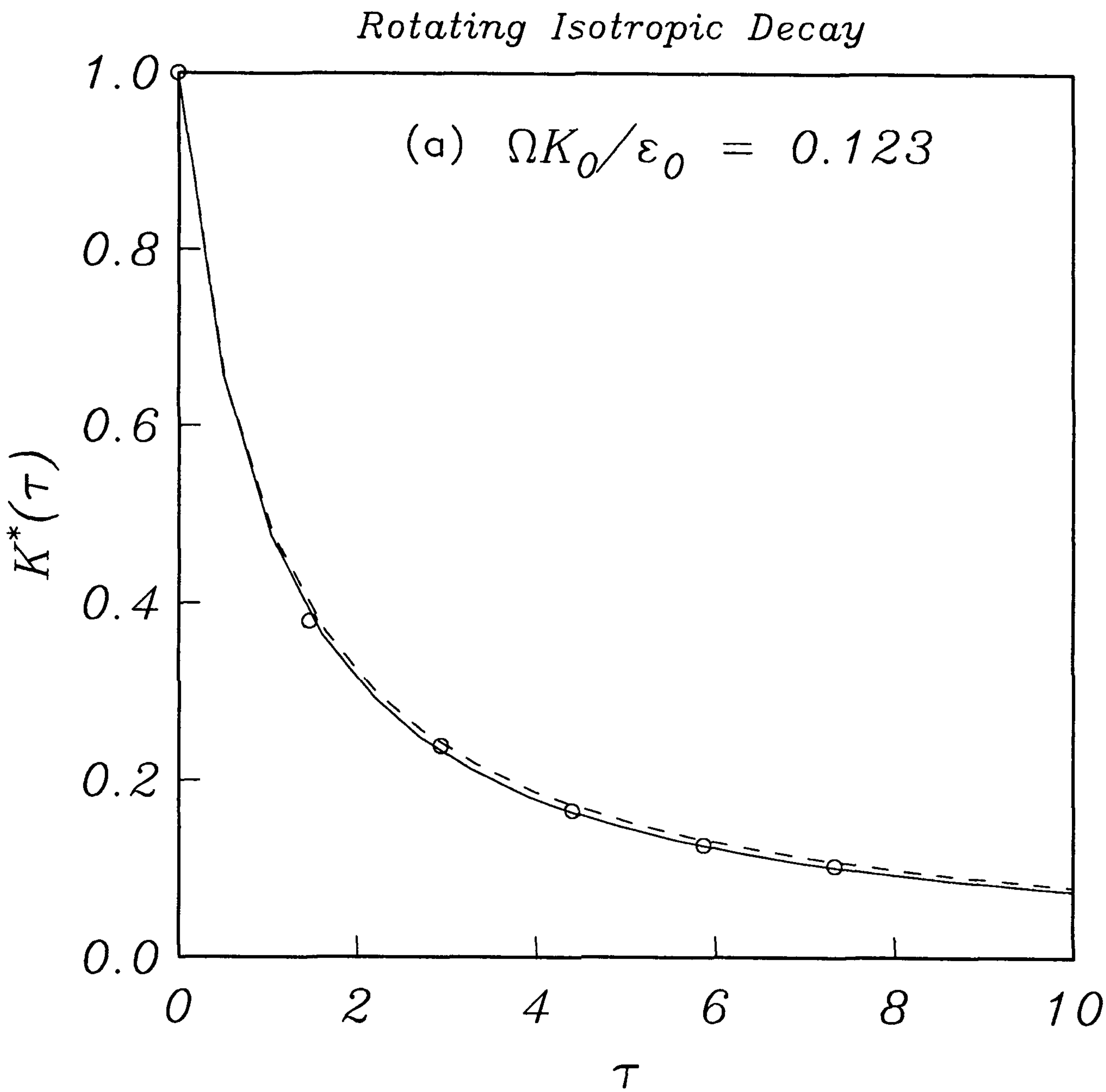
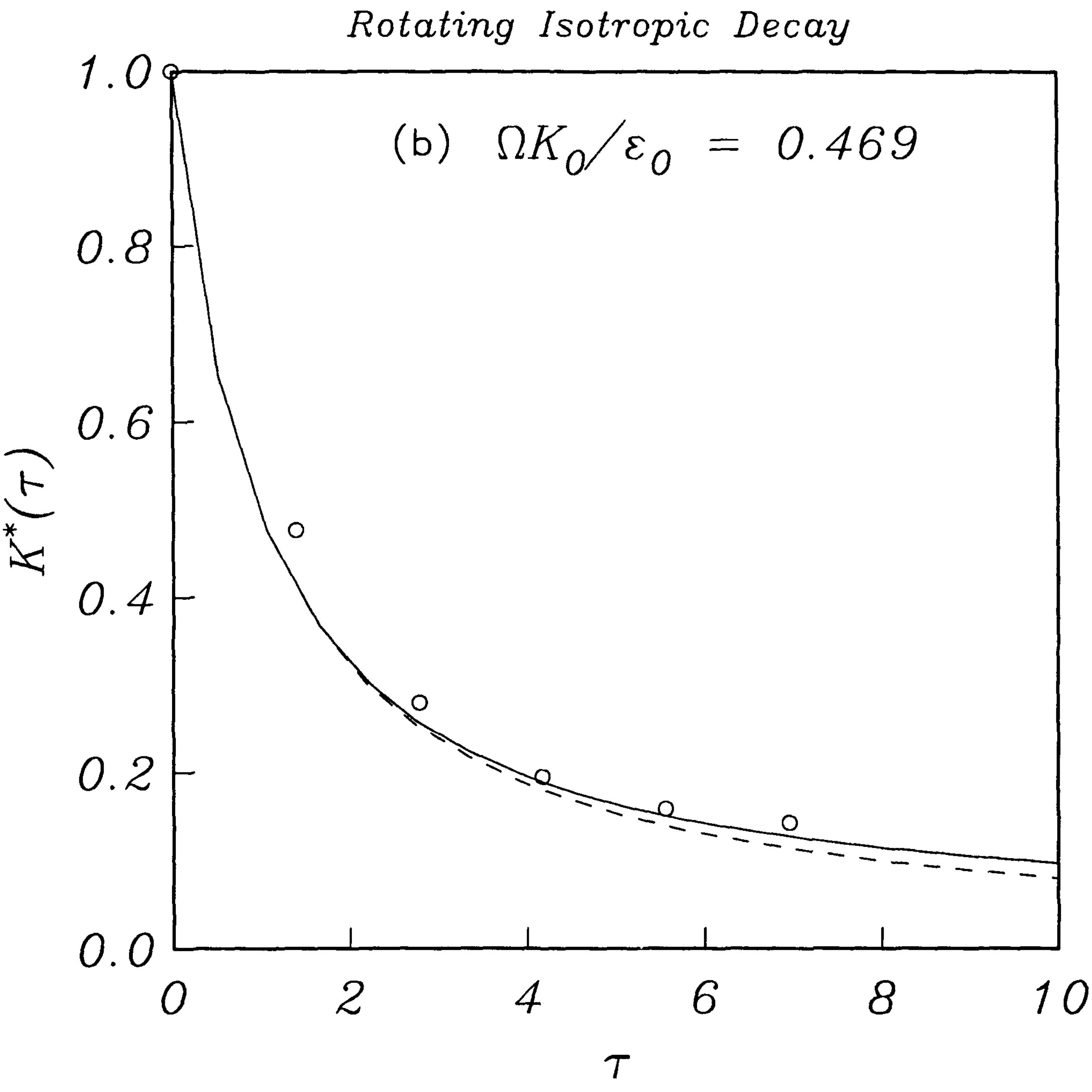


Figure 1. The decay of turbulent kinetic energy in rotating isotropic turbulence: —— Bardina model; - - - standard model; o experimental data [15]; • simulations [14]. (a) $\Omega K_0/\varepsilon_0 = 0.123$, (b) $\Omega K_0/\varepsilon_0 = 0.469$, (c) $\Omega K_0/\varepsilon_0 = 69.5$.



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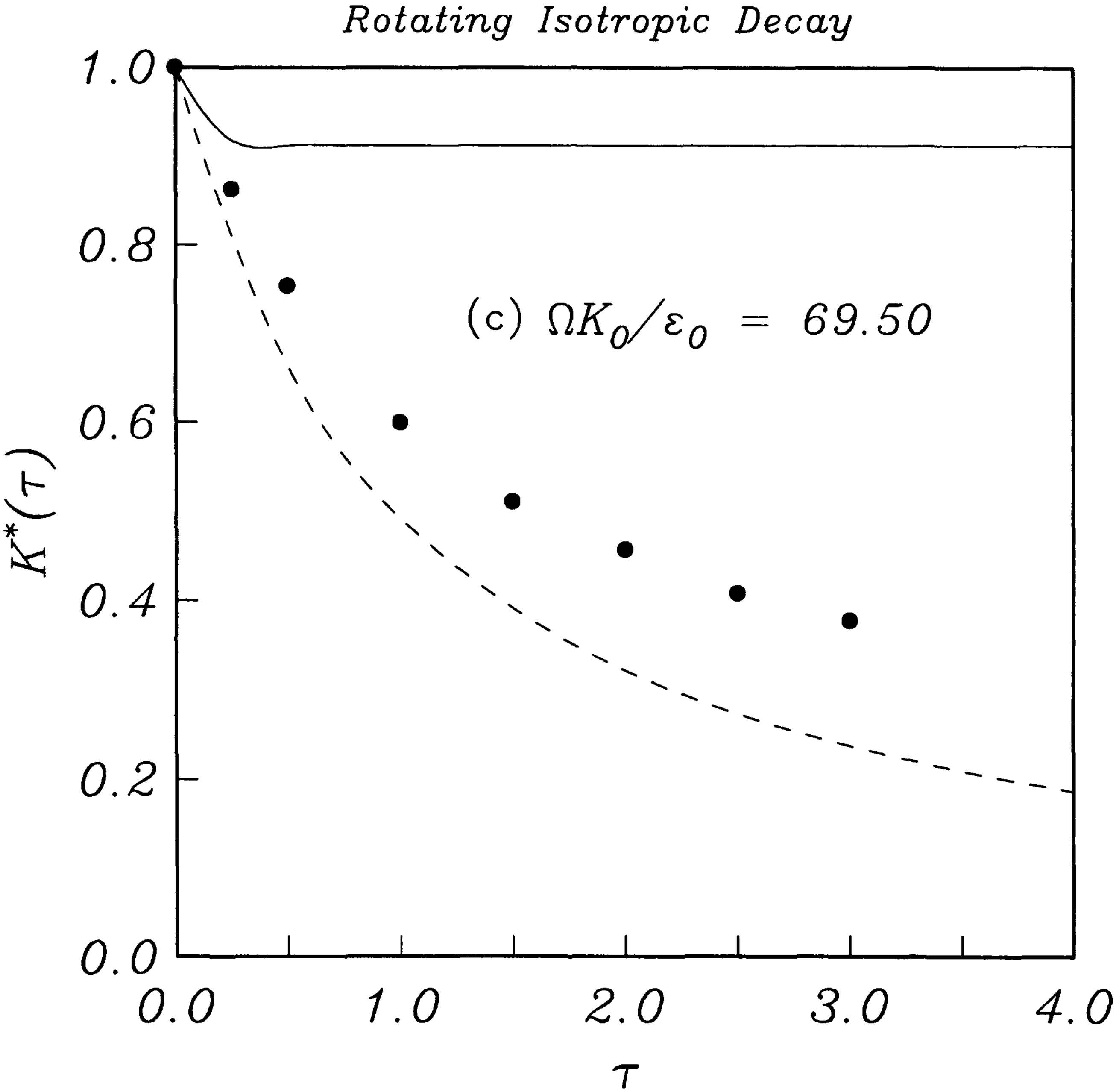

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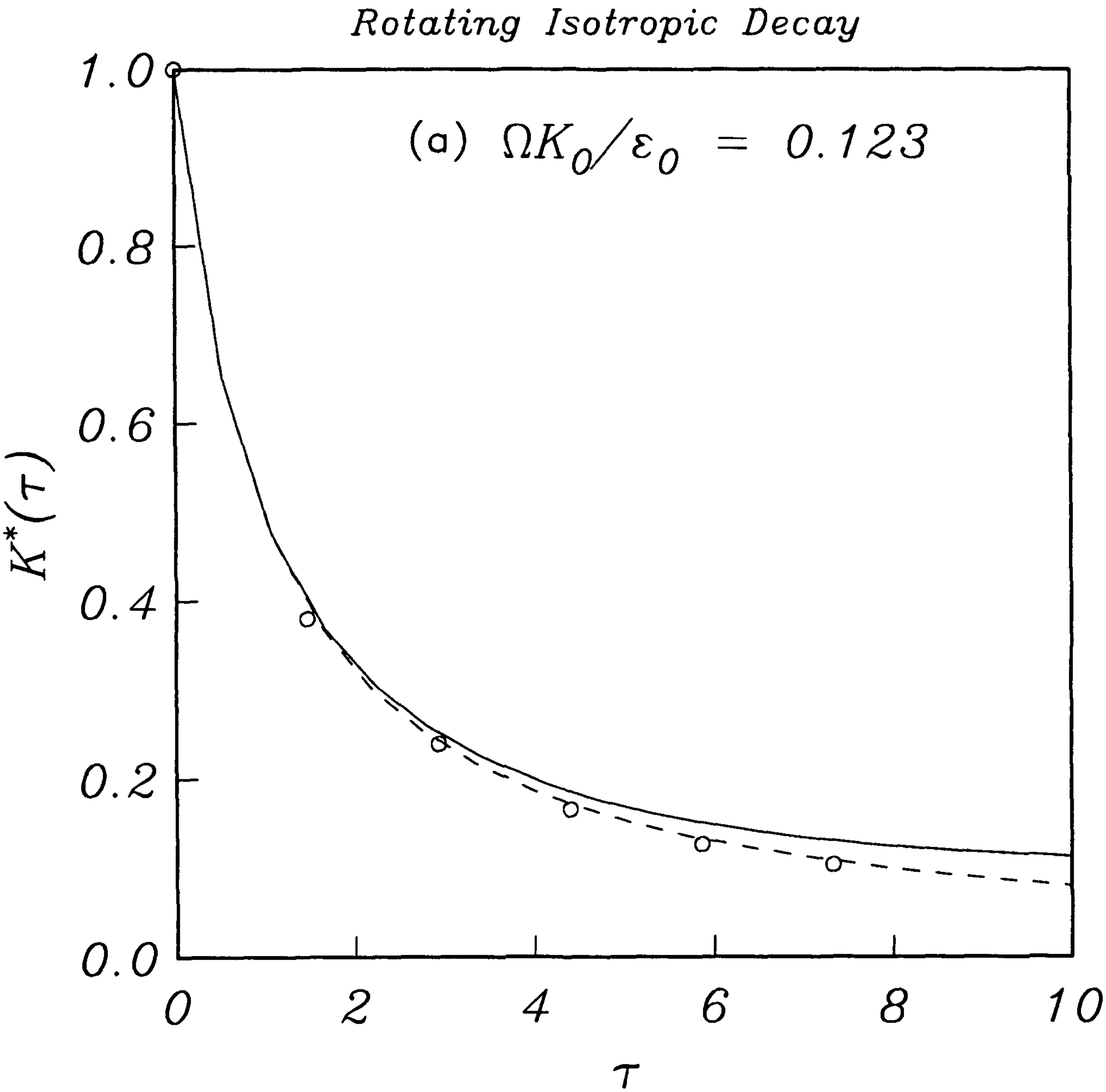
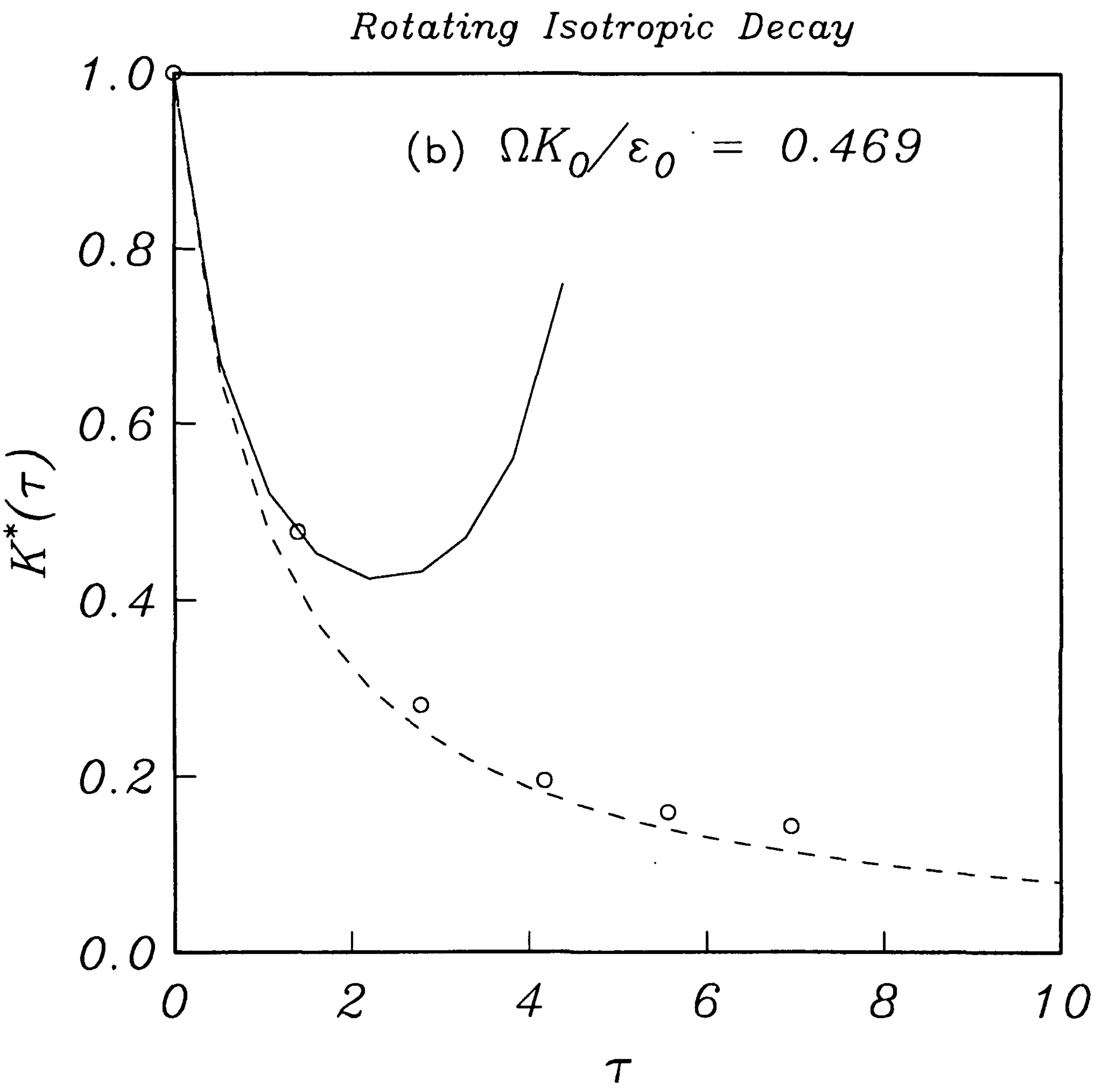


Figure 2. The decay of turbulent kinetic energy in rotating isotropic turbulence: —— Hanjalic and Launder model; - - - standard model; o experimental data [15]. (a) $\Omega K_0/\varepsilon_0 = 0.123$, (b) $\Omega K_0/\varepsilon_0 = 0.469$.



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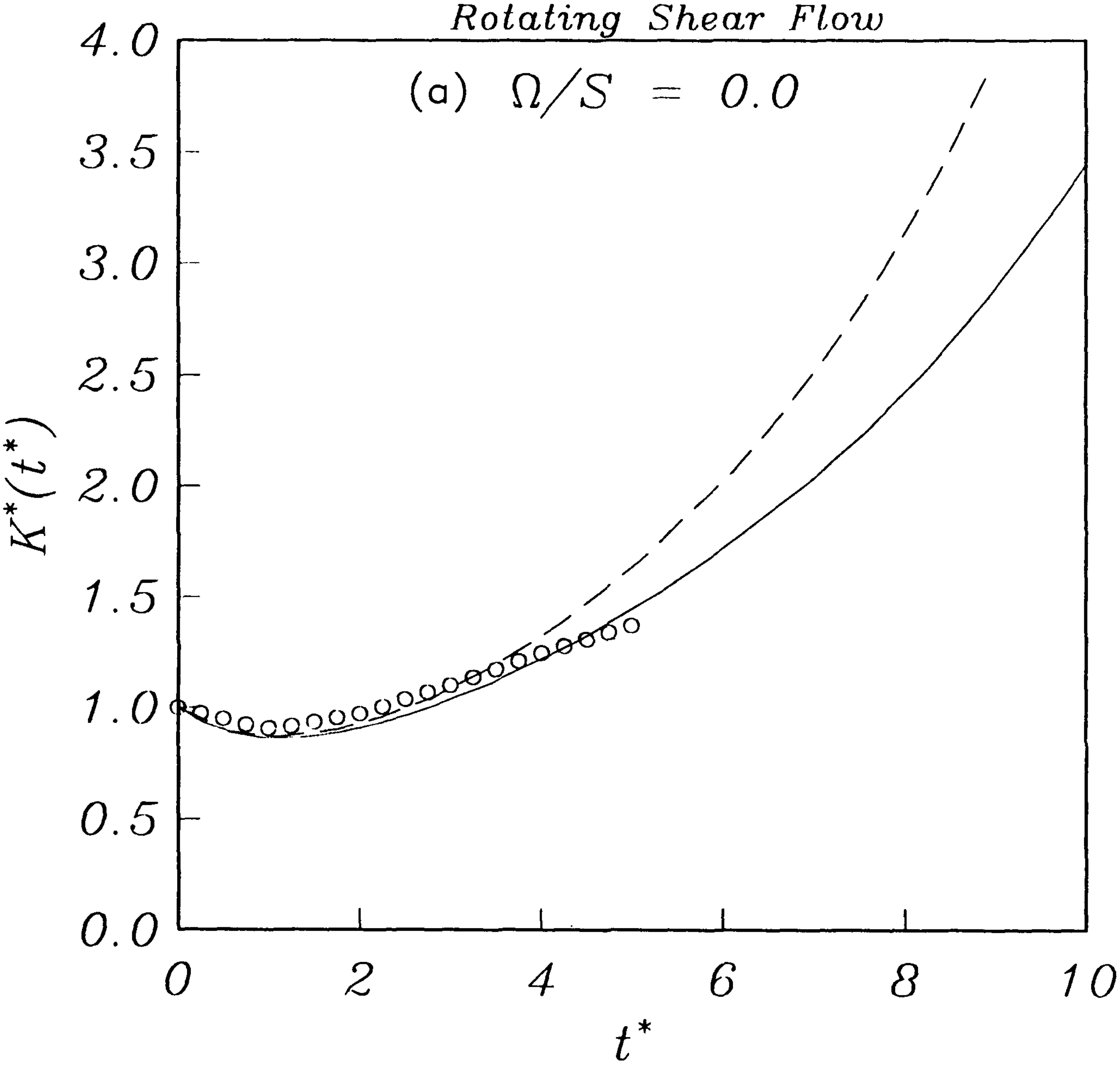
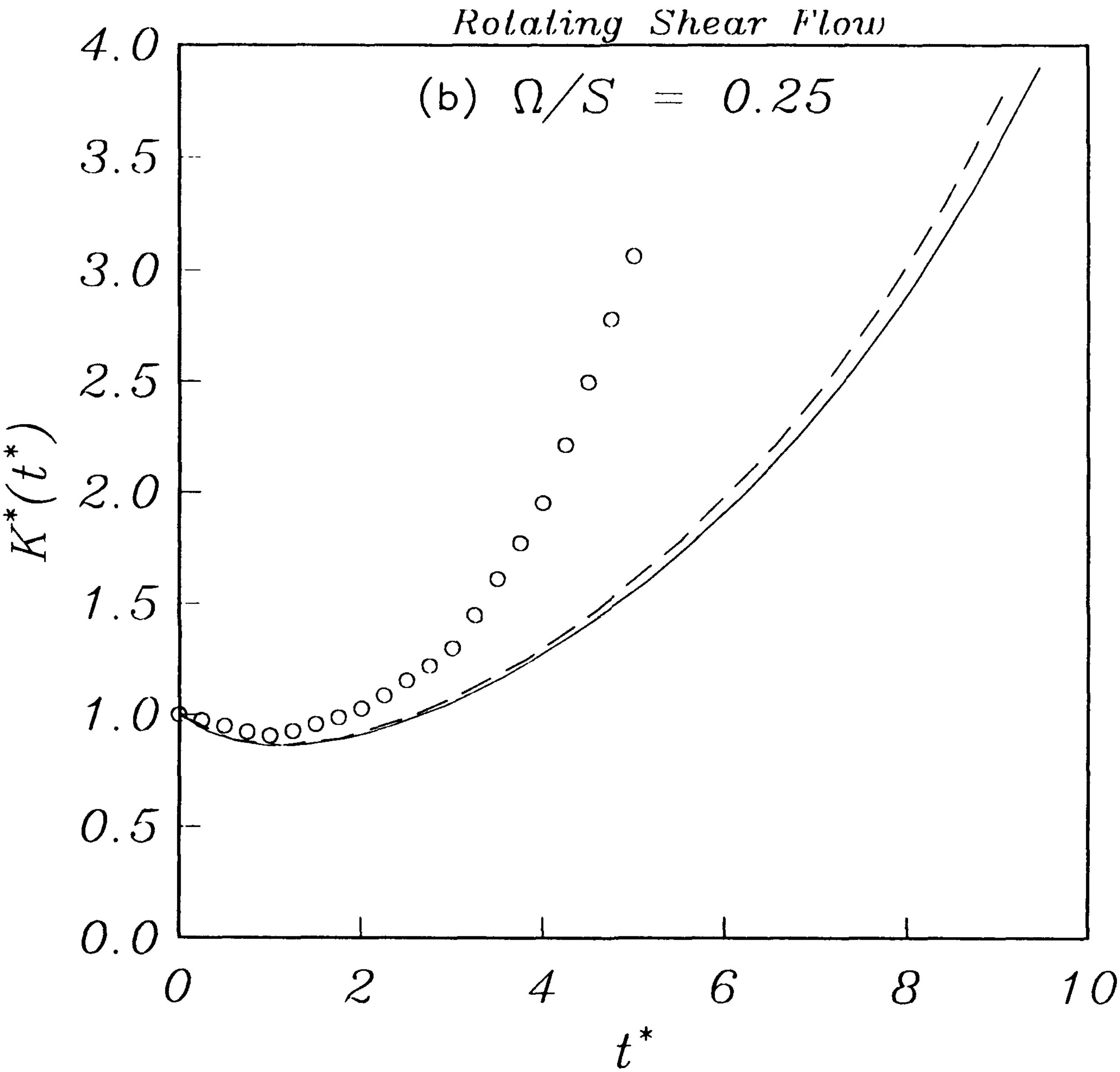


Figure 3. Time evolution of the turbulent kinetic energy in rotating shear flow for $\varepsilon_0/SK_0 = 0.296$: —— standard model; - - - Bardina model; o large eddy simulations [16]. (a) $\Omega/S = 0$, (b) $\Omega/S = 0.25$, (c) $\Omega/S = 0.5$.



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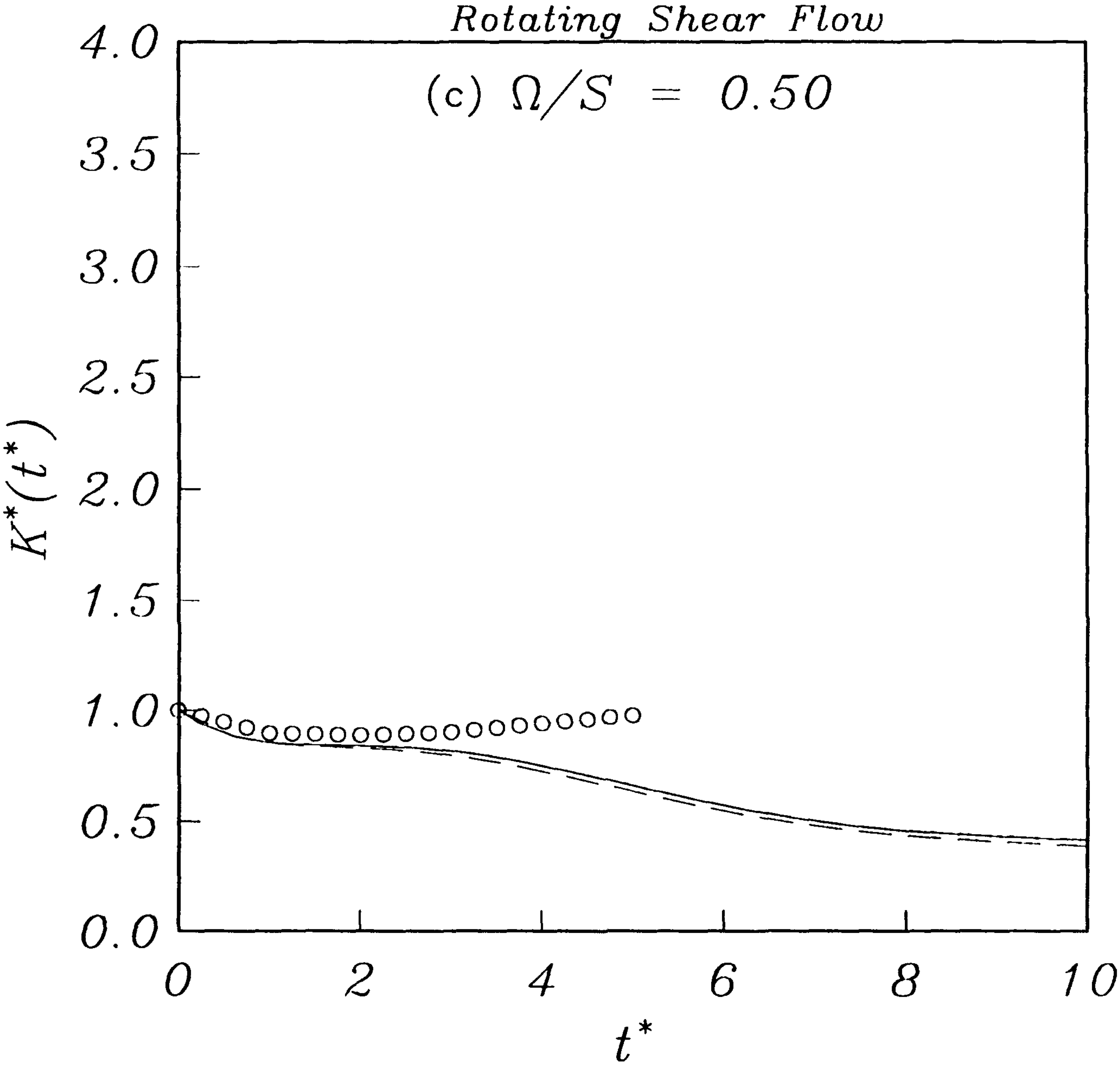

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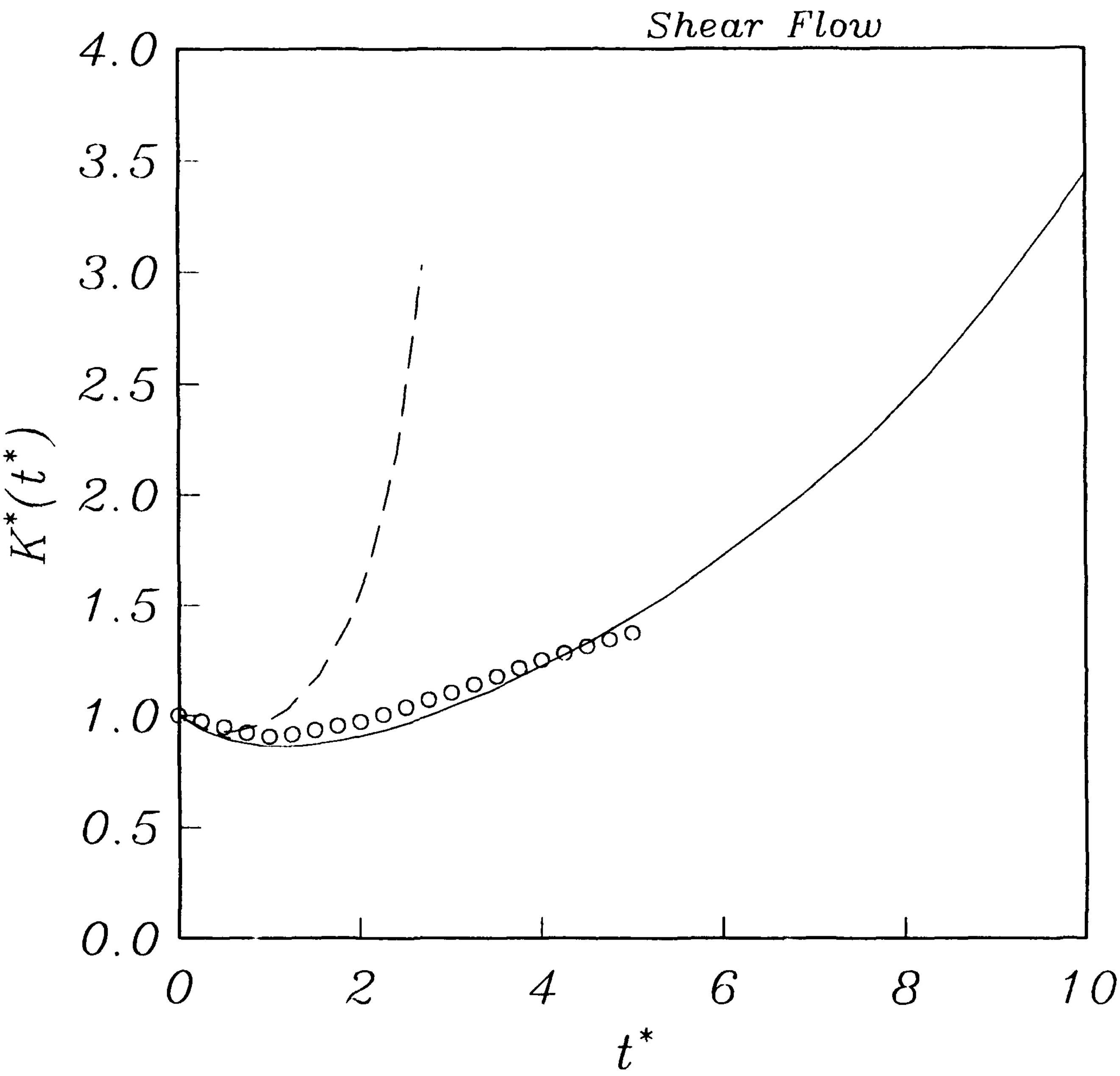


Figure 4. Time evolution of the turbulent kinetic energy in homogeneous shear flow for $\varepsilon_0/SK_0 = 0.296$: —— standard model; - - - Hanjalic and Launder model; o large eddy simulation [16].

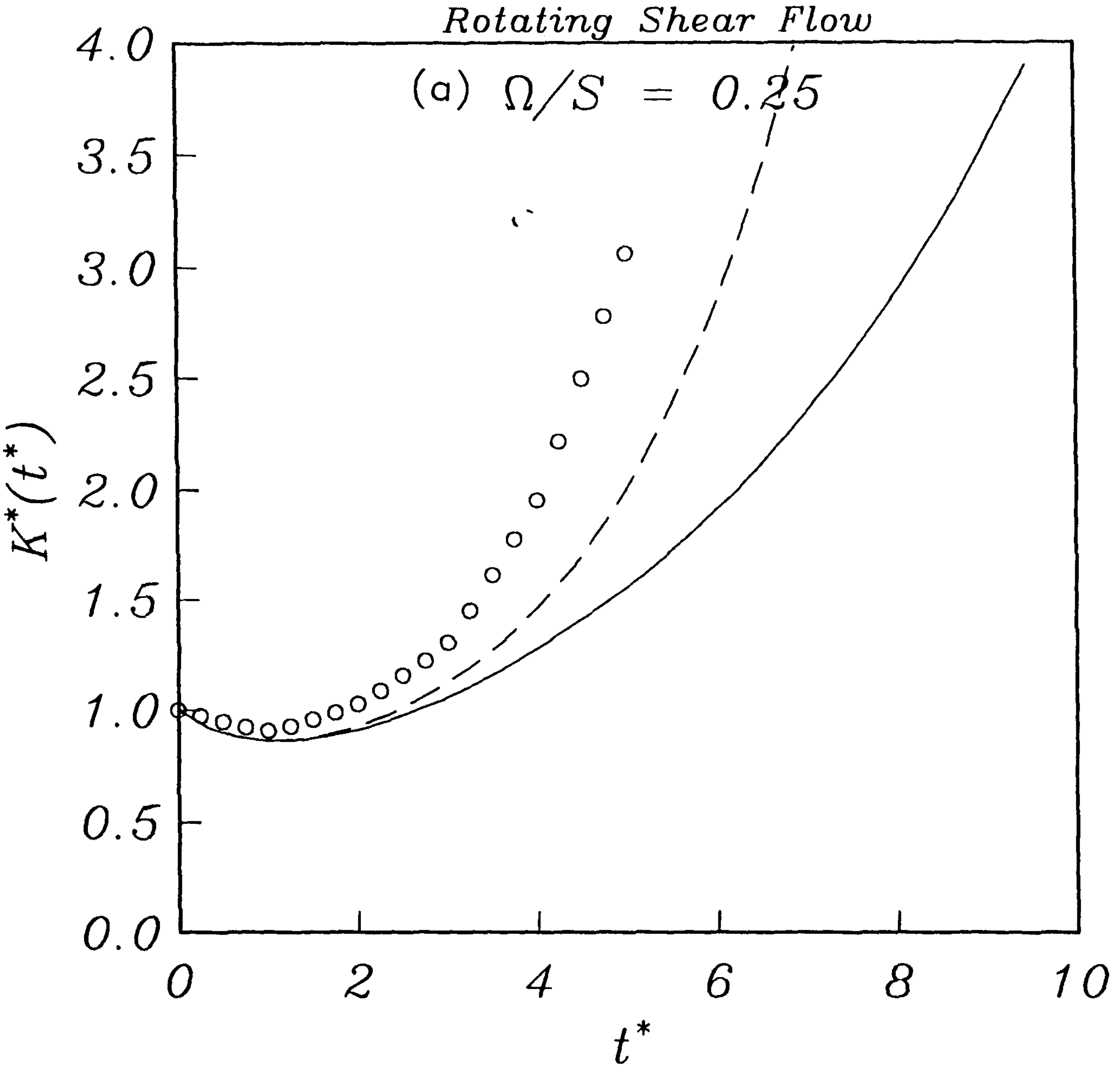
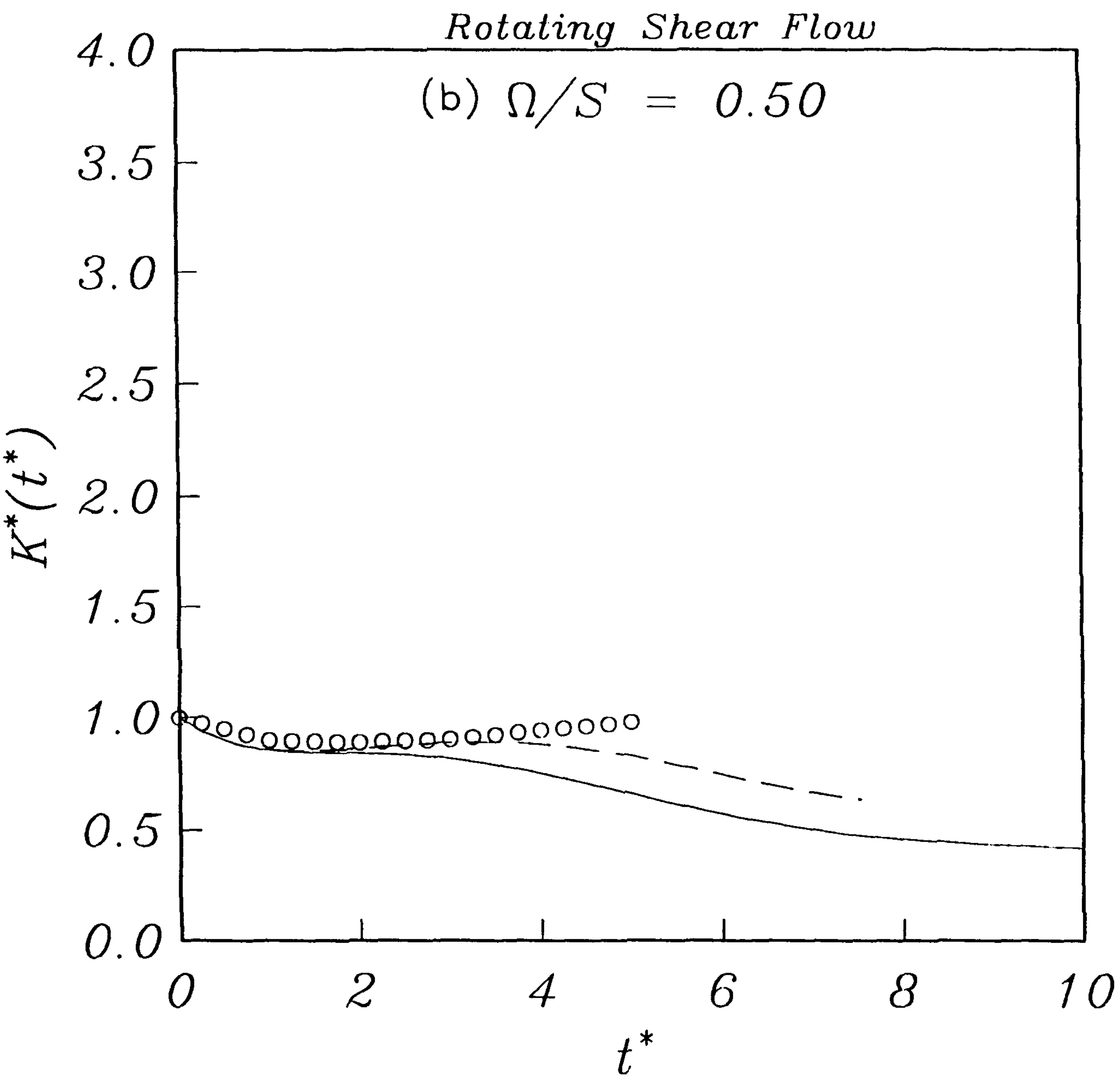


Figure 5. Time evolution of the turbulent kinetic energy in rotating shear flow for $\varepsilon_0/SK_0 = 0.296$: —— standard model; - - - Raj model; ∘ large eddy simulations [16]. (a) $\Omega/S = 0.25$, (b) $\Omega/S = 0.5$.



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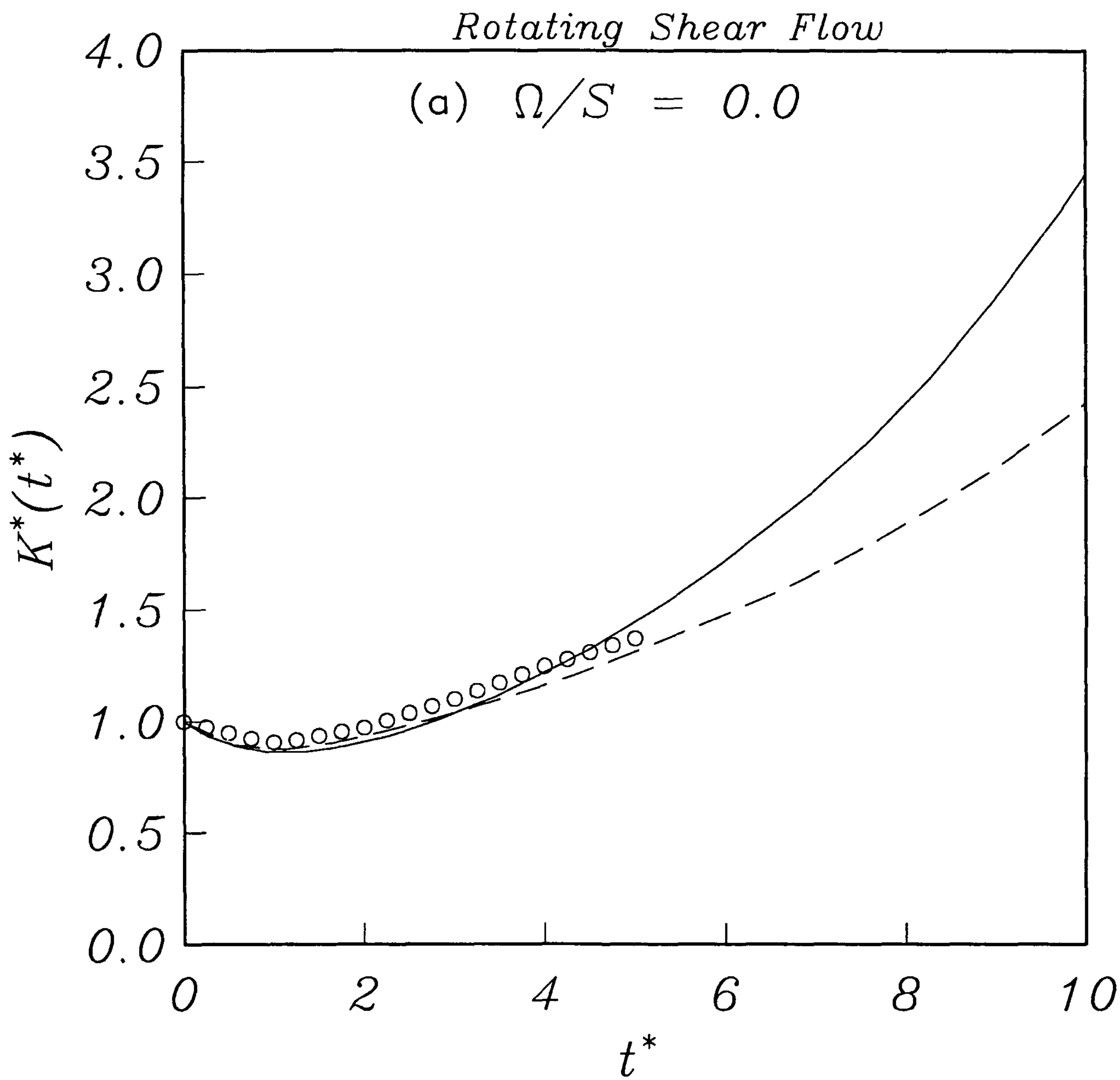
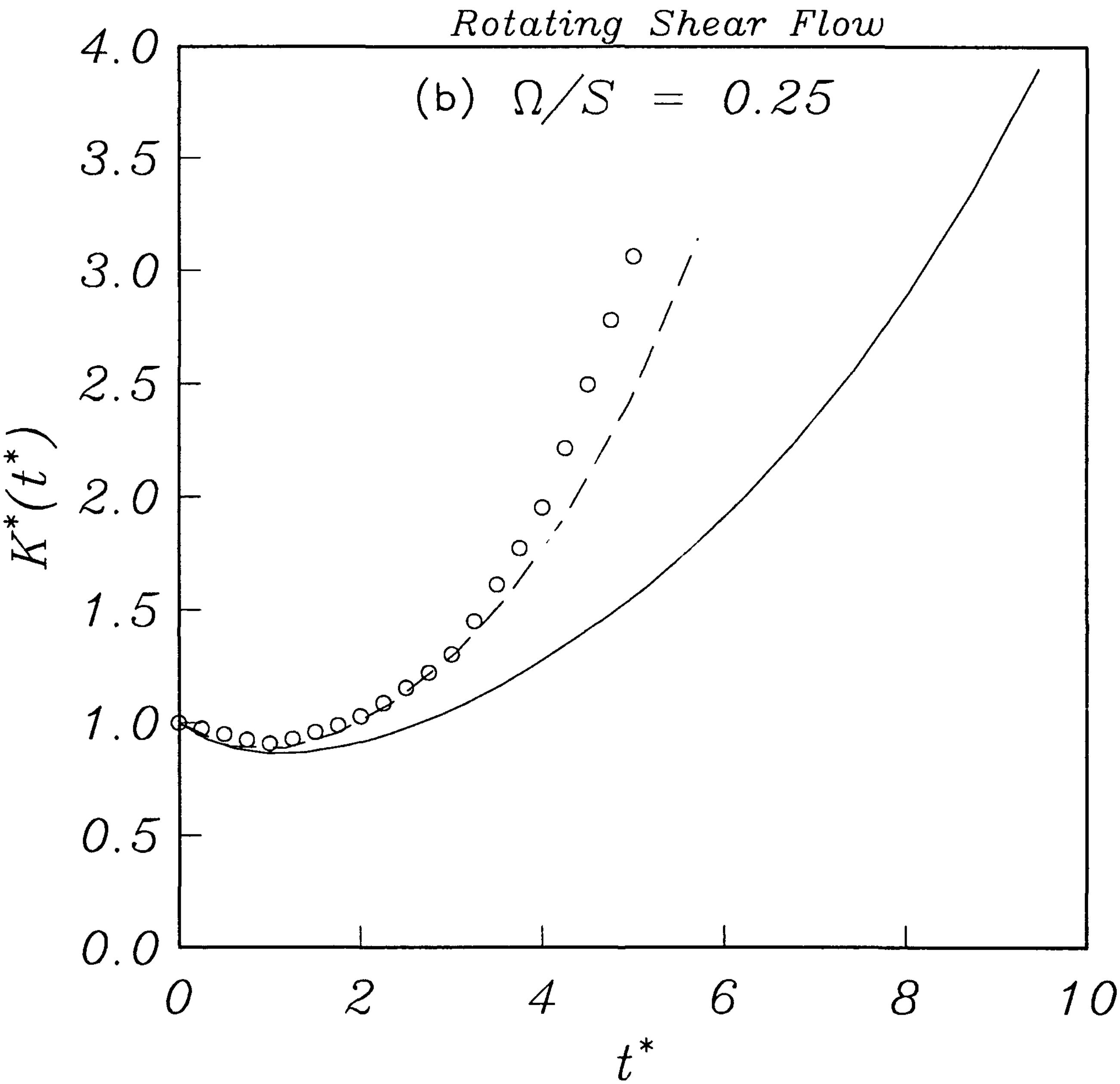


Figure 6. Time evolution of the turbulent kinetic energy in rotating shear flow for $\varepsilon_0/SK_0 = 0.296$: —— standard model; - - - tensor dissipation model [18]; o large eddy simulations [16]. (a) $\Omega/S = 0$, (b) $\Omega/S = 0.25$, (c) $\Omega/S = 0.5$.



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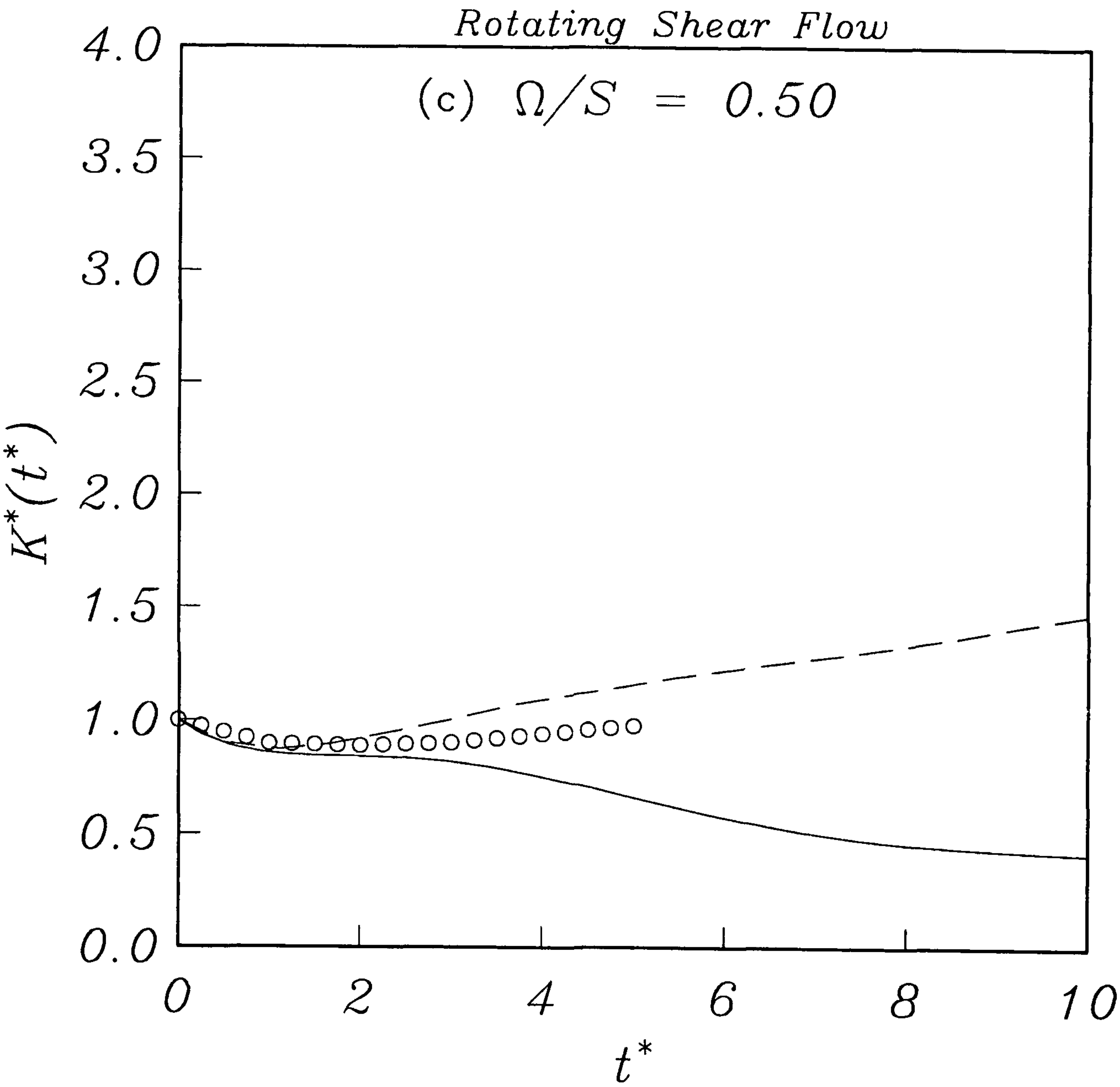

Figure 6. Time evolution of the turbulent kinetic energy in rotating shear flow for $\varepsilon_0/SK_0 = 0.296$: —— standard model; - - - tensor dissipation model [18]; o large eddy simulations [16]. (a) $\Omega/S = 0$, (b) $\Omega/S = 0.25$, (c) $\Omega/S = 0.5$.