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DATA ASSOCIATION ALGORITHMS FOR MULTIPLE TARGET TRACKING (U)

by

J. Chris McMillan and Sang Seok Lim

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ABSTRACT

Multi-target tracking (MTT) has many applications, and has therefore been the subject of considerable investigation. One key aspect of this problem, especially in a dense target environment, is the "scan-to-scan correlation" or "tracking data association" problem of assigning measurements to tracks. Various different approaches have been proposed to solve this problem. In this report, three algorithms for MTT data association are presented for comparison. First the nearest-neighbor standard filter (NNSF) algorithm is presented. Then two of the more promising extensions are presented: the multiple hypothesis test method (MHT), and joint probabilistic data association (JPDA) method. Whenever possible the same notation is used in presenting all three methods for ease of comparison. This report is intended to serve as a prelude to a comparative investigation of these three competing data association methods.

RÉSUMÉ

Les techniques de poursuites de cibles multiples ont plusieurs applications et ont donc fait l'objet de nombreuses recherches. L'un des aspect important du problème est l'association des données radar avec les pistes. Plusieurs approches ont déjà été proposées pour résoudre ce problème. Ce rapport compare trois algorithmes d'association des données. La méthode standard du voisin-immédiat (NNSF) est d'abord présentée. Deux variations prometteuse sont ensuite analysées: une méthode de test à hypothèses multiples (MHT) et une méthode d'association de données probalistique (JPDA). Ce rapport présente une analyse préliminaire des trois méthodes et sera suivi d'études plus approfondies.

EXECUTIVE SUMMARY

In this report three quite different algorithms for multiple target tracking in a cluttered environment are presented: Nearest Neighbour Standard Filter (NNSF), Multiple Hypothesis Test (MHT) and Joint Probabilistic Data Association (JPDA). The basic tracking equations and data association algorithms are presented, using a consistent notation, so that the different methods can be directly compared. The apparent advantages and dissadvantages of each are briefly explored, and suggestions made for further analysis.

Superficially the tradeoff among the three different approaches is simply one of accuracy vs. computational efficiency, with the NNSF method being the least accurate but most efficient, and the MHT method being the most accurate and least efficient. However this simplistic view overlooks the fact that the three methods differ fundamentally in their approach to data association.

The NNSF produces a single unambiguous data association solution at each point in time, based on the previous association and the current sensor information. This however may not be the best choice, especially because it does not make full use of all prior sensor data. Once an incorrect association is made, it seems unlikely that the solution would ever recover.

The MHT approach will maintain several (perhaps many) different possible data association solutions, and uses the history of sensor data to eliminate highly unlikely choices, eventually leaving only one best choice (hopefully). This should yield the best solution, however it does generally have periods of uncertainty. Thus if one could not wait for the solution, then it would be necessary to take special measures.

The JPDA approach does not explicitly make a data association decision. Instead this method applies all ambiguous measurements to each tracking filter with which it could reasonably be associated (with an appropriate weighting factor). Thus any given tracking filter will likely be assigned several "incorrect" measurements (hopefully with a low weighting) but will almost certainly also be assigned the "correct" measurement (hopefully with a high weighting). It is thus hoped that the incorrect measurements (hopefully random) will have a small cumulative effect.

Further investigation would be necessary to determine which, if any, of these data association methods is most appropriate for a given multi-target tracking problem.

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1.0 INTRODUCTION TO MULTIPLE TARGET TRACKING

1.1 The Problem

The multiple target tracking problem is encountered in many situations, whenever sensor data is available from one or more wide angle of view sensors (such as radar, electronic support measures (ESM), infra-red (IR), video, etc.), providing information on the position (such as range, azimuth, elevation etc.) of "targets" (aircraft, vessels, missiles, ground vehicles etc.). The sensors may also provide information useful for target identification purposes (size, velocity, luminosity, transmitter frequency, etc.). Although this information may be useful to a target tracking algorithm, the identification aspects of target tracking are beyond the scope of this report.

These target tracking sensors typically **scan** their field of view at regular intervals. There may be many **contacts** from such a scan, from the different objects in the sensors field of view, as well as some which may be just noise. The sensor measurements from all contacts within a scan are generally processed together, since they can safely be assumed to correspond to different objects (something which cannot be said of returns from different scans or from different sensors).

The basic tracking problem is to estimate the position and velocity of the target(s), using the available sensor data (from a sequence of scans). It is also normally desireable to be able to predict the targets' location some time in the near future. This is a problem because the sensor data normally has errors and/or ambiguities and the targets generally move between scans. The prediction is further complicated by unknown target manoeuvres. If there is only a single target, however, then Kalman filtering techniques provide an "optimal" solution. Although the nonlinearitites of this problem introduce some complications in the use of Kalman filtering, this problem has been extensively studied and suitable solutions exist.

The multi-target tracking problem however, is not simply a tracking problem when there are more than one target. As shall be seen below however, the possibility of more than one target significantly complicates the situation, because it introduces the problem of **associating** measurements with targets. When the distance between targets is comparable to the measurement error or to the error in target position prediction (from one scan period to the next), then this association problem can be very difficult. For example fighter aircraft flying in tight formation being tracked by a scanning radar.

1.2 Background

There currently is no satisfactory unified solution method for the Multi-target tracking problem. The single target tracking problem, however, has been studied in depth, and optimal estimation theory (Kalman filter theory etc.), does provide a satisfactory solution (see for example [1]). With this method, one can form the statistically optimal estimate of the target state vector (position and velocity) by recursively processing sensor measurements (radar, sonar, ESM etc.) taken from the target being tracked.

The multi-target tracking problem can be approached in the same way, as a number of stochastic estimation problems (one for each target). Unfortunately there is the added difficulty of deciding which measurements correspond to which targets (and which measurements don't correspond to any targets). This problem of associating measurements with targets is known as the "data association problem". Since this is not a problem of estimating the value of a random variable, it does not fall neatly into the realm of optimal estimation. However, it is amenable to stochastic treatment, and several approaches have been suggested. This report describes in brief three of the more promising approaches.

All three approaches are based on the intuitive premise that the probability of a particular measurement corresponding to a specific target is a function of the proximity of that measurement to the expected target location.

The simplest of the approaches described here is the "Nearest Neighbor" method, which at each time epoch (scan) makes a **complete assignment** decision. This complete assignment can be defined as a specific decision for each measurement:

1- that it came from one of the targets being tracked,
2- that it is noise, or
3- that it is from a new target.

Furthermore this allocation of measurements is made in such a way that no target is assigned more than one measurement from any given sensor scan. The nearest neighbor approach makes this complete assignment decision at each point in time by minimizing a global distance function which represents the closeness of each measurement to its assigned target. A problem with this approach is that if an incorrect assignment is made for a single measurement, then the tracking filter that processes it will make a poor prediction for the position of it's target for the next time epoch, likely leading to subsequent incorrect assignments, and thus the process could easily break down. The two other methods described here are different approaches to solving this problem. The nearest neighbour method is described in more detail in chapter

2 below.

The second approach to be described is the Multiple Hypothesis approach. As the name implies, this approach attempts to solve the problem (of not being able to correctly decide at each epoch which measurement corresponds to which target) by keeping all reasonably likely "complete assignments" (as defined above) as working hypothesis (with which to predict the next epoch's target positions). The global distance measure (of proximity of measurements to predicted targets) can then be used to assign a "probability" to each hypothesis at each epoch. These probabilities can then be combined over several epochs, with the expectation that incorrect hypothesis will lead to highly unlikely cumulative probabilities (ie. dead ends). In this way these hypothesis can be dropped and the correct hypothesis should eventually stand out alone. The problem with this method is that, unless the number of hypothesis carried forward from one epoch to the next is kept very low, then the "hypothesis tree" will grow extremely quickly, and a computationally unmanageable situation will arrise. This method is described in more detail in chapter 3 below.

The third method is the "Joint Probabilistic Data Association" or JPDA method, which takes a very different approach: assigning all measurements in the vicinity of a given target to that target, but weighting the measurements according to their proximity. Thus it is highly unlikely that the correct measurement will be ignored, but on the other hand it is an almost certainty that incorrect measurements will also have some unwanted influence. This JPDA method is described further in chapter 4 below.

1.3 State Space Description

For an object being tracked, the discretized equations of motion may be modeled by

$$x_{k+1} = \Phi_k x_k + G_k u_k \tag{1}$$

where x_k is an nx1 state vector (describing the position and velocity etc.) of the tracked objects at the k^{th} sample time, Φ_k is the transition matrix, and u_k is a mx1 state excitation vector to account for both maneuvres and modeling errors and is generally assumed to be white and Gaussian with zero mean and covariance Q_k. In a track-while-scan system, the k^{th} sample will occur approximately at time kT, where T is the scan interval of the sensor.

The observation equation representing valid sensor measurements of the objects being tracked has the form

$$y_k = H_k x_k + v_k \tag{2}$$

where y_k is the mx1 sensor measurement vector (also referred to as the sensor "reports") and v_k is white Gaussian observation noise with zero mean and covariance R_k. The observation equation for extraneous sensor reports resulting from thermal false alarms, clutter, and other targets is assumed to satisfy

$$y_k = H_k xe_{k|k-1} + w_k \tag{3}$$

where w_k is assumed white and uniformly distributed over some volume C of the measurement space centered about the predicted measurement

$$ye_{k|k-1} = H_k xe_{k|k-1} \tag{3a}$$

The number of such extraneous reports in any volume C obeys a Poisson distribution with mean BC, where B is the unnormalized extraneous reports density.

The tracking filter provides a state estimate $xe_{k|k}$ and one-scan-predicted state $xe_{k+1|k}$ given all measurement data up to the time k. The basic filter equations are described in the following section.

1.4 Basic Tracking Filter Equations

The tracking filter state estimate $xe_{k|k}$ and covariance $P_{k|k}$ update equations are calculated (as functions of the measurements y_k and the previous estimates $xe_{k|k-1}$ and $P_{k|k-1}$) as follows:

$$xe_{k|k} = xe_{k|k-1} + K_k[y_k - H_k xe_{k|k-1}] \tag{4}$$

$$K_k = P_{k|k-1}H_k^T[\ H_k P_{k|k-1} H_k^T + R_k\ ]^{-1} \tag{5}$$

$$P_{k|k} = [\ I - K_k H_k\ ] P_{k|k-1} \tag{6}$$

This requires knowledge of the measurement noise covariance $\mathbf{R}_k$ and the measurement matrix $\mathbf{H}_k$ relating the measurements to the state vector.

These estimates can then be propagated one scan into the future (prediction) by using the state transition matrix Φ_k as follows:

$$xe_{k+1|k} = \Phi_k xe_{k|k} \qquad (7)$$

$$\mathbf{P}_{k+1|k} = \Phi_k \mathbf{P}_{k|k} \Phi_k^T + G_k Q_k G_k^T \qquad (8)$$

1.5 Notation and Definitions

To facilitate comparison, the same notation is used wherever possible to describe the different approaches to data association. This notation, and related definitions are collected here for ease of reference:

k = time or scan index.

N[x,P] = probability density function for a zero mean Normally distributed (Gaussian) random vector x, with covariance matrix **P**

$$= \exp(-0.5\ x^T\ \mathbf{P}^{-1}\ x)/\sqrt{\{(2\pi)^n\ |\mathbf{P}|\}} \qquad (9)$$

(where n = dimension of x).

Nc = no. of measurements associated with confirmed tracks.

Nf = number of measurements associated with false targets.

Nn = number of measurements associated with new targets.

Nt = number of previously known (at time k-1) targets (confirmed tracks) within the cluster.

Nm; number of measurements.

P_D = probability of detection.

B_{ft} = density of false targets.

δ_{kt} = density of previously known targets that have been detected.

c = normalization constant given by summation of all the hypothesis probabilities within the cluster for one scan.

Pr(k) = probability of a hypothesis at time k given probability of the prior hypothesis.

$\Sigma_{b=1}^{L}$ = Summation over b from 1 to L.

$\Pi_{b=1}^{L}$ = Product over b from 1 to L.

L_k = total number of hypothesis for track i at time k in the absence of pruning:

$$L_k = (1+Nk)L_{k-1}$$

for Nk the number of sensor reports falling within the gate for track i at scan k (pruning may have reduced L_{k-1} between scans).

$Pa1_{k|k-1}$ = covariance of the estimation error given observations through scan k-1 and given track hypothesis "a1".

$Ka1_{k|k-1}$ = expected value of the estimation error given observations through scan k-1 and given track hypothesis "a1".

$Pa_{k|k-1}$ and $Ka_{k|k-1}$ are the Pa1 and Ka1 analogues corresponding to the hypothesis "a".

$S^{-1}{}_k$ = inverse of S_k which is the covariance of the innovation $y_k - ye_{k|k-1}$ given hypothesis "a1".

$xe_{k|k-1}$ =estimated state at scan k given hypothesis up to time k-1.

$P_{k|k}$ = covariance of the estimation error given observations up to scan k.

A_k = optimal tracking filter correction vector for MHT filter.

AA_k = optimal tracking filter gain for JPDA.

De = the expected track length.

$\#_{jt}$ = the event that an observation j belongs to a target t.

Z_k = the set of measurements up to time k.

P_D^t = the probability of detection of target t.

Nt(j) = number of targets associated with the observation j except for false alarms(false targets).

$Nc(t)$ = number of observations associated with the target t

$Nc'(t)$ = number of observations not associated with the target t.

Ncr = (Number of columns of Ω)*(Number of rows of Ω), where Ω is validation matrix given by eq.(25).

2.0 A NEAREST NEIGHBOR STANDARD FILTER ALGORITHM

2.1 Introduction

The nearest neighbor approach to data association determines a unique pairing so that at most one observation from each sensor scan can be paired with a previously established track. Also, a given observation can be used only once, either to update an existing track or to start a new track (which may be abandoned as noise if no subsequent observations match it). This method is based on likelihood theory, and the goal is to minimize an overall distance function that considers all observation-to-track pairings that satisfy a preliminary gating test.

2.2 NNSF Algorithm

In this section, a NNSF algorithm, based on the modified Munkres optimal assignment method (see reference [2]), is presented.

Step 1: Set k=1 (scan index) and all control parameters.

Step 2: Initialize the system parameters; Φ, Q, H, R.

Step 3: Simulate target trajectories.

Step 4: Simulate measurements (or receive the data from radar.)

Step 5: Validate the measurements using gating test: Form gate about

$$ye_{k|k-1} = H * xe_{k|k-1}$$

and select the Nk sensor reports to be used in filter updating. A measurement is valid if

$$\{y_k - ye_{k|k-1}\}^T S^{-1}_k \{y_k - ye_{k|k-1}\} \leq g^2 \qquad (10)$$

(this is a "g-σ" elliptical gate).

Step 6: Form the assignment matrix. The elements of the matrix are equal to the normalized distance function associated with the assignment of each of "Nobs" observations to each of "Ntr" tracks. If the gating relationship is not satisfied, the observation- to-track pairing can be given a very large distance to penalize this assignment.

Step 7: Solve the Assignment Matrix: Minimize the normalized distance function using the modified Munkres optimal assignment algorithm in section 2.3.

Step 8: Correlate the observations to the tracks according to the solution of optimal assignment matrix.

Step 9: Update and predict the state vectors (tracks) using (4) to (9).

Step 10: Output the predicted track positions.

Step 11: If k=kf (end of tracking mission), go to the Step 11. Otherwise, set k=k+1 and go to Step 4.

2.3 Modified Munkers Assignment Algorithm

The following method is based on the Munkres optimal assignment algorithm modified by Burgeois and Lassalle [12]. This method has an advantage (compared to older methods) for applications since the assignment matrix need not be square. For the convenience of presentation of the algorithm, the rows and columns of the matrix may be marked and referred to as covered. The zeros may be marked by being starred (*) or primed (′).

The Optimal Assignment Algorithm:

Step 1: Initially, no lines are covered and 0's are starred or primed.

Step 2: Let v = min.{Number of rows, Number of columns}.

Step 3: If (number of rows) > (number of columns), go to Step 6.

Step 4: For each row in the matrix, subtract the smallest element of the row from each element in the row.

Step 5: If the (number of columns) > (number of rows), go to Step 7.

Step 6: For each column in the matrix, subtract the smallest element of the column from each component of the column and go to Step 7.

Step 7: a) Find a zero, "Z", of the matrix.
b) If there is no starred zero in its row or its column,

star the zero (i.e. Z*). Repeat for all zeros of the matrix. Go to Step 8.

Step 8: a) Cover every column containing a starred zero Z*.
b) If v columns are covered, the locations of the Z* form the row-column associations (i.e. observation-to-track pairs). The algorithm is now completed. Otherwise, continue to the next step.

Step 9: a) Choose an uncovered zero and prime it (i.e. Z').
b) If there is no starred 0 in the row of Z', go to Step 10.
c) If there is a starred zero Z* in the row of Z', cover this row and uncover the column of Z*.
d) Repeat until all zeros are covered and then go to Step 11.

Step 10: a1) Let Zo denote the uncovered Z'. If there is no Z* in the column of Zo, go to Step (a6).
a2) Let Z1 denote the Z* in the column of Zo.
a3) Let Z2 denote the Z' in the row of Z1.
a4) Continue the steps a2 and a3 until a Z2 which has no Z* in its columns has been found.
a5) Un-star each starred zero of the sequence.
a6) Star each primed zero of the sequence.
b1) Erase all primes from primed zeros and uncover every line.
c1) Go to Step 8.

Step 11: a) Find the smallest uncovered element in the matrix and call it "m". "m" will be positive.
b) Add "m" to each covered row.
c) Subtract "m" from each uncovered column.
d) Go to Step 9 without altering stars, primes, or uncovered lines.

3.0 THE MULTIPLE HYPOTHESIS APPROACH

3.1 Introduction

In the sequel of this section, a track history "a" at scan k is defined by selecting a single sensor report y from each scan $j \le k$

$$\{y(Ij;j),\ j=1,2...k \mid 0 \le Ij \le Nj\}$$

where Nj is the number of reports at scan j and Ij=0 refers to the hypothesis that none of the sensor reports within the gate originated from the target. Hence the track history "a" is just the hypothesis that the entire sequence of measurements within "a" is correct; i.e. each sensor report y(Ij;j) originated from the target when Ij 0, while no sensor report was received when Ij=0, $1 \le j \le k$. The track history "a" at scan k is obtained from the track history "a1" at scan k-1 by selecting one of 1+Nk measurements and incorporating it into the measurement set specified by "a". In notational terms, a={a1, I}. Clearly one history "a1" at scan k-1 gives rise to 1+Nk histories "a" at scan k. Therefore, the total number of hypothesis (or histories) L_k at scan k is given by $(1+Nk)*L_{k-1}$.

3.2 The Tracking Filter Equations

The equations for target state estimation (the tracking equations) that are appropriate for use with MHT data association, are listed below. Section 1.5 above provides the notation. Appendix B explains the important hypothesis probability equation (11). The other equations come largely from Kalman filtering theory, and are very similar to the standard tracking equations of section 1.4 above.

Pr(k) = Hypothesis probability at scan k

$$= (1/c)\ (P_D)^{Nc}\ (1-P_D)^{Nt-Nc}\ (B_{ft})^{Nf}\ (B_{kt})^{Nn}$$

$$*\ \Pi_{j=1}^{Nc}\ N[y_k - H_k xe_{k|k-1}, S]$$

$$*\ Pr(k-1) \qquad (11)$$

$$Ka_{k|k-1} = Kal_{k|k-1} + Pal_{k|k-1}H_k^T Sal_{k|k-1}[y_k - ye_{k|k-1} - H_k Kal_{k|k-1}] \quad (12)$$

where

$$S^{-1}{}_{k|k-1} = H_k P_{k|k-1} H_k^T + R_k$$

$$Pa_{k|k-1} = Pal_{k|k-1} - Pal_{k|k-1}H_k^T S^{-1}{}_{k|-1} H_k Pal_{k|k-1} \quad (13)$$

or

$$Ka_{k|k-1} = Kal_{k|k-1} \quad (14)$$

$$Pa_{k|k-1} = Pal_{k|k-1} \quad (15)$$

if no measurement on the target was received in the gate.

$$A_k = \Sigma_{a=1}^{Lk}[Pr(k)Ka_{k|k-1}] \quad (16)$$

= optimal tracking filter correction vector at scan k

$$Ka_{k|k} = Ka_{k|k-1} - A_k \quad (17)$$

$$Pa_{k|k} = Pa_{k|k-1} \quad (18)$$

The state and covariance update:

$$xe_{k|k} = xe_{k|k-1} + A_k \quad (19)$$

$$P_{k|k} = \Sigma_{a=1}^{Lk}[Pr(k)\{Pa_{k|k-1} + Ka_{k|k-1}Kal_{k|k-1}\}] - A_k A_k^T \quad (20)$$

One scan prediction:

$$xe_{k+1|k} = \Phi_k xe_{k|k} \quad (21)$$

$$P_{k+1|k} = \Phi_k P_{k|k} \Phi_k^T + G_k Q_k G_k^T \quad (22)$$

$$Ka_{k+1|k} = \Phi_k Ka_{k|k} \quad (23)$$

$$Pa_{k+1|k} = \Phi_k Pa_{k|k} \Phi_k^T + G_k Q_k G_k^T \quad (24)$$

3.3 MHT Algorithm

This section outlines the basic algorithm for implementation of the Multiple Hypothesis Testing method for multi-target tracking, either in a simulation environment or in real time. First the algorithm is described verbally, followed by a top level flowchart.

For notations and definitions, see section 1.5. In an actual implementation the simulated quantities are received from sensors.

STEP 1: Set scan counter k=1 and all the control parameters.

STEP 2: Initialize the track and system parameters;

Φ, Q, H, R, $Pal_{1|0}$, $Kal_{1|0}$, Pr(0), $xe_{1|0}$, $P_{1|0}$

STEP 3: Simulate target trajectories.

STEP 4 Simulate measurements.

STEP 5: For each history (or hypothesis) "a1", $1 \leq a1 \leq L_{k-1}$, receive unconditional estimation parameters (from STEP 11):

$Pal_{k|k-1}$, $Kal_{k|k-1}$, Pr(k-1), $xe_{k|k-1}$, $P_{k|k-1}$

STEP 6: Form gate about

$$ye_{k|k-1} = H\ xe_{k|k-1}$$

and select the Nk sensor reports to be used in filter updating. A measurement is valid if

$$\{y_k - ye_{k|k-1}\}^T S^{-1}{}_k \{y_k - ye_{k|k-1}\} \leq g^2$$

(this is a "g-σ" elliptical gate).

STEP 7: (1) Identify new histories (or hypotheses) "a", for

$$1 \leq a \leq (1+Nk)*L_{k-1}.$$

(2) For each hypothesis, compute Pr(k) using equation (11). (see Appendix B for details.)

STEP 8: For each remaining history (or hypothesis) "a", and each track within "a"
Compute $Ka_{k|k-1}$ using equation (12).
Compute $Pa_{k|k-1}$ using equation (13).

STEP 9: For each history (or hypothesis) "a",
Calculate A_k using equation (16).
Calculate $Ka_{k|k}$ using equation (17).
Calculate $Pa_{k|k}$ using equation (18).

STEP 10: Compute new optimal updated estimates $xe_{k|k}$ using eq. (19). Compute error covariance $P_{k|k}$ using equation (20).

STEP 11: Predict for next scan data:
Compute $Ka_{k+1|k}$ using equation (23).
Compute $Pa_{k+1|k}$ using equation (24).
Compute $xe_{k+1|k}$ using equation (21).
Compute $P_{k+1|k}$ using equation (22).

STEP 12: GO TO STEP 5 for next scan data and/or output the results.

Figure 1 below provides a top level flow chart of this MHT process, showing the sequence of events. The important function of "pruning" the hypothesis tree (to reduce the number of hypothesis to consider) is described very briefly in appendix C. The clustering process, also to reduce the number of hypothesis to consider, is described in appendix D.

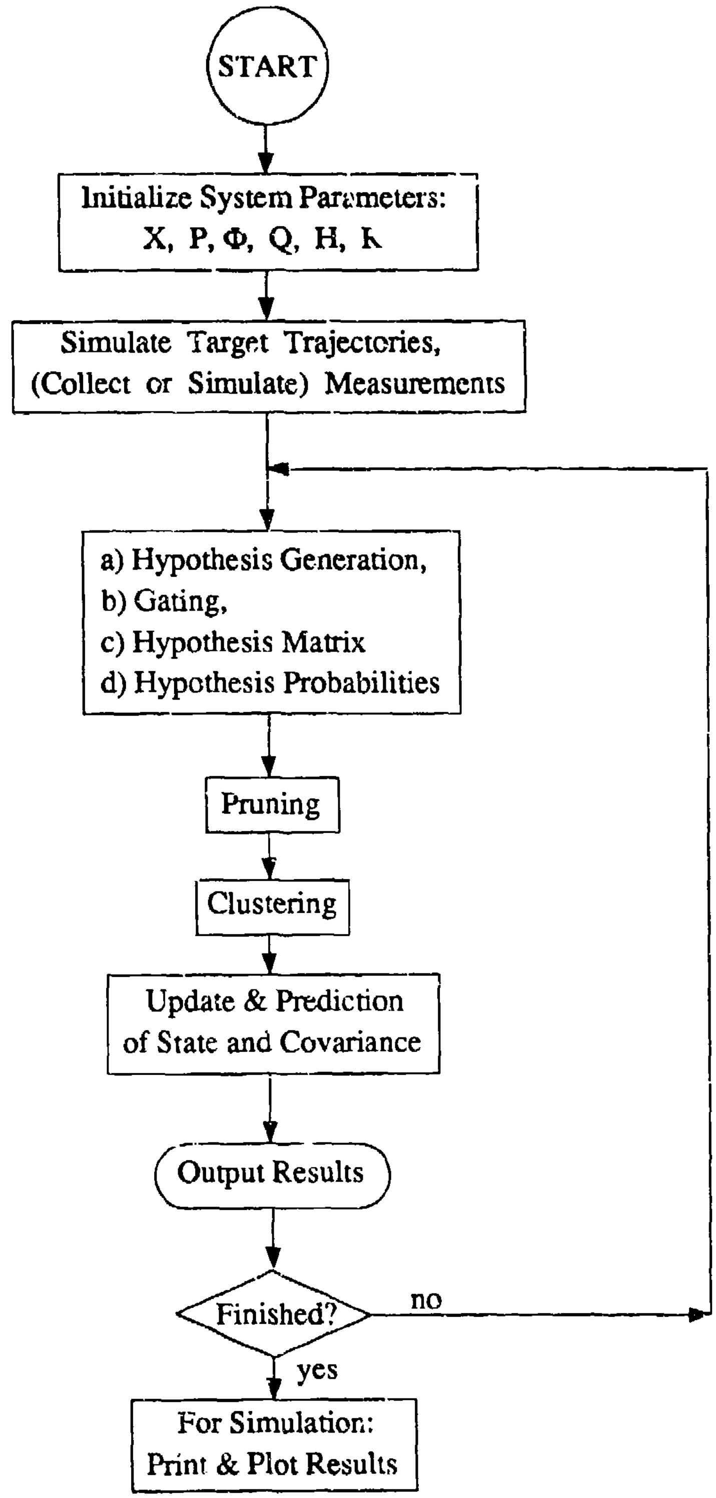

Figure 1. Multiple Hypothesis Test Flowchart.

4.0 JOINT PROBABILISTIC DATA ASSOCIATION

4.1 Introduction

In this section, a JPDA algorithm for multiple targets in Poisson clutter is presented. This all-neighbors approach incorporates all observations within a validation region about the predicted track position into the update of that track. Also, a given observation can be used to update multiple tracks. Track update is then based on a probabilistically determined weighted sum of all observations within the validation region. In fact, the method performs an averaging over observation-to-track data association hypothesis that have roughly comparable likelihood. Further, this is a target-oriented approach, in the sense that a set of established targets is used to form gates in the measurement space and to compute posterior probabilities, in contrast to the measurement-oriented algorithm such as MHT, where each measurement is considered in turn and hypothesized to have come from some established track, a new target, or clutter (false alarm).

4.2 A JPDA Algorithm

Step 1: Set k=1 (time index) and all the control parameters.

Step 2: Initialize the system parameters; Φ, Q, H, R.

Step 3: Either simulate the target trajectories and sensor measurements (including clutter etc.), or in a real implementation, receive and register (scale, time tag etc.) the sensor data.

Step 4: Validate the measurements using "g-σ ellipsoid" gating test (eq.(10)). Form the validation matrix $\Omega(\#)$: (examples can be found in [13]).

$$\Omega_{jt} = \{w(\#_{jt})\}, \quad j=1,2,\ldots,Nm; \; t=1,2,\ldots,Nt \qquad (25)$$

where

$$w(\#_{jt}) = \begin{cases} 1 & \text{if meas. } j \text{ is within the gate of target } t \text{ (event "}\#_{jt}\text{" occurs)} \\ 0 & \text{otherwise} \end{cases} \qquad (26)$$

Step 5: For each t=1,2,...,Nt compute the residual:

$$Y_{jt} = y(j,t) - ye_k \qquad (27)$$

Step 6: Compute covariance of Y_{jt} at k:

$$S(j,t) = H_k \, P_{k|k-1} \, H_k^T + R_k. \quad (28)$$

Step 7: Using Bayes' formula, compute the joint event prob.

$$P_{hyp}(\#) = Pr\{\# | Z_k\}$$

$$= (C^*/c) * \Pi_{j=1}^{Nt(j)}[-0.5*Y^T_{jt}S^{-1}Y_{jt} \; / \; \sqrt{\{(2\pi)^{Nm}|S(j,t)|\}} \;]$$

$$* \; \Pi_{t=1}^{Nc(t)}[P_D^{\,t}]$$

$$* \; \Pi_{t=1}^{Nc'(t)}[1-P_D^{\,t}] \quad (29)$$

where S^{-1} = inverse of $S(j,t)$ at scan k (30)

and where the numbers Nt(j), Nc(t) and Nc'(t) can be calculated from the validation matrix Ω.

Step 8: For each target t=1,2,...,Nt, compute

$$B(j,t) = \Sigma_{i=1}^{Ncr}[P_{hyp}(i)*w(j,t;i)] \quad (31)$$

and for t=0, compute

$$B(0,t) = 1 - \Sigma_{j=1}^{Nm}[B(j,t)] \quad (32)$$

Step 9: For each t=1,2,..., Nt, compute

$$Yt = \Sigma_{j=1}^{Nm}[B(j,t)*Y_{jt}]. \quad (33)$$

Step 10: Compute the filter gain AA_k:

$$AA_k = P_{k|k-1} \, H_k^T \, S^{-1}{}_k \quad (34)$$

$$S^{-1}{}_k = S^{-1}$$

Step 11: Update the state and covariance: for each t=1,2,...,Nt,

$$xe_{k|k} = xe_{k|k-1} + AA_k*Yt \quad (35)$$

$$P_{k|k} = P_{k|k-1} - AA_k \, S(j,t) \, AA_k^T \quad (36)$$

Step 12: Predict for the next scan data:

$$xe_{k+1|k} = \Phi_k \, xe_{k|k} \quad (37)$$

$$P_{k+1|k} = \Phi_k P_{k|k} \Phi_k^T + G_k Q_k G_k^T \quad (38)$$

$$ye_{k+1|k} = H_{k+1} \, xe_{k+1|k} \quad (39)$$

Step 13: If desired, calculate and output the estimated current target position, velocity etc. (at time k) from the current state estimate, $xe_{k|k}$, or the predicted target position etc., (at time k+1) using the predicted state estimate $xe_{k+1|k}$. The associated covariance may also be desired.

Step 14: Set k=k+1 and go to the Step 3.

Remark:

In the above algorithm, the joint event probabilities are computed using equation (29) which is derived under the assumption that the probability mass function (PMF) of the false measurements is given by the Poisson PMF (see equation (9-42) of [2]):

$$\mu(m) = \exp\{-\lambda V_k\}(\lambda V_k)^m / m! \qquad (40)$$

where λ is the spatial density of false measurements(i.e., the average number per unit volume) and V_k is the volume of the validation region. Thus λV_k is the expected number of false measurements in the gate and m is the number of false measurements.

If the PMF is uniformly distributed, then the expression for the probability should be modified accordingly (see equation (9.46) of [2]).

5.0 CONCLUSIONS

In this report three quite different algorithms for multiple target tracking in a clutter environment are presented: Nearest Neighbour Standard Filter (NNSF), Multiple Hypothesis Test (MHT) and Joint Probabilistic Data Association (JPDA). For further development towards implementation of the algorithm, the following comments and suggestions are offered.

Comments:

1) The main drawback of the NNSF algorithm is that the tracking performance or accuracy of the filter may become very poor in a dense target environment because of possible misassociation (choosing an incorrect measurement for processing by a target tracking filter). It's main advantage is that it is easy to implement and computationally feasible.

2) In the MHT method, the main drawback of the algorithm is that in a dense target environment the number of hypotheses can increase exponentially with each scan, leading to computational burden problems. Hence, for implementation, the development of an efficient way of pruning the hypotheses tree is necessary.

3) Another disadvantage of the MHT method is that the data association decision is often deferred, and thus a single best estimate for the target tracking solution is not always available. If target execution is desired before the MHT solution is resolved, this can be a problem. This problem can likely be overcome by providing a NNSF-type solution at all times (using the most probable current hypothesis: ie. using all measurements up to the current time).

4) One advantage of the MHT method is that it provides a systematic track initiation procedure.

5) Another advantage of the MHT method is that it is most likely to have the correct association solution as one of it's hypothesis (hopefully the hypothesis assigned highest probability).

6) The JPDA algorithm is a non-back-scan (or zero-scan) approach, meaning that all hypotheses are combined after computation of the probabilities, for each target at each time step. One problem with this method is that it implies that incorrect measurements are routinely (and purposely) used by the tracking filters, albeit hopefully with a lower weighting than the correct measurement. This method does however have the advantage of being more computationally efficient than an n-scan algorithm in a heavy

clutter environment such as sonar tracking.

7) Another difficulty with the JPDA method is in the implementation of track initiation. Since the MHT algorithm provides a systematic track initiation procedure, an effective way of combining these two methods may be of interest.

Suggestions:

1) Comparisons of the accuracy performance of these different algorithms for various target dynamic and clutter models.

2) Comparison of the computational burdens of these algorithms for various target densities and clutter models.

3) Investigation of ways and means of overcoming the shortcommings of these methods, especially pruning for the MHT method, and providing real time output from the MHT method.

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APPENDIX A: HYPOTHESIS TREE AND HYPOTHESIS MATRIX

Let Ω_k be the set of association hypotheses up to time k. This set is obtained from Ω_{k-1} and the latest set of measurements Y_k as follows. New hypotheses Ω_{k-1} first measurement $y1_k$, then augmenting the resulting set by associating $y2_k$, etc. The possible association for the i-th measurement yi_k are

a. It is the continuation of a previous history (or track).
b. It is a new track (or target).
c. It is a false alarm (or clutter).

Each target can be associated with at most one current measurement, which has to fall in its validation region.

Example:

For the configuration of targets and measurements shown in Fig.1, the hypothesis tree are formed using the rule mentioned above. In the hypothesis tree shown in Fig.2, each node represents the track qualities:
"0" is the false target or false alarm.
"1,2,..." are the confirmed or new targets.

The following figure 2 shows the hypothesis tree representation of the hypothesis formation technique outlined above. Each node of the tree represents an alternative hypothesis; further branches are added to each node as a new measurement point is considered.

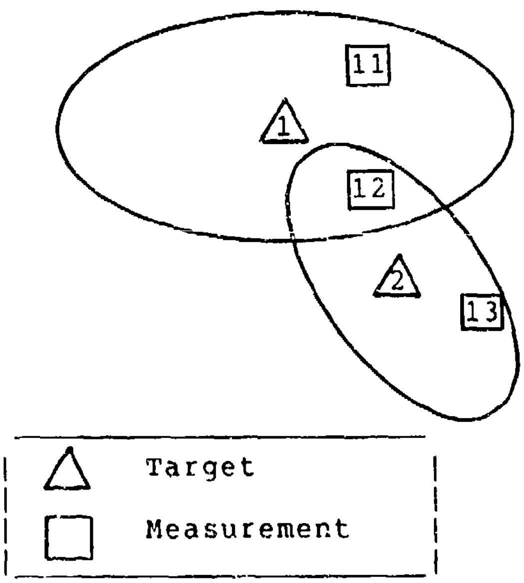
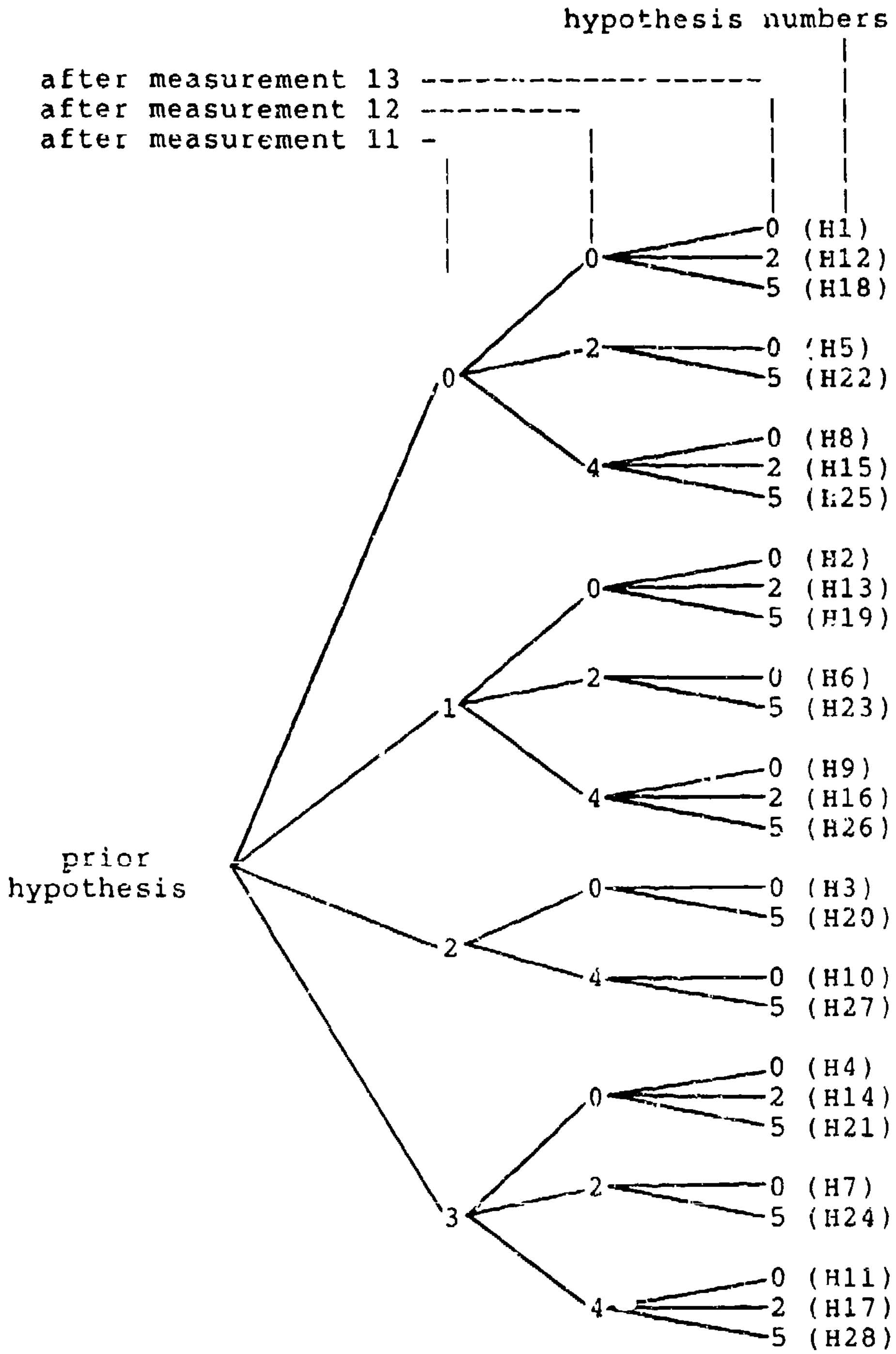


Figure A1. Configuration of Targets and Measurements.



0 : false target
1, 2: target 1 and 2
3,4,5:new targets

Figure A2. Hypothesis Tree for Configuration in Figure A1.

The hypothesis matrix corresponding to the tree can be formed as follows.

after the measurement 11	after the measurement 12	after the measurement 13	hypothesis numbers
0	0	0	H1
1	0	0	H2
2	0	0	H3
3	0	0	H4
0	2	0	H5
1	2	0	H6
3	2	0	H7
0	4	0	H8
1	4	0	H9
2	4	0	H10
3	4	0	H11
0	0	2	H12
1	0	2	H13
3	0	2	H14
0	4	2	H15
1	4	2	H16
3	4	2	H17
0	0	5	H18
1	0	5	H19
2	0	5	H20
3	0	5	H21
0	2	5	H22
1	2	5	H23
3	2	5	H24
0	4	5	H25
1	4	5	H26
2	4	5	H27
3	4	5	H28

Figure A3. Hypothesis matrix corresponding to the hypothesis tree.

APPENDIX B: HYPOTHESIS PROBABILITIES

In the equation for hypothesis probability in section 3.2 above (eq. (11)), an expression suggested by Reid (eq. (16) of ref. [11]) is used. Further, the formula is valid for type 1 sensors such as a scanning radar (which provide a "complete" picture of some coverage area in each data set, and hence can provide number-of-target information). If type 2 sensors such as ESM or a tracking radar are assumed (which in each report only provide information about individual targets), some modifications have to be done as given in [11]. Similar expressions for computation of hypothesis probabilities in the non-recursive form can be found in Bar-Shalom [2].

The equation (11) can be easily implemented since it is given by a recursive form. If all the prior hypotheses are first multiplied by $(1-P_D)^{Nt}$, then as a branch is created for each measurement and its hypothesized origin, the likelihood of the branch is found by multiplying the prior probability by either B_{ft}, B_{kt}, or

$$P_D * N[y_k - H_k xe_{k|k-1}, S]/(1-P_D)$$

as appropriate. After all such branches are generated the likelihoods are then normalized.

Example

In Figure B2, the hypothesis probabilities are computed for the hypothesis tree shown in figure B1, in terms of its prior hypothesis probability and the probability of each branch.

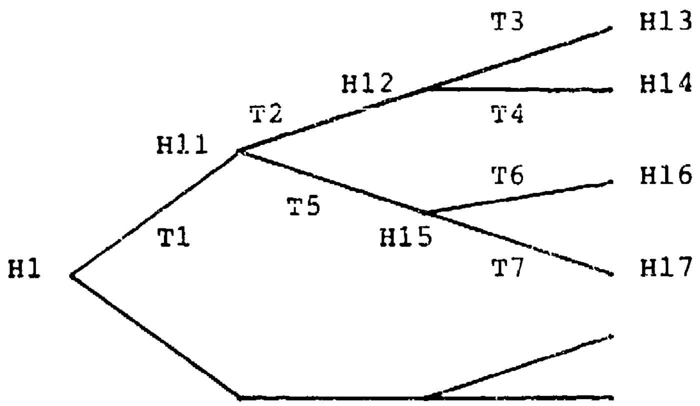


Figure B1. Hypothesis Tree.

```
Pr(H11) = Pr(H1 )*Pr(T1)
Pr(H12) = Pr(H11)*Pr(T2)
Pr(H13) = Pr(H12)*Pr(T3)
Pr(H14) = Pr(H12)*Pr(T4)
Pr(H15) = Pr(H11)*Pr(T5)
Pr(H16) = Pr(H15)*Pr(T6)
Pr(H17) = Pr(H15)*Pr(T7)

  |         |         |_____ Prob. of each track.
  |         |_______________ Prob. of prior hypothesis.
  |_________________________ Prob. after new measurements.
```

Figure B2. Hypothesis Tree Branch Probabilities.

In figure B2 the probability of each track is calculated using the simple rule in accordance with the track quality:

1/ If the measurement belongs to false alarm (or false target), the probability of the track(or branch) is given by

$$(B_{ft})*(1-P_D)/c.$$

2/ If the measurement "j" belongs to i-th track, the corresponding probability is given by $P_D*N[0,S(i)]$, where $N[a,b]$ is the Normal (Gaussian) density function associated with the assignment of the j-th observation to the i-th track of the prior history and is defined by

$$N[0,S(i)] = \frac{\exp(-d_{ij}*d_{ij}/2)}{\sqrt{(2\pi)^n|S(i)|}} \qquad (B1)$$

for the residual matrix $S(i)$ and normalized distance function d_{ij}.

3/ If an observation belongs to new target, the corresponding probability is computed by

$$B_{kt}*(1-P_D)/c.$$

Remark:

If the track deletion option is included, then the factor

$e^{-1/De}$ should be properly introduced in the probability calculations (as per reference [4] pp. 255-260).

APPENDIX C: HYPOTHESIS REDUCTION

Since, in the worst case, the number of hypotheses can grow exponentially with time, there is a clear need to limit this number. Some schemes for doing so include:

a. The first opportunity for limiting hypotheses is to require an observation to satisfy a gating relationship before any of the possible track associations are to be considered to be potentially valid.

b. The JPDA algorithm, by computing all the measurements at the current time, considers only the number of known targets with a single hypothesis per target.

c. The "N-scan-back" concept, by combining all histories that have common measurements from k-N to k.

d. An alternative method is to combine those hypotheses that similar effects, i.e., same number of targets but with slightly different state estimates. The mean and covariance of the resulting estimate is a combination of the individual estimates and covariance. At the same time, hypotheses with negligible probabilities are eliminated.

The method c has been found to be most effective in practice, and it provides an efficient approximation of the method d. The process of pruning hypotheses is highly dependent on the applications. Typical ways of pruning can be summarized as:

1) To remove hypotheses with probabilities that fall below some predetermined threshold. Disadvantage of this method is that it does not take into account the computational capacity.

2) To allow only a predetermined number, say M, of hypotheses to be maintained by ranking the hypotheses and choosing only the M most likely ones, as measured either by the probabilities or the score functions.

3) To rank and sum the probabilities of the more likely hypotheses. When this sum exceeds a threshold the remaining hypotheses are then eliminated.

APPENDIX D: CLUSTERING

A cluster is a group of hypotheses and associated tracks that does not interact with any other group of hypotheses (contained within other clusters). The hypotheses within a cluster will not share observations with the hypotheses within any other cluster. The basic purpose of clustering is to divide the large tracking problem into a number of smaller ones that can be solved independently. This can greatly reduce the number of hypotheses that must be maintained.

The steps in clustering are as follows. Initially, one cluster is set up for each confirmed target. Each new measurement is associated with a cluster if it falls in the validation region of any track from that cluster. A new cluster is initiated any time an observation is received that does not fall within the gates of any track contained in an existing cluster. The cluster is initiated on the observation using the alternatives (true target or false alarm) associated with its source. In order that clusters remain distinct, the gates of the tracks within the clusters must not overlap. Thus, when an observation falls within the gates of two or more tracks from different clusters, the clusters are merged. The merging must be done before the observation is processed. If an observation is associated with more than one cluster, then those clusters are combined into a super-cluster. If tracks within a cluster separate spatially and have no more common measurements, the corresponding cluster is subdivided accordingly into smaller clusters that can be handled separately.

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(U) Multi-target tracking (MTT) has many applications, and has therefore been the subject of considerable investigation. One key aspect of this problem, especially in a dense target environment, is the "scan-to-scan correlation" or "tracking data association" problem of assigning measurements to tracks. Various different approaches have been proposed to solve this problem. In this report, three algorithms for MTT data association are presented for comparison. First the nearest-neighbor standard filter (NNSF) algorithm is presented. Then two of the more promissing extensions are presented: the multiple hypothesis test method (MHT), and joint probabilistic data association (JPDA) method. Whenever possible the same notation is used in presenting all three methods for ease of comparison. This report is intended to serve as a prelude to a comparative investigation of these three competing data association methods.

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