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2

IMPACT OF CHANGES IN SUSPENDED-SEDIMENT LOADS ON THE REGIME OF ALLUVIAL RIVERS

Potamology Program (P-1)

Report 6

Prepared by

Environmental Laboratory
US Army Engineer Waterways Experiment Station
3909 Halls Ferry Road, Vicksburg, MS 39180-6199

Prepared for

US Army Engineer District, New Orleans
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Mueller, David S.; Dardeau, Elba A., Jr.

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This investigation was Phase IV of a four-phase study of the sediment regime of the Mississippi River Basin. Phase I, conducted under the earlier Lower Mississippi Valley Division (LMVD) Potamology Program (T-1), resulted in the publication of US Army Engineer Waterways Experiment Station Technical Report M-77-1, "Inventory of Sediment Sample Collection Stations in the Mississippi River Basin," in March 1977. The end product of Phase II was LMVD Potamology Program (P-1) Report 1, "Characterization of the Suspended-Sediment Regime and Bed-Material Gradation of the Mississippi River Basin," published in August 1981. The Phase II study identified a downward trend in Mississippi River suspended-sediment loads that apparently began around the middle of the 20th century. Phase III dealt with suspended-sediment sampling, analysis, and load-computation procedures used at key stations on major streams in the Mississippi River Basin and the possible correlation of these procedures with the downward trend in suspended-sediment

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loads. In this Phase IV study, existing regime equations were examined in light of their applicability to major alluvial river systems. The principal focus of the Phase IV study was the examination of the effects of changes in suspended-sediment loads as they relate to the regime of alluvial rivers.

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PREFACE

The study reported herein is a component of the Potamology Program (P-1) of the Lower Mississippi Valley Division (LMVD). The Potamology Program is conducted under the direction of the Commander, LMVD, and is a comprehensive study of physical forces that influence the flood-carrying capacity and navigability of the lower Mississippi River. The purpose of the Potamology Program is to define cause-and-effect relationships that result in short- and long-term changes in the stage-discharge relationships of the lower Mississippi River and to develop improved design concepts and criteria for construction of channel stabilization works that will improve flood control and navigation along this river.

The Potamology Program is composed of two major components: Sedimentation, Mississippi River Basin; and Aggradation and Degradation, Mississippi River. This study is one item under the Sedimentation component. Future studies will be directed toward development and utilization of a flow sediment model capable of detailed investigations of short- and long-term sedimentation trends locally and throughout the main stem Mississippi River.

The study reported herein was the responsibility of the US Army Engineer District, New Orleans (LMN), New Orleans, LA. The LMN contracted with the US Army Engineer Waterways Experiment Station (WES), Vicksburg, MS, for conduct of the study and preparation of the report. The study was conducted by WES from 1 June 1982 to 31 May 1983.

This investigation was Phase IV of a four-phase study of the sediment regime of the Mississippi River Basin. Phase I, conducted under the earlier LMVD Potamology Program (T-1), resulted in the publication of WES Technical Report M-77-1, "Inventory of Sediment Sample Collection Stations in the Mississippi River Basin," in March 1977. The end product of Phase II was LMVD Potamology Program (P-1) Report 1, "Characterization of the Suspended-Sediment Regime and Bed-Material Gradation of the Mississippi River Basin," published in August 1981. The Phase II study identified a downward trend in Mississippi River suspended-sediment loads that apparently began around the middle of the 20th century. Phase III dealt with suspended-sediment sampling, analysis, and load-computation procedures used at key stations on major streams in the Mississippi River Basin and the possible correlation of these procedures with the downward trend in suspended-sediment loads. In this Phase IV study, existing regime equations were examined in light of their applicability to major alluvial river systems. The principal focus of the Phase IV study was the examination of the effects of changes in suspended-sediment loads as they relate to the regime of alluvial rivers.

Publication has been delayed, with the intention of later including the findings of this study with those of follow-on efforts. However, the anticipated funding required to conduct follow-on efforts did not become available, and as a result, this report has remained unpublished until now.

ACKNOWLEDGMENTS

Messrs. Elba A. Dardeau, Jr., and Malcolm P. Keown, principal investigators, planned and structured the format of the Phase IV study and provided technical guidance, while Mr. David S. Mueller researched the scientific and engineering literature and summarized the findings required to meet Phase IV objectives. Messrs. Mueller and Dardeau prepared this report. Mr. Robert M. Russell, Jr., prepared the figures. Acknowledgment is also made to Mr. Billy J. Garrett, LMN, for his helpful guidance and suggestions during the study.

CONTENTS

CONVERSION FACTORS, NON-SI TO SI (METRIC) UNITS OF MEASUREMENT

Non-SI units of measurement used in this report can be converted to SI (metric) units as follows:

Multiply	By	To Obtain
cubic feet per second	0.02831685	cubic metres per second
Fahrenheit degrees	5/9	Celsius degrees or Kelvins*
feet	0.3048	metres
inches	25.4	millimetres
miles (US statute)	1.609347	kilometres
pounds (mass)	0.4535924	kilograms
pounds (mass) per cubic foot	16.01846	kilograms per cubic metre
tons (2,000 pounds, mass)	907.1847	kilograms

* To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use the following formula: $C = (5/9)(F - 32)$. To obtain Kelvin (K) readings, use: $K = (5/9)(F - 32) + 273.15$.

IMPACTS OF CHANGES IN SUSPENDED-SEDIMENT LOADS ON THE REGIME OF ALLUVIAL RIVERS

PART I: INTRODUCTION

Background

1. For nearly a century, fluvial geomorphologists and river engineers have attempted to unravel and define the complex natural and human-induced phenomena that have influenced the character of streams, particularly alluvial rivers. Despite extensive research, serious gaps in understanding these phenomena and a lack of clear consensus among researchers regarding their quantitative interrelations still remain.

2. The concept of a "graded condition" in streams can be traced to the late 19th century writings of G. K. Gilbert, who stated that "equilibrium of action in streams consists of a mutual adjustment between velocity, discharge, slope, and load" (Ritter 1986). In the early part of the 20th century, W. M. Davis (1902) introduced the terms "grade" and "graded slopes" to describe the balanced fluvial condition. The idea of a "graded river" was essentially unchallenged until Mackin (1948) redefined the concept to include slope adjustment "over a period of years."* Fluvial geologists and geomorphologists prefer to use the term "graded" or "poised" to describe such a channel that is in "quasi-equilibrium," while river engineers would say that such a channel is "in regime" (sometimes, "in regimen"). A stable channel adjusts its slope to provide the velocity and discharge required for transportation of the sediment load with a given particle-size distribution derived from the drainage basin. Ritter (1986) pointed out that even actively downcutting streams are still in "quasi-equilibrium" as defined by their hydraulic geometry. The discharge associated with the graded condition of a stream is often referred to as the dominant discharge, which is higher than the average discharge "because the sediment characteristics and grade of the stream are particularly influenced by higher stages" (Morris and Wiggert 1972).

3. Any change, whether natural or human induced, in the hydrologic, climatic, or geologic characteristics of a drainage basin can cause a channel (or a reach of a channel) to lose equilibrium. In most alluvial rivers, such

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* See paragraphs 69-79 for a complete discussion of Mackin's work.

changes occur continually; therefore, to transport sediment with maximum efficiency, the channel is always compensating for these changes by eroding its bed (degradation), depositing part of its sediment load (aggradation), or changing its course to increase or decrease its length (meandering). By means of such adjustments, a stream tends toward a certain hydraulic geometry and slope that has been described by some researchers as "dynamic equilibrium" (Winkley 1981).

4. For an alluvial channel to be in equilibrium, the energy input must equal the energy expended. Morris and Wiggert (1972) have expressed the relation as follows:

$$H = H_b + H_t + H_n + H_s + H_e \qquad (1)*$$

where

H = total energy available after precipitation (= elevation above mean sea level)

H_b = energy dissipated as heat through friction of flow

H_t = energy lost by turbulence (e.g., rapids, stream expansions, contractions, structures, etc.)

H_n = energy extracted by artificial means (e.g., turbines)

H_s = energy used to transport sediment and debris loads

H_e = energy expended in erosion or rocks, soils, etc.

In an ideal situation where a stream is graded and flowing at dominant discharge, only the sediment and debris supplied by the basin and nothing more will be transported; the H_e term would then become zero. However, such a system does not commonly exist in nature.

Purpose and Approach

5. Researchers do not specifically agree on the mechanics and effects of suspended-sediment transport as they relate to the character of an alluvial river. Since the end of the last century, many theories have been formulated. The purpose of this study was to survey pertinent regime equations to evaluate the effects of changes in suspended-sediment loads as they relate to the regime of alluvial rivers. This task was approached by researching the

* For convenience, symbols are listed and defined in the Notation (Appendix A).

hydraulic engineering literature, wherein pertinent investigations were identified. These studies, which dealt with the effects of changes in suspended-sediment loads on hydraulic parameters and bed-load transport capabilities in alluvial streams, are summarized and presented herein.

PART II: RESEARCH AND DEVELOPMENT OF REGIME EQUATIONS

6. A number of researchers, beginning with Robert Greig Kennedy of India in 1895, have attempted to quantitatively describe regime channels. This part of the report presents a chronological overview of the 21 selected investigations and provides discussions of the significant equations that generally relate changes in suspended-sediment loads to streamflow regime. The terms used in equations have been standardized (see Appendix A) insofar as practical with those used by the American Society of Civil Engineers (Vanoni 1975). Any discrepancies from the original references are explained in footnotes.

Kennedy (1895)

7. Kennedy has been credited as being the first person to develop equations for canals in regime. The observations upon which Kennedy based his results were made on the lower reaches of the Bari Doab canal system. The study area included a 90-mile* reach of the main canal and 12 distributing channels. The maximum discharge rate in the main canal was 1,700 cfs, whereas the 12 distributing channels had capacities ranging from 30 to 250 cfs. The channels of the Bari Doab canal system were thought to have formed permanent cross sections by silting and scouring during the many years of steady flow. These reaches had never been dredged; therefore, for a considerable time the sediment-transporting capability of each had been sufficient to handle the sediment introduced to the system. Each channel ". . . could have carried neither more nor less silt under the given conditions of width of bed, depth of channel, and velocity of flow" (Kennedy 1895). This canal system, believed to be in equilibrium, was found to have nearly rectangular cross sections with vertical sides of fine sediment and a horizontal bed of coarse sand. Kennedy (1895) postulated there should be a relation between bed width, depth, and velocity ensuring that a channel would transport its load without deposition.

8. Kennedy (1895) derived the mean velocity of each channel from the known "full supply" discharge and the measured cross sections. The "full supply" discharge was somewhat greater than the average discharge but less

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* A table of factors for converting non-SI units to SI (metric) units is presented on page 4.

than the flood discharge and was generally a constant ratio to the average in all the channels. The channels were very dissimilar with respect to width-depth ratios (b/d), which varied between about 4 and 15. By determining the line of best fit through a logarithmic plot of depth of flow versus mean velocity, Kennedy (1895) developed the following relation:

$$V_o = cd^m \tag{2}$$

where

V_o = mean velocity

c = 0.84

d = average depth

m = 0.64

He also concluded that bed width did not affect the above relation.

9. Kennedy (1895) stated that V_o could be called the "critical velocity" or that velocity at which silting is just prevented for the given depth. Thus, to avoid silting in any canal system, V_o must not be constant in all the channels, but rather must increase with depth according to Equation 2. When the soil properties permit, a velocity greater than V_o (but never a lesser velocity) may be used for design purposes. Kennedy stated that on different canal systems, the values of c and m could vary, but probably by only a small amount.

10. Kennedy's (1895) concepts of sediment transport probably resulted from the early tendency in canal design to make the channels narrow and deep. He postulated that

> sediment in a flowing canal is kept in suspension solely by the vertical components of the constant eddies, which can always be observed in any stream, boiling up gently to the surface. From the sides also, some such eddies may occur to a much smaller degree, but any such must be for the greater part horizontal, and of no silt-supporting power. In order, therefore, to obtain an expression for the silt-supporting power of the stream, it may safely be assumed that the quantity of silt supported will be proportional to the width of the bed, all other conditions remaining the same. It must also vary with the velocity of the stream V, say as V^{n-1}, since the greater the velocity the greater must be the force of the eddies, which become zero when the velocity is zero. There is a third variable, the depth, but this could affect neither the number nor the force of the eddies. The amount of silt supported in a stream may, therefore, be expressed by

> abV^{n-1} , where a is some constant unknown.* But whilst thus supported, the silt is being moved forward at velocity V , so that the amount of silt transported will be equal to abV^n . It is here presumed that all sediment is in suspension, but there is doubtless a small portion of the heavier silt simply rolling along the bed. This amount would vary as bV ; instead of as bV^n ; so that the value of n to include the rolling silt will be somewhat less than it would be if the suspended silt alone were considered. If it be assumed that the amount of silt supported is proportional to the upward pressure of the deflected currents of water, which varies simply as the square of the velocity, or as V^2 , the expression $n - 1 = 2$, or $n = 3$, is arrived at. From these considerations, therefore, allowing for rolling silt, a value something less than 3 would be derived for n .

The derivation, omitted here, can be found in Kennedy (1895).

11. The formula Kennedy derived, along with Kutter's flow formula that he used, both represented only 2 degrees of freedom. Kennedy did not derive a width formula, which meant that either a deep narrow channel or a wide shallow channel could be designed to carry the same discharge. Later Kennedy (1904) reassessed his research and developed a "rough-rule" relation for width-depth ratio (Graf 1971; Lindley 1919), thus satisfying three degrees of freedom. Kennedy further developed his "silt theory" in his "Hydraulic Diagrams" (Kennedy 1907).

Lindley (1919)

12. In 1895, Kennedy had made the first step toward determining regime theory. In 1917, F. W. Woods followed Kennedy's lead by publishing a list of channel dimensions in "Normal Data of Design for Kennedy Channels" (Woods 1917). Earlier, Kennedy (1907) had claimed that the degree of "silt charge" (or sediment concentration) is the same for all channels in any one system. Lindley (1919) pointed out that Kennedy's statements contained a fallacy and that neither Kennedy nor Woods (1917) presented any data to support what they had published; therefore, Lindley (1919) saw a need for retesting these earlier theories. He stated that he found "very few small channels of depths as great as Mr. Woods specifies, while some large channels considerably exceed his limit. Kennedy's rules were transgressed in design and in nature because, according to his Hydraulic Diagrams (Kennedy 1907), actual large channels should silt, and most small channels scour" (Lindley 1919). Lindley (1919)

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* Kennedy (1895) used A instead of a .

then suggested that

> channels carrying an average silt charge . . . had not attained regime, so that (Kennedy's) . . . selection of stable channels showed unconscious bias in favor of regimes of low silt charges. The writer has found great local and general variation of dimensions even on the same channel, and it is not certain to what extent Kennedy succeeded in choosing sites that represented fair averages on the channels he selected. It, therefore, appears necessary to retest Kennedy's theory for depths, and to fix the law of variation of widths . . . (using existing data and) . . . entirely empiric methods, so as to avoid bias in the choice or interpretation of data.

13. By examining data from so-called regime channels, Lindley (1919) found large variations to which he assigned the following probable causes:

- a. "Silt-clearance, berm cutting (as distinguished from mere trimming of irregularities), the use of bushing, and the working of silting tanks.*
- b. "Use of regulators in heading up more or less at different times.
- c. "Running of feeder channels with varying discharges.
- d. "Breaches which scour out the bed locally.
- e. "Differences between the silt-drawing power of offtakes, not at present measurable, but causing an uncertain modification of regime in the parent channel and setting an uncertain regime in the distributary.
- f. "The apparent tendency of silt to move in waves, so that a channel contains a succession of scouring and settling reaches.
- g. "Variation of silt-charge at different seasons in main canals, diminishing towards the lower ends of the system of channels.
- h. "Effect of different rugosities (or hydraulic roughness of the streambed) in allowing regime velocities to be attained with lesser or greater gradients."

14. Lindley (1919) analyzed data from hydraulic surveys made between 1915 and 1917 of the Lower Chenab Canal, a system of some 2,700 miles of channel having a combined discharge of 11,000 cfs. He analyzed widths between point bar deposits, depths, and gradients, averaged over 1-mile lengths, using

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* "Berm," as used by Lindley (1919), was a point bar deposit. "Bushing" probably refers to the use of planted vegetation to control erosion. "Silting tanks" would be synonymous with stilling basins.

1 mile of every 3- to 5-mile-long reach, with the exception of "irregular reaches" (Lindley 1919). Lindley then plotted average width against average depth on a logarithmic scale and noted the gradients of the channel at each point. Next, the widths, depths, and gradients of each branch system were grouped and averaged. The average width-depth relation could be expressed as

$$b_b = 3.8\ d^{1.61} \tag{3}*$$

where

b_b = bed width developed from water surface width under the assumption that side slopes are 1V:2H

This relation was valid for up to 9 ft of depth (Lindley 1919), with no indication of a change beyond that depth. Although the plotted gradients did not follow an exponential law, velocities, calculated using Kutter's relation with N = 0.225 , were found to follow the exponential law:

$$V_o = 0.95\ d^{0.57} \tag{4}$$

rather than the relation developed by Kennedy (1895) (Equation 2).

15. Lindley (1919) pointed out that Kennedy's velocities are " 'derived from the known full supply discharge and the measured cross-section area of the channel,' while the velocities of the present investigation are those assumed in designing (i.e., design procedures); and as long as Kutter's formula, with N = 0.0225 , is used, the new relation should be followed." Although he did not suggest that Kennedy's relation was wrong, Lindley (1919) did, however, point out that all existing formulas "assume a uniform rugosity in all channels covered by one value of coefficient." He contended that silt-formed channels probably demanded a regular variation of coefficient with size and with changing stages.

16. By combining the two relations between width and mean velocity with respect to depth, Lindley (1919) obtained the following relations between width and mean velocity:

$$\textit{for Kutter velocities} \qquad V_o = 0.59\ b_b^{0.35} \tag{5}$$

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* Lindley used b instead of b_b .

$$\textit{for Kennedy velocities} \qquad V_o = 0.50\, b_b^{0.40} \tag{6}$$

Theoretically, a certain velocity along the sides of a channel is required to maintain a balance between erosion and deposition. According to Lindley's formula, the wider the channel, the greater the mean velocity permitted. The depth will adjust itself to the velocity. Lindley (1919) defended this theory by saying:

> While it is known that greater or lesser silt-charges demand greater or lesser velocities for equal depth, the case of widths is not necessarily similar: berms forming the sides of channels are composed of such fine muddy silt, so easily suspended in flowing water, that it (the silt) flows equally all through a canal system, and berms formed of the same silt, at the same velocity, should be similar in character.

He, therefore, concluded that of the two channels carrying the same discharge, the one with the greater "silt charge" must be wider and shallower and must have a steeper gradient. Lindley (1919) had advanced the quantitative analysis of regime channels by defining a relation between velocity and width that added to the already established relations of velocity and depth.

Lacey (1930)

17. Prior to the time of Gerald Lacey, several researchers, including Lindley, had continued the work of Kennedy (paragraph 8) by correlating mean velocity and depth of flow; however, the coefficient c varied between 0.67 and 0.95 and the exponent m between 0.52 and 0.73, depending on the channel studied (Henderson 1966, Lacey 1930). Lacey (1930) stated that V_o was a function of both depth and a "silt factor" and that width was as important as depth in determining this velocity. He defined the silt factor as

$$f = \left(\frac{V_o}{V_{ok}} \right)^2 \tag{7}$$

where

f = silt factor

V_o/V_{ok} = ratio of the regime velocity actually obtained with a given depth to the velocity for the same depth that Kennedy's formula (Equation 2) would predict (Lacey 1930)

Interpreting d in terms of R, the hydraulic radius, he developed his first general formula:

$$V_o = 1.17\sqrt{fR} \tag{8}$$

This silt factor had a very simple scalar significance. The hydraulic radius of the channel would be $1/f$ times the hydraulic radius of a standard silt channel (Kennedy channel) flowing with the same mean velocity (Lacey 1930).

18. Lacey (1930) stated that if Lindley had dealt with mean bed width instead of bed widths, the product of width and depth (of flow) would have given a useful expression for the cross-sectional area of the channel. Furthermore, Lindley dealt with depth instead of the hydraulic radius, and because d steadily approaches R as the width of the stream increases, the structure of Lindley's formula was useful for designed channels. However, the formula did not permit extended application by extrapolation. Lindley's formulas for high discharges yielded bed widths that were admittedly too low, and, for this reason, his *suggestion that a channel of given discharge and* "silt charge" was uniquely determined remained to be proved (Lacey 1930).

19. Lacey successfully developed a formula that confirmed Lindley's theorem. The velocity-discharge relation presents itself in the simplest form possible if the cross-sectional area and the mean velocity are first considered. As shown in Figure 1, if two different channels, abc and a'b'c', have the same mean velocity but different "silt factors," then

$$fR = f'R'$$

and

$$fP = f'P'$$

where P and P' are the wetted perimeters of channels abc and a'b'c', respectively. If the above equalities are multiplied together, they can be reduced to

$$Af^2 = A'f'^2$$

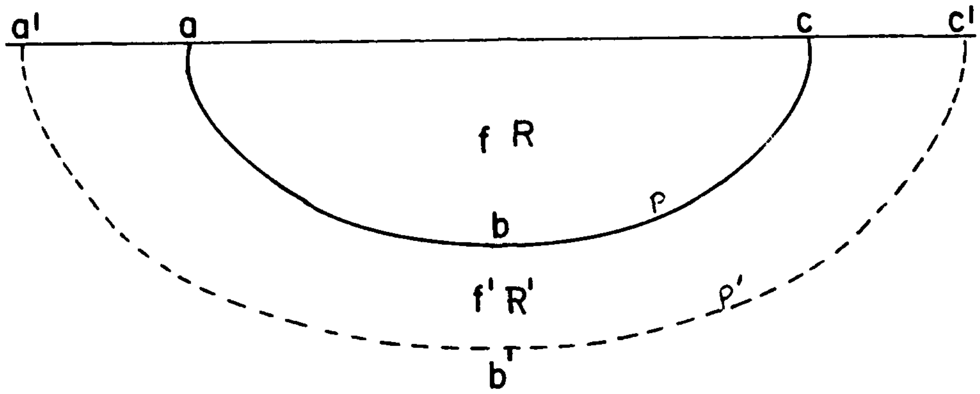

Figure 1. Channels abc and a'b'c' have the same mean velocity but different silt factors, f and f', respectively. Therefore, $fR = f'R'$ and $fP = f'P'$ (adapted from Lacey 1930)

where A and A' are cross-sectional areas for channels abc and a'b'c', respectively.

20. This relation assumes complete geometric similarity in the sections. Lacey (1930) stated that the sediment distributed around the wetted perimeter does not necessarily have to be of the same gradation throughout any one section. All that is necessary is that the grading of the sediment from a to b in the first section is identically similar in all respects to the grading from a' to b' in the second section. To test the relation of area, silt factor and velocity, Lacey (1930) plotted Af^2 against V_o and determined that

$$Af^2 = 3.8\ V_0^5$$

This could also be written,

$$Qf^2 = 3.8\ V_0^6$$

where Q = discharge. Finally Lacey (1930) developed an equation that related wetted perimeter to discharge:

$$P = 2.668\ Q^{0.5} \tag{9}$$

21. Equation 9 shows that for a given discharge, the wetted perimeter of a stable channel is constant and independent of the size of the material transported, with the silt size merely controlling the "shape." If channels with horizontal beds are considered, then "shape" is the ratio of bed width to depth. For channels having semielliptical cross sections, shape is the ratio of the major axis to the semiminor axis. The wetted perimeter is a constant for any given discharge, but the shape of the cross section depends on the size of the bed material (Figure 2). The coarser the bed material, the flatter the semiellipse and the greater the width of water surface. The finer the bed material, the more nearly the section approximates a semicircle. Thus, Lacey (1930) concluded that two elementary principles that govern the dimensions of stable channels in alluvium were:

> All stable silt-transporting channels flowing with the same mean velocity tend to assume the same "shape," and all stable silt-transporting channels of the same discharge have the same length of wetted perimeter irrespective of the particular grade of silt carried.

Lacey (1930) used the term "silt" very loosely, and in the above statement, "silt" could have also included other particle sizes.

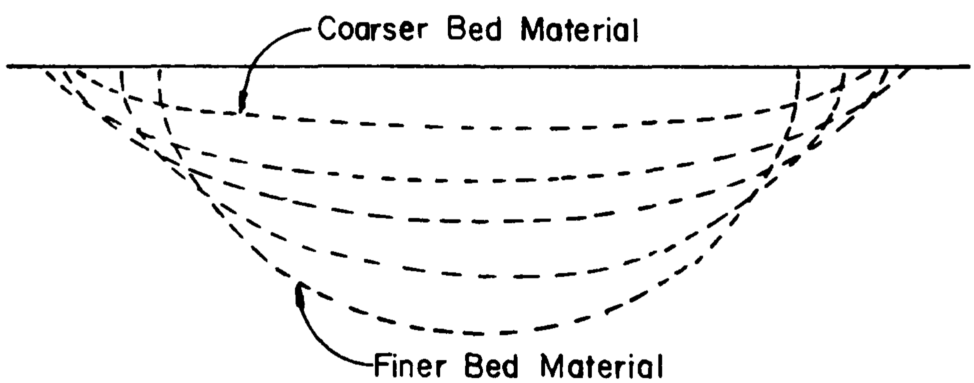

Figure 2. According to Lacey (1930), cross-sectional shape is a function of the size of the bed material. Coarser bed material yields a flatter semiellipse, whereas finer bed material yields an approximate semicircle (adapted from Lacey 1930)

22. The new formulas developed by Lacey covered a very large range of stable artificial channels. If truly general, these formulas should, when extrapolated, fit the maximum discharges of stable rivers. When the discharges of large alluvial rivers are considered, the wetted perimeter closely

approximates the width of the water surface; for the purpose of such an extrapolation, the difference in wetted perimeter and width is negligible, regardless of the precise shape that the channel assumes. From Equation 9, Lacey (1930) developed the following formula to determine the approximate width of a stable alluvial channel:

$$b_t = 2.67\ Q^{0.5} \tag{10)*$$

where

b_t = top width, ft

23. Lacey (1930) then attempted to relate the silt factor to Kutter's rugosity (roughness) coefficient N, developing the relation:

$$N = 0.022\ f^{0.2} \tag{11}$$

He stated that Equation 11 could apply equally to Manning's formula, with Manning's n essentially being interchangeable with N. By taking Manning's formula for velocity and substituting the appropriate formulas for the slope of the energy grade line, he obtained the following equation:

$$S_e = \frac{f^{1.51}}{2587\ Q^{1/9}} \tag{12)**$$

where S_e = slope of the energy grade line.

24. Lacey (1930) also researched the relation between the sediment size and the silt factor. He found that the average diameter of a sand grain in a sample of Upper Bari Doab Silt (standard $f = 1.0$) from the head reach of a distributary was 0.4 mm (probably slightly larger than the size of sand grains in Kennedy's channels). The diameter of the average large boulder transported by heavy floods on the Song River ($f \approx 40$) was approximately 25 in. He developed the following formula to provide a very rough estimate of the diameter in inches of the predominant size of sediment transported:

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* Lacey (1930) used W instead of b_t for top width.
** Lacey (1930) used S instead of S_e .

$$D = \frac{f^2}{64} \tag{13}$$*

Lacey (1930) stated that if he had been able to collect data on the size of a silt particle corresponding to a silt factor of about 0.4 (instead of 40), he could have derived a more reliable rule. With low silt factors, the silt has a remarkably uniform character, and a scalar relationship is, therefore, more easily obtained.

25. The general formulas developed by Lacey have a drawback that is independent of the gradation of sediment. Decreasing the discharge with a constant silt factor causes the cross section to approach a semicircle. Hence, at the limit

$$A = 6.2832\ R^2 = 3.8\ V_0^2 f^2$$

$$V_0 = 0.882\ fps\ (for\ any\ type\ of\ sediment)$$

26. For velocities less than this limiting value, the resultant wetted perimeter cannot contain the cross-sectional area. This fact can be considered either as a limitation of the formulas or, as Lacey (1930) concluded, as an argument that this velocity must be one of the physical constants of water because it is independent of the sediment size. Lacey (1930) summarized his conclusions as follows:

a. "For a given discharge and given silt factor, the Lindley theorem is true; that is, the cross-sectional area, wetted perimeter, and slope of a stable channel flowing in and transporting its own silt are uniquely determined.

b. "The wetted perimeter of a stable silt-transporting channel varies as the square root of the discharge and is independent of the type of silt transported, all stable channels of the same discharge having the same wetted perimeter, and the silt factor determining the shape.

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* Lacey (1930) used d instead of D .

c. "All stable silt-transporting channels of the same stable mean velocity have geometrically similar shapes, the silt factor determining the only difference, which is one of scale."

Lane (1935)

27. In 1935, E. W. Lane did extensive research in regime theory to aid in the design of the All-American Canal in California. He classified the two types of formulas previously developed for stable channels in India and in Egypt as:

a. Those giving an expression of velocity.

b. Those giving stable channel shapes.

28. The first type of formula, which is similar to that developed by Kennedy (1895) (Equation 2), indicates that critical velocity increases with depth. However, later experience showed that, as depth increases, the velocity at which bank erosion occurred is also reached. Kennedy believed that determining this limiting velocity was a matter of experience.

29. Several researchers contributed formulas of the second type. Woods (1917) developed relations of mean depth, velocity, and slope (see paragraph 12), whereas Lindley (1919) provided a relation of critical velocity to depth and bed width (Equations 5 and 6). Neither Woods nor Lindley suggested any modifications for the quantity or gradation of sediment. Lacey (1930) used channel shapes and velocities and introduced the effect of particle size but did not consider the quantity of the material transported. Lane (1935) analyzed the regime equations that had already been developed and concluded that none of the existing formulas would be safe to adopt for the design of sections of the All-American Canal. He stated that

> although these formulas no doubt provide workable relations for the conditions for which they were developed, these conditions have not been delineated sufficiently to enable them to be applied elsewhere. In general, also, they were developed empirically from a very limited range of conditions, and in most cases they omit important factors from consideration (Lane 1935).

30. Lane suggested analysis of the fundamental interrelations of a number of factors that he attributed to controlling the shape of streams in alluvial material. These factors included:

a. Hydraulic parameters (energy slope, roughness, hydraulic radius, depth, mean velocity, velocity distribution, and temperature).

b. Shape of the existing channel (width, depth, and steepness of side slopes).

c. Nature of material transported (size, shape, specific gravity, dispersion, quantity, as well as bank and subgrade material).

d. Other factors (alignment, uniformity of flow, and "aging"*).

31. Of the hydraulic factors, energy slope, roughness, hydraulic radius, and mean velocity are interdependent, and their relation can be determined quantitatively through the velocity formulas available at that time. The effect of the movement of material in suspension and by traction upon the roughness was not known; however, when compared with the uncertainty in other phases of the problem, the relations among these four elements are certain enough that further research into them would have probably been unjustified. Lane (1935) regarded the steepness of side slopes as relatively unimportant except that the steepest banks were subject to sloughing. He believed that both the velocity distribution and the mean velocity were of primary importance. Although temperature had been suggested as having an effect because it influences the viscosity of the water, its impact on cross-sectional shape was thought to be minimal in relation to other factors influencing stable channels. Lane and other researchers prior to him considered the size of material transported to be of major importance in controlling cross-sectional shape (with particle shape and specific gravity being secondary). In addition, he postulated that the quantity of material in motion, the scour resistance of the material comprising the banks and subgrade, the channel alignment, variable discharges, and "aging" were among other factors affecting cross-sectional shape of a stable channel (Lane 1935).

32 Lane (1935) stated that

> the velocity along the bottom of a stable channel must be sufficient to move the quantity of material supplied to it but not so great as to scour the subgrade. . . . The science of hydrodynamics has not yet progressed to the point where the relation between the velocity distribution adjacent to a surface can be related quantitatively to the drag of the water along the surface or to the velocity gradient adjacent to the surface.

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* By "aging," Lane (1935) could have been referring to the combined effects of such processes as armoring, cementation, and the interaction of organisms with the substrate (i.e., the bottom sediment of the streambed). "Aging" could also have referred to cross-sectional shape and character and development of the channel and floodplain.

He concluded that when high velocities extend close to a water surface, the pushing or dragging force of water on the bed was greater than if the velocities close to the surface were low (Lane 1935).

33. Researchers generally agreed that vertical currents stronger than the force of gravity kept material in suspension. This led Lane (1935) to state:

> A promising hypothesis for the capacity of a stream to transport material in suspension, therefore, should be that the capacity is proportional to its turbulence, which, in turn, is probably proportional to the energy expended. The concentration of a given quality [sic]* of solids which a stream could support, therefore, would be proportional to the energy expended per unit of volume of the water. This energy is proportional to the rate of fall of the water, which is equal to the product of the velocity and the slope. This is a kind of over-all relation, however, and silting may occur in one part of a ditch cross-section while other parts may be scouring.

34. Channels frequently carry a considerable amount of solid matter by dragging it along the bed, with either clear or sediment-laden water flowing above. The quantity of material moved depends on the velocity near the bottom of the channel. Therefore, a channel supplied with a heavy bed load requires high velocities to move the bed load through the reach and to avoid sediment deposition. Lane (1935) concluded that channels carrying bed load should have high ratios of bed width to depth to prevent the sides of the channel from being eroded by the high velocity required to carry a heavy bed load in a narrow channel. He stated that if the banks are composed of a scour-resistant material, the ratio of bed width to depth can be reduced.

35. Lacey (1930) had concluded that stable channels would be semi-elliptical, with the major axis horizontal; the ratio of the major axis to the semiminor axis would depend on the nature of the silt carried, and this ratio would be greater for coarser material. Lane's (1935) research did not support this conclusion, however. He used the "form factor," the ratio between the area of the channel section up to the water surface and the area of the enclosing rectangle, for his analysis. For an ellipse, the form factor would be 0.79; for a parabola, 0.67; for a triangle, 0.50; and for a rectangle, 1.00. Those canals studied by Kennedy carried a heavy load of graded sediment and were reported as having nearly vertical sides; therefore, according to Lane (1935), they would have a form factor of approximately 1.00:

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* Should be "quantity."

> Just what shapes for stable channels are produced by all varieties of conditions has not yet been determined, but the writer has observed that for channels carrying a heavy load of graded silt, ranging from colloids to fine sand, the sections have nearly horizontal beds composed of the fine sand and nearly vertical sides of silts and clays. Such channels have form factors of about 0.90. This is the condition on the canals of the Imperial Valley. This difference between the composition of the bed and bank material has also been observed in India. Similar conditions result in channels carrying considerable bed load and a moderate quantity of fine silt. For a channel carrying water containing a small quantity of silt at high velocity in a material containing a considerable number of cobbles, the cross-section is distinctly saucer-shaped, most of the section being covered with cobbles, but with a small silt berm at each edge. This condition was observed on some of the ditches on the Uncompahgre Project, in Colorado, and in the San Luis Valley, in Colorado. One ditch that was carefully measured, had a form factor of 0.85.

Lane (1935) believed that further study along these lines would disclose typical shapes and the reasons for a number of common conditions.

Lacey (1936)

36. In discussing Lane's (1935) article, Gerald Lacey emphasized that channel conditions and the problems associated with studying regime flow were much simpler in India than on the Colorado River. Lacey contended that certain elements of "flume traction"* could not be entirely dissociated from channel behavior on some of the Colorado canals. In channels with "flume traction," there was no limit to the surcharge that could be forced downstream, provided that slope was available and that the banks possessed some degree of erosion resistance. "In rivers flowing in deep alluvium, the surcharge is thrown down, and the river moves to a flank and picks up a fresh charge, which it sweeps forward" (Lacey 1936).

37. Lane (1935) had stated that the capacity of a stream to transport suspended sediment was probably proportional to its turbulence and to the energy expended. Lacey (1936), on the other hand, pointed out that, according to Gilbert (1914), the prime mode of sediment transport was saltation rather than suspension. With "flume traction" and a heavy surcharge, the turbulence necessary to drive the boundary layer forward would also occasionally result in a high suspended charge. Lacey (1936) believed that the ratio V^2/R for

* For a discussion of a laboratory investigation of "flume traction and transportation," see Chang (1939).

any grade of silt, irrespective of the concentration, was proportional to the turbulence. He found that fine silt with a heavy surcharge would probably demand the same value of V^2/R as would a coarse silt with a smaller charge in another locality.

38. Lacey (1936), building on Lane's (1935) research, correlated Lane's factors as dimensioned variables using the Buckingham theory (Buckingham 1914). This method consisted of determining dimensionless arguments from all the known independent variables and then correlating these arguments. A poor degree of correlation could indicate that important variables had been overlooked (Lacey 1936). In 1769, Antoine Chezy had selected four independent variables that resulted in the following equation (Clemens 1897):

$$\frac{S_e R}{\rho V_o^2} = \textit{the Chezy number} \qquad (14)^*$$

where

ρ = density of water

V_o = mean velocity

39. Osborne Reynolds added a fifth variable, the kinematic viscosity of water. The result was the classic Reynolds number (Reynolds 1883); thus

$$\frac{S_e R}{\left(\rho V_o^2\right)} = K \left(\frac{R V_o}{\upsilon}\right)^m \qquad (15)$$

where

K, m = constants

RV_o/υ = Reynolds number (dimensionless, in terms of hydraulic radius, with υ being the kinematic viscosity of water)

The acceleration due to gravity g does not enter this relation as an independent variable because Reynolds carried out his experiments with pipes.

40. In his earlier study, Lacey (1930) had suggested that the rugosity was implicit in the slope and the hydraulic mean depth for open channel and

* Lacey (1936) used i instead of S_e and V instead V_o .

that g had to be added as the sixth variable. From this addition, a third dimensionless variable, the Froude number, evolved:

$$\frac{S_e R}{\left(\rho V_o^2\right)} = K \left(\frac{RV_o}{\upsilon}\right)^m \left(\frac{V_o^2}{gR}\right)^{m'} \tag{16*}$$

where

m, m' = constants

Lacey (1936), addressing the nature in which the ratio V_o^2/R, enters the Froude number, concluded:

> The silt factor is merely a simple proportion to the ratio, V_o^2/R, and therefore, as long as dimensional analysis is used as an instrument in hydraulic analysis the ratio, V_o^2/R, must persist as a fundamental element, and with it the silt factor, which more appropriately could have been termed "turbulence."

41. Substituting for the energy gradient in terms of the geometric slope, density, and acceleration due to gravity yields, Lacey (1936) obtained the following equation:

$$\frac{S_{bg} R}{V_o^2} = K \left(\frac{RV_o}{\upsilon}\right)^{-1/3} \left(\frac{V_o^2}{gR}\right)^{2/3}$$

where

S_b = bed slope.**

If the silt factor is treated as being proportional to V_o^2/R, this equation can be solved and will prove the basis of Lacey's formulas with the exception of those involving discharge, thus leaving the silt factor in the ratio form as

$$V_o \propto g^{5/6} \upsilon^{-1/6} f^{-1/4} R^{3/4} S_b^{1/2}$$

---

* Lacey (1936) used p instead of m'.
** Lacey (1936) used S instead of S_b.

This relation is true for any regime channel in unconsolidated alluvium, whether fine sand, coarse sand, or boulders. The very low power of the kinematic viscosity should be noted, as should the complete disappearance of the "silt factor," which is inherent in the dimensions assumed by the channel. Rewriting the formula, then substituting $f^{1/4}$ for n and introducing the wetted perimeter as a variable gives (Lacey 1936):

$$\frac{P}{R} \propto \left(\frac{RV_o}{\upsilon}\right)^{1/3} \left(\frac{V_o^2}{gR}\right)^{1/3} \tag{17}$$

from which $P \propto Q^{1/2}$ and $P/R \propto V$ where V = velocity.

42. Earlier Lacey (1930) had proposed the formula:

$$P = 2.668\ Q^{1/2}$$

from which

$$P^2 = 7.12\ PRV_o$$

Then dividing through by PR yields

$$\frac{P}{R} = 7.12\ V_o \tag{18}$$

This is Lacey's fundamental shape formula. Lacey (1936) believed that Lane's (1935) work was unclear as to whether the Lacey shape formula had been confirmed because Lane had presented only shape versus particle diameter. Lacey (1936) believed that a final solution to the problem of silt transport would be found by investigation along the lines of dimensional analysis as he had outlined.

Shulits (1936)

43. Fluvial morphology deals with the forms of riverbeds and their shapes in plan and in profile. Knowledge of fluvial morphology is generally limited to qualitative reasoning and some site-specific empirical equations; however, a few quantitative principles have been developed theoretically. The major purpose of Samuel Shulits' (1936) paper was to examine these quantitative laws; however, he first made observations of the qualitative laws and their use.

44. Morphologic activity is most reflected by streambed slope, which can be either lengthened or shortened by the meandering processes of an alluvial river. Meandering is usually more prominent in the lower reaches of a river where the bed-material gradation shifts toward the fine fraction as a result of its travel downstream and where bed slope is small, thus suggesting a reciprocal relationship between size of material and slope. Therefore, slope, abrasion, and bed load are the primary principles of fluvial morphology (Shulits 1936).

45. Shulits (1936) stated that a natural river can decrease its slope by meandering or lengthening the present meanders. This decrease in slope is often caused by a decrease in bed load. On streams in which a dam has been built, degradation will occur below the dam in order to decrease the slope to account for the loss of the sediment trapped in the reservoir.

46. As particles move downstream, they are reduced in size by abrasion. Collision also reduces particle size, but to a much lesser extent. Sternberg (1875) developed a theoretical law for abrasion, which was later verified experimentally by Schoklitsch (1926, 1933, 1935). Sternberg (1875) had assumed that the reduction in particle weight was directly proportional to the work done in overcoming friction along the distance traveled. The Sternberg abrasion law is in the form:

$$W_b = W_{bo} e^{-c' \phi x} \qquad (19)*$$

* Shulits (1936) used P , P_o , and c instead of W_b , W_{bo} , and c' , respectively.

where

W_b = weight of bed sediment

W_{bo} = mean weight of bed sediment

e = base of natural logarithm

c' = constant of proportionality

ϕ = coefficient of friction

x = distance traveled downstream

Setting $\alpha = c\phi$ yields*

$$W_b = W_{bo} e^{-\alpha x}$$

where α = coefficient of abrasion. The α, which is not a constant for a given rock type, depends on the shape, tends to increase with the one-fourth power of the velocity of the particle, and is proportional to the diameter of the bed material (Shulits 1936).

47. Because the slope of a river varies directly with the decrease in bed-material size, Shulits (1936) stated that the next logical step would be to search for a relation between slope and bed-material size. Expressed mathematically

$$S = \sigma\, W_{bo}^{-\alpha x}$$

where S is the slope and σ is a constant, and where $x = 0$, and $S = S_o$ so that $\sigma W_{bo} = S_o$; then the equation can be rewritten

$$S = S_o e^{-\alpha x}$$

where S_o = slope at the head of the reach. Shulits (1936) found that this equation gave an approximate fit for the profile of the Colorado River between Boulder Dam and the Arizona-Sonora Boundary. If Z is the elevation of any point on the profile above a given datum, the following equation results:

---

* Shulits (1936) used a instead of α .

$$Z_o - Z = \left(\frac{S_o}{\alpha}\right) (1 - e^{-\alpha x})$$

This equation fits the profiles of the Middle Rhine, Maas, Mur, and Enns Rivers in Europe.

48. Sternberg (1875) had arrived at a similar formula by combining his abrasion law and the theoretical fact that the mean velocity just capable of moving a particle is proportional to the one-sixth power of its weight. This yielded a relation between mean velocity, particle size, and distance traveled by the grain. Substitution into the Chezy formula resulted in

$$Z_o - Z = 2\left(\frac{S_o}{\alpha}\right) (1 - e^{-\alpha x/2}) \tag{20}$$

This is also claimed to approximate the profiles for the European rivers.

49. Putzinger (1919) had assumed that the reduction in particle size was proportional to the work done by the tractive force in the distance traveled. His final slope equation was

$$Z_o - Z = K_1 (1 - e^{-Kx}) \tag{21}$$

where K_1 and K were constants. Shulits (1936), therefore, concluded with reasonable certainty that the slope of a river could be expressed by the general law in Equation 21. An investigation of bed load led Shulits to modify the Schoklitsch bed-load formula and combine it with the abrasion and slope formulas. The final result was

$$G = \left[\frac{KQ}{e^{(4/3)\alpha x}}\right] - \left(\frac{K_1 b}{e^{\alpha x/3}}\right) \tag{22}$$

where

G = bed load, lb/sec

Q = discharge, cfs

b = width, ft, of a rectangular channel having depth d

Shulits concluded that if the discharge and the width terms increase equally downstream, there would be no change in bed load; if the discharge term increases more rapidly than the width term, there would be an increase in bed load and a change in profile, probably degradation, and an increase in slope; and should the width term increase more rapidly, there would be a decrease in load and a corresponding change in regime.

Lacey (1946)

50. Lacey (1946) attempted to advance the theory of silt transported by "stable" channels* in alluvium. When testing the earlier regime relations using hydraulic data, he discovered two curious anomalies:

- a. Despite the great improbability of natural alluvial channels ever attaining regime, alluvial river data and boulder torrent data (i.e., data from steeply graded streams) confirmed the regime equations in many respects.
- b. Although much of the data that had been collected over a period of years from stable channels in the Punjab displayed general agreement, certain observations from the channels thought to be stable proved to be "discordant" (i.e., conflicting)** for no readily ascertainable reason.

51. By applying the principles of corresponding speeds and dynamic similarity and by introducing the terminal velocities of silt particles into his physical theory, Lacey (1946) succeeded in regrouping the fundamental variables of velocity, depth, and water surface slope to obtain two new parameters and to derive a "normal equation" applicable to all active alluvial channels in steady flow. This "normal equation" would be able to implicitly account for the degree of aggravation and scouring and for departure from "regime silt charge"† in the velocity, hydraulic radius, and water surface

* By Lacey's (1946) definition, "such channels neither silt nor scour." Engineers use the term "in regime" (sometimes, in regimen) to describe this condition, while fluvial geologists prefer to use "graded" or "poised" to describe a "stable" channel.

** Lacey (1946) pointed out in his paper that much of the "discordant" data could be reconciled when variables such as "silt charge" (concentration) and "shock" (the effect of roughening the sides and introducing irregularities into the bed of a given channel with a given discharge, sediment, concentration, and bed-material gradation) were taken into account.

† Lacey (1946) defined "regime silt charge" as the minimum transported silt load consistent with "full activity of the bed."

slope adopted by the channel, and in the actual particle-size gradation of the streambed (Lacey 1946).

52. Streams used to develop common relations should have similar characteristics. Because the strict definition of dynamic similitude cannot be applied to nonrigid boundary streams that transport not only water but also sediment, Lacey (1946) modified the definition. All that can be expected of similar alluvial channels, with respect to scale, is that they should be geometrically similar in plan and in cross section or "geometrically comparable."

53. Lacey (1946) stated that

> applying the theory of corresponding speeds to any two alluvial channels of assumed infinite width and constant depth, and employing the usual notation, the familiar expression
>
> $$\frac{T'}{T''} = \frac{L'V''}{L''V'}$$
>
> is obtained. This postulates that, in respect of longitudinal flow $(L.\ L',\ .\ .\ .)$, the particles of silt and water traverse equal longitudinal scalar distances in equal scalar times $(T, T'\ .\ .\ .)$. In addition to this requirement, it must be ensured that vertical distances are also traversed in equal scalar times. Particles of silt of the same grade and density fall through still water with the same constant terminal velocity v, and this can be determined by applying the modified Stokes' law. This velocity, therefore, when silt of the same grade is employed, is common to model and prototype. The author regards this velocity as fundamental and adopts it as a proportional to the vertical velocities of his models. Hence*
>
> $$\frac{T'}{T''} = \frac{L'V''}{L''V'} = \frac{d'v''}{d''v'}$$
>
> Employing the theorem that horizontal planes in the prototype remain horizontal in the model, and denoting the slopes by S' and S'', respectively,
>
> $$\left(\frac{d'}{L'}\right)\left(\frac{L''}{d''}\right) = \frac{S'}{S''} = \frac{V''v'}{V'v''}$$

* Lacey (1946) used D instead of d.

and finally

$$(VS) = Kv \tag{23}$$

where K is a constant. Thus, it follows that in all similar alluvial channels, the terminal velocities of the silt particles are directly proportional to the product (VS) .

54. Lacey (1946) believed that for a known temperature and a constant sediment concentration, the mean velocity V_o was a function of the average depth d , and the particle-size diameter D :

$$V_o = f\left(d_c, D\right) \tag{24}^*$$

There has been a wide spectrum of opinion related to the evaluation of this function. The most common evaluation is of the form

$$V_o = Kd_c^m$$

Lacey (1946) obtained an exponent of 0.5 in evaluating this equation.** He stated that for a constant gradation and concentration, mean velocity varies as the square root of the depth. Although the theory had not been proved, Lacey accepted the dynamic hypothesis that there is a fundamental equation for the velocity of sediment transport, which is identical to the equation for the velocity of wave propagation when the gradation and concentration remain constant. Following this theory and that of Equation 23 yields:

$$\left(d_c^{1/2} S\bullet\right) = K$$

where K is a constant (Lacey 1946).

---

* Lacey (1946) used F"( ) , d , and m instead of $f(\ )$, d_c , and D , respectively.

** Lacey (1946) used n instead of m .

55. Lacey's first parameter (VS^*) is directly proportional to the terminal velocity of the silt particles, whereas the second $d_c^{1/2}S^*$ is a function of the particle size, the density, and the sediment concentration. The first parameter represents the forces tending to restore the particles to the bed. The second parameter represents the forces tending to propel the particle forward. Lacey stated that these forces should be correlated, and he developed the normal equation in the form:

$$(VS^*) = f\left(d_c^{1/2}S^*\right)$$

He then expressed the normal equation in the exponential form:

$$(VS^*) = K\left(d_c^{1/2}S^*\right)^n$$

For channel flow, Lacey (1946) substituted the hydraulic mean depth d for the constant depth d_c employed in his physical theory (in which for constant depth, the values of d_c and d are identical for wide channels), writing the normal equation as

$$(VS^*) = K\left(d^{1/2}S^*\right)^n$$

Lacey believed that his physical theory had justified the form of his equation and, therefore, evaluated it empirically. The result was

$$(VS^*) = 1.6\left(d^{1/2}S^*\right)^{4/3} \tag{25}$$

Lacey (1946) called Equation 25 the "normal alluvial equation" and designated the graphical relation between (VS^*) and $d^{1/2}S^*$ as the normal diagram, stating,

> There is no doubt of the general trend of the diagram, and no statistical analysis is necessary in order to obtain a sufficiently accurate constant and power. It must be remembered that the

> observations vary enormously in weight, and even if they all carried the same weight, a statistical relationship, treating V , d , and S as independent variables, would be objectionable on physical grounds. Such a course of procedure would be equivalent to the abandonment of, say, the Reynolds number, in the analysis of data obtained from pipes.

The normal alluvial equation developed by Lacey (1946) applied only to subcritical flow, which yields upper limits of 253.0 and 43.5 for (VS^*) and $(d^{1/2}S^*)$, respectively.

56. From a study of the normal diagram, Lacey (1946) concluded that the new equation applies to all active channels in alluvium and is irrespective of the precise degree of scouring or silting, or variation in silt charge. All these variables are implicit in the actual values of V , d , and S adopted by the channel when associated with the actual gradation of the bed material, which must be sampled and measured at the same time as the other hydraulic elements.

57. Lacey (1946) maintained that the "full activity of the bed" (see footnote, paragraph 51), although random, must nevertheless be such that any reduction would lead to partial immobility and, therefore, quasi-rigidity of the channel. With regime silt charge defined, Lacey stated that for every regime channel transporting a regime silt charge, there is. for a fixed diameter of the bed material, a fixed value of the product (VS^*) . For normal nonregime channels, the actual bed material was not in conformity with the product (VS^*) . Therefore, this conformity of nonconformity allows "regime channels" to be distinguished from "nonregime channels."

58. Data published by the Lahore Research Institute (1937) confirmed Lacey's "physical theory" and the normal alluvial equation (Equation 25). Lacey pointed out that no suspended sediment samples had been taken in 1937, and no method or device had been developed to measure the "bed silt charge" (bed load). He then made a logarithmic plot of (VS^*) against particle-size diameter. Within the entire range of data, the modified Stokes' Law showed that the terminal velocity varied directly with the particle diameter. From this relation, he derived the following equation:

$$D = 0.73\ (VS^*) \tag{26}$$

If D denotes the diameter in inches, the equation becomes

$$D = 0.03\ (VS^*) \tag{27}$$

This is strictly a regime equation, and normal channels scouring or with a heavy sediment concentration would demand a lower value for the constant, such as 0.025.

59. Equation 24 could then be evaluated for the velocity of bed sediment, which should include the sediment concentration. Therefore,

$$V = f(d,\ D,\ C)$$

where

C = concentration or charge*

Substituting terminal velocity v as a measure of gradation yields

$$V = f(d,\ v,\ C) \tag{28}$$

From Equation 25, Lacey derived the following equation:

$$V = 1.42\ (VS^*)^{1/4} d^{1/2} \tag{29}$$

Lacey believed Equation 29 to be the correct fundamental equation. The expression was useful because (VS^*) was easy to assess for any system of channels. Substituting the terminal velocity for the product (VS^*) yields in terms of the physical theory,

$$V = K v^{1/4} d^{1/2} \tag{30}$$

---

* Lacey (1946) used s instead of C.

Lacey (1946) noted that Equation 30 was applicable only to a regime channel, with a "regime silt charge" (see footnote, paragraph 51) and should be applicable throughout the entire range of alluvial channel bed material.

60. Lacey (1946) could not improve on the correlation,

$$P = 2.66\ Q^{0.505}$$

by the introduction of the particle diameters as a variable. The original data presented by Lacey suggested that the coefficient is a function of the gradation because the Madras observations in fine silt gave a higher coefficient than Kennedy's (1895) observations for coarser material. Lacey attributed the scatter mainly to the fact that all the channels were artificial and excavated in soil of some tenacity and that some of the ratios of bed width to depth first adopted were arbitrary and could not be corrected. Lacey (1946), therefore, concluded that the sediment concentration was clearly of importance. He stated,

> It has frequently been stated that existing equations for the velocity of flow in alluvial silt transporting channels rest on no sound physical basis. The reason is not very far to seek. The problem is far removed from that of flow in pipes, for example, and no laboratory is large enough to contain it. It would appear that the academic worker must abandon his laboratory temporarily and seek a wider field of activity. The laboratory worker, however, can greatly assist a final solution by carrying out ad hoc experiments designed to prove the truth or falsity of statements such as the author has made in this paper.

Inglis (1947, 1948)

61. Sir Claude Cavendish Inglis, like others who preceded him, believed that alluvial channels have "a never-ceasing tendency to attain equilibrium, . . . a condition in which the load is carried with the minimum expenditure of energy for the existing conditions" (Inglis 1947). This theory holds true, whether flow conditions in the channel closely approximate equilibrium or vary widely, as in the case of meandering channels. Small adjustments to attain equilibrium are brought about by changes in the material exposed on the bed. Such changes can occur either by deposition "where there is an excess charge"

or by scour "where the charge is in defect" (i.e., where concentration is low) (Inglis 1947).

62. Inglis (1947) stated that the most efficient channels carry their load with the least expenditure of energy. He concluded that the most efficient shape for a rigid boundary channel is a semicircle, whereas Lacey (1930) had suggested that for mobile bed channels the most efficient section would approximate a "semiellipse."

63. In an ideal channel, conditions of discharge, concentration, and temperature remain constant; thus, true equilibrium is achieved. In nature, however, the discharge, concentration, and temperature of the water all vary, such that final equilibrium is never fully attained. Inglis (1947) suggested that the nearest approach to equilibrium is a year-to-year consistency for similar discharge conditions, which he called an "annual stability." But even this "annual stability" varies from year to year, owing to variations of concentration resulting from abnormal conditions and changes in the river. For the purpose of simplification, Inglis (1947) assumed that there was a dominant discharge and an associated charge and gradient to which a channel returns annually. At this dominant discharge, equilibrium was most closely approached and the tendency to change the least (Inglis 1947). Inglis modified Lacey's regime equations to meet dominant discharge conditions in rivers and to allow for a "super-regime charge" of material in movement. In this way, he hoped to visualize dominant flow conditions and to consider the effects of variations from dominant discharge.

64. Lacey's (1930) regime formulas, rewritten by Inglis in terms of discharge and "Lacey silt factors" (or, more appropriately, "sand factors"), were:

$$P = 8/3\ Q^{1/2} \tag{31}$$

$$R = 0.4725 \left(\frac{Q}{f}\right)^{1/3} \tag{32}$$

$$S = \frac{0.000547\ f^{1/6} f_{RS}^{3/2}}{Q^{1/6}} \qquad (33)*$$

According to Inglis (1947), Equation 32 shows that, with the same discharge and concentration, the depth varies inversely as the cube root of the gradation of the bed material and this relation holds equally well in straight or meandering channels. He also stated that in Equation 33, with a constant "sand factor" (i.e., silt factor), the slope required varies inversely as the one-sixth power of the discharge; however, for constant slope, the same factor would vary as the one-tenth power of the discharge (Inglis 1947). Therefore, Inglis (1947) concluded that for any discharge less than the dominant discharge, "the slope will be in defect for the dominant charge so that material will deposit, and, as a result, there will be a subdominant charge of material in movement." On the other hand, when the discharge exceeds the dominant discharge, the energy available exceeds that required to carry the dominant charge, and that excess energy is utilized in increasing the charge by scouring the bed, eroding the banks, and developing meanders (Inglis 1947).

65. Inglis (1947) also agreed with the general theories that had been published to that time stating,

> It is desirable to emphasize the fact that the behavior of a river throughout its length depends on the material washed into it, chiefly near its head and from tributaries. Where the charge is heavy or the material coarse, as is generally the case near the hills, the slope is steep and the channel is wide and shallow; where the material is fine, farther down the river, the channel is relatively deep and narrow.

66. Professor C. M. White presented a set of dimensional formulas of the type (Inglis 1948).

$$\frac{bg^{1/5}}{Q^{2/5}} = \alpha_1 \left(\frac{Q}{D^{5/2}g^{1/2}}\right)^{m_1} \left[\frac{D(C^1g)^{1/3}}{V^{2/5}}\right]^{m_2} \left(\frac{Q_s}{Q}\right)^{m_3} \qquad (34)**$$

where

V = velocity of water

Q_s = "quantity of solids conveyed" (suspended-sediment load)

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* At regime f and f_{RS} are numerically equal (Inglis 1947).

** Inglis (1948) used m instead of D, C instead of Q_s, and n_1, n_2, and n_3, instead of m_1, m_2, and m_3, respectively.

Inglis (1947) then worked on the assumption that the charge $C = Q_s/Q$ and the rate of fall of particles v were linked but that charge was not necessarily linked with grade. Therefore, he modified White's formulas as follows (Inglis 1947):

$$\frac{bg^{1/5}}{Q^{2/5}} = \alpha_1 \left[\frac{Q}{D_e^{5/2} g^{1/2}}\right]^{m_1} \left[\frac{Cv}{(\nu g)^{1/3}}\right]^{m_2} \qquad (35)*$$

where

D_e = effective diameter of bed sediment

υ = kinematic viscosity of water

Inglis (1948) believed that the justification for linking C with v and hence υg was twofold:

a. An investigation carried out at Poona, India, by Inglis showed that the rate of deposition of material in water was constant as long as the product Cv remained constant, as long as C varied inversely as v, where the material was quartz sand, with D being greater than about 0.15 mm and less than about 0.5 mm. For grains coarser than about 0.7 mm, $v \propto D^{0.5}$.

b. Experiments initiated by Inglis at Poona when he was Director of the Indian Waterways Experiment Station had shown that when the gradation of material was kept constant and the concentration varied, the exponents approximated closely to those in Inglis' equations.

67. Inglis (1948) stated that the dimensional components of his equations would become either zero or infinite when the charge became zero or if the bed slope and v became infinitely small. These conditions do not occur in practice because the state of flow changes from rough turbulent to smooth turbulent, causing a change of law.

68. Inglis (1948) compared the effect of charge in the formulas developed by Lacey with his own:

Lacey		Inglis	
$V = 0.7937\, Q^{1/6} f^{1/3}$	(36)	$V = \alpha_3 \dfrac{g^{7/18}}{\upsilon^{1/36}} Q^{1/6} (D_e\, Cv)^{1/12}$	(39)**

---

* Inglis (1948) used X and V_s instead of C and v.

** Inglis (1948) used m instead of D_e.

$$\frac{P}{R} = 5.65\ Q^{1/6} f^{1/3} \qquad (37)$$

$$\frac{b}{d} = \alpha_7 \frac{Q^{1/6} (Cv)^{7/12}}{g^{5/18} \nu^{7/36} D_e^{5/12}} \qquad (40)$$

$$\frac{1}{S} = \frac{1{,}828\ Q^{1/6}}{f^{5/3}} \qquad (38)$$

$$\frac{1}{S} = \alpha_5 \frac{g^{1/18} \nu^{5/36} Q^{1/6}}{\left(D_e C v\right)^{5/12}} \qquad (41)$$

where

α_3, α_7, α_5 = coefficients

In Lacey's formulas (Equations 36 and 37), V varies from P/R only in the coefficient. In Inglis' charge formulas (Equations 39-41), the charge C has only a slight effect on V, but has a very great effect on b/d and, hence, PR. The charge also has a marked effect on slope so that where charge is ignored, wide divergences are found even in regime channels.

Mackin (1948)

69. J. Hoover Mackin, a geologist, defined the term "graded stream" as "one which, over a period of years, slope is delicately adjusted to provide with available discharge and the prevailing channel characteristics, just the velocity required for transportation of all of the load supplied from the drainage basin" (Mackin 1948). He pointed out that although the slope usually decreases in a downvalley direction, the discharge, channel characteristics, and load do not vary systematically along the stream; therefore, the profile of a graded stream is not a simple mathematical curve. He stated that corrasive power of a stream and bedrock resistance to corrasion (i.e., mechanical erosion performed by moving agents, such as running water, wind, and glacial ice) determine the slope of the ungraded stream,* but have no effect on the profile of a graded stream. The profile of an aggrading stream** differs in

* Mackin (1948) defines "ungraded" as out of equilibrium during a transitional period when it is adjusting itself to changes in the drainage basin.

** Mackin (1948) defined an "aggrading stream" as "upbuilding approximately at grade" or increase in the elevation of the streambed while maintaining the same slope.

form from that of a graded stream, chiefly because of the difference in rate of downvalley decrease in particle size of the sediment load. Another difference between the two is that the aggrading profile is asymptotic with respect to horizontal line passing through the base level; however, the graded stream is not (Mackin 1948).

70. Mackin (1948) used the term "stream" to include specific reaches of concern. Many rivers have both graded and upgraded reaches. By using the expression "over a period of years," Mackin ruled out both short-termed (including seasonal) fluctuations and the exceedingly slow changes that accompany the progress of the erosion cycle. "Load" and "discharge" are not necessarily the most important factors controlling slope, but they are the only factors that are, in origin, independent of the stream. Slope stands alone because it is the only factor in the equilibrium that is automatically adjusted by the stream itself in such a direction as to accommodate changes in external controls that call for changes in velocity (Mackin 1948).

71. According to Mackin (1948), a graded stream responds to change in accordance with Le Chatelier's general law:

> If a stress is brought to bear on a system in equilibrium, a reaction occurs, displacing the equilibrium in a direction that tends to absorb the effect of the stress.

He pointed out that there are two possible approaches to the study of streams as agents of transportation:

a. In terms of relations between slope, discharge, channel form, and the size of grains comprising the load.

b. In terms of energy transformations.

He also believed that the two should not be combined.

72. "The energy of a stream between any two points is proportional to the product of the mass (of the water-sediment mixture) and the total fall between the two points" (Mackin 1948). This "total energy," which increases with an increase in discharge or slope, can be dissipated by any of the following:

a. Friction along the wetted perimeter of the channel (external friction loss).

b. Friction between the diverse threads of the turbulent current (internal friction losses).

c. Transportation of load.

External and internal friction losses occur whether or not the stream is engaged in transportation of load, and these losses increase with increased channel roughness, with irregularity in the alignment of the channel, and with an increase in wetted perimeter. Roughness, alignment, and cross-sectional form are referred to as "channel characteristics" and are determined by the "hydraulic efficiency" of the channel. Based on earlier work by Gilbert (1877) and Rubey (1933), Mackin (1948) emphasized that the share of the total energy used in the transportation of load could be as small as 3 percent. The fact that the energy dissipated in internal and external friction is overwhelmingly greater than that consumed in transportation means that if total energy determined by slope and discharge remain the same, then relatively slight changes in the channel characteristics cause very marked changes in the transporting power.

73. Most researchers have developed equations to describe the conditions of a graded stream, but Mackin (1948) believed that this approach was inadequate because equations are transposable. As set up in an equation, load is a function of velocity. Mackin (1948) stated that most engineers agree that velocity controls or determines the load carried by a stream. However, over a period of years, the load supplied to a stream is actually dependent, not on the velocity of the stream, but rather on the lithology, relief, vegetative cover, and erosional processes present in its drainage basin. A graded stream will maintain the particular slope that will provide just the velocity required to transport all of the supplied load. In this sense, velocity is determined by, or adjusted to, the load. Therefore, Mackin (1948) stated that "in the graded stream, load is a cause, and velocity is an effect; this relationship is not transposable."

74. He noted that several observations had shown large graded streams usually have lower slopes than smaller graded streams, the reason being that with increased channel size, there is usually an increase in cross-sectional area relative to the wetted perimeter and a corresponding relative decrease in friction. The result is that large streams commonly have higher velocities than smaller streams with the same slope. The load of a graded stream can increase in a downvalley direction relative to its discharge either because of subsurface loss of water (i.e., to the banks and bed) or as a result of the entry of heavily loaded tributaries (Mackin 1948).

75. Mackin (1948) believed graded profiles decreased in a downvalley direction between tributary confluences chiefly because of a downvalley

decrease in gradation of load due to processes within the stream. He disagreed with the common theory that the downstream decrease in the gradation of load is due to the downvalley decrease in slope, stating that "downvalley decrease in caliber of load is an important cause of downvalley decrease in the slope of the graded profile" and that "in the graded stream, the decrease in caliber is due primarily to attritional comminution of particles comprising the load" (Mackin 1948). The downvalley decrease in gradation of load due to attrition is more systematic than any other factor affecting the slope of a graded stream. However, the gradation of load does not vary systematically in graded streams joined by tributaries nor in graded streams in which the mineralogic composition of the load differs greatly in resistance to attrition.

76. Like slope, the other channel characteristics of a graded stream are developed by the stream itself. Slope and other channel characteristics vary from reach to reach, and any change in external controls usually results in changes in these variables. Therefore, Mackin (1948) revised the engineer's definition of a graded stream from one "in which slope is delicately adjusted," to one "in which slope and (other) channel characteristics are delicately adjusted" (Mackin 1948).

77. Griffith (1927) and Lacey (1930) had suggested that natural streams with a heavy bed load tend to flow in broad, shallow channels. Mackin (1948) stated that the two most important reasons for this shape are:

a. A large bed load requires high bed velocity and widening by bank erosion that must continue until velocity at the banks is reduced to the point where the resistance to erosion of the bank-forming materials equals the erosive force applied to them.

b. Shoaling by deposition will accompany widening of the narrow channel by erosion because the particles of the bed load tend to lodge, and move, and lodge again, the velocity required each time to set them in motion being greater than that required to keep them in motion, and because a higher velocity is required to set in motion a particle of the bed than one of the sloping banks.

The cross-sectional shape most efficient for transport varies with slope, with amount and gradation of the load, and especially with the proportions of the total load carried in suspension and moved along the bed (Mackin 1948). Cross-sectional shape is also dependent upon the resistance of the banks to erosion; therefore, for any set of slope-debris "charge" (concentration) factors, there will be one critical degree of erodibility of the bank-forming materials, such that a channel that was originally too narrow will quickly develop a cross section with depth-width relations that provide "the maximum silt-carrying capacity" (Mackin 1948). In less erodible bank-forming materials, the stream could develop the maximum efficiency section over a period of time for the given slope-debris "charge" factors. However, if the bank-forming materials are more erodible than the critical degree, the stream will adopt and maintain a section that is wider and shallower than the ideal transportation section. Mackin (1948), therefore, concluded

> A statement to the effect that a flow of water charged with debris must necessarily develop for itself that form of section that will give it a maximum debris-carrying capacity, is an invalid generalization because it ignores erodibility of the banks.

He also noted that stability is no guarantee of maximum efficiency (Mackin 1948).

78. In affirming the validity of Le Chatelier's general law (paragraph 71), Mackin (1948) described the manner in which a system in equilibrium would react to certain stress brought to bear on it. He commented on increase in load, decrease in load, change in discharge, rise of base level, and lowering of base level, as follows (Mackin 1948):

a. Increase in load. "A once-graded stream responds to an increase in load primarily by steepening its declivity below the point of influx. The steepening is accomplished by deposition of part of the excess load in the channel at the point of the influx with a consequent upbuilding of the channel at that point and the formation of a steepened part immediately below. Steepening of any segment permits increased transport of load through that segment to the next segment which is in turn the site of deposition and steepening. Thus the effect of an increase in load is registered by the downvalley movement of a wave deposition, large or small depending on the rate and manner of addition of the load, and deposition must continue throughout the stream below the point of influx until the slope is everywhere adjusted to the transport of all the debris delivered to it."

b. Decrease in load. "A decrease in load may be thought of as occurring at any point in the stream. It may be stated for the time being that the stream simply makes up for deficiency in the load supplied from above by picking up additional load from its channel floor. The net result is downcutting, with a consequent lowering in declivity downvalley from the point where the change occurred. Downcutting must continue until the profile is reduced to that slope which will provide just the velocity required to transport the reduced load."

c. Change in discharge. "If a segment of a graded stream receives all or most of its load at the upper end, changes in discharge call for readjustments in the slope in much the same way as changes in load. A decrease in discharge requires an increase in declivity because the load, remaining the same, must move faster through a smaller cross section, and because, as indicated earlier, decrease in the cross-sectional area of the channel involves a relative increase in frictional retardation of flow and, hence, a decrease in velocity. The stream affected by a decrease in discharge, being unable to transport all the load supplied to it on its former slope, deposits some of the load and thereby steepens the slope, the process continuing until the reduced stream is able, by reason of increased velocity on the steepened slope, to transport all the load shed into it. The opposite adjustment occurs in the case of increased discharge."

d. Rise of base level. "A rise of base level is equivalent to the rising of a barrier across the path of the graded stream. Each unit of increase in the height of the barrier trends to flatten the declivity immediately upstream. The stream, unable because of decreased declivity to carry all of the load through the flattened segment, deposits in the segment, thus increasing the declivity and transferring the flattening upvalley. Continuation of the process results in upstream propagation of a wave or, better, of an infinite number of small waves of deposition. If the barrier is raised slowly the stream may maintain itself in approximate adjustment during the process; its rate of aggradation is determined by the rate at which the barrier is elevated. If the barrier is raised rapidly or instantaneously a lake is formed; the distal part of the delta is then the 'flattened part' of the profile, and the rate of aggradation is determined by the rate of delta building into the lake. In either case the successive profiles developed during the period of readjustment will differ markedly in form from the original profile and from the eventual completely readjusted profile. The final readjusted profile will tend toward parallelism with the original profile, differing in this respect from the cases treated above. But, because of secondary effects of aggradation . . . precise parallelism will usually not be achieved; the only generalization that can be made is that the new profile will be everywhere adjusted to the new prevailing conditions."

e. Lowering of base level. "Lowering in base level is, insofar as the response of the stream is concerned, essentially the same as the lowering of a barrier in its path. Each small lowering of the control point steepens the gradient immediately upstream. Accelerated velocity in the steepened portion results in downcutting, and the steepening is propagated upvalley. Downcutting must continue until the slope is again completely adjusted to supply just the velocity required to transport all of the debris shed into the stream; as in the last case, and with the same qualification, the final readjusted profile will tend to parallel the original profile."

79. These changes can be classified into two categories: (a) changes in downvalley controls and (b) changes in upvalley controls. In general, a change in upvalley controls (load and discharge) results in a change in slope, whereas a change in downvalley controls (base level) will cause a change in the level of the profile, but not necessarily in the slope. Mackin (1948), therefore, had suggested many new ideas in the study of graded or regime rivers. He developed many general statements but avoided manipulation of the data into equation form.

Blench (1951)

80. Thomas Blench stated that a river is "in regime" if its mean measurable behavior during a certain time interval does not differ significantly from its mean measurable behavior during comparable times before or after the given interval. He pointed out that regime theory originated on the great canal systems of India and Pakistan where discharges are controlled and meanders prevented; however, this theory is not generally applicable to a "regime-type river"* (Blench 1951).

81. Blench (1951) believed that the present generalized theory would not exist without the Lacey theory. Lacey (1930) had used a single silt factor f (Equation 9), thereby effectively obtaining results for a physically possible set of channels whose relative importance of side action to bed action had been averaged out to some arbitrary value. This permitted him to work in terms of wetter perimeter P and hydraulic radius R. Blench (1951) found the exponents of Lacey's regime formulas to be consistent with the laws of rigid boundary hydraulics. As width goes to infinity, depth approaches the hydraulic radius and width approaches the wetted perimeter; therefore, because most rivers are wide, Blench replaced P with b and R with d and, leaving Lacey's exponents the same, obtained:

$$\frac{V_o^2}{d} = f_b \tag{42}$$

* Defined by Blench (1951) as "one that has formed a major part of every cross section from material that has been transported or could be transported at some stage of flow."

where f_b = a bed factor related to the nature of the sediment load, which is constant as long as the factors relevant to the bed load are constant. He believed this formula to be exact when the discharge and bed load were constant and regime conditions had been attained. Therefore, Equation 42 provides the mean for defining a dynamically satisfying bed factor (Blench 1951).

82. Blench's second generalized regime equation was

$$\frac{V_o^3}{b} = f_s \tag{43}$$

where f_s = side factor that measures the tractive force. The width term b in Equation 43 multiplied by average depth equals cross-sectional area. Equation 43, like Equation 42, can be taken as exact for "perfect regime conditions," but is also practically applicable to real regime conditions (Blench 1951). Blench developed this formula while attempting to determine the reason for the universal agreement of the exponent (0.5) in the Lacey (1930) formula (Equation 10)

$$P = 2.67\ Q^{1/2} \tag{44}$$

83. There is, however, significant disagreement over the coefficient 2.67, which Blench (1951) contended should be replaced by a variable dependent upon the nature of the bed sediment. His line of reasoning was as follows (Blench 1951):

> Universal acceptance of the exponent 1/2 (in Equation [44]*) based on data covering a very wide range, suggests that the expression PV_oR has a physical significance. Replacing P and R by b and d, respectively, the expression can be written as
>
> $$\frac{V_o^2/d}{V_o^3/b}$$
>
> in which the numerator has physical significance. Therefore, there should be physical significance in the denominator. Since V_o^2/d is a force per unit mass, V_o^3/b is probably in some way a measure of tractive force on the sides. $P^2 \upsilon V_o^3/b$ measures the square of the tractive force intensity on the sides (except for a multiplying constant)

* Equation and figure numbers enclosed in brackets within quoted material refer to this report.

provided that there is a laminar film at the sides (that is, provided the sides are technically "smooth").

84. Blench, therefore, concluded that the sides of the channels observed must have been smooth and that the Lacey formula represents nothing more than an average of the relative importance of bed to sides as measured by the ratio f_b/f_s. This meant that all Lacey channels were regime channels, but not all regime channels were Lacey channels. Logically, but not practically desirable, f_s should be replaced by υf_s (Blench 1951).

85. The third equation of Blench's generalized regime theory was

$$\frac{V_o^2}{gdS_c} = C' \frac{V_o b^{1/4}}{\upsilon} \qquad (45)*$$

where

S_c = channel slope expressed as a fraction

C' = nondimensional constant

$\frac{V_o b}{\upsilon}$ = Reynolds number in terms of width (see Equation 15)

Algebraic manipulation of Equations 43, 44, and 45 yields

$$d = 3\sqrt{\frac{SQ}{f_b^2}} \qquad (46)$$

$$b = \sqrt{\frac{BQ}{f_s}} \qquad (47)$$

$$S = \frac{f_b^{5/6} s^{1/12} Q^{-1/6}}{\frac{C'g}{\upsilon^{1/4}}} \qquad (48)$$

According to Blench (1951), the value of $C'g/\upsilon^{1/4}$, with υ averaging 10^{-5} is 2,080. The designer must, however, determine the value of f_b that nature will impose on his canal system, and there is no scientific formula to assist in making the decision. There is a rough empirical formula, adapted from an equation proposed by Lacey (1936) (who was careful to emphasize its empiricism)

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* Blench (1951) used C instead of C', and S instead of S_c.

$$f_b = 2\sqrt{D_b} \qquad (49)$$

in which D_b is the mean diameter of the bed material (in millimetres) to be expected in the canal. Scientifically, Equation 49 is unsatisfactory. Practically, however, it fits many cases quite well, probably because the quantities in movement are very minute in most canal applications (Blench 1951).

86. When applying regime formulas derived on canals to rivers, an allowance for "disturbing effects" (i.e., turbulence) and variable discharges must be made "in terms of engineering common sense and the formulas regarded with suspicion until supported by results" (Blench 1951). Some apparently satisfactory applications were found to be (Blench 1951):

- a. Determination of width between incised banks.
- b. Determination of scour between bridge piers.
- c. Determination of scour downstream from piers, along groins, and at spur heads.
- d. Estimation of aggradation upstream from reservoirs.
- e. Estimation of degradation downstream from reservoirs.
- f. Estimation of dredging.
- g. Determination of model scales.

Leopold and Maddock (1953)

87. Geomorphology, the branch of geology dealing with landforms and their genesis and history, had been classically treated almost exclusively in a qualitative manner. By the early 1950s, the US Geological Survey had vast amounts of streamflow data from all over the United States, which consisted of concurrent measurements of mean velocity, width, shape, area of cross section, and discharge. Suspended-sediment data covering a 10-year period were also available. Luna B. Leopold and Thomas Maddock, Jr., (1953) noted that these data were not collected for the study of fluvial morphology and, as a result, were incomplete in terms of water surface slope, bed roughness, and bed load.

88. Leopold and Maddock (1953) investigated changes in velocity, depth, and top width for an increase in discharge at a particular cross section of a river. In their paper, the term "depth" referred to mean depth resulting from dividing the cross-sectional area of flow by the top width. "Velocity" referred to a mean velocity obtained by dividing the discharge by the

cross-sectional area of flow. Although the authors realized that this was not the most useful measure of velocity, especially when dealing with sediment transport, it was the only measure of velocity for which a large quantity of data was available.

89. They found that the relation of width, depth, and velocity to discharge could be approximated by straight lines on logarithmic plots. Some scatter was attributable to temporary scour and fill of the streambed excluding any unnatural effects; the data analyzed indicated that through ranges of discharge up to the bank-full stage. The relations of width, depth, and velocity to discharge could be described by simple power functions:

$$b = a_1 Q^{m_1} \tag{50}$$

$$d = a_2 Q^{m_2} \tag{51}$$

$$V_o = a_3 Q^{m_3} \tag{52}$$

where m, m_2, m_3, a_1, a_2, and a_3 are numerical constants.*

90. Because width, depth, and velocity are all functions of discharge,

$$Q = bdV$$

therefore,

$$Q = \left(a_1 Q^{m_1}\right)\left(a_2 Q^{m_2}\right)\left(a_3 Q^{m_3}\right)$$

or

* Leopold and Maddock (1953) used W instead of b; a, c, and k, instead of a_1, a_2, and a_3, respectively; and b, f, and m instead of m_1, m_2, and m_3, respectively.

$$Q = a_1 a_2 a_3 Q^{m_1+m_2+m_3}$$

thus

$$m_1 + m_2 + m_3 = 1.0$$

and

$$a_1 a_2 a_3 = 1.0$$

The constants, a_1 , a_2 , and a_3 represent the intercepts of the lines on the logarithmic plots and are, respectively, values of width, depth, and mean velocity at a discharge of unity. Because in actual data, the discharges are greater than unity or, better expressed, the product of the values of width, depth, and mean velocity at a given discharge must equal that discharge. Width and depth vary widely from one cross section to another at a given discharge; therefore, more work was necessary before suggesting a range of values for the coefficients a_1 , a_2 , and a_3 . The average values for the exponents m_1 , m_2 , and m_3 , which are biased toward semiarid conditions because they were computed for 20 river cross sections on a variety of rivers in the Great Plains and Southwest, are (Leopold and Maddock 1953):

$$m_1 = 0.26$$
$$m_2 = 0.40$$
$$m_3 = 0.34$$

From these computations, Leopold and Maddock (1953) noted that depth increases faster than width for an increase in discharge. The relative rates of increase of width and depth are functions of the channel shape (Leopold and Maddock 1953).

91. Next, they plotted width, depth, and velocity corresponding to the mean annual discharge at cross sections from all parts of a river system and showed that discharge increases in the downstream direction because of the increase in drainage area. The depth, width, and velocity all tend to increase as power functions of the discharge. The general alignment of points on the downstream graphs indicates that, in a given river basin where all

cross sections are experiencing the same frequency of discharge, the corresponding values of depth, width, and mean velocity at different cross sections (even on different tributaries) having the same discharge tend to be similar, regardless of the location within the watershed (Leopold and Maddock 1953).

92. Leopold and Maddock (1953) point out that most geomorphologists are under the impression that the velocity of a stream is greater in the headwaters than in the lower reaches because the river is steeper in the upper reaches. However, Manning's equation,

$$V = \frac{1.49}{n} R^{2/3} S_e^{1/2} \qquad (53)^*$$

shows that the depth of flow (or hydraulic radius) is more influential than slope on change in velocity. Leopold and Maddock (1953) concluded that the rate of increase of depth downstream tends to overcompensate for the decreasing slope and to provide a net increase of velocity at mean annual discharge in the downstream direction of a river. When they plotted width, depth, and velocity against the mean annual discharge at various cross sections, the result was a simple power function. The exponents and coefficients varied from basin to basin, but the average exponent values for the basins studied were:

$$m_1 = 0.5$$
$$m_2 = 0.4$$
$$m_3 = 0.1$$

93. Thus, the changes in hydraulic characteristics downstream in a river system have been considered using only the mean annual discharge. They then made comparisons between the data already presented and similar data for infrequent flows. At each gaging station, in addition to the mean annual discharge, they studied flows that were equaled or exceeded 1, 4, 10, 30, and 50 percent of the time. For each discharge frequency, curves representing the change of hydraulic factors with discharge in the downstream direction were plotted. Leopold and Maddock (1953) stated

* Leopold and Maddock (1953) used 1.5 for 1.49 and substituted d for R and S for S_e in Manning's equation.

> The channel characteristics of natural rivers are seen to constitute, then, an interdependent system which can be described by a series of graphs having simple geometric form. The geometric form of the graphs describing these interactions suggests the term "hydraulic geometry." Channel characteristics of a particular river system can be described in terms of the slopes and intercepts of the lines in the geometric patterns. . . . Though the listing of slopes and intercepts of these lines may not provide any visual picture of a river basin, comparison of the values of these factors among rivers has useful aspects.

94. Leopold and Maddock (1953) also studied the effect of suspended sediment on hydraulic geometry. By the early 1950's, suspended-sediment samples were being taken on a regular basis at a number of locations on major streams. The samples in any given cross section could then be analyzed for concentration and, with the mean discharge for that same day, used to estimate the passing daily suspended-sediment load. Plots of the relation of suspended-sediment load versus discharge could then be made. These plots were characterized by a large scatter of points; however, for long periods of record, this relation can be approximated by a simple power function of the form (Leopold and Maddock 1953):

$$Q_s = aQ^j \tag{54}$$ *

where

Q_s = suspended-sediment load, tons per day

a, j = numerical constants, with the range of j being on the order of 2 to 3

95. They concluded that typically at a given cross section, the suspended-sediment load should increase more rapidly than the discharge. Consideration of the physical characteristics of surface runoff and observations of bed scour led to the tentative conclusion that the observed increase in sediment concentration resulted primarily from erosion of the watershed rather than from scour of the bed of the main stream in the reach where the measurement was made. There were not enough suspended-sediment stations on any given stream to analyze downstream changes in concentration. Leopold and Maddock (1953) stated that in long reaches of a river through more or less homogeneous

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* Leopold and Maddock (1953) used L instead of Q_s and p instead of a .

topography or lithology, the input of various tributaries did not appear to have any great effect on the turbidity of the water (Leopold and Maddock 1953).

96. Leopold and Maddock (1953) contended that certain aspects of the effect of suspended sediment on channel shape were best exemplified by consideration of the condition of constant discharge. For each station, width was plotted against suspended load on a semilogarithmic graph (Figure 3) (with suspended load being plotted on the logarithmic axis), and at each point the corresponding velocity was entered. Velocity isopleths slanted steeply downward to the right, while depth isopleths slanted gently downward to the right. The limitations of the graph were:

- a. The velocity field was only roughly defined by values in the available data.
- b. Only the suspended-sediment load was considered.
- c. There was a large scatter of points on any average curve showing relation of suspended sediment to discharge.

Figure 3 shows that there was an increased capacity for suspended load at constant discharge by decreasing width at a constant velocity or by increasing velocity at constant width.

97. The slope j of the curve relating suspended load to discharge (Equation 54) depends on the rate of increase of velocity with discharge defined by the exponent m_3 (Equation 52), and because $m_1 + m_2 + m_3 = 1$,

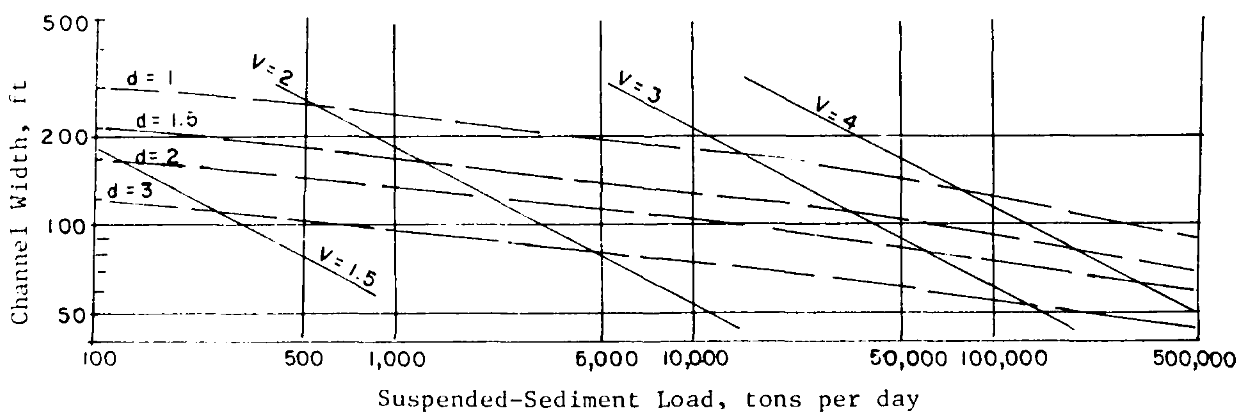

Figure 3. Average relation of suspended-sediment load to channel width for a constant discharge of 500 cfs. Velocity V isopleths are in feet per second, while depth d isopleths are in feet (adapted from Leopold and Maddock 1953)

when m_1 and m_3 are specified, then m_2 is also specified. Therefore, if the ratio of m_3 to m_2 is known and the value of m_1 is known, then the values of m_3 and m_2 are also known. From these relations, Leopold and Maddock (1953) developed Figure 4.

> From Figure [4], a number of useful concepts may be inferred. The first concerns the interaction of velocity, depth, and the suspendedsediment load. For any given rate of increase of width with dischargethat is, for any given value of m_1, the diagram indicates that the value of j increases with an increase in the m_3 to m_2 ratio. That is, the rate of increase of suspended-sediment load with discharge is a function of the ratio:
>
> $$\frac{\text{rate of increase of velocity with discharge}}{\text{rate of increase of depth with discharge}}$$
>
> Slopes of the lines representing depth, width, and velocity respectively, in relation to discharge are, according to the implications contained in Figure [4], functions of the slope of the line representing the relation of suspended sediment to

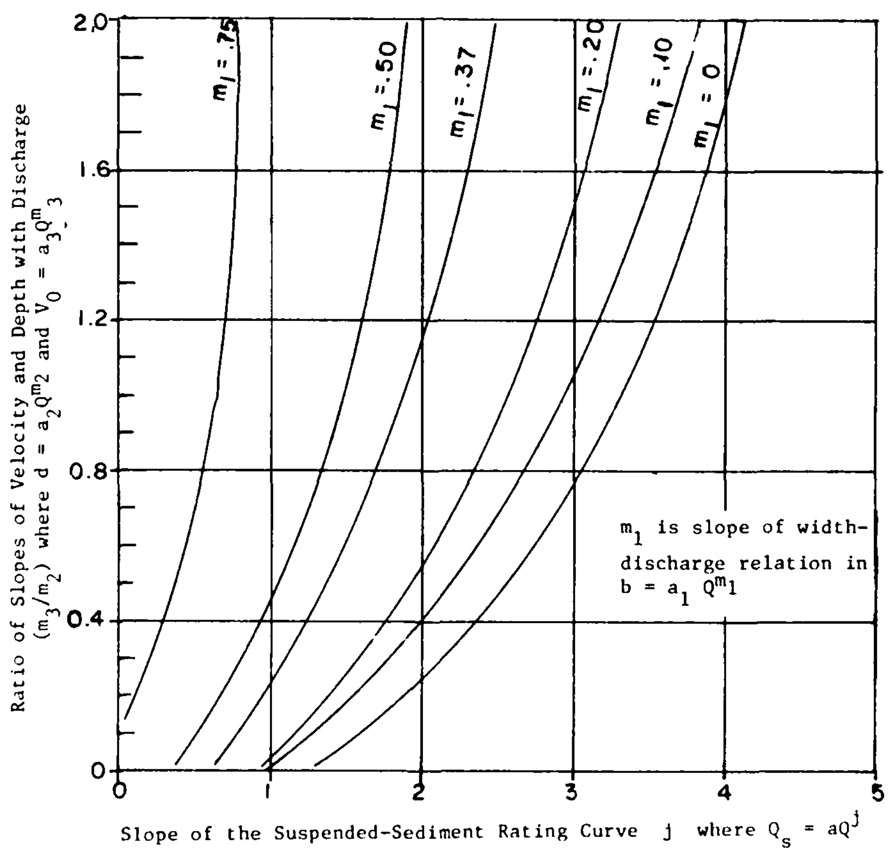

Figure 4. Relation of the rates of increase of width, depth, and velocity and suspended-sediment load expressed by values of m_1, m_3/m_2, and j (adapted from Leopold and Maddock 1953)

discharge. For a given value of b, the steeper the slope of the suspended sediment discharge line, the greater will be the slope of the velocity-discharge line (Leopold and Maddock 1953).

98. According to Leopold and Maddock (1953), the suspended-sediment concentration should decrease in the downstream direction. A value of $j = 1.0$ would mean that suspended load and discharge are increasing at the same rate. Therefore, the value of j for natural rivers should be somewhat less than 1.0. Using the mean values for m_1, m_2, and m_3, Figure 4 correctly yields $j = 0.8$. Therefore, Figure 4 appears to apply to average conditions both at a station and downstream. By their analysis, Leopold and Maddock (1953) stated, "For a given width, and at a given discharge, an increase in suspended-sediment load requires an increase in velocity and a reduction in depth." From the study of graphs developed by Gilbert (1914) from flume data, they concluded,

> At constant discharge, an increased velocity at constant width is associated with an increase of both suspended load and bed load in transport. At constant velocity and discharge, an increase in width is associated with a decrease of suspended load and an increase in bed load in transport (Leopold and Maddock 1953).

99. Past researchers had placed too much emphasis on bed load, and the role of suspended load in determining the observed characteristics of natural streams had been underestimated (Leopold and Maddock 1953). From analysis of rivers in the western United States during floods, Leopold and Maddock (1953) determined that the changes in the riverbed occurred simultaneously with changes in the rate of change of suspended-sediment concentration.

> It is the thesis of this paper that the observed changes in the stream bed resulted from changes in the sediment load brought into the measuring reach from upstream. It is postulated that the hydrodynamic factors involved tend to promote a mutual adjustment between channel shape and the sediment load carried into the reach. The change in sediment load, which results in a change of channel shape, involves both bed load and suspended load. However, because only the suspended load is measured, it is necessary to use the data on the suspended fraction of the load as an index to how the total sediment load interacts with the hydraulic variables. That the suspended fraction is a meaningful index is demonstrated by the fact that the relations between the channel shape factors and suspended load, which are derived from measurements of a number of different rivers, appear to apply in principle to channel changes at a given station during an individual flood. Specifically, with no change of channel width, a decrease of suspended load at a given discharge was accompanied by an increase in depth by bed scour that resulted in a decrease in velocity Figure [4]. The decrease in velocity provides the adjustment of capacity for carrying the load of the particular size distribution

> supplied by the watershed. In response to a decrease in load, the channel shape became adjusted through scour to the lower capacity required for quasi-equilibrium.

100. Changes in channel shape occur in response to changes in sediment load brought into the reach (Leopold and Maddock 1953). This postulate required proof that the observed change in suspended-sediment load was not the result of the observed change of velocity in the reach, but rather the cause. If the high suspended-sediment concentrations resulted from the scouring action of high velocities, the implication was that high velocity in a given reach scours the channel in that reach. The increase in sediment in transport resulting from the local bed scour should then account for the observed increase in sediment concentration. Under such an assumption, increasing velocity should be associated with bed scour and decreasing velocity with aggradation. Leopold and Maddock (1953) concluded that the scour and fill of the bed of the main stem of an alluvial river during a flood appeared to be a rapid adjustment of channel shape in response to a varying sediment load. Vanoni (1941) had shown that an increase in suspended load tended to decrease channel roughness and cause an increase in velocity. Buckley (1922) had also found similar results. The effect of suspended-sediment concentration on channel roughness was also discussed by Thomas (1946), who concluded that an increase in concentration resulted in lower values of Mannings's n.

101. Any large changes in suspended-sediment concentration that occurred with changes in discharge in a given stream should be reflected by substantial changes in roughness. Leopold and Maddock (1953) had found a change in velocity-depth relations that coincided with a change in suspended load-discharge relations at a discharge to be well below the peak on the Colorado and San Juan Rivers. Such local adjustments in velocity-depth relations resulted primarily from changes in roughness associated with changes in sediment load, rather than from changes in the slope of the water surface. They found that a change in roughness caused by a change in suspended-sediment load was also evident 350 miles downstream from Hoover Dam at Yuma, AZ. Since construction, a quasi-equilibrium has been established at Yuma by change in channel characteristics and not by changes in slope. Changes in slope that have occurred in upstream reaches near dams could, in time, affect the Yuma reach.

102. Comparing regime canals to natural rivers, Leopold and Maddock (1953) stated:

> The comparison of regime canals and natural rivers is valid only in the downstream direction. Canals carrying water high in

sediment concentration would not normally be expected to maintain the regime condition except at the discharge for which they were designed. The lack of a varying discharge at any particular canal cross section precludes a proper comparison with the at-a-station relation of rivers. Failure to recognize this would lead to fallacious reasoning, as in a discussion by Stevens (1937). (See also Lane (1935), in which he compared the width-discharge formula of a particular river gaging station with the Lacy (1930) width-discharge formula for canals.)

Another fact that demonstrates the analogy between regime canals and rivers in the downstream direction lies in a consideration of slope. In the design of canals, it has usually been found necessary to provide greater slope in the small laterals than in the large trunk canals. Thus, canals in regime have a longitudinal profile convex upward because the flow is reversed as compared with natural rivers; that is, the discharge in a canal system decreases downstream. This is analogous except for the direction of flow to the condition in natural rivers, which are steeper in the headwaters where the discharge is small than downstream where the flow is great.

Concurrent changes in m_3, m_2, and j during individual floods (Figure 4) define, at least in general, the mutual adjustments that occur locally when suspended-sediment concentration changes. Because Figure 4 describes relations both local and downstream for average values of the factors observed in rivers and approximately fits regime canals, any point in the graph was considered by Leopold and Maddock (1953) to represent the concurrent values of m_3, m_2, and j required for equilibrium. Leopold and Maddock, however, noted that the exact form and positions of the lines in Figure 4 must be considered subject to adjustment as more data become available.

103. Leopold and Maddock (1953) also found that graded reaches of a river showed a width-depth-velocity-discharge relation similar to that of reaches not known to be graded. They stated that this similarity indicated that the same factors operating to maintain equilibrium in a graded river are also sufficiently active in nongraded reaches to provide specific consistent patterns in the hydraulic variables, both locally and in a downstream direction. Stream-gaging data, however, were not available in sufficient quantity, in a close geographic network, or over a long enough period to allow differentiation between graded and nongraded reaches by hydraulic geometry alone, though such differentiation could theoretically be possible.

104. From their research, Leopold and Maddock (1953) concluded the following with regard to the longitudinal profile of a river:

Channels of different shapes have characteristic distributions of velocity in the cross section. With the same discharge, a very shallow and wide channel tends to have a greater average shear in the vertical plane than in the horizontal. Conversely a very deep and narrow channel tends to have greater average shear on the bank than on the bed. The relative erodibility of bed and bank allows the channel shape to develop a velocity distribution which is in approximate equilibrium with the particular erodibility characteristics. In this manner the increase of width discharge (value of m_1) probably is determined. Why this exponent (m_1) should so consistently be nearly equal to 0.5 in the downstream direction for widely different rivers is not known and constitutes an important unsolved problem.

Different concavities of longitudinal profile are associated with different relations of velocity and depth to discharge, as required by the hydraulics of open channels. However, a particular rate of increase of both velocity and depth downstream is necessary for maintenance of approximate equilibrium in a channel, inasmuch as the drainage area produces sediment and water in a characteristic manner.

Roughness varies with sediment load and with particle size but the interaction of opposing tendencies results in a small range of values of roughness downstream in most rivers. The suspended load and its change downstream characteristic of natural rivers require a particular rate of increase of velocity and depth downstream. Under the conditions of a nearly constant roughness, to provide the required velocity-depth relations slope must generally decrease downstream; it is for this reason that the longitudinal profile of nearly all natural streams carrying sediment is concave to the sky.

On the basis of their work, Leopold and Maddock (1953) agreed with Mackin (1948), stating that the wording in the definition of regime should be changed from "in which slope is delicately adjusted" to one in which slope and all other channel characteristics are delicately adjusted.

Lane (1955)

105. Fluvial morphology, as developed by geomorphologists, had been largely a qualitative or descriptive science that suffered somewhat from a lack of quantitative relations. Lane (1955) wanted to present geomorphology from an engineering viewpoint because the concept of equilibrium in streams as developed by the geomorphologist was very useful to engineers.

106. Lane (1955) was a proponent of the energy concept; that is, when a stream is capable of carrying more sediment than is supplied to it, bank erosion and scour occur. Scour increases the slope of the tributary streams, thus allowing the streams to introduce more material to the main stem. The more a stream scours into the surrounding land, the greater the available

load. Lane stated that most streams eventually reach either the ocean or some other base level as they cut down their beds, decreasing their slopes and diminishing their ability to transport the sediment brought to them. As the amount of material brought down to the stream increases and the stream's ability to carry it decreases, the rate at which sediment is brought to the stream becomes equal to the rate at which the stream carries it away, and equilibrium is attained. Because of the rapid variations of flow and sediment supply, a condition of equilibrium in which the load exactly equals the capacity of the stream rarely exists, except for a short duration. For most practical engineering purposes, however, the beds of a large number of streams are in equilibrium (Lane 1955).

107. Lane (1955) defined equilibrium as follows:

> For engineering purposes, a section of a stream may, therefore, be said to be in equilibrium if, although it may continually fluctuate between aggradation and degradation over a long period of years in terms of human history, the net amount of change is not sufficiently large to be detected by quantitative measurements.

Most alluvial streams not affected by the works of man are in equilibrium for most practical engineering purposes (Lane 1955). Lane also contended that if equilibrium existed in a reach and one of the conditions were changed, then over time the conditions of equilibrium would change throughout the entire reach. In long reaches, there is frequently more than one change at a time; however, one of the changes commonly overshadows the effects of the other changes with regard to the new equilibrium conditions.

108. In most cases, there was not a sufficient amount of data to make a quantitative analysis of equilibrium change; therefore, Lane (1955) developed the following expressions as a qualitative guide:

$$Q_b D \approx Q S_b \qquad (55)*$$

> Here Q_b is the quantity of sediment, D is the particle diameter or size of the sediment, Q is the water discharge, and S_b is the slope of the stream. This is an equation of equilibrium and if any of the four variables is altered, it indicates the changes which are necessary in one or more of the others to restore equilibrium. For example, if a stream with its sediment load is flowing in a condition of equilibrium and its sediment

* Lane (1955) used Q_s instead of Q_b, d instead of D, Q_w instead of Q, and S instead of S_b.

load is decreased, equilibrium can be restored if the water discharge or the slope is decreased sufficiently or if the diameter of the sediment is increased the proper amount. This equation is not an exact mathematical equation as it will not give the quantitative values of the variables involved which bring about equilibrium, but it is helpful to indicate qualitatively the changes which will take place in a stream when a change of any one of the variables occurs.

The sediment discharge Q_b in this equation is the coarser part of the sediment load or more exactly the bed-material load, since this is the part of the load which largely molds the bed formation. In most cases, the quantity of the fine load of silt and clay sizes can change almost indefinitely without materially affecting the river profile (Lane 1955).

Chien (1955)

109. Until the mid-1950's, the basic equations as proposed by Lacy (1946) remained essentially the same, although others had attempted to express them in different forms with various interpretations. The formulas as proposed by Lacey involved factors that were based essentially on experience alone, making use of the theory difficult for those who lacked this experience. Ning Chien, therefore, decided to evaluate these factors quantitatively in terms of the physical conditions imposed on the channel and to show the possible limitations of Lacey's (1946) regime theory.

110. Inglis (1947) had rewritten Lacey's (1946) three regime formulas (Equations 31-33), and later, Chien (1955) expressed the Lacey formulas, as follows:

$$\frac{V_o^2}{R} = 1.325\ f_{VR} \tag{56}$$

$$R^{1/3} S^{2/3} = 0.0052\ f_{RS} \tag{57}$$

$$P = aQ^{1/2} = 2.67\ Q^{1/2} \tag{58}$$

where

f_{VR}, f_{RS} = silt factors*

a = coefficient

---

* Inglis (1947) had used f instead of f_{VR} .

He stated that the above questions "can be applied only to channels which contain a bed of loose sediment of the same type that is moved along the bed and which are essentially at equilibrium" (Chien 1955).

111. Chien's (1955) study was structured around four premises, the first two concerning "the behavior of alluvial channels, in general," and the second two dealing with "Lacey's theory, in particular."

<u>a</u>. "The equilibrium state of an alluvial channel is attained by adjusting the dimensions of the cross section and the slope of the channel in accordance with the conditions imposed on the channel by the drainage basin. These conditions, which are independent of the channel itself, include the sediment inflow, the discharge, and the gradation and size of the material which composes the channel bed and the banks. It has long been recognized that the sediment inflow is a primary factor governing the stability of an alluvial channel.

<u>b</u>. "The alluvial channel differs from the fixed-boundary channel by the fact that the boundary is molded by the flow and may change with the flow. The frictional resistance of an alluvial channel flow cannot be described generally by one formula with universal constants, such as the Manning's formula. It must be interpreted as a composite effect of the following parts:

(1) Resistance of the sediment grains which compose the channel bed.

(2) Resistance of banks.

(3) Resistance of sand bars and other irregularities.

Each of these follows different and independent laws.

<u>c</u>. "Equations [56-58] are empirical relationships based on field observations only. They should not be visualized as fixed laws with universal application.

<u>d</u>. "Lacey's regime theory is the product of sixty years' experience gathered in India and Pakistan, and it serves today as a useful tool in designing irrigation canals in that area."

112. Chien (1955) hoped to find a philosophical basis for Lacey's regime theory determining why the sediment load was not included. He believed that Lacey's omission of sediment load as an explicit variable could be explained by either of the following two possibilities:

<u>a</u>. "The silt factors . . . actually include implicitly the sediment load together with the bed material size.

<u>b</u>. "Although the dimensions and the slope of an alluvial channel must depend on the sediment inflow, a certain combination of these variables may depend very little on the sediment load" (Chien 1955).

113. Chien (1955) used graphical solutions relating channel depth and slope with a unit discharge and sediment load taken from his earlier work (Chien 1954) as the basis for his analysis. These graphical solutions were constructed according to the Einstein bed-load function, the only working tool available at that time to completely describe alluvial channel flow and sediment transport. He first ignored bank friction and investigated velocity, hydraulic radius, slope, discharge, bed-material size, and silt factors. Plotting the silt factors against sediment concentration, he developed the following equations (Chien 1955):

$$f_{VR} = 0.061 \left(\frac{q_T}{q}\right)^{0.715} \tag{59}$$

$$f_{RS} = 1.18 \left(\frac{q_T}{q}\right)^{0.052} \tag{60}$$

where

q_T = total sediment transport rate (including both the bed load and the suspended load) per unit width

q = discharge per unit width

The silt factor f_{VR} depends strongly on the sediment load and is the combination of slope and hydraulic radius and is practically independent of the sediment transport rate (Chien 1955).

114. Chien (1955) then stated

> In the wider range of variables, especially of the sediment size, the trend of f_{RS} remains the same. With sediment size D as a variable and expressed in millimetres, Equation [60] becomes
>
> $$f_{RS} = 2.2\, D^{0.45} \left(\frac{q_T}{q}\right)^{0.052} \tag{61}$$
>
> for $q_T/q < 200$ ppm. The change of f_{VR} with sediment concentration not only follows a somewhat different pattern, but also is quite erratic. For other than conditions in India and Pakistan, f_{VR} depends also on the hydraulic characteristics of the channel. This makes the general application of Lacey's regime theory extremely difficult. It is not just the question of whether one has the proper training or experience in estimating the values of silt factors, but rather the experience gathered in northern India and Pakistan may fail to apply when the conditions are substantially different from those existing in that area.

He found that bank friction had little effect on the overall trends according to which the silt factors vary with the sediment concentration. He did, however, note a slight shift in relative position of the f_{VR} and f_{RS} curves for small channels (Chien 1955).

115. Chien (1955) noted two important points:

a. "The sediment load used in this study refers only to the bed-material load, the material which has been found abundant in the bed. The actual sediment concentration of the canal flow, including that of the wash-load, may be considerably higher.

b. "In using Einstein's bed-load function, the effect of bar resistance is determined from river measurements. There is some evidence indicating that below a certain limit, the bar resistance is affected by the width of the channel. In a narrow channel, the bars do not develop to their full sizes, due to the confinement by the banks. This, in effect, will bring the values of f_{VR} and f_{RS} closer together at small sediment concentrations, but will not materially change the conclusions derived from the present analysis."

116. The findings of Chien's (1955) study can be summarized as follows:

a. "The silt factors in Lacey's regime theory have been correlated with the conditions imposed on the channel by the watershed according to the Einstein bed-load function.

b. "For the channels in the Indian plain area from which the regime theory was originally derived, f_{VR} depends on the sediment concentration of the flow while f_{RS} varies primarily with the bed material size. Using these relationships, both the Einstein bed-load function and Lacey's regime theory give the same depth and slope of an alluvial channel in equilibrium.

c. "The general application of Lacey's regime theory to other than conditions in India and Pakistan is questionable as f_{VR} then depends also on the characteristics of the flow, making its selection extremely difficult."

Schumm (1960)

117. Lack of a simple quantitative expression for the physical properties of alluvium had hindered the study of hydraulics and morphology of streams. Stanley A. Schumm (1960) pointed out that studies of the behavior of sediment grains under different physical environments revealed a great change in sediment character within the 0.05- to 1.0-mm range. Schumm (1960) discussed the effect of sediment character on the shape of alluvial streams. He pointed out "to relate any one aspect of stream morphology to one other variable . . . was an oversimplification of the problem and that the reader

should be aware of the other important factors in stream morphology" (Schumm 1960).

118. Schumm chose the No. 200 sieve (0.074 mm) as the upper limit for silt and clay. He stated

> Assuming that the shape of a stream channel depends on the resistance of sediment composing the perimeter of the channel and the erosion potential of stream discharge, then sampling of bank and channel sediment at stable channel cross sections is necessary. Data were assembled for 90 cross sections, most of which were at or near Geological Survey gaging stations. Generally, the gaging stations are located at stable reaches of the river. However, they often are located at bridges which might be assumed to affect the shape of the cross section. In such situations, wherever practical, the samples were collected some distance upstream or downstream from the bridge. Sometimes, however, the depth of water necessitated obtaining samples and water depth from the bridge by using a small clam-shell type dredge (Schumm 1960).

Generally, the channel and banks were sampled only to a depth of 1 in., with 10 and 20 sample points taken across the stream; in some cases, a 4-in.-deep sample was collected. In the laboratory, the samples were subjected to a standard grain-size analysis. The samples were first sieved, and if they contained more than 20-percent silt-clay, they were prepared for hydrometer analysis (Schumm 1960).

119. Schumm (1960) described the character of the sediment comprising the perimeter of each channel in terms of its weighted mean percent silt-clay, which he calculated as follows:

$$M = \frac{(s_c)\ (b_f) + (s_b)\ (2d_m)}{b_f + 2d_m} \tag{62}$$ *

where

M = weighted mean percent silt-clay

s_c = percentage of silt-clay found in the channel alluvium

b_f = bank-full width

s_b = percentage of silt-clay found in the bank alluvium

The shape of each cross section, expressed as a dimensionless width-depth ratio, was plotted against M. The result of this plot was the equation:

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* Schumm (1960) used S_c, W, S_b, and D instead of s_c, b_f, s_b, and d_m, respectively.

$$\frac{b_f}{d_m} = 255\ M^{-1.08} \tag{63}$$*

Neither the percentage of silt-clay in the banks nor in the channel alone shows a correlation with the width-depth ratio. Equation 63 assumes that channels containing little silt-clay are relatively wide and shallow, whereas those composed predominantly of silt-clay are relatively narrow and deep. A plot of the median grain size against percentage of silt and clay in the channel samples showed no correlation, indicating that median grain size has no relation to channel shape for the range of channels investigated. Schumm (1960) stated

> It is hazardous to attempt to extrapolate the relationships of (width-depth and weighted mean percent silt-clay) as a general law for alluvial channel shape, but it is interesting to note that Rubey's (1952) comparison of the Illinois and Mississippi Rivers reveals differences similar to those noted between the sections characterized by high M and low M in this study. . . . The reason for the relation between channel shape and percentage of silt and clay is found in studies made by hydraulic engineers. Lane (1935) states, '. . . the greater the width-depth ratio the greater will be the ratio of the velocity acting on the bottom to that acting on the sides of a channel.' Where a narrow trench is cut in an alluvial valley, the tractive forces acting on the channel sides will be great, causing widening of the channel. Widening will continue until the resistance of the banks to scour prevents it. If the material in which the channel is cut is highly cohesive (has a high percent of silt-clay), the channel will be narrow, but if the alluvium lacks cohesion (has a small percent of silt-clay), the channel will widen to a greater extent.

120. Schumm (1960) also noted that the shape of the channels seemed to be independent of the discharge, and the absolute size of the channel (i.e., width and depth) was related to mean discharge (Leopold and Maddock 1953), but the ratio of width to depth was apparently determined by sediment type for the channels Schumm sampled. When he plotted depth versus M, Schumm found a large scatter of points. Because of the lack of correlation between channel depth and M Schumm (1960) concluded that perhaps channel width is more sensitive to changes in M than to changes in depth.

121. Schumm (1960) also investigated the effect of M on downstream changes in channel dimensions. He found that if M remained constant downstream, the width and depth of the channel increased at a uniform rate

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* Schumm (1960) used F instead of b_f/d_m for the channel shape factor (width-depth ratio).

with discharge, excluding the influence of other variables, and the width-depth ratio remained constant. If M increased, downstream channel depth increased more rapidly than width (which might even decrease); therefore, the width-depth ratio decreased. If M decreased, downstream channel width increased more rapidly than depth (which might even decrease), and width-depth ratio increased.

122. This study by Schumm (1960) suggests that M represents the resistance to erosion or general behavior of sediment in a stream channel containing only small amounts of gravel. Neither the mean annual discharge nor the mean annual flood affects this relation significantly, at least for the channels that Schumm sampled.

Schumm (1968)

123. Schumm (1968) conducted a study of the Murrumbidgee River in Australia and its associated paleochannels in the Riverine Plain

> to obtain additional data that would be relevant to the hypothesis that the sediment load largely determines the shape and pattern of alluvial channels and would, in turn, support the proposed classification of alluvial channels.

Ten readily accessible representative reaches of the Murrumbidgee River were selected for study. Five of these reaches were at or near existing gaging stations. At each location, a cross section was surveyed, and bed and bank sediment samples were collected.

124. Particle-size determinations were made by seiving and by hydrometer analysis. The percentage of sediment smaller than 0.074 mm (i.e., silt-clay) in the perimeter can be expressed by Equation 62. The modern Murrumbidgee River channel has a low width-depth ratio, a low gradient, a moderately high senuosity, and a small sediment load.

125. Dury (1965) had shown that meander wavelength was a better criterion of bank-full discharge than the meander belt width. He subsequently presented the following equation:

$$\ell = 30\ Q_B^{0.5} \tag{64}$$*

* Schumm (1968) has used Q_b instead of Q_B .

where

ℓ = meander wavelength, ft

Q_B = bank-full discharge, cfs

As is usually the case with empirical equations, there is a scatter of data points. Schumm (1968) pointed out that

> Dury (1965) recognized that this scatter may reflect the influence of other variables, such as sediment type, and Hack (1965) and Alexandre (1962) suggested that the sediment transported by streams is a significant factor which can influence the dimensions of the meander pattern.

To test this hypothesis, Schumm (1968) performed a multiple regression analysis* to determine the relation of meander wavelength, bank-full discharge, and percentage of silt-clay in the channel perimeter using data from 28 US rivers, three Murrumbidgee River sections, two "prior-stream" sections, and an "ancestral river" section of the Murrumbidgee,** which resulted in the following equation:

$$\ell = 438 \frac{Q_B^{0.43}}{M^{0.47}} \tag{65}$$

126. Another multiple-regression analysis of the data collected from the rivers in the United States and the Murrumbidgee River yielded the following relation among meander wavelength, mean annual discharge, and percentage of channel silt-clay:

$$\ell = 1,890 \frac{Q_m^{0.34}}{M^{0.74}} \tag{66}$$

where Q_m = mean annual discharge, cfs. The same set of data also yielded the following relation (Schumm 1968):

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* Schumm (1968, 1969) provided statistical data for all variables that he investigated.

** Schumm (1968) stated that the Riverine Plain of the Murrumbidgee River contains traces of "old aggraded and abandoned river channels." He termed the youngest set of this as "ancestral river channels," while the older set has been referred to as "prior-stream" channels.

$$\ell = 234 \frac{Q_{ma}^{0.48}}{M^{0.74}} \tag{67}$$

where Q_{ma} = mean annual flood, cubic feet per second

127. To determine the influence of discharge and the type of sediment load on channel morphology, Schumm (1968) combined earlier data from US rivers with those collected at the Wagga Wagga, Narrahdera, and Darlington Point cross sections on the Murrumbidgee River. A multiple regression of that data yielded the following relations:

$$b_f = 37 \frac{Q_m^{0.38}}{M^{0.39}} \qquad (68)^*$$

$$b_f = 2.3 \frac{Q_{ma}^{0.58}}{M^{0.37}} \tag{69}$$

128. The same set of data was again used to determine the effect of discharge and type of sediment load on the channel depth. When these two variables were used in a multiple regression analysis, the following equation was obtained:

$$d_m = 0.59\ M^{0.34} Q_m^{0.29} \tag{70}$$

When the mean annual flood was substituted for the mean annual discharge, the following equation resulted:

$$d_m = 0.09\ M^{0.34} Q_{ma}^{0.42} \tag{71}$$

The same data showed a relation between width-depth ratio and sediment load of the form:

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* Schumm (1968) used b instead of b_f .

$$\frac{b_f}{d_m} = 106\ M^{-0.78} \qquad (72)*$$

This was just slightly different from Equation 63. The addition of the mean annual discharge and the mean annual flood resulted in the following relations:

$$\frac{b_f}{d_m} = 56\ \frac{Q_m^{0.10}}{M^{0.74}} \qquad (73)$$

$$\frac{b_f}{d_m} = 21\ \frac{Q_{ma}^{0.18}}{M^{0.74}} \qquad (74)$$

Schumm (1968) concluded that

> although the relation between width-depth ratio and M differs from that obtained previously, the general relationship that bed-load channels are wide and shallow, whereas suspended-load channels are narrow and deep, pertains.

129. Schumm (1968) then introduced the following equation:

$$S_b = 59.5\ M^{-0.38} Q_m^{-0.32} \qquad (75)**$$

If the valley slope was also used, the equation became

$$S_b = 1.3\ \frac{S_v^{0.94}}{M^{0.23} Q_m^{0.02}} \qquad (76)$$

where S_v = valley slope. Equation 76 demonstrated that on a given valley slope, a straight channel would begin to meander if the mean annual discharge of M increased (bed-load decreases). Schumm (1963) had earlier found M

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* Schumm (1968) used F instead of b_f/d_m.
** Schumm (1968) used S_c instead of S_b.

and sinuosity p (ratio of channel length to valley length) to be related as follows:

$$p = 0.94\ M^{0.25} \tag{77)*$$

However, the data in his 1968 study changed the relation to the form:

$$p = 1.05\ M^{0.18} \tag{78}$$

Based on his analysis, Schumm (1968) concluded that the morphology of a stable alluvial river channel reflects the hydrologic, climatic, and geologic characteristics of the drainage basin.

Schumm (1969)

130. One objective of Schumm's (1969) work was to demonstrate that, with time, river regulation could cause a complete metamorphosis of river morphology. Although the immediate result of dam construction is local and generally manifested by channel degradation, geologic evidence suggests that the influence of regulatory and diversion structures can cause a complete change of river channel morphology throughout the length of the river system (Schumm 1969).

131. Schumm (1969) stated that the dimensions, slope, gradient, and pattern of stable alluvial rivers could be controlled by the quantity of water and quantity and type of sediment moved through their channels. To investigate these interrelations, he collected data on the channel geometry and sediment characteristics of 36 stable alluvial rivers in the field and obtained hydrologic data from the records of nearby gaging stations. The channels he selected were located in semiarid to subhumid regions of the Great Plains in the United States and of the Riverine Plain in New South Wales, Australia. These channels had shown no progressive adjustments during the previous 10 years of record.

132. Schumm (1969) found no relation between channel morphology and the particle size of the bed and bank sediments, possibly reflecting the small range in bed-material size of the channels studied. Although data on the

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* Schumn (1968, 1969) used P instead of p.

total sediment load were available for only five cross sections, the percentage of silt and clay (particles less than 0.074 mm in diameter) in the sediments forming the perimeter of the channels was found to be inversely related to the percentage of the total sediment load that was bed-material load at mean annual discharge using the following relation:

$$M = \frac{55}{Q_t} \tag{79}$$

where Q_t = percentage of total sediment load that is bed load. Schumm (1969) stated that M could be used as an index of the ratio of bed-material load to total sediment load and that the percentage of silt and clay can be the same for rivers of greatly diverse size and discharge.

133. The width-depth ratio and the sinuosity p were found to be significantly related to the type of sediment load M as follows:

$$\frac{b_f}{d_m} = 225\ M^{-1.08} \tag{80}$$

$$p = 0.94\ M^{0.25} \tag{81}$$

The results showed that both channel shape and sinuosity are determined largely by the type of sediment load moved through the channels. Schumm (1969) stated that

> . . . with data acquired in the field and from gaging station records multiple regression equations were obtained for channel width b_f ; depth d_m ; width-depth ratio b_f/d_m ; and meander wavelength ℓ ; as a function of channel silt-clay M ; and mean annual discharge Q_m ; or mean annual flood Q_{ma} (Schumm 1969).*

After reexamining the equations he developed in 1968 (Equations 68-76), Schumm (1969) then presented the following revised equations:

$$b_f = 37\ \frac{Q_m^{0.38}}{M^{0.39}} \tag{82}$$

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* Schumm (1969) used w instead of b_f for channel width.

$$b_f = 2.3 \frac{Q_{ma}^{0.58}}{M^{0.37}} \tag{83}$$

$$d_m = 0.6\ M^{0.34} Q_m^{0.29} \tag{84}$$

$$d_m = 0.09\ M^{0.35} Q_{ma}^{0.42} \tag{85}$$

$$\frac{b_f}{d_m} = 56 \frac{Q_m^{0.10}}{M^{0.74}} \tag{86}$$

$$\frac{b_f}{d_m} = 21.4 \frac{Q_{ma}^{0.18}}{M^{0.74}} \tag{87}$$

$$\ell = 234 \frac{Q_m^{0.48}}{M^{0.74}} \tag{88}$$

$$\ell = 1,890 \frac{Q_{ma}^{0.34}}{M^{0.74}} \tag{89}$$

$$S_b = 60\ M^{-0.38} Q_m^{-0.32} \tag{90}$$

134. Schumm (1969) then pointed out

> The empirical equations presented above tell several things of general interest. The shape (width-depth ratio) and sinuosity of an alluvial channel are primarily determined by the type of sediment load or the ratio of bed-material load to total load moved through the channels, whereas channel width, depth, gradient, wavelength, and amplitude are significantly influenced by both discharge and type of sediment load. The equations demonstrate that for most changes of hydrologic regimen, which involve both a change in discharge and type of sediment load, many aspects of channel morphology will change. In addition, the reaction of a channel to a change of discharge and type of load may result in changes of channel dimensions contrary to those indicated by the

> standard regime equations. That is, it is conceivable that under certain circumstances with a decrease of discharge, depth will decrease and width will increase.

Schumm made this statement because Equations 82-90 were empirical (i.e., they yielded directions rather than magnitudes of change).

135. Equations 82-85 and 88-89 show that channel width, depth, and meander wavelength were directly related to discharge, while Equation 90 indicates that the slope is inversely proportional to discharge. From this Schumm concluded:

$$Q \sim \frac{b_f,\ d_m,\ \ell}{S} \qquad (91)^*$$

Mean annual discharge or mean annual flood can be substituted for Q .

136. Equations 82-83 and 88-90 demonstrate that channel width, meander wavelength, and slope are inversely related to the type of sediment load M , while Equations 81, 84, and 85 show a direct relationship between channel depth, sinuosity, and M .

> The percentage of silt-clay in the perimeter of a channel reflects the nature of the sediment load moving through that channel expressed as the percentage of bed-material load in total load, Q_t , but for a given channel or for channels of equal discharge, M probably can be related inversely to the quantity of bed-material load that moves through the channel; therefore, $1/Q_b$ can be substituted for M in Equations 80 through 90 if discharge is constant (Schumm 1969). Schumm then developed the following relation:

$$Q_b \sim \frac{b_f,\ \ell,\ S}{d_m,\ P} \qquad (92)^{**}$$

The width-depth ratio was not included in Equation 92, although this variable was highly dependent on M because both width and depth appear separately in the equation, nevertheless, the relation between b_f/d_m and M proved useful for interpreting changes of channel width and depth in subsequent relations.

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* Schumm (1969) used Q_w instead of Q for discharge, w instead of b_f for width, and L instead of ℓ for meander wavelength.

** Like Lane (1955), Schumm (1969) used Q_s instead of Q_b .

137. Schumm (1969) then stated the following:

To discuss in more detail the effects of changing discharge and sediment load on channel morphology, a plus or minus exponent will be used to indicate how, with an increase or decrease of discharge or bed-material load, the various aspects of channel morphology will change. For the relatively straightforward cases of an increase or decrease in discharge or bed-materia' load alone, Equations [93] through [96] are obtained.

$$Q^{+} \simeq \frac{b_f^{+} d_m^{+} \ell^{+}}{S^{-}} \tag{93}$$

$$Q^{-} \simeq \frac{b_f^{-} d_m^{-} \ell^{-}}{S^{+}} \tag{94}$$

$$Q_b^{+} \simeq \frac{b_f^{+} \ell^{+} S^{+}}{d_m^{-} P^{-}} \tag{95}$$

$$Q_b^{-} \simeq \frac{b_f^{-} \ell^{-} S^{-}}{d_m^{+} P^{+}} \tag{96}$$

An increase or decrease of discharge Q alone could be caused by diversion of water into or out of a river system. An increase of Q_b can result from increased erosion in the catchment area which can be induced by deforestation or by an increase in area under cultivation. A decrease of Q_b could result from improved land use or a program of soil conservation.

An increase or decrease of discharge changes the dimensions of the channel and its gradient, but an increase or decrease of bed-material load at constant mean annual discharge changes not only channel dimensions but also the shape of the channel (width-depth ratio) and its sinuosity.

In nature, however, it would be rare that a change in discharge or sediment load could occur alone. Generally, any change in discharge will be accompanied by a change in the type of sediment load and vice versa. This is because these two variables are dependent on other factors which influence both runoff and sediment yield.

When, however, the effects of changing water discharge and sediment load are combined, we can no longer assume that M is related to ℓ/Q_b because discharge is changing. Therefore, M can be used only as an index of the ratio of bed-material load to total sediment load or of the percentage of bed-material load Q_t in the equations that follow.

Using the plus or minus exponents to indicate an increase or decrease in a variable, four combinations of changing discharge and sediment load can be considered. For example, if both discharge and the percentage of bed-material load increase, perhaps as the result of diversion of water from a bedload channel into a suspended-load channel, Equation [97] suggests the nature of the resulting channel changes

$$Q^{+}Q_t^{+} \simeq \frac{b_f^{+}\ell^{+}\left(\frac{b_f}{d_m}\right)^{+}}{P^{-}} S^{\pm}d^{\pm} \tag{97}$$

Equation [97] indicates that, with an increase in both discharge and percentage of bed-material load, width, meander wavelength, and width-depth ratio should increase and sinuosity should decrease. The influences of increasing discharge and percentage of bed-material load on channel depth and gradient are in opposite directions, and it is not clear in what manner gradient and depth should change. However, by including width-depth ratio in Equation [97], an estimate of the *direction of* change of depth can be obtained. Width-depth ratio is predominantly influenced by type of load [Equations 80, 86, and 87] and, therefore, it increases in Equation [97]. This suggests that depth will remain constant or decrease because both width and width-depth ratios increase. Channel gradient will probably increase because sinuosity decreases, thereby straightening the channel and increasing its slope.

When both Q_t and Q decrease, as could result from dam construction, the reverse of Equation 97 pertains as follows

$$Q^{-}Q_t^{-} \simeq \frac{b^{-}\ell^{-}\left(\frac{b_f}{d_m}\right)^{-}}{P^{+}} S^{\pm}d_m^{\pm} \tag{98}$$

When as common in nature, the changes in Q and Q_t are in opposite directions, the following relations are obtained:

$$Q^{+}Q_{t}^{-} \simeq \frac{d_{m}^{+}P^{+}}{S^{-}\left(\frac{b_{f}}{d_{m}}\right)^{-}} b_{f}^{\pm}\ell^{\pm} \qquad (99)$$

$$Q^{-}Q_{t}^{+} \simeq \frac{d_{m}^{-}P^{-}}{S^{+}\left(\frac{b_{f}}{d_{m}}\right)^{+}} \left(\frac{b_{f}}{d_{m}^{\pm}\ell^{\pm}}\right) \qquad (100)$$

The situation expressed in Equation [99] could result from a combination of controls. For example, dam construction with impoundment of sediment and diversion of water into the channel from another source. The situation of Equation [97] could result from increased water use and increased land use, thereby, decreasing discharge but increasing the percentage of bed-material load (Schumm 1969).

138. Schumm (1969) stated that the above interrelations qualitatively suggest that channel metamorphosis could occur with changes of discharge and the gradation of sediment load. The magnitude of changes could be calculated using Equations 80 through 90 if the magnitudes of the changes in discharge and the percentage of bed-material load in the total sediment load are known.

Blench (1970)

139. In discussing regime theory, Thomas Blench (1970) pointed out that the most publicized equations to evolve from India had been those developed by Lacey (1930, 1934). In recent years, the Lacey equations had "undergone repeated assault and test by research institutes and by individuals" (Blench 1970). A number of modifications had been made to the original equations, but Blench (1970) believed that "the only alterations justified by fact are in interpretation and in the unmeasurable overall silt factor by resolution into components "

140. Blench (1970) emphasized that his equations were restricted to the conditions from which they were derived. He split the overall silt factor in Lacey's equations into parts that expressed the effects of bed material, cohesive sides, concentration of bed material, and effective viscosity of water. Additionally, Blench replaced width and depth by the wetted perimeter and hydraulic radius because they gave slightly better correlations and simplified the calculations.

The findings inherent in the Lacey original analysis are stated herein, along with three independent equations, which are applied to the flow of a steady water-sediment discharge in a canal (with the following constraints):

a. The Froude number in terms of a representative depth d (which may be hydraulic radius or depth from water surface to mean sand bed) will tend to acquire a value dependent on the nature of the water-sediment complex. Therefore, a practical bed factor, f_b, can be defined by

$$f_b = \frac{V_o^2}{d} \qquad (101)*$$

in which f_b depends on D, C_B, ρ_s, ρ_f. υ and factors defining the nature of the bed-load material and quantity and nature of the suspended load. In Equation [101], V_o = the mean velocity of flow; D = particle diameter; C_B = the bed-load charge defined as a multiple of weight per second per unit of weight of fluid per second; ρ_s = a mean particle mass density; ρ_f = the fluid mass density; and υ = the kinematic viscosity of fluid.

b. Cohesive erodible-depositable sides appear to act as hydraulically smooth, and equilibrium can be achieved within a range of values of a practical side factor f_s defined by

$$f_s = \frac{V_o^3}{b} \qquad (102)**$$

The upper limit is set by erodibility, and the lower by depositability from suspension. The term b = width defined by $b = A/d$ in which A = cross-sectional area of flow. This definition ensures that $Vbd = Q$ in which Q = the discharge.

c. The Manning equation should be amended to read

$$V_o = n^{-1}d^{3/4}S^{1/2} \qquad (103)\dagger$$

in which n = an absolute rugosity dependent largely on f_b and somewhat on f_s (Blench 1970).

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* Blench (1970) used F_b instead of f_b and V instead of V_o, C instead of C_B, which is expressed in parts per hundred thousand by weight.

** Blench (1970) used F_s instead of f_s.

† Blench (1970) used N instead of n.

141. From these findings, Blench derived the following set of equations:

$$b = \sqrt{\frac{f_b Q}{f_s}} \tag{104}$$

$$d = \left(\frac{f_s Q}{2}\right)^{1/3} \tag{105}$$

$$S_e = \left[\frac{f_b^{5/6} f_s^{1/12}}{KQ^{1/6}\left(\frac{1 + C_B}{233}\right)}\right] \tag{106*}$$

$$S_e = \frac{f_b^{7/8}}{Kb^{1/4} d^{1/8}\left(1 + \frac{C_B}{233}\right)} \tag{107}$$

$$S_e = \frac{f_b^{11/12}}{Kb^{1/6} Q^{1/12} \; 1 + \frac{C_B}{233}} \tag{108}$$

in which

$$K = \frac{3.63g}{\upsilon^{1/4}} \tag{109}$$

He reported that there was a strong parallel between determination of Manning's n for a rigid boundary and f_b and f_s for the noncohesive mobile sand bed and erodible cohesive sides of a canal. The evaluation of Manning's roughness coefficient and the silt factors of Lacey, although

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* Blench (1970) used S instead of S_e .

guides, are given, based largely on experience. Blench (1970) concluded that his method offered the following advantages:

a. "Equations derived from data of self-adjusting channels of the type and size for which design is required.

b. "Provisions of the three necessary and sufficient independent equations for finding equilibrium values at b, d, and S_e.

c. "Consolidation of the unmeasurables of the water-sediment complex and of the cohesive erodible-depositable sides into two overall parameters f_b and f_s that can be calculated from the running conditions of existing channels in equilibrium and used by engineers with field experience.

d. "Simplicity."

Maddock (1972)

142. Maddock (1972) introduced two equations relating regime velocity to hydraulic roughness and transport of sediment in alluvial channels that move appreciable discharges of sediment,

$$V_o = \beta \frac{\left(g^{1/2}\gamma^{1/2}qS_e\right)^{1/2}}{\left(\gamma_s'D\right)^{1/4}} \left(\frac{\rho v^2}{\gamma_s' d}\right)^{1/8} \tag{110*}$$

$$V_o S = 10^{-3} f(D) C^{3/4} \tag{111**}$$

where

β dimensionless coefficient related to a characteristic energy expenditure

g = acceleration due to gravity

γ unit weight of water, pcf

q = unit of discharge, cfs

γ_s' submerged unit weight of sediment, pcf

ρ mass density of water, mass per cubic foot

* Maddock (1972) used V instead of V_o, B instead of β, d instead of D, and ω instead of ν.

** Maddock (1972) used $\phi(\ )$ instead of $f(\ )$ and Δ_γ instead of γ_s'.

v = fall velocity of a characteristic particle size D , fps

$f(D)$ = function of the characteristic particle size D

C = concentration of the total sediment discharge, ppm

The values for $f(D)$ (Figure 5) were derived from flume experiments (Maddock 1972).

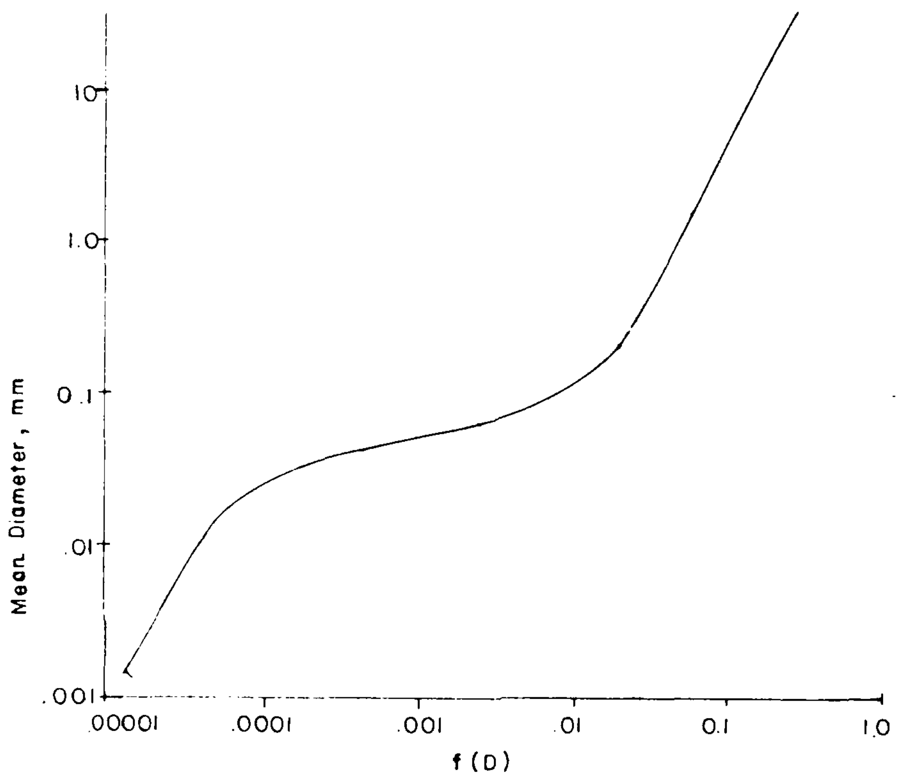

Figure 5. Relation between mean diameter of moving sediment particles and $f(D)$ in Equation 111 derived from representative flume experiments (adapted from Maddock 1972)

143. Dynamic equilibrium is usually the result of opposing tendencies. Maddock (1972) concluded that in alluvial channels the opposing tendencies are for the friction factor and the bed shear stress to be constant or for the joint variance of the friction factor and the bed shear stress to be minimized. The limit of this relation is

$$\frac{\gamma D S_e}{\rho V^2} = \frac{D_f}{\gamma D S_e} \qquad (112)*$$

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* Maddock (1972, 1976) used S instead of S_e .

where S_e = slope of the energy grade line. This relation had been common to many earlier studies of the hydraulics of alluvial channels. Maddock (1972) stated

> The relation is not unique because there is a coefficient, β, associated with a suite (or group) of bed forms having similar energy-dissipation characteristics. If velocity, depth, and slope are dependent variables, if discharge of water and sediment are independent variables, and if the bed is formed in the same way--i.e., by deposition in a channel of uniform width--then β is a constant. If the response of the dependent variables to the independent variables changes, then β is not a constant but may have a systematic variation that changes with the independent variables.
>
> Coefficient β is equal to 4.7, and an average or regime value of β is from 4 to 5.0; some obviously unstable streams have values as low as 2, and the Marala-Ravi Link Canal in Pakistan has a value of about 7. Streams having variable discharges of sediment and/or water that are associated with degradation and aggradation--as at the heads of canals and in the headwaters of drainage basins--usually have values of β of about 3.5 to 4. Conversely, the lower ends of long canals of uniform width and discharge tend to have high values of β. As the area of a drainage basin increases, the distance between major tributaries usually increases. As a result, the variation in the sediment discharge tends to decline, perturbations are dampened, and the value of β increases to about 5 or 5.5. The value of β declines slightly as the sinuosity increases. In Equation [110], the value of 4.7 represents a straight channel, self-formed by aggradation, so that it has an above average value of β.

144. Equation 110 represents a dynamic equilibrium that should not be regarded as a precise, but rather as a most probable relation that has an expected statistical variation about a mean value. Maddock (1972) reported that there was a maximum concentration of sediment for a given sediment size and for a given rate of expenditure of energy per unit mass (Equation 111). Figure 5 indicates that at least in the smaller particle diameters, mixtures having a median particle size D could have a lower value of $f(D)$ than that of equigranular material. For sediment having median particle sizes greater than about 0.5 mm, $f(D)$ is proportional to the square root of the characteristic particle diameter. Maddock (1972) pointed out that Equation 111, like Equation 110, should not be "considered deterministic. Both equations represent tendencies toward minima."

145. Maddock (1972) stated that the relations are not affected by the complexities of the discharge of the sediment and water spectrum as long as the bed is deformed and the channel is self-formed. He noted that Equation 111 was not applicable to shallow flows having low sediment

concentrations. Equations 110 and 111 can be combined to yield two other equations

$$S_e = \frac{[f(D)]^{2/3} C^{1/2} (\gamma_s' D)^{1/6} \left(\gamma_s' \dfrac{D}{\rho v^2} \right)^{1/12}}{100\, \beta^{2/3} g^{1/6} \gamma^{1/6} q^{1/3}} \tag{113}$$

$$V_o = \frac{\beta^{2/3} g^{1/6} \gamma^{1/6} q^{1/3} [f(D)]^{1/3} C^{1/4}}{10 \left(\gamma_s' D\right)^{1/6}} \left(\frac{\rho v^2}{\gamma_s' D} \right)^{1/12} \tag{114}$$

In Figure 5, $f(D) \propto \sqrt{D}$ for particle sizes greater than 0.5 mm; assuming that

$$f(D) \propto \left(\frac{\gamma_s' D}{\rho} \right)^{1/2}$$

in Equations 106 and 107

$$S_e \propto \frac{\left(\gamma_s' D\right)^{1/2} C^{1/2}}{\beta^{2/3} \gamma^{1/3} \rho^{1/6} q^{1/3}} \left(\frac{\gamma_s' D}{\rho v^2} \right)^{1/12} \tag{115}$$

and

$$V_o \propto \beta^{2/3} g^{1/3} q^{1/3} C^{1/4} \left(\frac{\rho v^2}{\gamma_s' D} \right)^{1/12} \tag{116}$$

146. One of the complications in using the fall velocity as a parameter is the effect of temperature on the fall velocity of spheres and the effect in streams. Very strong temperature effects have been observed in data from the Missouri River and lesser effects in other data. Maddock (1972) commented:

> Equation [111] is the most important equation in this paper, and therefore, its limitation must be understood--that is, there must be a prescribed amount of sediment moving on the bed before there can be a maximum amount of sediment transported for a given

rate of energy expenditure per unit of mass. This is a basic, yet unrecognized, limitation of the regime theory and is also a limitation on the application of the equations presented herein.

Blench (1969) had stated that (VS) is constant for a given particle size and sediment concentration; he was not concerned with the relation among (VS), size of sediment, and concentration. Inglis (1967) did not emphasize the (VS) relations in any way, but a combination of his equations yielded

$$(VS) \propto D^{1/2}C^{1/2}v^{1/2}$$

Lacey (1946) had introduced (VS) as a parameter, in which $(VS) \sim D$, but did not recognize the importance of sediment concentration or the variability of (VS) with different particle sizes and temperatures.

147. If β is held constant in Equation 114, then the equation is a regime equation. In an earlier paper, Maddock (1969) stated that for the equations of Blench (1969), Einstein and Barbarossa (1952), Engelund (1966), Inglis (1968), and Lacey (1946), this would also hold true; however, only Inglis (1968) had used the same form in his equation as that used in Equation 114 (Maddock 1972).

148. Equation 114 was derived from Equations 110 and 111 and if β is constant, Equation 114 can be expressed in the form

$$V_o = \frac{\beta g^{1/4}\gamma^{1/4}d^{1/2}\ [f(D)]^{1/2}C^{3/8}}{31.6\left(\gamma_s'D\right)^{1/4}}\left(\frac{\rho v^2}{\gamma_s'D}\right)^{1/8} \tag{117}$$

Equation 117 confirms Blench's statement that the Froude number in terms of a representative depth remains the same for all channels having the same concentrations and particle-size distribution (Maddock 1972). Maddock then restated Equation 113, in the following form:

$$b_t = \frac{10^6\beta^2 g^{1/2}\gamma^{1/2}}{[f(D)]^2C^{3/2}\left(\gamma_s'D\right)^{1/2}}\left(\frac{\rho v^2}{\gamma_s'D}\right)^{1/4} QS^3 \tag{118*}$$

---

* Maddock (1972) used b instead of b_t.

Commenting on Equation 118, he stated

> The equation is descriptive of the behavior of a channel at a given point. The fact that width is proportional to the cube of the slope, if other factors are constant, was shown by Maddock in studies of the Rio Grande near Vinton, TX. The equation describes the instability of a channel, as represented by a change in β, when a change in discharge is not accompanied by a change in sediment concentration or size.
>
> Note that if the slope remains constant, in order for β to remain constant, the width must be proportional to Q unless there is a compensating change in the size or concentration of the sediment. Streams have a tendency to widen because the sediment concentration frequently fails to increase sufficiently during floods. After a flood, β becomes larger for the normal discharge in the widened channel having a preflood slope and concentration. This generally leads to an unstable channel that tends toward aggradation or bar building. Therefore, in most instances the reduction of width is an effective procedure in channel rectification. Using Equation [111], it can be determined that normal flow in the rectified channel will have the same velocity as that in the wide channel regardless of the increase in depth. The narrow channel is subject to attack--too low a value of β --at high discharges and, therefore, must be protected (Maddock 1972).

149. Maddock (1972) found that the greatest effect of a change in sediment size occurred in the coarse silt and fine sand sizes. A change in particle size from 0.05 to 0.10 mm decreases the sediment concentration about 16 times for the same value of the velocity-slope product (see Figure 5). In contrast, a change in particle size from 5 to 10 mm decreases the sediment concentration about 1.6 times for the same value of the velocity-slope product. Secondly, he reached some unexpected conclusions regarding the effect of temperature on Equations 110 and 111.

> In Equation [110], the only changing variable is v, which is affected by temperature. Therefore, the presumption would be that all the variations in v are taken up by β and S_e. In the actual experiment, however, a temperature reduction from 86° F to 35° F increased velocity about 9 percent and slope 54 percent. Assuming that the fall velocity of the median grain size is proportional to a sphere of the same size, the fall velocities for a 0.16-mm-diam grain (the size of the sediment in the flume) are about 2 cm per second and 1.03 cm per second at temperatures of 86° F and 35° F, respectively, or a difference of about two times. Introducing the observed variations into Equation [110], the change in the coefficient, β is 1.05.
>
> The small change in the coefficient, β, is not unexpected because the slope, in a sense, is unconstrained in its formation. However, in Equation [111], where V_0 is constant, S_e is related to $f(D)$ and the sediment concentration. The unknown is which change controls the adjustment--a change in β or a change

in $f(D)$. In this experiment, most of the change was in $f(D)$ (Maddock 1972).

150. From the temperature effects studies on the Missouri River at Omaha (US Army Engineer Division, Missouri River 1969), Maddock (1972) observed

> If during falling temperatures, the observed increase in sediment concentration exceeds the increase in sediment concentration required to maintain a given velocity, then there is an increase in velocity of the stream. If the observed increase is less than the required increase, the velocity falls, and the bed-form factor ß declines.

Maddock (1972) then concluded

> One of the reasons that little progress has been made in understanding the behavior of alluvial channels is that many commonly accepted assumptions are in error, such as:
>
> a. The amount of sediment of a specific size that is transported is directly related to the amount of that sediment present in the bed.
>
> b. For every unit discharge of water on a given slope, there is some fixed discharge of sediment of a given size.
>
> c. The way an experiment is conducted in a flume has no influence on the results.
>
> d. A recirculating flume is equivalent to a natural channel.
>
> e. Specific sequences of bed forms follow only specific changes in discharge.
>
> f. The channel friction factor is deterministic and can be evaluated by the equivalent of a Moody diagram, in which only velocity, depth, slope, and sediment size are parameters--that is, the value of any one of the parameters may be determined from the other three.
>
> g. It is necessary to divide the roughness into grain and form roughness.
>
> The reason for the above assumptions is that stream behavior is controlled by constraints, either natural or artificial, on the response of dependent variables to changes in independent variables. As long as the constraints are consistent, the responses are consistent. Thus, the response under the constraint of constant depth or constant slope is different from the response when depth and slope are free to vary.

Simons et al. (1975)

151. Simons et al. (1975) stated that understanding river system behavior depends upon the competent knowledge of:

a. Geologic factors, including soil conditions.

b. Hydrologic factors, including possible changes in streamflows, runoff, and the hydrologic effects of changes in land use.

c. Geometric characteristics of the stream, including the probable geometric alterations that will be activated by the changes that development will impose on the channel.

d. Hydraulic characteristics such as depth, slope, and velocity of streams and the changes that may be expected in these characteristics in space and time.

e. Ecological/biological changes that will result from physical change and in turn will induce or modify physical changes.

They listed seven major factors affecting alluvial stream channel forms:

a. Stream discharge.

b. Sediment load.

c. Longitudinal slope.

d. Bank and bed resistance to flow.

e. Vegetation.

f. Geology, including types of sediments.

g. Works of man.

152. Although it "exercises its greatest influence on river form, character, and resistance," bed load is very difficult to estimate (Simons et al. 1975). According to Simons et al. (1975), three major variables that affect the amount of bed load a stream can carry are:

a. Size of the bed material.

b. Slope of the stream or average velocity.

c. Nature of the channel (depth, size, shape, and roughness of bed and banks).

As an aid in evaluating the effect of these variables on bed load, Lane and Borland (1951) had provided the following criteria:

a. Smaller concentrations of suspended material usually imply higher percentages of bed load.

b. The ratio of bed load to suspended load is usually larger for low or medium stages than for high stages.

c. Streams with wide shallow channels carry a higher proportion of sediment as bed load than do streams with deep narrow channels.

d. Streams with a high degree of turbulence tend to have smaller amounts of bed load.

e. The nature of the source of sediments influences the magnitude of the bed-load correction; that is, the occurrence of large quantities of coarse material in the watershed, so located that they can be moved easily into the channel, is indicative of higher percentages of bed load.

153. Simons et al. (1975) defined "graded streams" (see paragraph 2) as "those that have slopes, such that their energy is just sufficient to transport the material through the system that is delivered to the streams." They stated that ". . . this concept can only be applied as an average condition extending over a period of years" (Simons et al. 1975).

154. Lane (1957), who investigated the interrelation of slope, discharge, and channel pattern in meandering and braided streams, showed that when

$$S_b Q^{1/4} \leq 0.0017$$

for sand bed channels,* they will tend toward a meandering pattern. Similarly, when

$$S_b Q^{1/4} \geq 0.01$$

the stream tends toward a braided pattern. The region between these two values can be considered a transitional range where streams are classified as intermediate. This intermediate category describes many of the rivers in the United States (Simons et al. 1975).

155. Lane (1955) had studied the changes in river morphology in response to varying discharge and sediment load. Additionally, Leopold and Maddock (1953), Schumm (1971), and Santos-Cayudo and Simons (1972) investigated channel response to natural and imposed changes. These studies support the following general relations:

a. Depth of flow is directly proportional to discharge and inversely proportional to sediment load.

b. Channel width is directly proportional to both discharge and sediment load.

c. Channel shape, expressed as a width-to-depth ratio, is directly related to sediment load.

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* Simons et al. (1975) used S instead of S_b .

d. Channel slope is inversely proportional to discharge and directly related to both sediment load and particle size D_{50} .

e. Sinuosity is directly proportional to valley slope and inversely proportional to sediment load.

f. Transport of bed material is directly related to stream power and concentration of fine material and inversely related to the fall diameter of the bed material D_{50} (Simons et al. 1975).

From this, evolved Lane's qualitative relation

$$QS_b \propto Q_b D_{50}$$

The above relations were used to develop Table 1, in which Simons et al. (1975) qualitatively described the response of alluvial channels to changes in a number of variables.*

Maddock (1976)

156. According to Maddock (1976), unique relations governing the interactions of pertinent variables describing the flow of water in alluvial channels were difficult (if not impossible) to state. He did, however, recognize that "values of certain combinations of the variables are constrained within limits that are defined by whether the variables are dependent or independent. Furthermore, variables that are independent in some situations are dependent in others" (Maddock 1976). Therefore, in the short-term field situation, slope, discharge, suspended-sediment load, and width are independent variables, while mean velocity and mean depth are dependent variables. However, in a long-term field situation, discharge and suspended-sediment load are independent variables, whereas slope, width, mean velocity, and mean depth are dependent variables (Maddock 1976).

157. Maddock (1976) stated that the pertinent variables in an alluvial stream flowing with uniform depth and velocity are discharge, sediment concentration, depth, mean velocity, and slope of the energy grade line (usually assumed to be the same as the slope of the water surface), the kinematic viscosity of the fluid, the mass density of the fluid, the mass density of the

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* Simons et al. (1975) used Q_s instead of Q_b .

Table 1
Qualitative Response of Alluvial Channels*

Variable	Change in Magnitude of Variable	Effect on: Regime of Flow	River Form**	Resistance to Flow	Energy Slope	Stability of Channel	Area	Stage
Discharge	+	+	M → B	±	-	-	+	+
	-	-	B → M	∓	+	+	-	-
Bed-material size	+	-	M → B	+	+	±	+	+
	-	+	B → M	-	-	±	-	-
Bed-material load	+	+	B → M	-	-	+	-	-
	-	-	M → B	+	+	-	+	+
Wash load	+	+		-	-	±	-	-
	-	-		+	+	±	+	+
Viscosity	+	+		-	-	±	-	-
	-	-		+	+	±	+	+
Seepage force	Outflow	-	B → M	+	-	+	+	+
	Inflow	+	M → B	-	+	-	-	-
Vegetation	+	-	B → M	+	-	+	+	+
	-	+	M → B	-	+	-	-	-
Wind	Downstream	+	M → B	-	+	-	-	-
	Upstream	-	B → M	+	-	-	+	+

* Adapted from Simons et al. (1975).
** M = meandering, B = braided.

sediment particles, the median size of the sediment particles, the standard deviation of the sediment sizes, the characteristic fall velocity of the particles, and the acceleration due to gravity.

158. Maddock (1976) realized the effect of bed forms were very significant:

> There are any number of observations on natural streams that, slope being constant, a range of velocities may be observed for the same mean depth. These different velocities are accompanied by different bed forms and different sediment concentrations.

He introduced a resistance equation:

$$\beta^2 = \frac{V_o}{(gdS_e)^{1/2}} \frac{\left[(W_b - 1)D\right]^{1/2}}{(dS_e)^{1/2}} \left[\frac{(W_b - 1)gD}{v^2}\right]^{1/4} \qquad (119)*$$

where W_b = weight of bed sediment. He stated that ". . . the near constancy of β under unconstrained, or self-formed, conditions is the only reason why presently used equations have any engineering application" (Maddock 1976). In addition, Maddock (1976) pointed out

> while β is essentially constant for a self-formed channel, it may vary between channels. For example, in natural channels or canals the movement of sediment is not uniform. Where perturbations exist, the value of β for a bank-full discharge tends to be low. However, if there are long distances between tributaries of major streams or in a long canal of uniform section, these perturbations tend to be dampened and the value of β is higher if conditions are propitious for the formation of a flatter slope

159. At a specific location on a natural stream, β is a variable depending on the relative discharge of water and sediment. Under these conditions, high values of β are associated with channel aggradation and low values are associated with channel degradation and scour. Values of V_o, d, and S_e can be used to compute β and predict the potential degree of channel instability. Analysis of experimental data led to the following concentration equation (Maddock 1976):

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* Maddock (1976) used V instead of V_o, D instead of d, s instead of W_b, d instead of D, ω instead of v.

$$C = \left\{ 10^3 \left[\frac{V_o S_e}{f(D)} - \frac{K(W_b - 1) g^{1/2} D}{f(D) d^{1/2}} \right] \left[\frac{(W_b - 1) gD}{v^2} \right]^{1/4} \right\}^{4/3} \qquad (120)$$

Maddock (1976) plotted several sets of sample data to demonstrate the excellent correlation of Equation 120. He commented that the relation between $f(D)$ and D was

> not that of particles falling in still water but represents the summation of what goes in the stream, and there is no reason to suppose a still water fall velocity should not be modified by the effect of turbulence, particle interference, sorting and even variation in velocity distributions in the fluid. Like particle fall velocities, $f(D)$ responds to changes in temperature (Maddock 1976).

160. Maddock (1976) stated that

> K is a dimensionless parameter that has a normal value of about 60. However, when the mean values of V_o or d are used in an equation, certain velocity distributions are implicit. As long as there are a large number of particles moving over the bed or within the fluid, a statistically characteristic velocity distribution may be achieved, and K is unimportant. When the number of moving particles declines, either from a decrease in concentration or from an increase in size at the same concentration, the moving sediment follows those locations of high velocity and consequently, sediment moves at relatively lower mean velocities. The effect of this is to reduce K to as little as 30. The skewed velocity distribution may be caused by a considerable portion of the bed being covered by stationary sediment particles, in which case grain roughness controls. However, such a velocity distribution can also result from bed configuration. Therefore with few sediment particles in movement, an accurate knowledge of the velocity distribution is necessary to determine the rate of sediment transport. Evaluation of K is dependent on estimates of fall velocity and sediment size which are hard to make.

Consequently, although a K value of 60 had been used in Maddock's paper, values as low as 50 should not be surprising. He arrived at two conclusions (Maddock 1976):

a. Every sediment transport equation that has been based on field or laboratory observations has an area of application.

b. The sorting of transported sediment is an important method of adjustment to changes in independent variables.

Hey (1978)

161. Researchers have attempted "to predict the hydraulic geometry of stable alluvial channels" (Hey 1978). The various approaches have led to many different equations, and, therefore, different solutions to stable channel design. Some researchers have suggested that the problem is indeterminate despite the observed regularity of channel shapes and patterns. Richard David Hey examined the reasons for the diversity of opinion and presented a framework that

> will enable a general deterministic model to be established. This model, based on an understanding of the processes that operate in alluvial channels, indicates that it is possible to predict the bank-full hydraulic geometry of unstable as well as stable channels (Hey 1978).

162. The early empirical work by Kennedy (1895) and Lacey (1930, 1934, 1946) resulted in a set of equations that were applicable only to the area for which they were derived. Henderson (1963) and Simons and Albertson (1963) later proved that bank sediment size is an important independent variable that should have been included in those equations.

163. Hey (1978) remarked that "the uniqueness of these empirical regime equations led American engineers to develop a more rigorous semitheoretical procedure for channel design process" (e.g., Lane 1955). This approach has been referred to as tractive force design because "every particle lining (the channel) perimeter is at or below the threshold of motion" (Hey 1978). In such "fixed bed models," there is no provision for bed load.

164. Both engineers and geologists later recognized that a self-adjusting channel has 5 degrees of freedom because it can adjust its overall width, depth, velocity, slope, and sinuosity in response to erosion and deposition (Hey 1974, Kellerhals 1967); however, Hey (1978) stated that "these five variables are insufficient to uniquely define the hydraulic geometry of alluvial channels." He provided three reasons to support his case:

a. Width and average depth are not sufficient to describe cross-sectional geometry. Hey (1978) stated that wetted perimeter, hydraulic radius, and maximum flow depth d provide "a unique definition of cross-sectional shape" and, significantly, they have a greater relevance than width and depth.

b. Bed forms must be taken into account and their size and shape must be predicted to obtain a determinant solution (Hey 1978, Yalin 1965).

c. Sinuosity, or the ratio of channel length to valley length, "does not uniquely define the plan geometry" (Hey 1978). Many patterns are possible for a given value of sinuosity. Plan geometry can, however, be defined "provided arc length (i.e., the channel distance between successive points) is also specified" (Hey 1978).

165. Hey (1978) next described and derived the following process equations:

a. Continuity equation

$$V_o = f(Q, R, P) \tag{121}*$$

b. Sediment transport equation

$$S = f(P, R, d_{ma}, Q_{si}, D_b, \sigma_b^2, D_r, \sigma_r^2, D_l, \sigma_l^2) \tag{122}**$$

where

d_{ma} = maximum flow depth

Q_{si} = sediment discharge (input)

D_b = mean bed sediment diameter

σ_b^2 = standard deviation of bed sediment diameter

D_r = mean right-bank sediment diameter

σ_r^2 = standard deviation of right-bank sediment diameter

D_l = mean left-bank sediment diameter

σ_l^2 = standard deviation of left-bank sediment diameter

c. Bed-form equations

$$\lambda_a = f(R, P, d_{ma}, S_b, D_b, \sigma_b^2, D_r, \sigma_r^2, D_l, \sigma_l^2, Q_{si}) \tag{123}$$

$$\Delta_a = f(R, P, d_{ma}, S_b, D_b, \sigma_b^2, D_r, \sigma_r^2, D_l, \sigma_l^2, Q_{si}) \tag{124}$$

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* Hey (1978) used V_a instead of V_o, R_a instead of R, and P_a instead of P.

** Hey (1978) used σ instead of σ^2 for all three of his standard deviations.

where

λ_a = dune wavelength

Δ_a = dune height

d. Bank competence equations

$$P = f(R_f,\ d_m,\ S_b,\ D_1,\ \sigma_1^2,\ D_b,\ \sigma_b^2,\ \lambda,\ \Delta,\ Q_{si}) \qquad (125)^*$$

$$d_m = f(P_f,\ R_f,\ S,\ D_r,\ \sigma_r^2,\ D_b,\ \sigma_b^2,\ \lambda,\ \Delta,\ Q_{si}) \qquad (126)$$

where

R_f = bank-full hydraulic radius

d_m = maximum bank-full flow depth

λ = bank-full dune wavelength

Δ = bank-full dune height

P_f = bank-full wetted perimeter

e. Meander equations

$$p = f(S_b,\ S_v) \qquad (127)$$

$$z = f(P_f,\ R_f,\ d_m) \qquad (128)$$

$$R = f(P,\ d_{ma},\ S_b,\ V_o,\ D_b,\ \sigma_b^2,\ D_r,\ \sigma_r^2,\ D_1,\ \sigma_1^2,\ Q_{si},\ \lambda_a,\ \Delta_a,\ p,\ z) \qquad (129)$$

where z = meander arc wavelength.

166. To define the bank-full hydraulic geometry of alluvial channels, Equations 121-129 must be solved simultaneously to resolve "the interlinkage between variables" and to establish "the necessary dependent and independent variables to give a unique solution" (Hey 1978). The following relations describe this interlinkage:

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* Hey (1978) used S instead of S_b and D_b instead of d_m.

$$V = f(Q, Q_{si}, D_b, \sigma_b^2, D_r, \sigma_r^2, D_l, \sigma_l^2)$$

$$R_f = f(Q, Q_{si}, D_b, \sigma_b^2, D_r, \sigma_r^2, D_l, \sigma_l^2)$$

$$S_b = f(Q, Q_{si}, D_b, \sigma_b^2, D_r, \sigma_r^2, D_l, \sigma_l^2)$$

$$\lambda = f(Q, Q_{si}, D_b, \sigma_b^2, D_r, \sigma_r^2, D_l, \sigma_l^2)$$

$$\Delta = f(Q, Q_{si}, D_b, \sigma_b^2, D_r, \sigma_r^2, D_l, \sigma_l^2)$$

$$P_f = f(Q, Q_{si}, D_b, \sigma_b^2, D_r, \sigma_r^2, D_l, \sigma_l^2)$$

$$d_m = f(Q, Q_{si}, D_b, \sigma_b^2, D_r, \sigma_r^2, D_l, \sigma_l^2)$$

$$p = f(Q, Q_{si}, D_b, \sigma_b^2, D_r, \sigma_r^2, D_l, \sigma_l^2, S_v)$$

$$z = f(Q, Q_{si}, D_b, \sigma_b^2, D_r, \sigma_r^2, D_l, \sigma_l^2)$$

167. In discussing the process (Equations 121-129) and response relations (paragraph 166), Hey (1978) stated that because the process equations define the interaction between variables at a section and the response of the system to the imposed conditions, the response relations have been developed for unstable as well as stable channels. When used for design purposes, an iterative solution would have to be adopted to ensure channel stability. After the channel had been designed, the sediment output from the section would be calculated and compared with the input value. Any imbalance between the two would be corrected by altering the bank sediment and repeating the process until the two values were in equilibrium. Although bed sediment size could also be adjusted to achieve this purpose, this would be unwise because it must be compatible with the size of the input load. Provided that discharge and sediment input are invariant, as in most artificial channels, this simple model can be used for design purposes.

168. Commenting further on the response relations, and citing his earlier work, Hey (1978) continued

> Although these equations (i.e., relations) can be used to design a stable channel, they cannot be used to predict channel response consequent upon a change in flow regime or sediment transport rates. Under these conditions upstream and downstream feedback mechanisms will operate and all the variables will adjust as the channel develops towards a new equilibrium. For example, a change in flow regime, as a result of urbanization or river regulation, will affect the sediment transport characteristics of the river and the size of the bed sediment. As the channel adjusts its capacity both upstream and downstream the flood regime will change and this will modify the flow variability (Hey 1975a). By widening the perspective to incorporate upstream and downstream responses, casuality can no longer be identified and all the variables are interdependent. Prediction of channel response in these circumstances is not only dependent on detailed understanding of the process equations but also the rates and mode of operation of the feedback mechanisms. Given this information, it will then be

possible to simulate the spatial and temporal response of the channel to a given stimulus (Hey 1975b, 1976).

169. Hey (1978) believed that there were two possible methods of developing general response equations to define the bank-full hydraulic geometry of stable and unstable channels

> The ideal method depends on establishing precise mathematical equations for each adjustment process and then simultaneously solving them to obtain the response equations. Unfortunately, our lack of knowledge regarding these processes precludes this particular approach. Alternatively, they (response equations) can be established by submitting field data to multiple regression analysis using the dependent and independent variables enumerated in [paragraph 166].*

Data from a large variety of rivers would have to be used to ensure the resulting equations have general application (Hey 1978).

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* Paragraph numbers enclosed in brackets within quoted material refer to this report.

PART III: SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

Kennedy (1895)

170. Kennedy has been credited as being the first person to develop equations for describing canals in regime. By fitting a straight line through a logarithmic plot of depth of flow versus mean velocity, Kennedy (1895) developed the following relation (Equation 2):

$$V_o = cd^m \qquad (2\ \text{bis})$$

where

V_o = mean velocity

c = 0.84

d = average depth, ft

m = 0.64

He also defined V_o as the critical velocity or the velocity at which silting is just prevented (for the given depth). Due to the consistency of Kennedy's data, he originally stated that bed width had no justification for being correlated with mean velocity and depth of flow. This meant that either a deep narrow channel or a wide, shallow channel could be designed to carry the same discharge. Kennedy (1904) later reassessed his research and developed a "rough-rule" relation for width-to-depth ratio (Graf 1971, Lindley 1919).

Lindley (1919)

171. Lindley (1919) retested the early regime theories developed by Kennedy (1895) and Woods (1917). From hydraulic surveys made between 1915 and 1917 of the Lower Chenab Canal, Lindley analyzed widths between berms, depths, and gradients, averaged over 1-mile reaches. The average width-depth relation could be expressed as (Equation 3):

$$b_b = 3.8\ d^{1.61} \qquad (3\ \text{bis})$$

where b_b = bed width developed from surface width under the assumption that the side slopes are 1V:2H . This relation was valid for up to 9 ft of depth, with no indication of a change beyond that point. Velocities calculated using Kutter's relation, with $N = 0.225$, were found to follow the exponential law (Equation 4):

$$V_o = 0.95\ d^{0.57} \tag{4 bis}$$

rather than Kennedy's (1895) Equation 2. By combining the two expressions relating width and mean velocity to depth, Lindley (1919) obtained the following relations between width and mean velocity (Equations 5 and 6):

for Kutter velocities,

$$V_o = 0.59\ b_b^{0.35} \tag{5 bis}$$

for Kennedy velocities,

$$V_o = 0.50\ b_b^{0.40} \tag{6 bis}$$

He concluded that if two channels carry the same discharge, the one with the greater "silt charge" (sediment concentration) must be wider and shallower and have steeper banks. Lindley (1919) had advanced the quantitative analysis of regime channels by defining the relation between velocity and width, which added to the already established relation of velocity and depth.

<u>Lacey (1930)</u>

172. Lacey (1930), suggesting that mean velocity is a function of width, depth, and a "silt factor" f, defined the silt factor as (Equation 7):

$$f = \left(\frac{V_o}{V_{ok}}\right)^2 \tag{7 bis}$$

where V_o/V_{ok} = ratio of the regime velocity actually obtained with a given depth to the velocity for the same depth that Kennedy's (1895) formula (Equation 2) would indicate (Lacey 1930). From the above relation, Lacey developed his first general equation (Equation 8):

$$V_o = 1.17 \sqrt{fR} \qquad \text{(8 bis)}$$

where R = hydraulic radius . To test the relation of cross-section area, silt factor, and velocity, Lacey plotted Af^2 against V_o . The results of this plot and that from Equation 7 led to Equation 9, which relates wetted perimeter to discharge:

$$P = 2.668\ Q^{0.5} \qquad \text{(9 bis)}$$

where

P = wetted perimeter

Q = discharge, cfs

This equation shows that, for a given discharge, the wetted perimeter is constant and independent of the particle size transported, with particle size merely controlling the channel shape. Lacey also attempted to relate the silt factor to Manning's n or Kutter's N and to sediment size. The general formulas developed by Lacey have one restriction. For velocities less than 0.882 fps, the resultant wetted perimeter could not contain the cross-sectional areas. This restriction can be considered either a limitation of the formulas or, as Lacey (1930) concluded, an argument that this velocity must be one of physical constants of water because Equations 7, 8, and 9 do not take sediment size into consideration.

Lane (1935)

173. Lane (1935) classified the formulas previously developed for stable channels in India and Egypt into two types:

a. Those giving an expression of velocity.

b. Those giving stable channel shapes.

The existing regime equations had been developed empirically, and Lane contended that the equations omitted important factors that restricted their use to the channels and conditions from which they were developed. Lane (1935) contended that the velocity distribution, mean velocity, and quantity of material in motion were of primary importance. He concluded that the capacity of a stream to transport suspended material is proportional to its turbulence and that channels carrying bed load should have high ratios of bed width to depth. He suggested that analysis of the fundamental interrelations of a number of factors was needed to understand the mechanisms that control the shape of streams in alluvial material:

a. Hydraulic parameters (bed slope, roughness, hydraulic radius, depth, mean velocity, velocity distribution, and temperature).

b. Shape of the existing channel (width, depth, and steepness of side slopes).

c. Nature of material transported through the channel (size, shape, specific gravity, dispersion, and quantity, as well as bank and subgrade material).

d. Other factors (channel alignment, uniformity of flow, and "aging") (Lane 1935).

Lacey (1936)

174. Lacey (1936), building on Lane's (1935) research, correlated Lane's factors as dimensioned variables using the Buckingham π theory. This method consisted of determining dimensionless arguments from all the known independent variables and then correlating these arguments. A poor degree of correlation would indicate that important variables had been overlooked. From this type of analysis, Lacey (1936) developed the following relation (Equation 17):

$$\frac{P}{R} \propto \left(\frac{RV_o}{\upsilon}\right)^{1/3} \left(\frac{V_o^2}{gR}\right)^{1/3} \qquad \text{(17 bis)}$$

where

υ = kinematic viscosity of water

g = acceleration due to gravity

This is similar to Lacey's fundamental shape formula (Equation 18):

$$\frac{P}{R} = 7.12\ V_o \qquad \text{(18 bis)}$$

Lacey (1936) concluded that a final solution to the problem of sediment transport would be found by investigations using dimensional analysis.

Shulits (1936)

175. Shulits (1936) stated that the extent of knowledge of fluvial morphology had generally been limited to qualitative reasoning and some site-specific empirical equations. He discussed the few quantitative principles that had been developed theoretically. As particles move downstream, they are reduced in size by abrasion and collision. Based on Sternberg's abrasion law and the fact that the slope of a river varies directly with a decrease in bed-material size, Shulits developed an exponential equation (Equation 19) to approximate the slope of a river:

$$W_b = W_{bo} e^{-c' \phi x} \qquad \text{(19 bis)}$$

where

W_b = weight of bed sediment

W_{bo} = mean weight of bed sediment

e = base of natural logarithm

c' = constant of proportionality

ϕ = coefficient of friction

x = distance traveled downstream

After further investigation on several European rivers, Shulits (1936) concluded with reasonable certainty that the slope of a river could be expressed by Equation 21:

$$Z_o - Z = K_1(1 - e^{-Kx}) \qquad \text{(21 bis)}$$

where

Z_o = mean elevation

Z = elevation of any point on the profile above a given datum

K_1, K = constants

He modified the Schoklitsch bed-load formula and combined it with the abrasion and slope formulas to obtain Equation 22:

$$G = \left[\frac{KQ}{e^{(4/3)\alpha x}}\right] - \left(\frac{K_1 b}{e^{\alpha x/3}}\right) \qquad (22\ \text{bis})$$

where

G = bed load, lb/sec

α = coefficient of abrasion

Therefore, if the discharge and the width terms increase equally downstream, there would be no change in bed load. However, if the discharge term increases more rapidly than the width term, there will be an increase in bed load and a change in longitudinal profile, probably degradation, and an increase in slope. Should the width term increase more rapidly, there would be a decrease in bed load and a corresponding change in channel profile (Shulits 1936).

Lacey (1946)

176. Lacey (1946) attempted to advance the theory of silt transported by "stable" channels in alluvium. When testing the earlier regime relations using hydraulic data, he discovered two curious anomalies:

a. Despite the great improbability of natural alluvial channels ever attaining regime, alluvial river data and boulder torrent data (i.e., data from steeply graded streams) confirmed the regime equations in many respects.

b. Although much of the data that had been collected over a period of years from stable channels in the Punjab displayed general agreement, certain observations from the channels thought to be stable proved to be "discordant" (i.e., conflicting) for no readily ascertainable reason.

By applying the principles of corresponding speeds and dynamic similarity and by introducing the terminal velocities of silt particles into his physical theory, Lacey (1946) succeeded in regrouping the fundamental variables of velocity, depth, and water surface slope to obtain two new parameters from which he derived Equation 25, a "normal equation":

$$(VS^{*}) = 1.6\left(d^{1/2}S^{*}\right)^{4/3} \qquad (25\ \text{bis})$$

where (VS^{*}) = product of velocity and slope. Lacey stated that the new equation applies to all active channels in alluvium and is irrespective of the precise degree of scouring or silting, or variation in silt charge. All of these variables are implicit in the actual values of V , d , and S adopted by the channel when associated with the gradation of the bed material, which must be sampled and measured at the same time as the other hydraulic elements. In addition, Lacey pointed out that, for every stable channel transporting "regime silt charge," there is, for a fixed diameter D of the bed silt particles, a fixed value of the product (VS^*) . This led to Lacey's development of Equation 26:

$$D = 0.73(VS^*) \tag{26 bis}$$

where D = diameter, millimetres, and Equation 27 (for D , inches):

$$D = 0.03(VS^*) \tag{27 bis}$$

Lacey (1936) concluded that the silt charge (sediment concentration) was clearly of importance.

Inglis (1947,1948)

177. Inglis, like those who preceded him, stated that alluvial channels have a never-ceasing tendency to attain equilibrium, a condition in which the load is carried with the minimum expenditure of energy for the existing conditions. Small adjustments to attain equilibrium are brought about by changes in the material exposed on the bed. Such changes could occur either by deposition "where there is an excess charge" (or sediment concentration), or by scour "where the charge is in defect" (i.e., where concentration is low) (Inglis 1947). Inglis (1947) suggested that the nearest approach to equilibrium is a year-to-year consistency of similar discharge conditions, which he called an "annual stability." He developed a set of equations to describe river equilibrium and then compared them with the equations developed by Lacey. In Lacey's formulas, V varies from P/R only in the coefficient. In Inglis' equations, the concentration has only a slight effect on V but has a very great effect on b/d and, hence, P/R ; concentration also has a marked effect on slope so that where concentration is ignored, wide divergences on slope are found, even in stable channels. For the purpose of simplification, Inglis (1947) assumed that there was a dominant discharge and an associated concentration and gradient to which a channel returns annually. Inglis concluded that "for any discharge less than the dominant discharge, the slope will be 'in defect' (i.e., insufficient) to handle the dominant

concentration, so that material will be deposited, and, as a result, there will be subdominant charge (or lower concentration) of material in movement." On the other hand, when the discharge exceeds the dominant concentration, the excess energy is utilized in increasing the concentration by scouring the bed, eroding the banks, and developing meanders (Inglis 1947).

Mackin (1948)

178. Mackin (1948) defined a graded (or stable) river as one which, over a period of years, delicately readjusts its slope "to provide, with available discharge and the prevailing channel characteristics, just the velocity required for transportation of all of the load supplied from above." He pointed out that because the discharge, channel characteristics, and load do not vary systematically along the stream, the graded profile is not a simple mathematical curve. Mackin (1948) also believed that a graded stream responds to change in accordance with LeChatelier's general law:

> If a stress is brought to bear on a system in equilibrium, a reaction occurs, displacing the equilibrium in a direction that tends to absorb the effect of the stress.

179. Mackin (1948) stated that although the study of streams as agents of transportation can be approached either in terms of relationship between slope, discharge, channel form, and the size of grains comprising the load, or in terms of energy transformations, the two approaches should not be combined. From work by Gilbert (1877) and Rubey (1933), Mackin (1948) emphasized that the share of total energy utilized in the transportation of load could be as small as 3 percent. Because the energy dissipated in internal and external friction is overwhelmingly greater than that consumed in transportation, the total energy determined by slope and discharge remaining the same, relatively slight changes in the channel characteristics cause very marked changes in the transporting power. Many researchers had developed equations to describe the conditions of a graded stream, but Mackin believed that this approach was inadequate because equations are transposable. As set up in an equation, load is a function of velocity. However, over a period of years, the load supplied to a stream is actually dependent on the lithology, relief, vegetative cover and erosional processes in operation in its drainage basin and in the graded stream. The most efficient cross-sectional shape for transport varies with the slope, the amount and gradation of load, the resistance of the banks to erosion, and especially with the proportions of the total load carried in suspension and moved along the bed (Mackin 1948). Mackin classified changes

in regime into changes in downvalley controls and changes in upvalley controls. In general, a change in upvalley controls (load and discharge) results in a change in slope, whereas a change in downvalley controls (base level) will cause a change in the level of the longitudinal profile, but not necessarily in the slope. Mackin developed many general statements but avoided manipulation of the data into equation form.

Blench (1951)

180. Blench (1951) stated that a river is "in regime" if its mean measurable behavior during a certain time interval does not differ significantly from its mean measurable behavior during comparable times before or after the given interval. He pointed out that regime theory originated on the great canal systems of India and Pakistan where discharges are controlled and meanders are prevented; however, this theory is not generally applicable to a "regime-type river" (Blench 1951). Blench found the exponents of Lacey's (1930) regime formula (Equation 9) to be consistent with the laws of rigid boundary hydraulics. He reasoned that as width goes to infinity, depth approaches the hydraulic radius and width approaches the wetted perimeter. Because most rivers are wide, Blench replaced wetted perimeter with width b and hydraulic radius R with depth d, and leaving Lacey's exponents the same, arrived at Equation 42, which defines a bed factor f_b related to the nature of the bed load:

$$\frac{V_o^2}{d} = f_b \qquad (42\ \text{bis})$$

Blench's (1951) second regime equation (Equation 43) defines a side factor f_s that measures the square of the tractive force intensity over the sides.

$$\frac{V_o^3}{b} = f_s \qquad (43\ \text{bis})$$

Blench (1951) developed Equation 43 while attempting to determine the reason for the universal agreement of the exponent 0.5 in Lacey's (1930) formula (Equation 10):

$$b_t = 2.67\, Q^{0.5} \qquad (10\ \text{bis})$$

where b_t = top width. However, he contended that the coefficient 2.67 should be replaced by a variable dependent upon the nature of the bed sediment. He concluded that the Lacey formula represents an averaging of the relative importance of bed to sides as measured by the ratio f_b/f_s meaning that all Lacey channels were regime channels but that not all regime channels are Lacey channels. Equation 45, developed by Blench, became his third regime equation, thus completing the theory:

$$\frac{V_o^2}{gdS_c} = C' \frac{V_o b^{1/4}}{\upsilon} \qquad (45\ \text{bis})$$

where

S_c = channel slope expressed as a fraction

C' = a nondimensional constant

Blench emphasized that when applying regime formulas derived on canals to rivers, an allowance for "disturbing effects" (i.e., turbulence) and variable discharges must be made "in terms of engineering common sense and the formulas regarded with suspicion until supported by results" (Blench 1951).

Leopold and Maddock (1953)

181. Geomorphology in the past has been classically treated almost exclusively in a qualitative manner. Leopold and Maddock (1953) believed that the qualitative approach had been constructive; however, they proposed that a quantitative analysis should be attempted. First, they investigated changes in velocity, depth, and top width for an increase in discharge at a particular cross section of a river. Leopold and Maddock (1953) found that if width, depth, and velocity were plotted on logarithmic paper, their relations to discharge could be approximated by straight lines and, therefore, described by three simple power functions (Equations 50, 51, and 52):

$$b = a_1 Q^{m_1} \qquad (50\ \text{bis})$$

$$d = a_2 Q^{m_2} \tag{51 bis}$$

$$V_o = a_3 Q^{m_3} \tag{52 bis}$$

where m_1 , m_2 , m_3 , a_1 , a_2 , a_3 are numerical constants. When the width, depth, and velocity were plotted against the mean annual discharge at various cross sections along the river, the result was again simple power functions, but with different coefficients. Leopold and Maddock (1953) also studied the effect of suspended sediment on the hydraulic geometry. Plots of the relation of suspended-sediment load to discharge were characterized by a larger scatter of points. For long periods of record, the relation can be approximated by a simple power function (Equation 54) (Leopold and Maddock 1953):

$$Q_s = aQ^j \tag{54 bis}$$

where

Q_s = suspended-sediment load, tons per day

a, j = numerical constants

182. They concluded that, at a given cross section, the suspended-sediment load should increase more rapidly than discharge (Leopold and Maddock 1953). Consideration of the physical characteristics of surface runoff and observations of bed scour led to the tentative conclusion that the observed increase in sediment concentration resulted primarily from erosion of the watershed rather than from scour of the bed of the main stem in the reach where the measurement was made. For each station, width was plotted against suspended-sediment load on a semilogarithmic graph, and at each point the corresponding velocity was entered. Isopleths of velocity and depth were then drawn. This graph showed that there was an increased capacity for suspended-sediment load at constant discharge by decreasing width at a constant velocity or by increasing velocity at constant width. Some authorities had contended that despite a large suspended load in a river, only the bed load was of real significance in fluvial morphology; they assumed that it placed the greatest tax on the energy of the stream. However, Leopold and Maddock (1953) believed that too much emphasis had been placed on bed load and that the role of

suspended load in determining the observed characteristics of natural streams had been underrated. They postulated that changes in channel shape occur in response to a change in suspended-sediment load brought into the reach. However, adequate stream-gaging data were not available in a sufficiently close geographic network, or over a long enough period of record, to allow differentiation between graded and nongraded reaches by hydraulic geometry alone, though such differentiation could theoretically be possible.

183. The effect of suspended-sediment concentration on channel roughness has been discussed by many researchers, most of whom concluded that an increase in concentration resulted in lower values of Manning's n. Leopold and Maddock (1953) found a change in velocity-depth relations that coincided with a change in suspended load-discharge relations at a discharge well below the peak on the Colorado and San Juan Rivers. Such local adjustments in velocity-depth relations result primarily from changes in roughness associated with changes in sediment load, rather than from changes in slope of the water surface. On the basis of their work, Leopold and Maddock (1953) agreed with Mackin (1948), stating that the wording in the definition of regime should be changed from a state "in which slope is delicately adjusted" to one in which *slope and all other channel characteristics are* delicately adjusted (Leopold and Maddock 1953).

Lane (1955)

184. Lane (1955) was a proponent of the energy concept; that is, erosion of the bed occurs when a stream is capable of carrying more sediment than is supplied to it. This scour increases the slope of the tributary streams, which allow them to introduce more material to the main stem. He stated that a stream is in equilibrium if, "although it may continually fluctuate between aggradation and degradation over a long period of years in terms of human history, the net amount of change is not sufficiently large to be detected by quantitative measurements" (Lane 1955). Lane believed that most alluvial streams not affected by the works of man were in equilibrium for most practical engineering purposes. He also contended that if equilibrium existed in a reach and one of the conditions were changed, then, over time, the conditions of equilibrium would change throughout the entire reach. Because, in most cases, data are insufficient to make a quantitative analysis of equilibrium change, Lane developed an expression relating bed-material load, particle diameter, discharge, and slope of the stream (Equation 55):

$$Q_b D \approx Q S_b \quad (55\ \text{bis})$$

where

Q_b = bed-material load

D = particle diameter

S_b = bed slope

Chien (1955)

185. The formulas proposed by Lacey (1946) involved factors based essentially on experience alone, making use of the theory difficult for those who lacked experience. Chien (1955) decided to evaluate those factors quantitatively in terms of the physical conditions imposed on the channel and to show the possible limitations of Lacey's (1946) regime theory. Chien thought that Lacey's omission of sediment load as an explicit variable could be explained by either of the following two possibilities:

> a. "The silt factors . . . actually include implicitly the sediment load together with the bed-material size.
>
> b. "Although the dimensions and the slope of an alluvial channel must depend on the sediment inflow, a certain combination of these variables may depend very little on the sediment load" (Chien 1955).

Chien's study showed that, according to the Einstein bed-load function, the silt factors in Lacey's regime theory are the result of the conditions imposed on the channel by the watershed. In addition, he found that for conditions in India and Pakistan, the silt factor f_{VR} depends on the sediment concentration, while the silt factor f_{RS} varies primarily with the bed-material size. However, for other conditions f_{VR} also depends on the hydraulic characteristics of the channel. Therefore, Chien (1955) concluded that general application of Lacey's regime theory is questionable.

Schumm (1960)

186. Schumm (1960) analyzed the effect of sediment character on the shape of alluvial streams. He pointed out ". . . to relate any one aspect of stream morphology to one other variable . . ." was an oversimplification of the problem and the reader should be aware of the other important factors in stream morphology. Schumm (1960) described the sediment comprising the perimeter of each channel in terms of its weighted mean percent silt-clay, as calculated from Equation 62:

$$M = \frac{(s_c)(b_f) + (s_b)(2d_m)}{b_f + 2d_m} \qquad (62\ \text{bis})$$

where

M = weighted mean percent silt-clay

s_c = percentage of silt-clay found in the channel alluvium

b_f = bank-full width

s_b = percentage of silt-clay found in the bank alluvium

The shape of each cross section was expressed as a width-depth ratio and plotted against M. The result was the relation expressed in Equation 63.

$$\frac{b_f}{d_m} = 255\ M^{-1.08} \qquad (63\ \text{bis})$$

This relation showed that channels containing little silt-clay are relatively wide and shallow, whereas those composed predominantly of silt-clay are relatively narrow and deep. He noted that the shape of the channels seemed to be independent of the discharge, although the size of the channel was related to mean discharge. This study of Schumm (1960) suggests that M represents the resistance to erosion or general behavior of sediment in a stream channel containing only small amounts of gravel. A lack of correlation between channel depth and M led Schumm to conclude that perhaps channel width is more sensitive to changes in M than to changes in depth.

Schumm (1968)

187. Schumm (1968) conducted a study of the Murrumbidgee River in Australia and its associated paleochannels in the Riverine Plain. The percentage of sediment smaller than 0.74 in the perimeter of a channel was expressed by Equation 62. Schumm (1968) used multiple regression analysis to obtain equations for the following relations, whose terms are transposable:

a. Bank-full discharge, percentage of silt clay in channel perimeter, and meander wavelength (Equation 65):

$$\ell = 438\ \frac{Q_B^{0.43}}{M^{0.47}} \qquad (65\ \text{bis})$$

where

ℓ = meander wavelength, ft

Q_b = bank-full discharge, cfs

b. Mean annual discharge, mean annual flood, silt-clay percentage, and meander wavelength (Equations 66 and 67):

$$\ell = 1{,}890 \frac{Q_m^{0.34}}{M^{0.74}} \quad (66\ \text{bis})$$

$$\ell = 234 \frac{Q_{ma}^{0.48}}{M^{0.74}} \quad (67\ \text{bis})$$

where

Q_m = mean annual discharge, cfs

Q_{ma} = mean annual flood, cfs

c. Discharge, silt-clay percentage, and channel width (Equations 68 and 69):

$$b_f = 37 \frac{Q_m^{0.38}}{M^{0.39}} \quad (68\ \text{bis})$$

$$b_f = 2.3 \frac{Q_{ma}^{0.58}}{M^{0.37}} \quad (69\ \text{bis})$$

d. Discharge, silt-clay percentage, and channel depth (Equations 70 and 71):

$$d_m = 0.59\ M^{0.34} Q_m^{0.29} \quad (70\ \text{bis})$$

$$d_m = 0.09\ M^{0.34} Q_{ma}^{0.42} \quad (71\ \text{bis})$$

e. Silt-clay percentage and width-depth ratio (Equation 72):

$$\frac{b_f}{d_m} = 106\ M^{-0.78} \qquad (72\ \text{bis})$$

f. Discharge, silt-clay percentage, and width-depth ratio (Equations 73 and 74):

$$\frac{b_f}{d_m} = 56\ \frac{Q_m^{0.10}}{M^{0.74}} \qquad (73\ \text{bis})$$

$$\frac{b_f}{d_m} = 21\ \frac{Q_{ma}^{0.18}}{M^{0.74}} \qquad (74\ \text{bis})$$

g. Discharge, channel slope, and silt-clay percentage (Equation 75):

$$S_b = 59.5\ M^{-0.38} Q_m^{-0.32} \qquad (75\ \text{bis})$$

where S_b = bed slope

h. Discharge, channel slope, valley slope, and silt-clay percentage (Equation 76):

$$S_b = 1.3\ \frac{S_v^{0.94}}{M^{0.23} Q_m^{0.02}} \qquad (76\ \text{bis})$$

where S_v = valley slope

i. Sinuosity and silt-clay percentage (Equation 78):

$$p = 1.05\ M^{0.18} \qquad (78\ \text{bis})$$

where p = sinuosity.

Based on his analysis, Schumm (1968) concluded that the morphology of a stable alluvial river channel reflects the hydrologic, climatic, and geologic characteristic of the drainage basin.

Schumm (1969)

188. Schumm (1969) sought to demonstrate that, with time, river regulation could cause a complete metamorphosis of river morphology. He found no relation between channel morphology and the size of the bed and bank sediments, possibly reflecting the small range in bed-material size of the channels studied. The width-depth ratio and the sinuosity were found to be significantly related to M (Equations 80 and 81).

$$\frac{b_f}{d_m} = 225\, M^{-1.08} \tag{80 bis}$$

$$p = 0.94\, M^{0.25} \tag{81 bis}$$

He revised his own equations, stating that channel width, depth, and meander wavelength are directly related to discharge (Equations 82-85 and 88-89, herein):

$$b_f = 37\, \frac{Q_m^{0.38}}{M^{0.39}} \tag{82 bis}$$

$$b_f = 2.3\, \frac{Q_{ma}^{0.58}}{M^{0.37}} \tag{83 bis}$$

$$d_m = 0.6\, M^{0.34} Q_m^{0.29} \tag{84 bis}$$

$$d_m = 0.09\, M^{0.35} Q_{ma}^{0.42} \tag{85 bis}$$

$$\ell = 234\, \frac{Q_m^{0.48}}{M^{0.74}} \tag{88 bis}$$

$$\ell = 1,890 \frac{Q_{ma}^{0.34}}{M^{0.74}} \qquad (89\ bis)$$

Equation 90, on the other hand, indicates that the slope is inversely proportional to discharge:

$$S_b = 60\ M^{-0.38} Q_m^{-0.32} \qquad (90\ bis)$$

In addition, Equations 82-83 and 88-90 demonstrate that channel width, meander wavelength, and slope are inversely related to M, while Equations 81, 84, and 85 show a direct relation between channel depth, sinuosity, and M. From these ideas suggested by the empirical equations, Schumm developed 10 proportions to qualitatively describe river morphology. These proportions related discharge, bed-material load, total load, width, depth, meander wavelength, slope, width-depth ratio, and sinuosity. Schumm (1969) stated that his interrelations qualitatively suggest that channel metamorphosis could occur with changes of discharge and type of sediment.

Blench (1970)

189. Blench (1970) split the overall silt factor in Lacey's equations into parts that expressed the effects of bed material, cohesive sides, concentration of bed material, and effective viscosity of water. Blench also replaced width and depth by the wetted perimeter and hydraulic radius because they gave slightly better correlations and simplified the calculations. From his findings, Blench developed a set of equations relating width, Froude number, discharge, slope, acceleration due to gravity, and kinematic viscosity of water. He emphasized that the equations were restricted to the conditions from which they were derived. Blench (1970) concluded that there was a strong parallel between determination of Mannings' n for a rigid boundary and f_b and f_s for the noncohesive mobile sand bed and erodible cohesive sides of a canal. Like Manning's n, f_b and f_s are evaluated on the basis of experience.

Maddock (1972)

190. Maddock (1972) introduced two equations relating regime velocity to hydraulic roughness and to transport of sediment in alluvial channels that move appreciable discharges of sediment (Equations 110 and 111):

$$V_o = \beta \frac{\left(g^{1/2}\gamma^{1/2}qS_e\right)^{1/2}}{(\gamma_s{}'D)^{1/4}}\left(\frac{\rho v^2}{\gamma_s{}'d}\right)^{1/8} \tag{110 bis}$$

$$V_oS = 10^{-3}f(D)C^{3/4} \tag{111 bis}$$

where

β = dimensionless coefficient related to a characteristic energy expenditure

γ = unit weight of water

q = unit discharge

γ_s' = submerged unit weight of sediment, pcf

ρ = mass density of water

v = fall velocity of a characteristic particle size D

$f(D)$ = function of the characteristic particle size D

C = concentration of the total sediment discharge, ppm

191. Maddock (1972) stated neither equation can be considered deterministic, but rather, they represent tendencies toward minima. He noted that Equation 111 was not applicable to shallow flows having low sediment concentrations. Maddock (1972) stated that Equation 111 was the most important equation in his paper but that its limitations should be understood. There must be a prescribed amount of sediment moving on the bed before there can be a maximum amount of sediment transported for a given rate of energy expenditure per unit mass; Maddock considered this to be a basic, yet unrecognized, limitation of the regime theory. In Equation 110, if β is a constant, then the equation is a regime equation and includes the work of Einstein-Barbarossa, Lacey, Inglis, Blench, and Engelund (Maddock 1969).

192. Maddock (1972) combined Equations 110 and 111 to yield two other equations (Equations 113 and 114):

$$S_e = \frac{[f(D)]^{2/3}C^{1/2}\left(\gamma_s{}'D\right)^{1/6}\left(\dfrac{\gamma_s{}'D}{\rho v^2}\right)^{1/12}}{100\ \beta^{2/3}g^{1/6}\gamma^{1/6}q^{1/3}} \tag{113 bis}$$

$$V_o = \frac{\beta^{2/3} g^{1/6} \gamma^{1/6} q^{1/3} [f(D)]^{1/3} C^{1/4}}{10 \left(\gamma_s{}' D\right)^{1/6}} \left(\frac{\rho v^2}{\gamma_s{}' D}\right)^{1/12} \qquad \text{(114 bis)}$$

If β is held constant in Equation 114 and the equation is rewritten as (Equation 117):

$$V_o = \frac{\beta g^{1/4} \gamma^{1/4} d^{1/2} [f(D)]^{1/2} C^{3/8}}{31.6 \left(\gamma_s{}' D\right)^{1/4}} \left(\frac{\rho v^2}{\gamma_s{}' D}\right)^{1/8} \qquad \text{(117 bis)}$$

Equation 117 confirms Blench's statement that the Froude number in terms of a representative depth remains the same for all channels having the same concentration and particle-size distribution (Maddock 1972).

193. Maddock (1972) found that the greatest effect of a change in the size of sediment occurred in the coarse silt and fine sand size and that temperature affected his equations to a degree larger than expected (Maddock 1972).

Simons et al. (1975)

194. Simons et al. (1975) identified seven major factors affecting alluvial stream channel forms:

a. Stream discharge.

b. Sediment load.

c. Longitudinal slope.

d. Bank and bed resistance to flow.

e. Vegetation.

f. Geology.

g. Works of man.

Although bed load "exercises its greatest influence on river form, character, and resistance," it is very difficult to estimate. The authors provide the following statements (originally developed by Lane and Borland (1951)) about bed load:

a. Smaller concentrations of suspended material usually imply higher percentages of bed load.

b. The ratio of bed load to suspended load is usually larger for low or medium stages than for high stages.

c. Streams with wide, shallow channels carry a higher proportion of sediment as bed load than do streams with deep, narrow channels.

d. Streams with a high degree of turbulence tend to have smaller amounts of bed load.

e. The nature of the source of sediments influences the magnitude of bed-load correction.

Simons et al. (1975) concluded that the studies by Lane (1955), Leopold and Maddock (1953), Schumm (1971), and Santos-Cayudo and Simons (1972) supported the following general relations:

a. Depth of flow is directly proportional to discharge and inversely proportional to sediment load.

b. Channel width is directly proportional to both discharge and sediment discharge.

c. Channel shape, expressed as a width-to-depth ratio, is directly related to sediment discharge.

d. Channel slope is inversely proportional to discharge and directly proportional to both sediment discharge and grain size D_{50}.

e. Sinuosity is directly proportional to valley slope and inversely proportional to sediment load.

f. Transport of bed material is directly related to stream power and concentration of fine material and inversely related to the fall diameter of the bed material D_{50}.

From these concepts, Simons et al. (1975) developed a qualitative analysis of the variables affecting alluvial rivers to aid in the assessment of reaction to changes.

Maddock (1976)

195. Maddock (1976) emphasized that unique relations governing the interactions of pertinent variables describing the flow of water in alluvial channels were difficult (if not impossible) to state. His realization of the important influence of bed forms led him to introduce Equation 119, a resistance equation defining β:

$$\beta^2 = \frac{V_c}{(gdS_e)^{1/2}} \frac{\left[(W_b - 1)D\right]^{1/2}}{(dS_e)^{1/2}} \left[\frac{(W_b - 1)gD}{v^2}\right]^{1/4} \qquad (119\ \text{bis})$$

He stated that ". . . the near constancy of β under unconstrained or self-formed conditions is the only reason why presently used equations have any engineering application" (Maddock 1976). At a specific location on a natural stream, β is a variable depending on the relative discharge of water and sediment. Under these conditions, high values of β are associated with channel aggradation, and low values are associated with channel degradation and scour. Values of V_o, d, and S can be used to compute ß and predict the potential degree of channel instability. After Maddock (1976) analyzed his experimental data, he developed Equation 120:

$$C = \left\{ 10^3 \left[\frac{V_o S_e}{f(D)} - \frac{K(W_b - 1) g^{1/2} D}{f(D) d^{1/2}} \right] \left[\frac{(W_b - 1) g D}{v^2} \right]^{1/4} \right\}^{4/3} \qquad \text{(120 bis)}$$

This equation shows an excellent correlation when plotted with several sets of data. From his research, Maddock (1976) arrived at two conclusions:

a. Every sediment transport equation based on field or laboratory observation has an area of application.

b. The sorting of transported sediment is an important method of adjustment to changes in independent variables.

Hey (1978)

196. Prior to 1978, a number of researchers attempted "to predict the geometry of stable alluvial channels" (Hey 1978). The various approaches led to many different equations and different solutions to stable channel design. Some researchers suggested that the problem is indeterminate despite the observed regularity of channel shapes and patterns. Hey (1978) pointed out that uniqueness of the Indian empirical regime equations resulted in American engineers' developing a semitheoretical procedure for channel design. However, the American theory required the bed to be on the threshold of motion; therefore, no bed load is present. Contrary to previous theories, Hey contended that a river possessed nine degrees of freedom; he, therefore, derived a set of nine process equations and nine response relations (which show the interrelation between a number of variables).

197. To define the bank-full hydraulic geometry of alluvial channels, the process equations must be solved simultaneously to resolve "the interlinkage between variables and establish the necessary dependent and independent variables to give a unique solution" (Hey 1978). Because the process equations define the interaction between variables at a section and the response of the system to the imposed conditions, the response relations have been developed for unstable as well as stable channels. These equations and relations can be used to design stable channels; however, because of upstream and

downstream feedback mechanisms, they cannot be used to predict channel response dependent upon a change in flow regime or sediment transport rates. Hey (1978) believed that there were two possible methods of developing general response equations to define the bank-full hydraulic geometry of stable and unstable channels. The ideal method would be to establish precise mathematical equations for each adjustment process and then solve them simultaneously. However, at the current state of knowledge, this is nearly impossible. Therefore, multiple regression analysis of field data from a large variety of rivers is a practical solution to developing general equations (Hey 1978).

Conclusions

198. A number of researchers who have attempted to quantitatively describe regime channels have found that the lack of data, variation in discharge, and heterogeneous nature of the banks make analysis of large alluvial rivers difficult. Therefore, most studies have resulted in empirical equations derived from a limited amount of data obtained on small streams or canals where the variables affecting regime characteristics are simplified. As early as 1935, Lane made a statement that demonstrated the limitation of empirical equations in describing regime:

> Although these formulas, no doubt, provide workable relations for the conditions for which they were developed, these conditions have not been delineated sufficiently to enable them to be applied elsewhere. In general, also, they were developed empirically from a very limited range of conditions, and in most cases they omit important factors from consideration (Lane 1935).

Later, Blench (1951) cautioned the engineer about applying regime formulas for conditions that vary from those for which they were derived. When applying regime formulas derived on canals to rivers, an allowance for disturbing effects and variable discharges must be made "in terms of engineering common sense and the formulas regarded with suspicion until supported by results" (Blench 1951).

199. Due to the limited use of empirical equations, several researchers, including Lane (1935, 1955, 1957); Mackin (1948); Schumm (1960, 1963, 1968); and Simons et al. (1975) developed qualitative equations; however, equations are transposable, meaning that an equation relating sediment load and velocity can be restated to show that load is dependent on velocity. The sediment load actually is a function of the lithology, relief, vegetative

cover, and erosional processes, so that, over a period of years, the velocity is adjusted to carry the load supplied to the stream. This relation is not transposable. The qualitative relations suggest a direction of response of the river due to a change in one or more of the variables. These suggested responses should be treated only as guides.

200. The studies presented in Part II show that researchers cannot agree on the exact nature of interrelations between the variables affecting the regime of alluvial rivers; however, most of them do agree that the following variables are involved and that they are interrelated:

a. Geology--sources of erodible materials.

b. Hydrology--including precipitation, streamflow, and runoff.

c. Geometry--including channel cross-sectional shape, alignment, sinuosity, bed slope, and aging.

d. Hydraulics--including water and energy slopes, depth of flow, roughness, velocity distribution, and temperature.

e. Nature of material transported--including gradation, mineralogy, shape, specific gravity, and quantity of bank and bed material available.

201. In addition, changes in land use and man's activities (e.g., construction) alter the natural rate of runoff and soil loss, which, in turn, affects the regime of alluvial rivers. Even though investigators agree that changes in suspended-sediment load influence the regime of alluvial rivers, the exact role that this parameter plays remains to be determined. More research is needed to study this influence as well as the interrelations of suspended-sediment loads and the other variables affecting the regime of alluvial rivers.

Recommendations

202. There are no generally accepted quantitative relations describing the response of river regime to changes in sediment load. The future development of quantitative relations will most probably be an approach suggested by Hey (1978):

> The ideal method depends on establishing precise mathematical equations for each adjustment process and then simultaneously solving them to obtain the response equations. Unfortunately, our lack of knowledge regarding these processes precludes this particular approach. Alternately, they can be established by submitting field data to multiple regression analysis. . . .

At present, discharge, sediment, and channel geometry data are widely available for many streams, including the Mississippi River (Keown, Dardeau, and Causey 1981). In addition, improvements for flood control, navigation, water supply, erosion control, and river training are well documented. Hydraulic and meteorologic data are now routinely collected and transmitted via geostationary satellites to ground stations for rapid processing. Further, Landsat 4 provides continuous sun-synchronous coverage of the Earth every 16 days. Band 7 of Landsat 4's Multispectral Scanner, which is sensitive to radiation in the near-infrared portion of the electromagnetic (EM) spectrum (0.8 to 1.1 μm), is useful for mapping water bodies and delineating land-water interfaces. Landsat 4 also has a seven-band Thematic Mapper (TM). Band 1 of the spacecraft's TM, covering part of the visible EM spectrum (0.45 to 0.52 μm), is designed for water body penetration, making it useful for estimating suspended-sediment surface concentrations. The French satellite SPOT also has potential for sediment studies in either the multispectral or panchromatic mode. In the multispectral mode, the reflected infrared band (0.79 to 0.89 μm) is most useful for mapping land-water interfaces, while the green band (0.50 to 0.59 μm) shows the most promise for water body penetration.

203. Because all of these information sources are potentially adaptable to stream regime analysis and because automated processing techniques can provide the means to handle the large quantities of available data, the following process is recommended:

- a. A multiple regression approach in conjunction with a qualitative analysis be used to assess the interrelations of the identified parameters that influence stream regime.
- b. The results of this sensitivity analysis, in turn, be used to evaluate the impact of changing sediment loads on a basin's regime or on the character of a specific stream.

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APPENDIX A: NOTATION

a, a_1, a_2, a_3	Coefficients
A, A'	Cross-sectional area
b	Width of a rectangular channel having depth d, ft
b_b	Bed width developed from water surface width under the assumption that side slopes are 1V:2H
b_f	Bank-full width
b_t	Top width
c	Coefficient
c'	Constant of proportionality
C	Concentration or charge, ppm
C'	Constant
C_B	Bed-load concentration (or charge)
d, d', d''	Average depths of flow, ft
d_c	Constant depth
d_m	Maximum bank-full flow depth
d_{ma}	Maximum flow depth
D	Particle diameter
D_b	Mean diameter of bed material, mm
D_e	Effective diameter of bed material, mm
D_l	Mean left-bank sediment diameter
D_r	Mean right-bank sediment diameter
e	Base of natural logarithm
f_b	Bed factor
f_s	Side factor
f, f_{RS}, f_{VR}	Silt factors
$f(\ )$	Function of ( )
F	Froude number

g	Acceleration due to gravity
G	Bed load, lb/sec
H	Total energy available after precipitation (= elevation above mean sea level)
H_b	Energy dissipated as heat through friction of flow
H_e	Energy expended in erosion of rocks, soil, etc.
H_n	Energy extracted by artificial means (e.g., turbines)
H_s	Energy used to transport sediment and debris loads
H_t	Energy lost by turbulence (e.g., rapids, stream expansions, contractions, and structures)
i	Energy gradient
j	Integer
K, K_1	Constants
ℓ	Meander wavelength
L, L', L''	Lengths of flow or longitude scalar distance, ft
m, m_1, m_2, m_3, m'	Constants
M	Percent silt-clay fraction in perimeter of channel
n	Manning's n
N	Kutter's rugosity coefficient
p	Sinuosity
P, P'	Wetted perimeter
P_f	Bank-full wetted perimeter
q	Discharge per unit width
q_T	Total sediment transport rate per unit width
Q	Discharge, cfs
Q_a	Annual discharge
Q_b	Bed-material load
Q_m	Mean annual discharge, cfs

Q_{ma}	Mean annual flood, cfs
Q_s	Suspended-sediment load
Q_{si}	Sediment discharge (input)
Q_t	Percentage of total sediment load that is bed load
Q_β	Bank-full discharge, cfs
R	Hydraulic radius
R_f	Bank-full hydraulic radius
S, S', S''	Slopes
s_b	Percentage of silt-clay in bank alluvium
s_c	Percentage of silt-clay in channel alluvium
S_b	Bed slope
S_c	Channel slope expressed as a fraction
S_e	Slope of the energy grade line
S_o	Slope at the head of the reach
S_v	Valley slope
S_w	Slope of the water surface
$S*$	Slope, parts per thousand
T, T', T''	Scalar times
v, v', v''	Terminal velocities
V, V', V''	Velocities, fps
(VS)	Lacey's (1946) product of velocity times slope
V_o	Mean velocity or regime velocity, fps
V_o/V_{ok}	Ratio of the regime velocity actually obtained with a given depth to the velocity for the same depth that Kennedy's formula (Equation 2) would predict
W_b	Weight of bed sediment
W_{bo}	Mean weight of bed sediment
x	Distance traveled downstream

X	Integer
z	Meander arc wavelength
Z	Elevation of any point on the profile above a given datum
Z_σ	Mean elevation
α	Coefficient of abrasion
α_1, α_3, α_5, α_7	Coefficients
β	Dimensionless coefficient related to a characteristic energy expenditure
γ	Weight of water
γ_s'	Submerged unit weight of sediment, pounds per cubic bolt
Δ	Bank-full dune height
Δ_a	Dune height
λ	Bank-full dune wavelength
λ_a	Dune wavelength
υ	Kinematic viscosity of water
ρ	Density of water
ρ_f	Fluid mass density
ρ_s	Mean particle mass density
σ	Constant
σ_b^2	Standard deviation of bed sediment diameter
σ_l^2	Standard deviation of left-bank sediment diameter
σ_r^2	Standard deviation of right-bank sediment diameter
ϕ	Coefficient of friction