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EFFECT OF CELL SIZE ON RADAR CLUTTER STATISTICS

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ABSTRACT

The objective of this study is to investigate through computer simulation the effects of a change in cell size on signal statistics in backward, forward and perpendicular directions of observation when a plane wave is incident at 70 degrees from the vertical on a randomly rough surface. It is found that in the backward direction the signal has a Weibull-like distribution and its mean and variance both decrease with an increase in the cell size starting with a cell less than 1.5 of a wavelength in dimension. When the cell size is around 7.3λ x 2.5λ, (λ is the electric wavelength), the signal distribution approaches Rayleigh. In forward scattering the signal distribution is found to be closely Gaussian, and as cell size increases the mean increases as expected. Variance increases also, but only slightly. In the perpendicular direction the signal distribution is again closely Weibuli. Its mean and variance did not show any particular trends with a change in cell size.

1. INTRODUCTION

The purpose of this study is to determine the dependence of clutter statistics from a randomly rough surface on cell size by computer simulation. Radar backscatter calculations are performed by solving numerically the integral equation given by (1) for the surface current density, **J(r)**, on a perfectly conducting surface.

$$\mathbf{J(r)} = 2\hat{\mathbf{n}} \times \mathbf{H}^i + (1/2\pi)\int \hat{\mathbf{n}} \times [\ \nabla G \times \mathbf{J(r')}\ ]\ dS' \qquad (1)$$

where $\hat{\mathbf{n}}$ is the unit normal vector to the surface and $\mathbf{H}^i$ is the incident magnetic field. Once the surface current density is known over a surface patch of size A_o the far zone scattered field can be computed from

$$\mathbf{E}(\hat{\mathbf{r}}) = -C\eta\hat{\mathbf{r}} \times \int_{A_o} \hat{\mathbf{r}} \times \mathbf{J}_p \exp(jk\hat{\mathbf{r}}.\mathbf{r})\, dx\, dy \qquad (2)$$

where $C = (-jk/4\pi R) \exp(-jkR)$; R is range; $\hat{\mathbf{r}}$ is the unit vector pointing in the direction of observation; A_o is the cell size; η is the intrinsic impedance of free space; $\mathbf{J}_p$ is equal to either $\mathbf{J}_v$ or $\mathbf{J}_h$ for vertical and horizontal polarizations respectively and k is the wave number. Signal statistics here refer to the statistics of the scattered field amplitude.

To carry out the above calculations it is necessary to have a rough surface. In this study an anisotropically rough surface was generated on the computer. Its surface height statistics are given in Fig. 1 and two orthogonal cuts along principal directions of its autocorrelation function are shown in Fig. 2. The correlation lengths are approximately 8 cm and 10 cm respectively.

For the statistics shown in this report the incidence angle is chosen to be 70 degrees; polarization is taken to be horizontal and the wavelength is chosen to be 18 cm. Three azimuthal observation directions are chosen: backward, forward and perpendicular to the incidence direction. In each case three different cell sizes are considered and for each cell size 1100 scattered field amplitude samples are calculated. Thus, 3x1100 field samples are obtained for each azimuth direction.

To better understand the signal distribution, comparisons are made between the calculated distribution curves and four different distribution functions in backward and perpendicular observations: Rayleigh, Gamma, Weibull and lognormal. In forward scattering the Rayleigh function is replaced by the Gaussian function in making the comparisons. The K-distribution function is not used here because it is for describing composite targets with a large variance such as a pole or a building above a rough ground surface. A single-scale random surface does not have a large variance and hence its distribution will not fit the K-distribution. For ease of reference a summary of the properties of these distribution functions are given in Section 2. The comparisons between these distribution functions and simulated distribution data are given in Section 3 and conclusions are given in Section 4.

2. SIGNAL DISTRIBUTION FUNCTIONS

The basic properties of the probability density functions to be used for comparisons are summarized below.

(a) Gaussian density function

The Gaussian density function is given by

$$f(x;\lambda,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[- \left(\frac{x-\lambda}{\sigma}\right)^2 / 2 \right]$$

where σ is the standard deviation; and λ is the mean value of the random variable x.

(b) Lognormal density function

$$f(x;\lambda,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[- \left(\frac{\ln x-\lambda}{\sigma}\right)^2 / 2 \right]$$

Here, λ,σ are parameters of the lognormal function. They are related to the mean and variance of the random variable x as follows:

$$\lambda = \frac{1}{2}\ln\left[\frac{\text{mean}^4}{\text{variance} + \text{mean}^2}\right]$$

$$\sigma = \left\{\ln\left[\frac{\text{variance}}{\text{mean}^2} + 1\right]\right\}^{1/2}$$

For the lognormal function the random variable x and the parameter σ should be greater than zero and λ lies on the real axis.

(c) Gamma density function

$$f(x;\lambda,\sigma) = \frac{1}{\sigma\Gamma(\lambda)}\left[\frac{x}{\sigma}\right]^{\lambda-1}\exp\left[-\left(\frac{x}{\sigma}\right)\right]$$

where Γ(λ) is the Gamma function; σ is a scale parameter and λ is a shape parameter related to the mean and variance of the random variable x as follows:

$$\lambda = (\text{mean})^2 / \text{variance}$$

$$\sigma = \text{variance} / \text{mean}$$

All the gamma density function parameters, λ,σ, and the random variable x itself are greater than zero.

(d) Weibull density function

$$f(x;\lambda,\sigma) = \frac{\lambda}{\sigma}\left[\frac{x}{\sigma}\right]^{\lambda-1}\exp\left[-\left(\frac{x}{\sigma}\right)^{\lambda}\right]$$

where x,σ,λ are larger than zero and the parameters λ,σ can be determined in terms of the mean and variance of the random variable, ln (1/x), as follows:

$$\lambda = \frac{\pi}{\sqrt{6\,\text{variance}}}$$

$$\sigma = \frac{1}{\exp[-(0.57722/\lambda) + \text{mean}]}$$

(e) Rayleigh density function

$$f(x;\lambda,\sigma)=\left[\frac{x}{\sigma^2}\right]\exp\left[-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2\right]$$

where x is greater than or equal to zero and σ is greater than zero. There is only one parameter in the Rayleigh function and it is related to the variance of the random variable x or its mean as follows:

$$\sigma=\sqrt{\frac{2\text{variance}}{(4-\pi)}} = \text{mean}\sqrt{\frac{2}{\pi}}$$

It is interesting to note that the Rayleigh density function may be looked upon as a special case of the Weibull density function in which λ is equal to 2 and σ is replaced by $\sigma\sqrt{2}$.

3. COMPUTATIONAL PROCEDURE

To find the surface current density at a point within a cell of size, n x m, equation (1) is solved by iteration using the Kirchhoff current density as the estimate of the unknown current density inside the integral. This operation is carried out for each surface point and hence n x m integrations are performed over the cell. Then, equation (2) is used to find the far zone scattered field. This gives one sample of the signal distribution. To construct a histogram 1100 samples were generated and the range of signal amplitude is divided into 62 intervals [see Fig.3a]. This calls for 1100 x [(n x m) +1] integrations and 1100 resolution cells to obtain an estimate of the density function of a signal distribution. With 1100 samples the simulated data points still show significant scattering. However, the trend of the density function is discernible through smoothing [Gan,1987].

4. RESULTS AND COMPARISONS

To determine which density function is the closest to the smoothed curve in Fig. 3b we show comparisons of the smoothed curve with four different density functions when observation is in the backscattering direction. Visually, the Weibull density function provides the best fit. A quantitative evaluation is given in Table 1 where it is shown that the Weibull density function has the smallest rms error among the four density functions. Similar evaluation and comparisons for larger cell sizes are shown in Figs. 4 and 5. A significant point to note is that the Weibull distribution continues to perform the best and its parameter λ increases from 1.65 to 1.98 as the cell size increases (see Tables 1-3). This means that it is approaching the Rayleigh distribution with an increase in cell size. As the cell size increases, the we show comparisons of the smoothed curve with four different density functions when observation is in the backscattering direction. Visually, the Weibull density function provides the best fit. A quantitative evaluation is given in Table 1 where it is shown that the Weibull density function has the smallest rms error among the four density functions. Similar evaluation and comparisons for larger cell sizes are shown in Figs. 4 and 5. A significant point to note is that the Weibull distribution continues to perform the best and its parameter λ increases from 1.65 to 1.98 as the cell size increases (see Tables 1-3). This means that it is approaching the Rayleigh distribution with an increase in cell size. As the cell size increases, the mean and variance of the backscattered signal decrease. This may be due to the fact that the small cell size is less than one and a half wavelengths in dimension and that as cell size increases energy is more directed towards the specular direction.

Figures 6-8 show similar studies as in Figures 3-5 but is for forward reception. Here, the four statistical functions chosen for comparison are normal, Gamma, lognormal and Weibull. Visually, the normal function fits the best and rms errors for each case is shown in Tables 4-6. In each case the normal function gives the smallest rms error which decreases with an increase in the cell size. The mean value increases with the cell size as expected. The slight increase in variance appears to indicate that the effect due do the increase in signal level is larger than that due to narrowing in the forward beam.

The cases with reception in the perpendicular direction are shown in Figures 9-11. Here, the Weibull distribution provides again the best fit to the histograms. As cell size increases the distribution also approaches Rayleigh. No particular trends are apparent for the mean and variance of the signal as cell size increases.

REFERENCE

Z. Gan,"Statistical analysis of radar signals scattered from the sea surface via computer simulation," M.S., University of Texas at Arlington, December 1987.

FIGURE LEGENDS

1. Surface height distribution of the generated random surface.
2. Surface correlation function of the generated surface.
3. (a) Signal distribution and (b) signal model comparisons in the backscattering direction with the cell size, 24 cm x 24 cm.
4. (a) Signal distribution and (b) signal model comparisons in the backscattering direction with the cell size, 72 cm x 24 cm.
5. (a) Signal distribution and (b) signal model comparisons in the backscattering direction with the cell size, 132 cm x 45 cm.
6. (a) Signal distribution and (b) signal model comparisons in the forward scattering direction with the cell size, 24 cm x 24 cm.
7. (a) Signal distribution and (b) signal model comparisons in the forward scattering direction with the cell size, 72 cm x 24 cm.
8. (a) Signal distribution and (b) signal model comparisons in the forward scattering direction with the cell size, 132 cm x 45 cm.
9. (a) Signal distribution and (b) signal model comparisons in the perpendicular scattering direction with the cell size, 24 cm x 24 cm.
10. (a) Signal distribution and (b) signal model comparisons in the perpendicular scattering direction with the cell size, 72 cm x 24 cm.
11. (a) Signal distribution and (b) signal model comparisons in the perpendicular scattering direction with the cell size, 132 cm x 45 cm.

Figure 1

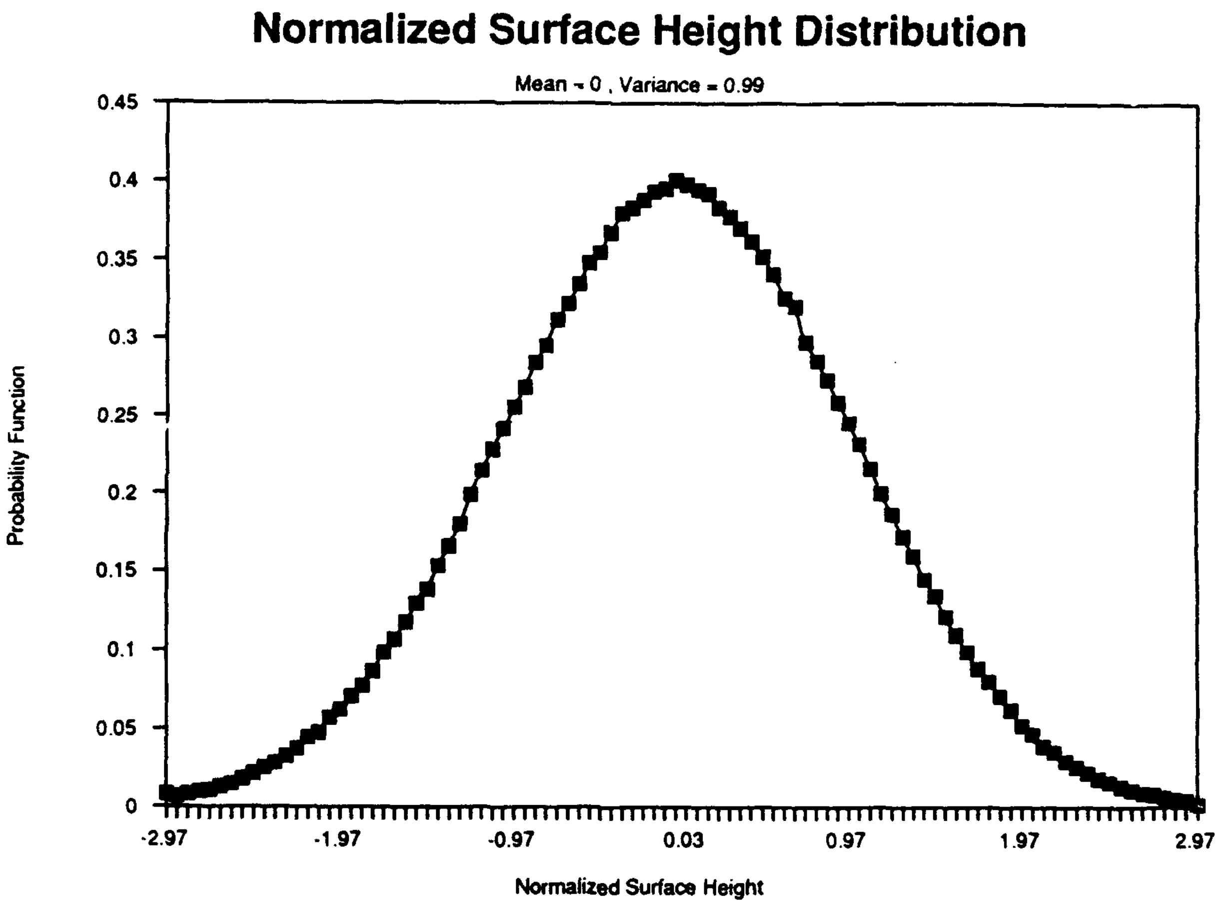
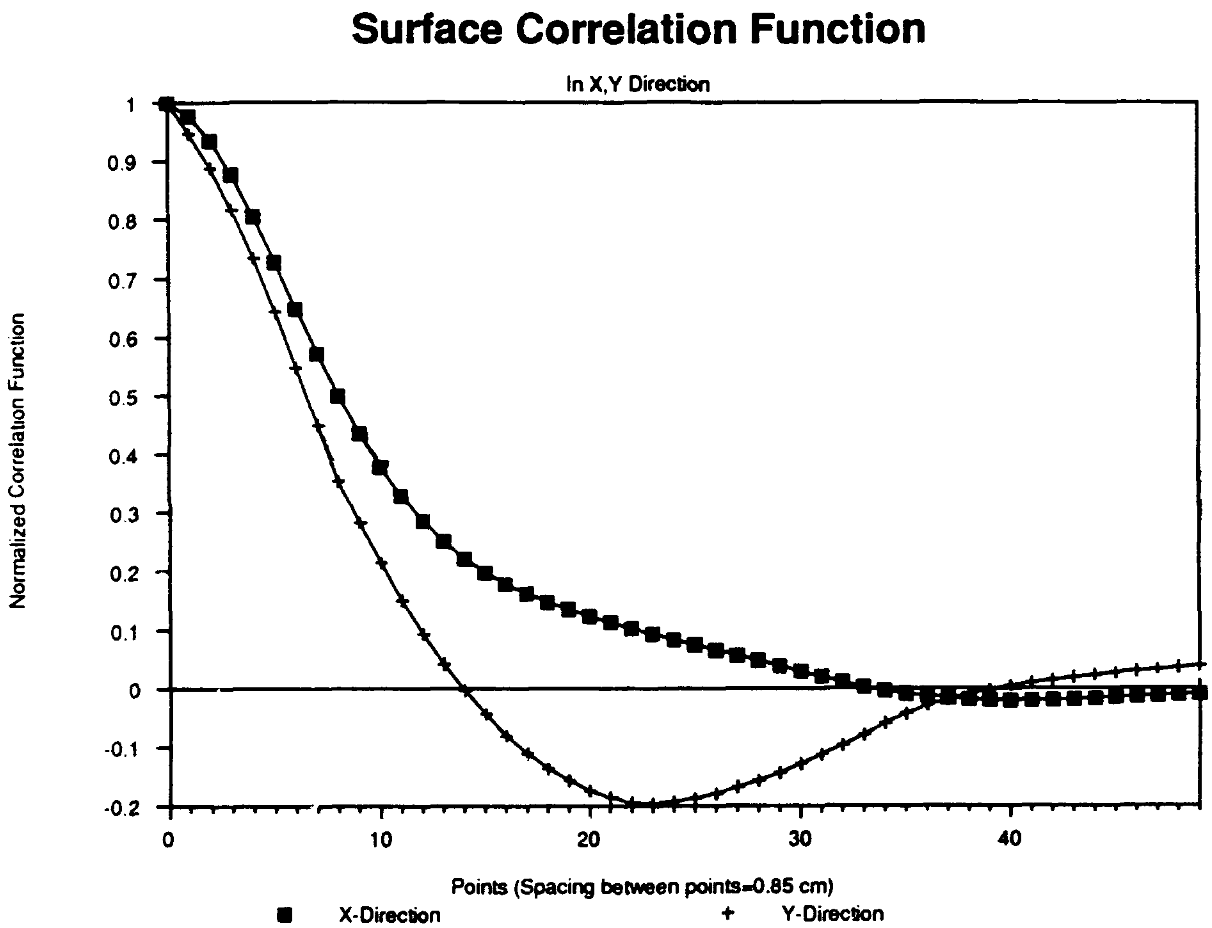
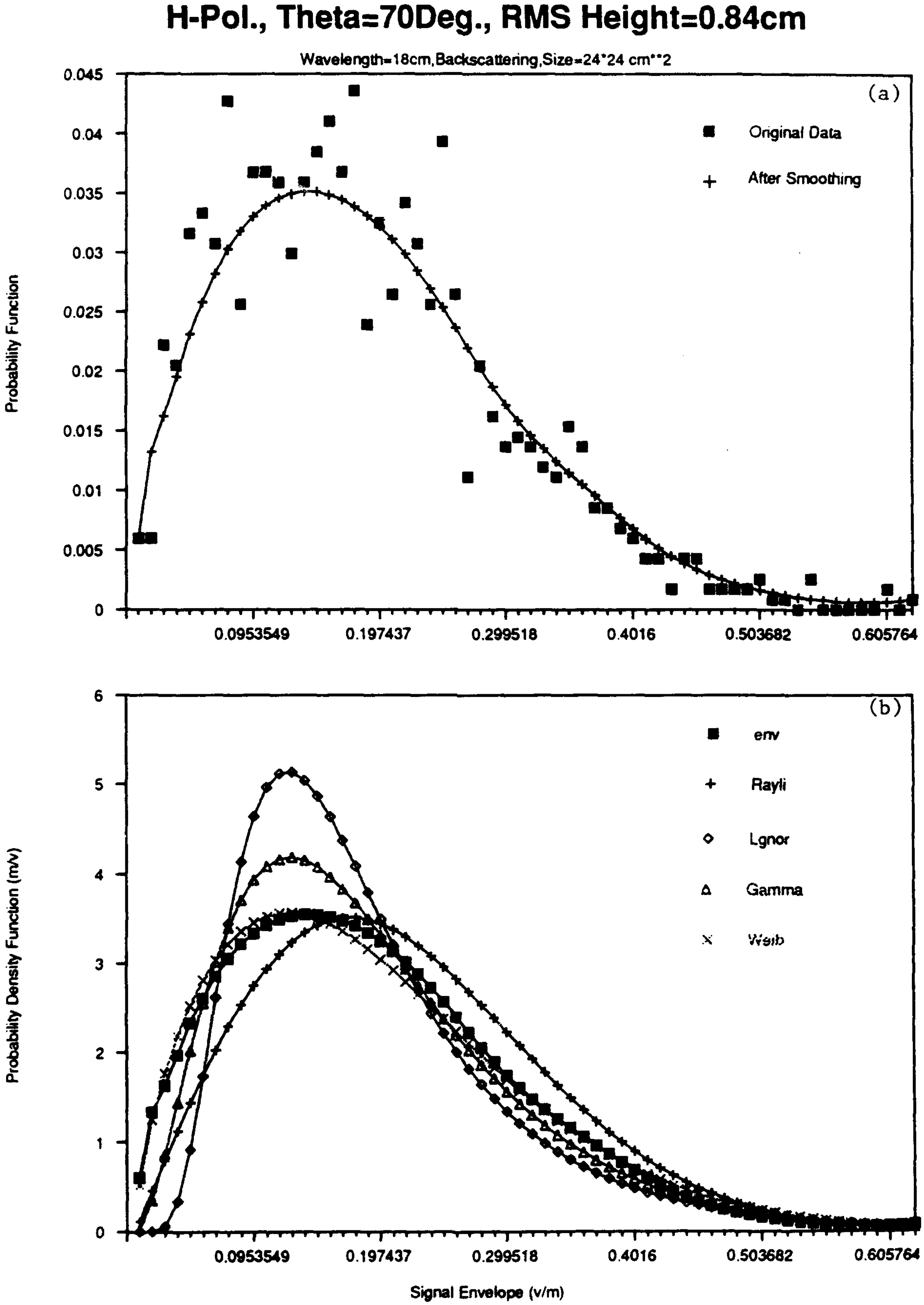

Normalized Surface Height Distribution
Mean ~ 0 , Variance = 0.99
Probability Function
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-2.97
-1.97
-0.97
0.03
0.97
1.97
2.97
Normalized Surface Height

Figure 2



Surface Correlation Function
In X,Y Direction
Normalized Correlation Function
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
10
20
30
40
Points (Spacing between points=0.85 cm)
X-Direction
Y-Direction

Figure 3



H-Pol., Theta=70Deg., RMS Height=0.84cm
Wavelength=18cm,Backscattering,Size=24*24 cm**2
(a)
Original Data
After Smoothing
Probability Function
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0.0953549
0.197437
0.299518
0.4016
0.503682
0.605764
(b)
env
Rayli
Lgnor
Gamma
Weib
Probability Density Function (m/v)
6
5
4
3
2
1
0
0.0953549
0.197437
0.299518
0.4016
0.503682
0.605764
Signal Envelope (v/m)

Figure 4

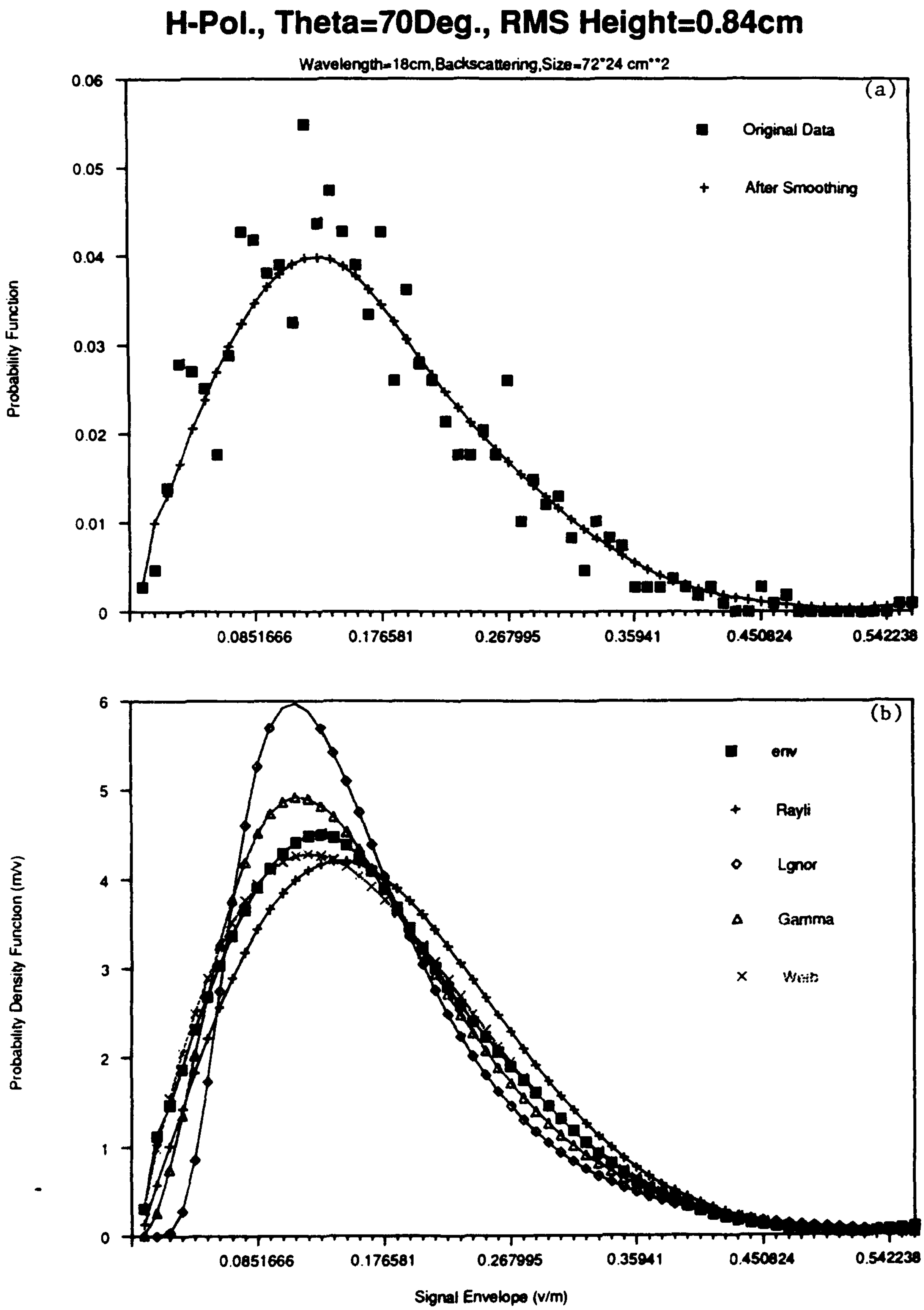

H-Pol., Theta=70Deg., RMS Height=0.84cm
Wavelength=18cm,Backscattering,Size=72*24 cm**2
(a)
Original Data
After Smoothing
Probability Function
0.06
0.05
0.04
0.03
0.02
0.01
0
0.0851666
0.176581
0.267995
0.35941
0.450824
0.542238
(b)
env
Rayli
Lgnor
Gamma
Probability Density Function (m/v)
6
5
4
3
2
1
0
0.0851666
0.176581
0.267995
0.35941
0.450824
0.542238
Signal Envelope (v/m)

Figure 5

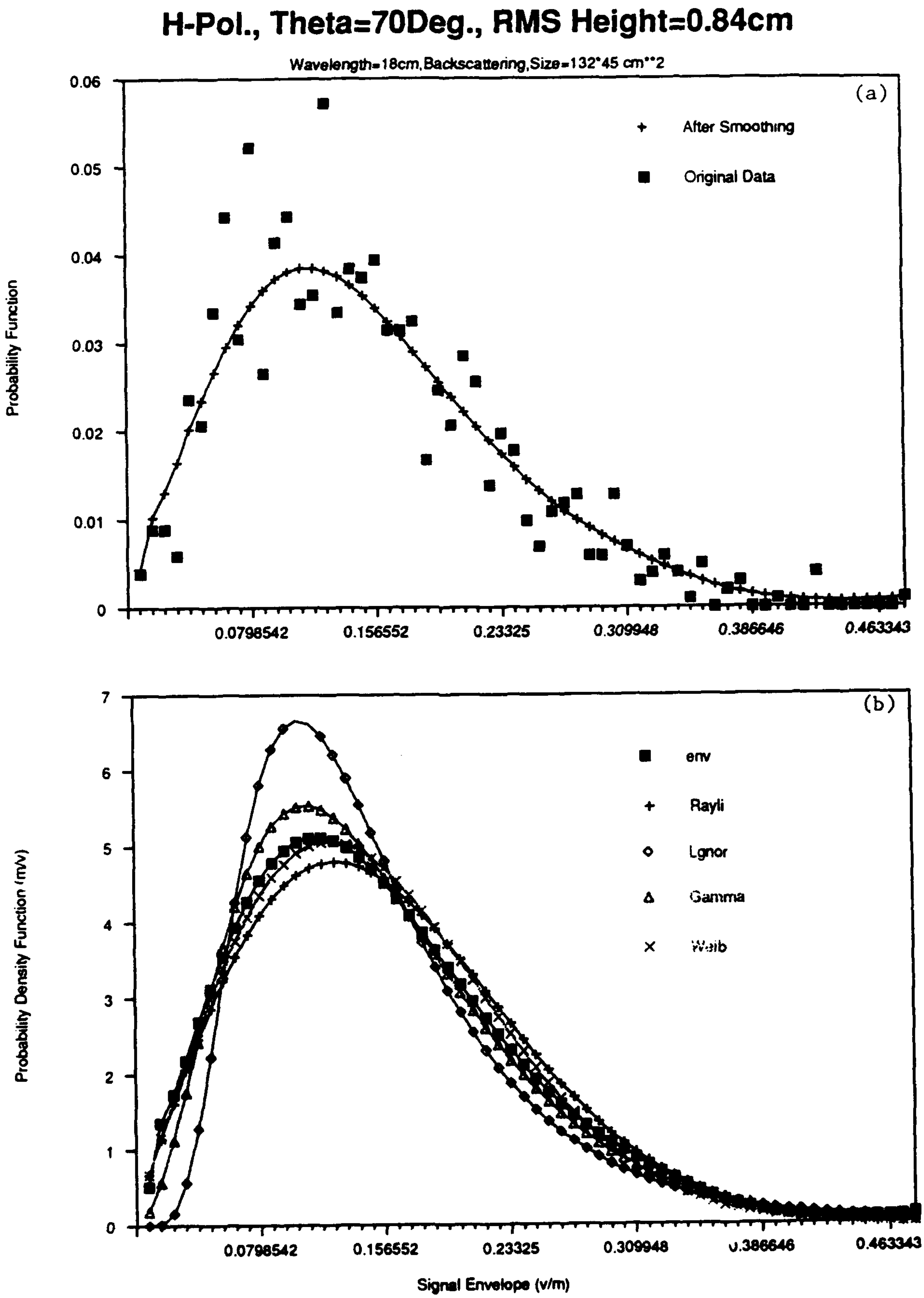

H-Pol., Theta=70Deg., RMS Height=0.84cm
Wavelength=18cm,Backscattering,Size=132*45 cm**2
(a)
After Smoothing
Original Data
Probability Function
0.06
0.05
0.04
0.03
0.02
0.01
0
0.0798542
0.156552
0.23325
0.309948
0.386646
0.463343
(b)
env
Rayli
Lgnor
Gamma
Weib
Probability Density Function (m/v)
7
6
5
4
3
2
1
0
0.0798542
0.156552
0.23325
0.309948
0.386646
0.463343
Signal Envelope (v/m)

Figure 6

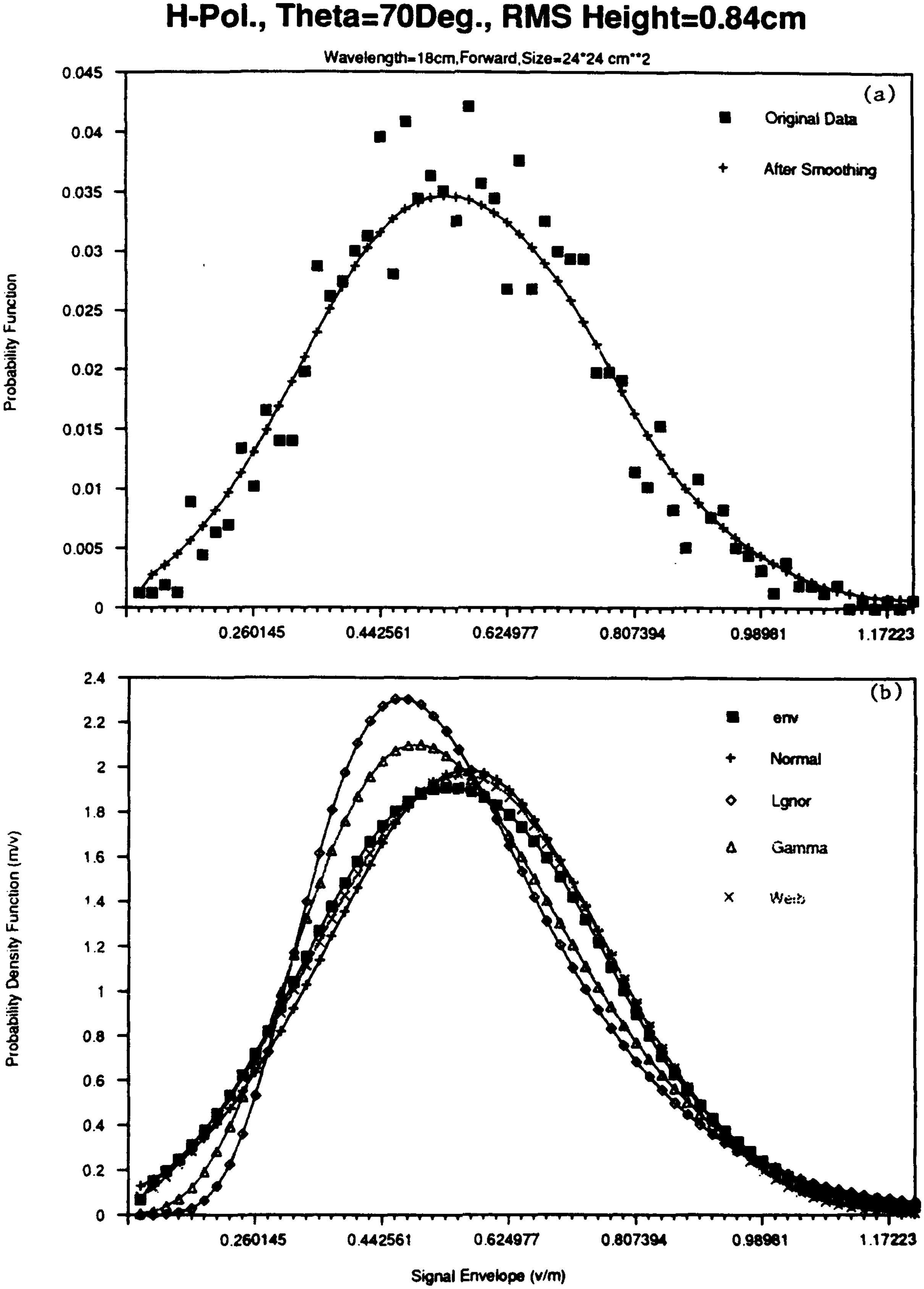

H-Pol., Theta=70Deg., RMS Height=0.84cm
Wavelength=18cm,Forward,Size=24*24 cm**2
(a)
Original Data
After Smoothing
Probability Function
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0.260145
0.442561
0.624977
0.807394
0.98981
1.17223
(b)
env
Normal
Lgnor
Gamma
Probability Density Function (m/v)
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.260145
0.442561
0.624977
0.807394
0.98981
1.17223
Signal Envelope (v/m)

Figure 7

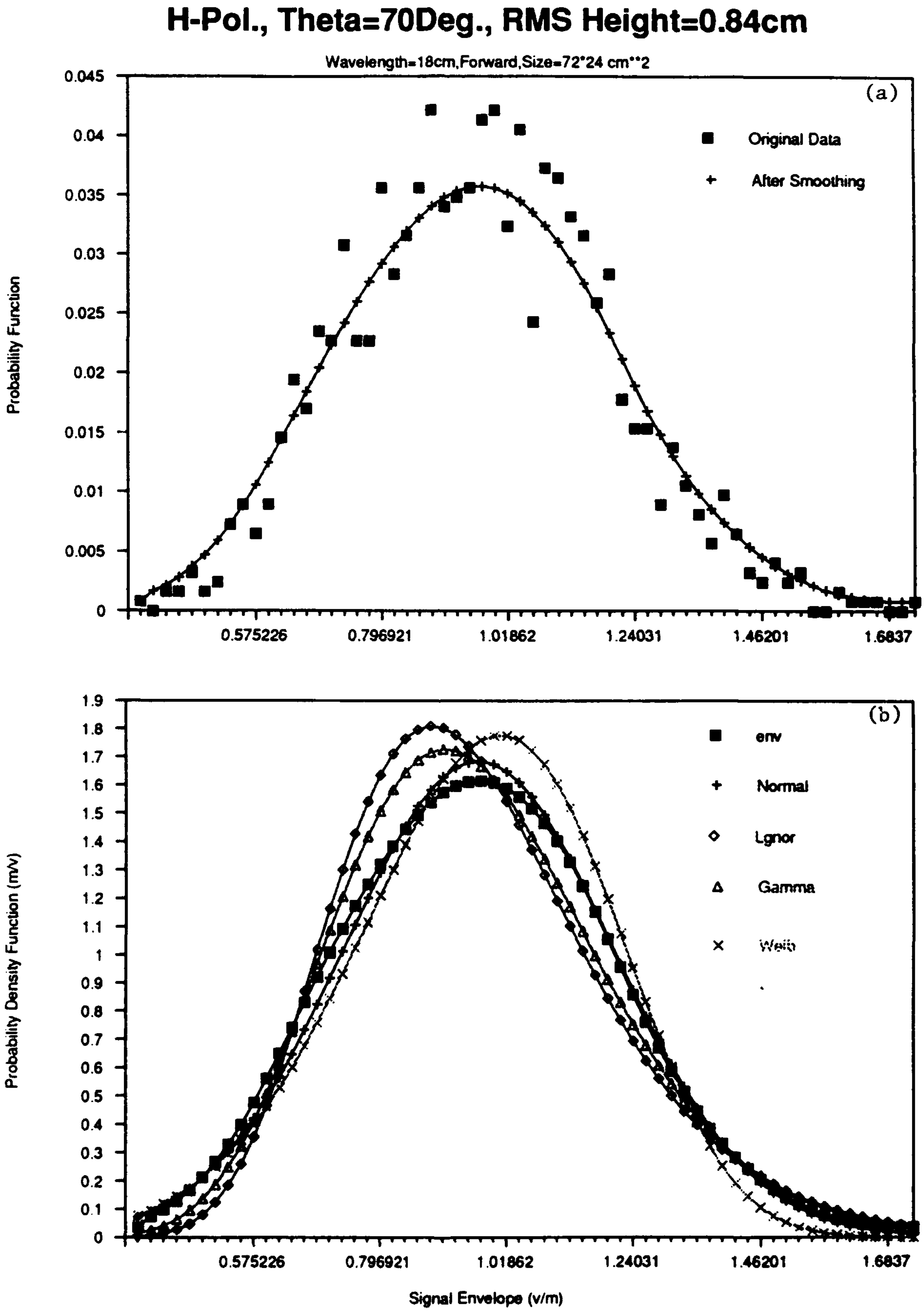

H-Pol., Theta=70Deg., RMS Height=0.84cm
Wavelength=18cm,Forward,Size=72*24 cm**2
(a)
Original Data
After Smoothing
Probability Function
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0.575226
0.796921
1.01862
1.24031
1.46201
1.6837
(b)
env
Normal
Lgnor
Gamma
Probability Density Function (m/v)
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.575226
0.796921
1.01862
1.24031
1.46201
1.6837
Signal Envelope (v/m)

Figure 8

H-Pol., Theta=70Deg., RMS Height=0.84cm

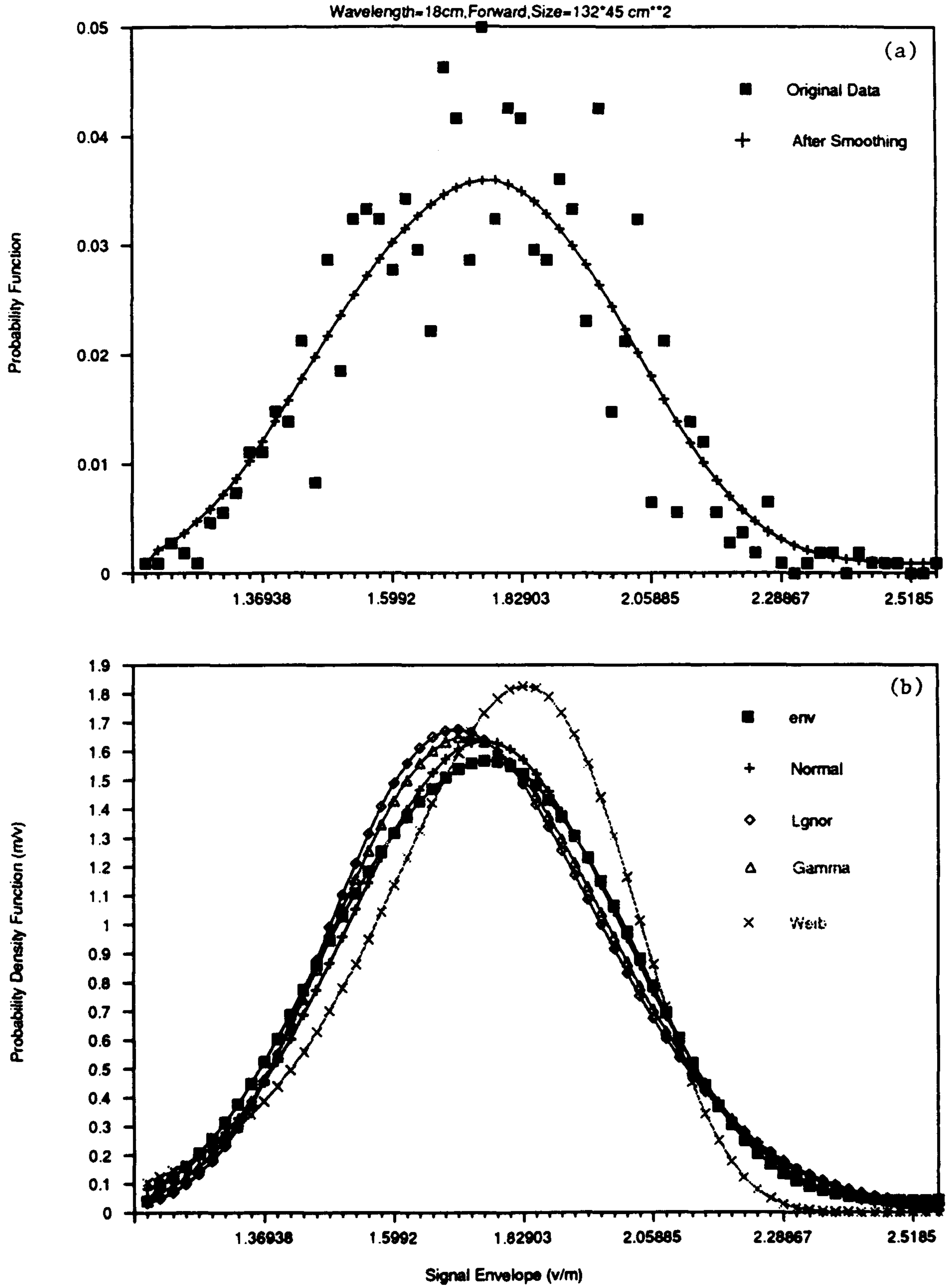


Figure 9

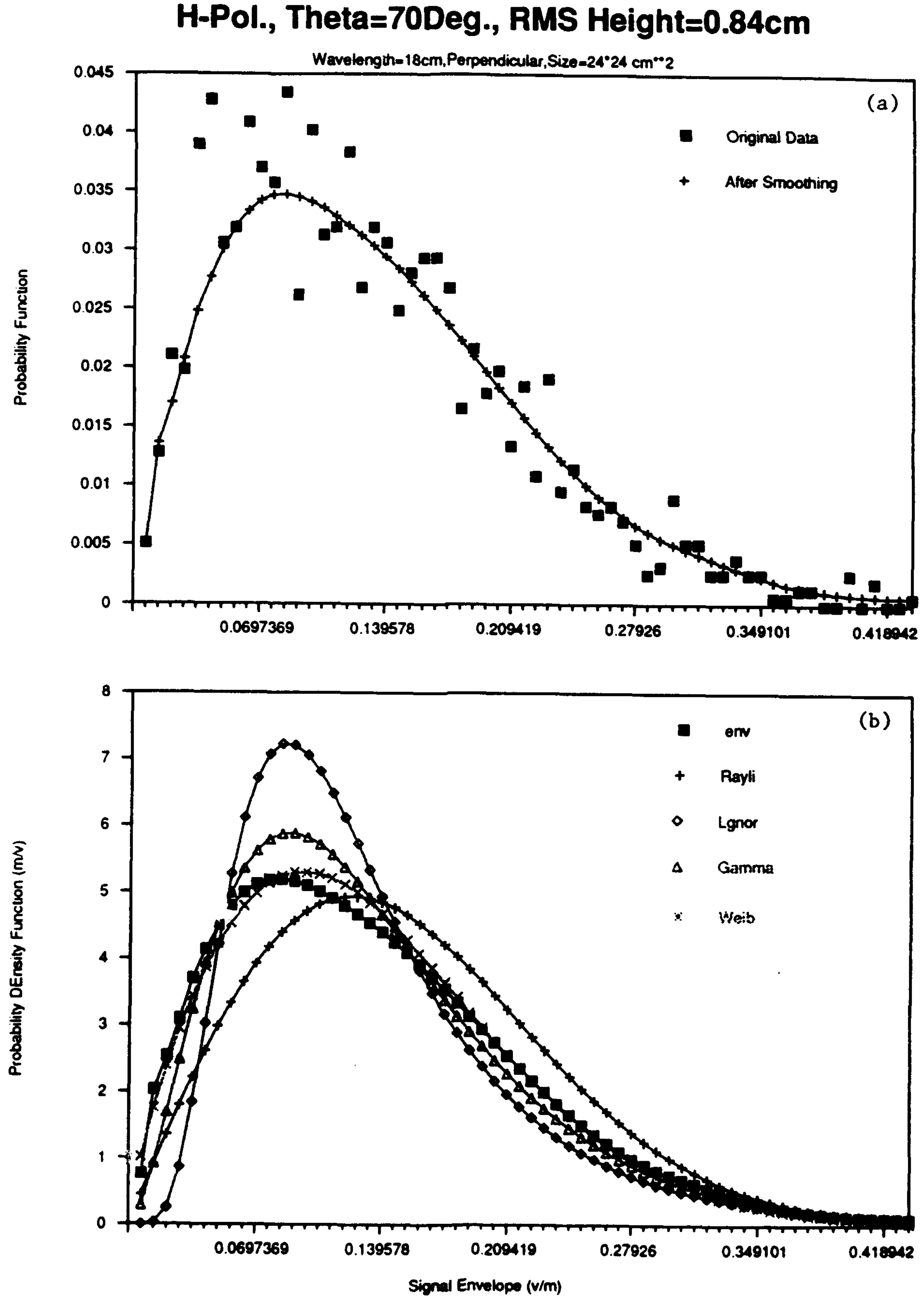

H-Pol., Theta=70Deg., RMS Height=0.84cm
Wavelength=18cm,Perpendicular,Size=24*24 cm**2
(a)
Original Data
After Smoothing
Probability Function
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0.0697369
0.139578
0.209419
0.27926
0.349101
0.418942
(b)
env
Rayli
Lgnor
Gamma
Weib
Probability DEnsity Function (m/v)
8
7
6
5
4
3
2
1
0
0.0697369
0.139578
0.209419
0.27926
0.349101
0.418942
Signal Envelope (v/m)

Figure 10

H-Pol., Theta=70Deg., RMS Height=0.84cm

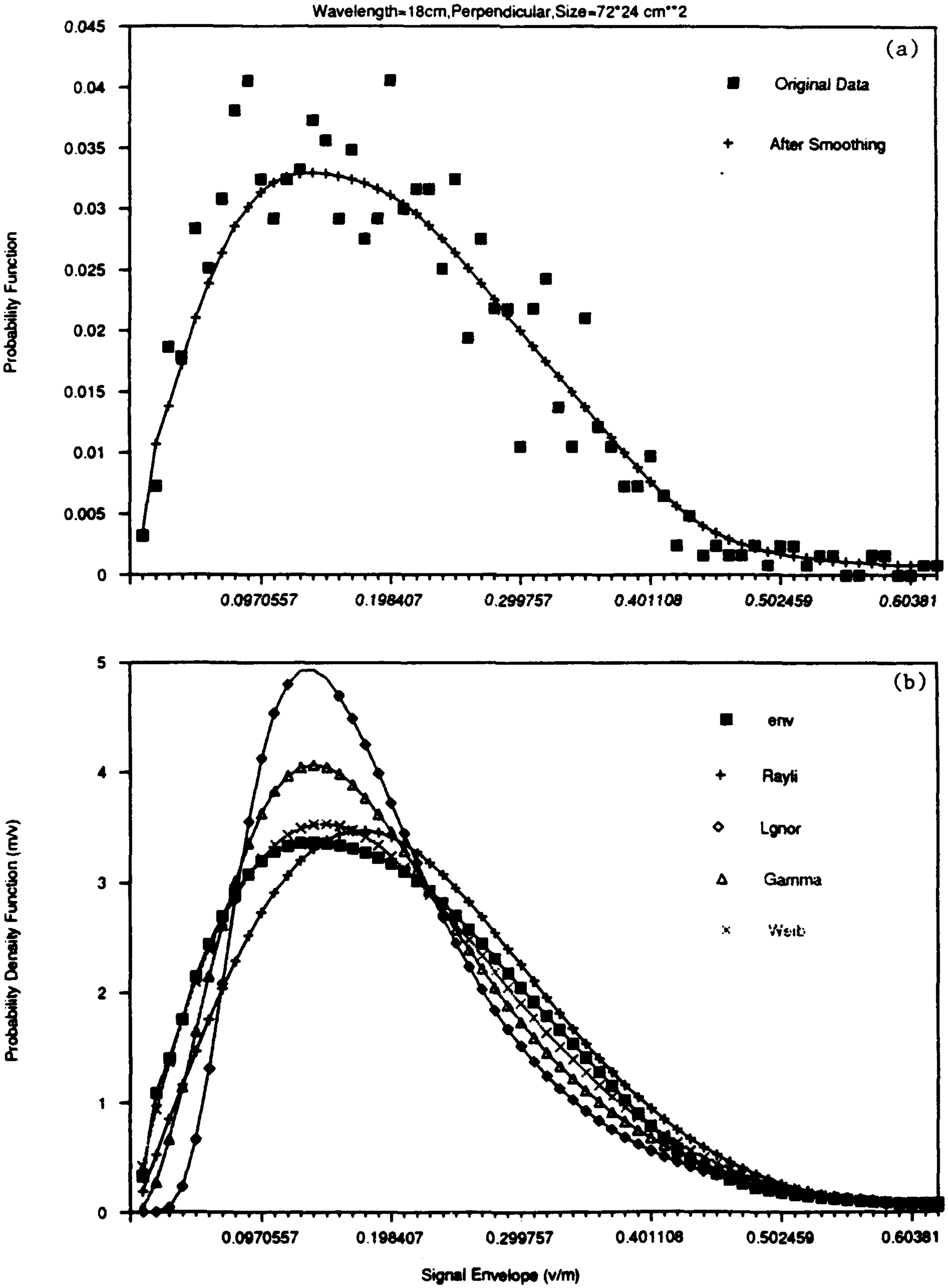
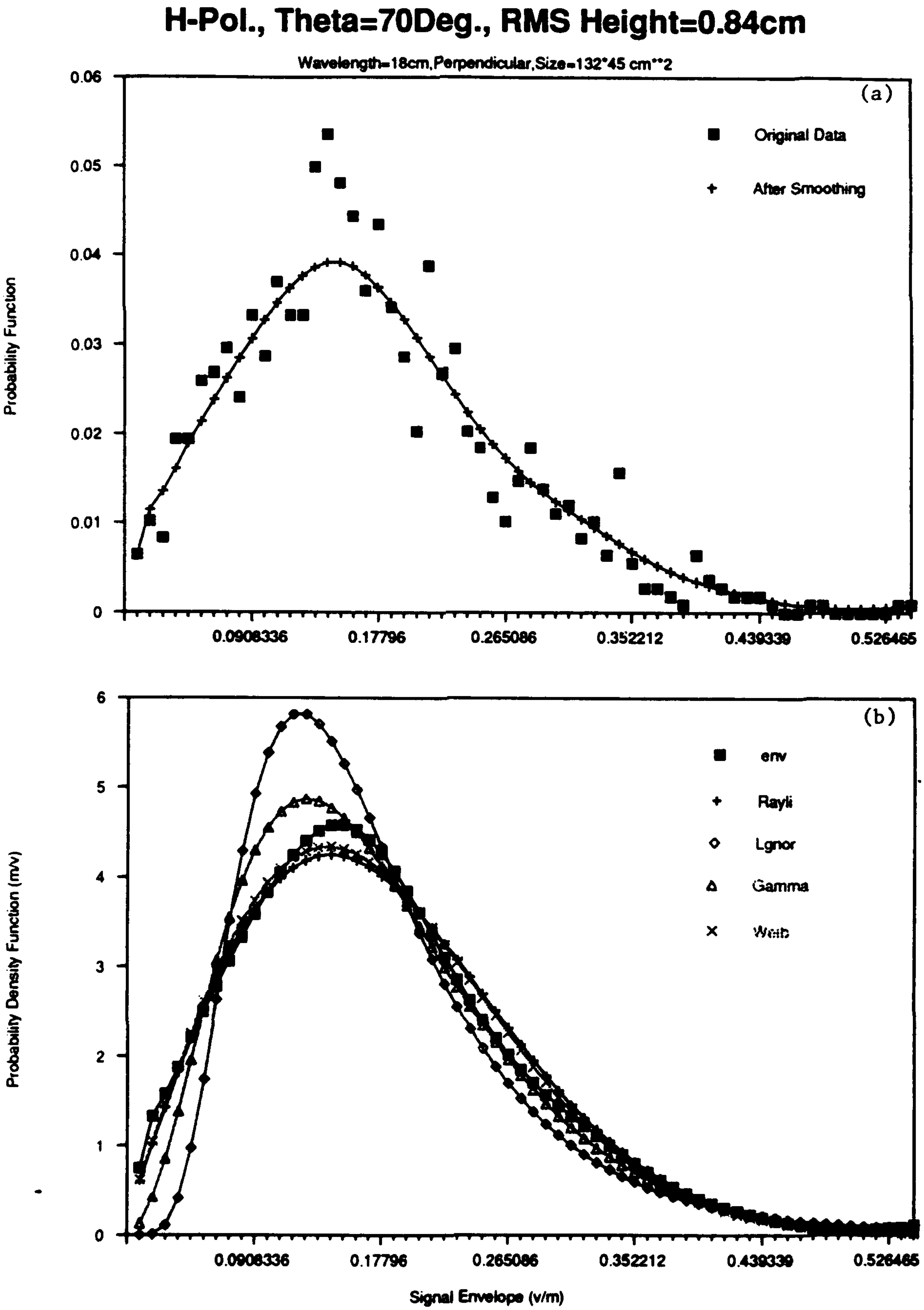


Figure 11



H-Pol., Theta=70Deg., RMS Height=0.84cm
Wavelength=18cm,Perpendicular,Size=132*45 cm**2
(a)
Original Data
After Smoothing
Probability Function
0.06
0.05
0.04
0.03
0.02
0.01
0
0.0908336
0.17796
0.265086
0.352212
0.439339
0.526465
(b)
env
Rayli
Lgnor
Gamma
Weib
Probability Density Function (m/v)
6
5
4
3
2
1
0
0.0908336
0.17796
0.265086
0.352212
0.439339
0.526465
Signal Envelope (v/m)

Table 1

H - Polarization, Incident Angle = 70 Degree,
Cell Size = 24*24 cm^2, Wavelength = 18 cm,
Backscattering.

Models	RMS Error (m/v)	The Model Parameter	
		λ	σ
Rayleigh	0.39		0.17
Lognormal	0.68	-1.8	0.55
Gamma	0.31	2.87	0.067
Weibull	0.11	1.65	0.215
mean= 0.19(v/m)		variance= 0.013$(v/m)^2$	

Table 2

H - Polarization, Incident Angle = 70 Degree,
Cell Size = 72*24 cm^2, Wavelength = 18 cm,
Backscattering.

Models	RMS Error (m/v)	The Model Parameter	
		λ	σ
Rayleigh	0.3		0.14
Lognormal	0.69	-1.92	0.52
Gamma	0.26	3.16	0.53
Weibull	0.11	1.82	0.19
mean= 0.17(v/m)		variance= 0.009 $(v/m)^2$	

Table 3

H - Polarization, Incident Angle = 70 Degree,
Cell Size = 132*45 cm^2, Wavelength = 18 cm,
Backscattering.

Models	RMS Error (m/v)	The Model Parameter	
		λ	σ
Rayleigh	0.22		0.12
Lognormal	0.66	-2.05	0.50
Gamma	0.26	3.42	0.04
Weibull	0.15	1.98	0.173
mean= 0.15 (v/m)		variance= 0.0068$(v/m)^2$	

Table 4

H - Polarization, Incident Angle = 70 Degree,
Cell Size = 24*24 cm^2, Wavelength = 18 cm,
Range = 750 cm, Forward.

Models	RMS Error (m/v)	The Model Parameter	
		λ	σ
Rayleigh	0.73		0.29
Lognormal	0.25	-0.63	0.37
Gamma	0.15	8.36	0.068
Weibull	0.04	3.25	0.63
Normal	0.07	0.57	0.195
mean= 0.56 (v/m)		variance= 0.038 $(v/m)^2$	

Table 5

H - Polarization, Incident Angle = 70 Degree,
Cell Size = 72*24 cm^2, Wavelength = 18 cm,
Range = 750 cm, Forward.

Models	RMS Error (m/v)	The Model Parameter	
		λ	σ
Rayleigh	0.9		0.356
Lognormal	0.14	-0.059	0.237
Gamma	0.10	16.7	0.058
Weibull	0.11	4.98	1.05
Normal	0.05	0.97	0.23
mean= 0.97 (v/m)		variance= 0.05 $(v/m)^2$	

Table 6

H - Polarization, Incident Angle = 70 Degree,
Cell Size = 132*45 cm^2, Wavelength = 18.0 cm,
Range = 750 cm, Forward.

Models	RMS Error (m/v)	The Model Parameter	
		λ	σ
Rayleigh	0.91		0.37
Lognormal	0.09	-0.5	0.137
Gamma	0.06	52.1	0.034
Weibull	0.17	9.16	1.85
Normal	0.04	1.76	0.245
mean= 1.75(v/m)		variance= 0.06$(v/m)^2$	

Table 7

H - Polarization, **Incident Angle = 70 Degree,**
Cell Size = 24*24 cm^2 **Wavelength = 18 cm,**
Range = 750 cm, **Perpendicular.**

Models	RMS Error (m/v)	The Model Parameter	
		λ	σ
Rayleigh	0.66		0.12
Lognormal	0.9	-2.14	0.55
Gamma	0.35	2.84	0.047
Weibull	0.15	1.79	0.15
mean= 0.15(v/m)		variance= 0.006$(v/m)^2$	

Table 8

H - Polarization, **Incident Angle = 70 Degree,**
Cell Size = 72*24 cm^2 **Wavelength = 18 cm,**
Range = 750 cm, **Perpendicular.**

Models	RMS Error (m/v)	The Model Parameter	
		λ	σ
Rayleigh	0.27		0.17
Lognormal	0.69	-1.88	0.54
Gamma	0.33	3.12	0.065
Weibull	0.11	1.82	0.226
mean= 0.18(v/m)		variance= 0.012 $(v/m)^2$	

Table 9

H - Polarization, Incident Angle = 70 Degree,
Cell Size = 132*45 cm^2, Wavelength = 18 cm,
Range = 750 cm, Perpendicular.

Models	RMS Error (m/v)	The Model Parameter	
		λ	σ
Rayleigh	0.15		0.164
Lognormal	0.64	-1.86	0.50
Gamma	0.29	3.54	0.05
Weibull	0.11	2.01	0.20
mean= 0.17(v/m)		variance= 0.008$(v/m)^2$	