

UNCLASSIFIED

2



DEPARTMENT OF
DEFENCE

AD-A231 414

106 JUL 87

AR-006-443

Communications Division

DTIC
ELECTE
JAN 29 1991
S E D

ELECTRONICS RESEARCH LABORATORY

91 1 28 041

UNCLASSIFIED

UNCLASSIFIED

2

AR-006-443



ELECTRONICS RESEARCH LABORATORY

Communications Division

DTIC
ELECTE
JAN 29 1991
S E D

RESEARCH NOTE
ERL-0530-RN

USING A DISCRETE FOURIER TRANSFORM TO ESTIMATE PHASE IN INTERFEROMETRIC DIRECTION FINDING

by

Gareth J. Parker

SUMMARY

This document discusses the use of the Discrete Fourier Transform to estimate the phase of particular frequency components of a sampled signal. Attention is paid to the limitations of the Discrete Fourier Transform in general. Particular emphasis is placed on estimating phase for use in Interferometric Direction Finding.

© COMMONWEALTH OF AUSTRALIA 1990

DEC 90

COPY No. 16

APPROVED FOR PUBLIC RELEASE

POSTAL ADDRESS: Director, Electronics Research Laboratory, PO Box 1600, Salisbury, South Australia, 5108. ERL-0530-RN

UNCLASSIFIED

This work is Copyright. Apart from any use as permitted under the Copyright Act 1968, no part may be reproduced by any process without written permission from the Australian Government Publishing Service. Requests and enquiries concerning reproduction and rights should be directed to the Manager, AGPS Press, GPO Box 84, Canberra ACT 2601.

Accession For	
NTIS GRA&I	☒
DTIC TAB	☐
Unannounced	☐
Justification	
By _____	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	

CONTENTS

	Page No
1 INTRODUCTION	1
2 INTERFEROMETRIC DIRECTION FINDING.....	1
3 THE DISCRETE FOURIER TRANSFORM	2
3.1 Relative phase in the Fourier Transform.....	2
3.2 The need for windowing.....	3
3.3 The effect of windowing.....	4
3.3.1 The boundary effect.....	5
3.3.2 Multiple frequency components	8
3.3.3 Experimental justification.....	8
3.3.4 The effect of windowing on the phase from the discrete Fourier Transform	12
3.3.5 The effect of frequency error on the phase from the discrete Fourier Transform	13
4 DIRECTION FINDING AND THE DISCRETE FOURIER TRANSFORM	16
4.1 Single frequency component signals.....	16
4.2 Multi-component signals.....	18
5 RESULTS	18
6 CONCLUSION	19
7 ACKNOWLEDGEMENTS	21
8 REFERENCES	23
9 BIBLIOGRAPHY.....	25

FIGURES

1 Two element interferometer.....	1
2 An example of the relativity of phase of a sinusoid	3
3 Linearity of magnitude over frequency using a rectangular window	6
4 Linearity of magnitude over frequency using a Hanning window.....	6
5 Magnitude of FFT of SIN, $f = 32$, using a rectangular window.....	9
6 Phase of FFT of SIN, $f = 32$, using a rectangular window.....	10
7 Magnitude of FFT of SIN, $f = 32$, using a Hanning window.....	10
8 Phase of FFT of SIN, $f = 32$, using a Hanning window.....	11
9 Magnitude of FFT of SIN, $f = 32$, using a triangular window	11
10 Phase of FFT of SIN, $f = 32$, using a triangular window	12
11 Phase of FFT of SIN, $f = 32$, with zero frequency error, using a triangular window	14
12 Phase of FFT of SIN, $f = 32$, with frequency error = 0.1, using a triangular window	14
13 Phase of FFT of SIN, $f = 32$, with frequency error = 0.2, using a triangular window.....	15
14 Phase of FFT of SIN, $f = 32$, with frequency error = 0.5, using a triangular window	15
15 Linearity over frequency of phase difference error.....	16
16 Linearity over actual phase difference of calculated phase difference.....	17

ABBREVIATIONS

DF	Direction Finding
DFT	Discrete Fourier Transform
FFT	Fast Fourier Transform

1 INTRODUCTION

The principle of Interferometric Direction Finding is based on the difference in phase of a signal frequency, measured at two physically separated antennas. This paper explores the possibility of using a Discrete Fourier Transform (DFT) to obtain phase from a signal, and hence the prospect of using the DFT in Direction Finding (DF) applications.

2 INTERFEROMETRIC DIRECTION FINDING

The bearing of a radio emitter may be determined by phase interferometry [1]. This involves the use of two or more spatially separated antennas in an array. The phase difference between the signals received at each antenna can be used to determine the angle of arrival.

Consider, as an example, the two element interferometer shown in Figure 1.

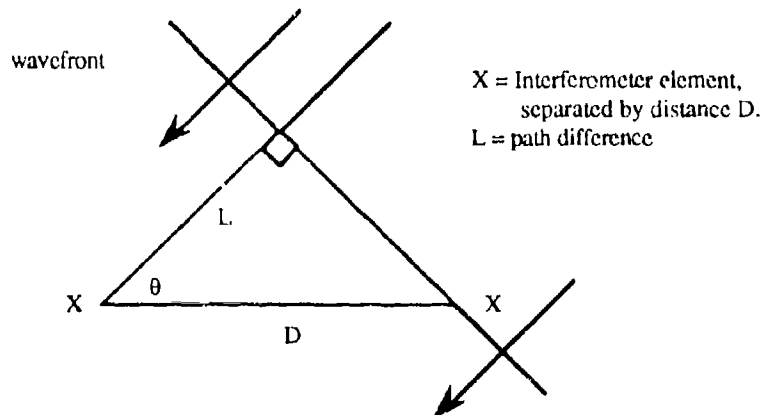


Figure 1 Two element interferometer

Due to the difference in path length, the phase difference of the signal on arrival at each antenna is

$$\phi = \frac{2\pi L}{\lambda} \Rightarrow L = \frac{\lambda \phi}{2\pi}$$

The bearing of the emitter, θ , may then be computed as

$$\theta = \arccos(L/D)$$

NOTE This is a simple arrangement and will result in an ambiguity in the direction of arrival, about the antenna baseline. In practice, at least three elements placed in a triangle, or four in a crossed-baseline arrangement are required to remove the ambiguity.

The phase difference between the signals received at each antenna must also be constant in time. Thus, the frequency of the component received at each antenna must be exactly the same. In practice, this means that when a receiver employs a frequency conversion stage prior to sampling, the same local oscillator must be used for all antenna channels.

3 THE DISCRETE FOURIER TRANSFORM

The Fourier Transform, $X(f)$ of a continuous signal, $x(t)$ is defined as

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft} dt$$

The analogous DFT of the signal, sampled at N points, T seconds apart is given by

$$\begin{aligned} X(k) &= \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N} \\ &= \sum_{n=0}^{N-1} x(n) [\cos(2\pi kn/N) - j\sin(2\pi kn/N)] \end{aligned}$$

where k and n are the discrete equivalents to f and t , related by $t = nT$ and $f = k/NT$.

NOTE Both n and N are required to be integers, while k, t, T and f are real numbers.

$X(k)$ is a sequence of complex numbers giving the discrete spectrum of $x(n)$. The quantisation steps in the spectrum will be referred to as frequency "bins". From these complex numbers, the magnitude and phase of each point in the spectrum may be obtained using simple trigonometry.

The Fast Fourier Transform (FFT) is simply a method of computing the DFT with much fewer calculations than the direct method, and is thus much faster to compute. Most algorithms for the FFT require that the number of points, N , be restricted to a power of two.

3.1 Relative phase in the Fourier Transform

As phase is a relative quantity, it is important to know what the phase of the transform $X(f)$ is measured with respect to.

The phase of a particular frequency component of a waveform, as determined using the Fourier Transform is measured relative to the first sampling point, with the equation describing the waveform being a COSINE relationship, as shown in Figure 2.

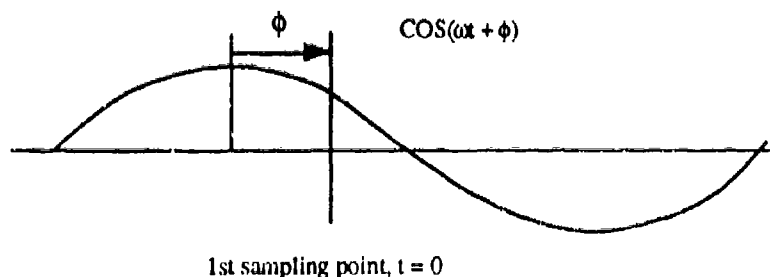


Figure 2 An example of the relativity of phase of a sinusoid

As an example, the following functions have "phases" resulting from the Fourier Transform as follows:

$$\begin{aligned}
 \text{SIN}(Ft + 0) &\Rightarrow \text{DFT phase} = -90^\circ, \text{ at } f = F \\
 \text{SIN}(0t + 90) &\Rightarrow \text{DFT phase} = 0^\circ \text{ (DC signal)} \\
 \text{SIN}(Ft + 90) &\Rightarrow \text{DFT phase} = 0^\circ, \text{ at } f = F
 \end{aligned}$$

3.2 The need for windowing

The DFT operates on a finite length data stream, or sample period. If the frequency of a particular component of a waveform is such that an integral number of periods of that component is contained in the sample period, then that component has no associated frequency error. This means that when the DFT of the waveform is taken, the frequency of that component falls exactly into one frequency bin. If however, the period does not have this property, then the component will not fall exactly into one bin, and that component is said to have a frequency error.

The frequency error of a component is the amount by which the frequency of that component deviates from the frequency of the closest bin. All real-world signals exhibit frequency error.

The effects of the frequency error on the magnitude of the spectrum have been well documented [2,3] and are generally known as Spectral Leakage and the Picket Fence Effect. Spectral Leakage occurs when the frequency of a component does not fall exactly into one bin, and refers to the spreading of the energy from one frequency bin to adjacent bins.

The Picket Fence Effect draws an analogy between the output of the DFT of a waveform, and a bank of analogue bandpass filters, each filter being centred at one of the frequency bins in the discrete spectrum. Ideally, each "filter" should have a square frequency response, but in practice, the responses are rounded, and the result is that the spectrum appears to be viewed through a picket fence.

The Spectral Leakage and Picket Fence effects are worst when the frequency lies midway between two frequency bins. i.e. a frequency error of 0.5.

In order to reduce these effects, "windows" are applied to the sampled data prior to performing the DFT. Windowing is the multiplication of the waveform by a discrete weighting function - the window.

3.3 The effect of windowing

Consider a signal $x(t)$, windowed by the window function $w(t)$, to produce the signal $y(t)$. The signal $x(t)$ has a Fourier Transform $X(f)$, and $w(t)$ has the Fourier Transform $W(f)$.

$$y(t) = x(t) \times w(t)$$

Then, $Y(f) = X(f) * W(f)$ (* denoting convolution)

$$= \int_{-\infty}^{+\infty} X(\lambda) W(f-\lambda) d\lambda$$

$$\text{Let } X(f) = |X(f)| e^{j\phi(f)}$$

$$W(f) = |W(f)| e^{j\theta(f)}$$

$$\text{So } Y(f) = \int_{-\infty}^{+\infty} |X(\lambda)| e^{j\phi(\lambda)} |W(f-\lambda)| e^{j\theta(f-\lambda)} d\lambda$$

$$= \int_{-\infty}^{+\infty} |X(\lambda) W(f-\lambda)| e^{j[\phi(\lambda) + \theta(f-\lambda)]} d\lambda \quad \dots (1)$$

In the discrete case, this integral becomes a summation.

$$Y(k) = \sum_{\lambda=0}^{N-1} |X(\lambda) W(k-\lambda)| e^{j[\phi(\lambda) + \theta(k-\lambda)]}$$

As an example, consider a sinusoidal signal, $x(t)$, sampled at N points and having some discrete time window $w(n)$ applied to it.

$$x(t) = A \cos(2\pi f_0 t + \phi)$$

It can be shown that the continuous Fourier Transform of $x(t)$ is given by

$$\Rightarrow X(f) = \frac{A}{2} \delta(f-f_0) e^{j\phi} + \frac{A}{2} \delta(f+f_0) e^{-j\phi}$$

where $\delta(f)$ is the delta function.

It follows that the DFT of $x(n)$ is then

$$X(k) = \frac{A}{2} \delta(k-k_0) e^{j\phi} + \frac{A}{2} \delta(k+k_0) e^{-j\phi}$$

The DFT, $W(k)$ of $w(n)$ can be expressed in the form

$$\begin{aligned} W(k) &= |W(k)| e^{j\theta(k)} \\ Y(k) &= \sum_{\lambda=0}^{N-1} X(k-\lambda) W(\lambda) \\ &= \sum_{\lambda=0}^{N-1} \frac{A}{2} (\delta(k-\lambda-k_0) e^{j\phi} + \delta(k-\lambda+k_0) e^{-j\phi}) |W(\lambda)| e^{j\theta(\lambda)} \\ &= \sum_{\lambda=0}^{N-1} e^{j(\phi+\theta(\lambda))} \frac{A}{2} \delta(k-\lambda-k_0) |W(\lambda)| + \sum_{\lambda=0}^{N-1} e^{j(-\phi+\theta(\lambda))} \frac{A}{2} \delta(k-\lambda+k_0) |W(\lambda)| \end{aligned}$$

But, $\delta(k-\lambda-k_0) = 0$ at all λ , except at $k-\lambda-k_0=0$, when $\delta = 1$, and $\delta(k-\lambda+k_0) = 0$ at all λ , except at $k-\lambda+k_0=0$, when $\delta = 1$.

$$\therefore Y(k) = \frac{A}{2} |W(k-k_0)| e^{j(\phi+\theta(k-k_0))} + \frac{A}{2} |W(k+k_0)| e^{j(-\phi+\theta(k+k_0))} \dots (2)$$

3.3.1 The boundary effect

Equation 2 consists of two terms which arise as a result of the window function acting on the positive and negative frequency components of $X(k)$ respectively.

It is customary to simplify the equation by ignoring the effect of the second term. However, blind use of this simplification may lead to incorrect results, as the contribution of the second term to positive frequency points is non-zero in the general case. The real part of the positive frequency spectrum appears to be reflected about the zero frequency point, while the imaginary part of the spectrum is reflected with sign inversion. For a finite length sample sequence, this reflection also occurs about the point $N/2$. Hence, the points close to $N/2$ are also similarly affected.

The extent of the contribution of negative frequencies will be largest for signals with a frequency close to 0 or $f_s/2$ ($k=N/2$), windowed by functions with large sidelobes in the frequency domain.

Figure 3 shows the linearity of the magnitude from a 128 point FFT of a discrete-frequency signal with a frequency error of 0.4. Each point corresponds to the magnitude of the output of an FFT performed on a sinusoid at that frequency. A rectangular time window, which has a sinc-like Fourier Transform with large sidelobes was used throughout. It can be seen that the lowest and uppermost 12 discrete frequencies suffer errors of more than 1 % in magnitude. A similar sized phase error is also incurred.

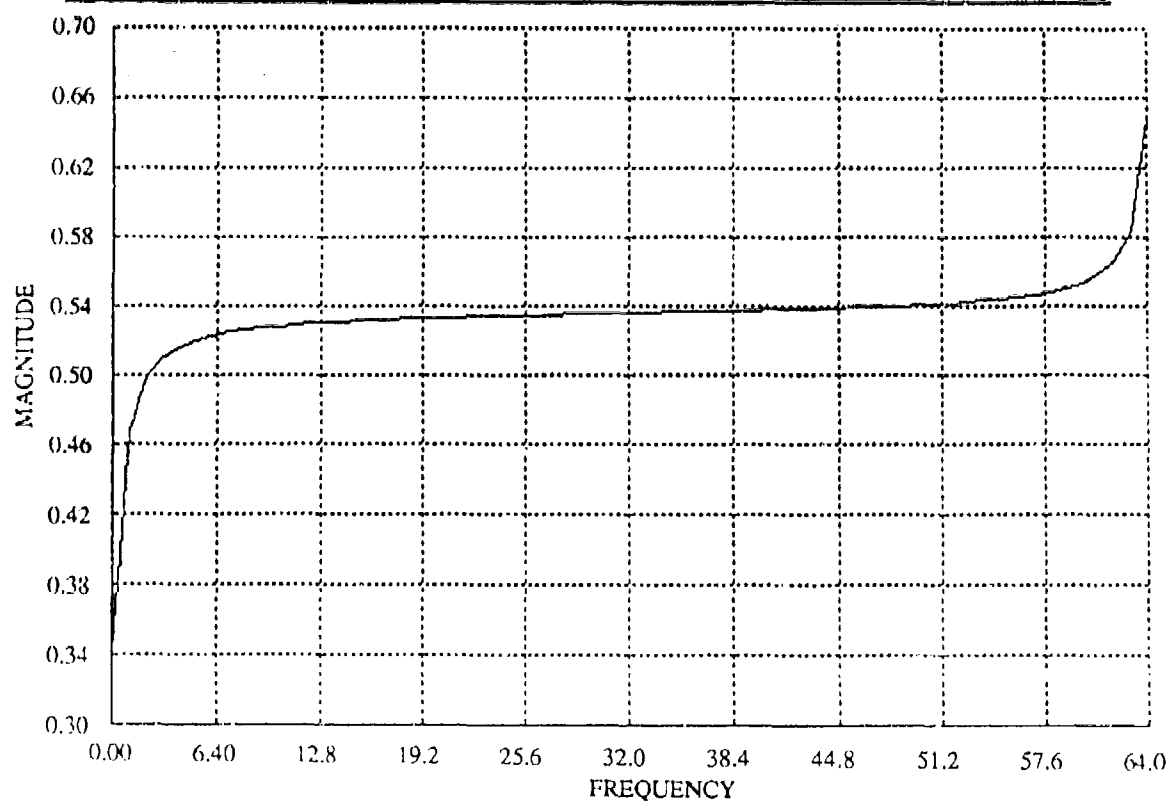


Figure 3 Linearity of magnitude over frequency, using a rectangular window

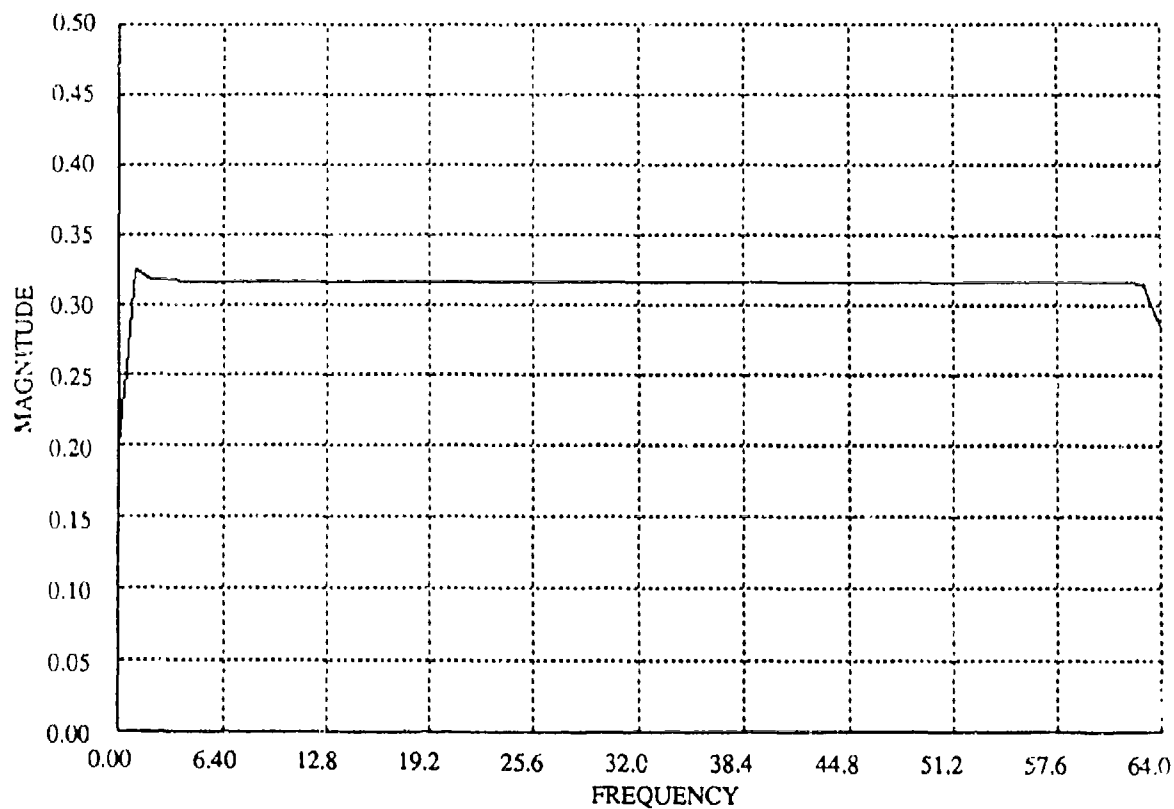


Figure 4 Linearity of magnitude over frequency, using a Hanning window

Figure 4 shows the linearity improvement when using a Hanning time window. This window has a much narrower Fourier Transform and as such only the first and last two points suffer errors above 1 %.

The effect may thus be reduced by applying suitable windows and/or considering points far away from the spectral extremes. Using longer length transforms allows a greater number of these points to be considered. The effect may be eliminated by applying the Hilbert Transform to the sampled data to obtain the "analytic" signal. Such a signal contains only positive frequency components.

For simplicity, the remainder of the paper will assume negative frequency components have negligible effect, unless otherwise stated. Under such conditions, equation 2 simplifies to

$$Y(k) = \frac{A}{2} |W(k-k_0)| e^{j(\phi+\theta(k-k_0))} \dots\dots\dots(3)$$

From equation 3, it can be seen that for a single frequency waveform, the magnitude of the spectrum will consist of an amplitude scaled Fourier Transform of the window function, centred at the frequency of the signal component k_0 .

If the signal has no frequency error, then the spectrum of the window will be centred exactly on a frequency bin. Thus the best window to use in this circumstance is the rectangular time window. As this has a Fourier Transform of a sinc-like function, there will be nulls at all bins other than at its centre. The transform then will consist only of one line, at the frequency of the signal.

By considering equation 3 again, it can be seen that the phase of the spectrum will be the phase of the Fourier Transform of the window, centred at the signal frequency, plus the phase of the signal component.

Again, for a signal with no frequency error, using a window with nulls at most frequency bins other than its centre will result in the phase from the DFT being a good estimate of the true phase of the spectrum. This is because if the leaked amplitude response is very small at a particular bin, then the contribution to the overall phase at that bin will be small.

NOTE If the magnitude is zero (or very small) at a frequency bin, then phase does not exist for that point and can be arbitrarily set. In these investigations, the phase in this case was set to zero.

3.3.2 Multiple frequency components

Most typical signals used in DF are real signals, containing no imaginary component. They may therefore be treated as a set of frequency components, each having particular amplitude and phase.

For the case where the signal $x(t)$ comprises more than one frequency component, the resultant Fourier Transform will be a superposition of the Fourier Transforms of the constituent components.

$$\text{ie. } Y(k) = \sum_i \frac{A_i}{2} |W(k-k_{oi})| \exp j(\phi_i + \theta(k-k_{oi})) \dots\dots\dots(4)$$

Although it would be tempting to think so, this does not mean that the resultant phase and magnitudes are each themselves superpositions of the constituent phases and magnitudes.

The real case, with no frequency error will have the Fourier Transform, evaluated at some particular frequency k_n , given by

$$Y(k_n) = \frac{A}{2} \angle (\phi_n + \theta(o)) + \frac{A}{2} \sum_{i \neq n} |W(k_n-k_i)| \angle (\phi_i + \theta(k_n-k_i)) \dots\dots\dots(5)$$

If the window used is rectangular, then equation (5) simplifies to the expected result $Y(k_n) = \frac{A}{2} \angle \phi_n$.

This indicates that with a rectangular window, and no frequency error, each component has a distinct phase, and no component affects any other component.

3.3.3 Experimental justification

The result for magnitude is expected, and is well known. The phase, on the other hand, seems intuitively satisfying, but it must be shown that this result agrees with results from actual FFT's. Fourier Transforms have been performed on 128 point sampled sinewaves at frequency $f_0=32$, and using three windows - Rectangular, Triangular, and Hanning, with frequency responses as follows:

Rectangular

$$W(k) = e^{-j2\pi k/N(\frac{N-1}{2})} \frac{\sin\left(\frac{2\pi k}{2}\right)}{\left(\sin\frac{2\pi k/N}{2}\right)}$$

Triangular

$$W(k) = \frac{2}{N} e^{j2\pi k/N(N-1)/2} \left[\frac{\sin\left(\frac{k\pi}{2}\right)}{\sin(\pi k/N)} \right]^2$$

Hanning

$$w(n) = \sin^2\left(\frac{n\pi}{N}\right)$$

$$W(k) = e^{j\pi k/N(N-1)/2} \{0.5D(k) + 0.25[D(k-1/N) + D(k+1/N)]\}$$

$$\text{where } D(k) = e^{j\pi k/N} \frac{\sin(\pi k)}{\sin(\pi k/N)}$$

The magnitude and phase resulting from performing an FFT on the sampled waveform using each of the above windows, has been plotted and appears in Figures 5 to 10. The theoretical phase, as calculated using equation (3) is marked with an "X".

NOTE There is only one non-zero point in the spectra produced using the Rectangular window, and three points using the Hanning window - the centre bin, and the two adjacent bins. This is because nulls from the frequency response of the windows fall in all other frequency bins.

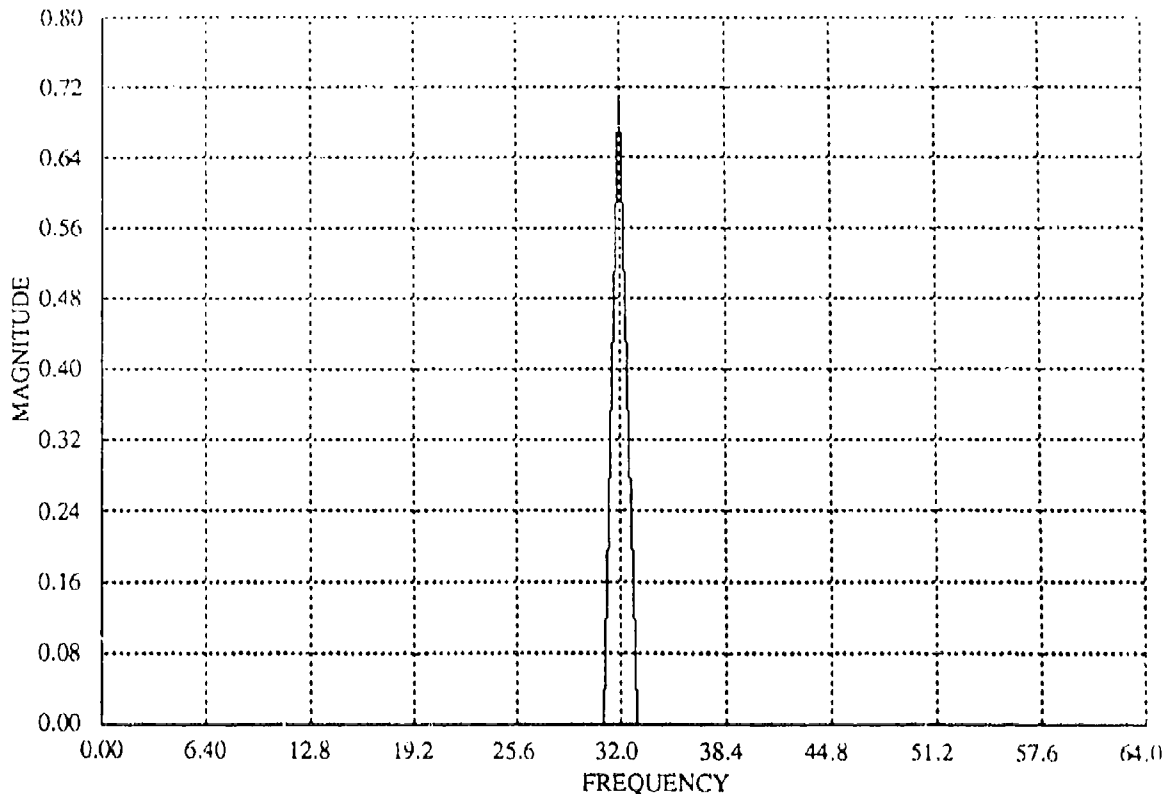


Figure 5 Magnitude of FFT of SIN, $f = 32$, using a rectangular window

The Rectangular and Hanning cases have shown exact correlation with the theoretical values.

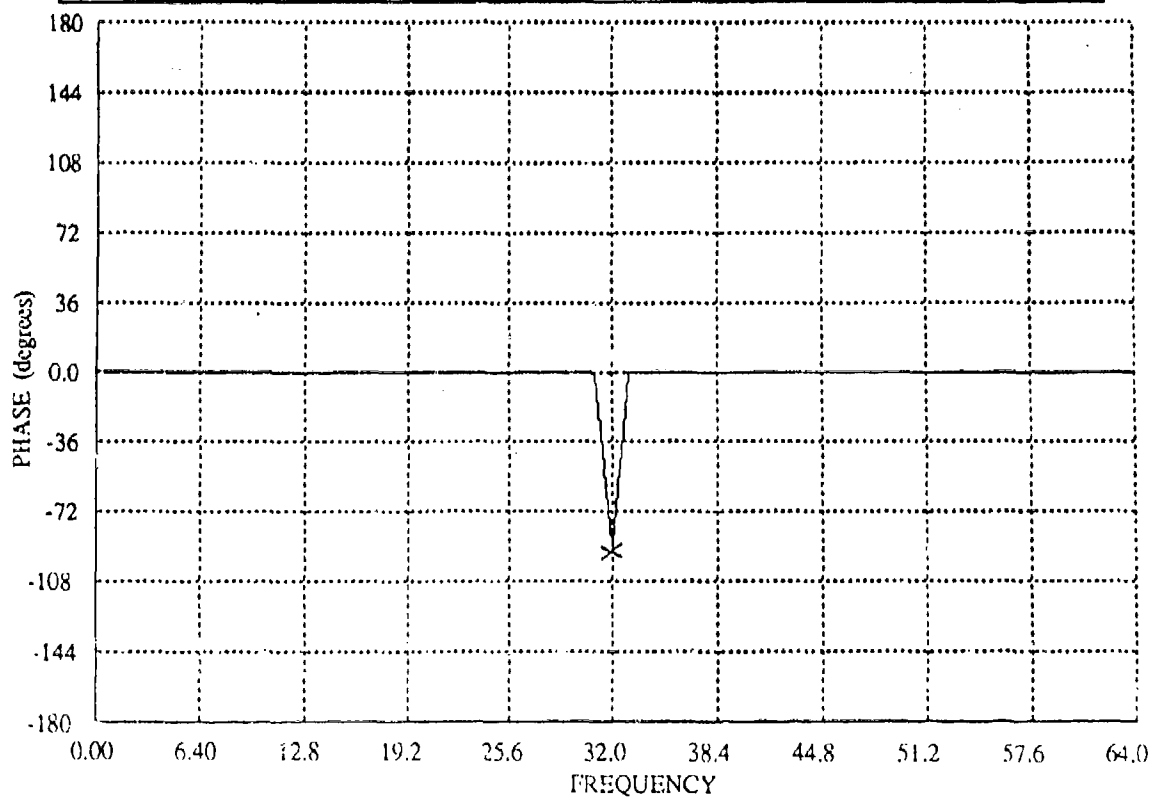


Figure 6 Phase of FFT of SIN, $f = 32$, using a rectangular window

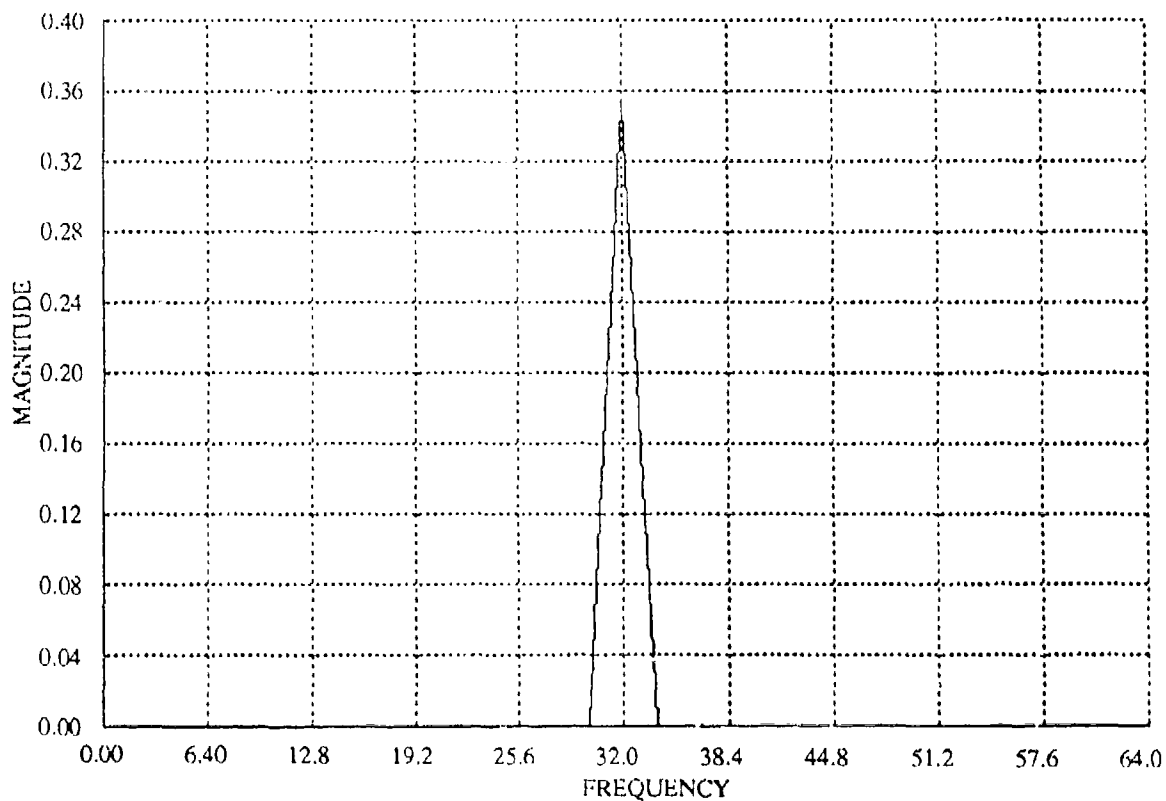


Figure 7 Magnitude of FFT of SIN, $f = 32$, using a Hanning window

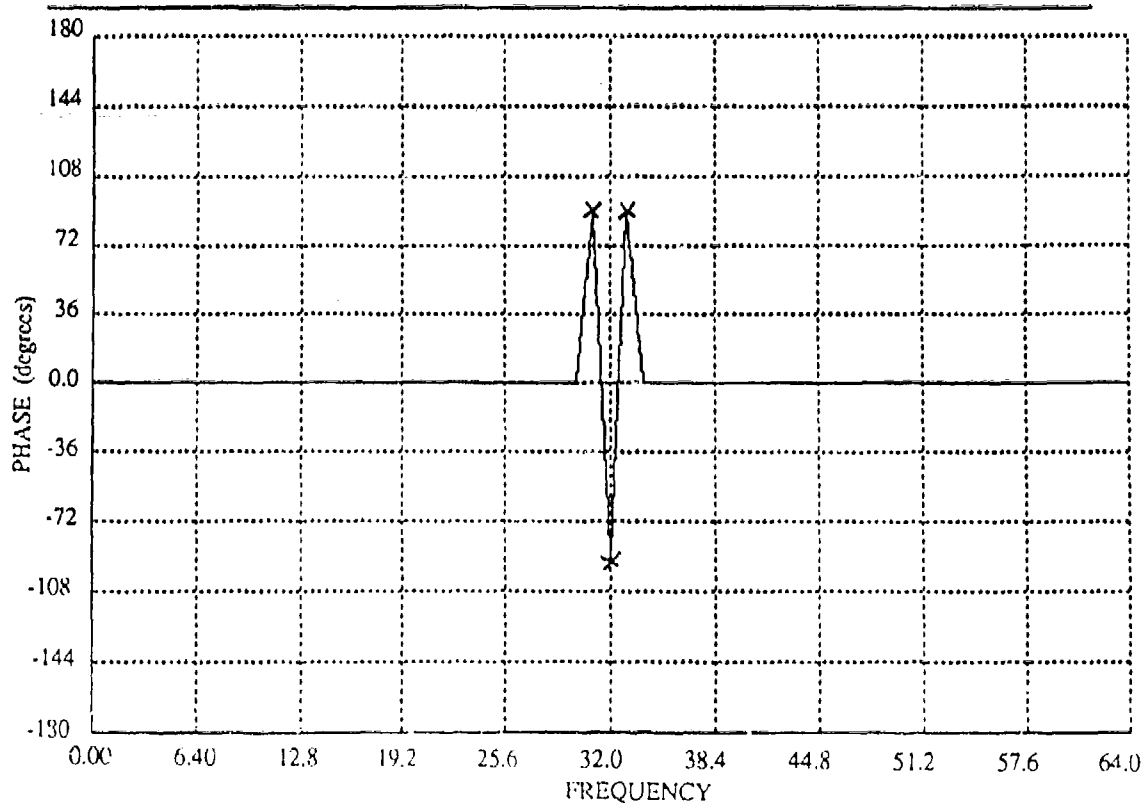


Figure 8 Phase of FFT of SIN, $f = 32$, using a Hanning window

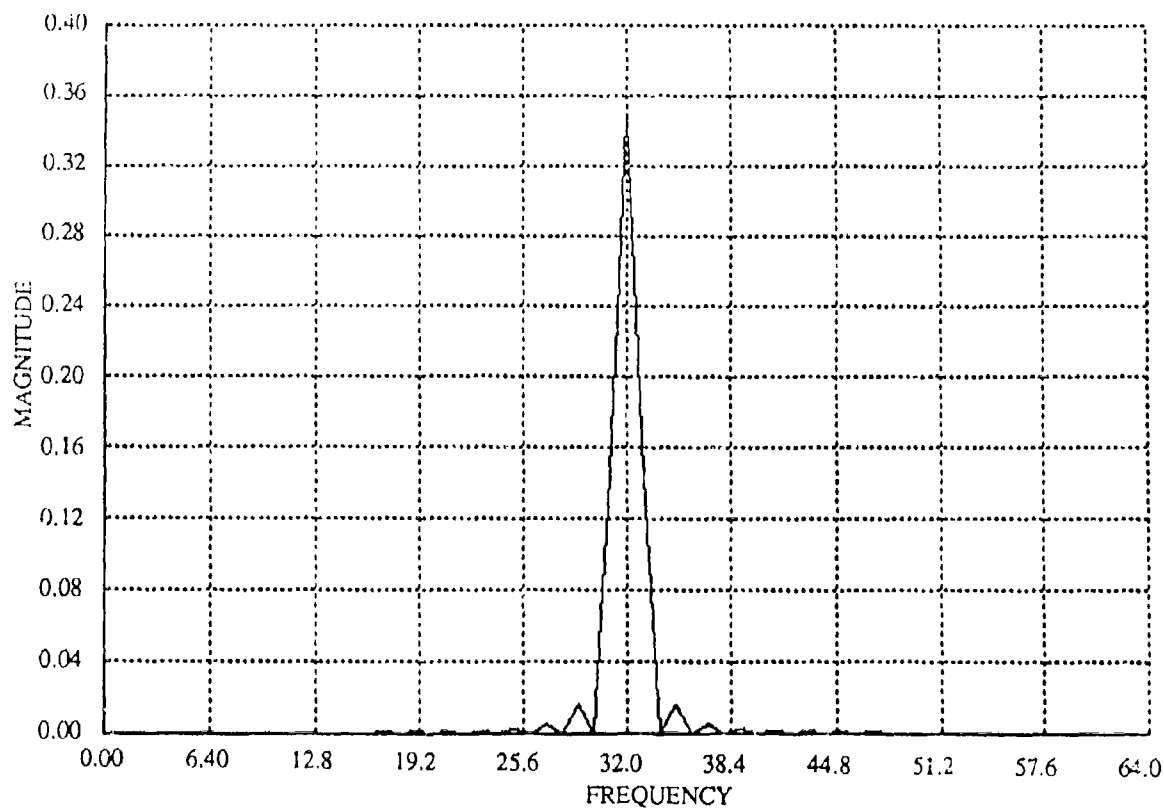


Figure 9 Magnitude of FFT of SIN, $f = 32$, using a triangular window

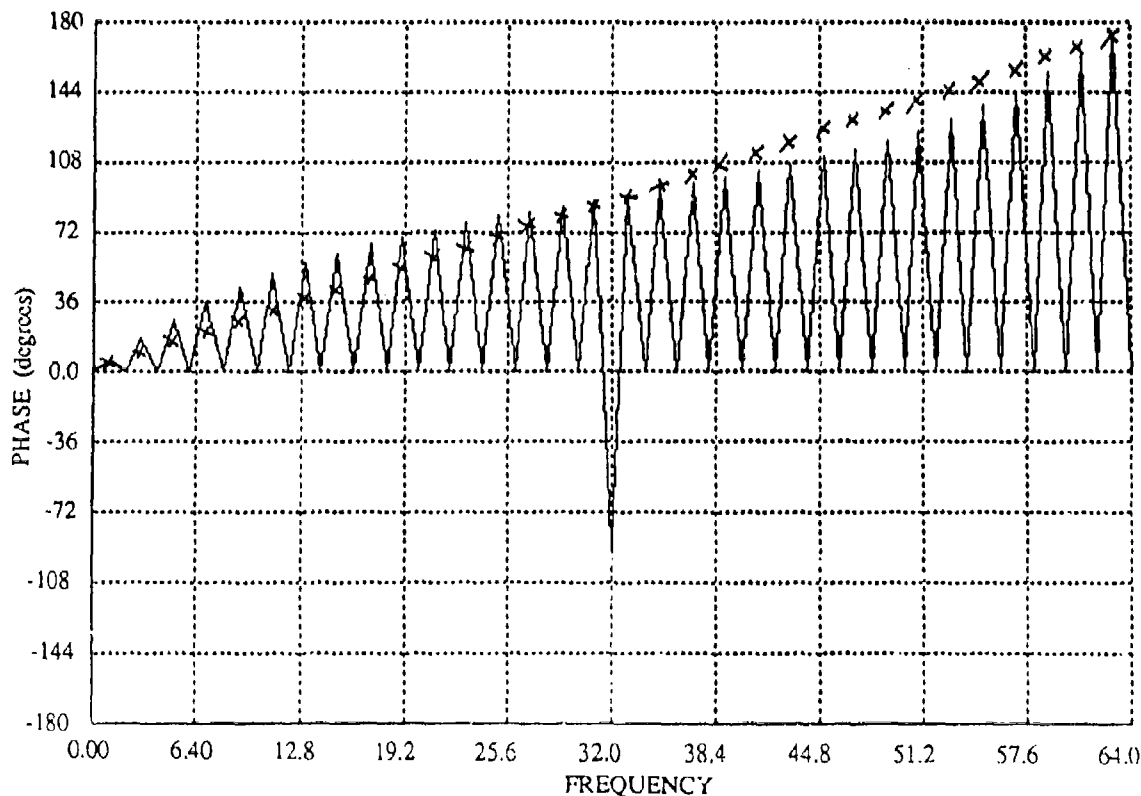


Figure 10 Phase of FFT of SIN, $f = 32$, using a triangular window

The Triangular case has shown very good correlation for frequencies near f_0 , but accuracy falls off as frequency moves further from f_0 . This is to be expected, as the magnitude, at distant frequencies is very small, and thus a ratio of two very small numbers is taken to obtain phase. Any error in either the real or imaginary component will cause large phase error.

The Rectangular and Hanning windows do not suffer from the same errors as the Triangular window because the centres of the frequency bins fall at the nulls of the magnitude spectra of these windows.

3.3.4 The effect of windowing on the phase from the Discrete Fourier Transform

Considering equation (5), it can be seen that, for the purely real case with no frequency error, the phase at k_n is dependant on the following factors:

- 1 the phase of the component at frequency k_n .
- 2 the phase of all other frequency components k_i .
- 3 the phase at f_n of the window function, centred at each of all other frequency components k_i .

NOTE The extent to which factors 2 and 3 above affect the phase at k_n , is proportional to the magnitude of the window function centred at k_i , at k_n . To reduce the effects of factors 2 and 3, the window function should be chosen such that $W(k)$ is minimal at frequency k_n . When there is no frequency error the optimal window to use is the Rectangular window, because the Fourier Transform of this window has nulls at all frequency bins other than its centre.

3.3.5 The effect of frequency error on the phase from the discrete Fourier Transform

Most typical signals have components with frequency error. In this case, equation (2) still holds, with k_n and k_i being the exact frequencies of the components - not the frequency bins in which they fall.

This has a detrimental effect on the use of most window functions. For example, consider the rectangular window. In the case of no frequency error a rectangular window has spectral nulls at all bins but the centre. However, with frequency error, the spectrum of the window will no longer be centred exactly at one bin. The result is that the nulls no longer fall at bin centres - there will now be finite magnitude levels at the bins. Hence there will also be a phase contribution at most bins.

As stated in paragraph 3.3.4, the effects of the contribution by window functions to magnitude and phase can be minimised by reducing the magnitude of the Fourier Transform of the window function at the appropriate frequency bins. This implies the use of windows with low sidelobe levels.

An example of the effect of frequency error on the phase of the spectrum resulting from a DFT is shown in Figures 11 to 14. Using a sampling frequency of 128 Hz, a 128 point transform has been performed on a sinusoid of frequency $f_0 = 32$ Hz. The sinusoid has been windowed by using a triangular window.

Figure 11 shows the phase of the spectrum with no frequency error. At $k=32$, the phase can be seen to be the correct -90° . Figure 12 shows the phase of the spectrum with the sinusoid having a frequency error of 0.1 (ie. $k_0 = 32.1$). The phase at $k = 32$ is seen to be approximately -72° .

Figures 13 and 14 show that the phase at $k = 32$ is reduced to -48° and 0° for frequency errors of 0.2 and 0.5 respectively. Thus, it can be seen that the phase at 32 becomes progressively less correct as the frequency error increases.

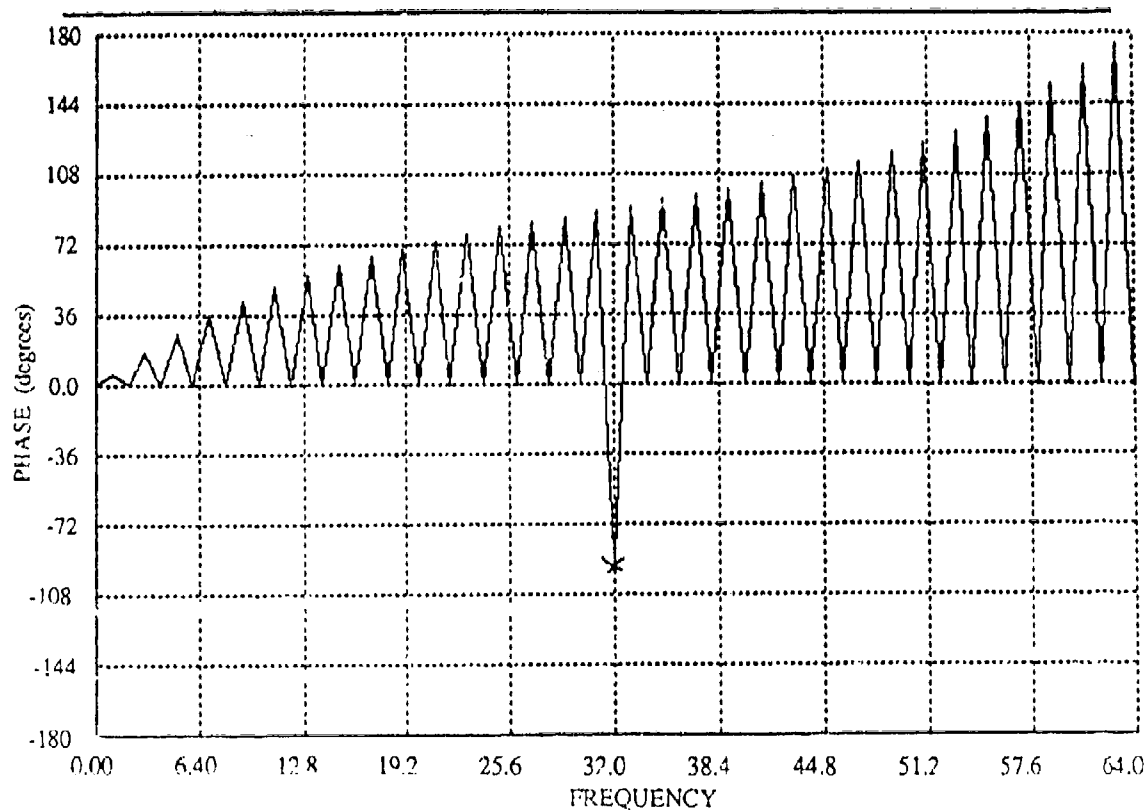


Figure 11 Phase of FFT of SIN, $t = 32$, with zero frequency error, using a triangular window

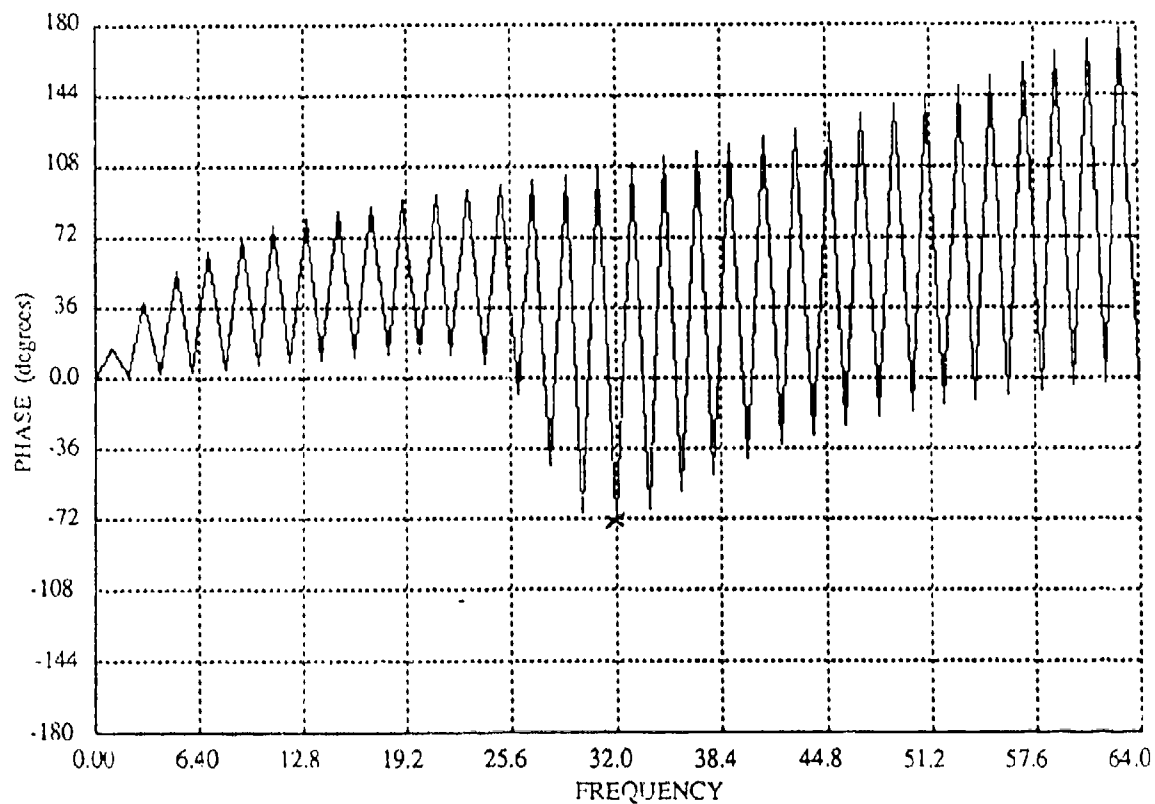


Figure 12 Phase of FFT of SIN, $f = 32$, with frequency error = 0.1, using a triangular window

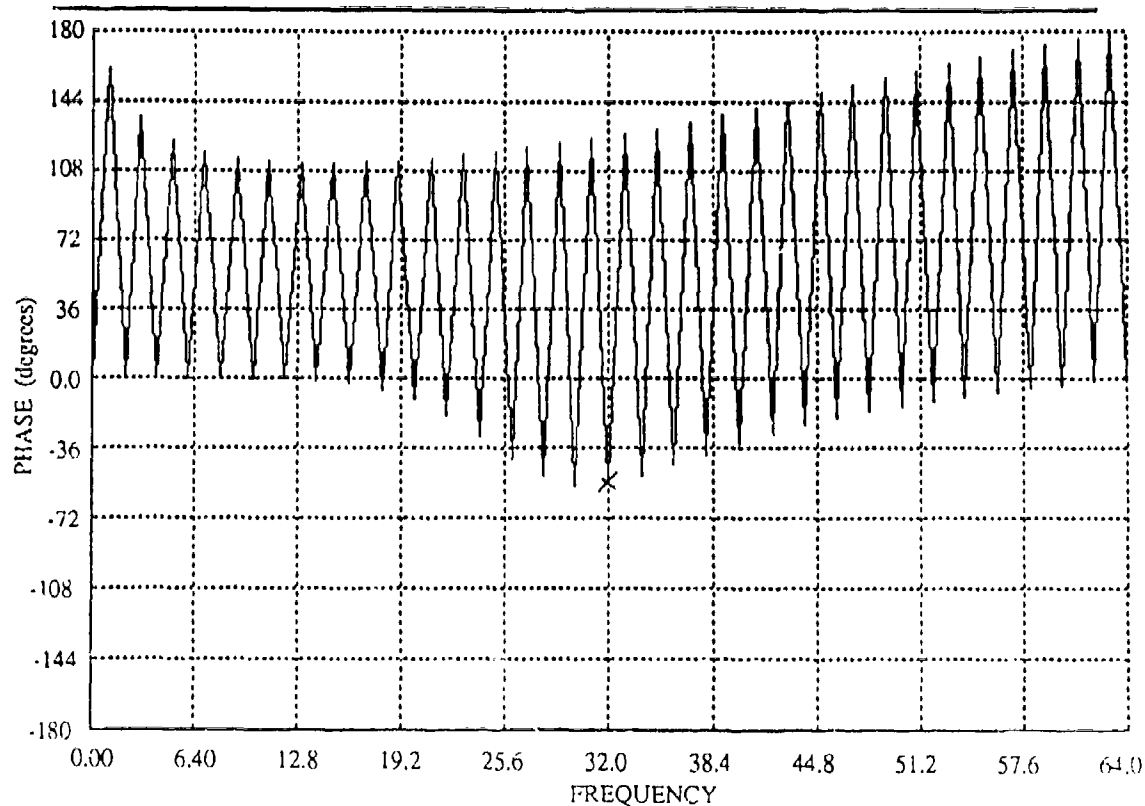


Figure 13 Phase of FFT of SIN, $f = 32$, with frequency error = 0.2, using a triangular window

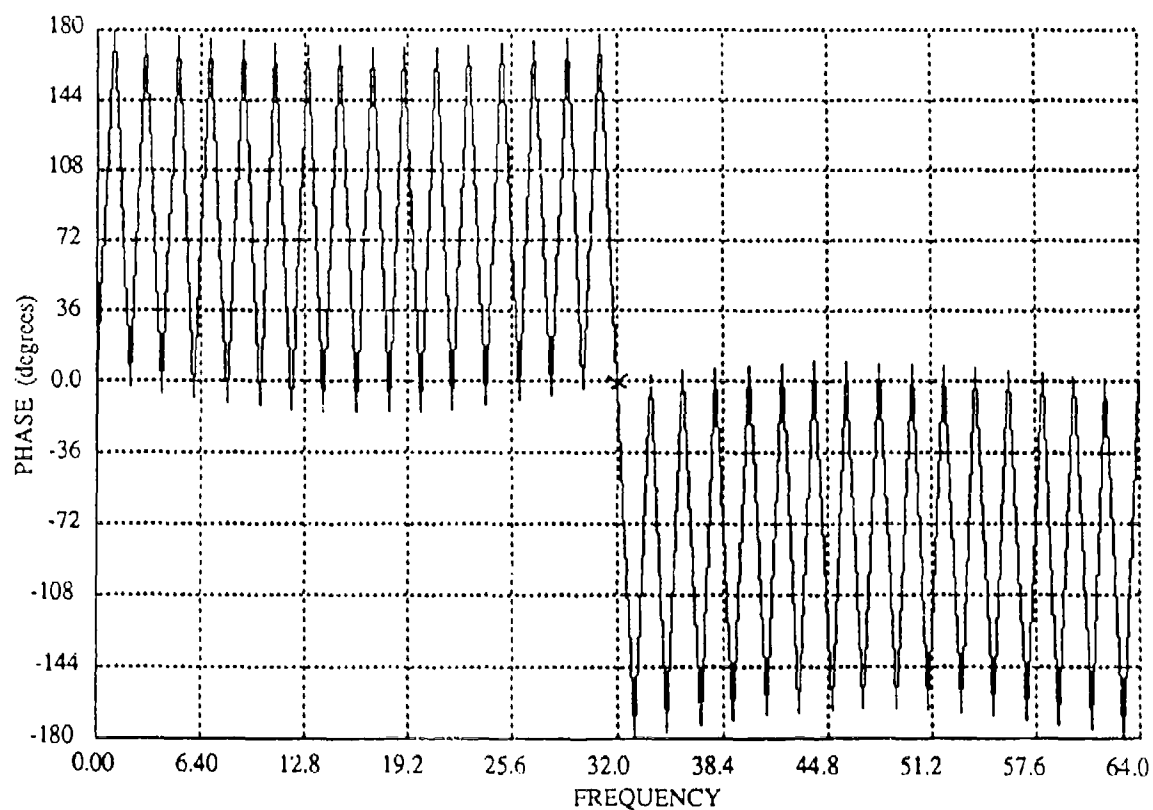


Figure 14 Phase of FFT of SIN, $f = 32$, with frequency error = 0.5, using a triangular window

4 DIRECTION FINDING AND THE DISCRETE FOURIER TRANSFORM

When using the DFT for DF applications the difference in phase between a frequency component of a signal received at one antenna and that received at another antenna must be obtained with high accuracy.

The DF calculation using physically separated antennas may be simulated by performing a DFT on a waveform, altering the phase of the frequency components of that waveform (to simulate wave propagation) and then performing another DFT on the new wave.

4.1 Single frequency component signals

Examination of equation (3) reveals that, for a single component signal, a DF type phase difference calculation should yield exact results, as the window dependant term will cancel in subtraction.

To verify this observation, a simulation was performed which involved constructing a 4096 point discrete sinewave at frequency k_0 , zero phase, and then performing an FFT on it. The phase of the sinewave was then altered by 90° , and the FFT was repeated. The phase difference from each FFT at the frequency k_0 was obtained, and the percentage error calculated. The simulation was performed for all values of k_0 between 0 and 2048, and the percentage error plotted against frequency as shown in Figure 15.

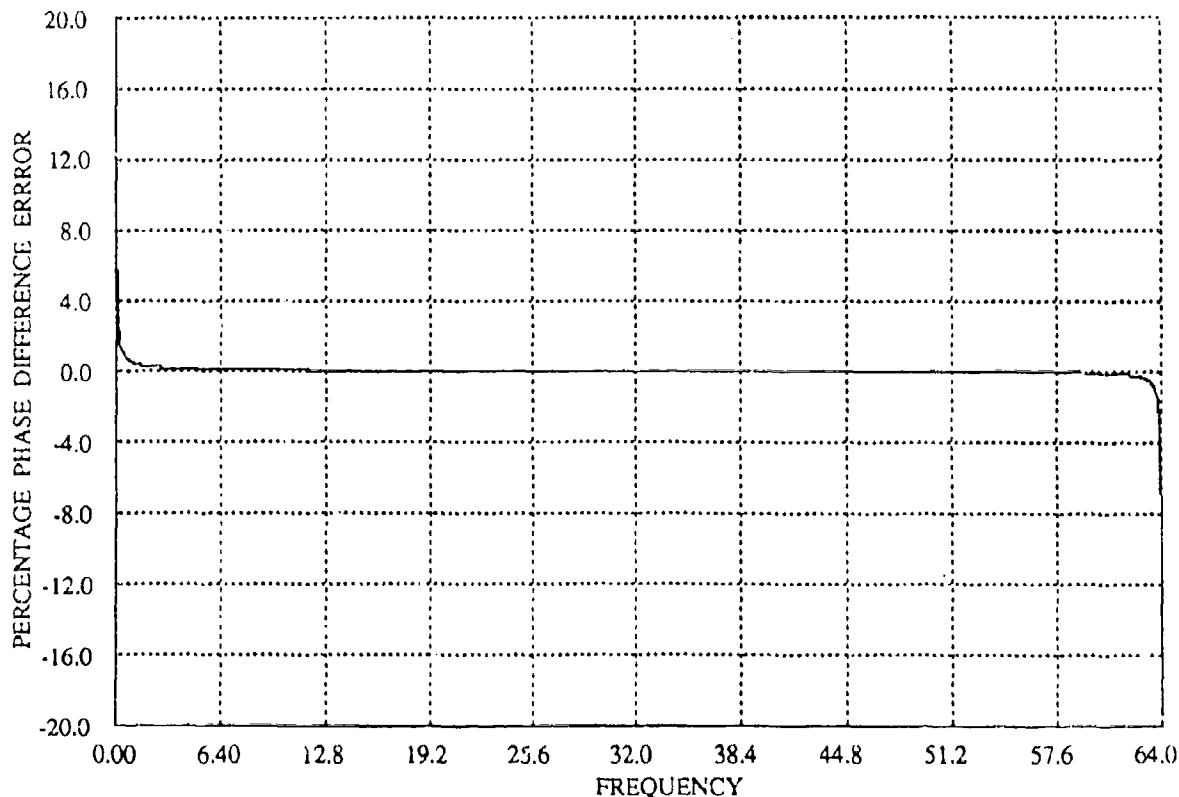


Figure 15 Linearity over frequency of phase difference error

A rectangular time window was used with each frequency k_0 having a frequency error of 0.4, as this gave the most conservative results. Figure 15, shows that the error is constant and near zero at all points except those close to the ends of the spectrum. The deviation at these points is due to the Boundary Effect, and may be minimised as explained in paragraph 3.3.1.

The nonlinearity towards the edges of the spectrum means that the bandwidth over which accurate relative phase estimation may be obtained is slightly less than the expected bandwidth of 0 to $f_s/2$. If multiple bands are to be used to analyse a wide bandwidth, then some overlapping of these bands may be necessary to obtain the desired linearity.

A second simulation was performed to verify the linearity over actual phase difference of the calculated phase difference. Some 128 point, 32 Hz sinewaves were generated at integral phase differences from 0 to 360°. FFTs were performed on the sinewaves, and the calculated phase differences were plotted against actual phase differences, refer Figure 16. Again, a rectangular time window was used, and the sinewaves had a frequency error of 0.4, to allow the most conservative results to be obtained. The plot shows almost perfect linearity.

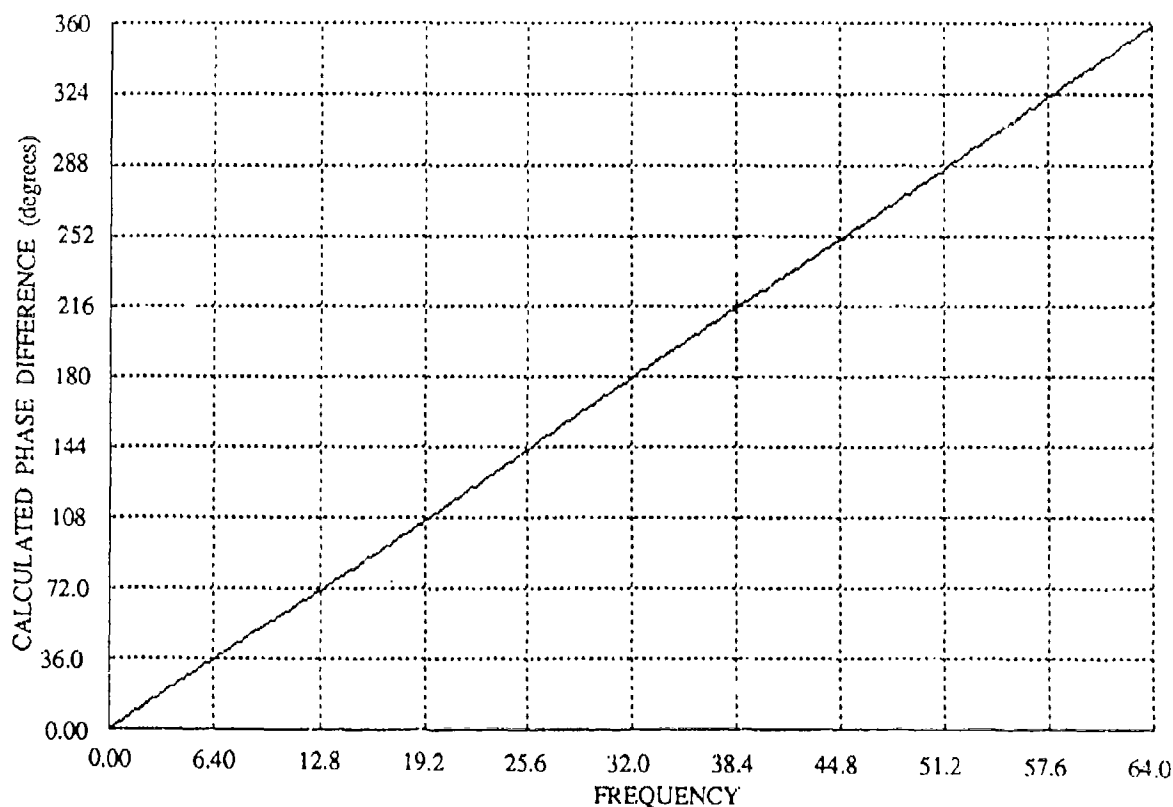


Figure 16 Linearity over actual phase difference of calculated phase difference

4.2 Multi-component signals

Referring back to equation (4), one could draw a similar conclusion to that of paragraph 4.1, that the window dependant term will cancel on subtraction in a DF type phase calculation of a multi-component signal. However, this assumption is incorrect. The window dependant term of equation (4) is also dependant on ϕ_i , the phase of the i th frequency component of the signal. The parameter ϕ_i will not be the same when measured at spatially separated antennas, due to propagation of the signal. Thus, the window dependant term does not cancel, and so, the resulting phase difference calculation is not exact.

The use of windows with low sidelobes reduces the error due to the phase contribution by other "distant" frequency components, k_i . However, most common windows such as the Hanning window, have a main lobe in the frequency domain that is wider than one frequency bin, and thus any adjacent or nearby frequency components will still contribute to the phase at the frequency of interest, k_n . This is tolerable, provided the amplitudes of these components are small. However, if the amplitude of adjacent or nearby components is of the same order as the amplitude of k_n , the phase corruption is too great to allow any reasonable accuracy to be obtained with a DF type phase calculation.

Thus, simple direction finding using the DFT is possible only for narrowband signals. The tolerable bandwidth is dependant on qualities of the window function such as main lobe width and sidelobe amplitude levels.

5 RESULTS

The following experimental observations have been made on the phase obtained using the DFT to process purely real signals. These observations are qualified by equations (2) and (4). It is expected that these observations should apply also to the case of complex signals (ie. those possessing both real and imaginary parts).

- 1 A DF type relative phase measurement of a single frequency component signal, with or without frequency error will achieve correct results, provided that allowance be made for the Boundary Effect - the contribution of negative frequency components to the positive part of the spectrum.
- 2 A DF type relative phase measurement of a multiple frequency component signal, with frequency errors, will not achieve exact phase estimation. Phase corruption by other frequency components, as well as that due to the negative part of the spectrum occurs. However, the use of windows with low magnitude sidelobes, and narrow main lobes will ensure a reasonably accurate estimation.

6 CONCLUSION

The Discrete Fourier Transform is capable of providing an accurate estimate of relative phase, as required for its use in direction finding. Windowing sampled data prior to performing the DFT reduces both the Boundary Effect and the effects of frequency error on relative phase measurements, and thus greatly enhances accuracy. Results of the investigation have indicated that the use of the Discrete Fourier Transform for direction finding is feasible.

7 ACKNOWLEDGEMENTS

The author wishes to thank Ashleigh Quick and Martin Gill for their assistance in various areas of this investigation.

8 REFERENCES

1. Gething, P.J.D. Radio Direction Finding and the Resolution of Multi-component Wave-Fields, Peter Peregrinus, 1978, pp130-132.
2. Harris, F.J. "On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform". IEEE Proc., vol.66, No.1, pp.51-83, January 1978.
3. Bergland, G.D. "A guided tour of the fast Fourier Transform". IEEE Spectrum, pp41-52, July 1969.

9 BIBLIOGRAPHY

1. Quick, A.G. and Gill, M.C. "Algorithms of Three Antenna Interferometric Direction Finders". A.T.R., vol.22, No.1, pp61-69, 1988.

DISTRIBUTION

	Copy No.
Defence Science and Technology Organisation	
Chief Defence Scientist)	
Central Office Executive)	1
Counsellor, Defence Science, London	Cnt Sht
Counsellor, Defence Science, Washington	Cnt Sht
Scientific Adviser, Defence Central	Cnt Sht
Scientific Adviser, Defence Intelligence Organisation	Cnt Sht
Scientific Adviser, Navy	Cnt Sht
Scientific Adviser, Air Force	Cnt Sht
Scientific Adviser, Army	Cnt Sht
Electronics Research Laboratory	
Director	2
Chief, Communications Division	3
Research Leader, Communications	4
Research Leader, Communications Countermeasures	5
Head, Communications Technology	6
Attention: Mr G. J. Parker	7-8
Head, Communications Surveillance Systems	9
Graphics and Documentation Support	10-11
Libraries and Information Services	
Director of Publications, Australian Government Publishing Service	12
OIC, Technical Reports Centre, Defence Central Library Campbell Park	13
Manager Document Exchange Centre, Defence Information Services	14
Defense Technical Information Center, United States	15-16
Defence Research Information Centre, United Kingdom	17-18
Director Scientific Information Services, Canada	19
Ministry of Defence, New Zealand	20
National Library of Australia	21
Defence Science and Technology Organisation Salisbury, Main Library	22-23
Librarian Defence Signals Directorate, Melbourne	24
British Library Document Supply Centre	25
Australian Defence Force Academy	26
Spares	
Defence Science and Technology Organisation Salisbury, Main Library	27-32

DOCUMENT CONTROL DATA SHEET

Page Classification

UNCLASSIFIED

Privacy Marking/Caveat

1a. AR Number 006-443	1b. Establishment Number ERL-0530-RN	2. Document Date DEC 90	3. Task Number	
4. Title USING A DISCRETE FOURIER TRANSFORM TO ESTIMATE PHASE IN INTERFEROMETRIC DIRECTION FINDING		5. Security Classification	6. No. of Pages	28
		☐ U ☐ U ☐ U Document Title Abstract	7. No. of Refs.	3
		S (Secret) C (Conf) R (Rest) U (Unclass) * For UNCLASSIFIED docs with a secondary distribution LIMITATION, use (L.)		
8. Author(s) Gareth J. Parker		9. Downgrading/Delimiting Instructions		
10a. Corporate Author and Address Electronics Research Laboratory PO Box 1600 SALISBURY SA 5108		11. Officer/Position responsible for Security..... Downgrading..... Approval for Release..... DERL		
10b. Task Sponsor				
12. Secondary Release of this Document APPROVED FOR PUBLIC RELEASE Any enquiries outside stated limitations should be referred through DSTIC, Defence Information Services, Department of Defence, Anzac Park West, Canberra, ACT 2600.				
13a. Deliberate Announcement NO LIMITATIONS				
13b. Casual Announcement (for citation in other documents)				
☒ No Limitation ☐ Ref. by Author & Doc No only				
14. DEFTEST Descriptors Discrete Fourier Transform Direction Finding Interferometers Signal Analysis		15. DISCAT Subject Codes 1703 1201		
16. Abstract This document discusses the use of the Discrete Fourier Transform to estimate the phase of particular frequency components of a sampled signal. Attention is paid to the limitations of the Discrete Fourier Transform in general. Particular emphasis is placed on estimating phase for use in Interferometric Direction Finding.				

16. Abstract (CONT.)		
17. Imprint Electronics Research Laboratory PO Box 1600 SALISBURY SA 5108		
18. Document Series and Number ERL-0530-RN	19. Cost Code 603855	20. Type of Report and Period Covered REPORT NOTE
21. Computer Programs Used NONE		
22. Establishment File Reference(s)		
23. Additional information (if required)		