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An International Conference organized by A.F.A.

CURVES AND SURFACES

USARDSG(UK)

· ABSTRACTS ·

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CHAMONIX MONT-BLANC
FRANCE

June 21-27, 1990.

An International Conference organized by A.F.A.

CURVES AND SURFACES

ABSTRACTS

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CHAMONIX MONT-BLANC
FRANCE

June 21-27, 1990.

Organization:

Association Française d'Approximation (A.F.A.)

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- Institut National des Sciences Appliquées de Toulouse.
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- Laboratoire de Modélisation et de Calcul (Grenoble),
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- Elf-Aquitaine,
- IBM-France,
- CCSA-CELAR,

PROGRAM

The program will include eleven one-hour invited talks covering the main topics of the conference, five mini-symposia, and a number of research talks.

Invited talks:

M. Attéia, Université Paul Sabatier, Toulouse (France),
Spline Manifolds.

M. Barnsley, Iterative Systems, Norcross (U.S.A.),
Approximation of Images.

W. Dahmen, Freie Universität Berlin (Germany),
Convexity Preserving Properties of Bernstein-Bézier Representations of Polynomials.

J. Gregory, Brunel University, Uxbridge (U.K.),
Parametric Surfaces in Computer Aided Geometric Design.

Y. Meyer, Université de Paris 9 -Dauphine (France),
Wavelets and Applications.

R.Q. Jia, University of Oregon, Eugene (U.S.A.).
Surface Compression and Quasi-interpolants.

C.A. Micchelli, IBM, Yorktown Heights (U.S.A.),
Power of 2: Wavelets, Stationary Subdivision, and its Adjoint.

F. Natterer, Westfälische Wilhelms-Universität Münster (Germany),
2D Sampling in Tomography.

L.L. Schumaker, Vanderbilt University, Nashville (U.S.A.),
Data Dependent Least Squares Fitting by Splines on Triangulations.

F. Utreras, Universidad de Chile, Santiago (Chile),
Variational Approach to Shape Preservation.

G. Wahba, University of Wisconsin, Madison (U.S.A.),
Additive and Interaction Splines, and the Estimation of Multiple Smoothing Parameters.

Mini-symposia:

Geometric continuity; organizer: B. Barsky (U.S.A.),
Optimal recovery and information based complexity; organizer: M. Kon (U.S.A.),
Data storage and reduction; organizer: T. Lyche (Norway),
Quasi-interpolants; organizer: C. Chui (U.S.A.),
Radial Functions; organizer: N. Dyn (Israel).

Partial

CONTENTS

Polygonal Approximation for Implicitly Defined Surfaces

Abstract: We give a simplicial algorithm which is especially adapted for modelling surfaces via polygonal pieces. The types of surfaces which are handled are of the form

$$B := \{x \in \mathrm{R}^3 : H(x) = 0\} \quad \text{where} \quad H : \mathrm{R}^3 \to \mathrm{R}^1$$

is a piecewise smooth map. For a compact surface B the first stage of the algorithm terminates automatically with an approximation having no holes or overlaps. Features of the second stage include adaptive local mesh refining and mesh smoothing. Graphic output is in the form of wire diagrams.

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Choosing Nodes for Parametric Curve Fitting using Local Information

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ABSTRACT

The problem of approximating a set of ordered points applies to numerous problems in CAD, such as creating curves from data points, approximating a curve by another of smaller degree, or calculating an approximate offset curve. It is usually solved by first computing parameter values (nodes) that are associated to data points, and then finding the curve's coefficients (or control points) by solving a linear system using a least squares method.

The node choice (pamaretrization) is of major importance since it has a pronounced influence on the resulting curve's shape. The well known "chord length" parametrization, where parameters are proportional to distances between data points, is often used, since it is natural, simple, and quick to compute. However, results are barely acceptable when the data points are irregularly spaced. Namely, unwanted loops or wiggles appear between the points.

Two new parametrizations are proposed, which avoid such problems by minimizing the curve's length. The first method uses the same expression as the recent "' ntripetal" method (ie. proportional to the square root of data point distances), though it is obtained differently. The second method, based on a similar approach, takes the direction changes of data points into account as well.

Both methods are based on local data, as with the "chord length" method, and are therefore easy to use (eg. one need only calculate distances, and scalar products for the second method). Moreover, they lead to better results. They are also independant of the parametric curve type used in the application.

Comparisons between different methods are detailed for various data examples, pointing out the advantages and drawbacks of each. It is also shown how these parametrizations techniques can be used to create NURBS curves, using the conjugate gradient method to find the NURBS' control point coordinates and rational weights.

Nonparametric analysis of changes in change-point hazard rate models : A point process approach.

by

A. Antoniadis and G. Grégoire

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ABSTRACT

This paper discusses the estimation of parameters in hazard rate models with a change-point. When the change-point location is known, a nonparametric estimator of the amount of change is obtained using kernel smoothing methods in semiparametric models under random censoring. Consistency and asymptotic limit distribution of these estimators are obtained.

When the change-point is unknown, a consistent estimator of its location is obtained by "cross-validation" techniques. The performance of the estimators on finite samples are checked via simulation and an application to real data illustrates our approach.

Ajustement de fonctions splines sur des surfaces

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On étudie le problème suivant : soit f^0 une fonction donnée sur des ouverts ω_i polygonaux inclus dans un ouvert Ω borné de R^2, construire une fonction f régulière sur Ω "approchant" f^0 sur les ω_i. Ce problème apparaît dans divers traitements de la géophysique où il est nécessaire de raccorder des surfaces.

Nous proposons comme solution une D^m-spline discrète d'ajustement (cf R. Arcangéli[1] pour cette définition), réalisant le minimum d'une fonctionnelle quadratique sur un espace d'éléments finis. La fonctionnelle minimisée est la somme :

- d'une quantité approchant, à l'aide de formules d'intégration numérique, le terme de fidélité aux données suivant : $\sum_i \int_{\omega_i} (v - f^0)^2 dxdy$

et

- d'un terme de lissage : $\sum_{\substack{\alpha = (\alpha_1,\alpha_2) \\ \alpha_1+\alpha_2 = m}} \int_\Omega (\partial^\alpha v)^2 dxdy$, pondéré par un paramètre d'ajustement, où $\partial^\alpha v$ désigne la dérivée partielle d'ordre α de v au sens des distributions et où $m \geq 2$ est un entier convenable.

La solution f est obtenue en résolvant un système linéaire bande symétrique et défini positif, dont la taille ne dépend que de la dimension de l'espace d'éléments finis utilisé.

On donne une estimation de l'erreur d'approximation de f^0 par f sur les ouverts ω_i ainsi que des résultats numériques pour des fonctions tests f^0 dans des cas de géométrie simple.

[1] R. ARCANGELI, Cours de D.E.A., Pau, à paraître.

The SHILP Modeling and Display Toolkit*

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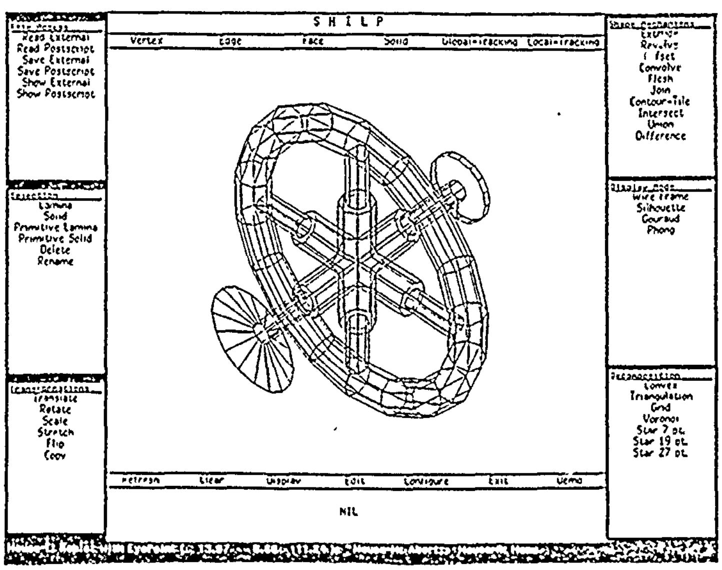
Abstract

We are crafting several tools for creating, editing and displaying solid models defined with algebraic boundary surfaces. Curves and surfaces can be represented in both implicit and rational parametric form, in either power or Bernstein polynomial bases. The current functionality of the toolkit includes restricted extrude, revolve and offset operations, edit operations on planar lamina and polyhedral solids, fleshing of wireframes with interpolating surfaces, and color rendering of solids. For the purpose of finite element generation, we allow the decomposition of arbitrary polyhedra with holes into convex pieces or tetrahedra. The creation and editing interface and tools run in vanilla Common Lisp and FORTRAN on Symbolics 3620's. The only non-portable portions of the code pertain to the graphics interface. The color rendering utilities have been primarily developed in C for HP and SGI workstations.

In this talk we shall describe the algorithmic and mathematical infrastructure of SHILP.



*Supported in part by NSF grant DMS 88-16286 and ONR contract N00014-88-K-0402

Methods of Knot Insertion for B-spline Curves

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Boehm's knot insertion algorithm and the Oslo algorithm are the methods commonly used to insert new knots into B-spline curves. There exist, however, a wealth of other possible methods. These alternative methods may be more attractive than Boehm's or the Oslo algorithm in a few cases, and, if nothing else, serve to enrich the theory of knot insertion. In this talk I will list and briefly explain some of these alternative methods.

Constructing a Triangle Facet Surface Approximation

to a Voxel Map

Günter Baszenski and Larry L. Schumaker

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We present a method to approximate a three dimensional object in voxel representation by a polyhedron with triangular facets. This piecewise linear approximation could be used as a first stage to construct a smooth surface by filling in the facets with parametric triangular patches.

Our construction process goes as follows:

We take slices of the voxel map fro several fixed height coordinates. On these slices the map induces discrete bivariate cross sections of the object. For each of these we construct a polygonal approximation. The polygon contours on neighboring slices are then connected by initial triangulations which we impose according to an error measure by swapping triangular edges.

G^1 and G^2 continuity for rational patches

The notion of a rational B - surface (R.B.S.) defined by a network of "mass vectors "(" vecteurs massiques " in french) was introduced and developed in 1988 by Fiorot and Jeannin.

This paper is devoted to the problems involved in the construction of a smooth connection between R.B.S. Its scope will be wider than methods already known in the case of polynomial patches.

More specifically , some results will be presented , concerning the G^1 continuity between two adjacent patches given either by a rectangular network of "mass vectors" or by a triangular one.

These results define the constraints required by the "mass vectors" to ensure such links.

A generalisation towards G^1 continuity constraints around a common corner of several R.B. patches is given.

Finally some methods for G^2 continuity are studied.

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MODELISATION DE COURBES DIGITALISEES PAR DES NURBS

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Un premier traitement des images binaires est l'analyse en composantes connexes, qui permet de décrire l'image par ses contours d'une façon rapide tout en gardant la forme de l'image le mieux possible(courbure...) d'où l'interêt de déterminer les points caractéristiques de l'image(points simple:faible déformation de la courbe, points anguleux:forte déformation de la courbe...).

L'approximation polygonale ou vectorisation ne permet pas de retrouver la structure initiale de la courbe. De plus une telle méthode d'approximation nécessite l'enregistrement d'innombrables données pour atteindre une satisfaisante apparence de continuité, et cela n'est pas facile à manier. Ce traitement est utile pour un deuxième traitement : approximation par des courbes splines utilisant la méthode des moindres carrés régularisés. Avec cette méthode on obtient des résultats satisfaisants en un temps court mais cela ne permet pas de représenter des coniques.

Les représentations par les B-splines rationnelles non uniformes(NURBS) : Ce type de modélisation est actuellement source de nombreuses études du fait de son caractère général pour représenter des primitives (segment de droite, conique ainsi que toute courbe B-spline) en gardant le même modèle, ce qui permet d'avoir une homogénéité dans la structure de données. Sur une courbe on détecte les différentes primitives ; avec les NURBS on obtient une approximation de la courbe en effectuant une approximation de chaque primitive et en étudiant les problèmes de connexion de ces primitives après approximation.

A VECTOR SPLINE APPROXIMATION

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Université Paul Sabatier

TOULOUSE, FRANCE

L. Amodei

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TOULOUSE, FRANCE

ABSTRACT

We introduce a new family of Spline functions solutions of the minimization problems:

$$P_{\alpha,\,\beta}\quad \begin{cases} \text{Min}\quad \left(\alpha \displaystyle\int_{\mathbb{R}^2} \|\nabla \text{ div } V\|^2 dx\, dy \;+\; \beta \displaystyle\int_{\mathbb{R}^2} \|\nabla \text{ rot } V\|^2 dx\, dy\right) \\ V \in \mathcal{X} \text{ and } V(X_i) = V_i,\ i=1,\ldots,N. \end{cases}$$

By means of the divergence (div) and rotational (rot) operators, the coupling between the vector function components is taken into account. This formulation is particularly well adapted for geophysical fluid flow interpolations (ex. horizontal wind fields).

High-Speed Random Algorithms for
Curve and Surface Generation

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and

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Many algorithms for generating curves and surfaces today involve a recursive tree traversal. These include sub-division refinement methods for generating B-splines and Beziér curves, line averaging methods for interpolants, and algorithms for wavelets and solutions to dilation equations. We show how ergodic theory can be used in a very general setting to produce random algorithms which generate the same curves and surfaces as the recursive ones. These images become attractors of random dynamical systems, and evolve simply as the trajectory of a *single* orbit. The random algorithms are very fast, involving only affine arithmetic, and are efficiently and highly parallelizable.

Méthode d'intersection d'une surface paramétrique polynomiale et d'un ensemble de 1/2 droites.

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La complexité de la méthode du lancer de rayons (synthèse d'images) est liée aux calculs d'intersection des rayons avec les objets de la scène. Si les objets sont définis par des modèles surfaciques mettant en oeuvre des surfaces paramétrées (Bézier, B splines, etc ..) les algorithmes d'intersection connus sont soit extrémement coûteux, soit peu robustes ou peu précis. Diverses méthodes : tessellation, Newton, ... donnent des solutions sans être réellement satisfaisantes.

Nous présentons une méthode numérique basée sur des outils de géométrie algébrique (implicitisation et inversion) proposé par [*]. L'algorithme proposé sépare, pour chaque carreau de Bézier, les calculs liés à la surface (pré-calcul) et ceux faisant intervenir le rayon.

Implicitisation :

Etant donnée une surface S paramétrée sur [0,1]x[0,1] polynomiale (x(u,v), y(u,v), z(u,v)), l'implicitisation consiste à trouver une fonction F telle que $(x,y,z) \in S$ implique $F(x,y,z) = 0$.

Nous avons comparé les différentes techniques (résultant de Sylvester, Cayley - Dixon) et étudié la "minimalité" de l'équation implicite obtenue. Dans notre cas, nous n'avons pas intérêt à calculer l'expression de F de façon complète. L'examen de la méthode de Cayley -Dixon conduit, pour l'application qui nous concerne, à associer à la surface S considérée des tables pré-calculées contenant les éléments nécessaires de son équation implicite en vue de l'étape suivante.

Equation d'intersection :

Le problème de l'intersection se ramène à la résolution d'une équation à une seule variable qui est le paramètre rayon, ce qui nous permet de trier plus natuellement les racines dans notre contexte. Cette étape nous fournit une racine à laquelle correspond un point $M(\overline{x},\overline{y},\overline{z})$ vérifiant $F(\overline{x},\overline{y},\overline{z}) = 0$.

Inversion :

Il s'agit de savoir si ce point M appartient à la surface paramétrée initiale (problème de la "minimalité"), et si oui, de déterminer les paramètres (u,v) de ce point. Ceci est en général obtenu par triangularisation d'un système linéaire (la validité de cette méthode est discutée).

Les résultats du logiciel résultant de cette étude seront présentés.

[*] T.W. SEDEBERG and D.C. ANDERSON and R.N. GOLDMAN
"Implicit Representation of Parametric Curves and Surfaces"
Computer Vision, Graphics and Image Processing 28, 72-84 (1984)

Finite Element Interpolation with Weighted Smoothing

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By a smoothed finite element interpolant we mean a finite element interpolant that has been chosen to minimize some "smoothing" semi-norm. Often the bending energy or thin-plate spline semi-norm is used for this purpose. We discuss the effect of using a weighted semi-norm in this smoothing procedure and give some examples showing how the weights can be chosen to reduce the under- and over-shoot of some interpolants.

Smooth Surface Reconstruction and Energy-based Segmentation

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The problem of using function values to construct a smooth multi-dimensional surface, e.g., a surface of minimal norm, arises in many areas. A common technique is to use a reproducing kernel (semi-)Hilbert space setting, where the reproducing kernels are known. Duchon derived the kernels for a continuous parameter family of such spaces using (semi-)Sobolev norms, say $D^m H^\eta$, where $1 - m < \eta$. For $\eta = 0$, these reverts to the traditional m^{th} order Sobolev spaces. However, for other values, we obtain interesting surfaces of intermediate smoothness. We begin by discussing these classes (and some of their less-obvious properties), and give example reconstruction from a number of these classes. We then discuss psychological experiments where subjects rate the quality of reconstructions from different classes. Segmentation of data from multiple smooth surfaces is another common problem, and a much more difficult one. We present a heuristic approach to segmentation based on the "energy", or approximate norm, of the optimal (single surface) reconstruction of subsets of data. For those classes where $\eta = 1/2$, we derive a closed form (over-)estimate of the energy, i.e., the m^{th} Sobolev semi-norm, of the spline of minimal norm. We then discuss heuristics using th' approximation to achieve low energy segmentations. We present some examples, showing the recovery of multiple 2D surfaces from real and synthetic depth data. We include an example where the multiple smooth surfaces are overlapping in x and y.

* Supported in part by NSF grant CCR8809022 and DARPA Grant #N00039-84-C-0165.

Approximation d'une Nappe de Points

par une Surface NURBS* à Bords Contraints

Frédéric Brossard et Mounib Mekhilef

Ecole Centrale de Paris

Un des problèmes particuliers à la restitution et à la construction des formes libres en CAO est celui de la détermination d'une surface minimale de raccordement avec des conditions de tangence et de courbure.

Etant donné une nappe de points, il s'aggit de trouver les paramètres définissant le carreau NURBS qui approche au mieux cette nappe. Ces paramètres sont:

- Le réseau polygonal caractéristique (densité, répartition, ...)
- Le poids de chaque pôle.
- Les éléments du vecteur nodal.

Par ailleurs, cette nappe doit vérifier certains critères tels que:

- Passage obligé par un bord parametré ou non.
- Tangence et/ou courbure imposés le long d'une partie du contour.

Après un bref rappel du contexte dans leauel nous nous plaçons, nous exploitons le nombre de degrés de liberté offert par les B-splines rationnelles non-uniformes de degré quelconque pout la mise en équation du problème. Nous déterminons, ensuite, le nombre de degrés de liberté optimal pour l'approximation par la méthode des moindres carrés, associée éventuellement à une méthode de recherche de minimum d'une fonction erreur. Dans une deuxième partie, nous présentons le traitement d'un cas d'école et celui d'un cas industriel sur des examples de pièces de carrosserie.

* NURBS: Non-Uniform Rational B-Spline

Convergence Orders of Interpolation with Multiquadric and Related Radial Functions

MARTIN BUHMANN

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In this talk we will describe the convergence analysis of different approaches to interpolating an n-variate function f, say, from the linear space spanned by translates of a function $\sqrt{\|\cdot\|^2 + c^2}^{\beta}$: $\mathbb{R}^n \mapsto \mathbb{R}$ where $\beta > -n$ is not an even integer and c is a nonzero parameter. Here, the data points at which we are interpolating are lying on a regular grid and the translates of the radial function are taken along these data points. We study the convergence orders that occur when we are interpolating to differentiable f while the spacing of the grid points tends to zero. and the approaches we will analyse differ in the choice of the parameter c in relation to the spacing of the data points on the grid. The main topic of the talk is a comparison of the ensuing convergence properties.

The results in this talk are largely based on joint work with Nira Dyn (Tel Aviv University).

A Geometrical Analysis of 3D Anatomical Structures

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Groupe Traitement d'Images - Département Mathématiques et Systèmes de Communication - Enst de Bretagne

Summary

In this study, we address the problem of the geometrical description of 3D objects formed by 2D cross-sectional adjacent slices. The original approach we propose in this preliminary work is based on the use of a complete and stable set of Fourier Descriptors (FD) of 2D curves that are used to define invariant features under elementary 3D geometrical transformations.

Such a shape description enables us to give a reduced representation of 3D contours by means of features which are directly related to 2D shape information such as curvature, length, etc...

Although we illustrate this approach using the specific application of bone shapes, a wide variety of 3D anatomical structures can be described by the proposed method, as it provides a general theoretical framework for the geometrical description of 3D objects formed by adjacent cross-sectional slices.

A slice is represented here by its boundary, which is itself described by a planar closed curve, expressed as a function of the arc length "s". The considered function is the radius function $\rho(s)$ which measures the length of the line connecting the contour to its centroid. Then, Fourier Descriptors are obtained from this curve description.

Invariancy under rotation, translation and starting point will be achieved by considering a combination of the previous FDs (invariants called I_k).

An application of the method is proposed for the description of a 3D bone structure (e.g. an ulna, a radius) reconstructed by a set of 2D Computerized Tomography (CT) images. The method of segmenting each slice is briefly presented; it is based on a sequence of gray-level thresholding and morphological filtering which produces accurate closed contours.

In this case, each slice can be represented by a set of 5 parameters: $|I_1|$, $|I_2|$, $|I_3|$, $|I_4|$, $|I_5|$. Each slice k is associated to a radius ρ_k, thus all slices are represented by one disc: the angle step is $2\pi k$ divided by the number of slices. Each invariant I_k is associated to a concentric circle. The value of $|I_k|$ is centered on the circle k. Each bone is modelled by a diagram composed of 5 signals. This diagram can be reduced because many slices contain poor geometrical information: this can be seen from the diagram.

This polar diagram provides interesting features. First, it gets a compact planar representation of rather complex 3D structures. Therefore, the possible inversion of the representation is very important for data compression: the whole 3D closed surface can be reconstructed using the reduced information contained in the diagram. Finally, one can use this reconstruction as an input of CAD/CAM systems for applications in orthopedia such as synthetic prostheses.

We suggest some ideas for future applications and work such as segmentation of 3D structures, identification of geometrical features and statistical analysis.

C^k Continuity of Rational Patches

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France

In the framework of Fiorot and Jeannin's results concerning the control of rational curves and surfaces by massic vectors, we propose a study of the construction, with C^k smoothness, of piecewise rational B-surfaces defined by rectangular or triangular nets of massic vectors.

The required conditions are explained in terms of massic vectors, but in the specific case of polynomial surfaces we find again the well known results.

"A Bivariate Interpolation Algorithm for Data which is Piecewise Monotone"

Ralph E. Carlson

A bivariate, monotone interpoiation algorithm was developed by Carlson and Fritsch using piecewise bicubic Hermite functions. These functions have been useful in solving other shape preserving interpolation problems such as those in which the underlying data are monotone in only one variable. The purpose of this talk is to describe a new algorithm which can be used to solve bivariate problems in which the underlying data are piecewise monotonic. This algorithm eliminates the "ringing" which is frequently present in other interpolants, such as bicubic splines, when steep gradients are located adjacent to flat spots.

An Optimal Interpolation Method for Solving
Nonlinear Boundary Value Problems

by

A.K. Chan and C.K. Chui
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Abstract

The locally supported cardinal interpolants with optimal order of approximation are applied to converting a nonlinear partial differential equation to a nonlinear difference equation. The nonlinearity is kept to the minimum due to the cardinality nature and the convergence of the numerical solutions is enhanced by the optimal order of approximation. Examples in nonlinear optics will be discussed.

Quasi-Interpolants From a Faber Series Approach to Cardinal Interpolation

C. K. Chui[†], J. Stöckler[‡], and J. D. Ward[†]

[†] Texas A&M University, College Station, U.S.A.
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Abstract: For a compactly supported function φ in $\mathbb{R}^d$ we study quasi-interpolants based on point evaluations at the integer lattice. We restrict ourselves to a special class where the coefficient sequence λf for given data f is computed by applying a univariate polynomial q to the sequence $\varphi\,|_{\mathbb{Z}^d}$, where powers mean discrete convolutions, and taking the convolution with the data $f\,|_{\mathbb{Z}^d}$. Such operators appear in the well known Neumann series formulation of quasi-interpolation. A criterion for the polynomial q is given such that the corresponding operator defines a quasi-interpolant.

With view on the cardinal interpolant, which is well defined if the symbol of φ does not vanish, we choose q as the partial sum of a certain Faber series. This series can be computed recursively. By our approach we omit the restriction that the range of the symbol of φ must be contained in a disk of the complex plane excluding the origin, which is necessary for convergence of the Neumann series. Furthermore, for symmetric φ we prove that the rate of convergence to the cardinal interpolant is superior to the one obtainable from the Neumann series.

SCHEMAS D'APPROXIMATION POUR DES MODELES NUMERIQUES DE TERRAINS

Approximation Schemes for Digital Terrains Modelling

Y.Correc †, Y. Lafranche * and A.Le Méhauté ‡

Résumé

Le calcul de modèles numériques de terrain (données matricielles) à partir de courbes de niveau, profils ou relevés divers (données vectorielles éparses) est un problème d'approximation classique. Pour le résoudre, certaines méthodes font appel à des schémas plus ou moins globaux dont le prix de la continuité est souvent une déterioration plus ou moins importante de la forme du terrain représenté. Remarquant que les hypothèses sous-jacentes à l'utilisation de ces schémas ne s'appliquent pas souvent en réalité au terrain naturel, on s'oriente vers des schémas plus robustes, basés sur la triangulation du semis de points constituant les données . Leur mise en œuvre révèle d'autres défauts, liés à la nature physique de la surface à modéliser. La prise en compte de celle-ci amène à combiner une triangulation et une subdivision, ce qui permet d'éliminer une grande partie des défauts constatés, et d'obtenir ainsi une solution satisfaisante du problème initial.

Abstract

The computation of digital terrain models (raster data) from contour lines, profiles or various surveys (sparse vector data) is a standard approximation problem. Its solution involves more or less global schemes, the continuity of which must be paid by some damage in the terrain shape. Pointing out the fact that, often, underlying assumptions do not actually apply to natural terrain, we suggest moving towards local schemes, based on the triangulation of scattered data.But their use exhibits some other shortcomings, connected with the physical character of the surface. We can overpass most of them by using a combination of triangulation and subdivision methods, obtaining then a satisfactory answer to the problem.

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QUALITE DES MODELES NUMERIQUES DE TERRAINS

On Digital Terrain Models Quality

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Résumé

La réalisation de modèles numériques de terrains (MNT) comporte plusieurs étapes, toutes génératrices d'erreurs, depuis la saisie des données géographiques initiales, jusqu'au calcul final de la matrice des altitudes.

L'utilisateur du MNT , ne disposant pas d'informations précises sur ces étapes, cherche malgré tout, par une analyse a posteriori, à évaluer la qualité du produit en termes d'erreurs globales et locales, en comparant le résultat à des données de référence. L'approche statistique classique n'étant pas satisfaisante, deux méthodes sont proposées : la première exploite les défauts des schémas d'approximation existants, et permet une estimation rapide et intuitive des grandeurs cherchées par une réduction de la dimension du problème. La seconde, plus systématique, fait appel à une technique d'ajustement par optimisation, sans calcul de dérivées, pour séparer les composantes globales (déplacement du modèle) et locales (bruit résiduel, déformation du modèle) de l'erreur.

Abstract

Error generation may occur at every step involved in the production of digital terrain models (DTM), from digitalisation of raw geographical data to computation of the terrain elevation matrix.

The DTM user's problem is that he has no precise information about any of these steps, but he wants to assess the product' s quality, in terms of global and (or) local errors, by means of a posteriori analysis and comparison with reference data.

As a statiscal approach is not satisfactory, we suggest two methods: the first one takes advantage of the approximation schemes deficiencies, to get a quick estimate of the studied values, from a reduced dimension problem. The second one uses least squares fit to separate the global (displacement) and local (buckling) components of error.

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Data Reduction using
box spline refinement and decomposition techniques

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Abstract. The combination of refinement and decomposition techniques for splines gives rise to several applications. With special emphasis on box spline surfaces and data reduction we will present examples using these techniques to solve different problems within image processing and geometric modelling.

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LISSAGE DE DONNEES GEOMETRIQUES PLANES

Considérons N points du plan : $M_i = (x_i, y_i)$, $1 \leqslant i \leqslant N$. Pour construire une courbe proche de ces points, on procède en général de la façon suivante :

1 - Construire une paramétrisation des données : $M_i = (x(t_i), y(t_i))$, $1 \leqslant i \leqslant n$, avec $t_i \in \mathbb{R}$,

2 - Choisir le type de courbes lissantes (Splines, Bézier ... etc...),

3 - Choisir un critère de lissage (moindres carrés, énergie minimum ...).

Peut-on éviter la première étape (paramétrisation des données) lorsqu'il n'existe pas de paramétrisation naturelle des points ? Pour satisfaire un tel programme, nous prenons une classe de courbes données par leur équation implicite :

$$P_a(x,y) = 0$$

avec

$$P_a(x,y) = \sum_{0 \leqslant \alpha+\beta \leqslant d} a_{\alpha\beta}\, x^\alpha y^\beta \quad \text{et} \quad \Sigma\, a^2_{\alpha\beta} = 1.$$

Le critère de lissage est pris au sens des moindres carrés

$$\operatorname*{Inf}_{a \in S_D} \sum_{i=1}^{N} P_a(x_i,y_i)^2 - \varepsilon \left(\frac{\partial P_a}{\partial x}(x_i,y_i)^2 + \frac{\partial P_a}{\partial y}(x_i,y_i)^2 \right)$$

où S_D est la sphère unité de $\mathbb{R}^D$, $D = (d+1)(d+2)/2$, $\varepsilon > 0$ donné. Nous montrons qu'en général une telle courbe est unique et étudions de nombreux exemples.

The Generation of an Aerodynamical Propeller Blade

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ABSTRACT

In this paper it will be shown how a design process for a simple propeller can be developed using the surface design method of Bloor and Wilson [1]. This technique considers a surface as the solution of a suitably chosen elliptic partial differential equation, where the desired shape is obtained by an appropriate choice of boundary conditions. The boundary conditions are given as functions of two parameters, whose isoparametric lines form a coordinate system within the surface.

To form the blade the method is used as described by Bloor and Wilson [2], whereby an aerofoil shape (which incorporates camber and twist) is considered as one boundary condition, with the other boundary condition taken as the tip of the blade which is a distance d from the base. Derivative boundary conditions must also be imposed along the curves which may be used to change the blades shape. The solution to this boundary value problem generates the blade. It will be shown how a number of blades can then be blended onto a cylindrical boss of circular cross-section to produce a blade propeller.

It will be shown how, by varying the various parameters in the problem, different attributes of the blade can be altered. For example by altering the derivative conditions and the length, it will be seen that the blade can be altered from a high aspect ratio aircraft blade to a low aspect ratio marine blade, and hence it will be shown how these parameters affect the aerodynamics of the blade.

In the case of the high aspect ratio blade it will be shown how aerodynamical considerations can be used to influence the shape of the blade and so obtain a design which is in some sense optimal.

1 Bloor, M I G & Wilson, M J 'Generating Blend Surfaces using Partial Differential Equations' CAD Vol 21 No 3 (1989) pp 165-171

2 Bloor, M I G & Wilson, M J 'Using Partial Differential Equations To Generate Free-Form Surfaces' CAD, to appear March 1990

Fast Computation of Cross-Validated Robust Splines and Other Non-linear Smoothing Splines

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Robust splines and other penalized likelihood estimates are well-known extensions of the ordinary smoothing spline, to estimate a smooth function g from approximate observations $z_i = g(t_i) + \varepsilon_i$, $i = 1, \cdots, n$. In the one-dimensional case, such estimates are minimizers over $H^m[a,b]$ of

$$E(f) = \int_a^b (f^{(m)}(t))^2 dt + \rho \sum_{i=1}^{n} \phi(f(t_i), z_i),$$

where ϕ is a given function corresponding to the data error model. For example, using the Huber function yields a spline which is robust against outliers in the data. Solving such a minimization problem (say with the Newton's method) generaly involves a sequence of reweighted ordinary spline problems.

The choice of the smoothing parameter ρ is crucial. The "leave-one-out" (or cross-validation) score, i.e. $\sum_{k=1}^{n} \phi(f_\rho^{[k]}(t_k), z_k)$ where $f_\rho^{[k]}$ minimizes $E(f) - \rho\phi(f(t_k), z_k)$, is an attractive criterion to measure the goodness of a given ρ. However computing this score requires to solve n minimization problems, and is thus very expensive. We propose an extension of the generalized cross-validation score and show that this can be easily implemented by a fast Monte-Carlo approximation technique extending the one proposed by Girard in the linear case. For large n, computing this score involves at most 2 minimization problems. Some interesting numerical results for robust splines and Poisson type data are presented to justify the extension proposed.

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Key words. smoothing splines, robust splines, penalized likelihood estimates, cross-validation, Monte-Carlo techniques

Surface Compression

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Each continuous function f on $\mathbb{R}^2$ can be decomposed into a series $\sum_{I \in \mathcal{D}} c_I(f)\varphi_I$ with φ a box spline and $\varphi_I(x) := \varphi(2^k x - j)$ a translated dilate of φ associated to the cube $I := j2^{-k} + 2^{-k}[0,1]^2$. We use the above wavelet decomposition to introduce algorithms for surface compression. Roughly speaking, higher frequency terms from the wavelet decomposition are used where the surface is rough and only low frequency terms are used where the surface is smooth. Convergence results and numerical examples are given. This is joint work with Björn Jawerth and Brad Lucier.

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QUASI-INTERPOLANTS DE TYPE DE SZÀSZ-MIRAKYAN

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Le but de cette note est d'étendre certains résultats établis par P. Sablonnière (Oberwolfach, Février 1989) pour les opérateurs de Bernstein aux opérateurs de Szàsz-Mirakyan. Soit $\varphi(t) = e^{ct}$, $c > 0$, $t \in \mathbb{R}_+$ et
$C_\varphi(\mathbb{R}_+) = \{f \in C(\mathbb{R}_+) : \|f\|_\varphi = \sup \{ |f(t)| / \varphi(t), t \in \mathbb{R}^+ \} < +\infty\}$

<u>L'opérateur de Szasz-Mirakyan</u> S_n de $C_\varphi(\mathbb{R}_+)$ dans $C[a,b]$ est défini par :

$$S_n f(x) = e^{-nx} \sum_{k \geq 0} f(k/n)(n\,x)^k / k\,!$$

C'est un opérateur linéaire positif et un automorphisme de $\mathbb{P}_n$. On peut considérer S_n et $R_n = S_n^{-1}$ comme opérateurs différentiels à coefficients polynomiaux : $S_n = \sum_{j=0}^{n} \beta_j^n D^j$ et $R_n = \sum_{j=0}^{n} \alpha_j^n D^j$, où α_j^n et $\beta_j^n \in \mathbb{P}_j$ sont calculables par récurrence.

On définit, pour $0 \leq k \leq n$, les <u>quasi-interpolants gauches</u> par :

$S_n^{(k)} = \sum_{j=0}^{k} \alpha_j^n D^j S_n$; on vérifie aisément que $S_n^{(k)}$ peut être étendu à $C_\varphi(\mathbb{R}_+)$ et qu'il est exact sur $\mathbb{P}_k$.

En particulier $S_n^{(o)} = S_n^{(1)} = S_n$ et $S_n^{(2)} = S_n - \frac{x}{2n} D^2 S_n$.

On démontre les résultats suivants :

1) Pour $[a,b] \subset \mathbb{R}_+$, il existe $M > 0$ tel que

$$\|S_n^{(2)}\| = \sup \{ \|S_n^{(2)} f\|_{C[a,b]} \ ; \ \|f\|_\varphi \leq 1 \} \leq M$$

pour tout $f \in C_\varphi(\mathbb{R}_+)$ et tout $n \geq 2$.

2) $S_n^{(2)} f$ converge uniformément vers f sur $[a,b]$ pour tout $f \in C_\varphi(\mathbb{R}_+)$.

3) Si $D^3 f$ et $D^4 f$ sont définies :

$$\lim n^2 [S_n^{(2)} f(x) - f(x)] = -\frac{1}{3} x D^3 f(x) - \frac{1}{8} x^2 D^4 f(x)$$

Dimensions of Certain

C^1-Finite Element Spaces

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Consider a volume $\Omega \subset \mathbb{R}^3$ which accepts a partition Δ into right triangular prisms obtained by integer translates of the planes: $\{ue^2+ve^3 : u,v \in \mathbb{R}\}$, $\{ue^1+ve^3 : u,v \in \mathbb{R}\}$, $\{ue^2 + ve^1 : u,v \in \mathbb{R}\}$,$\{u(e^1 - e^2) + ve^3 : u,v \in \mathbb{R}\}$, $\{u(e^1 + e^2) + ve^3 : u,v \in \mathbb{R}\}$, where $e^1 = (1,0,0)$, $e^2 = (0,1,0)$, $e^3 = (0,0,1)$. $S_k^1(\Delta)$ denotes the space of piecewise polynomials of total degree k over Δ which are C^1 continuous on Ω. We determine interalia the dimension of the space $S_k^1(\Delta)$ (c.f. L.L. Schumacker, On the dimension of piecewise polynomials in two variables, in *Multivariate Approximation Theory*, Birkhauser Verlag, 1979, 396-412).

We also determine the dimension of certain C^1-rational finite element spaces of Wachspress type. Construction of convenient basis functions for certain rational C^1-rational finite element spaces has been discussed in (E.L. Wachspress, C^1-rational finite elements, *Math. Comp. & Appl.*, to appear, 1990). These basis elements are linearly independent. However, it seems that the problem of determining the dimension of the spaces of C^1-rational finite elements of Wachspress type has not been studied. We study here the dimension of C^1-rational finite element spaces of Wachspress type with pieces of degree $(k,1)$ for any positive integer $k > 1$.

M.DI NATALE - L.GOTUSSO - R.PAVANI - D.ROUX

STATISTICAL CONTROL OF THE THEORETICAL SMOOTHING PARAMETER OF A METHOD FOR INVERSION OF FOURIER SERIES

A recent regularization method due to L. De Michele, M. Di Natale and D.Roux concerns the ill-posed problem of reconstructing a periodic integrable function f when the sequence of its Fourier coefficients is known. This method is stable also in the case of noisy data. In particular the method gives a good pointwise approximation of f at the Liptschitz points.

Moreover, for large classes of functions, evaluations of the pointwise difference between f and the approximating function $\tilde{f}$ are given. These evaluations depend on the error δ of the data and on the value of the parameter σ which controls the smoothing. They suggest also a standard method of the choice of σ.

It is of interest for the applications to verify if the theoretical value is also a good value of σ for some classical test functions.

We performed this control both from a qualitative and a quantitative point of view. This last investigation was fulfilled by large statistical experiments. The obtained results are presented in some tables and graphics.

A General Method of Treating Degenerate Bezier Patches

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ABSTRACT

Rectangular Bezier patches have been widely used in CAD/CAM applications for free-form surface representations. In most modelling situations, the patches are defined over a topologically rectangular mesh. But this kind of mesh does not allow the modelling of more complex shapes such as spheres and the intersection junction between two cylindrical surfaces. To represent such forms, a solution often used consists of employing patches with some degenerate boundaries. A Bezier patch is called degenerate when its control points are defined in such a way that the geometric features, such as the normal or curvature vectors, cannot be analytically defined everywhere on the patch. Degenerate patches may also result from common modelling situations where the control points are not suitablly defined.

Degenerate patches give rise to two important issues to be resolved before they may be fully used in a general surface modelling system :

1. how to compute the geometric features over the entire surface of a degenerate patch ;
2. what conditions must be satisfied when defining degenerate patches so as to maintain geometric continuities with adjacent patches (degenerate or not).

Previously published solutions address only specific cases. In this paper, we propose a general approach to deal with all degenerate situations in a unified manner. Both the non-rational and rational cases are considered. We first develop a method of computing geometric features over Bezier patches which works for all degenerate cases as well as for the normal case. Then, we analyze the constraints to be satisfied by degenerate patches in order to guarantee geometric continuities with adjacent patches. A geometric interpretation of these constraints is provided. Finally, some practical issues concerning the use of degenerate patches are discussed.

Unifying Rectangular and Triangular Bezier Patches In Free-Form Surface Modelling

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ABSTRACT

The use of rectangular surface patches play a predominante role in most CAD systems. This is certainly due to their conceptual simplicity and ease of use. While they are well adpated to model surfaces of an intrinsic rectangular structure, they are not the 'natural choice' for modelling more complex shapes.

Triangular patches are attractive for surface modelling because they are more suitable for representing any surface shape, and they provide the possibility of more local shape control. An emerging trend in free-form surface modelling is to unify triangular and rectagular models in one single system to better exploit the power of each.

The surface geometric continuity is often desired. When a smooth surface is modelled with a piecewise representation using triangular and rectangular patches, it is crucial to be able to control the geometric continuity between adjacent triangular and/or rectangular patches.

Previous work deals with the simplest case of smooth connection between two adjacent patches, triangular or rectangular in any combination. However, the general smooth connection problem between any number, L, of rectangular and any number, M, of triangular patches meeting at a common corner in any combination has never been investigated. In this paper, we develop a general solution to the G1 smooth connection problem around such a mixed N-patch corner ($N = L + M$) for Bezier patch representations, both non-rational and rational. We deduce the constraints guaranteeing G1 continuity around a mixed patch corner, and show how they are interrelated. Then, we discuss how to satisfy these G1 constraints with control points of triangular or rectangular Bezier patches. The results show that for an N-patch corner, the relationships between these constraints depend on the parity of N, independant of the combination of triangular and rectangular patches. In addition, we analyze the available degrees of freedom in the G1 constraints, which can be used to control the surface shape in the neighborhood of the corner. These results play an important role in the design of a piecewise G1 continuous surface representation method which unifies the use of triangular and rectangular Bezier patches. A number of such solutions are examined.

Conditions for Regular B-spline Curves and Surfaces

Nira Dyn (speaker); David Levin & Itai Yad-Shalom

University of Tel Aviv

Sufficient conditions for the regularity of a B-spline curve are derived in terms of geometrical quantities defined by the control points. These conditions exclude cusps and loops in the curve and are extendable to tensor-product B-spline surfaces. For the quadratic and cubic B-spline curves necessary and sufficient conditions are formulated.

Multiple Knots and Degree Elevation

Geometric Spline Curves

Dipl.-Math. Mathias Eck

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In the past years the interest in geometric continuity of curves (and surfaces) has been growing. In 1985 W. Boehm gave a simple and elegant B-spline like construction of curvature continuous B-spline curves (often called γ-splines). He proceeded with the generalization for curvature and torsion continuous B-splines of degree four. Later in 1989, Eck and Lasser extended Boehm's concept to B-spline curves of general degree with geometric continuity of higher order (Frenet-frame continuity). They pointed especially out the quintic case and showed how to modify these curves to get 'contact of order 4'.

For the usual C^r-continuous B-spline curves it is well known that the choice of multiple knots yields a reduction of continuity at the corresponding point. Using this fact, a degree elevation of usual B-splines can be obtained if the control polygon is chosen appropiately.

In our presentation these algorithms are adapted to the case of geometrically continuous B-spline curves of degree 3, 4 and 5. Some special effects are described as well as some practical applications.

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A Comparison of Curve Approximation Methods

The speaker will examine the specific problem of approximating a given parametric curve with a piecewise-cubic parametric curve. This is an important problem in many areas of CAD/CAM, and several techniques have been proposed for constructing such an approximation. Each cubic segment may be the complete interpolant to points and tangent vectors on the original curve, or we may attempt to gain a better fit by adjusting the lengths of the tangents at the endpoints of the approximating cubic. Likewise, there are different methods for determining how to divide the original curve's parameter space into subintervals of definition for the piecewise-cubic in order to respect tolerance. This may be done on a trial and error basis, recursively subdividing intervals over which error exceeds tolerance, or by doing some a priori analysis on the original curve. And error analysis may be done by comparing curves at the same parameter values, or at the (possibly different) parameters at which they come closest to each other.

Strengths and weaknesses of these different techniques will be explored both in theory and in specific examples of curves which arise commonly in design and manufacture.

APPROXIMATE SOLUTION FOR THE INITIAL VALUE PROBLEM $Y^{(3)}=F(X,Y)$, USING DEFICIENT SPLINE POLYNOMIALS

BY

THARWAT FAWZY and MAGDY AHMED

(Suez-Canal Univ. Ismailia,Egypt)

Abstract: A multistep method for approximating the solution of the initial value problem $y^{(3)}=f(x,y)$ using deficient spline polynomials is presented. The existence and uniqueness of the spline approximant as well as the consistency relations are investigated. The convergence problems is discussed in Part II.

In this part, we introduce the convergence theorems for the method introduced in Part I. We prove that if the spline approximant is of degree m, then the error is $O(h^{m+1-i})$ in $y^{(i)}(x)$ where m=6 and 7 and $i=0(1)m$.

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Splines and Digital Signal Processing

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A discrete spline defined on a non-uniform knot sequence can be generated as the output of a finite impulse response (FIR) digital filter. We first derive the Z-transform of a discrete polynomial spline using the derivative properties of piecewise polynomials. Two filter structures are provided. The first structure is obtained from the factorial basis functions while the second corresponds to the B-spline basis. Both are useful for computation and analysis of discrete splines. The filter inputs are the control vertices and the corresponding knot sequences. The filter outputs are the discrete spline values and all order differences.

We also show that splines can be extremely useful in the design and implementation of general FIR filters. We discuss an efficient procedure for the design of interpolated FIR (IFIR) filters with linear phase. This approach uses a B-spline function defined on a uniform knot sequence as an interpolator for a sparsely sampled frequency selective filter. The frequency selective filter is designed on an optimal subinterval of the normalized frequency domain using the alternation theorem and the Remez exchange algorithm. The technique provides a filter implementation with a minimum number of multiplications.

We provide a generalization of the IFIR filter by showing that an FIR filter whose unit sample response is a spline defined on a non-uniform knot sequence can also be implemented in two stages. The first stage is an MA filter with as many nodes as there are knots. The second stage is an AR filter which performs simple recursive summation. Because only the first stage requires multiplications, the filters can be implemented very efficiently. This is of particular importance in multi-dimensional image processing and machine vision applications. Several examples are provided to illustrate that complexity improvements of greater than an order of magnitude can be obtained with no loss in accuracy.

Rational Curves and Surfaces, Rational Splines.

We give a brief survey about a new description of rational curves and surfaces, resp. rational spline curves and surfaces and related properties. Using Bernstein polynomials or B-splines we control these curves and surfaces by a set of mass vectors (*vecteurs massiques* in french). Some examples are given as well as algorithms derived from the polynomial case. Different properties are developped for instance smoothness conditions or the behavior of the controlling mass polygon by projective or affine transformations.

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Numerically Stable Algorithms in Computational Geometry

Steven Fortune

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Can geometric algorithms be implemented using floating point arithmetic? A geometric algorithm uses a sequence of primitive tests on continuous data to produce a combinatorial output. Computing a primitive exactly may not be feasible because of the large precision required for the calculation. Computing the primitive approximately, say with floating point arithmetic, may invalidate the correctness of the algorithm, since the primitive may give the wrong answer for some inputs.

A geometric algorithm (implemented using floating point arithmetic) is "robust" if (1) it gives the correct answer if all primitives give the correct answer and (2) no matter what floating point rounding occurs, the computed answer is the correct answer for some perturbation of the input. An algorithm is "stable" if it is robust and the required perturbation is small. A small perturbation is one that is a small function of the problem size n and the machine precision epsilon, say O(n epsilon).

Recent research has shown that it is possible to construct provably stable algorithms for some problems in two-dimensional computational geometry. These algorithms include computing convex hulls, maintaining triangulations of point sets, and computing arrangements of lines. The proofs of stability combine techniques from error analysis, graph theory, and algorithm analysis.

On The Power of *A Posteriori* Estimation for Function Approximation

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A posteriori estimation, or the use of error criteria and adaptation criteria in numerical algorithms, have not received much mathematical exploration in the past. Upon derivation of such a criterion, it is usually analyzed only experimentally, i.e., tested on a set of examples.

We show that, rather surprisingly, carefully structured criteria for *a posteriori* estimation can enable function approximations to possess interesting and useful mathematical properties which they do not possess *a priori*. For example, we show that a simple adaptive procedure can produce a piecewise local polynomial approximation (e.g., piecewise Lagrange polynomial interpolation) which approximates the spline interpolation. More precisely, a simple criterion for adaptive point allocation can produce a piecewise local polynomial approximation that is close to the spline interpolation to the same function on the same mesh, to within any prescribed positive tolerance.

The relation of this approach to probabilistic (Bayesian) estimation is also discussed.

The Algebraic Structure of Sets of Functions with Fixed Connection Matrices

Ronald N. Goldman

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Different types of geometric continuity for parametric curves can be specified in terms of certain distinct classes of connection matrices. We look at the algebraic structure of the set of all scalar valued functions with a fixed connection matrix. We shall show that this set is closed under either multiplication or division if and only if the connection matrix is a reparametrization matrix. We conclude that reparametrization is the most general form of geometric continuity for which the shape parameters remain invariant under projection. We go on to show that Frenet frame continuity is also preserved under projection, even though the shape parameters change.

Fast Knot Insertion

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Abstract: Standard knot insertion schemes for B-spline curves, such as Boehm's algorithm or the Oslo algorithm, are based on convex combinations and are closely related to the de Boor evaluation algorithm. Here we present an new knot insertion scheme, akin to forward differencing, but numerically more stable, which, when more than just a few knots are inserted, is faster than the standard knot insertion techniques. Unlike the standard knot insertion algorithms, the fast knot insertion scheme is related not to evaluation but rather to differentiation. We shall derive the fast knot insertion algorithm using blossoming and then show how it is related to other classical algorithms for differentiation and integration of B-spline curves.

Best approximation of circle segments by Bezier curves

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Abstract. In a recent paper [1], some high accuracy schemes for approximating circular segments with cubic Bezier curves were given. In this talk we will show that these schemes are best possible in the sense that the squared distance to the circle is minimized with respect to the class of approximations in consideration. We may also discuss extensions to polynomial degrees higher than three.

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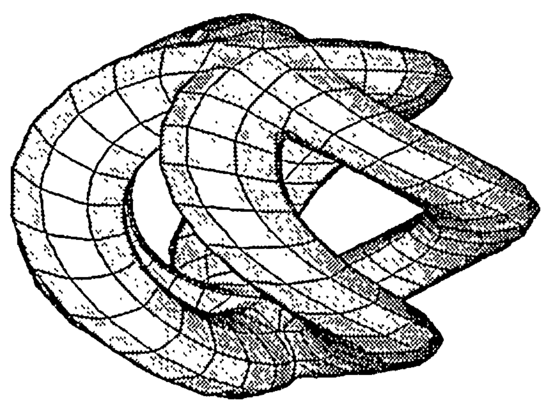
* speaker

G-Splines

K. Höllig

Abstract. We describe a new type of splines which allows to model smooth surfaces with arbitrary topological structure. In particular it is possible to incorporate "singular vertices" into a network of tensor product B-spline surfaces. The key observation is that, with an appropriate choice of the smoothness constraints, the class of admissible parametrizations forms a linear space and therefore standard tools from linear algebra are applicable. In particular the construction of bases, interpolation and blending schemes is reduced to finite dimensional matrix problems which are independent of the global structure of the mesh. This is illustrated with several examples.



g-spline surface

Keywords: splines, computer-aided design, geometric continuity

On Schoenberg's Exponential Euler Spline Curves

Kurt Jetter

(joint work with S.D. Riemenschneider and N. Sivakumar)

Given the univariate cardinal B-spline M_n, of order $n \in \mathbb{N}$, the exponential Euler spline is defined for any $0 \neq z \in \mathbb{C}$ by

$$\Phi_n(x; z) = \sum_{j=-\infty}^{\infty} z^j M_n(x - j), \; x \in \mathbb{R}.$$

For $z = e^{iu}$ we also write

$$\varphi_n(x, u) := \Phi_n(x; e^{iu}).$$

I. J. Schoenberg studied these functions in several papers, last not least in his CBMS-lectures on "Cardinal Spline Interpolation". We study these functions in more detail, and show

- Properties of the curves $x \mapsto \varphi_n(x, u)$;
- Application to cardinal interpolation with shifted B-splines;
- Monotonicity of $\varphi_n(x, u)$ with respect to the parameters x, u, or n.

We also point to how Smith and Ward's "metric condition" may be derived from our analysis.

Surface Compression and Quasi-interpolants

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This work deals with surface compression by using quasi-interpolants. Let ϕ be a compactly supported function on $\mathbb{R}^s$. Together with ϕ we have its dyadic dilates $\phi(2^k\cdot)$, $k \in \mathbb{Z}$, and their translates $\phi_I := \phi(2^k \cdot -j)$, $j \in \mathbb{Z}^s$. Here, we index these functions by the dyadic cube $I = j\,2^{-k} + 2^{-k}\Omega$ with $\Omega := [0,1)^s$. We shall use the notation $\mathcal{D}_k$ to denote the set of dyadic cubes I whose sidelength $\ell(I)$ is 2^{-k} and by $\mathcal{D}$ the union of the $\mathcal{D}_k$, $k \in \mathbb{Z}$. In order to compress a surface, DeVore, Jawerth and Lucier decomposed it into wavelets:

$$f = \sum_{I \in \mathcal{D}} a_I \phi_I,$$

and chose a finite expansion among all such finite sums with at most n terms. Let Σ_n denote the nonlinear manifold of all such functions with at most n of the coefficients $a_I \neq 0$. Earlier DeVore, Jawerth and Popov characterized functions with a given degree of nonlinear approximation from Σ_n under the following three assumptions about ϕ:

(i) ϕ satisfies the Strang-Fix conditions of a certain order;

(ii) each given ϕ_I can be written as a finite linear combination of the functions ϕ_I at the next dyadic level;

(iii) the multiinteger translates of ϕ are locally linearly independent.

A close look into their assumptions reveals that the first and second assumptions are essential. However, the third assumption restricts seriously the choice of ϕ. For example, the well-known Zwart element, which is a box spline with four directions, does not satisfy the linear independence requirement. In this talk we shall demonstrate that the third assumption could be removed, and, under assumptions (i) and (ii), characterize functions with a given degree of nonlinear approximation from Σ_n.

Elastica and Minimal Energy Splines

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A spline is a long thin strip of metal, wood, or plastic bent elastically to fair a smooth curve. The equation of spline curve (elastica) can be obtained by minimizing its strain energy which is proportional to the intergal of the square of the curvature taken along the elastica. We call such a spline "minimal energy spline". When the deformation of an elastica is small, one may drop the high order term for the curvature, and obtain the celebrated cubic spline.

A minimal energy spline has a prescribed length together with the constraints of arbitrary-angles or zero-curvatures at the end-points. The zero-curvatures at both end-points are corresponding to the natural boundary conditions. The minimal energy splines are curvature continuous curves. Each segment of a minimal energy spline is infinitely smooth and has linear curvature relationship.

Some results of plane minimal energy spline curves are readily extended to space curves. A linear curvature property for space minimal energy splines does not appear to hold.

Minimal Cost Approximation of Functions

from Noisy Information *

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We find the minimal information cost $mc(\epsilon)$ of obtaining en ϵ approximation of a function in s variables with r continuous derivatives, assuming that information consists of its perturbed values. We determine the optimal (up to a constant) number of these values, optimal precisions with which they should be obtained, as well as the best information and algorithm. In particular, if information cost is measured by a number of binary bits required for representing information, then $mc(\epsilon)$ is proportional to $\epsilon^{-\frac{s}{r}} log_2(\frac{1}{\epsilon})$.

* Joint results with I. Plaskota from the University of Warsaw

MAKING A CLEAN SWEEP (SURFACE)

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A *swept surface* S(u,v) is constructed by sweeping a profile curve P(u)=(x(u),y(u)) along a rail curve R(v). For every constant v, S(u,v) is a copy of P(u) in a plane normal to R(v). The result depends on the choice of local coordinate basis {A(v),B(v)} for that plane. The surface is then defined as S(u,v) = R(v) + x(u)A(v) + y(u)B(v). In general, S is neither polynomial nor rational, even when P and R are polynomial.

Published methods (e.g. [1],[2],[3]) for constructing swept surfaces have applied surface approximation methods for fitting the theoretical surface with a standard piecewise polynomial (or rational) surface. The local coordinate basis has been based on the Frenet frame of R. Two problems have limited the functionality of these methods:

* The Frenet frame tends to twist about the rail in a bad way, resulting in a bad surface ([3]).

* It is difficult to obtain a tight fit with surface approximation methods. This is noticeable particularly at the surface boundary, when an adjacent surface is to be continued ([2]).

This paper presents a new approach to the construction of swept surfaces, completely eliminating these problems:

* Approximation is done at the curve level, where tighter fitting is possible, fitting A(v) and B(v). The theoretical surface is thus fitted with a NURBS surface within any practical tolerance.

* A different frame, other than Frenet's, defines our local coordinate basis. It doesn't twist about the rail, hence the surface is torsion-free in the following sense: for a fixed u, the constant parameter curve S(u,v) and the rail curve R(v) have parallel tangents at every v.

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A Parallel algorithm for surface/plane intersection*

A. Kaufmann**

For a surface described in the Bézier/Bernstein representation, the surface/surface intersection using the subdivision algorithm is defined as a parallel process. Thus, the performances of the algorithms are interesting to study on a parallel computer. As the surface/plane intersection introduces several difficulties of surface/surface intersection, in a more simple case, we reduce the study to this problem.

Parallel algorithms for a given problem are quite dependent on the computer structure. Thus, we reduce this study to distributed memory computers. I.e.each processor has its own memory, and there is no shared memory. So, the computer must have an interconnection network between its processors. The particular network we use is a hypercube, or any network (as a ring) deduced from a hypercube.

The aim of this work is to find *intersection curves,* approximated by intersection of polyhedra in the subdivision process. Thus, we must keep links between polyhedra in order to build intersection curves. Two logical data structure are possible: quadtree and neighbouring links. For the implemented algorithm, we choose the second structure with a NEWS (North-East-West-South) definition. Moreover, the set of polyhedra is considered, in the parameter-space, as a sparse matrix with column storage. Hence, we split by polyhedra columns. Thus, a given column can only be splitted in two columns. The problem can be described by two binary trees:

1- an up-down tree for the subdivision,

2- a bottom-top tree for the merging of each piecewise local curve. The piecewise local curves are curves obtained by the intersection of columns (which are located in a given processor) and the plane.

Hence, in the distributed algorithm, we must find a compromise between the subdivision tree, which can use more and more processors, and the merging tree which grows with the number of processors.

To conclude, we introduce results of a first distributed algorithm for several examples. And we introduce another possible algorithm (hopefully better) for surface/plane intersection.

* A. Kaufmann. *A parallel algorithm for surface/plane intersection* Research Report RR-794-M Nov. 1989. IMAG Univ. J. Fourier. Grenoble (France).

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Fillets and Intersections of Surfaces Defined by Rolling Balls

In CAD/CAM systems several separate surfaces are needed to describe something like a specific part of a car body. To connect two surfaces, two methods are frequently used:

1. Trimming with surface/ surface intersection curves
2. Fillets between two surfaces defined by rolling balls.

The paper describes an algorithm which allows to calculate such fillets. The case of intersection curves can be achieved by setting the ball radius $r=0$ and using the same algorithm as for fillets.

All surfaces are trimmed B-spline surfaces. The intersection curves and the fillet surfaces are approximated by B-splines again.

The tangent directions of the boundary curves and the middle curve of the ball fillet are derived by means of differential geometry using the fundamental forms of the two surfaces. To guarantee a result which is exact down to a given tolerance, we describe an estimation algorithm for the length of the calculated spline segments.

The presented algorithm is implemented in the CAD/CAM system SYRKO used for car body design and maufacturing at Mercedes-Benz. Hence, the theory is supplemented by practical experience and examples.

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Construction of exponential tension B-splines of arbitrary order

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We consider the following generalizations of Schweikert's tension splines, namely smooth functions piecewise in span$\{\sinh(\rho_i x), \cosh(\rho_i x), 1, x, \ldots, x^{k-3}\}$, where k is the order. The tension parameter ρ_i may vary from subinterval to subinterval.

An algorithm for the construction of B-splines for these spaces is given. The algorithm is based upon a differentiation formula for general exponential tension splines. This differentiation formula yields a recurrence relation between those B-spline coefficients for which linear combinations with B-splines equal the different powers of x. The result is explicit expressions for such splines on each subinterval. As a corollary we obtain explicit formulas for polynomial splines on each subinterval by letting all tension parameters tend to zero.

We show that when the tension parameters tend to zero then the tension B-splines of order k tend to the polynomial B-splines of the same order, but when the tension parameters tend to infinity, the tension B-splines of order k tend to the polynomial B-splines of order $k-2$ situated centrally on the B-spline support, and tend to zero on the first and last subinterval.

Finally, we have a generalization of Boehm's formula for inserting one knot.

Probabilistic and Worst Case Complexity in Function Approximation

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We study the complexity (of function approximation and other linear problems) in a probabilistic setting, where we are given a preassigned probability $1 - \delta$ with which we guarantee that the approximation error is smaller than ϵ. A basic question is whether this is really different from the standard worst case setting, where the accuracy ϵ must be attained for *all* functions, not just most. We show that for small δ, one is dealing with the standard worst case setting, in a number of senses.

Precisely, suppose we have a probablity measure μ on the Banach function space F, and our probabilistic tolerances are in terms of μ: we demand our approximation be accurate within ϵ for a "large" set of functions, namely, a class whose μ-probability exceeds $1 - \delta$. We show that as $\delta \to 0$ the complexity of approximation converges to the standard worst case approximation picture. We also show that for linear approximation problems, measure 0 sets in function space are inconsequential in terms of the complexity of approximating the most intractable functions, for all "reasonable" measures μ.

More abstractly, let S be a linear mapping between two linear (Banach) spaces F and G (S is the identity in the case of function approximation). Let $N : F \to G$ be an information operator (i.e., a finite rank operator giving, say, the values of a function $f \in F$ at a finite set of points). Let μ be a measure on a bounded convex set in F, giving a probability distribution over functions to be approximated. We study (probabilistically) the complexity of approximating S with a composition of the form $\phi \circ N$ with N of finite rank. Here δ again is a small probability with which we are allowed to break an approximation tolerance ϵ. We investigate the $\delta \to 0$ behavior of the parameters of the problem (e.g., error of approximation, complexity), in particular their relation to those of the worst case problem.

Heinrich has recently considered the problem of function approximation in Sobolev spaces from partial information, and showed that, if an a priori Gaussian probability distribution is assumed on functions, the probabilistic complexity of approximation essentially converges to the worst case picture. We generalize this to the extent that arbitrary Hilbert spaces of functions can be considered, and the problem need not be identification of a function, but may involve computing an arbitrary linear functional or operator.

Linearity of Algorithms and a Result of Ando

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and

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In this paper, we prove that a Hilbert structure is necessary as well as sufficient for linearity of the following classes of Banach space approximation algorithms: spline, interpolatory, strongly optimal, and almost strongly optimal. In the context of information-based complexity, this provides a converse to the well-known result that a Hilbert structure is sufficient for such linearity properties. Furthermore, we point out a theorem of Ando and its application to establishing necessary and sufficient conditions for linearity of algorithms on L^p spaces.

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Rational Approximation of the Step,

Filter and Impulse Functions *

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Modern design of electronic filters, such as low pass, high-pass and band-pass filters includes the construction of a rational function which satisfies the desired specifications for cut-off frequencies, pass-band gain, transition band-width and stop-band attenuation. Thus rational function approximation to the χ^*-function, ($\chi^*(x) = 1$ if $-1 < x < 1$, 0 if $|x| > 1$) and the *sgn* function ($sgn(x) = 1$ if $x > 0$, -1 if $x < 0$) are needed. Elementary analytical methods lead to rational functions, such as $(\epsilon + t_n(x))^{-1}$ or $(1 + x^{2n})^{-1}$ where $t_n(x)$ is the n-th degree Chebyshev polynomial. More advanced methods employ elliptic functions.

We aim to derive new rational function approximations for the Heaviside, the filter and the impulse functions. Our motivation of these derivations is the simplicity of the results: the formulas depend on a single parameter N which determines the degree and accuracy. While our approximations do not yield the optimal ones, they are close to optimal, and they offer an advantage of simplicity of functional expression.

* Joint results with Y. Ikebe and F. Stenger

A PSEUDO-CUBIC WEIGHTED SPLINE CAN BE C^2 OR G^2

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The classical weighted spline introduced by Ph. Cinquin (1981), (see also K. Salkauskas (1984) and T.A. Foley (1986)) consists in minimizing $\int_a^b w(t)\,(x''(t))^2\,dt$ under the conditions $x(t_i) = y_i$, $i=1,\ldots,n$, where the function w is piecewise constant on the subdivision $a<t_1<t_2<\ldots<t_n<b$. The solution is a cubic spline but it is not C^2.

We consider here the minimization of $\int_a^b \frac{(x''(t))^2}{q(t)}\,dt$ where q is continuous and piecewise polynomial of degree one on the subdivision. The values $q_i = q(t_i)$ act as form parameters. The solution is a C^2 quartic spline but surprisingly, it has in fact all advantages of the cubic spline. Namely:

- computing the solution leads to a tri-diagonal linear system,
- computing the corresponding smoothing spline leads to a block 2x2 tri-diagonal system,
- the associated B-spline is based on 4 intervals of the subdivision (like the classical cubic B-spline).

The properties of this new weighted spline will be developped and its efficiency for interpolating, smoothing or designing will be illustrated on a selection of examples.

Finally, a weighted spline with G^2 continuity is described, which has 3 form parameters at each knot.

2-D Conservative Approximation by Rational B-Splines

B.I. Kvasov & S.A. Yatsenko

Let the initial data be defined as the set of points, ordered along the not intersecting, possibly curvilinear cross-sections of a three dimensional body. In the paper the Gordon's type algorithm [1] is proposed for the construction on the basis of data of the approximating surface of the C^2 class with the preservation of geometrical features of the initial data (convexity, monotonicity, etc.) along the finite system of curvilinear coordinate lines, forming a regular mesh on the surface, topologically equivalent to rectangular one.

For the construction of the curves along the initial cross-sections the algorithm of conservative interpolation by rational splines [2] is used The storage and the computing of rational splines is realized by means of their representation through the B-splines. To obtain a 2-D spline in the orthogonal direction the system of rational local approximating splines, generalizing the standard local approximation by cubic splines, is constructed. The capacities of the algorithm are illustrated by test 3-D examples.

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ELEMENTS FINIS COMPOSITES DE TYPE PS DE CLASSE C^r

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Soit τ une triangulation d'un domaine polygonal borné Ω de $\mathbb{R}^2$, dont l'ensemble des sommets est $\mathbb{A} = \{A_i ; i \in I\}$ avec $I \subset \mathbb{N}$.

Soit $\mathbb{P}_n^r(\Omega,\tau) = \{ f \in C^r(\Omega) : f|t \in \mathbb{P}_n \ \forall\, t \in \tau \}$, où $\mathbb{P}_n$ est l'éspace des polynômes à deux variables de degré total inférieur ou égal à n.On considère le problème d'Hermite suivant :

$H^r(\mathbb{A},u) = \{$ trouver $v \in C^r(\Omega) : D^\alpha v\,(A_i) = D^\alpha u\,(A_i)\ ;\ i \in I$ et $|\alpha| \le r \}$,où u est une fonction assez régulière donnée.

Ce problème admet une solution dans $\mathbb{P}_n^r(\Omega,\tau)$ ssi $n \ge 4r+1$(Zenisěk [3]).

Soit τ_6 la sous-triangulation de τ obtenue en subdivisant chaque triangle $t \in \tau$ en 6 micro-triangles suivant le procédé de Powell- Sabin [1] et :

$\mathbb{P}_n^r(\Omega,\tau_6) = \{ f \in C^r(\Omega) : f|t \in \mathbb{P}_n \ \forall\, t \in \tau_6 \}$

Sablonnière [2] a construit une solution de $H^r(\mathbb{A},u)$ dans $\mathbb{P}^r_{3r-1}(\Omega,\tau_6)$.Si τ est en plus une triangulation régulière de type 1 (réseau tridirectionnel) et si chaque triangle $t \in \tau$ est subdivisé en 6 micro-triangles par ses médianes alors nous avons le résultat suivant :

THEOREME : il y a une solution de $H^r(\mathbb{A},u)$ dans $\mathbb{P}_n^r(\Omega,\tau_6)$ pour $n = 2r+1$ si r est pair et $n = 2r$ si r est impair et les degrés des polynômes sont minimaux.

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The Strang-Fix conditions for functions with non-compact support

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In 1969 Strang and Fix gave a series of results which characterised approximation orders from certain finite element subspaces. After some considerable time, it was shown that their result had certain deficiencies, which were finally made good by de Boor and Jia.

Let the Sobolev space $W_p^k(\mathbb{R}^m)$ be defined to be the set of all functions u for which the quantity

$$\|u\|_{k,p} = \sum_{j \le k} |u|_{j,p} \quad : \quad |u|_{j,p} = \sum_{|\alpha|=j} \|D^\alpha u\|_p$$

is finite. Here D^α denotes the usual multivariate derivative. Let Φ be a finite subset of $W_p^0(\mathbb{R}^m)$ consisting of functions of compact support. Then Φ provides 'local L_p-approximation of order k' if for each $u \in W_p^k(\mathbb{R}^m)$ there exist weights c_ϕ^h so that

$$\left\| u - \sigma_h \sum_{\phi \in \Phi} \phi * c_\phi^h \right\|_p \le const\ h^k |u|_{k,p}$$

and

$$c_\phi^h(j) = 0 \quad \text{whenever } \operatorname{dist}(jh, \operatorname{supp} u) > r.$$

Here const and r are independent of h and u. In addition, the operator σ_h is the usual dilation, $(\sigma_h f)(t) = f(t/h)$, $t \in \mathbb{R}^m$.

We shall present two things. Firstly, a slight rephrasing of the second of these 'de Boor-Jia' conditions, and secondly, a version of this theorem which applies to functions which do not have compact support

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Frame splines: Moving Orthogonal Frames along a B-spline Curve

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Abstract

In this article we will present a new type of B-spline curves, which can be used for approximating positional data together with additional data. We use orientations as additional data, but other information (e.g. collor data) could also be used.

A frame is a description of a location with the orientation of the local coordinate system in three-space. For tool path generation along a trajectory orthogonal frames have to be calculated at the working points. When the tool path has to be highly accurate this results in large amounts of data.

Instead of determining all these points we can use B-spline techniques to create a spline on control frames. These Frame splines are 12-dimensional B-splines defined over orthogonal coordinate frames. However not every set of orthogonal control frames results in orthogonal frames along the positional curve.

We have developed a method for generating Frame splines from 3D B-splines by degree elevation and knot insertion. Control frames can also be generated for a predescribed frame course (e.g. tangential to the curve).

When the control frames are extended with tolerance information the tool paths can vary in accuracy when necessary.

We will show that Frame spline techniques can easily be extended to surfaces.

Transform Image Coding
Through Wavelet Decompositions

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In this talk, reporting joint work with Ronald A. DeVore and Björn Jawerth, we present a unified mathematical analysis of the error for transform coding methods for image compression. We analyze methods that use orthogonal wavelet transforms, pyramid encoding, dyadic box splines, etc. Based on this analysis we propose new methods for transform coding that are optimal within a particular class of methods and within a certain mathematical framework. We will focus on the following questions:

If a compression algorithm introduces differences between the original image and the compressed image, how should one measure the difference between the two?

Which images can be compressed well, or how can one judge the smoothness of images?

Computational examples will be given of several methods amenable to our analysis.

Approximation and Interpolation by Translates

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Consider approximation and/or interpolation by means of functions of the form

$$s(x) = p(x) + \sum_{j=1}^{N} a_j h(x - x_j) \tag{1}$$

where the set $\mathcal{A} = \{x_1, \dots, x_N : N \leq \infty\}$ is a collection of distinct points in R^n, h is some prescribed function on R^n, and the coefficients a_j and the polynomial p are chosen appropriately. Such methods have attracted wide attention recently.

We address certain theoretical questions naturally associated with approximants and interpolants of form (1) and related forms.

For example, when h is conditionally positive definite interpolants of form (1) are solutions to a variational problem and consequently enjoy appropriate properties. In particular, if $h(x) = -\sqrt{1 + |x|^2}$, $\mathcal{A}$ is the intersection of a dilated integer lattice with the unit ball B centered at the origin, i.e. $\mathcal{A} = (dZ^n) \cap B$, and s interpolates f on $\mathcal{A}$ then for appropriate f and all x in B

$$|f(x) - s(x)| \leq \lambda^{c/d} \tag{2}$$

where $0 < \lambda < 1$ and $c > 0$ are constants independent of d. The class of f's for which (2) holds is too complicated to describe here; suffice it to say that it includes entire functions of exponential type. In the general case $\mathcal{A}$ need not be "regularly spaced" and B can be taken to be a fairly general open set.

This and/or other related material will be presented.

Basis Functions for Rational Continuity

Dinesh Manocha & Brian A. Barsky (speaker)

University of California at Berkeley

The parametric or geometric continuity of a rational polynomial curve has often been obtained by requiring the homogeneous polynomial curve associated with the rational curve to possess parametric or geometric continuity, respectively. Recently this approach has been shown overly restrictive. We make use of the necessary and sufficient conditions of rational parametric continuity for defining basis functions for the homogeneous representation of a rational curve.

These functions are represented in terms of shape parameters of rational continuity, which are introduced due to these exact conditions. The shape parameters may be varied globally, affecting the entire curve, or modified locally thereby affecting only a few segments. Moreover, the local parameters can be represented as continuous or discrete functions. Based on these properties, we introduce three classes of basis functions which can be used for the homogeneous representation of rational parametric curves.

Detecting Cusps and Inflection Points in Curves

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Many algorithms in computer graphics, geometric modeling, robotics and vision use parametric curves for object representation. For computational reasons a polynomial or rational parametrization is used. It is often desirable to analyze these curves for undesirable features like cusps and inflection points. Previously known methods to analyze such features are limited to cubic curves and in many cases are for planar curves only. We present a general purpose method to detect cusps in polynomial or rational space curves of arbitrary degree. If a curve has no cusps in its domain of definition, it has a regular parametrization and our algorithm computes that.

Geometrically, a cusp is a discontinuity in the unit tangent vector. Since the curve is everywhere differentiable, the discontinuity in the unit tangent vector occurs only if the first derivative vector vanishes. The vanishing of the first derivative vector is necessary but not a sufficient condition for the existence of cusps. We show that if a curve has a proper parametrization then the vanishing of the first derivative vector is necessary as well as a sufficient condition for the existence of cusps. We present a simple algorithm to compute the proper parametrization of an improperly parametrized polynomial curve and reduce the problem of detecting cusps in a rational curve to that of detecting cusps in a polynomial curve. Finally, we use the regular parametrizations to analyze for inflection Points.

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2D AND 3D SEGMENTATION BASED ON DIFFERENTIAL EQUATIONS AND "SPLINE SNAKES"

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In 2D or 3D image analysis, segmentation is often considered as a discrete problem: the primarily continuous edge detectors are then discretized. A continuous modelling by bi- or tri-cubic spline functions provides a stable evaluation of differential operators such as gradient or laplacian. Moreover tracking the surface of an object can be proved to be equivalent to finding a stable manifold of a system of differential equations. Finding this manifold turns out to be a particular case of surface intersection problems. A similar method can be applied to detect particular points of the surface such as local extrema.

The major advantages of this method are as follows: segmentation and surface tracking are obtained simultaneously, complex structures in which branching problems may occur can be described, and the information brought by the segmentation step allows to model the surface easily with surface patches. This algorithm was successfully tested on 3D medical images provided by Computer Tomography and Magnetic Resonance Imaging.

A possible extension of our method is to include a model of the object of interest. This model is described with spline functions and will be iteratively modified to fit with the real object. These deformations are the result of a strengh field originating from the initial image. Each point of the model is submitted to a strengh that is transferred to the control vertices of the model. Thus a differential equation describing the evolution from the initial model to the real object can be defined and solved. This approach is very similar to the so-called "snakes" method (hence the name of "spline snakes") but the model is described by a limited number of control vertices. Besides, this method can easily be modified for an adaptative approximation of the shape and applied to 2D or 3D segmentation.

Estimation Theory for Dynamic Systems with Bounded Uncertainty: a Survey

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Many different problems such as linear and nonlinear regressions, parameter and state estimation of dynamic systems, state space and time series prediction, interpolation,smoothing, function approximation have a common general structure that here is referred to as *generalized estimation problem.* In all these problems one has to evaluate some unknown variable using available data (often obtained by measurements on a real process). Available data are always associated with some uncertainty and it is necessary to evaluate how this uncertainty affects the estimated variables.

Obviously the solution of the problem depends on the type of assumptions that are made on the uncertainty. The cases most investigated so far are unquestionably related to the assumption that uncertainty is given by an additive random noise with (partially) known probabilty density function (pdf). However, in many situations the very random nature of uncertainty may be questionable. For example the real process generating the actual data may be very complex (large scale, nonlinear, time varying) so that only simplified models can be practically used in the estimation process. Then the residuals of the estimated model have a component due to deterministic structural errors, and treating them as purely random variables may lead to very disappointing results.

An interesting alternative approach, referred to as *set membership or Unknown But Bounded (UBB)* error description, has been pioneered by the work of Witsenhusen and Schweppe in the late 60's. In this approach uncertainty is described by an additive noise which is known only to have given upper and lower bounds. Motivation for this approach is the fact that in many practical cases the UBB error description is more realistic and less demanding than the statistical one. However despite the appeal of its features, the UBB approach has not yet reached a wide diffusion. An important reason for this is certainly the fact that until the first 80's reasonable results and algorithms had been obtained only for uncertainty bounds of integral type (mainly l_2), while in practical applications pointwise bounds (l_∞) are mainly of interest.

Real advances have been obtained in the last few years for the pointwise bounds case, leading to theoretical results and algorithms which can be properly applied to practical problems where the use of statistical techniques is questionable.

The purpose of this talk is to review these results and to present them in a unified framework in order to contribute to a better understanding of the present state of the art in the field and to stimulate further basic and applied researches.

IMAGES LIKE SURFACES: PARALLEL LEAST SQUARES APPROXIMATION METHODS

L. Bacchelli Montefusco **, C. Guerrini *, L. Puccio**

Parallel methods for approximating surfaces [1] may reach high efficiency when used for dealing with images. In fact, the numerical handling of images of practical size is not possible with traditional scalar computers, due to the very large dimensions of the problems involved. Parallel methods are thus required which take advantage of the different architectures of modern multiprocessors in order to obtain good distribution of the work-load among the processors.

In this paper we have considered the "continuous-object discrete-image" model of the image restoration problem [2] with completely general assumptions concerning the blurring function, and have sought an approximate solution of the integral equation

$$g_{i,j} = \iint_{-\infty}^{+\infty} h_{i,j}(\xi,\mu)\, f(\xi,\mu)\, d\xi\, d\mu + e_{i,j} \quad , i,j = 1,2,..N \qquad (1)$$

in the finite dimensional polynomial spline space $S^k{}_{2k-1}[\Delta_1 x \Delta_2]$ by means of regularized least-square method [3]. The numerical solution of this problem is obtained with two different strategies, taking into account the available architectures: shared memory and distributed memory multiprocessors. For shared resources we have exploited parallelism by evaluating the matrix of the linear system arising from a suitable discretization of (1) in parallel, dividing it into blocks of columns of comparable sizes, and then adopting parallel techniques for its solution. For distributed memory parallel machines we propose a domain decomposition method which greatly reduces communication and synchronization costs and allows realization of a very efficient coarse-grained asynchronous parallel algorithm. The methods have been tested on a CRAY Y-MP/432 and an Intel iPSC/2, respectively, and timings and efficiency results have been given for several real images.

**KEYWORDS : image restoration, polynomial splines, domain decomposition, shared-distributed memory, Hypercube.

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Local Kriging Interpolation:

Application to Scattered Data on the Sphere

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The purpose of this paper is to show how the C^m local kriging interpolation scheme presented by Montès (1989) may be easily applied to scattered data on the surface of the sphere if distances are taken on geodesic arcs. The main steps of the method are: (a) triangulation of the data point locations on the sphere, (b) at each node, selection of an adjacency level and building of the subset of data points adjacent to it, (c) kriging of the basis functions defined by the data subsets, (d) selection of a suitable weighting function, (e) search of the element (spherical triangle) containing the point p to be interpolated, (f) kriging of the three basis functions related to the nodes of the element containing p, (g) weighting the kriged values at p to obtain the desired interpolation value at p. The continuity of the interpolated surface depends on the continuity of the generalized covariance and that of the weighting function. Some preliminary results obtained with this method are shown. The application of this method to the interpolation of scattered data on non-spherical surfaces is possible. The main advantage of this method with respect to the existing ones is that the generation of gradients as additional data is not necessary, but, if gradient exist, they may be used in the kriging process.

A Procedure for Determining Starting Points for a Surface/Surface Intersection Algorithm

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Iterative methods for calculating the intersection curve(s) of two given parametric patches require a pair of initial points (one point on each surface) lying in a sufficiently small neighborhood of the curve.

In this talk we present a method for computing suitable initial points for such algorithms. Arbitrarily close points can be obtained by defining a sequence P_n of sets of points on the surface satisfying

$$\lim_{n \to \infty} \{ \max_{p \in P_n} \min_{x \in C} ||p - x|| \} = 0,$$

where C denotes the set of all intersection points. We illustrate this with several numerical examples.

Positivity Preserving Interpolation
with Quadratic Splines

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A scheme for interpolation to non-negative data with nonnegative C^1 quadratic splines is discussed. A generalization of this to the bivariate setting is then outlined, where the C^1 piecewise quadratics are taken over the Powell-Sabin split triangulation. This makes use of our necessary and sufficient condition for the nonnegativity of a bivariate quadratic function on a triangle in terms of its Bézier ordinates.

Discrete Simplex Splines

M. Neamtu

Abstract

A convenient way of approaching the problem of subdividing polynomial splines is to define the so called *discrete splines*. This has been successfully done in the univariate case and also in the multivariate case for polynomial splines on uniform grids - so called *box splines*. In this talk we elaborate on how one could define discrete analogs of *simplex splines* i.e., splines that can be exploited on non-uniform grid partitions.

ABSTRACT

REGRESSION OF DATA ON A PIECEWISE CONSTANT SURFACE

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Preliminary examination of satellite images of Arctic sea ice indicates that, in many cases, the ice sheet moves in discrete blocks, interacting at their boundaries in shear, convergence and divergence. Automatic pattern recognition techniques are used to geometrically match temporally proximate pairs of images and so derive the vector displacement field. The numerical analysis problem is then to fit a piecewise constant model to this two dimensional data.

A variety of techniques for solving this problem are discussed. Geometric image analysis methods can be used as well as boundary parameterization and optimization of a residual norm. Procedures are evaluated in terms of performance and compatibility with the data.

CURVE AND SURFACE SMOOTHING

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In computer aided geometric design, we often face the fact that although the overall shape of a parametrically-defined curve (or surface) is satisfactory, a closer look reveals unwanted oscillations. The interactive repositioning of control points is the usual tool to achieve a smoother shape but it is both tedious and unreliable. An automatic non-interactive method for curve smoothing based on filtering techniques is presented. A simple linear low pass filter, defined by a fixed interval width in the parameter space, is applied to the curve.

It is shown that filtered polynomial spline curves are also polynomial spline curves with a possibly different knot sequence. The filtered curve is smoother than the original one, has a higher order of continuity and its order is one unit higher than the starting curve. A very simple algorithm is derived to evaluate the control points of the filtered curve as convex combinations of the control points of the original one. The evaluation of the coefficients of the convex combinations is done by applying the Oslo knot insertion algorithm. No knot insertion procedure is needed in the usual case of a uniformly spaced knot sequence, if the filter window width is multiple of the knot interval.

The curve filtering procedure is the base to filter surfaces defined by tensorial product. Examples of the application of the method to ship hull design are presented. Restrictions and possible extensions of the proposed method are discussed.

On visually smooth interpolation schemes in R^3

P.R.Pfluger, Univ. of Amsterdam & M.Neamtu, Univ.of Twente, The Netherlands

We consider the problem to construct a surface which interpolates to positional data $X = \{x_i\}_{i=1}^{K}$ in R^3. We will assume that a triangulation of X is known (we then have a piecewise linear interpolating surface) and that the *direction* and the *orientation* of the normal vectors at the points x_i are given. In the case we have no information on the normal vectors we can use estimates from a least squares fit through the given points $x_i \in X$.

The surface will be constructed locally on every triangle of the given triangulation, such that the global surface has a continously varying normal (with orientation) of unit length. The surface will be visually C^1 continuious.

The method depends on the choice of a map M which maps a standard triangle in R^2 on the local patches in R^3. The vertices of the patches are given by the triangulation of X. First we investigate conditions on the map M which guarantee overall visual smoothness. Then we try to represent M by properly chosen functions. It is our goal to make a judicious choice of the free parameters in the representation.

We will theoretically compare our approach to the methods proposed by G.M.Nielson, G.Farin and B. R. Piper and we hope to present some numerical results.

A New Curve Tracing Algorithm and its Application to the Computation of Surface Intersections and Exact Aspect Graphs of Parametric Surfaces

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Abstract:

We consider the problem of tracing a curve defined implicitly in $\Re^n$ by a system of $n-1$ polynomial equations in n variables. The main difficulties are finding a sample point for every real branch and marching through singularities. We give a numerical prediction/correction procedure that tackles these difficulties. Its input is the defining equations of some curve Γ, and its output is a graph whose nodes are points on Γ that are either singular or extremal in some direction, say x_0, and whose arcs are the sampled smooth branches between these points. This graph is similar to Arnon's s-graph representation of plane curves, but its construction does not require cylindrical algebraic decomposition.

The algorithm is divided into four steps: (1) compute all extremal points of Γ in the x_0 direction (this includes all singular points); (2) compute all the intersections of Γ with the hyperplanes orthogonal to the x_0 axis at the extremal points; (3) for each interval of the x_0 axis determined by these hyperplanes, intersect Γ and the hyperplane orthogonal to the x_0 axis at the mid-point of the interval; (4) march numerically from the intersections found in step 3 to those found in step 2 by predicting new points through Taylor expansion and correcting them through Newton iteration.

Step 3 provides (actually several) sample points for each real curve branch, while step 4 trivially avoids singularities by marching only within intervals where the curve is extrema-free and therefore non-singular. Steps 1 to 3 involve solving systems of n polynomial equations in n unknowns. This is done by using the continuation method. Step 4 involves the inversion of systems of linear equations. This is done by LU decomposition.

The algorithm has been implemented and applied to the computation of intersection curves of complex surfaces, including singularities such as nodes and tacnodes. Several examples are presented. The aspect graph is a data structure describing all possible shapes (aspects) that the visual contours of an object may assume from different viewpoints. For a solid represented by a collection of rational parametric patches, it can be shown that computing the aspect graph is equivalent to tracing a set of curves in high-dimension spaces. Our algorithm has been applied to this problem, and preliminary results are presented.

APPROXIMATION SPLINE DE COURBES "ANGULEUSES"

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Soient $\{x_i, y_i\}_{i=1, N}$ les points successifs d'une fonction échantillonnée de façon dense. A partir de cet échantillon on détermine n (n<<N) "points caractéristiques de la courbe", pour construire une subdivision : n_1 points "anguleux" (discontinuité de la dérivée) et n_2 points "simples" (continuité des dérivées) Pour déterminer une spline d'ordre k, en utilisant la méthode des moindres carrés généralisés, les points "anguleux" peuvent être traités de deux façons :

— On choisit les nœuds $\{t_i\}_{i=1 \text{ à } m+k}$ de la spline à partir des n valeurs $\tau_1,\ldots,\tau_n$, des points caractéristiques, en prenant des nœuds multiples pour les points anguleux, puis on détermine les coefficients $\{a_i^*\}_{i=1 \text{ à } m}$ des $B_{i,k}$ associées à $\{t_i\}$ en minimisant la fonctionnelle :

$$I_d(a) = \sum_{j=1}^{N} [\sum_{i=1}^{m} a_i B_{i,k}(x_j) - y_j]^2 + \mu \sum_{i=1}^{m} \sum_{l=1}^{m} a_i a_l \sum_{r=1}^{n-1} \int_{\tau_r}^{\tau_{r+1}} (B_{ik}^{(d)} B_{lk}^{(d)})^2 \quad \text{où } d \leq (k/2) \text{ et } \mu > 0 .$$

Les coefficients $\{a_i^*\}$ sont solutions des équations normales où la matrice est (2k-1) diagonale.

— On peut utiliser les splines d'inf-convolution en se donnant une subdivision $\{t'_i\}_{i=1 \text{ à } n2+k}$ et les $B_{i,k}$ associées et une subdivision $\{t''_j\}_{j=1 \text{ à } n1+2}$ (points anguleux) et les $B_{j,2}$ associées. On calcule b* et c* minimisant :

$$E_d(b,c) = \sum_{j=1}^{N} [\sum_{i=1}^{n2} b_i B_{i,k}(x_j) + \sum_{s=1}^{n1} c_s B_{s,2}(x_j) - y_j]^2 + \mu \sum_{i=1}^{n2} \sum_{l=1}^{n2} b_i b_l \int_{x_1}^{x_N} (B_{ik}^{(d)} B_{lk}^{(d)})^2 .$$

On utilise une modification de l'algorithme direct de P.J. Laurent pour obtenir la solution.

On compare ces deux méthodes : qualité des résultats obtenus, complexité des calculs et extension aux courbes planes paramétriques.

"End conditions of univariate multiquadric interpolation"

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Let s be the interpolant to the equally spaced values $\{f(kh) : k = 0, 1, \ldots, N\}$ from the linear space that is spanned by the functions $\{\{\phi(x - kh) : 0 \leq x \leq 1\} : k = 0, 1, \ldots, N\}$, where ϕ is the multiquadric $\{\phi(r) = (r^2 + c^2)^{1/2} : r \in \mathcal{R}\}$ for some positive constant c and where $h = 1/N$ for some positive integer N. We consider the magnitude of the error $\{f(x) - s(x) : 0 \leq x \leq 1\}$ as $h \to 0$ when the underlying function $\{f(x) : 0 \leq x \leq 1\}$ has a Lipschitz continuous first derivative. We find that $\|f - s\|_\infty$ is $O(h)$ unless f satisfies two end conditions in which case the error is $O(h^2)$. Because the interpolation operator is bounded, these properties are also obtained by the best approximation. However, even in cases when $\|f - s\|_\infty$ is bounded below by a positive multiple of h, it is shown that the pointwise error $|f(x) - s(x)|$ is $O(h^2)$ for any fixed x. Some modifications to the set of approximating functions that provide $O(h^2)$ uniform accuracy are suggested. This work was done in collaboration with R.K. Beatson.

Interpolation for Bivariate Functions of Bounded Variation

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Consider bivariate functions $f : [a,b] \times [c,d] \to I\!R$ of bounded variation in the sense of Hardy-Krauss, i.e., $f(x,\cdot), f(\cdot,y) \in BV$ for some fixed $x \in [a,b], y \in [c,d]$ and

$$\sup \sum_k \sum_m |f(x_{k+1},y_{m+1}) - f(x_{k+1},y_m) - f(x_k,y_{m+1}) + f(x_k,y_m)| < \infty$$

For certain univariate interpolation processes including Lagrange interpolation on Jacobi nodes, trigonometric Lagrange interpolation and spline interpolation we construct the blending and the tensor-product operator. Then we discuss the bivariate L^p-error for these approximation processes. The results can be extended to functions with mixed derivative of bounded variation.

AN ANALYSIS OF THE BUTTERFLY SUBDIVISION SCHEME OVER UNIFORM TRIANGULATIONS

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ABSTRACT

The Butterfly Scheme, which was first introduced by Dyn, Gregory and Levin, is an Interpolatory Recursive Subdivision Algorithm which is defined upon arbitrary triangular networks. When the initial network is a uniform, there are several ways to analyse the convergence properties of the limit surfaces. One method involves the analysis of the generating polynomial of the scheme as in a paper by Dyn, Levin and Micchelli. An equivalent approach is the study of its difference and divided difference schemes using matrix analysis. This paper presents the latter method which is just a generalization of the binary subdivision scheme analysis for curves. The main task is to show that all the divided difference schemes of the butterfly scheme produce continuous surfaces. From this we show that the limit surface of the basic butterfly scheme is C1. Finally, some graphic examples are given to show the smoothing processes of the scheme.

KEYWORDS: RECURSIVE SUBDIVISION, ITERATION MATRIX, DIVIDED DIFFERENCES, BERNSTEIN-BEZIER POLYNOMIALS, SURFACE INTERPOLATION.

Ewald Quak and Larry L.Schumaker

Penalized least squares methods
for the construction of
bivariate polynomial spline functions

The method of smoothest spline interpolation is based on finding a bivariate polynomial spline function which interpolates given data values at the vertices of a prescribed triangulation, minimizing an energy expression to fix all free parameters which are not yet determined by the given interpolation and smoothness conditions.

If the prescribed data values are strongly influenced by measurement errors, interpolation is no longer useful and should be replaced by a sum of least squares Motivated by univariate penalized least squares methods, a spline fit can be constructed by minimizing a functional that is a combination of a sum of least squares and an energy functional, where a smoothness parameter $\lambda > 0$ controls the interaction of the two terms, i.e

for a given triangulation Δ consisting of the triangles T_j, a spline space $S_d^r(\Delta)$ of piecewise polynomials of degree d and smoothness r with respect to Δ and prescribed values f_i, $i = 1, \ldots, V$, for verrices v_i, a spline function $s^* \in S_d^r(\Delta)$ is determined so that

$$\rho_\lambda(s^*) = \min_{s \in S_d^r(\Delta)} \rho_\lambda(s) ,$$

where

$$\rho_\lambda(s) := \lambda\mathcal{E}(s) + I(s) ,$$

$$\mathcal{E}(s) := \sum_{j=1}^{N} \mathcal{E}_{T_j}(s) ,$$

$$\mathcal{E}_{T_j}(s) := \int\int_{T_j} [(s_{xx})^2 + 2(s_{xy})^2 + (s_{yy})^2] dx dy$$

and

$$I(s) = \sum_{i=1}^{V} [s(v_i) - f_i]^2 .$$

Algorithms and numerical results for these spline fits will be presented

Polyharmonic Cardinal B-Splines

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We generalize the notion of B-spline to the thin plate splines and to other d-dimensional polyharmonic-splines; for regular nets, we give the main properties of these "B-splines": Fourier transform, decaying when $||x|| \to \infty$, integration property, link with the polynomial B-splines, $\mathcal{P}_1$ reproduction ... We show that, in some sense, B-splines may be considered as a regularized form of the Dirac distribution.

Then, we generalize the notion of polyharmonic cardinal B-spline defined above to obtain "B-splines" on a regular net which are halfway between "elementary B-splines" and the cardinal interpolating spline function. We give the main properties of these functions: Fourier transform, decaying when $||x|| \to \infty$, integration, $\mathcal{P}_k$ reproduction (for $k \leq 2m-1$) of the associated B-spline approximation, etc. We show that, in some sense, quasi-interpolating polyharmonic B-splines may be considered as a finer regular approximation of the Dirac distribution than polyharmonic B-splines are.

Exploring Cubic Bézier Curves with Straightedge and Triangle

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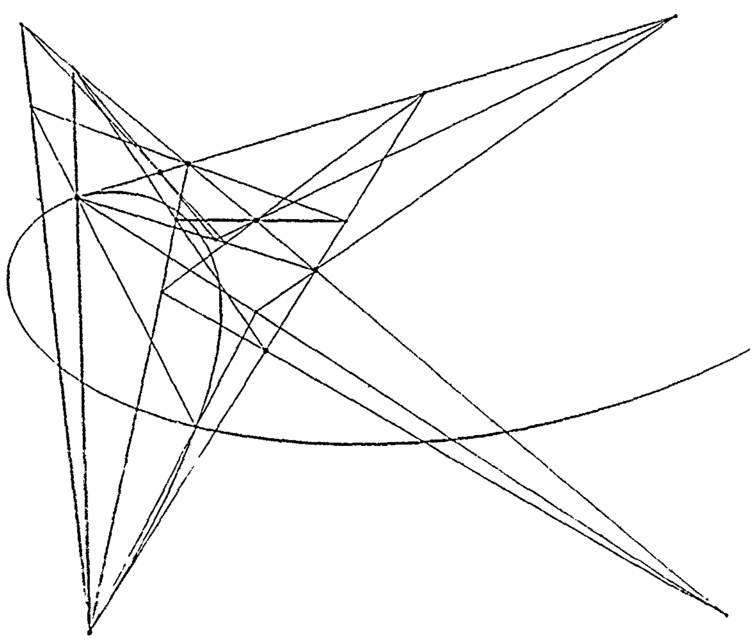
January, 1990

Abstract

Given the four Bézier points of a cubic polynomial curve in the plane, we derive explicit geometric constructions to solve three problems with a straightedge and triangle. As a warmup exercise, we use the de Casteljau Algorithm to construct a point on the cubic. Second, we construct the line joining the cubic's three points of inflection, one of which is a point at infinity and the other two of which may be complex. Third, we construct the cubic's double point, which is either a crunode (a self-intersection), an acnode (an isolated point, not adjacent to any other real point), or a cusp.

We also generalize these constructions to the case where the given cubic is rational, instead of polynomial. In the rational case, the four Bézier points of the input cubic must be supplemented with some extra points, acting as sliders, that encode the corresponding weights. Designing an encoding scheme that avoids degenerate cases turns out to be an exercise in Descriptive Geometry.



Algorithms for Local Convexity of

Bézier Curves and Surfaces

Thomas Rando and John Roulier

Theory and algorithms are presented which determine whether a Bézier curve or Bézier surface is locally convex. Furthermore, an algorithm for producing control points which guarantee local convexity of a Bézier curve or surface which satisfies given constraints is presented. Both of these algorithms are based on examining the control points and not on general curve or surface interrogation. The theory is based in part on previous results of one of the authors involving locally convex planar Bézier curves.

Polynomial n-sided patches

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The practical need for interpolating smooth surfaces on the basis of a curve network of irregular topology leads to the problem of n-sided patch interpolation. Attempts to overcome the problem involve rational patches or polynomial patches with a high degree (six in general). The paper describes the construction of low degree polynomial n-sided patches which are local interpolations of the boundary curves. Different ways of constructing the cross derivatives and also the patch equations corresponding to these cross derivatives are discussed. These constructions contain different numbers of free geometric parameters which can be used as designer handles in a CAD system. Finally the geometric properties of the patches are summarized.

CONSTRUCTION AND INTERACTIVE MODIFICATION OF TETRAHEDRAL MESHES

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A flexible generator of tetrahedral meshes capable to manage the interactive modification (by refinement) of the mesh is presented and discused. Sequences of nested meshes as needed in multigrid context can also be generated Numerical experiments performed show that the meshes constructed are mon-degenerate and smooth.

Polynomial Basis Functions for Curved Elements Using Hyperbolas

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A conforming polynomial second order basis for the three sided two dimensional finite elements with one curved side is constructed in such a way that the curved side is approximated by an arc of hyperbola. The basis is used to calculate approximate solutions of Laplace's equations over the unitary disk with Dirichlet boundary conditions. The basis has the property that it remains conforming when the curved side reverts to a straight line segment. The calculations of the typical integrals are made directly in the original domain of interest without the use of a non-linear transformation that is required in the high order transformation methods. Various tesselations of the problem domain were done and the numerical experiments show that the results are completely satisfactory for all the examples considered.

An Alternative to the h-convergence Test

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One of the tests normally applied to any approximation scheme is the order of convergence as the grain of the data becomes smaller. This test normally applies in the limit as the data becomes uniformly infinitely dense. It does not give a great deal of information about the actual accuray of finite density.

An alternative, applicable to schemes which are linear in the data ordinates, which provides more information is to consider the accuracy of fit to data whose ordinates are derived from trigonometric functions of various frequencies.

This gives the traditional h-convergence as the order of accuracy as the frequency tends to zero. However, it also provides a more quantitative assessment for non-zero frequencies, and also directional information in the bivariate and trivariate cases.

Recursive division construction C^2 at the singular points

Malcolm SABIN

Fegs Ltd. (UK)

About ten years ago recursive division surface definitions were proposed by Catmull and Clark and by Doo and Sabin. The Catmull-Clark construction was an extension of the regular bicubic B-spline, the Do-Sabin one of the biregular quadratic B-spline.

Although the quadratic construction gave C^1 continuity everywhere, the cubic had interesting fractional power behaviour at the singular points, which meant that despite retaining the C^1 property, the curvature at the singular points could be either unbounded or identically zero.

In this paper a modification to the Catmull-Clark construction is described which gives the cubic C^2 continuity everywhere.

QUASI-INTERPOLANTS DE TYPE BERNSTEIN

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Résumé

Les quasi-interpolants introduits par l'auteur (Oberwolfach, Février et août 1989) sont de la forme $Q_n^{(k)} f = \sum_i \mu_{in}^{(k)}(f)\, b_i^n$.

Les $\{b_i^n\}$ sont une base de Bernstein de polynômes sur un simplexe (resp. un hypercube) de degré total (resp. partiel) inférieur ou égal à n. Les formes linéaires $\{\mu_{in}^{(k)}(f)\}$ utilisent des valeurs ponctuelles ou des moments de f ou de ses dérivées.
Pour $0 \le k \le n$, l'opérateur $Q_n^{(k)}$ reproduit les polynômes de degré total (resp. partiel) inférieur ou égal à k. On contruit explicitement plusieurs familles de tels opérateurs et on donne des résultats sur leurs normes et leurs propriétés de convergence pour des fonctions régulières.

Abstract

The quasi-interpolants introduced by the author (Oberwolfach, February and August 1989) have the general form $Q_n^{(k)} f = \sum_i \mu_{in}^{(k)}(f)\, b_i^n$.

The $\{b_i^n\}$ are a Bernstein basis of polynomials on a simplex (resp. hypercube) of total (resp. partial) degree at most n. The linear forms $\{\mu_{in}^{(k)}(f)\}$ use values or moments of f or its derivatives. For $0 \le k \le n$, the operator $Q_n^{(k)}$ reproduces exactly polynomials of total (resp. partial) degree at most k. We construct explicitly several families of such operators and give some results on their norms and their convergence properties for regular functions.

ELEA : A Tool for 3-D Surface Regression Analysis
In Propellant Grains

E. Saintout, D. Ribereau, P. Perrin (1) - A.Y. Le Roux (2)

This paper presents a computer code to determine the surface regression properties as functions of burn distance and local burning rate (normal vector and burning rate value at each point) in propellant grains.
Burn area, moment of inertia, center of gravity, mass flow rate, and a representation of burning surface geometry are available at each time step.

To build the propellant grain geometry, we use a specific solid modeler which offers the standard C.A.D. Boolean functions (join, cut and intersection), and besides the traditionnal set of primitives as cylinders, spheres... it contains some abilities as fin or star forms.
Propellant grain geometry and restrictor are generated in the same time; Initial burning surface is defined by unrestricted boundary surface.

To compute surface regression, the initial burning surface is divided in triangular elements by an automatic mesh generator.

Equations (Hamilton-Jacobi similar type) which describe the surface growing normally to itself according to the local burning rate are then solved on the mesh at each step of the regression. More precisely, the update of the current mesh is as follows: each node of the mesh is displaced along its normal vector proportionnally to its own burning rate; the junctions between this updated mesh and the restricted surfaces, and the crossings between parts of the mesh are controlled; the mesh is corrected if necessary and becomes the new current burning surface mesh.
We may use every law of burning rate which is function of the point, of the local normal vector and of the mass flow rate.

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Error Estimates for Simplified Representations of Curves and Surfaces

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Surfaces and curves in Computer-Aided Design are normally represented as vector-valued linear combinations of certain nonnegative scalar functions which sum up to one (partition of unity). The direct evaluation of the functions is often replaced by a sequence of refinement steps (subdivision, knot insertion) on the set of vector-valued coefficients (the control net), until some "flatness test" is satisfied. Normally, the refined control net is interpreted as a representation of a simplified curve or surface, e.g.: a piecewise linear or bilinear curve or surface ("control polygon"). Then the refinement process shows quadratic convergence to the curve or surface defined by the original control net.

We prove some numerically accessible estimates for the error between two interpretations of a control net, one of which normally is a "simplification" of the other. They are useful, for instance, as safe "flatness tests" to stop the refinement process properly.

If the refined control net is interpreted as a piecewise quadratic or cubic curve or surface, higher convergence orders of refinement processes are possible. In this way some variations of refinement processes can be defined which converge better than quadratically with respect to the refinement parameter.

Applications cover partitions of unity by polynomials, B-splines, and rational functions in one or several variables.

Some Extensions of the Problem of Best Interpolating Spline Curves

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The problem of best parametric interpolation extends the classical variational problem of interpolating scalar data to the case of vector-valued data. In addition the objective functional is minimized with respect to the nodes in order to obtain an optimally parametrized spline curve. Quite complete results on existence, characterization and uniqueness have been obtained by Marin, Pinkus, P.W. Smith and myself for polynomial splines in the case of scalar data and in the general cubic case. Here extensions of these results to Tchebycheffian splines and to the curve fitting problem are considered. The motivation for the first extension is that it preserves more geometric properties of the data (e.g. lying on a circle). In the second extension the non-linearity of problem causes new difficulties.

Adaptive G_1 Approximation of Range Data Using Triangular Patches

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An adaptive surface fitting algorithm is proposed for the approximation of a sampled surface described by an array of 3D points distributed on a rectangular mesh. The G_1 smooth piecewise approximation of the data is obtained by using an adaptive Delaunay triangulation technique combined with a modified triangular Bernstein-Bézier patch model.

The adaptive Delaunay triangulation technique allows us to obtain progressively a refined polyhedral approximation of the raw data. Beginning with a very coarse approximation, the process determines the 3D points corresponding to the worst approximation by the planar triangles of the current polyhedral surface according to a given error measure, and then adds them to an incremental Delaunay triangulation to produce a finer approximation. The Delaunay triangulation process is in fact executed on a 2D plane onto which the 3D sampled points are projected in the form of a regular rectangular mesh, the sampling step being taken as unit of distance. The 3D triangles are obtained by back-projection. This triangulation technique fully exploits the data structure inherent in range images and is very efficient.

Two of the authors have recently proposed a modified triangular Bernstein-Bézier patch model with duplicated inner control points. This model allows a very simple G_1 smooth connection between adjacent patches, especially those around an N-patch corner. The G_1 constraints can be solved locally by using patches of low degree. By combining this model with the adaptive Delaunay triangulation technique, a G_1 piecewise approximation of the range data can be obtained. This combination can be realized in two ways: 1) a postprocessing way in which the final polyhedral approximation is smoothed to get a G_1 piecewise surface; 2) an embedding way in which the new model is directly used at the end of each step of the adaptive triangulation process to provide a better measure of the approximation error.

† Now with Thomson Digital Image, 20 - 22, rue Hegesippe Moreau, 75018 Paris, FRANCE

Universal Splines and Geometric Continuity

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In this talk we develop the concept of universal splines and apply this new concept to the study of geometrically continuous spline curves of arbitrary degree. This yields geometric constructions for both the spline control points and the Bézier points and gives algorithms for computing locally supported basis functions and for knot insertion. As a result of our development we obtain a generalization of polar forms to geometrically continuous spline curves. The presented algorithms have been coded in Maple, and concrete examples illustrate the approach.

PROCEDURAL SURFACE INTERPOLATION WITH GREGORY PATCHES

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A local G1-continuous surface interpolation method is described. A set of interpolation points in R3 connected by edges into a topology-defining network of quadrilateral and/or triangular meshes is supplied by the user. A geometrically smooth surface is constructed as a union of bicubic quadrilateral and/or quartic triangular Gregory patches defined over the given meshes, in which each original edge is replaced by a cubic boundary curve. Our procedure determines surface normals at interpolation points, the Bézier control points for cubic boundaries, and finally the internal control points for the patches. The method uses an intuitive geometric, rule-based approach to find a 'good' default solution. It produces surfaces of high visual quality even for very irregular sets of data where traditional techniques fail. This default solution can be overridden or further refined by user-supplied normals and boundary curves. Moreover, along any boundary curve the shape of the two adjoining patches can be locally controlled with *shape parameters*, such as *tilt, bulge* and *shear* which modify the cross-boundary derivative vector in different ways. This gives extra control to the user while preserving first order geometric continuity between the patches.

Efficient Computation of Multiple Knots Nonuniform Spline Functions

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This paper presents simple canonical representations of general spline functions which lead to orders of magnitude reduction in curve and surface rendering. This approach eliminates the redundancy found in the traditional representations. Because of the derivative properties of piecewise polynomial functions, it can be shown that the rth derivative of an rth order B-spline function is a set of weighted impulses located at the knots defining the spline function. The weights of these impulses are uniquely defined by the order of the B-spline and the locations of the knots. We provide recurrence relations to compute these values and present new high speed algorithms for the generation of curves and surfaces.

We show how this representation extends to multiple knot splines and to nonuniform rational B-spline functions (NURBs). This representation also leads to efficient algorithms for the implementation of motion transforms eg. translation, rotation, scaling.

Computational complexity analyses are provided to demonstrate the improvement of our technique over de Boor's linear combination algorithm and forward differencing approaches.

Geometric Modelling

in Numerical Grid Generation

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During the last few years, numerical grid generation has evolved as a critical link in the events leading to numerical solution of the partial differential equations of fluid mechanics. The accuracy of the numerical algorithm depends not only on the formal order of approximation, but also on the distribution of grid points in the computational domain. The grid employed can have a profound influence on the quality and convergence rate of the solutions.

A multitude of techniques and computer codes have been developed for generating computational grids in arbitrary regions. However, in most of these codes and methodologies, the evaluation of the geometry input and realization of mapping between physical and computational space allowing appropiate zonal/block strategies are 'ong, very laborious, and extremely time consuming. Geometry-grid generation is considered as the most time and cost critical part in a typical application.

A systematic procedure for grid generation which can provide computational grids for a wide range of geometries related to internal/external flow considerations is presented. The development of associated grid generation codes (GENIE & EAGLE) along with respective geometric modeling module is discussed. Applications-oriented complex computational examples are presented, demostrating the requirements and limitations associated with geometrical modelling.

Reparametrization of Polynomial Curves and Rational Curves.

Procedures of change of variables, or *reparametrization*, applied to polynomial curves in Bezier form or rational curves in BR form are described. These procedures are devised to give us on output the following *standard* representation: the (BR) form defined on the unit interval $[0,1]$. Indeed, we study the following problem: Given a polynomial curve (resp. rational curve) $\mathcal{C}:\mathcal{I}\to\mathrm{R}^m$ in Bezier form (resp. in (BR) form) and $f:[0,1]\to\mathcal{I}$ a polynomial or a rational map, find the Bezier or (BR) form of the composed map $\mathcal{C}of$. The above reparametrization is successively choosen linear, homographic, quadratic, and more generally polynomial or rational. Algorithms are provided: they accept Bernstein forms (Bezier curves as well as (BR) curves) as input, and use such forms in intermediate computations, and generate them on output.

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Simplicial Methos for Manifolds and Applications

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Using techniques from Combinatorics, Topology and Optimization we will show how to approximate implicitely defined manifolds by piecewise linear manifolds and apply to.

1. geometric modelling;
2. domain decomposition;
3. implicit ordinary differential equations.

The results will be displayed using computer graphics pictures (35 mm. slides). Perspectives of the method presented will be given.

A BUILDING METHOD FOR HIERARCHICAL COVERING SPHERES OF A GIVEN SET OF POINTS

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In Computational Geometry, there are a lot of problems related (or solved using) Voronoi tesselation
Some of them are for example, finding the nearest neighbor, the convex hull, etc.

Building the Voronoi tesselation or its related or dual constructions is for those examples a very important step, and for that reason, there are many algorithms used for it.

A good approach is to build the set of Covering Spheres, dual to the Voronoi tesselation. The Covering Spheres are, if the space is of n dimensions, the ones determined by at least n+1 points of a given set of m (m>n+1) , such that the sphere does not contain inside it any point of the whole set.

There are also many ways to build these spheres, our algorithm uses a hierarchical and recursive structure as follows:
1st Step : A first auxiliary sphere containing all the set is determined.
2nd Step : (Beginning of the recursive algorithm) A point of the set is introduced and a unflagged sphere containing it is identified.
Start the search with the first n+1 spheres and, if the sphere contains the point, is flagged as subdivided, continue the search with the spheres produced by the subdivision.
3rd Step : Find the spheres that contain the point to be inserted.
4th Step : Flag the above spheres and create new ones with the point introduced determining them

All the spheres are stored, in that way hierarchy is determined beeing the root a sphere and the leaves the ones inside it.

The advantages of the method are shown in the paper including an example and demonstrations about the speed and easiness of the algorithm. All those advantages are due to the combination of the hierarchical scheme with the concept of the covering Spheres.

An Algorithm for Smoothest Interpolation

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In a recent paper [2] we proved uniqueness of the smoothest interpolant with free nodes of interpolation from the Sobolev space W_2^3. Here we propose an algorithm for finding this extremal function, interpolating given values and having minimal L_2-norm of the third derivative. It is surprising that a complicated nonlinear system which we obtain using the characterization of the smoothest interpolant with free nodes given by Pinkus [1], can be solved applying univariate bisection method.

References

[1] Pinkus, A., On smoothest interpolant, *SIAM J. on Numerical Analysis*, 19(1988), No. 6, pp 1431-1441.

[2] Uluchev, R., Smoothest interpolation with free nodes in W_p^r (submitted to *J. Approximation Theory*).

B-SPLINE SURFACE FITTING FOR REAL-TIME SHAPE DESIGN

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ABSTRACT

High-speed interactive geometric design prescribes specific requirements for the implemented algorithms. Besides real-time image shading for NURBS and a designer-oriented user-interface, fast shape determination on basis of sparse data must be supported.

We present a method to approximate NURBS or B-spline surfaces to loosely ordered data by fast multi-stage curve fitting. To ensure a minimal number of control points while confining the curve within a given tolerance, optimal knots are determined for each curve individually.

In the second stage, knots are harmonized to enable the tensor-product description. The fits may (but need not) be constrained to specific knot placement schemes, allowing the results to be applicable at e.g. strictly uniform B-spline modelers. The performance of the system will be outlined and further extensions will be discussed.

Exact Conversion of Trimmed Composite Bézier Surfaces into Composite Bézier Surfaces Representations

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Trimmed surfaces consist of a number of succesive connected curves which trim one or more (composite) original surface(s). If geometrical data are exchanged between disimilar CAD/CAM systems, serious problems occur with trimmed surfaces if the area between these curves is not explicitely defined. This paper investigates how an explicit definition of the surface within Bézier trimming curves can be gained if these curves trim a composite Bézier surface. Further, an algorithm is given which meets the requirements formulated for data exchange purposes.

Catmull-Rom Spline Surfaces

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Catmull-Rom spline curves generalize the usual notion of a spline curve by replacing control verticies (which may be thought of as constant functions) with functions defined on the paramater interval and taking values in R^2 or R^3; they have the advantage of having local control and the capablity of being either interpolating or approximating.

This paper extends these notions to surfaces and develops a class of surface types which posseses advantages analogous to those of Catmull-Rom curves. Specific examples are given of low degree surfaces that are both interpolatory and possess local control. In these examples control verticies are replaced with affine maps generated by portions of a polygonal mesh. The resulting surface closely and smoothly approximates the mesh and is constrained to pass through verticies of the mesh.

CONSTRUCTION DE SURFACES B-SPLINES NON UNIFORMES PAR APPLICATION DES METHODES DE COONS ET GORDON

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Ce travail a été réalisé sur EUCLID-IS chez MATRA DATAVISION

L'objectif est de modèliser des surfaces à partir de courbes sous trois contraintes :

- la surface obtenue respecte l'esthétique des courbes données par l'utilisateur,
- la méthode proposée est simple à utiliser,
- la surface respecte les données à la précision voulue par l'utilisateur.

Type de données à traiter.

La forme générale des données du problème est un treillis rectangulaire de courbes. Comme cas particuliers, on trouve l'interpolation de sections et le remplissage d'un contour rectangulaire. La demande était de se limiter à des courbes et surfaces polynomiales ou polynomiales par morceaux.

Modèles existants.

Parmi toutes les representations mathématiques de surfaces, la plupart des systèmes de CAO utilisent deux grandes méthodes de représentation des surfaces complexes.

D'une part les formes à pôles polynomiales ou rationnelles (surfaces B-splines et carreaux de Bezier). Ce type de représentation est réputé pour sa souplesse d'usage mais ne répond pas directement à notre problème.

D'autre part les surfaces obtenues par équations mélangeantes sur les courbes (surfaces de Coons et Gordon). Ce type de définition est particulièrement adapté au problème et n'est pas limité quant à la définition des courbes données, mais il présuppose que ces sections soient des isoparamètriques.

Méthode mise en œuvre.

A partir de sections polynomiales par morceaux, on calcule leurs representations B-splines polynomiales non- uniformes. Après avoir évalué, par mixage, une base B-spline commune dans chaque direction parametrique on modifie le paramètrage initial des sections afin de satisfaire l'hypothèse de base de la méthode de Gordon.

Les fonctions mélangeantes sont ensuite definies par des courbes B-splines d'interpolation.

La surface finale est une surfaces B-spline polynomiale non-uniforme dont les pôles sont calculés en développant les équations de Coons ou Gordon sur les fonctions B-splines définissant les sections et les fonctions mélangeantes.

TITLE: Norms of Inverses and Condition Numbers for Matrices Associated with Scattered Data

SPEAKER: J. D. Ward Texas A&M

ABSTRACT

In this talk, we discuss interpolation matrices arising in connection with translates of radial basis functions. In particular, we give a general method for obtaining bounds both on the norm of the inverse of the interpolation matrix and on the condition number of that matrix. We apply our method to obtain these bounds in several cases,including those associated with functions generated either by completely monotonic functions or integrals of such functions. These estimates depend only on the minimal separation distance for the data and the dimension s of the ambient space R^s.

An Equational Characterization of Geometric Continuity Between Algebriac Surfaces

Joe Warren

Rice University

This talk will describe necessary and sufficient conditions for geometric continuity of any desired order between a pair of algebraic surfaces that meet at a common point. This characterization involves a set of equations that are linear in the coefficients of the defining polynomials for the surfaces. Next, this characterization will be extended to include a necessary and sufficent characterization of geometric continuity between a pair of algebraic surfaces that meet along a common curve. Again, this characterization involves a set of equations that are linear in the coefficients of the defining polynomials. Finally, an application of these results to the problem of surface fitting will be discussed.

Base Points and Rational Bèzier Surfaces

Joe Warren
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A triangular rational Bèzier surface of degree n can be expressed in the form

$$
\begin{aligned}
x &= \sum_{i,j,\geq 0, i+j\leq n} x_{ij} w_{ij} B_{ij}(s,t), \\
y &= \sum_{i,j,\geq 0, i+j\leq n} y_{ij} w_{ij} B_{ij}(s,t), \\
z &= \sum_{i,j,\geq 0, i+j\leq n} z_{ij} w_{ij} B_{ij}(s,t), \\
w &= \sum_{i,j,\geq 0; i+j\leq n} w_{ij} B_{ij}(s,t)
\end{aligned}
$$

where the B_{ij}'s are the bivariate Bernstein basis functions of degree n. The four-tuple (x, y, z, w) denotes the point $(\frac{x}{w}, \frac{y}{w}, \frac{z}{w})$ in affine space. In this formulation, the points (x_{ij}, y_{ij}, z_{ij}) may be interpreted as forming a Bèzier control net with associated weights w_{ij}. The relationship of a Bèzier control net and its corresponding rational surface patch is well-understood.

Those values of s and t for which f_1, f_2, f_3, and f_4 simultaneously vanished are referred to as *base points* of the parameterization. In the rational Bèzier formulation, setting one of the weights w_{00}, w_{0n}, or w_{n0} to zero introduces a base point at the vertex of the underlying parametric domain triangle. Finding the image of a base point under this parameterization involves computing the limit of $(\frac{x}{w}, \frac{y}{w}, \frac{z}{w})$ as the a curve in parameter space approaches the base point. As this approaching curve varies, the limit (and therefore the image of the point) varies along a parametric curve. For rational Bèzier surfaces, this image curve is directly related to the Bèzeir control net. This paper will describe a technique for deriving this curve from the Bèzeir control net and use this technique to create multiple sided patches defined over a triangular domain. This paper will conclude by discussing conditions for C^0 and C^1 continuity between such patches.

Conditions for Geometric Continuity of Curves and Surfaces

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Developing conditions for geometric continuity of surface patches and applying rational surface representations are two important research directions in CAGD therefore much interest has been devoted to them.

Necessary and sufficient conditions for geometric C^p-continuity ($p = 1, 2, ..., n$) attached to combinations of rectangular and triangular polynomial B'ezier patches have been discussed by several authors (BOEHM/SHOUSHAN, DEGEN, DEROSE, HOSCHEK /LIU, LIU, WASSUM).

In out presentation necessary and sufficient conditions for geometric C^p-continuity ($p = 1, 2, ..., n$) of neighbouring rational B'ezier patches are determined as a generalization of these results.

Practical applications based on special sufficient geometric C^1/ C^2-conditions are described as well as geometric interpretations.

Chebyshev approximation by curves in $\mathbb{R}^n$

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Motivated by an application in railway engineering we consider the problem to determine a curve in $\mathbb{R}^n$ which approximates a given set of points y_j uniformly. We want to find

$$\min_a \max_j \min_t \| y_j - x(t,a) \| .$$

Optimality conditions for this problem and connections to alternation properties in approximation theory are discussed. The special case of approximation by a straight line is considered in some detail. An analysis of the second order optimality condition is given and several computational methods are compared.

The eventual aim is to find approximations with curvature constraints. Approximation by a straight line is the first step towards the solution of this problem.

Convergence Orders for Multivariate Interpolation of Scattered Data by Radial Basis Functions

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Via a variational formulation which differs from the theory of Madych and Nelson, the error of multivariate interpolation of scattered data by radial basis functions can be bounded by a certain integral involving the Fourier transform of the interpolated function. A perturbation theorem allows a local transition between regular and scattered data problems, yielding convergence orders of interpolation by radial basis functions including $\phi(r) = r^\beta$, $\phi(r) = (c^2 + r^2)^{\beta/2}$, and $\phi(r) = \exp(-\beta r^2)$ for the usual values of β.

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C^2 Transfinite Interpolation on a Triangle with One Curved Side

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We know that in CAGD, sometimes, technology of designing C^2 surface is required (for example, the design of a concealed airplane). So it is important to present an efficient C^2 interpolation scheme for arbitrary triangles, specially, for the curved triangle.

In this paper, for the curved triangle with one curved side and two straight sides, we provide a side-vertex interpolation scheme which interpolates to the triangle.

For a straight triangle, the scheme is based on the combination of interpolation operators which consist of univariate Hermite interpolation operator along lines joining a vertex and its oposite side.

For a curved triangle, we transform the curved triangle into a straight triangle using a differentiable homeomorphic transformation between the curved and straight triangle. And we provide the error analysis for two schemes.

ON BIVARIATE OSCULATORY INTERPOLATION

Liang Xue-zhang, Jilin University, China

Li Lou-qing, Hubei University, China

Let Π_n denote the space of real bivariate polynomials of total degree $\leq n$. In 1965 the first one of the authers has given the following theorem :

Theorem. If $\{Q_i \mid 1 \leq i \leq s\} \subset R^2$ is a unisolvent interpolating set of nodes for Π_n (where $s=\frac{1}{2}(n+1)(n+2)$), and if none of these nodes is on the irreducible curve of degree k : $l(x,y)=0$ (either k=1 or k=2; k=1 means a straght line; k=2 means a conic) , Then $\{Q_i \mid 1 \leq i \leq s\}$ with the $(2n+3)k-1$ points being distinct and selected freely in the irreducible curve must constitute a unisolvent set of nodes for Π_{n+k} .

By the theorem we have further proposed two processes of constructing the properly posed set of nodes for bivariate Lagrange interpolation : the Line-superposition Process and the Conic-superposition Process . The purpose of this paper is to generalize the Line-superposition Process and set up a new bivariate osculatory interpolation, the Order-raising Process . The new method expands and develops the osculatory interpolation schemes proposed by Le Mehaute in 1981 and by Hakopian in 1984 .

Some Geometric Properties of the Convex Hulls of the Rational Cubic Bezier Curve Segment under the de Casteljau Algorithm

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Waseda University

[Abstract]

The rational cubic Bezier curve segment is subdivided into two sub-segment at a specified parameter value by the de Casteljau algorithm.

Consider the convex hull by the four control vertices of the curve defined in homogeneous coordinates. The shape of the convex hull is, in general, a tetrahedron.

First, we present recursive relations with respect to:

(1) single vertices,
(2) lines passing through two vertices,
(3) planes passing through three vertices and
(4) the tetrahedron made by the four vertices

of the convex hull before and after the subdivision. These relations require only additions and shift operations.

Next, we present a new convergent property of the convex hulls during subdivision process. That is, the ratio of i th maximum deviation from the curve with respect to i-1 th's converges to 1/4. This property is expected to be utilized for efficient termination of iterative sub-division process.

INTERPOLATION DE LAGRANGE PAR DES SPLINES QUADRATIQUES SUR UN QUADRILATERE DE $\mathbb{R}^2$.

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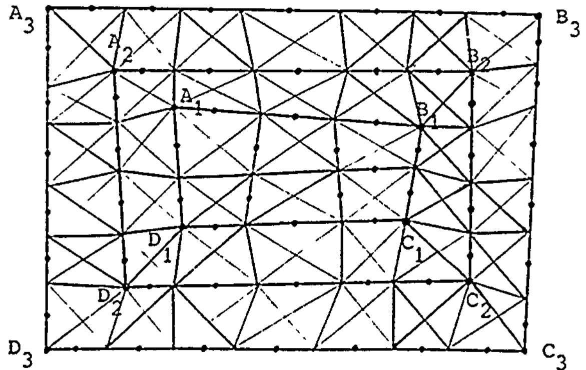
On étudie un problème d'interpolation de Lagrange par des splines quadratiques sur une triangulation τ d'un quadrilatère Q de $\mathbb{R}^2$ (voir figure).

1°) - Cas où Q est un quadrilatère quelconque :

Pour $T \in \tau$, on note $\mathbf{P}_2(T)$ l'ensemble des polynômes de degré total au plus égal à 2, définis sur T et $S_2^1(Q)$ l'ensemble des fonctions $s \in C^1(Q)$ telles que $s/T \in \mathbf{P}_2(T)$ pour tout $T \in \tau$ (splines quadratiques).



Etant donné $I \in \mathbb{N}$ fixé, Q est formé de I macro-quadrilatères emboités $(A_iB_iC_iD_i)$ $(1 \leq i \leq I)$. Le macro-quadrilatère central est formé de n' rangées de n micro-quadrilatères chacune (sur la figure n' = 2 ; n = 3). Les régions $(Q_i \setminus Q_{i-1})$ $(2 \leq i \leq I)$, sont des couronnes de micro-quadrilatères, chacun d'eux étant subdivisé en 4 triangles par ses diagonales. Les points d'interpolation sont les sommets des macro-quadrilatères, les milieux des segments portés par leurs frontières et quelques points choisis convenablement à l'intérieur du quadrilatère central $(A_1B_1C_1D_1)$.

Théorème 1.

Etant donné f, fonction définie sur Q, il existe une spline unique $s \in S_2^1(Q)$ interpolant f aux points choisis.

2°) Cas où Q est un carré.

Etant donné N entier naturel impair, Q est formé de N^2 micro-carrés, tous identiques au carré central. On appelle Π_N l'opérateur qui a f associe son interpolant spline $\Pi_N(f) = s$ appartenant à $S_2^1(Q)$.

Proposition.

En posant : $\|\Pi_N\| = \sup\limits_{\|f\|_Q \leq 1} \|\Pi_N(f)\|$ avec $\|f\|_Q = \sup\limits_{x \in Q} |f(x)|$ on a les résultats :

$$\|\Pi_1\| = 3$$

$$\|\Pi_N\| \leq 2N-1 \quad (N \geq 3, N \text{ impair}).$$

Théorème 2.

Pour $f \in C^3(Q)$ on a :

$$\|f - \Pi_N(f)\|_Q \leq c \cdot h^2$$

où c est une constante ne dépendant que des normes des dérivées partielles $\|D^{kl}f\|_{Q'}$ $(k+l = 3)$ sur un domaine Q' contenant Q tel que dist(Q, Q') = h/2 et h = 1/N.

On the Convexity of the Parametric Bezier Surfaces over Triangles

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Abstract

The convexity of surfaces is an interesting mathematical topic with application to modeling object. This paper give sufficient condition for convexity of parametric Bezier surfaces over triangles.

Let T be a given closed triangle in the uv-plane. For point $p(u,v) \in T$. Let (r,s,t) be barycentric coordinates of p with respect to T. A triangular Bezier patch is defined by

$$B_n(u,v)=B_n(r,s,t)=\sum_{\substack{i+j+k=n \\ i,j,k\geq 0}} C_{ijk}\phi^n_{ijk}(r,s,t)$$

where

$$\phi^n_{ijk}(r,s,t)=\frac{n!}{i!\ j!\ k!}\ r^i\ s^j\ t^k$$

Lemma 1. $B_n(u,v)$ is convex if and only if the Gaussian curvature $K>0$ for all $(u,v)\in T\backslash\partial T$.

Lemma 2. The positivity of Gaussian curvature of $B_n(u,v)$ is independent with the linear transform of u,v.

From Lemma 1 and 2, we can transform convexity of $B_n(u,v)$ into positivity of a new Bernstein polynomial over T and obtain a sufficient conditions.

Further, we obtain weak sufficient conditions with subdivision and degree elevation.

The Applications of Bivariate Interpolating Splines

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A kind of bivariate interpolating splines has been constructed on triangulated region. We have used this kind of splines to find the numerical solutions of partial differential equations and to fit surfaces. In this paper we only discuss the application in surface fitting. Some examples are given. The advantages of this kind of splines are:

1.- Because it is polynomial spline, the calculation is very simple.

2.- The approximation order is high. If $F(x,y) \in C^{2r}(D)$ denotes the exact expression of the surface, and $S(x,y)$ is the spline of degree $2n-1$ interpolating the values $F(x_i, y_j), (i = 0,1,...,m_1; j = 0,1,...,m_2)$ then on D

$$||S_{x^\alpha y^{\gamma-\alpha}} - F_{x^\alpha y^{\gamma-\alpha}}|| \leq Ah^{2r-\gamma}, \qquad 0 \leq \alpha \leq r; \quad r = 0,1,...,2n-1.$$

where A is a constant independent from x, y and n, $h = \max_{i,j}(x_{i+1} - x_i, y_{j+1} - y_j)$.

3.- The convexity can be decided by the partial derivatives of $S(x,y)$ on the grid points.

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