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EQUIVALENT CIRCUITS FOR RESONATORS AND
TRANSDUCERS DRIVEN PIEZOELECTRICALLY

ARTHUR BALLATO
ELECTRONICS TECHNOLOGY AND DEVICES LABORATORY

OCTOBER 1990

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INTRODUCTION

Mechanical analogs were considered necessary in the last century for visualizing electromagnetic phenomena. Mechanisms were familiar objects, while electricity and light represented terrae incognitae. The situation was reversed in 1914 when Butterworth first used an equivalent electrical circuit to represent a mechanically vibrating system. This was followed by Van Dyke's independent discovery in 1925 that the same circuit characterized the impedance behavior of a piezoelectric resonator. Mason subsequently introduced, in the 1930s, acoustic transmission lines, mechanical ports, and piezoelectric transformers, thereby extending the circuit to encompass electro-mechanical conversion devices of wide generality. Today, the Mason equivalent circuit is universally used for bulk and surface acoustic wave device characterizations. It has also given rise to a variety of alternative formulations such as analog networks, KLM circuits, and systems models. The pervasive nature of electronics technology in the modern world has now resulted in a situation where an electrical network is often more familiar than the actual mechanical structure that it represents. The outlook has turned completely around!

A powerful motivation for using equivalent circuits to characterize piezoelectric devices is that the using equipment is often itself completely electrical in nature. The whole ensemble thus becomes subject to analysis or synthesis from a single perspective, i.e., network theory. In this report are described a brief history of circuit equivalents through the Mason circuit, followed by a presentation of newer descriptions of piezoelectrically vibrating structures, specifically, analog networks and KLM circuits.

BUTTERWORTH-VAN DYKE AND MASON MODELS

The use of equivalent electrical networks to represent mechanical systems is pervasive today, particularly with respect to the depiction of piezoelectric resonators and transducers. This is a complete reversal of the situation existing less than one hundred years ago. One has only to read the works of Maxwell [1] and Kelvin [2] to see how ardently mechanical explanations were sought for electrical phenomena. Other analogies are treated in references [3] and [4].

Electrical network elements are schematic representations of differential equations representing electrical variables; some of these are shown in Fig. 1. All elements but the transmission line (TL) have no spatial extent associated with them. Analogies between systems are based on the isomorphism of the associated differential equations.

The situation where electrical phenomena were interpreted in mechanical terms was reversed in the case of electrically driven vibrators when Butterworth [5], and, independently, Van Dyke

[6],[7] established the equivalence of the mechanically vibrating system to a certain purely electrical network, when both were viewed at the electrical input terminals.

The Butterworth-Van Dyke (BVD) [8]-[11] circuit consists of a series string of capacitor, resistor, and inductor, all shunted by a second capacitor. It represents the input immittance of a piezoelectric vibrator in the vicinity of a single resonance. When the vibrator is not freestanding, but is instead loaded at its mechanical ports for operation as a transducer, the representation is that of the three-port shown in Fig. 2. This representation is due to W. P. Mason [12]. The elements are constant in value.

Mason later gave an exact three-port circuit, [13], shown in Fig. 3, for the case of a one-dimensional resonator, driven piezoelectrically in a single mode, with the applied field normal to the direction of the wave propagation. This network contains three lumped elements that are transcendental functions of frequency. He stated that these were necessary to take into account the acoustic wave propagation in the crystal. Mason's contributions have been pervasive in the area of equivalent circuits of bulk and surface wave resonators and transducers [14]-[22].

It was soon realized that a distinction had to be made according to whether the direction of the driving electric field was along or normal to the direction of acoustic wave propagation. In the latter case the electrical input circuit [23] consists only of a shunt capacitor, usually denoted C_0 ; in the former case, the input circuit consists additionally of a series capacitor of value $-C_0$. The case where field and propagation are perpendicular is referred to as "crossed-field," or "unstiffened"; when they are parallel it is known as "in-line," or "stiffened." In the case of a simple thickness mode excited in a crystal plate, the two situations are called "lateral-excitation," or LE (field perpendicular to the thickness direction along which the waves propagate), and "thickness-excitation," or TE. In the TE case, the self-generated field of the wave produces a reaction that leads to the negative C_0 ; this field also produces an effective elastic stiffness that is increased over the isagric value, hence the name "stiffened." This increase is often, but not always, absent when LE is used; the appellation "unstiffened" is therefore a misnomer and is to be used with caution.

Redwood and Lamb [24]-[28] recognized that the three lumped transcendental elements employed by Mason could be replaced by the more graphic transmission line (TL) representation shown in Fig. 4. This figure is drawn for the "in-line" case. The TL schematic chosen by Redwood and Lamb is that of a coaxial cable. An alternative schematic is that of the two-wire TL, also known as a Lecher line; the lumped-circuit equivalents of the two-wire line are shown in Fig. 5. Fig. 4 is redrawn in the two-wire

manner in Fig. 6. The Lecher line was used extensively by H. Hertz in his electromagnetism experiments. When the transmission-line schematic is thus redrawn it is easy to see that the piezoelectric excitation occurs in series with the forces appearing at the mechanical ports. One may, in fact, split the piezo transformer into two parallel pieces and place them at the TL ends as seen in Fig. 7. The mechanical forces have here been set to zero as indicated by the short circuits placed at the mechanical ports.

ANALOG NETWORKS

Figure 7 places in evidence the concept of excitation taking place by the imposition of piezo tractions at the surfaces of the crystal plate, instead of volumetrically distributed throughout the crystal due to the interaction of the electric field and the dipoles. The piezoelectric portion of the constitutive relation between the elastic stress field and the electric field intensity is

$$T_{ij} = - e_{kij} E_k. \quad (1)$$

The force-tractions acting are the derivatives:

$$F_j = T_{ij,i} = - e_{kij} E_{k,i}. \quad (2)$$

The electric field suffers a delta-function discontinuity at the TE electrodes; this can be interpreted as a piezo-traction impressed at the surfaces.

In Fig. 8 are shown the physical interpretations of the various circuit elements that comprise the electrical equivalent circuit of a crystal resonator with electrodes that have mass, but whose mass can be considered to be lumped at the surface. Figure 9 gives the complete network for a TE plate supporting a single piezoelectrically excited simple thickness mode. Also shown is the BVD lumped equivalent valid in a small frequency region about a single resonance.

Figure 10 introduces additional symbols associated with the BVD circuit, and shows the effect of placing a "load" capacitor, C_L , in series with the crystal resonator. This is usually the means employed to adjust the resonator frequency of operation in an oscillator. The combination can be exactly expressed in terms of a four-element BVD circuit having modified values. Strictly speaking, the simple BVD network shown is obtained directly only in the LE case, where no negative capacitor is present. Otherwise, the negative C_0 appears in series with the C_1 , and the modified capacitance ratio r' is just $r - 1$. Often $r \gg 1$, so the modification is slight. In the LE case, it is not desirable to place the load capacitor in series; the load capacitor is placed in parallel, and the resultant modified elements are easily determined. The immittance behavior of the BVD network is shown as a function of frequency in Fig. 11.

When, as in Fig. 9, more than one mode is driven, the piezo transformers are extended to the other TLs in the manner shown in Fig. 12. Figure 13 gives the network for a single driven mode, with physical interpretations of the various elements superimposed.

The resonator equivalent circuit for all three modes driven, with no forces applied at the mechanical ports, may be bisected by the methods of network theory to give the representation of Fig. 14. Each TL is of length equal to one-half of the plate thickness; the piezo transformer turns ratios are functions of the piezoelectric coupling coefficients, k_m , for mode m .

By a partial-fractions expansion about the poles of the TL impedance, each mode may be represented exactly with lumped elements, as shown in Fig. 15. The circuit elements differ, of course, for each mode. This is an extended version of the BVD circuit, with each shunt branch realizing a single harmonic of the mode.

Use of transmission lines to represent acoustic waves in distributed structures leads to a "building-block" approach to equivalent circuit realizations [29]. This is illustrated in Fig. 16 showing a thin-film piezo layer deposited on a substrate; the circuit realization is given in Fig. 17.

In the brief discussion above, the TL circuits have been introduced without a consideration of the specific analogies between the electrical and mechanical variables. A more detailed treatment leads to the introduction of "normal coordinates" [30] to represent the TL variables, since, e.g., the three motions in an infinite plate supporting simple thickness motion represent the eigenmodes of the system, and each is accorded its own TL. At the boundaries, one must transform between the normal coordinate basis particular to the crystal type and its orientation, and the laboratory coordinate system. This is accomplished in network terms by the transformer arrangement of Fig. 18, where the turns ratios are the eigenvector components of each mode [29],[31]. The complete representation of a thickness mode plate is shown in Fig. 19.

By interconnecting transformers of the kind shown in Fig. 18, one is able to represent the juxtaposition of plates with various boundary conditions. Two media joined by welded boundaries (stress and displacement continuous across the interface) are represented in Fig. 20. Often the connections are simplified considerably, because of either the relative crystal orientations, or because of the materials, or both.

One of the virtues of the analog form is the ease with which the building-block approach can be used to accommodate more difficult situations. One such is Mindlin's problem, where

mechanical surface tractions applied to a rotated-Y-cut quartz plate produce not only mechanical-acoustic motion, but also electromagnetic radiation [32]-[34]. Figure 21 models this situation.

A systems approach to equivalent networks has been taken by Hayward [35],[36]. In this method, Laplace transforms and linear systems theory are used to produce a circuit representation that is cast in signal-flow graph form.

Additional equivalent networks have been developed to model magnetostatic waves [37] and bulk acoustic wave resonators of piezoelectric semiconductors [38]. This latter network is given in Fig. 22. The procedure can be applied in finite element simulations [39]. When a resonator is coated with electrodes that are of a thickness such that wave propagation within the electrodes cannot be neglected, the lumped inductors seen in Figs. 8 and 9 must be replaced by TLs. An example is shown in Fig. 23.

KLM EQUIVALENT CIRCUITS

The Krimholtz, Leedom, & Matthaei (KLM) equivalent circuits [40]-[44] are quite different in nature from all of the foregoing, and have the following advantageous features:

- ▲ They are exact, in the one-dimensional approximation, and for a single mode type (for example, a ceramic resonator stack).
- ▲ An arbitrary number of stacked transducers is accommodated, but each must be acoustically identical.
- ▲ A simple network representation is produced, having:
 - series reactance or shunt susceptance (dispersive);
 - piezo transformer or immittance inverter (dispersive);
 - acoustic transmission lines (nondispersive).
- ▲ Mechanical ports are not in series with piezo voltage.
- ▲ Arbitrary spatial variation of piezo excitation is allowed:
 - "in-line" (stiffened modes)
 - "crossed-field" (unstiffened modes)
- ▲ Simplifying algorithms exist for certain excitations.

The KLM networks have the following disadvantageous features:

- ▼ Single modal type allowed, with density and stiffness constant, or averaged.
- ▼ Members of the stack are all connected either in series, or all in parallel.
- ▼ Circuit components are dispersive; this is unphysical.
- ▼ Networks are equivalent only at the three ports:
 - no building-block approach is possible.
- ▼ Complexity is traded from network schematic to component values:
 - Fourier transforms $F(k)$, spatial, of excitation

TABLE 1. KLM CIRCUITS

Excitation	Electrical Input	TL Feed	Spatial Variation
In-Line "stiffened"	Series C and X_1 o transformer, ϕ_p o Z-inverter, K	parallel series	even function odd function
Crossed-Field "unstiffened"	Shunt C, B_1 o Y-inverter, J o transformer, ϕ_s	parallel series	even function odd function

$F(k)$ = spatial Fourier transform of the excitation function;
 K is proportional to $F(k)$; J and ϕ_s are proportional to
 k times $F(k)$; ϕ_p is proportional to $1/F(k)$; X_1 and
 B_1 are proportional to $H(|F(k)|^2)$.

- Hilbert transforms $H(|F(k)|^2)$

- ▼ Frequency domain operation only; $e^{i\omega t}$
- no time domain, transient operation.

These features are summarized in Table 1.

Figure 24 shows the KLM circuit for a thickness-stretch plate of thickness l with electrodes on the major surfaces.

CONCLUSION

A brief review has been given of the genesis of equivalent networks for piezoelectric resonators and transducers. This led to a consideration of various types of representations, such as lumped element, multimode, analog, systems, and KLM types.

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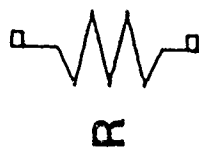
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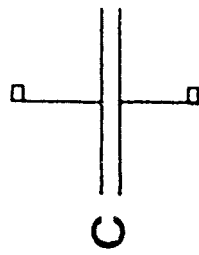
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Resistor



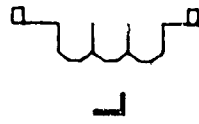
$$V = IR$$

Capacitor



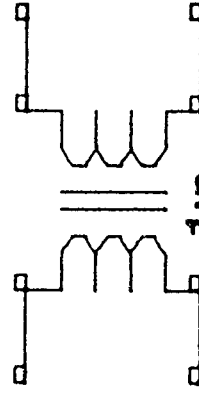
$$I = C \dot{V}$$

Inductor



$$V = L \dot{I}$$

Transformer

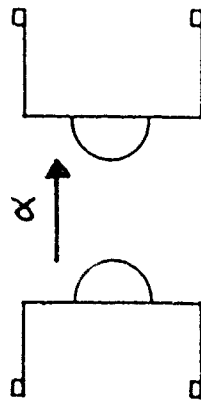


$$V_2 = nV_1; I_1 = -nI_2$$

$$V_1 I_1 = 0$$

11

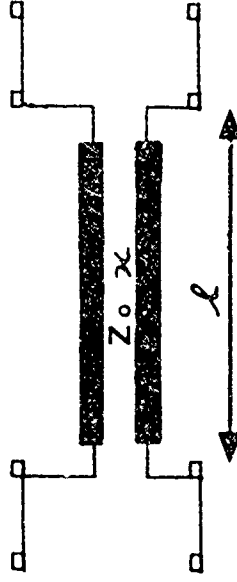
Gyrator



$$V_1 = -\alpha I_2; V_2 = \alpha I_1$$

$$V_1 I_1 = 0$$

Transmission Line



$$V_{,1} = -j \omega Z_0 I$$

$$I_{,1} = -j \omega \frac{1}{Z_0} V$$

Figure 1. Network elements.

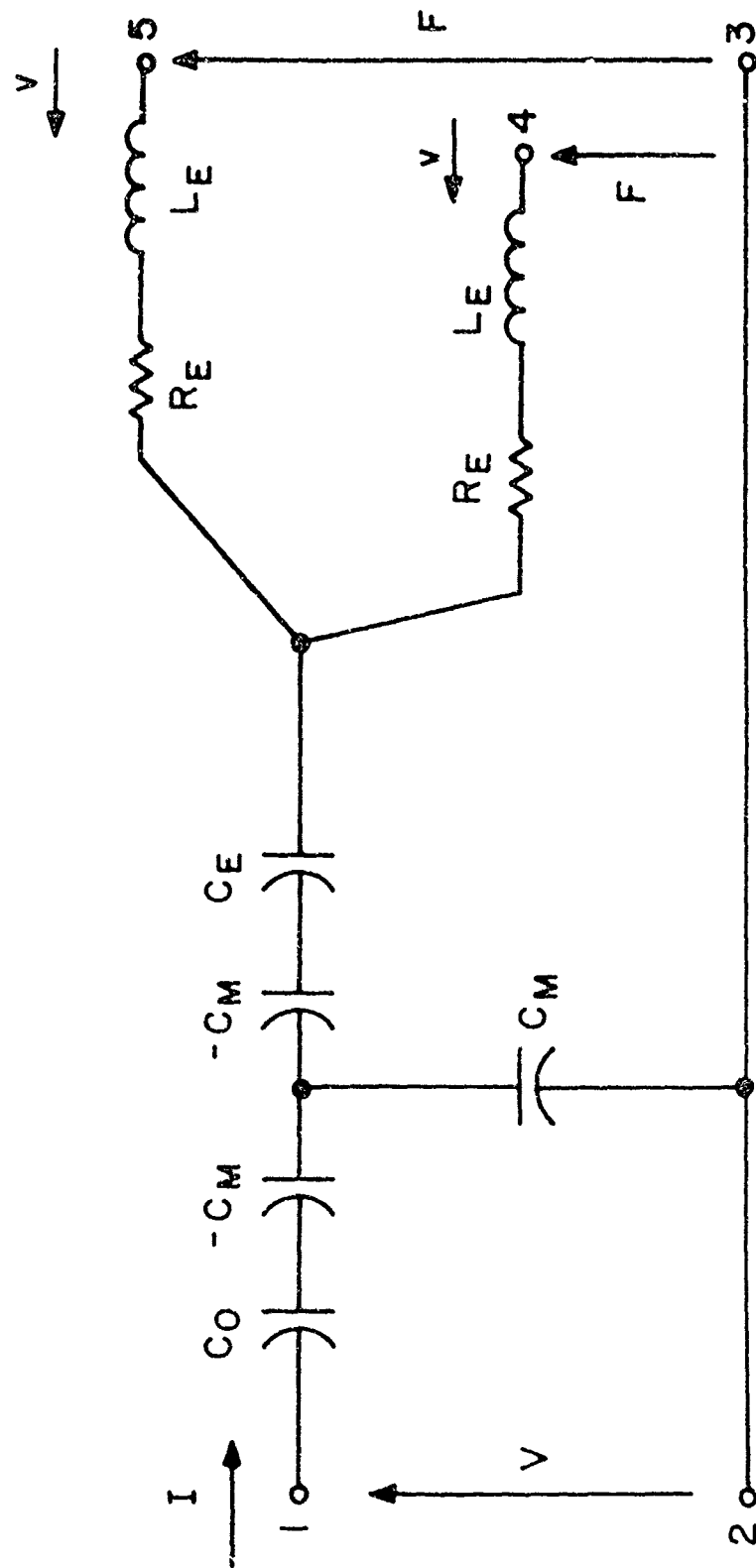


Figure 2. Mason: Three-port, single resonance, equivalent circuit.

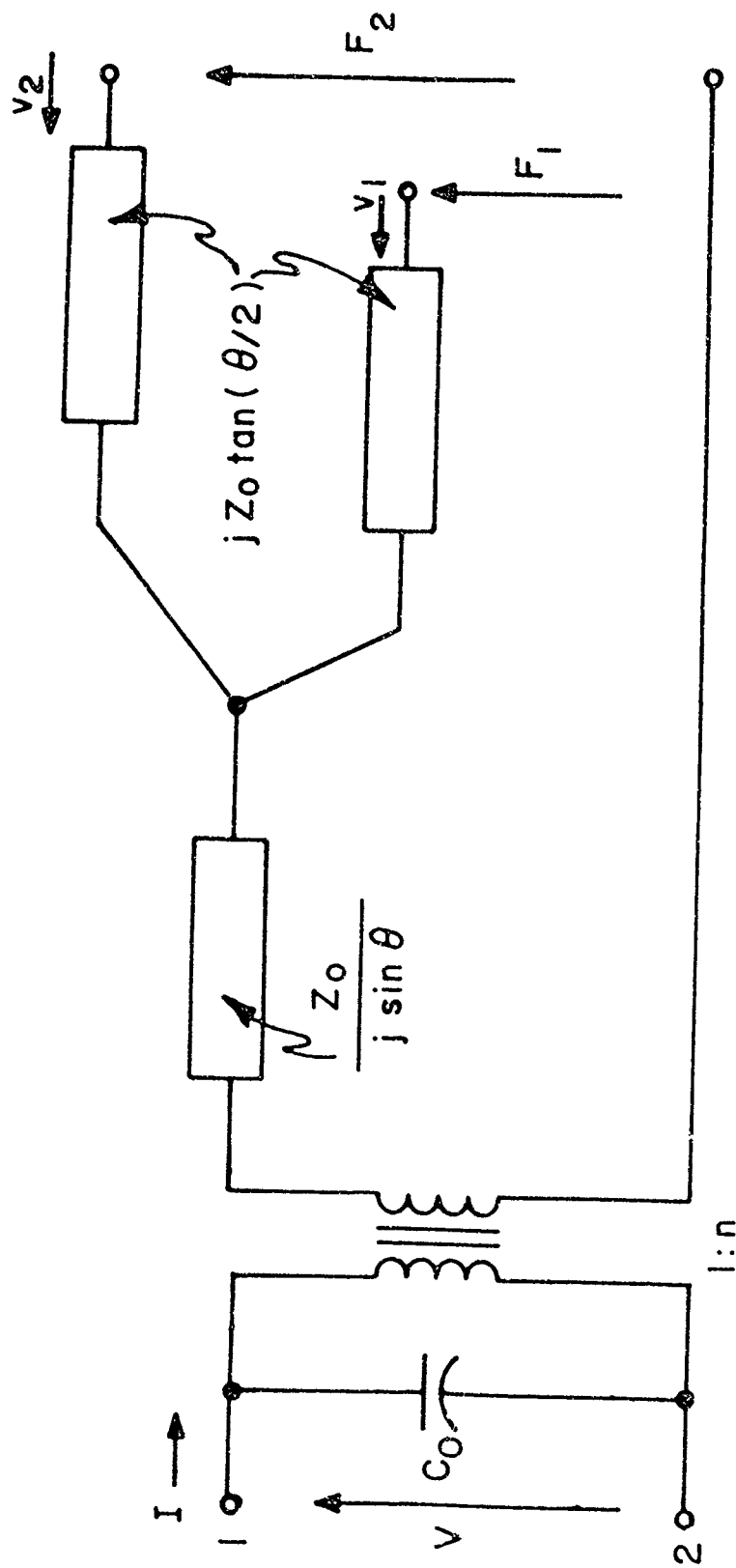
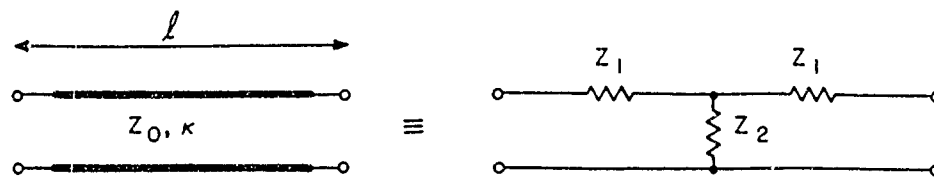


Figure 3. Mason: Exact, three-port, single mode-type, equivalent circuit. Applied field normal to wave propagation.



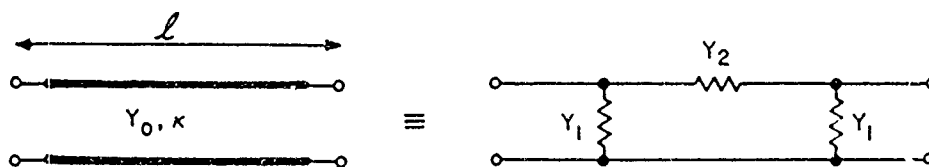
Figure 4. Redwood and Lamb: Transmission-line schematic of Mason's exact, three-port, single mode-type, circuit. Applied field parallel to wave propagation.



$$Z_1 = \frac{Z_0}{j \sin \theta} (\cos \theta - 1) = j Z_0 \tan(\theta/2)$$

$$Z_2 = \frac{Z_0}{j \sin \theta} ; \theta = \kappa l$$

LUMPED, TEE, FORM OF A TRANSMISSION - LINE SECTION.



$$Y_1 = \frac{Y_0}{j \sin \theta} (\cos \theta - 1) = j Y_0 \tan(\theta/2)$$

$$Y_2 = \frac{Y_0}{j \sin \theta} ; \theta = \kappa l$$

LUMPED, PI, FORM OF A TRANSMISSION - LINE SECTION

Figure 5. Transmission line lumped equivalents.

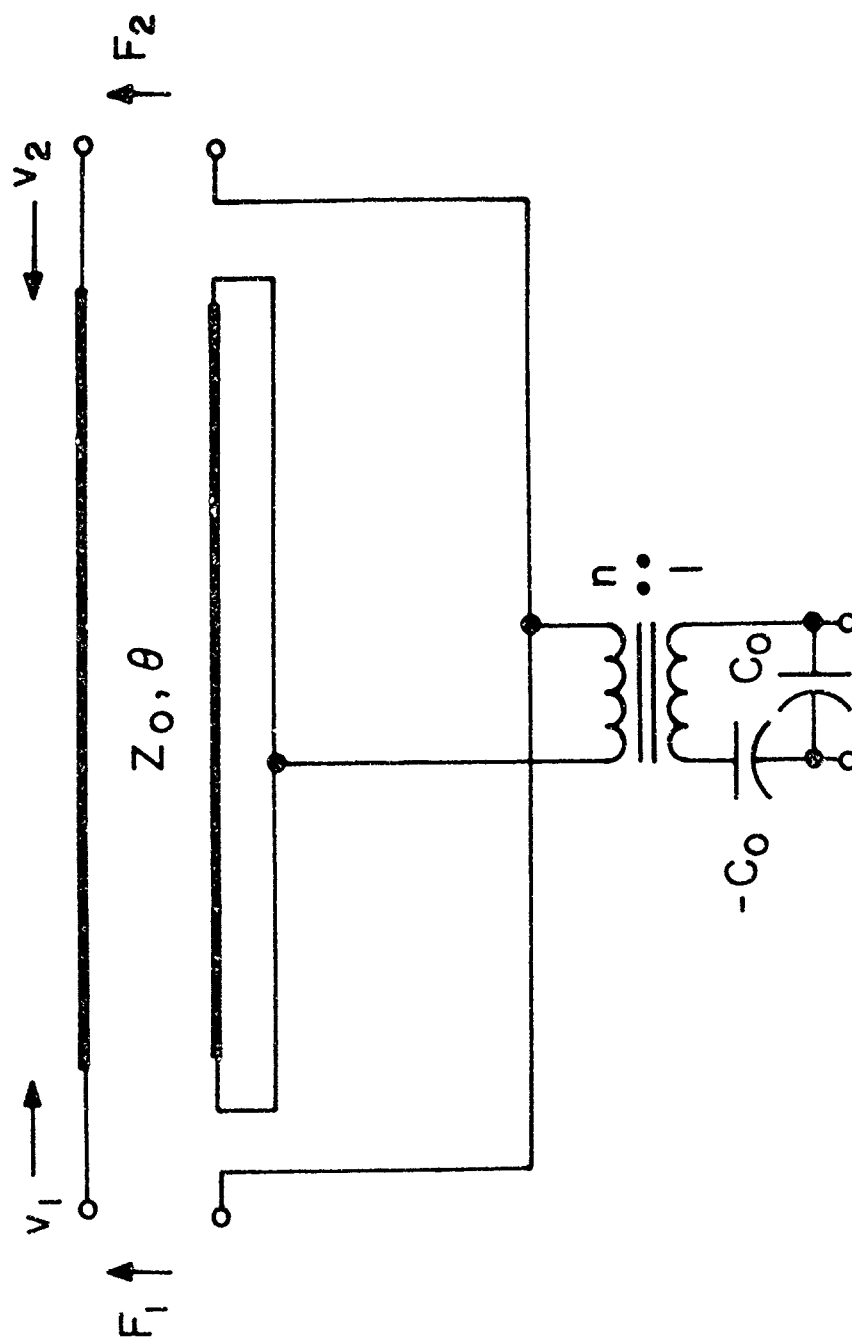


Figure 6. Two-wire transmission line schematic of Mason's exact, three-port, single mode-type, circuit. Applied field parallel to wave propagation.

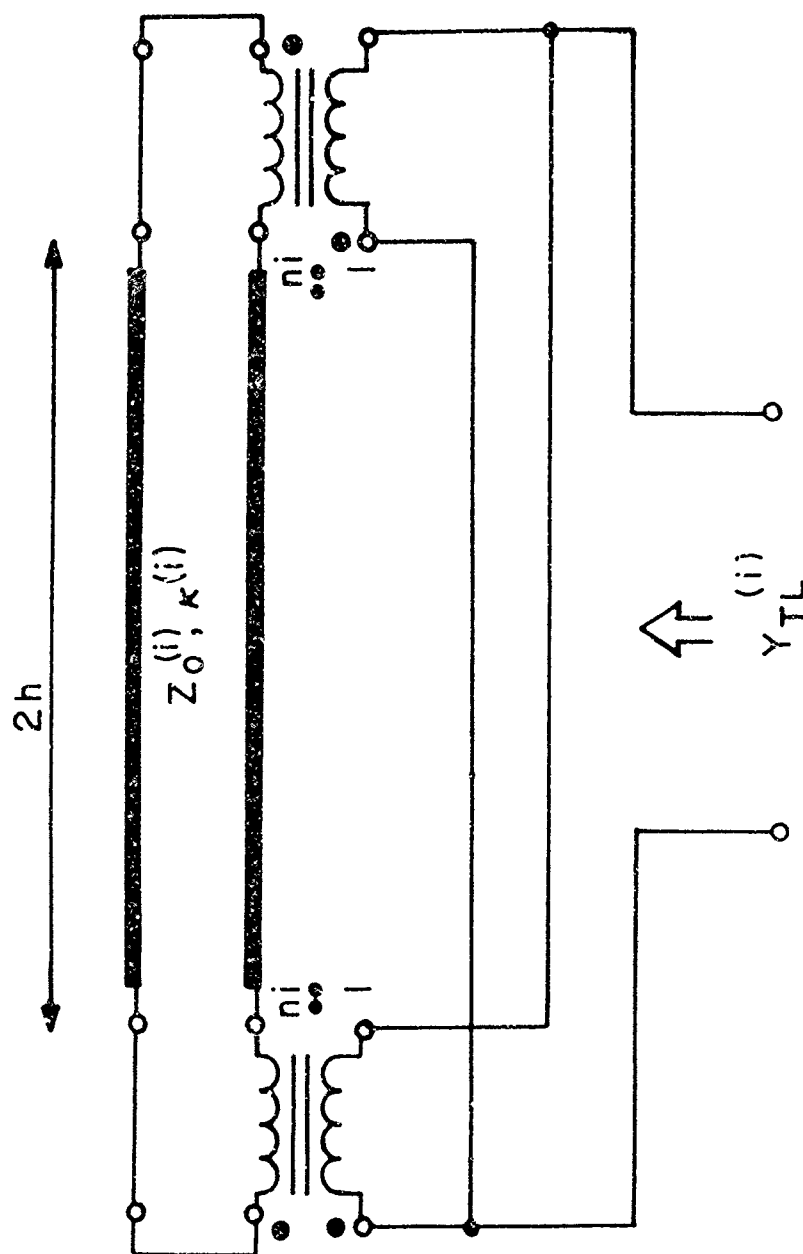


Figure 7. Traction-free plate. Representation of single thickness mode, electrical input circuit omitted.

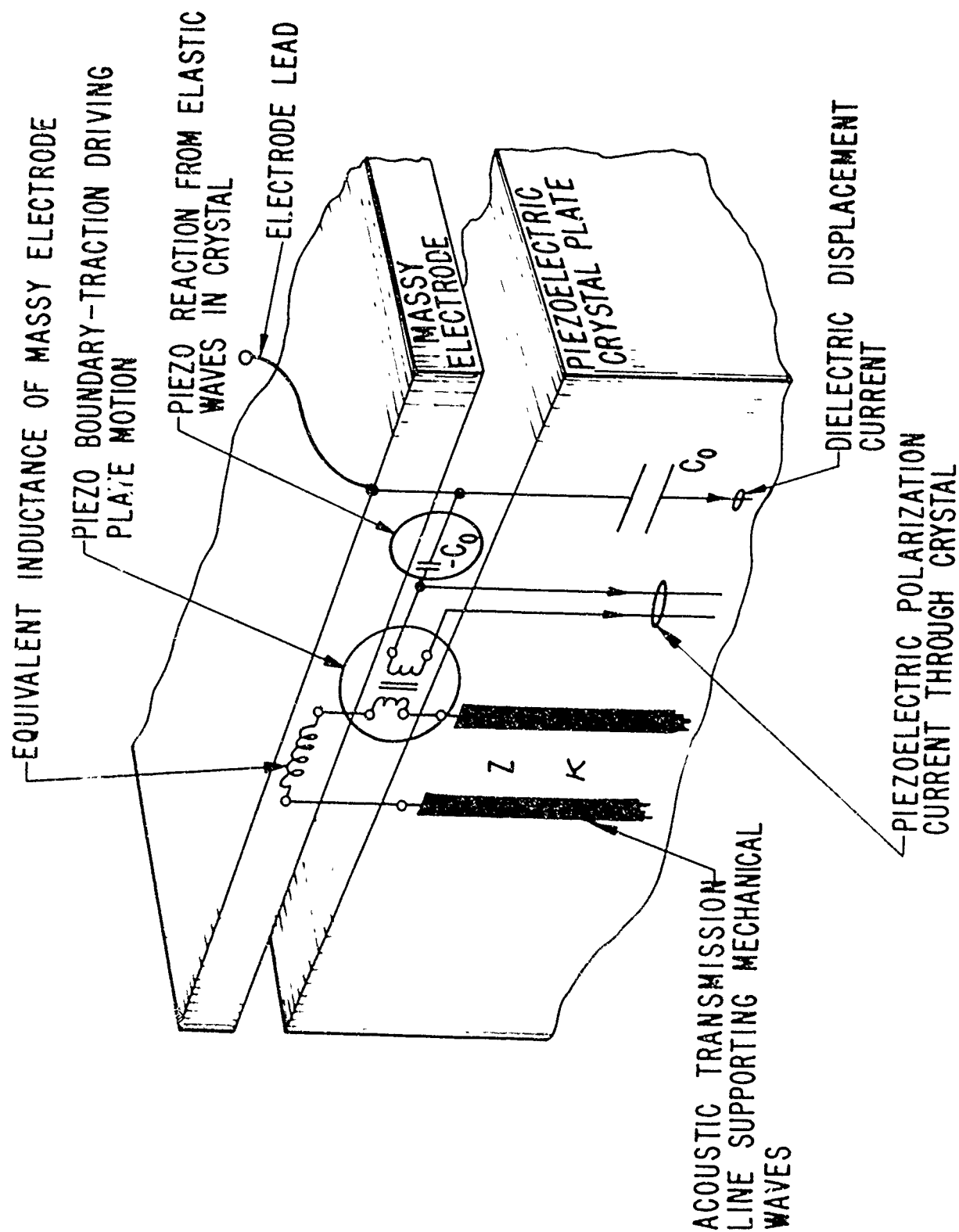


Figure 8. Exploded view of crystal and massy electrode, with electrical analog network superposed.

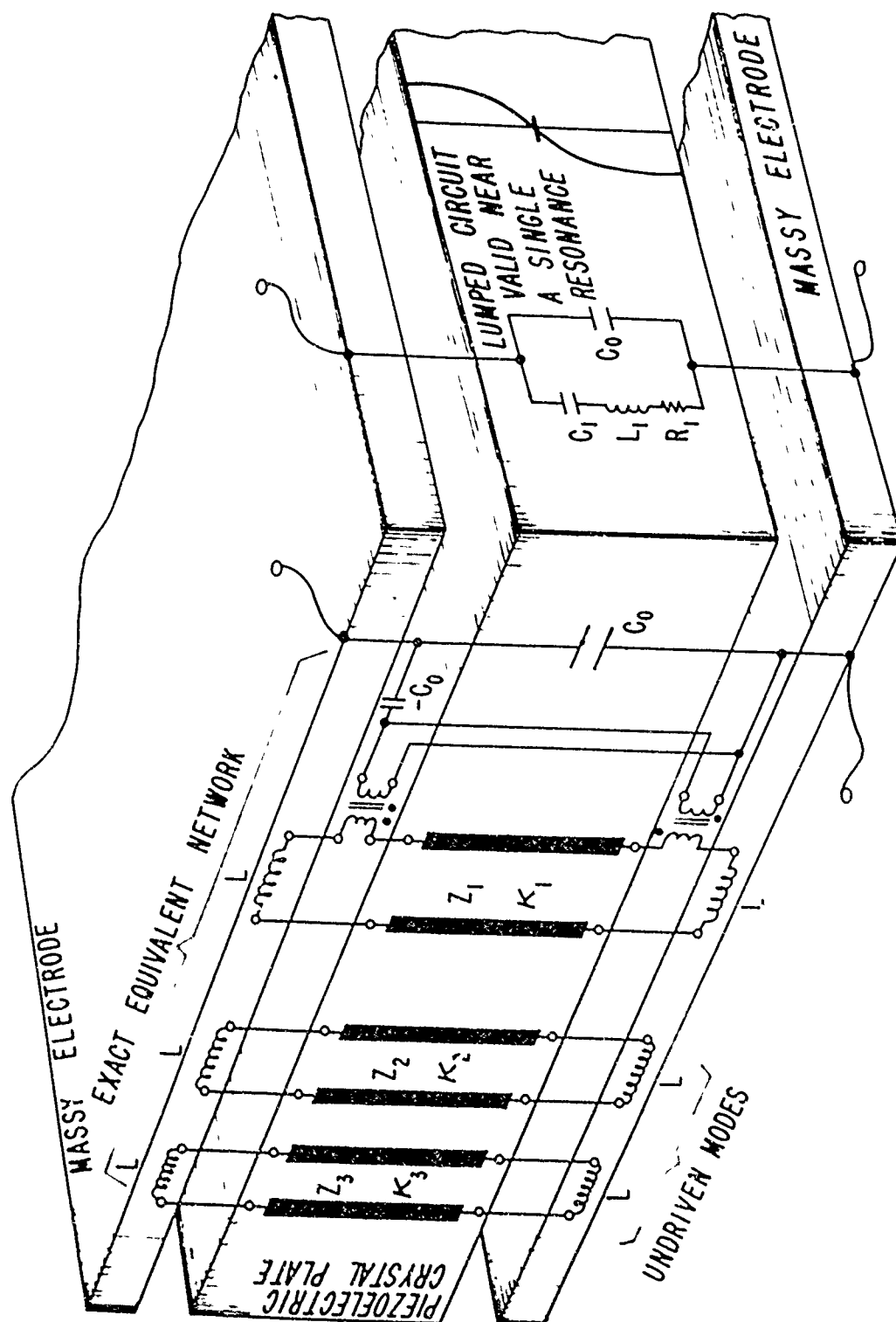
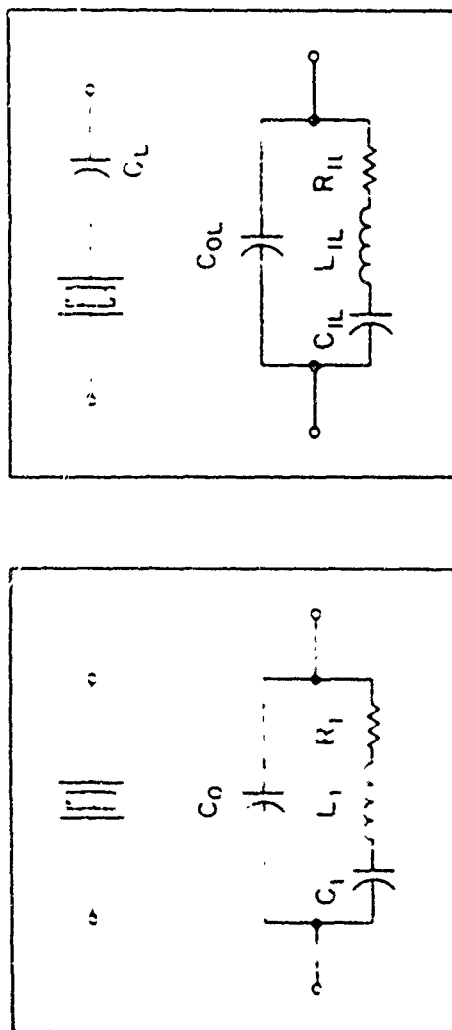


Figure 9. Exploded view of a crystal plate with mass electrodes, for the case of a single thickness mode piezoelectrically driven. Superposed are the exact and lumped approximate equivalent electrical networks.



$$f_g = 1/2 \pi \sqrt{L_1 C_1}$$

$$r = C_0 / C_1$$

$$\alpha = C_0 / (C_0 + C_L)$$

$$f_L = f_g \sqrt{1 + \alpha / r}$$

$$r_1 = r / (1 - \alpha)$$

$$C_{OL} = C_0 (1 - \alpha)$$

$$C_{IL} = C_1 (1 - \alpha)^2 / (1 + \alpha / r)$$

$$R_{IL} = R_1 / (1 - \alpha)^2$$

$$L_{IL} = L_1 / (1 - \alpha)^2$$

Figure 10. Butterworth-Van Dyke networks.

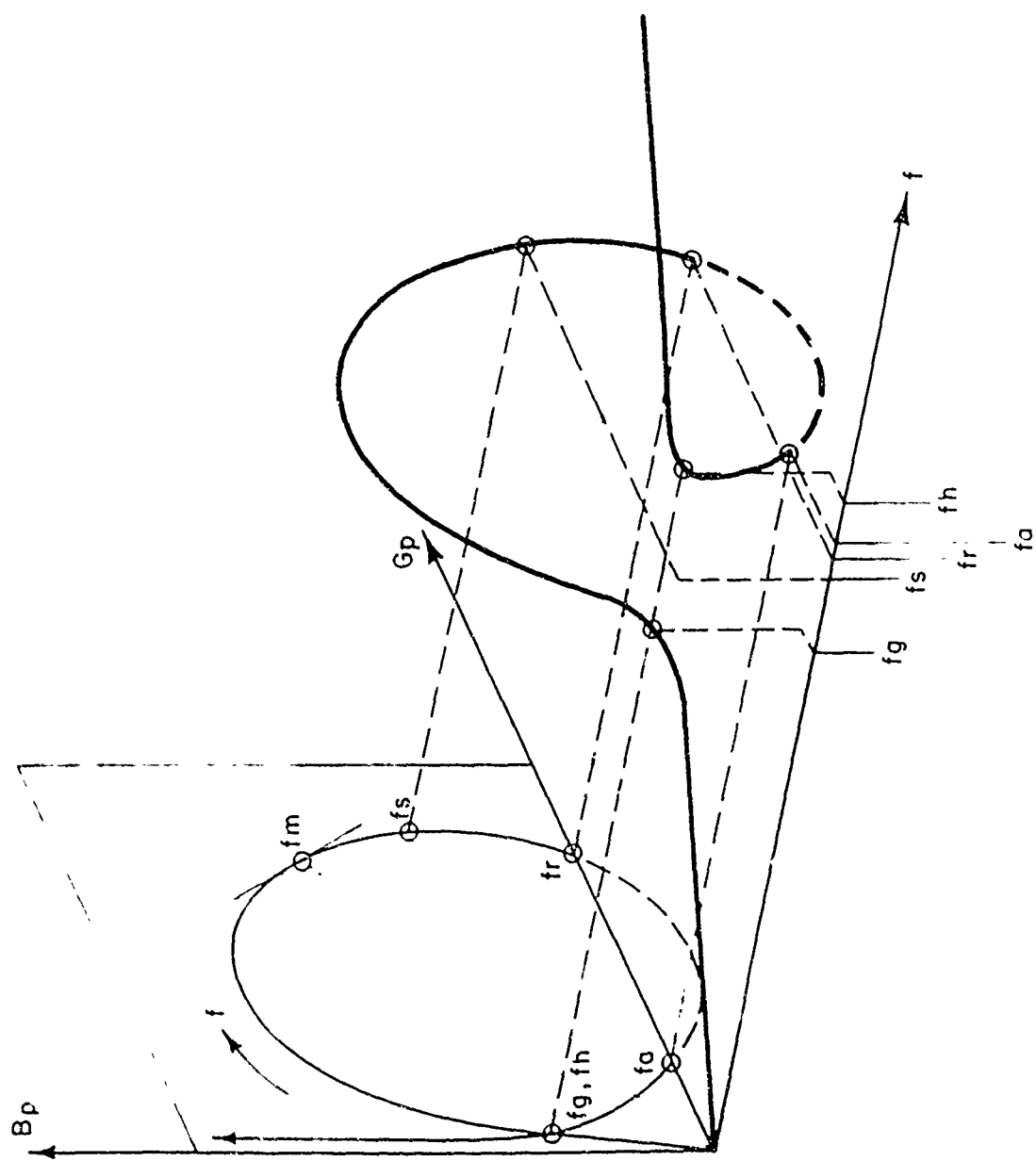


Figure 11. Immittance behavior of the Butterworth-Van Dyke circuit as a function of frequency.

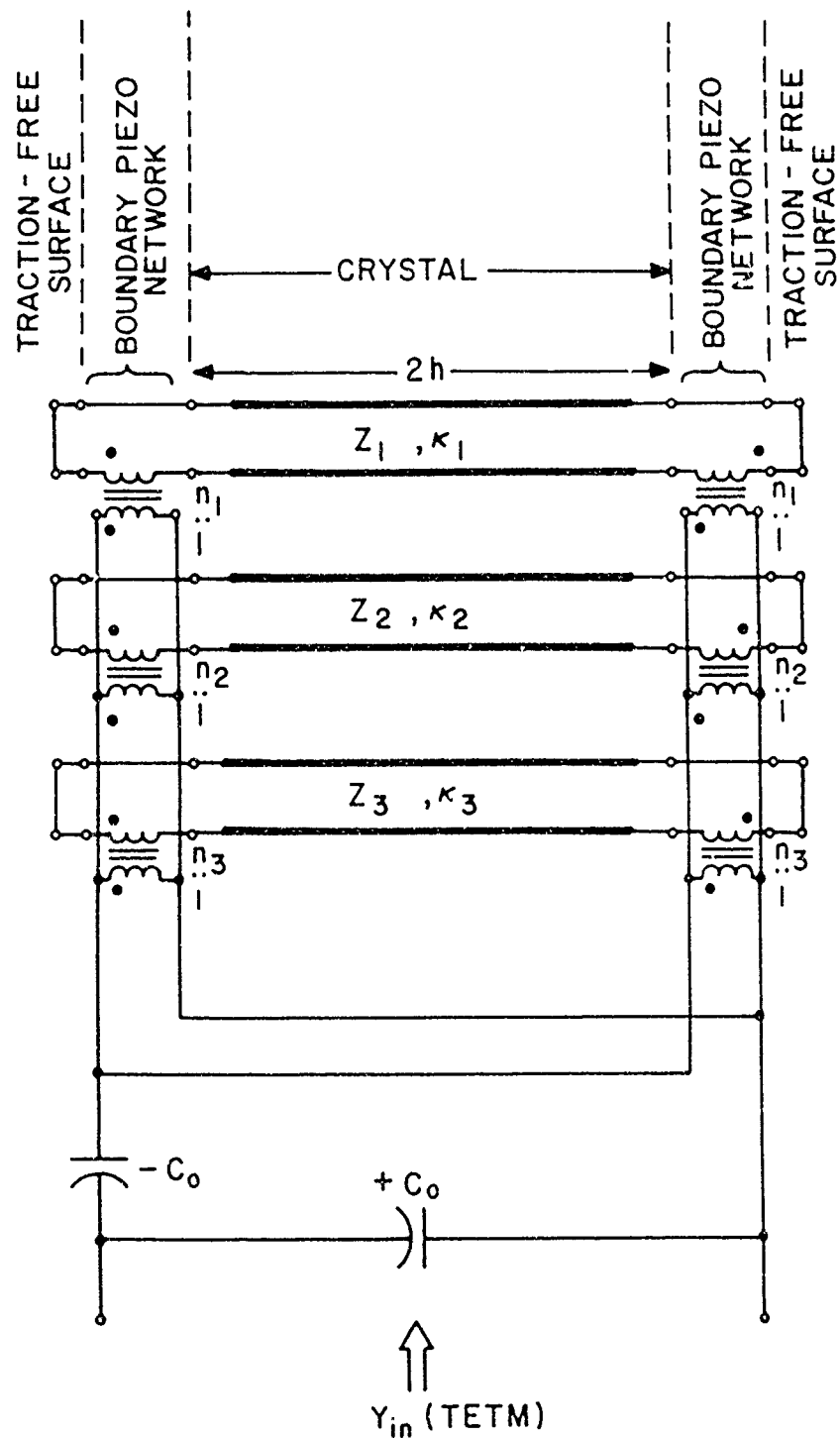


Figure 12. Equivalent network analog representation of traction-free plate, TETM.

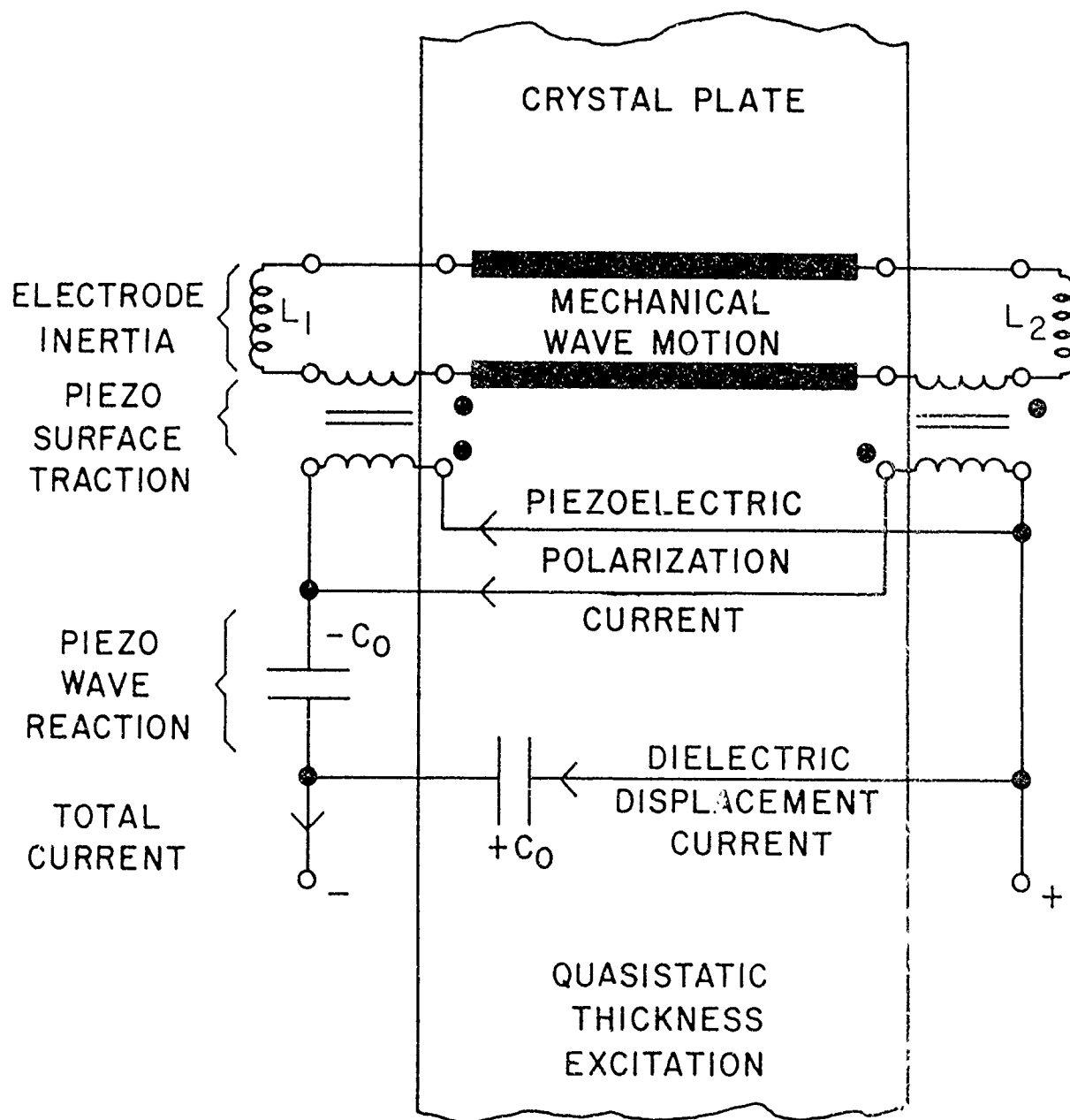
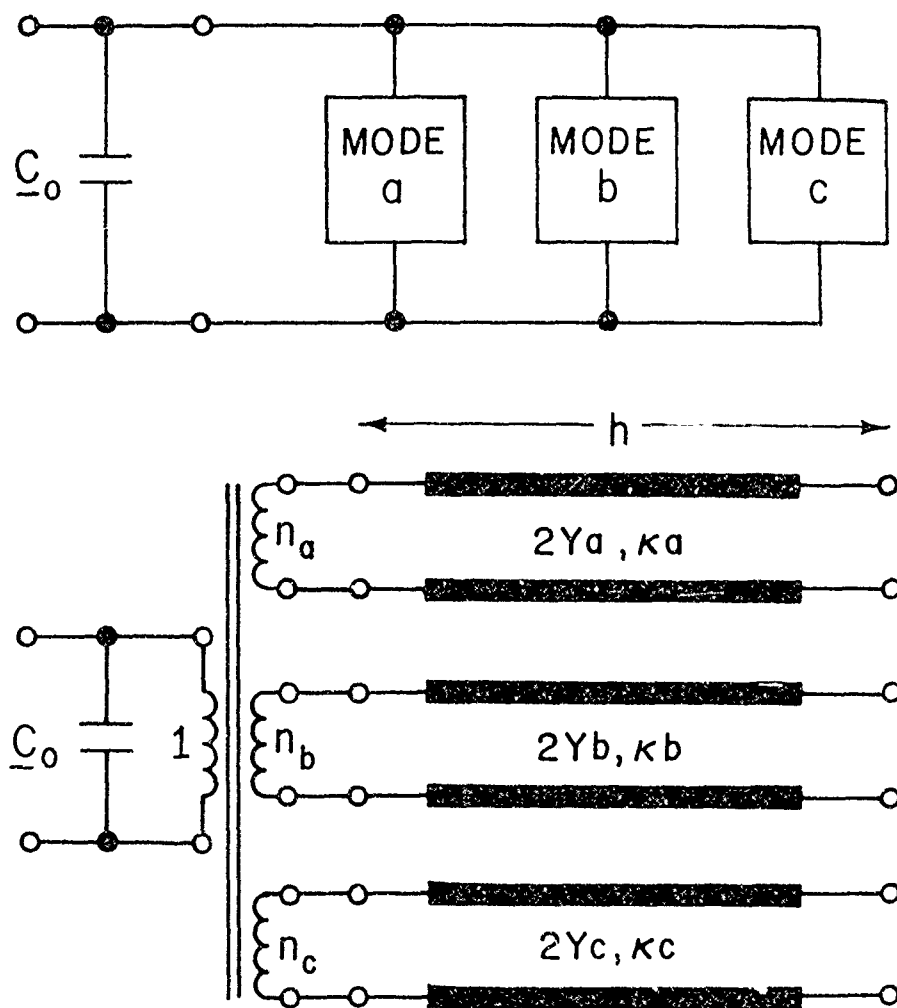


Figure 13. Analog equivalent network of a plate resonator.



$$n_m^2 = C_0 k_m^2 f_{Rm} (Y_m / 2)^{-1}$$

Figure 14. Analog network in bisected form.

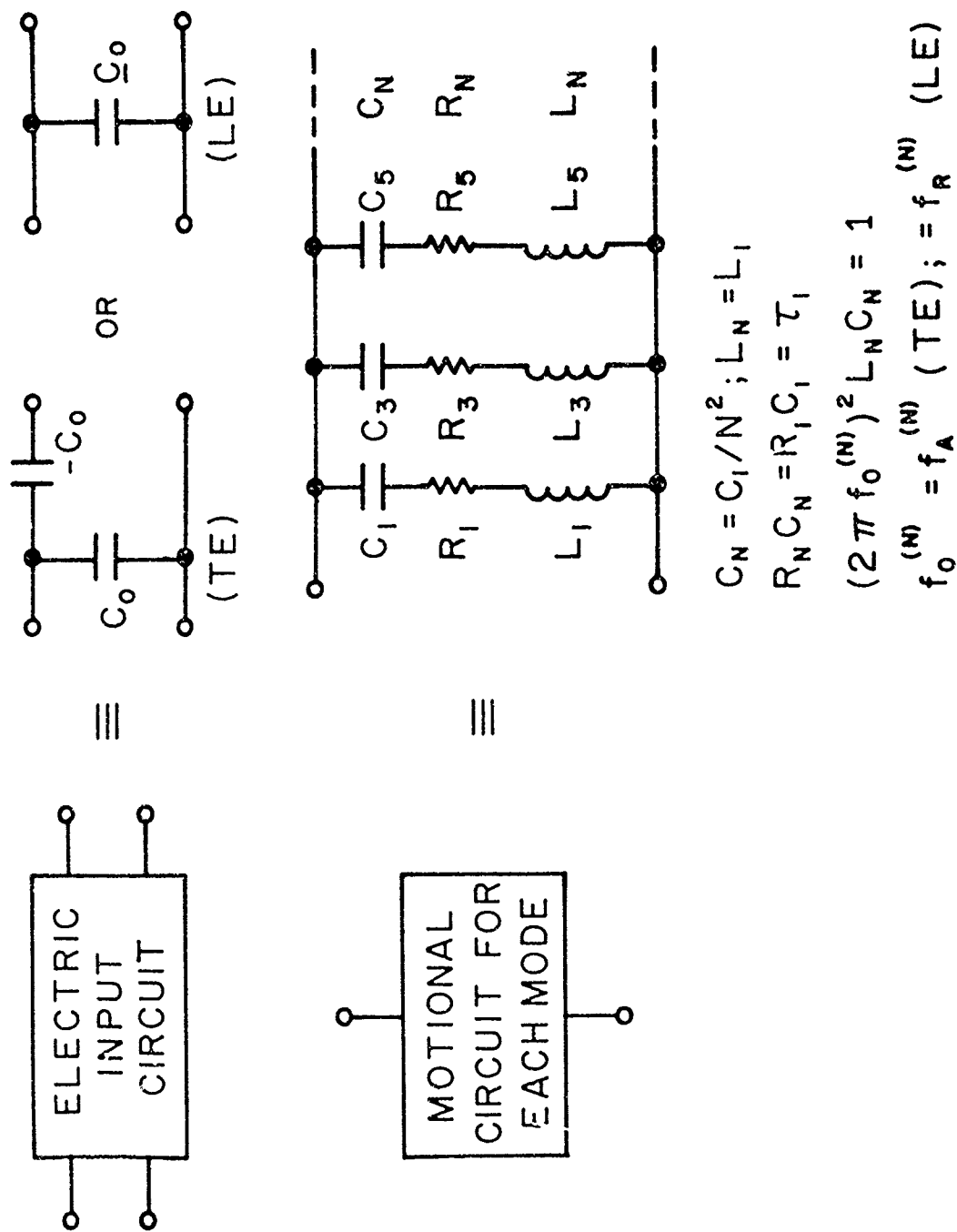


Figure 15. Equivalent network for simple thickness modes of plates.

THIN-FILM LAYER OF HIGH-COUPLING
PIEZOELECTRIC CRYSTAL DRIVES THE
COMPOSITE STRUCTURE.

ELECTRODES

HIGH Q
SUBSTRATE

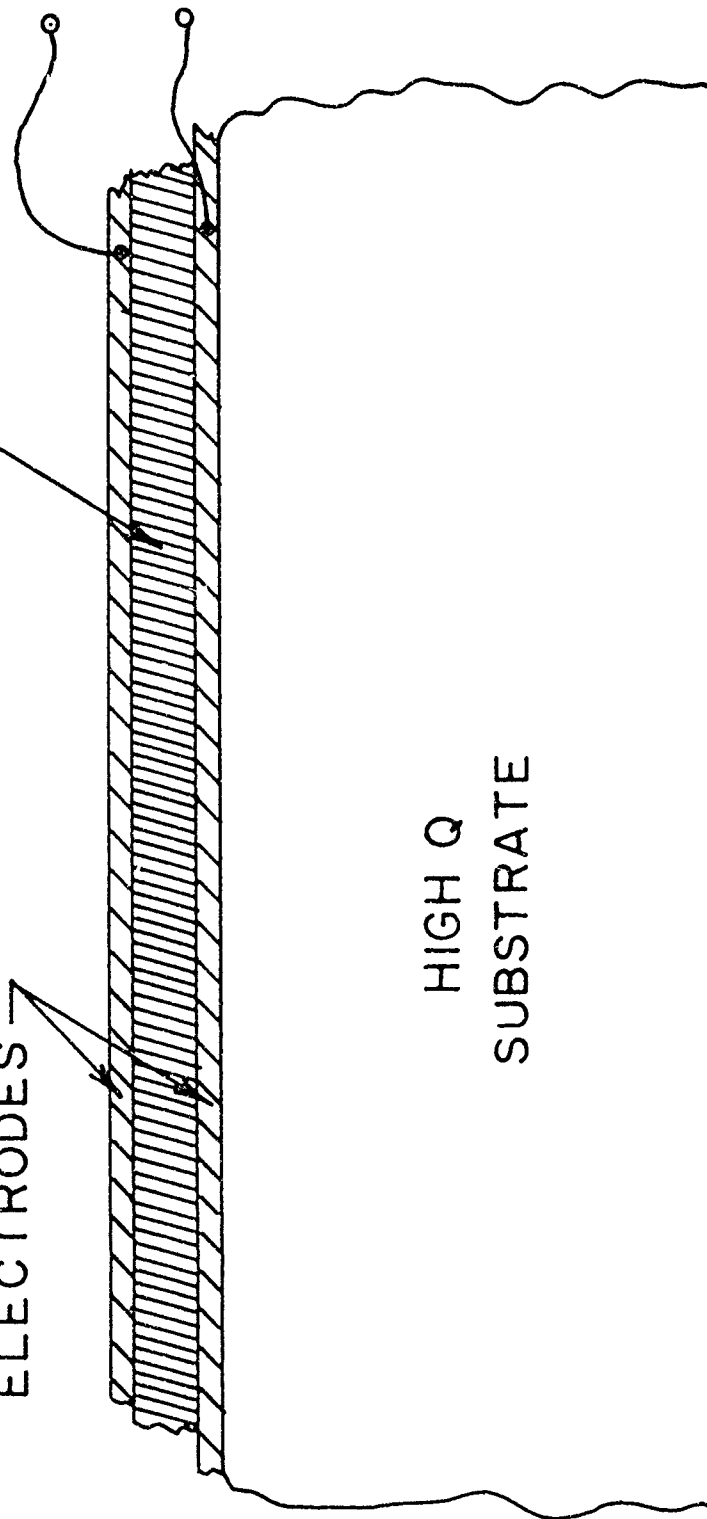


Figure 16. Composite resonator structure.

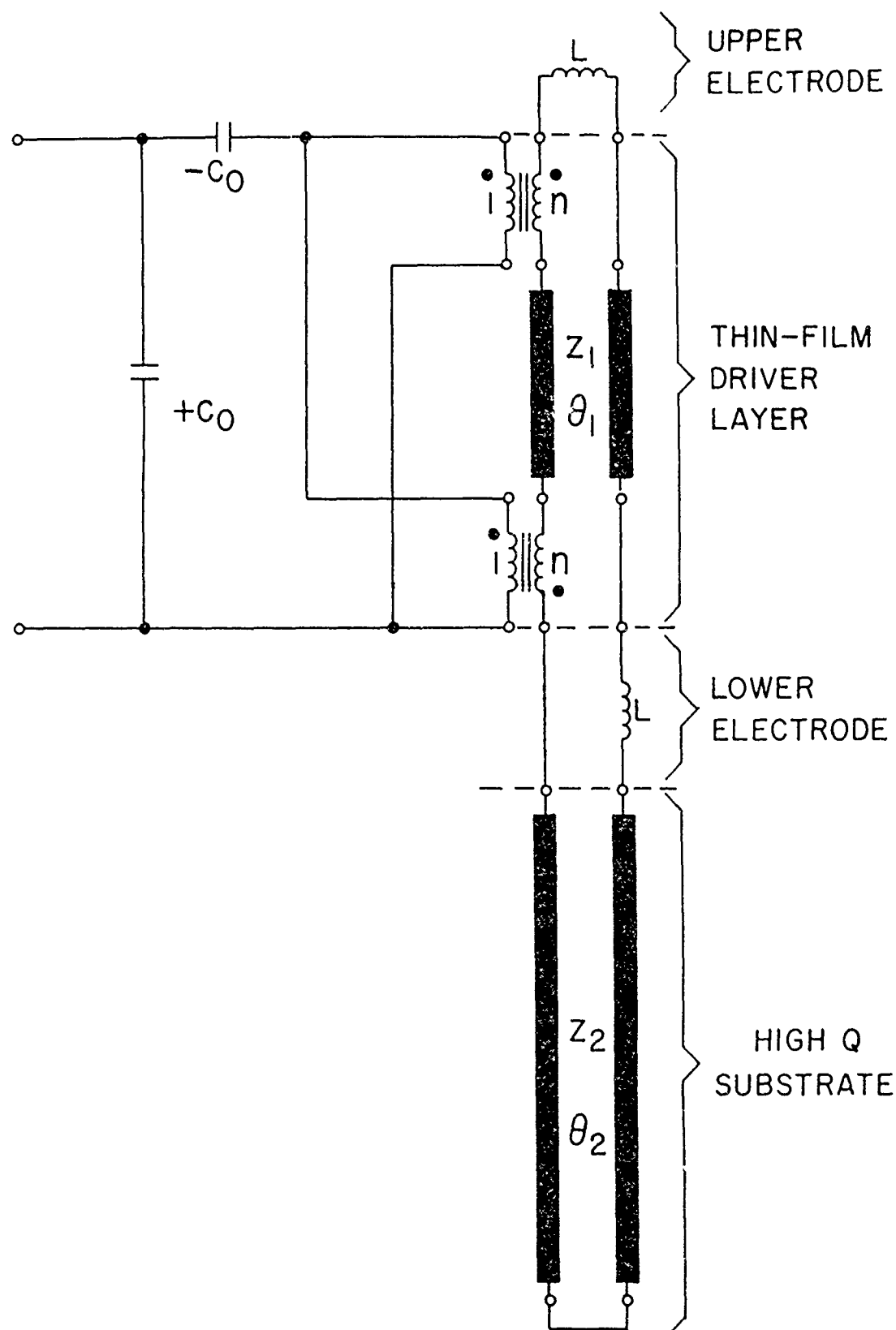


Figure 17. Equivalent network of composite resonator.

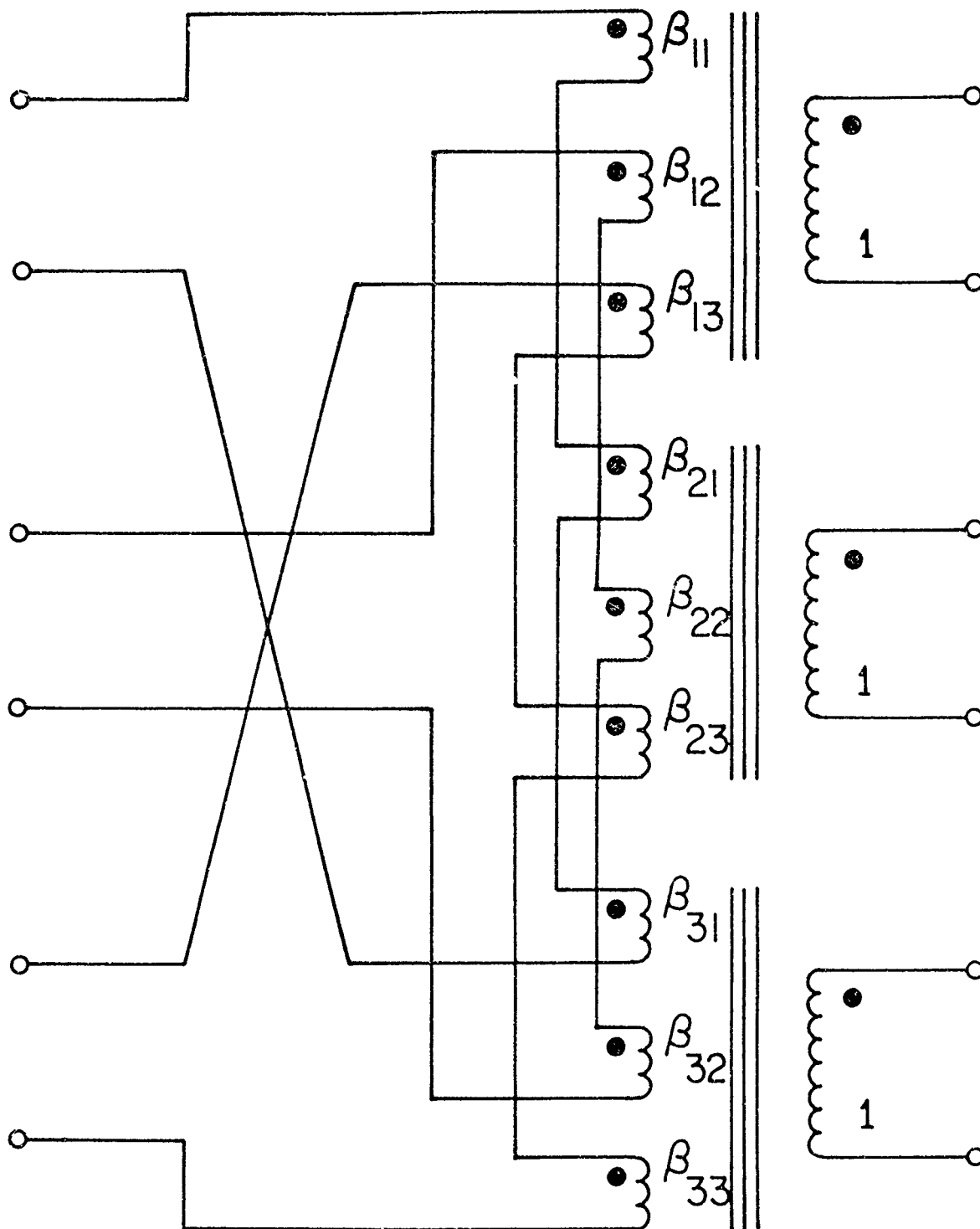


Figure 18. Network realizing an orthogonal transformation.

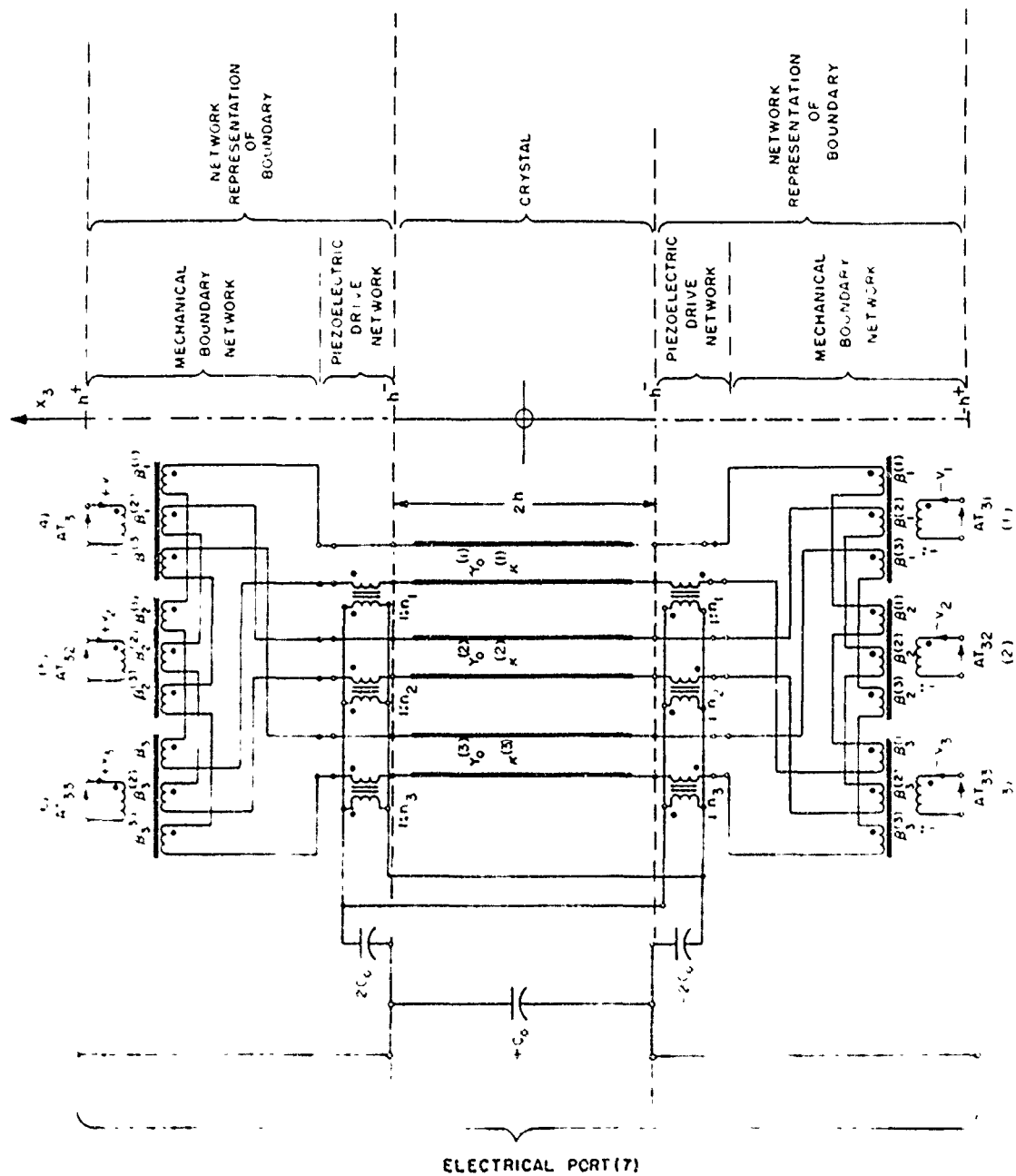


Figure 19. Exact analog representation of TETM plate, for arbitrary boundary conditions.

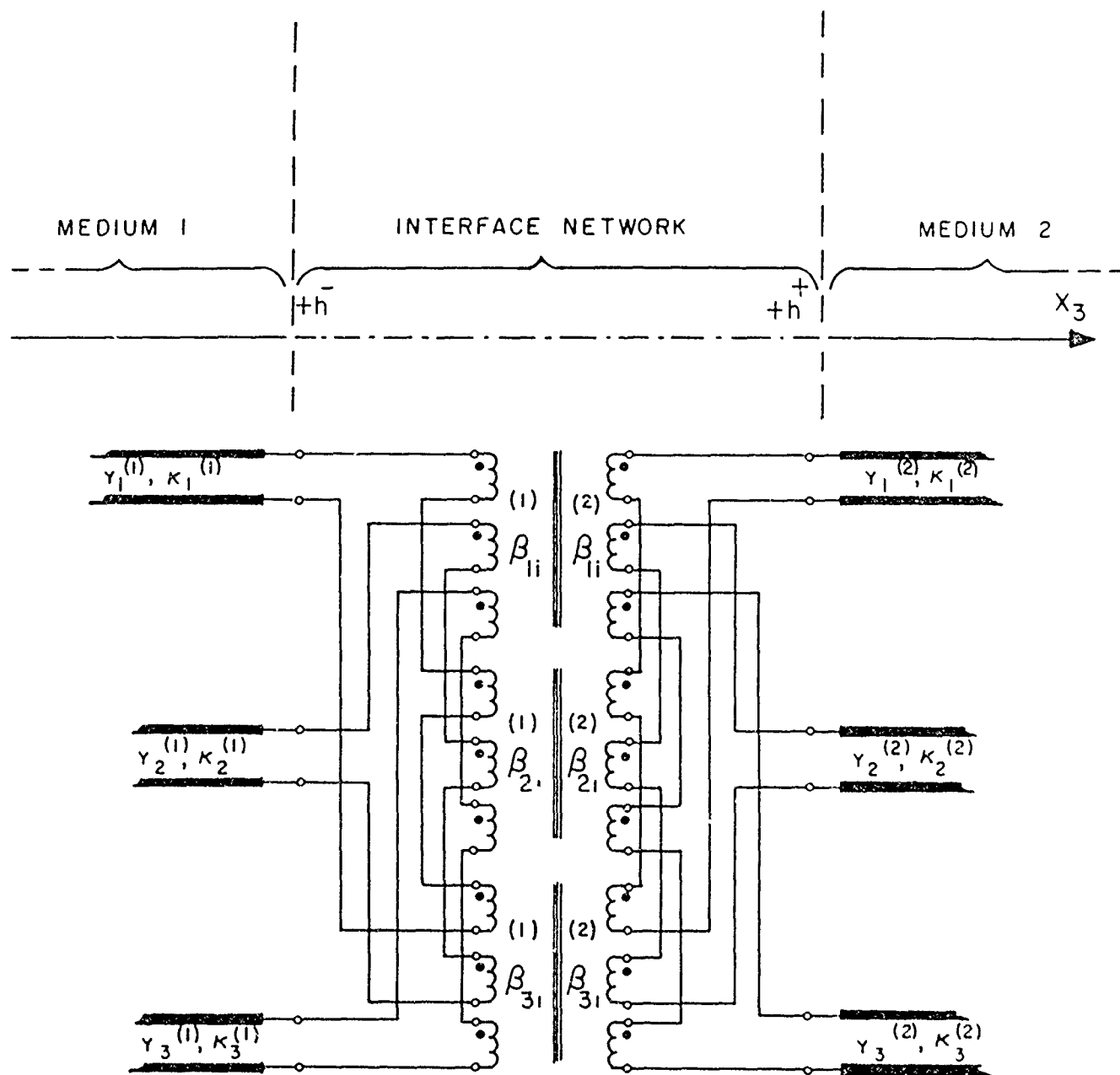


Figure 20. Exact representation of mechanical coupling at the interface of two arbitrarily anisotropic media. Plane acoustic wave propagation normal to boundary. Piezoelectric drive transformers omitted for clarity.

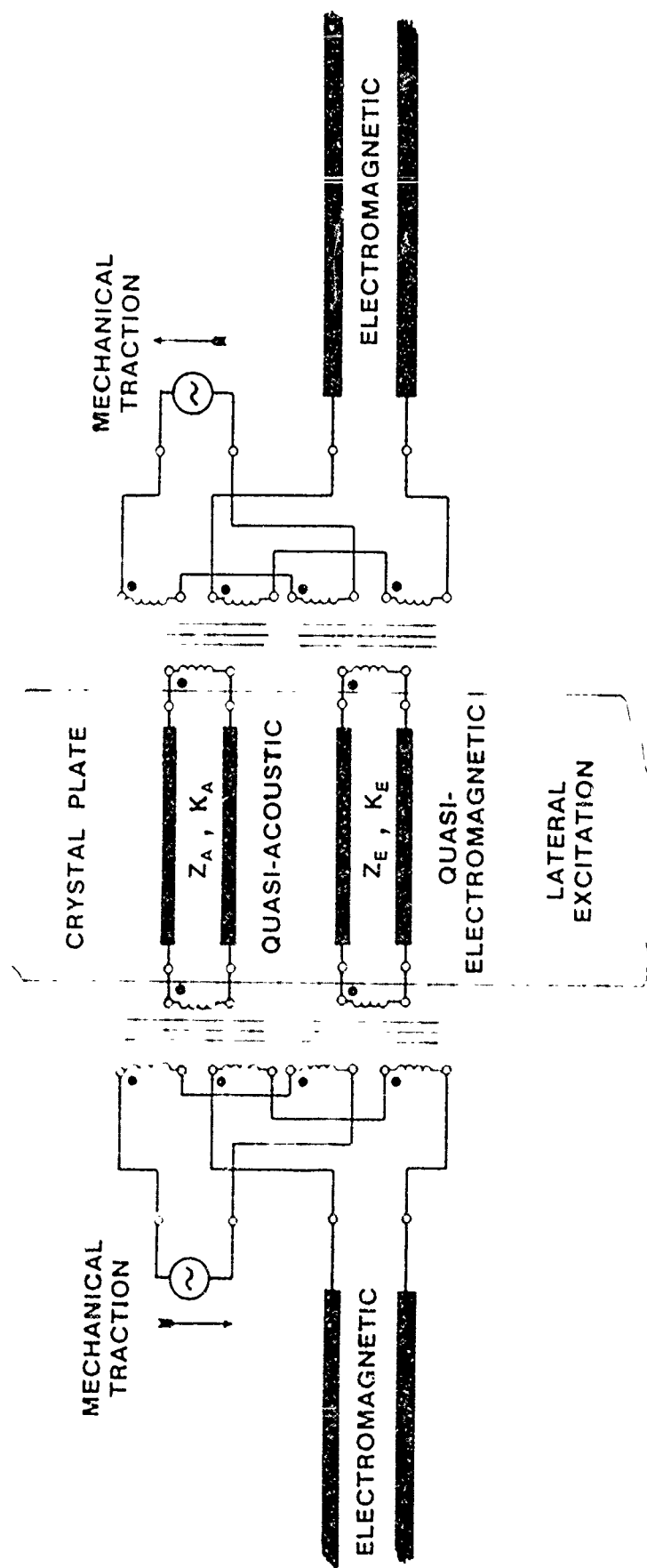


Figure 21. Analog network modeling Mindlin's problem.

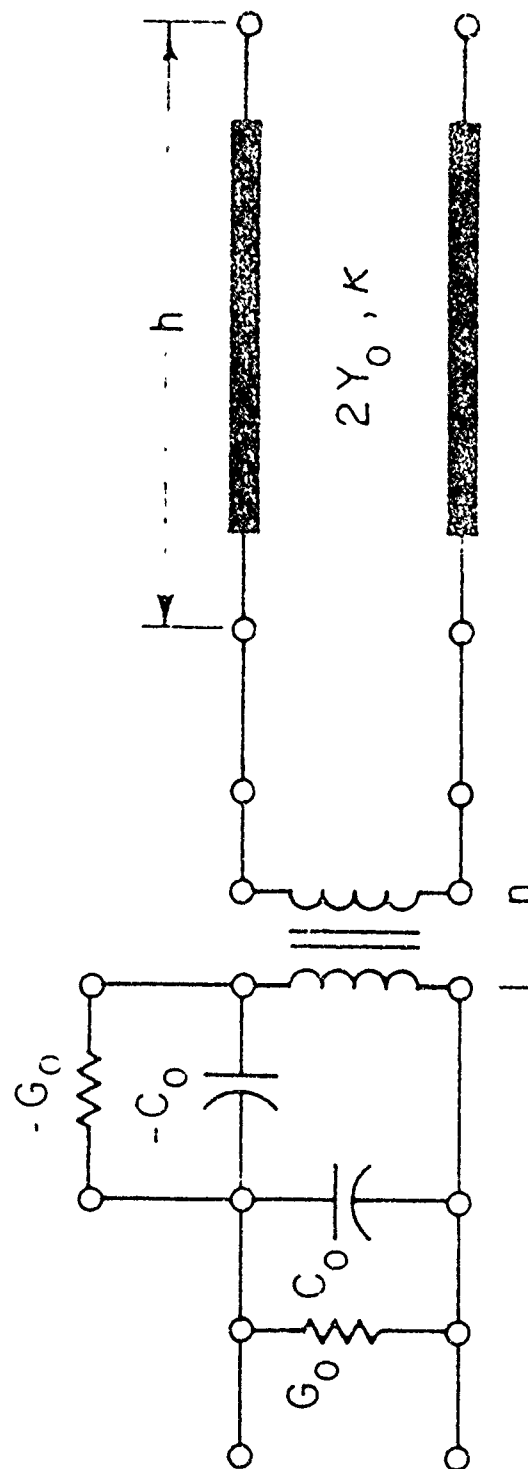


Figure 22. Analog network modeling a semiconductor vibrator.

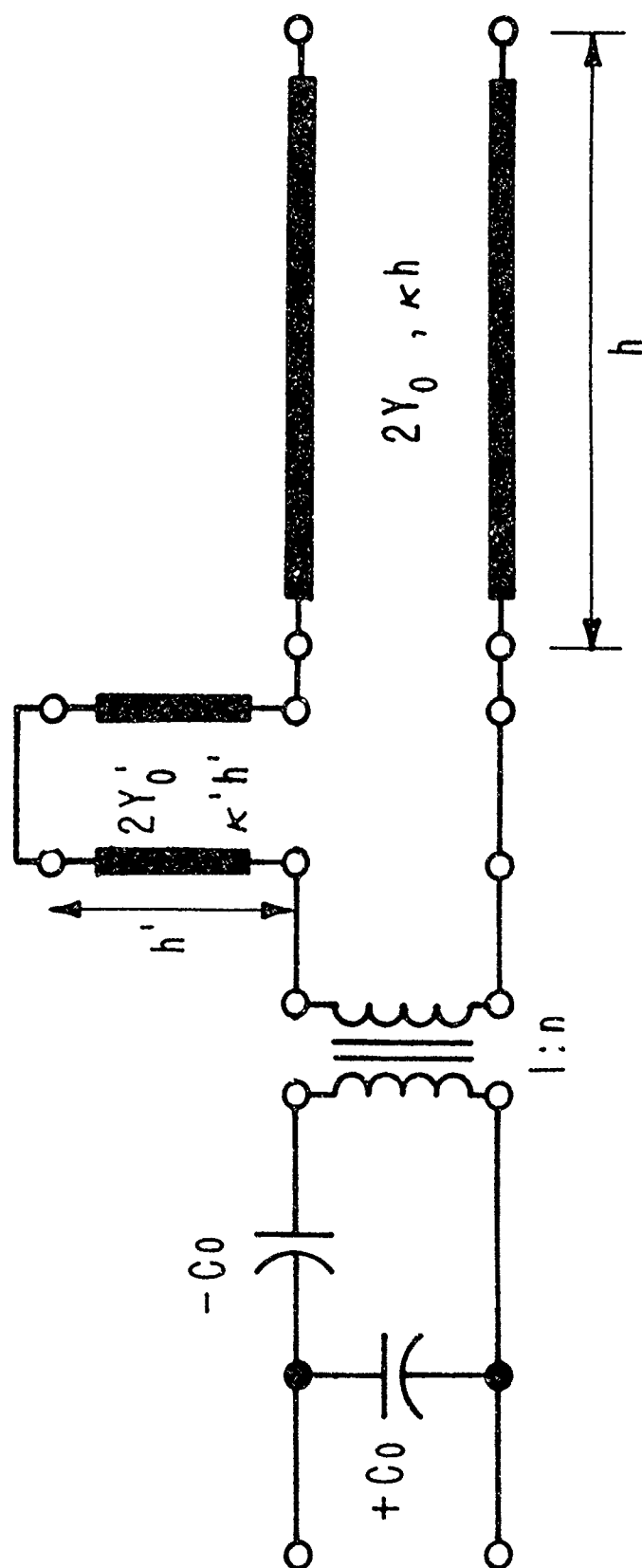


Figure 23. Analog network modeling a resonator with thick electrodes.

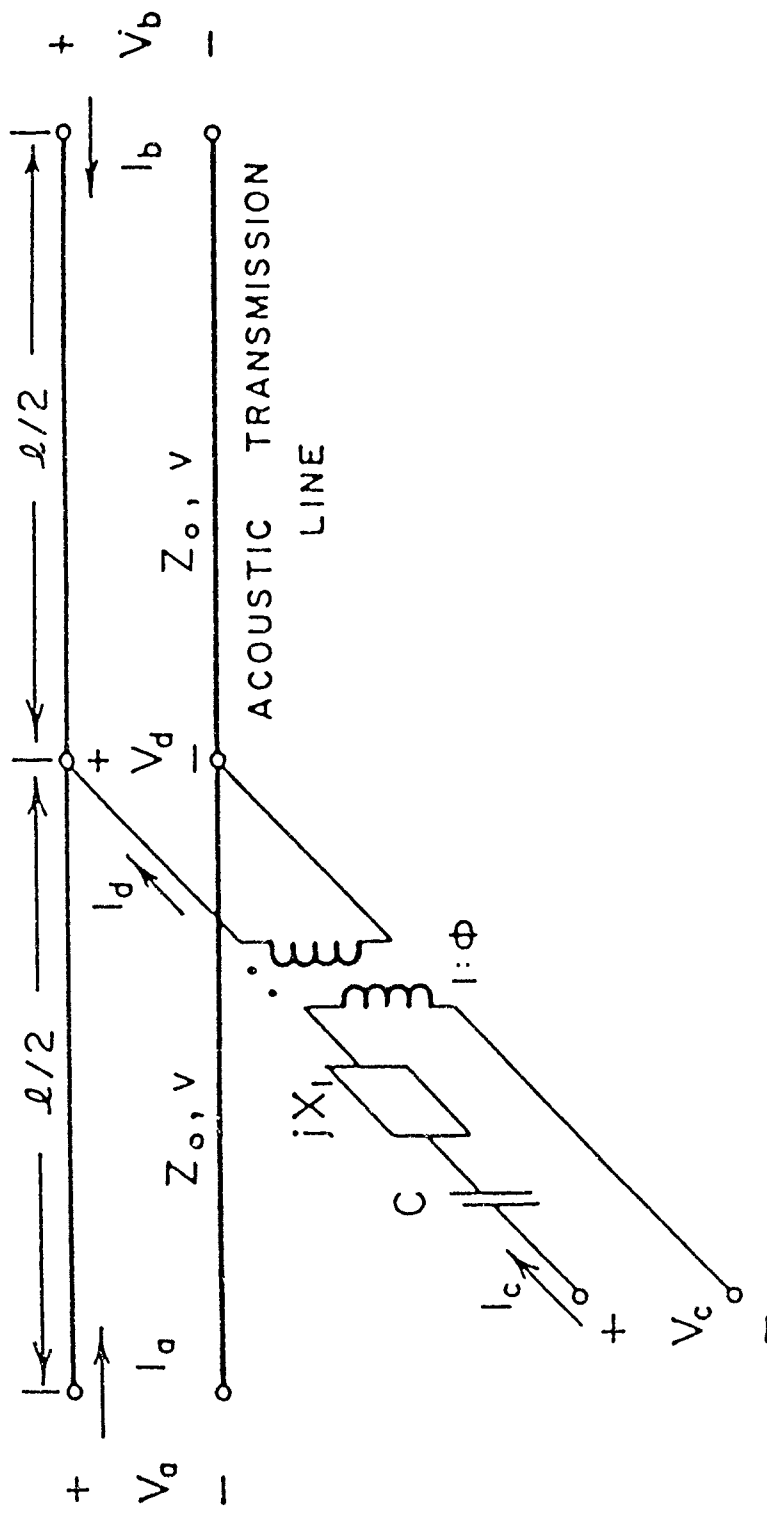


Figure 24. KLM equivalent circuit for thickness-expander plate and end-electroded length-expander bar with even excitation.

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