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THE MANY-ON-MANY GENERAL RENEWAL LANCHESTER COMBAT MODEL

FINAL REPORT

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this study non-Markovian models of some stochastic combats were developed and analyzed in order to obtain more realistic models of firefight than the extant deterministic and exponential Lanchester models.

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POSITION, POLICY, OR DECISION, UNLESS SO DESIGNATED BY
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A. Statement of the Problem Studied

1. Background

The principal motivation for this work is the development of more realistic small-to-moderate size firefight models than the extant deterministic and exponential Lanchester square law models. It is important to note here that what we call exponential model has been called stochastic Lanchester in the literature to date. We believe substituting the adjective "exponential" for "stochastic" is a more accurate description because the classical stochastic Lanchester model merely assumes that the interkilling time is an exponential random variable. On the other hand, the models we define allow an arbitrary interkilling time random variable.

Obviously the deterministic and exponential Lanchester models are greatly simplified descriptions of the firefight. Common sense and field experiments dictate that: 1) the real firefight is stochastic, and 2) the exponential assumption is a gross simplification of the actual interkilling time and is only used because its "no memory" property greatly simplifies the analysis. Both the deterministic and exponential lanchester models have been studied in great detail, and excellent expositions devoted to them may be found in references [3], [8], and [9]. Theoretical solutions to the stochastic combat models defined in this paper may be found in references [1] and [6] and are for the one-on-one (reference [1]) and two-on-one (reference [6]).

2. Statement of Problem

Our specific objectives, paraphrased here from the STATEMENT OF WORK in our January 1986 proposal to ARO, are as follows:

1. Develop a virsatile many-on-many combat simulation model capable of providing

important measures of combat outcome. Specify and evaluate the statistical procedures required to achieve this objective.

2. In parallel with the simulation effort, develop an analytical-numerical procedure to provide the desired measures of combat outcome.

3. Conduct analytical and Monte Carlo studies on the superposition of renewal processes with the objective of developing a satisfactory approximate model using a nonhomogeneous Poisson process approximation of the superposition.

3. Models

The basic firefight model we are considering in the study has two sides A and B conducting a continuous engagement satisfying the following assumptions:

1. There are initially a_o on the A side and b_o on the B side with breakpoints a_f and b_f respectively.

2. Every member of A side picks a B opponent at random (all are visible and in range).

3. Each marksman fires until killed or until his target is killed and resumes firing immediately on a survivor, selected at random, in one of two distinct ways described below and denoted as Versions 1 and 2.

4. The interkilling time, random variable for each marksman does not change from kill-to-kill.

5. All fire independently.

6. The ammunition supply is unlimited.

7. Similar assumptions apply to the B side.

8. The battle continues until one side reaches its breakpoint.

The two modes of resuming firing on a survivor are:

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1.	Version 1.	Consider a given marksman firing at a target. Whether his target is killed by him or the other member of his side, he resumes afresh the interkilling process on the survivor.

2.	Version 2.	If the marksman's target is killed by him he starts afresh the interkilling process on a survivor. If, on the other hand, his target is killed by another member on the side, his remaining time to a killing is carried over to his next target.

The jargon we use to describe Versions 1 and 2 are "reslect on" and "reslect off" respectively.

In addition to the strategy of random selection of target, we have considered two additional strategies, namely, 1) concentrated fire in which all firers aim at one target, and 2) distributed fire. The side using distributed fire spreads its fire across the opponents so as to equalize, as much as possible, the number of firers per target. E.g., if the side using distributed fire has ten firers and the opponent six, there would be two firers on each of four opponents and one firer on each of two opponents. If, on the other hand, there are a total of twelve opponents, there would be one firer on each of ten opponents and no firers on each of two opponents.

We distinguish homogeneous and heterogeneous combats. Homogeneous combat is defined as one in which all combatants on a side have the same interkilling time random variable. In heterogeneous combat we allow the interkilling time random variables on each side to differ.

It should also be noted that the interkilling time random variable is a consequence of a complex process involving a sequence $T_1, T_2,...$ of interfiring time random variables and the associated sequence of round-to-round kill probabilities $p_1, p_2,...$. In the simplest case, one would have a constant round-to-round kill probability with the T_is independently, identically distributed.

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B. Summary of the Most Important Results

1. Notation

Notation used throughout the remainder of this paper are:

a_0 = the initial random on side A (at time 0),

a_f = breakpoint for side A, i.e., the number on side A at the time A side loses (breaks and runs),

b_0 = initial number on side B (at time 0),

b_f = breakpoint for side B, i.e., the number on side B at the time B side loses (breaks and runs),

$A(t)$ = random variable, number alive on side A at time t,

$B(t)$ = random variable, number alive on side B at time t,

$p_{ab}(t)$ = $P[A(t) = a, B(t) = b]$, a state probability function,

$m_A(t)$ = $E[A(t)]$, expected value of A(t),

$m_B(t)$ = $E[B(t)]$, expected value of B(t),

$\sigma_A(t)$ = $D[A(t)]$, standard deviation of A(t),

$\sigma_B(t)$ = $D[B(t)]$, standard deviation of B(t),

$P[i]$ = probability i side wins, i = A,B,

T_D = random variable, time duration of combat,

μ_{T_D} = expected value of TD,

σ_{T_D} = standard deviation of TD,

ν_i = mean interkilling time on side i, i = A,B,

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$r_A \; = \; 1/\nu_A \; =$ A's kill rate (attrition coefficient for side B),

$r_B \; = \; 1/\nu_B \; =$ B's kill rate (attrition coefficient for side A),

and whenever the single shot kill probability and interfiring time random variables are the same from round-to-round we use the notation,

$p_i \; =$ the constant kill probability of all contestants on side i, i = A,B,

$\mu_i \; =$ mean interfiring time on side i, i = A,B, so that

$$r_A \; = \; 1/\nu_A \; = \; p_A/\mu_A \; ,$$

$$r_B \; = \; 1/\nu_B \; = \; p_B/\mu_B \; .$$

2. Figures of Merit

Overall battle figures of merit include:

1. Expected value and standard deviation of the number of survivors on side A, $m_A(\infty)$ and $\sigma_A(\infty)$ respectively,

2. Expected value and standard deviation of the number of survivors on side B, $m_B(\infty)$ and $\sigma_B(\infty)$ respectively,

3. Expected value and standard deviation of T_D the time duration of combat, μ_{T_D} and σ_{T_D} respectively.

4. The probabilities of win for sides A and B, P[A] and P[B] = 1 - P[A] respectively. Time varying figures of merit are $p_{ab}(t)$, $m_A(t)$, $\sigma_A(t)$, $m_B(t)$ and $\sigma_B(t)$.

3. Simulation Results

3.1 Simulation Models

Two different simulation models were developed. One uses the interfiring time random variable in simulating the combat and is used only when the interfiring time random variable and kill probability do not change from round-to-round. Each side must be homogeneous and use the random-selection-of-target strategy in either the "reselect on" or "reslect off" model.

The second one uses the interkilling time random variable and allows each side 1) to be either homogeneous or heterogeneous, 2) use either the random selection, or concentrated, or distributed strategies and 3) be operated in the "reselect on" or "reselect off" mode.

Unless information on number of rounds fired is needed, in which case we must perforce use the interfiring time model, it is always desirable to use the interkilling time model because we move from killing-to-killing rather than from firing-to-firing and therefore consume less CPU time. However, this may not always be possible if we are unable to generate values of the interkilling time random variable.

The models are of the standard discrete event type. The random number generator which we used to produce pseudorandom sequences has the prime modulus multiplicative generator

$$X_i = 16807 \, X_{i-1} \bmod (2^{31} - 1) \; ,$$

$$U_i = X_i/(2^{31} - 1) \; .$$

This particular generator, due to Lewis, Goodman, and Miller (reference [7]), has been studied theoretically and tested empirically (see references [4] and [5]) and has been judged to perform acceptably.

The verification for both the interfiring time and interkilling time models was done in two phases. First, each model was checked manually to ensure its proper functioning. The second phase of model verification involved comparing the simulation outputs of the interfiring and

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interkilling models against known analytical results. The simulations were run for one million replications each to ensure that the estimates of all eight overall combat measures were reliable. In all cases, the relative error between the simulation output and the analytical value was less than .3% indicating that indeed the coding is correct.

3.2 Statistical Procedures

Four statistical procedures were developed for use in conjunction with the models. A limited empirical evaluation of them was conducted toward establishing their validity.

The first is a single sample procedure and provides simultaneous confidence intervals for any desired set of figures of merit with preassigned confidence level; the second, on the other hand, is a double sample procedure which again provides simultaneous coverage but with control in the sense that the relative precision for each confidence interval is bounded.

The second two techniques are concerned with estimating differences between two models with respect to overall battle parameters. An example of this latter problem arises if one wished to ascertain how well a more realistic model, such as one developed in this study, is approximated by an exponential Lanchester model. We answer this question by estimating the actual relative differences (which we call relative errors) that obtain and make a judgement based on these results. Thus, the third procedure is a single sample one and provides simultaneous confidence intervals for the relative differences of the figures of merit with preassigned confidence level; the fourth procedure provides the simultaneous coverage with control on the confidence interval lengths by bounding the absolute precision of each.

3.3 Two Examples

3.3.1 Deficiency of the Exponential Lanchester Approximation

The fourth statistical procedure is used on a firefight for which the following hold:

1. Side A - lognormal interfiring time with a kurtosis of .50, μ_A = 1.0, p_A = .10.

2. Side B - lognormal interfiring time with a kurtosis of .50, μ_B = 1.0, p_B = .90.

Table 1 below shows the point estimates of the relative errors that are obtained when the actual combat is approximated by the appropriate exponential Lanchester. The double sample procedure was used to get the absolute precision of the relative errors of the eight overall combat parameters to be less than or equal to two percentage points. Note the substantial differences between the hypothesized model and its exponential approximation. It should also be noted that, in general, increasing the combat size, i.e., increasing a_0 and b_0 exacerbates the relative errors. The importance of this observation stems from the fact that in the literature there is the mistaken notion (documented in reference [2]), arising from a misunderstanding of

	a_0=5,a_f=3 b_0=3,b_f=2	a_0=5,a_f=0 b_0=3,b_f=0	a_0=10,a_f=7 b_0=5,b_f=3	a_0=10,a_f=0 b_0=5,b_f=0	a_0=20,a_f=14 b_0=10,b_f=6	a_0=20,a_f=0 b_0=10,b_f=0	a_0=100,a_f=0 b_0=50,b_f=0
$E[T_D]$	41.89	29.20	42.31	21.34	41.78	32.90	41.92
$\sigma[T_D]$	-25.79	-28.44	-22.01	-3.83	-7.98	6.63	21.80
$E[A(\infty)]$	6.97	-.10	4.17	10.06	3.19	31.30	73.25
$\sigma[A(\infty)]$	10.82	-7.88	19.90	-1.58	24.83	3.21	35.36
$E[B(\infty)]$	-3.98	-16.33	-6.25	-23.26	-8.36	-72.57	-189.92
$\sigma[B(\infty)]$	1.66	-3.45	-5.62	-7.04	-10.72	-20.21	-29.28
$P[A]$	19.51	12.18	22.44	15.79	35.49	37.21	73.69
$P[B]$	-20.14	-4.10	-16.15	-10.37	-23.21	-32.06	-71.43

Table 1. Relative error estimates (%) when using an exponential Lanchester approximation to a combat for which side A and B have log normal interfiring times.

a mathematical theorem, that the exponential approximation holds in the limit. This is not the case.

3.3.2 Comparison of Three Fire Allocation Strategies

The simulation model was used to study distributed, concentrated, and random fire allocation in up to four-on-four homogeneous, nonexponential stochastic combats with "reselect on" and zero breakpoints. It should be noted that an exponential model would show no differences in these strategies. Win probability is used as a measure of relative effecitveness for each strategy against the others.

In Table 2 below are shown some of the results obtained when each combatant's interkilling time random variable was based on having a constant round-to-round kill probability and a gamma(2) interfiring time random variable. The standard deviations of the estimates are so small that the win probabilities shown are correct to three significant figures. An examination of this table shows that the best strategy is the distributed one followed by the random strategy. Note also that as the battle size increases the effect of strategy becomes more pronounced as one would expect.

Battle Size	Distributed vs. Random		Random vs. Concentrated		Distributed vs. Concentrated	
2-on-2	.509	.491	.508	.492	.517	.483
3-on-3	.521	.479	.536	.464	.558	.442
4-on-4	.531	.469	.575	.425	.608	.392

Table 2. Win probabilities when single shot kill probability = .9 and mean interfirng time = .9.

4. Theoretical Results

Theoretical results have been obtained for the following battles:

1. Both sides homogeneous with either "reselect on" or "reselect off," continuous interkilling time random variables, arbitrary a_o, a_f, b_o and b_f.

2. Both sides heterogeneous with either "reselect on" or "reselect off," continuous interkilling time random variables, arbitrary a_o, a_f, b_o and b_f.

The algorithm which has been developed for each of these cases systematically produces an expression for each of the state probabilities, $p_{ab}(t) = P[A(t) = a, B(t) = b]$. Each of these state probabilities is an n-dimensional integral, where n = the number of kills corresponding to that state; thus $n = (a_o - a) + (b_o - b)$. In general, numerical integration must be used to get the actual values. These state probabilities are then used to compute time varying characteristics or the eight overall battle characteristics as listed in Table 1.

As an example, suppose $a_o = 2$, $a_f = 0$, $b_o = 2$ and $b_f = 0$. Table 3 below lists all the possible states. Figure 1 below shows, for a specific battle, how the state probabilities vary with time while Figure 2 shows, for the same battle, some time varying characteristics of the number of survivors.

| Transient States | Absorbing States | |
	A wins	B wins
(2,2), (1,2)	(2,0)	(0,2)
(2,1), (1,1)	(1,0)	(0,1)

Table 3. States (a,b) in 2-vs-2 combat with zero breakpoints.
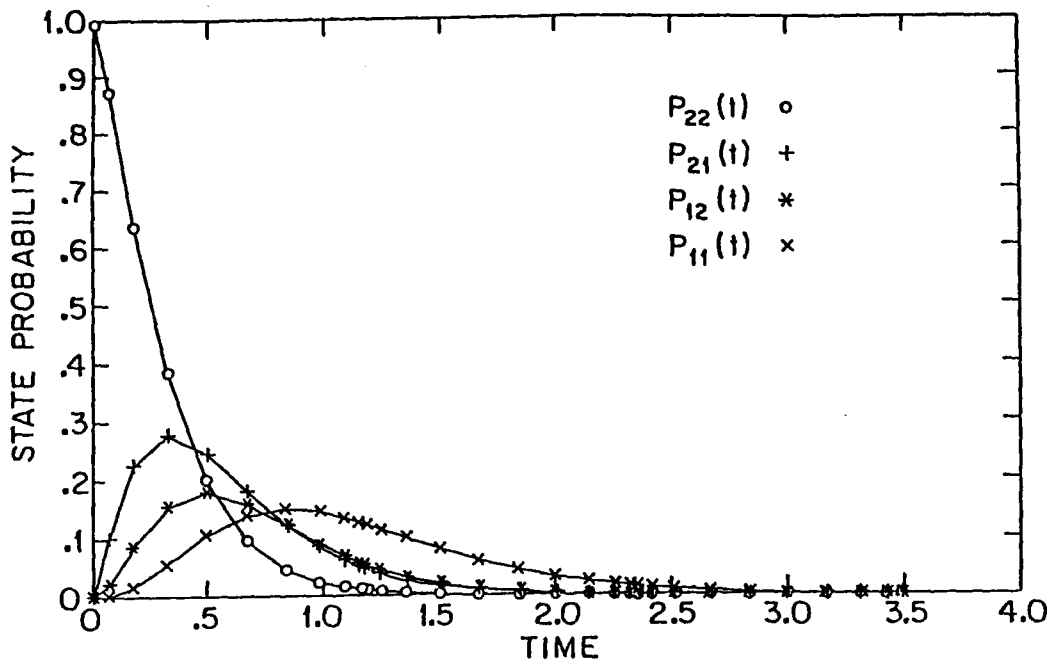
(a) Transient States
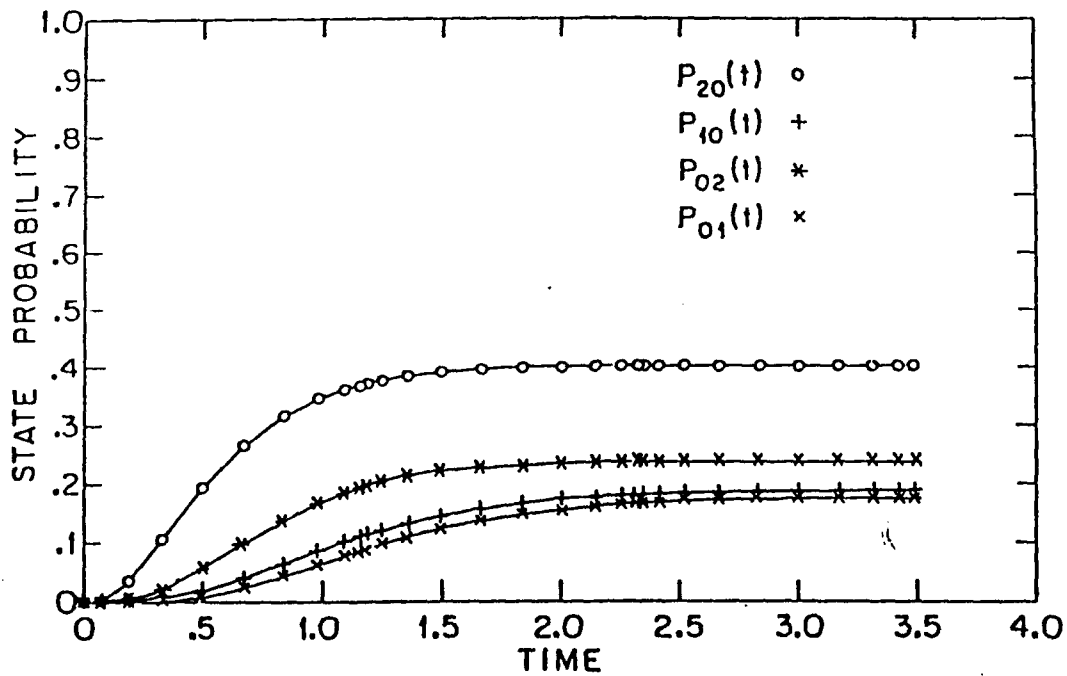
(b) Absorbing States

Figure 1.   State probabilities versus time for a 2-on-2 battle: A side interfiring
time random variable gamma(2) with $1/\mu_A = 10$, $p_A = 1/10$, $r_A = 1$;
B side interfiring time random variable gamma(2) with $1/\mu_B$, $p_B = 9/10$,
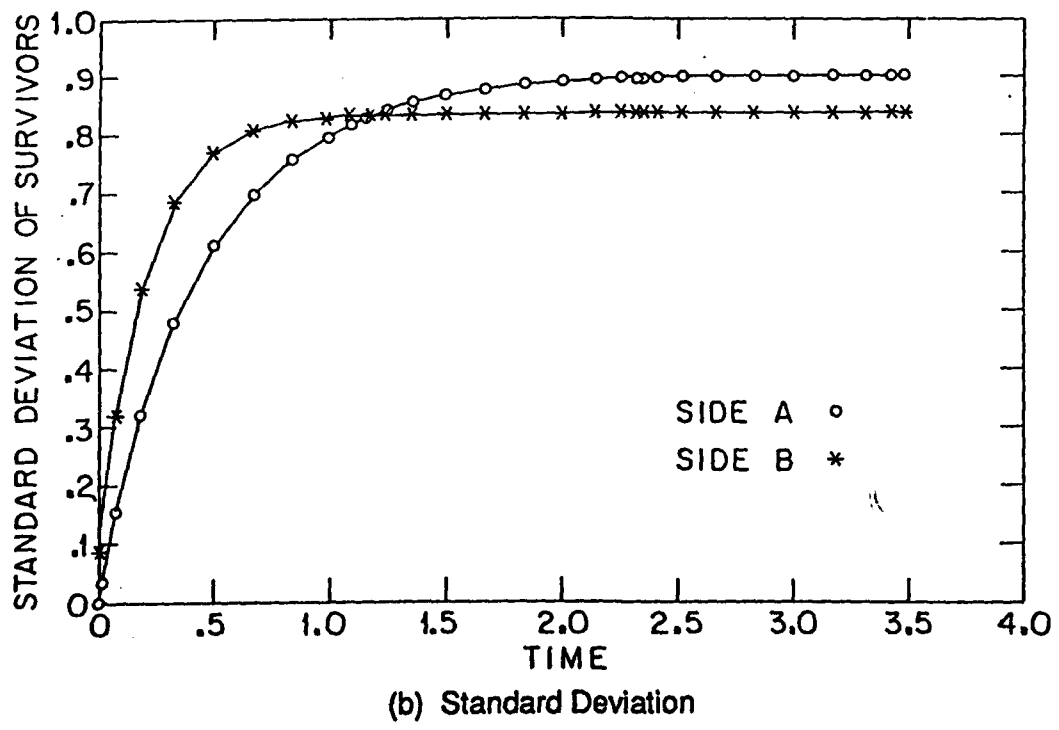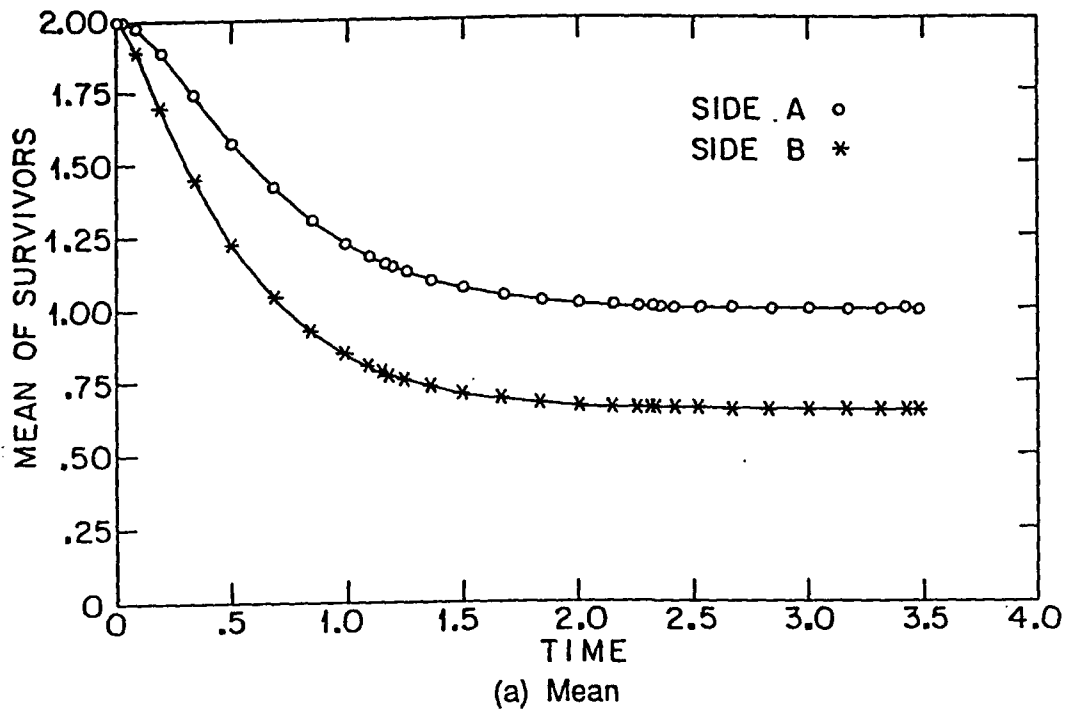$r_B = 1$; "reselect on."

Figure 2.   Mean and standard deviation of number of survivors versus time for
the 2-on-2 combat of Figure 1.

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5. A Nonhomogeneous Poisson Process Approximation

An individual combatant in a "reselect off" firing mode is clearly a renewal process which may terminate by either: 1) being killed or 2) the battle terminates by one of the sides reaching its breakpoint. Furthermore, the evolution of each of the combatnats is independent of the others. Thus, we have a situation in which each side may be looked upon as a superposition of independent, identical but terminating renewal process. We have approximated this superposition by a nonhomgeneous Poisson process which in turn enables us to write forward differential-difference Kolmogorov equations for the system. Numerical integration of these equations for state probabilities consumes substantially less computer time than the multiple integrals involved in the exact solution. Our studies to date show that this approximation produces acceptable approximations.

C. Publications and Technical Reports

Publications

1. Gafarian, A. V. and Manion, K. R., "Some Two-on-Two Homogeneous Stochastic Comnbats," <u>Naval Research Logistics</u>, 36, pp. 721-764 (1989).

Technical Reports

1. Gafarian, A. V. and Manion, K. R., "The General Stochastic Two-on-Two Combat Model," Industrial and Systems Engineering Department, University of Southern California, June 1987, 60 pp.

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Ph.D. Dissertations

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2. Parkhideh, S., "General Solutions to Some Heterogeneous and Homogeneous Stochastic Combats," Industrial and Systems Engineering Department, University of Southern California, August 1990, 98 pp.

D. Participating Scientific Personnel

1. Gafarian, A. V., Principal Investigator.

2. Hong, Y. G., Graduate Student, received Ph.D. in Industrial and Systems Engineering, May 1989.

3. Parkhideh, S., Graduate Student, received Ph.D. in Industrial and Systems Engineering, December 1990.

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[9]   Taylor, J. G., <u>Lanchester Models of Warfare</u>, Vol. II, research monograph published by the Military Applications Section of the Opeations Research Society of America, March 1983, 819 pp.