Application of Fourier-Fresnel Imaging to Neutral - Atom Interferometry

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In this note we provide a brief summary of the use of Fourier-Fresnel imaging to neutral atom interferometry. We shall see that in applications of neutral-atom interferometry not requiring the interferometer to have an open (multiply connected) topology, the use of this imaging has significant advantages, notably ease of alignment, significantly increased through-put flux, ability to work with very short wavelength (and/or high velocity atoms).

Many of the results presented here are not new, and were recognized by previous authors to have important application in electron interferometry and microscopy. Indeed, Cowley and Moodie remark that range of applications of their work to light is quite limited. The results are primarily interesting in that they span the boundary between trapezoidal Moire fringes and sinusoidal wave-interference fringes. Their limited applicability, perhaps accounts for the relative obscurity of many of the results to typical curricula of modern-day optics. The emphasis in this note is thus to put existing results (along with some additions that are needed for clarity) in a form suitable for use by techniques available for neutral-atom interferometry.

1. Possible Layouts for a Neutral-Atom Interferometer.

Figure 1. depicts two commonly discussed configurations for a neutral particle interferometer. In these, a source of neutral particles, is collimated into a narrow beam by a sequence of two slits, a distance A_c apart, each with a slit width w_c. The particles are preferably cold, slow, and possessing long deBroglie wavelength, λ. Following collimation, they pass through a sequence of three gratings (spaced respectively A_1 and A_2 apart), and thence to a detector. Since it is generally counter-productive to place significant spacing between the second collimating slit and the first grating, these elements are shown as combined into a

single finite extent grating.

The results of analyses of these configurations have frequently concluded that although the collimator may operate in the Fresnel regime ($w_c \gg \sqrt{A_c \lambda}$), there is no interference visible at the third grating unless the second grating is divided into two distinct parts, with each part accepting only one fully separated diffraction order from the first grating while blocking all others; and in addition, the third grating accepts only one fully separated diffraction order from each of the parts of the second grating. Typical transmission of such a configuration is about 10^{-3}. Moreover, widening the collimator acceptance angle w_c/A_c to increase the throughput flux translates into a corresponding broadening of the diffracted beams, now causing the orders to overlap. As a result, the throughput of the collimator must also be limited with the total system throughput quite low.

An additional disadvantage of such a system is that it is difficult to align. Gratings appropriate for matter-wave deBroglie wavelengths are unsuitable for electromagnetic radiation, unless the radiation is in the soft x-ray region with a wavelength comparable to the matter-wave deBroglie wavelength.

The purpose of the present note is to indicate that, contrary to previous analyses, it is not necessary to separate the diffraction orders if the open topology is not needed for a specific application, such as one requiring exploitation of the quantum topological phase. A price one must pay for acceptance of all diffraction orders is that the input atomic beam must be more nearly monochromatic. In actuality, this price was already being paid with the separated order configurations, since use of a significantly non-monochromatic beam with them will increase each diffracted beam's angular width, and again lead to order overlap with the corresponding necessity of further reduction of w_c and consequent throughput.

2. Historical Background of Fourier-Fresnel Imaging.

In 1836 Talbot[1] reported the results of an experiment that were quite surprising. The explanation of these results was

[1] H. Talbot, Phil. Mag. 9, 401 (1836).

provided by Lord Rayleigh[2] in 1881. The reciprocal effect (with interchanged source and detector) was observed experimentally by von Lau in 1948.

A diagram of the Talbot and von Lau experimental configurations is shown in Figure 2. In the Talbot configuration, monochromatic light from a point source is focused parallel by a lens and passes through two transmission gratings (Ronchi rulings) to an extended detector. Clearly, when the gratings have zero spacing (i.e. their planes are in contact) and are oriented so that their slits are parallel, they will form a Moire pattern. As one is translated parallel to the other in its own plane in a direction perpendicular to the slits, the transmitted light intensity will vary in a periodic trapezoidal fashion (triangular when the slit width to spacing ratio is 1/2). When the gratings are separated and the translation is again performed, the trapezoidal pattern tends to wash out. Talbot's surprising result was that at grating separations that are integral multiples of a characteristic length L_T, given by

$$L = n\, 2\, d^2/\lambda \equiv n\, L_{TR}, \text{ with } n = 0, 1, 2, \ldots \text{ is an integer,}$$

and d being the gratings' period, the Moire pattern reappears with high contrast. Moreover, when the integer n is odd, the pattern is trapezoidal dependence is shifted by 1/2 period.

A similar effect occurs in von Lau's configuration when an extended source is used along with a point detector at the focus of a lens following the second grating. Moreover, in von Lau's configuration, the image plane of the lens ~~becomes~~ displays an intensity pattern that is identical to the grating transmission function.

Since the fundamental Talbot-Rayleigh length L_{TR} is a function of the wavelength λ, the Talbot and von Lau effects clearly are examples of wave interference. Moreover, since the second grating accepts light in many diffraction orders simultaneously, a full analysis of these experiments must employ Fresnel, not Fraunhofer diffraction. Moreover, the scaling of L_{TR} is intriguing for neutral atom interferometry, since it has convenient values when calculated in terms of available atomic

[2] Lord Rayleigh, Phil. Mag. 11, 196 (1881).

deBroglie wavelengths and micro-fabricated gratings. However, neither the Talbot nor von Lau configurations are directly suitable for matter - wave interferometry because of the required presence of a lens.

Cowley and Moodie[3] provided a theoretical and experimental analysis in 1957 of the images of a point source formed by a transmission grating. They found that when a condition on the geometry is satisfied (similar to the Talbot-Rayleigh condition given above), the image plane of the grating displays an intensity pattern that is a magnified exact replica of the imaging grating. When the point source is replaced by a grating in their analysis, they produce an intermediate configuration which contains the Talbot and von Lau configurations as limiting cases (source and/or detector moved to infinity). Note that these configurations do not involve the use of a lens, and as such are eminently suitable for use in matter-wave interferometry (an application of their results that they noted).

Winthrop and Worthington[4] reanalyzed the Cowley and Moodie configuration in 1965 and discovered the existence of additional high contrast images that do not satisfy the Talbot-Rayleigh condition, and do not provide exact replicas of the imaging grating. They called these additional images Fresnel images, reserving the term Fourier images for those that form exact magnified images. Fresnel images of an intensity distribution that consists of a multiplicity of magnified replicas (aliases) of the original grating.

3. Application of Fourier - Fresnel imaging to neutral - atom Interferometry

We now summarize and apply the above results to a configuration suitable for matter-wave interferometry. Consider a sequence of three broad transmission gratings, whose planes are spaced a distance R apart. The middle grating has equally spaced open (vacuum) slits of width s with periodic spacing d, while the

[3] J.M.Cowley and A.F.Moodie, Proc. Phys. Soc. (London) 70, 486,497,505 (1957).

[4] J.T.Winthrop and C.R.Worthington, JOSA, 55, 373 (1965).

first an third gratings have open slits of width 2s with periodic spacing 2d. A convenient ratio of $s/d = 0.1$ will suffice for our purposes, but this choice is not critical. The sequence is diffusely illuminated by nearly monochromatic but weakly collimated neutral-atom deBroglie waves. In this configuration, unlike the configurations discussed earlier for neutral atom interferometers, the middle grating is not split into two parts.

Let us define a characteristic wavelength for this configuration $\lambda_{TR} \equiv d^2/R$. The first grating forms a series of small equally spaced incoherent sources, each one illuminating the second grating. All possible Fresnel diffraction orders of the second grating then reach the third grating. Following the above results, when the atomic debroglie wavelength $\lambda = \frac{m}{n} \lambda_{TR}$, then each source point on the first grating will form an image on the third grating consisting of a series of n stripes per 2d, each of width s. (When $n = 1$ holds, the images are Fourier images. When $n > 1$ holds the images are Fresnel images, by Winthrop and Worthington's definition.) If every n'th stripe is positioned on an open slit of the third grating, then transmission will occur. If the third grating is slowly translated across its own pattern, then the transmission will vary periodically with spatial period of $2d/n$. When λ / λ_{TR} is not a ratio of two small integers, periodic transmission does not occur. The periodic transmission signal can then easily be measured using standard phase sensitive detection (lock-in) techniques.

Since each source point of the first grating provides exactly the same pattern, the periodic set of sources provided by the first grating will increase the intensity transmitted by the third grating in proportion to the number of sources. Correspondingly, since the sizes of the second and third gratings may be increased without limit, ~~and~~ a very high detector flux will thus be obtained.

An additional interesting effect can be observed when one varies λ. Transmission resonances can be observed when $\lambda \approx \lambda_{TR} m/n$ holds. If one monitors the aforementioned periodic transmission at the n-th harmonic, there will occur a resonance whenever m is approximately integral. The width and shape of these resonances

(Resonances with both m and n odd have opposite phase.)

depends on (s/d), m and n. When the atomic beam has a dominantly particle-like character (as opposed to wave-like) the n = 1 resonance reappears. Thus, in the high velocity limit (i.e. beam cooler/decelerator turned off), the three gratings form only geometric shadows, and a simple Moire pattern results. This wave-particle transition occurs when the deBroglie wavelength is sufficiently short that any point on the third grating is illuminated by at most one slit.

The above configuration will also work as a sensitive inertial sensor, in the same sense as the earlier considered interferometers. The path from any source point to any image point forms a set of nested diamond shape interferometers, each with its own inertial sensing capability. However, each such interferometer has a different area. Thus if significant gravitational or Coriolis force is applied to the atoms, the various interferometers within the nest will get out of phase and the interference will disappear. Thus the magnetic-field servo system discussed earlier by the author can be used to keep the atoms sensing essentially the equivalent of an inertial frame, and the inertial signals obtained from the servo system error signal.

Finally, let us discuss a further advantage of the system thus described. That is its ease of alignment. It was noted above that in the high velocity limit (i.e. beam cooler/decelerator turned off), the gratings form only geometric shadows, and a simple Moire pattern results. Using the same phase sensitive detection technique, one then can align the gratings to this Moire pattern by maximizing the n = 1 harmonic signal. In addition, since the grating period d can be much larger than that required for the separated beam configurations, these gratings can now be incorporated as elements of an *in-situ* optical interferometer, and the coarse alignment performed with light.

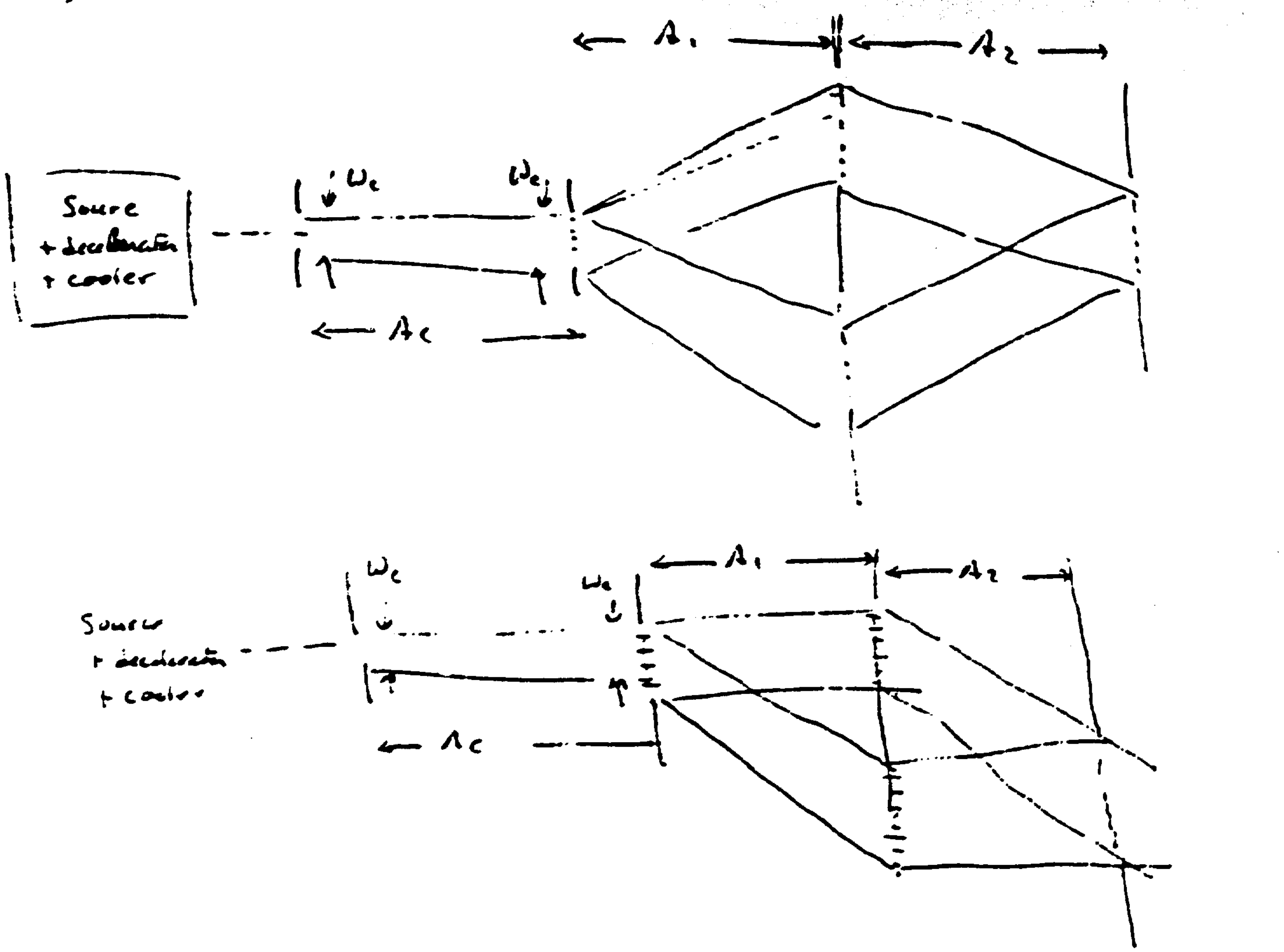
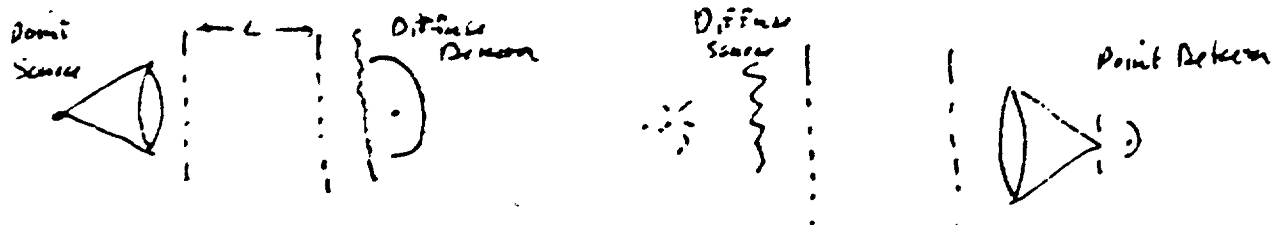


Fig 1. Configurations in Separated beam matter-wave Interferometers



Talbot experiment

von Laue Experiment

Fig 2