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NATO Advanced Study Institute

on Waveguide Optoelectronics

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DESIGN AND MODELLING OF PASSIVE AND ACTIVE OPTICAL WAVEGUIDE DEVICES

Roel BAETS

Laboratory of Electromagnetism and Acoustics (LEA)

University of Gent, Belgium

LEA

RAD 6414-EE-02

DAJA45-90-M-0130

USARD+SG(UK)

OUTLINE

1. MODES IN WAVEGUIDES
2. LONGITUDINALLY VARIABLE STRUCTURES
3. COUPLED MODE THEORY
   - COUPLERS
   - STRUCTURES WITH GRATINGS
4. COUPLING LIGHT INTO WAVEGUIDES
5. WAVEGUIDES IN LASER CAVITIES
6. SOME GENERALITIES ABOUT WAVEGUIDE MODELLING AND DESIGN

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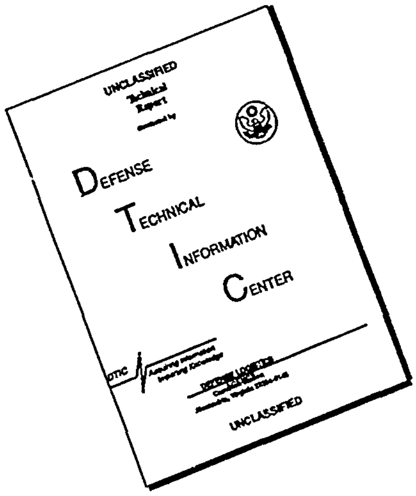

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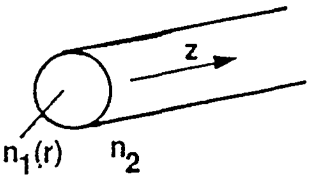
WAVE PROPAGATION MODEL
DEFINITION OF PROBLEM



$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t} \qquad \nabla \times \overline{H} = \frac{\partial \overline{D}}{\partial t}$$

$$\nabla \cdot \overline{D} = 0 \qquad \nabla \cdot \overline{B} = 0$$

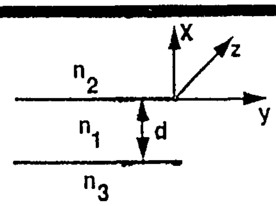
$$\overline{D} = \varepsilon_0 n^2 \overline{E} \qquad \overline{B} = \mu_0 \overline{H}$$

Find solutions of the form (modes) :

$$\overline{E} = \overline{E}(r, \phi) \exp[j(\omega t - \beta z)]$$

$$\overline{H} = \overline{H}(r, \phi) \exp[j(\omega t - \beta z)]$$

WAVE PROPAGATION MODEL
SLAB WAVEGUIDE



Two independent sets of solutions :

E_y, H_x, H_z (TE)

H_y, E_x, E_z (TM)

$$\text{TE}: \frac{d^2 E_y}{dx^2} + (n^2 k^2 - \beta^2) E_y = 0 \qquad \left(k = \frac{\omega}{c}\right)$$

$$\begin{cases} E_y = A\, e^{-\delta x} & x \geq 0 \\ E_y = A \cos \kappa x + B \sin \kappa x & 0 \leq x \leq -d \\ E_y = (A \cos \kappa d - B \sin \kappa d)\, e^{\gamma (x+d)} & x \leq -d \end{cases}$$

with $\delta = \sqrt{\beta^2 - n_2^2 k^2}$ $\qquad \kappa = \sqrt{n_1^2 k^2 - \beta^2}$ $\qquad \gamma = \sqrt{\beta^2 - n_3^2 k^2}$

WAVE PROPAGATION MODEL
SLAB WAVEGUIDE

Continuity of E_y and H_z ($\sim \frac{\partial E_y}{\partial x}$) at interfaces

→ eigenvalue equation

$$\mathrm{tg}\,\kappa d = \frac{\kappa(\gamma+\delta)}{\kappa^2-\gamma\delta} \longrightarrow$$ discrete number of solutions for β

→ $\beta(\lambda, n_1, n_2, n_3, d)$

WAVE PROPAGATION MODEL
SLAB WAVEGUIDE

NORMALISATION

Waveguide characterized by

$$v = k_0 d \sqrt{n_1^2 - n_2^2}$$ v-number (normalized frequency or thickness)

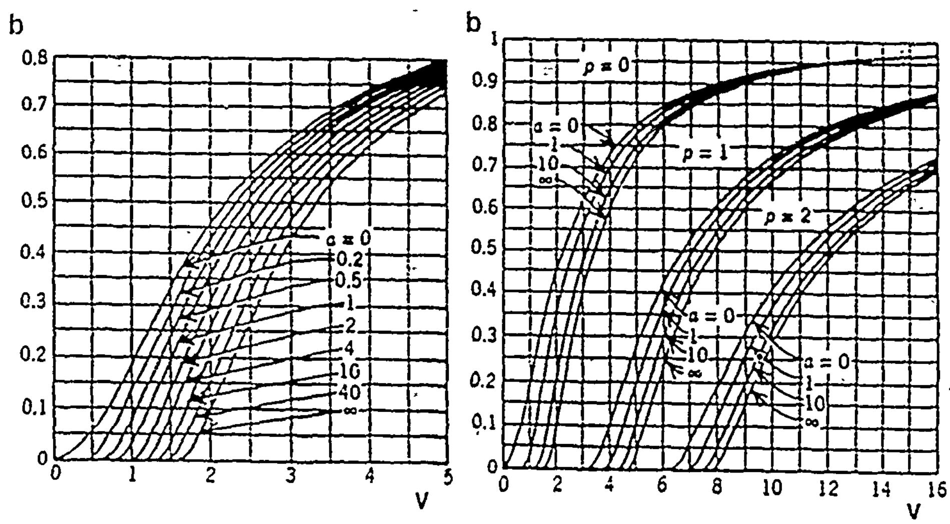
$$a = \frac{n_2^2 - n_3^2}{n_1^2 - n_2^2}$$ asymmetry factor

Mode characterized by

$$\beta = k_0 n_{eff}$$

$$b = \frac{n_{eff}^2 - n_2^2}{n_1^2 - n_2^2}$$

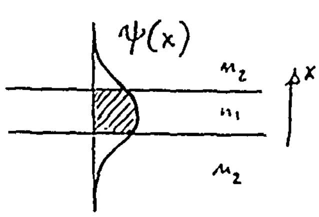
WAVE PROPAGATION MODEL
SLAB WAVEGUIDE



FUNDAMENTAL TE MODE

3 LOWEST ORDER TE MODES

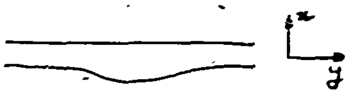
CONFINEMENT FACTOR Γ



$$\Gamma = \frac{\int_{core} |\psi|^2 dx}{\int_{-\infty}^{\infty} |\psi|^2 dx}$$

If the core layer shows gain g [cm^{-1}]
then the modal gain is Γg

EFFECTIVE INDEX METHOD



$$n(x,y) = n_0(x) + \Delta n(x,y)$$

$$\nabla_T^2 E_y + (k_0^2 n^2(x,y) - \beta^2) E_y = 0$$

Assumption : $E_y(x,y) = F(x,y) \cdot G(y)$ where fast variations along y are taken up in $G(y)$

$$\rightarrow \frac{\partial F}{\partial y} \simeq 0$$

$$\rightarrow \underbrace{\frac{1}{G}\frac{d^2 G}{dy^2}}_{\text{function of } y} + \underbrace{\frac{1}{F}\frac{\partial^2 F}{\partial x^2} + k_0^2 n^2}_{\text{function of } x \text{ and } y} - \beta^2 = 0$$

$$\rightarrow \frac{1}{F}\frac{\partial^2 F}{\partial x^2} + k_0^2 n^2 = \gamma^2(y)$$

$$\frac{1}{G}\frac{d^2 G}{dy^2} - \beta^2 = -\gamma^2(y)$$

$$\gamma \triangleq k_0 n_{eff}(y)$$

EFFECTIVE INDEX METHOD

Approximation of $n_{eff}(y)$ by perturbation method

$$n(x,y) = n_0(x) + \Delta n(x,y)$$

$F_0(x)$ satisfies : $\frac{d^2 F_0}{dx^2} + k_0^2 (n_0^2 - n_{eff,0}^2) F_0 = 0$ (1)

$F(x,y)$ satisfies : $\frac{\partial^2 F}{\partial x^2} + k_0^2 (n^2 - n_{eff}^2) F = 0$ (2)

$$n^2 \approx n_0^2 + 2\Delta n\, n_0$$

$$\int_{x\text{-axis}} (1) F - (2) F_0 \longrightarrow \left(n_{eff}^2(y) - n_{eff,0}^2\right) \int_{-\infty}^{\infty} F_0 F \, dx = \int_{-\infty}^{\infty} 2\Delta n\, n_0 F F_0 \, dx$$

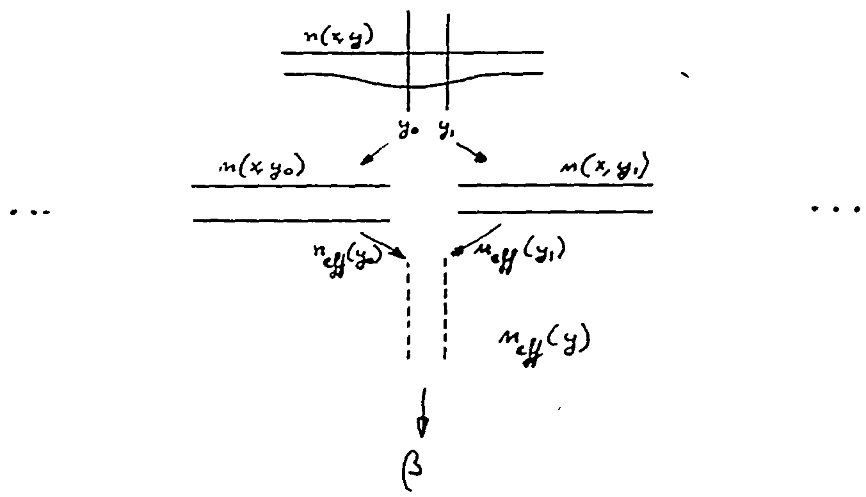
$$\longrightarrow n_{eff}^2(y) = n_{eff,0}^2 + \frac{\int_{-\infty}^{\infty} 2\Delta n\, n_0 F F_0 \, dx}{\int_{-\infty}^{\infty} F_0 F \, dx}$$

$$\approx n_{eff,0}^2 + 2 \frac{\int_{-\infty}^{\infty} \Delta n\, n_0 F_0^2 \, dx}{\int_{-\infty}^{\infty} F_0^2 \, dx}$$

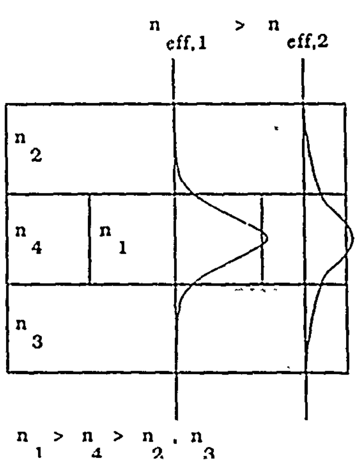
$$\left\{\begin{array}{l} \dfrac{\partial^2 F}{\partial x^2} + \left(k_0^2 n^2(x,y) - k_0^2 n_{eff}^2(y)\right) F = 0 \\[2ex] \dfrac{d^2 G}{d y^2} + \left(k_0^2 n_{eff}^2(y) - \beta^2\right) G = 0 \end{array}\right.$$

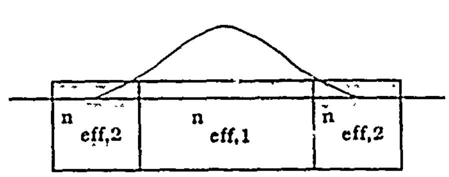
1D wave equation (to be solved for each y)

1D wave equation (to be solved once)



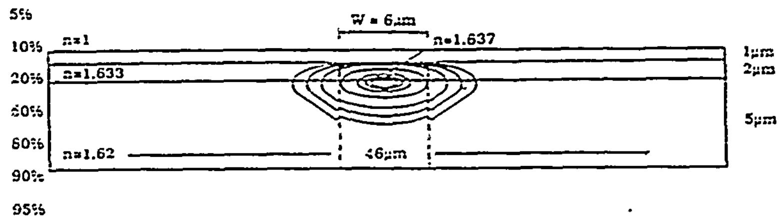
EFFECTIVE INDEX METHOD



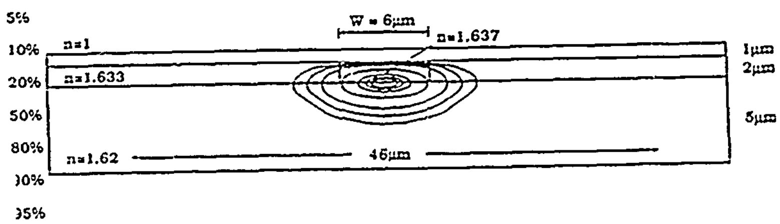


FUNDAMENTAL WAVEGUIDE MODE

A. EFFECTIVE INDEX METHOD



B. FINITE DIFFERENCE METHOD



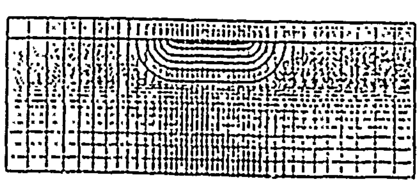
FINITE DIFFERENCE METHOD

BASIC EQUATION :

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + (k_0^2 \varepsilon_r - \beta^2)\phi = 0 \qquad (1)$$

* A finite cross section is defined by enclosing the guide in a rectangular box, where $\phi = 0$ on the side walls

* In this box a graded mesh is defined



FINITE DIFFERENCE METHOD

Continuity conditions :

$$\left.\begin{array}{l}\phi \\ \dfrac{\partial \phi}{\partial n}\end{array}\right\} \text{continous at boundary between } S_\upsilon$$

Substitution of the discreted form of $\nabla_t^2 \phi$ and the continuity conditions into (1) leads to :

$$\frac{2}{w(e+w)}\phi_W + \frac{2}{s(n+s)}\phi_S - \left(\frac{2}{w(e+w)} + \frac{2}{e(e+w)} + \frac{2}{s(n+s)}\right.$$

$$\left.\frac{2}{n(n+s)}\right)\phi_P + \frac{2}{e(e+w)}\phi_E + \frac{2}{n(n+s)}\phi_N \qquad (5)$$

$$+ k_0^2 \frac{wn\varepsilon_1 + ws\varepsilon_2 + es\varepsilon_3 + en\varepsilon_4}{(e+w)(n+s)}\phi_P - \beta^2 \phi_P = 0$$

FINITE DIFFERENCE METHOD

Equation (5) holds for each node point P. The resultant eigenvalue equation is of the form

$$[[A] - \beta^2 [U]]\ [X] = 0 \qquad (6)$$

with

$$[X] = [\phi_1, \phi_2, ..., \phi_{NTOT}]^T \qquad (7)$$

[U] is the unit matrix, and NTOT is the total number of mesh points. The matrix [A] is a real, but generally not symmetric, sparse matrix. Eigenvalues and corresponding eigenvectors of [A] are found by a simultaneous iteration algorithm.

BEAM PROPAGATION METHOD
TWO DIMENSIONAL

Problem :

Calculation of the propagation of a given input field E_0 (x,z) through a medium with a refractive index n(x,z)

Assumptions :

1. Scalar wave equation

$$\nabla^2 E + k^2 n^2 (x,z) E = 0 \qquad (1)$$

BEAM PROPAGATION METHOD
TWO DIMENSIONAL

2. Refractive index variation can be written as :

$$n(x,z) = n_0(x) + \Delta n(x,z) \qquad (2)$$

where $\Delta n \ll n_0$ and n_0 (x) chosen so that the solutions of :

$$\nabla^2 \Phi + k^2 n_0^2 (x) \Phi = 0 \qquad (3)$$

are known eigenfunctions :

$$\Phi_n (x) e^{-jk_n z} \qquad (4)$$

In practice . n_o = constant

BEAM PROPAGATION METHOD
TWO DIMENSIONAL

3. Neglect the influence of the reflected fields on the forward propagating beam :

- no large abrupt change of $n(x,z)$ as a function of z
- no periodic reflections that add up coherently

This assumption yields for a field $\varepsilon(x,z)$ propagating in n_0

$$\varepsilon = \sum_{n=1}^{\infty} B_n \Phi_n e^{-jk_n z} \qquad (5)$$

Boundary value problem transformed into an initial value problem

⇒ a stepwize solution feasible

4 Paraxial fields

BEAM PROPAGATION METHOD
TWO DIMENSIONAL

Assume

$$E(x, z_0 + \Delta z) = \varepsilon(x, z_0 + \Delta z) \,.\, e^{\Gamma} \qquad (7)$$

where

$$\varepsilon(x, z_0 + \Delta z)$$

is propagated in the homogeneous medium n_0,
e^{Γ} is a correction factor due to $\Delta n(x,z)$

$\varepsilon(x,z)$ satisfies

$$\nabla^2 \varepsilon + k^2 n_0^2 \varepsilon = 0 \qquad (8)$$

ALGORITHM FOR ONE PROPAGATION STEP

$E(y, z_0)$
$\downarrow$
FOURIER TRANSFORM
$\downarrow$
$E^F(k_y, z_0)$
$\downarrow$
PROPAGATION OF EACH PLANE WAVE COMPONENT
$(\times\ e^{-jk_z \Delta z})$
$\downarrow$
$E^F(k_y, z_0 + \Delta z)$
$\downarrow$
INVERSE FOURIER TRANSFORM
$\downarrow$
$\tilde{E}(y, z_0 + \Delta z)$
$\downarrow$
CORRECTION FOR Δn
$(\times\ e^{-jk_0 \Delta n\, \Delta z})$
$\downarrow$
$E(y, z_0 + \Delta z)$

BOUNDARY CONDITIONS

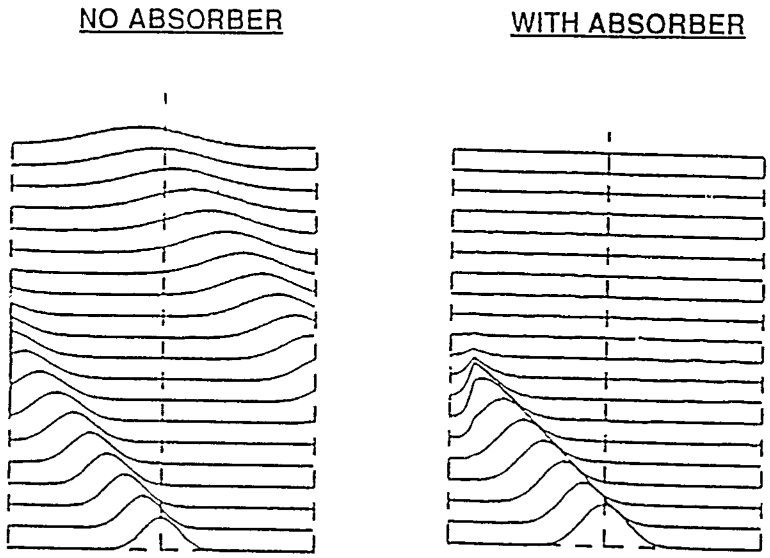
FFT is

- discrete in both x and k_x space
- limited window in both dimensions

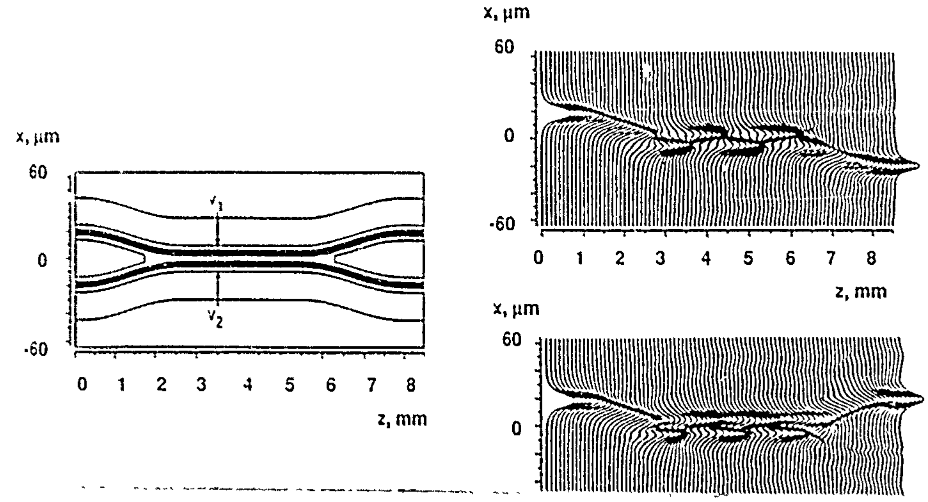
FFT is F.T. of the periodic extension of the field

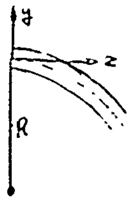
Radiation condition is simulated by absorbing region at the edges.
(- windowing filter)

GAUSSIAN BEAM
HOMOGENEOUS MEDIUM



BPM EXAMPLE : DIRECTIONAL COUPLER





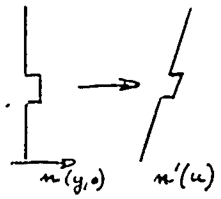
BENDS

conformal transformation : $w = u + jv = R \ln \frac{y + jz + R}{R}$

→ - waveguide in (u, v) plane is v-independent

- wave equation becomes new wave equation with $n'(u) = \exp(u/R)\, n(u)$

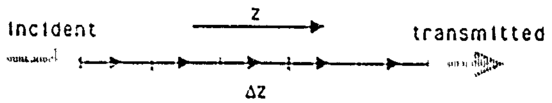
$$\approx \left(1 + \frac{u}{R}\right) n(u) \quad \text{for } u \ll R$$



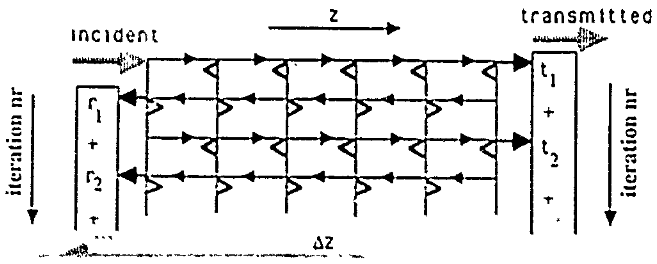
⇒ curved waveguide can be analysed as straight waveguide with modified refractive index profile

ITERATION PRINCIPLE OF THE UNIDIRECTIONAL & BIDIRECTIONAL BPM

UNIDIRECTIONAL BPM

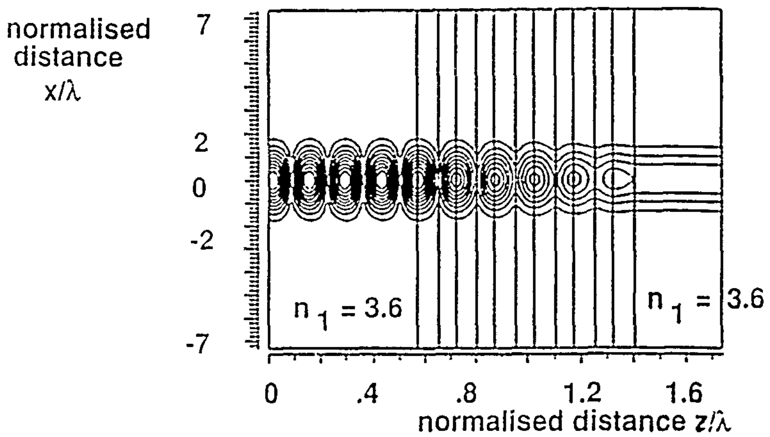


BIDIRECTIONAL BPM



REFLECTIONS OF A GAUSSIAN BEAM INCIDENT ON A BRAGG REFLECTOR

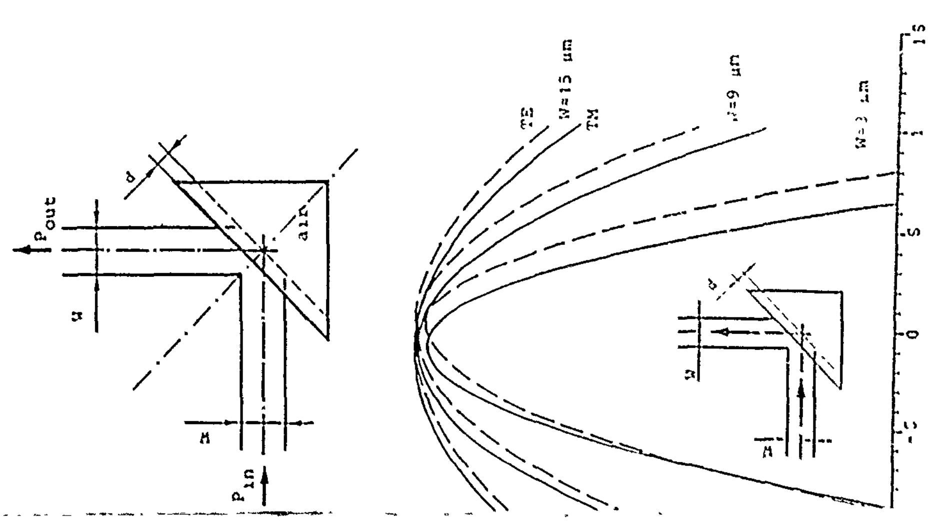
simulated by the bidirectional BPM at $\lambda = 0{,}864\ \mu m$



- total field amplitude $|E(x,z)|$ is normalised to 1
- contour lines with interval 0.1 are plotted

S AT 90° CORNER BENDS

BPM ANALYSIS

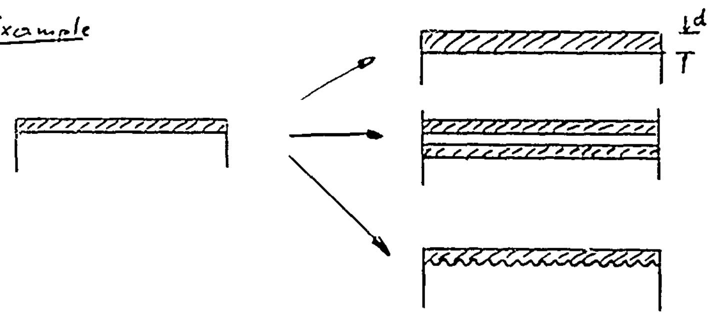


COUPLED MODE THEORY

ANALYSIS OF DIELECTRIC STRUCTURE WHICH IS A PERTURBATION OF A SIMPLER DIELECTRIC STRUCTURE

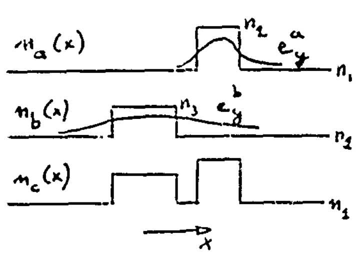
→ <u>COUPLED MODE THEORY</u>

<u>Example</u>



Unperturbed	Perturbed
$\Psi = \sum A_i \Psi_i$	$\Psi = \sum A_i(z)\, \Psi_i$
	$A_i(z)$?

DIRECTIONAL COUPLER



unperturbed waveguide "a"

unperturbed waveguide "b"

perturbed waveguide (both for "a" and "b")

$$E_y = A(z)\, e_y^a(x)\, e^{j(\omega t - \beta_a z)} + B(z)\, e_y^b(x)\, e^{j(\omega t - \beta_b z)}$$

$$\downarrow \qquad\qquad \downarrow$$

$$P^a_{pert} = e^{j\omega t} \varepsilon_0\, e_y^a(x) A(z) [n_c^2 - n_a^2] e^{-j\beta_a z} \quad \Big| \quad P^b_{pert} = e^{j\omega t} \varepsilon_0\, e_y^b(x) B(z) [n_c^2 - n_b^2] e^{-j\beta_b z}$$

$$\rightarrow \quad \frac{dA}{dz} e^{j(\omega t - \beta_a z)} = \frac{j}{4\omega} \frac{\partial^2}{\partial t^2} \int_{-\infty}^{\infty} [P^a_{pert} + P^b_{pert}]\, e_y^a(x)\, dx$$

$$\begin{cases} \dfrac{dA}{dz} = -j\kappa_{ab}\, B\, e^{+j(\beta_a-\beta_b)z} - j M_a A \\ \dfrac{dB}{dz} = -j\kappa_{ba}\, A\, e^{j(\beta_b-\beta_a)z} - j M_b B \end{cases}$$

with

$$\kappa_{\substack{ab\\ba}} = \frac{\omega\varepsilon_0}{4}\int_{-\infty}^{\infty}\left(n_c^2 - n_{\substack{b\\a}}^2\right) e_y^a\, e_y^b\, dx$$

$$M_{\substack{a\\b}} = \frac{\omega\varepsilon_0}{4}\int_{-\infty}^{\infty}\left(n_c^2 - n_{\substack{a\\b}}^2\right)\left(e_y^{\substack{a\\b}}\right)^2 dx$$

Change of variables: $A = A' e^{-jM_a z}$ and $B = B' e^{-jM_b z}$

$$\begin{cases} \dfrac{dA'}{dz} = -j\kappa_{ab}\, B'\, e^{j(\beta_a+M_a-\beta_b-M_b)z} \triangleq -j\kappa_{ab}\, B'\, e^{2j\delta z} \\ \dfrac{dB'}{dz} = -j\kappa_{ba}\, A'\, e^{j(\beta_b+M_b-\beta_a-M_a)z} \triangleq -j\kappa_{ba}\, A'\, e^{-2j\delta z} \end{cases}$$

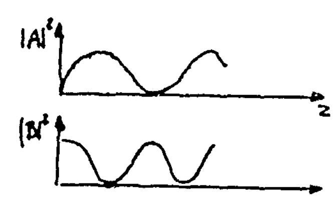
$$2\delta = \beta_a + M_a - \beta_b - M_b$$

CASE 1 : $\delta = 0$ (symmetry)

$\kappa_{ab} = \kappa_{ba} = \kappa$

$|A|^2 = P_0 \sin^2(\kappa z)$

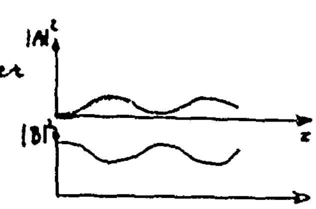
$|B|^2 = P_0 \cos^2(\kappa z)$



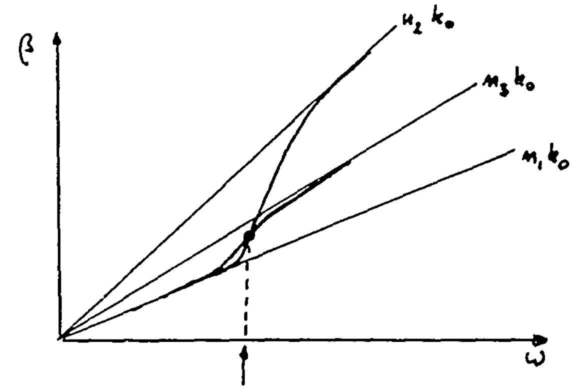
Power transfer length : $\boxed{L = \dfrac{\pi}{2\kappa}}$

CASE 2 : $\delta \neq 0$ $\rightarrow$ No complete power transfer

$$\frac{|A|^2_{max}}{P_0} = \frac{|\kappa_{ba}|^2}{\kappa_{ba}\kappa_{ab} + \delta^2}$$



DIRECTIONAL COUPLER



$k_o = \frac{\omega}{c}$

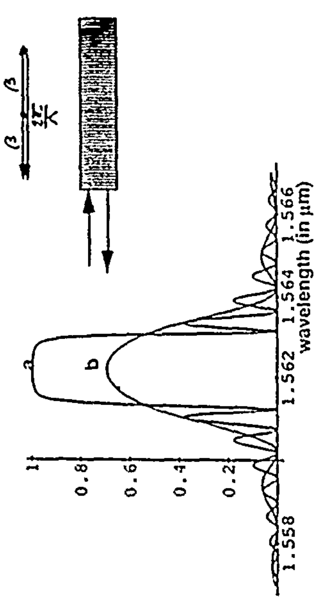
Frequency where coupling is strong

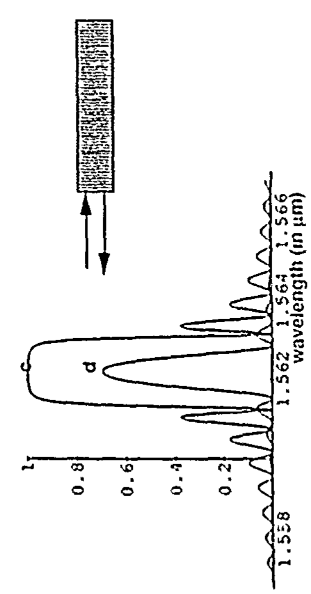
WAVEGUIDES WITH GRATINGS

(BRAGG REFLECTORS)

ANALYSED WITH COUPLED MODE THEORY

Strong coupling if $\beta - \frac{2\pi}{\Lambda} = -\beta$





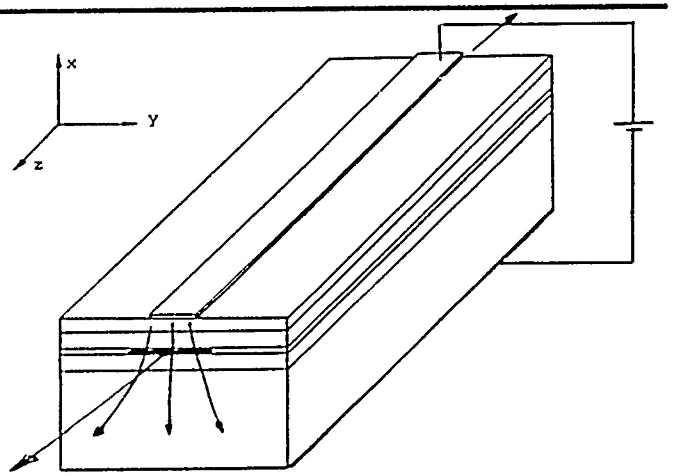
a. $L = 600\ \mu m$, $\kappa = 60\ cm^{-1}$

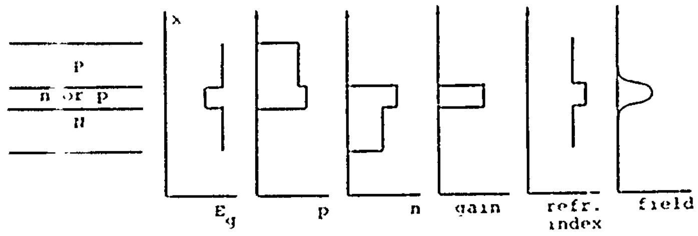
b. $L = 300\ \mu m$, $\kappa = 60\ cm^{-1}$

c. $L = 600\ \mu m$, $\kappa = 30\ cm^{-1}$

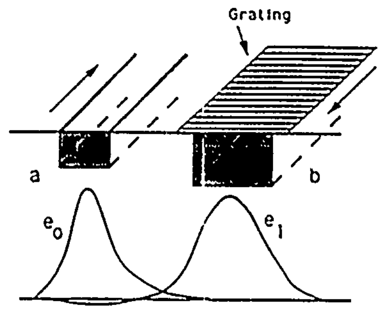
d. $L = 600\ \mu m$, $\kappa = 20\ cm^{-1}$

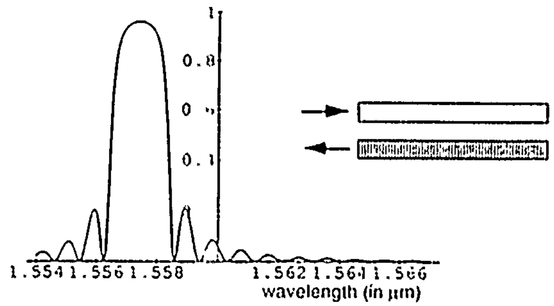
SEMICONDUCTOR LASER DIODES
Some basics

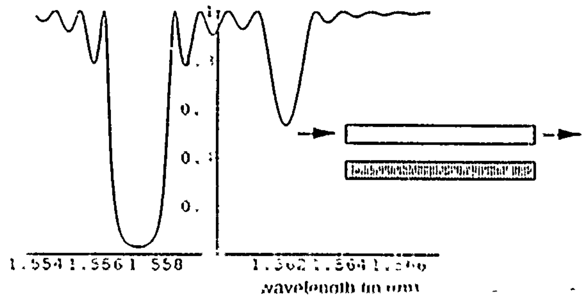




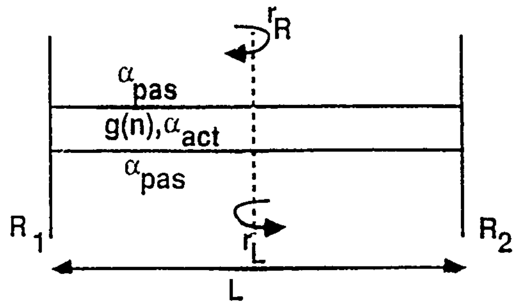
IMEC-RUG







LASER DIODES
Cavity Resonance



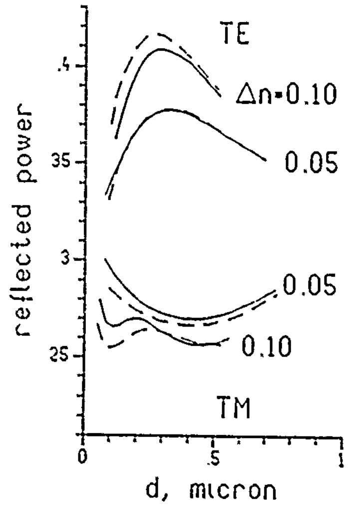
FABRY-PEROT CAVITY

RESONANCE : $\underbrace{r_L(n,\lambda)\cdot r_R(n,\lambda)}_{\text{roundtrip gain}} = 1$

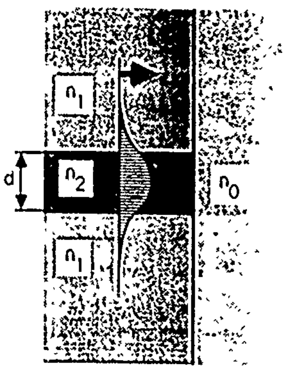
FABRY-PEROT : $g(n) = \alpha + \frac{1}{2L} \ln \frac{1}{R_1 R_2}$ (amplitude)

$$\frac{\lambda}{n_r} = \frac{2L}{m}$$, m integer (phase)

BIDIRECTIONAL BPM : REFLECTION FROM A SEMICONDUCTOR LASER FACET



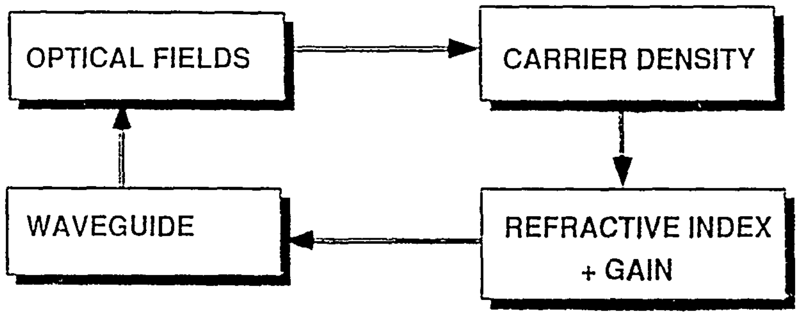
—— bidirectional BPM
······· exact solution



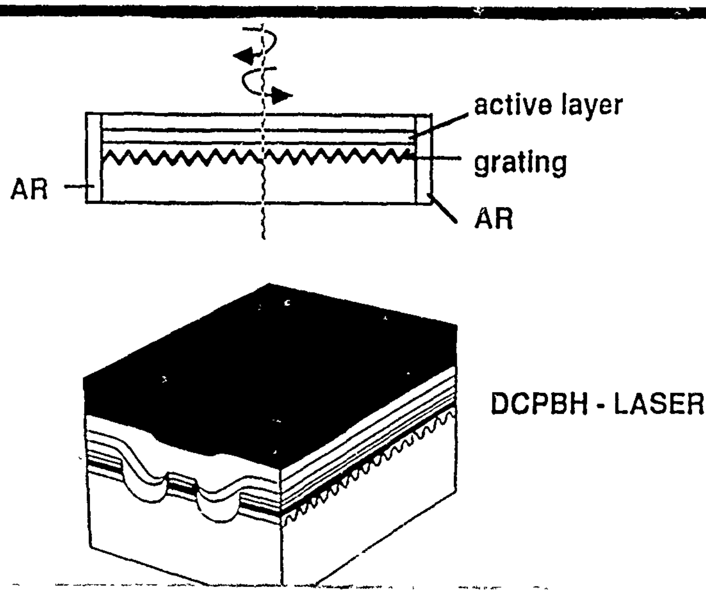
Laser facet configuration

SELF CONSISTENT MODELLING

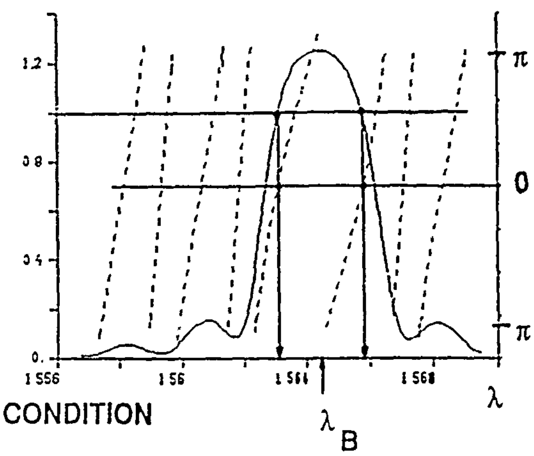
EXAMPLE : ELECTRICAL - OPTICAL INTERACTION



SLM LASERS
DISTRIBUTED FEEDBACK (DFB) LASER



SLM LASERS
DFB LASER



RESONANCE CONDITION

PHASE : - no mode at λ_B
- modes symmetrically around λ_B

→ two lines in spectrum

SLM LASERS
SINGLE MODE DFB LASERS

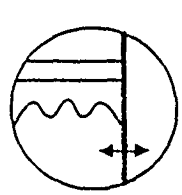
1. ONE OR TWO NON-AR-COATED FACETS

→ ASYMMETRIC LASER

→ 2 MODES DO NOT HAVE THE SAME THRESHOLD GAIN

→ ONE LINE IN SPECTRUM

PROBLEM WITH FACETS IN DFB LASERS :

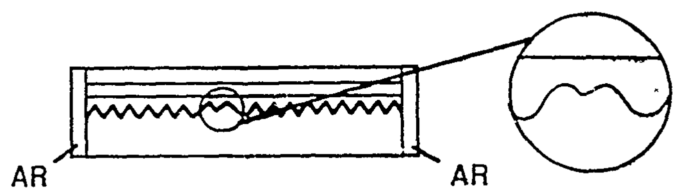


position of facets with respect to grating can not be controlled technologically

→ some lasers good, some bad
→ yield problem

SLM LASERS
SINGLE MODE DFB LASERS

2. $\lambda/4$ PHASE SHIFTED LASERS



→ PHASE RESONANCE AT λ_B

→ ONE LASING PEAK AT λ_B

LONGITUDINAL SPATIAL HOLE BURNING

P(z)

⇓

N(z)

⇓

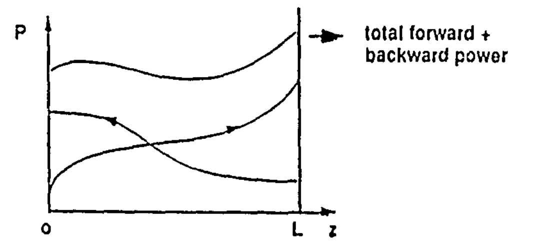
$\Delta n_r(z)$

⇓

Bragg Deviation

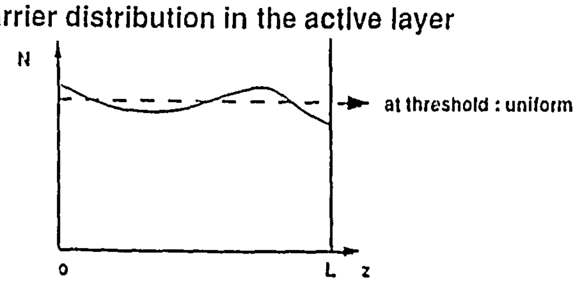
$\frac{2\pi}{\lambda} n_{eff}(z) - \frac{\pi}{\Lambda}$

Internal Power Distribution in a DFB laser



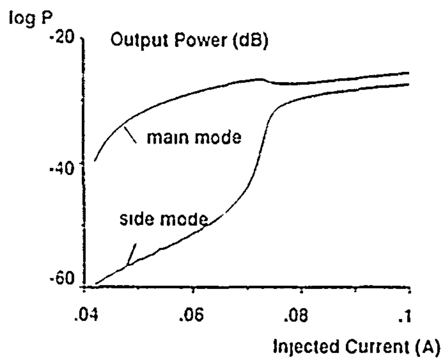
⇓ stimulated recombination

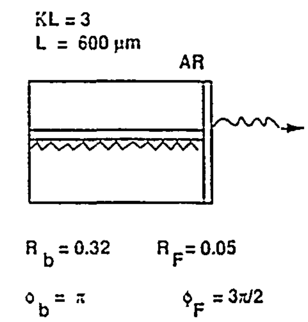
Carrier distribution in the active layer



DC ANALYSIS OF DFB LASER

<u>EXAMPLE</u> : UNSTABLE SINGLE MODE BEHAVIOUR DUE TO SPATIAL HOLE BURNING



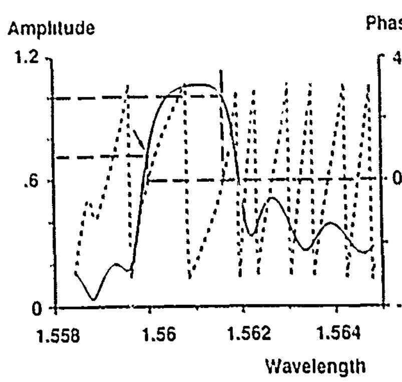


$I_{th} = 39.56$ mA, $\lambda_{th} = 1.5616$, $\Delta\alpha L = 0.25$

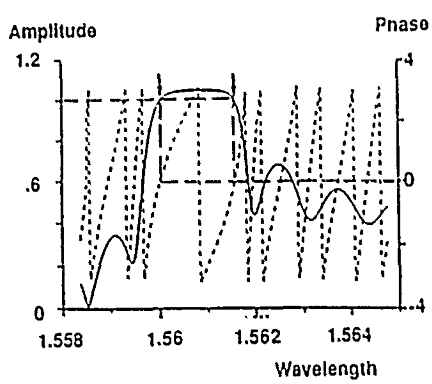
DC ANALYSIS OF DFB LASER

<u>ROUNDTRIP GAIN OF UNSTABLE DFB LASER</u>

At threshold main mode
$I = 39.6$ mA



At threshold side mode
$I = 70$ mA



SOME CHARACTERISTICS OF OPTICAL WAVEGUIDE MODELS

① Rigorous solution of a waveguide problem is difficult

$\Downarrow$

People introduce simplifications

BUT: Validity of simplifications not clearly established

Simplifications very specific to specific structures
$\Rightarrow$ many different models

② Very few problems can be solved analytically

$\Downarrow$

Numerical solution methods

BUT:
- numerical solution does not easily allow for design and optimisation within certain parameter space
- numerical solution introduces numerical error

Validation ??

Simple model + well established numerical procedures	$\xleftrightarrow{?}$	rigorous model + less established numerical procedures

③ It is difficult to compare modelling results with experimental results quantitatively

Because :
- modelling result based on simplified model
- experimental structure not sufficiently well defined

④ All models are limited to single (or few) device level

No higher level "circuit-like" models available.

⑤ Most software implementations are produced by universities

- maintenance + servicing + updating ??

CONCLUSIONS

- Optical waveguide modelling requires simplification.
  Care is needed to judge the appropriateness of the simplifications
- Modelling is good for
  - demonstrating conceptual ideas theoretically
  - explain experimental observations
  - t is inadequate for accurate design and optimisation

An evolution is needed towards integrated modelling and CAD-tools that contain a number of models with a flexible interface in between them and with sufficient intelligence to protect the user against use beyond the validity range of the models.