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EXPANSION OF INTEGRAL EQUATIONS ARISING IN SCATTERING THEORY (U)

S.T. MCDANIEL

Technical Memorandum
File No. 90-59
2 March 1990

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TABLE OF CONTENTS

LIST OF FIGURES

I. INTRODUCTION

There are presently only two basic analytical methods applicable to scattering from random rough interfaces -- the Rayleigh or small wave-height approximation and the Kirchhoff or physical optics approximation. Both of these methods are valid only for a limited range of parameters. Composite-roughness theory, which combines these two approaches in their respective regions of validity, also has a defect; its predictions are dependent on an arbitrary parameter.

The Rayleigh approximation arises from assuming that the scattered field can be written in terms of waves propagating away from the surface, even in the hollows between adjacent peaks on the surface. The field on the bounding interface is not explicitly computed. The Kirchhoff approximation, on the other hand, arises from retaining only the lowest order term in the solution of an integral equation of the second kind for the surface field. This approximate surface field is then used to compute the scattered field via the Helmholtz integral.

In addition to the integral equation of Maue[1] and Meecham[2], which is traditionally used when discussing the Kirchhoff approximation for scattering from pressure release surfaces, there exist other surface field formulations: namely, Uretsky's[3] integral equation of the first kind and a variant of the Maue-Meecham equation introduced by Dashen[4]. In this article, the solution of these three different integral equations is obtained in the form of a series by first expanding the kernel of the integral equation in a small parameter and then applying the method of successive approximations.

Section II introduces the problem addressed, scattering from a one-dimensional periodic pressure release surface and reviews the result obtained when the Rayleigh hypothesis is invoked. In Section III, the three integral equations are solved and expressions are obtained for the reflection coefficients. Uretsky's equation yields a result identical to the Rayleigh series to third order. The two integral equations of the second kind, however, yield results that differ from each other and the Rayleigh series at third order. Only one of these series, that based on Uretsky's integral equation, has the necessary attributes to represent a meaningful physical solution. The reflection coefficients arising from the two integral equations of the second kind are not reciprocal and also are unbounded when the grazing angle associated with the reflected order vanishes. Uretsky's equation may also be cast into an integral equation of the second kind and the convergence of its solution demonstrated by a method due to *Urusovskii*[5]. However, if the kernel of this integral equation is expanded, this proof is inapplicable. This is also true for the other two integral equations leading to the disparate results.

Section IV summarizes these findings with the conclusion that Uretsky's integral equation is the preferred starting point for the development of improved low-frequency scattering formulations.

II. THE SMALL WAVE HEIGHT APPROXIMATION

Figure 1 depicts the problem of interest -- scattering of an incident plane wave of wavelength $\lambda = 2\pi/k$

$$p_{inc}(x,z) = \exp[ik(\alpha_0 x - \gamma_0 z)], \tag{1}$$

by a one-dimensional periodic pressure release surface. Our nomenclature closely follows Holford's[6] with an incident grazing angle ϕ_0 so that $\alpha_0 = \cos\phi_0$ and $\gamma_0 = \sin\phi_0$. Far from the surface, the total field may be written

$$p(x,z) = p_{inc}(x,z) + \sum_n R_n \exp[ik(\alpha_n x + \gamma_n z)], \tag{2}$$

where R_n is the reflection coefficient for the n^{th} order and

$$\alpha_n = \alpha_0 + (nK/k),$$

$$\gamma_n = \begin{cases} (1 - \alpha_n^2)^{1/2} \; ; & |\alpha_n| \le 1 \\ i\,(\alpha_n^2 - 1)^{1/2} \; ; & |\alpha_n| > 1, \end{cases} \tag{3}$$

where the fundamental surface wavelength $\Lambda = 2\pi/K$. Note that in Eq. (2) when the limits on the summation are omitted, the limits are understood to be $-\infty$, $+\infty$. This same simplification will also be used for infinite integrals.

Rayleigh assumed that Eq. (2) held not only in the far field but also on the boundary $z = \zeta(x)$. Equating $p[x, \zeta(x)] = 0$, then yields

$$\exp[-ik\gamma_0\zeta(x)] = \sum_n R_n[iKnx + ik\gamma_n\zeta(x)]. \tag{4}$$

To obtain a solution to this equation R_n is expanded in a small parameter $\epsilon \sim O(kh)$

$$R_n = R_n^{(0)} + \epsilon\, R_n^{(1)} + \epsilon^2\, R_n^{(2)} + \ldots \quad . \tag{5}$$

The exponentials in Eq. (4) with arguments $\sim O(\epsilon)$ are also expanded, permitting an iterative solution for the $R_n^{(i)}$:

$$R_n^{(0)} = -\delta_{n,0},$$

$$R_n^{(1)} = 2ik\gamma_0 h_n,$$

$$R_n^{(2)} = 2k^2\gamma_0 \sum_m \gamma_m\, h_{n-m}\, h_m$$

$$R_n^{(3)} = (ik^3\gamma_0/2) \sum_{m,p} h_{n-p}\, h_{p-m}\, h_m\, [-(\gamma_n^2 + \gamma_0^2)/3 + \gamma_m^2 + \gamma_p^2 - 4\gamma_m\gamma_p], \tag{6}$$

with $h_m = {}_0\int^{\Lambda} \frac{dx}{\Lambda} \zeta(x) \exp(-iKmx)$. The identity

$$n^2 \sum_{m,p} h_{n-p}\, h_{p-m}\, h_m = 3 \sum_{m,p} h_{n-p}\, h_{p-m}\, h_m\, (p^2 - m^2), \tag{7}$$

which can be verified by interchanging the indices has been used to cast the third order reflection coefficient into a reciprocal form.

III. SURFACE INTEGRAL FORMULATIONS

A. The Maue-Meecham Integral Equation

Maue[1] and Meecham[2] derived an integral equation of the second kind for the normal derivative of the pressure on the boundary

$$\psi(x) = 2\hat{\psi}(x) - \int \psi(x')\, K_1\, (x',x)dx', \tag{8}$$

where $\psi(x) = \frac{i}{k} \left(- \frac{\partial p}{\partial z} + \zeta'(x) \frac{\partial p}{\partial x}\right)$,

$$\hat{\psi}(x) = -[\gamma_0 + \alpha_0 \zeta'(x)]\, p_{inc}[x,\zeta(x)],$$

$$K_1(x',x) = \frac{ik}{2} \frac{H_1^{(1)}(k\rho)}{\rho} [\zeta(x') - \zeta(x) - (x' - x)\zeta'(x)],$$

and $\rho = \{(x' - x)^2 + [\zeta(x') - \zeta(x)]^2\}^{1/2}$.

The Kirchhoff approximation results if only the first term on the right hand side of Eq. (8) is retained.

On applying the operator ${}_0\int^{\Lambda} \frac{dx}{\Lambda} \exp(-ik\alpha_m x)$ to both sides of Eq. (8), one obtains the set of equations

$$\psi_m + \sum_n V^{(1)}_{m,n}\, \psi_n = 2\hat{\psi}_m \tag{9}$$

where

$$V^{(1)}_{m,n} = \frac{ik}{2} \int_0^{\Lambda} \frac{dx}{\Lambda} \int d\tau\, K_1\, (\tau, x)\, \exp[ik\alpha_n \tau + iK(n-m)x], \tag{10}$$

$$\hat{\psi}_m = \int_0^{\Lambda} \frac{dx}{\Lambda}\, \hat{\psi}(x)\, \exp(-ik\alpha_m x),$$

and $\tau = x' - x$.

To obtain the desired series solution, the Hankel function appearing in the kernel of the integral equation is expanded in the series[7]

$$(1+r)^{-\upsilon/2}\, H^{(1)}_{\upsilon}\, [z(1+r)^{1/2}] = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \left(\frac{rz}{2}\right)^m H^{(1)}_{\upsilon+m}\, (z), \tag{11}$$

which is absolutely convergent for

$$|r| = [\zeta(\tau+x) - \zeta(x)]^2/\tau^2 < 1.$$

The integrations indicated in Eq. (10) may then be performed (see the Appendix) to obtain

$$\begin{aligned} V^{(1)}_{m,n} = {} & i\, h_{m-n}\, [k\, (\gamma_m - \gamma_n) + (m-n)\, K\, \alpha_n/\gamma_n] \\ & + (ik^2/2) \sum_{q,p} h_{m-q}\, h_{q-p}\, h_{p-n}\, \{ K[(m-q)\, \alpha_q \gamma_q \\ & + (p-n)\alpha_n \gamma_n - 2(q-p)\alpha_p \gamma_p] - (k/3)\, (\gamma_n^3 - \gamma_m^3 + 3\gamma_q^3 - 3\gamma_p^3)\} \\ & + \ldots\ . \end{aligned} \tag{12}$$

It is of interest to note that the leading term in this expansion can be written in the form

$$\frac{1}{k} \left[\frac{\alpha_n(\alpha_m + \alpha_n)}{\gamma_n(\gamma_m + \gamma_n)^2} + \frac{1}{(\gamma_m + \gamma_n)}\right] \int_o^{\Lambda} \frac{dx}{\Lambda} \exp[iK(n-m)x]\zeta''(x), \tag{13}$$

from which it is evident that this is a curvature correction.

If ψ_n is expanded in powers of ϵ

$$\psi_n = \psi_n^{(0)} + \epsilon\, \psi_n^{(1)} + \epsilon^2\, \psi_n^{(2)} + \ldots\ ,$$

and the exponentials of argument $\sim O(\epsilon)$ are also expanded, Eq. (9) may be solved for the $\psi_n^{(j)}$

$$\psi_n^{(0)} = -2\,\gamma_0\,\delta_{n,0}$$

$$\psi_n^{(1)} = 2ik\,\gamma_0\gamma_n h_n$$

$$\psi_n^{(2)} = k^2\gamma_0 \sum_m h_{n-m}\,h_m\,(2\,\gamma_m\gamma_n - \gamma_n^2)$$

$$\psi_n^{(3)} = ik^3\gamma_0 \sum_{m,p} n_{n-m}\,h_{m-p}\,h_p\,[\gamma_n(\gamma_m^2 - \gamma_n^2/3 - 2\gamma_m\gamma_p)$$

$$+\gamma_p\,(1+2\alpha_p^2 + \alpha_p\alpha_n - 2\alpha_n\alpha_m - 2\alpha_m\alpha_p)]. \qquad (14)$$

The identities

$$\sum_{m,p} h_{n-m}\,h_{m-p}\,h_p\,(p) = \frac{1}{2}\sum_{m,p} h_{n-m}\,h_{m-p}\,h_p\,(m)$$

$$= \frac{1}{3}\sum_{m,p} h_{n-m}\,h_{m-p}\,h_p\,(n)$$

are used in obtaining Eq. (14).

With ψ_n known, the reflection coefficients are found using

$$R_m = \frac{1}{2\gamma_m}\sum_n \psi_n\,{}_0\!\int^{\Lambda}\frac{dx}{\Lambda}\exp[iK(n-m)x - ik\gamma_m\zeta(x)]. \qquad (15)$$

The R_m are expanded as in Eq. (15) and the exponential factor with argument $\sim 0\,(\epsilon)$ is likewise expanded. For $0 \le j \le 2$, the $R_n^{(j)}$ are identical to the Rayleigh reflection coefficients. For $j = 3$, the result differs

$$R_n^{(3)} = R_n^{(3)}(\text{Rayleigh}) + \frac{ik^3\gamma_0}{\gamma_n}\sum_{p,j} h_{n-p}\,h_{p-j}\,h_j$$

$$\times[\gamma_p(\alpha_n^2 + \alpha_n\alpha_p - 2) - \gamma_j\,(\alpha_n + \alpha_n\alpha_j - 2) + (\gamma_n^3 - \gamma_0 + \alpha_0\gamma_0\alpha_n)/3]. \qquad (16)$$

The additional term in Eq. (16) is not reciprocal and also becomes unbounded as γ_n vanishes.

B. Dashen's Formulation

Dashen[4] introduced the integral equation

$$\psi(x) = -2\gamma_0 \left(\frac{ds}{dx}\right)^2 p_{inc}[x,\zeta(x)] - \left(\frac{ds}{dx}\right)^2 \int \psi(x')K_2(x',x)dx', \qquad (17)$$

where $K_2(x',x) = \frac{ik}{2} \frac{H_1^{(1)}(k\rho)}{\rho} [\zeta(x') - \zeta(x)],$

and $\left(\frac{ds}{dx}\right)^2 = 1 + [\zeta'(x)]^2.$

Performing the same operations as in subsection A, the set of equations

$$\psi_m + \sum_n V^{(2)}_{m,n} \psi_n = 2\tilde{\psi}_m, \qquad (18)$$

is obtained. In this case

$$\tilde{\psi}_m = {}_0\!\int^\Lambda \frac{dx}{\Lambda} \exp(-iKmx) \left(\frac{ds}{dx}\right)^2 \exp[-ik\gamma_0\zeta(x)],$$

and

$$V^{(2)}_{m,n} = \frac{ik}{2}\ {}_0\!\int^\Lambda \frac{dx}{\Lambda} \left(\frac{ds}{dx}\right)^2 \int d\tau\, K_2(\tau,x) \exp[ik\alpha_n\tau + iK(n-m)x].$$

With the use of Eq. (11), $V^{(2)}_{m,n}$ may be expanded

$$V^{(2)}_{m,n} = ik(\gamma_m - \gamma_n) h_{m-n} - ikK^2 \sum_{j,p} h_{m-j} h_{j-p} h_{p-n} (j-p)$$

$$x(m-j)(\gamma_p - \gamma_m) + \frac{ik^3}{6} \sum_{j,p} h_{m-j} h_{j-p} h_{p-n} (\gamma_n^3 - \gamma_m^3 + 3\gamma_j^3 - 3\gamma_p^3)$$

$$+ \ldots \quad . \qquad (19)$$

The first term in Eq. (19) may be rewritten as

$$-\frac{(\alpha_m+\alpha_n)}{(\gamma_m+\gamma_n)}\ {}_0\!\int^\Lambda \frac{dx}{\Lambda} \zeta'(x) \exp[iKx(n-m)],$$

and is a slope correction.

The surface field transform ψ_n can again be expanded in powers of ϵ. For Dashen's equation, the result differs at third order from Eq. (14). In this case,

$$\psi_n^{(3)} = -2ik^3\gamma_0 \sum_{m,p} h_{n-m}\, h_{m-p}\, h_p \Big[\frac{\gamma_n^3}{6} + \frac{\gamma_p^3}{3}$$

$$+ \gamma_o (\gamma_p^2 - \gamma_m^2 + \gamma_n^2/3 - \gamma_0^2/3) + \gamma_n\gamma_m\gamma_p - \gamma_n\gamma_m^2/2$$

$$+ (\gamma_p/3)(\alpha_p\alpha_0 - \alpha_p\alpha_n + \alpha_0\alpha_n - 1)\Big] \qquad (20)$$

The corresponding reflection coefficient

$$R_n^{(3)} = R_n^{(3)}(\text{Rayleigh}) + \frac{ik^3\gamma_0}{2\gamma_n} \sum_{m,p} h_{n-m}\, h_{m-p}\, h_p$$

$$\times[2\gamma_n\gamma_0(\gamma_0-\gamma_n)/3 - 4\gamma_p\gamma_n^2/3 - \gamma_n^2\gamma_m - \gamma_p^3$$

$$-2\gamma_p(\alpha_p\alpha_0 - \alpha_p\alpha_n + \alpha_n\alpha_0 - 1)/3], \qquad (21)$$

differs from that of subsection A, but, nevertheless, suffers from the same deficiencies.

C. Uretsky's Integral Equation

Uretsky's[3] integral equation is obtained by evaluating the Helmholtz integral on the boundary

$$\int \psi(x')\, K_0(x,x)dx' = -2p_{inc}[x,\zeta(x)], \qquad (22)$$

where $K_0(x',x) = (k/2)\, H_0^{(1)}(k\rho)$. This integral equation reduces to the system of equations

$$\sum_n V_{m,n}^{(0)}\, \psi_n = -2\bar{\psi}_{mj} \quad , \qquad (23)$$

where

$$\bar{\psi}_m = {}_0\int^{\Lambda} \frac{dx}{\Lambda} \exp(-ik\alpha_m x)\ p_{inc}[x,\zeta(x)], \quad (24)$$

and

$$V^{(0)}_{m,n} = {}_0\int^{\Lambda} \frac{dx}{\Lambda} \int d\tau K_0(\tau,x)\ \exp[ik\alpha_n\tau + iK(n-m)x]. \quad (25)$$

Again, using Eq. (11), $V^{(0)}_{m,n}$ may be written as a series

$$V^{(0)}_{m,n} = \delta_{m,n}/\gamma_m - k^2 \sum_p h_{m-p}\ h_{p-n}\ (\gamma_m+\gamma_n-2\gamma_p) + \ldots \quad (26)$$

The second term in Eq. (26) can be manipulated to show that it depends on both the square of the surface slope and the product of the curvature and height.

The surface field transform differs at third order from that of subsection A

$$\psi^{(3)}_n = ik^3\gamma_0\gamma_n \sum_{m,p} h_{n-p}\ h_{p-m}\ h_m\ [-\gamma_0^2/3 + \gamma_m\gamma_n + \gamma_m^2 - 2\gamma_p\gamma_m], \quad (27)$$

and the reflection coefficients are identical to the Rayleigh reflection coefficients. In a related study[8], it is shown, for the case of a pressure release sinusoidal surface, that the reflection coefficients resulting from the expansion of Uretsky's equation are identical to the Rayleigh result at every order in the expansion.

With Eq. (24), Eq. (23) can be rewritten

$$\psi_m + \sum_n \bar{V}_{m,n}\ \psi_n = -2\gamma_m\bar{\psi}_m , \quad (28)$$

where

$$\bar{V}_{m,n} = \gamma_m V^{(0)}_{m,n} - \delta_{m,n}. \quad (29)$$

For the solution of Eq. (28) to converge as the number of equations retained becomes large, $\bar{V}_{m,n}$ must be a completely continuous operator in the space of square summable sequences, and the right hand side of this equation must be an element of this space, or[9]

$$\sum_{m,n} |2\gamma_m \bar{\psi}_m|^2 < \infty, \tag{30}$$

and

$$\sum_{m,n} |\bar{V}_{m,n}|^2 < \infty. \tag{31}$$

From Eq. (29), this latter requirement will be satisfied if

$$\sum_{m,n} |\gamma_m V^{(0)}_{m,n}|^2 < \infty. \tag{32}$$

Urusovskii[5] has shown that $\gamma_m V^{(0)}_{m,n}$ is bounded. Thus with m=n+p, Eq. (32) becomes

$$\sum_{\substack{n,p \\ |n|,|p| \geq N < \infty}} |\gamma_{n+p} V^{(0)}_{n+p,n}|^2 < \infty . \tag{33}$$

With the assumption that $\zeta(x)$ is four times differentiable, Eq. (25) may be integrated by parts, twice with respect to both x and τ, whence

$$|\gamma_{n+p} V^{(0)}_{n+p,n}| \leq \frac{M|p+n|}{|p|^2 |n|^2} \quad \text{as} \quad |p|,|n| \rightarrow \infty, \tag{34}$$

where $M < \infty$. This is sufficient to satisfy Eq. (33). Two integrations by parts of Eq. (34) are sufficient to demonstrate that Eq. (30) holds. Thus, Kantorovitch's method of reduction[9] is applicable to Uretsky's formulation as well as the two integral equations of the second kind. Note, however, that this proof is inapplicable to the expansions of $V_{m,n}$ given in Eqs. (12), (19), and (26).

D. Monostatic Backscattering Strength

The expressions given for the reflection coefficients are of sufficient complexity that it is difficult to ascertain their dependence on various parameters. If one takes ensemble averages, the resulting expressions are simpler. Let us first consider the quantity

$$Q_{n,j} = < R_n^{(1)} R_j^{*(1)} + R_n^{(2)} R_j^{*(2)} > + < R_n^{(1)} R_j^{(3)*} + R_n^{(3)} R_j^{(1)*} > , \qquad (35)$$

which is needed to compute the backscattering strength.

For a Gaussian distribution of surface heights

$$<h_p h_q h_r h_s> = W(Kp)\ W(Kr)\ \delta_{p,-q}\ \delta_{r,-s}$$

$$+ W(Kp)\ W(Kq)\ \delta_{p,-r}\ \delta_{q,-s} + W(Kp)\ W(Kq)\ \delta_{p,-s}\ \delta_{q,-r}$$

where $W(Kp)$ is the rough surface wave number spectrum normalized such that

$$< \zeta^2(x) > = \sum_p W(Kp).$$

The first term on the right hand side of Eq. (35) is the same for all of the formulations

$$Q_{n,j}^{(1)} = 4k^2 \gamma_0^2 \delta_{n,j} \{W(2k\alpha_0) + k^2 \sum_m W\ [k(\alpha_m - \alpha_0)]$$

$$\times\ W\ (2k\alpha_0 + Km)\ [\gamma_m \gamma_m^* + \mathrm{Re}(\gamma_m \gamma_{-m}^*)]\}, \qquad (36)$$

for monostatic backscatter, where $\mathrm{Re}(x)$ denotes the real part of x. For the Rayleigh and Uretsky formulations, the second term on the right hand side of Eq. (35) is

$$Q_{n,j}^{(2)} = 8k^4 \gamma_0^2 \delta_{n,j}\ W(2k\alpha_0)\ \mathrm{Re}[\sum_p W(Kp)\ (\gamma_p^2 - 2\gamma_0 \gamma_p - \gamma_p \gamma_{-p})].$$

For the Maue-Meecham and Dashen formulations, respectively, one obtains

$$Q_{n,j}^{(2)} = Q_{n,j}^{(2)}\ (\text{Rayleigh}) - 16\alpha_0^2 ik^4 \gamma_0 \delta_{n,j}\ W(2k\alpha_0)$$

$$\times\ \mathrm{Re}[\sum_p W(Kp)\ (\gamma_0 - \gamma_p)],$$

and

$$Q^{(2)}_{n,j} = Q^{(2)}_{n,j}\ (\text{Rayleigh}) + 2k^4\gamma_0\delta_{n,j}\ W(2k\alpha_0)$$

$$\times\ \mathrm{Re}\{\sum_p W(Kp)\ [2\gamma_p(\gamma_0-\gamma_p)^2 + 8\alpha_0(\gamma_p\alpha_p-\gamma_p\alpha_0-\gamma_0\alpha_0)/3]\}.$$

The scattering strength is then given by

$$S = \frac{k}{K}\ \gamma_0^2\ (Q^{(1)}_{n,j} + Q^{(2)}_{n,j}).$$

Figure 2 compares the backscattering strength predicted by the different formulations. The wave number spectrum assumed for this computation is

$$\frac{1}{K}\ W(Kp) = \frac{.01}{(Kp)^3}\ ; \qquad .05/m < Kp < 100/m.$$

IV. CONCLUSIONS

The key to solving a general scattering problem is first to obtain a solution for the relevant field parameter evaluated on the scattering surface. For all but very simple cases, which can be treated numerically, this entails an approximation which thence limits the region of applicability of the result obtained.

This article has considered three different formulations of the boundary integral equation for plane wave incidence on a periodic pressure release surface. In the low-frequency limit, the three equations yield different results. Only Uretsky's integral equation yields physically acceptable reflection coefficients and these are identical to those obtained using the Rayleigh or small wave height approximation.

Whereas the Rayleigh approximation is in good agreement with exact solutions for scattering at low frequencies and grazing angles[10], when the frequency or grazing angle is significantly increased, this approximation yields scattering strength predictions considerably higher than those experimentally observed. It appears that Uretsky's integral equation is the only viable starting point for developing a general scattering theory applicable over a greater range of parameters.

ACKNOWLEDGEMENT

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Appendix

All of the integrals considered are evaluated using the following prescription. The Hankel functions are first expressed in the form

$$H_{\upsilon}^{(1)}(z) = \frac{1}{\pi} \int \frac{dp}{(k^2 - p^2)^{1/2}} \exp\{\frac{iz\,(k^2 - p^2)^{1/2}}{k} - i\upsilon \sin^{-1}[\frac{(k^2 - p^2)^{1/2}}{k}]\}. \tag{A-1}$$

Terms of the type $[\zeta(\tau+x) - \zeta(x)]$ are expressed as

$$\int_0^1 d\alpha \sum_m h_m\,(iKm\tau)\,\exp[iKm(x+\alpha\tau)], \tag{A-2}$$

and those of the form $[\zeta(\tau+x) - \zeta(x) - \tau\zeta'(x)]$ as

$$\int_0^1 \beta d\beta \sum_m h_m(iKm\tau)^2 \exp\{iKm[x + (1-\beta)\tau]\}. \tag{A-3}$$

By assigning k a small positive imaginary part δ, the order of integrations can be reversed and the integration over τ performed. The integration over p in Eq. (A-1) may then be performed by closing the contour in the upper half plane, at which point one can set $\delta = 0$. The remaining integrations are straight forward.

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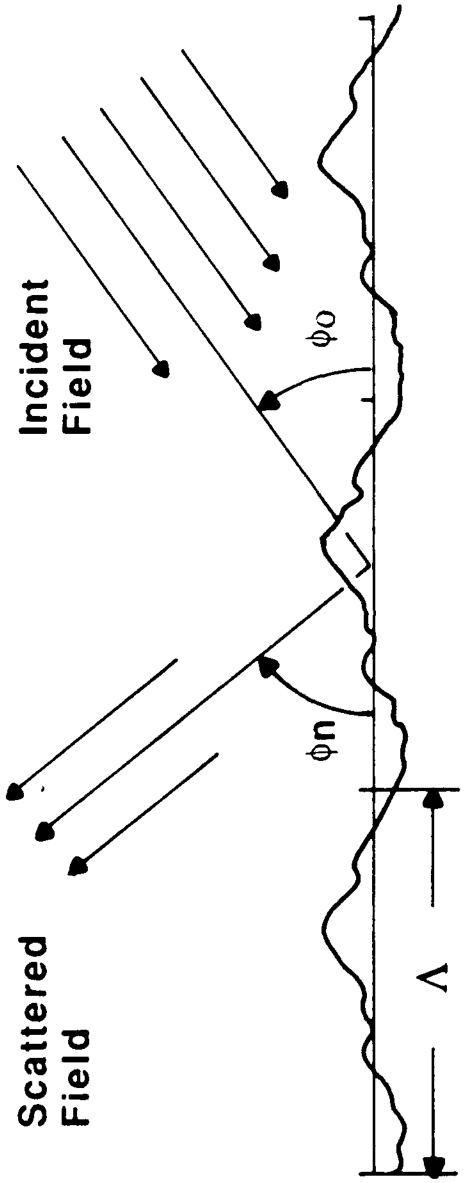

Figure 1. Scattering geometry.

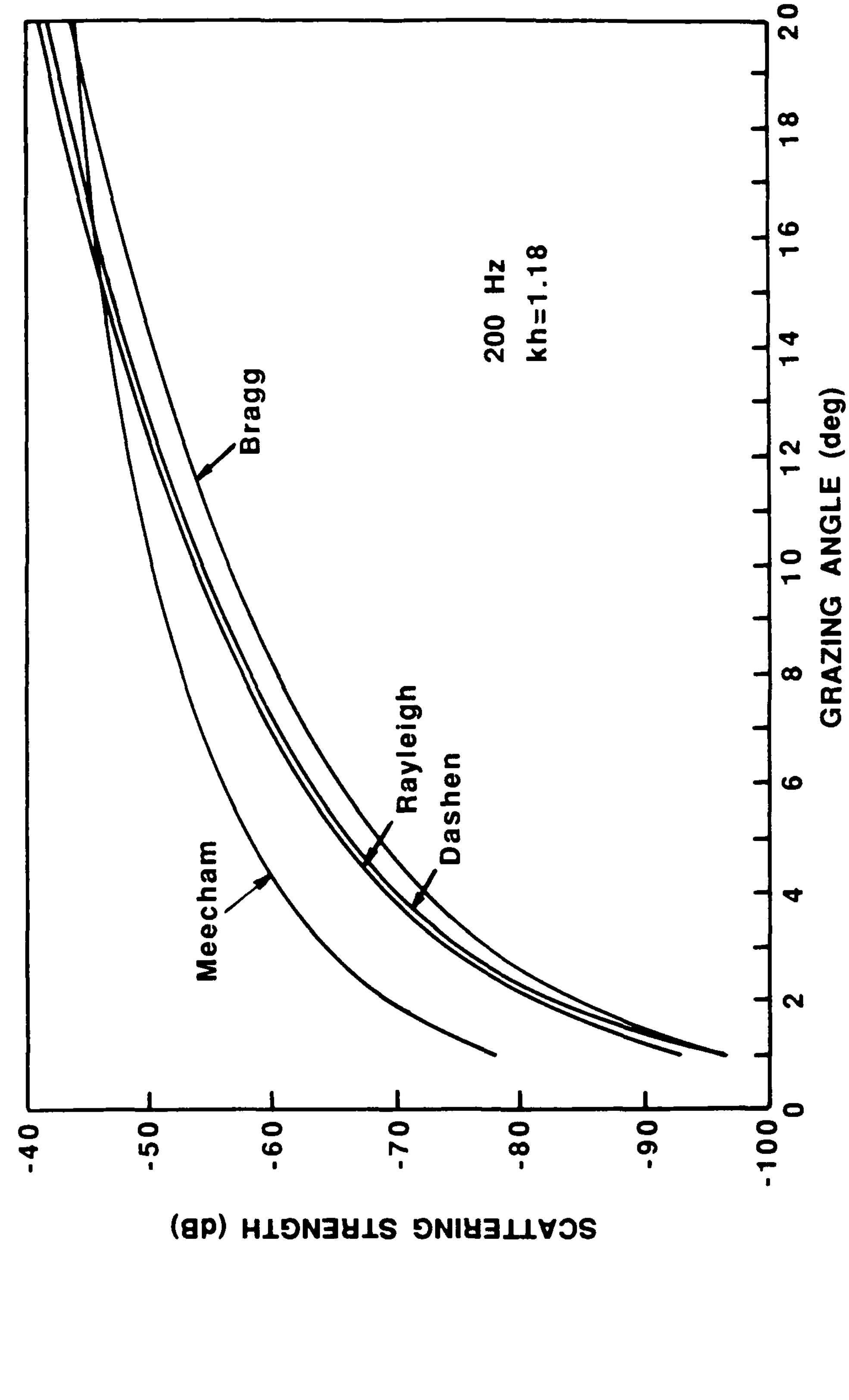

Figure 2. Monostatic backscattering strength predicted by expansion of the three integral formalisms.

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