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ADVANCED MODELS FOR CLUTTER

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13. ABSTRACT (Maximum 200 words)

This report summarized three important results obtained during the reporting period. First, it is shown that the Luneburg-Kline asymptotic series for the current induced on a conducting rough surface always produces an error in the shadow zone. That is, the L-K series yields zero net current in the shadow zone for all terms in the asymptotic series. This shortcoming limits the usefulness of the L-K series for recovering diffraction effects. The method of smoothing is used to determine the fluctuating component of the field scattered by a randomly positioned collection of discrete scatterers to the level of the distorted wave Born approximation. Of particular importance here is the role of the coherent or average field in the medium as a source for the fluctuating field. Finally, it is shown how to go about developing an improved high frequency approximation for scattering from rough surfaces. In particular, it is shown that the shadowed Kirchhoff approximation should comprise the iterate in the basic integral equation for the current because it contains an infinite resummation of all of the shadowing effects.

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1. Introduction

The primary intent of this research is to develop new models for estimating clutter levels from natural terrain with particular application to surveillance radar. Specific attention under this contract is to be directed toward estimating the effects of subsurface volume scattering. For radar frequencies above s–band, most natural surface materials do not permit much penetration into the subsurface volume. However, for frequencies in the UHF range, there is the possibility that some penetration of the incident electromagnetic energy into the subsurface volume will be possible. A good example of this was provided by NASA's Shuttle Imaging Radar (SIR–B) mission which detected ancient stream beds beneath the desert in Africa.

The penetration of the radar signal into the volume beneath the surface is important because of the potential scattering of this energy by discontinuities in the dielectric constant of the subsurface material. These abrupt changes could be caused by rocks, moisture, pockets, tunnels, and vegetation roots. If these materials scatter energy back up through the surface, they comprise yet another source of clutter which must be accounted for. This is the motivation for considering the possibility of subsurface scattering and attempting to model it.

The second chapter presents the results of our efforts to use a Luneburg–Kline representation to improve on the physical optics approximation for the current induced on a rough surface. We find the L–K representation to be flawed because of its failure to properly represent the current in the optical shadow regions of the rough surface. In particular, the L–K representation predicts that the current will be exactly zero in the shadow regions irrespective of frequency.

The third chapter discusses a distorted wave Born approximation for the fluctuating field in a discrete random media. This is the approximation that we will be using to model the scattering by subsurface objects, so it is very important to this research.

The fourth chapter discusses a way to improve on the standard physical optics approximation for the field scattered by a rough surface in the high frequency limit. This is the method that should be used rather than the Luneburg–Kline representation.

2. An Inherent Limitation of the Luneburg–Kline Representation for the Current on a Conducting Body

2.1 Background

In the field of scattering from randomly rough surfaces, there is a very pressing need to develop tractable high frequency approximations which are an improvement on physical optics. Of course, the first candidate that comes to mind is the Fock theory of shadow boundary diffraction [2.1]. However, because of its dependence upon certain canonical geometries, it is not obvious how the Fock theory could be beneficially applied to the randomly rough surface problem. This same statement applies to most of the other theories that provide some improvement on physical optics [2.2].

Recently, Lee [2.3] corrected and extended the earlier work of Schensted [2.4] in applying a Luneburg–Kline formalism to scattering from a perfectly conducting body. Ansorge [2.5] used the same methods developed by Lee to obtain results for scattering from dielectric bodies. What Lee was able to do was to obtain a k_0^{-1} correction to the physical optics approximation for the current induced on a perfectly conducting body by an incident plane wave. Although the results were very complicated from an algebraic point of view, they did not require any special canonical scatterer geometry. Lee obtained these results by satisfying boundary conditions on the surface for each field vector expansion coefficient in the Luneburg–Kline series of inverse powers of wavenumber (k_0). Lee noted that the derivation of higher order correction terms was possible but algebraically quite involved.

Lee's results have clearly shown that the Luneburg–Kline formalism is capable of providing wavenumber dependent corrections to the physical optics current induced on a conducting scatterer. The next most important question that needs to be addressed concerns the limitations of this approach and its subsequent results. That is, are there any fundamental limitations of this approach and what are the significance of these relative to the accuracy of the results produced by the technique? The purpose of this paper is to investigate these questions.

The approach to be taken in this paper will be somewhat different from the one employed by Lee because he assumed a well–formed shadow zone and no multiple scattering [2.3]. These assumptions are not necessary if one starts with an integral equation for the current induced on the surface such as the magnetic field integral equation (MFIE). Thus, the methodology to be used in this paper is to expand the current, normalized by the k_0–dependent phase factor contained in the incident field, in a Luneburg–Kline series and then to use the MFIE to solve for the series coefficients. It will be shown that each coefficient satisfies an integral equation of the second kind in which the Born term requires knowledge of all lower order coefficients. Furthermore, these integral equations can all be solved exactly because of the particular form of the kernel. The solution shows what appears to be a fundamental limitation of the Luneburg–Kline representation for the induced surface current, namely, that all the expansion coefficients are identically zero in the shadow zone. That is, none of the field penetration into the shadow zone that is known to occur for k_0 finite can be recovered using the Luneburg–Kline formalism. This deficiency is traced to the asymptotic nature of the series and its inability to represent creeping wave type currents in the shadow zone. The conclusion of this paper is that the Luneburg–Kline representation for the current induced on a perfectly conducting body is incapable of accurately predicting the current in the shadow zone and is therefore limited in its capabilities irrespective of the number of terms retained in the series.

2.2 Analysis

The problem to be addressed here is the determination of the electric current density $\vec{J}_s$ induced on the surface $z = \zeta(\vec{r}_t)$ by an incident magnetic field $\vec{H}^i$. (The generalization of these results to a arbitrarily shaped closed body is not difficult.) The surface $z = \zeta(\vec{r}_t)$ separates free space $(z > \zeta)$ from a perfectly conducting medium $(z \leq \zeta)$. The unit vector $\hat{k}_i$ specifies the direction of travel of the incident field. The electric surface current density $\vec{J}_s$ must satisfy the magnetic field integral equation (MFIE) as follows;

$$\vec{J}_s(\vec{r}) = 2\hat{n}(\vec{r})\times\vec{H}^i(\vec{r}) + 2\hat{n}(\vec{r})\times \int \vec{J}_s(\vec{r}_o) \times \nabla_o G(\vec{r} - \vec{r}_o) ds_o \tag{2.1}$$

In (1), $\hat{n}(\vec{r})$ is the unit normal to the surface at the point $\vec{r} = \vec{r}_t + \zeta(\vec{r}_t)\hat{z}$ and is given by

$$\hat{n}(\vec{r}) = [-\zeta_x\hat{x} - \zeta_y\hat{y} + \hat{z}]/(1 + \zeta_x^2 + \zeta_y^2)^{1/2} \tag{2.2}$$

where $\zeta_x = \partial\zeta/\partial x$ and $\zeta_y = \partial\zeta/\partial y$ are the x and y surface slopes at the point $\vec{r}$ on the surface. $G(\vec{r} - \vec{r}_o)$ is the free space Green's function, i.e.

$$G(\vec{r} - \vec{r}_o) = \exp(-jk_o|\vec{r} - \vec{r}_o|)/4\pi|\vec{r} - \vec{r}_o|, \tag{2.3}$$

and ∇_o is the conventional three–dimensional gradient evaluated on the surface ($z = \zeta$ and $z_o = \zeta_o$). Noting that the area integration over the surface can be converted to one over the $z = 0$ plane through

$$ds_o = (1 + \zeta_x^2 + \zeta_y^2)^{1/2} \; d\vec{r}_{t_o}$$

where $d\vec{r}_{t_o} = dx_o dy_o$, (1) can be rewritten as follows;

$$\vec{J}(\vec{r}) = 2\vec{N}\times\vec{H}^i + 2\vec{N}\times\int \vec{J}(\vec{r}_o)\times\nabla_o G(|\vec{r} - \vec{r}_o|)d\vec{r}_{t_o} \tag{2.4}$$

where

$$\vec{J}(\vec{r}) = \vec{J}_s(\vec{r})(1 + \zeta_x^2 + \zeta_y^2)^{1/2} \tag{2.5}$$

and

$$\vec{N} = -\zeta_x\hat{x} - \zeta_y\hat{y} + \hat{z} \tag{2.6}$$

If $\vec{J}(\vec{r})$ can be determined from (2.4), the scattered field can be found from the following integral expression;

$$\vec{H}^s(\vec{R}) = \nabla\times\int \vec{J}(\vec{r})G(|\vec{R} - \vec{r}|)d\vec{r}_t \tag{2.7}$$

As a preparatory step to introducing the Luneburg–Kline expansion, the k_o dependence introduced by the incident field is removed. That is, with

$$\vec{J}(\vec{r}) = \vec{L}(\vec{r}) \exp(-j\vec{k}_i \cdot \vec{r}), \tag{2.8}$$

$$\vec{H}^i(\vec{r}) = \vec{h}_i \exp(-j\vec{k}_i \cdot \vec{r}), \tag{2.9}$$

and $\vec{h}_i$ a constant, (2.4) may be rewritten as follows;

$$\vec{L}(\vec{r}) = 2\vec{N} \times \vec{h}_i + 2\vec{N} \times \int \vec{L}(\vec{r}_o) \times\nabla_o G(|\vec{r} - \vec{r}_o|) \exp(j\vec{k}_i \cdot \Delta\vec{r})d\vec{r}_{t_o} \tag{2.10}$$

where $\Delta\vec{r} = \vec{r} - \vec{r}_o$. The purpose of (2.8) is to remove the known high frequency behavior or dependence on k_o from the current. That is, as $k_o \to \infty$ it is known that $\vec{L}(\vec{r})$ is independent of k_o. The modified current $\vec{L}(\vec{r})$ is expanded in a Luneburg–Kline series, i.e.

$$\vec{L}(\vec{r}) = \sum_{n=o} \frac{\vec{j}_n(\vec{r})}{k_o^n}. \tag{2.11}$$

where the vector expansion coefficient $\vec{j}_n(\vec{r})$, n = 0, 1, ..., are independent of the electromagnetic wavenumber k_o. (2.11) is next substituted into (2.10) and it is assumed that term by term integration is permissible so that the following results;

$$\sum_{n=o} \vec{j}_n(\vec{r})k_o^{-n} = 2\hat{N} \times \vec{h}_i + 2\hat{N} \times \sum_{n=o} k_o^{-n} \int \vec{j}_n(\vec{r}_o) \times \nabla_o G \exp(j\vec{k}_i \cdot \Delta\vec{r})d\vec{r}_{t_o} \tag{2.12}$$

In order for (2.12) to be satisfied, the integral term must also have a Luneburg–Kline expansion. That is, if

$$\vec{T}_n(\vec{r}, k_o) = \int \vec{j}_n(\vec{r}_o) \times \nabla_o G \exp(j\vec{k}_i \cdot \Delta\vec{r})d\vec{r}_{t_o} \tag{2.13}$$

then it may be written as follows;

$$\vec{T}_n(\vec{r}, k_o) = \sum_{m=o} \vec{T}_{nm}(\vec{r})/k_o^m \tag{2.14}$$

where the $\vec{T}_{nm}$ expansion coefficients are independent of k_o. Combining (2.13) and (2.14) yields

$$\vec{T}_{n0} + \frac{\vec{T}_{n1}}{k_o} + \frac{\vec{T}_{n2}}{k_o^2} + ... = \int \vec{j}_n(\vec{r}_o) \times \nabla_o G(|\Delta\vec{r}|) \exp(j\vec{k}_i \cdot \Delta\vec{r})d\vec{r}_{t_o} \tag{2.15}$$

so that the vector coefficients can be determined as follows;

$$\vec{T}_{n0} = \lim_{k_o \to \infty} \int \vec{J}_n(\vec{r}_o) \times \nabla_o G(|\Delta\vec{r}|) \exp(j\vec{k}_i \cdot \Delta\vec{r}) d\vec{r}_{t_o} \tag{2.16a}$$

$$\vec{T}_{n1} = \lim_{k_o \to \infty} k_o \left[\int \vec{J}_n \times \nabla_o G \exp(j\vec{k}_i \cdot \Delta\vec{r}) d\vec{r}_{t_o} - \vec{T}_{n0} \right] \tag{2.16b}$$

$$\vec{T}_{nm} = \lim_{k_o \to \infty} k_o^m \left[\int \vec{J}_n \times \nabla_o G \exp(j\vec{k}_i \cdot \Delta\vec{r}) d\vec{r}_{t_o} - \sum_{p=0}^{m-1} \frac{\vec{T}_{np}}{k_o^p} \right] \tag{2.16c}$$

Substituting (2.15) into (2.12) yields

$$\sum_{n=o} \vec{J}_n(\vec{r}) k_o^{-n} = 2\hat{N} \times \left[\vec{h}_i + \sum_{n=o} \sum_{m=o} \vec{T}_{nm}(\vec{r}) k_o^{-n-m} \right] \tag{2.17}$$

so that equating like powers of k_o yields

$$\vec{J}_o(\vec{r}) = 2\hat{N} \times \{\vec{h}_i + \vec{T}_{00}\} \tag{2.18a}$$

$$\vec{J}_1(\vec{r}) = 2\hat{N} \times \{\vec{T}_{10}(\vec{r}) + \vec{T}_{01}(\vec{r})\} \tag{2.18b}$$

$$\vec{J}_2(\vec{r}) = 2\hat{N} \times \{\vec{T}_{11}(\vec{r}) + \vec{T}_{20}(\vec{r}) + \vec{T}_{02}(\vec{r})\} \tag{2.18c}$$

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Substituting from (2.16) into (2.18) yields the following sequence of integral equations for the vector expansion coefficients;

$$\vec{J}_o(\vec{r}) = 2\hat{N} \times \vec{h}_i + 2\hat{N} \times \lim_{k_o \to \infty} \int \vec{J}_o(\vec{r}) \times \nabla_o G(|\Delta\vec{r}|) \exp(j\vec{k}_i \cdot \Delta\vec{r}) d\vec{r}_{t_o} \tag{2.19a}$$

$$\vec{j}_1(\vec{r}) = 2\hat{N} \times \lim_{k_o \to \infty} \{k_o[\int \vec{j}_o(\vec{r}_o) \times \nabla_o G(|\Delta\vec{r}|) \exp(j\vec{k}_i \cdot \Delta\vec{r}) d\vec{r}_{t_o} - \vec{T}_{00}]\}$$

$$+ 2\hat{N} \times \lim_{k_o \to \infty} \int \vec{j}_1(\vec{r}) \times \nabla_o G(|\Delta\vec{r}|) \exp(j\vec{k}_i \cdot \Delta\vec{r}) d\vec{r}_{t_o} \qquad (2.19b)$$

•   •

•   •

•   •

This sequence of integral equations has a number of interesting properties. First, except for the source or Born term, all the integral equations *are identical* in form in that they appear as

$$\vec{j}_p(\vec{r}) = 2\hat{N} \times \vec{s}_p(\vec{r}) + 2\hat{N} \times \lim_{k_o \to \infty} \int \vec{j}_p(\vec{r}) \times \nabla_o G(|\Delta\vec{r}|) \exp(j\vec{k}_i \cdot \Delta\vec{r}) d\vec{r}_{t_o}$$

$$p = 0,1,2, \ldots \qquad (2.20)$$

Also, since the source term, $2\hat{N} \times \vec{s}_p(\vec{r})$, in (2.20) depends on $\vec{j}_o$, $\vec{j}_1$, ..., and $\vec{j}_{p-1}$, the integral equations are *recursive*. Thus, if $\vec{j}_o$ can be determined then it should be possible to determine *all* higher order vector coefficients. Because the vector expansion coefficients, $\vec{j}_n(\vec{r})$, are independent of k_o, it is further noted that the source term $\vec{s}_p(\vec{r})$ is determined almost completely by a Luneburg–Kline series representation for the following integral

$$\int \nabla_o G(|\Delta\vec{r}|) \exp(j\vec{k}_i \cdot \Delta\vec{r}) d\vec{r}_{t_o} .$$

For example, from (2.19a) it follows that

$$\vec{s}_o(\vec{r}) = \vec{h}_i \qquad (2.21a)$$

while from (2.19b)

$$\vec{s}_1(\vec{r}) = \lim_{k_o \to \infty} k_o \left[\int \vec{j}_o(\vec{r}_o) \times \nabla_o G(|\Delta \vec{r}|) \exp(j\vec{k}_i \cdot \Delta \vec{r}) d\vec{r}_{t_o} \right.$$

$$\left. - \lim_{k_o \to \infty} \int \vec{j}_o(\vec{r}_o) \times \nabla_o G(|\Delta \vec{r}|) \exp(j\vec{k}_i \cdot \Delta \vec{r}) d\vec{r}_{t_o} \right] \quad (2.21b)$$

It should be noted that the presence of $\vec{j}_o(\vec{r}_o)$ under the integral signs in (2.21b) has a very minimal effect because the dominant terms are the ones which depend upon k_o.

One very important point about the sequence of integral equations in (2.19) is that *their solutions are known.* In fact, (2.19a) is essentially the magnetic field integral equation in the high frequency ($k_o \to \infty$) limit, so its solution is given by [2.6]

$$\vec{j}_o(\vec{r}) = \begin{cases} 2\vec{N} \times \vec{h}_i & (\vec{r} \textit{ not shadowed}) \\ 0 & (\vec{r} \textit{ in shadow}) \end{cases}$$

There is also the possibility of a multiple scattering contribution to $\vec{j}_o(\vec{r})$ from other points on the surface [2.6]. Thus, for the n<u>th</u> vector expansion coefficient, the solution is (except for the contributions of multiple ray bounces on the surface)

$$\vec{j}_n(\vec{r}) = \begin{cases} 2\vec{N} \times \vec{s}_n & (\vec{r} \textit{ not shadowed}) \\ 0 & (\vec{r} \textit{ in shadow}) \end{cases} \quad (2.22)$$

The complete surface current density is thus obtained by combining (2.5), (2.8), (2.11), and (2.22), i.e.

$$\vec{J}_s(\vec{r}) = (1 + \zeta_x^2 + \zeta_y^2)^{-1/2} \exp(-j\vec{k}_i \cdot \vec{r}) \sum_{n=o} \frac{\vec{J}_n(\vec{r})}{k_o^n} \tag{2.23}$$

The reader is cautioned not to expect a one–to–one relationship between a specific $\vec{J}_n$ coefficient in (2.23) and the nth order iterate of the original integral equation in (2.4). It can be shown that a direct relationship does exist for n = 0 and 1 but it breaks down for $n \geq 2$. This results is a consequence of the fact that any one iterate contains, in general, all orders of $1/k_o$.

2.3 Discussion

One of the immediate consequences of (2.23) is that the current on any shadowed portion of the surface is identically zero. This is obviously a high frequency approximation, but the analysis presented above makes no *explicit* approximations and, in fact, appears to be exact. Clearly, an exact analysis cannot lead to an approximate result. What is happening in this case is that the L–K series is dong the best job that an *asymptotic* series can do in representing the current in the shadowed parts of the surface. The failure of the L–K asymptotic series is linked to the fact that the current in the shadow zones of the surface *cannot be represented* by an asymptotic series of the L–K form. To prove this, recall that the definition of an asymptotic series such as (2.11) is that if S_m represents the partial sum of the first m + 1 terms * then

$$\lim_{k_o \to \infty} k_o^m [\vec{L}(k_o,\vec{r}) - \vec{S}_m] = 0 \tag{2.24}$$

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* $\vec{S}_m = \sum_{n=0}^{m} \vec{J}_n(\vec{r})/k_o^n$

for all values of m. In the limit as $k_0 \to \infty$, the current in the shadow zones of the surface is zero. Thus, from (2.24) with m = 0, $\vec{S}_0 = 0$ or $\vec{J}_0(\vec{r}) = 0$. For m = 1, (2.24) yields

$$\lim_{k_0 \to \infty} (k_0 L) = \lim_{k_0 \to \infty} (k_0 \vec{S}_1)$$

but $\vec{S}_1 = \vec{J}_1 / k_0$ because $\vec{J}_0 = 0$, so this leads to

$$\vec{J}_1 = \lim_{k_0 \to \infty} (k_0 \vec{L}) \tag{2.25}$$

It is well known that the current in the shadow zone has the form of a creeping wave [2.1]

$$\vec{L} \approx \sum_{\ell=1} \vec{C}_\ell \exp\{-j[k_0 \delta + \exp(-j\pi/3)\beta_\ell (k_0/2)^{1/3} \int_0^\delta \kappa^{2/3} d\delta \tag{2.26}$$

where C_ℓ are the launching amplitudes, δ is the distance measured along the surface, β_ℓ is a root of the Airy integral [2.1], and κ is the curvature of the surface at the distance δ measured along the surface. Thus, in the limit as $k_0 \to \infty$, each term in the series of (2.26) exhibits an exponential dependence on k_0. Furthermore, since the real part of this dependence leads to an *exponential* decay with distance, it is clear that

$$\vec{J}_1 = \lim_{k_0 \to \infty} (k_0 \vec{L}) = 0$$

and, in fact *all* of the higher $\vec{J}_n$'s will vanish. Thus, the only acceptable asymptotic series for the current in the shadow region is the *null* series. The reason for this is contained in the definition of the asymptotic series, e.g. (2.24). That is, the only series of the form

$$\vec{S}_m = \sum_{n=o}^{m} \vec{J}_n(\vec{r})/k_o^n$$

which can satisfy (2.24) for all m *in the shadowed* regions of the surface is the null series. Note that it is the constraint imposed by the *form* of the asymptotic series which dictates the end result. *Thus, other than the null series result, the current in the shadow region does not have an asymptotic series representation.* This is an important result because it is the first time (to the author's knowledge) that the failure of an L–K series in the shadow region has been both demonstrated and explained.

While the L–K series does not lead to an *exact* result for the surface current density, it still holds the potential for providing a tractable improvement to a pure geometrical optics solution. This fact is demonstrated by Ansorge's [2.5] calculations for scattering by a dielectric sphere using the ray optics field approach. The attractiveness of the current approach as developed here is due in large part to the fact that the complete L–K representation can be developed *entirely* from an integral of known functions, i.e.

$$\int \nabla_o G(|\Delta\vec{r}|) \exp(j\vec{k}_i \cdot \Delta\vec{r}) d\vec{r}_{t_o} .$$

While a complete L–K series development for this integral is prohibitive, it may be possible to obtain the terms up to and including k_o^{-2} [2.7]. One of the primary advantages of obtaining the k_o^{-1} and k_o^{-2} corrections to the k_0^0 asymptotic expansion of this integral is the recovery of some of the cross polarizing properties of a rough surface in the high frequency (but not optical) limit.

The results obtained in this paper raise an interesting point. Usually, the first frequency dependent correction to physical optics comes from the non–zero width of the transition zone between the illuminated and shadowed regions on the scatterer and the propagation of creeping waves into the shadow zone. If the L–K series produces a result that is identically zero in the shadow zone, what is the physical source of the k_0^{-n}, n=1, 2, ... terms in the illuminated zone? It would appear that this may be due to the transition zone encroaching into the illuminated zone as the frequency is decreased. This is the kind of question that needs to be answered if there is to be a full understanding of the Luneburg–Kline asymptotic representation.

It should be noted that $\vec{s}_1(\vec{r})$ in (2.21b) has essentially been computed by Chaloupka and Meckelburg [2.8] in a very clever application of integration by parts. Their results should provide the basic ingredients to study the question raised in the previous paragraph.

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3. An Application of Smoothing To Wave Propagation Through Discrete Random Media

3.1 Background

The distorted wave Born approximation (DWBA) is one means for improving on the standard Born approximation and it is particularly useful when the latter fails [3.8]. It is based on splitting the unknown into the sum of two parts in such a way that one part is very accurately known while the second, and yet to be determined part, is small compared to the known term. This method had its origins in nuclear physics [3.8] and has recently been used in electromagnetic wave propagation through particulate media to obtain a first order estimate of the incoherent power scattered by a randomly positioned collection of objects [3.6, 3.9, 3.7].

In the em propagation problem, the total field is split into the sum of an average or coherent field and an incoherent or fluctuating part having zero mean. For sparsely populated media, the average field is known to be the solution of the Foldy–Twersky integral equation [3.4]. Similarly, the fluctuating field in this limit is small compared to the average field provided one does not progress too far into the particulate medium. Thus, assuming that one can solve the Foldy–Twersky equation for the average field and that one restricts interest to those regions of parameter space where the fluctuating field is small compared to the mean field, it should be possible to use the DWBA to estimate the fluctuating field and, hence, the incoherent scattered power.

In the previous em propagation studies [3.6, 3.9, 3.7], it was assumed that the DWBA was equivalent to embedding the particulate scatterers in the average medium, illuminating them by the average field, and then letting them scatter into the average medium, (the effective parameters of the average medium were obtained from the wavenumber of the average field). There is a problem with this approach; the average medium only has meaning relative to the average field. That is, scattering is the process whereby the average field is converted into the fluctuating or incoherent field and this latter field does not propagate in the average medium. It would seem then that the previous studies claiming to use the DWBA have, in essence, double counted for the average medium. The error stems from postulating how the DWBA applies to the problem rather than deriving this result from basic scattering relationships.

The purpose of this paper is to show that the DWBA follows directly from an application of the method of smoothing. In particular, the DWBA will be shown to be equivalent to the lowest order smoothing approximation for the fluctuating field. Furthermore, this result will confirm the suspicion that previous results have essentially double counted for the average medium. Given the conditions under which the DWBA is applicable, a similarly constrained equation for the average field will also be obtained. In the absence of correlations between scatterers, this equation reduces to the Foldy–Twersky integral equation. All of these results, for both the average and fluctuating fields, are obtained by a straightforward application of the method of smoothing to the discrete media and do not require the introduction of an equivalent continuous random media [3.5].

3.2 Analysis

The approach to be used herein is the application of the method of smoothing to the integral equation describing the scattering process in the discrete random medium. It should be noted that the same analytical procedures apply equally well to the problems of propagation in a continuous random medium or scattering by a random interface. This generality of the procedure follows from the fact that smoothing can be applied to any Fredholm integral equation of the second kind describing a stochastic process [3.3].

Consider a volume V in which there are N randomly located objects having possibly random volumes (V_n), orientations (Ω_n), and relative dielectric constants $\left\{\epsilon_{r_n}\right\}$ and where $n = 1, 2, \ldots, N$. The total electric field at any point in space ($\vec{E}_t$) may be expressed as the sum of the incident field ($\vec{E}_i$) which would exist in the absence of the objects and the scattered field due to the objects ($\vec{E}_s$), e.g.

$$\vec{E}_t(\vec{r}) = \vec{E}_i(\vec{r}) + \vec{E}_s(\vec{r}) \tag{3.1}$$

The scattered field may be related to the total field inside each object as follows;

$$\vec{E}_s(\vec{r}) = L\bar{\bar{K}}_\Sigma \vec{E}_t^i(\vec{r}_n + \vec{r}_o) \tag{3.2}$$

where L is the three dimensional integral over all space, i.e.,

$$L = \int\int\int d\vec{r}_o$$

and the dyadic operator $\bar{\bar{K}}_\Sigma$ is given by

$$\bar{\bar{K}}_\Sigma = -k_o^2 \sum_{n=1}^{N} \left[\epsilon_{r_n} - 1\right] S_n\left[\vec{r}_o, V_n, \Omega_n\right] \bar{\bar{G}} \left[\vec{r} - \vec{r}_n - \vec{r}_o\right] \tag{3.3}$$

The i superscript on $\vec{E}_t$ denotes the total field interior to the scatterer. In (3.3), k_o is the free space wavenumber ($= 2\pi/\lambda_o$) while $S_n(\vec{r}_o, V_n, \Omega_n)$ is the support of the n th scattering object whose centroid is located by the position vector $\vec{r}_n$. S_n is given by

$$S_n\left[\vec{r}_o, V_n, \Omega_n\right] = \begin{matrix} 1, & \vec{r}_o \text{ inside } V_n \\ 0, & \vec{r}_o \text{ outside } V_n \end{matrix} \tag{3.4}$$

and depends only on the volume (V_n) and orientation (Ω_n) of the nth object and the body centered position vector $\vec{r}_o$. The dyadic $\bar{\bar{G}}$ is given by

$$\overline{\overline{G}} = -\,\text{P.V.}\left[\overline{\overline{I}} + k_o^{-2}\nabla_o\nabla_o\right] g\left[\vec{r} - \vec{r}_n - \vec{r}_o\right] + \overline{\overline{I}}\delta\left[\vec{r} - \vec{r}_n - \vec{r}_o\right]/3k_o^2 \tag{3.5}$$

where $\overline{\overline{I}}$ is the unit dyad, $\delta(\cdot)$ is the three dimensional delta distribution, and P.V. denotes the principle value which excludes a small spherical volume centered at $\vec{r} - \vec{r}_n - \vec{r}_o = 0$. The g in (3.5) is the free space scalar Green's function,

$$g\left[\vec{r} - \vec{r}_n - \vec{r}_o\right] = \exp\left[-jk_o|\vec{r} - \vec{r}_n - \vec{r}_o|\right]/4\pi|\vec{r} - \vec{r}_n - \vec{r}_o| \tag{3.6}$$

Substituting (3.2) into (3.1) yields the following integral equation of the second kind for the total field in the presence of the scatterers;

$$\vec{E}_t(\vec{r}) = \vec{E}_i(\vec{r}) + L\,\overline{\overline{K}}_\Sigma\left[\vec{r}, \vec{r}_n + \vec{r}_o\right] \vec{E}_t^i\left[\vec{r}_n + \vec{r}_o\right] \tag{3.7}$$

The first step in applying the method of smoothing to (3.1) is to decompose $\vec{E}_t$ into the sum of an average or mean value $\left[<\vec{E}_t>\right]$ and a zero mean fluctuating part $\left[\delta\vec{E}_t\right]$. That is, with

$$\vec{E}_t = <\vec{E}_t> + \delta\vec{E}_t \tag{3.8}$$

(3.7) becomes

$$<E_t> + \delta\vec{E}_t = \vec{E}_i + L\overline{\overline{K}}_\Sigma < \vec{E}_t^i> + L\overline{\overline{K}}_\Sigma\,\delta\vec{E}_t^i \tag{3.9}$$

and averaging this equation yields

$$<\vec{E}_t> = \vec{E}_i + LP\left\{\bar{\bar{K}}_\Sigma <\vec{E}_t^i>\right\} + LP\left\{\bar{\bar{K}}_\Sigma\, \delta\vec{E}_t^i\right\} \tag{3.10}$$

where P = < > is the averaging operator. It should be noted that in (3.10),

$$P\left\{\bar{\bar{K}}_\Sigma < \vec{E}_t^i>\right\} \neq <\bar{\bar{K}}_\Sigma> <\vec{E}_t^i>$$

because it is implied that when $<\vec{E}_t^i>$ appears next to $\bar{\bar{K}}_\Sigma$, it contains a dependence on the random positions of the scattering objects, as (3.7) shows explicitly. Subtracting (3.10) from (3.9) gives the following result;

$$\delta\vec{E}_t = L(1-P)\,\bar{\bar{K}}_\Sigma <\vec{E}_t^i> + L(1-P)\,\bar{\bar{K}}_\Sigma\, \delta\vec{E}_t^i \tag{3.11}$$

The fluctuating part of the total field is also equal to the fluctuating part of the scattered field because

$$\vec{E}_t = \vec{E}_i + \vec{E}_s \tag{3.12}$$

and the incident field, $\vec{E}_i$, has no fluctuating part so that $\delta\vec{E}_t = \delta\vec{E}_s$.

If the point of observation $\vec{r}$ is successively taken to be inside each of the N scatterers, (3.10) and (3.11) will result in 2N coupled integral equations for the fields $<\vec{E}_t^i>$ and $\delta\vec{E}_t^i$ inside each of the N scatterers. A solution of these equations gives the fields inside all of the scatterers and then the average and fluctuating fields outside of the scatterers can be computed from (3.10) and (3.11). The need to deal with interior and exterior fields, even in a statistical description, is simply a consequence of the boundary value nature of this problem and has been noted previously [3.2].

The goal of this research is to obtain an approximate solution of (3.11) and then find the corresponding equation for $<\vec{E}_t>$ that is self-consistent with this level of simplification. The most straightforward approximation is to ignore the term $L(1-P)\bar{\bar{K}}_\Sigma \delta\vec{E}_t^i$ in (3.11) because it appears to be the source of multiple scattering in so far as $\delta\vec{E}_t$ is concerned. We use the word appears because, without this term, $\delta\vec{E}_t$ will indeed be determined by $<\vec{E}_t^i>$ alone but $<\vec{E}_t^i>$ depends on $\delta\vec{E}_t^i$ through (3.10). To resolve this quandary, it is necessary to substitute for $\delta\vec{E}_t^i$ in (3.10) using (3.11), i.e.

$$<\vec{E}_t> = \vec{E}_i + LP\left\{\bar{\bar{K}}_\Sigma <\vec{E}_t^i>\right\} + LP\left\{\bar{\bar{K}}_\Sigma L(1-P)\bar{\bar{K}}_\Sigma <\vec{E}_t^i>\right\}$$

$$+ LP\left\{\bar{\bar{K}}_\Sigma\left[L(1-P)\bar{\bar{K}}_\Sigma \delta\vec{E}_t^i\right]\right\} \tag{3.13}$$

Now, if the suspicious multiple scattering term $L(1-P)\bar{\bar{K}}_\Sigma \delta\vec{E}_t^i$ is ignored in both (3.11) and in (3.13), there results

$$\delta\vec{E}_t \approx (1-P)\, L\bar{\bar{K}}_\Sigma <\vec{E}_t^i> \tag{3.14}$$

and

$$<\vec{E}_t> \approx \vec{E}_i + P\left\{L\bar{\bar{K}}_\Sigma\left[1 + L(1-P)\bar{\bar{K}}_\Sigma\right]<\vec{E}_t^i>\right\} \tag{3.15}$$

which are both to the same order of approximation and therefore self-consistent relationships. Note that in (3.15) $<\vec{E}_t>$ is determined independent of $\delta\vec{E}_t$ which means that, indeed, (3.14) is a single scattering approximation for $\delta\vec{E}_t$ insofar as $\delta\vec{E}_t$ is concerned.

This last phrase "in so far as $\delta\vec{E}_t$ is concerned" must be added because we are dealing with two field quantities here and we must be precise in defining exactly what we mean by the term multiple scattering.

It is clear from the way in which (3.11) and (3.13) were simplified to (3.14) and (3.15) that the latter two equations for $\delta\vec{E}_t$ and $<\vec{E}_t>$ will be valid whenever

$$|(1-P)\, L\bar{\bar{K}}_\Sigma <\vec{E}_t^i>| \; > \; |(1-P)\, L\bar{\bar{K}}_\Sigma \delta\vec{E}_t^i| \tag{3.16}$$

This inequality says that the fluctuating part of a one–time–scattered (by all objects) average interior field must be large compared to a one–time–scattered (by all objects) interior fluctuating field. It would seem that if $|<\vec{E}_t^i>| >> |\delta\vec{E}_t^i|$ then this inequality is surely satisfied. However, there may be instances where the average and fluctuating fields are comparable in magnitude but (3.16) is still satisfied. The point of this discussion is to emphasize that it is (3.16) that must be satisfied in order for (3.14) and (3.15) to be valid.

3.3 Discussion

Eqn. (3.14) represents the lowest order non–trivial approximation for the fluctuating field scattered in and out of the random media. This field quantity is the source of the incoherent power due to the randomness in the media. According to (3.14) $\delta\vec{E}_t$ is obtained by taking the fluctuating part of the field resulting from N objects scattering the interior coherent or average field into free space. This result reconfirms the physical intuition that the fluctuating field is caused entirely by scattering of the coherent field. Contrary to what is postulated in some earlier works, the scattering is into free space and not into an average medium. Eqn.(3.14) is the distorted wave Born approximation (DWBA) for the fluctuating field in a random media comprising discrete scatterers. If (3.14) is used to compute the incoherent power scattered by a half–space filled with randomly positioned scatterers, it can be shown that whenever there is a phase retarding or an attenuation effect of the randomness it is halved relative to the erroneous double counting previously reported. It should be remembered that (3.14) is a single scattering approximation for the fluctuating field. Hence, the DWBA is a single scattering approximation for the fluctuating field and, consequently, the incoherent power. Hopefully, this result answers the question of whether the DWBA is a single or multiple scattering approximation.

An interesting sidelight of this study is the equation that was obtained for the average or coherent field, i.e., eqn. (3.15). This equation was obtained by retaining the same order terms as were kept in the DWBA for $\delta\vec{E}_t$. This does not mean that all the terms in (3.15) are of equal importance. For example, if only one interaction between $\bar{\bar{K}}_\Sigma$ and $<\vec{E}_t^i>$ is retained, i.e.

$$<\vec{E}_t> \approx \vec{E}_i + PL\bar{\bar{K}}_\Sigma <\vec{E}_t^i> \tag{3.17}$$

the resulting equation is the Foldy–Twersky integral equation for the average field. To show this, it is first necessary to split the averaging operator into the product of an average over the random positions of the scatterers and the conditional average over all other random properties of the objects such as dielectric constant, volume, and orientation, i.e.

$$P = << \cdot \,|\vec{r}_n> >_{\vec{r}_n}$$

Thus, (3.17) becomes

$$<\vec{E}_t> = \vec{E}_i + L << \bar{\bar{K}}_\Sigma|\vec{r}_n> <\vec{E}_t^i> >_{\vec{r}_n} \tag{3.18}$$

because only $\bar{\bar{K}}_\Sigma$ depends on the other random parameters. From the definition of $\bar{\bar{K}}_\Sigma$, i.e., (3.3), the conditional average becomes

$$<\bar{\bar{K}}_\Sigma|\vec{r}_n> = -k_o^2(<\epsilon_r> - 1) < S\,(r_o,<V_n>, <\Omega>)> \sum_{n=1}^{N} \bar{\bar{G}}\left[\vec{r} - \vec{r}_n - \vec{r}_o\right] \tag{3.19}$$

where $<\epsilon_r>$, $<V_n>$, and $<\Omega>$, and $<S(\cdot)>$ represent the mean relative constant, volume, orientation, and spatial support, respectively, for the N scattering objects. Assuming a uniform probability density function for the $\vec{r}_n$, n = 1, 2, ...N, i.e.,

$$P(\vec{r}_n) = \begin{matrix} 1/V & \vec{r}_n \text{ inside } V \\ 0 & \vec{r}_n \text{ outside } V \end{matrix} \tag{3.20}$$

and substituting (3.19) and (3.20) in (3.18) yields

$$<\vec{E}_t> = \vec{E}_i - k_o^2\left[<\epsilon_n> - 1\right] L<S> \int_V \sum_{n=1}^{N} \bar{\bar{G}}\left[\vec{r} - \vec{r}_n - \vec{r}_o\right] <\vec{E}_t^i\left[\vec{r}_n + \vec{r}_o\right]> d\vec{r}_n/V$$

or

$$<\vec{E}_t> = \vec{E}_i - k_o^2\left[<\epsilon_n> - 1\right] L<S>\left[\frac{N}{V}\right]\int_V \bar{\bar{G}}\left[\vec{r} - \vec{r}_n - \vec{r}_o\right]< \vec{E}_t^i\left[\vec{r}_n + \vec{r}_o\right]> d\vec{r}_n \tag{3.21}$$

because all N integrations are identical and the series can be summed. Since

$$L<S> = \int_{<V_n>} d\vec{r}_o$$

(3.21) becomes

$$<\vec{E}_t> = \vec{E}_i - k_o^2\left[< \epsilon_r> - 1\right] \rho \int_{<V_n>} \int_V \bar{\bar{G}}\left[\vec{r} - \vec{r}_n - \vec{r}_o\right] <\vec{E}_t^i\left[\vec{r}_n + \vec{r}_o\right]> d\vec{r}_n d\vec{r}_o \quad (3.22)$$

where $\rho = N/V$ is the density of scatterers in the volume V. This is identical to the Foldy–Twersky equation; see eqn. (3.4) of [3.2].

The next term in (3.15), i.e.

$$PL\bar{\bar{K}}_{\Sigma}L(1-P)\bar{\bar{K}}_{\Sigma} <\vec{E}_t^i>$$

clearly involves correlations between scatterers so it must be akin to the Lax–Twersky approximation for $<\vec{E}_t>$ [3.1, 3.10]. This term and its meaning is presently under study.

3.4 Summary

The original intent of this work was to rigorously derive a low order approximation for the incoherent or fluctuating field in a discrete random medium using techniques that did not require the use of an equivalent continuous random medium or were not largely numerically oriented. The purpose of the work was twofold with the primary reason being to check some earlier calculations supposedly based on the distorted wave Born approximation. A secondary purpose was to attempt to develop a relatively simple, self consistent, low order approximation for both the fluctuating and average fields. The method of smoothing was chosen to accomplish this because of its ability to straightforwardly provide accurate low order approximate solutions.

The results obtained for the fluctuating field, in the lowest order of approximation, were found to be equivalent to the DWBA but without the double counting for the average medium erroneously postulated in earlier efforts. A consequence of this result is that the incoherent power scattered from a volume of randomly located discrete scatterers is larger than predicted by these earlier results. This is because the double counting for the attenuation effects of the average medium effectively masks the scattering from the objects deep in the medium. A secondary result of this study of the fluctuating or incoherent field was that it could be clearly shown that the DWBA does not contain any multiple scattering of the fluctuating field. This result answers the old question of does the DWBA include multiple scattering by first forcing one to carefully define multiple scattering.

Finally, if the equation for the average or coherent field is developed to exactly the same order of approximation as used in the fluctuating field, an apparently new integral equation is obtained. If scatterer–to–scatterer correlations are small or can be ignored, this equation is shown to reduce to the Foldy–Twersky integral equation. If correlations cannot be ignored, the result appears somewhat similar to the Lax–Twersky equation but this situation is till under investigation.

In regard to a secondary purpose of this paper, it was found that the method of smoothing is a very simple means for analyzing the propagation characteristics of discrete random. This statement applies to both the average and fluctuating fields. Furthermore, it is not necessary to use an equivalent continuous media representation for the discrete scattering process because the discrete character of the problem is easily dealt with by the method of smoothing.

3.5 References

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4. A Distorted Wave Born Approximation for High Frequency Scattering From Rough Surfaces

4.1. Background

Recently, it has been shown [4.1] that, for random surface scattering, first order smoothing is capable of providing an improvement to the classic Rayleigh–Rice first order boundary perturbation result. In particular, first order smoothing yields a result which depends only on the dominance of the average or coherent scattered field over the zero mean fluctuating field. Consequently, it is valid for small surface height (relative to the electromagnetic wavelength) but arbitrary slopes, curvatures, etc. The absence of any restrictions on the surface height derivatives is why the first order smoothing result is superior to the boundary perturbation approximation; in the case of small slopes, the former reduces to the latter [4.1]. It was also noted in [4.2] that whenever it is the fluctuating scattered field to be determined, first order smoothing and the distorted wave Born approximation (DWBA) produce equivalent results. This is true for scattering by random surfaces or by volumes comprising randomly varying discrete or continuous constitutive properties.

For random media problems, first order smoothing is essentially a low frequency approximation because it is dependent upon the dominance of the coherent or average propagating or scattered field. At high frequencies, the coherent field becomes vanishingly small so first order smoothing totally breaks down in its accuracy. However, there is the possibility of still using the distorted wave Born approximation (DWBA) to obtain an accurate description of the scattering process in this limit. The key to success in using this approximation is to start with an <u>appropriate</u> Born approximation; it need not be the Born term appearing in the integral equation for the unknown scattered field or the current.

The purpose of this note is to apply the above methodology to scattering by an arbitrarily roughened planar conducting surface in the high frequency limit. By selecting the appropriate Born term to be the exact high frequency scattering limit, which is known, it will be possible to develop an approximation which resurrects frequency dependent diffraction effects. This approximation comes from the Born term in the integral equation for the difference between the unknown quantity and its (known) high frequency limiting behavior. The key element in this DWBA methodology is accurate knowledge of the behavior of the scattered field in the ray optic limit ($k_0 = 2\pi/\lambda_0 \to \infty$). In the case of random surface scattering, the high frequency problem is less complicated than the low frequency limit because there is no need to worry about the average or coherent scattered field since it is usually vanishingly small as $k_0 \to \infty$.

The net result of this analysis is a rigorous methodology for taking known high frequency scattering results and extending them down to lower frequencies.

4.2. Analysis

To illustrate the method, the problem of scattering by an arbitrarily roughened planar interface will be considered. Above the interface defined by $z = \zeta(x,y)$ is free space while beneath it ($z < \zeta$) is a perfectly conducting medium. The current induced on the surface satisfies the magnetic field integral equation given by

$$\vec{J}_s(\vec{r}) = \vec{J}_s^i(\vec{r}) - 2\hat{n}(\vec{r}) \times \int \nabla_0 g(\vec{r} - \vec{r}_0) \times \vec{J}_s(\vec{r}_0) ds_0 \tag{4.1}$$

where

$$\vec{J}_s^i(\vec{r}) = 2\hat{n}(\vec{r}) \times \vec{H}_i(\vec{r}), \tag{4.2}$$

$$g(\vec{r} - \vec{r}_0) = \exp(-jk_0|\vec{r} - \vec{r}_0|)/4\pi|\vec{r} - \vec{r}_0|,$$

k_0 is the wavenumber, and $\hat{n}(\vec{r})$ is the unit normal to the surface at the point $\vec{r}$

$$\hat{n}(\vec{r}) = (-\zeta_x\hat{x}-\zeta_y\hat{y}+\hat{z})/\sqrt{1+(\nabla\zeta)^2}. \tag{4.3}$$

and $\vec{H}_i$ is the incident magnetic field. It is convenient to convert the integral in (4.1) to one over the z=0 plane and to multiply both sides of (4.1) by the factor $\exp(jk_{sz}\zeta)$; the resulting current can then be Fourier transformed with respect to x and y to yield the far zone scattered field. The factor k_{sz} is the z–component of the vector wavenumber pointing in the scattering direction. With these manipulations, (4.1) can be written in the following operator form;

$$\vec{J}(\vec{r}) = \vec{J}^i(\vec{r}) + L\overline{\overline{G}}(\vec{r}, \vec{r}_o) \cdot \vec{J}(\vec{r}_o) \tag{4.4}$$

where

$$\vec{J}(\vec{r}) = \vec{J}_s(\vec{r}) \exp(jk_{sz}\zeta) \sqrt{1+(\nabla\zeta)^2} \tag{4.5a}$$

$$\vec{J}^i(\vec{r}) = \vec{J}_s^i(\vec{r}) \exp(jk_{sz}\zeta) \sqrt{1+(\nabla\zeta)^2} \tag{4.5b}$$

$$L = \iint_{-\infty}^{\infty} (\cdot) dx_o dy_o \tag{4.5c}$$

$$\overline{\overline{G}}(\vec{r}, \vec{r}_o) = -2\vec{N}(\vec{r}) \times \nabla_o g(\vec{r} - \vec{r}_o) \exp\left[jk_{sz}(\zeta-\zeta)\right] \times \tag{4.5d}$$

and

$$\vec{N}(\vec{r}) = -\zeta_x\hat{x}-\zeta_y\hat{y}+\hat{z} \tag{4.5e}$$

where ζ_x and ζ_y are the x and y–components of surface slope. Eqn. (4.4) is the integral equation that will be considered since its solution essentially determines the scattered field.

4.2.1 <u>The Distorted Wave Born Approximation (DWBA)</u>

The key step in developing the distorted wave Born approximation (DWBA) is to write the unknown current $\vec{J}$ as the sum of a known term ($\vec{J}_a$), whose range of validity is also well known, and an unknown term ($\epsilon\vec{J}$) which is small in a region where $\vec{J}_a$ is very dominant [4.3], e.g.

$$\vec{J} = \vec{J}_a + \epsilon\vec{J} \tag{4.6}$$

This sum is substituted in (4.4) to produce the following integral equation for $\epsilon\vec{J}$;

$$\epsilon\vec{J} = \left\{(\vec{J}^i - \vec{J}_a) + L\bar{\bar{G}}\cdot\vec{J}_a\right\} + L\bar{\bar{G}}\cdot\epsilon\vec{J} \tag{4.7}$$

The Born or source term in this equation is

$$(\vec{J}^{\,i} - \vec{J}_a) + L\bar{\bar{G}}\cdot\vec{J}_a \tag{4.8}$$

substituting this approximation in (4.6) for $\epsilon\vec{J}$ yields what is usually called the DWBA, i.e.,

$$\vec{J} \approx \vec{J}^i + L\bar{\bar{G}}\cdot\vec{J}_a \tag{4.9}$$

It is interesting to note that (4.9) is potentially very different from a first order interative solution of (4.4) such as

$$\vec{J}^i + L\bar{\bar{G}}\cdot\vec{J}^{\,i} \tag{4.10}$$

This is because $\vec{J}_a$ and $\vec{J}^i$ may be different or they may have differing domains of validity. The DWBA hinges on the accuracy of $\vec{J}_a$ within a given region of parameter space and it attempts to extend this region by iterating $\vec{J}_a$. The first order iteration in (4.10) is generally less accurate or even well understood because one seldom knows when $\vec{J}^i$ by itself is accurate. Of course, when $\vec{J}^i \approx \vec{J}_a$ then both methods produce the same approximation. Another point of note about (4.9) occurs when $\vec{J}_a$ and $\vec{J}^i$ differ significantly. In this case, it will probably take many interations of (4.4) to produce the same effect as the single term $L\bar{\bar{G}}\cdot\vec{J}_a$ in (4.9). Thus, the DWBA is effectively a resummation of the standard iterative solution of (4.4), i.e.

$$\vec{J}_a \approx \sum_{n=0}^{N} [L\bar{\bar{G}}\cdot]^n \vec{J}^i \tag{4.11}$$

where N may be infinite or finite.

The DWBA is not without its own set of drawbacks. For example, having both an accurate solution and knowing the region of parameter space over which the solution is valid are very difficult conditions to satisfy. Further compounding the problem is the need to know just how much (4.9) will extend the region of validity of $\vec{J}_a$. Nevertheless, it appears to be a method for which a bit of knowledge can go a long way. As proof of this, one need only turn to low frequency scattering by a rough surface where the DWBA is much more robust than the boundary perturbation method [4.1], as previously noted. Finally, although (4.9) was derived for a roughened planar surface, it is clear that it is much more general than this. That is, (4.9) is valid for scattering by any arbitrarily shaped body if the operator L in (4.5c) is replaced by an integration over the x and y extent of the scatterer.

4.2.2 Application to High Frequency Scattering

In the limit of $k_0 \to \infty$, it is well known [4.4, 4.5] that the exact solution of (4), and also (4.9), comprises the sum of two currents. The first of these is the shadowed Kirchhoff result, e.g.

$$\vec{J}_{sh}(\vec{r}) = s_i \vec{J}^i(\vec{r}) \tag{4.12}$$

where s_i is unity if the point $\vec{r}$ on the surface is illuminated by the incident field and zero if it is shadowed by another part of the surface. The second contribution is due to multiple ray bounces on the surface and it will be denoted as simply $\vec{J}_m$ because it has a rather complicated form in general [4.5]. It can be obtained by solving (4.4) via iteration in the limit as $k_0 \to \infty$ and considering only the stationary phase points in the evaluation of the integral term which give rise to multiple scattering [4.5]. Thus, the appropriate high frequency asymptotic solution for the current is given by

$$\vec{J}_a = \lim_{k_0 \to \infty} \vec{J} = s_i \vec{J}^i + \vec{J}_m \tag{4.13}$$

This result is the exact solution of both (4.4) and (4.9) for the limiting case of $k_0 \to \infty$.

Substituting (4.13) in (4.9) yields the following high frequency DWBA for the current induced on the rough surface

$$\vec{J} = \vec{J}^i + L\overline{\overline{G}} \cdot s_i \vec{J}^i + L\overline{\overline{G}} \cdot \vec{J}_m \tag{4.14}$$

in terms of the known limiting forms for the current. In this case, it seems clear that the DWBA is taking the exact limiting behavior for the current and extending it to lower frequencies. Thus, the integrals in (4.14) should be evaluated accurately as possible and certainly not by asymptotic means (as $k_0 \rightarrow \infty$) for this would just lead to the obvious result in (4.13). There is another interpretation of (4.14) which is probably more meaningful for the rough surface scattering case. Taking the limit as $k_0 \rightarrow \infty$ can also be viewed as being equivalent to dealing with rough surfaces which have no frequency components that are larger than the electromagnetic wavenumber k_0. Solving (4.14) by non–asymptotic means has the effect of allowing some frequency components on the surface which are larger than (the now finite) k_0. Unfortunately, it is not possible to predict just how much high frequency ($>k_0$) structure is accurately accounted for by (4.14). This is a limitation common to most finite order iteration schemes.

The DWBA is clearly superior to the first order interative Born result in (4.10) when dealing with near grazing incidence. In addition, (4.14) may be one of the few approximate results capable of estimating high frequency scattering from surfaces having correlation lengths which are the order of k_0 [4.6]. (For these surfaces, the term high frequency denotes a large height to wavelength ratio). This is because the DWBA is based on an exact resummation of multiple scattering effects in the limit as $k_0 \rightarrow \infty$. In contrast, the integral term in the first order iterative Born approximation in (4.10) may not even converge to a meaningful result due to all of the multiple scatterings taking place on the surface.

4.3. Conclusions

The distorted wave Born approximation is an attractive alternative to the first order iterative Born approximation under certain conditions. The purpose of this paper is to point out these conditions when one is interested in high frequency scattering from roughened planar conducting surfaces. To accomplish this purpose, a DWBA is developed for scattering from arbitrary surfaces and contrasted to the first order iterative Born approximation. In the case of high frequency scattering, it is shown that the DWBA is capable of extending a ray optic solution into the finite frequency domain. The potential for this approximation to predict scattering from very rough surfaces is also of note. Consequently, it may well be capable of dealing with the backscattering enhancement problem [4.6] associated with such surfaces.

4.4 References

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5. Summary

The goal of this research is to advance the understanding of clutter and to thereby lead to better models for predicting clutter. The first chapter points out and explains a basic limitation of the Luneburg–Kline expansion for the current on any perfectly conducting body or surface. Although not specifically shown, this result also applies to imperfectly conducting surfaces. It is found that the Luneburg–Kline representation in inverse powers of the electromagnetic wavelength always leads to zero current in the optical shadow zone of the surface. Hence, the Luneburg–Kline representation is limited in its ability to produce the correct diffraction effects associated with finite frequencies. This means that there is an uncertainty associated with the use of the Luneburg–Kline expansion in that it is not clear when it starts to produce false results. Given this limitation, it is not clear that the L–K representation is worth further development or study.

The second chapter investigates ways to most easily and straightforwardly represent the scattering by a volume distribution of randomly located, shaped, and electrically constituted objects. The primary finding of this chapter is that the method of smoothing is a very robust approach for providing both low and high order approximations for the average or mean field and the fluctuating field. The average field is important because it acts as the source of the fluctuating field which, in turn, is the source of the incoherent scattered power. The smoothing approach shows that, to lowest order, the fluctuating field is formed by all the objects in the medium scattering the mean incident field into free space. This result is contrary to previous postulated results wherein the fluctuating field scatters into the average medium. The importance of this latter result is its prediction that objects deep in the scattering volume will be more significant contributors to the backward scattering process than previously thought.

Finally, the last chapter investigates how one goes about obtaining an improvement to the Kirchhoff approximation for the surface current. First, it is pointed out that the pure Kirchhoff approximation is neither a low frequency or a high frequency asymptote. That is, it lacks the effects of shadowing to make it a true high frequency result, and it does not contain the proper polarization dependence for a low frequency limit. Consequently, if one starts with the Kirchhoff approximation, it is difficult, at best, to estimate just how accurate such a starting point is. The material in the last chapter recommends starting with the shadowed Kirchhoff approximation because it is an exact result in the optical limit. In fact, iteration of this approximation appears to be one way for recovering some lower frequency diffraction effects. Of course, it is also recognized that the inclusion of shadowing at the level of the current is easy to write down, but it is difficult to actually implement. Nevertheless, this chapter does provide some rationale for improving on the standard Kirchhoff approximation.

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