

DTIC FILE COPY



WRDC-TR-90-3027

AD-A230 223

A Theoretical Approach to Analysis and Design of Efficient Reduced Control for Space Structures

Dr Hayrani Oz

The Ohio State University  
Dept of Aeronautical & Astronautical Engineering  
Columbus, Ohio 43210

October 1990

DTIC  
LECTE  
DEC 24 1990  
S D

Final Report for the Period October 1986 to October 1990

Approved for public release; distribution unlimited.

FLIGHT DYNAMICS LABORATORY  
WRIGHT RESEARCH AND DEVELOPMENT CENTER  
AIR FORCE SYSTEMS COMMAND  
WRIGHT PATTERSON AIR FORCE BASE, OHIO 45433-6553

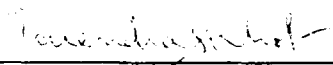
90 12 21 008

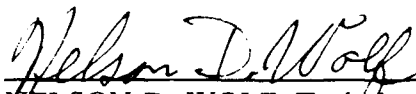
NOTICE

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility or any obligation whatsoever. The fact that the Government may have formulated or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication, or otherwise as in any manner, as licensing the holder or any other person or corporation; or as conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.


This report is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

  
\_\_\_\_\_  
NARENDRA S. KHOT  
Project Engineer  
Design & Analysis Methods Group

  
\_\_\_\_\_  
NELSON D. WOLF, Technical Manager  
Design & Analysis Methods Group  
Analysis & Optimization Branch

FOR THE COMMANDER

  
\_\_\_\_\_  
DAVID K. MILLER, Lt Col, USAF  
Chief, Analysis & Optimization Branch  
Structures Division

"If your address has changed, if you wish to be removed from our mailing list, or if the addressee is no longer employed by your organization please notify WRDC/FIBRA, Wright-Patterson AFB OH 45433-6553 to help us maintain a current mailing list".

Copies of this report should not be returned unless return is required by security considerations, contractual obligations, or notice on a specific document.

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE 15 Oct 90	3. REPORT TYPE AND DATES COVERED Final Oct 86 - Oct 90		
4. TITLE AND SUBTITLE A Theoretical Approach to Analysis and Design of Efficient Reduced Control for Space Structures			5. FUNDING NUMBERS PR 2302 TA N5 WU 02	
6. AUTHOR(S) Dr Hayrani Oz				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Department of Aeronautical & Astronautical Engineering The Ohio State University Columbus, Ohio 43210			8. PERFORMING ORGANIZATION REPORT NUMBER N/A	
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) Flight Dynamics Laboratory (WRDC/FIBRA) Wright Research Development Center Wright-Patterson AFB OH 45433-6553			10. SPONSORING / MONITORING AGENCY REPORT NUMBER WRDC-TR-90-3027	
11. SUPPLEMENTARY NOTES Narendra S. Khot, WRDC/FIBRA, (513) 255-6992				
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution unlimited.			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) The report develops concepts of efficiency for the control of structural dynamic systems. The efficiencies defined are unique nondimensional measures of structure-control system performance forming a common basis of evaluation of such systems. Developments are presented both for distributed parameter systems and spatially discrete, typically finite element models of structural systems. Internal characterizations of power efficiencies of structural control systems are given by developing an efficiency modes analysis of controllers. The significance of relationship of controller efficiency modes to structural modes is studied. Based on these concepts, efficient model/controller reduction approaches are developed. The efficiencies of systems with dynamic compensators including reduced order efficiency-state estimators are also developed. Illustrative examples based on the ACOSS-4 tetrahedral structure and the CSDL-ACOSS-6 (Model 2) as typical large space structures, are given. The research establishes that efficiency of structural control systems is a fundamental concept that must be addressed in any structure control system design. The efficiency measures also establish the performance of $\infty$ -dimensional systems on the basis of finite dimensional control design models.				
14. SUBJECT TERMS Multivariable Controls; Control Structure Interaction, Reduced Order System, Efficiency			15. NUMBER OF PAGES 270	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT	

FOREWORD

This final report was prepared by the Ohio State University, for the Analysis and Optimization Branch (FIBR) of the Wright Research and Development Center. The work was performed under Contract F33615-86-C-3212 which was initiated under Project No. 2032, "Structures", Task No. N5, "Structural Dynamics and Controls". The objective of this contract was to develop mathematical algorithms for designing efficient reduced order controls with particular reference to the large space structures. The program manager was Dr H. Oz of the Department of Aeronautical and Astronautical Engineering. In FIBR, Dr N. S. Khot was the project monitor.



Accession For	
NTIS CRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution /	
Availability Codes	
Dit	Availa a/or Special
A-1	

## TABLE OF CONTENTS

Section	Page
1.0 INTRODUCTION . . . . .	1
2.0 EFFICIENCY OF STRUCTURE-CONTROL SYSTEMS . . . . .	5
2.1 Introduction . . . . .	5
2.2 Control Objectives for the DPS . . . . .	5
2.3 Global Control Power for the DPS . . . . .	8
2.4 Globally Optimal Control for the DPS . . . . .	11
2.5 Suboptimal Control for the DPS . . . . .	14
2.6 Control Powers for Discrete-Systems . . . . .	16
2.7 Efficiency of a Structure-Control System . . . . .	20
2.8 Illustrative Examples: Control of ACOSS-4 Tetrahedral Truss . . . . .	26
Analysis via Efficiencies . . . . .	26
Efficient Model Reduction . . . . .	30
3.0 EFFICIENCY MODES ANALYSIS OF STRUCTURE CONTROL SYSTEMS . . . . .	44
3.1 Introduction . . . . .	44
3.2 Computation of Control Power Matrices for Efficiency Quotients . . . . .	44
3.3 Efficiency Eigenvalue Problem . . . . .	46
3.4 Efficiency Modes and Principal Control Directions of a Structure-Control System . . . . .	50
3.5 Principal Efficiency Components - A Link Between Controller Design and Initial Conditions . . . . .	53
3.6 Illustrative Examples on ACOSS-4 . . . . .	56
Efficiency Modes Analysis . . . . .	56
Effect of Initial Disturbance on Efficiencies . . . . .	57
4.0 EFFICIENCY MODEL/CONTROLLER REDUCTION . . . . .	71
4.1 Introduction . . . . .	71
4.2 Decomposition of Efficiency Quotients . . . . .	72
4.3 Controller Reduction in the Efficiency State Space Efficiency Components Truncation . . . . .	80
4.4 Efficiency Filtering . . . . .	84
4.5 Efficient Modal Controller Reduction . . . . .	87
4.6 Further Considerations for Efficient Model/Controller Reduction . . . . .	90
4.7 Illustrative Examples on ACOSS-4 . . . . .	94
Decomposition of Efficiency Quotients . . . . .	94
In/Efficiency-Filtering; Controller Reduction . . . . .	95
Efficient Model Selection (Reduction) . . . . .	96
5.0 EFFICIENCY OF SYSTEMS WITH COMPENSATOR DYNAMICS . . . . .	106
5.1 Introduction . . . . .	106
5.2 Efficiency of Structure-Control Systems With Compensator Dynamics-Compensator Efficiency . . . . .	107
5.3 Reduced-Order Efficiency State Estimators . . . . .	113
5.4 Residual Dynamic Response to Control Power Spillover . . . . .	117

TABLE OF CONTENTS (Concluded)

Section	Page
6 OTHER APPLICATIONS OF CONTROLLER EFFICIENCY AND APPLICATION TO CSDL-ACOSS-6 . . . . .	122
6.1 Introduction . . . . .	122
6.2 Inefficient Control With Truncated Low Frequency Modes . . . . .	122
6.3 Actuator Selection via Efficiencies . . . . .	124
6.4 Effect of LQR Weighting Matrices on Efficiencies . . . . .	126
6.5 Application to a Large Space Structure: CSDL-ACOSS-6 . . . . .	127
7 CONCLUSIONS . . . . .	153
REFERENCES . . . . .	155
APPENDIX A: PROOF OF FUNCTIONAL $S^R$ AS CONTROL POWER . . . . .	157
Truster Jets . . . . .	160
Proof-Mass Actuators . . . . .	162
APPENDIX B: CLOSED-FORM SOLUTION FOR $S^*$ . . . . .	164
APPENDIX C: COMPUTER RESULTS OF EXAMPLES . . . . .	166
ILLUSTRATED IN SECTIONS 3, 4 AND 6	
Nomenclature for Computer Outputs . . . . .	166
Examples 3.1 and 4.1 . . . . .	169
Example 3.2 . . . . .	176
Examples 3.3 and 3.6 . . . . .	183
Examples 3.4 and 4.2 . . . . .	190
Example 4.4 . . . . .	204
Example 4.5 . . . . .	221
Example 6.3 . . . . .	234
Example 6.4 . . . . .	246

## LIST OF ILLUSTRATIONS

Figure		Page
Figure 2.1:	ACOSS-4 Tetrahedral Structure . . . . .	33
Figure 2.2a:	Percent Global efficiencies for ACOSS-4. . . . .	34
Figure 2.2b:	Percent Relative Model efficiencies for ACOSS-4 . . . . .	35
Figure 2.2c:	Model efficiency coefficients for ACOSS-4 . . . . .	36
Figure 2.3.a:	Control Powers for n=2 . . . . .	37
Figure 2.3b:	Control Powers for n=8 . . . . .	38
Figure 2.4a:	Responses for n=2 of natural control (continuous input), suboptimal control and evaluation model with m=2, e*=1.27%, e=1.27% $\mu=1$ , sq*=77.67%, sq=98.73% . . . . .	39
Figure 2.4b:	Responses for n=8 of natural control (continuous input), and suboptimal control and evaluation model with m=2, e*=5.06%, e=56.8% $\mu=11.2$ , sq*=8.3%, sq=43.2% . . . . .	40
Figure 2.4c:	Responses for n=12 of natural control (continuous input) and suboptimal control with m=6, e*=31.8%, e=100% $\mu=3.13$ , sq=sq*=0 . . . . .	41
Figure 2.5a:	Response of evaluation model with efficient model reduction applied for n=8, m=2, e*=14.7%, e=96.58%, $\mu=6.57$ , sq*=0.24, sq=3.45% . . . . .	42
Figure 2.5b:	Response of evaluation model with efficient model reduction applied for n=8, m=4, e*=44.7%, e=93.93%, $\mu=2.1$ , sq*=0.14, sq=6.07% . . . . .	43
Figure 3.1:	Globally optimal system LOS for n=2 (modes: 11,12); example 3.1 . . . . .	59
Figure 3.2:	Suboptimal system LOS for n=2 (modes: 11,12), m=1 (input #4); example 3.1 . . . . .	60
Figure 3.3:	Evaluation model LOS for n=2 (modes: 11,12) m=1 (input #4); example 3.1 . . . . .	61
Figure 3.4:	Globally optimal system LOS for n=2 (modes: 11,12); example 3.3 . . . . .	62

Figure 3.5: Suboptimal system LOS for  $n=2$  (modes: 11,12);  $m=4$  (#1-4);  
example 3.3 . . . . . 63

Figure 3.6: Evaluation model LOS for  $n=2$  (modes: 11,12),  $m=4$  (#1-4);  
example 3.3 . . . . . 64

Figure 3.7: Globally optimal system LOS for  $n=8$  (modes: 1-8)  $m=2$  (#1-2);  
example 3.4 . . . . . 65

Figure 3.8: Suboptimal system LOS for  $n=8$  (modes: 1-8),  $m=2$  (#1-2);  
example 3.4 . . . . . 66

Figure 3.9: Evaluation model LOS for  $n=8$  (modes 1-8),  $m=2$  (#1-2);  
example 3.4 . . . . . 67

Figure 3.10: Globally optimal system LOS for  $n=8$  (modes: 1-8),  $m=4$  (#1-4);  
example 3.5 . . . . . 68

Figure 3.11: Suboptimal system LOS for  $n=8$  (modes: 1-8),  $m=4$  (#1-4);  
example 3.5 . . . . . 69

Figure 3.12: Evaluation model LOS for  $n=8$  (modes: 1-8),  $m=4$  (#1-4); . 4);  
example 3.5 . . . . . 70

Figure 4.1: Globally optimal system LOS for  $n=8$  (modes: 1-8);  
example 4.4; controller reduction via efficiency filtering  
for  $n_{pr}=4$  . . . . . 97

Figure 4.2: Suboptimal system LOS for  $n=8$  (modes: 1-8),  $m=4$  (#1-4);  
example 4.4; controller reduction via efficiency filtering  
for  $n_p=4$  . . . . . 98

Figure 4.3: Evaluation model LOS for  $n=8$  (modes: 1-8),  $m=4$  (#1-4),  
example 4.4; controller reduction via efficiency filtering  
for  $n_p=4$  . . . . . 99

Figure 4.4: Globally optimal system LOS,  $n=8$  (modes: 1-8), example 4.4;  
controller reduction via efficiency filtering for  $n_p=8$  . 100

Figure 4.5: Suboptimal system LOS,  $n=8$  (modes: 1-8),  $m=4$  (#1-4),  
example 4.4; controller reduction via efficiency filtering  
for  $n_p=8$  . . . . . 101

Figure 4.6: Evaluation model LOS,  $n=8$  (modes: 1-8),  $m=4$  (#1-4),  
example 4.4; controller reduction via efficiency filtering  
for  $n_p=8$  . . . . . 102

Figure 4.7: Globally optimal system LOS,  $n=8$  (modes: 3-6, 9-12);  
example 4.5; efficient model selection reduction . . . . . 103

Figure 4.8: Suboptimal System LOS,  $n=8$  (modes: 3-6, 9-12);  $m=4$  (#1-4),  
example 4.5; efficient model selection (reduction) . . . . . 104



Figure 4.9:	Evaluation model LOS, n=8 (modes: 3-6, 9-12) m=4 (#1-4), example 4.5; efficient model selection (reduction) . . . . .	105
Figure 6.1:	Globally optimal system LOS, n=4 (modes: 9-12), example 6.1; response of an inefficient system $e^*=8.39\%$ , $e=45.67\%$ . . . . .	130
Figure 6.2:	Suboptimal system LOS, n=4 (modes: 9-12) m=2 (#1,2), example 6.1; response of an inefficient system $e^*=8.39\%$ , $e=45.67\%$ . . . . .	131
Figure 6.3:	Evaluation for Model LOS, n=4 (modes: 9-12), m=2 (#1-2), example 6.1; response of an inefficient system $e^*=8.39\%$ , $e=45.67\%$ . . . . .	132
Figure 6.4:	Globally optimal system LOS, n=4 (modes: 9-12), example 6.2; response of an inefficient system $e^*=0.2\%$ , $e=21.33\%$ . . . . .	133
Figure 6.5:	Suboptimal system LOS, n=4 (modes: 9-12), m=1 (#5), example 6.2; response of an inefficient system, $e^*=0.2\%$ , $e=21.33\%$ . . . . .	134
Figure 6.6:	Evaluation model LOS, n=4 (modes: 9-12) m=1 (#5), example 6.2; response of an inefficient system, $e^*=0.2\%$ , $e=21.33\%$ . . . . .	135
Figure 6.7:	Globally optimal system LOS, n=8 (modes: 1-8) m=1 (#1) for actuator selection via efficiencies . . . . .	136
Figure 6.8:	Suboptimal system LOS, n=8 (modes: 1-8), m=1 ( 1) for actuator selection via efficiencies . . . . .	137
Figure 6.9:	Evaluation model LOS, n=8 (modes: 1-8), m=1 ( 1) for actuator selection via efficiencies . . . . .	138
Figure 6.10:	Globally optimal system LOS, n=8 (modes: 1-8), for actuator selection via efficiencies . . . . .	139
Figure 6.11:	Suboptimal system LOS, n=8 (modes: 1-8), m=12; for actuator selection via efficiencies . . . . .	140
Figure 6.12:	Evaluation model LOS, n=8 (modes: 1-8), m=12; for actuator selection via efficiencies . . . . .	141
Figure 6.13:	Globally optimal system LOS, n=8 (modes: 1-8), effect of LQR weighting on efficiency, $r=0.05$ , $e^*=16.89\%$ , $e=56.93\%$ . . . . .	142
Figure 6.14:	Suboptimal system LOS, n=8 (modes: 1-8), m=1; effect of LQR weighting on efficiency $r=0.05$ , $e^*=16.89\%$ , $e=56.93\%$ . . . . .	143

Figure 6.15:	Evaluation model LOS, n=8 (modes: 1-8), m=1; effect of LQR weighting on efficiency $r=0.05$ , $e^*=16.89\%$ , $e=56.93\%$ . . .	144
Figure 6.16:	Globally optimal system LOS, n=8 (modes: 1-8); effect of LQR weighting on efficiency, $q=100$ , $e^*=7.81\%$ , $e=56.93\%$ . . .	145
Figure 6.17:	Suboptimal system LOS, n=8 (modes: 1-8); effect of LQR weighting on efficiency, $q=100$ , $e^*=7.81\%$ , $e=56.93\%$ . . .	146
Figure 6.18:	Evaluation model LOS, n=8 (modes: 1-8); effect of LQR weighting on efficiency, $q=100$ , $e^*=7.81\%$ , $e=56.93\%$ . . .	147
Figure 6.19:	CSDL - ACOSS - 6 (model 2) structure . . . . .	148
Figure 6.20:	Globally optimal system, square x-y plane deflection at node 37, n=8, (modes: 12,13,17,21,22,24,28,30), example 6.3 . . . . .	149
Figure 6.21:	Suboptimal system, square x-y plane deflection at node 37, n=8, (modes: 12,13,17,21,22,24,28,30), m=4 (#18-21), example 6.3 . . . . .	150
Figure 6.22:	Globally optimal system, square x-y plane deflection at node 37, n=10, (modes: 7-16), example 6.4 . . . . .	151
Figure 6.23:	Suboptimal system, square x-y plane deflection at node 37, n=10, (modes: 7-16), m=10 (#1-10), example 6.4 . . . . .	152

LIST OF TABLES

Table		Page
Table 2.1:	Efficiencies for ACOSS-4 with $n=2$ . . . . .	32
Table 2.2:	Efficiencies for ACOSS-4 with $n=8$ . . . . .	32
Table 2.3	Efficiencies and Control Powers for ACOSS-4 with an 8th order model obtained via Efficient Model Reduction Approach . . .	32

## 1.0 INTRODUCTION

The purpose of this research is to study a nondimensional measure of structure-control system performance which has the potential to characterize both quantitatively and qualitatively the designer's ability in dealing with some of the problem areas such as assessment of control spillover effects, model/controller order reduction, input configuration, and the interaction between structural and control variables from the structure-control system point of view. The nondimensional measure is defined as the efficiency of the system.

The concept of efficiency is widely encountered in thermal and thermomechanical sciences. In these fields an efficiency is defined as a nondimensional ratio of two scalars which represent energy or energy related quantities. Typically, one of the scalars characterizes a theoretically ideal, but physically unrealizable process, and the other characterizes an actual physically realized process. The difference between these two scalars represents waste of the total available energy or energy related quantity in realizing the actual physical process and is regarded as an irreversibility inherent in the physical system. Many different forms of efficiency are defined in thermomechanical sciences depending on how the theoretically ideal and the physically realized actual processes are identified in a certain application. Some examples are propulsion efficiency, heat engine and heat pump efficiency, adiabatic compressor efficiency, brake efficiency, overall efficiency, etc. (Ref. 9). In thermomechanical disciplines an important objective is to design a system with high or maximum efficiency consistent with the physical constraints. In practical engineering terms, the thermomechanical systems which use (or convert) the largest fraction of the total available energy in realizing the physical function of that system, thus causing the minimum waste of resources, are desired.

The utility of efficiency concepts as analysis and design tools in thermomechanical systems is well established and provides valuable physical insight to the working of the system. The question arises whether such a time tested concept in thermomechanical discipline can be extended to the field of distributed parameter structure-control systems and yield comparable practical value and physical insight for the analysis and design of such systems. This investigation is a quest in that direction and presents a conceptual framework to establish the usefulness of efficiency concept for structure-control systems.

For structure-control systems, the efficiency concept is defined as the ratio of two control functionals pertaining to the particular structure-control design. The control functionals are judiciously defined to represent the average control powers consumed during the control period. For the purpose of defining efficiencies, four quadratic control power functionals are relevant. Out of these four control powers a relative model efficiency and a global efficiency can be defined for a structure-control system with maximum possible percent efficiency of 100 for each. The two efficiencies are related by a modal efficiency coefficient.

Relative model efficiency is defined as the ratio of a quadratic modal control power functional to a quadratic real control power functional associated with any control design model. Physically, relative model efficiency represents the fraction of real control power expended on the distributed-parameter system (DPS) usefully absorbed by the control design model. The relative model efficiency can be defined for control of the DPS by both spatially discontinuous (discrete) and spatially continuous input configurations. If the dimension of the control design model and the DPS system are identical then the relative model efficiency by definition becomes 100%. Consequently, the relative model

efficiency is predominantly an indicator of the effect of finite dimensionalization of the control design model in using the available control power and configuration.

On the other hand, global efficiency is defined as the ratio of the real control power of a globally optimal control based on a spatially continuous input profile to the real control power of achieving the same control objectives with a spatially discontinuous input profile. The definition is based on dynamically similar control systems (Ref. 1). Physically, the global efficiency is related to the degree of extra control power associated with a spatially discontinuous input configuration in controlling a DPS versus accomplishing the same task with a spatially continuous input profile and therefore, it should be a predominant indicator of the effectiveness of the input configuration to control DPS.

Modal efficiency coefficient is a proportionality constant between the global efficiency and the relative modal efficiency. Thus, with these quantities, the effect of finite dimensionalization of the control design model and the effect of nature of spatial discretization of the input profile in controlling a DPS can be studied.

The concept of efficiency is introduced to address particular issues of DPS control. A good physical understanding of the concept is best brought about by considering a DPS formulation. Therefore, the basic aspects of the concept will be presented from the perspective of a DPS formulation. On the other hand, in practice, for a complex structure only a spatially discrete formulation is explicitly available. Since such a spatially discrete formulation acts as a surrogate for DPS formulation, the transition of the definitions of efficiency to spatially-discrete structural dynamics is also presented. In particular, since spatially discrete models are almost always obtained via the Finite Element

method (FEM), specific interpretations of the concepts for the finite element models are given.

The premise of the concepts presented in this report is that a structure-control system must strive to achieve the highest possible efficiencies. In that, the control system should be designed such that as much of the real control power as possible should be channeled to the reduced-order control design model leaving little, or, of possible, no control power spillover for the truncated dynamics. This objective requires maximization of the relative model efficiency. Furthermore, any control design, characterized by a spatially discontinuous input profile, should try to approximate the performance of the globally optimal solution characterized by a dynamically similar spatially continuous input profile. This object requires maximization of the global efficiency. Based on the proposed concepts, an efficient model reduction approach can be proposed in that for a given input configuration, the components which contribute least to the efficiencies can be discarded.

The efficiencies will be shown to be functions of control parameters such as the number of control inputs and locations, method of control, and model order; as well as structural parameters such as natural frequencies and mode shapes. All relevant parameters can be altered to change the efficiency of hybrid structure-control system. An important aspect of the results is that the performance of the entire infinite-dimensional ( $\infty$ -D) DPS can be studied based on quantities computed from the finite-dimensional control design model alone without any knowledge of the truncated dynamics. The concepts introduced are valid irrespective of the theory or method by which the controls are designed. The efficiency concept can be used both for analysis and design of the structure-control system.

## 2.0 EFFICIENCY OF STRUCTURE-CONTROL SYSTEMS

### 2.1 Introduction

The concept of efficiency for structure-control systems is introduced in this chapter. The developments presented in this chapter constitute the central foundation and concepts around which this investigation revolves. A model efficiency and a global efficiency are defined for the structure-control system from the point of view of control power consumed during control. The efficiency concept is illustrated to be a useful tool in understanding the interaction between the control variables and structure variables and is shown to provide insight to the behavior of the structure-control system. This chapter and the illustrations of the theory presented herein focus on the analysis of a structure-control system. The illustrative examples evaluate performances of several linear quadratic regulator designs for the ACOSS-4 structure (Ref. 10).

In Section 2.2, control objectives for a distributed parameter system (DPS) are stated. Section 2.3 defines a global control power for the DPS and Section 2.4 presents the globally optimal control for the DPS. Suboptimal control of the DPS is discussed in Section 2.5. Section 2.6 generalizes the concepts of control power to spatially discrete systems, in particular finite element models (FEM) of structural systems are noted. The definition of efficiency of a structure-control system is introduced in Section 2.7. Also, presented in Section 2.7 is the methodology for the use of efficiency as an analysis and design tool. Finally illustrative examples are given in Section 2.8 for the control of ACOSS-4 tetrahedral truss structure.

### 2.2 Control Objectives for the DPS

We consider the DPS equation of motion

$$m(p)\ddot{u}(p,t) + \mathcal{L}[u(p,t)] = f(p,t) \quad (2.1)$$



where  $m$ ,  $u$ ,  $\mathcal{L}$ , and  $f$  are mass distribution, displacement field vector, stiffness operator matrix, and the input field vector, respectively.  $p$  denotes a position vector, which will be suppressed in the following for convenience. In general, (2.1) will represent the three-dimensional partial differential equations of motion. For convenience, we shall assume  $f > 0$ .

The eigenvalue problem of (2.1) is

$$\omega_r^2 m \phi_r = \mathcal{L} \phi_r \quad (2.2)$$

with the orthogonality relations

$$\int_{D(p)} \phi_r^T m \phi_s dD(p) = \delta_{rs} \quad , \quad \int_{D(p)} \phi_r^T \mathcal{L} \phi_s dD(p) = \omega_r^2 \delta_{rs} \quad , \quad r, s = 1, 2, \dots \quad (2.3)$$

where  $\omega_r$  is the natural frequency,  $\phi_r$  is the corresponding vector of eigenfunctions, and  $D(p)$  is the structural domain. Introducing the modal expansion

$$u = \sum_r \phi_r(p) \xi_r(t) \quad (2.4)$$

(2.1) is transformed to

$$\ddot{\xi}(t) + \omega_r^2 \xi_r(t) = f_r(t) \quad r = 1, 2, \dots \quad (2.5)$$

in the modal configuration space where  $\xi_r$  is a modal configuration coordinate and  $f_r(t)$  represents a modal input coordinate. Similar to modal expansion (2.4) of the displacement field vector, a modal expansion for the input field vector can be written

$$f(p, t) = \sum_r m(p) \phi_r(p) f_r(t) \quad (2.6)$$

From (3) and (6) the modal input coordinate  $f_r$  can be obtained as

$$f_r(t) = \int \phi_r^T(p) f(p, t) dD \quad (2.7)$$

The control objective on the  $\infty$ -D DPS is to insure proper allocation of  $n$  pairs of eigenvalues of a set of  $(n)$  modes from Equation (2.5). Another objective is to minimize control spillover effects so that the response of the control design will not be degraded by excessive control spillover. In improving stability characteristics of time-invariant linear systems, the premise of all control methods is to obtain desirable eigenvalue locations either directly or indirectly. The phrase "eigenvalue allocation" is used here in a general sense to address both direct and indirect means of allocation. As examples, a direct eigenvalue allocation technique is the Simon-Mitter Algorithm or any other technique in which desired eigenvalue locations are imposed as explicit constraints in obtaining the control gains; on the other hand, an indirect eigenvalue allocation technique is to use the linear quadratic regulator (LQR) theory by which eigenvalue positions are obtained indirectly as a by-product of optimizing a performance measure. Hence, the LQR approach is sometimes categorized as an optimal eigenvalue allocation technique. No implication is intended in this paper as to the use or necessity of direct eigenvalue allocation techniques in understanding and applying the proposed concepts of efficiency. In the sequel, the emphasis will be on the generic meaning of the phrase "eigenvalue allocation" as a qualifying phrase for the function of the control system which may have been designed by either a direct or indirect allocation whatever the case may be.

From the control objectives stated above, one would infer the ideal control to be the one by which  $(n)$  pairs of closed-loop eigenvalues are located as

desired with the minimum control power, and control spillover is eliminated completely. The solution for such an ideal control for the  $\infty$ -D DPS (2.1) will be stated in Sec. (2.4).

### 2.3 Global Control Power for the DPS

Whatever one's favorite control design may be to satisfy the control objectives, one can define and compute a global control evaluation functional for the  $\infty$ -D DPS in the form

$$S = \int_0^{\infty} \int_D m^{-1}(p) f^T(p, t) f(p, t) dD dt \quad (2.8)$$

(2.8) represents the total quadratic control power expended on the actual DPS. The control power (2.8) is dimensionally the average power consumed by the control design over the control period. Because (2.8) is the control power on the entire DPS, it is recognized as a global quantity. Since the global control power defined by (2.8) is computed from the physical input field vector, it will also be referred to as the real control power  $S^R$  where superscript R denotes "real," hence

$$S^R = \iint m^{-1}(p) f^T(p, t) f(p, t) dD dt \quad (2.9)$$

$S^R$  can be computed for any given  $f(p, t)$  regardless of the details of the control design technique by which it is computed. By a similar motivation, we define a global modal control power functional

$$S_{\infty}^M = \sum_{r=1}^{\infty} \int_0^{\infty} f_r^2(t) dt \quad (2.10)$$

where M denotes that the quadratic controls are the modal control input coordinates and the subscript  $\infty$  implies that all modes, hence the entire DPS, are considered in the computation. In contrast to  $S^R$ , which involves the real input field,  $S^M$  seems to represent an abstract quantity since modal inputs are used in its definition. However,  $S^R$  and  $S_{\infty}^M$  are related.

Substituting the modal expansion (2.7) into definition (2.9) and using the orthogonality relations one obtains

$$S^R = \iint m^{-1} \sum_{r=1}^{\infty} m \phi_r f_r \sum_{s=1}^{\infty} m \phi_s f_s dD dt = \sum_{r=1}^{\infty} \int_0^{\infty} f_r^2(t) dt \quad (2.11)$$

hence

$$S^R = S_{\infty}^M \quad (2.12)$$

Identity (2.12) defines an invariance property. The global control power for the DPS is frame indifferent; it is the same whether one studies it in the real-space of (2.9) or the modal-space of (2.10).

The global modal control power  $S_{\infty}^M$  can be decomposed into

$$S_{\infty}^M = \sum_{r=1}^n \int_0^{\infty} f_r^2(t) dt + \sum_{r=n+1}^{\infty} \int_0^{\infty} f_r^2(t) dt \quad (2.13)$$

$$S_{\sigma}^M = S_C^M + S_U^M \quad (2.14)$$

and from (2.12)

$$S^R = S_C^M + S_U^M \quad (2.15)$$

where the definitions of  $S_C^M$  and  $S_U^M$  should be evident from (2.13) and (2.14).  $S_C^M$  is the portion of the control power  $S^R$  channeled for control of the (n) modes and  $S_U^M$  is the remaining control power channeled into, or better said, spilled over to the uncontrolled modes. If the set of modes (n) are referred to as the control design model,  $S_C^M$  will be termed design model control power. Similarly,  $S_U^M$  will be referred to as control power spillover.

Since  $S^R$ ,  $S_U^M$ ,  $S_C^M$  are positive definite quantities:

$$S^R \geq S_C^M \quad (2.16)$$

where the equality is satisfied if and only if  $S_U^M = 0$ ; that is, when there is no control power spillover.

An important feature of the control of the  $\infty$ -D DPS is imbedded in inequality (2.16). Because  $S^R$  is computed by using the real input  $f(p,t)$  applied to the actual structure, and  $S_C^M$  is computed by using the modal inputs to the finite-dimensional control design model, inequality (2.16) relates how the control design model performance stands relative to the actual DPS. It is clear that any mismatch between  $S^R$  and  $S_C^M$  would automatically mean that some of the control power is lost to the residual modes,  $S_U^M \neq 0$ . On the other hand, in accordance with the control objectives, the ideal control system for the DPS would yield  $S_U^M = 0$ , that is, it would minimize the power spillover

It remains to address the specifics of how one might realize a minimum global control power to achieve the control objectives. To this end, we shall assume linear state-feedback control.

#### 2.4 Globally Optimal Control for the DPS

In the absence of other objectives and design constraints, it is reasonable to try to achieve the control objectives stated in Sec. (2.2) with the minimum amount of global control power. Hence one can state the optimization problem

Minimize  $S_o^M$ , or equivalently,  $S^R$

subject to:

$$\rho(\pm i\omega_r) \rightarrow \bar{\rho}(\bar{\alpha}_r \pm i\bar{\beta}_r) \quad r = 1, 2, \dots, \infty \quad (2.17)$$

This is an optimization problem for the  $\infty$ -D DPS. The uncontrolled eigenvalues  $\pm i\omega_r$  are relocated to specified positions  $\bar{\alpha}_r \pm i\bar{\beta}_r$  where  $\rho(\cdot)$  represents an eigenvalue spectrum. There are no restrictions on the constraint values  $\bar{\alpha}_r$  and  $\bar{\beta}_r$ . The case where only  $n$  pairs of eigenvalues are relocated to new positions and the remaining residual pairs are not moved, is a special case of the above formulation since we can always write

$$\rho(\pm i\omega_r) \rightarrow \bar{\rho}(\bar{\alpha}_r \pm i\bar{\beta}_r) \quad r = 1, 2, \dots, n \quad (2.18)$$

$$\rho(\pm i\omega_r) \rightarrow \bar{\rho}(\pm i\omega_r) \quad r = n+1, \dots, \infty$$

The solution of the minimum global control power is known to be (Refs. 1,2)

$$f_r^*(t) = (g_{r_1}^* \xi_r^*(t) + g_{r_2}^* \dot{\xi}_r^*(t)) \quad (2.19)$$

$$r = 1, 2, \dots, \infty$$

$$g_{r_1}^* = \omega_r^2 - (\bar{\alpha}_r^2 + \bar{\beta}_r^2), \quad g_{r_2}^* = 2\bar{\alpha}_r \quad (2.20)$$

where (\*) denotes the optimum quantities. Substituting the solution (2.19) into (2.6) one obtains the optimum input field vector which can achieve the desired eigenvalue locations with minimum control power (Ref. 3):

$$f^*(p, t) = \int (G_1^*(p, p') u(p', t) + G_2^*(p, p') \dot{u}(p', t)) dD(p') \quad (2.21)$$

where  $G_1^*$  and  $G_2^*$  are identified as symmetric optimum distributed control influence (Kernel) functions

$$G_1^*(p, p') = \sum_{r=1}^{\infty} g_{r_1}^* m(p) \phi_r(p) \phi_r(p') m(p') = G_1^*(p', p) \quad (2.22)$$

$$G_2^*(p, p') = \sum_{r=1}^{\infty} g_{r_2}^* m(p) \phi_r(p) \phi_r(p') m(p') = G_2^*(p', p)$$

We observe the following characteristics of the optimal control solution (2.21): the optimal modal input coordinate  $f_r^*$  is a feedback of only the corresponding  $r$ -th modal coordinate, therefore optimal modal control coordinates are independent of each other. This feature of feedback control has come to be known as independent modal-space control (Ref. 4). The corresponding optimal input

field  $f^*(p,t)$  is spatially continuous since the modal synthesis of spatially continuous functions  $m(p)\phi_r(p)$  is a spatially continuous function. Without proof, we also state that controlled DPS under the optimal spatially continuous feedback input has the same eigenfunctions as the uncontrolled DPS, preserving its natural properties (Refs. 1,2). Therefore, the optimal control (2.19-2.21) has also been referred to as Natural Control (Ref. 2).

Specifically, if the desired closed-loop eigenvalues are given as the set (2.18) from (2.19, 2.20) we compute  $g_{r1}^* = g_{r2}^* = 0$  for  $r=n+1, \dots, \infty$  which yields  $f_r^* = 0, r=n+1, n+2, \dots, \infty$ .

Upon substituting this result into (2.21) we get

$$F^*(p,t) = \int \{G_{1n}^*(p,p')u(p',t) + G_{2n}^*(p,p')\dot{u}(p',t)\} dD(p') \quad (2.23)$$

where  $G_{1n}^*$  and  $G_{2n}^*$  are the same as (2.22) except that the summation ends at  $n$ . The point is that the solution (2.23) does not represent a model truncation, instead the required summations end at  $n$  because the remaining terms have been computed to be zero.

The above procedure indicated that optimal control can be found with virtually no effort for any eigenvalue set for the  $\infty$ -D DPS. It remains to check the control spillover effect over the uncontrolled modes  $r=n+1, \dots$ . To this end, we substitute the form of  $f^*(p,t)$  given by (2.23) into (2.7) which yields upon recognizing the first orthogonality in (2.3),

$$f_s(t) = 0 \quad s = n+1, \dots, \infty$$

Perfect spillover elimination from residual modes is also achieved by the optimal control (2.23). We must point out that the spillover inputs  $f_s(t)$  vanish due to



appearance of  $\phi_r(p)$  in  $f(p,t)$  regardless of the functional form of the modal control inputs  $f_r(t)$ ,  $r=1,2,\dots,n$ ; that is whether modal inputs are independent or not. Therefore, spillover control is ultimately not a matter of what the temporal behavior of the control inputs is, but is a matter of spatial distribution of control. Any other spatial distribution of input would not yield perfect spillover elimination at least theoretically.

Last but not the least important feature of the optimal control is that the solution is unique (Refs. 1,2), therefore it is globally optimal, and it controls the  $\infty$ -D DPS accomplishing the control objectives ideally.

The global control power for the optimal control for the DPS can be evaluated by substituting (2.21) or (2.23) into (2.8)

$$S^* = \int m^{-1} f^{*T}(p,t) f^*(p,t) dD \quad (2.24)$$

which yields from (2.12)

$$S^R = S_{\omega}^M = S^* \quad (2.25)$$

## 2.5 Suboptimal Control for the DPS

By definition, any control input field  $f(p,t)$  of the form

$$f(p,t) \neq f^*(p,t)$$

will have a higher real control power than  $S^*$ . The most common suboptimal control is the one that seems most practical to implement; it is the point (or localized) input distribution

$$f(p,t) = \sum_{k=1}^m \delta(p-\Delta p_k) F_k(t) \quad (2.26)$$

where  $\delta$  is the spatial Dirac delta function  $\Delta p_k$  is the domain of the influence of the local k-th input and  $F_k(t)$  is the total input over  $\Delta p_k$ .

The real control power for the suboptimal control is again computed by using (2.26) in (2.9). Denoting the total input vector of m inputs by  $F=[F_1 F_2 \dots F_m]^T$ , it can be shown that (Ref. 5)

$$S^R = \int F(t)^T R F(t) dt \quad R = \text{diag}[m(p_k) \Delta p_k]^{-1} \quad (2.27)$$

and from identity (2.12) the total modal control power corresponding to suboptimal control profile (2.26) is

$$S_C^M = \int F^T R F dt$$

Furthermore, the design model control power for (n) pairs of relocated eigenvalues is

$$S_C^M = \sum_{r=1}^n \int f_r^2(t) dt = \int f_n^T(t) f_n(t) dt = \int F^T B_n^T B_n F dt \quad (2.28)$$

where  $f_n(t)$  is the n-component design model modal input vector generated by the control (2.26) and

$$f_n(t) = B_n F \quad B_{nrk} = [\phi_{rk}(p_k)] \quad r = 1, 2, \dots, n$$

$k=1, 2, \dots, m$   $\phi_{rk}(p_k)$  is the area under  $\phi_k$  over  $\bar{\Delta p}_k$ .

The control power spillover due to localized inputs F can be evaluated as

$$S_{ii}^M = \sum_{r=n+1}^{\infty} \int f_r^2(t) dt = \int f_v^T(t) f_v(t) dt = \int F^T B_v^T B_v F dt \quad (2.29)$$

where

$$f_u(t) = B_u F, \quad B_{u_{rk}} = [\phi_{rk}(p_k)] \quad r = n+1, \dots, k = 1, 2, \dots, m$$

$B_n$  and  $B_u$  are the respective partitions of the B matrix for the n-th order control design model and the uncontrolled dynamics.

One does not need to compute the control spillover performance according to (2.29) if  $S^R$  and  $S_C^M$  are already available. For, from (2.27) and (2.28)

$$S_u^M = S^R - S_C^M = \int F^T [R - B_n^T B_n] F dt \quad (2.30)$$

in which  $F(t)$  is directly available from the control design model once it is selected. Equation (2.30) indicates that for a given DPS (hence the mass distribution and therefore the matrix R are known) when a control design model is selected, the control spillover performance can be determined solely on the basis of the control design model. No knowledge of the uncontrolled dynamics is needed. This points out the usefulness of the judicious definition of the control evaluation functional S in the form of (2.9).

The control power spillover for the suboptimal point input profile (2.26) cannot vanish, therefore from (2.15) we deduce

$$S^R > S_C^M$$

and since  $S^R$  as given by (2.27) is suboptimal

$$S^R > S^*$$

## 2.6 Control Powers for Discrete-Systems

Quite often, instead of partial differential equations, a large set of spatially discretized ordinary differential equations are assumed to describe the

dynamics of the DPS adequately, such as the finite element models (FEM) of complex structural systems. Extension of the previous definitions and results to such cases will be useful. We assume that the structural system is described by

$$M\ddot{q}(t) + Kq(t) = Q(t) \quad (2.31)$$

instead of (2.1), where  $q(t)$  and  $Q(t)$  are each  $N$ -component generalized coordinates and the generalized forces vectors, respectively.  $M$  and  $K$  are  $N \times N$  symmetric, positive definite mass and stiffness matrices, respectively. Equation (2.31) is usually referred to as the  $N$ -dimensional evaluation model replacing the  $\infty$ -D DPS, (2.1).

Denoting by  $E$  the modal matrix associated with the system (2.31), the modal transformations and the orthogonality relations

$$q = E\xi \quad , \quad f(t) = E^T Q(t) \quad E^T M E = I \quad , \quad E^T K E = [\omega^2]_N \quad (2.32)$$

again yield the modal equations of motion (2.5) with the exception that this time  $r=1,2,\dots, N$  represents the complete system. Again, by definition the total modal control power is

$$S_N^M = \sum_{r=1}^N \int f_r^2(t) dt = \int f^T(t) f(t) dt \quad (2.33)$$

where  $N$  denotes the total system as opposed to  $\infty$  in (2.13) for the DPS Equation (2.1). The corresponding real control power, after recognizing the invariance property (2.12) is

$$S^R = S_N^M = \int f^T f dt = \int Q^T E E^T Q dt \quad (2.34)$$

Noting from the orthogonality relations that  $EE^T = M^{-1}$ , for a discrete-system the equivalent of  $S^R$  in (2.9) is

$$S^R = \int Q^T M^{-1} Q dt \quad (2.35)$$

Hence, the generalized input vector  $Q(t)$  plays the role of  $f(p,t)$ .

In particular, if (2.31) represents the FEM equations of motion, the generalized loads vector  $Q(t)$  is the vector of joint loadings. If  $F(t) = [F_1 F_2 \dots F_N]^T$  is the real joint inputs vector of  $f(p,t)$  one can write

$$Q = DF \quad (2.36)$$

where  $D$  is the joint loads distribution matrix. If  $f(p,t)$  is spatially continuous over the whole structural domain, then by necessity, it will yield an input at each joint along every joint degree of freedom. It follows that the equivalent of a spatially continuous input  $f(p,t)$  in a FEM setting is tantamount to having a full generalized loads vector  $Q$ . On the other hand, if the input  $f(p,t)$  is spatially discontinuous, such as in (2.26), that will be tantamount to having a  $Q$  with some zero components. If there are  $m$  independent joint inputs  $F_1, \dots, F_m$  as in (2.26) substitution of (2.36) into (2.34) yields

$$S^R = \int F(t)^T R F(t) dt \quad R = D^T M^{-1} D \quad (2.37)$$

Hence, the weighting matrix  $R$  in (2.37) is the FEM equivalent of the weighting matrix  $R$  in (2.27) for the partial differential equations of motion.

It is clear from the form of  $R$  that, given any  $F(t)$ ,  $S^R$  describes the control power for the entire evaluation model. Even if  $F(t)$  may have been designed by considering only a reduced  $\{n\}$ th order modal model of the  $N$ -th order system (2.31), when  $S^R$  is computed, it will be, according to (2.37), the global

control power for the total N-th order evaluation model, not for the n-th order reduced-control design model.

For an n-th order control design model, it is easy to see that the counterparts of (2.13) and (2.27)-(2.29) are

$$S_N^M = S_C^M + S_U^M \quad (2.38)$$

where

$$S_C^M = \int F^T D^T E_n E_n^T D F dt, \quad S_U^M = \int F^T D^T E_u E_u^T D F dt \quad (2.39)$$

$$S_U^M = S^R - S_C^M = \int F^T D^T [M^{-1} - E_n E_n^T] D F dt \quad (2.40)$$

and  $E_n$  and  $E_u$  are the control design model and the uncontrolled model modal matrices, respectively. Here again, from (2.40), for a given physical system (hence the evaluation model mass matrix  $M$  is known) and the n-th order control design model, control spillover performance can be ascertained solely on the basis of the control design model. Specifically, if a FEM is used, the modes that are poorly computed will be inconsequential from the control point of view as long as those modes are in the uncontrolled set.

Finally, it remains to ascertain the counterparts of the globally optimal control and the control power (2.23)-(2.25) for the system of (2.31). From the previous discussions and the nature of the globally optimal control, it is straight forward to obtain

$$S^* = \int Q^* T M^{-1} Q^* dt = \int F^{*T} D^* T M^{-1} D^* F^* dt \quad (2.41)$$

$$Q^* = M E f^*(t) = D^* F^* \quad , \quad F^* = D^{*-1} M E f^*(t) \quad (2.42)$$

where  $f^*(t)$  is the vector of  $N$ -independent modal inputs  $f_r^*$   $r=1,2,\dots,N$  precisely as computed according to Equations (2.19), (2.20). For a FEM,  $Q^*$  is the full generalized inputs vector tantamount to having  $N$  joint inputs  $F_r^*$ ,  $r=1,2,\dots,N$ , that is as many inputs as the total number of degrees of freedom. In other words,  $D^*$  for the globally optimal control must be a full rank  $N$  joint loads distribution matrix.

The forms of  $Q^*$  and its interpretation makes it clear that a FEM not having inputs at each joint along each joint degree of freedom would correspond to a discontinuous input profile and have a suboptimal performance.

## 2.7 Efficiency of a Structure-Control System

Implementable control designs for large flexible structures such as complex truss-like configurations planned for the space station will inevitably employ spatially discontinuous suboptimal input profiles consisting of a large number of distributed point force and torque actuators. In view of the control objectives and the features of the control powers we discussed heretofore, it would be desirable for any implementable structural control system to channel as much of the real control power as possible to the control design model. In other words, the power spillover  $S_{ij}^M$  should be minimized by the control design. An equally desirable feature of the control design would be to keep the total control power as small as possible, that is, to keep the real control power of the design as near to the globally optimal control power as possible. These aspects, by necessity, bring about the concept of efficient structure-control designs. An important element of the structure-control design process must be

to find the most efficient structure-control combination for the control objectives.

We define the percent global efficiency of a structure-control system as

$$e^* \% = \frac{S^*}{S^R} \times 100 \leq 100\% \quad (2.43)$$

where  $S^R$  is the real control power of any suboptimal control design with the closed-loop eigenvalue spectrum  $\{\rho\} = \{\rho\}$ .  $S^*$  is the globally optimal control power corresponding to the same eigenvalues (cost  $\bar{J}_0$ ). This is to say that the global efficiency is based on the comparison of dynamically similar (Ref. 1) globally optimal and suboptimal control designs for the desired closed-loop eigenvalues. Since  $S^R$  is suboptimal, the upper bound of global efficiency is 100%.

Next, we define the percent relative model efficiency of a structure-control system to be

$$e \% = \frac{S_c^M}{S^R} \times 100 \leq 100\% \quad (2.44)$$

The relative model efficiency is an indicator of the percentage of the real control power channeled into the control design model, the balance indicating the control power spillover. This efficiency is determined solely by using the properties of the particular control design model. There is no reference to the corresponding globally minimum control. Therefore, we refer to  $e$  as the relative model efficiency. A less than perfect model efficiency automatically implies control power wasted to uncontrolled modes. However, as a 100%  $e$  means no



control spillover, it will not guarantee a 100% global efficiency as  $S^R$  and  $S^*$  may still be different.

The relative model efficiency and the global efficiency are related by

$$e = \mu e^* \quad \mu = \frac{S_c^M}{S^*} \quad (2.45)$$

where  $\mu$  is defined to be the modal efficiency coefficient.

Complimentary to the above definitions, one can also introduce the global and the model spillover quotients

$$sq^* = \frac{S_U^M}{S^*} = \frac{(S^R - S_c^M)}{S^*} = \frac{1}{e^*} - \mu, \quad sq = \frac{S_U^M}{S^R} = 1 - e < 1 \quad (2.46)$$

$sq$  indicates the portion of the real control power lost as control spillover cost.  $sq^*$  indicates the control power spillover of the suboptimal design as a fraction of the globally minimum control power that would be expended on the entire DPS. Studies show that the control power spillover a suboptimal control profile can incur, can be many times more than it would take to control the entire system with a spatially continuous optimal input.

Given an initial modal state disturbance  $x_0$  for a stable control system,

$$x_0 = [\xi_1(0) \dot{\xi}_1(0) \dots \xi_n(0) \dot{\xi}_n(0)]$$

The control power for infinite time control are given by

$$S^R = x_0^T P^R x_0, \quad S_c^M = x_0^T P_c^M x_0, \quad S^* = x_0^T P^* x_0 \quad (2.47)$$

where  $P^R$ ,  $P^M$  and  $P^*$  are the real, modal, and the globally optimal (natural) control power matrices.  $P^R$  and  $P^M$  can be obtained as the solutions of the associated Lyapunov equations for any suboptimal control design discussed in the preceding sections. The relevant equations are presented in Chapter 3. The natural control power matrix  $P^*$  is given in closed form in Ref. (6) and in Appendix B.

The global and relative model efficiencies of any control design can now be computed by using the cost matrices

$$e^* = \frac{x_0^T P^* x_0}{x_0^T P^R x_0}, \quad e = \frac{x_0^T P_C^M x_0}{x_0^T P^R x_0}, \quad \mu = \frac{x_0^T P_C^M x_0}{x_0^T P^* x_0} \quad (2.48)$$

Each one of the efficiencies, through the power matrices, depends on:

- \* The number, type and locations of localized inputs  $F_k(t)$
- \* The particular control design technique used to compute the actual spatially discontinuous feedback input  $F$
- \* The order  $n$  of the control design model and the closed-loop eigenvalue spectrum  $\{\bar{\rho}\}$
- \* Structural parameters through the appearance of modal frequencies and mode shapes
- \* The initial modal disturbance state  $x_0$ .

For the analysis and design of structural-control systems via efficiencies, one would typically take the following steps: 1) For any set of selected system variables and parameters mentioned above (such as a given  $n$ -th order design model and a control input configuration) obtain a control law by whatever technique or theory deemed appropriate. 2) For the control inputs obtained in step 1, compute

the corresponding  $S^R$  and  $S_C^M$  defined by Eqs. (2.27), (2.28) or Eqs. (2.37) and (2.39), and calculate the relative model efficiency  $e$  and the model spillover quotient  $sq$  as defined by Eqs. (2.44) and (2.46), respectively. 3) For different values and/or sets of variables and parameters repeat steps 1 and 2, compare the corresponding model efficiencies, simulate if necessary, and identify satisfactory designs consistent with the designer's criteria and constraints. In applying steps 1-3, one should recognize that there is no need for explicit knowledge of the closed-loop eigenvalues if studies based on relative model efficiencies are all that is desired. However, in addition, if global efficiencies  $e^*$  and global spillover quotients  $sq^*$  are also desired for further consideration, one must then proceed with the following steps: 4) For the controls designed in step 1, compute the corresponding closed-loop eigenvalues  $(\bar{\rho})$  if they are not already available. Otherwise, this step is not needed. 5) For the spectrum  $(\bar{\rho})$  found in step 4, compute the modal control gains and the modal inputs given by Eqs. (2.19) and (2.20) of the globally optimal spatially continuous control which is dynamically similar to the control design of step 1. 6) In accordance with Eqs. (2.25), (2.11) and (2.12) obtain the globally minimum real control power  $S^*$  possible for the eigenvalue spectrum  $(\bar{\rho})$  elicited by the control design of step 1. Closed-form solution for  $S^*$  is given in Appendix B for any defined  $(\bar{\rho})$ . 7) Calculate the global efficiency  $e^*$  and global spillover quotient  $sq^*$  of the control design of step 1, as defined by Eqs. (2.43) and (2.46) by using  $S^R$  from step 2 and  $S^*$  from step 6. If desired, compute the model efficiency coefficient defined in Eq. (2.45). 8) For different values and/or sets of variables and parameters repeat steps 4-7, compare the corresponding global efficiencies and spillover quotients, simulate if necessary, and identify a satisfactory designs. 9) Study the results of steps 3-8 collectively to evaluate the control designs.

The efficiency approach to structure control will liberate the engineer from the need of detailed knowledge of the unmodeled modes. Because the behavior of an infinite dimensional system can be studied and understood by means of computing its efficiencies, which require explicit knowledge of only the finite number of modeled control modes. This feature should make the efficiency approach to control design a practical tool.

With such an approach, it is possible to determine the optimal control input distribution and even the optimum eigenvalue distribution for a given  $n$ -order control design model. For a given input field, the efficiencies can be used to determine the order  $n$  of a control design where model orders that yield high efficiencies can be selected. In addition, because the efficiencies are dependent on the input distribution, closed-loop eigenvalues, and other structural and control parameters, different order control models could become more efficient simply by changing the structure-control system's configuration and parameters. In all of these, the objective then should be to maximize the global and relative efficiencies.

In particular, for  $N$ -th order discrete evaluation models,  $S_n^M$  represents the control power consumed in controlling  $n < N$  modes, while  $S^R$  represents the power consumed by all  $N$  modes. Therefore, the relative efficiency  $e$  becomes a valid nondimensional measure of the effects of model order reduction. A similar statement holds true for global efficiency. Based on these observations, a closed-loop Efficient Model Reduction Technique can be formulated in that one can propose to retain in the control design model the modes to which the relative model and/or global efficiencies are most sensitive for any given input configuration.

In contrast to design, efficiencies can be used to evaluate the merits of a given control design since they reflect the effects of many variables of the control problem. In this chapter, we shall illustrate the analyses of some Linear Quadratic Regulator (LQR) control designs based on the efficiency concept. Also, the proposed concept of Efficient Model Reduction Technique will be demonstrated.

## 2.8 Illustrative Example: Control of ACOSS-4 Tetrahedral Truss Structure Analysis via Efficiencies

As a demonstration of the utility of the efficiency concept, the performances of various LQR control designs (step 1) for the ACOSS-4 structure shown in Fig. 2.1 were evaluated for different order modal control design models and different number of inputs (steps 3 and 8). The inputs were located at the pods of the structure. A twelfth order ( $N=12$ ) evaluation model (2.31), obtained via FEM, was considered. For a given  $n$ -th order control design and number of inputs  $1 \leq m \leq 6$ , the control designs were based on the minimization of the LQR performance measure (step 1)

$$J = \frac{1}{2} \int_0^{\infty} (x^T W_x x + F^T W_c F) dt$$

$$W_x = \text{block-diag} \begin{bmatrix} 1 & 0 \\ 0 & \omega_r^{-2} \end{bmatrix} \quad , \quad W_c = [1]$$

where the LQR control design weighting matrix  $W_c$  has no relationship to the weighting matrix  $R$  uniquely defined for efficiency purposes. The LQR design approach essentially is an indirect eigenvalue allocation. Instead of requiring

explicit eigenvalue allocation one can implicitly admit the desired eigenvalues ( $\bar{\rho}$ ) to be those of the LQR solutions for specific choices of weighting parameters  $q$  and  $r$ . For each LQR steady-state Riccati equation solution, the closed-loop eigenvalues were computed and assigned to be the set  $\{\bar{\rho}\}$  (step 4) where upon the corresponding globally optimal control power  $S^*$  was computed (step 6) for the set  $\{\bar{\rho}\}$ . For each LQR solution,  $S^R$  and  $S_C^M$  were computed by solving the associated Lyapunov equations for the closed-loop system (step 2). In the simulations, the  $2n$ -th order initial modal state  $x_0$  was assumed to be  $x_0 = [1 \ \omega_1 \ 1 \ \omega_2 \ \dots \ 1 \ \omega_n]$  and the uncontrolled modes were initially undisturbed.

For control design models of order  $n=2, 4, 6, 8, 10, 12$  and the input numbers  $m=1, 2, 3, 4, 5, 6$  (steps 3 and 8);  $e^*$ ,  $e$ ,  $\mu$ ,  $sq^*$ , and  $sq$  for the chosen LQR weighting matrices,  $W_x$  and  $W_c$ , were computed (steps 2 and 7). The results are shown in Figs. 2.2 (steps 3 and 8). The model selections were made by starting from the lowest structural modes to the higher ones. For brevity, efficiencies and spillover quotients are tabulated only for  $n=2$  and  $n=8$  (Tables 2.1 and 2.2). Efficiencies of other design models and actuator configurations can be inferred from the efficiency curves.

From Figs. 2.2a and b, we observe the interactions among the efficiencies, the order of the control design model, and the input configuration. For a given number of inputs the model efficiency increases with the order of the control design model. However, this is not necessarily true for the global efficiency. For one and two inputs the global efficiency seems to increase with model order, but for three inputs increasing the model order beyond  $n=4$  decreases the global efficiency. We also observe that for a given control design model, increasing the number of inputs increases the global efficiency, but this is not necessarily true for the model efficiency. Indeed for a sixth order design model ( $n=6$ ),

three or more inputs cause a decrease in the model efficiency. Therefore, the third, fourth, fifth, and sixth actuators are located poorly with respect to the truncated modes ( $n > 7-12$ ) such that more of control energy is lost as control power spillover to cause a drop in the model efficiency. A similar observation is made for 1 and 2 inputs in regards to model efficiency. Hence, it appears that the first 3 actuators represent a critical number of inputs for this particular structure.

The curves of  $sq^*$  and  $sq$  would describe more vividly the effect of model truncation. However, because  $sq^*$  and  $sq$  are related to  $e^*$ ,  $\mu$  and  $e$ , for brevity these curves are not shown. Tables 2.1 and 2.2 list some values of the spillover quotients. From the  $sq^*$  values given in Table 2.1 one reads, for example, that for  $m=1$  and  $n=2$  the amount of control power lost to model truncation is 152 times the total control power that would be required to control the entire DPS with a spatially continuous optimal input profile (natural control). The control powers  $S^R$ ,  $S_C^M$  for the LQR designs and the corresponding dynamically similar natural control powers  $S^*$  for  $n=2, 8$  are shown in Figs. 2.3a and b. The power plots show that natural control power  $S^*$  can be significantly lower than the real power  $S^R$  obtained by using LQR designed controls. The distances among the power curves are indicators of the global and model efficiencies and the modal efficiency coefficient.

The LQR values of  $S^R$  decrease monotonically to limit values with increasing number of inputs. The dynamically similar natural control powers  $S^*$  increase monotonically to different limit values. We conjecture that the two designs will not converge because of fundamental differences in their design concepts. The natural control is a distributed partial differential equation control solution according to Equations (2.1), (2.21) and (2.22). On the other hand, the LQR

solution is a discrete control solution based on the a priori reduced-order (truncated) model of the dynamic system. In order for the two optimal solutions to converge to different limits, their conceptual framework must be inherently different. This suggests that other than LQR discrete closed-loop laws can be formulated as direct approximations to the closed-loop distributed natural control solution with control powers between the LQR and natural control powers (Ref. 3).

The response profiles for a sensor collocated with the first input are given in Fig. 2.4 for different design model orders and inputs. The response profiles corresponding to the natural (globally optimal) control with continuously distributed input, suboptimal control of the  $n$ -th order control design model with  $m$  point inputs and the evaluation model, which includes the control spillover effects of the suboptimal control, are superposed in Figs. 2.4a, b and c for comparison purposes. It is seen that almost identical responses can be obtained with drastically different control powers. The similarity between the responses of the (suboptimal) control design model and the evaluation model for  $n=8$ ,  $m=2$  in Fig 2.4b may suggest that model truncation is insignificant. This is true from an output viewpoint. However, there still exists a considerable inefficiency in the control design due to 43.2% control power wasted ( $s_q=43.2\%$  in Table 2.2 for  $n=8$ ,  $m=2$ ) to truncated modes from an input viewpoint. This inefficiency can hardly be ignored. One would also want the control design to be efficient in its control power, therefore assessment of spillover effects based on response alone without considering the control powers would be premature.

Fig. 2.4c shows the response of the evaluation model both for natural control with continuously distributed inputs and LQR control with  $m=6$  point inputs. Both responses are identical. Because there is no mode truncation in



the evaluation model the relative model efficiency is 100%. On the other hand, the global efficiency is about 32% reflecting the fact that the LQR solution with 6 point inputs uses about 3 times more control power than it one were to use a spatially continuous input profile designed for independent control of all 12 modes. In spite of the more control power, the LQR solution cannot produce a response better than that of natural control. In the LQR solution with 6 point inputs for 12 modes intermodal coupling of the controlled responses is inevitable. In this case, it is this coupling of the controlled modal responses that causes excessive use of control effort without producing an improvement in the controlled response over that of natural control. This truly reflects the inefficiency of the control design model.

#### Efficient Model Reduction

An efficient model reduction concept would truncate the modes to which the model or global efficiency is least sensitive. We shall demonstrate the model reduction technique based on the model efficiency. A similar procedure can be based on the global efficiency. However, for brevity we do not demonstrate this alternate approach.

We use the model efficiency curves in Fig. 2.2b to find an eighth order reduced model with two and four point inputs. For two inputs ( $m=2$ ) from Fig. 2.2b, we note that the smallest increments in the model efficiency are caused by Modes 1, 2, 7 and 8. Hence we retain Modes 3-6, 9-12 as the control design model for the given two inputs. Similarly, for the four input configuration ( $m=4$ ) from Fig. 2.2b, we note that Modes 1, 2, 11 and 12 have the least contributions to the model efficiency. Hence, we truncate these modes and retain Modes 3-10. The efficiencies and control powers of the new eighth order control design models for

two and four inputs are shown in Table 2.3. A comparison of these results to the control design model, which was based on the lowest eight structural modes (Table 2.2), shows that the new control design models have significantly better efficiencies and the effect of model truncation for the new reduced modes are insignificant. For example, for ( $m=2$ ) although the natural control power  $S^*$  has increased from 8 to 24 due to the mode selection based on model efficiency, the total actual control power  $S^R$  remained almost the same (158 vs. 162), but the modal control power  $S_C^M$  rose from 90 to 157 which indicates that the new control design model absorbs almost all of the actual control effort yielding a 97% model efficiency. The response profiles for the new eighth order control design models are shown in Figs. 2.5a and b. Again, in these figures responses of the corresponding natural control, suboptimal control, and evaluation model are superposed. They are hardly different from each other, the responses of the suboptimal control and the evaluation model had almost undetectable overshoots at the peaks in comparison to natural control. Therefore, the curves were not labeled and only the response of the evaluation models are shown in Figs. 2.5a and b. Among all responses natural control always achieved lower amplitudes than the others. Finally, one can now compare the responses of the 8th order reduced order models with  $m=2, 4$  to the response of the 12th order evaluation model in Fig. 2.4c with  $m=6$  inputs.

Table 2.1: Efficiencies for ACOSS-4 with n=2

m	e**	e%	$\mu$	sq*	sq%
1	0.647792	1.337612	2.06488	152.3056	98.66239
2	1.271105	1.271092	0.99999	77.67170	58.72891
3	1.818071	1.859323	1.02269	53.98065	98.10468
4	2.971592	2.961611	0.99664	32.65535	97.03839
5	3.606529	3.392659	0.99615	26.73133	96.40734
6	4.642842	4.642761	0.99998	20.53855	95.35720

Table 2.2: Efficiencies for ACOSS-4 with n=8

m	e**	e%	$\mu$	sq*	sq%
1	2.70735	56.92516	21.0262	15.91033	43.07480
2	5.05710	56.82256	11.2362	8.53799	43.17744
3	19.45213	63.55967	3.2675	1.87333	36.44030
4	29.19059	69.40089	2.3775	1.04825	30.59911
5	39.41595	69.00026	1.7506	0.78648	30.99740
6	47.56904	72.06676	1.5150	0.58721	27.93324

Table 2.3: Efficiencies and Control costs for ACOSS-4 with an 8th order model obtained via Efficient Model Reduction Approach

Control Design Mode s		m	e**	e%	$\mu$	sq*	sq%	S <sup>R</sup>	S <sub>C</sub> <sup>M</sup>	S <sub>U</sub> <sup>M</sup>	s*
3-6, 9-12		2	14.7	96.55	6.57	0.24	3.45	162.07	156.47	5.6	23.82
3-10		4	44.7	93.93	2.10	0.14	6.07	60.67	56.99	3.68	27.09

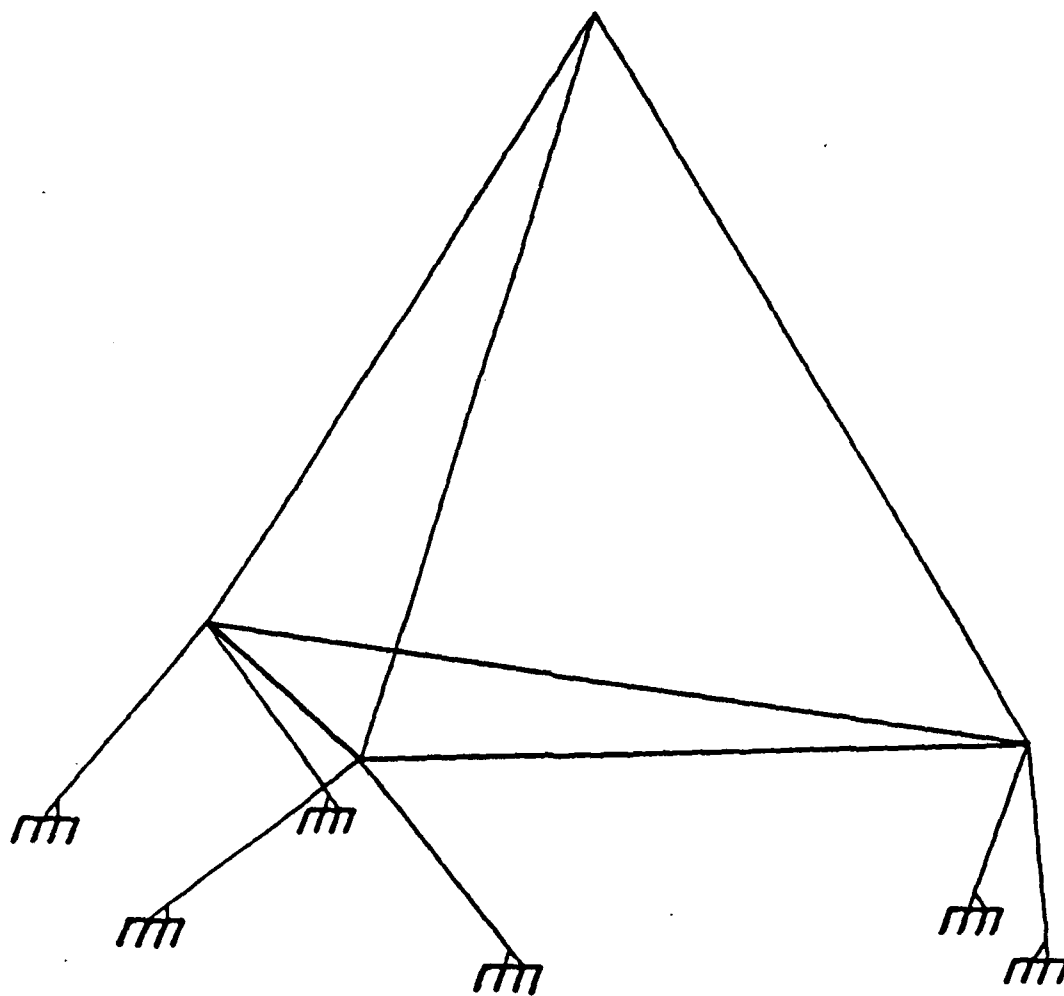


Figure 2.1: ACROSS-4 Tetrahedral Structure

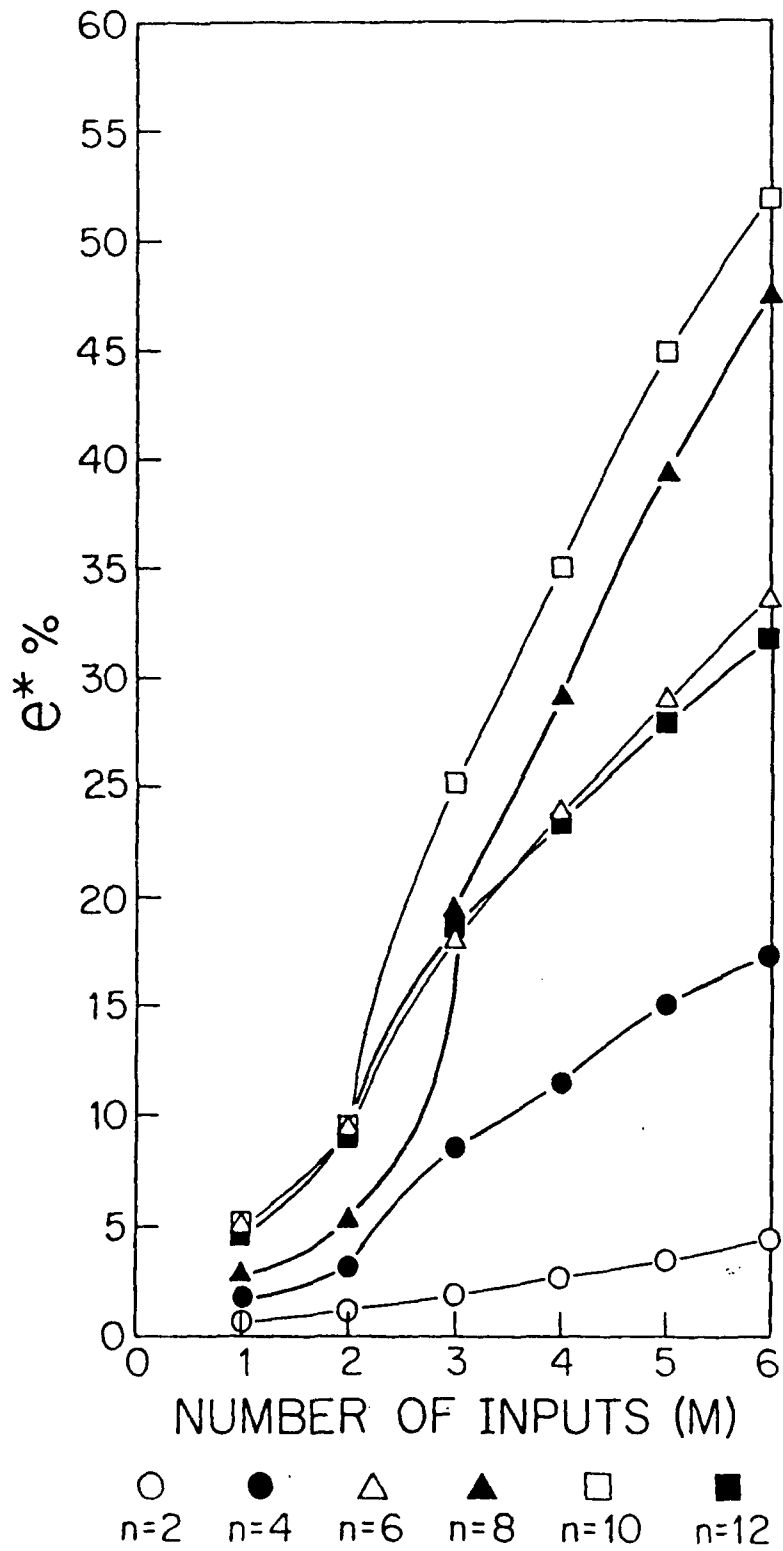


Figure 2.2a: Percent Global efficiencies for ACROSS-4.

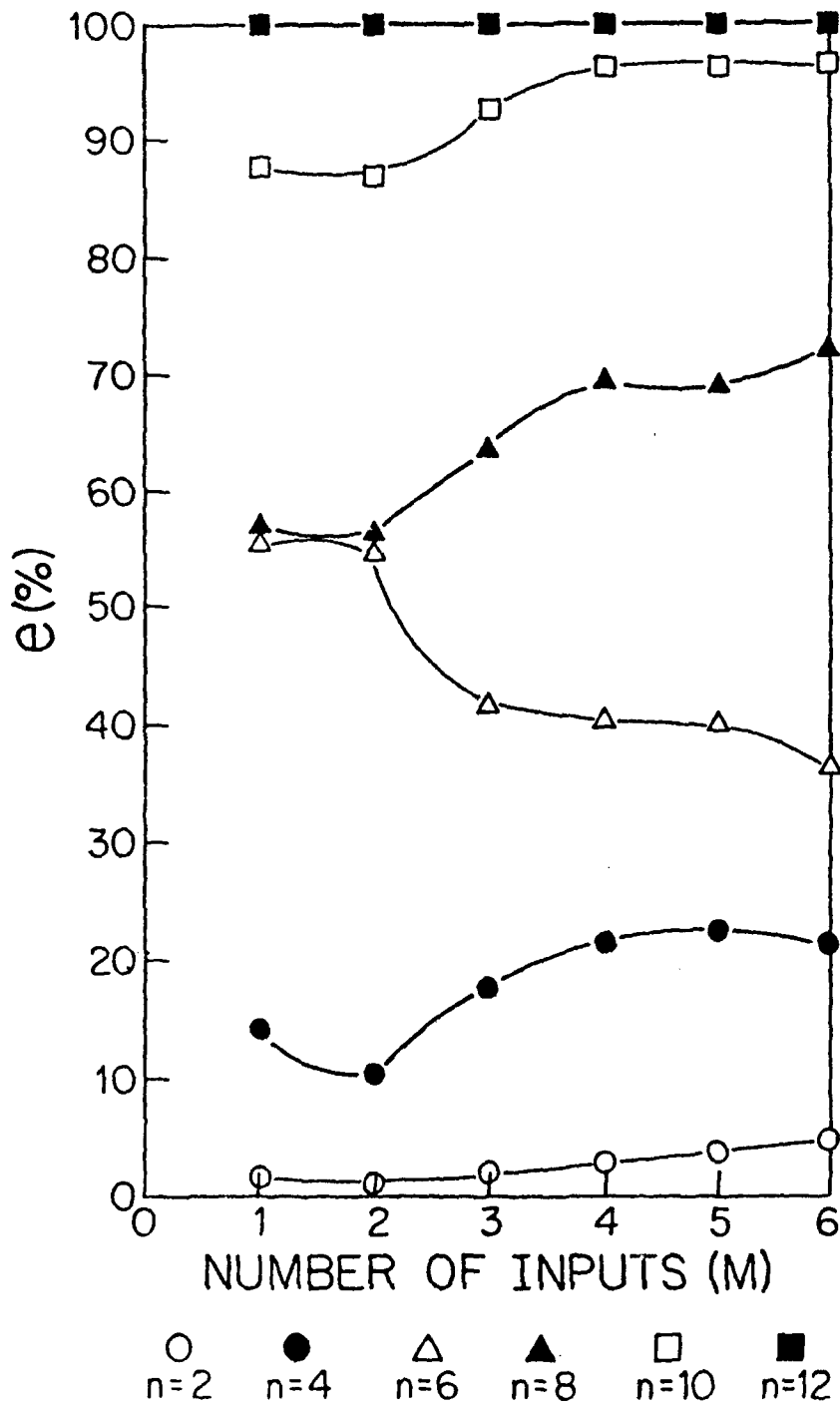


Figure 2.2b: Percent Relative Model efficiencies for ACROSS-4

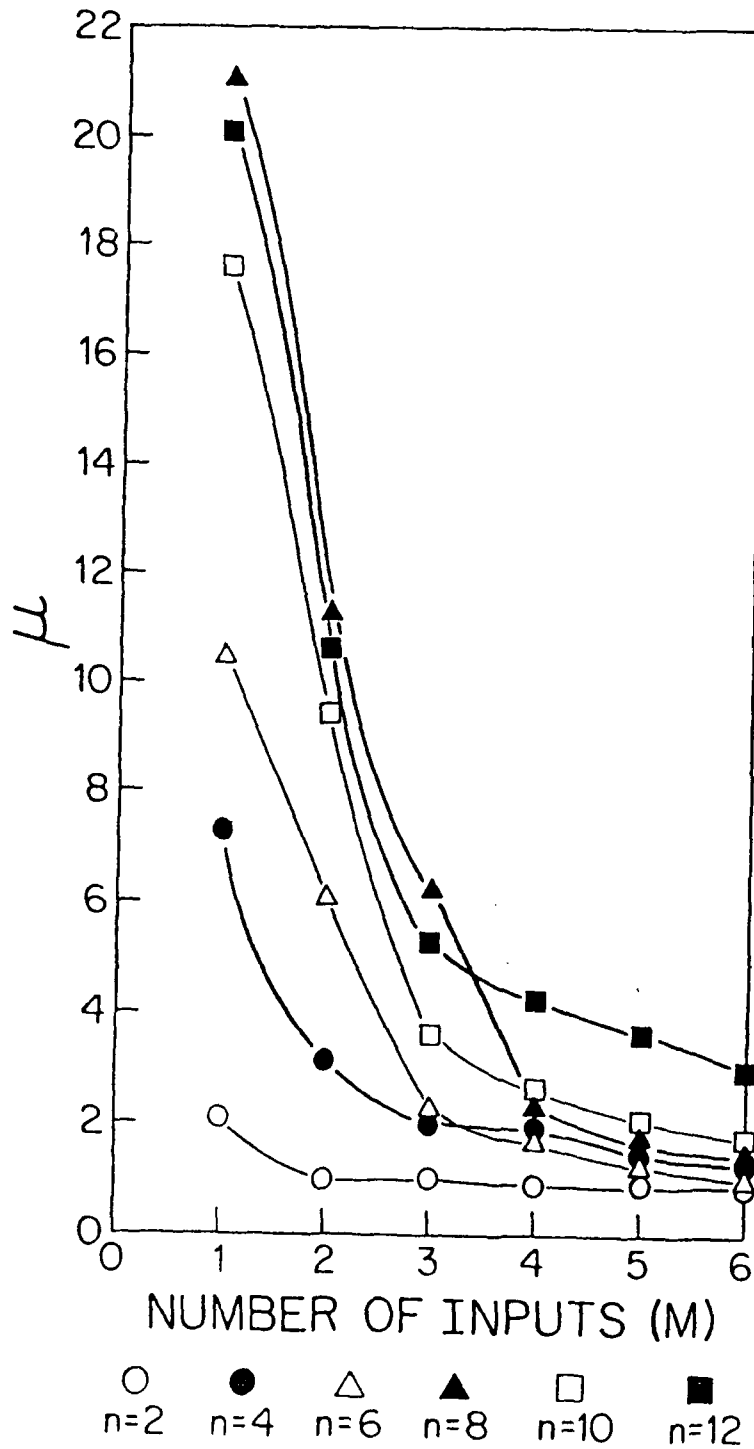


Figure 2.2c: Model efficiency coefficients for ACOSS-4

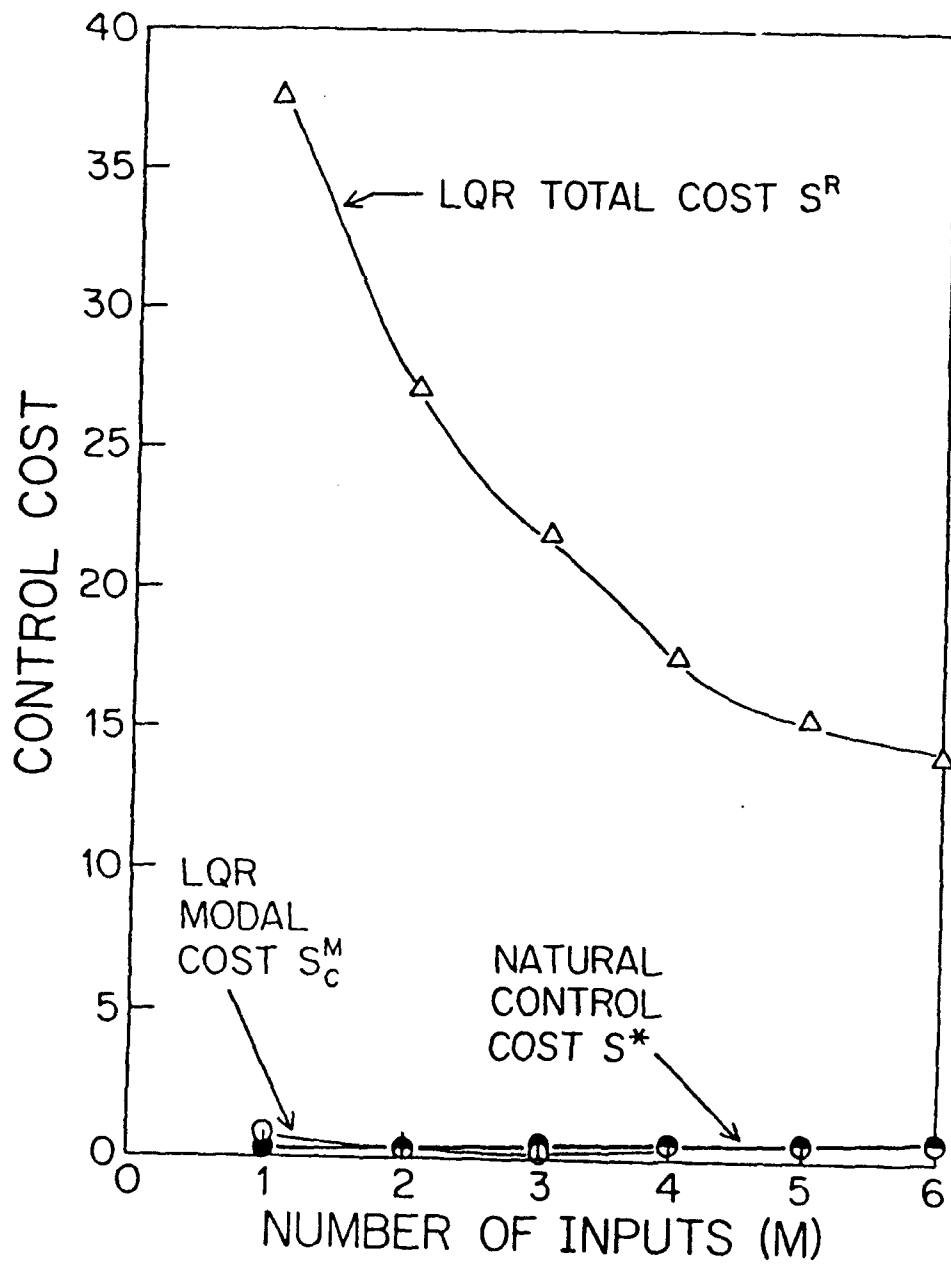


Figure 2.3.a: Control Powers for n=2



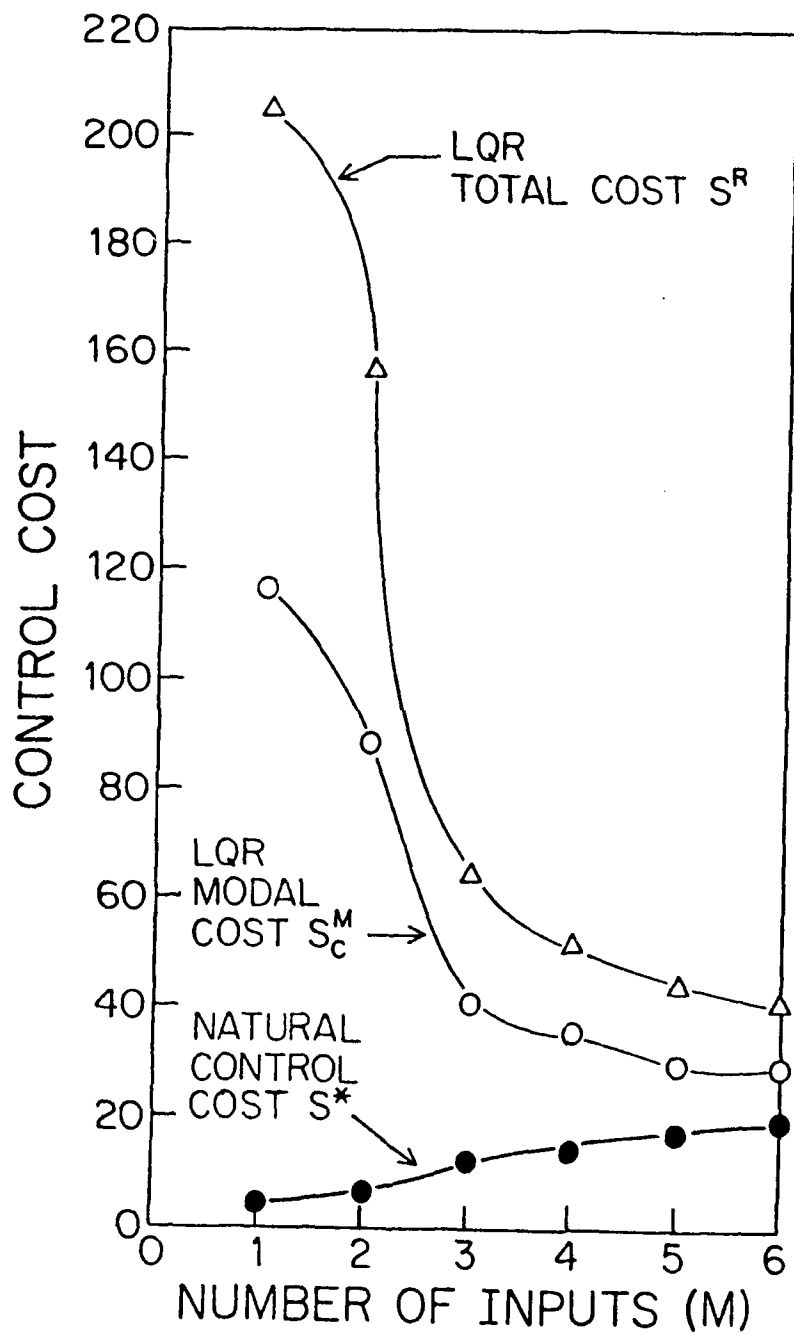


Figure 2.3b: Control Powers for n=8

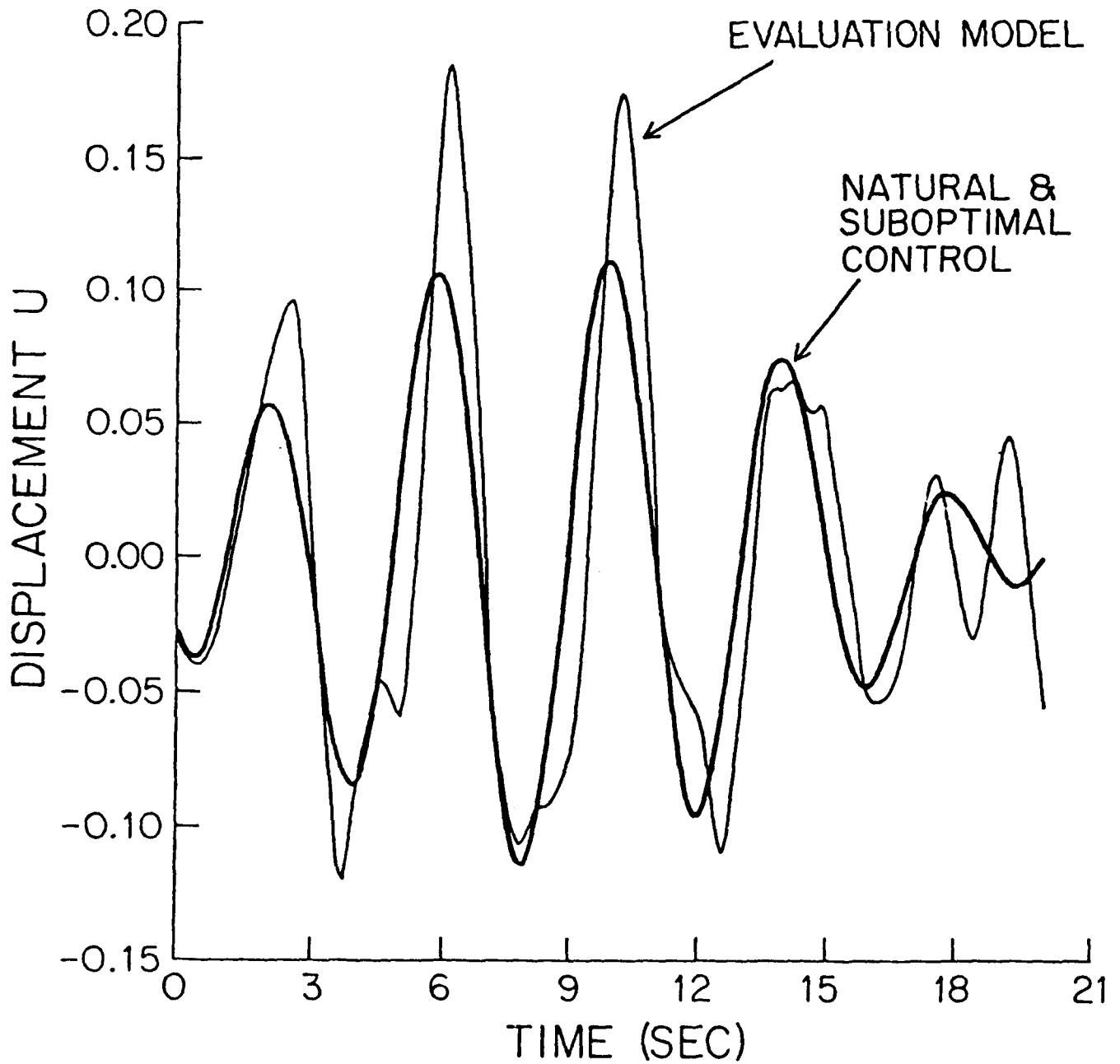


Figure 2.4a: Responses for  $n=2$  of natural control (continuous input), suboptimal control and evaluation model with  $m=2$ ,  $e^*=-1.27\%$ ,  $e=-1.27\%$   $\mu=1$ ,  $sq^*=77.67\%$ ,  $sq=98.73\%$

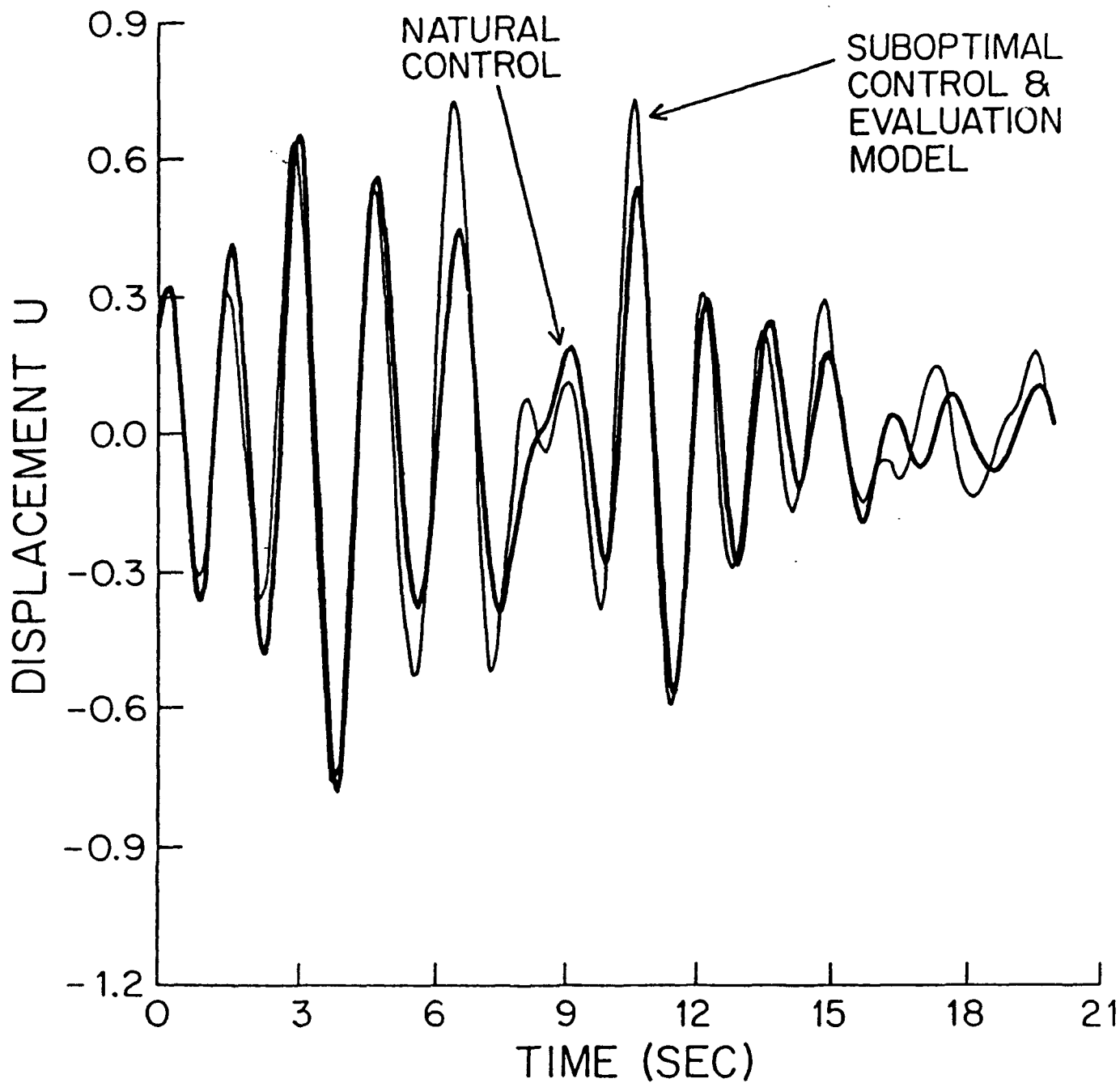


Figure 2.4b: Responses for  $n=8$  of natural control (continuous input), and suboptimal control and evaluation model with  $m=2$ ,  $e^*=-5.06\%$ ,  $e=-56.8\%$   $\mu=11.2$ ,  $sq^*=-8.3\%$ ,  $sq=43.2\%$

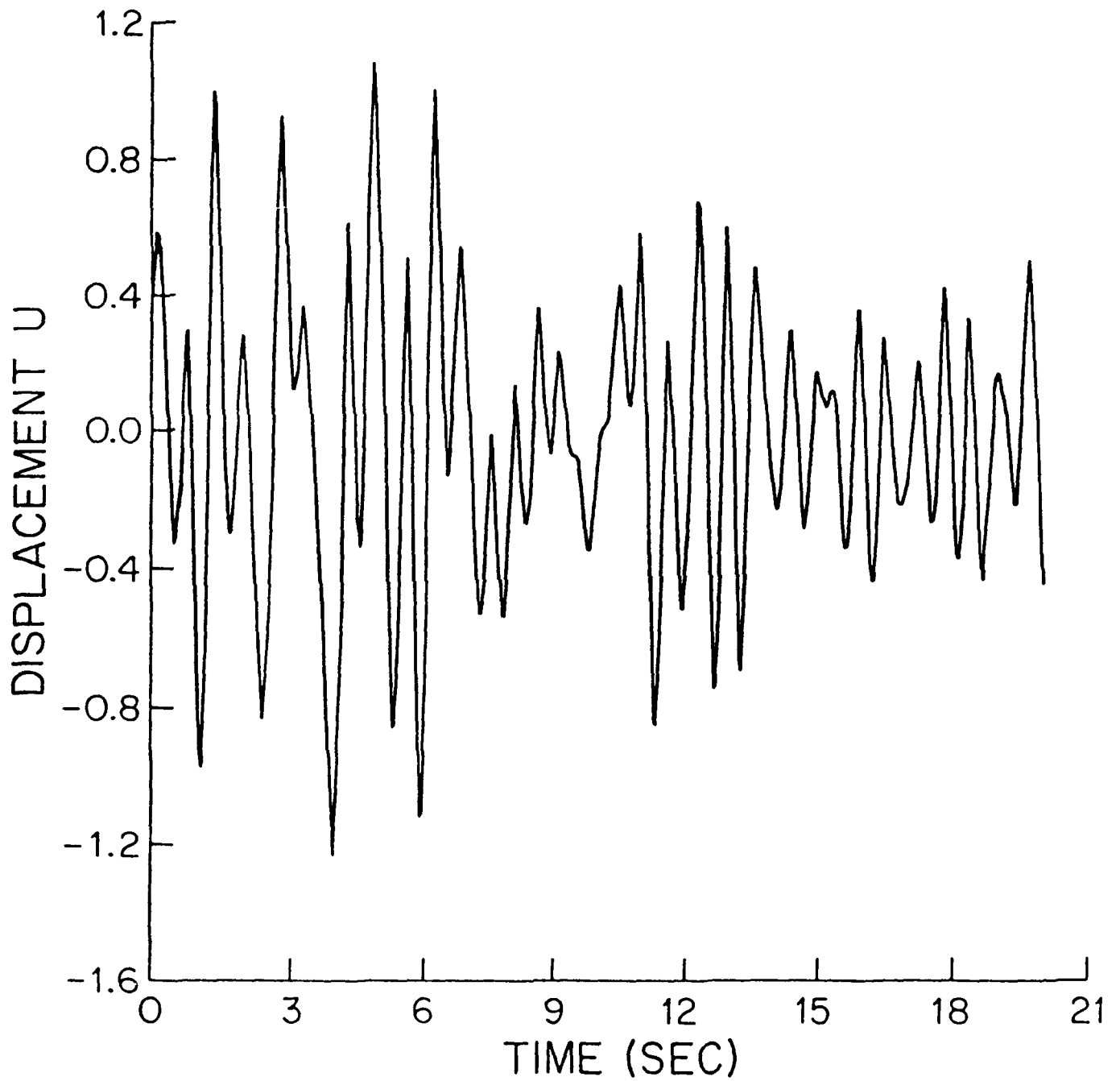


Figure 2.4c: Responses for  $n=12$  of natural control (continuous input) and suboptimal control with  $m=6$ ,  $e^*=31.8\%$ ,  $e=100\%$   $\mu=3.13$ ,  $sq=sq^*=0$

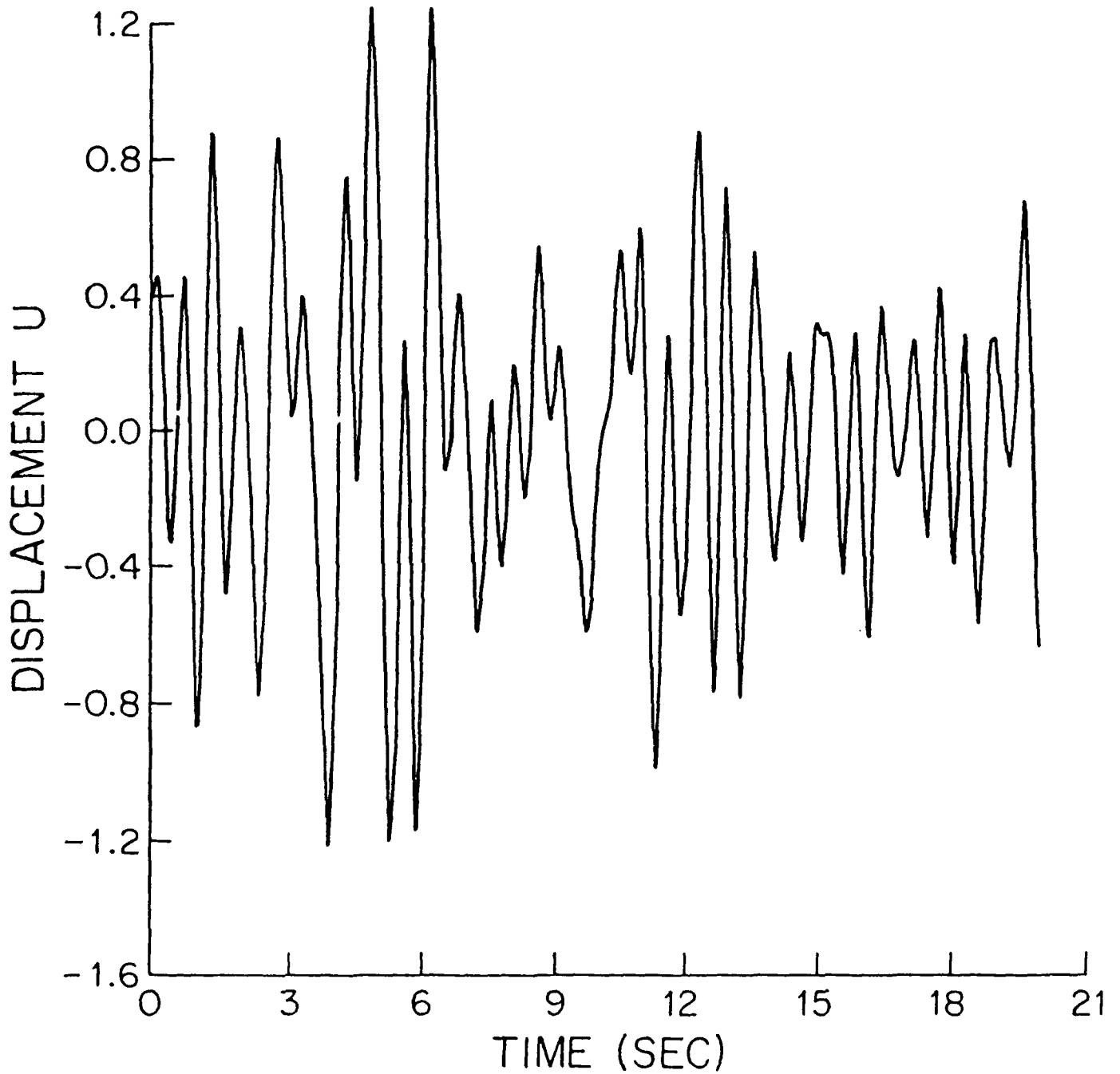


Figure 2.5a: Response of evaluation model with efficient model reduction applied for  $n=8$ ,  $m=2$ ,  $e^*=14.7\%$ ,  $e=96.58\%$ ,  $\mu=6.57$ ,  $sq^*=-0.24$ ,  $sq=3.45\%$

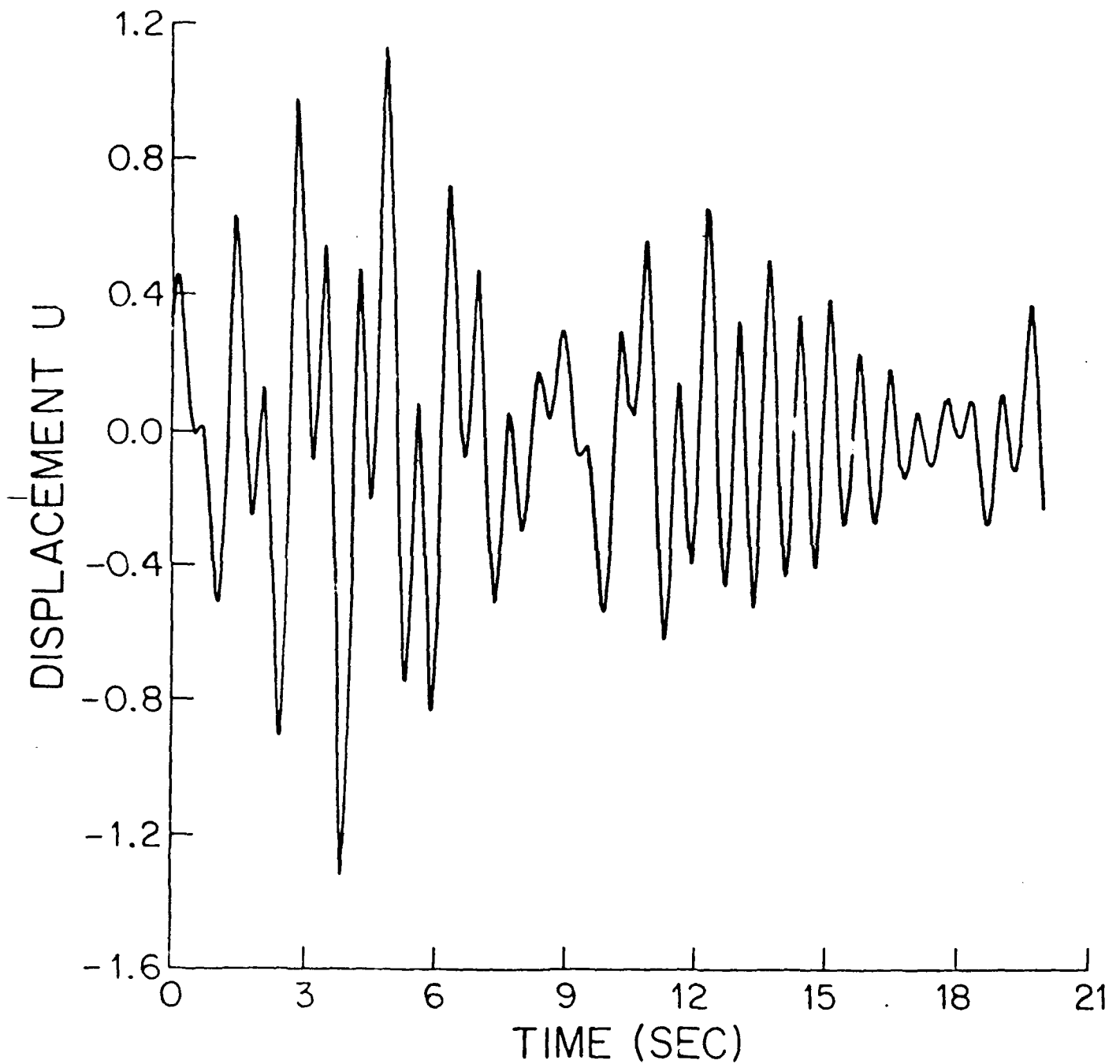


Figure 2.5b: Response of evaluation model with efficient model reduction applied for  $n=8$ ,  $m=4$ ,  $e^*=44.7\%$ ,  $e=93.93\%$ ,  $\mu=2.1$ ,  $sq^*=0.14$ ,  $sq=6.07\%$

### 3.0 EFFICIENCY MODES ANALYSIS OF STRUCTURE CONTROL SYSTEMS

#### 3.1 Introduction

In this chapter, we delve into the structure of the efficiency quotients to gain further insight to the structure-control problem. The developments of this chapter center around the fact that the efficiency quotients presented in Chapter 2 are the ratios of quadratic functionals of positive definite symmetric control power functions. Therefore, each one of the quantities  $e$ ,  $sq$ ,  $e^*$  and  $sq^*$  represents a Rayleigh's quotient. It is warranted, then, to exploit the features of a Rayleigh's quotient to perform an efficiency modal analysis for the structure-control system and study its implications.

In Section 3.2, we discuss the computation of power matrices for efficiency quotients. Section 3.3 is central to this chapter. The efficiency eigenvalue problem is studied in this section, a number of spectral properties of the control power matrices are discovered, and an efficiency state-space transformation is defined. In Section 3.4, the definitions of "controller modes" and "Principal Controller directions" are given. In addition, an efficiency ellipsoid is identified, the surface of which represents all initial disturbances which result in a given efficiency of the system. Section 3.5 introduces the concept of principal efficiency components and discusses its role to quantify the structure of a link between the controller design and initial conditions. Illustrative examples, again using the ACOSS-4 tetrahedral structure studies in Chapter 2, are given in Section 3.6.

#### 3.2 Computation of Control Power Matrices for Efficiency Quotients

Consider the  $N$ -th order evaluation model of the structural dynamic system given by Eq. (2.31)

$$M\ddot{q} + Kq = Q \quad (3.1)$$

where  $q$  and  $Q$  are the generalized coordinates and generalized input vectors, respectively. Noting the transformation from a physical control inputs vector  $F$  to the generalized inputs  $Q$  as

$$Q = DF$$

and the modal properties given by Eqs. (2.32) for the system (3.1), Eq. (3.1) can be replaced by the modal system

$$\ddot{\xi}_N + [\omega^2]_N \xi_N = B_N F \quad , \quad B_N = E_N^T D \quad (3.2)$$

where  $\xi$  is the  $N$ -dimensional evaluation model displacements vector of Eq. (3.1). Typically, we shall assume that Eqs. (3.1) represents a FEM evaluation model of the structural system. Introducing the  $2n$ -dimensional modal states vector for a control design model,  $2n < 2N$ :

$$x = [\xi_1 \dot{\xi}_1 \xi_2 \dot{\xi}_2 \dots \xi_n \dot{\xi}_n]^T \quad (3.3)$$

we have the modal-state representation of Eqs (3.1) and (3.2) for any smaller order control design model

$$\dot{x} = Ax + BF \quad (3.4)$$

$$A = \text{block-diag} \begin{bmatrix} 0 & 1 \\ -\omega_r^2 & 0 \end{bmatrix}, \quad B = [0 \ b_1 \ 0 \ b_2 \ 0]^T \quad r=1,2,\dots,n < N \quad (3.5)$$

where  $b_r$  is the  $r$ -th row vector of  $B_N$ . The control powers  $S^R$ ,  $S_C^M$  and  $S^*$  given by Eqs. ((2.37)-(2.42)), restated here for ready reference,

$$S^R = \int F^T R^R F dt \quad , \quad R^R = D^T M^{-1} D \quad (3.6)$$

$$S_C^M = \int F^T R_C^M F dt \quad , \quad R_C^M = D^T E_n E_n^T D \quad (3.7)$$

$$S^* = \int F^{*T} R^* F^* dt \quad , \quad R^* = D^{*T} M^{-1} D^* \quad (3.8)$$

become, for a stable closed-loop system,

$$S^R = x_0^T P^R x_0 \quad , \quad S_C^M = x_0^T P_C^M x_0 \quad , \quad S^* = x_0^T P^* x_0 \quad (3.9)$$

for which, the state feedback control input is:



$$F = Gx \quad , \quad F^* = G^*x \quad (3.10)$$

The gain matrix  $G$  is the control gain matrix available from any control design method. The gain matrix  $G^*$  is the gain matrix of the globally optimal solution given by Eqs. (2.42), (2.19) and (2.20) as discussed in Section 2.6.

The control power matrices  $P^R$  and  $P_C^M$  are obtained by solving the Lyapunov Equations

$$A_{CL}^T P^R + P^R A_{CL} + G^T R^R G = 0 \quad (3.11)$$

$$A_{CL}^T P^M + P^M A_{CL} + G^T R_C^M G = 0 \quad (3.12)$$

$$A_{CL} = A + BG \quad (3.13)$$

and a closed-form solution for  $P^*$  exists, as given in Appendix B.

Noting Eq. (2.38) and the forms ((3.9), (3.11) and (3.12)), we also have for the truncated modes

$$S_U^M = x_0^T (P^R - P_C^M) x_0 = x_0^T P_U^M x_0 \quad (3.14)$$

Hence, the control power wasted on the uncontrolled dynamics is readily available following solutions of Eqs. (3.11) and (3.12) which are based on the controlled dynamics alone, therefore, no knowledge of truncated dynamics is required.

It must be noted that the weighting matrices  $R$  in Eqs. (3.6)-(3.8) and, hence, the control powers, are uniquely defined for computation of structural-control system efficiency. Having obtained the power matrices, one can now compute  $e$ ,  $e^*$ ,  $sq$  and  $sq^*$  associated with any control design according to the definitions (2.43)-(2.46).

### 3.3 Efficiency Eigenvalue Problem

From the definitions of  $e$ ,  $sq$ ,  $e^*$  and  $sq^*$ , one notes that they are non-dimensional quotients involving positive definite numerator and denominator power matrices and the initial state disturbance  $x_0$ . Therefore, an efficiency quotient

is also a Rayleigh's quotient, and an efficiency eigenvalue problem can be defined to analyze the structure-control system.

In the following, we only need to define a numerator and a denominator power matrix without necessarily referring to  $e$ ,  $e^*sq^*$  and  $sq$  specifically and use  $e$  as a generic symbol to represent any one of them. This should not present any difficulty as the context will make it clear whether we are dealing with a specific or a general quotient. Hence, consider the general form of an efficiency quotient

$$e = \frac{x_0^T P_N x_0}{x_0^T P_D x_0} \quad P_N, P_D > 0 \quad (3.15)$$

where subscripts N and D denote numerator and denominator power matrices. For a particular structure control system design both  $P_N$  and  $P_D$  are readily available as given in Section 3.2.

The efficiency of the given structure-control design system is then dependent on the nature of initial state disturbance  $x_0$ . To see how the initial state interacts with the particular system, we address the eigenvalue problem associated with the efficiency quotient (3.15).

Introducing the transformation

$$x = P_D^{-1/2} z \quad (3.16)$$

an efficiency quotient (3.15) becomes

$$e = \frac{z_0^T E z_0}{z_0^T z_0} \quad , \quad E = P_D^{-1/2} P_N P_D^{-1/2} \quad (3.17)$$

where  $E$  is the nondimensional symmetric positive definite efficiency matrix.

Next, we consider the standard eigenvalue problem associated with the matrix  $E$

$$E v_r = \lambda_r v_r \quad r=1, 2, \dots, 2n \quad (3.18)$$

where  $v_r$  is a real eigenvector with positive real eigenvalue  $\lambda_r$ , and form the modal matrix  $V$  of  $E$

$$V = [v_1 \ v_2 \ \dots \ v_{2n}] \quad (3.19)$$

The modal matrix  $V$  is normalized to satisfy

$$V^T V = I \quad , \quad V^T E V = \Lambda \quad (3.20)$$

where

$$\Lambda = \text{diag} [\lambda_1 \ \lambda_2 \ \dots \ \lambda_{2n}] \quad (3.21)$$

and  $I$  is a  $2n \times 2n$  identity matrix.

Introducing a second transformation

$$z = V \epsilon \quad (3.22)$$

substitute it into Eq. (3.17) and use the normality conditions (3.20) to obtain the quotient  $e$  in the form

$$e = \frac{\epsilon_0^T \Lambda \epsilon_0}{\epsilon_0^T \epsilon_0} \quad (3.23)$$

We shall refer to the new states  $\epsilon$  in Eq. (3.23) as the efficiency states. Combining Eqs (3.16) and (3.22) we have the transformation

$$x = T \epsilon \quad , \quad T = P_D^{-1} V \quad (3.24)$$

In terms of the efficiency states, the quotient  $e$  (3.23) can be expanded in the form

$$e = \sum_{i=1}^{2n} c_i^2 \lambda_i \quad , \quad c_i^2 = \frac{\epsilon_{0i}^2}{\epsilon_0^T \epsilon_0} \quad , \quad 0 \leq c_i \leq 1 \quad (3.25)$$

in which we shall refer to  $c_i^2$  and  $\lambda_i$  as the  $i$ -th efficiency coefficient and the  $i$ -th characteristic efficiency, respectively.

Next, introducing the inverse transformation of (3.24)

$$\epsilon = L x \quad L = T^{-1} \quad (3.26)$$

into Eq. (3.23) and considering the original form of efficiency quotient in terms of modal states  $x$ , Eq. (3.15), we obtain the following relationships

$$P_N = L^T \Lambda L \quad , \quad P_D = L^T L \quad (3.27)$$

The modal-state dynamics, Eq. (3.4) can also be represented in terms of the efficiency states by using the transformation (3.24)

$$\dot{\epsilon} = A^e \epsilon + B^e F \quad (3.28)$$

$$A^e = L A T \quad , \quad B^e = L B \quad (3.29)$$

The feedback control law for  $F$  in Eq. (3.10) becomes

$$F = G^e \epsilon \quad , \quad G^e = G T \quad (3.30)$$

In terms of  $\epsilon$ -states the closed-loop system becomes

$$\dot{\epsilon} = A_{CL}^e \epsilon \quad , \quad A_{CL}^e = A^e + B^e G^e = L A_{CL} T \quad (3.31)$$

where  $A_{CL}$  is given by Eq. (3.13).

By using Eqs. (3.27) and (3.31) in Eqs. (3.11) and (3.12), the Lyapunov Equations associated with the  $\epsilon$ -state space representation are obtained as:

$$A_{CL}^{eT} \Lambda + \Lambda A_{CL}^e + G^{eT} R_C^M G^e = 0 \quad (3.32)$$

$$A_{CL}^{eT} I + \Lambda I_{CL}^e + G^{eT} R^R G^e = 0 \quad (3.33)$$

Hence,  $\Lambda$  is recognized as the numerator power matrix and the identity matrix  $I$  is the denominator power matrix associated with the definition of an efficiency quotient in the  $\epsilon$ -state space. That is

$$e = \frac{x_0^T P_N x_0}{x_0^T P_D x_0} = \frac{\epsilon_0^T T^T P_N T \epsilon_0}{\epsilon_0^T T^T P_D T \epsilon_0} = \frac{\epsilon_0^T \Lambda \epsilon_0}{\epsilon_0^T \epsilon_0} \quad (3.34)$$

From Eq. (3.34) or inverting Eq. (3.27) we obtain the orthonormality relationships

$$T^T P_N T = \Lambda \quad , \quad T^T P_D T = I \quad (3.35)$$

Thus, the transformation  $T$  diagonalizes both the numerator and denominator power matrices simultaneously and matrix  $T$  is orthonormal with respect to the real

power control matrix  $P_D$ . This completes the eigenvalue problem of the efficiency quotient (3.15).

### 3.4 Efficiency Modes and Principal Controller Directions of a Structure-Control System

In Section (3.2) we showed that the transformation matrix  $T$  defined by Eq. (3.24) can be used to transform from the modal state space ( $x$ -space) to a new efficiency state-space ( $\epsilon$ -space) to describe the system dynamics by Eq. (3.28). Furthermore, just as the structural modal matrix  $E_N$  diagonalizes the structural mass and stiffness matrices according to Eq. (2.32), the matrix  $T$  diagonalizes similarly both of the control power matrices  $P_N$  and  $P_D$  according to Eq. (3.35). By analogy, we shall refer to the  $T$  matrix as the efficiency modal matrix, and to its columns  $t_r$  as the efficiency modes of the control system.

$$T = [t_1 \ t_2 \ \dots \ t_{2n}] \quad (3.35)$$

Next, recognizing the efficiency coordinates  $c_i$  in Eq. (3.25) as the normalized efficiency coordinates associated with an initial disturbance  $\epsilon_0$  on the efficiency state

$$c_i = \epsilon_{i0}/|\epsilon_0| \quad (3.36)$$

and introducing

$$\epsilon_i = L_i^T x \quad (3.37)$$

where  $L_i$  is the  $i$ -th row vector of  $L=T^{-1}$ , the efficiency Equation (3.25) can be written in the form

$$\sum_{i=1}^{2n} c_i^2 \frac{\lambda_i}{e} = 1 \quad , \quad c_i = \frac{L_i^T x_0}{|\epsilon_0|} \quad (3.38)$$

Equation (3.38) represents the equation of an ellipsoid in the  $2n$ -dimensional space with principal axes of length  $\sqrt{e/\lambda_i}$  ( $i=1, \dots, 2n$ ) and  $c_i$  is the coor-

dinate along the  $i$ -th principal axis of all initial disturbances which yield a specific efficiency  $e$ . Furthermore, the direction of the  $i$ -th principal axis is given by the  $i$ -th eigenvector  $V_i$  of the efficiency matrix  $E$  where  $V_i$ 's are orthonormal in the usual sense as given by (3.20) associated with the eigenvalue problem of Eq. (3.18). With respect to the efficiency eigenvalue problem associated with the original modal coordinates  $x$ , Eq. (3.15), the  $i$ -th principal axis of the ellipsoid (3.38) is given by the  $i$ -th column  $t_i$  of the transformation matrix  $T$ , Eq. (3.24) which is the modal matrix of quotient (3.15). We note that the directions  $t_i$  are orthonormal not in the usual sense but with respect to the denominator control power matrix  $P_D$ , given in Eq. (3.35).

Since  $0 < e \leq 1$  (excluding the case when  $e = \text{sq}^*$ ), different initial conditions  $x_0$  yielding different efficiencies simply will result in rescaling of the lengths of the principal axes of the ellipsoid, largest length in the  $i$ -th direction given by  $\sqrt{1/\lambda_i}$ . The largest and the smallest possible length of the axes are then  $\sqrt{1/\lambda_{\min}}$  and  $\sqrt{1/\lambda_{\max}}$ , respectively.

We shall refer to the ellipsoid (3.38) as the efficiency ellipsoid, alternately, recognize the efficiency modal vectors  $t_i$  as the principal controller directions. We note that the efficiency modes, characteristic efficiencies and the shape of the efficiency ellipsoid are completely and only defined by the particular control system design and the structural system design embedded in the matrices  $G$ ,  $A$  and  $B$  in Eqs. (3.4) and (3.10). Initial disturbances  $x_0$  then determine the size of the efficiency ellipsoid acting merely as a scaling factor to yield a specific efficiency for the structure-control system.

Because a quotient  $e$  in the form of Eq. (3.15) represents a Rayleigh's quotient and since  $0 < e \leq 1$ , we observe the following properties for efficiencies:

- a) The characteristic efficiencies  $\lambda_i$  are bounded by  $0 \leq \lambda_i \leq 1$  for  $e$ ,  $e^*$  and  $sq$ , for the quotient  $sq^* \lambda_i \geq 0$ .
- b) Efficiencies have stationery values at  $\lambda_i$  for initial disturbances  $x_0 = t_r \quad r=1,2,\dots,2n$ . Specifically, the minimum efficiency a structure-control system can achieve is  $e_{\min} = \lambda_{\min}$ , and will occur if  $x_0 = t_{\min}$ , that is if the initial disturbance coincides with the eigenvector (controller efficiency mode) associated with  $\lambda_{\min}$ , that is if the  $x_0$  is completely aligned with the direction of the principal axis along  $t_{\min}$ . We shall refer to  $\lambda_{\min}$  as the fundamental efficiency. Similarly, the upper bound of the efficiency of the system is given by  $e_{\max} = \lambda_{\max}$ , corresponding to  $x_0 = t_{\max}$ .

In particular, if the quotient (3.15) is the relative model spillover quotient

$$sq\% = 100 * \frac{S_U^M}{S^R} = 100 * \frac{x_0^T P_U^M x_0}{x_0^T P^R x_0} \quad (3.39)$$

it then represents the percent inefficiency (fraction of control power wasted on residual dynamics) of the control system. Noting that

$$P^R = P_C^M + P_U^M \quad (3.40)$$

and denoting the eigenvalues of the relative efficiency quotient

$$e = \frac{x_0^T P_C^M x_0}{x_0^T P^R x_0} \quad (3.41)$$

by  $\lambda^e$  and the eigenvalues of the spillover quotient (3.39) by  $\lambda^s$  we write the eigenvalue problem

$$[\lambda^e P^R - P_C^M] t_r = 0 \quad r=1,2,\dots,2n \quad (3.42)$$

substituting (3.40) into (3.42) we obtain

$$[(1-\lambda_r^e)P^R - P_{ij}^M]t_r = 0 \quad (3.43)$$

which constitutes the eigenvalue problem for the relative model spillover quotient sq. Hence, the eigenvalues of the spillover quotient sq (or equivalently phrased, controller power inefficiency) are

$$\lambda_r^s = 1 - \lambda_r^e \quad r=1,2,\dots,2n \quad (3.44)$$

with the same eigenvectors  $t_r$  as that of the relative model efficiency e in Eq. (3.42). Similar to Eq. (3.25), for the model spillover quotient we can write the expansion

$$sq = \sum_{i=1}^{2n} c_i^2 \lambda_i^s = \sum_{i=1}^{2n} c_i^2 (1-\lambda_i^e) = \sum_{i=1}^{2n} c_i^2 - \sum_{i=1}^{2n} c_i^2 \lambda_i^e \quad (3.45)$$

which yields, after recognizing  $\sum_{i=1}^{2n} c_i^2 = 1$

$$sq = 1 - e \quad (3.46)$$

as was given in Eq. (2.46).

Note that although there may be infinitely many truncated modes in the system, what happens from the point of view of control power used is described completely in the 2n-dimensional space spanned by the controller efficiency modes  $t_r$ ,  $r=1,2,\dots,2n$ , for the infinite dimensional system.

### 3.5 Principal Efficiency Components - A Link Between Controller Design and Initial Conditions

The ultimate efficiency of a control system is dictated together with the structure-control design embedded in the matrices A, B and G and the initial disturbance  $x_0$ . To see how these two factors interact, consider the expansion of an efficiency quotient given by (3.25) and write



$$e = \sum_{i=1}^{2n} e_i \quad , \quad e_i = c_i^2 \lambda_i \quad (3.47)$$

where  $e_i$  represents the efficiency of the controller in the  $i$ -th principal controller direction. We shall refer to  $e_i$  as the  $i$ -th principal efficiency component. Obviously, from Eq. (3.47), the principal efficiency  $e_i$  is the product of the corresponding characteristic efficiency  $\lambda_i$  and the coefficient  $c_i^2$ . For a given structure, the characteristic efficiency  $\lambda_i$  is, however only a function of the control gain matrix  $G$  and the controller configuration represented in matrix  $B$  via Eqs. (3.15), (3.11), (3.12) and (3.13). On the other hand, the efficiency coordinate  $c_i$ , Eq. (3.38) is a function of both the initial state  $x_0$  and the controller design via  $L = T^{-1}$  which in turn is a function of only  $G$  and  $B$ , again by virtue of Eqs. (3.15) and (3.11), (3.12) and (3.13). Expressing the functional dependencies of the quantities involved in the principal efficiency components in (3.47), one can write

$$e_i = c_i^2 \lambda_i = \frac{x_0 L_i(A, B, G) L_i(A, B, G) x_0}{L^T(A, B, G) L(A, B, G)} \lambda_i(A, B, G) \quad (3.48)$$

and observe the effect of both structure and control design on both  $c_i$  and  $\lambda_i$ . Common sense requires that an efficient controller must have high principal efficiency components in the principal controller directions, in other words, the products  $c_i^2 \lambda_i$  must be high in each direction. Although a control design might have a large characteristic efficiencies, it will not necessarily have a high efficiency unless the projection(s)  $c_i$  of the initial disturbance  $x_0$  is (are) significant along the directions of large  $\lambda_i$ . Considering that both  $c_i$  and  $\lambda_i$  are functions of control design as shown by Eq. (3.48), it may, therefore, be the control design itself which will either enhance or degrade its efficiency. Hence, it is of significance not only to have controllers with large

characteristic efficiencies but also to have their principal ( $\lambda_1$ ) directions aligned favorably with the initial disturbances  $x_0$ , thus extracting large projections of power so that they will yield high efficiencies in those directions.

Traditionally, in the design of a control system (such as in the LQG-type designs) there is no avenue to bring in the initial disturbance information to the computation of the control gain matrices. After having obtained a so called "optimal" controller via LQG algorithms, one does not really know how the control system will interact with any initial disturbance until a simulation is done. It is possible that an optimal controller (in the sense of the theory used to design it) will be an extremely inefficient controller if it does not "see" or "receive" the initial disturbance properly along its principal controller directions.

On the other hand, the efficiency modes analysis of a structure-control system clearly reveals the internal link by which the controller-design and the initial disturbances  $x_0$  interact. In fact, after identifying the modal matrix  $T$  and  $\lambda_i$ , that is the principal directions and the characteristic efficiencies of a controller one can readily for any initial disturbance  $x_0$  form the projections  $c_i$  via Eq. (3.38) to examine how they pair up with the respective characteristic efficiencies  $\lambda_i$  and obtain an a priori (before simulation) information about the controller performance. Certainly, the observation of the link between the controller design and the initial disturbance is a constructive one so that given an initial disturbance and a structure, the objective would be to design a control gain which will pair up significant  $\lambda_i$  with significant projections  $c_i$ .

### 3.6 Illustrative Examples on ACOSS-4

The structure is the ACOSS-4 structure considered in Chapter 2, Fig. 2.1. In this case, the structure is subjected to an initial disturbance of unit displacement in the x direction at Node 2. Input configurations are selected from a set of twelve available actuators located at the pods of the structure. In all cases, the controls were designed by using LQR theory with unit control weighting ( $r=1$ ) and state weighting  $q=\omega_r^2$  for the modal displacements and  $q=1$  for the modal velocities. The evaluation model has  $N=12$  modes yielding a 24th order state-space evaluation model.

#### Efficiency Modes Analysis

Example 3.1: The control design model has two structural modes, Modes 11 and 12,  $n=2$ (Modes 11,12) one input was used  $m=1$  (#4) which was actuator number 4. Figures labeled SIM39 (Figures 3.1 through 3.3) give the line of sight error simulation results for this case.

Example 3.2  $n=2$  (Modes:11,12),  $m=2$ (#3,#4), that is two actuators #3 and #4 were used for control.

The nomenclature for the computer outputs of the examples presented in this section and the computer outputs are given in Appendix C. Simulations of the globally optimal distributed input system, labeled "Globally Optimal System", "Suboptimal system" with  $m$ -point inputs and the "evaluation model" response with  $m$ -point inputs are presented for the examples listed below. A designation SIM (number) associated with an example indicate that figures labeled with SIM 9 Number show the line of sight error for that example. Line of sight error is computed at the vertex (Node 1) of the structure, according to the expression

$LOS=U_{x1}^2+U_{y1}^2$ , where  $U_{x1}$ ,  $U_{y1}$  are nodal displacements in the x and y directions at the vertex.

Example 3.3:  $n=2$  (Modes:1,1,1,2),  $m=4$  (#1-4), SIM41 (Figures 3.4 and 3.6).

Example 3.4:  $n=8$  (Modes:1-8 natural order),  $m=2$  (#1,2), SIM1 (Figures 3.7, 3.8 and 3.9).

Example 3.5:  $n=8$  (Modes:1-8 natural order),  $m=4$  (#1-4), SIM10 (Figures 3.10, 3.11 and 3.12).

Example 3.6  $n=2$  (Modes:1-8 natural order),  $m=2$  (#1,2). this case presents the results of the (in) efficiency modal analysis of the quotients with  $P_N=P_U^M$ , SIM1 (Figures 3.7, 3.8, and 3.9).

#### Effect of Initial Disturbances on Efficiencies

Example 3.7 Consider the characteristic efficiencies for the global efficiency  $e^*$  and the relative model efficiency  $e$  of Example 3.4 ( $n=8(1-8)$ ,  $m=2$ ) listed in Appendix C. For  $e^*$ , we observe that with a mere change of  $x_o$ , this system can have a maximum global efficiency of  $\lambda_{max}^*=53.76\%$ , corresponding to the 2nd characteristic efficiency listed. This would occur when  $x_o=t_2$ , that is when the initial state is the second column of the efficiency modal matrix T listed. The worst efficiency will occur when  $x_o=t_{14}$  corresponding to a characteristic efficiency of  $\lambda_{min}^*=\lambda_{14}^*=0.773\%$  as listed in the computer outputs in Appendix C. For the model efficiency we note that all characteristic efficiencies are  $59.79\% \geq \lambda \geq 54.04\%$ . Hence, again the corresponding initial disturbances which will culminate in respective efficiencies can be identified from the columns of

the modal matrix  $T$  listed in Appendix C. It is noted that regardless of  $x_0$ , the model efficiency can not change more than 5.7% within that bracket. Therefore, in this case the model efficiency is very robust to changes in  $x_0$ .

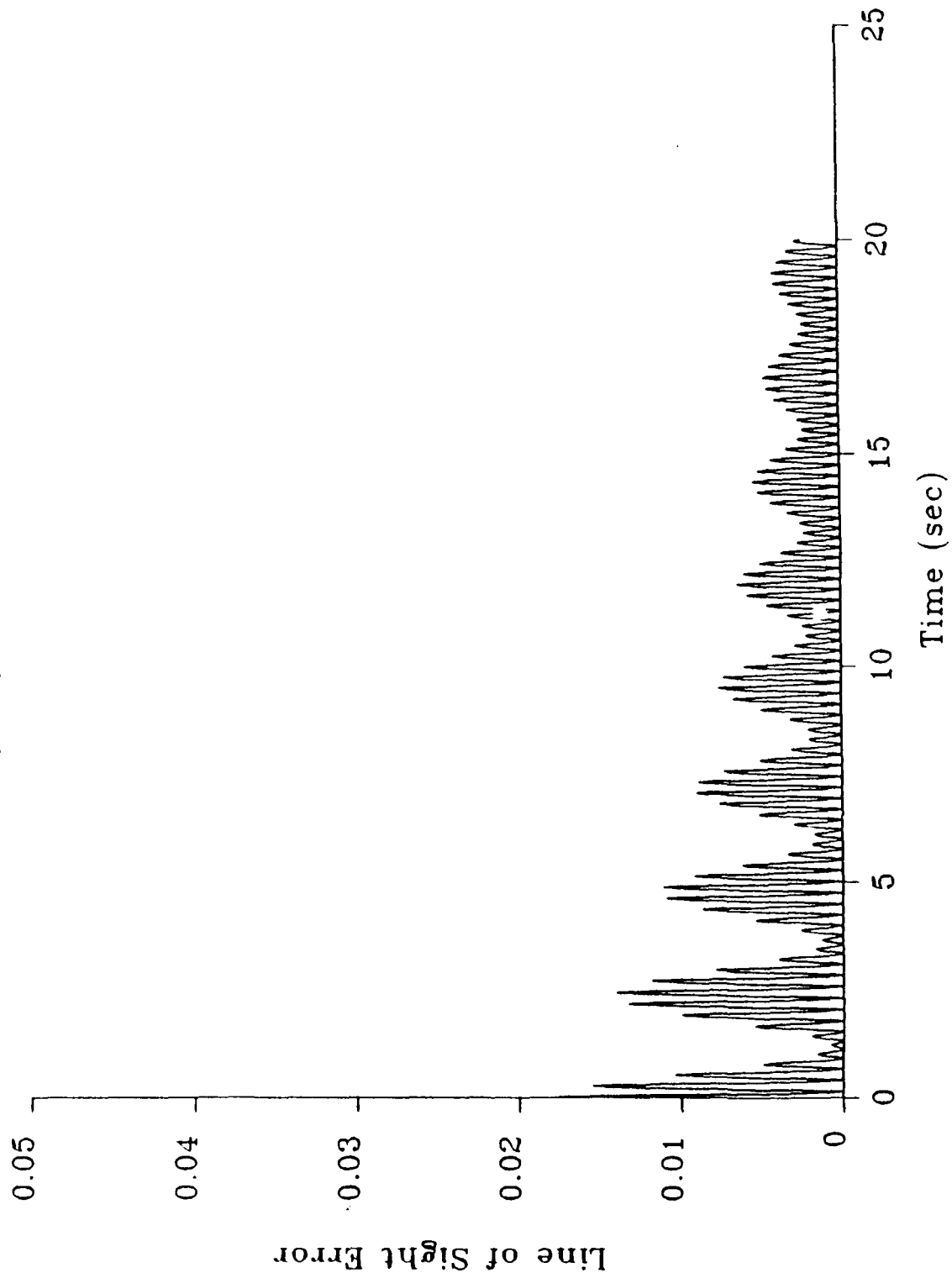


Figure 3.1 Globally optimal system LOS for  $n=2$  (modes: 11,12); example 3.1

Suboptimal System SIM39

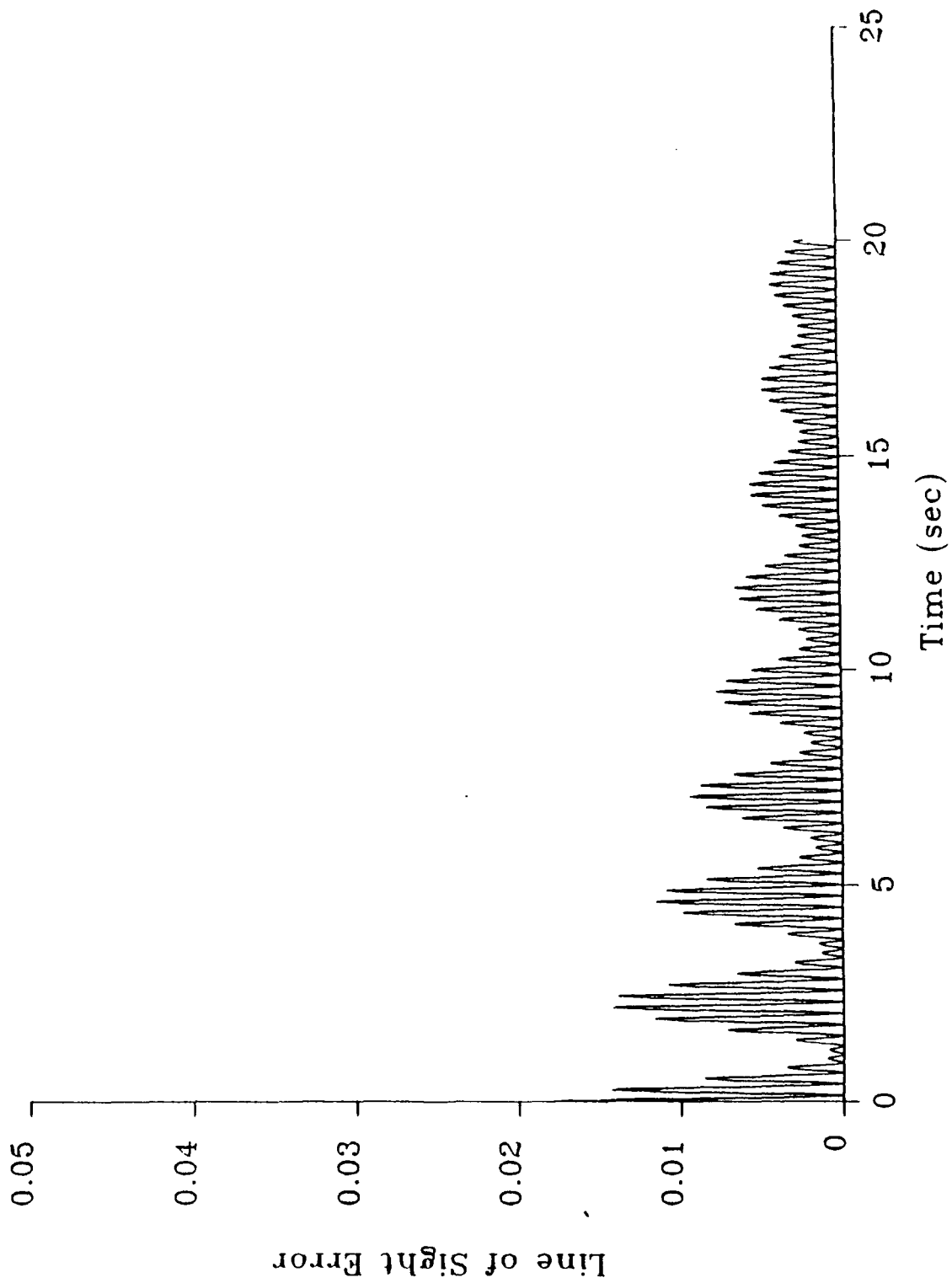


Figure 3.2: Suboptimal system LOS for  $n=2$  (modes: 11,12),  $m=1$  (input #4); example 3.1

Evaluation Model SIM39

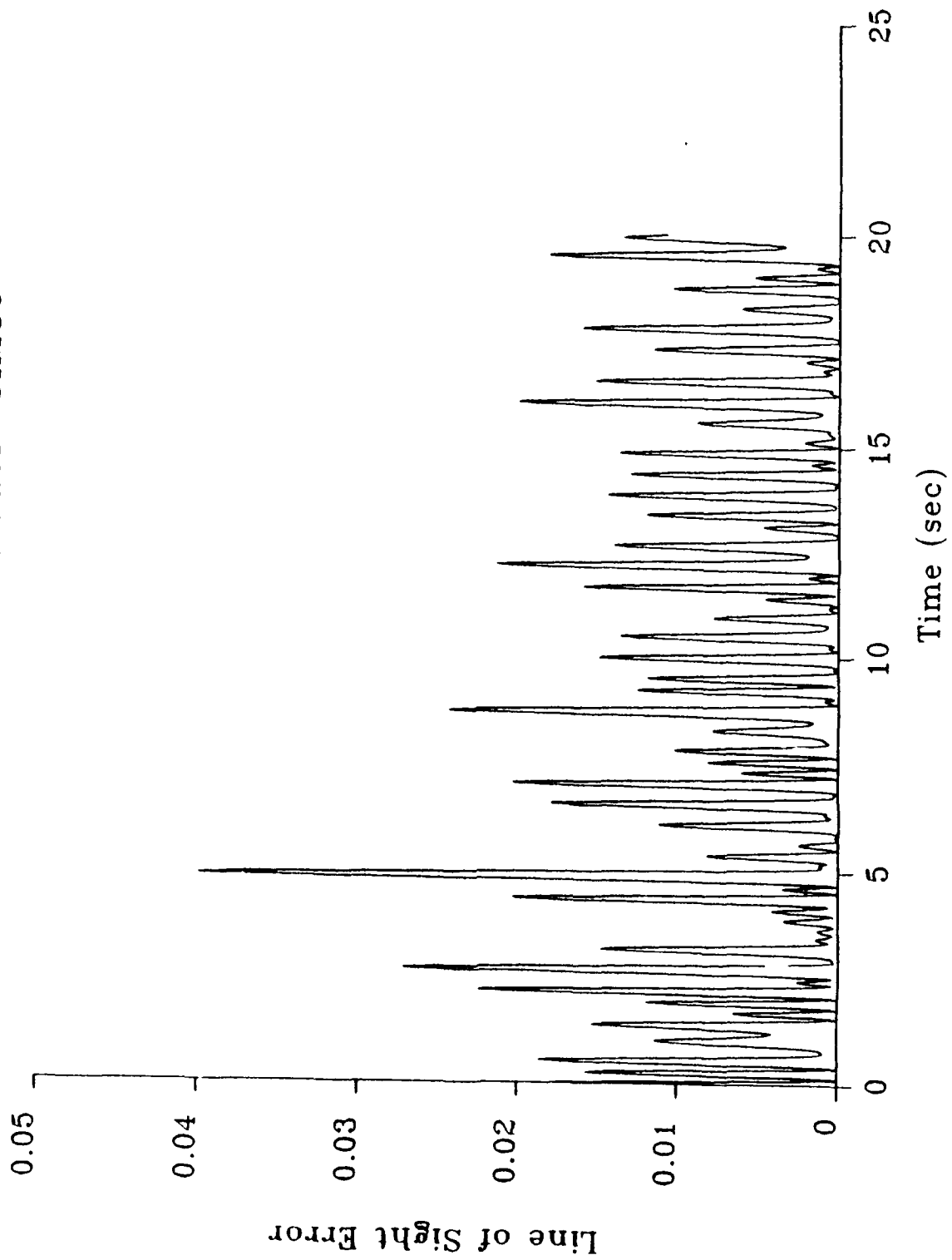


Figure 3.3: Evaluation model LOS for  $n=2$  (modes: 11,12)  $m=1$  (input #4); example 3.1



Globally Optimal System SIM41

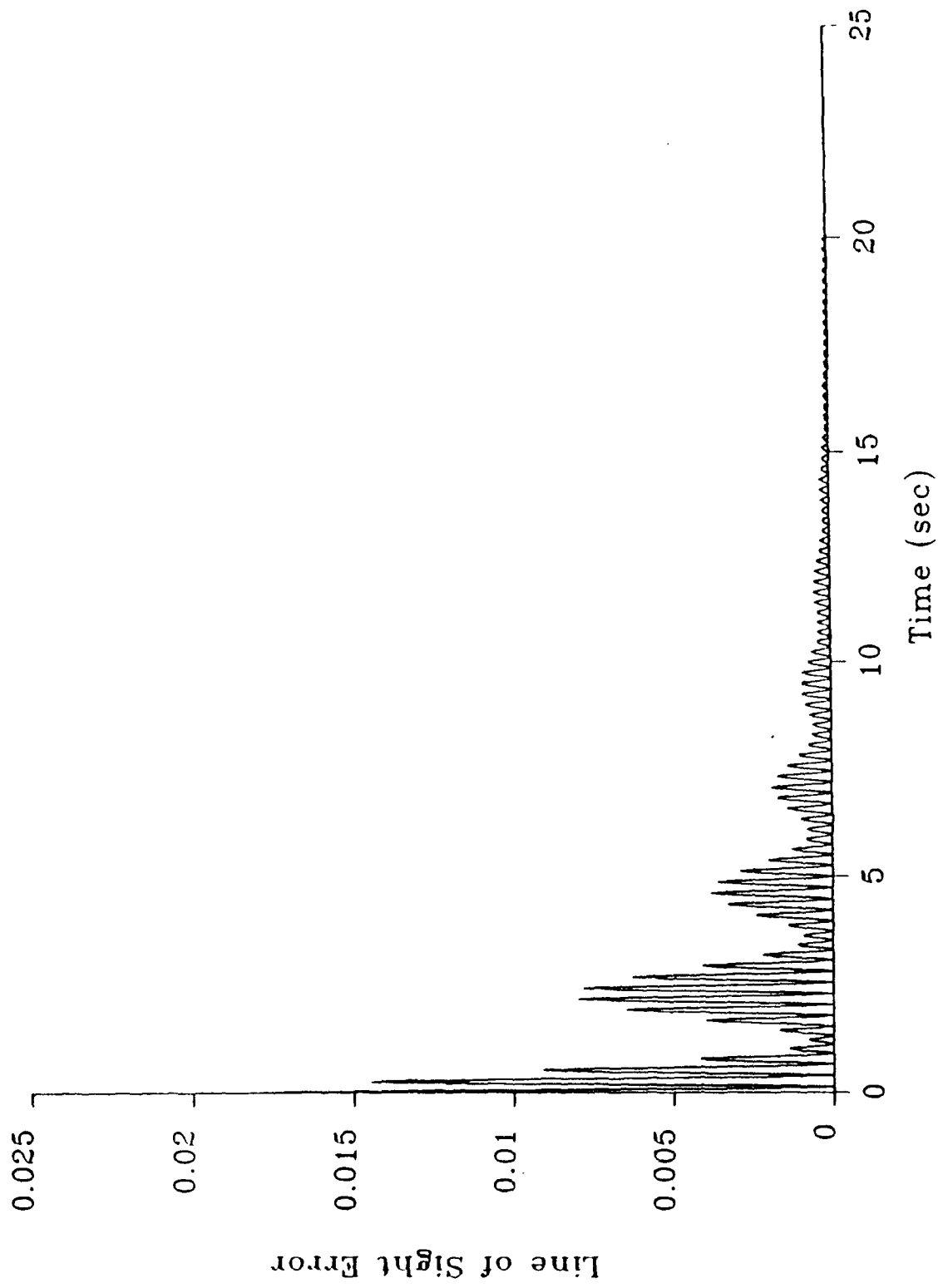


Figure 3.4: Globally optimal system LOS for  $n=2$  (modes: 11,12); example 3.3

# Suboptimal System SIM41

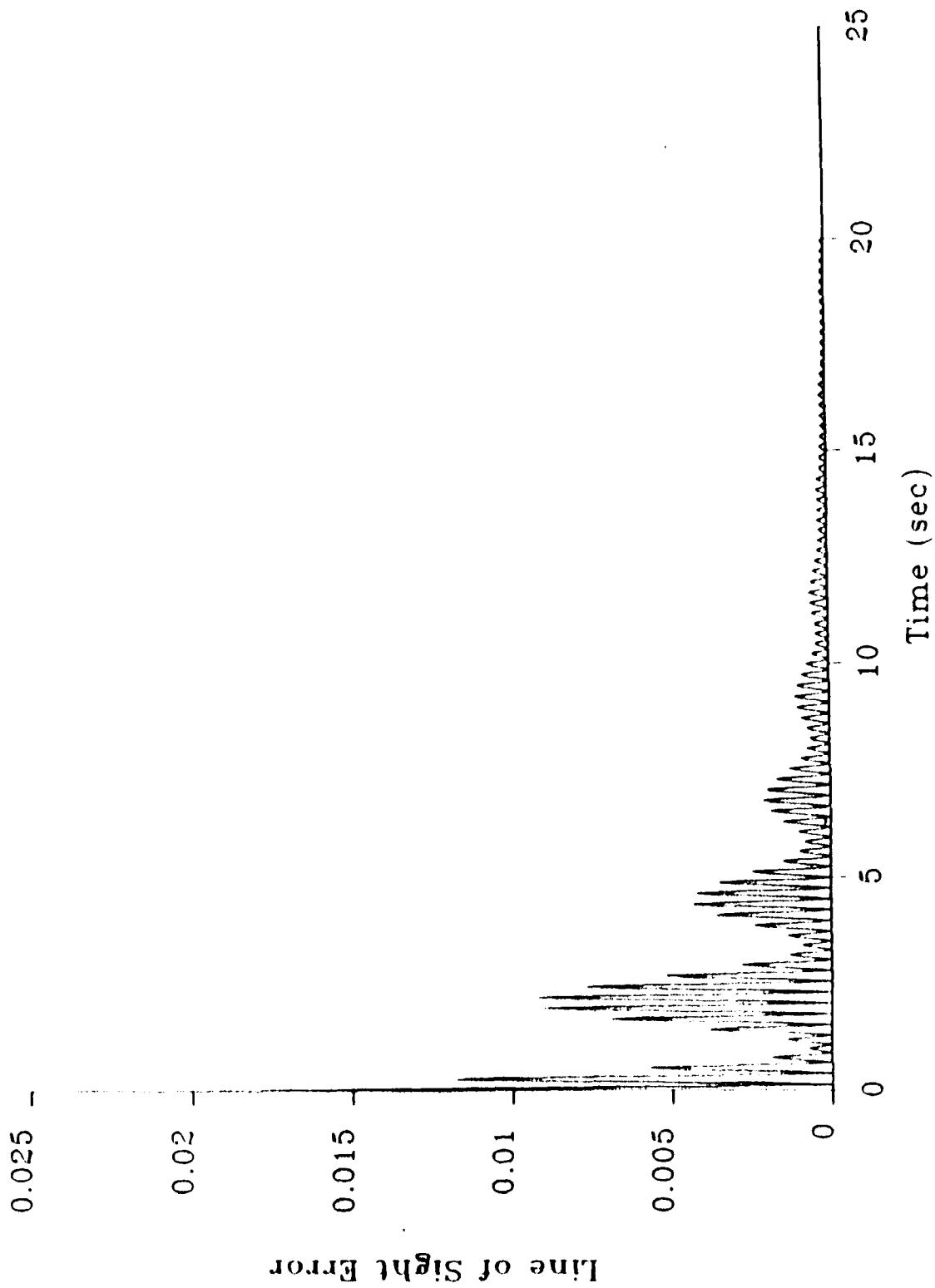


Figure 3.5: Suboptimal system LOS for  $n=2$  (modes: 11,12);  $m=4$  (#1-4); example 3.3

# Evaluation Model SIM41

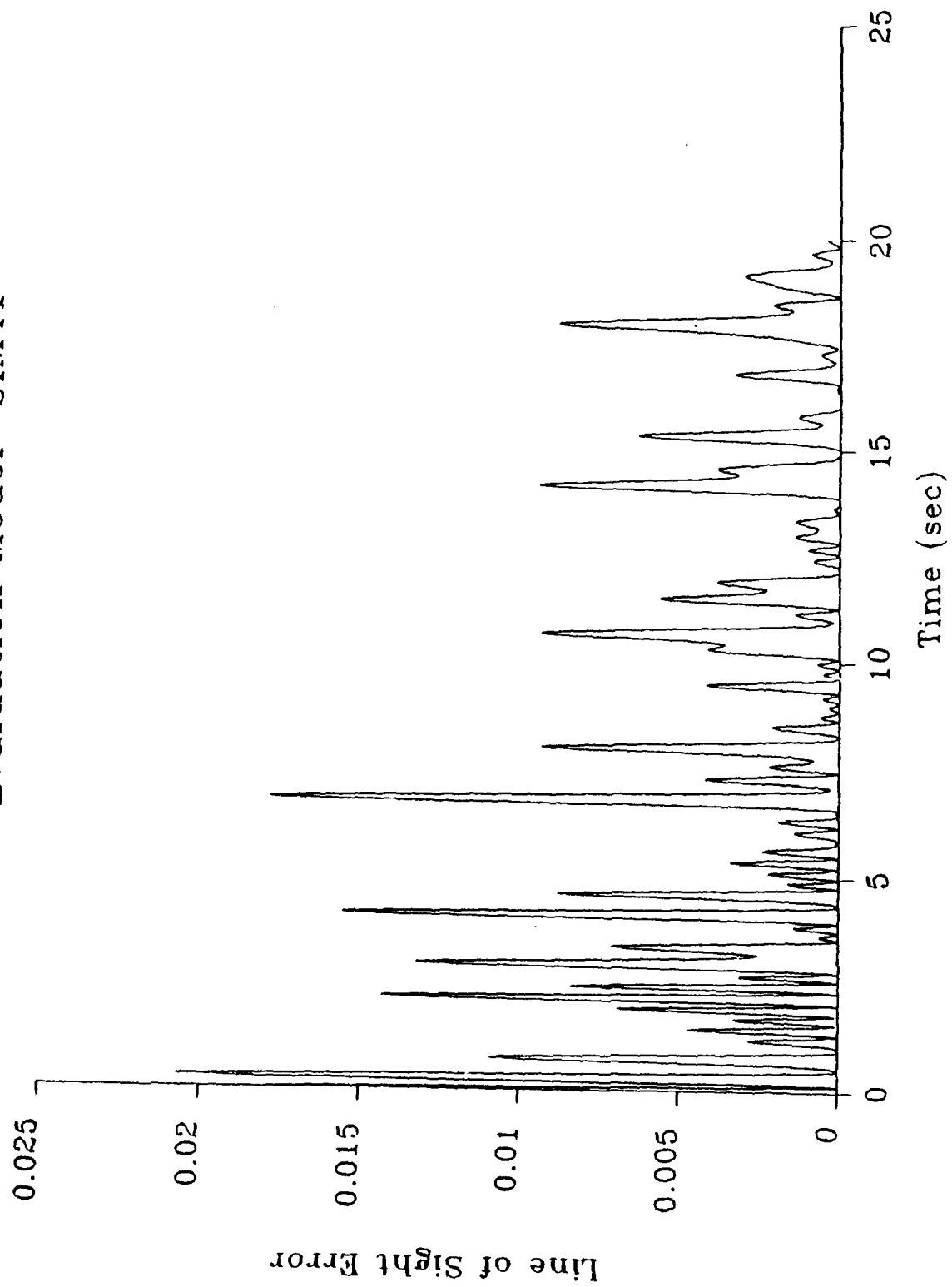


Figure 3.6: Evaluation model LOS for  $n=2$  (modes: 11,12),  $m=4$  (#1-4); example 3.3

Globally Optimal System SIM1

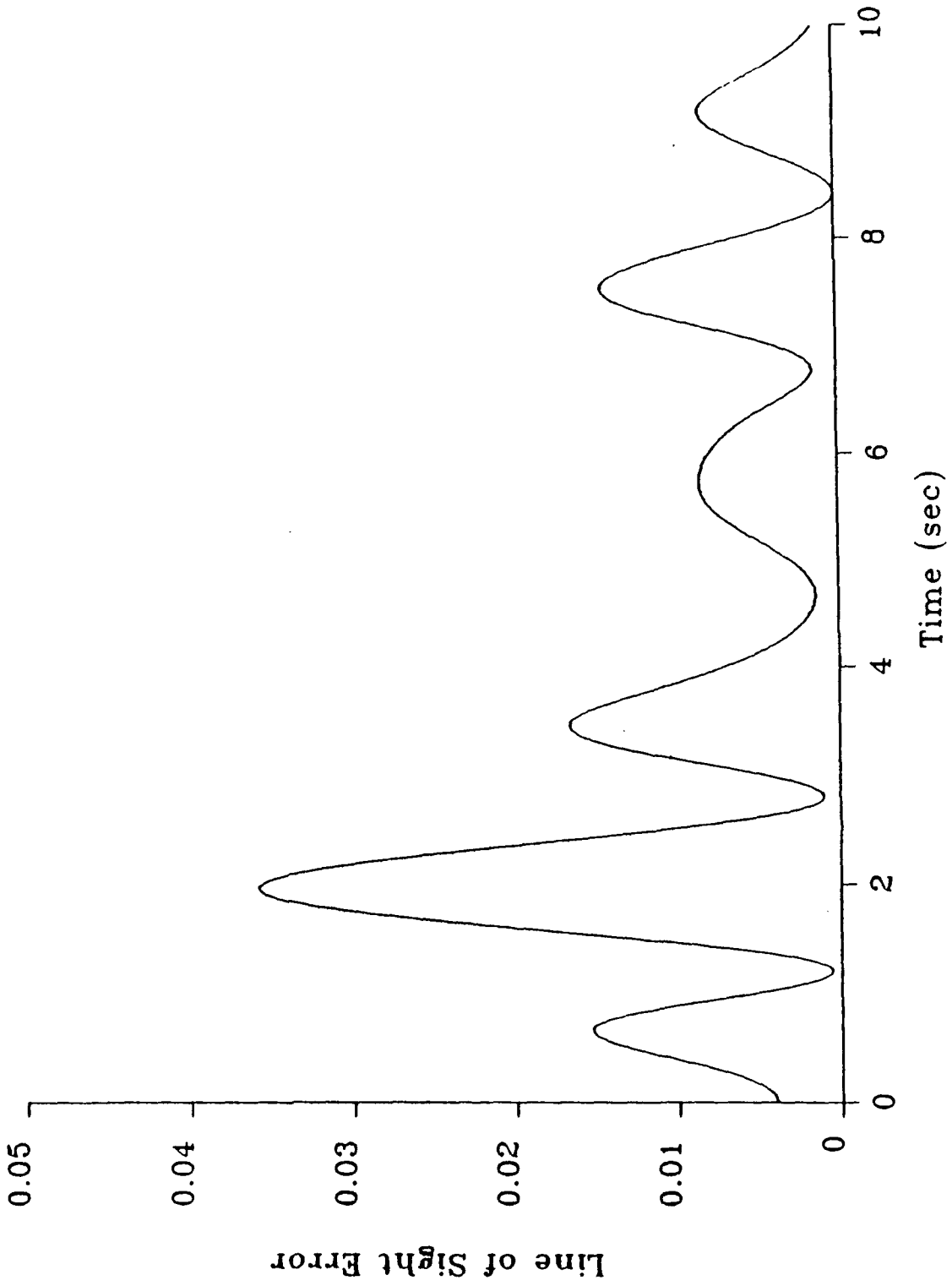


Figure 3.7: Globally optimal system LOS for  $n=8$  (modes: 1-8)  $m=2$  (#1-2); example 3.4

# Suboptimal System SIM1

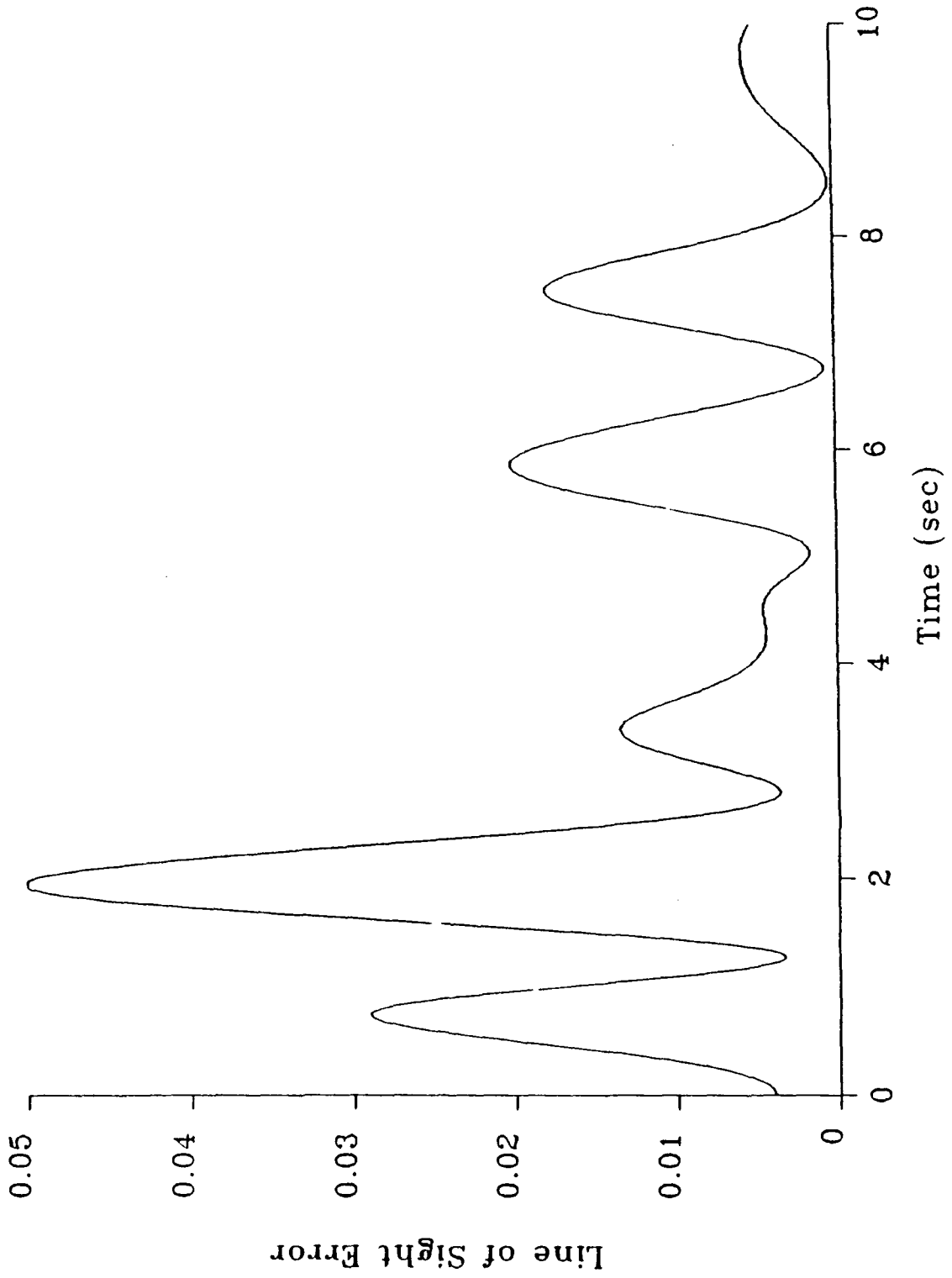


Figure 3.8: Suboptimal system LOS for  $n=8$  (modes: 1-8),  $m=2$  (#1-2); example 3.4

# Evaluation Model SIM1

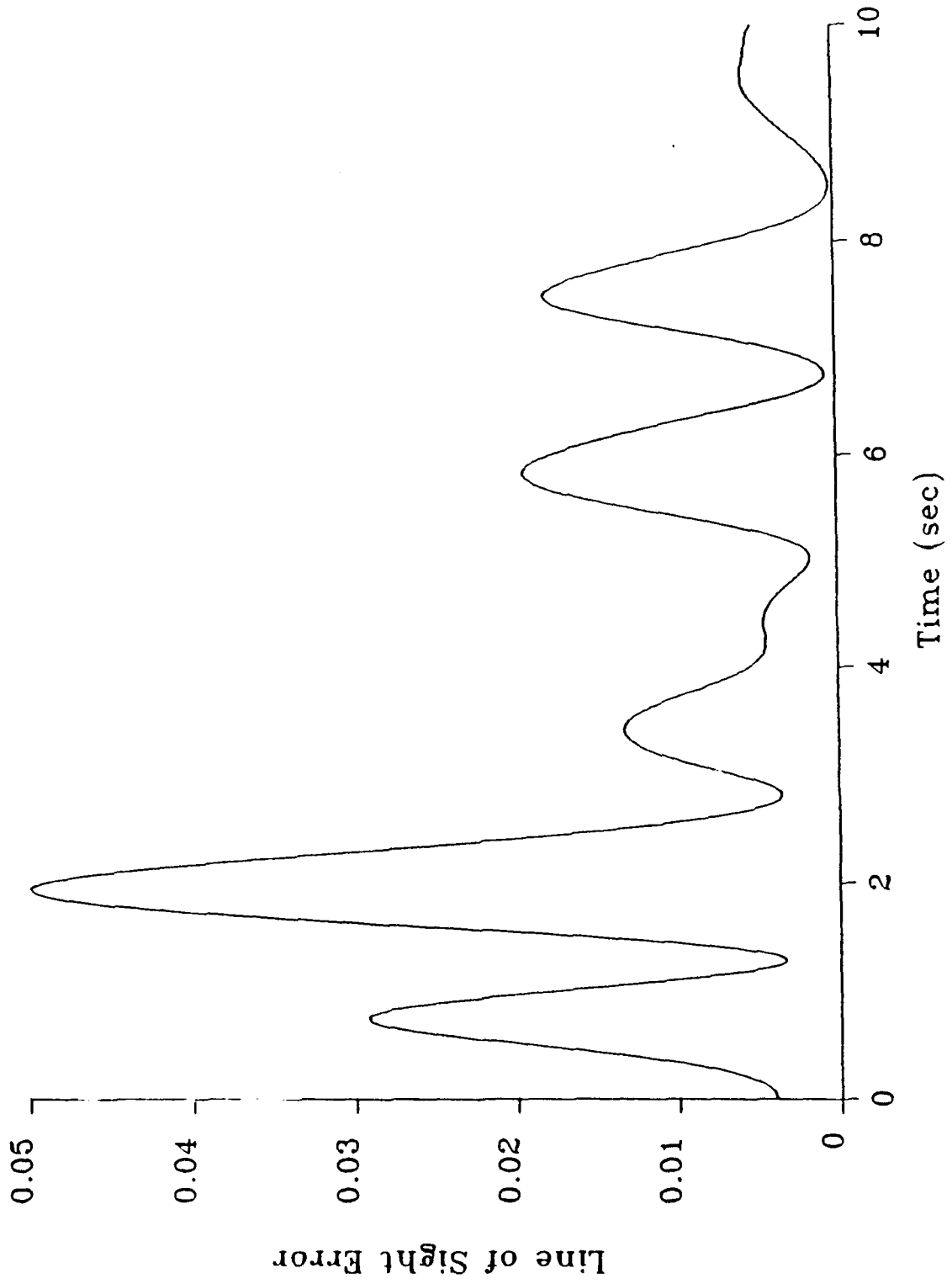


Figure 3.9: Evaluation model LOS for  $n=8$  (modes 1-8),  $m=2$  (#1-2); example 3.4

Globally Optimal System SIM10

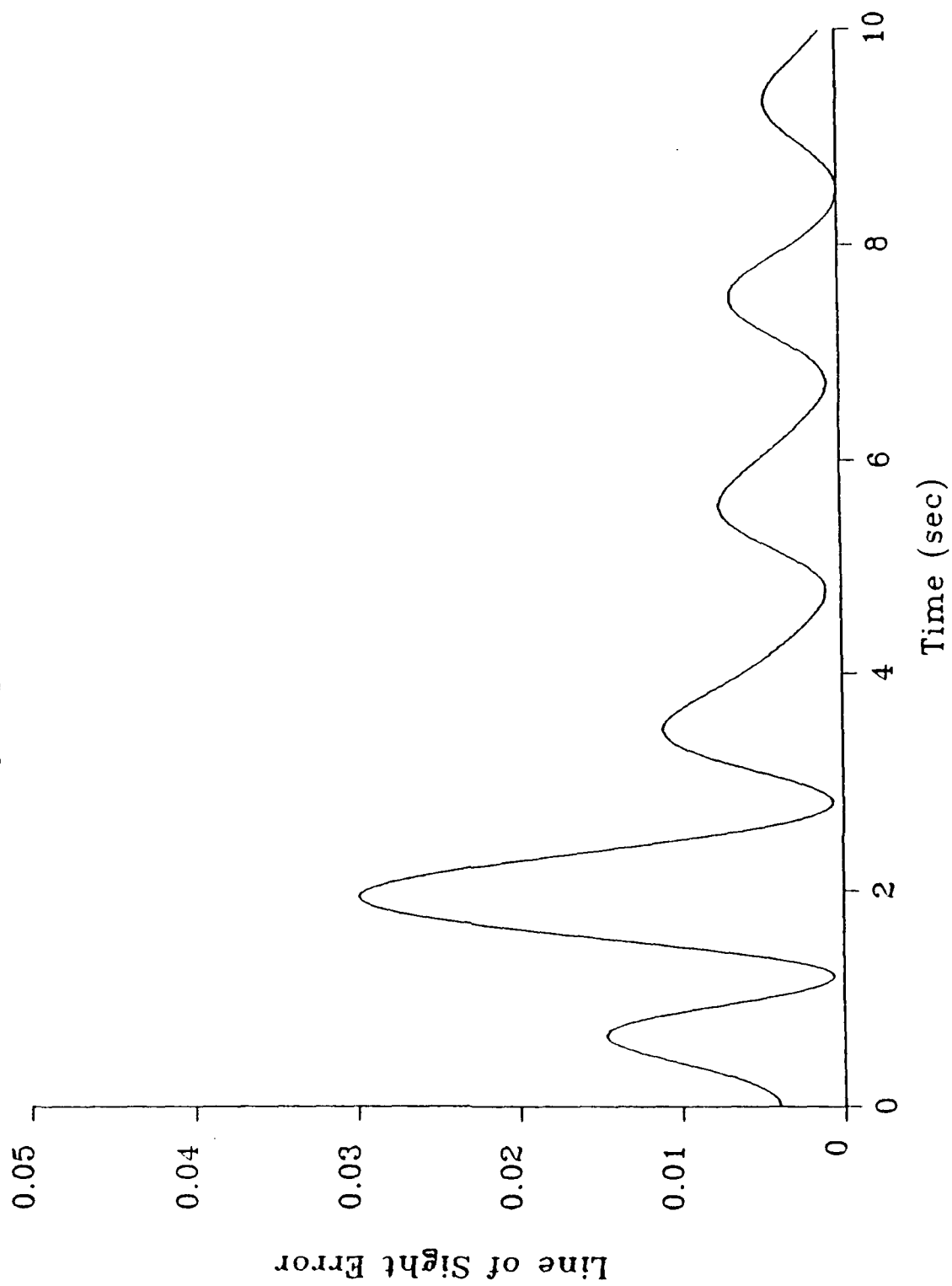


Figure 3.10: Globally optimal system LOS for  $n=8$  (modes: 1-8),  $m=4$  (#1-4); example 3.5

Suboptimal System SIM10

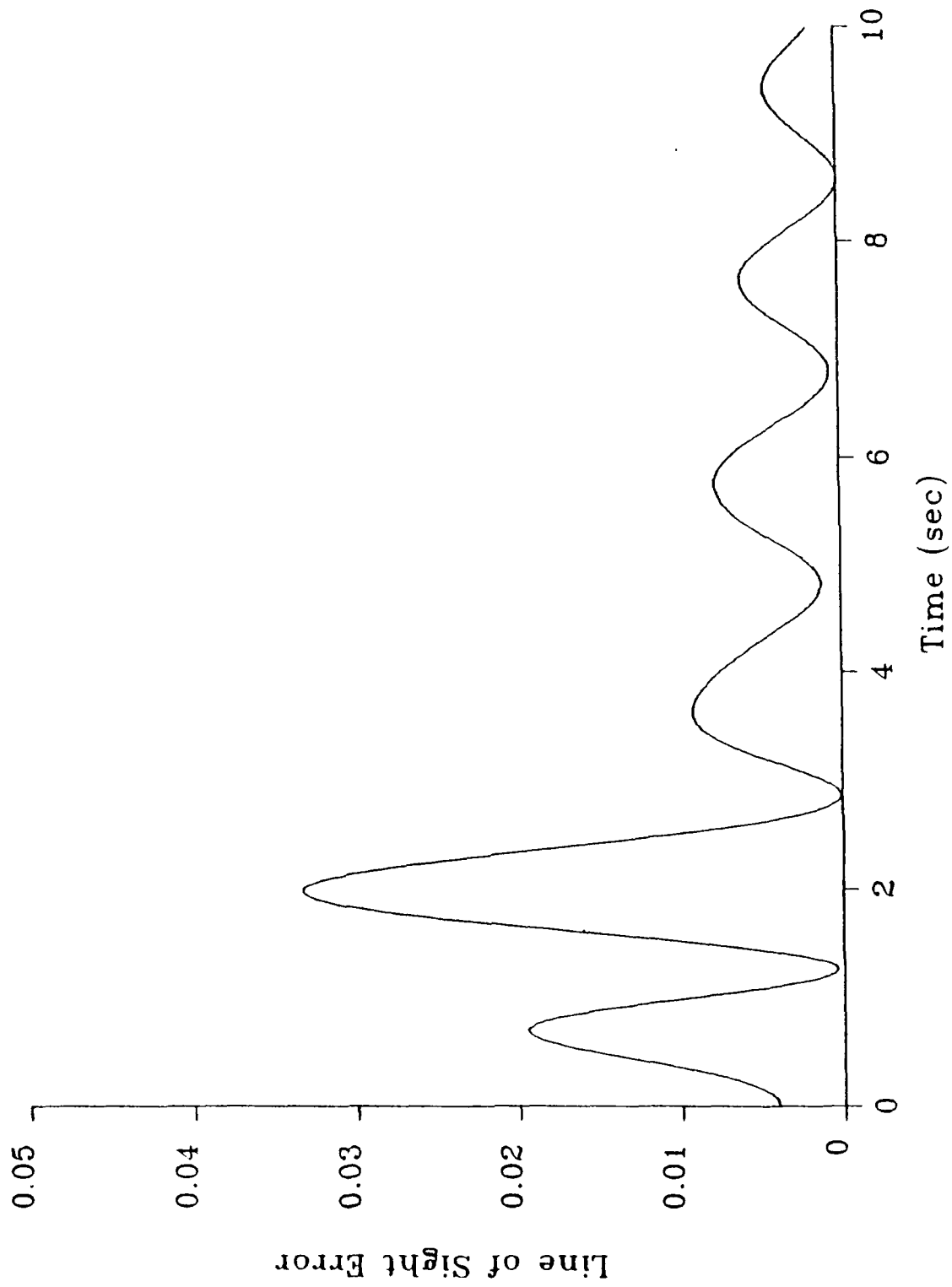


Figure 3.11: Suboptimal system LOS for  $n=8$  (modes: 1-8),  $m=4$  (#1-4); example 3.5



Evaluation Model SIM10

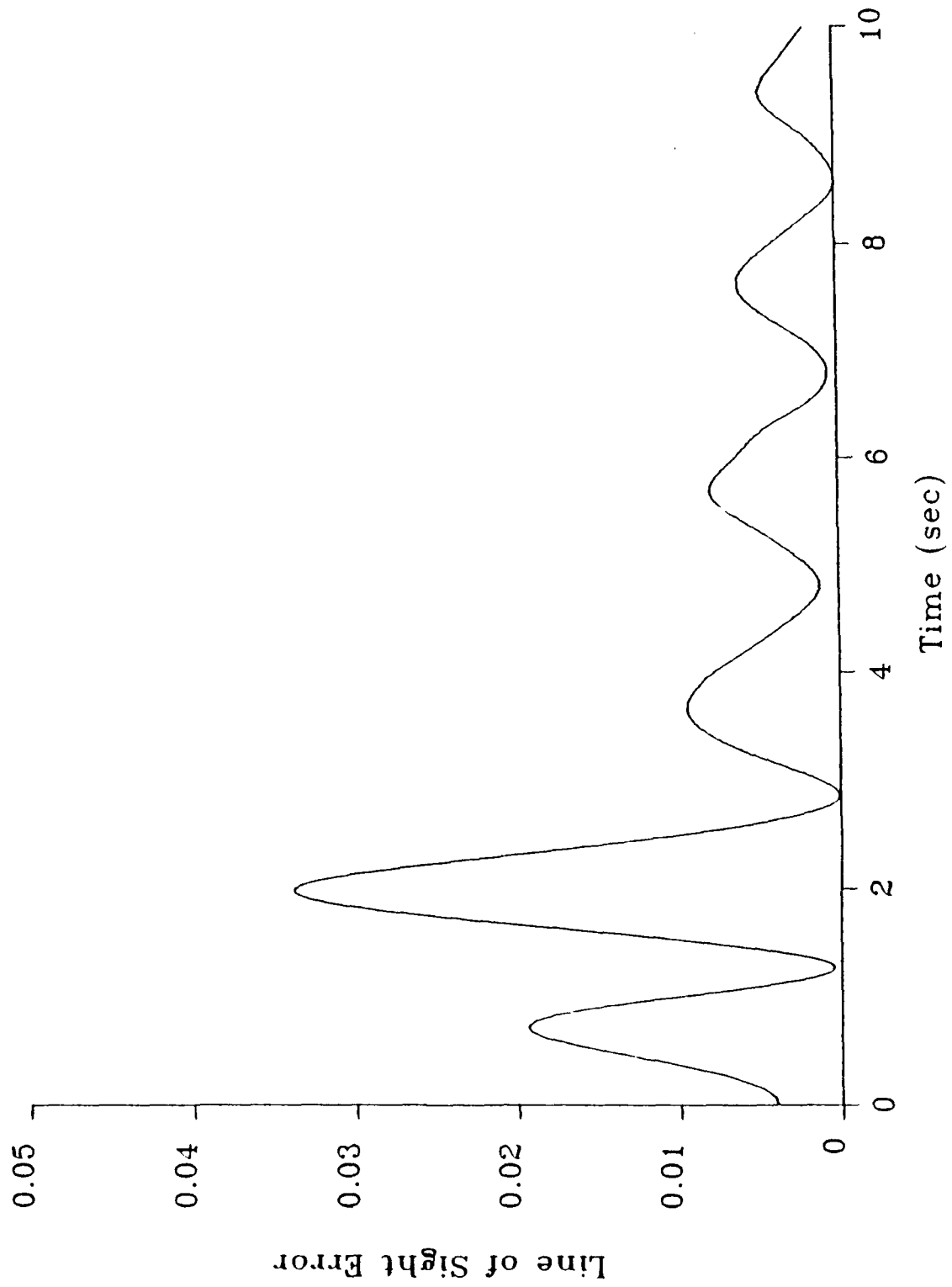


Figure 3.12: Evaluation model LOS for  $n=8$  (modes: 1-8),  $m=4$  (#1-4); example 3.5

## 4.0 EFFICIENCY MODEL/CONTROLLER REDUCTION

### 4.1 Introduction

In the efficiency analysis of a  $2n$ -th order structure-control system design model, we identified  $2n$ -controller efficiency modes which led to the efficiency state ( $\epsilon$ -state) description of the original design model in the structural modal space ( $x$ -state). The two modal descriptions are fundamentally distinct in their nature. As one is all too familiar with structural modes of the system characterized by the  $x$ -states, the controller efficiency modes characterized by the  $\epsilon$ -states constitute a new concept where the latter describe principal controller directions signifying avenues of efficient use of available control power. From a structure-control system perspective, how these two sets of modes relate to each other becomes a measure of efficiency of the control task. A certain degree of misalignment of these controller modal directions and the structural modal directions result in controller's not "seeing" the structure directions retained in the model properly, thus not channeling the control power to the structure and wasting it to the unrepresented structure truncated dynamics. Conversely, certain structural directions may be "seen strongly or in full view" by the principal controller directions leading to efficient channeling of control power to the structure in that direction.

For analysis and design purposes via the efficiency concept, the above observations need to be expressed in meaningful mathematical terms. To this end, one needs to correlate the structural modal states  $x$  and the efficiency modal states  $\epsilon$ . Quantifying such correlations can help the designer to identify structural modal states with weak or strong contributions to a set of critical principal controller directions. Once this is accomplished, one can then proceed to

delete structural modal states and/or controller efficiency modal states which are weakly correlated.

The interrelationship between the structural modes and controller modes can be structured and quantified by decomposition of efficiency quotients by defining decomposition matrices for the numerator and denominator control power matrices. These decompositions, in turn, pave the way to proposing relevant model controller reduction criteria based on the efficiency concept.

In Section 4.2 we introduce decomposition matrices for the numerator and denominator control power matrices by using the spectral properties of the matrices given in Chapter 3. Next component efficiencies associated with structural modal states are defined for the principal efficiency components. Certain features of these components efficiencies are discussed requiring the need to define coupled and decoupled component efficiencies. In Section 4.3, the first controller reduction method in the efficiency state space is presented. Section 4.3 leads to the subject of Section 4.4, efficiency filtering. Section 4.5 presents the methodology for efficient modal controller reduction which relies on the decomposition discussed in Section 4.2. Also, in Section 4.5 several important observations are made to be able to use the efficiency concepts properly in the analysis and design studies. Finally, in Section 4.6, we give illustrative examples using the ACOSS-4 structure.

#### 4.2 Decomposition of Efficiency Quotients

Associated with any of the quantities  $e$ ,  $e^*$ ,  $sq$  and  $sq^*$ , we define  $2n$ -dimensional numerator and denominator matrices  $[N]$  and  $[D]$  with elements

$$[N] = N_{ij} = [X_0 L_i L_i^T \lambda_i]_{jj} \quad i, j=1, 2, \dots, 2n \quad (4.1)$$

$$[D] = D_{ij} = [Y_0 L_i L_i^T]_{jj} \quad (4.2)$$

where  $X_0$  is the initial disturbance covariance matrix

$$X_0 = x_0 x_0^T \quad (4.3)$$

Next by forming the total sums  $N$  and  $D$ , one can verify that

$$N = \sum_i^{2n} \sum_i^{2n} N_{ij} = x_0^T P_N x_0 \quad (4.5)$$

$$D = \sum_i^{2n} \sum_j^{2n} D_{ij} = x_0^T P_D x_0 \quad (4.6)$$

$$e = N/D \quad e = \{e, e^*, sq, sq^*\} \quad (4.7)$$

Each element  $N_{ij}$  and  $D_{ij}$  represents the power contribution of  $j$ -th structural modal state  $x_j$  in the  $i$ -th principal controller direction  $\epsilon_i$  for the respective numerator and denominator terms. We shall refer to matrices  $[N]$  and  $[D]$  as the numerator and denominator control power decomposition matrices, respectively. The quotient  $e$  in terms of sums of their elements represents an efficiency decomposition. Although through the controller design one has the power matrices  $P_N$  and  $P_D$  available in the beginning, their representations in terms of the elements  $N_{ij}$  and  $D_{ij}$  unveils the internal structure of the power associated with the controller. This exposition of the internal structure is, however, possible only through the efficiency modal analysis. Specifically through this modal analysis, each element  $N_{ij}$  and  $D_{ij}$  quantifies the correlation between the  $j$ -th structural mode  $x_j$  and the  $i$ -th controller mode.

Next, we define the partial sums

$$N_j = \sum_{i=1}^{2n} N_{ij} \quad , \quad D_j = \sum_{i=1}^{2n} D_{ij} \quad j=1,2,\dots,2n \quad (4.8)$$

as the power contributions (extraction) of  $j$ -th structural modal state  $x_j$  to (from) all principal controller directions. Similarly, we define power contributions (extraction) of all structural modal states to (from) the  $i$ -th principal controller direction:

$$N_i = \sum_{j=1}^{2n} N_{ij} \quad , \quad D_i = \sum_{j=1}^{2n} D_{ij} \quad i=1,2,\dots,2n \quad (4.9)$$

Considering definitions (4.9) and the definition of principal efficiency component  $e_i$ , Eq. (3.47), it follows that

$$e_i = \frac{N_i}{\sum_i D_i} = \frac{N_i}{D} = c_i^2 \lambda_i \quad i=1,2,\dots,2n \quad (4.10)$$

Where we recall that  $e_i$  represents a component with respect to the  $i$ -th principal controller direction. Noting Eq. (3.25), one can also regard  $e_i$  as the contribution to efficiency of the  $i$ -th efficiency state  $\epsilon_i$ . Similarly, one can identify efficiency components with respect to the structural modal directions and define

$$e_j = \frac{N_j}{D} = \frac{N_j}{\sum_j D_j} \quad , \quad e = \sum_j e_j \quad j=1,2,\dots,2n \quad (4.11)$$

We shall refer to  $e_j$  as the  $j$ -th (structural) component efficiency and emphasize the difference in the terminology that is being used here that  $e_j$  is the component efficiency and  $e_i$  is the principal efficiency component, respectively, the former is with respect to structural modal components  $x_j$  and the latter with respect to controller efficiency modal components  $\epsilon_i$ .

Strictly speaking, although definitions of components  $e_i$  and  $e_j$  as in Eqs. (4.10), (4.11) are mathematically correct since their sums over respective

indices yield the total efficiency  $e$ , from a physical perspective certain features of these definitions must be considered cautiously. In both definitions, the denominator is common for all components  $e_i$  and  $e_j$ . So in effect, the significance of a component is solely determined by the numerator power contributions  $N_i$  and  $N_j$ . For that matter, the use of  $D$  which is the real control power  $S^R$  as a common denominator is of no consequence, as the relative contributions of  $N_j$  (or  $N_i$ ) terms to total efficiency would not change if we were to use any other quantity as a common denominator.

Indeed, in this case the function of efficiency as a relevant nondimensional quantity is subordinated to the contributions to the dimensional numerator control powers. Therefore, from the efficiency point  $e_j$  and  $e_i$  component descriptions may not be proper characterizations. In case of component efficiencies  $e_j$  for the structural modal states  $x_j$ , the characterization is completely out of contact with the concept and purpose of efficiency quotient as we show next.

Considering the definition of  $N_j$ , Eqs. (4.8) and (4.1) and the spectral decomposition of control powers matrices in (3.27), we can write

$$N_j = [X_0 \sum_{i=1}^{2n} L_i \lambda_i L_i^T]_{jj} = [X_0 P_N]_{jj} = e_j D \quad j=1, 2, \dots, 2n \quad (4.12)$$

summing over all component efficiencies:

$$eD = \sum_j^{2n} N_j = \text{Trace } X_0 P_N = x_0^T P_N x_0 = S_C^M \quad (4.13)$$

Thus, component efficiency characterization  $e_j$  in this sense is no different that component cost analysis of a modal power  $S_C^M$  of the control design model as defined in Ref. (7) and all the benefit and the physical reason in introducing a denominator control power matrix to yield the nondimensional efficiency is

lost. Furthermore, the components  $N_j$  given by Eq. (4.12) are computed exactly as given in Ref. (7) associated with which there is certainly no concept of efficiency.

A similar situation exists for the efficiency components  $e_i$ , however, at a conceptually less critical level. Considering the definition of  $N_i$ , Eqs. (4.9), (4.2) and (3.27), we can write

$$N_i = \sum_{j=1}^{2n} (X_0 L_i \lambda_i L_i^T)_{jj} \quad i=1, 2, \dots, 2n \quad (4.14)$$

which upon recognizing  $x_0^T L_i = \epsilon_{i0}$  and  $\sum_j (x_0 L_i^T)_{jj} = \epsilon_{i0}$  yields

$$N_i = \lambda_i \epsilon_{i0}^2 = e_i D.$$

Summing over  $i$

$$\sum_i N_i = \sum_i \lambda_i \epsilon_{i0}^2 = eD = S_C^M = \epsilon_0^T \Lambda \epsilon_0 \quad (4.15)$$

This is again a decomposition of the numerator modal power of the control design model regardless of the function of denominator control power  $D$  as the nondimensionalization factor. The form of  $S_C^M$  as in Eq. (4.15) is similar to the cost decoupling concept discussed in Refs. (7 and 12). Here, the efficiency states  $\epsilon$  play a role similar to the cost decoupling coordinates. Hence, principal efficiency components  $e_i$  characterization of  $e$  is also a characterization of the numerator control power  $S_C^M$  alone. However, because this characterization can be affected only in terms of efficiency coordinates  $\epsilon_i$  after solving for the efficiency modal matrix  $T$ , the function of denominator control power in the concept of efficiency is accounted for indirectly. In contrast, in the  $e_j$  characterization there was no need for the efficiency concept. It must be noted, however, that the identification of the cost decoupling coordinates as in Ref. (7

and 12) is only a recognition of the algebraic similarity. The mathematical and the physical foundations under which the decoupling features are brought about in this investigation and in Ref. (7 and 12) are fundamentally different.

From the above discussion, to be consistent with the definitions of efficiency there evolves the need to define component efficiencies  $e_j$  for the contributions of structural modal states  $x_j$  which takes into account the denominator contributions  $D_j$  as well as the numerator contributions  $N_j$ . To this end, we define two component efficiencies. The first one is:

$$e_j = \frac{N_j D + D_j N}{2D^2} \quad e = \sum_j^{2n} e_j \quad (4.16)$$

which we shall refer to as the coupled component efficiency. The coupled designation is used to note that in the definitions of  $N_j$  and  $D_j$ , terms involving products of  $x_{j_0}$  with all other states appear. Hence, a state  $x_{j_0}$  may have a high component efficiency contribution not because of itself alone but also because of the contribution of its coupling terms with another state or states. The second definition of component efficiency disregards all coupling terms among the states, and we refer to it as the decoupled component efficiency:

$$\begin{aligned} e_j &= n_j/d_j \\ n_j &= \bar{x}_{j_0}^T P_{N_{jj}} \bar{x}_{j_0} \\ d_j &= \bar{x}_{j_0}^T P_{D_{jj}} \bar{x}_{j_0} \\ \bar{x}_j^T &= [\xi_j \dot{\xi}_j] \quad j=1, \dots, n \end{aligned} \quad (4.17)$$

where  $P_{N_{jj}}$  and  $P_{D_{jj}}$  are the 2x2 block diagonal partitions of the numerator and denominator power matrices, respectively. In the definition of component efficiency, we retained the pair of a modal displacement and its velocity as a component for obvious reasons. This is also to be observed in all component effi-



ciencies. The decoupled component efficiency  $e_j$  would be exactly the efficiency of the system if all initial states but the modal states  $j$  (displacement and velocity) were zero, hence, they indeed represent proper component contributions to the overall efficiency. We also recognize that  $e_j < 1$ .

Next, noting that

$$N = \sum_j n_j + \sum_{i \neq j} \sum \bar{X}_{io}^T P_{Nij} \bar{x}_{oj} \quad i, j=1, 2, \dots, n \quad (4.18)$$

and

$$D = \sum_j d_j + \sum_{i \neq j} \sum \bar{X}_{io} P_{Dij} \bar{x}_{oj}$$

where the second summations are the off block-diagonal power cross-coupling terms. In general, these off-diagonal contributions are sign variant, and therefore, one might expect them to vanish in a statistical sense if sufficient modal states are disturbed initially. Thus, we can write an expected total efficiency in terms of the block-diagonal decoupled powers as

$$E(e) \cong \frac{\sum_j^n n_j}{\sum_j^n d_j} \quad (4.19)$$

where  $E$  denotes expectation. Further, we caution that

$$e \neq \sum_{j=1}^n e_j.$$

Many studies that have been conducted during the course of this investigation verify that the expectation of overall efficiency in terms of decoupled component

contributions is quite relevant and off-block diagonal contributions to both the numerator and control powers are insignificant.

In the definitions of both the coupled and decoupled component efficiencies ( $e_j$ ) one notes that the numerator and denominator component contributions  $N_j$ ,  $D_j$ ,  $n_j$  and  $d_j$  can be computed exactly directly from the numerator and denominator power matrices  $P_N$  and  $P_D$  without invoking any of the results of the efficiency modes analysis. In these definitions of the exact component contributions via the power matrices summation over all principal controller directions  $i=1,2,\dots,2n$  is implicit. However, in reality not all principal controller direction contributions are significant, that is the least efficient controller directions need not be taken into account, therefore, it suffices in general to consider only the partial sums involving dominant controller modes in Eqs. (4.1), (4.2) and (3.27)

$$N_j \cong [X_0 \sum_{i=1}^{n_p} P_{Nj}]_{jj} \quad , \quad D_j \cong [X_0 \sum_{i=1}^{n_p} P_{Dj}]_{jj}, \quad n_p < 2n \quad j=1, \dots, 2n \quad (4.20)$$

where

$$P_{Ni} = \lambda_i L_i L_i^T \quad , \quad P_{Di} = L_i L_i^T \quad (4.21)$$

are controller power spectral components of the power matrices available only after an efficiency modal analysis. Similarly, for approximations to  $n_j$  and  $d_j$ :

$$P_{Njj} \cong \sum_i^{n_p} [P_{Ni}]_{jj} \quad \text{and} \quad P_{Djj} = \sum_i^{n_p} [P_{Di}]_{jj}, \quad j=1,2,\dots,2n \quad (4.22)$$

We are now in a position to propose model/controller reduction criteria based on the concept of efficiency. To this end, we shall utilize the component efficiencies  $e_j$  and principal efficiency components  $e_i$  discussed in this section.

#### 4.3 Controller Reduction in the Efficiency State Space - Efficiency Components Truncation

We use the transformation (3.26) from the modal space to the efficiency space to represent the closed-loop system in the efficiency space as given by Eqs. (3.28)-(3.31).

A controller reduction in the  $\epsilon$ -space can be affected by retaining only  $n_p$  components of  $\epsilon$  associated with the dominant controller modes with highest  $n_p$  principal efficiency components  $e_i$ , Eqs. (3.47) that is  $\{e_i\}_1 = \{e_1 > e_2 > \dots e_{n_p}\}$ ,  $n_p < 2n$ . Rearranging the controller modal matrix  $T$  in accordance with the set  $\{e_i\}_1$  we write:

$$x = T_1 \epsilon_1 + T_2 \epsilon_2 \quad (4.23)$$

where  $T_1$  are the controller modes corresponding to the characteristic efficiencies  $\lambda_i$  associated with the set  $\{e_i\}_1$  and subscript 2 denotes the truncated states. The decreasing order of principal efficiency components  $\{e_i\}$  does not necessarily correspond to highest  $n_p$  characteristic efficiencies  $\lambda_i$ . Similarly, the partitioned form of the closed-loop system in the efficiency space is:

$$\begin{bmatrix} \dot{\epsilon}_1 \\ \dot{\epsilon}_2 \end{bmatrix} = \begin{bmatrix} A_{11}^e + B_1^e G_1^e & A_{12}^e + B_1^e G_2^e \\ A_{21}^e + B_2^e G_1^e & A_{22}^e + B_2^e G_2^e \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} \quad (4.24)$$

where the matrices  $A^e$ ,  $B^e$  and  $G^e$  are given by Eqs. (3.28)-(3.31). A reduced order controlled system is obtained from the above by considering only the dynamics associated with  $\epsilon_1$

$$\dot{\bar{\epsilon}}_1 = [A_{11}^e + B_1^e G_1^e] \bar{\epsilon}_1 = A_{CL11}^e \bar{\epsilon}_1 \quad \bar{\epsilon}_{10} = \epsilon_{10} \quad (4.25)$$

for which the feedback control law is obtained by rendering  $G_2^e = 0$  in the full order control law Eq. (3.30).

$$F = G_1^e \bar{\epsilon}_1 \quad (4.26)$$

However, considering the full-order  $2n$ -th order design model Eq. (4.24) with  $G_2^e=0$ , the actual system dynamics with the reduced-order controller would be,

$$\begin{bmatrix} \dot{\epsilon}_1 \\ \dot{\epsilon}_2 \end{bmatrix} = \begin{bmatrix} A_{11}^e + B_1^e G_1^e & | & A_{12}^e \\ \hline A_{21}^e + B_2^e G_1^e & | & A_{22}^e \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} \quad (4.27)$$

Thus, in the actual system, the reduced-order states  $\epsilon_1$  and the truncated states  $\epsilon_2$  retain their dynamic coupling. Because the simulations of (4.25) and (4.27) will yield different responses for  $\epsilon_1$ , an overbar is used for  $\epsilon_1$  in (4.25) in ignoring the dynamic coupling of  $\epsilon_1$  with the truncated efficiency states  $\epsilon_2$ . The implicit assumption in this approach is that the system (4.27) remain stable with the reduced-order controller. This assumption seems reasonable since the reduction criterion is based on retaining the states associated with the most efficient controller directions, thus, the anticipation is that significant control power still remains with the reduced-order controller. Put in other words, the truncated controller directions associated with the  $\epsilon_2$  states with low principal efficiency components  $e_i$  will have high inefficiencies, thus, their elimination will save control authority. On the other hand, the stability of the reduced model Eq. (4.25) is guaranteed as we will note shortly. In addition, the efficiency of the reduced order model (4.25) with the reduced-order controller (4.26) would be of interest. Assuming that the  $A_{CL11}^e$  is stable, the Lyapunov Equations associated with the modal power of the design model  $S_C^M$  and the real power  $S^R$  become

$$A_{CL11}^{eT} P_o^M + P_o^M A_{CL11}^e + G_1^{eT} R_C^M G_1^e = 0 \quad (4.28)$$

$$A_{CL11}^{eT} P_o^R + P_o^R A_{CL11}^e + G_1^{eT} R^R G_1^e = 0 \quad (4.29)$$

Next, considering the  $n_p \times n_x$  upper left hand partitions of Equations (3.32) and (3.33), we obtain

$$A_{CL11}^{eT} \Lambda_{11} + \Lambda_{11} A_{CL11}^e + G_1^{eT} R_C^M G_1^e = 0 \quad (4.30)$$

$$A_{CL11}^{eT} I_{11} + I_{11} A_{CL11}^e + G_1^{eT} R^R G_1^{eT} = 0 \quad (4.31)$$

Comparing Eqs. (4.28)-(4.31) we conclude:

$$P_{\epsilon}^M = \Lambda_{11} \quad , \quad P_{\epsilon}^R = I_{11} \quad (4.32)$$

where

$$\Lambda_{11} = \text{diag}[\lambda_1^e \lambda_2^e \dots \lambda_n^e] \quad (4.33)$$

and  $I_{11}$  is the  $n_p$ -dimensional identity matrix. Conversely, associated with the closed-loop matrix  $A_{CL11}^e$ , since Eqs. (4.30), (4.31) guarantee  $\Lambda_{11}$  and  $I_{11}$  as positive-definite solutions, it follows that  $A_{CL11}^e$  is a stable matrix. Thus, we have shown that the reduced-order modal matrix  $T_1$  still simultaneously diagonalizes the modal and real power matrices  $S_C^M$  and  $S^R$  of the reduced-order model (4.25) without perturbing the original characteristic efficiencies associated with the  $\epsilon_1$  states. The  $n_p$ -th order efficiency of the reduced system becomes

$$e_{n_p} = \frac{\epsilon_{10}^T \Lambda_{11} \epsilon_{10}}{\epsilon_{10}^T \epsilon_{10}} = \frac{n_p}{\sum_{i=1,2,\dots} c_i^2 \lambda_i^e} \quad (4.33)$$

The change in the efficiency of (4.25) in comparison to the original system (4.24) is

$$\Delta e = e - e_{n_p} = \sum_{i=n_p+1}^{2n} c_i^2 \lambda_i^e - \sum_{i=n_p+1}^{2n} e_i \quad (4.34)$$

Hence, the efficiency of the reduced-system order in Eq. (4.25) will decrease by the amount of truncated efficiency components of the original system of Eq. (4.24).

This method of controller reduction in the  $\epsilon$ -space, because of the diagonal forms of the power matrices  $\Lambda_{11}$  and  $I_{11}$  is akin to the controller reduction via cost decoupled coordinates discussed in Ref. (7).

The controller reduction in the efficiency space is attractive from the point of view of that the reduced-order controller (4.26) requires no new computations for the control design since it is readily obtained by disregarding the control gains associated with the  $\epsilon_2$ -states in the original control law (3.30). Furthermore, the efficiency of the reduced system (4.25) requires no new computation either, since it is readily given by truncating the efficiency components  $e_i$  of the original system at  $i=n_p$  in (4.33). The method also has some disadvantages. The reduced order system (4.25) neglects dynamic coupling with the  $\epsilon_2$ -states which has to be assessed although one hopes that truncation of the least efficient states  $\epsilon_2$  results in truncation of least important dynamic coupling terms. Because of ignoring this dynamic coupling which is referred to as dynamic spillover between  $\epsilon_1$  and  $\epsilon_2$ , the sole assessment of the reduced-order controller (4.26) can only be obtained by simulation of the  $2n$ -th order system (4.27) instead of the  $n_p$ -th order system. However, in this case one notes that the reduced control to be simulated in (4.27) has the form  $F=G_1^o \epsilon_1$  instead of  $F=G_1^o \bar{\epsilon}_1$ . If one were to evaluate (4.27) by using the latter form of reduced control using feedback of  $\bar{\epsilon}_1$ , this would constitute an open-loop control with respect to the  $\epsilon_1$  and  $\epsilon_2$  dynamics and the system (4.27) would still remain neutrally stable. The problem is that although the  $\epsilon$ -states (or any other cost/power decoupling states as presented in Refs. 11, 12) diagonalize the power matrices  $P_N$  and  $P_D$  they remain dynamically coupled. The truncation of dynamically coupled states always leads to uncertainty of the behavior of the retained states in the actual system. In contrast, truncation of dynamically decoupled states still insures that the retained states in the actual system behave exactly as they do in the reduced-order model of the system. Certainly structural modal coordinates have precisely this feature and this should point

out why they are ever so desirable in the analysis and synthesis of structural dynamic systems. This being the case, it makes practical sense only to consider the reduced order controller form as  $F=G_1^* \bar{\epsilon}_1$  instead of using feedback of  $\bar{\epsilon}_1$  in (4.25) which leads one to consider Eq. (4.27) which in turn constitutes a closed-loop system. Finally, because the efficiency  $e_p$ , Eq. (4.33) of the reduced-order system given by (4.25) is obtained without regard to dynamic spillover between  $\epsilon_1$  and  $\epsilon_2$  states, the true efficiency of the dynamics of  $\epsilon_1$  states in (4.27) when the dynamic spillover is considered will be different from  $e_p$ , ta (4.33). This effect can indeed be significantly different in some cases so that the simplistic optimism provided by Eq. (4.34) to anticipate the change in the efficiency becomes unrealistic.

The evaluation of the stability, efficiency and the response of the system (4.27) will necessitate a recycle of the stability and efficiency analysis. This being the case, it proves more convenient to readdress the  $\epsilon$ -state truncation approach in terms of the original modal states  $x$  of which the structure-control designer has a better understanding. In this form, when studied in the modal space, the re-evaluation of the  $\epsilon$ -space controller reduction is tantamount to an efficiency filtering method. This we show in the next section.

#### 4.4 Efficiency Filtering

Implementation of the reduced-controller in the form of Eq. (4.26) requires estimation of the  $\hat{\epsilon}_1$  states which will theoretically be different than those states  $\epsilon_1$  in (4.27) with dynamic spillover considered. However, from an implementation point of view, there is no advantage gained in feeding back the  $\bar{\epsilon}_1$  as discussed in the previous section, as opposed to feeding back the actual  $\epsilon_1$

states with which there is an advantage over  $\bar{\epsilon}_1$  for the reasons just discussed.

Thus, we consider the reduced feedback controller

$$F = G_1^o \epsilon_1 \quad (4.35)$$

Then, Eq. (4.27) represents the actual system in the  $\epsilon$ -space. However, the control law (4.35) and the system (4.27) can be transformed back to the original modal  $x$ -space. Write the control law (4.35) as

$$F = [G_1^o \ 0] \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} \quad (4.36)$$

and note

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} = T^{-1} x = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} x, \quad [G_1^o \ G_2^o] = C [T_1 \ T_2] \quad (4.37)$$

Introducing the partitions into (4.36), we obtain

$$F = GT_1 L_1 x = \bar{G} x \quad (4.38)$$

which yields the new closed-loop dynamics

$$\dot{x} = [A + B\bar{G}]x = [A + BGT_1 L_1]x \quad (4.39)$$

Therefore, the  $\epsilon$ -space truncation of  $\epsilon_2$  states is equivalent to full  $x$ -state feedback through the new feedback gain matrix  $\bar{G}$  which simply represents a projection of the original  $G$  matrix through the matrix  $T_1 L_1$  which is readily available as soon as the  $\epsilon_1$  partition is decided. The matrix  $T_1 L_1$  essentially filters out the contribution of the least efficient  $\epsilon_2$  states to the  $x$ -states. Hence, we refer to this method as efficiency filtering.

Alternately, noting that

$$T_1 L_1 = I - T_2 L_2 \quad (4.40)$$

the closed-loop system becomes:

$$\dot{x} = [A + BG(I - T_2 L_2)]x = [A_{CL} - BGT_2 L_2]x \quad (4.41)$$

or

$$\dot{x} = [A_{CL} + \Delta G]x, \quad \Delta G = BGT_2 L_2 \quad (4.42)$$



hence, the closed-loop systems of Eqs. (4.39) and (4.42) are equivalent and they can also be viewed as the perturbation of the original system (3.31) through the modal gain error matrix  $\Delta G$ . Because all matrices involved in  $\Delta G$  are known, the gain perturbation is a structured perturbation. The stability and the performance robustness (perturbation in  $P_C^M$  and  $P_R$ ) of the perturbed system (4.42) can now be studied by the methods of Refs. (12) and (13). From the performance bounds on  $P_C^M$  and  $P^R$ , the robustness of efficiency to filtering the  $\epsilon_2$  states can easily be inferred. We remind that, the study of stability and performance of efficiency filtered closed-loop system (4.42) in terms of the x-state is equivalent to the study of the  $\epsilon$ -state Equation (4.27) by using the reduced-controller (4.35).

The  $\epsilon$ -state truncation and the efficiency filtering technique as a way of obtaining and evaluating reduced order controllers do not seem attractive since the approach anticipates a lower efficiency in the reduced system. Furthermore, the truncation of efficiency states represents a further degree of abstraction in physical understanding of the controller-reduction in contrast to truncation of modal states. Also, there is the problem of dynamic spillover associated with the approach, therefore, it is plausible to search for controller reduction criteria which will culminate in an increase in the efficiency of the reduced-order system and the reduction is affected in terms of the original modal (x-space) states. At the very least, if no significant degradation of the efficiency is experienced in such a reduction process, the method would be preferable over the  $\epsilon$ -state reduction method. The underlying philosophy of such a method is then to identify and truncate modal coordinates  $x$  in the design model which have significant inefficiency associated with them so that their deletion will make more efficient use of control power available to the modes retained thereby

resulting in an increase in the efficiency of the controller despite a reduced-order system.

#### 4.5 Efficient Modal Controller Reduction

The premise of the controller reduction methodology we shall propose in this section is that the structural modal states  $x_j$  that contribute least to a given set of principal efficiency components  $e_i$  along the associated principal controller directions are to be deleted. Conversely, the modal coordinates which contribute most to a given set of spillover components  $sq_i$  along the associated principal controller directions are to be deleted. It turns out the two views do not necessarily yield the same set of truncated modal states.

The method involves computation of coupled or decoupled component efficiencies  $e_j$  as defined by Eqs. (4.16) and (4.17) for an initial, structural-control design model and performing the  $2n$ -th order efficiency modal analyses for one or more of the quotients  $e$ ,  $e^*sq$  and  $sq^*$ . Upon studying the characteristic eigenvalues and the principal efficiency components of these quotients along the principal controller directions the significant components of relevant efficiency and/or spillover quotients are recognized to retain  $n_p \leq 2n$  principal directions in the computation of component efficiencies  $e_j$  according to Eqs. (4.20)-(4.22) where the summation over  $i$  involves  $n_p$  controller directions. Note that depending on which quotients are studied, the quotient  $e_j$  in Eqs. (4.20)-(4.22) could specifically represent any of quantities  $e_j$ ,  $e_j^*$ ,  $sq_j$  and  $sq_j^*$ .

After computing the component efficiencies, one then retains a desired number  $n_R$  of the structural components with the highest component efficiencies  $e_j$  and/or  $e_j^*$ . Conversely, if the above analyses are based on spillover quotients one retains  $n_R$  structural modal coordinates with the least component spillover

quotients  $sq_i$  and  $sq_i^*$ . As shown in Section (3.3)  $e$  and  $sq$  have the same eigenvectors, that is the same principal controller directions and their eigenvalues are related as  $\{\lambda^e\}$  and  $\{\lambda^s\} = \{1 - \lambda^e\}$ . Therefore, if  $e$  and  $sq$  are considered, modal analysis of either one will be sufficient. If, however,  $e^*$  and  $sq^*$  are desired, no immediate relationship between their eigenvectors and eigenvalues are recognized, so modal analyses for both of them must be performed. Furthermore, recall that since  $e$ ,  $e^*$  and  $sq$  cannot be greater than unity their characteristic eigenvalues cannot be greater than unity. The eigenvalues of the global spillover quotient  $sq^*$  can be greater than unity. After selecting the  $n_R$  structural modal coordinates as suggested, two avenues are available. Restating here for convenience the original control design model

$$\dot{x} = Ax + BF \quad , \quad F = Gx$$

on which the efficiency analyses are performed, the first avenue is to retain only the  $2n_R$  states  $x_R$  and truncate the remaining modal states and implement the reduced order model

$$\dot{x}_R = A_R x_R + B_R F \quad , \quad F = G_R x_R \quad 2n_R < 2n \quad (4.43)$$

without redesigning the feedback control law where  $A_R$ ,  $B_R$  and  $G_R$  are the partitions of  $A$ ,  $B$  and  $G$  corresponding to the  $2n_R$  states  $x_R$ . This is, in general, the most common spirit of controller reduction. To assess the effectiveness of the reduced order model the stability and new efficiency of the system (4.43) must be studied. In general, the characteristic efficiencies and the modes of the controller will change. But, if the reduced-order model identified is a good one its efficiency features will have changed over the original system in the direction that the analysis anticipates, that is truncation of least efficient or most inefficient modal states should leave behind at least a more efficient reduced model.

The second avenue available for controller reduction, rather than truncating the original control law, is to redesign the feedback control law for the set of modal states  $x_R$

$$\dot{x}_R = A_R + B_R F_R \quad , \quad F_R = G_{RR} x_R \quad (4.44)$$

where  $G_{RR}$  is the new redesigned control gain matrix for the reduced-order model for the reduced structural system. For example, if the initial control design is obtained via LQR it will be repeated for the system with the performance measure

$$2J = \int (x_R^T W_{xR} + F_R W_{CR} F_R) dt \quad (4.45)$$

Of course, the control redesign is a conservative approach. It guarantees the stability of the reduced order model and its efficiency can be reanalyzed. Intuitively one would stand a better chance of improving it with the control redesign.

The redesigning of the controls is viewed here as a more realistic and, in fact, practical approach to the model/controller reduction. The reduced order model (4.44) is smaller than the original control design model for which algorithmic and theoretical capability exists to begin with. Consequently, application of the same capability to a reduced order system by using the same tools should not be a problem. Furthermore, the process of identifying a good reduced order model is an off-line procedure and performing control redesigns and efficiency modal reanalyses do not require more computational resources than available to perform the original  $2n$ -th order design. Countless, aimless redesigns are not advocated here to identify a good reduced model that satisfies the system constraints. On the contrary, the efficiency modal analysis presented heretofore, does lay a foundation for the designer for a very much directed and

purposeful avenue to do the redesigns efficiently so that at least one perhaps only a few redesigns will finish the job.

The final point that must be noted is that both types of reduction concepts discussed in this section and in Section 4.4, that is the efficient, modal control reduction and the efficiency-filtering can be used simultaneously. The truncation of modal structural states based on the criterion of this section to retain a  $2n_R$ -order structural dynamics model can be viewed as a closed-loop model reduction. On the other hand, the truncation of efficiency-states to obtain a control input as the feed back of a reduced number of  $n_p$ -efficiency states can be viewed as the closed-loop controller reduction. The control law design can thus, be designed for a  $2n_R$ -th reduced-order design model in the form of reduced  $n_p$ - $\epsilon_1$ -states' feedback, thus, affecting a simultaneous model/controller reduction technique.

In the following section, we discuss a number of additional aspects to guide the designer in using the efficiency approach. The process readily produces more efficient designs as we will present in the next section.

#### 4.6 Further Considerations for Efficient Model/Controller Reductions

Certain other features of the approaches discussed in this chapter, must be noted. Consider the pair  $e, sq$  for the design model. Their components  $e_i = c_i^2 \lambda_i^0$  and  $sq_i = c_i^2 (1 - \lambda_i^0)$  have the property that the indices  $i$  in the set  $(e_i)$  with the highest  $n_p$  components will not necessarily correspond to the indices  $i$  in the set  $(sq_i)$  with the lowest  $n_p$  components. This anomaly depends on the relative values of the coefficients  $c_i^2$  which are normalized square initial disturbances along the principal controller directions. As this may seem disheartening, one cannot, however, change the fact that efficiency or the inefficiency of the structure

control system can be a strong function of the initial disturbance  $x_0$ . Some disturbances are better handled by a given system design whereas others result in poor efficiency performance.

In choosing the  $n_p$  efficiency components  $e_i$ , we do not necessarily consider the highest  $n_p$  of them although this seems like a logical choice. Further care must be taken to identify the set  $\{e_i, i=1, \dots, n_p\}$ . A principal efficiency component  $e_i$  may be high either because of a high  $c_i^2$  and low  $\lambda_i^e$ , or low  $c_i^2$  and high  $\lambda_i^e$  or both. Recalling that a spillover quotient  $sq_i$  is given by (3.45), in the first case, the corresponding spillover coefficient  $sq_i$  will also be high, in the second case,  $sq_i$  will be low and in the third case it will be relatively lower. All depends on relative separations of  $c_i^2$  and  $\lambda_i^e$ , so retaining a high  $e_i$  may also mean retaining a high spillover quotient  $sq_i$ . The word of caution is that before dominant  $e_i$ 's are taken into account corresponding  $sq_i$ 's must also be scrutinized. Similar caution must be taken in case of  $e_i^*$  and  $sq_i^*$ .

Along the same lines, if an initial control design has a low efficiency, it makes little sense to select modes based on their relative contributions to an already low efficiency. Instead, one should analyze their contributions to a high spillover quotient and retain or delete them based on that account. Conversely, one should bring in new modes into the control design model that were discarded in the original analysis.

The model reduction process that is being discussed here seems to assume that the controller configuration (number of inputs and their locations) is fixed. In reality, however, model reduction (or mode selection) process and the controller configuration are interdependent. For design purposes, one would normally try to achieve a desired level of efficiency for a reduced model order of  $2n_R$  and with an upper bound on the number of inputs  $m \leq m_{\max}$ . Hence, because

the efficiencies are also a function of controller configuration, the controller reduction process should also utilize the controller configuration changes. The ultimate goal is, for the given initial disturbance, to identify the best reduced order model with the best input configuration to go with it to achieve a high efficiency for the structure-control system.

In the above discussion, it is the initial disturbance represented in the efficiency coefficients  $c_i^2$  responsible for not matching the set  $\{e_i\}$  vs.  $\{sq_i\}$  of  $\{e_i^*\}$  and  $\{sq_i^*\}$ . Hence, for a given control design, selection of the best principal controller directions will change if the initial disturbance changes. An alternate approach to selecting the set  $n_p$  of principal controller directions which avoids the initial disturbance is to consider the controller directions associated with the  $n_p$  highest characteristic efficiencies  $\{\lambda_i^0 \ i=1, \dots, n_p\}$ . Another feature of this choice is that the highest set  $\{\lambda_i^0\}$  will now correspond to the  $n_p$  lowest of characteristic spillover values  $\{\lambda_i^0 = 2, \dots, n_p\}$ .

Recall from Section 3, that from the properties of a Rayleigh's quotient efficiency is bracketed by  $\lambda_{\max} \geq e \geq \lambda_{\min}$ . It follows that a control system whose efficiency will be least sensitive to the initial disturbance  $x_0$  must have a minimal separation between  $\lambda_{\max}$  and  $\lambda_{\min}$ , that is if the characteristic efficiencies are dense in a short interval. Ideally, if  $\lambda_{\max} = \lambda_{\min}$ , the control system will be uniformly efficient, better said equalized in all principal directions and will have the same efficiency regardless of  $x_0$ . This would be desirable feature. In addition, if  $\lambda_{\min}$  can be made high then the system will also have a high efficiency regardless of  $x_0$ . The price that must be paid to make the structure-control system efficiency insensitive to  $x_0$  is that all principal controller directions will now become equally significant, hence, truncation of efficiency states will not be possible as each  $\epsilon$ -state and its principal direc-

tion will contribute equally to the efficiency. Therefore, one will not be able to discriminate among the principal controller directions to affect their truncation in the  $\epsilon$ -space.

Another feature related to this last observation is the sensitivity of the stability of the system to the principal controller directions. Since stability of a linear system does not depend on the initial conditions, the stability of a system truncated based on the efficiency will depend not on the magnitudes of the truncated efficiency components  $e_i(i=n_p+1, \dots, 2n)$  but on the modal properties  $\lambda_i$  and  $t_i$  associated with the truncated  $e_i$ . The more significant the modal properties of the truncated principal controller directions are, the easier it will be to destabilize the system via efficiency truncation. Again, because a poor  $x_0$  can associate poor component efficiencies  $e_i$  with significant controller modes, efficiency reduction based on truncation of controller modes of poor  $e_i$  will conflict with maintaining closed-loop stability. Stability robustness, then, to efficiency-state truncation should be good if the truncated controller modes of poor  $e_i$  are not significant. On the other hand, if the insensitivity of the efficiency of the structure-control system to initial disturbance is desired, since this will necessitate equalization of the characteristic efficiencies, each controller mode will become equally important. Consequently, insensitivity to  $x_0$  of the efficiency of the system, in the presence of a desire to controller mode truncation, will come at the expense of system with a sensitive stability.

In all of these endeavors, the efficiency and spillover quotients and the modal analyses serve as the design directors qualifying and quantifying the merits of design changes to achieve the best possible performance with high efficiency. In the next section, a number of illustrations are presented utilizing



all that is discussed in this chapter. Again, the particular structure is the ACOSS-4 tetrahedral structure used in the previous chapters.

#### 4.7 Illustrative Examples on ACOSS-4

The same situation as in Section 3.6 is considered.

##### Decomposition of Efficiency Quotients

For the sake of brevity, the decomposition of the numerator and denominator control power matrices are presented only for a 4-th order control design model. The efficiency components  $e_1$  and coupled and decoupled component efficiencies of the structural modes are also shown in Appendix C.

Example 4.1: This is the same case as in Examples 3.1. The numerator/denominator control power decomposition matrices [N] and [D] are given on in Appendix C for both global and model efficiencies. Efficiency components are  $e_1$ ,  $e_1^*$  and component efficiencies  $e_j$ ,  $e_j^*$  for coupled (Method 1) and decoupled (Method 2) cases are also shown.

Example 4.2: The case of Example 3.4 is given with the same information listed under Example 4.1.

##### Efficiency Decomposition Using $n_p$ -principal Controller Modes

Example 4.3: The same system of Example 4.2 is considered. However, only the six ( $n_p=6$ ) highest characteristic global and relative model efficiencies were used in the decomposition of numerator/denominator control power matrices and the component efficiencies of the

structural modes were based on the 6 retained dominant controller directions.

#### In/Efficiency - filtering; Controller Reduction

Example 4.4: Example 3.5 with  $n=8$  (natural order) modes and  $m=4$  was considered. The original model had  $e^*=30.75\%$ ,  $e=80.65\%$ ,  $sq^*=0.628$  and  $sq=19.35\%$ . Although the model efficiency  $e$  is good, the global efficiency  $e^*$  seems relatively poor. Hence, an inefficiency of 70% is associated with the global efficiency which is reflected in the value of  $sq^*$ . (Note that by definition  $sq^*=1-e^*$ ). Since there is more inefficiency associated with  $e^*$  than efficiency, the modal analysis was done on  $sq^*$ . The efficiency-filtering was performed on the modes of  $sq^*$  by truncating the highest  $n_p$  global spillover efficiency components  $sq_i^*$   $i=1,2,\dots,n_p$ . Up to  $n_p$  spillover components could be filtered without destabilizing the original control design. The original gain matrix  $G$  was simply projected through  $\bar{G}=GT_2L_2=G(1-T_1L_1)$ , as in Eq. (4.41) to obtain the filtered gain matrix  $\bar{G}$ . Note that the  $n_p$  principal global spillover components were deleted, not retained, so it is necessary to retain the remaining components represented by subscript 2. Conversely, the projection  $(1-T_1L_1)$  eliminates the highest contributing  $n_p$  states represented by subscript 1. Simulation results for  $n_p=4$  and  $n_p=8$  are shown by figures SIM32 (Figures 4.1, 4.2 and 4.3) and SIM34 (Figures 4.4, 4.5 and 4.6), respectively. The efficiencies of both of these  $sq^*$ -filtered cases were recomputed. For  $n_p=2$ , the new efficiency quotients

were  $e^*=26.71\%$ ,  $e=4.32\%$ ; for  $n_p=4$  the new values were  $e^*=40.9\%$ ,  $e=73.18\%$ . Hence, sq\*-filtering did improve the global efficiency over the original design by about 1/3. Other pertinent details are shown on the computer outputs listed in Appendix C. Note that the simulation results show hardly any difference from the original design SIM10 (Figures 3.10, 3.11, and 3.12). The consequence of this particular study is that the 16th order design model can be controlled by the feedback of only 8 efficiency states, hence, resulting in a 50% reduction in the controller size.

#### Efficient Model Selection (Reduction)

Example 4.5 The model efficiency of the 16-th order control design model of the first 8 natural modes with four inputs ( $n=8$  natural,  $m=4$ ) which was studied in Example 3.5 and Example 4.4 was  $e=80.25\%$ . In an effort to improve the relative model efficiency  $e$  further by selecting a different set of 8 modes other than the lowest 8 natural modes, the component efficiencies of Example 3.5 were scrutinized. The component efficiencies for both  $e$  and  $e^*$  are listed in Appendix C. Modes 1, 2, 7 and 8 have the lowest coupled component efficiencies  $e_j^*$  and  $e_j$  according to Method 1. Hence, these modes were replaced with the modes that were not in the original set. A redesign was done for the new 16th order design model with the new selected set  $n=\{\text{Modes: } 3-6, 9-12\}$ . The model efficiency increased to  $e=97.21\%$ , although the global efficiency was reduced to  $e^*=10.27\%$ . Simulation results are shown in Figures SIM35 (Figures 4.7, 4.8 and 4.9).

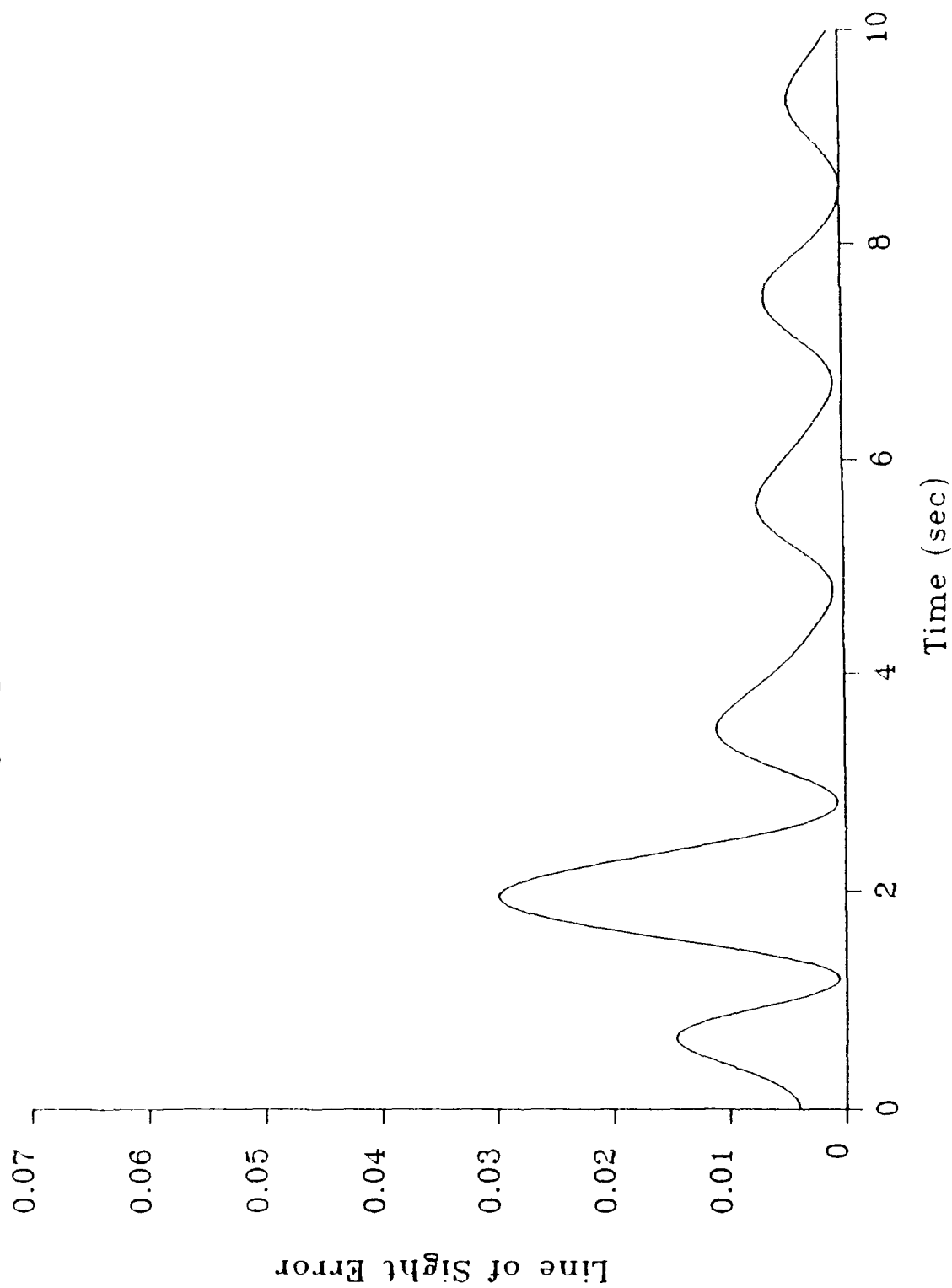


Figure 4.1: Globally optimal system LOS for  $n=8$  (modes: 1-8); example 4.4; controller reduction via efficiency filtering for  $n_p=4$

Suboptimal System SIM32

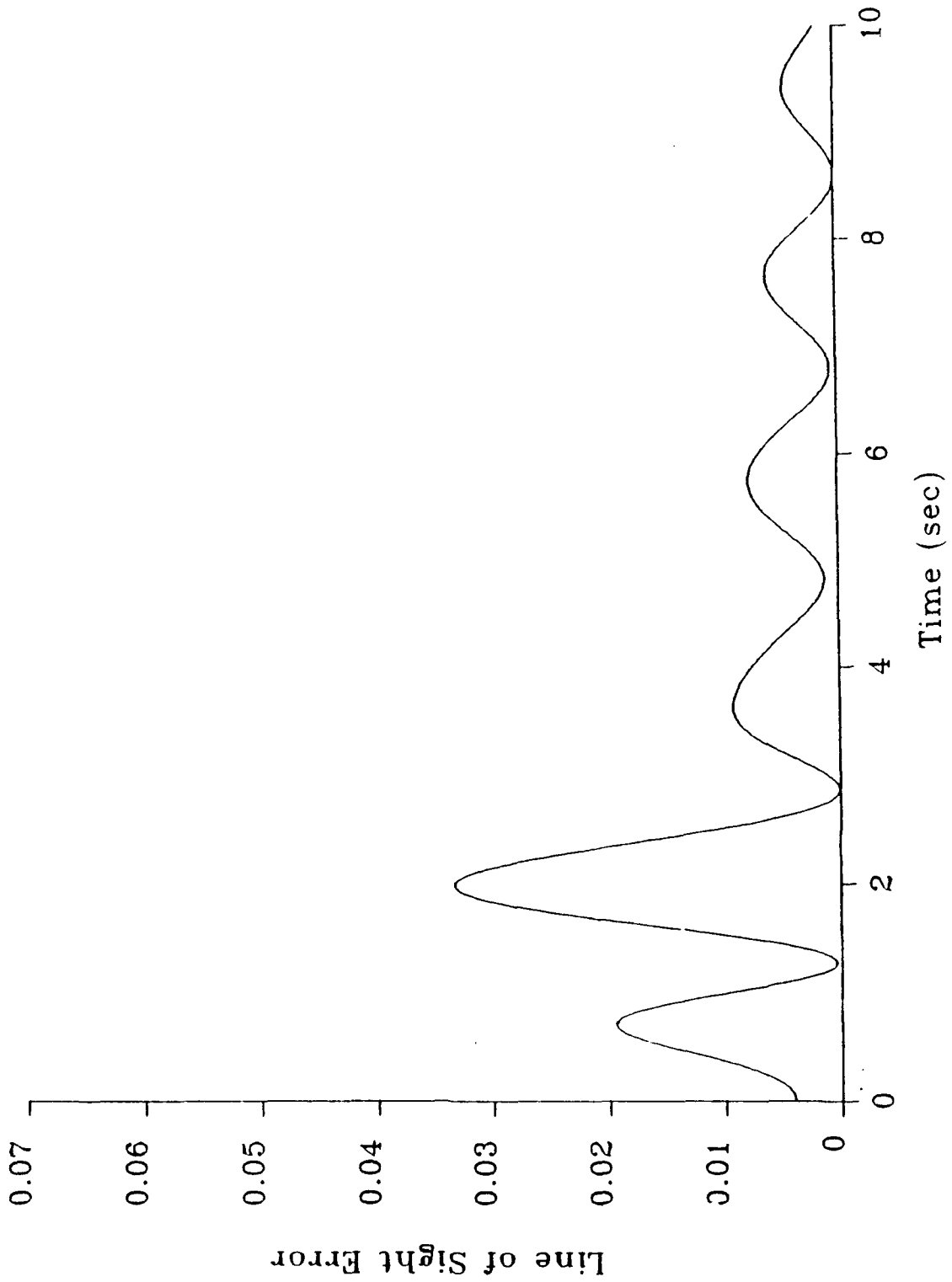


Figure 4.2: Suboptimal system LOS for  $n=8$  (modes: 1-8),  $m=4$  (#1-4); example 4.4; controller reduction via efficiency filtering for  $n_p=4$

Evaluation Model SIM32

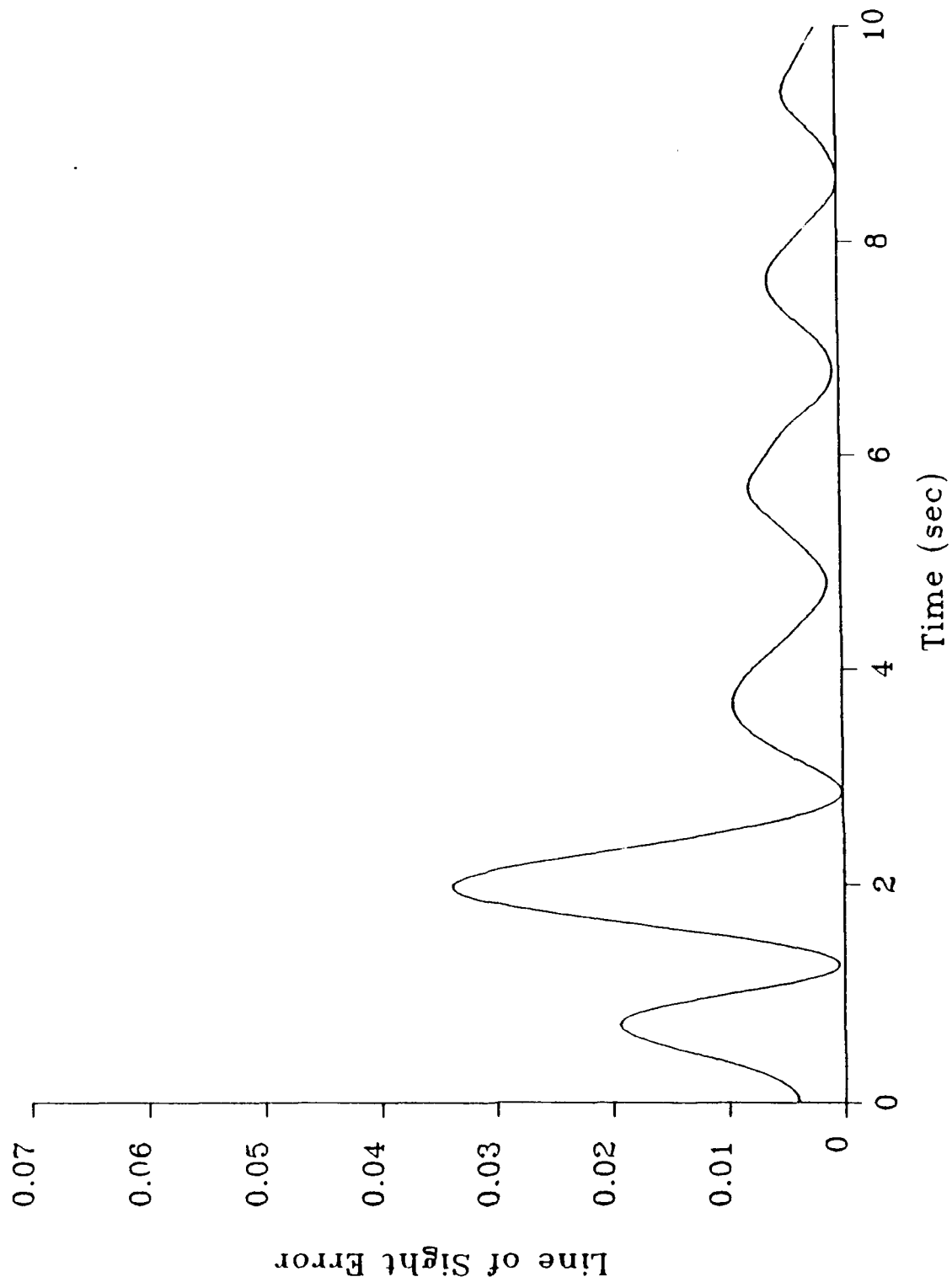


Figure 4.3: Evaluation model LOS for  $n=8$  (modes: 1-8),  $m=4$  (#1-4), example 4.4; controller reduction via efficiency filtering for  $n_p=4$

Globally Optimal System SIM34

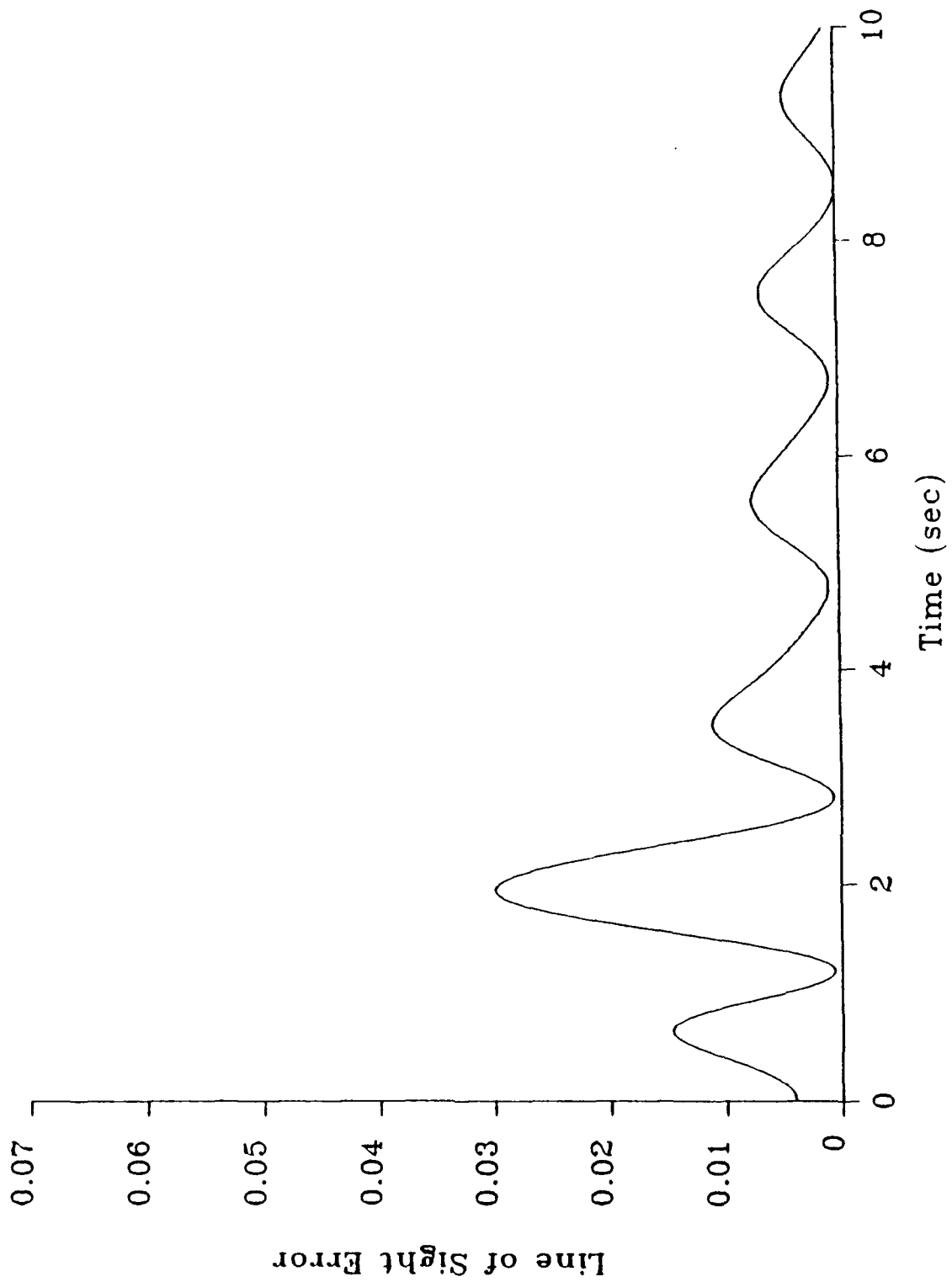


Figure 4.4: Globally optimal system LOS,  $n=8$  (modes: 1-8), example 4.4; controller reduction via efficiency filtering for  $n_p=8$

# Suboptimal System SIM34

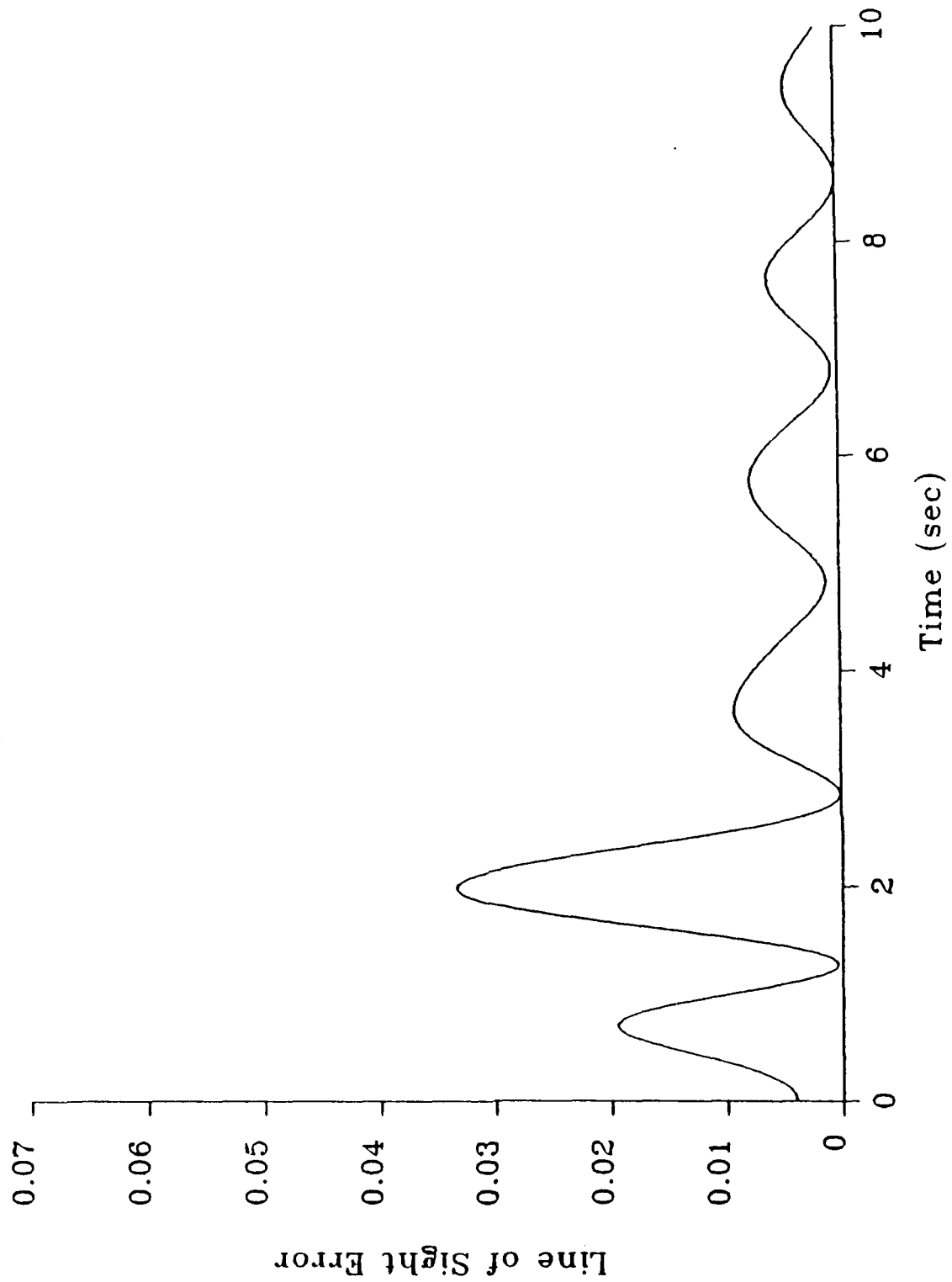


Figure 4.5: Suboptimal system LOS,  $n=8$  (modes: 1-8),  $m=4$  (#1-4), example 4.4; controller reduction via efficiency filtering for  $n_p=8$



# Evaluation Model SIM34

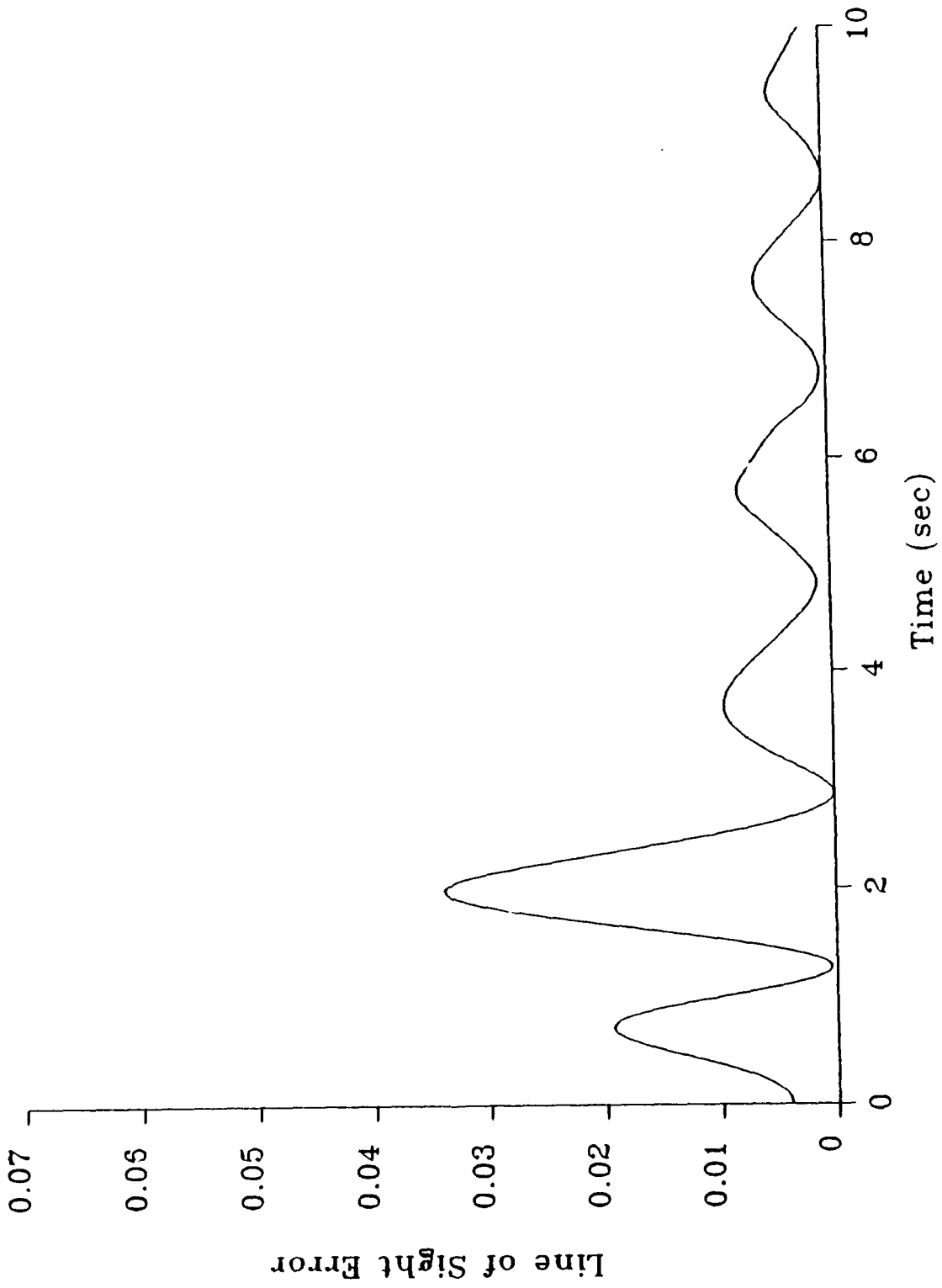


Figure 4.6: Evaluation model LOS,  $n=8$  (modes: 1-8),  $m=4$  (#1-4), example 4.4; controller reduction via efficiency filtering for  $n_p=8$

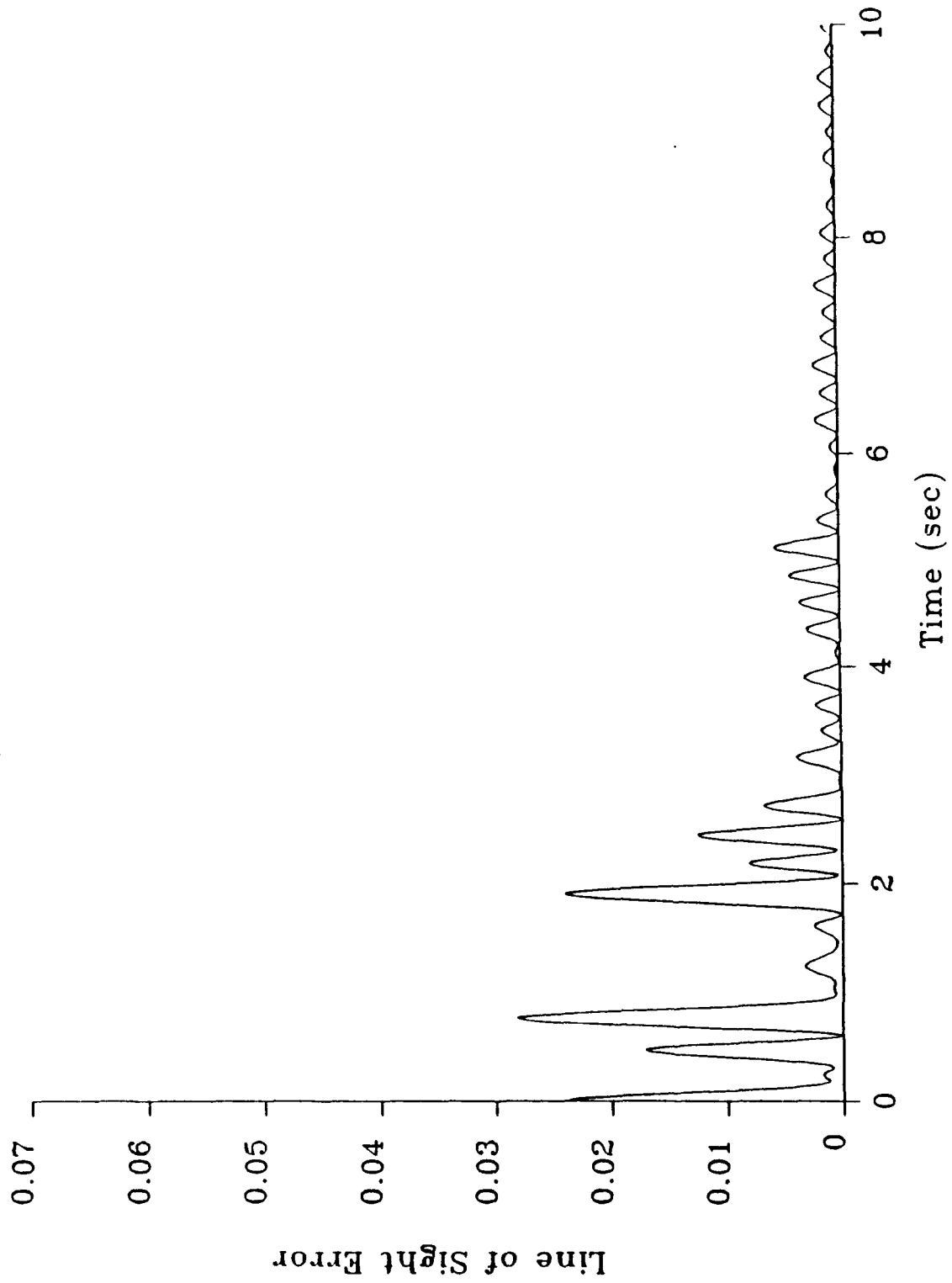


Figure 4.7: Globally optimal system LOS, n=8 (modes: 3-6, 9-12); example 4.5; efficient model selection reduction

Suboptimal System SIM35

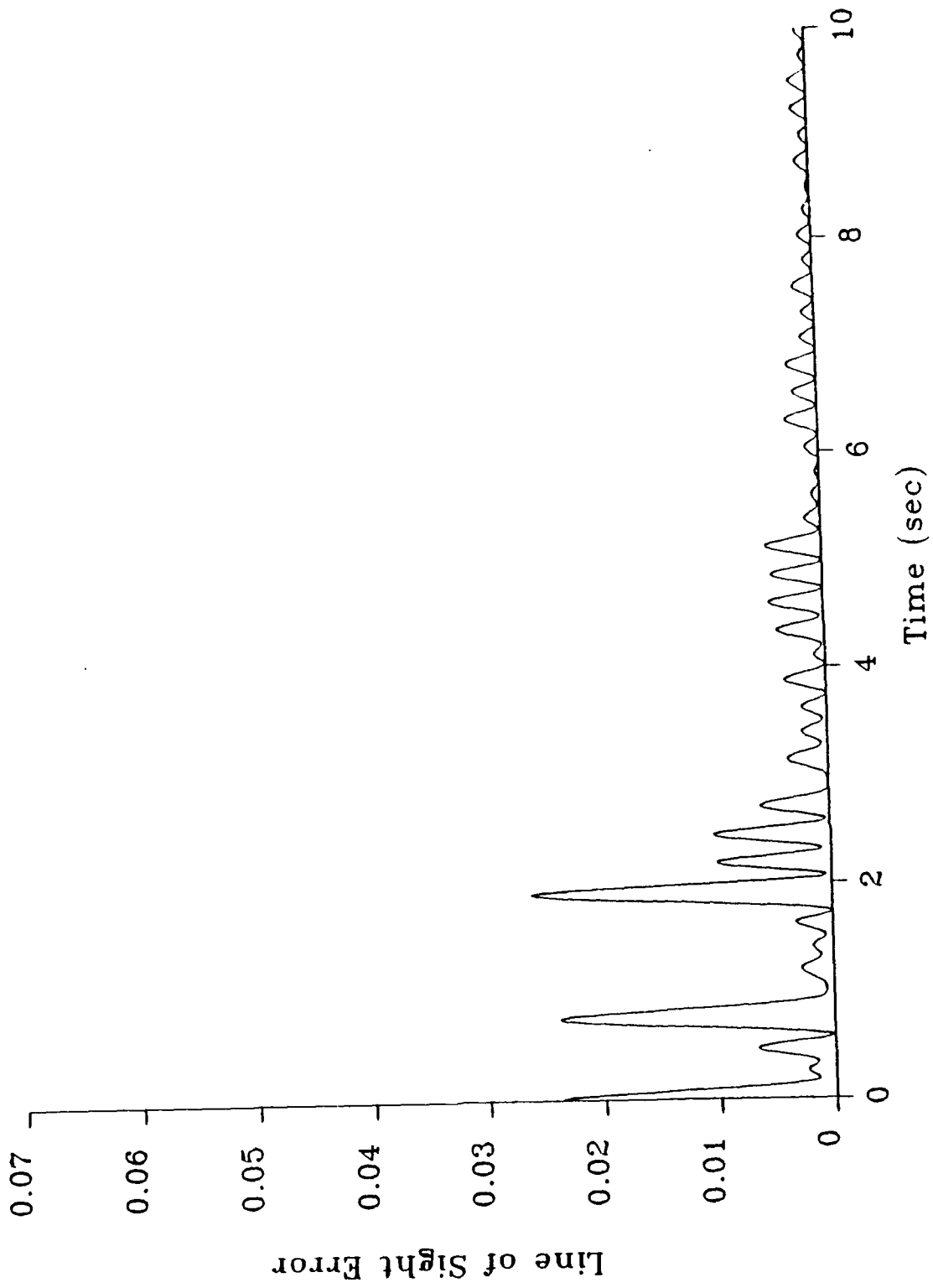


Figure 4.8: Suboptimal System LOS,  $n=8$  (modes: 3-6, 9-12);  $m=4$  (#1-4), example 4.5; efficient model selection (reduction)

Evaluation Model SIM35

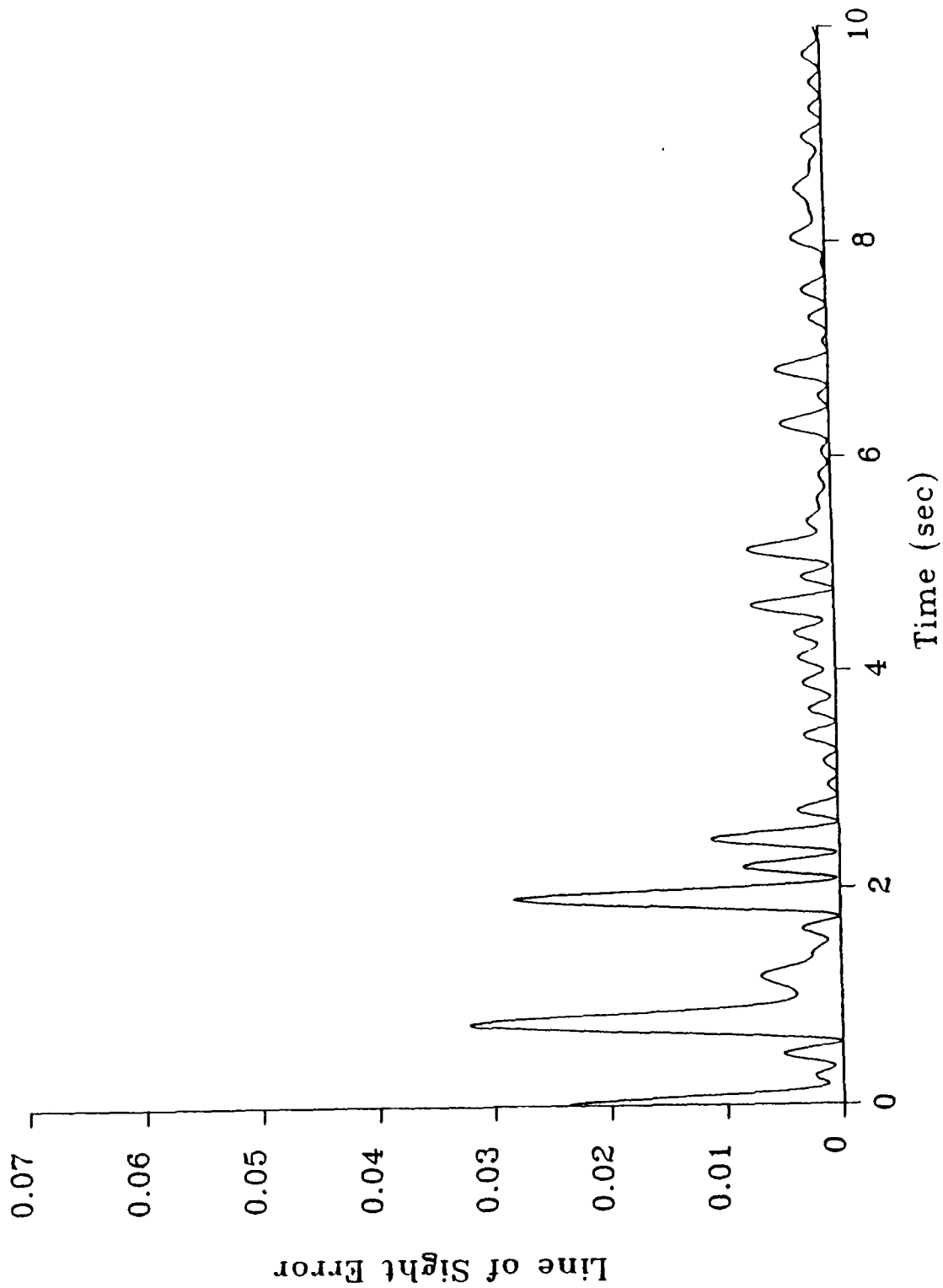


Figure 4.9: Evaluation model LOS,  $n=8$  (modes: 3-6, 9-12),  $m=4$  (#1-4), example 4.5; efficient model selection (reduction)

## 5.0 EFFICIENCY OF SYSTEMS WITH COMPENSATOR DYNAMICS

### 5.1 Introduction

The developments presented in the previous chapters were based on a static state feedback control law  $F-Gx$ . In this chapter, we consider the case whereby the feedback control law is obtained via a dynamic compensator and extend the efficiency concepts studied in Chapters 2-4 to consider the effect of compensator dynamics. The extension of the previous results, in this case, is straightforward and the idea of compensator efficiency within the structure-control system arises naturally.

In Section 5.2, efficiency quotients are revisited including the compensator dynamics. Two definitions of compensator efficiency are given following the component efficiency definitions introduced in Section 4. A most common dynamic compensator is the state-estimator in the control loop. In Section 5.3, we, therefore, pay special attention to the design of a reduced-order state estimator relevant to the subject of this research. In particular, an  $n_p$ -th order efficiency state estimator design is given where  $n_p$  efficiency states  $\epsilon_1$  are estimated to implement the reduced-order control law  $F=G_1^o \epsilon_1$  discussed in Sections 4.3 and 4.4. Here  $n_p$  is the number of dominant principal controller directions which will have been identified from an efficiency modes analysis of the system, hence,  $n_p < 2n$ . Finally, in Section 5.4 we consider formally the quantitative and qualitative effect of control power spillover on the response of the residual dynamics not considered in the  $2n$ -th order control design model. Although this topic may have been discussed in Chapter 2, we postponed its presentation until after the discussion of compensator dynamics, so that it will seal appropriately all the developments presented heretofore and conclude this

investigation by underlining the critical nature of the efficiency of structure-control systems.

No illustrative examples are presented in this chapter

## 5.2 Efficiency of Structure - Control Systems with Compensator Dynamics - Compensator Efficiency

In the previous chapters we assumed that the control law was a simple state-feedback. In this chapter, we generalize the concepts to the case where the control law is obtained via a dynamic compensator. Again, the system is described by the modal state-space equations

$$\dot{x} = Ax + BF \quad (5.1)$$

The controller is in this case defined by the following dynamic compensator equations with an  $n_c$ -dimensional state vector  $x_c$ .

$$\dot{x}_c = A_c x_c + B_c F + K_c y \quad (5.2)$$

$$F = G_c x_c + H_c y \quad (5.3)$$

where the  $m_y$ -dimensional system output  $y$  is given by

$$y = Cx \quad (5.4)$$

Then the closed-loop system is described by

$$\dot{\bar{x}} = \bar{A}_{CL} \bar{x} \quad , \quad \bar{x} = [x^T \ x_c^T]^T \quad (5.5)$$

$$\bar{A}_{CL} = \left[ \begin{array}{c|c} A+BH_cC & BG_c \\ \hline B_cH_cC+KC & A_c+B_cG_c \end{array} \right] \quad (5.6)$$

and the control law

$$F = \bar{G}\bar{x} \quad , \quad \bar{G} = [H_cC \ G_c] \quad (5.7)$$

Since the definitions of the global efficiency and the model efficiency are defined in terms of only inputs  $F$  to the dynamic system not on how they are

computed or derived their definitions Eqs. (3.6-3.8), are still valid, hence, using the form of F as given by Eq. (5.3) in Eqs. (3.6), (3.7) we obtain

$$\bar{S}^R = \int F^T R^R F dt = \bar{x}_0^T \bar{P}^R \bar{x}_0 \quad (5.8)$$

$$\bar{S}^M = \int F^T R_C^M F dt = \bar{x}_0^T \bar{P}_C^M \bar{x}_0 \quad (5.9)$$

where  $\bar{P}$  are the  $(n+n_c)$ -dimensional power matrices obtained from the Lyapunov Equations:

$$\bar{A}_{CL}^T \bar{P}^R + \bar{P}^R \bar{A}_{CL} + \bar{G}^T R^R \bar{G} = 0 \quad (5.10)$$

$$\bar{A}_{CL}^T \bar{P}^M + \bar{P}^M \bar{A}_{CL} + \bar{G}^T R_C^M \bar{G} = 0 \quad (5.11)$$

in which  $R^R$  and  $R_C^M$  are still as given by Eqs. (3.6) and (3.7)

$$R^R = D^T M^{-1} D \quad , \quad R_C^M = D^T E_n E_n^T D$$

The model efficiency  $e$  and the spillover quotient  $sq$  are computed exactly as described in Chapter 3 by using

$$P_N = \bar{P}^M \quad , \quad P_D = \bar{P}^R \quad (5.12)$$

and all that is discussed in Chapters 3 and 4 will apply equally well to control with compensator dynamics. The global efficiency  $e^*$ , however, needs a little careful consideration. Recall that this definition involved control of the DPS with a continuously distributed input profile, where the optimal control was the independent feedback of the modal states of the DPS structure. An important feature of this optimal DPS control is that it is dynamically similar to the dynamics of the control design model of the structural modes in the closed-loop system. Hence, even in the case of a dynamic compensator, we shall insist on the similarity of the DPS distributed control design to the structural modes alone, regardless of the modes in the closed-loop system due to the additional compensator dynamics. Thus, if  $\{\rho\} = \{\bar{\rho}_S, \bar{\rho}_C\}$  is the eigenvalue spectrum of the closed-loop system, Eq. (5.5), with the compensator, as a union of the structural eigenvalue spectrum  $\{\bar{\rho}_S\}$  and the compensator eigenvalue spectrum  $\{\bar{\rho}_C\}$ , the control

power  $S^*$  of the optimal distributed solution will be found by using  $(\rho_S)$  in the equations given in Section 2.4.

The global efficiency will then be

$$e^* = \frac{S^*}{S^R} \quad (5.13)$$

Although at this point, it is not clear whether  $S^*$  is greater or less than  $S^R$  with a dynamic compensator the quotient  $e^*$  will still preserve its function as a practical concept since it will relate the power performance of the actual design represented via  $S^R$  to an ideal baseline design  $S^*$  being a measuring stick. Specifically, the function of the compensator will be highlighted in such a setting. Thus writing the control power  $S^*$

$$2S^* = x_0^T P^* x_0 \quad (5.14)$$

the global efficiency  $e^*$  will be given by

$$e^* = \frac{x_0^T P^* x_0}{\bar{x}_0^T \bar{P}^R \bar{x}_0} \quad (5.15)$$

where  $P^*$  is again computed as in Appendix B for the spectrum  $(\rho^*) = \{\bar{\rho}_s\}$ . Alternately, augmenting  $P^*$  with zeros

$$P^* = \begin{bmatrix} P^* & O_{n \times n_c} \\ O_{n_c \times n} & O_{n_c \times n_c} \end{bmatrix}$$

$e^*$  can be written as

$$e^* = \frac{\bar{x}_0^T \bar{P}^* \bar{x}_0}{\bar{x}_0^T \bar{P}^R \bar{x}_0} \quad (5.17)$$



Next, partition the power matrices  $\bar{P}_C^M$  and  $\bar{P}^R$  corresponding to the structural modal states  $x$  and the compensator states  $x_c$

$$\bar{P}^R = \begin{bmatrix} P_S^R & P_{SC}^R \\ P_{SC}^{RT} & P_C^R \end{bmatrix}, \quad \bar{P}_C^M = \begin{bmatrix} P_S^M & P_{SC}^M \\ P_{SC}^{MT} & P_C^M \end{bmatrix} \quad (5.18)$$

where  $P_S$  and  $P_C$  are  $n \times n$  and  $n_c \times n_c$  dimensional, and  $P_{SC}$  are  $n \times n_c$  dimensional matrices. The subscripts S, C and SC correspond to the structural modal states, compensator states and the coupling between the structural and compensator states, respectively. The model efficiency  $e$  can now be written as in Section 4.2.

$$e = N/D$$

where

$$\begin{aligned} N &= x_0^T P_S^M x_0 + 2x_0^T P_{SC}^M x_{c0} + x_{c0}^T P_C^M x_{c0} \\ D &= x_0^T P_S^R x_0 + 2x_0^T P_{SC}^R x_{c0} + x_{c0}^T P_C^R x_{c0} \end{aligned} \quad (5.19)$$

and  $x_0$  corresponds to the initial disturbance of structural states. In practice, the question of how to choose the compensator initial conditions  $x_{c0}$  arise. From the above decomposition it is clear what the function of  $x_{c0}$  can be in terms of efficiency of the controller. For  $x_{c0}=0$ , the efficiencies become

$$e = \frac{x_0^T P_S^M x_0}{x_0^T P_S^R x_0}, \quad e^* = \frac{x_0^T P^* x_0}{x_0^T P_S^R x_0} \quad (5.20)$$

where  $P_S^R$  and  $P_S^M$  are the  $n \times n$  upper left partitions of  $\bar{P}^R$  and  $\bar{P}^M$  obtained from the  $(n+n_c)$ -dimensional Lyapunov Equations (5.10 and 5.11). In the case of  $e^*$  the care that had to be observed in defining it by Eq. (5.13) resolves itself automatically regardless of augmentation of  $P^*$  with zeros in Eq. (5.16) and it would

be plausible to require independent feedback of structural modal states to yield  $P^*$ .

On the other hand, if the compensator initial states are non-zero, the efficiencies  $e$  and  $e^*$  will be different than what will be obtained by Eqs. (5.20). Indeed, a careful choice of  $x_{c0}$  can enhance the efficiency of the control system with the participation of coupling terms. The case studies we presented in earlier in Chapter 4 demonstrated that coupling terms in the control powers could be beneficial.

This brings us to the point of defining compensator efficiency of the control system as a contribution to the overall efficiencies  $e$  and  $e^*$ . This can be done easily considering the developments presented in Section 4 on component efficiencies. The concepts presented therein can be generalized to this case by associating the subscript  $j$  in the equations with the structural modal states and compensator states, hence, we consider  $j=x,c$  and rewrite Eq. (4.1) as:

$$e = \sum_j e_j = e_s + e_c \quad (5.21)$$

where  $e_j=e_s$  and  $e_j=e_c$  follow from Eqs. (4.16) and (4.17)

$$e_s = \frac{ND_s + N_s D}{2D^2} \quad , \quad e_c = \frac{ND_c + N_c D}{2D^2} \quad (5.22)$$

$$D_s = \text{Trace}[\bar{P}^R \bar{X}_o]_s \quad , \quad D_c = \text{Trace}[\bar{P}^R \bar{X}_o]_c \quad (5.23)$$

$$N_s = \text{Trace}[\bar{P}^M \bar{X}_o]_s \quad , \quad N_c = \text{Trace}[\bar{P}^M \bar{X}_o]_c \quad (5.24)$$

$$\bar{X}_o = \bar{x}_o \bar{x}_o^T \quad (5.25)$$

Similarly for  $e^*$

$$e^* = \sum_j e_j^* = e_s^* + e_c^* \quad (5.26)$$

$$e_s^* = \frac{N^*D_s + N_s^*D}{2D^2}, \quad e_c^* = \frac{N^*D_c + N_c^*D}{2D^2} \quad (5.27)$$

$$N_s^* = \text{Trace}[P^*X_0] = S^* = N^*, \quad N_c^* = 0$$

$D_s$ ,  $D_c$  and  $D$  are as given by Eqs. (5.23), (5.24) and (5.19). We shall refer to  $e_c$  and  $e_c^*$  as the compensator efficiencies.

Alternately, decoupled compensator efficiencies can be defined as in Section 4.2. It is easy to see that, in this case

$$e_j = \frac{n_j}{d_j}, \quad e_j^* = \frac{n_j^*}{d_j} \quad j=s,c \quad (5.28)$$

$$n_s = x_0^T P_S^M x_0, \quad n_c = x_{c0}^T P_C^M x_{c0} \quad (5.29)$$

$$d_s = x_0^T P_S^R x_0, \quad d_c = x_{c0}^T P_C^R x_{c0} \quad (5.30)$$

$$n_s^* = x_0^T P^* x_0, \quad n_c^* = 0$$

and the expected efficiencies are as in Eq. (4.19)

$$E(e) = \frac{n_s + n_c}{d_s + d_c}, \quad E(e^*) = \frac{n_s^*}{d_s + d_c} \quad (5.31)$$

which rely on the expectation that the cross-coupling terms between the structural and compensator states will average out to zero.

We must emphasize that the developments resented here do not depend on the specific control theory used to design the controls. In fact, the efficiency of the system will serve to evaluate the control design in a most essential way.

For example, the compensator can again be designed as optimal compensators in the sense of Linear Quadratic Gaussian (LQG) designs or as fixed order optimal compensators via Minimum Energy Optimal Projection (MEOP) technique (Ref. 14). The efficiency of all such design will still need to be considered as addressed in this investigation.

### 5.3 Reduced-Order Efficiency-State Estimators

The feedback control law

$$F = Gx$$

as given in the previous chapters requires the knowledge of structural modal states of the  $2n$ -th order control design model. Typically, a full order state-estimator might be used to implement the feedback law, Eq. (3.10) by using an estimate  $\hat{x}$  of the structural states. The  $2n$ -th order state-estimator constitutes a dynamic compensator discussed in Section 5.2, Eqs. (5.2 and 5.3) and has the form

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}F + Ky \tag{5.32}$$

$$F = G\hat{x} \tag{5.33}$$

where  $\hat{\cdot}$  denotes estimator matrices to be found and  $G$  is the  $m \times 2n$  dimensional control gain matrix designed for the system by any suitable approach. The solution for the estimator in terms of the compensator notation of Section 5.2 is known to be

$$\hat{A} = A - KC = A_c, \quad \hat{B} = B = B_c, \quad G = G_c, \quad H_c = 0 \tag{5.34}$$

where  $K$  is the estimator gain chosen to assign a desired set of estimator (compensator) eigenvalues  $\{\rho_c\}$ . In this case, the separation principle holds and the

4-nth order closed-loop eigenvalues will be  $(\bar{\rho}) = (\bar{\rho}_s \bar{\rho}_c)$  where  $\bar{\rho}_s$  are the eigenvalues of the closed-loop matrix  $A_{CL}=A+BG$  for the structural dynamics.

Hence, all of the developments in Section 5.2 apply to this full-state estimator and the impact of the estimator design on the overall system efficiency can be studied by following the procedure given therein. In this vein, the estimator efficiency concept surfaces as the compensator efficiency.

In practice, instead of full-order estimator, a reduced-order estimator of order  $n_c=2n-m_y$  can be designed where  $m_y$  is the number of measurements. This procedure is standard text book material and does not deserve any special attention here.

Another estimator which would be especially relevant to the theme of this investigation would be an efficiency-state estimator, that is an estimate  $\hat{\epsilon}$  of the efficiency state  $\epsilon$  introduced in Chapter 3. Since the efficiency state  $\epsilon$  is a  $2n$ -dimensional vector, a full-order  $\epsilon$ -state estimator will not be of interest. Since it will merely be related to the full-order modal state estimator by the efficiency modal matrix

$$\hat{x} = T\hat{\epsilon} \quad (5.35)$$

On the other hand, a reduced-order estimator in the  $\epsilon$ -space will be of interest if we recall the efficiency-filtering approach discussed in Section 4.4. We showed in Section 4.4 that a reduced-order control law, Eq. (4.35)  $F=G_1^o\epsilon_1$  can be used where  $\epsilon_1$  are the dominant efficiency states identified along the  $n_p < 2n$  principal controller directions. In Section 4.4, it was also shown that the control law was equivalent to a projection (filtering) of the  $2n$ -dimensional structural states, implying essentially a full-order state feedback of the form

$$F = GT_1L_1x$$

as given by Eq. (4.38).

However, the reduced-control law can be implemented in reality as the feedback of  $n_p$  retained efficiency states  $\epsilon_1$  if an estimate  $\hat{\epsilon}_1$  of these states are available from an  $n_p$ -th order efficiency-state estimator. It turns out that design of such a reduced  $n_p$ -th order  $\epsilon_1$ -estimator is possible. Recalling that  $\epsilon_1$  and  $x$  are related via

$$\epsilon_1 = L_1 x$$

where  $L_1$  is the  $n_p \times 2n$  upper partition of the  $T^{-1}$  matrix, we recognize that  $\epsilon_1$  are  $n_p$  linear combinations of the structural modal states. An estimator, which can estimate these specific linear combinations of the  $x$ -states will be precisely what is needed to implement the reduced order efficiency state feedback control law (4.35). The design of estimators of linear combinations of states was presented in Ref. (15) in a different context and can be adapted in our case to design an  $n_p$ -th order  $\epsilon_1$ -estimator.

Assume the  $\epsilon_1$ -estimator in the form

$$\dot{\hat{\epsilon}}_1 = \hat{A} \hat{\epsilon}_1 + \hat{B}F + Ky \quad (5.36)$$

$\hat{A}$ ,  $\hat{B}$  and  $K$  are  $n_p \times n_p$ ,  $n_p \times m$  and  $n_p \times m_y$  dimensional matrices to be found. Multiplying the modal structural dynamic equations on the left by  $L_1$

$$L_1 \dot{x} = L_1 A x + L_1 B F \quad (5.37)$$

Subtract Eq. (4.37) from Eq. (4.36)

$$\dot{\hat{\epsilon}}_1 - L_1 \dot{x} = \hat{A}(\hat{\epsilon}_1 - L_1 x) + (K C - L_1 A + \hat{A} L) x + (\hat{B} - L_1) B F \quad (5.38)$$

where the term  $\hat{A} L_1 x$  was added and subtracted on the right hand side. If  $K$  and  $\hat{B}$  are chosen so as to satisfy

$$\hat{B} = L_1 \quad (5.39)$$

$$KC - L_1A + \hat{A}L = 0 \quad (5.40)$$

and introducing the estimation error vector

$$\epsilon_E = \hat{\epsilon}_1 - L_1x \quad (5.41)$$

Eq. (5.38) reduces to

$$\dot{\epsilon}_E = \hat{A}\epsilon_E \quad (5.42)$$

which has the solution

$$\epsilon_E = e^{\hat{A}t} \epsilon_{EO} \quad , \quad \epsilon_{EO} = \epsilon_E(0) \quad (5.43)$$

Hence, if the estimator matrix  $\hat{A}$  is chosen, such that it has eigenvalues with negative real parts, then  $\epsilon_E(t) \rightarrow 0$  from which we obtain

$$\lim_{t \rightarrow \infty} \hat{\epsilon}_1 = \lim_{t \rightarrow \infty} L_1x$$

and the  $\epsilon_1$  will be estimated exactly as the desired combination of states  $x$  for large  $t$ .

If we make the simple choice

$$\hat{A} = \text{diag}[\alpha_i] \quad \alpha_i < 0 \quad i=1, \dots, n_p \quad (5.44)$$

equation (5.40) can be solved for the estimator gain matrix  $K$  by rewriting it as  $2n$ -simultaneous equations

$$C^TK^T = A^TL_1^T - L_1^T\hat{A}^T \quad (5.45)$$

where the  $2n \times m_y$   $C^T$  matrix is assumed to have rank  $m_y$ ,  $m_y < 2n$ . Denoting the  $m_y$ -dimensional row vectors of the estimator gain matrix  $K$  by  $k_i^T$  and the  $n_p$  column vectors of the right hand side of Eq. (5.45) by  $a_i$ ,  $i=1,2,\dots,n_p$ , we write Eq. (5.45) in the form

$$C^T k_i = a_i \quad i=1,2,\dots,n_p \quad (5.46)$$

The solutions to Eq. (5.46) can be obtained by singular value decomposition of the measurement matrix  $C^T$

$$C^T = U \Sigma V^* \quad (5.47)$$

where  $U$  and  $V$  are  $2n \times 2n$  and  $m_y \times m_y$  unitary matrices;  $*$  denotes complex conjugate transpose, and  $\Sigma$  is the  $2n \times m_y$  matrix of singular values

$$\Sigma = \begin{bmatrix} \Sigma_0 \\ 0 \end{bmatrix} \quad \Sigma_0 = \text{diag } \sigma_j \quad j=1,2,\dots,m_y \quad (5.48)$$

The solution for the estimator gains  $k_i$  can be shown to be

$$k_i = V \Sigma_0^{-1} U_1^* \alpha_i \quad i=1,\dots,n_p \quad (5.49)$$

where  $U_1^*$  is the  $m_y \times 2n$  upper partition of  $U^*$ . It follows that the estimator gain matrix  $K$  will be given by

$$K^T = V \Sigma_0^{-1} U_1^* (A^T L_1^T - L_1^T \hat{A}^T). \quad (5.50)$$

This completes the design of  $\epsilon_1$ -estimator.

#### 5.4 Residual Dynamic Response to Control Power Spillover

Consider the control-design model and the uncontrolled system dynamics

$$\dot{x} = Ax + BF \quad F = Gx \quad (5.51)$$

$$\dot{x}_U = A_U x_U + B_U F \quad (5.52)$$

The response of the closed loop control design model is

$$x = \phi_{CL}(t, t_0) x_0 \quad , \quad \phi_{CL}(t) = e^{A_{CL} t} \quad (5.53)$$



where  $\phi_{CL}$  is the closed-loop transition matrix for the control design model. Denoting the control period over which  $x_0$  is regulated by  $T=(t_f-t_0)$ , the spillover excitation of the uncontrolled modes can be measured by the total square response during  $T$ . The total squared modal displacement response of the uncontrolled dynamics will be given by

$$Y_{ij}^2 = \sum_{r=n+1}^N \int_{t_0}^t \xi_{ru}^2 dt \quad (5.54)$$

The uncontrolled modal displacements due to control spillover excitation are given by the convolution integrals

$$\xi_{ru} = \omega_{ru}^{-1} b_{ru}^T \int_{t_0}^t \text{Sin} \omega_{ru}(t-\tau) Gx(\tau) d\tau \quad (5.55)$$

Substituting (5.55) and (5.53) into (5.54), we obtain

$$Y_{ij}^2 = x_0^T \left[ \int_{t_0}^t \phi_{CL}^T(\tau', t_0) B_{ij}^T Q_y(t, \tau, \tau') B_{ij} \phi_{CL}(\tau, t_0) d\tau d\tau' \right] x_0 \quad (5.56)$$

where  $B_{ij}$  is the input influence matrix of the truncated dynamics as given in Eq. (2.29). The time dependent weighting matrix  $Q_y$  is given by

$$\begin{aligned} Q_y &= [\omega_u^{-2}] [S^2(t, \tau, \tau')] \quad (5.57) \\ [\omega_u^{-2}] &= \text{diag} [\omega_{ru}^{-2}] \quad r=n+1, \dots, \\ [S^2(t, \tau, \tau')] &= \text{diag} [\text{Sin} \omega_{ru}(t-\tau') \text{Sin} \omega_{ru}(t-\tau)] \end{aligned}$$

The weighting matrix  $Q_y$  obviously involves the cross correlations of the impulse response functions of the residual modes and is inversely proportional to the square of the uncontrolled natural frequencies. Equation (5.56) will then, noting that after integrations effectively an additional  $\omega_{ru}^{-2}$  will evolve so that the results will be inversely proportional to  $\omega_{ru}^{-4}$  yield

$$Y_U^2 = x_0^T U_y (\omega_U^{-4}) x_0 \quad (5.58)$$

where  $U_y$  must be a positive definite resultant matrix of the weighted integration in Eq. (5.56) since  $Y_U^2$  is positive definite and the argument  $\omega_U^{-4}$  is shown to highlight the nature of dependency of the results on residual natural frequencies.

Next, considering the spillover control power  $S_U$  defined by Eq. (2.2g) and the Eqs. (5.51-5.53)

$$S_U = x_0^T \left[ \int_{t_0}^t \int_{t_0}^t \int_{t_0}^t \phi_{CL}^T(\tau', t_0) G^T B_U^T B_U G \phi_{CL}(\tau, t_0) d\tau d\tau' dt \right] x_0 \quad (5.59)$$

which must yield

$$S_U = x_0^T P_U^M x_0 \quad P_U^M = P^R - P_C^M \quad (5.60)$$

where the power matrices  $P^R$  and  $P_C^M$  are available as required for efficiency analysis. If the control law is derived via a dynamic compensator, that is if Eqs. (5.5) and (5.7) are used instead of (5.51), as described in Section 5.2, it is straight forward to see that the counterparts of Eqs. (5.58) and (5.59) are obtained by replacing  $x_0$  with  $\bar{x}_0$ ,  $\phi_{CL}$  with the  $\bar{\phi}_{CL}$ , the closed-loop transition matrix of Eq. (5.50) and  $G$  with  $\bar{G}$ . The weighting matrices  $B_U^T B_U$  and  $B_U^T Q_y B_U$  will remain the same. Consequently,  $\bar{P}_U = \bar{P}^R - \bar{P}_C^M$  will take the place of  $P_U^M$  in Eq. (5.60).

Hence, recognizing that the above derivation will remain the same quantitatively whether a compensator dynamics is included or not we return to Eq. (5.58), (5.59) for  $Y_U$  and  $S_U$ . Comparing  $Y_U$  with  $S_U$  we note that  $Y_U$  is a weighted version of the control power wasted in the uncontrolled modes where the weighting matrix is  $Q_y$  for  $Y_U^2$  and unity for  $S_U$ . Hence, the residual response and the wasted con-

control power are correlated. The significance of this result is that a measure of residual dynamic response is available through the wasted power  $S_u$ , which is in turn available from the efficiency analysis directly without ever involving the residual dynamics in the computations. It follows that within the efficiency concepts presented in this investigation, evaluation model response simulations are not necessary to infer degradation of the control design model behavior.

Since  $Q_y$  is inversely proportional to the fourth power of uncontrolled natural frequencies, it follows that if natural frequencies larger than unity are in the uncontrolled dynamic model the net effect will be that the matrix  $U_y$  in Eq. (5.58) will be considerably smaller than the control power matrix  $P_{ij}^M$ . Therefore, it may take considerable control power spillover or considerable inefficiency in the control design model before any significant degradation of response of control design model due residual excitation can be detected. This is certainly not surprising as it is well known that high frequency modes are difficult to excite, hence, considerable control power is bled by those modes even for minimal excitation of them if the structure control design is not efficient. To put it more bluntly, it is more critical to align the controller directions with the structural modes in the design model to obtain high efficiencies if higher modes are not in the control design model.

The above analysis proves the assertion made in Section 2.8 that performance of the structural-control system must not be judged on response studies alone. Although no residual dynamics response can be detected in the system outputs, the controller can still be wasting control power operating inefficiently. In space missions, ultimately, the power used inefficiently by the control system has to be generated on board the spacecraft and therefore, has the potential to impact the design of power subsystem and other subsystems and even the mission

significantly. The efficiency results presented up to this point, will establish a most important interdisciplinary link of controller design to other important disciplines of space systems design. A good structural control design must therefore, strive for an efficient controller which can be accomplished only after an efficiency analysis is carried out.

Furthermore, if an inefficient controller is used to control a model with high frequencies in the design model which will require considerable control power and less than unity low frequency modes are in the uncontrolled set, it would take only a small amount of inefficiency to produce considerable residual mode excitation and degradation of the system response. In such cases design of efficient controllers will become even a more critical task.

## 6.0 OTHER APPLICATIONS OF CONTROLLER EFFICIENCY & APPLICATION TO CSDL-ACOSS-6

### 6.1 Introduction

In this chapter, we present some illustrations of the variety of structure-control system designs that can be addressed via the efficiency concepts. Specifically, in following the subject of Section 5.4 on "Residual Dynamic Response to Control Power Spillover," we give a vivid illustration of the performance of an inefficient controller in Section 6.2.

Another important topic in the design of structure-control system is to determine a good input configuration, that is how many actuators to use and where to put them, as well as quantifying the effectiveness of actuators. In Section 6.3, we give illustrations of how efficiencies can be used to select the input configuration.

Another application of the efficiency concept can be to choose the weighting matrices (or parameters) in the application of the LQR theory. Because the efficiencies are defined regardless of the particular control theory used in the design of the control law, the opportunity exists that the LQR weighting matrices can be chosen to improve the controller efficiency. This is done in Section 6.4.

As a final analysis and design study by using the efficiency technique, in Section 6.5 we present results of efficiency modes analysis of a representative large flexible spacecraft configuration, the Charles Stark Draper laboratory ACOSS-6 (Model 2) structure.

### 6.2 Inefficient Control with Truncated Low Frequency Modes

It was shown in Section 5.4, that the total square modal response of truncated dynamics of a system was a weighted version of the control power spill-

over and the weighting was inversely proportional to the fourth power of the natural frequencies of the truncated modes. Hence, the efficiency or inefficiency of a controller becomes critical with low frequency truncated modes, also from a response point of view.

As an illustration of this, we refer the reader to Examples 3.1 and 3.3 with respective simulations given in figures SIM39 (Figures 3.1, 3.2 and 3.3) and SIM41 (Figures 3.4, 3.5 and 3.6), in Chapter 3. Both of these controllers had low efficiencies,  $e^*=0.173\%$ ,  $e=4.31\%$  and  $e^*=6.2\%$ ,  $e=28.59\%$ . The control design model was 4-th order and consisted of high frequency modes (#11 and 12), the lowest frequencies were left in the truncated dynamics. Note that the degradation in the response will be as serious as the magnitude of the lowest frequency in the truncated dynamics. For the ACOSS-4 structure the lowest natural frequency is  $\omega_1=1.34$  rad/sec., so it too has an attenuating effect on the control power spillover (proportional to  $1/\omega_1^4$ ), nevertheless, not as strong as the other truncated modes. Certainly if  $\omega_1 < 1$ , the control power spillover effect would be magnified in the residual response. However,  $\omega_1=1.34$  was still low enough to accentuate the residual response degradation due to an inefficient controller. In both SIM39 (Figures 3.1-3.3) and SIM41 (Figures 3.4-3.6) evaluation model responses are significantly degraded.

Two other examples of evaluation model response degradation due to inefficient controllers are presented below:

Example 6.1:  $n=4$  (Modes: 9-12),  $m=2$  (#1,#2). The system has  $e^*=8.39\%$ ,  $e=45.67\%$ .

SIM28 shows the responses (Figures 6.1 through 6.3).

Example 6.2:  $n=4$  (Modes: 9-12),  $m=1$  (#5). The system has  $e^*=0.202\%$ ,  $e=21.33\%$ . SIM30 shows the responses (Figures 6.4 through 6.6). Clearly the more inefficient system of Example 6.2 has serious degradation of response.

### 6.3 Actuator Selection Via Efficiencies

The full-order 12 mode ACOSS-4 evaluation model was considered in conjunction with a full set of 12 actuators,  $n=12$ ,  $m=12$ . Because there is no model truncation, the relative model efficiency of the designs are  $e=100\%$ . However,  $e^*$  is not necessarily  $100\%$  since the 12 inputs do not represent a continuously distributed input field although they may come close to representing it. The number of inputs were decreased from  $m=12$  to  $m=1$ , in each case  $e^*$  was computed, the changes in  $e^*$  with the deletion of each input being a reflection of the effectiveness of that input. The following results were obtained:

#### ACTUATOR EFFECTIVENESS ON ACOSS-4 EVALUATION MODEL

<u>m=Actuator Number</u>	<u>%e*Global efficiencies</u>	<u><math>\Delta e^*</math></u>	<u>Actuator Rank</u>
12	70.88	-0.59	12
11	71.47	0.28	11
10	71.19	7.13	3
9	64.06	6.74	4
8	57.32	13.85	2
7	43.47	31.73	1
6	11.74	0.7	10
5	11.04	0.78	8
4	10.26	0.72	9
3	9.54	1.16	7
2	8.38	4.10	6
1	4.28	4.28	5

First, note that  $e^*$  is valid indicator of the quality of the input configuration and the effectiveness of an actuator can be measured by its contribution to  $e^*$ . Hence, each actuator was ranked according to its effectiveness. The most effec-

tive 5 actuators were (#7,8,10,9,1). For this model, the above ranking of the actuators is in agreement with the actuator ranking by computing the work done by each actuator during the control period (Ref. 16). A second study was done by using a reduced-order model  $n=1-8$ . In this case, since  $e$  is less than 100%, it too was considered in judging the actuator effectiveness along with the global efficiency\*.

ACTUATOR EFFECTIVENESS ON ACROSS-4 WITH A REDUCED-ORDER MODEL

<u>m</u>	<u>% e*</u>	<u>% e</u>	<u>Rank*</u>	<u><math>\Delta e^*</math></u>	<u><math>\Delta e\%</math></u>	<u>Rank</u>
1	5.69	56.92	8	5.69	56.92	1
2	8.41	55.26	12	2.72	-1.66	10
3	25.79	75.67	1	17.38	20.41	2
4	30.76	80.65	9	4.97	4.98	5
5	45.30	70.62	2	14.54	-10.03	12
6	53.09	60.84	5	7.79	-9.78	11
7	60.48	76.66	6	7.39	15.82	3
8	67.85	84.21	7	7.37	7.55	4
9	72.40	83.47	10	4.55	-0.74	9
10	81.19	87.29	3	8.79	3.82	6
11	89.36	87.58	4	8.17	0.29	7
12	93.08	87.28	11	3.72	-0.30	8

Again, note that  $e^*$  is a consistent indicator of actuator effectiveness whereas  $e$  is anomalous, both rankings are different and in this case because the control design model changed, ranking of actuators according to  $e^*$  has also changed from the previous example set when the evaluation model was considered. Simulation results of  $n=8, m=1$ ; and  $n=8, m=12$  are shown in figures labeled SIM8 and SIM18 (Figures 6.7 through 6.12).



#### 6.4 Effect of LQR Weighting Matrices on Efficiencies

The global and model efficiencies were computed for different state and control weighting parameters  $q$  and  $r$  as discussed in Section 2.8, for the weighting matrices  $W_x = \text{block-diag } [q]$ ,  $W_c = r[1]$  for a given control design model and input configuration,  $n=8$  (natural) and  $m=1$ . The results are shown below:

EFFECT OF LQR WEIGHTING PARAMETERS

<u>r</u>	<u>q</u>	<u>e*</u>	<u>e</u>
1.0	$\omega_r^2, 1$	5.69	56.93
0.1	$\omega_r^2, 1$	12.12	56.93
0.05	$\omega_r^2, 1$	16.89	56.93
0.01	$\omega_r^2, 1$	27.29	56.93
1	1	4.99	56.93
1	0.5	4.99	56.93
1	0.1	4.99	56.93
1	10	5.26	56.93
1	100	7.81	56.93
1	1000	31.86	56.93

SIM20 and SIM26 (Figures 6.13 through 6.18) show the responses for  $r=0.05$  and  $q=100$ , respectively. The above example shows that the model efficiency  $e$  is insensitive to control and state weighting, which might be expected since  $e$  is the ratio of two control powers  $S^M$ ,  $S^R$  for the same control design, hence, changing  $q$  and  $r$  changes both by the same proportion. On the other hand,  $e^*$  is based on two dynamically similar control designs but with different input profiles hence,  $e^*$  should be sensitive to changes in  $q$  and  $r$ .

These examples are based on a rather simple variation of the LQR weighting matrices. The effect of changes in the full weighting matrices, for example, in the case of different frequency shaped weighting matrices, may produce not so simple trends. In the above examples, the insensitivity of the model efficiency coefficient to weighting matrices can be used to advantage in the sense that it

will be possible to adjust the response time of the system by changing  $q$  and  $r$  without changing (a good) model efficiency.

#### 6.5 Application to a Large Space Structure: CSDL-ACOSS-6 (Model 2)

Next, a realistic large space structure was considered. The structure shown in Fig. (6.19) is the Charles Stark Draper Laboratory model known as ACOSS-6 (Model 2) (ref. 17). The finite Element evaluation model that was considered was produced by NASTRAN at the Flight Dynamics Laboratory at WRDC/WPAFB. The model that was studied for efficiency had 294 degrees of freedom. Efficiency of two different control design models employing different sets of inputs were considered. There were available 21 actuators located at positions shown in reference 18. Efficiency of two designs were analyzed.

Example 6.3: In this case, the control design model had 8 modes,  $n=\{12, 13, 17, 21, 22, 24, 28, 30\}$  with 4 inputs  $m=\{18, 19, 20, 21\}$ . Actuator numbers are in consistence with the designation listed in Ref. (18). The modes chosen were designated significant for LOS studies (ref. 19). The simulation results of square x-y plane deflection at node 37 are shown in figures labeled SIMV1 (Figures 6.20 and 6.21). The global efficiency was  $e^*=0.176\%$  and the relative model efficiency was  $e=0.657\%$ .

Example 6.4: The control design model had 10 modes  $n=\{7-16\}$  and 10 inputs  $m=\{1-10\}$ . The same deflection as in example 6.3 is shown in figures labeled SIMV2 (Figures 6.22 and 6.23). The efficiencies of this system were  $e^*=0.236\%$  and  $e=0.401\%$ .

Because the first six modes obtained from the structural eigenvalue problem were not reliable none of them were taken in the control design models in both of the examples. Such poorly known modes have no consequences for the reliability of efficiency analysis as long as they are in the truncated dynamics of the system as discussed in Section 2.6. The computer results of efficiency modes analysis are given in Appendix C.

In examples 6.3 and 6.4 for the ACROSS-6 structure the responses of the evaluation model were not shown since this would require simulation of a 588 th order state model, clearly beyond any reasonable expectation. However, in light of the illustrations and theoretical developments presented thus far, the controls designed for the ACROSS-6 structure are extremely inefficient. Almost 99.5% of control power will excite the truncated dynamics. To make this inefficiency even more critical, there are some modes with natural frequencies less than unity left in the truncated dynamics. Hence, as demonstrated in Example 6.2 for the ACROSS-4 structure, in this case for ACROSS-6 too, serious degradation of the responses will occur in the actual (evaluation model) structure instead of the control design model responses shown in Figures (6.21) and 6.23). However, no simulation of the evaluation model is necessary based on the qualitative conclusion that the efficiency approach affords. (It is understood, however, that a simulatable model larger than the control design model and smaller than the 588-th order evaluation model can be used to underline these statements. The challenge here is to resist this temptation).

The efficiency obtained above for a realistic large space structure are extremely low indeed showing the need and demonstrating the merits of a full-scale assessment of the need for an efficiency analysis. As shown in Appendix A, the efficiencies defined in this research are literally the power efficiencies

of the control system. A 0.6% model efficiency simply means 6 energy units/sec. out of 1000 were consumed by the controller, the rest simply being wasted. The wasted power will have been derived on board the spacecraft to serve no useful control purpose. Such inefficiencies in the system cannot be tolerated. It should be recalled that although the control design model is a much smaller model, the information that is obtained from them in the form of power efficiency pertains to the behavior of the full-order evaluation model.

Globally Optimal System SIM28

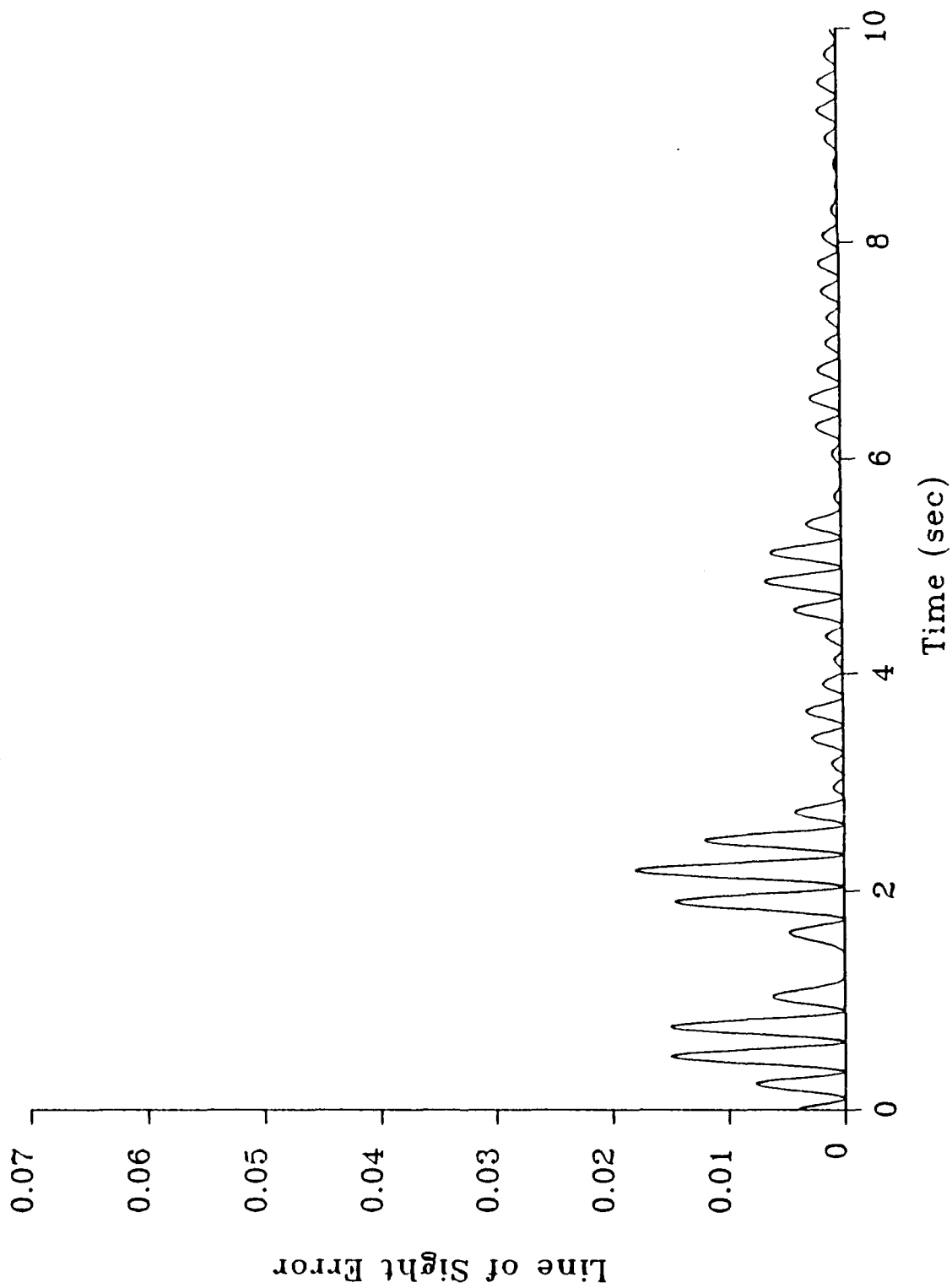


Figure 6.1: Globally optimal system LOS,  $n=4$  (modes: 9-12), example 6.1; response of an inefficient system  $e^*=8.39\%$ ,  $e=45.67\%$

Suboptimal System SIM28

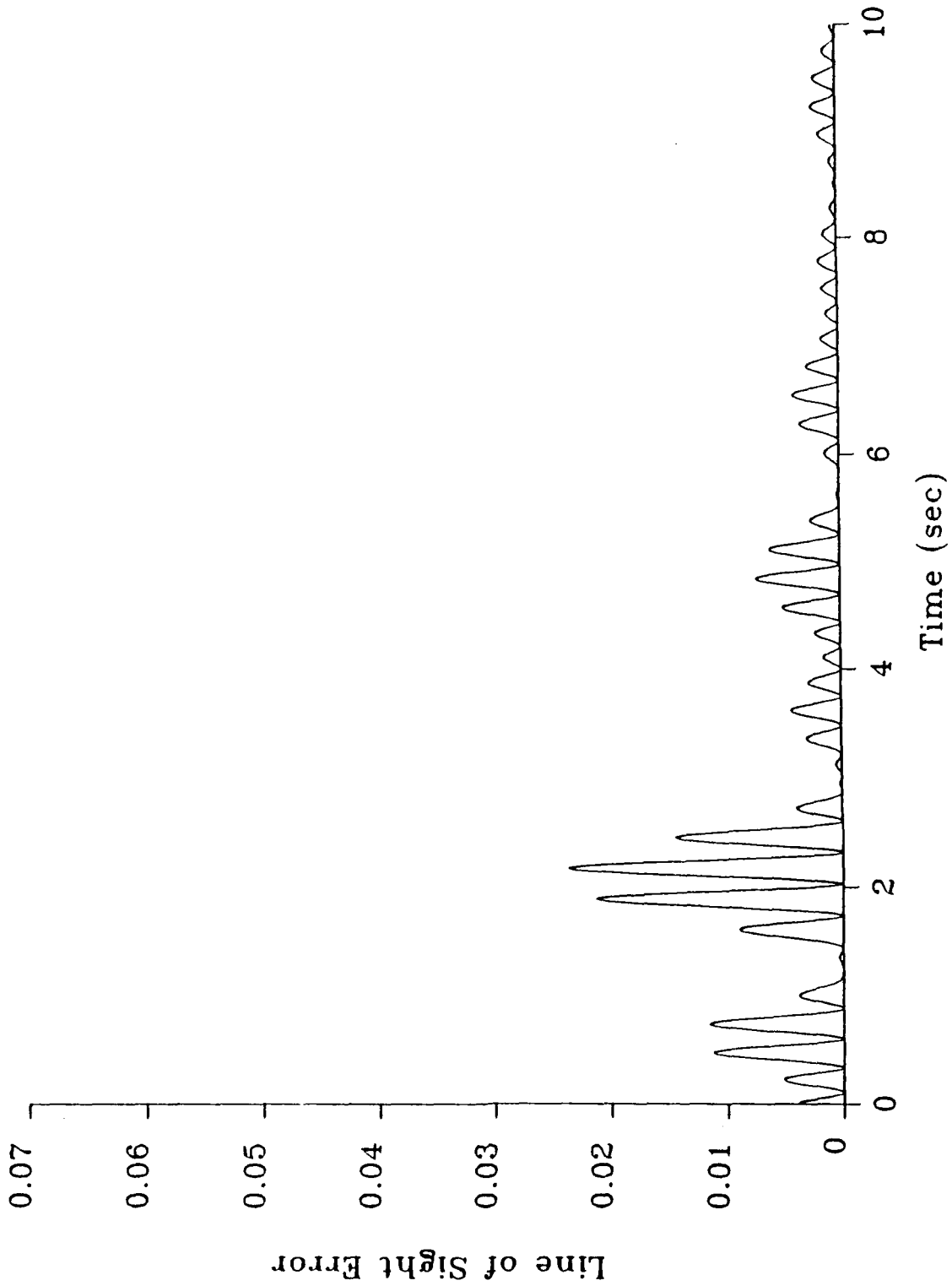


Figure 6.2: Suboptimal system LOS,  $n=4$  (modes: 9-12)  $m=2$  (#1,2), example 6.1; response of an inefficient system  $e^*-8.39\%$ ,  $e=-45.67\%$

Evaluation Model SIM28

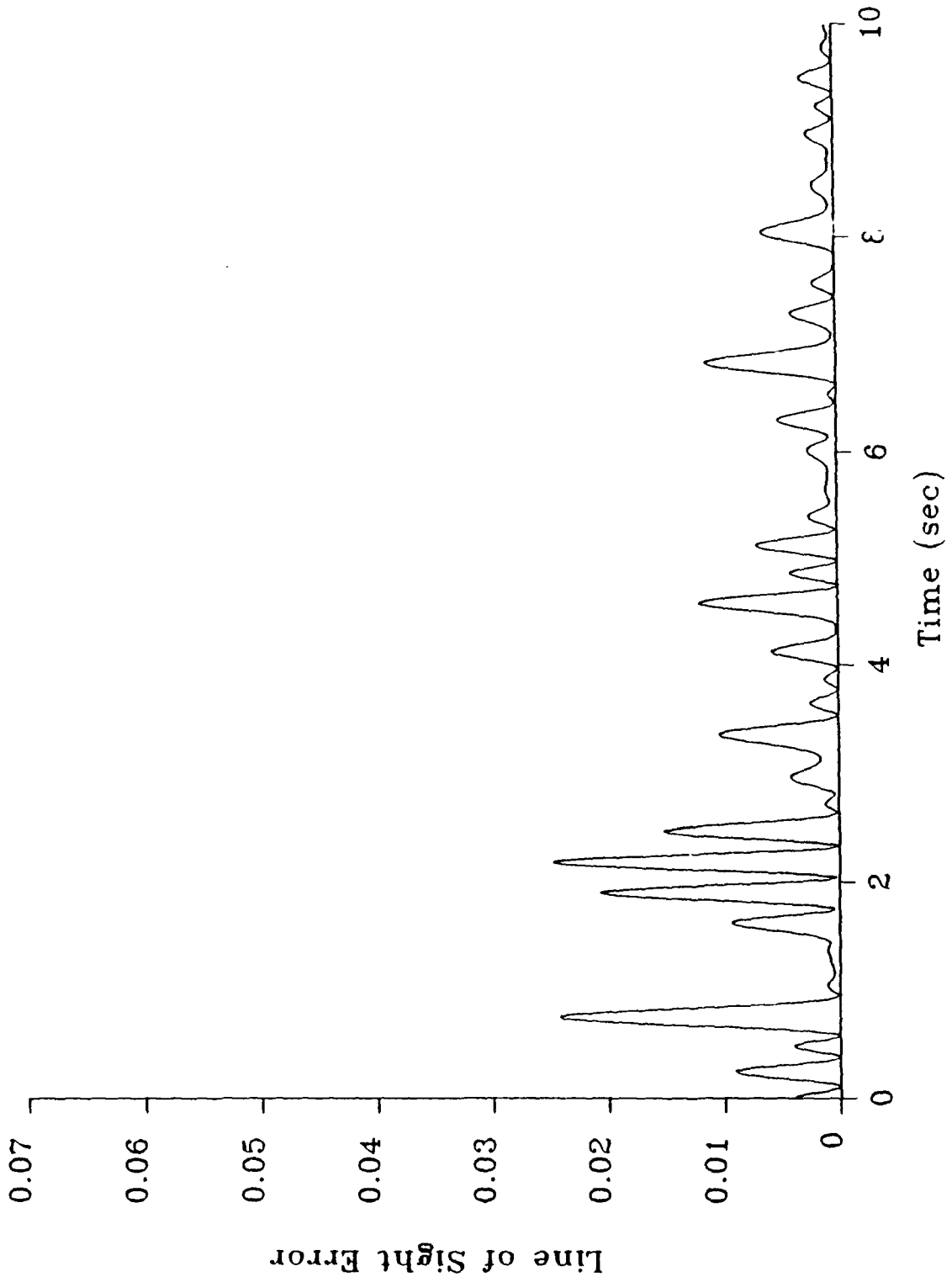


Figure 6.3: Evaluation for Model LOS,  $n=4$  (modes: 9-12),  $m=2$  (#1-2), example 6.1; response of an inefficient system  $e^* \approx 8.39\%$ ,  $e \approx 45.67\%$

Globally Optimal System SIM30

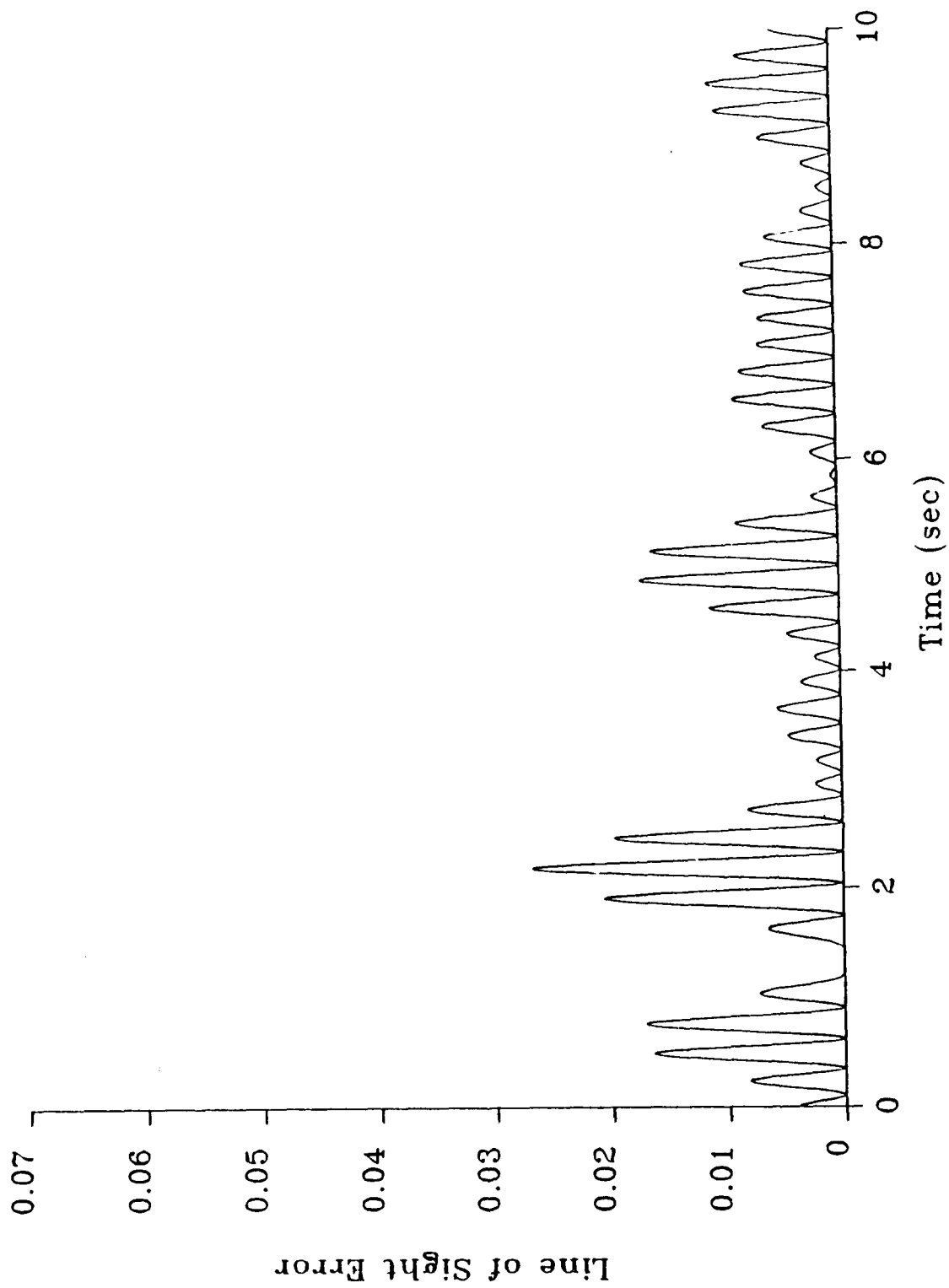


Figure 6.4: Globally optimal system LOS, n=4 (modes: 9-12), example 6.2; response of an inefficient system  $e^* = 0.2\%$ ,  $e = 21.33\%$



Suboptimal System SIM30

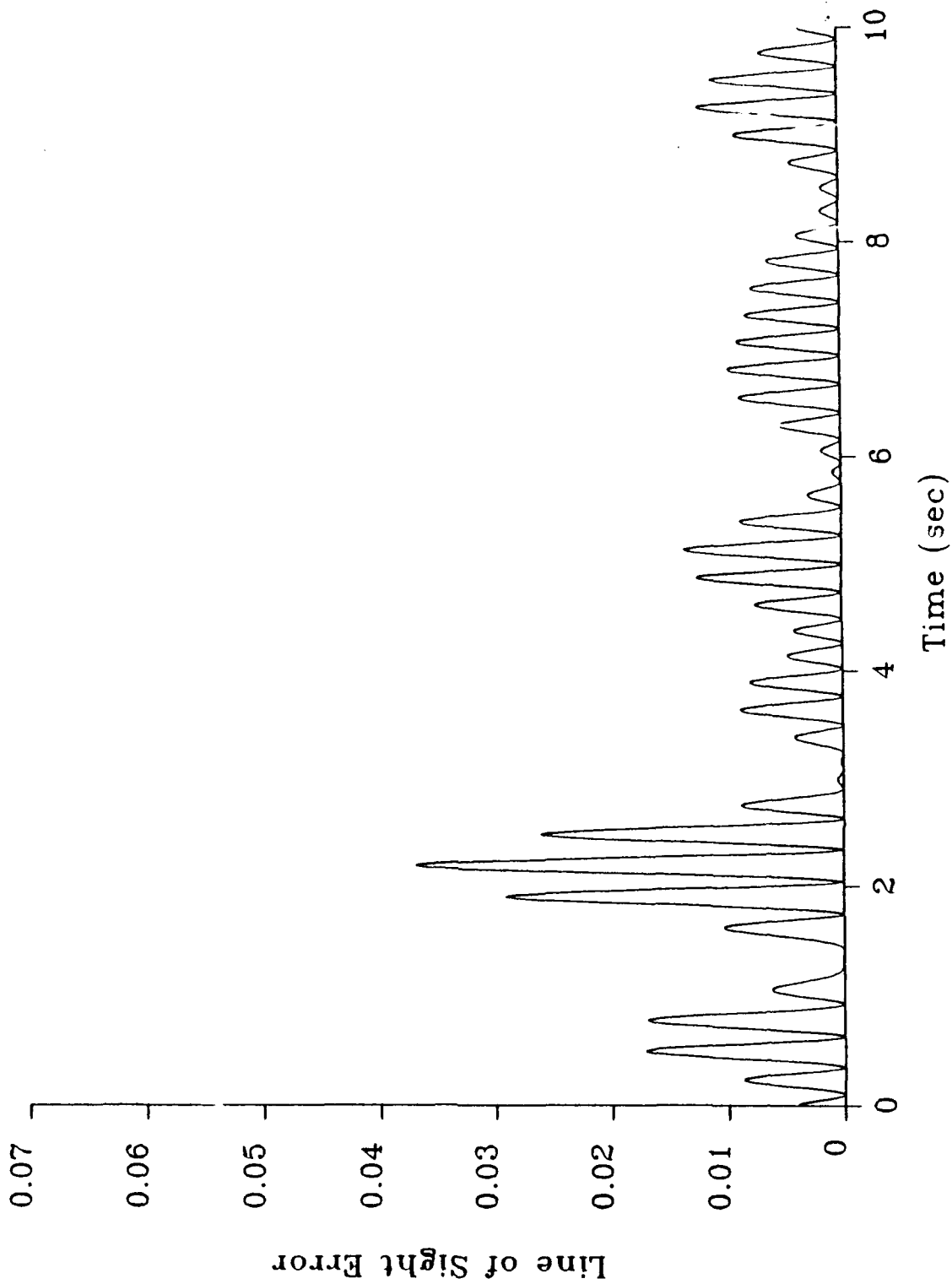


Figure 6.5: Suboptimal system LOS,  $n=4$  (modes: 9-12),  $m=1$  (#5), example 6.2; response of an inefficient system,  $e^*=-0.2\%$ ,  $e=-21.33\%$

Evaluation Model SIM30

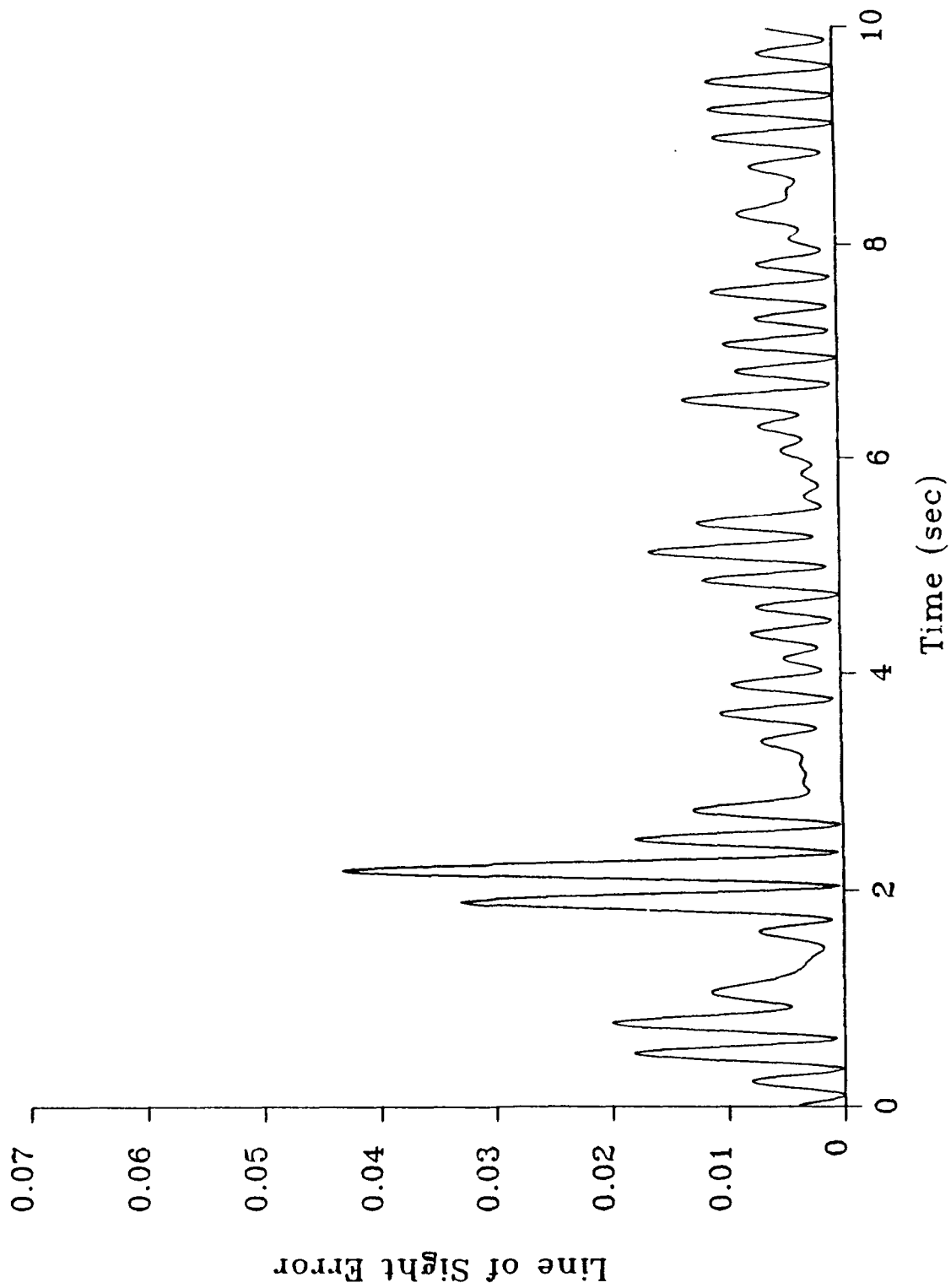


Figure 6.6: Evaluation model LOS,  $n=4$  (modes: 9-12)  $m=1$  (#5), example 6.2; response of an inefficient system,  $e^*=0.2\%$ ,  $e=21.33\%$

Globally Optimal System SIM8

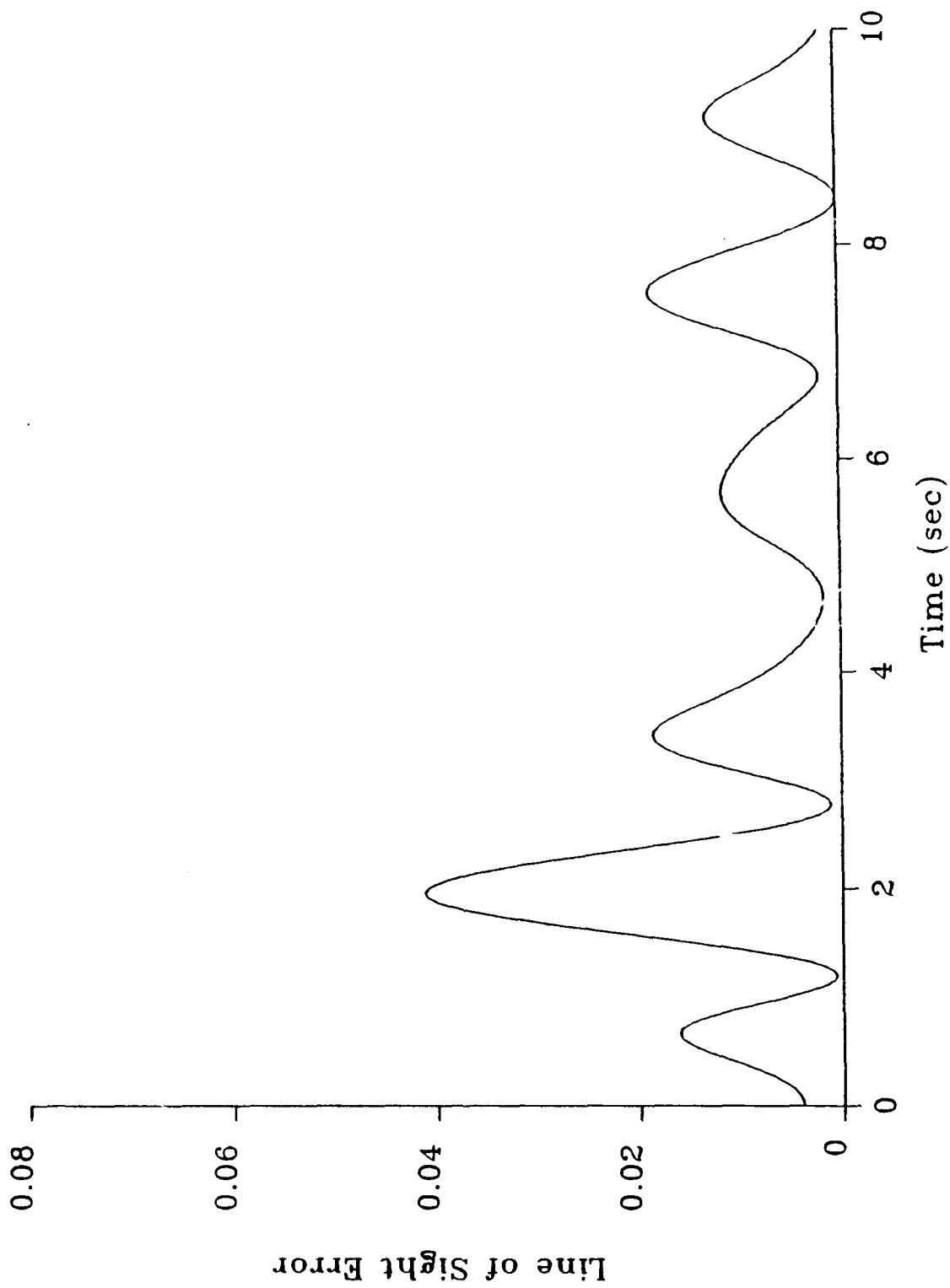


Figure 6.7: Globally optimal system LOS, n=8 (modes: 1-8) m=1 (#1) for actuator selection via efficiencies

# Suboptimal System SIM8

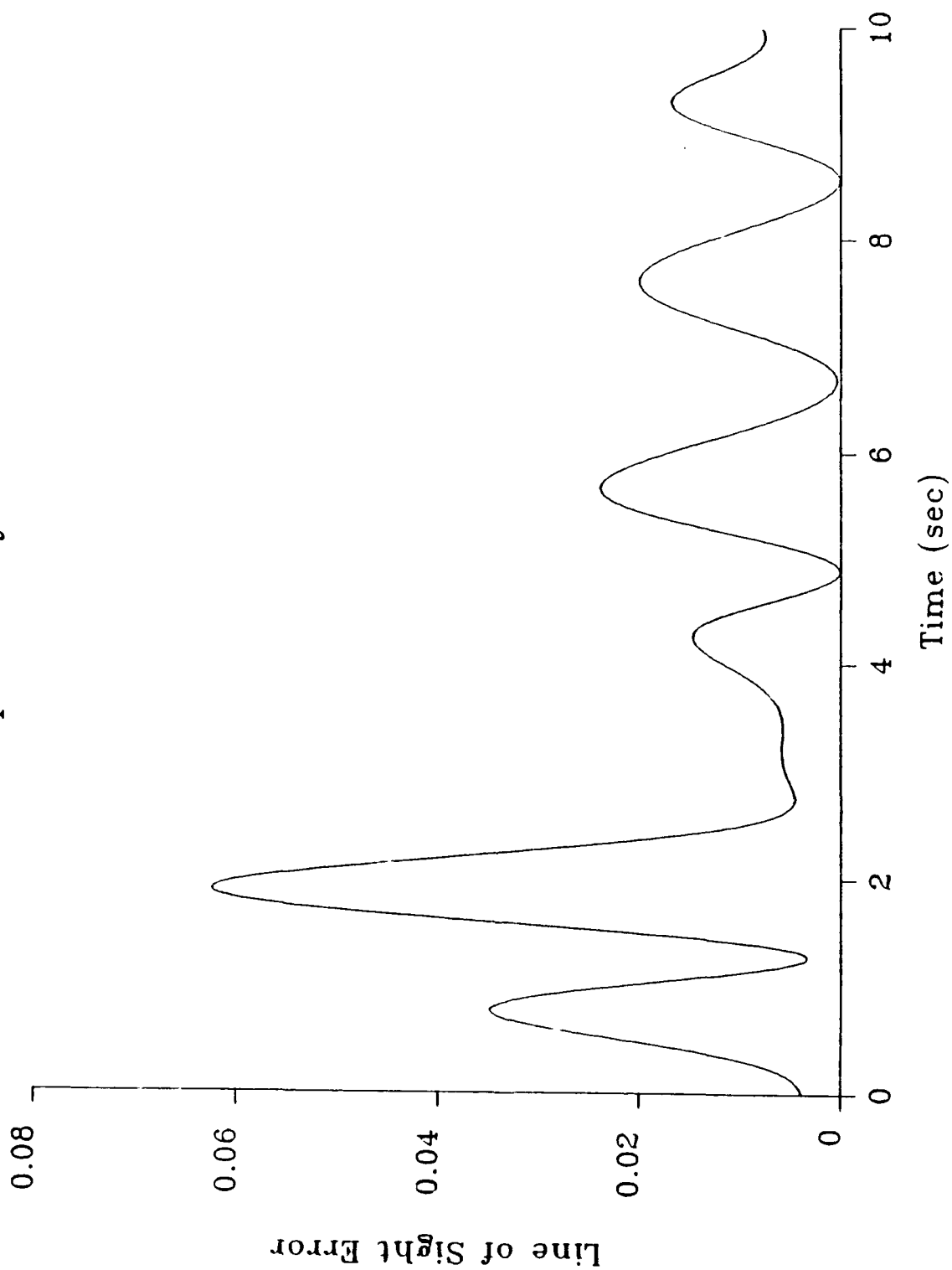


Figure 6.8: Suboptimal system LOS,  $n=8$  (modes: 1-8),  $m=1$  (1) for actuator selection via efficiencies

Evaluation Model SIM8

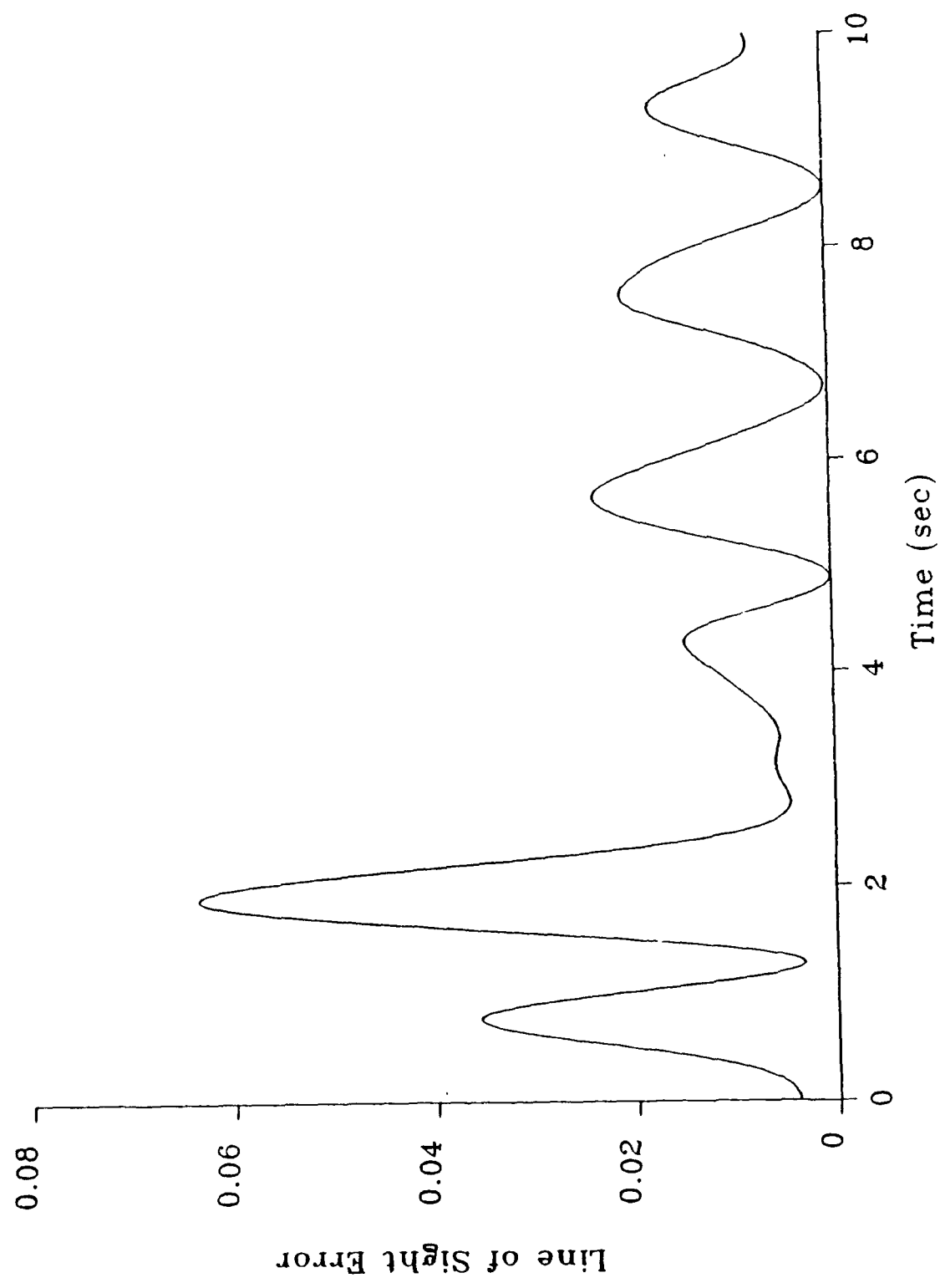


Figure 6.9: Evaluation model LOS, n=8 (modes: 1-8), m=1 (1) for actuator selection via efficiencies

Globally Optimal System SIM18

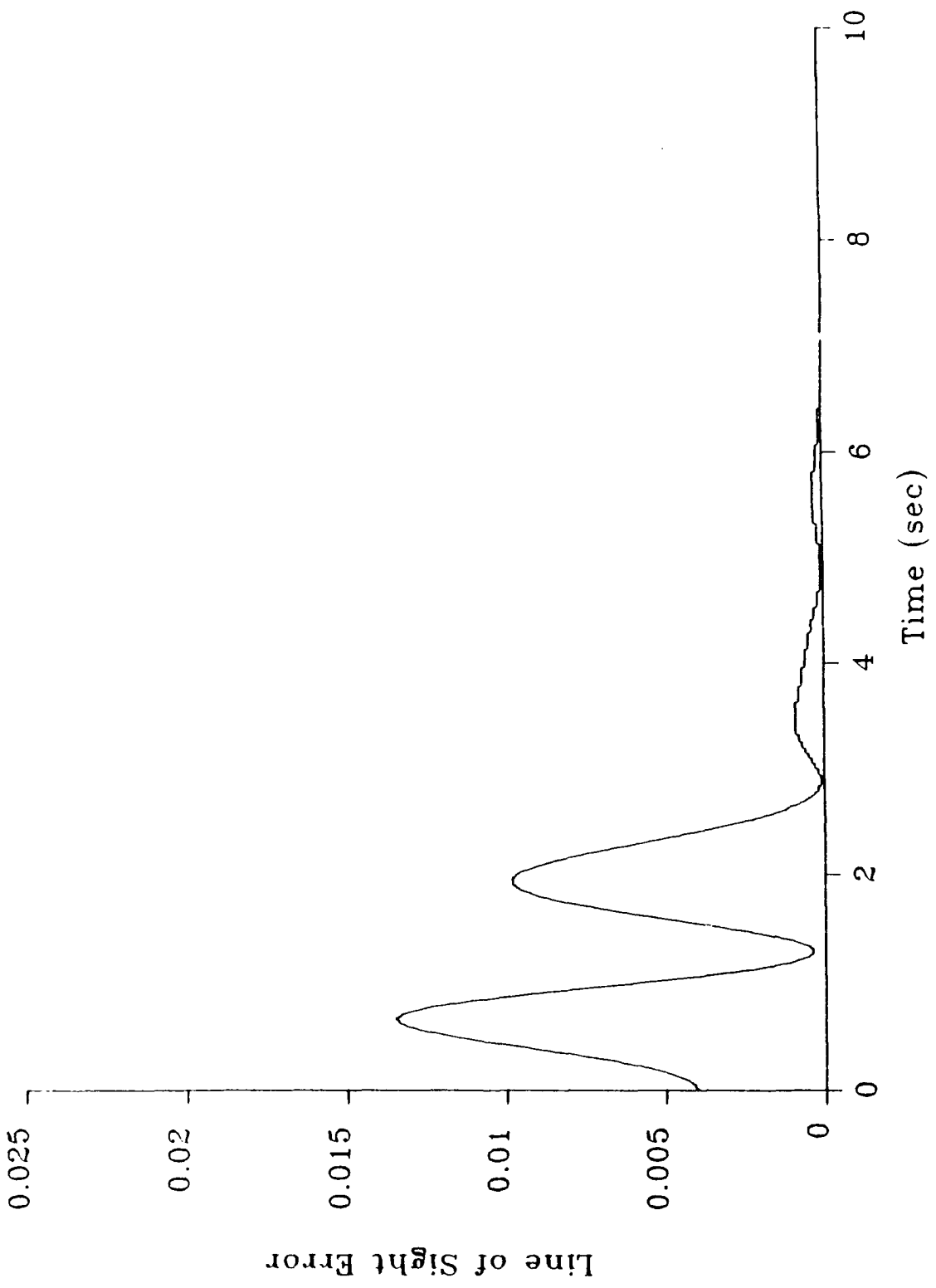


Figure 6.10: Globally optimal system LOS, n=8 (modes: 1-8), for actuator selection via efficiencies

# Suboptimal System SIM18

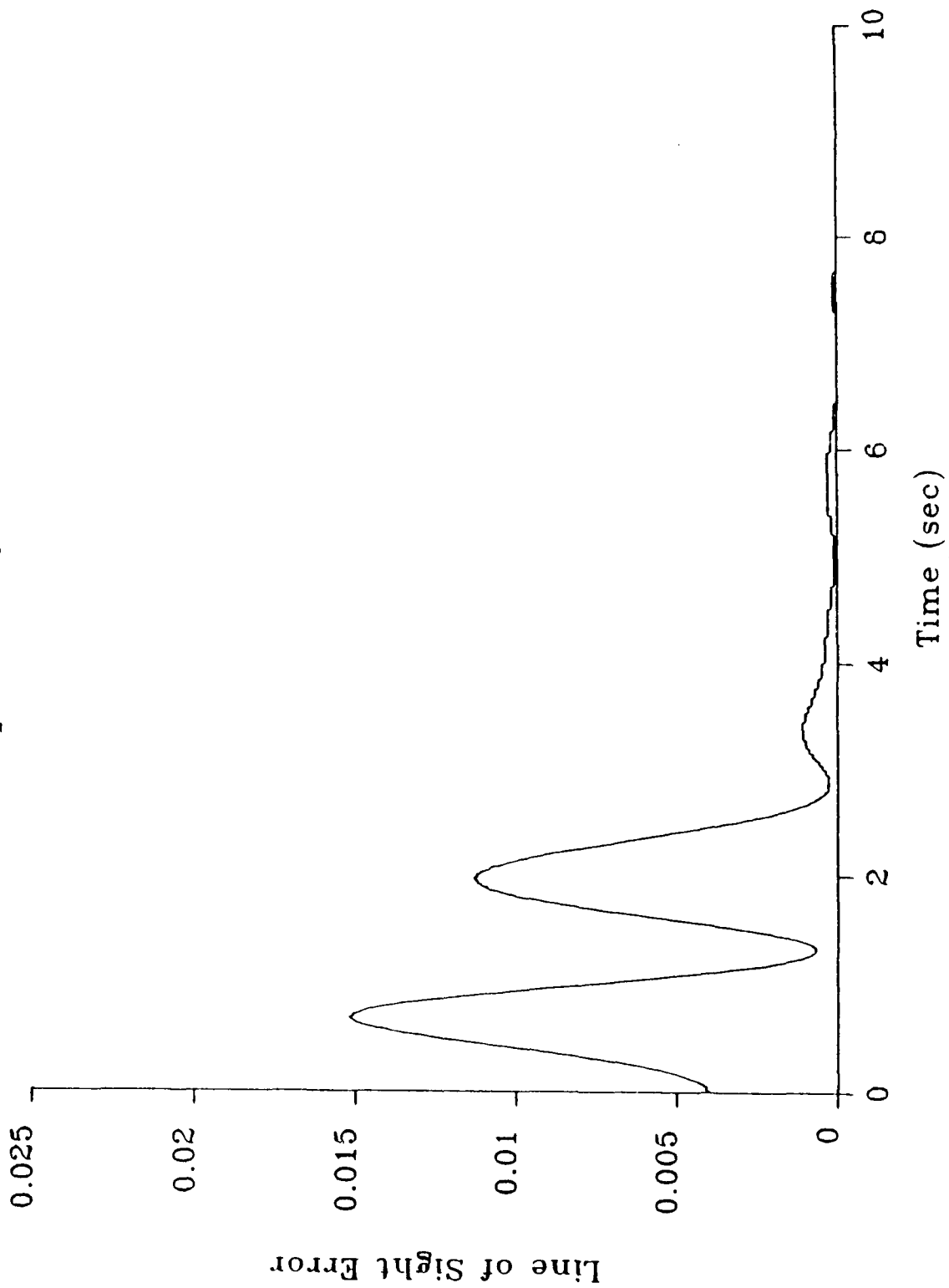


Figure 6.11: Suboptimal system LOS,  $n=8$  (modes: 1-8),  $m=12$ ; for actuator selection via efficiencies

Evaluation Model SIM18

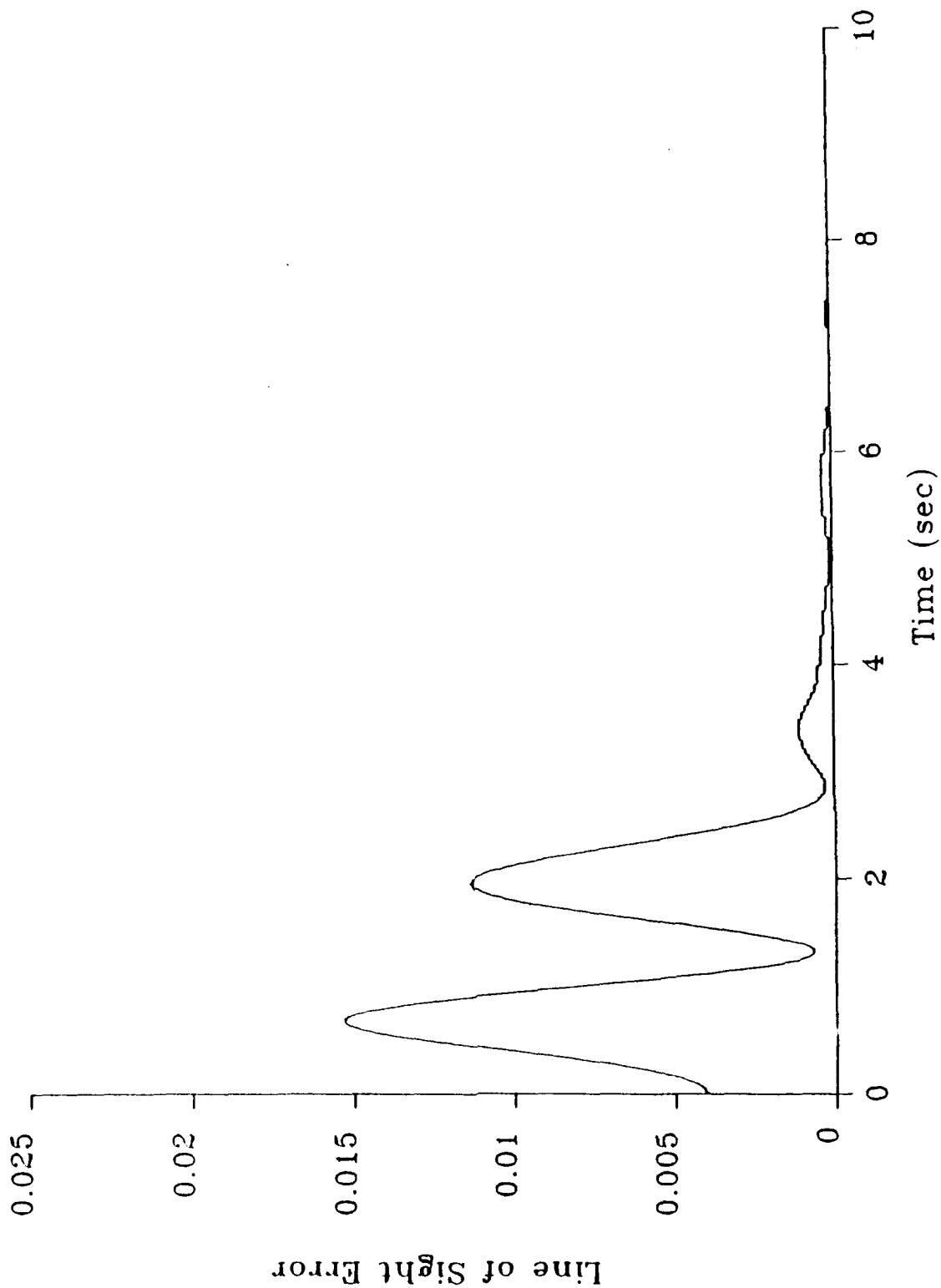


Figure 6.12: Evaluation model LOS,  $n=8$  (modes: 1-8),  $m=12$ ; for actuator selection via efficiencies



# Globally Optimal System SIM20

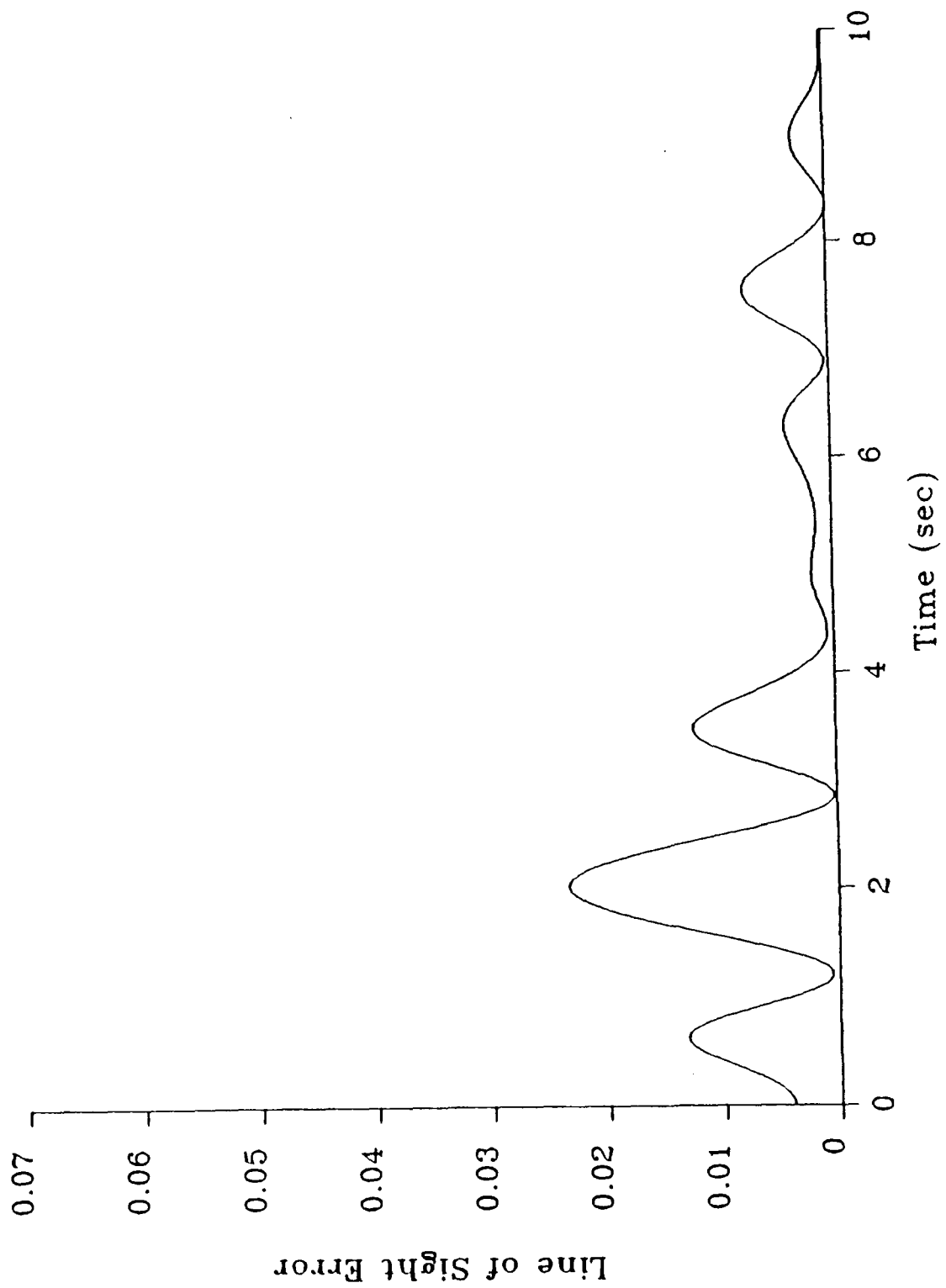


Figure 6.13: Globally optimal system LOS,  $n=8$  (modes: 1-8), effect of LQR weighting on efficiency,  $r=0.05$ ,  $e^*=16.89\%$ ,  $e=56.93\%$

Suboptimal System SIM20

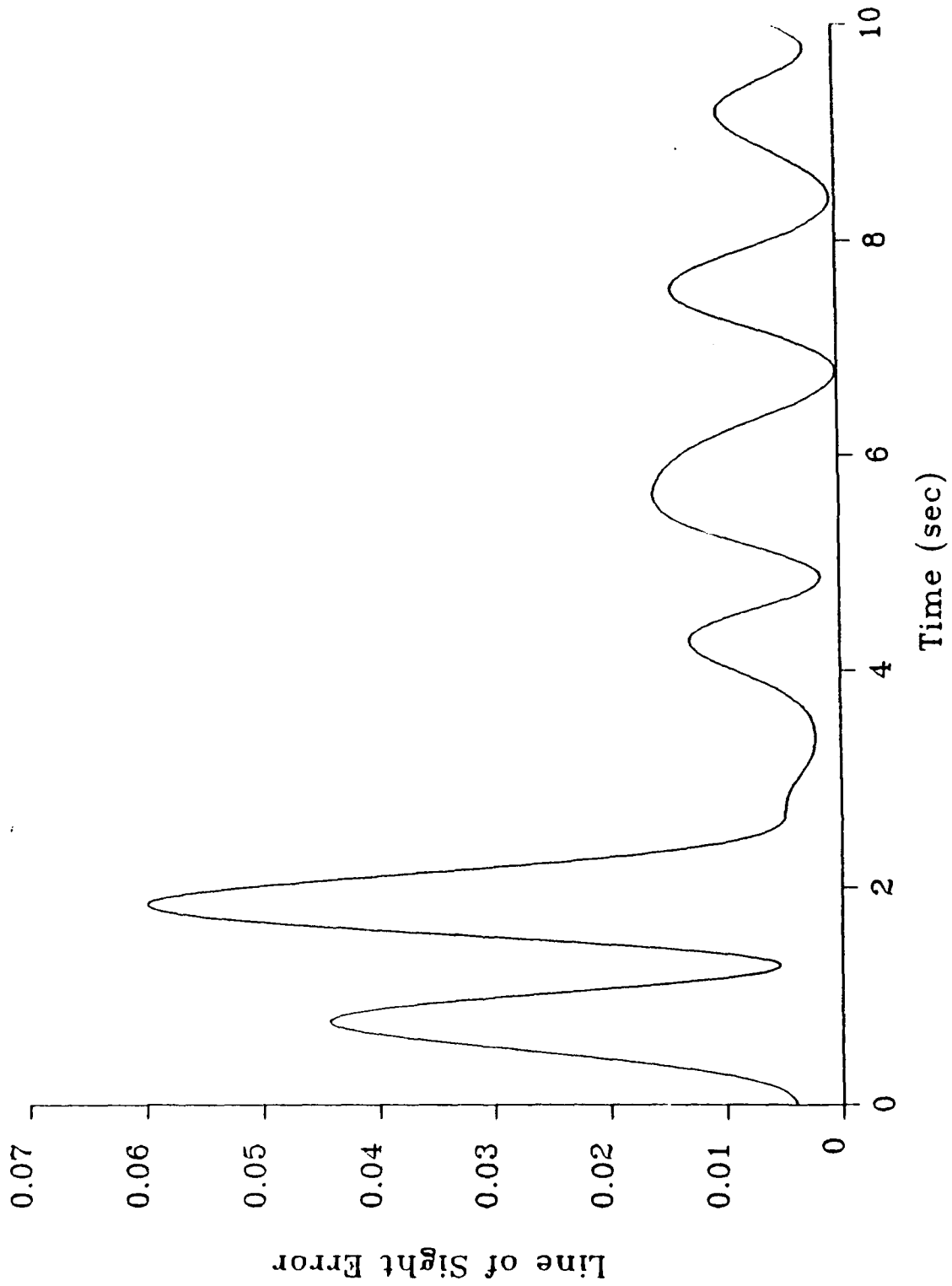


Figure 6.14: Suboptimal system LOS, n=8 (modes: 1-8), m=1; effect of LQR weighting on efficiency  $r=0.05$ ,  $e^*=16.89\%$ ,  $e=56.93\%$

# Evaluation Model SIM20

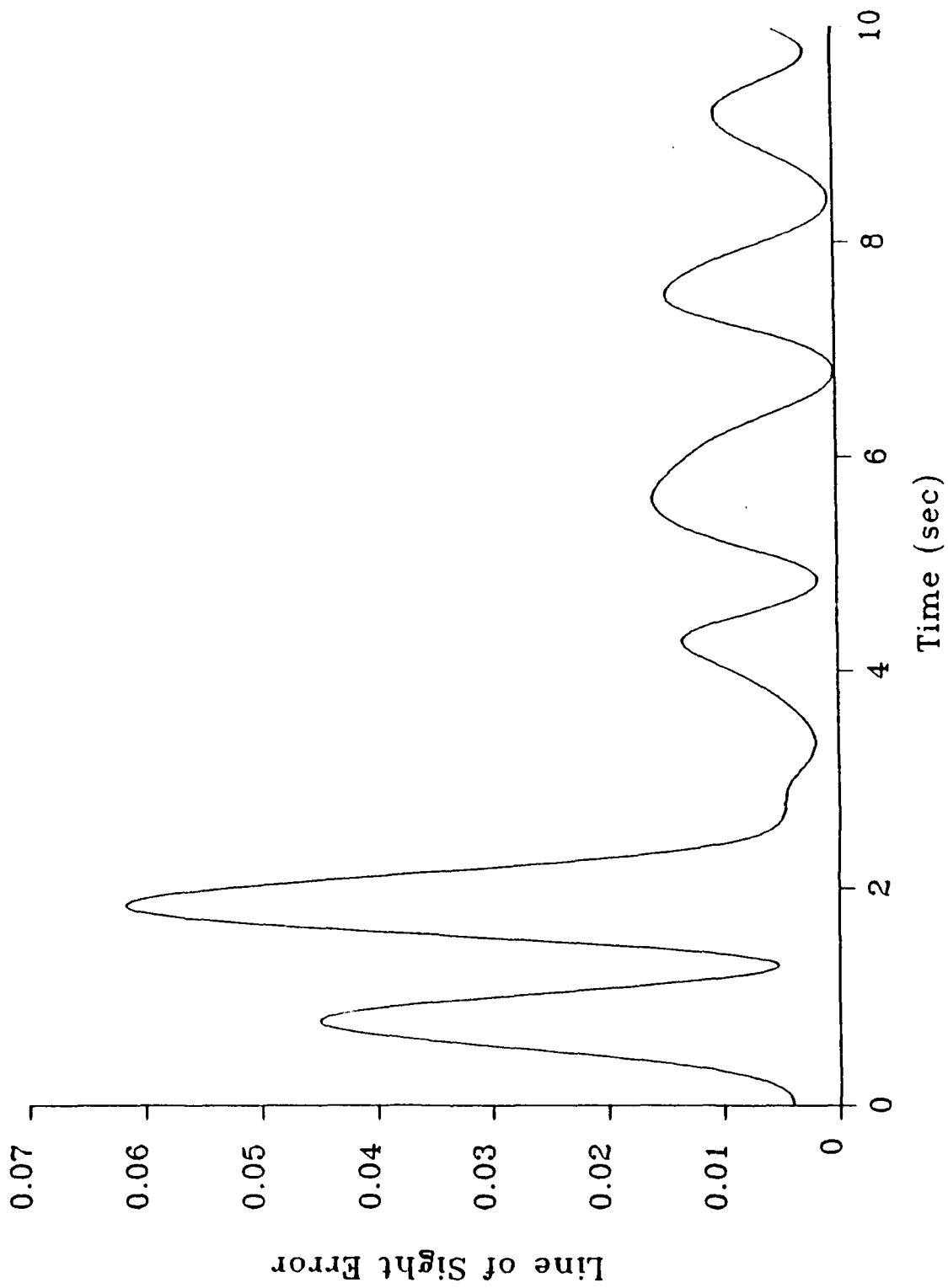


Figure 6.15: Evaluation model LOS, n=8 (modes: 1-8), m=1; effect of LQR weighting on efficiency  $r=0.05$ ,  $e^*=16.89\%$ ,  $e=56.93\%$

Globally Optimal System SIM26

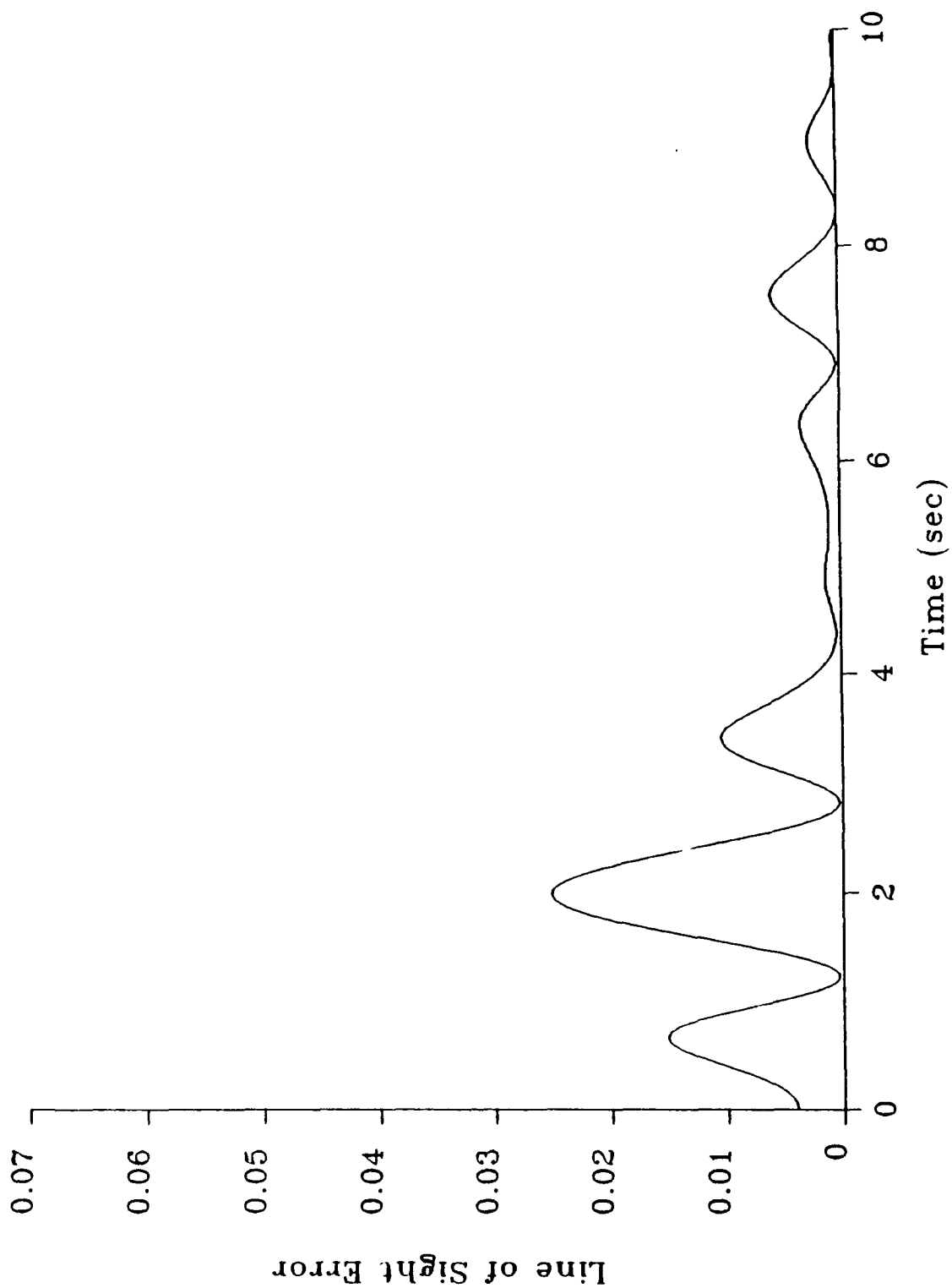


Figure 6.16: Globally optimal system LOS, n=8 (modes: 1-8); effect of LQR weighting on efficiency,  $q=100$ ,  $e^*=7.81\%$ ,  $e=56.93\%$

Suboptimal System SIM26

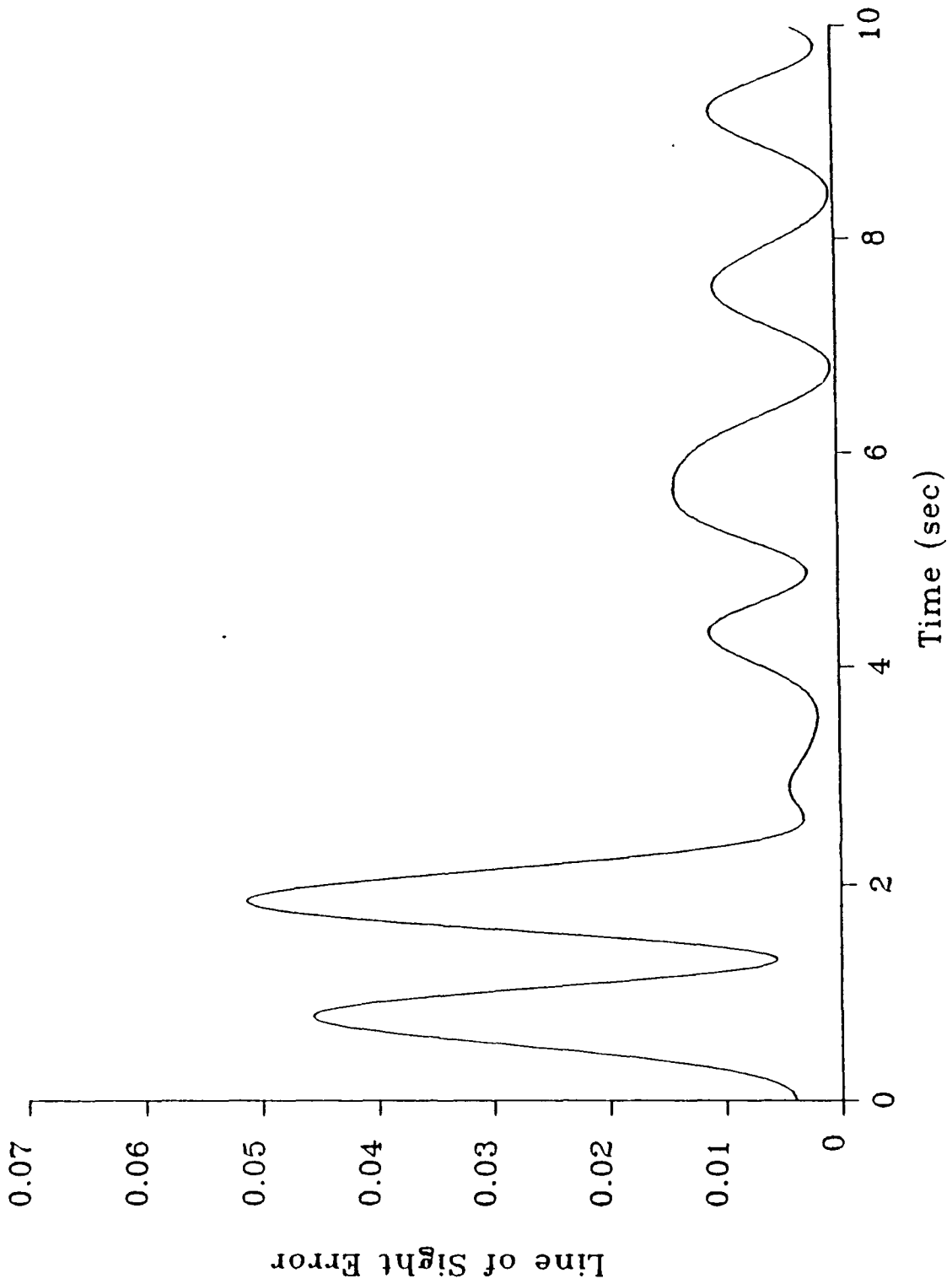


Figure 6.17: Suboptimal system LOS, n=8 (modes: 1-8); effect of LQR weighting on efficiency, q=100, e\*-7.81%, e=56.93%

# Evaluation Model SIM26

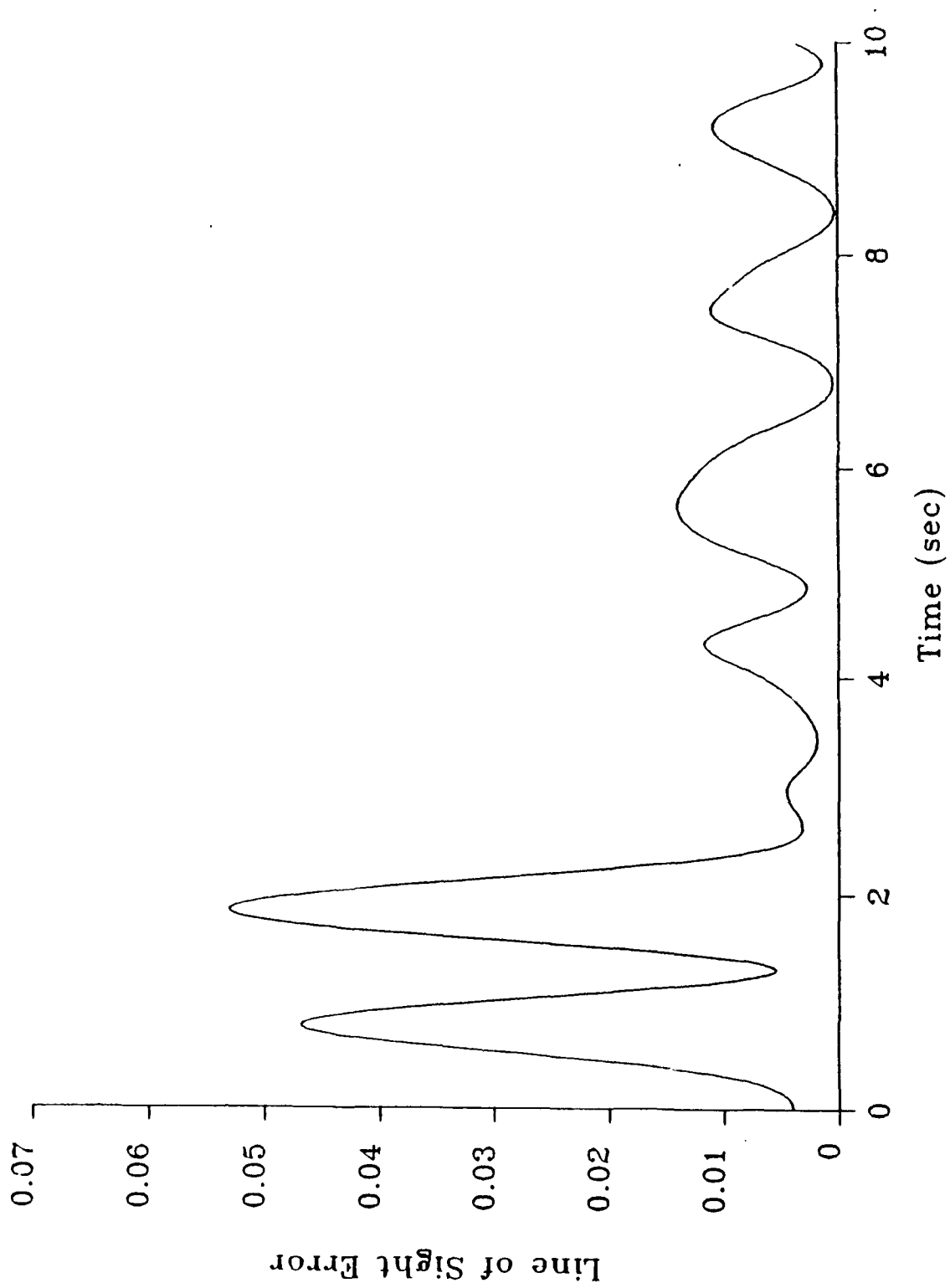


Figure 6.18: Evaluation model LOS, n=8 (modes: 1-8); effect of LQR weighting on efficiency, q=100, e\*=7.81%, e=56.93%

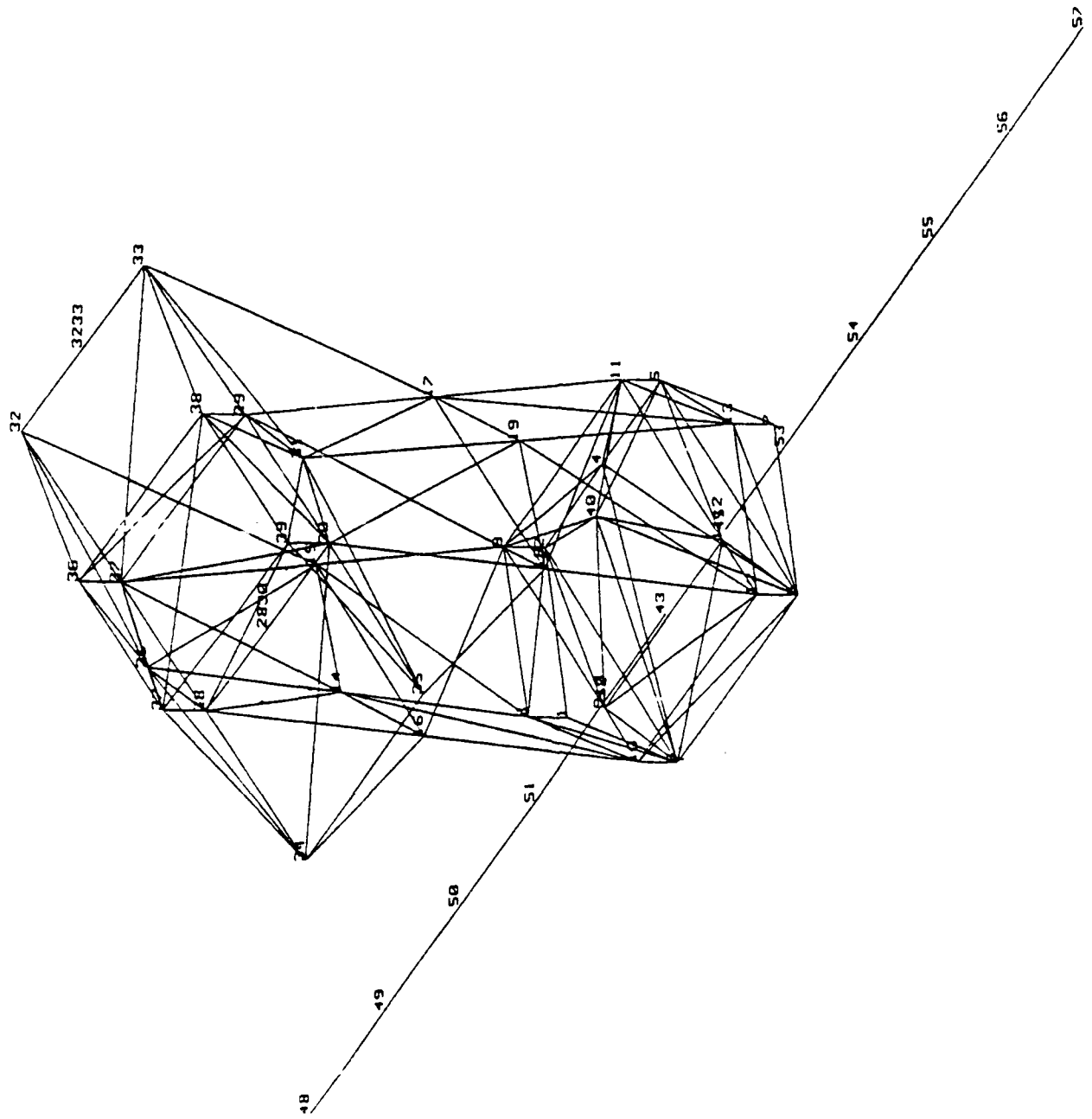


Figure 6.19: CSDL - ACOSS - 6 (model 2) structure

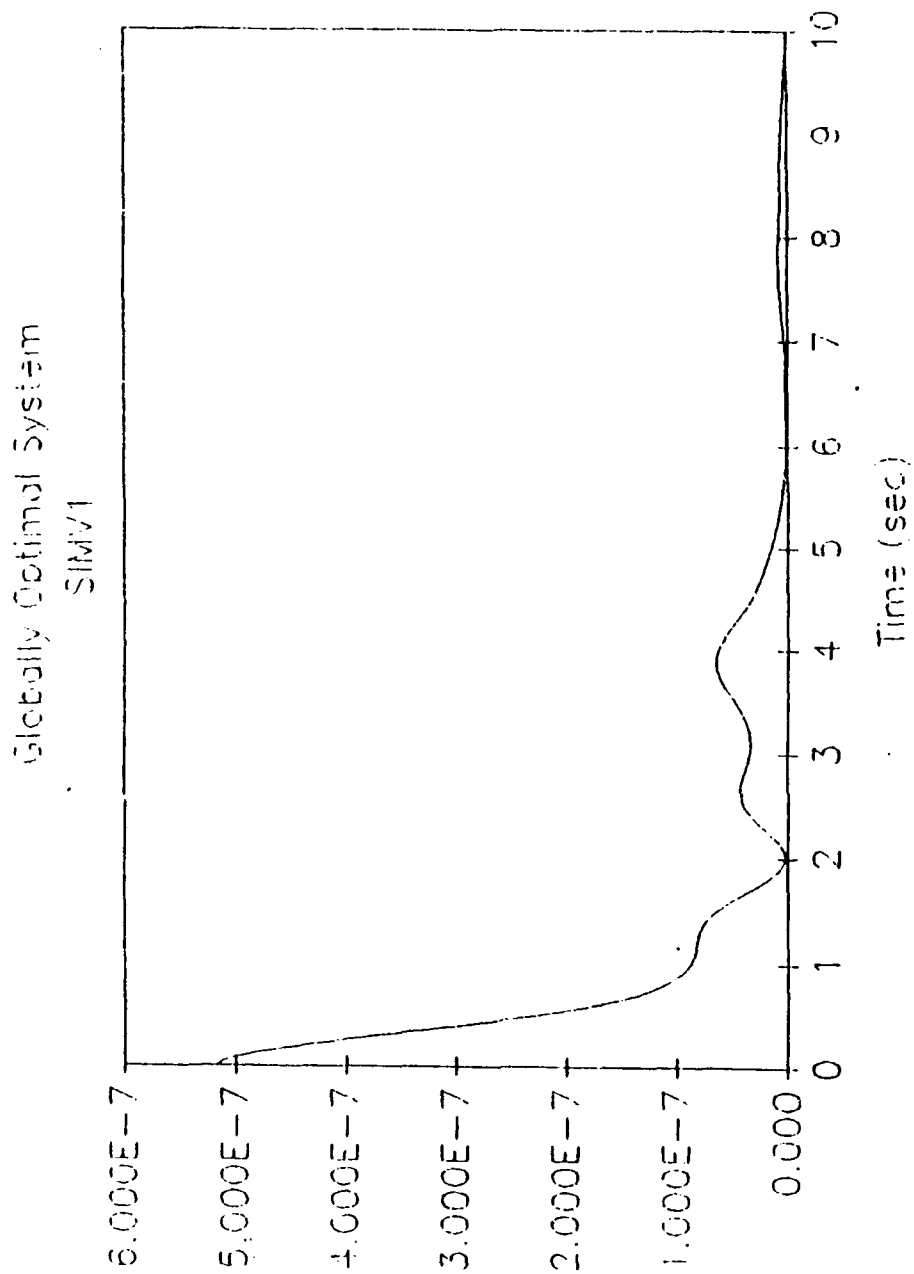


Figure 6.20: Globally optimal system, square x-y plane deflection at node 37, n=8, (modes: 12,13,17,21,22,24,28,30), example 6.3



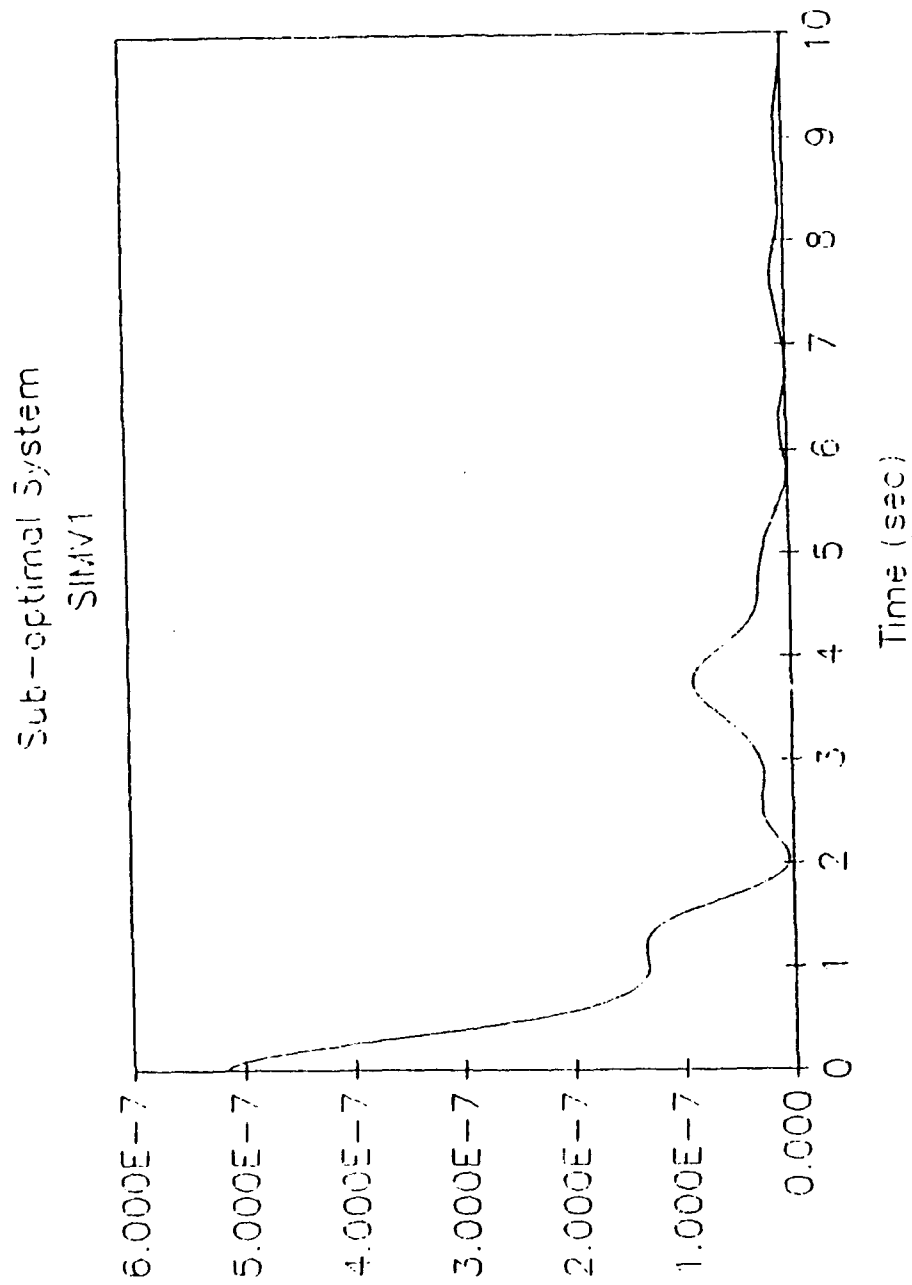


Figure 6.21: Suboptimal system, square x-y plane deflection at node 37,  $n=8$ , (modes: 12,13,17,21,22,24,28,30),  $m=4$  (#18-21), example 6.3

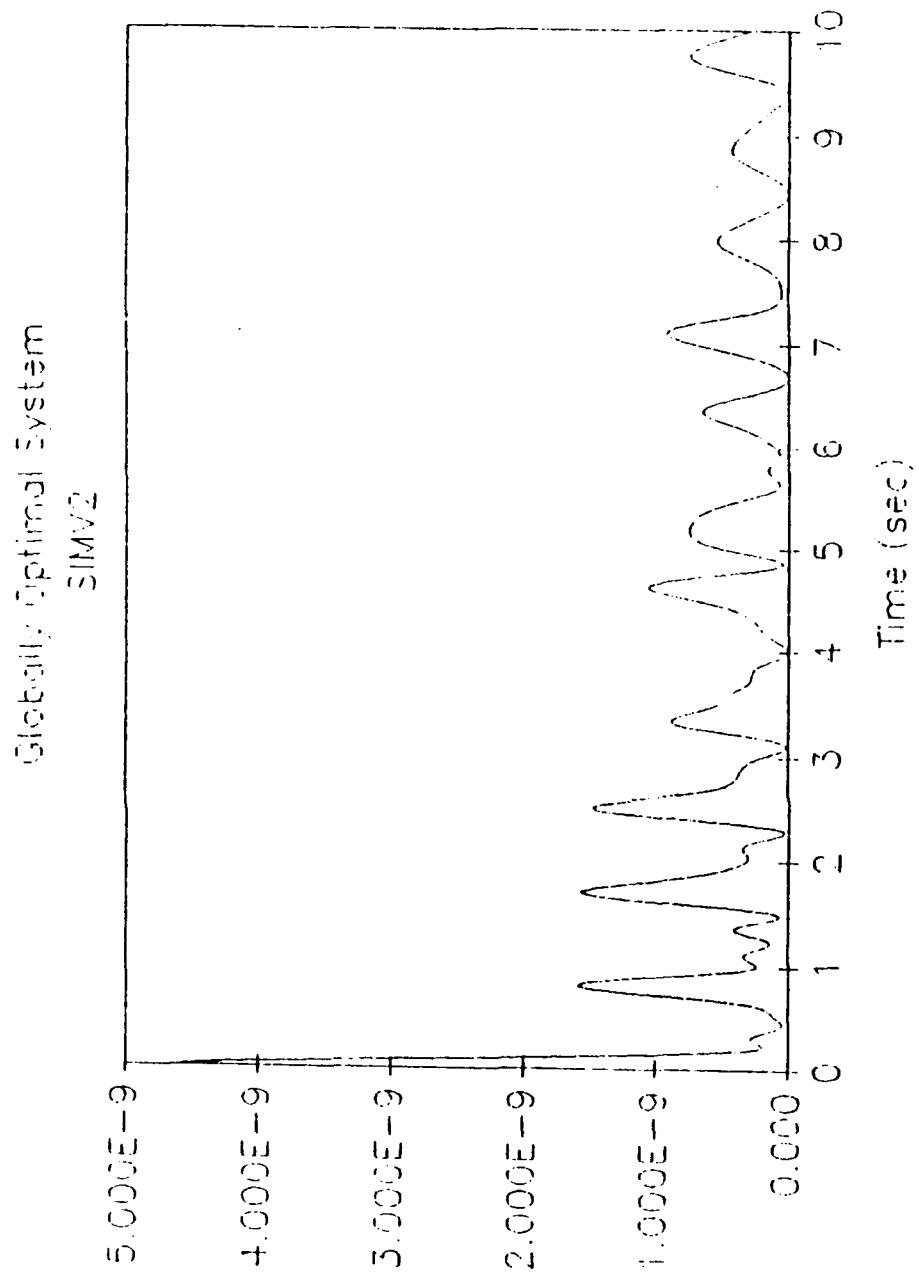


Figure 6.22: Globally optimal system, square x-y plane deflection at node 37, n=10, (modes: 7-16), example 6.4

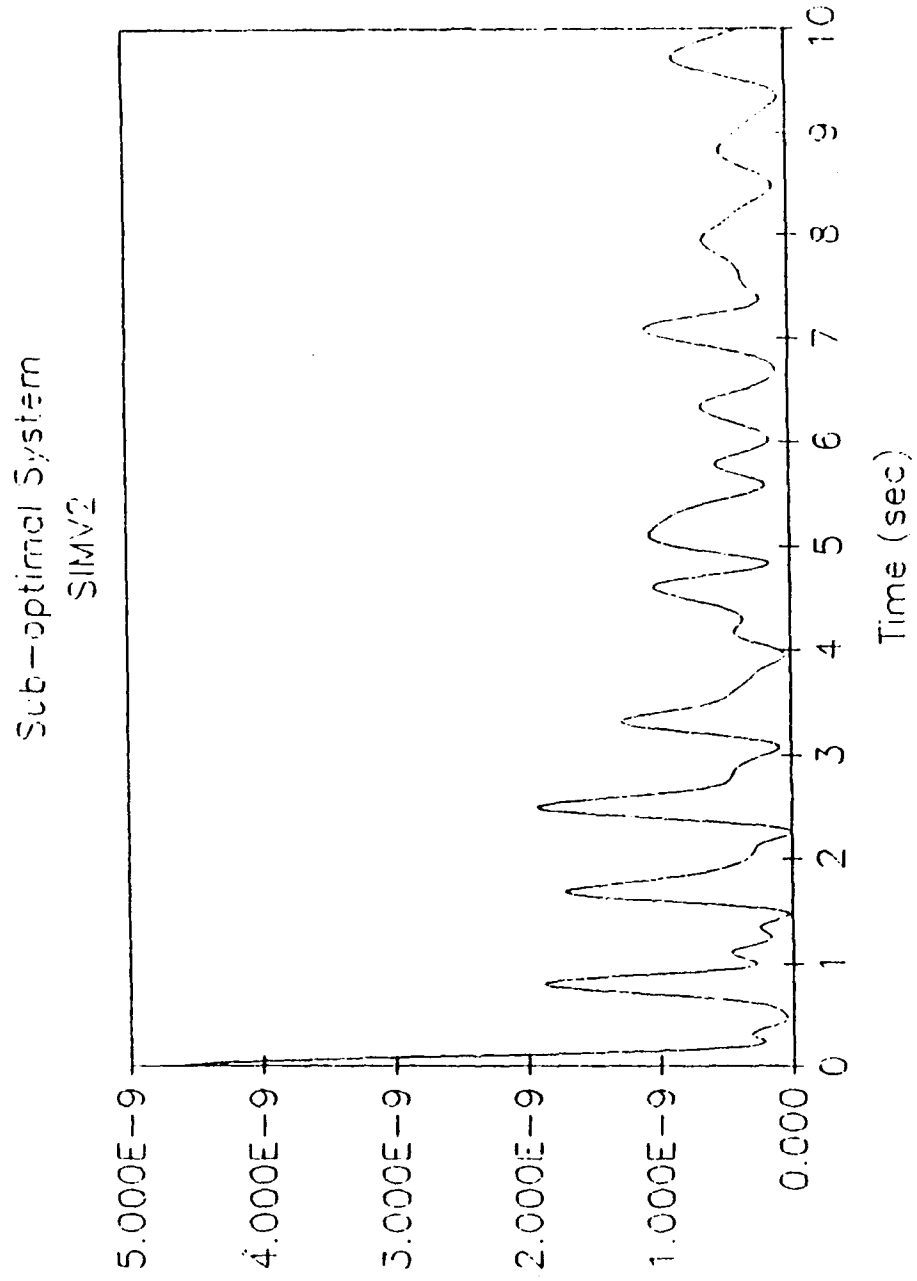


Figure 6.23: Suboptimal system, square x-y plane deflection at node 37, n=10, (modes: 7-16), m=10 (#1-10), example 6.4

## 7.0 CONCLUSIONS

The concept of efficiency of an engineering system is a time proven design and analysis tool in many disciplines. Structure-Control Systems should not pose an exception to this practice. This research undertook the task of defining an efficiency concept for structure-control systems, and established physical grounds for the efficiencies which would have the potential to address the issues in the control of large structural dynamic systems. We have shown that the global and relative model efficiencies proposed and developed in this research are the power efficiencies of the control system and therefore, they must be considered as fundamental figures of merit of a control system design along with other more familiar performance measures. No less important a feature of the efficiencies is that the performance of the evaluation model can be judged on the basis of the control design model alone without explicitly involving the full-order evaluation model, a feat made possible only by a unique judicious choice of the weighting matrices involved in the efficiency definitions.

The investigation developed the new concept of controller modes signifying characteristic avenues of efficient control power utilization. These efficiency modes are complementary to the familiar concept of structural modes and together they determine the effectiveness of any structural control design. Ultimately what counts is the degree of matching of these two sets of modes. The efficiency modes analysis leading to the recognition of principal controller directions led to a procedure to give structure to the internal workings of the structural control system through the control power matrices. We have attempted to exploit this internal characterization of the system by defining component efficiencies and efficiency components. Furthermore, we proposed and developed model/controller reduction approaches based on that structural information by correlating

the structural modes and the controller (efficiency) modes. We have illustrated the significance of the efficiency concepts by a host of examples on realistic structures. The end point of this investigation offers many other avenues of research. To mention only two: optimization of structural control system for efficiency subjected to other design constraints, design of control gains by a direct approach utilizing the system efficiency as the target design criterion. We have demonstrated that the Linear Quadratic Regulator Theory can produce highly inefficient control designs although they are regarded as optimal. However, the optimality of an LQR design is subjective and theory merely serves as a means of obtaining control gains in some sense. On the other hand, the definitions of power efficiencies of the control system constitute absolute non-dimensional unique performance measures. No subjectivity is involved in their definitions. The efficiency of all control systems regardless of the methodology by which they are designed can be analyzed by the approaches developed in this research and their merits can be established on a common ground.

## REFERENCES

1. Oz, H., "Dynamically Similar Control Systems and a Globally Optimal Minimum Gain Control Technique: IMSC," Journal of Optimization Theory and Applications, Vol. 59, No. 2, 1988, pp. 183-207.
2. Meirovitch, L., and Silverberg, L., "Globally Optimal Control of Self-Adjoint Distributed Systems," Optimal Control Applications and Methods, Vol. 4, pp. 365-386, 1983.
3. Oz, H., "Robust Approximations for Closed-loop Distributed Parameter Structures," The Ohio State University, Aeronautical and Astronautical Engineering Research Report on Dynamics and Control, No: AAE-RR-DC-102-1988.
4. Meirovitch, L., Baruh, H., and Oz, H., "A Comparison of Control Techniques for Large Flexible Systems," Journal of Guidance, Control and Dynamics, Vol. 6, No. 4, 1983, pp. 302-310.
5. Oz, H., "A New Concept of Optimality for Control of Flexible Structures," The Ohio State University, Aeronautical and Astronautical Engineering Research Report on Dynamics and Control, No: AAE-RR-DC-101-1988.
6. Oz, H., and Adiguzel, E., "Generalized Natural Performance Charts for Control of Flexible Systems," AIAA-84-1951-CP AIAA Guidance and Control Conference, Aug. 1984, Seattle, WA.
7. Yousuff, A., and Skelton, R., "Controller Reduction by Component Cost Analysis," IEEE Trans. on Auto. Control, Vol. AC-29, No. 6, 1984, pp. 520-530.
8. Oz, H., "Efficient Controller Reduction for Structure Control," NASA/DOD Control/Structures Interaction Technology 1987, AFWAL-TR-88-3052, pp. 136-144.
9. Hill, P.G., and Peterson, C.R., Mechanics and Thermodynamics of Propulsion, Addison-Wesley Publishing Company, Reading, MA, 1965.
10. RADC-TR 80-78, Vol I, ACOSS Four (Active Control of Space Structures) Theory, The Charles Stark Draper Laboratory, Inc., Cambridge, MA, April 1980.
11. Skelton, R.E., Dynamics Systems Control, John Wiley & Sons, Inc. 1988.
12. Yedavalli, R.K., "Time Domain Robust Control Design for Linear Quadratic Regulators by Perturbation Analysis," Proceedings IFAC Workshop on Model Error Concepts and Compensation, Boston, MA, June 1985, pp. 914-919.
13. Yedavalli, R.K., "Perturbation Bounds for Robust Stability in Linear State Space Models," International Journal of Control, Vol. 42, 1985, pp. 1507-1517.

14. Hyland, D.C., Bernstein, D.S., "The Optimal Projection Equations for Fixed-Order Dynamic Compensation," IEEE Transactions on Automatic Control, Vol. AC-29, No. 11, 1984, pp. 1034-1037.
15. Öz, H., Meirovitch, L., "Modal-Space Control of Distributed Gyroscopic Systems," Journal of Guidance and Control, Vol. 3, No. 2, 1978, pp. 140-150.
16. Khot, N.S., "Personal Communications," 1988-89.
17. RADC-TR-80-377, ACOSS Six (Active Control of Space Structures), The Charles Stark Draper Laboratory, Inc., Cambridge, MA, Jan. 1981.
18. Thyfault, D. "Decentralized Control of a Large Space Structure Using Direct Output Feedback", M.S. Thesis, Department of Aeronautical Engineering, Air Force Institute of Technology, Dec. 1983.
19. RADC-TR-82-21, ACOSS Five (Active Control of Space Structures) Phase IA., Lockheed Missiles and Space Company, Inc., March 1982.
20. Özgüner, Ü., Breitfeller, E., "Development of a Control Oriented Model of a Cantilevered Beam with End-Mass," Proceedings of the American Control Conference, Atlanta, GA, 1988.

APPENDIX A: PROOF OF FUNCTIONAL  $S^R$  AS CONTROL POWER

The control evaluation functionals  $S^R$ ,  $S^M$  and  $S^*$  which are uniquely defined for the efficiency analysis of structure-control systems represent power quantities, true rate of change of energy, a fundamental concept in the design of engineering systems. To see this, consider the starting point of this investigation in Chapter 2 which is the control functional  $S^R$  defined over the DPS:

$$S^R = \int f^T(p,t)m^{-1}(p)f(p,t)dD(p)dt \quad (A-1)$$

corresponding to the partial differential equations of motion.

$$m(p)\ddot{u}(p,t) + L(Up,t) = f(p,t) \quad (A-2)$$

First, perform a dimensional analysis. In Eq. (A-2)  $m(p)$  has the units of mass/dimD where dimD is the dimension of Domain of the Structure, mass is the mass or inertia distribution over D. For one-dimensional beam elements dimD=length, for plate elements dimD is an area etc... Consistently, the input function  $f(p,t)$  represents a force density over the domain D, it has the dimension Load/dimD where load can be a force or a moment. Substituting the dimensional equivalents of the terms appearing in the definition (A-1)

$$f(p,t) = \frac{\text{Load}}{\text{dimD}}, \quad m(p) = \frac{\text{mass}}{\text{dimD}}$$

$$S^R = \int \frac{\text{Load}}{\text{dimD}} \frac{\text{dimD}}{\text{mass}} \frac{\text{Load}}{\text{dimD}} \text{dimD} * d(\text{time})$$

$$S^R = \frac{\text{Load}^2 * \text{Time}}{\text{mass}} \quad (A-3)$$

and noting



$$\text{Load} = \text{mass} * \text{acceleration} = \text{mass} * \frac{\text{displacement}}{\text{time}^2}$$

Expression (A-3) yields after simplification

$$S^R = \frac{\text{Load} * \text{displacement}}{\text{Time}} = \frac{\text{Energy}}{\text{Time}} = \text{Power} \quad (\text{A-4})$$

Since  $S^M$ ,  $S^*$  are also derived from  $S^R$  by an invariant frame transformation,  $S^M$  and  $S^*$  are also power quantities.

Next, we consider the mechanical aspects of this power. To make this easier to see, consider the one-dimensional mechanical analogy of the DPS equation of motion (A-2), the familiar spring mass system where the spatial dimension has been (integrated) out.

$$m\ddot{u} + ky = f(t) \quad , \quad \frac{k}{m} = \omega^2 = \text{rad/sec}^2 \quad (\text{A-5})$$

for which  $S^R$  is

$$S^R = \int m^{-1} f^2(t) dt \quad (\text{A-6})$$

Multiply Eq. (A-5) by  $m^{-1}f(t)$  and integrate over time, and recognize that the right hand side becomes  $S^R$ :

$$\int f\ddot{u} dt + \int \omega^2 f u dt = S^R \quad (\text{A-7})$$

substituting  $\dot{u} = du/dt$

$$\int f d\dot{u} + \int \omega^2 f u dt = S^R \quad (A-8)$$

Again the left hand side has power units and specifically the first integral on the left is the area under the load versus velocity diagram. Alternately, write the equation of motion (A-5)

$$m(\ddot{u} + \omega^2 u) = f(t) \quad (A-9)$$

and define the effective rigid body acceleration that the forcing function  $f(t)$  imparts to the mass  $m$  as  $\ddot{u}_R = \ddot{u} + \omega^2 u$  where  $u_R$  is the effective rigid body displacement due to  $f(t)$ . Hence, (A-5) is equivalent to

$$m\ddot{u}_R = f(t)$$

Multiply this last expression by  $\dot{u}_R$

$$m\dot{u}_R\ddot{u}_R = f(t)\dot{u}_R \quad (A-10)$$

$$\frac{d}{dt} \frac{1}{2} m \dot{u}_R^2 = f \dot{u}_R \quad (A-11)$$

from which we recognize the effective rigid body kinetic energy

$$T^R = \frac{1}{2} m \dot{u}_R^2$$

and Eq. (A-10) is the more familiar power expression

$$\frac{d}{dt} T^R = f \dot{u}_R = P \quad (A-12)$$

The incremental power change associate with Eq. (A-12) is

$$f d\dot{u}_R = dP$$

The effective rigid body incremental velocity  $d\dot{u}_R$  can be written as

$$d\dot{u}_R = (u + \omega^2 u) dt \quad (A-13)$$

Multiplying on the left by  $f$  and recognizing (A-12)) we obtain

$$\int f(\ddot{u} + \omega^2 u) dt = \int f d\dot{u}_R - \int dP = P$$

where  $P$  is the total mechanical power. The integral on the left is  $S^R$  by virtue of Eq. (A-8), hence, we get

$$S^R = \int m^{-1} f^2 dt = \int f d\ddot{u}_R = P$$

We have thus also shown mechanically that  $S^R$  functional is a total power expression associated with the control input  $f$  in trying to move the structure with an effective rigid body velocity  $u_R$ . It must be recognized that  $u_R$  is not a fictitious mathematical expression. Both  $\ddot{u}_R$  and  $\dot{u}_R$  as derivatives are well defined quantities at all times in terms of  $u$  as given by Eq. (A-13). The concepts of effective rigid body velocity/acceleration are also plausible because this is what the controller attempts to impart to the structural mass via the thrust of control input  $f$ . The controller actuators at any instant will not recognize that the mass has flexibility. The effect of flexibility is inherent in the structure defined by the domain of  $m$ , and it manifests itself to result in an actual acceleration  $\ddot{u}$  by extracting  $\omega^2 u$  from  $\ddot{u}_R$

$$\ddot{u}_R - \omega^2 u = \ddot{u} \tag{A-14}$$

which provides the mechanism to bleed off control power into the flexible dynamics to store it as elastic energy in the structure.

Next, we consider some specific control hardware.

### Thruster Jets

For a thruster using an exhaust jet, the thrust force  $f(t)$  is the control input and given by

$$f = \dot{m}_p(t) V_{ex} \tag{A-15}$$

where  $V_{ex}$  is the exhaust jet velocity and  $\dot{m}(t)$  is rate of flow of propellant mass  $m_p$ . Form the control functional  $S^R$  for such a thruster

$$S^R = \int m^{-1} f^2(t) dt \quad (A-16)$$

and substituting (A-15) into (A-16) by assuming constant exhaust velocity

$$S^R = \frac{V_{ex}^2}{m} \int \dot{m}_p^2(t) dt \quad (A-17)$$

in which the propellant flow rate is the control variable. For simplicity, the change in the total mass  $m$  due to propellant flow was neglected. The kinetic energy rate of flow carried away by the exhaust jet is on the other hand

$$\dot{E}_p = \frac{d}{dt} (m_p V_{ex}^2) = \frac{1}{2} V_{ex}^2 \dot{m}_p(t) = P(t) \quad (A-18)$$

where  $P$  is the power of the exhaust jet at time  $t$ . Substituting from (A-18) into (A-17) for  $\dot{m}_p$  we get

$$S^R = \frac{4}{m V_{ex}^2} \int P^2(t) dt \quad (A-19)$$

Denote the root mean square power in the exhaust beam by  $P_{rms}$  and write

$$P_{rms}^2 T = \int P^2(t) dt$$

where  $T$  is the control (regulation) time. Next, introduce an average characteristic power  $P_c$  for the combination of the structure to be controlled and the nature of the thruster propellant

$$P_c = \frac{1}{2} m V_{ex}^2 / T = \frac{m g_o^2 I_{sp}^2}{2T} \quad (A-20)$$

where  $I_{sp}$  is the specific impulse of the thruster propellant and  $g_0$  is the gravitational acceleration on earth. Now the  $S^R$  can be rewritten as

$$S^R = \left( \frac{2P_{rms}}{P_c} \right) P_{rms} \quad \text{or} \quad S^R = \left( \frac{4P_{rms}}{W_0 g_0 I_{sp}^2} \right) P_{rms} \quad (\text{A-21})$$

in which  $W_0$  is the on earth weight of the structure.

Equations (A-21) clearly shows that  $S^R$  is proportional to the root mean square power of the exhaust beam power with a nondimensional proportionality constant which provides an interesting combination of control, structure and thruster propellant parameters.

#### Proof-Mass Actuators

As another controller example, we consider a proof-mass actuator of the type discussed in Ref. (20). The actuator dynamics given in Ref. (20) accounts for coupling with the structural dynamics and typically represents a compensator dynamics, a subject which was discussed in Section 5.2. For simplicity of this discussion, we shall disregard the interaction of the actuator with the structure and represent the force  $f(t)$  it exerts on the structure by the equation

$$f(t) = K_{amp} K_f V(t) \quad (\text{A-22})$$

where  $V(t)$  is the control voltage,  $K_f$  is a force coefficient (force/amper). The  $K_{amp}$  is given in terms of the armature current and voltage:

$$K_{amp}^{(s)} = I_{arm}^{(s)} / V(s) = \frac{1}{Ls + R} \quad (\text{A-23})$$

$s$  is the LaPlace variable,  $L$  and  $R$  are the inductance and resistance of the actuator. If the time constant  $L/R$  is small, then  $K_{amp} = R^{-1}$ . In this case the actuator force output is simply given by

$$f(t) = K_f I_{arm}(t) = K_f R^{-1} V(t) \quad (A-24)$$

Forming  $S^R$

$$S^R = \int_0^T \dot{l}^2(t) dt = \int_0^T K_f^2 R^{-2} V^2 dt = \int_0^T K_f^2 l^2 dt$$

Multiplying and dividing by  $R$  and recognizing that  $I^2 R$  is the electrical power  $P$  consumed by the actuator to control the structure we write,

$$S^R = \frac{K_f^2}{mR} \int_0^T R I^2 dt = \frac{K_f^2 T}{mR} P_m$$

for which we defined the mean power  $P_m = T^{-1} \int_0^T R I^2 dt$  representing the average electrical power of the proof-mass actuator over the control period. The coefficient of  $P_m$  can also be shown to be a nondimensional quantity relating the control, structure and actuator parameters as in the case of thrust jets discussed earlier.

The quantities  $S^M$  and  $S^*$  by definition have the same units as  $S^R$  and therefore, the efficiency quotients defined  $e$ ,  $e^*$  in this report are physically the power efficiencies of the controller.

APPENDIX B: CLOSED FORM SOLUTION FOR S\*

The S\* is the control power for the continuously distributed input field over DPS where the modal inputs are based in the independent modal-space control approach (Regs. 1, 4). The modal state-space dynamics for the 2n-th order design model is:

$$\dot{x}_r = \begin{bmatrix} 0 & 1 \\ -\omega_r^2 & 0 \end{bmatrix} x_r + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f_r^*(t) \quad r=1,2,\dots,n \quad (\text{B-1})$$

with the modal states  $x_r = [\xi_1, \xi_r]^T$ , and  $f_r^*(t)$  is the modal input given by

$$f_r^*(t) = g_{r1}^* \dot{\xi}_r + g_{r2}^* \xi_r$$

$$g_{r1}^* = \omega_r^2 - \rho_{r1}\rho_{r2} \quad , \quad g_{r2}^* = -(\rho_{r1}\rho_{r2})$$

$\rho_{r1}$  and  $\rho_{r2}$  are the r-th closed-loop modal eigenvalues corresponding to the open-loop structural eigenvalues  $\pm j\omega_r$ .

The control power S\* as defined in Chapter 2 can now be written as

$$S^* = \int f_r^{*2}(t) dt = \sum_{r=1}^n x_{r0}^T P_r^* x_{r0}$$

$P_r^*$  is the 2x2 optimal control power obtained as the solution of n 2x2 independent Lyapunov equations

$$A_r^T P_r^* + P_r^* A_r + G_r^{*T} G_r^* = 0 \quad r=1,\dots,n \quad (\text{B-2})$$

$$A_r = \begin{bmatrix} 0 & 1 \\ -\omega_r^2 & 0 \end{bmatrix} \quad , \quad G_r^* = [g_{r1}^* \quad g_{r2}^*]$$

The solution of these modal Lyapunov equations is easily obtained as

$$P_r^* = \begin{bmatrix} P_1^* & P_0^* \\ P_0^* & P_2^* \end{bmatrix} \quad (B-3)$$

$$P_1^* = \frac{\omega g_1^2 g_2^2}{2(g_1 - \omega^2)} + \frac{g_2(g_1 - \omega^2)}{2} - \frac{g_1^2}{2g_2} - g_1 g_2 \quad (B-4)$$

$$P_0^* = - \frac{g_1^2}{2(g_1 - \omega^2)} \quad (B-5)$$

$$P_2^* = \frac{g_1^2}{2g_2(g_1 - \omega^2)} - \frac{g_2}{2} \quad (B-6)$$

in which  $g_1 = g_{r1}^*$  ,  $g_2 = g_{r2}^*$  ,  $\omega = \omega_r$   $r=1, 2, \dots, n$ .



APPENDIX C: COMPUTER RESULTS OF EXAMPLES ILLUSTRATED IN SECTIONS 3, 4 & 6

Nomenclature for the computer outputs

The following consists of description of terminology encountered in the computer program that may not be self explanatory. Some output statements will not be of any interest to the reader as they are only for program control and debugging purposes. Hence, no explanation is given for them.

- \* Reduced principal efficiency components NP is the number of efficiency states considered,  $n_p=1,2,\dots, NP$ .
  
- \* Structural natural frequencies for the evaluation model are ordered in increasing order. The first n are in the control design model in increasing order, the rest are in the truncated model in increasing order. NE is the evaluation model order. N is the controlled structural modes.
  
- \* Init. Gen. Displacement and Init. Gen. Velocity are  $q(t_0)$  and  $\dot{q}(t_0)$  for the evaluation model.
  
- \*  $S^*$  is  $S^*$  , SM is  $S_c^M$  and SR is  $S^R$ .
  
- \* Principal Components Analysis of \_\_\_\_ efficiency is \_\_\_\_ efficiency modes analysis.

- \* PO is the  $n_p$  dimensional efficiency states initial conditions  $\epsilon$ . CE is squared efficiency coefficients  $c_i^2$ .  $i=1,2,\dots, NP$ , Eq. (3.36).
- \* Characteristic efficiencies are  $\lambda_i^e$  in random order. Efficiency vectors are the efficiency components  $e_i$ , total efficiency is the efficiency (not in percent).
- \* Eff... Num... Control Cost is  $x_0^T P_N x_0$   
 Eff... Denom... Control Cost is  $x_0^T P_D x_0$   
 Numerator/Denominator Component cost is  $(X_0 P_N)_{jj}$   $j=1,2,\dots,n$  for each structural mode.
- \* Unordered...Comp Eff (Num. comp. Costs/Denom) is  $(X_0 P_N)_{jj} / x_0^T P_D x_0$ .
- \* Unordered Comp..Efficiencies is Equation (4.16).
- \* Method 1 is the "coupled efficiencies" cf. Equation (4.16) of structural degrees of freedom.
- \* Method 2 is the "decoupled efficiencies" of Equation (4.17) of structural degrees of freedom.
- \* Mode (in modsel) gives in decreasing order coupled efficiencies  $e_j$  for structural modes. In modsel implies that the modes are designated as labeled in selected modes for control design. For example if 14th

and 31st modes are selected for control in modsel they are designated as the first and 2nd selected modes, respectively.

- \* NPth order efficiency (ENP) indicates that efficiencies are computed via Eq. (4.20) - (4.22) by considering NP efficiency components.
- \* Block-diag numerator and denominator Comp. costs are side by side  $n_j$  and  $d_j$  in Eq. (4.17). Each row corresponds to a  $j$ -th structural mode.
- \* Block-diag. eff...Num/Denom...Control cost is the numerator/denominator of Eq. (4.19). Unordered comp. efficiencies is Eq. (4.17) for each  $j$ .
- \* Block-diag..Comp..Eff.. is  $e_j$  in Eq. (4.17) in decreasing order for the modes in selected modes.
- \* NPth order block-diag. efficiency ENP is Eq. (4.19).



1 .000000 .000000 .000000 1.000000 .000000 .000000 .000000 .000000 .000000 .000000 .000000 .000000

INIT. GEN. VELOCITY

1 .000000 .000000 .000000 .000000 .000000 .000000 .000000 .000000 .000000 .000000 .000000 .000000

-----IN ROUTINE CONTROL-----  
 -----IN ROUTINE USERCPL-----

-----LINEAR QUADRATIC REGULATOR CONTROL DESIGN -----

OPTIMAL GAIN MATRIX

RG4 1 -1.37535482E-02 1.41421293E+00 -8.05713811E-01 -1.41283475E+00

CLOSED LOOP EIGENVALUES

REAL PART           IMAGINARY PART  
 --.10244383E+00      --.10254772E+02  
 --.10244383E+00      .10264772E+02  
 --.17511023E-01      --.12905104E+02  
 --.17511023E-01      .12905104E+02

RICCATI MATRIX

1                   .499999   2.035748   3.477907  
 2                   9.764363   -5.75956   .019071  
 3                   2.036747   -5.475956   .493051  
 4                   3.477907   .019071   .493051   57.118582  
 -----IN ROUTINE CLEIGP-----

COMPLEX CLOSED-LOOP EIGENVALUES

--.1024438E+00                   --.1028477E+02  
 --.1024438E+00                   .1025477E+02  
 --.1751102E-01                   --.1290510E+02  
 --.1751102E-01                   .1290510E+02  
 -----IN ROUTINE EFFNCY-----

DAMPING RATIO OF CONTROLLED MODES

DAMPING RATIOS :           .0099502           .0013569

IN ROUTINE FOR EFFICIENCY COMPUTATIONS

INITIAL MODEL STATE VECTOR:  
-.3033375E+00 .0000000E+00 .8117159E+00 .0000000E+00

PER CENT EFFICIENCIES

-----

EFFICIENCY COEFFICIENT:

.2350453E+02

GLOBAL EFFICIENCY

.1333403E+00

RELATIVE MODEL EFFICIENCY:

.6210925E+01

---GLOBALLY MINIMUM COST S#---: .2718747E+01

---MODEL COST S#---: .6860378E+02

---REAL COST SR---: .1587549E+04

GLOBAL SPILLOVER QUOTIENT: .5204443E+03

RELATIVE MODEL SPILLOVER QUOTIENT: .9567891E+00

-----IN ROUTINE SIMDAT-----

IN ROUTINE RESSIM

-----IN ROUTINE MOEFF-----

PRINCIPAL COMPONENTS ANALYSIS OF GLOBAL EFFICIENCY

P0

- 1 39.509565
- 2 14.371874
- 3 1.635957
- 4 .023735

C E

```

1  .383534
2  .014950
3  .013812
4  .000000

```

CHARACTERISTIC EFFICIENCIES

```

1  .01229
2  .042031
3  .01229
4  .041935

```

EFFICIENCY VECTOR

```

1  .001208
2  .00628
3  .000002
4  .000000

```

TOTAL EFFICIENCY = .00184

GL03.. EFF..MODAL MATRIX T

```

1  -.000001  .062252  .000000  -.000264
2  .000000  .027115  -.000015  .640254
3  .020508  .000014  .000880  .000001
4  -.011361  -.000007  .264648  .000220
-----IN ROUTINE EMOSEL-----
EFF..NUM...CONTROL COST : .291875E+01
EFF..DENOM..CONTROL COST : .158755E+04
EFFICIENCY CHECK AFTER DECOMP.: .183941E-02

```

OPERATOR COMPONENT COSTS:

.997230E+00 .192152E+01

ENHANCER COMPONENT COSTS:

```

.238626E+02 .150378E+00
UNORDERED..COMP EFF(NUM.COMP.COSTS)/DENOM.) :
.528117E-03 .111039E-02
UNORDERED COMP..EFFICIENCIES : .327878E-03 .151053E-02

```

```

*****
METHOD 1: INTERACTING CONTRIBUTIONS TO THE NP
EFF..NUMBER..COMPONENTS, IN DECREASING ORDER:

```

```

*****
MODE(IN MODEL) COMP..EFF..
2 .151053E-02
1 .327878E-03
NP TH ORDER EFFICIENCY END: .133341E-02

```

```

BLOCK-DIAG NUMERATOR & DENOMINATOR COMP. COSTS:
.997230E+00 6 .237431E+02
.192152E+01 6 .156366E+04

```

```

BLOCK-DIAG.EFF...NUM...CONTROL COST : .291875E+01
BLOCK-DIAG.EFF...DENOM...CONTROL COST : .158740E+04
UNORDERED COMP..EFFICIENCIES : .420007E-01 .122886E-02

```

```

*****
METHOD2: BLOCK-DIAGONAL CONTRIBUTIONS TO
NP EFFICIENCY COMPONENTS IN DECREASING ORDER:

```

```

*****
MODE(IN MODEL) BLOCK-DIAG..COMP..EFF..
1 .420007E-01
2 .122886E-02

```

```

NP TH ORDER BLOCK-DIAG.EFFICIENCY ENP : .193870E-02
-----IN ROUTINE MODEFF-----
PRINCIPAL COMPONENTS ANALYSIS OF RELATIVE EFFICIENCY

```

- 1 39.631383
- 2 -3.257025
- 3 2.120201
- 4 -1.378122

PJ



C E

1  
2  
3  
4

1	.019291
2	.006592
3	.002831
4	.031196

CHARACTERISTIC EFFICIENCIES

1  
2  
3  
4

1	.043211
2	.043211
3	.043211
4	.043211

EFFICIENCY VECTOR

1  
2  
3  
4

1	.042748
2	.002395
3	.000122
4	.000052

TOTAL EFFICIENCY = .04321

REL.. EFF..MOAL MATRIX Y

1 2 3 4  
1 -.009245 .019831 .059199 -.003099  
2 .020784 .005547 .035322 .638922  
3 .020194 -.000881 .003478 -.000841  
4 .025023 .250821 -.081403 .001723  
-----IN ROUTINE EMOSEL-----  
EFF..NUM..CONTROL COST : .684038E+02  
EFF..DENCH..CONTROL COST : .158765E+04  
EFFICIENCY CHECK AFTER DECOMP.: .432109E-01

OPERATOR COMPONENT COSTS:

.103137E+01 .675724E+02

Denominator Component costs

233682E+02 .130378E+04  
UNORDERED..COMP EFF(NUM.COMP.COSTS)/DENOM.) :  
.67919E-03 .425613E-01  
UNORDERED COMP..EFFICIENCIES : .67919E-03 .425613E-01

\*\*\*\*\*  
METHOD 1: INTERACTING CONTRIBUTIONS TO THE NP  
EFF..NUMBER.. COMPONENTS, IN DECREASING ORDER:

\*\*\*\*\*

MODE(IN MDSEL) COMP..EFF..  
2 .425613E-01  
1 .57919E-03  
NP TH ORDER EFFICIENCY ENP: .432109E-01

BLOCK-DIAG NUMERATOR & DENOMINATOR COMP. COSTS:

.102596E+01 6 .237431E+02  
.675570E+02 6 .155366E+04  
BLOCK-DIAG.EFF..NUM..CONTROL COST : .635930E+02  
BLOCK-DIAG.EFF..DENCH..CONTROL COST : .138740E+04  
UNORDERED COMP..EFFICIENCIES : .432109E-01 .432109E-01

\*\*\*\*\*  
METHOD2: BLOCK-DIAGONAL CONTRIBUTIONS TO  
NP EFFICIENCY COMPONENTS IN DECREASING ORDER:

\*\*\*\*\*

MODE(IN MDSEL) BLOCK-DIAG..COMP..EFF..  
1 .432109E-01  
2 .432109E-01  
NP TH ORDER BLOCK-DIAG.EFFICIENCY ENP : .432109E-01  
15.07.67.UCLP, C2, N1701M3, 0.512KUNS.



```

1  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
2  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
3  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
4  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
5  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
6  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
7  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
8  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
9  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
10 .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
11 .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
12 .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000

```

INIT. GEN. VELOCITY

```

1  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
2  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
3  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
4  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
5  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
6  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
7  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
8  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
9  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
10 .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
11 .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000
12 .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000  .000000

```

-----LINEAR QUADRATIC REGULATOR CONTROL DESIGN -----

OPTIMAL GAIN MATRIX

```

ROW 1  -2.91372557E-02  -1.00404635E+00  7.70322341E-01  6.63498174E-01
ROW 2  -9.83476376E-03  9.95571342E-01  -7.69564199E-01  -1.24605421E+00

```

CLOSED LOOP EIGENVALUES

REAL PART

```

-1.6549753E+00  -.10294772E+02
-1.6549753E+00  .10294772E+02
-1.9837371E-01  -.12905104E+02
-1.9837371E-01  .12905104E+02

```

IMAGINARY PART

```

-1.6549753E+00  -.10294772E+02
-1.6549753E+00  .10294772E+02
-1.9837371E-01  -.12905104E+02
-1.9837371E-01  .12905104E+02

```

RICCATI MATRIX

```

1  727.327457  .499996  2.561121  3.319765
2  .499996  5.875891  -5.226722  .025225
3  2.561121  -5.226722  .499996  .496640
4  3.319765  .025225  .496640  50.425053

```

COMPLEX CLOSED-LOOP EIGENVALUES

```

-1.654975E+00  -.1029477E+02
-1.654975E+00  .1029477E+02
-1.983737E-01  -.1290510E+02
-1.983737E-01  .1290510E+02

```

-----IN ROUTINE EFFNCY-----

DAMPING RATIO OF CONTROLLED MODES

DAMPING RATIOS : .0141455 .0015372

IN ROUTINE FOR EFFICIENCY COMPUTATIONS

INITIAL MODAL STATE VECTOR:  
-.3033375E+00 .0000000E+00 .8117159E+00 .0000000E+00

PER CENT EFFICIENCIES

-----

EFFICIENCY COEFFICIENT:

.3033375E+02

GLOBAL EFFICIENCY

.2575349E+00

RELATIVE MODEL EFFICIENCY:

.7894466E+01

---GLOBALLY MINIMUM COST Sa---: .3593351E+01

---MODEL COST Sa---: .1101502E+03

---REAL COST SR---: .1395297E+04

GLOBAL SPILLOVER QUOTIENT: .3576431E+03

RELATIVE MODEL SPILLOVER QUOTIENT: .9210555E+00

---IN ROUTINE SIMOAT-----

IN ROUTINE RESSIM

---IN ROUTINE MCOEFF-----

PRINCIPAL COMPONENTS ANALYSIS OF GLOBAL EFFICIENCY

1  
1 37.060355  
2 -.087373  
3 2.260434  
4 .013155

PJ

1 =

1	.33435e
2	.21191e
3	.20365e
4	.000000

CHARACTERISTIC EFFICIENCIES

1	.001577
2	.034737
3	.001577
4	.064732

EFFICIENCY VECTOR

1	.001555
2	.001015
3	.000005
4	.000000

TOTAL EFFICIENCY = .00252

GL08.. EFF..MODAL MATRIX T

1	2	3	4	
1	-.000024	.074198	.000000	-.000328
2	.000020	.003377	-.000020	.763099
3	.021824	.000024	.001331	.000002
4	-.017179	-.000019	.281616	.000389
-----IN ROUTINE EMOSEL-----				
EFF..NUM..	CONTROL COST :			.359335E+01
EFF..DENOM..	CONTROL COST :			.139529E+04
EFFICIENCY CHECK AFTER DECOMP.: .257535E-02				

NUMERATOR COMPONENT COSTS:

.141556E+01 .217579E+01

MINIMATOR COMPONENT COSTS:

```

.169722E+02 .137916E+00
UNORDERED.. COMP EFFNUM..COMP.COSTS)(GENM..) :
.101524E-02 .156010E-02
UNORDERED COMP..EFFICIENCIES : .523196E-03 .203215E-02

```

```

*****
METHOD 1: INTERACTING CONTRIBUTIONS TO THE NP
EFF..NUMER.. COMPONENTS, IN DECREASING ORDER:

```

```

*****

```

```

MODE(LIN MOOSEL) COMP..EFF..
2 .203215E-02
1 .523196E-03
NP TH ORDER EFFICIENCY END: .257535E-02

```

```

BLOCK-DIAG NUMERATOR & DENOMINATOR COMP. COSTS:

```

```

.1*15565*01 E .157132E+02
.217679E+01 E .137825E+04

```

```

BLOCK-DIAG.EFF..NUM...CONTRCL COST : .359335E+01

```

```

BLOCK-DIAG.EFF...JENDY...CONTRCL COST : .139496E+04
UNORDERED COMP..EFFICIENCIES : .8*7567E-01 .157939E-02

```

```

*****
METHOD 2: BLOCK-DIAGONAL CONTRIBUTIONS TO
NP EFFICIENCY COMPONENTS IN DECREASING ORDER:

```

```

*****

```

```

MODE(LIN MOOSEL) BLOCK-DIAG..COMP..EFF..
1 .847557E-01
2 .157939E-02

```

```

NP TH ORDER BLOCK-DIAG.EFFICIENCY END : .257535E-02
-----IN ROUTINE MOSEFF-----
PRINCIPAL COMPONENTS ANALYSIS OF RELATIVE EFFICIENCY

```

```

1 37.043525
2 -6.163113
3 2.244774
4 .833030

```

P0

C E

1  
2  
3  
4

1	.953470
2	.013421
3	.003511
4	.000497

CHARACTERISTIC EFFICIENCIES

1  
2  
3  
4

1	.073650
2	.085133
3	.073356
4	.085133

EFFICIENCY VECTOR

1  
2  
3  
4

1	.077547
2	.001370
3	.002335
4	.000043

TOTAL EFFICIENCY = .07894

REL.. EFF..MODAL MATRIX T

1  
2  
3  
4

1	.000067	.074187	.001336	.000015
2	-.017230	-.000159	.000327	.762912
3	.021816	-.000021	.001366	.000493
4	-.017623	-.000073	.281543	-.000350

-----IN ROUTINE EMGSEL-----  
EFF..NUM...CONTROL COST : .110150E+03  
EFF..BENCHM...CONTROL COST : .139529E+04  
EFFICIENCY CHECK AFTER DECOMP.: .769445E-01

OPERATOR COMPONENT COSTS:

.145462E+01 .109596E+03



Denominator Component Costs

.163761E+22 .1373-15+04  
UNORDERED.. COMP EFF(NUM.COMP.COSTS)/DENOM.) :  
.104263E+02 .773019E-01  
UNORDERED COMP..EFFICIENCIES : .993035E-03 .779-58E-01

\*\*\*\*\*  
METHOD 1: INTERACTING CONTRIBUTIONS TO THE NP  
EFF..NUMER.. COMPONENTS, IN DECREASING ORDER:

\*\*\*\*\*

MODE(IN MDSEL) COMP..EFF..  
2 .779-58E-01  
1 .993035E-03  
NP TH ORDER EFFICIENCY EMP: .769645E-01

BLOCK-DIAG NUMERATOR & DENOMINATOR COMP. COSTS:

.143932E+01 6 .157132E+02  
.108500E+03 6 .137825E+04  
BLOCK-DIAG..EFF..NUM..CONTROL COST : .110120E+03  
BLOCK-DIAG..EFF..DENOM..CONTROL COST : .139496E+04  
UNORDERED COMP..EFFICIENCIES : .861310E-01 .798541E-01

\*\*\*\*\*  
METHOD2: BLOCK-DIAGONAL CONTRIBUTIONS TO  
NP EFFICIENCY COMPONENTS IN DECREASING ORDER:

\*\*\*\*\*

MODE(IN MDSEL) BLOCK-DIAG..COMP..EFF..  
1 .861310E-01  
2 .789541E-01  
NP TH ORDER BLOCK-DIAG..EFFICIENCY EMP : .789413E-01  
15.03.04.UCLP, C8, N17J1H3, 0.512KLM5.



1 .000000 2 .000000 3 .000000 4 1.000000 5 .000000 6 .000000 7 .000000 8 .000000 9 .000000 10 .000000 11 .000000 12 .000000

INIT. GEN. VELOCITY

1 .000000 2 .000000 3 .000000 4 .000000 5 .000000 6 .000000 7 .000000 8 .000000 9 .000000 10 .000000 11 .000000 12 .000000

-----IN ROUTINE CONTROL-----  
 -----IN ROUTINE USERCTRL-----

-----LINEAR QUADRATIC REGULATOR CONTROL DESIGN -----

OPTIMAL GAIN MATRIX

ROW 1 -2.4728917E-01 9.32269282E-01 -1.14345814E+00 -9.80532237E-01  
 ROW 2 -2.47238608E-01 8.32906847E-01 -1.14550759E+00 -9.80541789E-01  
 ROW 3 -3.14605526E-02 -5.55576335E-01 7.29131982E-01 1.07580161E-01  
 ROW 4 -5.40347183E-03 5.50239340E-01 -7.28636431E-01 -2.08286834E-01

CLOSED LOOP EIGENVALUES

REAL PART      IMAGINARY PART  
 --.26383437E+00      --.10284814E+02  
 --.26383437E+00      .10294814E+02  
 --.11511354E+00      --.12905052E+02  
 --.11511354E+00      .12905052E+02

RICCATI MATRIX

1 402.275124 2 .499417 3 3.137735 4  
 2 .499417 3.805832 -4.945317 .052610  
 3 6.047545 -4.945317 .488931  
 4 3.137735 .052610 .488931 8.711809  
 -----IN ROUTINE CLEISG-----

COMPLEX CLOSED-LOOP EIGENVALUES

--.2638343E+00      --.1028481E+02  
 --.2638343E+00      .1029481E+02  
 --.1151135E+00      --.1290505E+02  
 --.1151135E+00      .1290505E+02

-----IN ROUTINE EFFNCY-----

DAMPING RATIO OF CONTROLLED MODES

DAMPING RATIOS : .025644 .0039197

IN ROUTINE FOR EFFICIENCY COMPUTATIONS

INITIAL MODAL STATE VECTOR:

-.3033373E+00 .0000000E+00 .8117159E+00 .0000000E+00

PER CENT EFFICIENCIES

-----

EFFICIENCY COEFFICIENT:

.610910E+01

GLOBAL EFFICIENCY

.6201973E+01

RELATIVE MODEL EFFICIENCY:

.2833627E+02

---GLOBALLY MINIMUM COST \$M---: .1520400E+02

---MODAL COST SM---: .7010426E+02

---REAL COST SR---: .2451517E+03

GLOBAL SPILLOVER QUOTIENT: .1151323E+02

RELATIVE MODEL SPILLOVER QUOTIENT: .7140373E+00

-----IN ROUTINE SIMDAT-----

IN ROUTINE RESSIM

-----IN ROUTINE MODEFF-----

PRINCIPAL COMPONENTS ANALYSIS OF GLOBAL EFFICIENCY

1	15.279254
2	-2.916153
3	1.542495
4	.707205

P0

C E

1  
2  
3  
4

1	.352257
2	.034689
3	.011005
4	.002040

CHARACTERISTIC EFFICIENCIES

1  
2  
3  
4

1	.053702
2	.280174
3	.053603
4	.280139

EFFICIENCY VECTOR

1  
2  
3  
4

1	.051138
2	.009719
3	.000590
4	.000572

TOTAL EFFICIENCY = .06202

SLDB.. EFF..MODAL MATRIX Y

1  
2  
3  
4

1	-.000187	.097364	.000011	-.023426
2	-.000133	.240108	-.002220	-.001862
3	.052617	.000519	.005663	-.000023
4	-.073134	.000695	.673449	.007642

-----IN ROUTINE EMOUSEL-----  
EFF..NUM..CONTROL COST : .152040E+02  
EFF..DENOM..CONTROL COST : .245152E+03  
EFFICIENCY CHECK AFTER DECOMP.: .520137E-01

OPERATOR COMPONENT COSTS:

.257051E+01 .126334E+02

Denominator Component COSTS

```

.353741E+01      .235514E+03
UNORDERED..COMP EFF(NUM.COMP.COSTS)/DENOM.) :
.105553E-01      .515329E-01
UNORDERED COMP..EFFICIENCIES :      .644923E-02      .555694E-01

```

```

*****
METHOD 1: INTERACTING CONTRIBUTIONS TO THE NP
EFF..NUMBER.. COMPONENTS, IN DECREASING ORDER:

```

```

*****

```

```

MODE(IN MDSEL)  COMP..EFF..
  2      .555694E-01
  1      .544929E-02
NP TH ORDER EFFICIENCY END:      .620187E-01

```

```

BLOCK-DIAG NUMERATOR & DENOMINATOR COMP. COSTS:
.257061E+01  5      .917572E+01
.126334E+02  6      .235253E+03

```

```

BLOCK-DIAG..EFF..NUM..CONTROL COST :      .152040E+02
BLOCK-DIAG..EFF..DENOM..CONTROL COST :      .244428E+03
UNORDERED COMP..EFFICIENCIES :      .280154E+00      .537014E-01

```

```

*****
METHOD2: BLOCK-DIAGONAL CONTRIBUTIONS TO
NP EFFICIENCY COMPONENTS IN DECREASING ORDER:

```

```

*****

```

```

MODE(IN MDSEL)  BLOCK-DIAG..COMP..EFF..
  1      .280154E+00
  2      .537014E-01

```

```

NP TH ORDER BLOCK-DIAG..EFFICIENCY END :      .622023E-01
-----IN ROUTINE MODEFF-----
PRINCIPAL COMPONENTS ANALYSIS OF RELATIVE EFFICIENCY

```

```

1      15.153113
2      -3.234243
3      1.558929
4      1.530032

```

PG

C E

1  
1 .237869  
2 .042663  
3 .009913  
4 .009549

CHARACTERISTIC EFFICIENCIES

1  
1 .283879  
2 .323446  
3 .323660  
4 .284267

EFFICIENCY VECTOR

1  
1 .266241  
2 .013801  
3 .003205  
4 .002715

TOTAL EFFICIENCY = .28596

REL.. EFF..MODAL MATRIX T

1 2 3 4  
1 .000755 .099900 -.002642 .007813  
2 -.100515 .027163 1.025133 .009083  
3 .052313 -.000330 .005164 .006122  
4 -.078934 -.053361 -.006140 .675713  
-----IN ROUTINE EMODEL-----  
EFF..NUM..CONTROL COST : .731043E+02  
EFF..OENCM..CONTROL COST : .245152E+03  
EFFICIENCY CHECK AFTER DECOMP.: .235963E+00

UMERATOR COMPONENT COSTS:

.309970E+01 .670056E+02

Denominator Component Costs

UNORDERED..COMP..EFFICIENCIES : .118925E-01 .274030E+00  
UNORDERED..COMP..EFFICIENCIES : .118925E-01 .274030E+00  
UNORDERED..COMP..EFFICIENCIES : .118925E-01 .274030E+00

\*\*\*\*\*  
METHOD 1: INTERACTING CONTRIBUTIONS TO THE NP  
EFF..NUMBER..COMPONENTS, IN DECREASING ORDER:

\*\*\*\*\*

MODE(IN MDSEL) COMP..EFF..  
2 .274080E+00  
1 .118925E-01  
NP TH ORDER EFFICIENCY ENP: .285963E+00

BLOCK-DIAG: NUMERATOR & DENOMINATOR COMP. COSTS:

.295564E+01 & .917572E+01  
.658725E+02 & .235253E+03  
BLOCK-DIAG..EFF..NUM..CONTROL COST : .693381E+02  
BLOCK-DIAG..EFF..BENOM..CONTROL COST : .244428E+03  
UNORDERED COMP..EFFICIENCIES : .323205E+00 .284259E+00

\*\*\*\*\*  
METHOD 2: BLOCK-DIAGONAL CONTRIBUTIONS TO  
NP EFFICIENCY COMPONENTS IN DECREASING ORDER:

\*\*\*\*\*

MODE(IN MDSEL) BLOCK-DIAG..COMP..EFF..  
1 .323205E+00  
2 .284259E+00  
NP TH ORDER BLOCK-DIAG..EFFICIENCY ENP : .295720E+00  
15.08.18.UCLP, C6, NIT01M5, 0.512KMS.







```

1 1 .00000 2 .00000 3 .00000 4 .00000 5 .00000 6 .00000 7 .00000 8 .00000 9 .00000 10 .00000 11 .00000 12 .00000

```

INIT. GEN. VELOCITY

```

1 1 .00000 2 .00000 3 .00000 4 .00000 5 .00000 6 .00000 7 .00000 8 .00000 9 .00000 10 .00000 11 .00000 12 .00000
-----IN ROUTINE CONTROL-----
-----IN ROUTINE USERCAL-----

```

-----LINEAR QUADRATIC REGULATOR CONTROL DESIGN -----

OPTIMAL GAIN MATRIX

```

RQ4 1 -1.72453089E-01 9.91905951E-01 2.35914776E-01 -9.89927902E-01 1.32255730E+00
      -8.8843879E-01 -3.42255032E-01 9.93682125E-01 -6.76357044E-01 -9.90029803E-01
      8.08153965E-01 9.91392065E-01 3.20262258E+00 7.27390459E-01 1.657791205E+00
      9.36693760E-01

```

```

RQ4 2 1.72290057E-01 -9.91527947E-01 2.35936232E-01 -9.89777196E-01 1.32543598E+00
      -8.93439389E-01 3.40983161E-01 -9.92929421E-01 -6.76155006E-01 -9.90231041E-01
      -8.07850049E-01 -9.91329482E-01 -3.20044329E+00 -7.26454237E-01 1.66122813E+00
      9.37563818E-01

```

CLOSED LOOP EIGENVALUES

REAL PART	IMAGINARY PART
-0.34988270E+00	-0.23973522E+01
-0.34988270E+00	0.33973522E+01
-0.24235256E+00	-0.29595929E+01
-0.24235256E+00	0.29595929E+01
-0.28298095E+00	-0.42031980E+01
-0.28298095E+00	0.42031980E+01
-0.67369323E-01	-0.16648202E+01
-0.67369323E-01	0.16648202E+01
-0.43061084E-01	-0.13420454E+01
-0.43061084E-01	0.13420454E+01
-0.66365050E-01	-0.47550516E+01
-0.66365050E-01	0.47550516E+01
-0.37815405E-01	-0.28916000E+01
-0.37815405E-01	0.28916000E+01
-0.40876707E-01	-0.46610355E+01
-0.40876707E-01	0.46610355E+01

-.60776707E-01 .05660393E+01

DIAGONAL MATRIX

	1	2	3	4	5	6	7	8	9	10	11	12
1	41.93773	.83250	.00049	.00980	.000613	-.000222	-.255526	-.527857	.000220	.000021	-.568494	-.203258
2	-.845962	-.074397	.000229	.000030	.000973	.000355	2.499013	.049256	-.000195	.000052	2.149722	-.122252
3	.493503	23.475605	-.001229	.000165	.000973	.000355	2.499013	.049256	-.000195	.000052	2.149722	-.122252
4	1.515700	-.231125	-.000549	.000129	.673044	-.985192	-.000450	.000413	-.912521	-.593365	-.000014	.000023
5	.006459	-.001229	41.365331	.673910	.219557	.962922	.219557	.962922	.219557	.962922	.219557	.962922
6	.000523	.000127	.962922	.219557	.219557	.962922	.219557	.962922	.219557	.962922	.219557	.962922
7	.003803	.000127	.475910	15.094936	2.745115	.354239	.001175	-.000197	2.553633	-.160072	-.0000130	.0000218
8	.001039	.000130	-.207492	.187845	.290217	.290217	.058713	-.005054	-.11702825	-.4213543	-.005677	-.0000619
9	.000613	.000873	.673044	2.745115	.290217	.290217	.058713	-.005054	-.11702825	-.4213543	-.005677	-.0000619
10	-.002106	.000541	5.665383	.668303	.290217	.290217	.058713	-.005054	-.11702825	-.4213543	-.005677	-.0000619
11	-.000222	.000355	-.985192	.394258	.005373	.005373	.005373	-.006803	5.973879	-1.036744	.001539	-.000331
12	-.000650	-.000157	-2.334400	.381024	.005373	.005373	.005373	-.006803	5.973879	-1.036744	.001539	-.000331
13	-.255526	2.499013	-.000450	-.001205	.058713	.005373	37.665083	.486657	.006653	.001662	-2.818985	-1.947329
14	-.5113816	-.804003	-.000154	.000177	-.005054	-.006803	.486657	4.324081	-.002007	.480358	-.000399	-.0000202
15	-.527857	.045256	.000413	-.000197	-.005054	-.006803	.486657	4.324081	-.002007	.480358	-.000399	-.0000202
16	2.248116	-.527903	-.000345	.000683	.005373	.005373	.005373	-.006803	5.973879	-1.036744	.001539	-.000331
17	.000451	.000195	-.912521	2.553633	-.11702825	5.973879	.006653	-.002007	35.985477	.480358	-.000399	-.0000202
18	.000021	.000052	-.593365	-.160672	-.4213543	-1.086744	.001452	.000625	.480358	3.034535	.000333	-.000015
19	-.001331	.000030	-.3335435	.216343	-.005677	.001539	-.2.893386	3.796911	-.000399	.000338	69.644007	.453058
20	-.558494	2.149722	-.000014	-.000130	-.005677	.001539	-.2.893386	3.796911	-.000399	.000338	69.644007	.453058
21	-21.084325	-7.491365	.003325	.003321	.000619	-.000351	-1.847329	-.254796	-.000203	-.000015	.663068	3.849973
22	8.918106	-1.259159	-.004125	.000552	-.000619	-.000351	-1.847329	-.254796	-.000203	-.000015	.663068	3.849973
23	-.846952	1.516700	.000523	-.001089	-.002155	-.000650	-5.113816	2.248116	.000451	-.001831	-21.084825	8.918106
24	556.933235	.028415	.001245	.002091	.000541	-.000157	-.804003	-.527903	.001050	.000090	-7.491360	-1.259159
25	-.074897	-.331126	.000127	.000130	.000541	-.000157	-.804003	-.527903	.001050	.000090	-7.491360	-1.259159
26	.000229	-.000549	.962922	-2.074992	5.665383	-2.334400	-.000154	-.000346	1.605675	-3.935406	.009325	-.0004125
27	.001245	-.001853	.349333	.378201	.668303	.381024	.000177	.010083	2.079657	.216343	.003321	.0000552
28	.000030	.000128	.219567	.137885	.668303	.381024	.000177	.010083	2.079657	.216343	.003321	.0000552
29	.002091	.000093	.378201	15.030719	.668303	.381024	.000177	.010083	2.079657	.216343	.003321	.0000552

COMPLEX CLOSED-LOOP EIGENVALUES

-.4306103E-01	-.1342045E+01
-.4306103E-01	-.1342045E+01
-.6736933E-01	-.1664820E+01
-.6736933E-01	-.1664820E+01
-.3781541E-01	-.2991500E+01
-.3781541E-01	-.2991500E+01
-.2+235.6E+00	-.2953535E+01
-.2+235.6E+00	-.2953535E+01

-----IN ROUTINE CLEIG-----

```

-.3397352E+01
-.3397352E+01
-.3397352E+01
-.2031138E+01
.4273178E+01
-.4661035E+01
.6661035E+01
-.4755052E+01
.755052E+01

```

-----IN ROUTINE EFFNCY-----

DAMPING RATIO OF CONTROLLED MODES

```

DAMPING RATIOS : .0320697 .0404333 .0130766 .0816140 .1024450 .011731 .0087695 .0140605

```

IN ROUTINE FOR EFFICIENCY COMPUTATIONS

INITIAL MODAL STATE VECTOR:

```

-.621411E-01 .000000E+00 .2526035E+00 .000000E+00 .6249160E+00 .000000E+00
-.351953E+00 .000000E+00 -.3241424E+00 .000000E+00 .000000E+00 .000000E+00
-.6878961E-01 .000000E+00 -.9402789E-01 .000000E+00 .000000E+00 .000000E+00

```

PER CENT EFFICIENCIES

-----

EFFICIENCY COEFFICIENT:

.6571710E+01

GLOBAL EFFICIENCY

.8403358E+01

RELATIVE MODEL EFFICIENCY:

.5525729E+02

-----GLOBALLY MINIMUM COST S4----- .1573100E+01

-----MODAL COST S4----- .1059513E+02

-----REAL COST S4----- .1989906E+02

GLOBAL SPILLOVER QUOTIENT: .5321219E+01

RELATIVE MODEL SPOILOVER QUOTIENT: .674271E+00  
 -----IN ROUTINE SIMDAT-----  
 IN ROUTINE RESIM  
 -----IN ROUTINE MODEFF-----  
 PRINCIPAL COMPONENTS ANALYSIS OF GLOBAL EFFICIENCY

P0

	I
1	-1.32133
2	-.986339
3	-1.259155
4	3.218630
5	-.307059
6	-1.826597
7	.924191
8	-.127331
9	-.711440
10	-.384204
11	.340402
12	-.315091
13	-.638591
14	-.240756
15	-.021123
16	-.000041

C E

	I
1	.087744
2	.039499
3	.079680
4	.520652
5	.004738
6	.167676
7	.042925
8	.000815
9	.025437
10	.007419
11	.005823
12	.004930
13	.003667
14	.002913
15	.000042
16	.000000

CHARACTERISTIC EFFICIENCIES

1  
 1 .37422  
 2 .53753  
 3 .25552  
 4 .00362  
 5 .36714  
 6 .06871  
 7 .01907  
 8 .52766  
 9 .00930  
 10 .01954  
 11 .01904  
 12 .01537  
 13 .00963  
 14 .00773  
 15 .00773  
 16 .26003

EFFICIENCY VECTOR

1  
 1 .03283  
 2 .02123  
 3 .02033  
 4 .00435  
 5 .00170  
 6 .01452  
 7 .00021  
 8 .00042  
 9 .00249  
 10 .00145  
 11 .00011  
 12 .00036  
 13 .00033  
 14 .00023  
 15 .00000  
 16 .00000

TOTAL EFFICIENCY = .08408

GLOBAL EFFICIENCY MATRIX T

	1	2	3	4	5	6	7	8	9	10	11	12
1	.00233	.00002	-.00253	-.00003	.00053	.00009	.00035	-.00001	-.01020	.00003	.00001	-.00001
2	.00321	.31375	-.03503	-.00019	.00002	-.00016	.00000	.00002	-.00391	.00001	.00000	.00000
3	.00235	.64593	.42167	.00047	.00000	.00053	.29645	.00103	-.00002	.11324	.02948	-.00000
4	.00000	-.00775	.00001	-.00087	.00002	-.00696	-.00311	-.01309	.00002	.01510	.35613	.39377

5	-.000033	.000000	-.000033	.000001	-.000004	-.000004	-.000004	-.000004	.000016	.000016	.000592	-.003513
	-.000011	-.015474	.000072	.144112	-.000004	-.000004	-.000004	-.000004	.000016	.000016	.000592	-.003513
	.000022	.000005	.000006	.000039	-.000043	-.000043	-.000043	-.000043	-.000055	-.000055	-.000502	.021198
6	.000035	-.028673	-.000119	.231705	-.000043	-.000043	-.000043	-.000043	-.000055	-.000055	-.000502	.021198
	.000019	-.000035	.000031	.000205	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
7	-.062222	.000019	.342453	-.000003	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
	-.000130	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
8	-.022630	.001017	.074748	-.000015	-.000000	-.000000	-.000000	-.000000	.000000	.000000	.000000	.000000
	.002305	.000000	.000000	.000022	1.014576	.000000	.000000	.000000	.000000	.000000	.000000	.000000
9	.000002	.061182	-.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
10	.000004	-.180016	.000011	-.000293	.000003	.000003	.000003	.000003	.000000	.000000	.000000	.000000
	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
11	.265130	.000000	.032662	.000000	.052440	.000000	.000000	.000000	.000000	.000000	.000000	.000000
	.000132	.000001	.000027	.000417	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
12	-.223935	.000011	-.033835	-.000001	1.037501	.000000	.000000	.000000	-.000000	-.000000	.000000	.000000
	.054015	-.000078	.000553	.163200	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
13	-.007931	.000000	-.004037	-.000010	-.008711	.000001	.000001	.000001	.000001	.000001	.000001	.000001
	.066070	.001314	.004548	-.005943	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
14	.055343	.000004	.020267	-.000040	-.042642	-.000031	-.000031	-.000031	-.000001	-.000001	.000000	.000000
	.373892	-.009243	.022312	-.027627	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
15	.000003	-.001579	.000002	-.001895	.000006	.000006	.000006	.000006	.000015	.000015	.000015	.000015
	.000003	.000000	.000001	.000003	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
16	-.000024	-.002693	-.000012	.003036	.000019	-.000019	-.000019	-.000019	.000010	.000010	-.000010	.000010
	.000071	.000001	.000002	.000010	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000

-----IN ROUTINE EROUSEL-----  
 EFF..NUM..CONTROL COST : .167310E+01  
 EFF..DEMO..CONTROL COST : .193981E+02  
 EFFICIENCY CHECK AFTER DISCOMP.: .840836E-01

UMERATJR COMPONENT COSTS:

.299267E+03	.115015E-01	.123633E+00
.262362E+00	.423741E+00	.833562E+00
.421412E-02	.133573E-01	

ENOMINATOR COMPONENT COSTS:



```

.43796E-01      .598799E+00      .139747E+02      .326331E-01
.112947E+01      .413553E+00      .246705E+01      .261583E-01
.546772E+00      .725975E+00      .521332E-02      .897555E-02
UNORDERED..COMP EFF=(NUM..COMP..COSTS)/DENOM..):      .186957E-02
.150400E-04      .398122E-03      .418916E-01      .156424E-02
.132333E-01      .212956E-01      .671731E-03      .115218E-01
.211785E-03      .671731E-03      .100052E-03      .837956E-01
UNORDERED COMP..EFFICIENCIES :      .125632E-02
UNORDERED COMP..EFFICIENCIES :      .100052E-03
UNORDERED COMP..EFFICIENCIES :      .125632E-02

```

```

*****
METHOD 1: INTERACTING CONTRIBUTIONS TO THE NP
EFF..NUMBER.. COMPONENTS, IN DECREASING ORDER:
*****

```

```

MODE(IN MODESEL)  COMP..EFF..
3      .326331E-01
5      .261583E-01
5      .115218E-01
4      .897555E-02
8      .186957E-02
2      .156424E-02
7      .125632E-02
1      .100052E-03
NP TH ORDER EFFICIENCY EMP:      .840836E-01

```

```

BLOCK-DIAG NUMERATOR & DENOMINATOR COMP. COSTS:
.299267E-03 E      .387037E-01
.119015E-01 E      .622764E+00
.123633E+00 E      .142992E+02
.262382E+00 E      .102151E+01
.423741E+00 E      .902857E+00
.833562E+00 E      .228093E+01
.421412E-02 E      .432641E+00
.133573E-01 E      .657577E+00
BLOCK-DIAG..EFF..NUM...CONTROL COST :      .167313E+01

```

```

BLOCK-DIAG..EFF..DENOM...CONTROL COST :      .201863E+02
UNORDERED COMP..EFFICIENCIES :      .773227E-02      .191107E-01
UNORDERED COMP..EFFICIENCIES :      .258857E+00      .527735E+00
UNORDERED COMP..EFFICIENCIES :      .974045E-02      .194412E-01
UNORDERED COMP..EFFICIENCIES :      .864612E-02
UNORDERED COMP..EFFICIENCIES :      .365441E+00

```

\*\*\*\*\*  
 METHOD: BLOCK-DIAGONAL CONTRIBUTIONS TO  
 NP EFFICIENCY COMPONENTS IN DECREASING ORDER:

\*\*\*\*\*  
 MDE(IN MDSEL) BLOCK-DIAG...COMP...EFF..  
 5 .527785E+00  
 6 .365441E+00  
 4 .258857E+00  
 6 .194412E-01  
 2 .191107E-01  
 7 .974045E-02  
 3 .804612E-02  
 1 .773227E-02

NP TH ORDER BLOCK-DIAG...EFFICIENCY ENP : .8259331E-01  
 -----IN ROUTINE MDEFF-----  
 PRINCIPAL COMPONENTS ANALYSIS OF RELATIVE EFFICIENCY

PO

1	2.971157
2	-2.025190
3	1.525633
4	-1.367178
5	-1.107671
7	-740037
7	-.537500
8	.501815
9	.392584
10	-.367271
11	.238765
12	.237931
13	.185292
14	-.087248
15	.071021
16	-.011489

CE

1	.443650
2	.206322
3	.117127
4	.039398
5	.051551
6	.027523
7	.014519
9	.012555
9	.007745
10	.006779

11 .002355  
 12 .002849  
 13 .001725  
 14 .003343  
 15 .005255  
 16 .000007

CHARACTERISTIC EFFICIENCIES

1  
 1 .540430  
 2 .540430  
 3 .540430  
 4 .597931  
 5 .597931  
 6 .597931  
 7 .597931  
 8 .540430  
 9 .540430  
 10 .597931  
 11 .597931  
 12 .597931  
 13 .540430  
 14 .540430  
 15 .540430  
 16 .597931

EFFICIENCY VECTOR

1  
 1 .237785  
 2 .115115  
 3 .063305  
 4 .056174  
 5 .036873  
 6 .016459  
 7 .008031  
 8 .006840  
 9 .004196  
 10 .004054  
 11 .001713  
 12 .001732  
 13 .000333  
 14 .000207  
 15 .000137  
 16 .000000

TOTAL EFFICIENCY = .55257

REL... EFF... MEDIAL MATRIA T

	1	2	3	4	5	6	7	8	9	10	11	12
1	-.000002	-.000001	-.000001	-.000001	-.000001	-.000001	-.000001	-.000001	-.000001	-.000001	-.000001	-.000001
2	.000005	.000005	.000007	.000007	.000007	.000007	.000007	.000007	.000007	.000007	.000007	.000007
3	-.000006	-.000006	-.000006	-.000006	-.000006	-.000006	-.000006	-.000006	-.000006	-.000006	-.000006	-.000006
4	-.000006	-.000006	-.000006	-.000006	-.000006	-.000006	-.000006	-.000006	-.000006	-.000006	-.000006	-.000006
5	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002
6	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002
7	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002
8	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002
9	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002
10	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002
11	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002
12	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002
13	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002	.000002
14	.000005	.000005	.000005	.000005	.000005	.000005	.000005	.000005	.000005	.000005	.000005	.000005
15	.000005	.000005	.000005	.000005	.000005	.000005	.000005	.000005	.000005	.000005	.000005	.000005
16	.000005	.000005	.000005	.000005	.000005	.000005	.000005	.000005	.000005	.000005	.000005	.000005

-----IN ROUTINE EMDSSEL-----  
 EFF...MUM...CONTROL COST : .109351E+02  
 EFF...DENOM...CONTROL COST : .198931E+02  
 EFFICIENCY CHECK AFTER DECOMP. : .532573E+00

NUMERATOR COMPONENT COSTS:

.261891E+01  
 .323546E+00  
 .22371E+00  
 .392317E+00

ENumerator COMPONENT COSTS:

.755311E+01  
 .17526E+01

```

.437346E-01      .59799E+00      .139747E+02
.112375E+01      .13653E+00      .246705E+01
.544772E+00      .725775E+00
UNORDERED..COMP EFF(CNUM*COMP.COSTS)/DENOM.) :
.131617E-02      .162652E-01      .379590E+00
.339369E-01      .112359E-01      .741410E-01
.163714E-01      .197164E-01
UNORDERED COMP..EFFICIENCIES :      .126619E-02      .383834E+00
UNORDERED COMP..EFFICIENCIES :      .325512E-01      .713257E-01
UNORDERED COMP..EFFICIENCIES :      .137499E-01

```

```

*****
METHOD 1: INTERACTING CONTRIBUTIONS TO THE NP
EFF..NUMBER..COMPONENTS, IN DECREASING ORDER:

```

```

*****

```

```

MODE(CIN MDISEL)  COMP..EFF..
3      .383834E+00
6      .713257E-01
4      .325512E-01
8      .197370E-01
2      .164470E-01
7      .157499E-01
5      .113615E-01
1      .126619E-02
NP TH ORDER EFFICIENCY ENP:      .552573E+00

```

```

BLOCK-DIAS NUMERATOR & DENOMINATOR COMP. COSTS:

```

```

.231645E-01 £      .387037E-01
.336598E+00 £      .622764E+03
.772955E+01 £      .142992E+02
.610853E+00 £      .102151E+01
.433941E+00 £      .802367E+00
.136400E+01 £      .223099E+01
.258715E+00 £      .632641E+00
.371529E+00 £      .687577E+00

```

```

BLOCK-DIAS.EFF..NUM...CONTROL COST :      .111275E+02

```

```

BLOCK-DIAS.EFF..DENOM..CONTROL COST :      .201935E+02
UNORDERED COMP..EFFICIENCIES :      .597991E+00      .540490E+00
UNORDERED COMP..EFFICIENCIES :      .597991E+00      .540490E+00
UNORDERED COMP..EFFICIENCIES :      .597991E+00      .540490E+00

```

\*\*\*\*\*  
METHOD: BLOCK-DIAGONAL CONTRIBUTIONS TO  
NP EFFICIENCY COMPONENTS IN DECREASING ORDER:

\*\*\*\*\*  
CODE(IN CODESEL) BLOCK-DIAG..COMP..EFF..  
1 .597991E+00  
6 .597991E+00  
7 .597991E+00  
4 .597991E+00  
3 .540650E+00  
8 .540650E+00  
2 .540650E+00  
5 .540650E+00

NP IN ORDER BLOCK-DIAG..EFF..CIENCY EMP : .551240E+00  
15.11.17.05LP, CR, N1701M3, 0.332KLNS.



1 1 .000000 2 .000000 3 .000000 4 1.000000 5 .000000 6 .000000 7 .000000 8 .000000 9 .000000 10 .000000 11 .000000 12 .000000

INIT. GEN. VELOCITY

1 1 .000000 2 .000000 3 .000000 4 .000000 5 .000000 6 .000000 7 .000000 8 .000000 9 .000000 10 .000000 11 .000000 12 .000000

-----IN ROUTINE CONTROL-----  
 -----IN ROUTINE USERCL-----

-----LINEAR QUADRATIC REGULATOR CONTROL DESIGN -----

OPTIMAL GAIN MATRIX

ROW 1	-7.03249497E-02	6.32484885E-01	4.15960513E-02	-6.62599131E-01	2.75396617E-01
	-3.42529726E-01	-2.16472434E-01	8.49677061E-01	-5.13950713E-01	-9.49736965E-01
	4.35609934E-01	7.30522447E-01	-5.77145560E-01	3.56038895E-02	4.88675737E-01
	2.22104481E-01				
ROW 2	5.78972084E-02	-6.88705489E-01	1.41864677E-01	-6.24042715E-01	-2.725966410E-01
	-8.54849464E-02	-1.33063134E-01	-9.14830706E-01	7.20930538E-02	-9.67704971E-01
	-5.99372083E-01	-6.60005489E-01	-1.08671560E+00	-6.37707224E-01	7.32215359E-01
	7.43317951E-01				
ROW 3	1.05030835E-01	-1.00471596E+00	1.16612022E-01	-1.46774704E-01	3.94238316E-01
	-1.32441288E+00	-4.96282332E-01	-5.04848164E-02	-1.50021922E-01	1.30677322E-01
	7.97341449E-01	7.06328753E-01	2.72726146E-01	-1.10721222E+00	-1.468453849E+00
	-3.40796559E-01				
ROW 4	2.56983595E-02	-3.24182592E-01	-1.09253492E-01	1.06437273E+00	6.61925994E-02
	2.90177173E-01	1.77899962E-01	6.32231335E-01	8.87340164E-02	-3.44260384E-01
	-2.10961246E-01	-6.79917941E-01	1.35313030E+00	-6.56968529E-01	-1.93950941E+00
	-9.98578560E-01				

CLOSED LOOP EIGENVALUES

REAL PART	IMAGINARY PART
-0.36737474E+00	-0.33979975E+01
-0.36737474E+00	0.33979975E+01
-0.47356953E+00	-0.47067222E+01
-0.47356953E+00	0.47067222E+01
-0.28885493E+00	-0.29523828E+01
-0.28885493E+00	0.29523828E+01
-0.40439391E+00	-0.42023438E+01
-0.40439391E+00	0.42023438E+01
-0.18793909E+00	-0.28994288E+01
-0.18793909E+00	0.28994288E+01



```

-1.037550JDE+00      -1.155+075JJE+01
-1.1037550JJE+00      .165+875JE+01
-.65524639E-01      -1.13-20793E+01
-.65524639E-01      .13-20793E+01
-.80592219E-01      -1.4709706E+01
-.86692219E-01      .4709706E+01
  
```

RICCATI MATRIX

	1	2	3	4	5	6	7	8	9	10	11	12
1	27.599853	494450	348236	-331714	-128134	-297617	-231757	-256923	.065234	-0.11268	-0.331565	-0.043236
2	-1.152940	-157668	-201287	-014261	1.376434	-008676	1.270144	-072172	.057347	.027537	.475669	.000726
3	1.879954	494450	348236	-331714	-128134	-297617	-231757	-256923	.065234	-0.11268	-0.331565	-0.043236
4	-1.152940	-157668	-201287	-014261	1.376434	-008676	1.270144	-072172	.057347	.027537	.475669	.000726
5	1.879954	494450	348236	-331714	-128134	-297617	-231757	-256923	.065234	-0.11268	-0.331565	-0.043236
6	-1.152940	-157668	-201287	-014261	1.376434	-008676	1.270144	-072172	.057347	.027537	.475669	.000726
7	1.879954	494450	348236	-331714	-128134	-297617	-231757	-256923	.065234	-0.11268	-0.331565	-0.043236
8	-1.152940	-157668	-201287	-014261	1.376434	-008676	1.270144	-072172	.057347	.027537	.475669	.000726
9	1.879954	494450	348236	-331714	-128134	-297617	-231757	-256923	.065234	-0.11268	-0.331565	-0.043236
10	-1.152940	-157668	-201287	-014261	1.376434	-008676	1.270144	-072172	.057347	.027537	.475669	.000726
11	1.879954	494450	348236	-331714	-128134	-297617	-231757	-256923	.065234	-0.11268	-0.331565	-0.043236
12	-1.152940	-157668	-201287	-014261	1.376434	-008676	1.270144	-072172	.057347	.027537	.475669	.000726
13	1.879954	494450	348236	-331714	-128134	-297617	-231757	-256923	.065234	-0.11268	-0.331565	-0.043236
14	-1.152940	-157668	-201287	-014261	1.376434	-008676	1.270144	-072172	.057347	.027537	.475669	.000726
15	1.879954	494450	348236	-331714	-128134	-297617	-231757	-256923	.065234	-0.11268	-0.331565	-0.043236
16	-1.152940	-157668	-201287	-014261	1.376434	-008676	1.270144	-072172	.057347	.027537	.475669	.000726

```

COMPLEX CLOSED-LOOP EIGENVALUES
-.6552464E-01      -1.13+2079E+01
-.6552464E-01      13+2079E+01
-1.037551E+00      -1.66+875E+01
  
```

```

-.1037551E+00      .1564875E+01
-.1479331E+00      -.2898429E+01
-.1479331E+00      .2898429E+01
-.2898429E+00      -.2952383E+01
-.2898429E+00      .2952383E+01
-.3673747E+00      -.3397998E+01
-.3673747E+00      .3397998E+01
-.4043989E+00      -.4202344E+01
-.4043989E+00      .4202344E+01
-.4735695E+00      -.4706722E+01
-.4735695E+00      .4706722E+01
-.8669222E-01      -.4709709E+01
-.8669222E-01      .4709709E+01

```

-----IN ROUTINE EFFNCY-----

DAMPING RATIO OF CONTROLLED MODES

```

DAMPING RATIO :      .0487651      .0621994      .0647058      .0973729      .1074937      .0957992      .1001101      .0134040

```

IN ROUTINE FOR EFFICIENCY COMPUTATIONS

INITIAL MODAL STATE VECTOR:

```

-.6214141E-01      .0000000E+00      .2526036E+00      .0000000E+00      .6249150E+00      .0000000E+00
-.3519592E+00      .0000000E+00      -.3241424E+00      .0000000E+00      -.4082741E+00      .0000000E-00
-.6878861E-01      .0000000E+00      -.9402789E-01      .0000000E+00      .0000000E+00

```

PER CENT EFFICIENCIES

-----

EFFICIENCY COEFFICIENT:

.2622305E+01

GLOBAL EFFICIENCY

.3075699E+02

RELATIVE MODEL EFFICIENCY:

.8065421E+02

-----GLOBALLY MINIMUM COST SUM----- .2475399E+01

-----MEDAL COST SUM----- .7015714E+01

-----REAL COST SP----- .3698509E+01

GLOBAL SPILLOVER QUOTIENT: .6289805E+00

RELATIVE MODEL SPILLOVER QUOTIENT: .1934575E+00

-----IN ROUTINE SIMCAT-----

IN ROUTINE RESSIM

-----IN ROUTINE MODEFF-----

PRINCIPAL COMPONENTS ANALYSIS OF GLOBAL SPILLOVER QUOTIENT

P0

1	-.761161
2	-.733384
3	-.560508
4	-.922959
5	.205792
6	.106238
7	-.466538
8	.083816
9	-.090123
10	-.036907
11	.068785
12	-.019324
13	-.029424
14	.055880
15	-.292535
16	-.098116

C E

1	.216553
2	.201036
3	.117423
4	.313402
5	.015830
6	.004219
7	.081374
8	.002625
9	.003035
10	.000503
11	.001763
12	.000140
13	.000324

14 .001187  
 15 .031987  
 16 .002598

CHARACTERISTIC EFFICIENCIES

1  
 2 .678933  
 3 .708031  
 4 .903094  
 5 .281659  
 6 2.218083  
 7 7.309411  
 8 .294864  
 9 7.611583  
 10 4.668067  
 11 18.718626  
 12 2.258960  
 13 18.247127  
 14 7.798900  
 15 .913139  
 16 .003503  
 17 .005061

EFFICIENCY VECTOR

1  
 2 .147025  
 3 .142334  
 4 .106050  
 5 .099684  
 6 .035111  
 7 .030836  
 8 .023995  
 9 .019987  
 10 .014173  
 11 .009530  
 12 .003995  
 13 .002547  
 14 .002524  
 15 .001068  
 16 .000112  
 17 .000018

TOTAL EFFICIENCY = .62899

GLOBAL SPG MODAL MATRIX T

	1	2	3	4	5	6	7	8	9	10	11	12
1	1j	1a	15	16								
1	-.003453	.001068	.001049	-.000292	-.074395	-.166637	-.000657	-.495625	.075484	-1.161235	.065339	2.482595
2	-.830144	-.025223	-.000977	.006318	.003070	1.128925	-.012155	.908-15	-.067291	3.243989	-.075251	1.580551
3	-.5.88J6	-.045949	-.030123	-.009354	-.000442	1.179758	-.005201	1.246010	-.032774	-.720329	-.071459	-.042490
4	-.106695	.028737	.002606	-.010080	.190393	-.397078	.029135	.620484	-.102351	-.053383	-.055752	1.024794
5	2.843081	-.007044	.038494	.015333	.604541	.008649	.014997	-.004035	-.030024	.003036	.051486	.001239
6	-.057579	.032057	.004285	.022741	-.160743	.001577	.128867	.143514	.056037	.005232	1.736413	-.003774
7	-.031125	1.136237	-.100122	.001535	.400156	-.011780	-.001594	-.002237	.002728	-.012865	-.004109	-.000949
8	-.011979	.130732	.432449	-.034493	.0240156	.012904	-.098664	.021305	.054350	.005453	1.131550	.021547
9	-.423343	.593170	.155473	.018151	.024467	.003891	.091138	.002159	-.006379	.001117	-.037285	.000722
10	.007983	1.234597	.074774	.004768	.029435	.003891	.091138	.002159	-.006379	.001117	-.037285	.000722
11	-.011570	.128947	.007009	.023377	.110535	-.005424	.010489	.032535	.008336	.002683	.104941	-.000509
12	.001269	-.757191	-.095354	.026546	-.000980	.001905	.121558	-.003171	.002871	.000867	.010885	-.000364
13	-.028211	.057474	.016816	.346645	-.000980	.001905	.121558	-.003171	.002871	.000867	.010885	-.000364
14	.001338	-.043016	.002340	.002612	-.034090	.009604	1.450229	-.009211	-.020320	-.000399	-.007791	-.001811
15	-.003607	.090159	-.018506	.003330	-.006713	-.014153	-.000755	.009122	-.151347	-.000411	.007535	.000451
16	-.003157	.005010	.007894	-.005219	-.012839	-.0416029	-.015978	.401129	.125642	.017353	-.033665	.006024
17	-.012493	-.019509	.238872	.098914	-.012839	-.0416029	-.015978	.401129	.125642	.017353	-.033665	.006024
18	-.029073	-.029697	.070707	.011354	.010725	-.002815	.007231	.018940	.405684	.009152	-.013123	-.002191
19	-.140619	.042043	-.473833	1.210172	.028848	1.932473	-.038357	-1.804271	.079097	-.086677	.089369	-.046477
20	-.002376	-.003492	-.010638	-.008995								
21	.006474	-.017534	.237295	.045195								
22	.022907	-.018967	.066064	-.033382								
23	.684262	-.066812	-.252925	1.237274								

-----IN ROUTINE EMODEL-----  
EFF..NUM..CONTROL COST : .168280E+01  
EFF..DEMAND..CONTROL COST : .267540E+01  
EFF..CIENCY CHECK AFTER DECOMP.: .528998E+00

OPERATOR COMPONENT COSTS:

.351563E-02	.123610E+00	.681859E+00
.169545E+00	.232456E+00	.398241E+00
.894497E-02	-.104273E-01	

ENOMINATOR COMPONENT COSTS:

```

.533979E-03      .151002E-01      .611043E+00
.211208E+00      .445563E+00      .118204E+01
.201357E-02      -.765393E-02
UNORDERED.. COMP EFF(NUM..COMP..COSTS)/DENOM..):
.131403E-02      .452026E-01      .254842E+00
.633718E-01      .868366E-01      .142938E+00
.334342E-02      -.339749E-02
UNORDERED COMP..EFFICIENCIES :      .719807E-03      .249939E-01      .199255E+00
UNORDERED COMP..EFFICIENCIES :      .682685E-01      .959370E-01      .210419E+00
UNORDERED COMP..EFFICIENCIES :      .190840E-02      .190840E-02      -.287609E-02

```

```

*****
METHD 1: INTERACTING CONTRIBUTIONS TO THE NP
EFF..NUMER..COMPONENTS, IN DECREASING ORDER:
*****

```

```

MODE(IN MODESEL)  COMP..EFF..
6      .210419E+00
3      .192260E+00
5      .953370E-01
4      .692685E-01
2      .249939E-01
7      .190840E-02
1      .719907E-03
8      -.287609E-02      .598630E+00
NP TH ORDER EFFICIENCY END:

```

BLOCK-DIAG NUMERATOR & DENOMINATOR COMP. COSTS:

```

.112007E-03 £      .149867E-04
.115490E+00 £      .154696E-01
.100000E+01 £      .608842E+00
.436994E+00 £      .308828E+00
.293417E+00 £      .409637E+00
.349403E+00 £      .117409E+01
.138203E-02 £      .270308E-03
.473773E-03 £      .119366E-03

```

BLOCK-DIAG.EFF..NUM...CONTROL COST : .219727E+01

```

BLOCK-DIAG.EFF..DENOM...CONTROL COST : .251728E+01
UNORDERED COMP..EFFICIENCIES :      .747373E+01      .746562E+01      .154247E+01
UNORDERED COMP..EFFICIENCIES :      .141501E+01      .716285E-00      .297593E+00
UNORDERED COMP..EFFICIENCIES :      .511281E+01      .396300E+01

```

\*\*\*\*\*  
 MTRC02: BLOCK-DIAGONAL CONTRIBUTIONS TO  
 NP EFFICIENCY COMPONENTS IN DECREASING ORDER  
 \*\*\*\*\*

\*\*\*\*\*  
 MTRC02: BLOCK-DIAGONAL CONTRIBUTIONS TO  
 NP EFFICIENCY COMPONENTS IN DECREASING ORDER  
 \*\*\*\*\*

```

.MC0E(IN M0USEL)  BLOCK-DIAG..CMP..EFF..
1      .747373E+01
2      .746562E+01
7      .511281E+01
8      .396900E+01
3      .164247E+01
4      .141501E+01
5      .716285E+00
6      .297593E+00
  
```

NP 1M ORDER BLOCK-DIAG.EFFICIENCY EMP : .872878E+00  
 -----IN ROUTINE EGRUJ-----

GLOBAL FILTERING  
 -----IN ROUTINE CLEIGP-----

```

COMPLEX CLOSED-LOOP EIGENVALUES
--.5721622E-01  -.1345591E+01
-.5721622E-01  .1345581E+01
-.8396655E-01  -.1662001E+01
-.8396655E-01  .1662001E+01
-.1723801E+00  -.2911917E+01
-.1723801E+00  .2911917E+01
-.2777074E+00  -.2919084E+01
-.2777074E+00  .2919084E+01
-.6455285E-01  -.3307799E+01
-.6455285E-01  .3307799E+01
-.6137389E-02  -.4164775E+01
-.6137389E-02  .4164775E+01
-.4697455E+00  -.4707505E+01
-.4697455E+00  .4707505E+01
-.1741002E-01  -.4762697E+01
-.1741002E-01  .4762697E+01
-----IN ROUTINE EFFNCY-----
  
```

DAMPING RATIO OF CONTROLLED MODES  
 CAMPING RATIOS : .0424832 .0504570 .0590947 .0947075 .0193116 .0014736 .0392934 .0336555

IN ROUTINE FOR EFFICIENCY COMPUTATIONS

INITIAL MODAL STATE VECTOR:

```

-.6214141E-01      .000000E+00      .2526036E+00      .0000000E+00      .0000000E+00
-.321952E+00      .000000E+00      -.324142E+00      .0000000E+00      .0000000E+00
-.6878361E-01      .000000E+00      -.9402789E-01      .0000000E+00      .0000000E+00
    
```

PER CENT EFFICIENCIES

-----

EFFICIENCY COEFFICIENT:

.1789869E+01

GLOBAL EFFICIENCY

.4088485E+02

RELATIVE MODEL EFFICIENCY:

.7317851E+02

---GLOBALLY MINIMUM COST SM---: .4729293E+01

---MODAL COST SM---: .8464813E+01

---REAL COST SM---: .1156735E+02

GLOBAL SPILLOVER QUOTIENT: .6360253E+00

RELATIVE MODEL SPILLOVER QUOTIENT: .2682149E+00

-----IN ROUTINE SIMDAT-----

IN ROUTINE BESSIM

-----IN ROUTINE MODEFF-----

PRINCIPAL COMPONENTS ANALYSIS OF GLOBAL SPILLOVER QUOTIENT

P0

```

1      .527438
2     -1.725428
3     -1.683463
4     -1.593971
5     -1.572446
6     -1.440027
7     -1.14073
9     -1.154169
    
```



9	-.070771
10	-.036119
11	-.033576
12	-.068109
13	-.007162
14	.005453
15	-.188859
16	-.201901

C E

1	.058836
2	.629503
3	.398772
4	.076114
5	.069290
6	.040961
7	.002752
8	.005026
9	.001059
10	.000276
11	.000269
12	.000981
13	.000011
14	.000006
15	.007544
16	.008619

CHARACTERISTIC EFFICIENCIES

1	2.371737
2	.205943
3	1.293920
4	1.383048
5	.863573
6	.773209
7	9.817460
8	2.098180
9	9.481030
10	21.468448
11	20.734019
12	2.526634
13	16.115556
14	15.034908
15	.004818
16	.003755

EFFICIENCY VECTOR

1  
 1 .13954  
 2 .12964  
 3 .12783  
 4 .10529  
 5 .06323  
 6 .03165  
 7 .02703  
 8 .01045  
 9 .01004  
 10 .00592  
 11 .00586  
 12 .00247  
 13 .00109  
 14 .00035  
 15 .00003  
 16 .00002

TOTAL EFFICIENCY = .65603

GLOBAL SPO MODAL MATRIX T

	1	2	3	4	5	6	7	8	9	10	11	12
1	-.029874	-.001222	-.011861	.011549	-.003271	-.007433	-.094594	.013869	-.642463	-.626899	2.916270	.005120
2	.589144	-.118141	.007882	-.000489	-.017641	-.034253	1.447740	-.020733	.018749	3.781289	.856306	-.025804
3	.002289	-.004042	.044945	.013467	-.017641	-.034253	1.447740	-.020733	.018749	3.781289	.856306	-.025804
4	.117295	.457151	-.007627	-.035800	.013042	.012806	1.889303	.005378	-.371180	-.675310	-.222010	-.019346
5	-.067244	-.004793	.007132	-.043311	.013042	.012806	1.889303	.005378	-.371180	-.675310	-.222010	-.019346
6	.270160	-.033842	-.010736	.000375	-.028304	.043930	.733758	-.045592	2.543671	-.433346	.890309	-.105324
7	.107252	.015368	-.097420	-.026705	-.028304	.043930	.733758	-.045592	2.543671	-.433346	.890309	-.105324
8	-.1969749	.002071	.012117	.043313	-.029322	.143685	.009233	-.300535	-.049499	-.001958	-.004424	-.143980
9	.008155	.014118	.027423	.004237	-.029322	.143685	.009233	-.300535	-.049499	-.001958	-.004424	-.143980
10	.705651	-.124524	-.114572	1.011451	-.338086	.011196	.071967	.738126	-.036331	-.003798	.003321	1.827789
11	-.088961	-.075741	.000887	-.120436	-.029039	-.131170	.008304	-.081626	.000546	-.013443	-.000525	-.197339
12	.176699	.016947	.374997	.398654	-.029039	-.131170	.008304	-.081626	.000546	-.013443	-.000525	-.197339
13	.002030	.004374	-.020647	-.003699	.461558	.048494	.001045	.599951	-.010311	-.003967	.018647	.329274
14	.748243	-.065201	1.103213	-1.040548	.240797	.322509	-.001071	-.002651	.003408	.002308	.001180	-.001299
15	-.039163	.008118	-.000651	.091793	-.314799	.202497	-.004379	1.824139	-.052923	.009818	-.012993	-1.133792
16	-.011087	.021445	-.003167	-.059480	.017874	.015735	-.000096	-.008892	-.001776	.000436	-.000040	-.062991
17	-.001112	.247194	.003712	-.008679	.147181	.008236	.000984	1.47181	-.008236	.000940	-.000304	-.041231
18	-.000239	.000818	.015737	-.004444	-.003382	-.011076	.001324	.006322	-.045550	.005206	.000328	.002131
19	1.41231	-.013570	-.539119	1.94382	-.003382	-.011076	.001324	.006322	-.045550	.005206	.000328	.002131
20	-.024325	-.015143	-.014791	-.042231	.017874	.015735	-.000096	-.008892	-.001776	.000436	-.000040	-.062991
21	-.004505	-.003161	-.007919	.017162	.147181	.008236	.000984	1.47181	-.008236	.000940	-.000304	-.041231
22	-.053946	-.010905	.111228	.270809	-.020415	-.041948	.012059	.002003	.051224	.025269	-.008462	-.022343
23	-.012738	-.041703	.061171	-.004119	-.000857	-.015029	.000793	.003090	.692176	-.017423	-.032374	.025942
24	.053353	-.278700	1.303483	-.535373	-.000857	-.015029	.000793	.003090	.692176	-.017423	-.032374	.025942
25	.051845	.010391	-.027589	.015088	-.000857	-.015029	.000793	.003090	.692176	-.017423	-.032374	.025942
26	.891142	-.027533	.043955	.269927	-.000857	-.015029	.000793	.003090	.692176	-.017423	-.032374	.025942

14    -.073672    -.097153    .004495    .082140    -.036413    -.143496    -.157412    .061311    .027727    -.779981    .068320    .228650  
 .166567    6.773393    1.347866    -.245483

-----IN ROUTINE MODEFF-----  
 PRINCIPAL COMPONENTS ANALYSIS OF RELATIVE SPOILOVER QUOTIENT

PO

1	1	-1.995067
2	1	1.780715
3	-1	-1.499371
4	1	1.164361
5	-1	-1.374718
6	1	.307945
7	-1	-.428973
8	-1	-.270057
9	-1	-.288056
10	-1	-.274486
11	1	.168407
12	-1	-.180523
13	1	.222543
14	-1	-.097206
15	-1	-.236711
16	-1	-.169031

CE

1	1	.360656
2	1	.274129
3	1	.194091
4	1	.117204
5	1	.012139
6	1	.008198
7	1	.015908
8	1	.006305
9	1	.007175
10	1	.006513
11	1	.002452
12	1	.002817
13	1	.004281
14	1	.000817
15	1	.004944
16	1	.002470

CHARACTERISTIC EFFICIENCIES

1	1	.265735
2	1	.243493

2 .275646  
 4 .265624  
 5 .455076  
 6 .607254  
 7 .234562  
 8 .428902  
 9 .306053  
 10 .250844  
 11 .609035  
 12 .432758  
 13 .276007  
 14 .432511  
 15 .002035  
 16 .002466

EFFICIENCY VECTOR

1  
 2  
 3  
 4  
 5  
 6  
 7  
 8  
 9  
 10  
 11  
 12  
 13  
 14  
 15  
 16

TOTAL EFFICIENCY = .26821

REL..SPO MODAL MATRIX T

	1	2	3	4	5	6	7	8	9	10	11	12				
1	.09524	.066748	.053634	.031249	.005536	.004978	.004686	.002704	.002196	.001634	.001493	.001219	.001182	.000353	.000010	.000006
2	-.021665	.017312	-.040862	.028581	-.015119	.010983	-.053331	-.033556	-.044898	-.004421	-.009930	.000195	.020048	.019983	-.001200	.005521
3	.017312	-.015993	-.010507	.057205	-.028371	.069035	.176387	-.003710	.40555	.009930	.000195	.020048	.019983	-.001200	.005521	.005521
4	-.040862	.015993	-.010507	.057205	-.028371	.069035	.176387	-.003710	.40555	.009930	.000195	.020048	.019983	-.001200	.005521	.005521
5	.028581	-.015119	.010983	-.053331	-.033556	-.044898	-.004421	-.009930	.000195	.020048	.019983	-.001200	.005521	.005521	.005521	.005521
6	-.015119	.010983	-.053331	-.033556	-.044898	-.004421	-.009930	.000195	.020048	.019983	-.001200	.005521	.005521	.005521	.005521	.005521
7	.010983	-.053331	-.033556	-.044898	-.004421	-.009930	.000195	.020048	.019983	-.001200	.005521	.005521	.005521	.005521	.005521	.005521
8	-.053331	-.033556	-.044898	-.004421	-.009930	.000195	.020048	.019983	-.001200	.005521	.005521	.005521	.005521	.005521	.005521	.005521
9	-.033556	-.044898	-.004421	-.009930	.000195	.020048	.019983	-.001200	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521
10	-.044898	-.004421	-.009930	.000195	.020048	.019983	-.001200	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521
11	-.004421	-.009930	.000195	.020048	.019983	-.001200	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521
12	.000195	.020048	.019983	-.001200	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521
13	.019983	-.001200	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521
14	-.001200	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521
15	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521
16	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521	.005521

6	-.013434	.040310	.040524	-.078818	.028933	.535718	.133452	-.065328	-.016997	.737353	-.103273	.002888
7	.002661	-.018833	-.088095	-.015253	-.019881	.013377	.009929	.245467	-.050851	.056414	.315040	-.011275
8	-.005236	.033077	-.000091	-.014914	-.015124	.943789	-.125170	-.013412	-.043219	-.436209	-.242693	.003745
9	-.011937	.073323	.066998	.011619	-.010104	.030392	-.013239	-.026712	.054284	-.034361	-.021548	.013473
10	-.018275	.001410	-.001205	.009436	.034713	-.090044	.152930	-.041293	.098121	.168566	-.088826	-.019916
11	-.018589	-.074634	-.043265	-.009824	.006126	.002505	.028695	.208623	-.017409	.019190	-.002481	-.018231
12	-.004798	-.100366	-.032469	-.049073	.021594	-.000753	-.085175	.036703	-.126059	-.076929	-.003584	-.066105
13	1.072572	.084315	-.043504	-.019044	-.007357	-.006121	.007667	-.009657	-.001145	.020945	.003150	.070498
14	.040574	-.003495	.184893	.114190	-.047355	.016836	-.035559	.002191	-.033339	-.073476	.018409	-.017243
15	.000617	-.001067	-.526887	.848721	.038822	-.000453	.019279	.003497	.003217	.035584	-.003933	-.090949
16	-.058812	-.115498	.004332	.009004	.070304	-.021708	.002873	-.037112	-.057386	.005704	.003865	.987692
	.072112	.423927	-.322893	.918396								
	-----IN ROUTINE EMCOSEL-----											
	EFF...NUM...CONTROL COST :	.310254E+01										
	EFF...DENOM...CONTROL COST :	.115673E+02										
	EFFICIENCY CHECK AFTER DECOMP.:											
	.268215E+00											

OPERATOR COMPONENT COSTS:

.176321E-01	.149263E+00	.102929E+01
.387844E+00	.722200E+00	.703174E+00
.317207E-02	-.365545E-01	

ENCINATOR COMPONENT COSTS:

```

.578143E-01      .383768E+00      .602329E+01
.150709E+01      .267415E+01      .655449E+01
.349302E-02      .399108E-03
UNORDERED.. COMP EFF(NUM.COMP.COSTS)/DENOM.) :
.152430E-02      .129039E-01      .809920E-01
.335292E-01      .624343E-01      .607896E-01
.274226E-03      -.316015E-03
UNORDERED COMP..EFFICIENCIES :      .143243E-02      .109012E-01      .911355E-01
UNORDERED COMP..EFFICIENCIES :      .342372E-01      .622201E-01      .600106E-01
UNORDERED COMP..EFFICIENCIES :      .177610E-03      -.153380E-03

```

```

*****
METHOD 1: INTERACTING CONTRIBUTIONS TO THE NP
EFF..NUMBER.. COMPONENTS, IN DECREASING ORDER:
*****

```

```

MODE(IN MODESEL)  COMP..EFF..
3      .911355E-01
5      .622201E-01
6      .600106E-01
4      .342372E-01
2      .109012E-01
1      .143243E-02
7      .177610E-03
8      -.153380E-03
NP TH ORDER EFF:CIENCY EMP:      .259961E+00

```

BLOCK-DIAG NUMERATOR & DENOMINATOR COMP. COSTS:

```

.477426E-02 £      .122923E-01
.155300E+00 £      .387384E+00
.737684E+00 £      .286307E+01
.925309E-01 £      .352527E+00
.483790E+00 £      .181239E+01
.682628E+00 £      .246886E+01
.170974E-02 £      .390180E-02
.242362E-02 £      .547422E-02

```

BLOCK-DIAG.EFF...NUM...CONTROL COST : .216086E+01

```

BLOCK-DIAG.EFF...DENOM...CONTROL CJST :      .791190E+01
UNORDERED COMP..EFFICIENCIES :      .383394E+00      .400893E+00      .257655E+00
UNORDERED COMP..EFFICIENCIES :      .262479E+00      .266054E+00      .276495E+00
UNORDERED COMP..EFFICIENCIES :      .437936E+00      .442732E+00

```

\*\*\*\*\*  
 METHOD: BLOCK-DIAGONAL CONTRIBUTIONS TO  
 UP EFFICIENCY COMPONENTS IN DECREASING ORDER

\*\*\*\*\*  
 MDS2(IN MDS2EL) BLOCK-DIAG..COMP..EFF..  
 8 .42732E+00  
 7 .43793E+00  
 2 .40893E+00  
 1 .38239E+00  
 6 .27649E+00  
 5 .26605E+00  
 4 .26279E+00  
 3 .25765E+00

AP IM CROR BLOCK-DIAG.EFFICIENCY EMP : .27311E+00  
 15.06.47.DUCLP, CB, N1701M3, 1.024KLMS.





1 .00000 .00000 .00000 1.00000 .00000 .00000 .00000 .00000 .00000 .00000 .00000 .00000

INIT. 354. VELOCITY

1 .00000 .00000 .00000 .00000 .00000 .00000 .00000 .00000 .00000 .00000 .00000 .00000

-----IN ROUTINE CONTROL-----  
 -----IN ROUTINE USERPR-----

-----LINEAR QUADRATIC REGULATOR CONTROL DESIGN -----

OPTIMAL GAIN MATRIX

ROW 1	3.53258359E-01	-3.17423452E-01	-2.80569498E-01	9.26633383E-01	-3.25321251E-01
	-5.59423787E-01	2.28662525E-01	7.136502745E-01	-1.25420310E+00	-7.55054859E-01
	5.03438475E-02	5.47322574E-01	1.22752877E+00	7.909224188E-01	-2.416222456E+00
	-9.58848164E-01				
ROW 2	-1.27810060E-01	-6.98685505E-02	4.15178868E-03	-9.30387609E-01	1.68746798E-01
	-9.51346452E-01	-6.09411036E-01	-5.63897524E-01	6.39466185E-01	-6.44731471E-01
	-1.80859033E+00	-8.74950135E-01	3.06922359E-01	8.74983137E-01	-2.31270652E+00
	-9.77556355E-01				
ROW 3	2.41958480E-01	-1.34192157E+00	-2.34463007E-01	-6.02992932E-02	-1.82739576E-01
	1.12851271E-01	6.45067482E-01	4.77596678E-01	-7.38572458E-01	-7.21963751E-01
	-3.5250029E-01	-3.54990298E-01	-5.38230565E-01	-5.46382547E-01	3.72538001E-01
	1.09530601E-01				
ROW 4	-3.34220556E-01	2.37371495E-01	1.95115327E-01	6.54014079E-01	-1.61491461E-02
	-3.57821587E-01	-9.3270733E-02	-7.13319363E-01	3.90051136E-01	6.62702595E-01
	5.82043742E-01	4.08567314E-01	7.04091258E-01	5.36421634E-01	-6.83589431E-01
	-2.05331731E-01				

CLOSED LOOP EIGENVALUES

REAL PART

IMAGINARY PART

-0.35710170E+00	-0.33979980E+01
0.35710170E+00	0.33979980E+01
-0.40423594E+00	-0.42021731E+01
-0.40423594E+00	0.42021731E+01
-0.23864217E+00	-0.29525863E+01
-0.23864217E+00	0.29525863E+01
-0.19373358E+00	-0.23380079E+01
-0.19373358E+00	0.23380079E+01
-0.32957765E+00	-0.95414577E+01
-0.32957765E+00	0.95414577E+01

-.35123312E+00  
 -.33123302E+00  
 -.2623333E+00  
 -.2623333E+00  
 -.2623333E+00  
 -.2623333E+00  
 -.11483370E+00  
 -.11483370E+00

RICCATI MATRIX

	1	2	3	4	5	6	7	8	9	10	11	12
1	63.793237	-.81369	-5.653985	-.261735	-1.121688	-.239360	2.240407	1.077914	-.854223	-.159978	.165762	-.035779
2	-.197281	-.047330	-.133972	-.007833	.436635	-.125959	-2.364056	.178658	1.584187	-.032310	.369560	-.005974
3	.481359	5.301563	.300503	-.597901	.006387	.184759	-.002434	.490157	2.022917	-.677376	.763099	-.070062
4	-.5653986	.300508	32.453065	.490157	2.022917	-.388464	-1.6448511	-.677376	.763099	-.070062	-.364232	-.216935
5	-.093773	-.019334	.227502	-.001500	-.527956	.212020	1.396199	-.130370	.513947	.028780	2.102690	-.023518
6	-.261735	-.597901	.490157	3.721240	-.022543	.003258	.492727	-2.228359	-.448599	.242315	-.205544	-.577888
7	-.121688	.436635	2.022917	-.527956	32.151713	.492727	-2.228359	-.448599	.242315	-.205544	-.577888	.043302
8	.198077	.229927	-.219740	-.147140	.492727	2.772522	.712144	-.152793	1.241331	-.001663	-.297305	-.024276
9	-.299360	-.125959	.388464	.212020	.492727	2.772522	.712144	-.152793	1.241331	-.001663	-.297305	-.024276
10	-.2083529	.018752	2.092703	-.030310	-.228359	.712144	45.970159	.476001	.280437	.373502	.455898	-.206337
11	2.240407	-2.364056	-1.643511	1.396199	-2.228359	.712144	45.970159	.476001	.280437	.373502	.455898	-.206337
12	-.972902	.154429	.462730	-.027232	-.448599	-.152793	.676001	2.539578	-1.471684	.033383	.952706	-.001518
13	1.097914	.178658	-.677376	-.130370	-.448599	-.152793	.676001	2.539578	-1.471684	.033383	.952706	-.001518
14	-.905995	-.006672	.190845	-.002020	.242315	1.241331	.280437	-1.471684	2.539578	.481156	-1.9056383	-2.050287
15	-.854223	1.584187	.753099	.513947	.242315	1.241331	.280437	-1.471684	2.539578	.481156	-1.9056383	-2.050287
16	1.278358	.951454	-1.305459	-.905796	-.205544	-.001663	.373502	.033383	.481156	3.087895	2.4616128	-.243329
17	-.169978	-.032310	-.070062	.028780	-.205544	-.001663	.373502	.033383	.481156	3.087895	2.4616128	-.243329
18	1.65762	.369560	-.364232	2.102690	-.677888	-.237305	.455898	.952706	-1.9056383	2.4616128	2.092708	-.092802
19	-10.969152	-1.689440	3.839492	.143569	.043302	-.024276	-.206337	-.001518	-2.050287	-.243329	.77960	2.854674
20	-.035779	-.006574	-.216935	-.029318	.198077	-2.093529	-.872802	-.905995	1.278358	-1.371201	-10.369152	2.084249
21	2.086249	-.116682	-.319382	.021119	.229927	.018752	.154429	-.006672	.951454	.014757	-1.633440	-.114692
22	-.197281	.595671	-.093773	.242838	.198077	-2.093529	-.872802	-.905995	1.278358	-1.371201	-10.369152	2.084249
23	.408937721	.498719	3.235686	3.176752	.229927	.018752	.154429	-.006672	.951454	.014757	-1.633440	-.114692
24	-.07330	-.006387	-.013334	-.012440	-.219740	-.022543	-.003258	-.492727	2.772522	.712144	45.970159	.476001
25	.488719	3.847995	-4.760496	.046478	-.219740	-.022543	-.003258	-.492727	2.772522	.712144	45.970159	.476001
26	-.138972	.184759	.227502	-.022543	-.219740	2.092708	.662730	.190885	-1.305269	2.055014	3.839482	-.139382
27	3.235686	-4.960496	*****	.464595	-.219740	2.092708	.662730	.190885	-1.305269	2.055014	3.839482	-.139382
28	-.007633	-.002434	-.001500	.003258	-.147140	-.030310	-.027232	-.002020	-.905796	-.023564	.143569	.027119
29	3.176752	.646478	.464595	3.733923	-.147140	-.030310	-.027232	-.002020	-.905796	-.023564	.143569	.027119

-----IN ROUTINE CLEIG0-----  
 COMPLEX CLOSED-LDOP EIGENVALUES  
 -.1397336E+00  
 -.1397336E+00  
 -.2898008E+01  
 -.2898008E+01  
 -.2896425E+00  
 -.2896425E+00  
 -.289586E+01  
 -.289586E+01

-.2830+22E+03      .2952586E+01  
 -.3671017E+00      -.3397993E+01  
 -.3571017E+00      .3397993E+01  
 -.4342359E+00      -.422173E+01  
 -.4042359E+00      .422173E+01  
 -.3295776E+00      -.8541453E+01  
 -.3295776E+00      .8541453E+01  
 -.3512330E+00      -.9249743E+01  
 -.3512330E+00      .9249743E+01  
 -.2523994E+00      -.1028437E+02  
 -.2523994E+00      .1028437E+02  
 -.1148337E+00      -.1290505E+02  
 -.1148337E+00      .1290505E+02

-----IN ROUTINE EFFNCY-----

DAMPING RATIO OF CONTROLLED MODES

DAMPING RATIOS :      .0553305      .0972953      .1074097      .0957549      .0385570      .0379543      .0255061      .0099019

IN ROUTINE FOR EFFICIENCY COMPUTATIONS

INITIAL MODAL STATE VECTOR:

.6293160E+00      .0000000E+00      .0000000E+00      -.3519592E+00      .0000000E+00      .0000000E+00      -.0000000E+00  
 -.4082741E+00      .0000000E+00      -.4250273E+00      -.4250273E+00      .0000000E+00      .0000000E+00      -.4483981E+00  
 -.3033375E+00      .0000000E+00      .8117159E+00      .8117159E+00      .0000000E+00      .0000000E+00      .0000000E+00

PER CENT EFFICIENCIES

-----

EFFICIENCY COEFFICIENT:

.3465899E+01

GLOBAL EFFICIENCY

.1027019E+02

RELATIVE MODEL EFFICIENCY:

.9721661E+02

---GLOBALLY MINIMUM COST SW---: .2813357E+02

---GOAL COST SW---: .2663578E+03

---REAL COST SR---: .2739839E+03

GLOBAL SPILLOVER QUOTIENT: .2710166E+00

RELATIVE MODEL SPILLOVER QUOTIENT: .2783392E-01

---IN ROUTINE SIMCAT-----

IN ROUTINE RESSIM

---IN ROUTINE MODEERR-----

PRINCIPAL COMPONENTS ANALYSIS OF GLOBAL EFFICIENCY

P O

1	15.067461
2	-3.062879
3	-2.756281
4	-3.035550
5	-1.701927
6	-1.395726
7	-1.167633
8	2.173935
9	-.938023
10	2.787144
11	-.991473
12	-.275385
13	-.682947
14	-.133150
15	-.141933
16	.112059

C E

1	.828619
2	-.034240
3	.027728
4	.033532
5	.010572
6	.007110
7	.004979
8	-.017345
9	.003211
10	.028353
11	-.002901
12	.000277
13	.000851

14 .000055  
 15 .000074  
 16 .000046

CHARACTERISTIC EFFICIENCIES

1  
 2 .053615  
 3 .635029  
 4 .521250  
 5 .281849  
 6 .429914  
 7 .522693  
 8 .701955  
 9 .155542  
 10 .551227  
 11 .052631  
 12 .335633  
 13 .689351  
 14 .156919  
 15 .549554  
 16 .280556  
 .336225

EFFICIENCY VECTOR

1  
 2 .044995  
 3 .014995  
 4 .014620  
 5 .009479  
 6 .004545  
 7 .003716  
 8 .003493  
 9 .002698  
 10 .001770  
 11 .001521  
 12 .000974  
 13 .000191  
 14 .000134  
 15 .000036  
 16 .000021  
 .000015

TOTAL EFFICIENCY = .10270

GL23.. EFF..MODEL MATRIX T

	1	2	3	4	5	6	7	8	9	10	11	12
1	.000042	-.002779	.001241	-.000409	.002230	-.001232	.017209	-.003128	.303853	-.000054	.063370	-.007338
2	-.055339	.002057	-.001137	.027370	-.014737	-.001767	.026074	-.003692	.153111	.000165	-.073159	.043740
3	.000033	.003440	.001113	.173935	-.001924	.000925	-.013597	-.012676	-.012610	.000050	.357659	.003937
4	-.000451	.006320	-.001343	.031224	.000299	-.029815	-.033745	-.050343	.131557	-.000042	-.103297	-.043355
5	.000044	-.002548	.003536	1.055569	.004279	-.007677	-.031648	.002244	.353343	.000006	.010024	.054395
6	-.002192	-.021642	-.000215	-.025216	-.001331	.025500	-.224524	-.007059	.072709	.000824	.075151	-.120270
7	-.000013	1.193247	-.007331	.025494	-.002904	.004459	.306327	-.004910	.020793	.000002	.007537	.035599
8	-.001037	.009154	-.015492	.000543	-.022150	.031142	-.135427	-.004742	-.165527	.000043	-.017934	1.273339
9	.000043	.093399	.001213	.034246	.009953	.061389	-.000582	.000387	.003168	.000012	-.000848	-.003999
10	-.000127	-.093163	-.007225	-.012538	1.035967	-.120580	.013362	-.001717	-.009579	.000765	.001164	.024652
11	-.000044	.003511	1.20100	.003512	.048690	.009523	.000863	-.000230	-.003807	.000012	.000605	.002579
12	-.000055	-.473456	-.087254	.035203	.034039	1.092259	-.015944	-.000791	.024433	-.000400	-.001695	-.025434
13	-.000102	.002554	-.003313	.099751	-.002709	-.003955	-.000632	.000128	.000156	-.000013	.001151	-.000063
14	-.000273	.000381	.012097	-.003685	.000027	.026279	.035542	-.125214	.050319	-.034071	.000050	.001075
15	.052146	-.000142	.000153	.000233	.000040	-.000032	.000053	-.000036	-.000046	.000042	-.000043	-.000064
16	-.124654	-.000423	.000874	-.001026	-.003425	.001837	.000692	.000443	-.000149	-.000149	-.000599	.000139
	-.000273	-.004564	.005950	-.000331								

-----IN ROUTINE EMCOSEL-----  
EFF..NUM...CONTROL COST : .281337E+02  
EFF..DENDM...CONTROL COST : .273984E+03  
EFFICIENCY CHECK AFTER DECOMP.: .102702E+00

OPERATOR COMPONENT COSTS:

.622520E+00 .311729E+00 .644647E+00  
.119050E+01 .435715E+01 .604329E+01  
.255575E+01 .126381E+02

EMULATOR COMPONENT COSTS:

```

.42502E+01      .12031E+01      .751153E+00
.14555E+01      .954771E+01      .124238E+02
.934375E+01      .234497E+03
UNORDERED..COMP EFF(NUM..COMP.COSTS)/DENOM.) :
.227211E-02      .113776E-02      .162290E-02
.434515E-02      .159030E-01      .220753E-01
.932312E-02      .660176E-01      .193298E-02      .734449E-03
UNORDERED COMP..EFFICIENCIES :      .974034E-02      .133673E-01
UNORDERED COMP..EFFICIENCIES :      .541523E-02

```

```

*****
METHOD 1: INTERACTING CONTRIBUTIONS TO THE NP
EFF..NUMBER..COMPONENTS, IN DECREASING ORDER:
*****

```

```

MODE(IN MODESEL)  COMP..EFF..
8      .663394E-01
6      .133673E-01
5      .974094E-02
7      .641523E-02
4      .253179E-02
1      .193298E-02
3      .953728E-03
2      .794495E-03      .102702E+00
NP TH ORDER EFFICIENCY END:

```

```

BLOCK-DIAG NUMERATOR & DENOMINATOR COMP. COSTS:
.622520E+00 £      .394947E+01
.311729E+00 £      .948802E+00
.444647E+00 £      .807673E+00
.119050E+01 £      .172214E+01
.435715E+01 £      .971231E+01
.604829E+01 £      .119335E+02
.255575E+01 £      .905377E+01
.126881E+02 £      .234296E+03
BLOCK-DIAG..EFF..NUM...CONTROL COST :      .281367E+02

```

```

BLOCK-DIAG..EFF...DENOM...CONTROL COST :      .272424E+03
UNORDERED COMP..EFFICIENCIES :      .157621E+00      .328550E+03      .530523E+00
UNORDERED COMP..EFFICIENCIES :      .691295E+00      .448622E+00      .506833E+00
UNORDERED COMP..EFFICIENCIES :      .282296E+00      .538126E-01

```

\*\*\*\*\*  
 METHOD: BLOCK-DIAGONAL CONTRIBUTIONS TO  
 NP EFFICIENCY COMPONENTS IN DECREASING ORDER:  
 \*\*\*\*\*

```

MODE(IN MODSEL)  BLOCK-DIAG..COMP..EFF..
  4      .591295E+00
  3      .550529E+00
  6      .505933E+00
  5      .448622E+00
  2      .328550E+00
  7      .282286E+00
  1      .157521E+00
  8      .536126E-01
  
```

NP 14 ORDER BLOCK-DIAG EFFICIENCY EMP : .103290E+00  
 -----IN ROUTINE MCODEFF-----  
 PRINCIPAL COMPONENTS ANALYSIS OF RELATIVE EFFICIENCY

P0

```

  1      13.459606
  2      5.553262
  3      4.518705
  4      -3.515792
  5      -2.386274
  6      -1.933356
  7      -1.317091
  9      2.265936
  9      1.551625
  10     -1.528066
  11     -1.053971
  12     -.908973
  13     -.661021
  14     .121570
  15     -.139709
  15     -.083612
  
```

CE

```

  1      .551210
  2      .112557
  3      .077950
  4      .045115
  5      .030405
  6      .013643
  7      .013414
  8      .018759
  9      .009956
  10     .003674
  
```



11	.064033
12	.002333
13	.000770
14	.000054
15	.000071
16	.000025

CHARACTERISTIC EFFICIENCIES

1	.977381
2	.961943
3	.978356
4	.945503
5	.991111
6	.934447
7	.950387
8	.676346
9	.981854
10	.987992
11	.954353
12	.994930
13	.956554
14	.986639
15	.546335
16	.945448

EFFICIENCY VECTOR

1	.644651
2	.110524
3	.076175
4	.042661
5	.030135
6	.013567
7	.013295
8	.012687
9	.003776
10	.009558
11	.003906
12	.002377
13	.000742
14	.000053
15	.000045
16	.000024

TOTAL EFFICIENCY = .97217

REL... EFF... MODEL MATRIX T

	1	2	3	4	5	6	7	8	9	10	11	12
1	-.000643	-.014305	.004543	.002305	.015435	-.060654	.015752	.272357	.004463	-.025960	-.012451	-.023039
2	.127735	.001163	-.035759	.012423	-.015733	-.065816	.032222	.163408	.034740	-.019533	.390761	.211696
3	-.033386	-.036764	.075415	.002917	.079038	-.105149	.047459	-.120028	.023090	-.033603	-.063017	-.042813
4	.262530	.131643	.034603	.014335	.003706	.003706	.093474	-.114657	.102953	-.520844	.735755	.302927
5	.156272	-.052225	-.359305	.003300	-.157275	-.043702	.093474	-.114657	.102953	-.520844	.735755	.302927
6	.225153	-.047203	-.001785	.011037	-.074156	.156124	-.015608	-.002013	-.001066	.003207	-.122447	.252662
7	.073704	.063677	.000547	.012731	-.135347	.787464	-.253839	.009936	-.119978	.321104	-.11793	-.573506
8	.027713	-.007313	-.032830	-.005544	.092555	-.053613	.003339	.023318	-.030705	.221339	.093535	.121040
9	-.006692	.063733	-.063971	-.003958	.209875	.532012	.134177	.203155	.066425	-.336423	-.053143	.154547
10	-.350734	.970877	.100102	-.024744	-.002072	.001239	.009190	-.001526	-.030937	-.006523	.019212	-.000512
11	.012452	-.004375	-.006597	.130054	-.097637	-.009430	-.010701	-.010203	.053612	.055245	.019918	-.000311
12	-.084565	-.027545	-.017034	1.110865	.008409	.008409	.007893	.000671	.098665	.031541	.003523	-.004311
13	-.001450	-.009753	-.003333	-.033010	-.050524	-.066780	.007893	.007727	.479437	.050752	.049837	-.021591
14	-.010783	.325917	.011523	.215626	.057624	.029523	.054443	-.000984	.009218	-.006839	.000961	-.003755
15	-.000073	-.036489	.000063	.009119	-.000637	.029523	.054443	-.000984	.009218	-.006839	.000961	-.003755
16	-.012333	.056502	-.013236	-.079036	-.562753	-.003975	.911451	.000999	-.141137	.202265	-.017159	-.107811
17	-.005201	.122723	-.033988	-.012585	-.003155	-.001985	.000623	.000110	.013050	-.000037	-.001350	.002567
18	.045574	.014413	.018038	-.000137	-.003155	-.001985	.000623	.000110	.013050	-.000037	-.001350	.002567
19	-.000205	-.003130	.000241	-.002215	.008323	-.002463	-.020000	-.001175	-.164356	.039845	.004219	.007484
20	-.222562	.089806	.616585	.019645	.008323	-.002463	-.020000	-.001175	-.164356	.039845	.004219	.007484
21	.004518	.032871	-.000136	.000338								
22	-----	IN ROUTINE EMDSSEL	-----									
23	EFF...NUM...	CONTROL COST :		.266358E+03								
24	EFF...DENOM...	CONTROL COST :		.273984E+03								
25	EFFICIENCY CHECK	AFTER DECOMP. :		.972166E+00								

NUMERATOR COMPONENT COSTS:

.292983E+01	.889113E+00	.739883E+00
.193099E+01	.895145E+01	.121029E+02
.925110E+01	.229473E+03	

DENOMINATOR COMPONENT COSTS:

```

.625202E+01 .120351E+01 .791153E+00
.191555E+01 .954771E+01 .124293E+02
.334375E+01 .234493E+03
UNORDERED...COMP EFF(NUM..COMP.COSTS)/GENCM.) :
.106935E+01 .224513E+02 .288296E-02
.704794E+02 .328199E-01 .441738E-01
.337551E-01 .837541E+00
UNORDERED COMP..EFFICIENCIES : .128904E-01 .375775E-02 .284509E-02
UNORDERED COMP..EFFICIENCIES : .592413E-02 .333473E-01 .441389E-01
UNORDERED COMP..EFFICIENCIES : .334596E-01 .834933E+00

```

```

*****
METHOD 1: INTERACTING CONTRIBUTIONS TO THE NP
EFF..NUMBER..COMPONENTS, IN DECREASING ORDER:
*****

```

```

MOJSE(IN MOJSEL) COMP..EFF..
8 .834803E+00
6 .441389E-01
7 .334596E-01
5 .333473E-01
1 .128904E-01
4 .692413E-02
2 .375775E-02
3 .284509E-02
NP TH ORDER EFFICIENCY EMP: .972166E+00

```

```

BLOCK-DIAG NUMERATOR & DENOMINATOR COMP. COSTS:

```

```

.284269E+01 E .334947E+01
.863028E+00 E .949602E+00
.794529E+00 E .807673E+00
.166379E+01 E .172214E+01
.920389E+01 E .971231E+01
.116597E+02 E .119335E+02
.896170E+01 E .905377E+01
.229273E+03 E .234296E+03

```

```

BLOCK-DIAG.EFF..NUM...CONTROL COST : .265298E+03

```

```

BLOCK-DIAG.EFF...GENCM...CONTROL COST : .272424E+03
UNORDERED COMP..EFFICIENCIES : .719765E+00 .909598E+00 .933725E+00
UNORDERED COMP..EFFICIENCIES : .955117E+00 .348249E+00 .973572E+00
UNORDERED COMP..EFFICIENCIES : .989821E+00 .978599E+00

```

\*\*\*\*\*  
 METHOD: BLOCK-DIAGONAL CONTRIBUTIONS TO  
 NP EFFICIENCY COMPONENTS IN DECREASING ORDER

\*\*\*\*\*

MODE(IN)	MODE(EL)	BLOCK-DIAG. COMP. EFF.
7		.929831E+00
3		.92725E+00
6		.979372E+00
8		.97859E+00
4		.96117E+00
5		.948269E+00
2		.90996E+00
1		.719765E+00

NP TH ORDER BLOCK-DIAG. EFFICIENCY ENP : .973948E+00  
 15.05.38.00LP, CE, N1701M5, 0.332KUNS.

### EXAMPLE 6.3

NUMBER OF REDUCED PRINCIPAL EFFICIENCY COMPONENTS NP= 16

NUMBER OF INPUTS M= 4

NUMBER OF RESIDUAL MODES NP= 20

ORDER OF REDUCED PRINCIPAL EFFICIENCY COMPONENTS NP= 16

NUMBER OF OUTPUTS N= 3

P, T<sub>1</sub> T<sub>2</sub> F<sub>1</sub> F<sub>2</sub>  
 T<sub>1</sub> T<sub>2</sub> F<sub>1</sub> F<sub>2</sub>  
 T<sub>1</sub> F<sub>1</sub> F<sub>1</sub> T<sub>1</sub> F<sub>1</sub> T<sub>1</sub>  
 SELECTED ADRES: 12 13 17 21 22 24 28 30  
 SELECTED INPUTS: 13 19 20 21  
 SELECTED OUTPUTS: 23 24 25 29  
 STRUCTURAL SQUARED NATL. FREQS:  
 -0.799211E+02 0.494074E+03 0.516347E+01 0.525732E+01  
 0.745318E+01 0.245991E+00 0.593323E+00 0.305931E+00  
 0.338592E+00 0.137218E+01 0.323758E+01 0.124735E+02  
 0.144159E+02 0.117122E+02 0.140174E+02 0.156570E+02  
 0.257745E+01 0.285711E+01 0.370742E+02 0.331799E+02  
 0.278971E+02 0.362761E+01 0.979541E+02 0.130379E+03  
 0.193331E+03 0.292591E+03 0.455112E+03 0.476415E+03  
 0.474949E+02 0.237915E+01 0.321148E+03 0.563647E+03  
 0.271132E+01 0.272378E+04 0.374922E+04 0.305412E+04  
 0.313531E+03 0.315591E+04 0.245073E+04 0.350209E+04  
 0.374325E+01 0.311122E+01 0.391037E+04 0.603046E+04  
 0.607374E+04 0.412122E+01 0.248354E+04 0.431743E+04  
 0.327611E+04 0.329132E+01 0.391122E+04 0.573331E+04  
 0.337421E+04 0.548134E+01 0.335353E+04 0.558932E+04  
 0.313131E+01 0.313131E+01 0.722433E+04 0.722433E+04  
 0.377013E+01 0.313131E+01 0.313131E+04 0.332533E+04  
 0.332533E+04 0.313131E+01 0.313131E+04 0.377013E+04  
 0.332533E+04 0.313131E+01 0.313131E+04 0.377013E+04  
 0.332533E+04 0.313131E+01 0.313131E+04 0.377013E+04  
 0.332533E+04 0.313131E+01 0.313131E+04 0.377013E+04  
 0.332533E+04 0.313131E+01 0.313131E+04 0.377013E+04  
 0.332533E+04 0.313131E+01 0.313131E+04 0.377013E+04  
 0.332533E+04 0.313131E+01 0.313131E+04 0.377013E+04  
 0.332533E+04 0.313131E+01 0.313131E+04 0.377013E+04  
 0.332533E+04 0.313131E+01 0.313131E+04 0.377013E+04  
 0.332533E+04 0.313131E+01 0.313131E+04 0.377013E+04











1 0.000000  
 2 0.000000  
 3 0.000000  
 4 0.000000  
 5 0.000000  
 6 0.000000  
 7 0.000000  
 8 0.000000  
 9 0.000000  
 10 0.000000  
 11 0.000000  
 12 0.000000  
 13 0.000000  
 14 0.000000  
 15 0.000000  
 16 0.000000

EFFICIENCY VECTOR

1 0.000000  
 2 0.000000  
 3 0.000000  
 4 0.000000  
 5 0.000000  
 6 0.000000  
 7 0.000000  
 8 0.000000  
 9 0.000000  
 10 0.000000  
 11 0.000000  
 12 0.000000  
 13 0.000000  
 14 0.000000  
 15 0.000000  
 16 0.000000

TOTAL EFFICIENCY = 0.00176  
 -----IN ROUTINE ENCODEL-----  
 EFF..NUM..CONTROL COST : 0.113002E-02  
 EFF..JENCH..CONTROL COST : 0.540603E+00  
 EFFICIENCY CHECK AFTER SECURF.: 0.176400E-02

OPERATOR COMPONENT COSTS:

0.23333E+04 0.593371E-04 0.193027E-21  
 0.412500E+04 0.150322E+03 0.850733E-03  
 0.574333E+10 0.410039E+03

EXMINATOR COMPONENT COSTS:

0.00000E+00 0.00000E+00 0.111349E+03  
 0.00000E+00 0.00000E+00 0.074012E+01  
 0.00179E+01 0.170357E+01  
 JNDORDERD COMP EFFICIENCIES(DENOM.):  
 0.00000E+00 0.00000E+00 0.285710E+11  
 0.00000E+00 0.00000E+00 0.132810E+02  
 0.00000E+00 0.00000E+00 0.37280E+03  
 UNORDERED COMP EFFICIENCIES : 0.218534E+03  
 UNORDERED COMP EFFICIENCIES : 0.93899E+05  
 UNORDERED COMP EFFICIENCIES : 0.562601E+04  
 UNORDERED COMP EFFICIENCIES : 0.78439E+03  
 UNORDERED COMP EFFICIENCIES : 0.528926E+01  
 UNORDERED COMP EFFICIENCIES : 0.115136E+03  
 UNORDERED COMP EFFICIENCIES : 0.134693E+00  
 UNORDERED COMP EFFICIENCIES : 0.13632E+00  
 UNORDERED COMP EFFICIENCIES : 0.39117E+01  
 UNORDERED COMP EFFICIENCIES : 0.021703E+02  
 UNORDERED COMP EFFICIENCIES : 0.17675E+01  
 UNORDERED COMP EFFICIENCIES : 0.113002E+02  
 UNORDERED COMP EFFICIENCIES : 0.066975E+00  
 UNORDERED COMP EFFICIENCIES : 0.111570E+03  
 UNORDERED COMP EFFICIENCIES : 0.33345E+03  
 UNORDERED COMP EFFICIENCIES : 0.93918E+03  
 UNORDERED COMP EFFICIENCIES : 0.233683E+07

\*\*\*\*\*  
 METHOD COUPLED MODAL CONTRIBUTIONS TO THE NP  
 EFF. NUMER., COMPONENTS, IN DECREASING ORDER:  
 \*\*\*\*\*

MODE(IN MOSEL) COMP EFF.  
 0 0.78439E+03  
 1 0.37280E+03  
 5 0.30459E+03  
 4 0.21453E+03  
 2 0.56260E+04  
 8 0.24350E+04  
 7 0.92899E+05  
 3 0.15914E+11  
 NP TH ORDER EFFICIENCY ENP: 0.176400E+02

BLOCK-DIAS NUMERATOR & DENOMINATOR COMP. COSTS:

0.29833E+04 0.35843E+00  
 0.58337E+04 0.52892E+01  
 0.12958E+25 0.11513E+03  
 0.15400E+04 0.13469E+00  
 0.15052E+03 0.13632E+00  
 0.85078E+03 0.39117E+01  
 0.67433E+10 0.02170E+02  
 0.41305E+09 0.17675E+01

BLOCK-DIAS EFF. NUM. CONTROL COST : 0.113002E+02

BLOCK-DIAS EFF. DENOM. CONTROL COST : 0.066975E+00  
 UNORDERED COMP EFFICIENCIES : 0.111570E+03 0.927553E+02 0.112553E+16  
 UNORDERED COMP EFFICIENCIES : 0.33345E+03 0.110412E+02 0.954678E+02  
 UNORDERED COMP EFFICIENCIES : 0.93918E+03 0.233683E+07

\*\*\*\*\*  
 METRIC: BLOCK-DIAG. EFFICIENCY EXP : 0.174022E-02  
 \*\*\*\*\*

\*\*\*\*\*  
 METRIC: BLOCK-DIAG. EFFICIENCY EXP : 0.174022E-02  
 \*\*\*\*\*

ADJ(CM VALUE) BLOCK-DIAG. EFFICIENCY EXP :  
 1 0.304273  
 2 -0.372289  
 3 -0.285321  
 4 -0.361602  
 5 -0.173639  
 6 0.332091  
 7 0.150655  
 8 0.132350  
 9 -0.398191  
 10 -0.075218  
 11 0.024091  
 12 -0.010375  
 13 0.005705  
 14 -0.000523  
 15 -0.000009  
 16 -0.000043

\*\*\*\*\*  
 METRIC: BLOCK-DIAG. EFFICIENCY EXP : 0.174022E-02  
 \*\*\*\*\*

\*\*\*\*\*  
 METRIC: BLOCK-DIAG. EFFICIENCY EXP : 0.174022E-02  
 \*\*\*\*\*

P0

1 0.304273  
 2 -0.372289  
 3 -0.285321  
 4 -0.361602  
 5 -0.173639  
 6 0.332091  
 7 0.150655  
 8 0.132350  
 9 -0.398191  
 10 -0.075218  
 11 0.024091  
 12 -0.010375  
 13 0.005705  
 14 -0.000523  
 15 -0.000009  
 16 -0.000043

C E

1 0.174022  
 2 0.215233  
 3 0.199289  
 4 0.254119  
 5 0.377056  
 6 0.172457  
 7 0.358793  
 8 0.627399  
 9 0.311051  
 10 0.000931

14 0.000000  
15 0.000000  
16 0.000000  
17 0.000000  
18 0.000000  
19 0.000000  
20 0.000000

CHARACTERISTIC EFFICIENCIES

1 0.000000  
2 0.000000  
3 0.000000  
4 0.000000  
5 0.000000  
6 0.000000  
7 0.000000  
8 0.000000  
9 0.000000  
10 0.000000  
11 0.000000  
12 0.000000  
13 0.000000  
14 0.000000  
15 0.000000  
16 0.000000

EFFICIENCY VECTOR

1 0.000000  
2 0.000000  
3 0.000000  
4 0.000000  
5 0.000000  
6 0.000000  
7 0.000000  
8 0.000000  
9 0.000000  
10 0.000000  
11 0.000000  
12 0.000000  
13 0.000000  
14 0.000000  
15 0.000000  
16 0.000000

TOTAL EFFICIENCY = 0.000000  
-----IN ROUTINE EXECUTION-----  
EFF. .... CUMULATIVE COST : 0.000000  
EFF. .... CUMULATIVE COST : 0.000000

EXHAUSTION COSTS AFTER DISCOUNT 0.3571502-02

GENERATOR COMPONENT COSTS:

0.3556000-03	0.1572275-04	0.7523335-11
0.3534000-03	0.1557812-04	0.3917421-03
0.2691000-03	0.1253318-03	

GENERATOR COMPONENT COSTS:

0.59475E+00 0.02405E+04 2.12249E+02  
 0.55255E+00 0.13502E+00 2.87012E+01  
 0.93170E+00 0.17697E+01  
 UNORDERED COMP EFFICIENCIES (COSTS/DENOM) :  
 0.55185E+00 0.24302E+07 0.11732E+10  
 0.55355E+00 0.24310E+02 0.13920E+02  
 0.55307E+00 0.17661E+03  
 UNORDERED COMP EFFICIENCIES : 0.19797E+02 0.64733E+04  
 UNORDERED COMP EFFICIENCIES : 0.13911E+02 0.17901E+02  
 UNORDERED COMP EFFICIENCIES : 0.52413E+04 0.18902E+03

\*\*\*\*\*  
 METHOD COUPLED MODAL CONTRIBUTIONS TO THE NP  
 EFF. NUMER. COMPONENTS, IN DECREASING ORDER:

\*\*\*\*\*

MODE(IN)	MODE(SL)	COMP	EFF.
1	0.19797E+02	0.19797E+02	0.64733E+04
5	0.17901E+02	0.17901E+02	0.17901E+02
4	0.13911E+02	0.13911E+02	0.52413E+04
6	0.11643E+02	0.11643E+02	0.11643E+02
8	0.18902E+03	0.18902E+03	0.18902E+03
7	0.52518E+04	0.52518E+04	0.52518E+04
2	0.64733E+04	0.64733E+04	0.64733E+04
3	0.12177E+10	0.12177E+10	0.12177E+10

NP IN ORDER EFFICIENCY EMP: 0.557180E-02

BACK-DIAS NUMERATOR & DENOMINATOR COMP. COSTS:

0.394513E-03	0.259434E+00
0.665514E-04	0.525936E-02
0.796907E-11	0.115136E-04
0.833012E-03	0.134595E+00
0.157610E-02	0.136122E+00
0.565855E-03	0.391172E-01
0.225914E-04	0.561705E-02
0.125940E-02	0.176751E-01

BACK-DIAS EFF. NUM. COSTS LIST: 0.42550E+02

0.59475E+00	0.02405E+04	2.12249E+02
0.55255E+00	0.13502E+00	2.87012E+01
0.93170E+00	0.17697E+01	
UNORDERED COMP EFFICIENCIES (COSTS/DENOM) :		
0.55185E+00	0.24302E+07	0.11732E+10
0.55355E+00	0.24310E+02	0.13920E+02
0.55307E+00	0.17661E+03	
UNORDERED COMP EFFICIENCIES :	0.19797E+02	0.64733E+04
UNORDERED COMP EFFICIENCIES :	0.13911E+02	0.17901E+02
UNORDERED COMP EFFICIENCIES :	0.52413E+04	0.18902E+03

THE FOLLOWING INFORMATION IS FOR THE INFORMATION OF THE OFFICE OF THE ATTORNEY GENERAL, THE BOARD OF THE COMMONWEALTH OF MASSACHUSETTS AND THE BOARD OF THE COMMONWEALTH OF CALIFORNIA TO THE EXTENT OF THE INFORMATION CONTAINED HEREIN IS NOT OTHERWISE AVAILABLE TO THE PUBLIC.

THIS INFORMATION IS NOT TO BE DISSEMINATED OUTSIDE THE BUREAU OF REVENUE AND TAXATION.

AMOUNT	ADDRESS	BLOCK
1	0-10173-01	0-10173-01
5	0-10074-01	0-10074-01
3	0-99-0113-02	0-99-0113-02
8	0-71-0003-02	0-71-0003-02
5	0-59-0193-02	0-59-0193-02
4	0-58-5913-02	0-58-5913-02
1	0-3-2245-01	0-3-2245-01
7	0-21-0123-02	0-21-0123-02

NO THE ORDER BLOCKS-DIG. EFFICIENCY EMP : 0.555312E-02  
 17.51.024000, CE, NITRHS, 1.554XENS.













1	0.000000
2	0.000000
3	0.000000
4	0.000000
5	0.000000
6	0.000000
7	0.000000
8	0.000000
9	0.000000
10	0.000000
11	0.000000
12	0.000000
13	0.000000
14	0.000000
15	0.000000
16	0.000000
17	0.000000
18	0.000000
19	0.000000
20	0.000000
21	0.000000
22	0.000000
23	0.000000
24	0.000000

C E

1	0.573457
2	0.301155
3	0.337532
4	0.140147
5	0.018071
6	0.052237
7	0.330713
8	0.073332
9	0.052352
10	0.000000
11	0.018535
12	0.050007
13	0.007422
14	0.005110
15	0.000000
16	0.000000
17	0.000000
18	0.051521
19	0.000000
20	0.000000
21	0.000000
22	0.000000
23	0.000000
24	0.000000

EFFICIENCY VECTOR

1	0.000000
2	0.000000
3	0.000000
4	0.000000
5	0.000000
6	0.000000
7	0.000000
8	0.000000
9	0.000000
10	0.000000
11	0.000000
12	0.000000
13	0.000000
14	0.000000
15	0.000000
16	0.000000
17	0.000000
18	0.000000
19	0.000000
20	0.000000
21	0.000000
22	0.000000
23	0.000000
24	0.000000

EFFICIENCY VECTOR

1	0.000000
2	0.000000
3	0.000000
4	0.000000
5	0.000000
6	0.000000
7	0.000000
8	0.000000
9	0.000000
10	0.000000
11	0.000000
12	0.000000
13	0.000000
14	0.000000
15	0.000000
16	0.000000
17	0.000000
18	0.000000
19	0.000000
20	0.000000
21	0.000000
22	0.000000
23	0.000000
24	0.000000





```

0.0554338E-04
0.221211E-01
0.553037E-01
0.111375E+00
0.102540E+04
0.112045E-03
0.127542E+04
0.454949E+04
0.111375E-01
0.112045E-03
0.127542E+04
0.454949E+04
0.355433E-04
0.451723E-04
0.428731E-04
0.773935E-04

```

```

*****
METHODS TO BE USED TO CALCULATE CONTRIBUTIONS TO THE NP
EFFICIENCY COMPONENTS, IN DECREASING ORDER:

```

```

*****
MODE(IN MODE) COMP. EFF.
  5 0.103354E-02
 10 0.724642E-03
  1 0.113545E-02
  2 0.113545E-04
  3 0.773935E-04
  4 0.561733E-04
  5 0.423731E-04
  6 0.208433E-04
  7 0.335433E-04
  8 0.274135E-04
  9 0.265433E-05
 11 0.978733E-11
NP TH ORDER EFFICIENCY EMP: 0.236531E-02

```

```

BLOGG-DIAS NUMERATOR & DENOMINATOR COMP. COSTS:
0.113354E-03 0.305555E+00
0.740740E-05 0.244333E-01
0.423731E-05 0.731691E-03
0.423731E-05 0.557575E-01
0.335433E-05 0.335433E+00
0.423731E-05 0.454949E-01
0.125045E-05 0.127400E-01
0.335433E-05 0.335433E-01
0.423731E-05 0.423731E-01

```

```

BLOCK-----4-----0.5150-0E+02
BLOCK-----1-----0.19594E+01
UNORDERED COMP. EFFICIENCIES : 0.304283E-02 0.305153E-03
UNORDERED COMP. EFFICIENCIES : 0.371549E-03 0.541730E-02
UNORDERED COMP. EFFICIENCIES : 0.331183E-02 0.275754E-02
UNORDERED COMP. EFFICIENCIES : 0.991257E-02 0.213410E-03
UNORDERED COMP. EFFICIENCIES : 0.453932E-02 0.121545E-16
UNORDERED COMP. EFFICIENCIES : 0.121545E-16 0.978005E-05

```

```

*****
METHODS FOR THE SOLUTION OF THE PROBLEM ARE:
NP EFFICIENCY INCREASING ORDER:
NP EFFICIENCY DECREASING ORDER:

```

```

*****
MODE(I) MODEL) BLOCK-1133-COMP. EFF.
  7 0.371549E-02
  10 0.541730E-02
  4 0.635005E-02
  5 0.751183E-02
  9 0.757768E-02
  1 0.121545E-02
  2 0.275754E-02
  3 0.305153E-03
  8 0.304283E-02
  12 0.371549E-03
  11 0.121545E-16

```

```

NP TH ORDER BLOCK-1133-EFFICIENCY IMP : 0.262982E-02
*****
PRINCIPAL COMPONENTS ANALYSIS OF RELATIVE EFFICIENCY

```

```

  1 -0.273699
  2 -0.595539
  3 -0.525397
  4 0.513370
  5 -0.207927
  6 -0.009299
  7 -0.177922
  8 -0.225097
  9 0.544479
  10 -0.259127
  11 0.175149
  12 -0.297723
  13 -0.119029
  14 -0.443359

```

12	0.00000
13	0.00000
14	0.00000
15	0.00000
16	0.00000
17	0.00000
18	0.00000
19	0.00000
20	0.00000
21	0.00000
22	0.00000
23	0.00000
24	0.00000

C E

1	0.00000
2	0.00000
3	0.00000
4	0.00000
5	0.00000
6	0.00000
7	0.00000
8	0.00000
9	0.00000
10	0.00000
11	0.00000
12	0.00000
13	0.00000
14	0.00000
15	0.00000
16	0.00000
17	0.00000
18	0.00000
19	0.00000
20	0.00000
21	0.00000
22	0.00000
23	0.00000
24	0.00000

CHARACTERISTIC ENERGIES

1	0.00000
2	0.00000
3	0.00000
4	0.00000
5	0.00000
6	0.00000
7	0.00000
8	0.00000
9	0.00000
10	0.00000
11	0.00000
12	0.00000

13  
 14  
 15  
 16  
 17  
 18  
 19  
 20  
 21  
 22  
 23  
 24

EFFICIENCY VECTOR

1  
 2  
 3  
 4  
 5  
 6  
 7  
 8  
 9  
 10  
 11  
 12  
 13  
 14  
 15  
 16  
 17  
 18  
 19  
 20  
 21  
 22  
 23  
 24

TOTAL EFFICIENCY = 0.0001  
 ----- IN RELATIVE MODEL -----  
 EFFICIENCY VECTOR COST 1 0.572117E+02  
 EFFICIENCY VECTOR COST 2 0.217747E+01  
 EFFICIENCY CHECK AFTER DECOM: 0.000000E+00

UNIT-A COMPONENT COSTS

0.141357E+03 0.237768E+05  
 0.471718E+02 0.952493E+03  
 0.256168E+02 0.275907E+03  
 0.224585E+02 0.527458E+03



