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ONE-DIMENSIONAL ANALYSIS OF A RADIAL SOURCE FLOW OF WATER PARTICLES INTO A VACUUM

A. Setayesh

Radex, Inc.
Three Preston Court
Bedford, MA 01730

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GEOPHYSICS LABORATORY
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
HANSCOM AIR FORCE BASE, MASSACHUSETTS 01731-5000

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An analytical investigation is made of a one-dimensional radial source flow of water into a vacuum. The equations that describe this model are a sixth-order, nonlinear, two-point boundary value problem. The problem is solved by the "shooting/splitting" technique. The results of this investigation are compared with the other works in the literature and the comparisons generally are in good agreement. Engineering parameters for the release of water into space are used to give some indication of water flow characteristics into a vacuum environment; these parameters were chosen to resemble those of a recent water release experiment from the space shuttle.

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1. INTRODUCTION

The flow of a liquid jet into a near vacuum environment has been studied by many investigators. Many of these studies are experimental but also provide some theoretical interpretation of the experiments. For example, see the works of Johnson and Hallett [1968], Kassal [1974], Fuchs and Legge [1979], Curry et al. [1985a,b], and Muntz and Orme [1987]. Johnson and Hallett [1968] investigated the behavior of supercooled water drops at various conditions, such as in carbon dioxide below 0.05 atmospheric pressure. Kassal's [1974] experiment showed that the spherical particles break in half during the freezing process and form bowl-shaped configurations in the vacuum chamber. Fuchs and Legge [1979] concluded that the bursting effect caused by the sudden boiling of the water produces vapor bubbles which decompose the jet. They stated that the formation of vapor bubbles was caused by a superheating of the water due to sudden pressure drop and cooling at the surface of the jet. Muntz and Orme [1987] discussed the possible events when liquid streams are released into a vacuum. They stated that for fluids having vapor pressures up to several torr in streams with a few hundred microns in diameter, it is possible to maintain stable and controllable streams. However, fluid streams having vapor pressures in the of tens of torr range with one millimeter and greater diameter are expected to burst.

Curry et al. [1985a, b] performed an experiment in a vacuum chamber (10^{-5} Torr), wherein water was released from an orifice with a diameter of 1.6 mm into the chamber. The vent plume was studied using a Mie scattering polarimeter and a high-speed flow visualization camera. They concluded that there are mainly two classes of particles which can be found in the water plume, viz. the large particles with diameters close to the orifice diameter (1.6 mm) and a cloud of very small particles whose diameters were about 0.15 μm. Pike et al. [1990] showed that the particle sizes as measured from the recent groundbased and onboard video images of a sunlit Shuttle Orbiter water dump are similar to those noted in vacuum-tank experiments (Curry et al. [1985a,b]).

Some mathematical modeling of such flows has also been done and compared with experimental data (Gale et al., 1964, Glenn 1969). Gale et al. [1964], in their mathematical study of liquids (including water) released into a vacuum, analyzed the effects of initial temperature, droplet diameter, and ambient pressure on the time required for cooling to the triple point and for subsequent freezing. Part of their mathematical analysis was supported by experimental data. Glenn [1969] studied a radial source flow of uniform-size liquid particles into a vacuum. A numerical procedure for the integration was developed for the models, and he discussed parametrically the effects on the flow structure of such variables as particle size, velocity, and temperature. The present investigation is inspired by the work of Glenn [1969] in which he assumed a one-dimensional jet flow of spherical particles of liquid water into vacuum. Glenn's [1969] work has been extended, and some revisions made to his work which are included in this report.

2. DESCRIPTION OF THE MODEL

The problem considered here is the mathematical modeling of stationary radial source flow into a vacuum. The equations describing this model are the one-dimensional continuity, momentum, and energy for the vapor and particles. The analysis constitutes a sixth-order, nonlinear, two-point boundary value problem. A number of assumptions have been made to simplify the problem. These include the following: The particles are liquid, spherical shaped, with uniform size and temperature. The wake of the particles does not influence the nearby particles. Re-condensation of vapor as a result of any subcooling during the expansion process is neglected. The heat transfer between particles and vapor occurs only by convection and radiation. The fluid is considered to be inviscid, except the part that can produce drag. From the above constraints, one can assume a simple liquid jet breakup into the vacuum environment.

The derivations of the equations that describe this model can be found in Appendix A. Glenn's work [1969] has been extended and the equations have been modified to investigate the behavior of particles after freezing, and to include heat radiation from the particle. The Clausius-Clapeyron equation is used for calculating the local vapor pressure in all cases considered. The problem has been solved with the engineering parameters for the release of water into space. These parameters were chosen to resemble those of a recent water release experiment from the space shuttle (Discovery, March 1989). Nevertheless, it should be mentioned here that the mechanism for water release and its breakup into space is an extremely complex process which cannot be fully described by this simple one-dimensional model. However, it may give some indication of the factors involved. Future work will include two, and perhaps, three dimensional models.

The governing equations that are used in this study are as follows:

$$\frac{d}{dr}[r^2 \rho_v (1-\alpha) u_v] = \dot{m}\, \alpha\, r^2 \tag{1}$$

$$\frac{d}{dr}[r^2 \rho_p\, \alpha\, u_p] = -\dot{m}\, \alpha\, r^2 \tag{2}$$

$$\rho_v (1-\alpha)\, u_v\, u_v' + (1-\alpha)\, P' - \alpha\, f_p = \dot{m}\, \alpha\, (u_p - u_v) \tag{3}$$

$$\rho_p\, \alpha\, u_p\, u_p' + \alpha\, P' + \alpha\, f_p = 0 \tag{4}$$

$$\begin{aligned} \rho_v (1-\alpha)\, u_v\, C_{pv}\, T_v' - u_v (1-\alpha)\, P' &= \alpha\, q_p + \alpha\, f_p\, (u_p - u_v) + \\ &\dot{m}\, \alpha\, [C_{pv} (T_p - T_v) + \frac{1}{2} (u_p - u_v)^2] \end{aligned} \tag{5}$$

$$\rho_p \, \alpha \, u_p \left\{ \left[C_{p_v} - \left(\frac{d\lambda_v}{d T_p} \right) \right] T_p' + \lambda_f \, \beta' \right\} - u_p \, \alpha \, P' = \tag{6}$$

$$- \alpha \, q_p - \dot{m} \, \alpha \left[\lambda_v + (1-\beta) \, \lambda_f \right]$$

Equations (1-6) constitute a sixth order system in six unknown quantities; vapor density ρ_v, vapor velocity u_v, particle velocity u_p, particle temperature T_p, vapor temperature T_v, and particle volume fraction α, and five additional variables which will be defined later.

The above equations are subject to the following constraints:

$$\begin{aligned} T > T_* \,; &\quad \beta = 1, \; \beta' = 0 \\ T_p = T_{**} \,; &\quad T_p' = 0 \\ T_p < T_{**} \,; &\quad \beta = \beta' = 0 \end{aligned} \tag{7}$$

Where T_{**} and T_* are the triple point and initial temperatures respectively, and β represents the particle liquid volume fraction.

The five additional unknowns, mass rate m, particle radius σ, local pressure P, drag force f_p, and total heat transfer by radiation and convection q_p, are defined as follows:

$$\dot{m} = \left(\frac{3\epsilon}{\sigma}\right) \left[P_0 \, (T_p) - P\right] (2\pi \, R \, T_p)^{-1/2} \tag{8}$$

$$\left(\frac{\sigma}{\sigma_*}\right)^3 = \left(\frac{r}{r_*}\right)^2 \left(\frac{\alpha}{\alpha_*}\right) \left(\frac{u_p}{u_{p*}}\right) \tag{9}$$

$$P = \rho_v \, R \, T_v \tag{10}$$

$$f_p = C_p \left(\frac{3}{8\sigma}\right) \rho_v \, (u_p - u_v) \, | u_p - u_v | \tag{11}$$

$$\begin{aligned} q_p = {} & \frac{3}{2} \left(\frac{k}{\sigma^2}\right) (Nu_p) \, (T_p - T_v) \; + \\ & \frac{3}{\sigma} \, \epsilon \, \sigma_{SB} \, (T_p^4 - T_v^4) \end{aligned} \tag{12}$$

For convenience, the equations can be normalized by the following nondimensional barred quantities:

$$\begin{aligned} & u_v = u_* \, \bar{u}_v \, , \qquad \alpha = \bar{\alpha} \, , \qquad T_v = \bar{T}_v \, T_* \\ & \rho_v = \rho_* \bar{\rho}_v \, , \qquad r = r_* \, \bar{r} \\ & u_p = u_* \, \bar{u}_p \, , \qquad P_0 = P_0 \, (T_p) = \bar{P}_0 \, P_* \\ & T_p = \bar{T}_p \, T_* \, , \qquad \rho_p = \rho_* \, \bar{\rho}_p = const \end{aligned} \tag{13}$$

The following quantities, based on the initial conditions, can be introduced in the problem:

$$\theta_1 = \left(\frac{R\ T_*}{u_*^2}\right)^{\frac{1}{2}}, \qquad \theta_2 = \frac{r_*}{\sigma_*}$$

$$\theta_3 = \frac{P_*}{\rho_*\ R\ T_*}, \qquad \theta_4 = 3\ (2\pi)^{-\frac{1}{2}}\ \epsilon\ \alpha_*^{\frac{1}{3}}$$

$$\theta_5 = \frac{3}{8}\ C_D\ \alpha_*^{\frac{1}{3}}, \qquad \theta_6 = \frac{3}{2}\ \alpha_*^{\frac{1}{3}}\ (Nu_p)\ \frac{k\ T_*}{\rho_*\ \sigma_*\ u_*^3}$$

$$\theta_7 = \frac{C_{p_v}\ T_*}{u_*^2} \tag{14}$$

$$\theta_8 = \left[C_{p_v} - \left(\frac{d\ \lambda_v}{d\ T_p}\right)\right]\frac{T_*}{u_*^2}$$

$$\theta_9 = \frac{\lambda_v}{u_*^2}, \qquad \theta_{10} = \frac{\lambda_f}{u_*^2}$$

$$\theta_{11} = \frac{3\ \alpha^{\frac{1}{3}}\ \sigma_{SB}\ \epsilon_r\ T_*^4}{\rho_*\ u_*^3}$$

Then the results can be written in matrix form:

$$\begin{bmatrix} \bar{u}_v(1-\bar{\alpha}) & \bar{\rho}_v(1-\bar{\alpha}) & 0 & 0 & 0 & -\bar{\rho}_v\bar{u}_v \\ 0 & 0 & \bar{\rho}_p\bar{\alpha} & 0 & 0 & \bar{\rho}_p\bar{u}_p \\ \theta_1^2\bar{T}_v & \bar{\rho}_v\bar{u}_v & 0 & 0 & \theta_1^2\bar{\rho}_v & 0 \\ \theta_1^2\bar{T}_v & 0 & \bar{\rho}_p\bar{u}_p & 0 & \theta_1^2\bar{\rho}_v & 0 \\ -\theta_1^2\bar{u}_v\bar{T}_v & 0 & 0 & 0 & \bar{\rho}_v\bar{u}_v(\theta_7-\theta_1^2) & 0 \\ -\theta_1^2\bar{u}_p\bar{T}_v & 0 & 0 & \bar{\rho}_p\bar{u}_p\Theta & -\theta_1^2\bar{u}_p\bar{\rho}_v & 0 \end{bmatrix} \begin{bmatrix} \bar{\rho}_v' \\ \bar{u}_v' \\ \bar{u}_p' \\ \bar{T}_p' \\ \bar{T}_v' \\ \bar{\alpha}' \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix} \tag{15}$$

The Θ in the sixth row and fourth column of the matrix is equal to θ_8, except at the triple point T_P = T.., where $\Theta = \theta_{10}$; and parameters A_1 through A_6 are defined as follows:

$$A_1 = \frac{\theta_1\ \theta_2\ \theta_4\ (\bar{\alpha})^{2/3}}{(\bar{u}_P)^{1/3}\ (\bar{T}_P)^{1/2}\ (\bar{r})^{2/3}} \left[\theta_3\ \bar{P}_0\ (\bar{T}_P) - \bar{\rho}_v\ \bar{T}_v\right] - \frac{2\ \bar{\rho}_v\ (1 - \bar{\alpha})\ \bar{u}_v}{\bar{r}} \tag{16}$$

$$A_2 = -\frac{\theta_1\ \theta_2\ \theta_4\ (\bar{\alpha})^{2/3}}{(\bar{u}_P)^{1/3}\ T_P^{1/2}\ (\bar{r})^{2/3}} \left[\theta_3\ \bar{P}_0\ (\bar{T}_P) - \bar{\rho}_v\ \bar{T}_v\right] - \frac{2\ \bar{\rho}_P\ \bar{\alpha}\ \bar{u}_P}{\bar{r}} \tag{17}$$

$$A_3 = \frac{(\bar{\alpha})^{2/3}\ (\bar{u}_P - \bar{u}_v)}{(\bar{r})^{2/3}\ (\bar{u})^{1/3}\ (1 - \bar{\alpha})} \times \left\{\theta_2\ \theta_5\ \bar{\rho}_v\ |\bar{u}_P - \bar{u}_v| + \frac{\theta_1\ \theta_2\ \theta_4}{(\bar{T}_P)^{1/2}} \left[\theta_3\ \bar{P}_0\ (\bar{T}_P) - \bar{\rho}_v\ \bar{T}_v\right]\right\} \tag{18}$$

$$A_4 = -\frac{\theta_2\ \theta_5}{(\bar{r})^{2/3}\ (\bar{u}_P)^{1/3}\ (\bar{\alpha})^{1/3}} \left[\bar{\rho}_v\ (\bar{u}_P - \bar{u}_v)\ |\bar{u}_P - \bar{u}_v|\right] \tag{19}$$

$$A_5 = \frac{(\bar{\alpha})^{2/3}}{(1 - \bar{\alpha})\ (\bar{r})^{2/3}\ (\bar{u}_P)^{1/3}} \left\{\frac{\theta_1\ \theta_2\ \theta_4}{(\bar{T}_P)^{1/2}} \left[\theta_3\ \bar{P}_0\ (\bar{T}_P) - \bar{\rho}_v\ \bar{T}_v\right] \times \right.$$
$$\left[\theta_7\ (\bar{T}_P - \bar{T}_v) + \frac{1}{2}\ (\bar{u}_P - \bar{u}_v)^2\right] + \left[\theta_2\ \theta_5\ \bar{\rho}_v (\bar{u}_P - \bar{u}_v)^2\ |\bar{u}_P - \bar{u}_v|\right] + \tag{20}$$
$$\left. \frac{\theta_2\ \theta_6\ (\bar{T}_P - \bar{T}_v)}{(\bar{\alpha})^{1/3}\ (\bar{r})^{2/3}\ (\bar{u}_P)^{1/3}} + \theta_2\ \theta_{11}\ (\overline{T_P^4} - \overline{T_v^4})\right\}$$

$$A_6 = -\frac{1}{(\bar{\alpha})^{1/3}\,(\bar{r})^{2/3}\,(\bar{u}_P)^{1/3}}\left\{\frac{\theta_1\,\theta_2\,\theta_4}{(\bar{T}_P)^{1/2}}\left[\theta_9 + (1-\beta)\,\theta_{10}\right] \times \right.$$

$$\left.\left[\theta_3\,\bar{P}_0\,(\bar{T}_P) - \rho_v\,\bar{T}_v\right] + \frac{\theta_2\,\theta_6\,(\bar{T}_P - \bar{T}_v)}{(\bar{\alpha})^{1/3}\,(\bar{r})^{2/3}\,(\bar{u}_P)^{1/3}} + \theta_2\,\theta_{11}\,(\overline{T_P^4} - \overline{T_v^4})\right\} \tag{21}$$

Here, the local vapor pressure as a function of particle temperature can be stated in terms of the Clausius-Clapeyron equation

$$\overline{P_o}(\overline{T_P}) = e^{-\frac{\theta_9}{\theta_1^2}\left(\frac{1}{\overline{T_P}} - 1\right)} \tag{22}$$

3. METHOD OF SOLUTION

The finite difference and shooting methods are two of a number of methods available in the literature to solve the two-point boundary value problems. Here the shooting method was chosen. Glenn [1968] used the simplest form of shooting method, called the " straight-shooting" scheme. In this method (which is also common in all other shooting methods), the two-point boundary value problem can be reduced to an initial value problem. The differential equations are integrated as an initial value problem, and the initial conditions are adjusted until the conditions at the other end point of the interval are satisfied. Further discussion of this method can be found in Glenn's work [1968].

4. DISCUSSION OF THE RESULTS

Figures 1 through 5 present comparisons of the current work to the work of Glenn [1969], and Fuchs and Legge [1979]. Figures 1 to 4 show the variation of the particle temperature, particle velocity, vapor temperature, and particle size (evaporation rate) with respect to radial distance r from the source for different sizes of the particles. Most of the initial conditions in these figures are similar to those which were used by Glenn [1969]. For example, the initial temperature, the initial velocity of the stream, and the source diameter are taken to be 93.65 °C, 15.24 m/s, and 6.35 mm, respectively. The parameters used for the purpose of comparison with the work of Glenn [1969], and Fuchs and Legge [1979], can be found in Tables 1 and 2, respectively. Figure 1 compares the temperature variation for different initial size particles. In Figure 2, the velocity changes are investigated and compared to the work of Glenn [1969]. Since the vapor velocity increased very rapidly after injection of the water into the vacuum, the smaller particles are most likely affected by this increase in vapor speed and their velocity will increase according

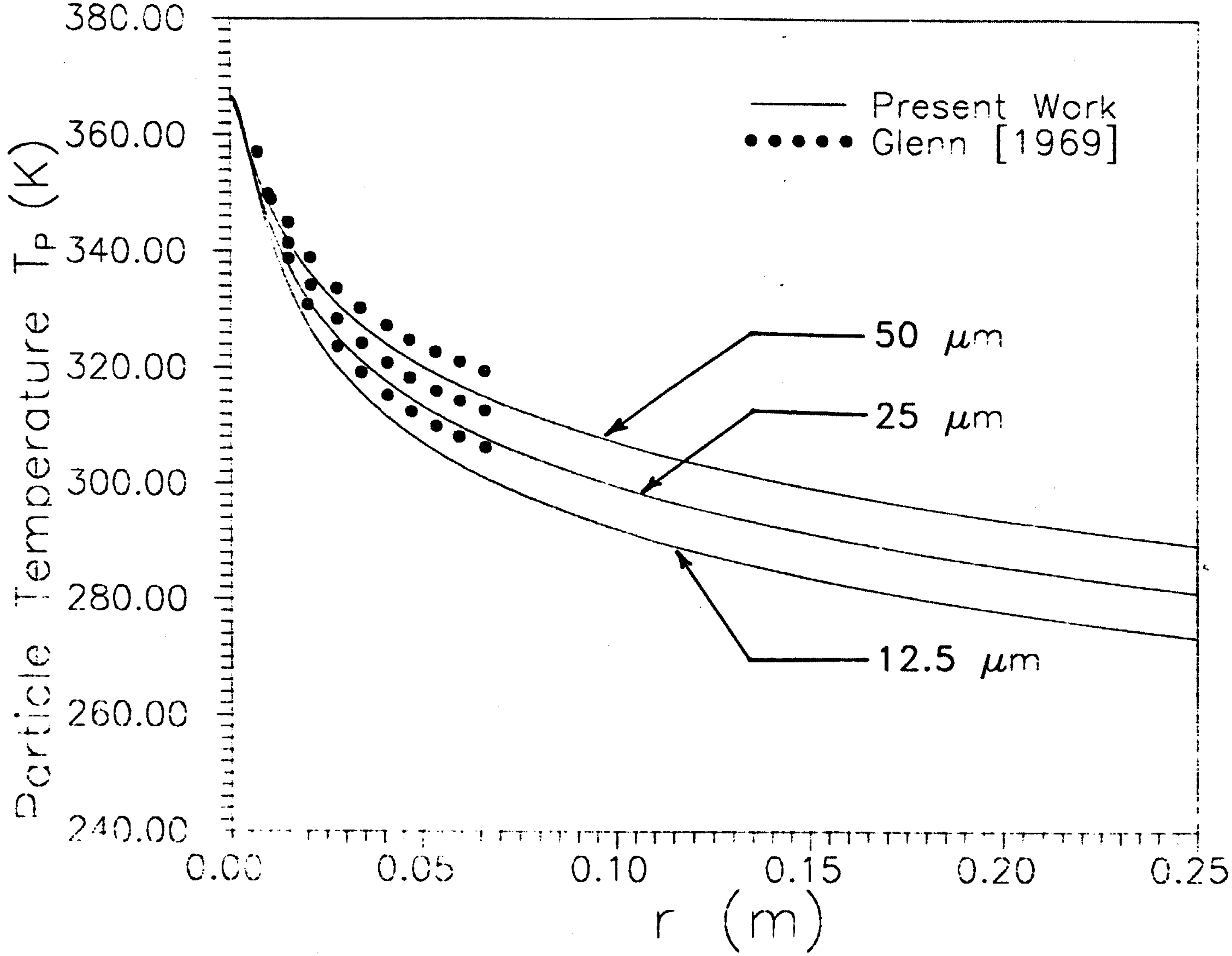

Figure [illegible]. Effect of initial Particle Size on Particle Temperature

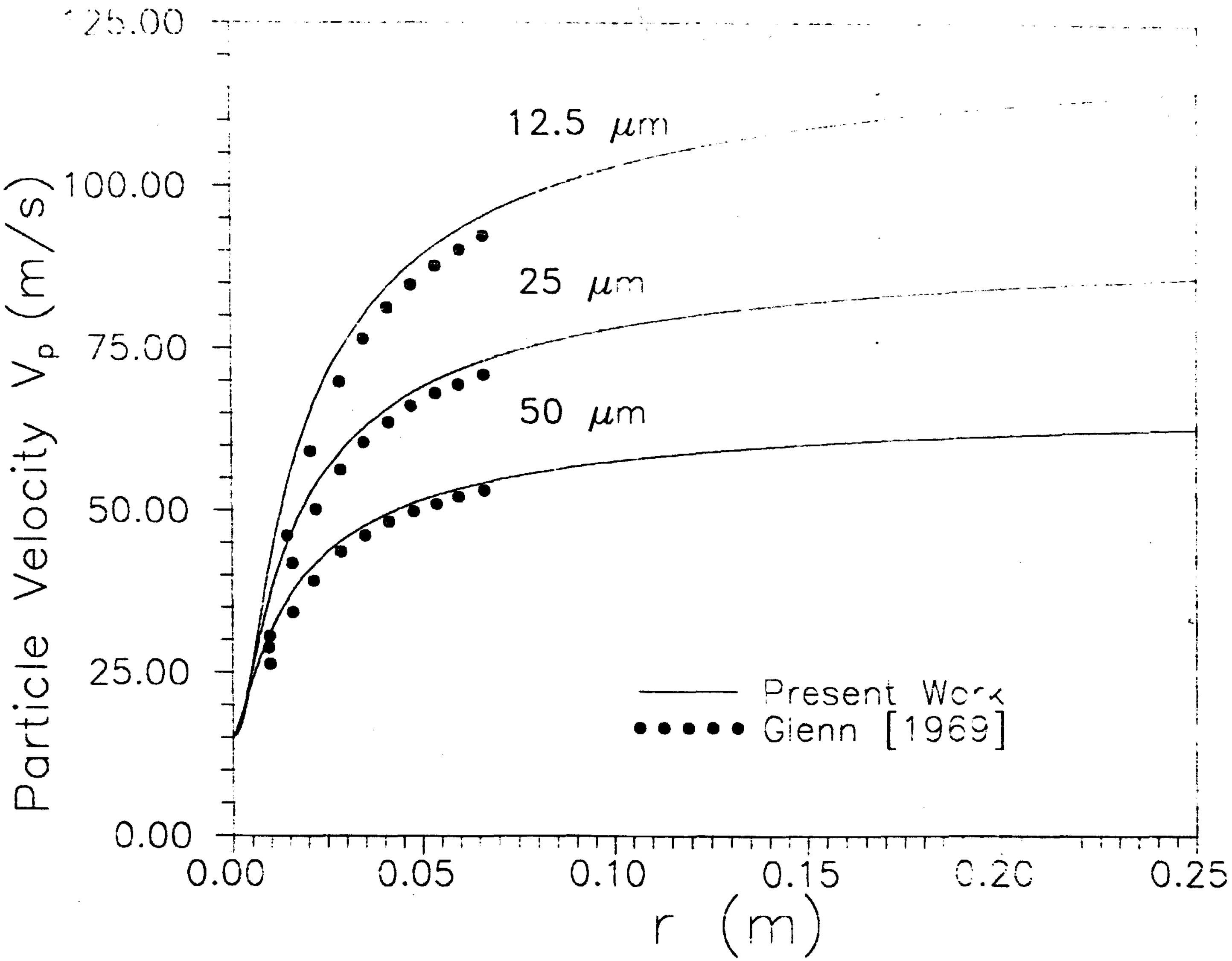

Figure 2. Effect of Initial Particle Size on Particle Velocity

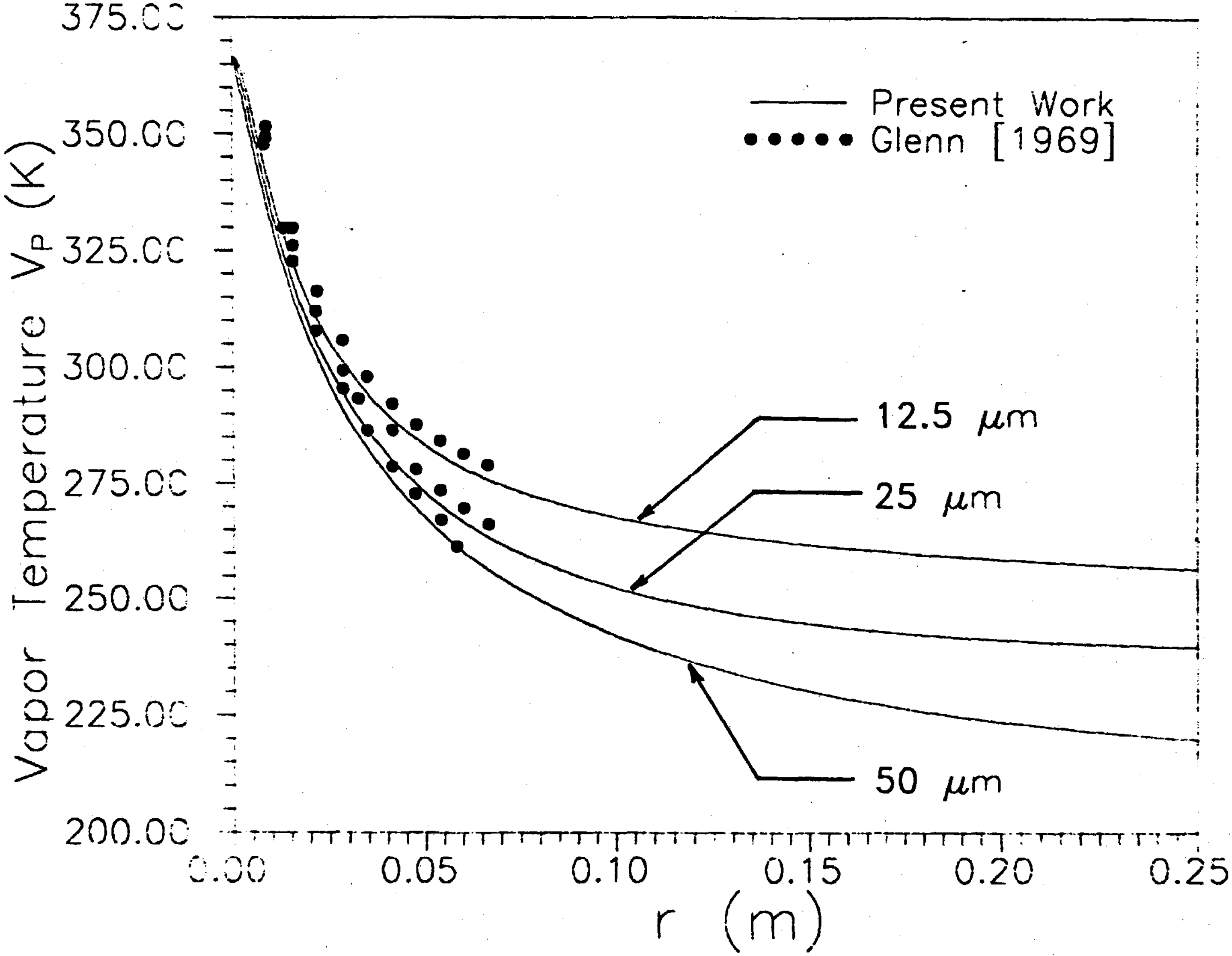

Figure 3. Effect of Initial Particle Size on Vapor Temperature

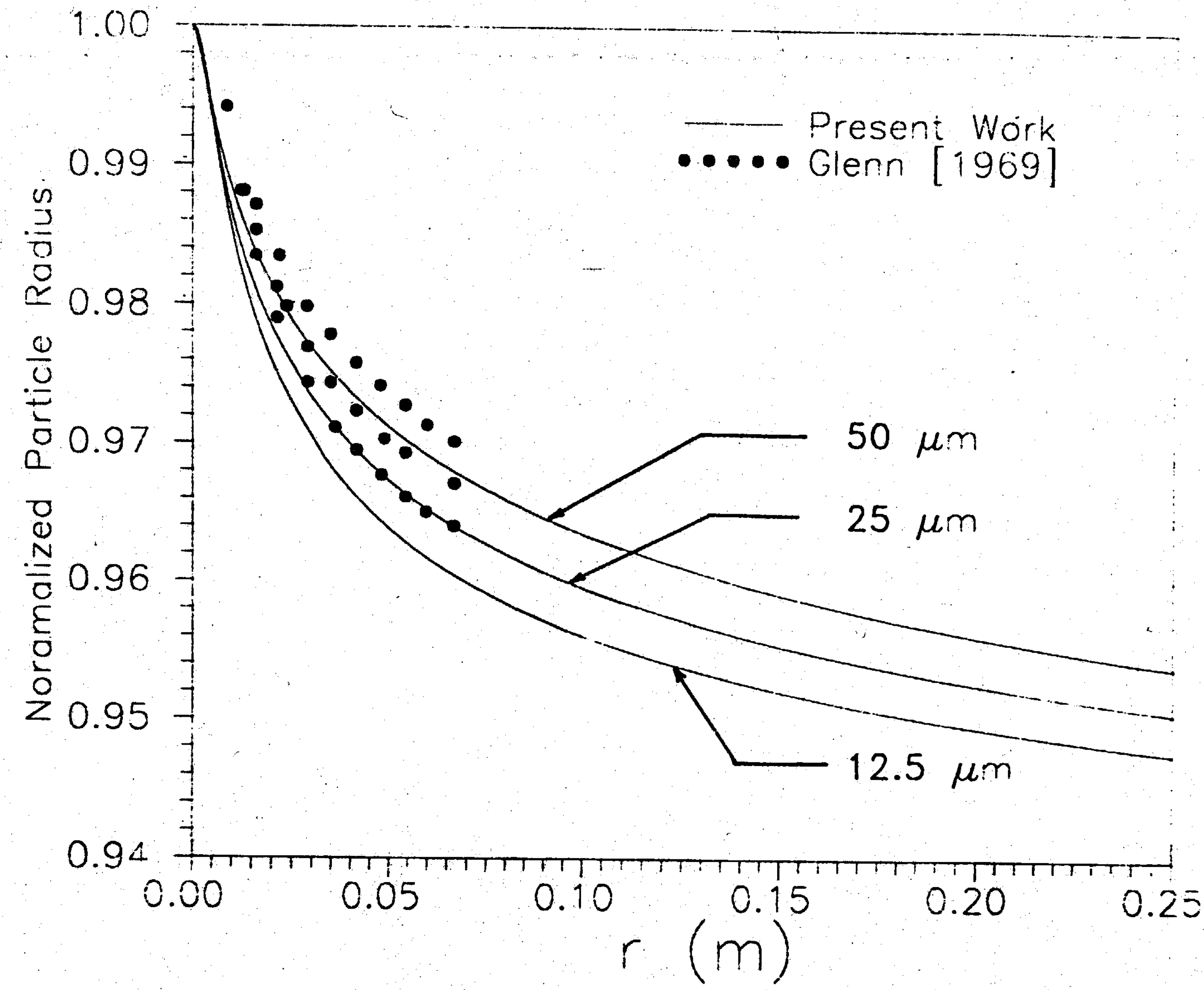

Figure 4. Particle size as a function of the distance

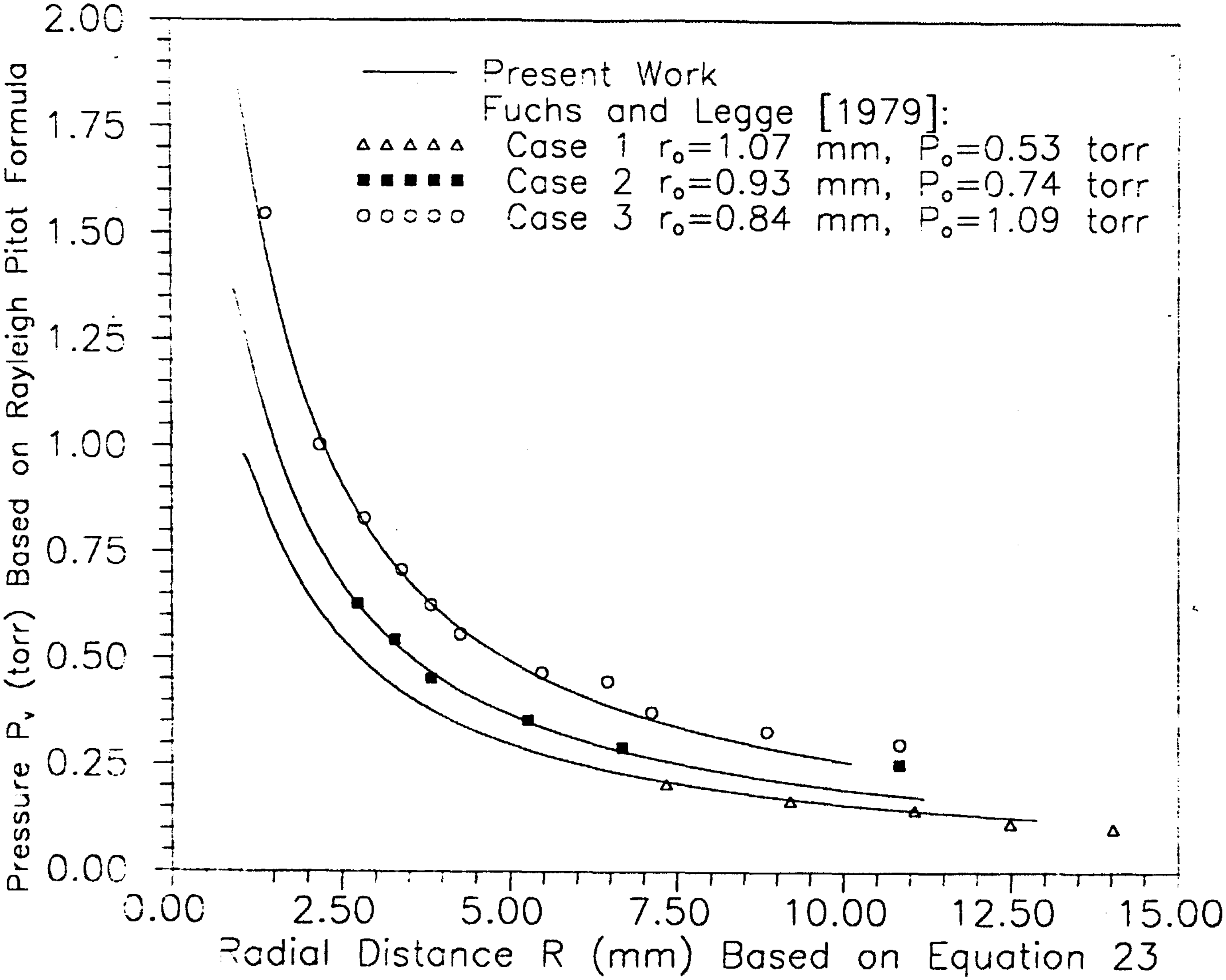

Figure 5. Pitot-Pressure measured at the vapor expansion flow radic to the jet

Table 1. Parameters Used for Figures 1-4

Initial Temperature (°K)	366.65
Initial Velocity (m/s)	15.24
Initial Source Radius (m)	3.175×10^{-3}
Initial Particle Radius (μm)	12.5, 25, or 50
Initial Vapor Pressure (N/m^2)	8.0 x 1[illegible]
Initial Vapor Density (kg/m^3)	0.47916
Initial Volume Fraction	0.74
Evaporation Coefficient	0.05
Drag Coefficient	0.9
Particle Nusselt Number	2.73
Vapor Thermal Conductivity (W/m °K)	0.024403
Vapor Specific Heat (J/kg °K)	1863.172
Heat of Evaporation (J/kg)	2.2741×10^{6}
Particle Density (kg/m^3)	962.8
Gas Constant for Water (J/kg °K)	461.94

Table 2. Parameters Used for Figure 5

Initial Temperature (^oK)	300.15
Initial Velocity (m/s)	12
Initial Source Radius (m)	1.0×10^{-4}
Initial Particle Radius (μm)	5
Initial Vapor Pressure (N/m^2)	4.0×10^3
Initial Vapor Density (kg/m^3)	0.025
Initial Volume Fraction	0.74
Evaporation Coefficient	0.05
Drag Coefficient	0.5
Particle Nusselt Number	2.73
Vapor Thermal Conductivity (W/m ^oK)	0.024403
Vapor Specific Heat (J/kg ^oK)	1863.172
Heat of Evaporation (J/kg)	2.386136×10^6
Heat of Fusion (J/kg)	3.340777×10^5
Particle Density (kg/m^3)	987.1
Gas Constant for Water (J/kg ^oK)	461.94

to the particle size. For example, the larger particles will be less affected by the vapor velocity. There is a further discussion with respect to the velocity increase of the particles in the later part of this work. Figure 3 shows the vapor temperature variation. The temperature drop in vapor is somewhat different from that found in the particles. Figure 3 shows that the vapor temperature decreases more rapidly for the larger particles than for the smaller particles. Figure 4 shows the size reduction of the particles due to evaporation, as a function of the radial distance. This figure indicates that the smaller particles evaporate much faster than larger particles near the source. As can be seen from Figures 1 through 4, and in particular Figure 2, the present results are in good agreement with the work of Glenn [1969].

Figure 5 investigates the changes in vapor pressure as a function of radial distance R, based on Equation (23). This equation relates the radial distance R to the Mach number and the specific heat ratio of vapor. The r_o is the fictitious sonic radius defined by Fuchs and Legge [1979]. Since the vapor flow is supersonic, the vapor pressure is calculated based on the Rayleigh-Pitot formula (Equation (24), where P_o is defined as a critical pressure at sonic r_o) which is considered by Fuchs and Legge [1979]. In this figure the experimental measurement of Fuchs and Legge [1979] compares very well with the calculated results of the present work. As can be seen from Figure 5, the curves start at some distance from the source, the distance that the vapor needs to travel to reach supersonic speed.

$$\frac{R}{r_o} = \frac{1}{M}\left(\frac{2+(\gamma-1)M^2}{\gamma+1}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}} \tag{23}$$

$$\frac{P_v}{P_o} = \left\{\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{(\gamma-1)}}\left(\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}\right)^{\frac{\gamma}{(\gamma-1)}}\left(\frac{2\gamma M^2-(\gamma-1)}{\gamma+1}\right)^{-\frac{1}{(\gamma-1)}}\right\} \tag{24}$$

Figures 6 through 12 show water flow characteristics near the source of release, into a vacuum environment, for the parameters listed in Table 3.

Figure 6 shows the variation of particle temperature as a function of initial particle size for three different particles. The particle sizes are well within the estimates for water release from the space shuttle Discovery for the flight of March 1989 (ranges within 0.25 μm to 1 millimeter sizes). The initial temperature of the liquid water is 70 °C (343.15 °K), which leaves with an initial velocity of 15 m/s. The smaller particles, such as 7.5 μm, freeze only about eight centimeters from the source, while the large particles, such as 250 μm, take much longer distances to freeze.

Figure 7 shows the evaporation rate of the particles as a function of distance from the source. The initial conditions of these curves are the same as for Figure 6. The particles with smaller radius tend to evaporate faster.

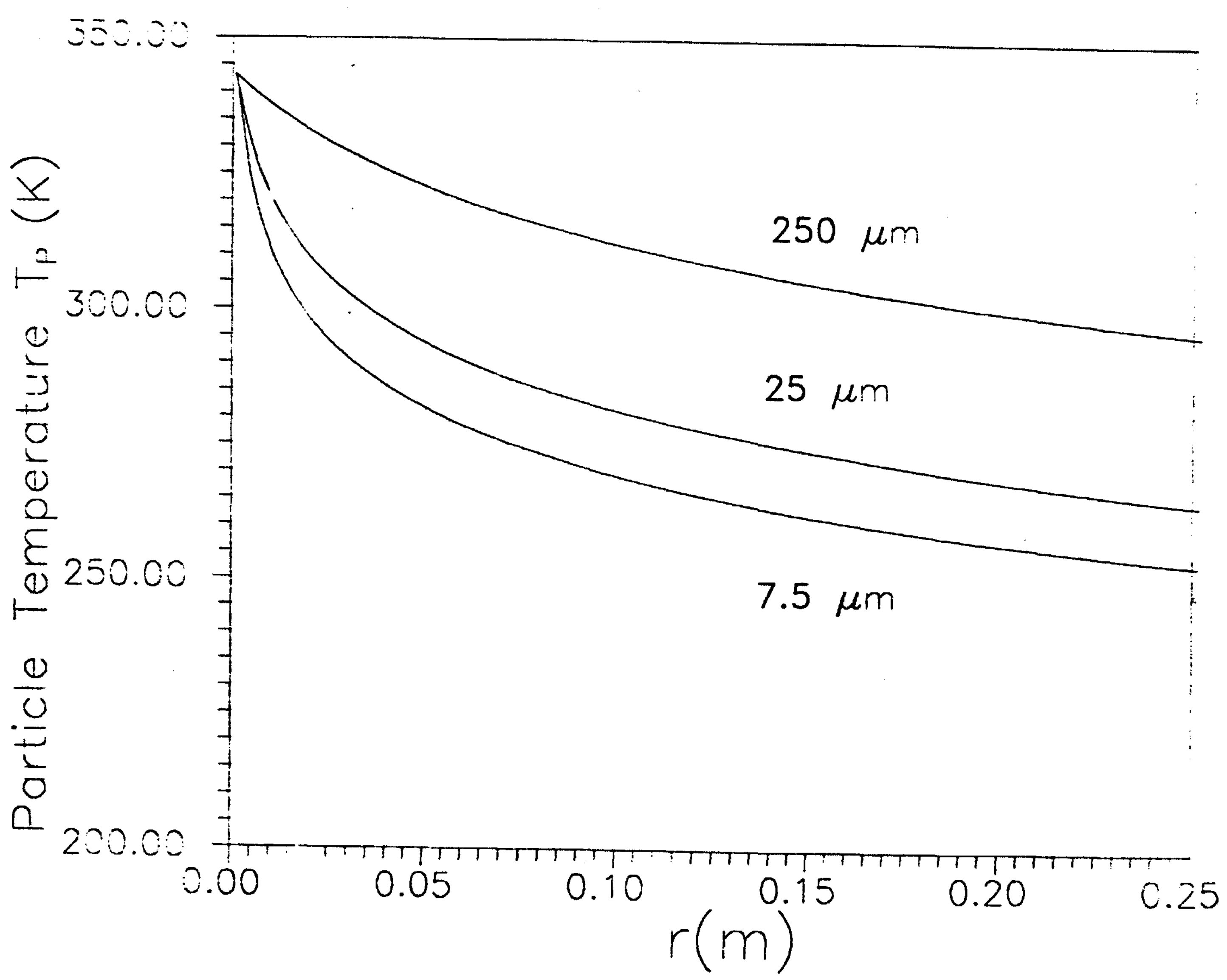

Figure 6. The Effect of Initial Particle Size on Particle Temperature

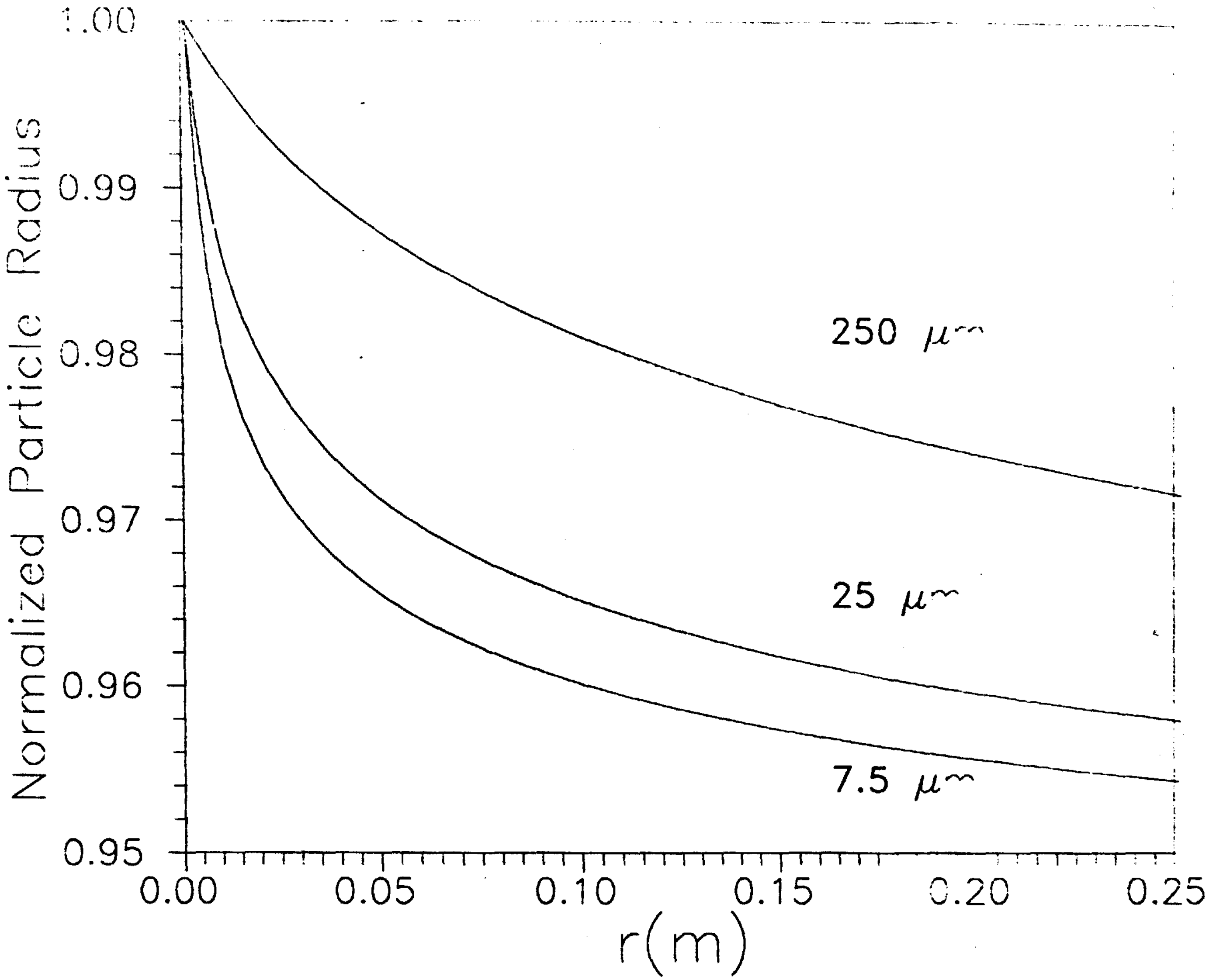

Figure 7. Particle Size as a Function of Distance

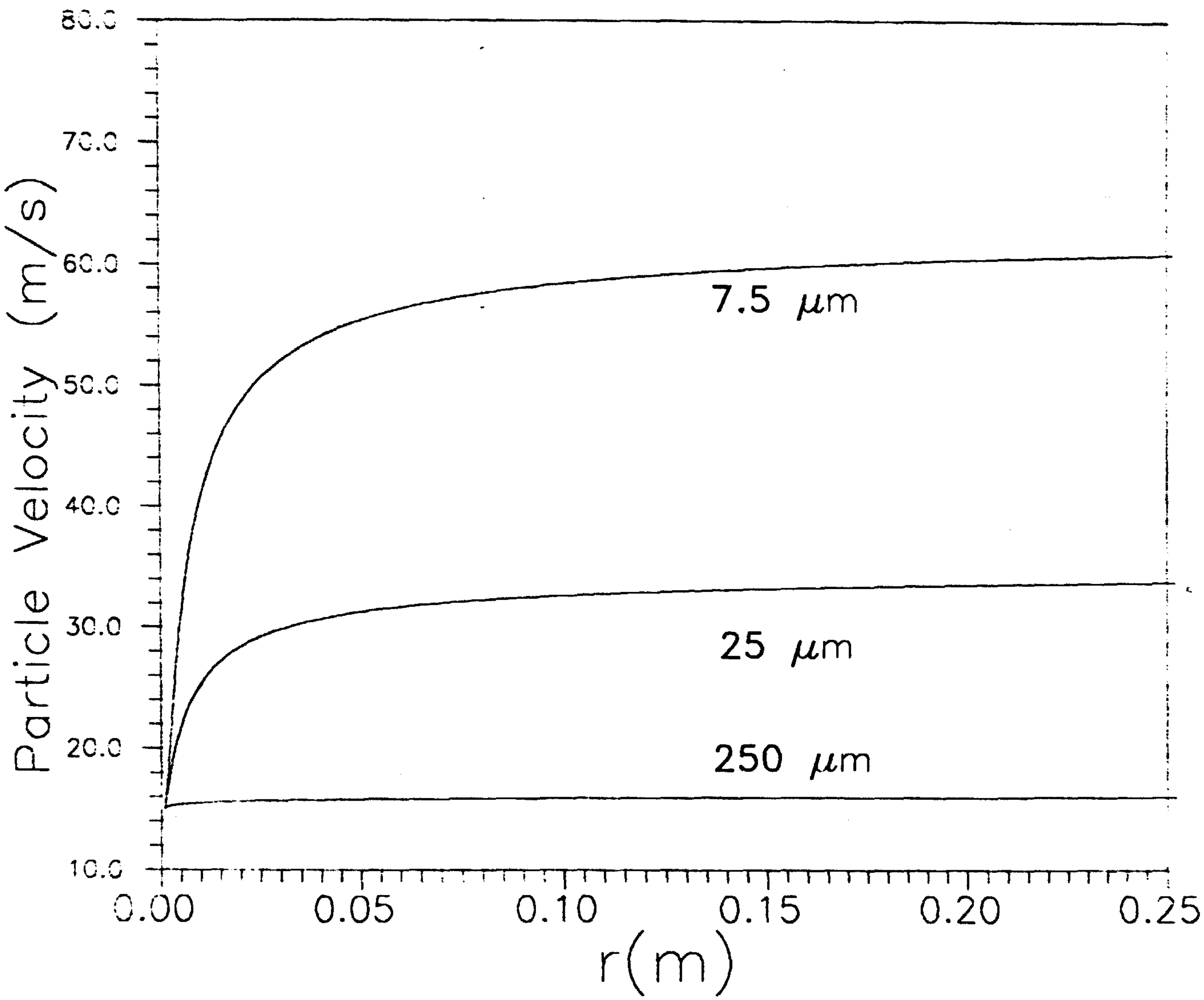

Figure 8. The Effect of Initial Particle Size on Particle Velocity

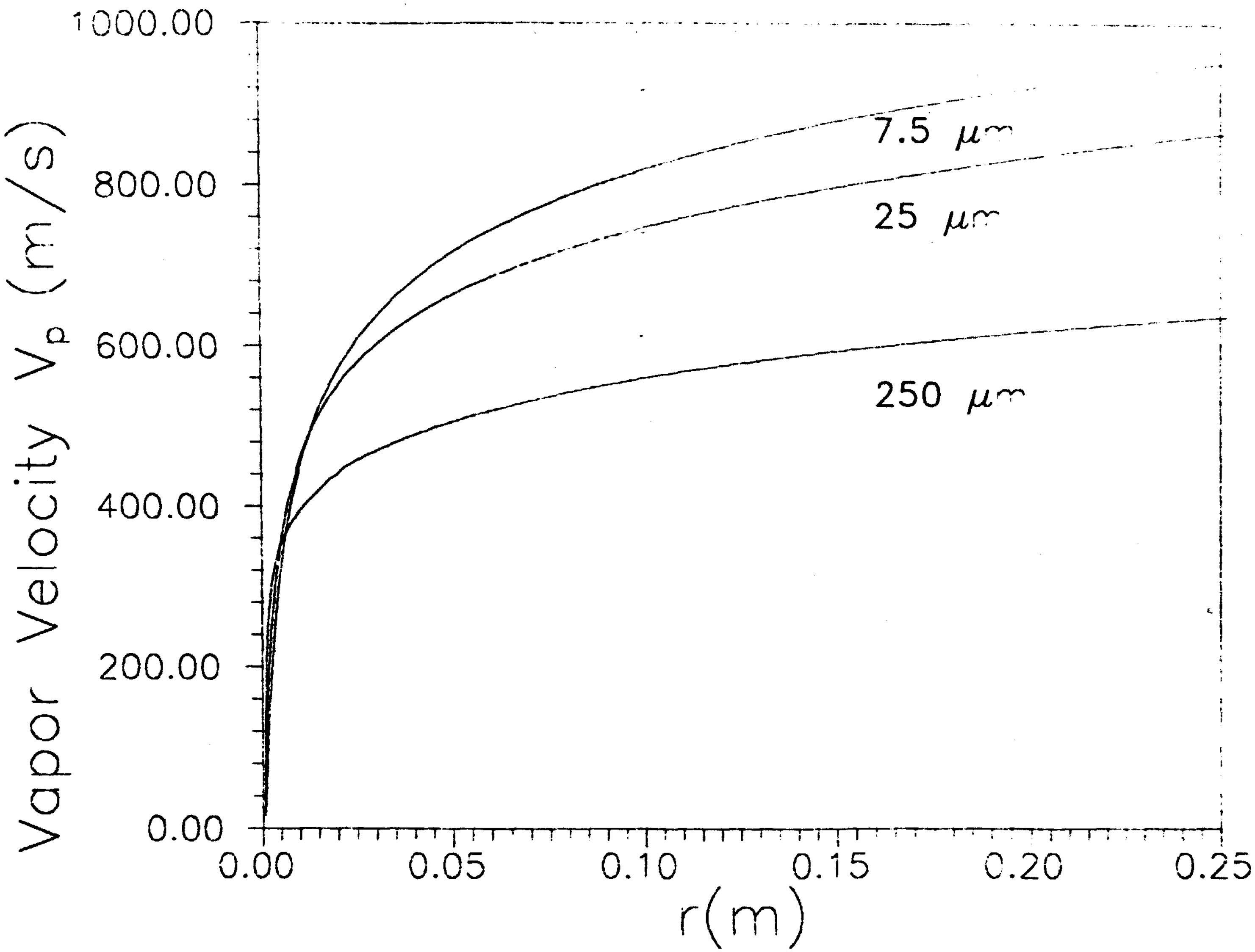

Figure 9. The Effect of Initial Particle Size on Vapor Velocity

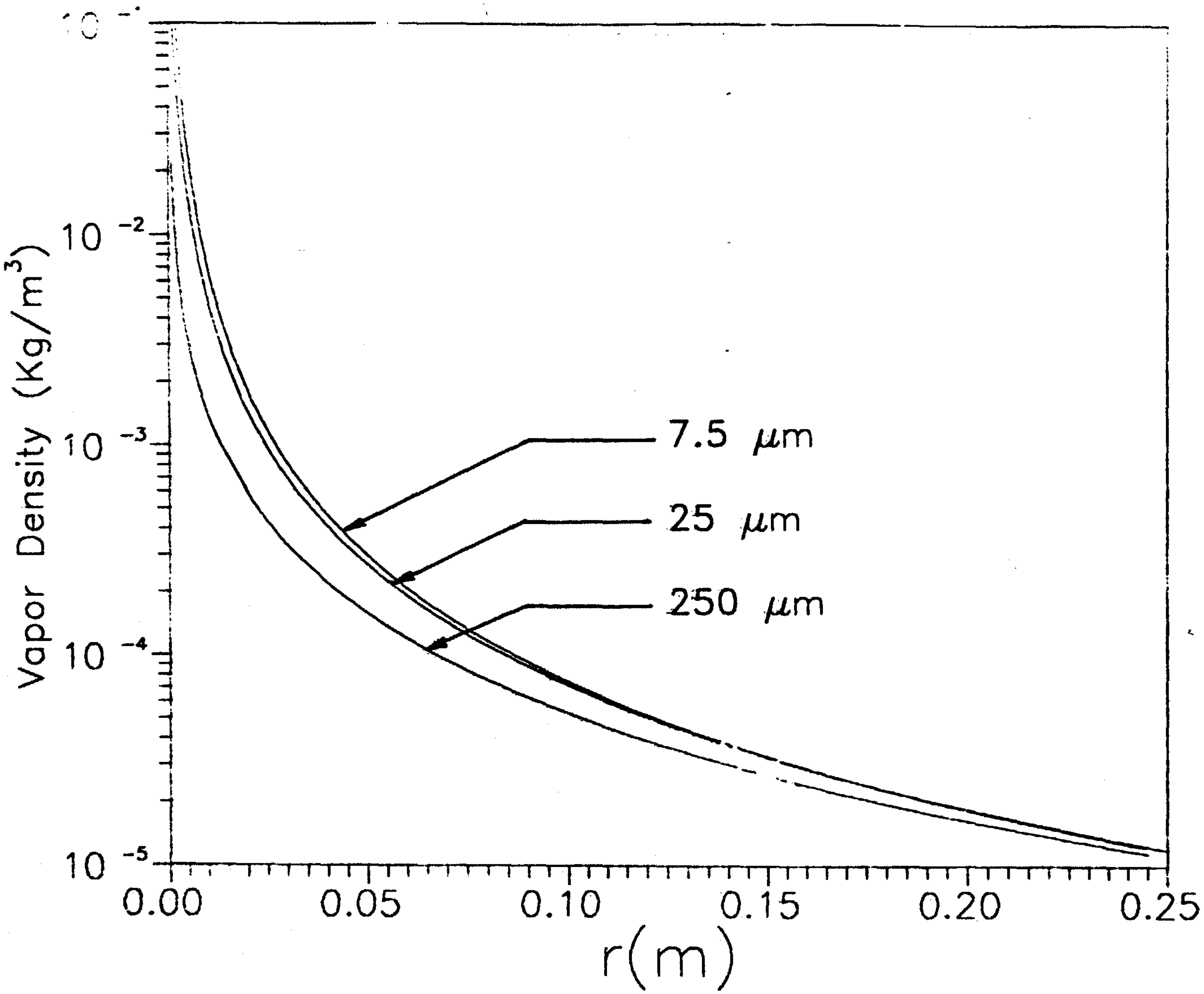

Figure 10. The Effect of Initial Particle Size on Vapor Density

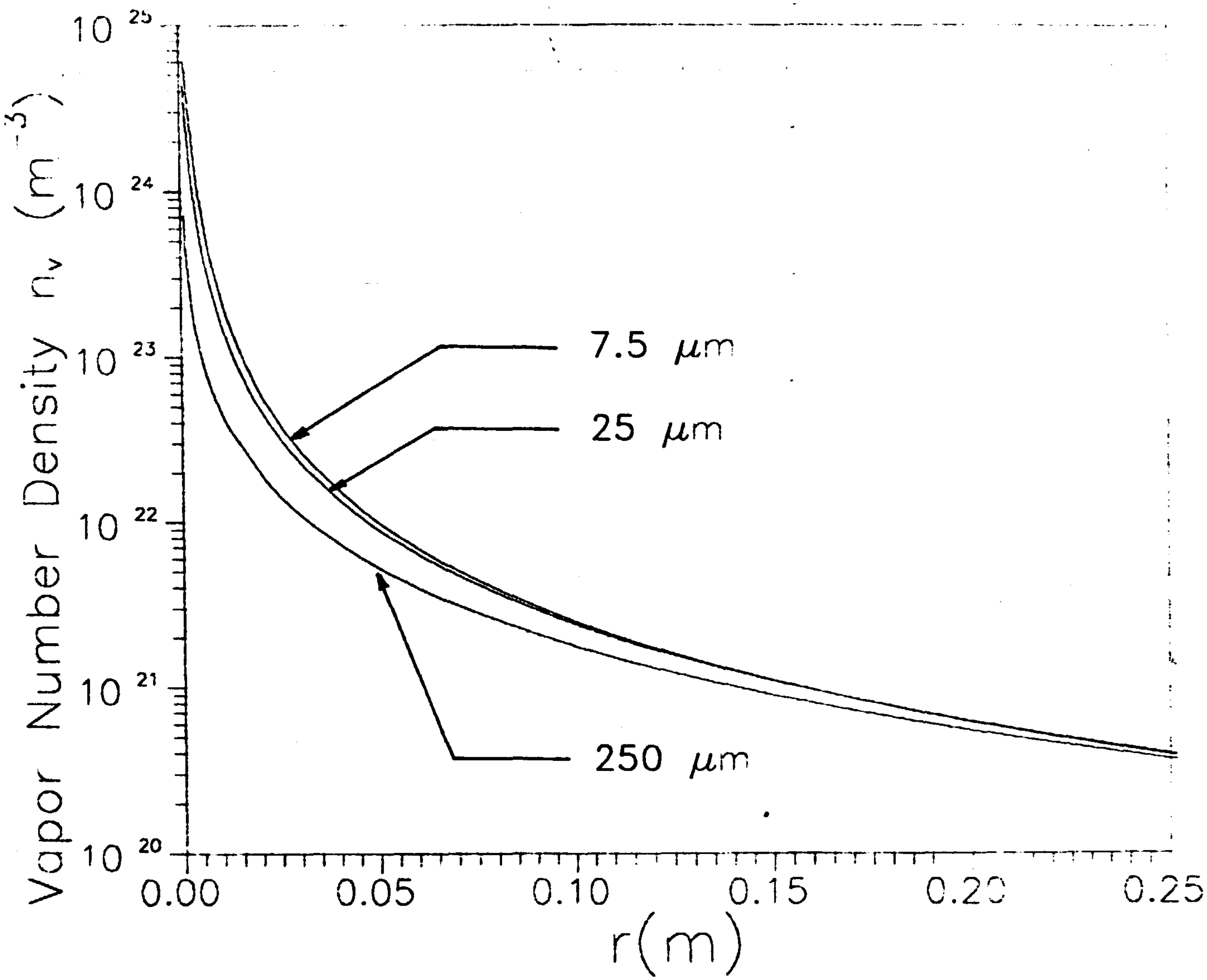

Figure 11. The Effect of Initial Particle Size on Vapor Number Density

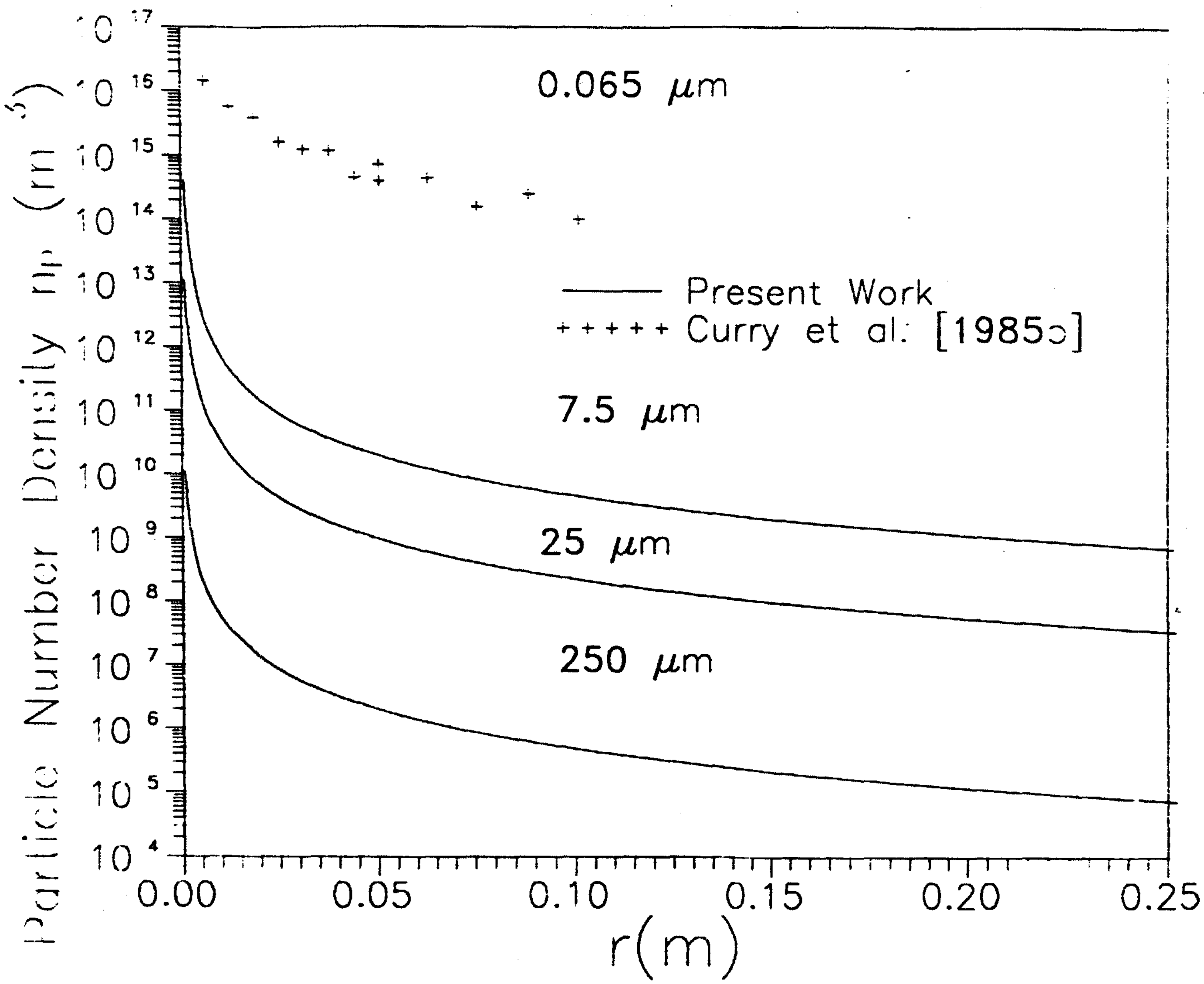

Figure 12. The Effect of Initial Particle Size on Particle Number Density

Table 3. Parameters Used for Figures 6-12

Initial Temperature (°K)	343.15
Initial Velocity (m/s)	15
Initial Source Radius (m)	7×10^{-4}
Initial Particle Radius (μm)	7.5, 25, or 250
Initial Vapor Pressure (N/m^2)	3.119×10^{4}
Initial Vapor Density (kg/m^3)	0.19833
Initial Volume Fraction	0.74
Evaporation Coefficient	0.05
Drag Coefficient	0.9
Particle Nusselt Number	2.73
Vapor Thermal Conductivity (W/m °K)	0.024403
Vapor Specific Heat (J/kg °K)	1863.172
Heat of Evaporation (J/kg)	2.33382×10^{6}
Heat of Fusion (J/kg)	3.34077×10^{5}
Particle Density (kg/m^3)	977.7
Gas Constant for Water (J/kg °K)	461.94
Stefan-Boltzman Constant ($W/m^2\ {}^{o}K^4$)	5.669×10^{-8}
Emissivity of Particle	0.9

Figure 8 shows how particle velocity changes as a function of distance from the source. For the large particles, the velocity changes very little from its initial velocity of 15 m/s. However, for smaller particles, the velocity changes very rapidly near the source and becomes almost constant far from the source. This phenomenon can be explained by the forces that influence particle speeds. As explained earlier, the vapor velocity increases very rapidly after ejection from the source. This can be seen in Figure 9 where, in less than one half millimeter from the source, the vapor velocity reaches sonic speed. Therefore, this sharp increase in vapor velocity can affect the velocity of the particles, especially the smaller ones. This situation can be seen in Figure 8 where the 7.5 μm particle velocity increases from 15 m/s to as much as 60 m/s. In contrast the 250 μm particle velocity increases very slightly (from 15 m/s to about 16 m/s). Glenn [1969] explained another factor, which is worth mentioning here. In Figure 7, it is clear that the evaporation rate of the smaller particles near the source is much faster than for the larger ones. Therefore, it is possible that the local back pressure which is created by evaporation, and as earlier discussed, vapor drag, could increase the velocity of the smaller particles.

Figure 10 shows that vapor density decreases very rapidly from about 0.2 kg/m^3 to 0.00001 kg/m^3 in only 0.25 meter distance from the source. Figure 11 is similar to Figure 10 except that the vapor density is presented in terms of vapor number density. Figure 12 shows particle number density variations for 7.5, 25, and 250 μm as a function of radial distance r. Figure 12 also shows a representation of the experimental measurements of Curry et al. [1985b] for a 0.13 μm diameter particle. As can be seen from Figure 12, the particle number density calculations of the present work are in good quantitative agreement with the experimental work of Curry et al. [1985b].

5. SUMMARY AND CONCLUSIONS

An analytical investigation was made of a one-dimensional radial source flow of water into a vacuum. The particles were assumed to be spherical, with uniform size and temperature. The heat transfer between particles and vapor was assumed to be by convection and radiation only. The equations that describe this model are a sixth-order, nonlinear, two-point boundary value problem. The problem was solved by the "shooting/splitting" technique. The results of this investigation were compared with the other works in the literature, and were generally in good agreement.

Data similar to those of recent shuttle orbiter water dumps into space were used to give some indication of water flow characteristics into a vacuum environment. Since the results of the present work are limited to a one-dimensional model, for a better understanding of the problem, the following goals are suggested for future study. A two-dimensional continuum model of analysis for a source flow of liquid water jet into a vacuum (for distances close to the source) is the next logical step. In order to investigate the complete water flow structure as it is released into space, kinetic models would be necessary.

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APPENDIX A. DERIVATION OF GOVERNING EQUATIONS

Under the assumptions stated in the text, and using the control volume analysis, the system of equations governing the flow are derived as follows:

A.1 CONTINUITY OF VAPOR AND PARTICLE

The net flux of vapor out of a spherical control volume is equal to the rate at which mass is generated by evaporation from the particles. With this analogy, the continuity equation of vapor can be stated in integral form as

$$\iint \rho_v \, (1-\alpha) \, u_{v_j} \, n_j \, dA = \iiint \dot{m} \, \alpha \, dV \tag{A1}$$

Where ρ_v, α, u_v, and $\dot{m}$ are the vapor density, the particle volume fraction, vapor velocity, and mass rate evaporated per unit volume of particles, respectively. n_j represents the unit vector.

Applying Gauss' theorem to the left side of the Equation (A1) yields:

$$\iint \rho_v \, (1-\alpha) \, u_{v_j} \, n_j \, dA = \iiint \frac{\partial}{\partial x_j} [\rho_v \, (1-\alpha) \, u_{v_j}] \, dV \tag{A2}$$

therefore, Equation (A1) can be stated as

$$\iiint \frac{\partial}{\partial x_j} [\rho_v \, (1-\alpha) \, u_{v_j}] \, dV = \iiint \dot{m} \, \alpha \, dV \tag{A3}$$

In the spherical coordinate system, with motion restricted to the radial coordinate,

$$\frac{d}{dr} [\rho_v \, (1-\alpha) \, u_v] + \frac{2}{r} [\rho_v \, (1-\alpha) \, u_v] = \dot{m} \, \alpha \tag{A4}$$

or simply:

$$\frac{d}{dr} [r^2 \rho_v \, (1-\alpha) \, u_v] = \dot{m} \, \alpha \, r^2 \tag{A5}$$

By analogy with Equation (A1), the continuity equations for particles can be stated as follows:

$$\iint \rho_p \, \alpha \, u_{p_j} \, \eta \, dA = -\iiint \dot{m} \, \alpha \, dV \tag{A6}$$

where ρ_p and u_{p_j} are particle density and velocity, respectively.

Proceeding as before,

$$\iint \rho_p \, \alpha \, u_{p_j} \, \eta \, dA = \iiint \frac{\partial}{\partial x_j} [\rho_p \, \alpha \, u_{p_j}] \, dV \tag{A7}$$

Thus, equation (A6) can be written as

$$\iiint \frac{\partial}{\partial x_j} [\rho_p \, \alpha \, u_{p_j}] \, dV = -\iiint \dot{m} \, \alpha \, dV \tag{A8}$$

or, in one-dimensional spherical coordinates

$$\frac{d}{dr} [r^2 \, \rho_p \, \alpha \, u_p] = -\dot{m} \, \alpha \, r^2 \tag{A9}$$

A.2 MOMENTUM EQUATION FOR VAPOR AND PARTICLE

The net force acting on the vapor within the control volume, plus the rate at which momentum is added to the vapor by the evaporating particles, must equal the net rate of momentum outflow. The momentum equation for the vapor can be stated as

$$-\iint P \, (1-\alpha) \, \eta_i \, dA + \iiint [\alpha \, f_p + P \frac{\partial}{\partial x_j} (1-\alpha) \, dV + \iiint \dot{m} \, \alpha \, u_{p_i} \, dV = \iint \rho_V \, (1-\alpha) \, u_{v_i} \, u_{v_j} \, \eta_j \, dA \tag{A10}$$

when f_p is the total drag force exerted on the vapor per unit volume. P represents the local pressure.

Again, applying Gauss' theorem twice in Equation (A10), the result becomes:

$$-\iiint\left[(1-\alpha)\frac{\partial P}{\partial x_j}-f_p\,\alpha-\dot{m}\,\alpha\,u_{p_j}\right]dV=$$

$$\iiint\left\{\rho_v\,(1-\alpha)\,v_j\,\frac{\partial}{\partial x_j}\,u_{v_i}+u_{v_i}\,\frac{\partial}{\partial x_j}[\rho_v\,(1-\alpha)\,u_{v_j}]\right\}dV \tag{A11}$$

Using the vapor Equation (A3) and substituting it into Equation (A11), the result then becomes

$$-\iiint\left[(1-\alpha)\frac{\partial P}{\partial x_j}-f_p\,\alpha-\dot{m}\,\alpha\,(u_{p_j}-u_{v_j})\right]dV=$$

$$\iiint\rho_v\,(1-\alpha)\,u_{v_j}\,\frac{\partial u_{v_i}}{\partial x_j}\,dV \tag{A12}$$

Restricting motion along the radial coordinate, (A12) reduces to:

$$\rho_v\,(1-\alpha)\,u_v\,u_v'+(1-\alpha)\,P'-\alpha\,f_p=\dot{m}\,\alpha\,(u_p-u_v) \tag{A13}$$

The momentum equation for the particle can be formulated in a manner similar to the analogy used for the vapor momentum equation and one obtains:

$$-\iint P\,\alpha\,n_j\,dA-\iiint\left(\alpha\,f_p-P\frac{\partial\alpha}{\partial x_j}\right)dV-\iiint\dot{m}\,\alpha\,u_{p_i}\,dV$$

$$=\iint\rho_p\,\alpha\,u_{p_i}\,u_{p_j}\,n_j\,dA \tag{A14}$$

Using Gauss' theorem and employing the particle continuity equation (A8), the momentum equation for the particle in one-dimensional radial coordinates becomes

$$\rho_p\,\alpha\,u_p\,u_p'+\alpha\,p'+\alpha\,f_p=0 \tag{A15}$$

A.3 ENERGY EQUATION FOR VAPOR AND PARTICULATE MATTER

From the first law of thermodynamics, the rate of heat addition to the vapor, plus the rate at which work is done on the vapor, plus the rate of energy addition due to the evaporating particulate matter, must equal the rate of energy outflow of the vapor,

$$\iiint q_p \, \alpha \, dV + \iiint \alpha \, f_p \, u_{p_i} \, dV - \iint P \, (1 - \alpha) \, u_{v_j} \, \eta \, dA \, +$$

$$\iiint \alpha \, \dot{m} \, (h_{v_{T_p}} + \frac{1}{2} u_{p_i} \, u_{p_i}) \, dV = \tag{A16}$$

$$\iint \rho_v \, (1 - \alpha)(e_{v_{T_v}} + \frac{1}{2} u_{v_i} \, u_{v_i}) \, u_{v_i} \, \eta \, dA$$

where q_p is the total rate of heat transfer per unit volume by convection and radiation from the particles to vapor. $h_{v_{T_p}}$ is enthalpy of the vapor at particle temperature and $e_{v_{T_v}}$ is internal energy of vapor at vapor temperature.

Equation (A16) can be written as

$$\iiint \alpha \left[q_p + f_p \, u_{p_i} + \dot{m} \, (h_{v_{T_p}} + \frac{1}{2} u_{p_i} \, u_{p_i}) \right] dV = \tag{A17}$$

$$\iint \rho_v \, (1 - \alpha)(h_{v_{T_v}} + \frac{1}{2} u_{v_i} \, u_{v_i}) \, u_{v_i} \, \eta \, dA$$

where $h = e + P/\rho$. Again, using Gauss' theorem, the right hand side of equation (A17) becomes

$$\iint \rho_v (1 - \alpha)(h_{v_{T_v}} + \frac{1}{2} u_{v_i} u_{v_i}) u_{v_i} \eta \, dA =$$

$$\iiint \frac{\partial}{\partial x_j} \left[\rho_v (1 - \alpha) u_{v_j} (h_{v_{T_v}} + \frac{1}{2} u_{v_i} u_{v_i})\right] dV =$$

$$\iiint \left\{\left[\rho_v (1 - \alpha) u_{v_j}\right] \frac{\partial}{\partial x_j} (h_{v_{T_v}} + \frac{1}{2} u_{v_i} u_{v_i}) + (h_{v_{T_v}} + \frac{1}{2} u_{v_i} u_{v_i}) \frac{\partial}{\partial x_j} \left[\rho_v (1 - \alpha) u_{v_j}\right]\right\} dV \tag{A18}$$

Using (A3)

$$\frac{\partial}{\partial x_j}\left[\rho_v (1 - \alpha) u_{v_j}\right] = \dot{m} \alpha \tag{A3}$$

and substituting the above equation into (A18), we have

$$\iiint \alpha \, [q_p + f_p u_{p_i} + \dot{m} (h_{v_{T_p}} + \frac{1}{2} u_{p_i} u_{p_i} - h_{v_{T_v}} - \frac{1}{2} u_{v_i} u_{v_i})] \, dV = \iiint \rho_v (1 - \alpha) u_{v_i} \frac{\partial}{\partial x_j} (h_{v_{T_v}} + \frac{1}{2} u_{v_i} u_{v_i}) \, dV \tag{A19}$$

If the vapor is now assumed to behave as a perfect gas, so that

$$h_v = \int C_{p_v} \, dT$$

where C_{p_v} is specific heat of the vapor and considered to be constant, then Equation (A19) can be reduced to the following equation

$$\rho_v (1 - \alpha) u_v C_{p_v} T_v' + \rho_v (1 - \alpha) u_v^2 u_v' =$$

$$\alpha [q_p + f_p u_p] + \dot{m} \alpha [C_{p_v} (T_p - T_v) + \frac{1}{2} (u_p^2 - u_v^2)] \tag{A20}$$

Now, multiplying the vapor momentum Equation (A13) by u_v and subtracting the result from (A20) yields:

$$\rho_v (1 - \alpha) u_v C_{p_v} T_v' - u_v (1 - \alpha) p' =$$

$$\alpha q_p + \alpha f_p (u_p - u_v) + \dot{m} \alpha \left[C_{p_v} (T_p - T_v) + \frac{1}{2} (u_p - u_v)^2 \right] \tag{A21}$$

The energy equation for particles by similar analogy with Equation (A16) can be formulated as follows:

$$-\iiint q_p \alpha \, dV - \iiint \alpha f_p u_{p_i} \, dV - \iint P \alpha u_{p_i} \eta \, dA -$$

$$\iiint \alpha \dot{m} (h_{L_{T_p}} + \lambda_v + \frac{1}{2} u_{p_i} u_{p_i}) \, dV =$$

$$\iint \rho_p \alpha \left[\beta (e_{L_{T_p}} + \frac{1}{2} u_{p_i} u_{p_i}) + \right. \tag{A22}$$

$$\left. (1 - \beta)(e_{S_{T_p}} + \frac{1}{2} u_{p_i} u_{p_i}) \right] u_{p_i} \eta \, dA$$

where $e_{L_{T_p}}$ and $e_{S_{T_p}}$, and β are the internal energy of the liquid and solid phases at particle temperature, and liquid volume fraction of the particles, respectively. λ_v is latent heat of vaporization at T_p and defined as

$$\lambda_v = h_v - h_L \tag{A23}$$

Then Equation (A22) becomes

$$\iiint -\alpha\left[q_p + f_p\, u_{p_i} + \dot{m}\,(h_{L_{T_p}} + \lambda_v + \frac{1}{2}\, u_{p_i}\, u_{p_i})\right] dV =$$

$$\int \rho_p\, \alpha\left[\beta\,(h_{L_{T_p}} + \frac{1}{2}\, u_{p_i}\, u_{p_i}) + \right. \tag{A24}$$

$$\left.(1 - \beta)(h_{S_{T_p}} + \frac{1}{2}\, u_{p_i}\, u_{p_i})\right] u_{p_i}\, \eta\, dA$$

where $h_{L_{T_p}}$ and $h_{S_{T_p}}$ are the enthalpy of the liquid and solid phases of the particles at particle temperature, respectively. Using Gauss' theorem, the right hand side of Equation (A24) can be stated as

$$\iint \rho_p\, \alpha\left[\beta\,(h_{L_{T_p}} + \frac{1}{2}\, u_{p_i}\, u_{p_i}) + \right.$$

$$\left.(1 - \beta)(h_{S_{T_p}} + \frac{1}{2}\, u_{p_i}\, u_{p_i})\right] u_{p_i}\, \eta\, dA =$$

$$\iiint \frac{\partial}{\partial x_j}\left\{\rho_p\, \alpha\, [\beta\,(h_{L_{T_p}} + \frac{1}{2}\, u_{p_i}\, u_{p_i}) + \right. \tag{A25}$$

$$\left.(1 - \beta)(h_{S_{T_p}} + \frac{1}{2}\, u_{p_i}\, u_{p_i})] u_{p_j}\right\} dV$$

Substituting (A25) into (A24), and with some mathematical manipulation, (A24) becomes

$$\iiint -\alpha\left[q_p + f_p\, u_{p_i} + \dot{m}\,(h_{L_{T_p}} + \lambda_v + \frac{1}{2}\, u_{p_i}\, u_{p_i})\right] dV =$$

$$\iiint \left[\rho_p\, \alpha\, u_{p_j} \frac{\partial}{\partial x_j}\,(h_{L_{T_p}} - (1 - \beta)\,\lambda_f + \frac{1}{2}\, u_{p_i}\, u_{p_i}) + \right. \tag{A26}$$

$$\left.(h_{L_{T_p}} - (1 - \beta)\,\lambda_f + \frac{1}{2}\, u_{p_i}\, u_{p_i})\, \frac{\partial}{\partial x_j}\, \rho_p\, \alpha\, u_{p_j}\right] dV$$

Where λ_f is heat of fusion and defined as

$$\lambda_f = h_{L_{T_p}} - h_{S_{T_p}}$$

Employing the particle continuity relation (A8),

$$\iiint -\alpha \left\{ q_p + f_p u_{p_i} + \dot{m} \left[\lambda_v + (1 - \beta) \lambda_f\right] \right\} dV =$$
$$\iiint \rho_p \alpha u_{p_j} \frac{\partial}{\partial x_j} \left[h_{L_{T_p}} - (1 - \beta) \lambda_f + \frac{1}{2} u_{p_i} u_{p_i} \right] dV \tag{A27}$$

Equation (A27) reduces to

$$\rho_p \alpha u_p \left[c_p T_p' - \lambda_v' + \lambda_f \beta' \right] + \rho_p \alpha u_p^2 u_p' =$$
$$- \alpha \left[q_p + f_p u_p + \dot{m} (\lambda_v + (1 - \beta) \lambda_f) \right] \tag{A28}$$

and, as before, multiplying the particle momentum equation (A15) by u_p and subtracting the result from (A28), yields

$$\rho_p \alpha u_p \left[\left(c_p - \frac{d\lambda_v}{dT_p} \right) T_p' + \lambda_f \beta' \right] - u_p \alpha P' =$$
$$- \alpha q_p - \dot{m} \alpha \left[\lambda_v + (1 - \beta) \lambda_f \right] \tag{A29}$$

Equation (A19) is subjected to the following constraints:

$$\begin{aligned} T_p > T_* ;\quad & \beta = 1, \quad \beta' = 0 \\ T_p = T_{**} ;\quad & T_p' = 0 \\ T_p < T_{**} ;\quad & \beta = \beta' = 0 \end{aligned} \tag{A30}$$

where T_* and T_{**} are the initial and triple point temperatures respectively.

Equations (A5), (A9), (A13), (A15), (A21), and (A29) constitute a sixth order system in the ten unknown quantities ρ_v, α, u_v, m, u_p, P, f_p, T_v, q_p, and T_p. Therefore four constitutive relations are required.

A.4 CONSTITUTIVE RELATIONS

The rate of mass evaporated per unit volume of particles is assumed to be defined by the Hertz-Knudsen equation:

$$\dot{m}=\frac{4\,\pi\,\sigma^2\,\epsilon\,(P_0-P)\,\left(\frac{M}{2\,\pi\,R\,T_p}\right)^{1/2}}{\frac{4}{3}\,\pi\,\sigma^3}=\frac{3\,\epsilon}{\sigma}\,(P_0-P)\left(\frac{M}{2\,\pi\,R\,T_p}\right)^{1/2} \tag{A31}$$

where ϵ, R, M, and P_0 are evaporation coefficient, universal gas constant, molar mass, and vapor pressure, respectively. The particle size, σ, can be related to particle velocity and volume fraction by assuming that no particles are either created or destroyed. The number density of particles n_p (number of particles per unit volume) can be written:

$$n_p=\frac{\alpha}{\frac{4}{3}\pi\sigma^3} \tag{A32}$$

and applying the aforementioned conservation theorem:

$$\frac{d}{dr}(r^2 n_p u_p)=0 \tag{A33}$$

substituting (A32) into (A33) and integrating. Equation (A33) then becomes

$$\frac{r^2\alpha u_p}{\frac{4}{3}\pi\sigma^3}=C \tag{A34}$$

where the constant C can be defined in terms of the constants at the source. Then (A34) becomes

$$\frac{r^2\alpha u_p}{\frac{4}{3}\pi\sigma^3}=\frac{r_*^2\alpha_* u_{p*}}{\frac{4}{3}\pi\sigma_*^3} \tag{A35}$$

where the subscript (*) refers to conditions at the initial source radius, $r = r_*$.

Equation (A35) can be written as

$$\left(\frac{\sigma}{\sigma_*}\right)^3=\left(\frac{r}{r_*}\right)^2\left(\frac{\alpha}{\alpha_*}\right)\left(\frac{u_p}{u_{p*}}\right) \tag{A36}$$

Next, assuming the vapor behaves as a perfect gas, the equation of state is:

$$P=\rho_v\frac{R}{M}T_v \tag{A37}$$

and the drag force per unit volume of the particles is taken to be of the form:

$$f_p=\frac{3C_D}{8\sigma}\rho_v(u_p-u_v)|u_p-u_v| \tag{A38}$$

where C_D is the drag coefficient and is, in general, a function of particle Reynolds and Mach numbers.

$$q_p=q_c+q_r \tag{A39}$$

Finally, the heat transfer rate by convection and radiation is given by q_c and q_r, respectively, and can be defined per unit volume of the particle as

$$q_c=\frac{4\pi\sigma^2h(T_p-T_v)}{\frac{4}{3}\pi\sigma^3} \tag{A40}$$

Here the heat transfer coefficient h can be defined as a function of the Nusselt number, thermal conductivity, and radius of the particle

$$h=\frac{Nu_pk}{2\sigma} \tag{A41}$$

therefore Equation (A40) becomes

$$q_c = \frac{3}{2}\frac{k}{\sigma^2} Nu_p (T_p - T_v) \tag{A42}$$

and

$$q_r = \frac{4\pi\sigma^2 \epsilon_r \sigma_{SB}}{\frac{4}{3}\pi\sigma^3}(T_p^4 - T_v^4) \tag{A43}$$

where σ_{SB} and ϵ_r are the Stefan-Boltzman constant and emissivity of the particle. Equation (A3) is simplified to the following equation:

$$q_r = \frac{3}{\sigma}\epsilon_r\, \sigma_{SB}\, (T_p^4 - T_v^4) \tag{A44}$$

Equations (A5), (A9), (A15), (A21), and (A29), together with Equations (A31), (A36), (A38), (A42), and (A44) constitute the complete system. Therefore, they can be consolidated into a sixth order system in the unknowns ρ_v, u_v, u_p, T_p, T_v, and α.

APPENDIX B. GLOSSARY

A Area

A_1-A_6 Intermediate parameters defined in Equations (16-21)

c_{p_v} Specific heat of vapor

C_D Drag coefficient

e Specific internal energy

f_p Drag force per unit volume

h Specific enthalpy

k Thermal conductivity of vapor

$\dot{m}$ Mass rate evaporated per unit volume of particles

M Molecular weight

n_p Particle number density

Nu_p Particle Nusselt number

P Local pressure

P_o Vapor pressure

q_c Convection heat transfer rate per unit volume of particle

q_p Convection and radiation heat transfer ($q_p = q_r + q_c$)

q_r Radiation heat transfer rate per unit volume of particle

R Universal gas constant, Radial distance in Equation (23)

r radial coordinate

T Temperature

u Velocity

V Volume

α	Particle volume fraction; particle volume/total volume
β	Particle liquid fraction; liquid particle volume/total particle volume
γ	Ratio of specific heats
ϵ	Evaporation coefficient
ϵ_r	Emissivity of the particle
θ_1	Dimensionless gas constant parameter
θ_2	Dimensionless initial radius ratio
θ_3	Dimensionless initial pressure ratio
θ_4	Dimensionless evaporation parameter
θ_5	Dimensionless drag parameter
θ_6	Dimensionless heat transfer parameter (convection)
θ_7	Dimensionless vapor specific heat
θ_8	Dimensionless parameter $\left[C_{p_v} - \left(\frac{d\lambda_v}{dT_p} \right) \right] \frac{T_*}{u_*^2}$
θ_9	Dimensionless heat of vaporation
θ_{10}	Dimensionless heat of fusion
θ_{11}	Dimensionless heat transfer parameter (radiation)
λ_v	Heat of vaporation
λ_f	Heat of fusion
ρ	Density
σ	Particle radius
σ_{SB}	Stefan-Boltzman constant

Subscripts

*	Initial condition
**	Triple point
v	Vapor
p	Particle
L	Liquid
S	Solid