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Noise Factor and Antenna Gains in the Signal/Noise Equation for Over-the-Horizon Radar

MITRE

Bedford, Massachusetts

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NOISE FACTOR AND ANTENNA GAINS IN THE SIGNAL/NOISE EQUATION FOR OVER-THE-HORIZON RADAR

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The predetection signal-to-noise ratio (SNR) of a radar or communication system is proportional to the power gain of the transmit antenna and the directive gain of the receive antenna, and is inversely proportional to the operating noise factor of the receiving system. The operating noise factor is approximately equal to the product of the external noise factor and the signal/noise processing factor when the system is external noise-limited, as is usually the case for over-the-horizon (OTH) radar. Unfortunately, the form of the signal/noise equation that is employed for some applications, particularly OTH radar, often does not explicitly yield these results rather than equivalent implicit results.

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ABSTRACT

The predetection signal-to-noise ratio (SNR) of a radar or communication system is proportional to the power gain of the transmit antenna and the directive gain of the receive antenna, and is inversely proportional to the operating noise factor of the receiving system. The operating noise factor is approximately equal to the product of the external noise factor and the signal/noise processing factor when the system is external noise-limited, as is usually the case for over-the-horizon (OTH) radar. Unfortunately, the form of the signal/noise equation that is employed for some applications, particularly OTH radar, often does not explicitly yield these results. It is recommended that the form of the signal/noise equation be amended to explicitly yield these results rather than equivalent implicit results.

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INTRODUCTION AND SUMMARY

The predetection signal-to-noise ratio (SNR) of a radar or communication system is proportional to the power gain of the transmit antenna and the directive gain of the receive antenna, and is inversely proportional to the operating noise factor of the receiving system. The operating noise factor is approximately equal to the product of the external noise factor and the signal/noise processing factor when the system is external noise-limited, as is usually the case for over-the-horizon (OTH) radar. Unfortunately, the form of the signal/noise equation that is employed for some applications, particularly OTH radar, often does not explicitly yield these results. It is recommended that the form of the signal/noise equation be amended to explicitly yield these results rather than equivalent implicit results.

The recommended form of the signal-to-noise equation that includes both internal and external system noise and signal/noise processing losses is discussed in Section 2. The recommended form conforms to the internationally-accepted definition of system operating noise factor but extended to include signal/noise processing. Other equivalent forms of the signal/noise equation are discussed in Section 3.

SECTION 2

RECOMMENDED FORM OF THE SIGNAL-TO-NOISE RADAR EQUATION

With reference to figure 1, the pre-detection signal-to-noise ratio s/n of a bistatic radar system is given by [1]

$$\frac{s}{n} = \overbrace{\frac{p_t g_t}{4\pi \ell_{p1} r_1^2} \quad \frac{\sigma_T}{4\pi \ell_{p2} r_2^2} \quad \frac{d_r \lambda^2}{4\pi}}^{s} \quad \underbrace{\frac{1}{k t_{ref} b f}}_{n} \qquad (2\text{-}1)$$

incident power density on target	fraction of reflected power returned	collecting area of receive antenna

where

s = available signal power at the output terminals of the equivalent lossless receiving antenna (W)

n = system available noise power, after signal processing, but before threshold detection, referred to the output terminals of the equivalent lossless receiving antenna (W)

= $k t_{ref} b f$

p_t = average power delivered to the transmit antenna (W)

= p_o / a_{nt}

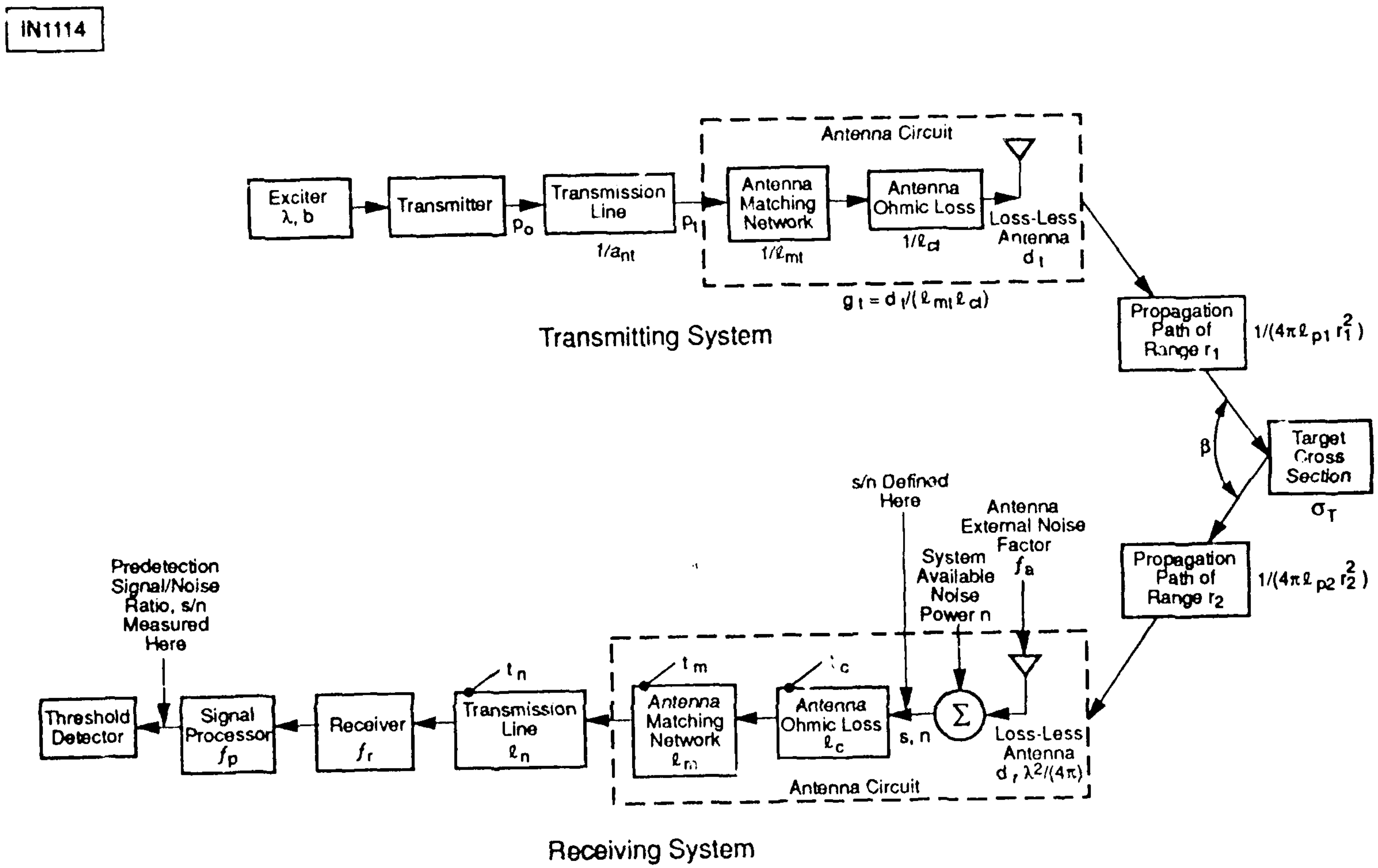

Figure 1. Signal/Noise Equivalent Circuit of a Bistatic Radar System

p_o = transmitter available average power (W)

a_{nt} = transmit transmission line loss factor accounting for the line absorption loss and the antenna reflection loss (numeric ≥ 1). The factor a_{nt} is evaluated in eq. (2-6).

g_t = transmit antenna power gain (numeric)

r_1 = free-space slant range from the transmit antenna to the target (m)

ℓ_{p1} = excess propagation loss factor, over that of free space loss factor $4\pi r_1^2$, from the transmit antenna to the target (numeric)

σ_T = bistatic target radar cross-section (m^2) for a bistatic angle β.

r_2 = free-space slant range from the target to the receive antenna (m)

ℓ_{p2} = excess propagation loss factor, over that of free space loss factor $4\pi r_2^2$, from the target to the receive antenna (numeric)

d_r = receive antenna directive gain (numeric)

λ = rf wavelength (m)

k = Boltzmann's constant = 1.38×10^{-23} (J/k)

t_{ref} = arbitrary reference noise temperature (k)

b = effective noise bandwidth of the receiving system[2] (Hz) = $(1 / g_o \int_{v_a}^{v_b} g_{ov} dv$

g_o = maximum gain of the receiving system gain g_{ν} within the frequency band $\nu_b - \nu_a$ of the principal response of the receiving system (numeric)

f = system operating noise factor of the receiving system (numeric).

We follow the convention that lower case letters denote numeric values of the parameters and that upper case letters denote the parameters when expressed in dB [i.e. 10 $\log_{10}$ (numeric value)]. The parameters of the transmitting system have been denoted by the subscript t to distinguish them from those of the receiving system.

For an OTH radar system, the bistatic slant ranges r_1 and r_2 are approximately equal and the bistatic angle $\beta \approx 0$. Futhermore, the terrestrial propagation path losses are usually included in the parameters g_t, d_r, and σ_T rather than in the parameters ℓ_{p1} and ℓ_{p2}. Accordingly,

$$r_1 \approx r_2 = r \tag{2-2}$$

$\ell_{p1} \approx \ell_{p2} =$ one-way ionospheric excess propagation loss factor including focusing gain by a spherical ionosphere (2-3)

$\sigma_T \approx$ monostatic target radar cross-section (m^2) in the backscatter direction ($\beta = 0$).

The earth multipath pattern (including the null on the radio horizon) and power absorption by the earth in the vicinity of the transmit and receive antennas are included in their respective power gains g_t and g_r by considering the earth as part of their respective antenna ground plane systems. The earth multipath pattern in the vicinity of the target is included in the target radar cross-section σ_T.

The system operating noise factor f, which includes both external and internal noises and is based on an international CCIR definition of noise factor[2] but extended to include signal/noise processing losses, is given by[1]

$$f = [f_a + (\ell_c - 1)(t_c / t_{ref}) + \ell_c(\ell_m - 1)(t_m / t_{ref})$$

$$+\ell_c\ell_m(\ell_n - 1)(t_n / t_{ref}) + \ell_c\ell_m\ell_n(f_r - 1)]f_p \tag{2-4}$$

where f_a is the receive antenna external noise factor integrated over the antenna pattern function (numeric); ℓ_c, ℓ_m, ℓ_n are the available loss factors of the receive antenna, matching network, and transmission line, respectively (numeric $\geq$ 1); t_c, t_m, t_n are the ambient temperatures (k) of the receive antenna, matching network, and transmission line, respectively, and f_r is the receiver noise factor (numeric $\geq$ 1) and f_p is the signal/noise processing factor (or simply, processing factor). If the ambient temperatures of the antenna, matching network, and transmission line are equal to the reference temperature t_{ref}, then eq. (3) reduces to

$$f = f_a - 1 + \ell_c\ell_m\ell_n f_r \;; \quad t_c = t_m = t_n = t_{ref} \tag{2-5}$$

It is convenient to set t_{ref} = 288k because measurements of atmospheric noise and man-made environmental noise are usually referenced to thermal noise at that temperature and because at that temperature $10 \log_{10} kt_{ref}$ = -204.0059 dBj is approximately a whole number.

The available loss factors ℓ_c, ℓ_m, ℓ_n and the receiver noise factor f_r are evaluated in Ref. [1] and will not be repeated here. The available loss factors ℓ_c, ℓ_m, ℓ_n are a function of the impedance parameters and source impedances of the respective circuits and are equal to the reciprocal of the respective circuit efficiencies. The receiver noise factor f_r is a function of the receiver source admittance and the characteristic noise parameters f_o, r_n, y_{no} of the receiver where

f_o = the minimum noise factor for any possible source impedance

r_n = empirical noise parameters, with the dimension of resistance, which is a measure of the noise factor sensitivity to a change in source impedance

y_{no} = source admittance for which $f_r = f_o$.

The transmit transmission line loss factor a_{nt} is given be

$$a_{nt} = \left|1 - \Gamma_t \exp(-2\gamma_t d_{nt})\right|^2 \ell_{nt} \tag{2-6}$$

where

Γ_t = voltage reflection coefficient at the transmit transmission line - matching network interface (numeric)

γ_t = complex propagation constant of the transmit transmission line (m^{-1})

d_{nt} = length of transmit transmission line (m)

ℓ_{nt} = available loss factor of the transmit transmission line (numeric) given by eq. (19) of Ref. [1].

The processing factor f_p (numeric ≥ 1) is the available power signal-to-noise ratio at the output of a signal processor with a matched filter to that for the same signal processor but with a weighted filter. The processing factor includes range and Doppler frequency processing and is restricted to processing before threshold detection. Threshold detection is defined at the point at which rf phase information is lost. The processing factor f_p , for a radar system with range and Doppler frequency processing, is given by

$$f_p = f_{range} \ f_{Doppler} \tag{2-7}$$

where (3), (4)

f_{range} = range processing factor (numeric ≥ 1)

$$= [\tau_r \int_o^{\tau_r} w_r^2(\tau)dt] \int_\tau / [\int_o^{\tau_r} w_r(t)dt]^2$$

$f_{Doppler}$ = Doppler frequency processing factor (numeric ≥ 1)

$$= [\tau \int_o^{\tau} w_D^2(\tau)dt] / [\int_o^{\tau} w_D(t)dt]^2$$

$w_r(t)$ = weighting function (numeric) of range processing filter

$w_D(t)$ = weighting function (numeric) of Doppler freqency processing filter

τ_r = range window processing time interval (s)

τ = coherent integration processing time interval (s)

For an external noise-limited system, the system operating noise factor given by eq. (2-4) reduces to

$$f \approx f_a f_p \ , \ f_a >> \ell_c \ell_m \ell_n f_r - 1 \tag{2-8}$$

where $\ell_c, \ell_m, \ell_n, f_r$ are system internal noise parameters generated by the receiving system hardware. Substituting eq. (2-8) into eq. (2-1), the predetection signal-to-noise ratio reduces to

$$\frac{s}{n}=\frac{p_t g_t}{4\pi\ell_{p1}r_1^2}\;\frac{\sigma_T}{4\pi\ell_{p2}r_2^2}\;\frac{d_r\lambda^2}{4\pi}\;\frac{1}{f_p}\;\frac{1}{\mathrm{kt_{ref}}bf_a}\;,\quad f_a >> \ell_c\ell_m\ell_n f_r - 1 \tag{2-9}$$

It should be noted in eq. (2-9) that for an external noise-limited system, the predetection signal-to-noise ratio is proportional to the transmit antenna power gain and receive antenna directive gain, and is inversely proportional to the antenna external noise factor. Eq. (2-9) is a particularly useful form of the signal/noise equation for OTH radar since OTH radar systems are usually designed to be external noise-limited. However, eq. (2-1) is a preferred form of the signal/noise equation for any radar system because it utilizes an internationally-accepted convention for defining the system operating noise factor that includes both externally and internally-generated noise.

The external noise factor f_a is given by (2),(5)

$$f_a = t_a / t_{ref} = (1/4\pi)\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} f_s(\theta,\phi)\, d_r(\theta,\phi)\, \sin\theta\, d\theta\, d\phi \tag{2-10}$$

where

$t_a =$ effective sky temperature (deg k)

$f_s(\theta,\phi) = t_s(\theta,\phi)/t_{ref} =$ external noise factor angular distribution (numeric)

$t_s(\theta,\phi) =$ sky temperature angular distribution (deg k)

The International Radio Consultative Committee [CCIR (French)] has published statistical values of f_a for atmospheric noise based on measurements in the frequency range 0.01-20 MHz as a function of location, hour, and season (6),(7) and man-made noise based on measurements in the frequency range 0.25-250 MHz as a function of type of location (8). CCIR claims that their values of f_a [denoted $(f_a)_{\mathrm{CCIR}}$] are normalized to correspond to those that would be measured with an electrically-short vertical monopole element mounted on a ground plane of infinite extent and infinite conductivity. The directive gain of such an antenna is given by [9]

$$[d_r(\theta,\phi)]_{CCIR} = \begin{cases} 3\sin^2\theta, & 0 \le \theta \le \pi/2 \ \ rad \\ 0, & -\pi/2 \le \theta < 0 \ \ rad \end{cases} \tag{2-11}$$

Substituting eq. (2-11) into eq. (2-0), the CCIR external noise factor $(f_a)_{CCIR}$ is given by

$$(f_a)_{CCIR} = \int_o^{2\pi}\int_o^{\pi/2} [f_s(\theta,\phi)]_{CCIR} \ 3\sin^3\theta \ \ d\theta d\phi \tag{2-12}$$

where

$[f_s(\theta,\phi)]_{CCIR}$ = CCIR external noise factor angular distribution (numeric).

A receiving system's external noise factor f_a, expressed in terms of CCIR external noise factor $(f_a)_{CCIR}$, is found from eqs. (2-10) and (2-12) to be

$$f_a = (f_a)_{CCIR} \cdot [f_a / (f_a)_{CCIR}]$$

$$= (f_a)_{CCIR} \frac{\int_o^{2\pi}\int_{-\pi/2}^{\pi/2} f_s(\theta,\phi) \ d_r(\theta,\phi) \ \sin\theta \ \mathrm{d}\theta \ \mathrm{d}\phi}{\int_o^{2\pi}\int_o^{\pi/2} [f_s(\theta,\phi)]_{CCIR} \ 3\sin^3\theta \ d\theta \ d\phi} \tag{2-13}$$

For isotropic external noise with a constant sky temperature angular distribution t_{so},

$$f_a = (f_a)_{CCIR} = f_{so} \quad , \quad f_s(\theta,\phi) = [f_s(\theta,\phi)]_{CCIR} = \text{constant} = f_{so} \tag{2-14}$$

where

$$f_{so} = t_{so} / t_{ref}$$

because, by definition of directive gain,

$$\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} d_r \sin\theta \ d\theta \ d\phi = \int_0^{2\pi}\int_{-\pi/2}^{\pi/2} (d_r)_{CCIR} \ \sin\theta \ d\theta \ d\phi = 4\pi \qquad (2\text{-}15)$$

It should be noted that in OTH radar systems the directive gain $d_r(\theta,\phi)$ generally has a null on the horizon ($\theta = \pi / 2 \ \ rad$) because of earth multipath. The directive gain of an OTH receiving antenna is therefore appreciably different at angles near the horizon from that given by eq. (2-11) which has a maximum on the horizon. The external noise factor f_a can therefore be appreciably different from $(f_a)_{CCIR}$ even if $f_s(\theta,\phi) = [f_s(\theta,\phi)]_{CCIR} \neq$ constant!

At least six of the parameters in the signal/noise equation (2-1) are dependent upon antenna characteristics. The average power p_t delivered to the transmit antenna is dependent upon the mismatch input impedance of the transmit antenna. The transmit antenna power gain g_t is a function of the radiation pattern and ohmic losses of the transmit antenna (including its ground plane system). The receive antenna directive gain d_r is a function of the radiation pattern of the receive antenna (including its ground plane system). The excess propagation loss factors ℓ_p, and ℓ_{p2} are functions of the gain patterns of the transmit and receive antennas, respectively. The system operating noise factor f is a function of the radiation pattern and ohmic losses of the receive antenna. A seventh parameter, the target radar cross-section σ_T is a function of the radiation patterns of both the transmit and receive antennas if earth multipath in the vicinity of the target is included as part of the target radar cross-section. In OTH radar systems, the parameters p_t, g_t, d_r, and f are strongly dependent upon antenna characteristics, whereas the parameters ℓ_p, ℓ_{p2}, and σ_T are weakly dependent upon antenna characteristics for a given mode of propagation. The parameters ℓ_{p1}, ℓ_{p2}, and σ_T are usually approximated by considering only the central ray in the scanned direction.

SECTION 3

OTHER EQUIVALENT FORMS OF THE SIGNAL-TO-NOISE RADAR EQUATION

For a radar system with Doppler frequency processing (as in the case of the OTH radar), the effective noise bandwidth b of the receiving system is given by

$$b = \text{Doppler frequency cell width} \approx 1/\tau \tag{3-1}$$

where τ is the coherent integration time (s). If the noise power spectral density is uniform within the bandwidth b, then the predetection signal-to-noise ratio is given by

$$\frac{s}{n} = \frac{e}{n_o f_p} \tag{3-2}$$

where

n_o = system available noise power spectral density, before signal processing, referred to the output terminals of the equivalent lossless receive antenna (W/Hz = J).

$$= k t_{ref} f / f_p = n / (b f_p) = n\tau / f_p \tag{3-3}$$

e = available signal energy, after signal processing, referred to the output terminals of the equivalent lossless receive antenna (J)

$$= s/b \approx s\tau \tag{3-4}$$

We further note that the receive antenna power gain g_r is related to the receive antenna directive gain d_r by

$$g_r = d_r / \ell_c = \eta_c d_r \tag{3-5}$$

where

ℓ_c = available loss factor of the receive antenna including ohmic losses of the elements and ground plane system (numeric $\geq$ 1).

η_c = $1/\ell_c$ = radiation efficiency of the receive antenna (numeric $\leq$ 1).

Substituting eqs. (2-2), (2-3), (3-3)-(3-5) into eq. (2-1), the predetection signal-to-noise ratio is given by

$$\frac{e}{n_o} = \frac{p_t g_t}{4\pi r^2} \frac{\sigma_T}{4\pi r^2} \frac{g_r \lambda^2}{4\pi} \frac{\tau}{k t_{ref} f_1 \ell} \tag{3-6}$$

where

$\ell = \ell_{p1} \ell_{p2} f_p$ = system loss factor (numeric $\geq$ 1)

$f_1 = (1/\ell_c f_p) f = (\eta_c / f_p) f$ = modified system operating noise factor (numeric)

Eq. (3-6) is an equivalent form of the radar equation that is commonly employed for OTH radar systems.(2-1) Please note that eq. (3-6) is equivalent to that of eq. (1) and reduces to eq. (2-9) for an external noise-limited system. However, the disadvantages of eq. (3-6) are that it is an implicit rather than explicit function of the receive antenna directive gain d_r and system operating noise factor f, and it utilizes a modified system operating noise factor f_1, rather than the internationally-accepted definition of system operating noise factor but extended to include signal/noise processing. For an external noise-limited system, f_1 reduces to $(1/\ell_c)\ f_a$ rather than f_a .

Another form of the radar equation that is commonly used for OTH radar system performance estimation is given by

$$\frac{e}{n_o} = \frac{p_{t1} d_t}{4\pi r^2} \frac{\sigma_T}{4\pi r^2} \frac{d_r \lambda^2}{4\pi} \frac{\tau}{n_o \ell} \tag{3-7}$$

where

p_{ta} = average power radiated by the transmit antenna (W)

= $(1/\ell_{ct}) p_t = \eta_{ct} p_t$

d_t = transmit antenna directive gain (numeric)

= $\ell_{ct} g_t = (1/\eta_{ct}) g_t$

ℓ_{ct} = available loss factor of the transmit antenna including ohmic losses of the elements and ground plane system (numeric ≥ 1)

η_{ct} = $1/\ell_{ct}$ = radiation efficiency of the transmit antenna (numeric ≤ 1).

Eq. (3-7) is equivalent to that of eq. (2-1) and reduces to eq. (2-9) for an external noise-limited system. However, the disadvantage of eq. (3-7) is that it is an implicit rather than explicit function of the average power P_t delivered to the transmit antenna and the system operating factor f.

It is recommended that eq. (2-1), rather than eqs. (3-6) and (3-7), be the form of the signal-to-noise radar equation that is employed for radar systems. The advantages of eq. (2-1) are that it employs an internationally-accepted definition of system operating noise factor but extended to include signal/noise processing and it yields for an external noise-limited system a predetection signal-to-noise ratio that is an explicit function of the transmit antenna power gain, receive antenna directive gain, and the antenna external noise factor.

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3. Nathanson, F. E., <u>Radar Design Principles</u>, New York, NY: McGraw-Hill, 1969, p. 522. Ref. [2] gives the reciprocal of f_{range} and $f_{Doppler}$ in eq. (2-7) because [2] defines loss factor as a gain instead of an attenuation.

4. Barton, D. K., and Ward, H. R., <u>Handbook of Radar Measurement</u>, Englewood Cliffs, NJ: Prentic-Hall, 1969, p. 348.

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9. Weiner, M. M., <u>Monopole Elements on Circular Ground Planes</u> (Artech House, Norwood, MA, 1987), p. 63, eq. (3.8.18). Please note that the exponent two in $\sin^2\theta$ is inadvertently missing.