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The Dimensions of Power as Illustrated in a Steady-State Model of Conflict

Jack Hirshleifer

July 1989

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**The Dimensions of Power as Illustrated in a  
Steady-State Model of Conflict**

**Jack Hirshleifer**

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**Prepared for  
The Pew Charitable Trusts**

**RAND**

### PREFACE

This Note contains a theoretical analysis of the determinants of power, defined as the ability to achieve one's goals in the presence of rivals. The analysis deals with contending decisionmakers who have mixed incentives: potential mutual gains from cooperation exist, but on the other hand each side may have an opportunity to profit from conflictual efforts aimed at capturing a larger share of that common gain. The most important human associations are all characterized by such mixed cooperative-conflictual incentives. Among the many examples are international relations, the clash of factions within alliances and committees, struggles between capital and labor, and even the contests for advantage that take place within the family. The Note should therefore be of interest to a wide range of researchers in political science, economics, and sociology.

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## SUMMARY

Power is the ability to achieve one's ends in the presence of rivals. Thus the struggle for power applies not only among nations but in industry, among animals, and even within families. I assume unitary rational decisionmakers interested solely in maximizing income. To this end, each contender strikes an optimal balance between *productive* activity and *conflictual* activity. In a steady-state model, the contenders are engaging in a continuing interaction having elements of both "war" and "peace."

The factors determining power can be grouped under three headings:

*Capabilities*: resources, and the utilization efficiencies for production or for conflict.

*Payoff functions*: the equations translating productive efforts into income and conflictual efforts into distributive shares thereof.

*Protocol*: the "rules of the game" determining whether, for example, the Cournot or Stackelberg or other solution concept is applicable.

It might be thought that in a power struggle the stronger side will grow ever stronger, leading to total subjection of the initially weaker opponent. On the contrary, in a linked-productivity model as assumed here, *the weaker side in terms of resources tends to have a comparative advantage in conflictual activity*. This leads to a Power Equalization Principle (PEP): antagonists achieve equal incomes despite initially unequal resource endowments (strict form of the PEP), or at any rate the distribution of income is less unequal than the distribution of resources (mild form of the PEP). Whenever the Cournot or the Stackelberg solution concept is applicable, either the strict or the mild form of the PEP will hold. However, where a contingent-threat or "hierarchical" solution concept is applicable, resource disparities amplify the power advantage of hierarchical position.

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## I. INTRODUCTION

Power, for the purposes of this analysis, is the ability to achieve one's ends in the presence of rivals. I will assume that all decisionmakers are rational and interested solely in maximizing income. To this end, each contender strikes an optimal balance between *productive activity* (aimed at generating income through cooperation with other parties) versus *conflictual activity* (aimed at appropriating the income produced by others, or else defending against opponents' efforts to do the same). For simplicity here, the analysis is limited to two-sided interactions.<sup>1</sup>

War and peace, or more generally conflict and settlement, are usually regarded as mutually exclusive. Rival nations are said to be at war or else at peace. And similarly for the ordinarily nonviolent struggles sometimes likened to war—strikes and lockouts, lawsuits, political campaigns, etc. The question usually posed is, Why do we sometimes observe conflict and sometimes cooperation? In contrast, in the paradigm employed here the antagonists do not “go to war” or “make peace” but instead arrive at a steady-state equilibrium typically involving elements of both struggle and accommodation.<sup>2</sup> In this context the question is, What determines *the absolute and relative levels of income achieved*? This is the problem of power.

Two broad, almost self-evident generalizations apply to mixed interactions involving elements of both conflict and cooperation:

- (1) The resources that the two sides devote to productive activity mainly determine the aggregate social income available.
- (2) The relative magnitudes of the contenders' commitments to conflictual activity mainly determine how that social income will be divided between them.

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<sup>1</sup>Apart from allowing for a third, fourth, or more players in the game, a fuller analysis would also recognize that even single decisionmaking entities such as tribes or nations are comprised of individuals with only partially concordant interests.

<sup>2</sup>In part, this distinction is one of perspective. Primitive tribes, at any particular moment, may be at war or peace with their neighbors. But viewed over a longer time span, each tribe may be observed to alternate between peace and war with a certain frequency—which corresponds to a chosen division of effort between peacefully exploiting its own territory or attempting to appropriate what others have produced.

The factors determining power can be grouped under three main headings:

- (1) **Capabilities:** the resources on each side, and the efficiency with which they can be utilized for productive or for conflictual ends.
- (2) **Payoff Functions:** these are the equations that translate productive efforts into income and conflictual efforts into distributive shares.
- (3) **Protocol:** the "rules of the game" determine, for example, whether the Cournot or Stackelberg or other solution concept is applicable, and whether one side or the other is in a more favorable strategic position.

The discussion that follows takes up these categories in turn.

## ii. ELEMENTS OF THE BASIC MODEL

The four-way diagram of Fig. 1 illustrates a simple general-equilibrium model of mixed conflict-cooperation interactions between two opponents. In the first (upper-right) quadrant, contender 1 can choose, within his or her initial resource endowment, between productive effort  $E_1$  and conflictual effort  $F_1$ . The diagonally opposite quadrant shows the corresponding options for contender 2. The upper-left quadrant shows how the respective fighting efforts  $F_1$  and  $F_2$  determine  $p_1$ , the share of aggregate income won by contestant 1, where of course  $p_2 = 1 - p_1$ . (Since the contours of equal probability are shown as rays emerging from the origin, as drawn  $p_1$  is a function only of  $F_1/F_2$ —a special assumption to be reviewed shortly.) And finally, the lower-right quadrant shows how the productive efforts  $E_1$  and  $E_2$  combine to generate different overall totals of income  $I$ . (The pictured “convex” shape of the iso-income contours reflects a degree of positive complementarity or favorable interaction between the parties’ productive efforts, another assumption to be examined below.)

The dashed rectangle in Fig. 1 illustrates one possible outcome, for given initial choices  $E_1, F_1$  on the part of decisionmaker 1 and  $E_2, F_2$  on the part of 2. The productive activity levels  $E_1$  and  $E_2$  together determine aggregate income  $I$ , while the conflictual commitments  $F_1$  and  $F_2$  together determine the respective shares  $p_1$  and  $p_2$ . The dotted rectangle illustrates a different outcome that might ensue when the two sides both choose to devote more effort than before to fighting. As drawn, the increases in  $F_1$  and  $F_2$  have canceled one another out so that  $p_1$  and  $p_2$  remain unchanged. Thus, the only effect of symmetrically increased fighting efforts here has been to reduce the amount of income available to be divided between the parties.

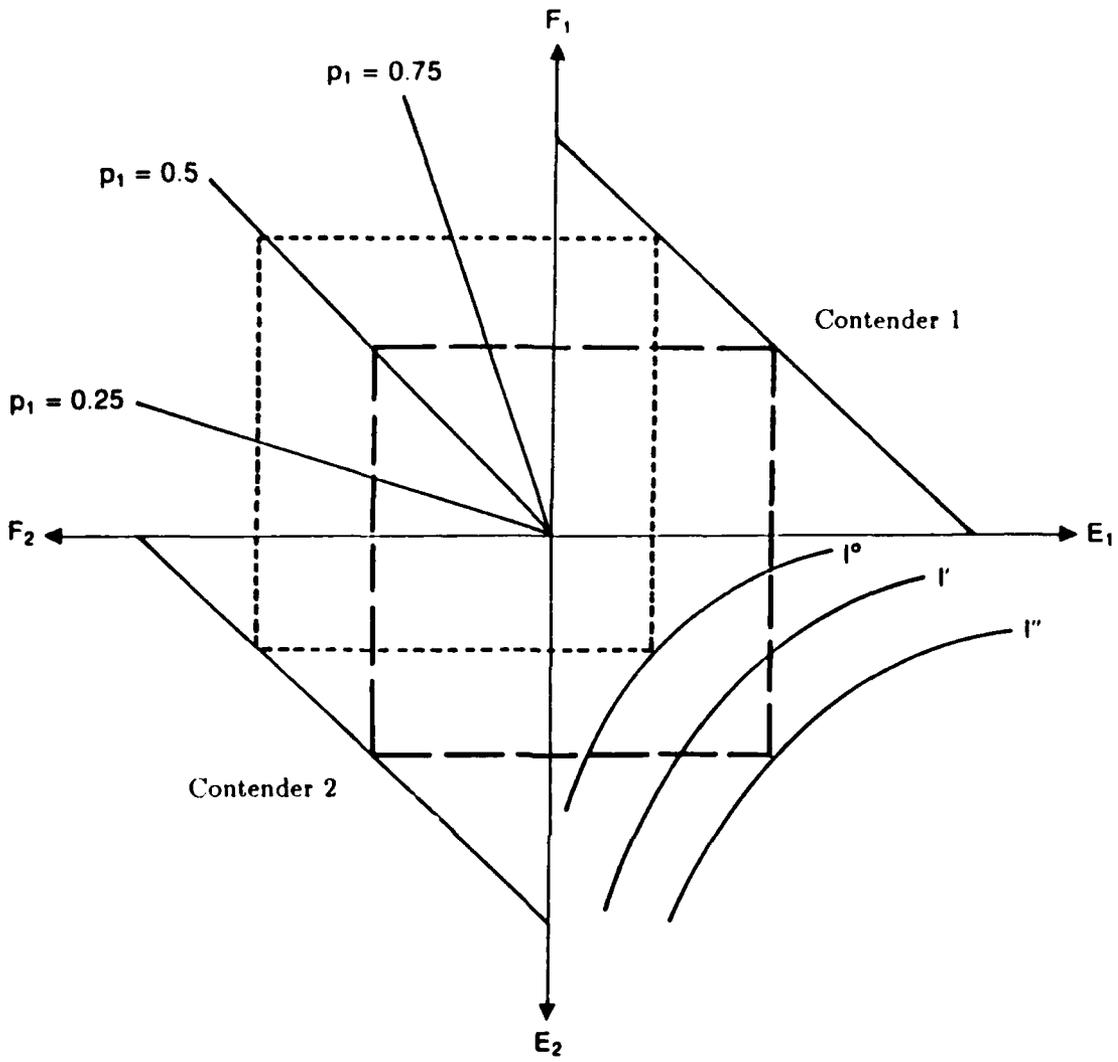


Fig. 1—Productive technology determines income  $I$ , and conflict technology determines fractional division  $p_1$

### III. REACTION CURVES AND COURNOT EQUILIBRIUM

In this section I postulate that the underlying strategic situation justifies the Cournot solution concept.<sup>3</sup> I will be deriving the Reaction Curves  $RC_1$  and  $RC_2$  showing each side's optimal conflictual or fighting effort  $F_i$  as a function of the opponent's varying  $F_j$ . The Cournot solution occurs at the intersection of the Reaction Curves, where each party's choice is a best response to the opponent's action taken as given.

The underlying equation system has four classes of logical elements.

First, each side must divide its resources  $R_i$  between productive effort  $E_i$  and fighting effort  $F_i$ , leading to accounting identities in the form of Resource Partition Functions:

$$E_1 + F_1 = R_1 \qquad \text{Resource Partition Functions} \qquad (1)$$

$$E_2 + F_2 = R_2$$

Second, there is an Aggregate Production Function (APF) that shows how the parties' productive efforts combine to determine the social income  $I$  available for division between them. Equation (2) is a simple version of an APF where  $s$ , the "productive complementarity index," is set at  $s = 1$ .<sup>4</sup> (A more general formulation will be provided below.)

$$I = E_1 + E_2 \qquad \text{Aggregate Production Function} \qquad (2)$$

The third element is the Contest Success Function (CSF), where by assumption here the outcome of the struggle depends only upon the ratio of the parties' conflictual efforts  $F_1$  and  $F_2$ .<sup>5</sup> In addition, as will be explained in more detail below, the "mass effect

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<sup>3</sup>The Cournot equilibrium concept has been used by Brock and Magee (1978) and Becker (1983) to analyze pressure-group politics, a special case of conflict as defined here. Later on I will be comparing the Cournot results with those obtained under other solution concepts.

<sup>4</sup>Given this assumption, the iso-income contours in the lower-right quadrant of Fig. 1 would be straight lines of slope  $135^\circ$  rather than the "convex" curves shown there.

<sup>5</sup>There are other significantly different ways of formulating the Contest Success Function. I have explored elsewhere some of the implications of making fighting success a function of the *numerical difference* between the commitments (Hirshleifer 1988a, 1988b). Another approach is to think in terms of a winner-take-all contest, where it is only the *rank order* of the commitments that counts (see, for example, Hillman and Riley, 1988).

parameter'' is set at  $m = 1$  and equal unit fighting effectiveness ( $a_1 = a_2$ ) has been assumed.

$$p_1 = F_1 / (F_1 + F_2)$$

Contest Success Functions (3)

$$p_2 = F_2 / (F_1 + F_2)$$

Finally, there are Income Distribution Functions. These represent something more than accounting identities, as there is an implicit assumption that neither party has any source of income other than what can be acquired through the appropriate struggle. (Thus, in this model there is no way to gain by opting out of the contest.)

$$Y_1 = p_1 I$$

Income Distribution Functions (4)

$$Y_2 = p_2 I$$

Decisionmaker 1's optimizing problem can be expressed:

$$\text{Max } Y_1 = p_1(F_1 | F_2) \times I(E_1 | E_2) \text{ subject to } E_1 + F_1 = R_1 \quad (5)$$

Using Eq. (3) for  $p_1$  and Eq. (2) for  $I$ , by standard constrained-optimization techniques the Reaction Curve  $RC_1$  may be expressed as:<sup>6</sup>

$$F_1 / F_2 = (E_1 + E_2) / (F_1 + F_2) \quad \text{Reaction Curve } RC_1 \quad (6a)$$

A corresponding equation applies of course for the rival contender:

$$F_2 / F_1 = (E_1 + E_2) / (F_1 + F_2) \quad \text{Reaction Curve } RC_2 \quad (6b)$$

<sup>6</sup>Player 1 maximizes  $Y_1 = p_1(F_1 | F_2) \times I(E_1 | E_2)$  subject to  $E_1 + F_1 = R_1$ , where  $p_1 = F_1 / (F_1 + F_2)$  and  $I = E_1 + E_2$ . Following the usual Lagrangian procedure, the first-order conditions are:

$$\begin{aligned} \partial Y_1 / \partial E_1 &= p_1 \times \partial I / \partial E_1 - \lambda = 0 \\ \partial Y_1 / \partial F_1 &= I \times \partial p_1 / \partial F_1 - \lambda = 0 \end{aligned}$$

which together imply:

$$I \times \partial p_1 / \partial F_1 = p_1$$

Taking the derivative and substituting leads to the equation in the text:

$$F_1 / F_2 = (E_1 + E_2) / (F_1 + F_2)$$

A similar analysis applies of course for player 2.

These two Reaction Curves<sup>7</sup> summarize the entire system of equations, and so can be solved simultaneously to generate the solution values:

$$F_1 = F_2 = (R_1 + R_2)/4 \quad \text{Interior Cournot solution, ratio form} \quad (7)$$

This solution is illustrated by the interior intersection of the solid Reaction Curves ( $RC_1^o$  and  $RC_2^o$ ) in Fig. 2 below.<sup>8</sup> And since  $F_1 = F_2$ , it follows that  $p_1 = p_2$ , so that:

$$Y_1 = Y_2 = (R_1 + R_2)/4 \quad \text{Incomes at interior Cournot solution} \quad (8)$$

This completes the simplest version of the general equilibrium model. A numerical example is provided on line 1 of Table 1, where by assumption the respective resources are  $R_1 = R_2 = 100$ . As is consistent with Eqs. (7) and (8), we see that half of the antagonists' summed resources are dissipated in conflictual effort ( $F_1 = F_2 = 50$ ), the half remaining being equally divided between the contenders as income ( $Y_1 = Y_2 = 50$ ).

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<sup>7</sup>In this simple case the Reaction Curves can also be put in explicit form:

$$\begin{aligned} F_1 &= \sqrt{F_2(R_1 + R_2)} - F_2 && RC_1, \text{ ratio form (explicit)} \\ F_2 &= \sqrt{F_1(R_1 + R_2)} - F_1 && RC_2, \text{ ratio form (explicit)} \end{aligned}$$

<sup>8</sup>The Reaction Curves also intersect at  $F_1 = F_2 = 0$ , but the zero-zero intersection is not a Cournot equilibrium. Owing to the ratio form of the Contest Success Function, the probabilities of success  $p_1$  and  $p_2$  are indeterminate when  $F_1 = F_2 = 0$ . It might at first seem that, at the origin,  $p_1 = p_2 = 1/2$ —since that is the value approached as  $F_1$  and  $F_2$  go to zero together. But if (say) player 1 chooses  $F_1 = 0$ , then player 2 would rationally respond by setting  $F_2$  equal to any small positive magnitude (since doing so discontinuously improves his fighting success from 50 percent to 100 percent). It follows, therefore, that the respective Reaction Curves are defined only over the open interval that does not include the singular point at the origin.

Table 1  
 COURNOT AND HIERARCHICAL EQUILIBRIA

Line No.	a1	a2	m	s	R1	R2	F1	F2	P1	P2	E1	E2	I	Y1	Y2	Solution
1.	1	1	1	1	100	100	50	50	.5	.5	50	50	100	50	50	Interior (Fig. 2)
2.					200	100	75	75	.5	.5	125	25	150	75	75	Interior (Fig. 2)
3.					400	100	123.6	100	.553	.447	276.4	0	276.4	152.8	123.6	Corner (Fig. 2)
4.	1.25	.75	1	1	100	100	43.6	56.4	.564	.436	56.4	43.6	100	56.4	43.6	Interior
5.					200	100	65.5	84.5	.564	.436	134.5	15.5	150	84.5	65.5	Interior
6.					400	100	106.1	100	.639	.361	293.9	0	293.9	187.7	106.1	Corner
7.	.75	1.25	1	1	100	100	56.4	43.6	.436	.564	43.6	56.4	100	43.6	56.4	Interior
8.					200	100	84.5	65.5	.436	.564	115.5	34.5	150	65.5	84.5	Interior
9.					400	100	140.7	100	.458	.542	259.3	0	259.3	118.7	140.7	Corner
10.	1	1	3	1	100	100	75	75	.5	.5	25	25	50	25	25	Interior
11.					200	100	111.4	100	.581	.419	88.6	0	88.6	51.4	37.1	Corner
12.					400	100	155.1	100	.789	.211	244.9	0	244.9	193.1	51.7	Corner
13.	1	1	1	1.5	100	100	50	50	.5	.5	50	50	141.4	70.2	70.7	Interior
14.					200	100	78.7	64.2	.551	.449	121.3	35.8	210.4	115.8	94.5	Interior
15.					400	100	125.6	80.7	.609	.391	274.4	19.3	347.4	211.5	135.9	Interior
16.	1	1	1	1	200	100	26.0	1	.963	.037	174.0	99	273.0	262.9	10.1	Hierarchical
17.					200	100	1	4.5	.182	.818	199	95.5	294.5	53.6	240.9	No. 1 is governor
																Hierarchical
																No. 2 is governor

#### IV. COURNOT SOLUTIONS WHEN CONTENDERS HAVE DIFFERING CAPABILITIES

In the preceding analysis the Contest Success Functions determining the relative shares  $p_1$  and  $p_2$  were entirely symmetrical between the two parties. So in a numerical example assuming equal initial resources (line 1 of Table 1), it is not surprising that the Cournot solution had the contenders achieving identical incomes. *Power differences* can only emerge from some asymmetry in the postulated conditions.

However, Eq. (8) tells us that the incomes  $Y_1$  and  $Y_2$  depend only upon the aggregate sum of the resources  $R_1 + R_2$ . Thus, regardless of the fractional distribution of resources between the parties, the outcome will be entirely symmetrical at any interior solution. This remarkable result will be called the Power Equalization Principle (PEP).

- **The Power Equalization Principle:** disparities of resources do not generally imply differences of power.

This rather startling proposition can be expressed in a strong and a weak form:

- **PEP (strong form):** regardless of the initial resource distribution, contending parties in mixed conflict-cooperation interactions will achieve exactly identical incomes.
- **PEP (weak form):** in mixed conflict-cooperation interactions, the final distribution of income will always have lesser variance than the initial distribution of resources.

Much of what follows is aimed at exploring the ranges of applicability of the strong and the weak forms of the Power Equalization Principle.

Since the decisionmakers or contenders analyzed here will normally be groups of individuals, larger resources may stem from greater per-capita endowments or simply from having more members. In what follows I will often speak of "wealthier" and "poorer" contenders, on the understanding that these terms refer to aggregate group resource availabilities rather than members' per-capita endowments. As will be seen, *the Power Equalization Principle is consistent with the obvious fact that numbers convey advantage*

*in battle.* (The reason greater resources need not translate into power is that the better-endowed side has a lesser *incentive* to devote effort to conflictual activity.)

### RESOURCE DIFFERENTIALS—INTERIOR VS. CORNER SOLUTIONS

What happens, specifically, when the parties have unequal resource endowments? Line 2 of Table 1 is a numerical example for a “moderate inequality” case:  $R_2$  remains 100 but now  $R_1$  has been increased to 200. As pictured by the long dashes representing Reaction Curves  $RC'_1$  and  $RC'_2$  in Fig. 2, although the resources devoted to fighting are larger than before, the amounts so dissipated remain equal ( $F_1 = F_2 = 75$ ). The richer party is now devoting absolutely more resources to peaceful productive effort ( $E_1 > E_2$ ), and indeed even relatively more. The equality of  $F_1$  and  $F_2$ , implying  $p_1 = p_2 = 1/2$ , also dictates that the parties achieve equal incomes once again:  $Y_1 = Y_2 = 75$ . So of the aggregate resource increment of 100 units, all initially accruing to side 1, half has been dissipated in increased fighting and the other half divided equally between the antagonists. Thus, the *strong form* of the Power Equalization Principle continues to apply.

An intuitive interpretation is as follows. The richer contender, player 1, can afford to spend more on each of the two types of activity and so will be dividing his increment of resources between  $E_1$  and  $F_1$ . His opponent, although no richer than before, now has both offensive and defensive incentives to shift toward spending *more* than before on fighting (making his  $F_2$  larger, which means of course that his  $E_2$  must be smaller). The offensive incentive for increasing  $F_2$  is that more social income is available to be seized. The defensive incentive is that the larger  $F_1$  means that player 2 must make his own  $F_2$  bigger just to maintain his previous level of income.

Underlying the Power Equalization Principle is the fact that, when a contender's resources are small relative to the opponent's, *the marginal yield of fighting activity is higher to begin with than the marginal yield of productive activity.* Specifically, supposing that contestant 2 is the poorer player, straightforward differentiation of  $Y_2 = p_2 I$  leads to:

$$\frac{\partial Y_2}{\partial E_2} = \frac{F_2}{F_1 + F_2} \quad \text{and} \quad \frac{\partial Y_2}{\partial F_2} = \frac{F_1(E_1 + E_2)}{(F_1 + F_2)^2} \quad (9)$$

When  $R_2$  is very small, then  $E_2$  and  $F_2$  must be small as well, in which case the marginal product of productive effort  $\partial Y_2 / \partial E_2$  goes toward zero while the marginal product of fighting  $\partial Y_2 / \partial F_2$  remains positive.

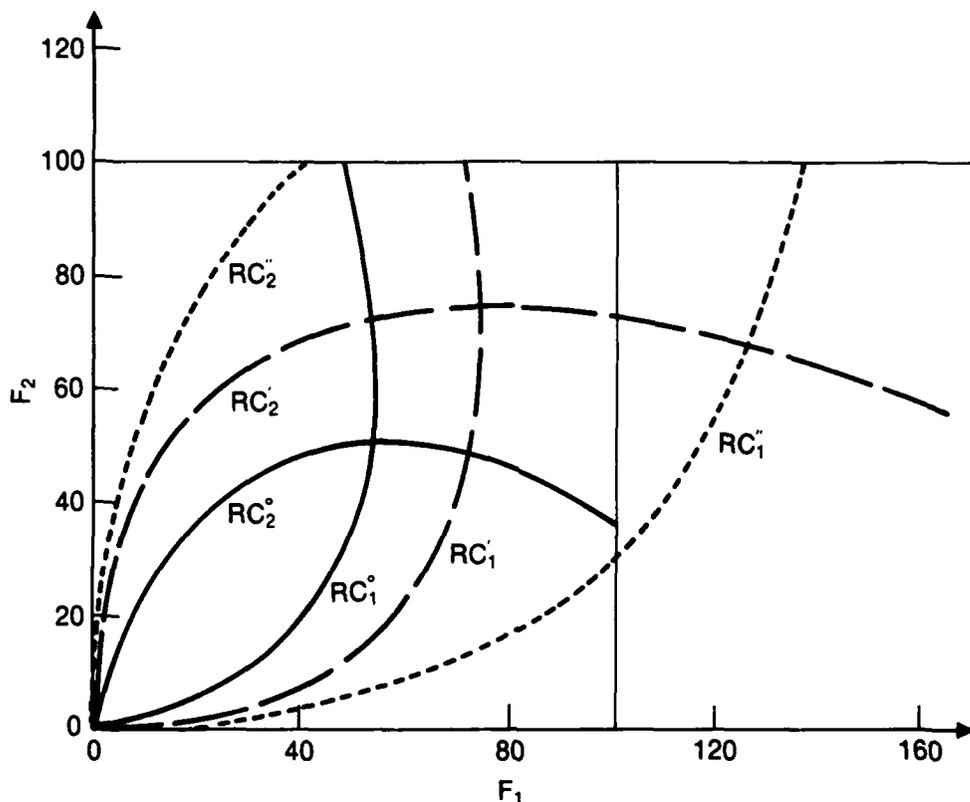


Fig. 2—Reaction Curves and Cournot Equilibrium—interior and corner solutions

*So conflict is a relatively more attractive option for the poorer side.<sup>9</sup> Fighting effort permits you to “tax” the opponent’s production, while your own production is “taxed” by his fighting effort. When your rival is richer, it becomes relatively more profitable to tax him (to capture part of his larger production) and relatively more burdensome to be taxed by him (to devote effort to production which will be largely captured by him anyway). The opposite holds, of course, when your antagonist is less endowed than you.*

Thus rational behavior in a conflict interaction is for the poorer side to specialize more in fighting, the richer side more in production. An example: in early historical periods, cities or empires with relatively advanced productive industry were regularly raided or preyed upon by nomadic tribes who specialized in developing their fighting prowess. The effect was to moderate the initial wealth disparity.

<sup>9</sup>Becker obtains a somewhat analogous proposition: “Politically successful groups tend to be small relative to the size of the groups taxed to pay their subsidies” (1983, p. 385).

But there is a limit to this process, which is reached once the poorer side comes to devote all of its resources to fighting. This situation, the "extreme inequality" case, is illustrated by line 3 of Table 1 where the resource disparity has become  $R_1 = 400$  versus  $R_2 = 100$ . The corresponding picture is represented by the Reaction Curves in Fig. 2 drawn in short dashes. Notice that the less-endowed side (player 2) has run into the upper bound upon resources available to be devoted to fighting.<sup>10</sup> Only when this is the case—when the ratio  $R_1/R_2$  of the endowed resources exceeds a certain critical value<sup>11</sup>—does  $F_1$  exceed  $F_2$ , implying that  $p_1 > p_2$ , so that the richer side is able to translate larger initial resources into larger final income.

Table 2 displays the solution values of this "base case" in terms of the underlying parameters. We can see that  $Y_1$  and  $Y_2$  depend solely and symmetrically upon the total resources  $R_1 + R_2$ . Thus, an increment of resources to one contender brings him no additional power, unless his opponent has reached an upper bound in terms of possible fighting effort.

When the limit  $F_2 = R_2$  on the poorer side's fighting effort is reached, the strong form of the Power Equalization Principle no longer holds. However, the weak form continues to apply. Thus the distribution of power (as measured by comparative attained income) is always more equal than the distribution of *resources*—exact equality holding at interior solutions. The underlying reason is, as has been seen, that larger aggregate resources raise the marginal product of productive effort compared with fighting effort. Fighting is a relatively more advantageous activity for the less-endowed (the poorer or the less numerous) contender.

## FIGHTING EFFECTIVENESS DISPARITIES

Capabilities may differ in dimensions other than resource endowments. Specifically, the antagonists may also differ in the unit *effectiveness* of resources devoted to conflict, to production, or both. The analysis here covers differences in unit fighting effectiveness only.

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<sup>10</sup>At the discontinuity, the Reaction Curve equation shown in the text of course no longer applies. The best-response condition, if contestant 2 is at his upper limit, is simply  $F_2 = R_2$ .

<sup>11</sup>The discontinuity occurs here at  $R_1/R_2 = 3$ . When the resource ratio is less than 3, the two sides set  $F_1 = F_2$  and divide the overall income equally. When the ratio exceeds 3, the poorer side comes as close as possible by setting  $F_2 = R_2$ , but still cannot achieve equality of income.

Table 2

COURNOT EQUILIBRIUM: BASE CASE ( $m = s = a_1 = a_2 = 1$ )

---

**Interior Solution**

$$RC_1 : \frac{F_1}{F_2} = \frac{E_1 + E_2}{F_1 + F_2}$$

$$RC_2 : \frac{F_2}{F_1} = \frac{E_1 + E_2}{F_1 + F_2}$$

**Solution values:**

$$F_1 = \frac{R_1 + R_2}{4} = F_2$$

$$p_1 = .5 = p_2$$

$$I = \frac{R_1 + R_2}{2}$$

$$Y_1 = \frac{R_1 + R_2}{4} = Y_2$$

**Corner Solution** [where  $S \equiv \text{sqrt}\{R_2(R_1 + R_2)\}$ ]

$$RC_1 : F_1 = S - F_2$$

$$RC_2 : F_2 = R_2$$

**Solution values:**

$$F_1 = S - R_2$$

$$F_2 = R_2$$

$$p_1 = \frac{S - R_2}{S}$$

$$p_2 = \frac{R_2}{S}$$

$$I = R_1 + R_2 - S$$

$$Y_1 = \frac{S - R_2}{S}(R_1 + R_2 - S)$$

$$Y_2 = \frac{R_2}{S}(R_1 + R_2 - S)$$

---

Generalizing to allow for different unit effectiveness coefficients  $a_1$  and  $a_2$ , the equations for the Contest Success Function become:<sup>12</sup>

$$\begin{aligned} p_1 &= a_1 F_1 / (a_1 F_1 + a_2 F_2) \\ p_2 &= a_2 F_2 / (a_1 F_1 + a_2 F_2) \end{aligned} \quad \begin{array}{l} \text{Contest Success Functions} \\ (a_1 \neq a_2) \end{array} \quad (10)$$

These equations naturally reduce to (3) above upon setting  $a_1 = a_2 = 1$ .

Table 3 shows the Reaction Curves and the solution values of the model at an interior Cournot equilibrium in terms of the underlying parameters and coefficients. Once again, when either side's resources are increased, that is good news for both contenders, but (at an interior solution) no relative advantage is conferred upon the side receiving the increment.<sup>13</sup>

Numerically, lines 4 through 6 of Table 1 show the interaction of wealth disparities with unit fighting effectiveness, when it is the *better-endowed* side that has superior effectiveness ( $a_1 = 1.25$  and  $a_2 = .75$ ). As can be seen by comparison with lines 1 through 3 where  $a_1 = a_2 = 1$ , the wealthier party is now able to commit less resources to fighting, and even so ends up considerably better off. The poorer opponent, correspondingly, devotes more resources to fighting (if not already at its upper bound) but nevertheless ends up worse off.

Lines 4 and 5 illustrate, for the first time, interior solutions with unequal incomes. That is, the strong form of the PEP does not seem to apply. However, this is a misleading impression, as may be seen by comparing lines 4 and 5 with lines 7 and 8 where the parameters have been interchanged so as to give the poorer-endowed side the higher fighting effectiveness. The results in terms of income are exactly reversed, which proves that it was not the *resource disparities* as such that generated the differing income levels  $Y_1$  and  $Y_2$ , but only the differing fighting effectiveness parameters  $a_1$  and  $a_2$ . At the corner solution of line 6, having the advantage both in terms of resources ( $R_1$  versus  $R_2$ ) and in terms of fighting effectiveness ( $a_1$  versus  $a_2$ ) leads contender 1 to a strongly superior outcome. By comparison, on line 9 we can see that even though player 2 is at his corner solution, despite the huge 4:1 resource inferiority he still ends better off owing to the unit fighting effectiveness advantage.

<sup>12</sup>The straightforward development is omitted.

<sup>13</sup>In the interests of brevity, and since no new principle is involved, the corner solutions are omitted from Table 3 and the tables following.

Table 3

COURNOT EQUILIBRIUM: VARYING FIGHTING EFFECTIVENESS ( $a_1 \neq a_2$ )

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**Interior Solution**

$$RC_1 : \frac{F_1}{a_2 F_2} = \frac{E_1 + E_2}{a_1 F_1 + a_2 F_2}$$

$$RC_2 : \frac{F_2}{a_1 F_1} = \frac{E_1 + E_2}{a_1 F_1 + a_2 F_2}$$

**Solution values:** [where  $a \equiv \sqrt{a_1} + \sqrt{a_2}$ ]

$$F_1 = \frac{\sqrt{a_2}(R_1 + R_2)}{2a}$$

$$F_2 = \frac{\sqrt{a_1}(R_1 + R_2)}{2a}$$

$$p_1 = \frac{\sqrt{a_1}}{a}$$

$$p_2 = \frac{\sqrt{a_2}}{a}$$

$$I = \frac{R_1 + R_2}{2}$$

$$Y_1 = \frac{\sqrt{a_1}}{a} \frac{R_1 + R_2}{2}$$

$$Y_2 = \frac{\sqrt{a_2}}{a} \frac{R_1 + R_2}{2}$$

---

Summing up: (i) even when differing fighting effectiveness is allowed for, resource disparities *alone* do not bring about power differences—except in a somewhat diluted way at corner solutions. (ii) Superiority in unit fighting effectiveness, for a given resource disparity, conveys a heavy power advantage. (iii) At corner solutions, there is a positive interaction between resources and fighting effectiveness; a better-endowed party can derive greater benefit from having superior unit fighting effectiveness.

## V. COURNOT SOLUTIONS FOR DIFFERING ANALYTICAL FORMS OF PAYOFF FUNCTIONS

The outcomes of mixed conflict-cooperation interactions may also depend upon the functional forms of the payoff equations. Specifically, we will be considering alternative forms of: (i) the Aggregate Production Function that governs the social return to productive effort, and (ii) the Contest Success Functions that determine the division of income between the parties.

### CONTEST SUCCESS FUNCTIONS

While maintaining the postulate that the payoffs are a function of the ratio of the fighting efforts,<sup>14</sup> the CSF equation can be significantly altered by adjusting certain numerical parameters. Equation (10) above displayed one type of modification, allowing for differential unit fighting effectiveness ( $a_1 \neq a_2$ ). For simplicity, so as to focus instead upon possible variations of the "mass effect parameter"  $m$ , henceforth equal unit effectiveness ( $a_1 = a_2$ ) will always be assumed. Then the generalized version of Eqs. (3), with  $m$  as parameter, is:

$$\begin{aligned}
 p_1 &= F_1^m / (F_1^m + F_2^m) \\
 p_2 &= F_2^m / (F_1^m + F_2^m)
 \end{aligned}
 \qquad \begin{array}{l}
 \text{Contest Success Functions} \\
 (m \text{ as parameter})
 \end{array}
 \qquad (11)$$

For the case where the opponent's fighting effort is  $F_2 = 100$ , Fig. 3 shows how the form of the CSF varies with increases in the mass effect parameter.

The analytical elements of the interior Cournot equilibrium are displayed in Table 4. Referring back to Table 1, lines 10 through 12 extend the previous numerical example, with the mass effect parameter set at  $m = 3$  in place of the previously assumed  $m = 1$ . The main implications of the higher mass effect parameter are: (i) the marginal product of fighting effort increases,<sup>15</sup> leading both sides to commit more resources to

<sup>14</sup>For alternative possible assumptions, see footnote 4 above.

<sup>15</sup>Generalizing Eqs. (9) for any  $m$ , the marginal products for player 2 become:

$$\frac{\partial Y_2}{\partial E_2} = \frac{F_2^m}{F_1^m + F_2^m} \quad \text{and} \quad \frac{\partial Y_2}{\partial F_2} = (E_1 + E_2) \frac{m F_1^m F_2^{m-1}}{(F_1^m + F_2^m)^2}$$

For  $m > 1$  there will be, as illustrated in Fig. 3, an initial range of increasing returns in  $\partial Y_2 / \partial F_2$ , the marginal product of fighting effort.

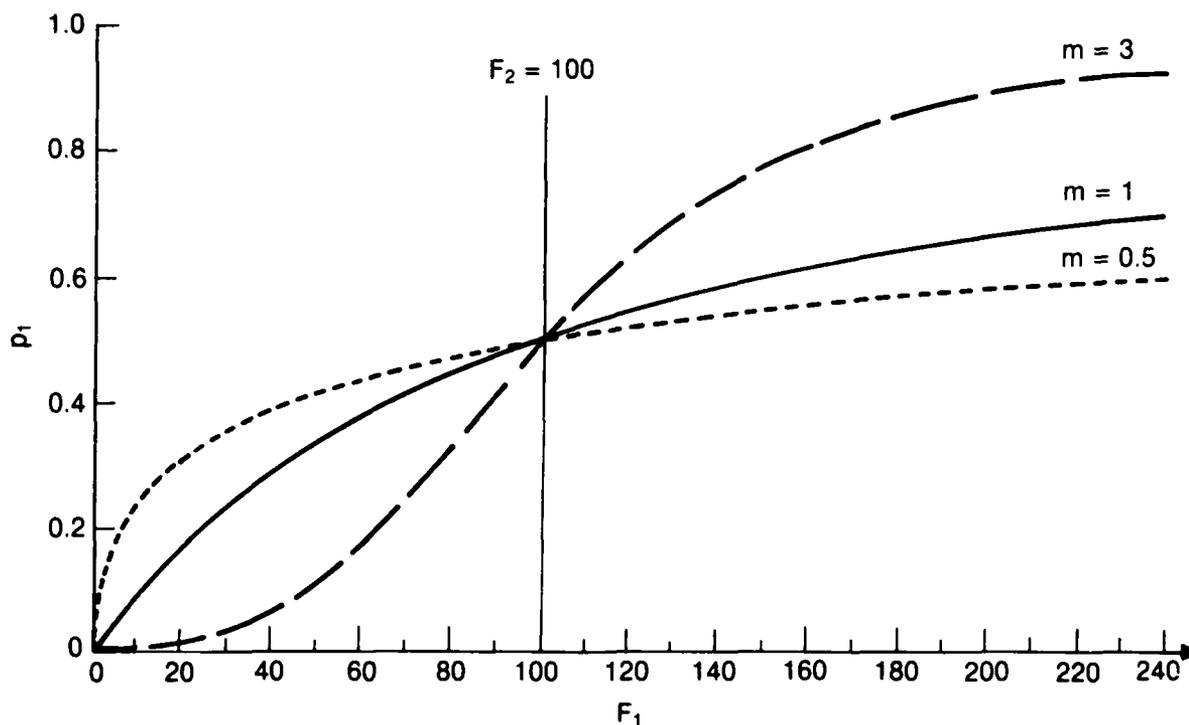


Fig. 3—Contest Success Functions  
(Ratio form)

conflict, with the evident consequence of reduced social income in aggregate. (ii) At interior solutions, the two contenders still achieve equal incomes: the *strong form of the PEP* holds as before. (iii) However, with higher values of  $m$ , the poorer-endowed side reaches its upper-bound constraint earlier (i.e., at a smaller resource disparity). In other words, the strong form of the Power Equalization Principle does not hold over as wide a range. (iv) Furthermore, when the upper constraint on the poorer-endowed player is binding, any given resource ratio translates into a larger power ratio  $p_1/p_2 = Y_1/Y_2$  as  $m$  increases. And indeed, after an increase in  $m$ , the wealthier party may end up absolutely better off despite the smaller total available to be divided.

Table 4

COURNOT EQUILIBRIUM: VARYING THE MASS EFFECT PARAMETER ( $m \neq 1$ )

**Interior Solution**

$$RC_1 : \frac{F_1}{F_2^m} = \frac{m(E_1 + E_2)}{F_1^m + F_2^m}$$

$$RC_2 : \frac{F_2}{F_1^m} = \frac{m(E_1 + E_2)}{F_1^m + F_2^m}$$

**Solution values:** [where  $M \equiv \frac{m}{m+1}$ ]

$$F_1 = .5M(R_1 + R_2) = F_2$$

$$p_1 = .5 = p_2$$

$$I = (R_1 + R_2)(1 - M)$$

$$Y_1 = .5(R_1 + R_2)(1 - M) = Y_2$$

While the sweep of the Power Equalization Principle is therefore somewhat attenuated as the mass effect parameter of fighting increases, it remains true that resource disparities are always converted only in a diluted way into power differences. The underlying reason remains the same. To wit, *the poorer side is always motivated to invest relatively more heavily in fighting effort.*

**AGGREGATE PRODUCTION FUNCTION**

A generalized form of Eq. (2) is:<sup>16</sup>

$$I = (E_1^{1/s} + E_2^{1/s})^s \quad \begin{array}{l} \text{Aggregate Production Function} \\ \text{(general } s) \end{array} \quad (12)$$

Suppose the productive complementarity index  $s$  increases. Then, for given amounts of resources devoted to production, it is easy to verify that not only will overall

<sup>16</sup>This is a member of the family of CES (constant elasticity of substitution) production functions. There are of course many other ways of generalizing Eq. (2).

income rise but, for each party alone, the marginal product of productive effort will rise compared with fighting effort. However, it is not immediately apparent what happens when the interaction of the two sides' decisions is taken into account.

The consequences of this interaction are indicated in Table 1, where comparison of lines 13 through 15 with lines 1 through 3 reveals the effect of increasing the complementarity index from  $s = 1$  to  $s = 1.5$ . The poorer side, as expected, now devotes relatively less effort to fighting, and in fact is no longer constrained by the upper bound upon fighting effort even at the largest wealth disparity shown in the table. But somewhat unexpectedly, perhaps, the wealthier side now tilts a bit more toward fighting. The reason is that the poorer side, having devoted more effort to production, now becomes a more attractive target. The upshot is that although both parties are better off when productive complementarity increases, the wealthier side gains more—not only is aggregate production greater, but the richer contender is in a position to capture a larger share.

The generalized equations for the Reaction Curves, allowing for  $s \neq 1$ , are shown in Table 5a. However, the solution values for the variables do not have convenient reduced-form expressions in terms of the underlying parameters. Table 5b therefore shows instead numerical approximations of the parametric derivatives of the solution values,  $\delta F_1 / \delta R_1$ .<sup>17</sup> These indicate how the equilibrium values on lines 13 through 15 of Table 1 respond to wealth variations.

To interpret Table 5b, take line 13a as an example. This displays how the solution values on line 13 of Table 1 (which applied for the symmetrical case of  $R_1 = R_2 = 100$ ) change when  $R_1$  increases by one unit ( $\Delta R_1 = 1$ ) with  $R_2$  held constant ( $\Delta R_2 = 0$ ). As shown,  $F_1$  and  $F_2$  both increase from their initially equal levels, but the former does so by a greater amount—so  $p_1$  rises and  $p_2$  falls. Thus both sides reap an income increment when the resources of one side increase, but the relatively enriched party does gain somewhat more. The implication is that when the productive complementarity index takes on values of  $s > 1$ , the strong form of the PEP no longer holds even at interior solutions.

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<sup>17</sup>These derivatives show how the equilibrium solution values change in response to variations of the parameters. While an explicit analytic solution can be formulated, direct numerical calculation is easier.

Table 5a

COURNOT EQUILIBRIUM: VARYING THE PRODUCTIVE  
COMPLEMENTARITY INDEX ( $s \neq 1$ )

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**Interior Solution**

$$RC_1 : \frac{F_1}{F_2} E_1^{\frac{1-s}{s}} = \frac{E_1^{1/s} + E_2^{1/s}}{F_1 + F_2}$$

$$RC_2 : \frac{F_2}{F_1} E_2^{\frac{1-s}{s}} = \frac{E_1^{1/s} + E_2^{1/s}}{F_1 + F_2}$$

---

Summarizing for Cournot solutions generally, increases in the mass effect parameter  $m$  or in the productive complementarity index  $s$  both weaken the Power Equalization Principle, though for somewhat different reasons. As  $m$  grows with  $s = 1$  held fixed, the incomes achieved remain equal at interior solutions. But the poorer contender is more likely to become bound by his resource constraint and so forced into a corner solution—which, furthermore, now has become relatively more rewarding to the better-endowed side. If it is  $s$  that rises above unity, even the interior solutions become asymmetrical in favor of the better-endowed rival. Nevertheless, despite these cases where the strong form of the PEP (exact equality of achieved incomes) no longer applies, the weak form of the PEP continues to hold: the achieved income disparity is never as great as the initial resource disparity between the antagonists.

Table 5b  
 NUMERICAL APPROXIMATIONS FOR PARAMETRIC  
 DERIVATIVES OF SOLUTION VALUES  
 (Lines 13-15 of Table 1)

Line No.	Variation	R <sub>1</sub>	R <sub>2</sub>	F <sub>1</sub>	F <sub>2</sub>	P <sub>1</sub>	P <sub>2</sub>	E <sub>1</sub>	E <sub>2</sub>	I	Y <sub>1</sub>	Y <sub>2</sub>
13.	Calculated at:	100	100	50	50	.5	.5	50	50	141.4	70.7	70.7
	$\Delta R_1 = 1$	+1	0	+ .321	+ .178	+ .0007	- .0007	+ .679	- .178	+ .707	+ .454	+ .252
	$\Delta R_2 = 1$	0	+1	+ .178	+ .321	- .0007	+ .0007	- .178	+ .679	+ .707	+ .252	+ .454
14.	Calculated at:	200	100	78.7	64.2	.551	.449	121.3	35.8	210.4	115.8	94.5
	$\Delta R_1 = 1$	+1	0	+ .262	+ .114	+ .0004	- .0004	+ .738	- .114	+ .680	+ .455	+ .226
	$\Delta R_2 = 1$	0	+1	+ .317	+ .412	- .0008	+ .0008	- .262	+ .588	+ .744	+ .249	+ .495
15.	Calculated at:	400	100	125.6	80.7	.609	.391	274.4	19.3	347.4	211.5	135.9
	$\Delta R_1 = 1$	+1	0	+ .213	+ .058	+ .0002	- .0002	+ .787	- .058	+ .699	+ .507	+ .192
	$\Delta R_2 = 1$	0	+1	+ .401	+ .573	- .0009	+ .0009	- .401	+ .427	+ .681	+ .092	+ .589

## VI. NON-COURNOT STRATEGIC PROTOCOLS

While the Cournot solution concept is valuable for ordering ideas, it is difficult to specify a plausible protocol leading to the Cournot equilibrium. Supposedly, each side is acting on the premise that it is free to choose while the opponent's decision is fixed. In other words, each decisionmaker assumes he has the last move. But they can't both be right.<sup>18</sup>

### STACKELBERG SOLUTIONS

In contrast with the Cournot assumption, under the Stackelberg protocol one party is the first-mover or leader and the other the second-mover or follower.<sup>19</sup> A rational follower will respond along his Reaction Curve to the leader's initial commitment. A rational leader will therefore select a prior choice so as to take advantage of this predictable response.

Does the power balance tilt in favor of the first-mover? One's first impression might be that it is more advantageous to be a leader than a follower. However, in certain circumstances, for example the position known as *Zugschwang* in chess, having the first move can be a definite handicap. In battle the first-mover is able to seize the high ground—but the second-mover, being in a position to observe the choice made, has the benefit of superior information. So the net balance of advantage remains unclear.

Within the model here, using the base-case parameter values  $s = a_1 = a_2 = 1$  but allowing  $m$  to vary, the rather startling result is obtained that, for all interior solutions, *having the first move in the Stackelberg sense makes no difference whatsoever!* That is, the Stackelberg solutions replicate the Cournot outcomes.

The leader (player 1, let us say) will be attempting to maximize income  $Y_1 = p_1 I$  along the Reaction Curve  $RC_2$  of the follower. But it is unnecessary to carry out any

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<sup>18</sup>On this see Brams and Wittman (1981) and Hirshleifer (1985). When each party allows in advance for rational responses of the other, they are led in certain circumstances toward the "perfect equilibrium" concept of Selten (1975). Another approach is to imagine that, in choosing larger or smaller efforts, each side postulates a "conjectured" non-zero response on the part of the opponent (Bresnahan, 1981). These solution concepts cannot be covered here.

<sup>19</sup>The possible source or explanation of leadership will be considered briefly below.

tedious maximization, since the following very simple condition holds for either party's income along the opponent's Reaction Curve:<sup>20</sup>

$$Y_1 = F_2/m \text{ along } RC_2 \text{ and } Y_2 = F_1/m \text{ along } RC_1 \quad (13)$$

So player 1 as leader will choose  $F_1$  in such a way that the opponent's best response will be the highest  $F_2$  along player 2's Reaction Curve  $RC_2$ . Correspondingly, player 2 as the leader would choose that  $F_2$  which leads to a maximal  $F_1$  in the  $RC_1$  equation. The next point to note is that, in all of the interior solutions,  $F_1$  and  $F_2$  are respectively maximized along their own Reaction Curves at the Cournot equilibrium.<sup>21</sup> (Geometrically, this means that the intersection of the Reaction Curves occurs when the  $RC_1$  curve has vertical slope and the  $RC_2$  curve has horizontal slope.) Therefore, setting aside certain amendments and qualifications called for when corner solutions obtain, or when parameters other than  $m$  are also allowed to vary, all the Cournot results carry over. And in particular, at least the weak form of the Power Equalization Principle always holds. Thus, *being a Stackelberg leader or follower has no implications whatsoever for power.*

#### HIERARCHICAL SOLUTIONS

In drastic contrast are the implications of another type of leadership, which I will call *hierarchical*. The hierarchical leader or "governor" ( $G$ ) is in a position to make quite a different kind of first move. Specifically, he can issue a coercive signal. Thus a *governor makes a first move by committing himself not to a single prior action but to a set of conditional posterior reactions.*<sup>22</sup> He specifies in advance how he will respond to whatever the subordinate does in the way of an action move. In effect then, what he chooses is his own Reaction Curve after allowing the opponent or "subordinate" ( $S$ ) to

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<sup>20</sup>For general  $m$ , the  $RC_2$  equation can be written:

$$F_2 = F_1^m \frac{m(E_1 + E_2)}{F_1^m + F_2^m} = mp_1 I$$

So, along  $RC_2$  we have  $Y_1 = F_2/m$ .

<sup>21</sup>This is equivalent to saying that the derivatives along the Reaction Curves, at the Cournot solution values for  $F_1$  and  $F_2$ , are  $dF_2/dF_1 = 0$  along  $RC_2$  and  $dF_1/dF_2 = 0$  along  $RC_1$ .

<sup>22</sup>The factors that may permit one side or the other to achieve the "governor" position are considered briefly below.

act as the Stackelberg leader. To avoid confusion with the ordinary Cournot reaction function, let us say that the governor selects a Hierarchical Response Function (*HRF*).

Of course, the governor wants to select the best possible *HRF* in terms of income ultimately achieved—one that will induce the subordinate (player *S*) to select a minimal fighting effort  $F_S = M$ . (Ideally, the governor would prefer to see the opponent choose  $M = 0$ , but this may not always be feasible.) To this end *G* must credibly *promise* to respond in a rewarding way if *S* chooses  $F_S = M$ , and must credibly *threaten* something unpleasant if any  $F_S > M$  is chosen.

The *threat* part of the *HRF* is most easily analyzed if the governor undertakes to respond to noncompliance with “massive retaliation.” That is, in the event that the subordinate chooses any  $F_S > M$ , the governor engages himself to set his  $F_G = R_G$ .<sup>23</sup> As for the promise part of the *HRF*, the amount offered for compliance need only be infinitesimally greater than what the subordinate could possibly achieve by defiance. Under these assumptions the *HRF* can be written:

$$F_G = \begin{cases} R_G, & \text{if } F_S > M \\ F_G^*, & \text{if } F_S = M, \end{cases} \quad \text{Hierarchical Response Function} \quad (14a)$$

where  $F_G^*$  is determined in:

$$Y_S(M | F_G^*) = Y_S(RC_S | R_G) \quad (14b)$$

Thus if he complies by setting  $F_S = M$ , the subordinate is offered an income level  $Y_S(M | F_G^*)$  just equal to the income level  $Y_S(RC_S | R_G)$  he could earn by making a best response along his Reaction Curve  $RC_S$  to the governor’s “massive retaliation” choice of  $F_G = R_G$ .

For the Contest Success Functions analyzed here, the subordinate cannot actually be induced to set  $F_S = M = 0$ . As indicated in Eq. (3) or (10) or (11), were he to do so,

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<sup>23</sup>This is the *maximal* threat. Alternatively, *G* might look for a *minimal* threat just sufficient to induce the desired behavior. In a full-knowledge situation the results would be the same either way, since with proper calculation on all sides the threat would succeed—no fighting ever taking place. Absent such perfect knowledge, the disadvantage of a minimal threat is that it leaves no margin for error if the opponent’s resolve or payoffs have been miscalculated. On the other hand, if there has been such a serious miscalculation that even the maximal threat does not work, much more damage than is necessary will be inflicted.

his  $p_S$  and therefore his  $Y_S$  would also be zero.<sup>24</sup> (This analysis rules out the possibility of side payments.) On the other hand, any positive  $M$ , no matter how small, is in principle achievable. In Table 1, line 16 represents an illustrative *HRF* solution for a minimal threshold level of  $F_S$  set arbitrarily at  $M = 1$ , assuming the richer contender is the governor. Line 17 shows a corresponding solution when the poorer-endowed party is in a position to act as governor.

It is evident from Table 1 that the coercive power implicit in the position of governor is a truly enormous source of advantage compared with any considered previously, quite overwhelming the previously observed tendency toward equalization of income through conflict processes. More consistent with previous results is the fact that *resource disparities alone* are only a minor source of coercive power in the hierarchical protocol. However, and also in parallel with previous observations, resource disparities tend to amplify the power implications of superior hierarchical position. Comparing lines 16 and 17 we see that the better-endowed contender can make more effective use of governorship if he is able to secure that position—the reason being that he can issue a more powerful maximal threat.

Another point of interest is that the hierarchical solutions, although highly asymmetrical, are much closer to being Pareto-efficient than those obtained under the Cournot or Stackelberg protocols. Furthermore, as between the two hierarchical equilibria, the one where the poorer-endowed side is governor is the more efficient—since, when the other side conforms, a poorer governor is less able to commit resources to appropriative effort.

In view of the advantage accruing to being the governor, why do we not always observe hierarchical equilibria? One major reason is that in issuing a threat or a promise the governor is pledging to behave in a way inconsistent with his later self-interest. A promise or a threat is an undertaking to do something you would not otherwise be motivated to do; you can't usefully threaten or promise to do something that you want to do anyway. So the question becomes, how can a governor's pledges be made credible? This is not quite so great a problem in an ongoing or steady-state model, since then all sides will be taking a long-term point of view. While there is always a short-term advantage for the governor to default upon his prior commitments, either to reward

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<sup>24</sup>In contrast, where the Contest Success Function depends not upon the ratio but upon the *numerical difference* of the fighting commitments, in certain circumstances the governor can induce complete submission ( $F_S = 0$ ) on the part of the subordinate (Hirshleifer, 1988a).

compliance or to punish noncompliance, doing so would destroy what is for him a very advantageous permanent arrangement.

Nevertheless, the hierarchical solution remains somewhat fragile. It depends ultimately upon the subordinate's faith that the governor will always resist the temptation to realize an immediate short-run gain. Such faith is hard to sustain in a world where information is imperfect and where payoffs and attitudes are continually changing.

The possible robustness of the hierarchical solution is related to the question of which of the rivals secures the governing position. I have argued elsewhere (Hirshleifer, 1987, and see also Frank, 1988) that the *emotions*, which economists have usually disregarded as mere awkward obstacles to fully rational behavior, may have survived as part of the human psyche because they serve the function of guaranteeing the execution of threats and promises. The "charismatic" quality we look for in our leaders, I will assert, is the capacity to transcend short-run self-interested motivations. If a governor is passionately driven by sentiments of magnanimity in response to submission and anger in the event of defiance, subordinates can be assured that threats and promises will indeed be carried out.

## VII. CONCLUSION: THE DETERMINANTS OF POWER

The underlying premise has been that individuals, groups, or nations are rarely if ever totally "at war" or totally "at peace." Instead, contending parties normally find it advisable to divide their efforts between productive activity and appropriative struggles. Within this general framework, this analysis explores alternative steady-state solutions—allowing for possible variations in the rivals' resources or other capabilities, in the functions rewarding productive or fighting efforts, and in the strategic protocol and associated solution concept. As a crucial maintained assumption in the models here, all the productive activity on either side falls into a simple *common pool* of income available for capture through conflictual activities.

What is termed here the Power Equalization Principle (PEP) has a strong and a weak form. When the strong form applies, in equilibrium the antagonists' terminal incomes are exactly equal, despite initial resource disparities. When only the weak form holds, it remains true that the distribution of power (in the sense of achieved income) is more equal than the initial distribution of resources. At least in its weak form, the PEP proved to be a remarkably robust generalization, applying under a wide range of assumptions about payoff parameters, effectiveness coefficients, and solution concepts. The underlying explanation derives from a comparison of the marginal products of productive versus conflictual activities, which reveals that the less-endowed side has a comparative advantage in fighting, the richer side in producing. Appropriative effort allows you to place a tax upon your opponent's productive effort, and it is more profitable to tax a rich opponent than a poor one. Hence the conflict process, while of course it dissipates income in aggregate, also tends to bring about a more equal distribution of whatever income remains.

However, one set of circumstances definitely overcomes the Power Equalization Principle, to wit, when the interaction of the parties takes place under a *hierarchical protocol*. That is, where one of the contenders (the "governor" ) is in a position to issue a credible threat-and-promise as to how he will respond to noncompliant versus submissive behavior on the part of the subordinate. In these circumstances the distribution of income will be heavily skewed in favor of the governor, far more so than the initial distribution of resources. And, in addition, larger resources amplify the power advantage of hierarchical position.

As in all attempts to model complex phenomena, the necessity of making a host of special assumptions limits the applicability of these results: (i) this analysis has considered only two-party interactions, and so has made no attempt to address issues like alliances and the balance of power.<sup>25</sup> (ii) Full information has been assumed throughout, so that factors like deception have been set aside.<sup>26</sup> (iii) The steady-state assumption rules out issues involving timing, such as arms races, economic growth, or (on a smaller time scale) signaling resolve through escalation. (iv) The simplified mathematical form of the Contest Success Function does not allow for differences between offensive and defensive weapons, between ground and naval forces, between battle-seeking and Fabian tactics, and so on. For the economist, special interest attaches to the distinction between capital-intensive and labor-intensive modes of warfare (Stockfish, 1976). (v) In the Aggregate Production Function assumed here, all income fell into a common pool available for capture. More generally, each side might have some private income secure from capture, and in fact would be making an optimizing choice between devoting resources to private production or else to the common pool. (vi) Cutting in the opposite direction, the underlying resources on each side were assumed invulnerable to destruction or seizure. In a more general steady-state model, each contender would have to be making investments at a certain rate over time in order to offset damage or losses. (vii) The effects of distance and other geographical factors have not been considered.

Even when generalized in the various ways indicated by the list above, the steady-state model will remain inappropriate for the analysis of conflicts dominated by a single overwhelming or irreversible event like a Pearl Harbor attack. It is more applicable to arms races or to continuing low-level combats—such as those between city dwellers and nomads, or among the small states of pre-imperial China—than to a possible nuclear armageddon. Or, outside the military domain, it will be found to shed light upon the mixed cooperative-conflictual processes we observe in capital-labor relations, in politics, and within families.

I will expand briefly on only one application, political redistribution of income. In modern politics, at least, redistribution is overwhelmingly from the rich to the poor. This might seem surprising. After all, starting from their initial resource advantage, the rich could, it appears, make themselves richer still by appropriating what others have

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<sup>25</sup>There is of course a vast literature on these questions. I shall cite here only Blainey (1973) and Bernholz (1985).

<sup>26</sup>On this see, for example, Tullock (1974, Chap. 10) and Brams (1977).

produced. The observed pattern of redistribution is entirely consistent with the analysis here. The Power Equalization Principle indicates that, for the less-endowed side, the marginal product of conflictual effort aimed at taxing the remainder of society tends to be greater than the marginal product of productive effort. So "populist" politics will be profitable for the poor. More generally, any group—whether initially rich or poor—suffering a relative impoverishment will predictably tend to shift its energies from the productive toward the conflictual end of the activity spectrum. City dwellers in India riot when bus fares are raised, and in times of agricultural depression Kansas farmers will be found "raising less corn and raising more hell."

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