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## Transient Solutions From Scattered Fields in a Waveguide

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### Abstract

We examine a transient signal that scatters from a submerged object traversing a waveguide using the object in a waveguide model developed at NOARL. We are concerned with shallow water waveguides using realistic elongated targets. We show how to employ different pulse forms for resonant and nonresonant targets to determine distinguishing characteristics in the pulse signals that scatter from the target via bistatic measurements. Some numerical examples are presented for both frequency and time domain solutions.

### Introduction

In this work our aim is to describe what happens when an acoustic signal interacts with an object in a waveguide. We are interested in elongated targets. Presently elongated targets are now being successfully treated by variations of Waterman's Extended Boundary Condition (EBC) method for impenetrable, fluid and elastic targets<sup>1-4</sup>, particularly for axisymmetric objects such as very elongated impenetrable spheroids as well as spheroidal elastic solids and shells. Familiarity with the subject of scattering for the free state problem and the many complications and numerical pitfalls that are encountered can not help but lead one to the conclusion that when these objects are placed in a bounded environment, complications proliferate possibly to the point that exact methods if possible to formulate are not numerically practical. It therefore seems at the very least judicious to attempt to construct a theory that describes what happens to a signal once it scatters from an object in a waveguide to be based on the free state solution; in particular to determine a suitable unifying procedure that allows one to couple the free state solution to the solution for propagation of a signal in a waveguide. Here we give an outline of a normal mode based method to treat the object in a waveguide problem based on a coupling scheme and present some numerical results calculated from the formulation. In order to do this we first show how it is possible to reproduce the near field due to scattering of the guided wave. Then we introduce the coupling scheme that enables one to describe the scattered signal in the waveguide. Finally we indicate an efficient method that allows one to effectively treat the transient signal in a waveguide. Numerical examples are then presented.

### The Near Field Problem in a Waveguide

We wish to determine the near field due to the interaction of a guided wave with a submerged object. This is the first step required to couple the free state solution with a waveguide. Thus, this phase of the problem takes on considerable significance. We can then use the near field solution in a variety of strategies to obtain an approximate solution for the field scattered/propagated from an object in a waveguide. Note that we do not state the problem as either a scatter or propagation problem because we can view the initial event (the interaction of the guided wave with the object) as a scattering event while we can view the course of the scattered signal as a propagation event once the waveguide takes effect. By employing the near field in this manner it is clear that we have divided the problem into different computational as well as conceptual segments namely propagation-scattering-propagation. In this section we outline how to treat the interaction of a signal propagating in a waveguide with a submerged object. To do this we first review the treatment of scattering from a target via the extended boundary equation (EBC) method which leads to a T-matrix that transforms an initial field into a scattered field (for the free state problem) in matrix form as follows:

$$f = Ta$$

where  $f$  and  $a$  are column vectors representing the expansion coefficients of the scattered (incident) fields and  $T$  is the transformation (a square matrix) that maps the incident field onto the scattered field. Let us restrict our argument (for the sake of simplicity) to that of an axisymmetric target such as a spheroid and deal initially with a plane incident wave. We will then generalize to the case of a guided wave based on a normal mode solution. We will see below that even for a fairly general range dependent waveguide this approach will be sufficient for general environments. To be specific we look at the form of a T-matrix for a rigid axisymmetric target. This has been derived earlier and presented in a computationally practical form. The T-matrix that maps the incident field onto the scattered field is:

$$T = -ReQ Q^{-1}$$

where  $Q_{ij} = \int_{\text{Re}} \phi_i(k\rho) \partial \phi_j(k\rho) / \partial n dS$ .

In the spherical polar representation  $\phi_i(k\rho) = h_i(k\rho) Y_{im}(\theta, \phi)$  where  $h_i$  and  $Y_{im}$  are outgoing spherical Hankel functions and spherical harmonics of order  $i$ , with  $m$  being the azimuthal index.

An example of the power of this method is represented in Fig. 1 which illustrates the angular distributions for a rigid spheroid with aspect ratio of 16 to 1 at a  $kL/2$  of 200 for incident angles of 0- 30- 60- and 90-degrees relative to the axis of symmetry.

The T-matrix was developed in a spherical representation, it is a second rank tensor in irreducible form, and thus has meaning only when operating on a vector also in a spherical representation. Since it is in irreducible form it can easily be rotated once the simplest form of T is devised which is one of its salient properties. On the other hand, the fact that it requires the vector that it operates on to be in a spherical representation while having its value for plane waves (which are easily expanded in a spherical representation) imposes restrictions on the form of the guided wave. However, we show in what follows how it is possible to represent a waveguide solution of a fairly general form in a spherical representation. The plane wave solution may be written in a partial wave representation via the Raleigh series as follows:

$$\text{EXP}\{ipk\cos(\theta)\} = \sum (2n+1)(i)^n j_n(\rho k) P_n(\theta)$$

where  $q$  is the angle between  $k$  and  $r$ . For the more general representation one can use the addition theorem for spherical harmonics to obtain the plane wave in coordinates relative to a fixed cartesian system which we must ultimately use. That expression is:

$$P_n(\theta) = 4\pi / (2n+1) \sum \sum Y_{mn}(\theta, \kappa_n \cdot \varphi, \kappa_n) Y_{mn}(\theta, \rho' \cdot n, \varphi, \rho' \cdot n)^*$$

where  $\theta$  is the angle between  $\rho$  and  $k$  and  $\theta, \eta_n$  and  $\varphi, \eta_n$  are the respective angles relative to the cartesian coordinates of  $\eta$  for mode  $n$ .

Our interest, however, is in the guided wave impinging on a bounded object. Here the wave that insonifies the object for a stratified environment is of the form:

$$U_0 = 1/2 (e^{i\pi/4} / d \sum \varphi_n(\gamma_n z_s) \varphi_n(\gamma_n z) e^{\kappa_n r} / (\kappa_n r))^{1/2}$$

where  $\varphi_n(\gamma_n z)$  the vertical solution is an eigenfunction with eigenvalue  $\gamma_n$  corresponding to the vertical wave number for mode  $n$ , while  $z_s$  and  $z$  are the values of vertical displacements at the source and at observation respectively,  $r$  is the distance from source to receiver in the horizontal plane and  $\kappa_n$  is the horizontal wave number for mode  $n$ . Here  $k^2 = \gamma_n^2 + \kappa_n^2$ . In this formulation we include only propagating modes and assume that the origin of the source is localized in space such that it may be approximated by a point source and that we are sufficiently far from the source that evanescent modes can be ignored. This procedure is, in fact, standard practice in the derivation of normal mode equations and proves to be quite adequate for many environments. We will limit discussion to layered fluid bottoms so that  $U$  symbolizes the velocity potential. For an elastic bottom we would have to formulate the problem using the displacement vector in order to satisfy the boundary conditions at the fluid-elastic interface.

Here we indicate with an example how to convert  $U$  to a spherical representation for the case in which we represent the environment by  $n$  isovelocity layers. Let us pick a particular layer for which the submerged object resides. Then  $U$  is of the form:

$$U_0 = 1/2 (e^{i\pi/4} / d \sum a_n \sin(\gamma_n z_s) \sin(\gamma_n z) e^{\kappa_n r} / (\kappa_n r))^{1/2}$$

where  $a_n$  is the expansion coefficient. We want to take advantage of the Rayleigh expansion of a plane wave so that we write

$$\sin(\gamma_n z) e^{\kappa_n r} = (\text{Exp}(i\gamma_n z + \kappa_n r) - \text{Exp}(-i\gamma_n z + \kappa_n r)) / 2i$$

where the  $(x, y, z), (r, z)$  are illustrated in Fig. 1. The coordinates  $(x', y', z')$  for the T-matrix must *preforce* be in the representation in Fig. 2 where  $z'$  is along the axis of symmetry of the spheroid (our choice of target). We write  $\cos(\alpha_{n+}) = (\gamma_n z + \kappa_n r) / k \rho$  and  $\cos(\alpha_{n-}) = (\gamma_n z - \kappa_n r) / k \rho$  so that we have

$$\sin(\gamma_n z) e^{\kappa_n r} = [\text{Exp}\{ik\rho \cos(\alpha_{n+})\} - \text{Exp}\{ik\rho \cos(\alpha_{n-})\}] / 2i$$

Thus, after use of the Rayleigh series and the above expression we have  $U$  in the following spherical representation:

$$U_0 = 1/2 (e^{i\pi/4} / d \sum_n \sum_L a_n (2L+1) (i)^L j_L(\rho k) [P_L(\alpha_{n-}) - P_L(\alpha_{n+})] / (\kappa_n r))^{1/2}$$

where we must use the spherical harmonic addition theorem obtain the most general form of the expression. The above expression has been derived tacitly assuming that the interaction between the guided wave and the submerged object occurs at a point. In fact, the interaction is extended in space and we must allow for this in the final development. The final form depends on the way we interface the object with the guided wave. Let us assume we have derived the above expression to be valid at the origin of the submerged object. Further, we wish to find the field at some vertical line or over some surface with distance  $r_0$  from the center of the object. This will be developed in the next section.

So far we have only dealt with the free state T-matrix. If the object is near a surface then we have ignored multiple interactions between the object and the interface. Although, for most problems multiple interactions will produce high angle propagation that will ultimately get absorbed into a realistic (attenuating) bottom and should not be a strong factor in calculation. This effect can be included in our formulation by employing the T-matrix at an interface presented elsewhere.

#### A method to couple the near field into a waveguide

We outline the basis for one of the most useful methods to describe scattering from an object in a waveguide which we refer to as Huygens method. This was considered earlier<sup>5</sup> but was not fully developed. We begin by allowing a guided wave to impinge on an object as it traverses a given region. The object scatters the guided wave in some manner. We determine the scattered field near the object via a free field transition matrix or one near an interface as described above. Let us choose a surface circumscribing the submerged object. The surface is arbitrary to the extent that it is smooth and its dimensions do not exceed the boundaries of the waveguide and should be such that the largest dimension of the surface is such that the highest angle mode does not interact with the boundary of the circumscribed region. We next require the near field on the surface as well as its normal derivative. To

obtain this we must make a transformation from the coordinates of the waveguide to that of one relative to the axis appropriate to the submerged object. Recalling that the mode angles  $\alpha_n$  were obtained relative to the horizontal ( and not the vertical as is usual in normal mode theory ), we choose  $\beta$  to designate the angle that the mode makes with the axis of symmetry of the spheroid in the horizontal plane. It is to be noted that the reference axis of the spheroid and that of the waveguide differ. In particular, in order to exploit the axial symmetry of the object we must choose the z axis in the object body reference along the axis of symmetry while we can choose x and y at our convenience. In the waveguide, z is in the downward direction while x and y are in the horizontal plane. The angle scheme chosen are such that  $\theta_n$  and  $j_n$  the angles associated with the particular mode n are the appropriate angles to be implemented in the spherical harmonics. They are, in particular, the angles of the incident mode n relative to the symmetry axis of the submerged object and the angle that the plane generated by the incident ray-mode n and the symmetry axis of the spheroid makes above ( below ) the horizon respectively. They are obtained from:

$$\tan(\phi_n) = \tan(\alpha_n) / \sin(\beta)$$

$$\cos(\theta_n) = \cos(\beta) \cos(\alpha_n)$$

where the surface is chosen at a suitable region circumscribing the object (suitably near the object) with origin at the center of the object. The surface field can be obtained from the expression

$$f(\rho, \theta, \phi) = \sum \sum \alpha_{mnn} T_{mnn}(\kappa_n) Y_{mn}(\theta_{\kappa_n}, \phi_{\kappa_n}) Y_{mn}(\theta_{r'n}, \phi_{r'n}) \eta(\kappa_n \rho) / (\kappa_n \rho)^{1/2}$$

where the  $\alpha$ 's are projection coefficients of the normal-mode functions onto the spherical (partial wave) solutions. This is the most general form of the scattered near field.

The surface integral representation of the scattered field which also satisfies the asymptotic conditions of the waveguide is based on a variation of Huygens principle, hence the name Huygens method. The integral form that expresses Huygens principle is evaluated over an outgoing surface in the direction of the scattered signal. It is:

$$U_s(r) = \int (f(r') \partial G(r, r') / \partial n - G(r, r') \partial f(r') / \partial n) dS$$

For the unperturbed waveguide  $U_0$  in normal-mode representation we have for the initial field:

$$U_0 = 1/2 (e^{i\pi/4} / d) = \sum \varphi_n(\gamma_n z_s) \varphi_n(\gamma_n z) e^{\kappa_n r} / (\kappa_n r)^{1/2}$$

which we use in the derivation of the above equation and use to compare with results that follow.

We require the Green's Function for the waveguide and it is:

$$G_0 = (i/2d) \sum \varphi_n(\gamma_n z_s) \varphi_n(\gamma_n z) H_0(\kappa_n r)$$

where  $H_0$  is an outgoing cylindrical Hankel function and  $J_n$ ,

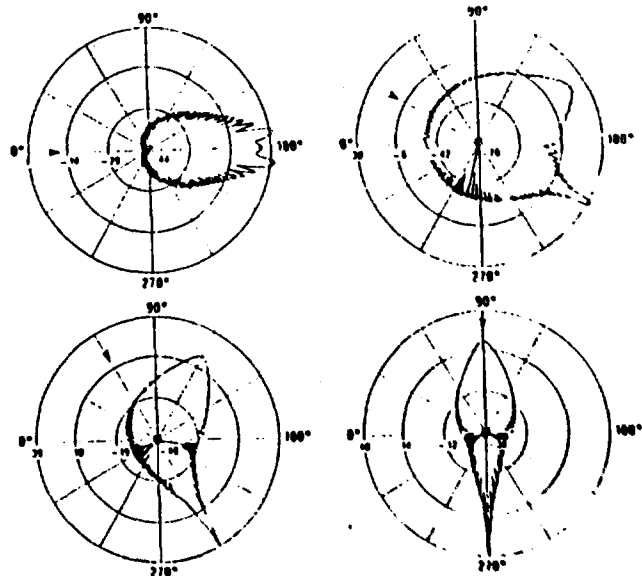
$\gamma_n$  and  $\kappa_n$  are the normal-mode functions and the vertical and horizontal eigenfunctions, respectively. By appropriate manipulation one arrives at an expression far from the object of the form

$$U_s(z, r) = \sum \alpha_n(r) \varphi_n(\gamma_n z) e^{\kappa_n r} / (\kappa_n r)^{1/2}$$

where

$$\alpha_n = \int \{ f(\rho', \theta, \phi) \partial \varphi_n(\gamma_n z') e^{\kappa_n r'(\theta)} / (\kappa_n r')^{1/2} / \partial n - \varphi_n(\gamma_n z') e^{\kappa_n r'(\theta)} / (\kappa_n r'(\theta))^{1/2} \partial f(\rho', \theta, \phi) / \partial n \} dS$$

Note that the above equation is in the form of a normal-mode solution, and therefore the scattered wave forms a guided wave suitably far from the object, as one would expect. This solution, in fact, is continuous throughout space and satisfies the boundary conditions of the confined environment that forms the guided wave. The above expression can be evaluated by a variety of strategies. We can: (1) integrate over a surface that circumscribes the target such as a spheroid or a sphere or we can: (2) integrate along a line in the outgoing direction. We will not labor here on conditions required for each of the surface strategies. We choose the second method here ( it is easier to implement and requires less time for calculation but is also of less general value ). In that case we must obtain the image solution of the near field so that the surface field is zero on the surface boundary. Otherwise, we would be required to integrate over the surface boundary which would be more time consuming than including the image field.



1. Free far field angular distributions for scattering from a spheroid of aspect ratio of 15 to 1 at a  $kl/2$  of 200. Incident angles are at 0-, 30-, 60-, and 90-degrees relative to the axis of symmetry.

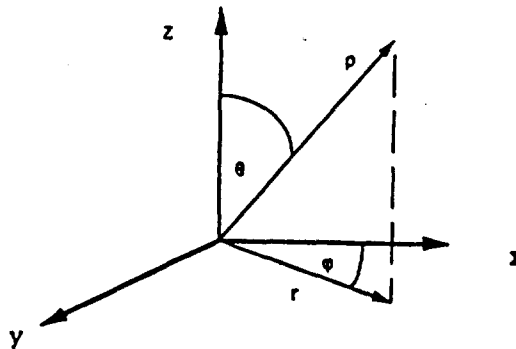


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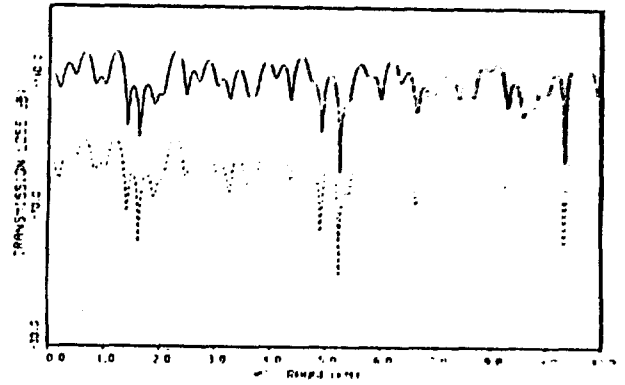
### Some Examples

We first test the Huygens method for a spheroid of length 50 m and width 10 m. The object is in a water column of 150 meters, 10km from a 200dB source at 100 Hz, 75 m below the surface with the object a distance 75m from the surface. The half space is composed of sand ( a density of 2 relative to that of water with a bulk speed of 1600 m/sec ). The half space has no attenuation for this example. The velocity profile in the water column is constant at 1500 m/sec. For this example the near field is fairly constant over the water column much like that due to a point source. Thus we expect that the scattered signal ( in this case we scatter broad side ) will behave much like a point source with origin at the center of the spheroid. Fig. 3 illustrates the transmission loss ( TL ) of the object scattered signal compared to that of the signal emanating from the origin of the object with a signal strength the same as that of the signal level at the object. The shape of the TL due to the scattered signal and that due to the point source are essentially the same consistent with the point source argument as well as our expectations. This observation was also made in a paper on the PE method<sup>6</sup>. The difference in magnitude between the two calculations is due to the fact that at this frequency most of the energy of the scattered field is diffracted into a broad region. We expect that with increasing frequency the field will become more focused in the forward direction and the difference in relative strength will diminish with increasing frequency. This is certainly borne out in calculations that we have done .

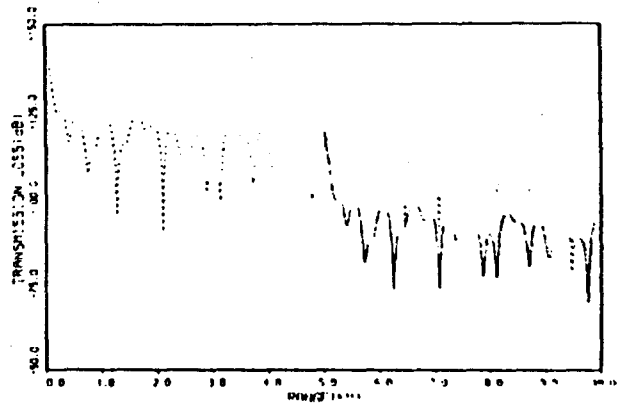
Fig. 4 illustrates the case of a cw solution of scattering from a waveguide for an object 100 by 10 m. The part of the waveguide under study here 200 m in depth over a 10 km range. The bottom has a sound speed of 1700 m/sec and has moderate attenuation. The velocity profile is allowed to vary consistent with a fall profile at 40 deg. latitude. The frequency of the 200dB signal is 250 Hz and interaction takes place with the target broadside at a distance about half way along the 10 km waveguide. We compare the initial signal with the scattered signal. The solid curve in Fig.4 is the TL due to the signal and the dashed curve is that due to the interaction of the guided wave with the object. We have compared this calculation with the sonar equation with results suggesting close agreement in magnitude. Fig. 5 illustrates the signal in a typical wave guide of a Gaussian pulse incident from a source 5 km away from a rigid 5 to 1 spheroid. The signal is at end-on incidence and the scattered signal is at 45 degrees relative to the axis of symmetry. The solution was constructed from the Fourier transform of the frequency domain solution. The frequency was such the only two modes are propagating and one can see the two modes as well as the direct signal progress in time. The appropriate parameters are listed in the figure.



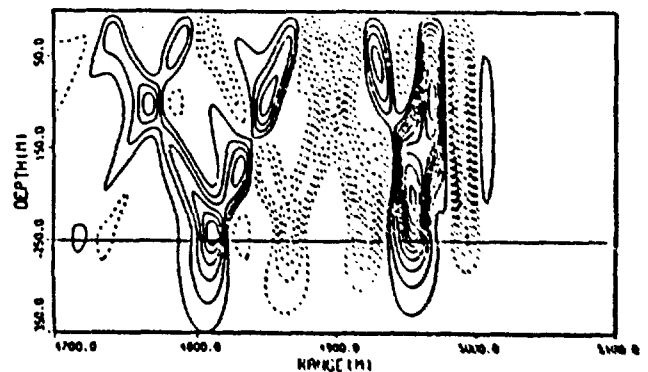
2. Coordinate frame of the object relative to the waveguide.



3. Comparison of a point source with origin at the center of a target with that due to Huygens for low frequency.



4. Comparison of the TL's due to a 10 to 1 rigid spheroid at 250 Hz to the incident field in a waveguide.



5. Time domain solution from a Gaussian pulse scattering from a 5 to 1 rigid spheroid at low frequency.

## Conclusions

We have outlined how it is possible to obtain the near field due to the interaction between a guided wave and a submerged object. The conditions were fairly general. Some details were presented on how to couple the near field to a normal mode solution of a waveguide via the Huygens method. Agreement between the method described here and other methods are

found to be quite good and based on that consideration we expect the method to be reliable predictors for a fairly broad class of problems. Moreover, although these examples were all performed for rigid spheroids the methods are just as easy to use for any class of targets including elastic shells.

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