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FEEDBACK CONTROL IN DISTRIBUTED PARAMETER SYSTEMS

Richard L. Rebarber
Department of Mathematics and Statistics
University of Nebraska
Lincoln, NE 68588-0323

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13. ABSTRACT (Maximum 200 words) In this project questions related to stabilization of vibrating systems by feedback were studied. A vibrating system is uniformly stabilized by a feedback if there is a uniform rate of decay for all vibrational modes of the closed-loop system, which is the system with feedback. Necessary conditions were found for certain kinds of feedback to uniformly stabilize a system which is originally undamped and has the feedback applied at the boundary. Sufficient conditions were found for the feedback to lead to a closed-loop system which is mathematically well-posed. It was also shown that in a large class of systems which are stabilized by feedback, if a small delay is introduced in the computation of the feedback, then the stability of the closed-loop system is destroyed. <i>Kryazev</i>					
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Final Technical Report
AFOSR Grant 88-0230
Feedback Control in Distributed Parameter Systems

I have been studying problems related to stabilization of distributed parameter systems by feedback control. The control systems are of the form $\dot{x}(t) = Ax(t) + Bu(t)$, where $x(t)$ and $u(t)$ are in Banach spaces X and U respectively, and A is the generator of a C_0 semigroup $S(t)$. A typical goal in Control Theory is to stabilize this system by choosing an operator K so that $u(t) = Kx(t)$ leads to a closed loop system which is exponentially stable.

During the period of the grant I have studied the following three problems:

- 1) Find necessary conditions for there to exist a compact K such that $A + BK$ is exponentially stable.
- 2) Given A and B , for what K is it true that $A + BK$ generates a C_0 semigroup?
- 3) Is the stabilization destroyed by an arbitrarily small time delay in the feedback loop? That is, if $A + BK$ is exponentially stable, is the closed loop system with feedback $u(t) = Kx(t - \epsilon)$ still exponentially stable?

The work I have done on the first two problems has mostly been described the following published papers:

[A] "Necessary Conditions for Exponential Stabilizability of Distributed Parameter Systems with Infinite Dimensional Unbounded Feedback," Systems and Control Letters, Vol. 14 (1990), pp.241-248.

[B] "Semigroup Generation and Stabilization by A^p -bounded Perturbations," Systems and Control Letters, Vol. 14 (1990), pp. 333-340.

[C] "Stability of Distributed Parameter Systems with Unbounded Control," Preprints for The Fifth IFAC Symposium on Control of Distributed Parameter Systems, Perpignan, France, June 1989, pp. 123-128.

Some of this work also appears in the following unpublished paper, written with Dr. Gareth Knowles, who is currently at Grumman Corporate Research Center:

[D] "Necessary Conditions for Exponential Stabilizability of Distributed Parameter Systems with Infinite Dimensional Control," with G. Knowles.

The work I have done on the third problem can be partly found in the following paper, which was cowritten with Dr. George Weiss, who is currently at Virginia Polytechnic Institute.

[E] "Robustness of Distributed Systems with Respect to Small Time Delays," with G. Weiss, to appear in the Proceedings of the 27th IEEE Conference on Decision and Control, 1990.

Dr. Weiss and I are in the process of writing up a paper for journal submission, entitled "Conditions for Robustness of the Stability of Feedback Systems with Respect to Small Delays in the Feedback Loop."

During the period of the grant I also made substantial revisions to the following paper:



A-1

[F] "Spectral Determination for a Cantilever Beam," IEEE Transactions on Automatic Control, Vol. 34, No. 5, 1989, pp. 502-510.

We conclude this report with a very brief description of the main results.

1) We say that $S(t)$ is μ -exponentially stable if for every $\gamma > \mu$ there exists $M(\gamma)$ such that $\|S(t)\| \leq M(\gamma)e^{\gamma t}$. The system is μ -stabilizable by $K : X \rightarrow U$ if $A + BK$ generates a μ -exponentially stable semigroup $S_K(t)$.

If $B : U \rightarrow X$ is a finite dimensional operator, it has been known for awhile that if the system is μ -stabilizable by a bounded feedback, then A must satisfy, for every $\gamma > \mu$, the following three conditions, where $\sigma(A)$ is the spectrum of A : (a) $\sigma_u(A, \delta) = \{\lambda \in \sigma(A) | \operatorname{Re}(\lambda) > \gamma\}$ consists of finitely many eigenvalues of finite multiplicity; (b) Let A_s be that "part" of A which has spectrum $\sigma(A) \setminus \sigma_u(A, \gamma)$. Then A_s generates a semigroup $S_s(t)$ such that $\|S_s(t)\| \leq Me^{\gamma t}$ for some M ; (c) Let A_u and B_u be the "parts" of A and B associated with $\sigma_u(A, \gamma)$. Then (A_u, B_u) is a controllable pair.

These results have been generalized in the above papers. In [D], the same necessary conditions are found when B is infinite dimensional and BK is compact. In [A] we allow the input operator to be unbounded (that is, with range in a larger space than X) and infinite dimensional, but "admissible" in a standard sense, and let K be compact. In the latter case this has been applied to show that a clamped plate with boundary control cannot be exponentially stabilized in the space of maximum regularity, even though the input operator is unbounded.

2) Regarding question 2, the paper [B] considers operators of the form $\tilde{A}_K = A + (\lambda_0 - A)^{-\eta}BK(\lambda_0 - A)^\eta$, where $\lambda_0 \in \rho(A)$, $\eta \in [-1, 1]$, and $A + BK$ generates a strongly continuous semigroup. Then under many circumstances, described precisely in [B], \tilde{A}_K also generates a strongly continuous semigroup which has the same growth as that generated by $A + BK$. Using this result, if the input operator is of the form $\tilde{B} = (\lambda_0 - A)^{-\eta}B$, where B is admissible, then we can indentify a class of \tilde{K} , some of which are unbounded, for which $A + \tilde{B}\tilde{K}$ generates a C_0 -semigroup. These results also have implications about stabilization. In [B], results of the following nature were obtained: If a cantilever beam is controlled by a moment force on the free end, and the state space is $H_0^2[0, 1] \otimes L^2[0, 1]$, then we cannot uniformly stabilize the beam with an $A^{-1/2}$ -bounded feedback. However, for any $\epsilon > 0$, there is an $A^{\epsilon-1/4}$ -bounded feedback which leads to a closed loop operator which generates a C_0 -semigroup, but the beam cannot be stabilized by any $A^{\epsilon-1/4}$ -bounded feedback.

3) In the past five years there have been several examples showing that exponential stabilization of distributed parameter systems can be destroyed by introducing arbitrarily small time delays into the feedback loop. Recent results by George Weiss and I show that this is a fairly general phenomenon.

The results in paper [E] are given in the frequency domain. Suppose the original system has a one-dimensional input/output space, and the open loop transfer function $H(s)$ is well posed and regular with feedthrough value d . If a time delay of length ϵ is introduced into the feedback loop, the closed loop transfer function is $G^\epsilon(s) = H(s)/(1 + e^{\epsilon s}H(s))$.

Suppose the system is uniformly stabilized by the feedback when there is no delay, so

$G^0 \in H^\infty$, the space of bounded analytic functions on $\{Re(s) > 0\}$. Further suppose that

$$\limsup_{\substack{|s| \rightarrow \infty \\ s \in C_0}} |H(s)| > 1, \quad (1)$$

$$|d| < 1. \quad (2)$$

Then for any $\delta > 0$, there is an $\epsilon \in (0, \delta)$ such that G^ϵ has poles in $\{Re(s) > 0\}$. This shows that if the open loop system is well posed and regular and (1) and (2) are satisfied, any exponential stabilization of H cannot be robust with respect to small time delays. Condition (1) will of course always occur when the open loop system has no damping, which is the case in all of the examples given previously.

We also have a simple positive result about robustness, and we can take the input/output space to be any Hilbert space Y , so the transfer functions are $L(Y)$ -valued. Suppose H is well posed and is analytic on $\{Re(s) > 0 \mid |s| > r\}$ for some $r > 0$. Suppose further that

$$\limsup_{\substack{|s| \rightarrow \infty \\ s \in C_0}} \|H(s)\| < 1.$$

Then if the closed loop system without delay is exponentially stabilized (that is, $G^0 \in H^\infty$), there is a $\delta > 0$ such that for any $\epsilon \in (0, \delta)$, $G^\epsilon \in H^\infty$. Thus under these conditions exponential stabilization is robust with respect to arbitrarily small time delays.

The grant has also aided my research by partially or fully funding the following professional travel:

The 27th IEEE Conference on Decision and Control, Austin, Texas, December 1988, where a talk was given.

SIAM Conference on Control in the 90's, San Francisco, May, 1989, where a talk was given.

The Fifth IFAC Symposium on Control of Distributed Parameter Systems, Perpignan, France, June 1989, where a talk was given.

The 1989 Conference on the Mathematical Theory of Systems and Networks, Amsterdam, June, 1989.

New Trends in Control of Distributed Parameter Systems, Institute for Mathematics and Applications, University of Minnesota, August 1989, where a talk was given.

SIAM Annual Meeting, Chicago, July, 1990.

I visited Dr. George Weiss in Blacksburg, Virginia for a week in June 1990, where I gave a talk and collaborated on research.

Dr. John Lagnese visited the University of Nebraska in October, 1989, and he gave a colloquium.

Dr. George Weiss visited the University of Nebraska in December, 1989, and he gave a seminar and collaborated with me on research.