US Army Corps of Engineers
 \\ \section*{BACKCALCULATION OF COMPOSITE \\ \section*{BACKCALCULATION OF COMPOSITE PAVEMENT LAYER MODULI} PAVEMENT LAYER MODULI}

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## PREFACE

This report was submitted as a doctoral dissertation to the University of Kentucky by the author, Dr. Mark Anderson, and is published herein in dissertation format. The dissertation was directed by Dr. Vincent Paul Drnevich, of the Department of Civil Engineering, University of Kentucky. Dr. Anderson received his degree (Doctor of Philosophy in Civil Engineering) from the University of Kentucky on 16 December 1988.

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Research completed at WES was conducted under the general supervision of Dr. W. F. Marcuson III, Chief, GL, Mr. H. H. Ulery, Jr., Chief, Pavement Systems Division (PSD), and Mr. J. W. Hall, Jr., Chief, Engineering Investigations, Testing, and Validation Group. Direct supervision of this research was provided by Dr. A. J. Bush III, PSD, who was the responsible Principal Investigator.

COL Larry B. Fulton, EN, is Commander and Director of WES. Dr. Robert W. Whalin is Technical Director.


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## CHAPTER I

## INTRODUCTION

## TERMINOLOGY

Conventional pavements are three-layer systems which are composed of an underlying subgrade material, a base layer, and a surface layer. The base layer is typically composed of a densely graded crushed rock material. There are two general conventional pavement types, rigid and flexible, which use Portland cement concrete (PCC) and asphaltic concrete (AC) as their surface layers, respectively. Full-depth pavements are similar to conventional pavements, but with the surface layer founded direct?y on the subgrade material.

Composite pavement is a term which has been used in various publications to describe a variety of pavement types. A composite pavement, in general, is a pavement which is composed of layers and/or materials not commonly found in conventional pavements. Examples of composite pavements include pavements with stabilized subgrade or subbase layers, semi-rigid or rigid bases, and multiple surface layers. When used without qualification within this document, the term composite pavement describes a full-depth Portland cement concrete pavement which has been overlain with asphaltic concrete.

When used without qualification in this document, the term nondestructive testing refers to load-deflection pavement evaluation testing using either a vibratory deflection device or an impulsive deflection device.

## BACKGROUND

## Historical

The use of nondestructive testing has been an increasingly cost effective tool in the evaluation of both airfield and highway pavements for the determination of structural condition. Results from nondestructive evaluations are used to estimate remaining life and allowable loads, as well as to provide data for design calculations. Evaluation methods have been developed (Bush and Alexander 1985) which give reasonable results for both rigid and flexible pavements. However, an increasingly large number of rigid pavements have been overlain with asphaltic concrete. These pavements have posed particular problems for evaluation and design.

Most design procedures for overlays of rigid airfield pavements, including the current Department of Defense (1986) procedure, use an equivalent thickness method. The typical method is to compute a design thickness of Portland cement concrete pavement and estimate an overlay thickness of asphaltic concrete which will provide equivalent support. The total overlay thickness depends not only on the existing rigid pavement layer thickness, but also on condition factors which are determined from a visual inspection of the condition of the Portland cement concrete slabs. This method was developed for the case of an initial overlay design for an exposed rigid pavement layer, and the method has deficiencies when used to design additional overlays. Overdesigns which lead to increased costs can occur when a sound Portland cement concrete layer with a nonstructural overlay is assumed to be badly cracked. Underdesigns which lead to pavement failures can occur when a badly
cracked Portland cement concrete layer with a structural overlay is assumed to be of good quality. When a Portland cement concrete pavement has already had an asphaltic concrete overlay, so that the condition of the underlying rigid layer cannot be determined by visual inspection, there are three general evaluation alternatives for the determination of design criteria. These alternatives are the use of historical data, the use of destructive testing, or the use of nondestructive testing.

The determination of pavement structural condition from historical data is ambiguous. Visual surveys have not been performed routinely on most airfield pavements. In most cases, historical data is limited to very general construction records. Use of construction data is complicated by the lack of knowledge about past overlays. A past overlay may have been designed for structural or nonstructural purposes, or both. Nonstructural overlays serve such purposes as improved skid resistance or to match the elevation of an adjoining pavement section. Structural overlays may be used to return a damaged pavement to the original design capability or to increase the structural capability when a change in mission has increased the design requirements. Long-term maintenance plans or stage designs may include periodic overlays which fulfill both structural and nonstructural needs. Overlay designs are further complicated when runways are extended or built in sections. It is therefore possible that an asphaltic concrete overlay over a rigid airfield pavement may be attributed to a combination of factors, some of which may not be structurally related. Judging rigid layer condition by the thickness of over?ay or other available historical data is a questionable practice which usually involves the use of "rules-of-thumb" or is based on the experience of the design engineer.

Destructive testing for the determination of the structural condition of an underlying rigid layer is preferred to the use of historical data, but has definite drawbacks. Small area tests, such as coring or sawing of rectangular prisms, may not be representative of the structural capability of the entire underlying rigid layer. It is not possible to accurately determine the condition of the underlying rigid layer by destructive testing without gross disturbance of relatively large areas by pavement surface removal or pavement trenching. This type of testing is costly, time consuming, and can lead to confusing results due to the sample disturbance which undoubtedly occurs during the removal process.

The preferred alternative for the determination of the condition of an underlying rigid pavement layer would involve nondestructive testing. Past nondestructive evaluations of military airfields have typically been accomplished by the dynamic stiffness modulus method (Green and Hall 1975), which is a good indicator of the overall structural capability of pavements. However, the dynamic stiffness modulus method does not give a clear indication of the relative structural behavior of the pavement layers. Recent developments in the nondestructive evaluation of pavements has led to the use of multilayer linear elastic modeling (Bush and Alexander 1985). Analysis is accomplished by modeling the pavement system as an equivalent elastic layered system. Utilization of this technique to evaluate pavement systems makes it possible to distinguish the relative behavior of the pavement layers. However, this differentiation is particularly difficult for rigid pavements with flexible overlays because both of the upper layers are relatively stiff compared to the underlying material. To be effective, a nondestructive testing
procedure for these pavement types must be able to identify the relative structural behavior of the Portland cement concrete and asphaltic concrete layers, so that an improved design method using multilayer linear elastic theory could be utilized.

Both evaluation and design procedures can be improved by the use of multilayer theories, and computer programs for this purpose have been developed (Department of the Navy 1986). However, for these design procedures to be effective, a fast and accurate method of determining layer moduli of elasticity must be available. The use of nondestructive testing for the estimation of layer moduli of elasticity is clearly the best alternative. Analysis of nondestructive testing data using multilayer linear elastic theory may be accomplished by existing methods for conventional pavement types, but an improved method is needed to separate the relative behavior of adjacent layers of asphaltic concrete and Portland cement concrete.

## Idealized Modeling

All common methods of nondestructive evaluation of pavement systems utilize an idealized mathematical model for comparison with the real pavement system. Figure 1 illustrates some of the differences between a typical idealized model and a real pavement system. In general, the properties of the real and idealized systems are not the same. Use of nondestructive test data to backcalculate layer properties implies that equivalent deflection responses are indicative of equivalent systems. In fact, this is not the case. Backcalculated moduli are effective moduli which apply only to the assumed idealized model. If layer properties from the idealized model are to be used in a
subsequent overlay design or remaining life prediction, it is essential that a consistent idealized modeling approach be used in the subsequent processes. As more sophisticated design procedures are developed and implemented, consistent evaluation models are required. Use of a dynamic structural response model can provide a more realistic model for comparison and subsequent design. However, the use of a dynamic model does not remove the need for consistency between evaluation and design procedures because the layer moduli backcalculated with a dynamic model are still effective moduli which apply only to the assumed dynamic idealized model.


IDEALIZED MODEL OF COMPOSITE PAVEMENT

ACTUAL BEHAVIOR

1) Multiple layers
2) Layers have variable thickness
3) Properties vary with depth
4) Interface friction varies
5) Anisotropic and stress dependent
6) Variable depth to rock
7) Pore pressures occur


ASSUMED BEHAVIOR

1) Few layers
2) Layers are horizontal
3) Layers are homogeneous
4) Interfaces are consistent
5) Isotropic and elastic
6) Constant depth to rock
7) Water table ignored

FIGURE 1. Idealized Modeling of Composite Pavement Systems

## OBJECTIVE

The overall objective of this study was to develop an evaluation algorithm for composite pavements which will utilize nondestructive testing techniques to produce values of in-place modulus of elasticity of the principle layers. This objective was accomplished by:

1. Reviewing current methods for nondestructive testing, structural response modeling, and backcalculation of layer moduli of elasticity.
2. Determining suitable nondestructive testing equipment for composite pavements for an implementation oriented method as well as a method utilizing a dynamic structural response model.
3. Developing an algorithm for the backcalculation of layer moduli of composite pavements from nondestructive testing data. Validation of the algorithm was accomplished by a sensitivity study which used theoretical data to compare known theoretical modulus values to backcalculated values. A User's Guide was prepared.
4. Verifying the evaluation method on field test sites. Nondestructive data were analyzed to produce backcalculated moduli values. The backcalculated moduli values were compared with moduli values from both field and laboratory testing.
5. Examining the acceptability of layer moduli values calculated using multilayer linear elastic (quasistatic) theory. The backcalculated moduli values were used in a
dynamic structural response model to predict dynamic deflections which were compared with measured dynamic deflections.
6. Developing a version of the algorithm which uses dynamic theory. This version was used to demonstrate the adaptability of the COMDEF method and also to demonstrate the importance of using dynamic theory in the analysis of nondestructive testing data.

The scope of this study was restricted to the development of a method for the estimation of layer moduli of elasticity of three layer pavement systems composed of a full-depth Portland cement concrete pavement which has been overlain with asphaltic concrete and is supported by a uniform subgrade material. Emphasis was given to development of a method which could be quickly implemented into routine evaluation procedures, but which provides a framework such that future improvements in theoretical modeling can be incorporated easily. The scope included a demonstration of the adaptability of the program by incorporation of a dynamic model within the backcalculation algorithm.

## CHAPTER II <br> CURRENT NONDESTRUCTIVE TESTING TECHNOLOGY

## LITERATURE REVIEW

Three separate computerized literature searches have been completed by the author on the subject of nondestructive testing of pavements. All references reviewed are listed in the Bibliography. Current technology is presented in three major sections. These are: (1) nondestructive testing devices, (2) pavement structural response models, and (3) methods of backcalculation. Several summary reports were utilized in the literature review portion of this study which provided information as well as identifying additional references for review. These include summary reports by Hall and Alexander (1985), Bush (1980 a,b), Lytton, et al. (1986), and Moore, et al. (1978).

## NONDESTRUCTIVE TESTING DEVICES

## Introduction

A number of nondestructive testing devices are available. These may be grouped into four general areas: (1) quasistatic deflection devices, (2) vibratory deflection devices, (3) impulsive deflection devices, and (4) wave propagation devices. Several summary reports were utilized in the review of nondestructive testing devices which gave good comparative descriptions of their relative capabilities (Bush 1980a, Smith and Lytton 1985, and Hall 1987).

## Quasistatic Deflection Devices

The nondestructive test device which most nearly represents a static loading condition is the plate bearing test. In pavements testing, the reaction for the plate bearing test is usually generated with a hydraulic jacking system applied against a large reaction mass. The reaction mass is typically a large truck or other heavy construction equipment. To reduce plate curvature under the heavy loadings required, a stacked series of decreasing diameter steel plates is used to transmit the load from the hydraulic jack to the largest diameter plate at the pavement surface. The typical data from this test is a series of measurements of the gross load and deflection of the plate as measured against a reference bar. An idealized illustration of the plate bearing test is shown in Figure 2. Complete details about the equipment and testing method may be found in various references (Lytton, et al. 1975, The Asphalt Institute 1978, and ASTM D 1196-64). Although this test has been used in various research and evaluation tests in the past, it is
not commonly used at the present time. The main reasons for the test falling into disfavor include the time required to set up and complete the testing, the difficulty in establishing a reference bar which provides a fixed datum, the limited amount of information which can be derived from gross load-deflection data, and the heavy equipment required on site to provide the reaction mass.


FIGURE 2. Idealized Plate Bearing Test

A second nondestructive testing device for the measurement of quasistatic deflection of pavements is the Benkelman Beam. While some modified beams have been used for this test, the standard Benkelman Beam is 12 feet ( 3.65 m ) long, with a pivot at one of the third points. The longer, 8 feet ( 2.44 m ), levered end rests on the pavement and deflects downward as the pavement deflects downward. The shorter, 4 feet ( 1.22 m ), levered end deflects upward as the pavement deflects downward, and this movement is measured by a dial indicator at the end of the
beam. The standard load is applied by a truck which has an 18 kip $(80 \mathrm{kN})$ load distributed on the dual wheels of a single axle. There are two standard test methods for the Benkelman Beam Test, the AASHTO method (AASHTO T 256-77) and The Asphalt Institute method (The Asphalt Institute 1983), which is also known as the Canadian Good Roads Association method. Both of the methods use a truck with the standard 18 kip ( 80 kN ) axle load, but each method uses a different standard truck tire. Both methods are rebound tests, with the initial reading taken while the pavement is loaded with the standard load and the final reading taken after removal of the load. An idealized illustration of the Benkelman Beam test is shown in Figure 3. The Benkelman Beam test has been used extensively in the past for research and evaluation testing. The reasons for past acceptance of this test include the relatively low expense for obtaining, maintaining, and transporting the equipment, the capability to apply realistic wheel loads during the testing process, and a relatively large base of historical data. The Benkelman Beam test is not commonly used at the present time. One problem with the Benkelman Beam test is that it is a relatively slow test compared with many newer methods. However, the major reason for the test falling into disfavor is the relative difficulty in the interpretation of the data. The lack of an independent datum makes interpretation difficult, particularly for cases which involve pavements which are at least moderately stiff. In these cases, the pivot point of the beam may be within the deflected area influenced. If there is displacement of the pivot point by the load, the measured deflection will be less than the actual total deflection due to the loading. In addition, the overall
deflected shape is difficult to determine with this test. Often, the results (as with the plate bearing test) are restricted to gross loaddeflection data.


FIGURE 3. Idealized Benkelman Beam Test

A third nondestructive testing device for the measurement of quasistatic response of pavements is the Curvature Meter. This device is used to predict the curvature of the deflected pavement surface under a typical axle load. The device consists of a reference bar supported at each end and a spring dial gauge at the center. The test predicts the curvature of the pavement under the tire load based on the known chord length (length of reference bar) and the middle ordinate (measured deflection) and is described in several references (Idaho Department of Transportation 1965, Guozheng 1982). An idealized illustration of the Curvature Meter test is shown in Figure 4. The Curvature Meter has
never been a commonly used test, even though it is inexpensive and easy to perform, due to the difficulty in interpreting the data. A standard test method has not been accepted, and reference bars of differing lengths give confusing results (Guozheng 1982). In addition, the pavement curvature measured in this test appears to be more indicative of the capability of the near surface layers and has not been shown to be a useful indicator of the relative structural capability of pavement layers.


FIGURE 4. Idealized Curvature Meter Test

A number of automated tests are available which use beamdeflection principles similar to the Benkelman Beam test. These include the LaCroix Deflectograph (Kennedy 1978), the British Pavement Deflection Data Logging Machine (Kennedy, et al. 1978), the California Traveling Deflectometer (Roberts 1977), and the CEBTP Curviameter (Paquet 1978). All of these tests provide automated testing using a beamdeflection principle and displacement transducers, except the CEBTP Curviameter which uses geophones to measure the displacement by electronic integration of velocity output. None of these testing devices have ever
achieved widespread acceptance. While automation of the beam-deflection principle has made the time per test comparable with newer nondestructive test methods, the problems with data interpretation remain. As with the standard Benkelman Beam test, the beam support may be influenced by the wheel load and it is difficult to define the overall deflected shape of the pavement under the load.

## Vibratory Deflection Devices

Vibratory deflection devices have gained a great deal of acceptance in pavement research and evaluation. Most common vibratory deflection devices have the same basis. A static preload is applied to the pavement, a haversine loading is superimposed on the static preload, and the peak deflections caused by the dynamic loading is measured at various radial distances away from the load by the electronic integration of the velocity outputs of geophones. The static preload is necessary to provide stability during the test, i.e. to hold down the testing device during the unloading portion of the sine loading. The magnitude of the preload is typically near the half-amplitude of the sinusoidal loading. A major advantage of vibratory deflection testing as compared to quasistatic deflection testing is the use of geophones to measure the pavement deflection. By electronically integrating the geophone velocity outputs, the need for a fixed datum as reference is eliminated. An inertial reference is utilized, so that the measured peak deflections are the deflections which can be attributed to the dynamic loading. An illustration of an idealized vibratory deflection test is shown in Figure 5.


FIGURE 5. Idealized Vibratory Deflection Test

The first vibratory deflection device to gain widespread acceptance, and the most commonly used, is the Dynaflect (Uddin, et al. 1983). The Dynaflect, manufactured by Geo-Log, Inc., of Granbury, Texas, generates its dynamic loading with a dual rotating mass system. This system uses two eccentric flywheels which rotate in opposite directions and are balanced so that their horizontal components cancel. The vertical components of the two rotating masses combine to produce a very smooth steady-state sinusoidal loading with a peak-to-peak amplitude of 1.0 kip ( 4.4 kN ) at a frequency of 8 Hz . With a typical static preload of about 2 kips ( 8.9 kN ), the typical total load varies from $1.5 \mathrm{kips}(6.7 \mathrm{kN})$ to 2.5 kips ( 11.2 kN ), and is applied to the pavement surface through a pair of 4 inch ( 0.1 m ) wide, 16 inch ( 0.4 m ) diameter wheels. Deflections are measured by the electronic integration of the outputs from five geophones ( $210 \mathrm{ohm}, 2.4 \mathrm{~Hz}$, shunted). The geophones are suspended from an automated placing bar and are typically spaced at 1 foot ( 0.3 m )
intervals, beginning at the center point between the loading wheels. A major advantage of the Dynaflect over common hydraulic vibratory deflection devices is its transportability, due to the relatively light weight of the loading trailer. Another advantage is the smooth loading function produced by the rotating mass loading system. The main disadvantage of the Dynaflect is the relatively small magnitude of dynamic load, which is not variable. For stiff pavements, the peak-to-peak dynamic load of 1 kip ( 4.4 kN ) produces deflections with such low magnitudes that small errors in the measurement of the deflections may create significant errors in subsequent data analysis. Another disadvantage is the fixed frequency of the Dynaflect loading ( 8 Hz ).

The most widely accepted hydraulic vibratory deflection device is the Road Rater (Sharpe 1978), which is manufactured by Foundation Mechanics, Inc., of El Segundo, California. The available models include the $400 \mathrm{~A}, 400 \mathrm{~B}, 2000$, and 2008. The model 400 A has the unique feature of a bumper mounted loading device. All of the other Road Rater models are trailer mounted. Series 400 Road Raters use two rectangular load plates of 4 inches ( 0.1 m ) by 7 inches ( 0.2 m ), spaced at 10 inches ( 0.3 m ) center to center. Series 2000 Road Raters use a circular load plate of 18 inches ( 0.5 m ) diameter. Static load for the model 400 A is vehicle dependent. Static loads for the models 400B, 2000, and 2008 are $2.4 \mathrm{kip}(11 \mathrm{kN}), 3.8 \mathrm{kip}(17 \mathrm{kN})$, and $5.8 \mathrm{kip}(26 \mathrm{kN})$, respectively. Dynamic load ranges are 0.5 kip to $1.0 \mathrm{kip}(2 \mathrm{kN}$ to 4 kN ) for the model 400A, 0.5 kip to $3.0 \mathrm{kip}(2 \mathrm{kN}$ to 13 kN ) for the model $400 \mathrm{~B}, 1 \mathrm{kip}$ to $5 \mathrm{kip}(4 \mathrm{kN}$ to 24 kN ) for the model 2000, and 1.2 kip to $8 \mathrm{kip}(5 \mathrm{kN}$ to 36 kN ) for the model 2008. Deflections are determined by the electronic integration of the velocity outputs from four geophones (590 ohm,
4.5 Hz , shunted), typically spaced on 1 foot ( 0.3 m ) centers from the center of the loading plate(s). An advantage of the Road Rater is that the dynamic load magnitude and frequency can be varied by the operator. Disadvantages include the limited dynamic load capabilities of the series 400 models and the high static loading of the series 2000 models.

A few other vibratory deflection devices have been used for pavement analysis, but these are typically custom designed and manufactured. Examples of custom devices include the Waterways Experiment Station "WES l6-kip Vibrator" (Hall 1973) and the Federal Highway Administration "FHWA Thumper" (May 1981). These devices have similarities to the other vibratory deflection devices, but have the obvious disadvantage of not being generally available to the engineering community.

## Impulsive Deflection Devices

Impulse deflection testing is typically accomplished by a devise known as the Falling Weight Deflectometer (FWD). The basis of the falling weight test is to lift a weight to a height above the pavement and drop it on a spring system which transfers the impulse to a load plate, and subsequently measuring the impulse force transmitted to the load plate and the peak deflections at various radial distances from the impact point. The test imparts a very small static preload to the pavement prior to the impulse. A rubber buffer system and a rubber pad under the load plate help to spread the loading function over a duration of about 30 milliseconds, approximating the passing of a moving wheel load. An idealized impulsive deflection (Falling Weight Deflectometer) test is illustrated in Figure 6. Numerous authors have noted the advantages of the Falling Weight Deflectometer. For example, the Falling

Weight Deflectometer provides the most realistic loading function of any nondestructive testing device compared to actual moving wheel loads (Hoffman 1983), is the fastest and most versatile nondestructive testing device (Bentsen, et al. 1988), and has been rated as the best overall pavement testing device (Lytton, et al. 1986).


FIGURE 6. Idealized Impulsive Deflection Test

In the United States, the most widely used impulsive deflection device is the Dynatest 8000 Falling Weight Deflectometer System (Ullidtz and Stubstad 1985), available through Dynatest Consulting of Ojai, California. Based on Bretonniere's (1963) early work, the Technical

University of Denmark, the National Danish Road Laboratory, and the Dynatest Group have gradually developed and employed the Falling Weight Deflectometer for use in the nondestructive testing of highway and airfield pavements. The Dynatest 7800 Falling Weight Deflectometer was introduced in 1977. This equipment showed promise but needed more automation to work well in a production testing situation. The Dynatest 8000 Falling Weight Deflectometer Test System was introduced in 1981. This system consisted of three parts, the Dynatest 8002 Falling Weight Deflectometer, the Dynatest 8600 System Processor, and the HewlettPackard HP-85 Portable Computer. The Dynatest 8000 Falling Weight Deflectometer System can produce a dynamic peak load of 25 kips (111 kN) and allows the simultaneous measurement of peak load and seven peak deflections. Deflections are calculated by the electronic integration of the outputs from seven geophones, typically spaced at 1 foot ( 0.3 m ) intervals from the load. The load is produced by dropping a weight on a loading plate of approximately 1 foot ( 300 mm ) diameter. The loading is transient and of short duration (about 30 milliseconds ). The load is adjustable both by varying the amount of weight dropped and by varying the height of drop. For testing of composite airfield pavements, the maximum dynamic load of 25 kips (111 kN) is normally used for nondestructive evaluation. This loading level simulates the load duration and stress levels produced by the passing of one wheel of a heavily loaded aircraft. Use of the highest load level also increases the magnitude of deflections. High deflection magnitudes decrease measurement round-off errors and improve measurement signal-to-noise ratios. In weakened areas, excessive deflections can occur when the maximum load is used. Deflections greater than the allowable value of $79 \mathrm{mils}(2 \mathrm{~mm})$
cannot be measured accurately by the Falling Weight Deflectometer geophones. The Falling Weight Deflectometer is equipped with an error message to indicate that a deflection has exceeded the allowable value. When this occurs, the load is reduced by adjusting the height of drop.

Dynatest has also developed a prototype device called the Heavy Weight Deflectometer (HWD) which is capable of providing impulsive loads of very high magnitudes. The Heavy Weight Deflectometer is intended for evaluating very stiff pavement systems such as rigid airfield pavements designed for higher magnitude traffic loadings (Bentsen, et al. 1988). The Heavy Weight Deflectometer has a maximum impulse load of 56 kips (250 kN). Other impulsive deflection devices available in this country include the PaveTech Falling Weight Deflectometer (Smith and Lytton 1983), the Phonix Falling Weight Deflectometer (Smith and Lytton 1983), and the KUAB Falling Weight Deflectometer (Bentsen, et al. 1988). None of these devices are as fast or versatile as the Dynatest 8000 Falling Weight Deflectometer System (Bentsen, et al. 1988), but all share similar advantages and disadvantages. The advantages of the Falling Weight Deflectometer are its speed of testing, transportability, low static preload, and its close approximation of the passing of a wheel load. A disadvantage of the Falling Weight Deflectometer is the complexity of the transient test dynamics. A typical Falling Weight Deflectometer load pulse is shown in Figure 7. The distorted shape is typical of the test, due to the complex load buffering system. Figure 8 shows the load magnitude spectrum in the frequency domain as obtained by a Fast Fourier Transform for the load pulse shown in Figure 7.


FIGURE 7. Typical Falling Weight Deflectometer Load Cell Output


FIGURE 8. Frequency Spectrum of Typical FND Load Pulse

By inspection of Figure 8, there are large magnitudes at all frequencies up to about 60 Hz and significant energy at all frequencies up to about 250 Hz for the Falling Weight Deflectometer test. In addition, the rubber buffer system and rubber pad beneath the load plate are difficult to model properly.

## Wave Propagation Devices

There are a number of wave propagation devices and techniques currently under study and development. None of these techniques have been developed sufficiently to be useful for routine pavement evaluation, but are worthy of mention due to the potential of these tests for future evaluation procedures. Typical wave propagation techniques for pavement evaluations use the principle of dispersion of Rayleigh waves. Dispersion is a change in wave velocity with frequency, or equivalently with wavelength. In a homogeneous, elastic half-space, the velocity of propagai. on of Rayleigh waves is independent of frequency. However, there is dispersion of Rayleigh waves in a layered medium. Rayleigh waves with longer wavelengths tend to propagate at deeper depths, so that the dispersion of the Rayleigh waves in a layered system is indicative of the relative material properties of the component layers. There are three common methods of generating Rayleigh waves for pavement evaluations. These are drop weight devices which provide very short rise times, vibratory devices, and strike hammers. An example of a drop weight device is the Texas Drop Hammer (Heisey 1981). An example of a vibratory device is the Air Force Nondestructive Pavement Testing Van (Steedman 1979). A variety of strike hammers have been used in a new technique termed the spectral analysis of surface waves (SASW) method,
with progressively larger hammers being used to generate longer wavelengths (Nazarian, et al. 1983). The majority of recent research on pavement evaluation by wave propagation has centered on the spectral analysis of surface waves technique. Measurement of dispersion is accomplished by monitoring the outputs of two transducers separated by a known distance. Spectral analysis is used to determine phase shift as a function of frequency. A dispersion curve of surface wave phase velocity as a function of frequency may be determined by the correct application of the Haskell-Thomson formulation for Rayleigh wave propagation in a multilayered medium (Haskell 1953, Thomson 1950). Recent advances in data reduction techniques have greatly improved the quality of the dispersion curves determined by the spectral analysis of surface waves method (Drnevich and Sayyedsadr 1987). Determination of layer properties requires inversion of the dispersion curve to obtain a relationship of wave velocity as a function of depth. This very difficult process is typically performed by making assumptions about the depth of propagation as a function of wavelength (Heisey 1981) or by applying a trial and error iteration to determine the inverted relationship (Nazarian, et al. 1983). Successful application of either of these methods requires a great deal of experience and judgment. Widespread acceptance of wave propagation techniques for pavement evaluation will require more sophisticated methods for the inversion process. The difficulty in obtaining a valid relationship for wave velocity as a function of depth is the biggest disadvantage of the spectral analysis of surface waves method. Another disadvantage is the time required to collect enough data to provide a well defined dispersion curve. The biggest advantage of this method, when sufficiently developed, is the
ability to determine layer properties when the layer thicknesses are unknown. Proper application of this method will allow layer thicknesses to be determined from the dispersion data by modeling the pavement system as a large number of thin layers and comparing the material properties of adjacent thin layers.

## PAVEMENT STRUCTURAL RESPONSE MODELS

## Introduction

Structural response models are used to predict the response of pavements to external loadings. A good structural response model should be able to predict the responses for a nondestructive test device and also be able to predict actual responses to service traffic. This combination allows a design engineer to predict design responses from a pavement evaluation within the same framework. This section provides a general discussion of currently available structural response models, which may be grouped into five general categories: (1) equivalent thickness models, (2) plate bending models, (3) multilayer linear elastic models, (4) time-independent continuum models, and (5) timedependent models.

## Equivalent Thickness Models

The concept of equivalent thickness (Odemark 1949) allows a multilayered pavement to be represented by a single layer of known elastic modulus. The formula used to calculate the equivalent thickness is:

$$
\begin{equation*}
T_{e q}=\sum_{i=1}^{n} c T_{i}\left(E_{i} / E_{e q}\right)^{1 / 3} \tag{1}
\end{equation*}
$$

where

$$
\begin{aligned}
& T_{e q}=\text { the equivalent pavement thickness } \\
& T_{i}=\text { actual thickness of the } i^{\text {th }} \text { pavement layer, } i=1, \ldots, n \\
& E_{i}=\text { actual modulus of the } i^{\text {th }} \text { pavement layer, } i=1, \ldots, n \\
& E_{e q}=\text { the assumed modulus of the equivalent pavement } \\
& c=0 \text { demark's constant } \approx 0.8 \text { to } 0.9
\end{aligned}
$$

The equivalent system is assumed to have the same responses as the actual pavement system, so that response predictions can be made quickly with classical Boussinesq theory (Boussinesq 1885).

Ullidtz (1973) developed a more advanced approach which allowed the use of a nonlinear (stress softening) elastic subgrade model by utilizing a set of deflection ratio curves. While this method is better than the traditional equivalent thickness method, it is still a relatively simple approach which can lead to significant errors in response predictions (Kuo 1979, Hung, et al. 1982).

The advantage of the methods which use equivalent thickness concepts is in the speed of calculation. Since the relationships are empirical and are represented by very simple equations, a solution can be found extremely quickly. However, the simplification of the problem makes these methods unreliable.

## Plate Bending Models

The classical differential equation for bending of a medium thickness plate (Timoshenko and Woinowsky-Krieger 1959) has been used to provide simple solutions for structural response of rigid pavements. This differential equation is:

$$
\begin{equation*}
\frac{E h^{3}}{12\left(1-\mu^{2}\right)}\left[\frac{\partial^{4} w(x, y)}{\partial x^{4}}+2 \frac{\partial^{4} w(x, y)}{\partial x^{2} \partial y^{2}}+\frac{\partial^{4} w(x, y)}{\partial y^{4}}\right]=p(x, y)-q(x, y) \tag{2}
\end{equation*}
$$

where

$$
\begin{aligned}
& E=\text { modulus of elasticity of the concrete slab } \\
& \mu=\text { Poisson's ratio of the concrete slab }
\end{aligned}
$$

$h=$ thickness of the concrete slab $w(x, y)=$ deflection of the slab at point $(x, y)$ $p(x, y)=$ externally applied load $q(x, y)=$ reaction of the idealized subgrade

Use of the differential equation for plate bending has been implemented in two different simple models. The first case is Westergaard's (1926) formulation for maximum stresses and deflections in a slab of infinite size on a Winkler foundation. The second case was formulated by Hogg (1938) and by Holl (1938) for the case of a slab of infinite size placed on a semi-infinite elastic subgrade.

The assumption of the Winkler foundation is a series of springs, each of which deflects in direct proportion to the load applied at that point. Westergaard formulated equations for maximum stress and displacement for three different loading conditions. These are the interior loading condition, edge loading condition, and corner loading condition. Influence charts (Pickett and Ray 1951) may be used to extend Westergaard's theory for the case of multiple loads. The mathematics of a plate on an elastic solid is more complicated than for the Winkler foundation and the solution for the maximum stress and displacement has been formulated (Hogg 1938, Holl 1938) only for the case of a single load at the interior of an infinite slab. Use of classical plate theory has been used in various algorithms and methods, including the Portland Cement Association (Packard 1973) design method for portland cement concrete pavements. As with equivalent thickness methods, the major advantage of simple models based on plate theory is in the speed
of calculation. However, these methods cannot model multilayer pavements without equivalent thickness techniques, and even then predict only the maximum stresses and displacements.

## Multilayer Linear Elastic Models

Recent developments in nondestructive evaluation of pavements has led to the use of layered elastic modeling (Bush and Alexander 1985). Analysis is accomplished by modeling the pavement system as an equivalent elastic layered system. Utilization of this technique makes it possible to model the behavior of multiple distinct pavement layers. The solution to a multilayered system was first formulated by Burmister (1943), and was limited to a two or three layer case. Burmister's work was extended into an $n$-layer case by Mehta and Veletsos (1959). The general method of solution is to assume axial symmetry and find a stress function which satisfies the governing differential equation for each of the layers. Numerical techniques are used to solve for a set of integration constants for the stress function which can be used to calculate predicted values of stress and displacement. The first commonly available multilayer program for structural response modeling of pavements was the program LAYER, developed by the Chevron Oil Company (Michelow 1963). LAYER had severe limitations, particularly that only a single load could be used, and that slip at the interface between layers was not allowed. LAYER was improved and has become commonly known as CHEVRON. CHEVRON allowed multiple loads, but still allowed no slip between layers. ELSYM5 was developed by Ahlborn (1972) and is based on the LAYER program. ELSYM5 included significant improvements, including a choice between full friction (continuity in displacement) or full slip
(continuity in stress) at the interface between layers. BISAR, developed by the Shell 0il Company (1978), further extended the state-of-the-art by allowing a range of slip conditions.

Multilayer elastic theory predicts the elastic response to static or quasistatic loads. However, use of an equivalent elastic system can be useful when analyzing dynamic loads and deflections. While the quasistatic analysis approach is certainly an empirical correlation when applied to dynamic nondestructive testing, a wealth of literature has indicated that the empirical predictions based on linear elastic models are useful for evaluation and design calculations. For response predictions of nondestructive testing loads, the typical method is to model the peak dynamic load as an equivalent quasistatic load and to assume that the calculated equivalent elastic deflection basin is a good approximation of the envelope of measured peak deflections. Research by Bodare and Orrje (1985) indicated that the theoretical dynamic modulus from a falling weight test approaches the theoretical static modulus for a homogeneous half-space when the time to peak load is relatively long. Although the time length which would be considered long is a function of various factors, including material properties and load radius, Bodare and Orrje (1985) presented an example using typical system properties which indicated that the needed time to peak was about 10 milliseconds , which is less than the time to peak of the Falling Weight Deflectometer. The multilayer linear elastic model tends to give larger deflections than measured dynamic peak values when the bottom layer is assumed to be semi-infinite. Past research (Bush 1980) has indicated that agreement between predicted elastic deflections and measured dynamic deflections is improved if a rigid layer is placed in the equivalent elastic system
at a depth of 20 feet. Although this assumption is an empirical correlation, other researchers (Roesset and Shao 1985, Mamlouk 1985) have supported the use of this assumption.

## Time-independent Continuum Models

Numerical application of continuum mechanics to pavement structural response modeling can allow more generalized descriptions to be included in the structural description. Continuum methods for making structural response predictions of pavements include the discrete element method, the finite element method, and the finite difference method.

Early development of continuum codes for rigid pavement analysis centered around the discrete element method. Newmark (1949) developed the use of discrete elements for plate analysis, and Hudson and Matlock (1966) extended the method to include rigid pavement models. The discrete element formulation for rigid pavement analysis implemented by Hudson and Matlock is a lumped parameter model which idealizes the rigid slab as a collection of rigid bars, torsional bars, and elastic joints, which is supported by vertical springs. The equilibrium equation for this lumped parameter system may be expressed in matrix form as:

$$
\begin{equation*}
[K]\{W\}=\{F\} \tag{3}
\end{equation*}
$$

where
[K] = the stiffness matrix
$(w)=$ the displacement vector
$(F)=$ the load vector

Use of the discrete element method offers a significant improvement over the simple plate models discussed previously, by allowing a more generalized description of the slab system. Historically, this method has the advantage of requiring much less computer capability than is required by many newer finite element codes. However, with the computers which are available at the current time, this advantage has become a smaller consideration.

The more popular method of time-independent continuum analysis of pavements systems has become the finite element method. Although there were other programs which pioneered the field of finite element analysis of pavements, such as KENTUCKY (Huang and Wang 1973), the use of the finite element method for structural response modeling of pavements was popularized by two codes developed at the University of Illinois at Urbana-Champaign. These time-independent continuum codes are called ILLI-SLAB (Tabatabaie-Raissi 1977) and ILLI-PAVE (Thompson 1982), and are used to make structural response predictions for rigid and flexible pavements, respectively. The finite element method breaks the pavement into a set of discrete bodies, or "finite elements." Each element has an element stiffness matrix, [k], which relates the element forces, ( $p$ ), to the element displacements, $\{\delta\}$. That is, for each element:

$$
\begin{equation*}
[k]\{\delta\}=\{p\} \tag{4}
\end{equation*}
$$

A global stiffness matrix, [K], is formulated by superposition of the eleme..: stiffness matrices, so that the generalized forces for the system, $(P\}$, may be related to the generalized displacements, $\{\Delta\}$. That is, for the system:

$$
\begin{equation*}
[K]\{\Delta\}=\{P\} \tag{5}
\end{equation*}
$$

Both ILLI-SLAB and ILLI-PAVE simulate a three-dimensional pavement system with a two-dimensional finite element algorithm. A reasonable degree of accuracy is achieved by the use of simplifying assumptions, such as axisymmetry, so that the three-dimensional behavior is approximated numerically by a set of elements, each of which is restricted to two-dimensional behavior. Three-dimensional codes provide a more realistic formulation, since each finite element can behave in a true three-dimensional sense. An example of a true three-dimensional finite element code which has been used for pavement structural response modeling is the SAP (Wilson 1969) program, which was used by Tabatabaie-Raissi (1977) in the development of ILLI-SLAB. While it is true that the three-dimensional finite element model is the more accurate from a theoretical viewpoint, the difference in complexity is significant. While programs such as ILLI-SLAB and ILLI-PAVE have had sufficient development so that they are reasonably fast and easy to use, current three-dimensional codes require a high degree of sophistication, not only in terms of the computer equipment required, but also sophistication of the user/operator. Errors in solutions can arise due to violation of computer memory requirements, as well as errors in mesh/element formulations or data manipulations. The additional accuracy of the three-dimensional method does not translate into a true benefit for routine evaluation, when the difficulty of use and increased potential for operator errors is considered. Three-dimensional finite element modeling is a method which has promise for the future, but whose use will be limited until there is more development which will make the programs easier to use by a typical design engineer on a variety of computer systems.

## Time-dependent Models

Responses of a pavement to a nondestructive test loading is a dynamic (time-dependent) problem. To provide the most accurate predictions of structural responses, the time-dependence must be considered. A number of methods which attempt to make true dynamic predictions of structural response are available.

Viscoelastic structural response models, such as the computer program VESYS (Kenis 1980), predict long-term responses to timeindependent loadings. The viscous response predicted by the program is related to performance models such as pavement rutting. The model used in VESYS is a multilayer viscoelastic model which is analogous to a linear elastic system. Although VESYS is often presented as a "dynamic" program, it is actually a special case of the multilayer linear elastic system. The responses are not true dynamic responses, but are indicative only of material creep properties.

One method which would provide a t،ue dynamic model with reasonable accuracy would be a time-dependent finite element model. However, even a two-dimensional finite element model with true timedependence becomes complex enough to preclude routine use. A threedimensional finite element model with time-dependence, such as DYNA (Hallquist 1976), typically requires a super-computer, such as a Cray, and a very experienced user/operator.

Another method for time-dependent analysis of pavement structural behavior is a time-dependent finite difference code. The finite difference method is a numerical application of continuum mechanics which utilizes many of the advantages of both the discrete element method and the finite element method. In the finite difference method, the
material is broken into a system of elements, each of which can experience complex behaviors, but the behavior of a given element is controlled by the elements in the vicinity of that element. A general purpose time-dependent finite difference program which has been used in limited study of pavement structural responses is the STEALTH (Hofmann 1981) program. Use of STEALTH is complicated, but requires much less computer capability and operator experience than comparable finite element codes, and can include more complicated behaviors than the lumped parameter models used in the discrete element method.

A method developed by Kausel (1981) utilizes Green's functions to predict the dynamic response of multilayered systems to single frequency harmonic vibrations. This method will be discussed at length in a later section. Roesset and Shao (1985) made dynamic predictions for pavement behavior when loaded by the Falling Weight Deflectometer based on this method. Roesset's method required a large number of solutions for dynamic responses at single frequencies and predicted the response to the Falling Weight Deflectometer loading by the use of an Inverse Fast Fourier Transform to create deflection time histories. Due to the complexity of the solution, Roesset limited his work to a very small number of demonstration cases. As shown in Figure 7, the Falling Weight Deflectometer creates complex waveforms. The process used by Roesset requires a Fast Fourier Transform of the load spectrum, application of that load spectrum to a unit load spectrum of single frequency deflections, and an Inverse Fast Fourier Transform with suitable boundary conditions applied to obtain a transient deflection time history. Small errors in one part of the analysis may cause large errors in the overall prediction. The disadvantage of Roesset's method for predictions 0 ;

Falling Weight Deflectometer response is that solution does not justify the complexity. Use of a continuum method which will allow a load time history to be input would be no more difficult than the method used by Roesset, and should give more accurate answers. The potential for Kausel's method in pavement structural response predictions is more likely in the area of vibratory deflection devices, as used in this study.

## METHODS OF BACKCALCULATION

## Introduction

A large number of computer programs for backcalculation of moduli from nondestructive testing data have been reported. However, close examination of the available literature indicated that all common existing algorithms could be grouped into three general methods. These are: (1) simplified methods, (2) gradient relaxation methods, and (3) direct interpolation methods. The method used in most common backcalculation algorithms is:

1. An ideal pavement system is assumed which corresponds to the pavement which was tested with the nondestructive testing device. It is typical to assume values of the layer thicknesses. The assumed values of layer thickness are based on construction records and/or on other tests, such as coring. It is also typical to assume all layer properties not related to stiffress (e.g., Poisson's Ratio, density).
2. A set of layer stiffness values are assumed. The stiffness values which must be assumed depend on the forward model which will be used. Typically, these values are either elastic or resilient modulus values.
3. A set of theoretical deflections are calculated with the forward model for structural response. The deflections are calculated so that the deflection locations and the load intensity match the sensor locations and the loading from the nondestructive test.
4. The calculated values of deflection are compared to the deflections measured by the nondestructive testing device. If the values agree within a reasonable tolerance, the values of layer stiffness assumed in the forward model are accepted as the actual pavement layer stiffness values.
5. If the calculated deflections do not agree with measured deflections within a reasonable tolerance, the values assumed for layer stiffness are adjusted so that an improved solution is determined. This process is referred to as the backward model.
6. Successive applications, or iterations, of the forward model (Step 3 above) and the backward model (Step 5 above) continue until a reasonable tolerance between calculated deflections and measured deflections is reached (Step 4 above) or until some other limiting factor is reached. Examples of limiting factors include a limit on the number of iterations or a limit on the overall execution time.

## Simplified Methods

A number of simplified approaches for the prediction of layer properties from load-deflection data have been developed. Examples of simplified methods include equivalent thickness methods and deflection basin geometry methods. An example of an equivalent thickness method is the computer program ELMOD, developed by Ullidtz (1973). An example of a deflection basin geometry method is the method of surface curvature
index (SCI) reported by Van der Loo (1982). These methods have as their major advantage the speed of calculation, but their usefulness is limited (Ullidtz 1973, Kuo 1979, Van der Loo 1982).

## Gradient Relaxation Methods

A number of very similar programs exist which use a gradient relaxation method pioneered by Michelow (1963) and developed at the Waterways Experiment Station (Bush and Alexander 1985) which was originally used in a program called CHEVDEF (Bush 1980), and has been used in a host of backcalculation programs with various names. In general, the backward models are the same for each of the backcalculation programs, with the only difference being the structural response model used as the forward model. For example, the forward models for the programs BISDEF, ELSDEF, and CHEVDEF are the structural response models BISAR, ELSYM5, and CHEVRON, respectively. The method searches for the best solution of layer moduli using iterative gradient relaxation. The solution is achieved by forming gradient matrices and solving for solutions which minimize the errors in the fitted basin. This method has the advantage of convergence in most cases with a small number of iterations. The specific method of solution is described below.

It is assumed that a relationship exists between deflection and layer moduli. The predicted deflection at a given sensor location $j$, $\Delta_{j}$, is assumed to be a function of the unknown layer moduli, that is:

$$
\begin{equation*}
\Delta_{j}=f\left(E_{1}, E_{2}, E_{3}, \ldots, E_{n}\right) \tag{6}
\end{equation*}
$$

where

$$
n=\text { number of unknown layer moduli of elasticity }
$$

so that the deviation at sensor location $j, \delta_{j}$, between the measured deflection, $D_{j}$, and the predicted deflection, $\Delta_{j}$, is given by:

$$
\begin{equation*}
\delta_{j}=D_{j}-\Delta_{j}=D_{j}-f\left(E_{1}, E_{2}, E_{3}, \ldots, E_{n}\right) \tag{7}
\end{equation*}
$$

The sum of the squares of the deviations for all of the sensor locations may be written as:

$$
\begin{equation*}
\sum_{j=1}^{m} \delta_{j}^{2}=\sum_{j=1}^{m}\left[D_{j}-f\left(E_{1}, E_{2}, E_{3}, \ldots, E_{n}\right)\right]^{2} \tag{8}
\end{equation*}
$$

where
$m=$ number of deflection measurements
To minimize the error with respect to the unknown values of moduli, the partial derivatives of the error function are taken with respect to the unknown moduli values. This gives a solution matrix of $n$ equations involving the unknown moduli values. The solution is calculated numerically by forming gradient equations which approximate the partial derivative relationships. The gradient equations are formed by the following method. A initial set of modulus values, $E^{0}$, is assumed and corresponding deflections $\Delta^{0}$ are computed. A second set of moduli values, $E^{1}$, are assumed. A new set of deflections is calculated for each of the combinations of moduli variations. That is, combinations where all but one of the moduli have values as in $E^{0}$ and one of the moduli is varied to a new value $E^{1}$. The deflection at a given sensor location $j$ may then be given as a function of the gradient equation and the unknown modulus of layer i. The general equation is:

$$
\begin{equation*}
\Delta_{j}=A_{j i}+S_{j i} \log _{10}\left(E_{i}\right) \tag{9}
\end{equation*}
$$

where

$$
\begin{aligned}
& \Delta_{j}=\text { the predicted deflection at sensor location } j \\
& E_{i}=\text { the unknown layer moduli of layer } i \\
& S_{j i}=\frac{\Delta_{i}^{0}-\Delta_{i}^{1}}{\log _{10} E_{i}^{0}-\log _{10} E_{i}^{1}} \\
& A_{j i}=\Delta_{i}^{0}-S_{j i} E_{i}^{1} \\
& E_{i}^{0}=\text { first assumed value of modulus for layer } i \\
& E_{i}^{1}=\text { modulus for layer } i \text { after the variation } \\
& \Delta_{i}^{0}=\text { predicted deflection at sensor location } j \text { for } E_{i}^{0} \\
& \Delta_{i}^{1}=\text { predicted deflection at sensor location } j \text { for } E_{i}^{1}
\end{aligned}
$$

An expression can be written for the deflection at sensor location $j$, $\Delta_{j}$, as a function of all the unknown moduli values, $E_{i}$. The equation must relate the following:

$$
\begin{equation*}
\Delta_{j}=\Delta_{j}^{0}+\left(\Delta_{j}^{0} \text { change due to moduli variations }\right) \tag{10}
\end{equation*}
$$

The general equation is:

$$
\begin{equation*}
\Delta_{j}=\Delta_{j}^{0}+\sum_{i=1}^{n} S_{j i}\left[\log _{10} E_{i}-\log _{1 饣} r:\right] \tag{11}
\end{equation*}
$$

The value of $\Delta_{j}^{0}$ can be expressed in terms of one of the unknown moduli (e.g., layer 3) as:

$$
\begin{equation*}
\Delta_{j}^{0}=A_{j 3}+S_{j 3} \log _{10} E_{3}^{0} \tag{12}
\end{equation*}
$$

Therefore the expression for $\Delta_{j}$ can be written as:

$$
\begin{equation*}
\Delta_{j}=A_{j 3}+S_{j 3} \log _{10} E_{3}^{0}+\sum_{i=1}^{n} S_{j i}\left(\log _{10} E_{i}-\log _{10} E_{i}^{0}\right) \tag{13}
\end{equation*}
$$

The expression for the summation of the squared deviations may be written as:

$$
\begin{equation*}
\sum_{j=1}^{n} \delta_{j}^{2}=\sum_{j=1}^{m}\left[D_{j}-A_{j 3}^{0}-S_{j 3} \log _{10} E_{3}^{0}-\sum_{i=1}^{n} S_{j i}\left(\log _{10} E_{i}-\log _{10} E_{i}^{0}\right)\right]^{2} \tag{14}
\end{equation*}
$$

The squared errors in deviation are minimized by taking the partial derivatives of the error expression with respect to each of the unknown moduli values. By setting the partial derivatives equal to zero, the following matrix equation may be obtained:

$$
\begin{equation*}
[B]\{E\}=\{C\} \tag{15}
\end{equation*}
$$

where, for $i$ and $k$ equal to the layer number, the matrix terms are:

$$
\begin{align*}
& C_{k}=\sum_{j=1}^{m} S_{j k}\left[D_{j}-A_{j 3}-S_{j 3} \log _{10} E_{3}-\sum_{i-1}^{3} S_{j i} \log _{10} E_{i}\right]  \tag{16}\\
& B_{k i}=\sum_{j=1}^{m} S_{j k} S_{j i}  \tag{17}\\
& E_{k}=\text { unknown layer moduli of layer } k \tag{18}
\end{align*}
$$

Solution of the matrix equation formed from the gradient expressions noted above provides the solution vector of layer moduli values.

The advantage of traditional gradient relaxation methods is that they provide closure to a solution in a relatively small number of iterations for most cases. The disadvantage is that a structural response model is required within the program, so that each iteration is time consuming.

## Direct Interpolation Method

A method developed by Uzan (1986) and implemented in the computer program MODULUS predicts the layer moduli of elasticity by direct interpolation from a set of solutions calculated from a structural response model which is chosen by the user. The method uses polynomial representations to directly predict the layer moduli of interest. A User's Guide is not yet available, so a complete description of the method is not presented here. The advantage of this method is that it is able to utilize input from a wide range of structural response models with a relatively small number of precalculated solutions. The disadvantage of this method is that for a small data set, the time required for the user to precalculate the solution matrix may actually take more time than a traditional approach. Accuracy of the method is not known at this time.

## CHAPTER III

## DEVELOPMENT OF THE COMDEF METHODOLOGY

## SELECTION OF A NONDESTRUCTIVE TESTING DEVICE

The Falling Weight Deflectometer (FWD), illustrated in Figure 9, was selected as the recommended loading device for the nondestructive testing of composite pavements. While some subjectivity was used in selecting this device, there appears to be strong justification for the choice. Selection of the Falling Weight Deflectometer for this project was based on a combination of factors which included the ability to simulate the passing of a wheel load, load capacity, transportability, ease of use, reproducibility of results, and availability. Both the Army and the Air Force plan to use the Falling Weight Deflectometer in the future for routine airfield pavement evaluations. The Air Force has already implemented the Falling Weight Deflectometer for routine airfield evaluation and the Army is phasing out the use of the WES 16 -kip Vibrator in favor of the Falling Weight Deflectometer. A Falling Weight Deflectometer was readily available for the project due to ongoing pavement analysis and equipment development sponsored by the Office of the Chief of Engineers, US Army and by the Engineering and Services Center, US Air Force.

FIGURE 9. The Falling Weight Deflectometer (FWD)

## SELECTION OF A STRUCTURAL RESPONSE MODEL

The structural response model chosen for the implementation version of COMDEF was the multilayer linear elastic model. There were several reasons for this choice. One reason was availability, as the Waterways Experiment Station is licensed for the use of the BISAR (Shell 1978) structural response model. Another reason was the author's experience with linear elastic modeling of pavements. However, the most important reason for the choice of the multilayer linear elastic model was to insure compatibility between the moduli backcalculated from the nondestructive testing and the moduli needed for subsequent design. Since most current design techniques utilize multilayer linear elastic theory, this appeared to be the best overall choice. A complete discussion of the theory used to make structural response predictions is presented in a subsequent section.

## STRUCTURAL RESPONSE BY MULTILAYER LINEAR ELASTIC THEORY

The proprietary FORTRAN program BISAR (Shell 1978) uses multilayer linear elastic theory to make structural response predictions for pavement systems. Since BISAR is proprietary, the discussion which follows is a general description of multilayer linear elastic theory, including the numerical techniques used to formulate the solutions. An earlier formulation, by Mehta and Veletsos (1959), was used as the primary reference. For purposes of this summary, the discussion is limited to the specific case of vertical surface deflections produced by a vertical disk load. Analogous solutions were developed for all stresses and displacements by Mehta and Veletsos.

The problem is to calculate vertical deflections at the surface of a multilayered system subjected to an axially symmetric static load applied uniformly to a circular area of known radius. The system is composed of an arbitrary number of layers, each of which can have different thickness and physical properties. It is assumed that the each layer is horizontal, homogeneous, isotropic, and linearly elastic. The lowermost layer in the system is assumed to be semi-infinite. It is typical to assume that at the interface between layers there is a continuous surface of contact which may be classified as either "rough" or "smooth." For a rough interface, no horizontal slip is allowed, and the shearing stress is continuous. For a smooth interface, slip is allowed and there is no shearing stress. BISAR allows a range of slip conditions by applying a weighting factor and superimposing both the smooth and rough conditions with appropriate weighted values.

Solution of the problem is based on classical Boussinesq (1885) theory. Boussinesq first formed the solution for a semi-infinite medium loaded at the surface by a concentrated normal load. In the 1940's, Burmister formulated the general solution for a three-layered system for the case of axially symmetric normal surface loading (Burmister 1943). The theory presented herein is an extension of Burmister's work.

The theory of elasticity is applied to the problem in terms of cylindrical coordinates $(r, \theta, z)$. The origin is defined as the point at the surface coincident with the vertical axis of symmetry of the load. One method of analysis of a multilayered system involves the expression of stresses and displacements in terms of a stress function (Love 1944). For the general case, the stress function approach is extremely complicated. In effect, a stress function is developed for each layer such that appropriate overall boundary conditions are satisfied at the surface, at infinity, and at each interface. However, for some particular load distributions, the stress functions may be determined readily. The case under consideration herein is one such case.

The differential equations of equilibrium for an elastic system loaded symmetrically with respect to the vertical axis are:

$$
\begin{align*}
& \frac{\partial \sigma_{r}}{\partial r}+\frac{\sigma_{r}-\sigma_{\theta}}{r}+\frac{\partial \tau_{r z}}{\partial z}=0  \tag{19}\\
& \frac{\partial \tau_{r z}}{\partial r}+\frac{\tau_{r z}}{r}+\frac{\partial \sigma_{z}}{\partial z}=0 \tag{20}
\end{align*}
$$

where

$$
\begin{aligned}
& r=\text { radial distance from axis of symmetry } \\
& \sigma_{r}=\text { normal stress in radial direction }
\end{aligned}
$$

$\sigma_{\theta}=$ normal stress in tangential direction
$\sigma_{z}=$ normal stress in vertical direction
$\tau_{r z}=$ shear stress at $r$ in vertical direction

Strains are related to the displacements by the relationships:

$$
\begin{align*}
& \epsilon_{r}=\frac{\partial u}{\partial r}  \tag{21}\\
& \epsilon_{\theta}=\frac{u}{r}  \tag{22}\\
& \epsilon_{z}=\frac{\partial w}{\partial z}  \tag{23}\\
& \gamma_{r z}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r} \tag{24}
\end{align*}
$$

where

$$
\begin{aligned}
& u=\text { radial displacement } \\
& w=\text { vertical displacement } \\
& \epsilon_{r}=\text { normal strain in radial direction } \\
& \epsilon_{\theta}=\text { normal strain in tangential direction } \\
& \epsilon_{z}=\text { normal strain in vertical direction } \\
& \gamma_{r z}=\text { shear strain at } r \text { in vertical direction }
\end{aligned}
$$

The compatibility equations are:

$$
\begin{align*}
& \nabla^{2} \sigma_{\theta}+\frac{2}{r^{2}}\left(\sigma_{r}-\sigma_{\theta}\right)+\frac{1}{1+\mu} \frac{1}{r} \frac{\partial \theta}{\partial r}=0  \tag{25}\\
& \nabla^{2} \sigma_{r}-\frac{2}{r^{2}}\left(\sigma_{r}-\sigma_{\theta}\right)+\frac{1}{1+\mu} \frac{\partial^{2} \theta}{\partial r^{2}}=0  \tag{26}\\
& \nabla^{2} \sigma_{z}+\frac{1}{1+\mu} \frac{\partial^{2} \theta}{\partial z^{2}}=0  \tag{27}\\
& \nabla^{2} \tau_{r z}-\frac{1}{r^{2}} \tau_{r z}+\frac{1}{1+\mu} \frac{\partial^{2} \theta}{\partial r \partial z}=0 \tag{28}
\end{align*}
$$

where

$$
\begin{aligned}
& \nabla^{2}=\text { the Laplace operator }=\frac{\partial^{2}}{\partial r^{2}}+\frac{1}{r} \frac{\partial}{\partial r}+\frac{\partial^{2}}{2 z^{2}} \\
& \mu=\text { Poisson's ratio } \\
& \theta=\sigma_{r}+\sigma_{\theta}+\sigma_{z}
\end{aligned}
$$

The general solution to the problem was formulated by Love (1944) in terms of a stress function, $\varphi$. In Love's formulation, the stresses are expressed as:

$$
\begin{align*}
& \sigma_{r}=\frac{\partial}{\partial z}\left[\mu \nabla^{2} \varphi-\frac{\partial^{2} \varphi}{\partial r^{2}}\right]  \tag{29}\\
& \sigma_{\theta}=\frac{\partial}{\partial z}\left[\mu \nabla^{2} \varphi-\frac{1}{r} \frac{\partial \varphi}{\partial r}\right]  \tag{30}\\
& \sigma_{z}=\frac{\partial}{\partial z}\left[(2-\mu) \nabla^{2} \varphi-\frac{\partial^{2} \varphi}{\partial z^{2}}\right]  \tag{31}\\
& \theta=(1+\mu) \frac{\partial}{\partial z} \nabla^{2} \varphi  \tag{32}\\
& \tau_{r z}=\frac{\partial}{\partial r}\left[(1-\mu) \nabla^{2} \varphi-\frac{\partial^{2} \varphi}{\partial z^{2}}\right] \tag{33}
\end{align*}
$$

It can be verified by substitution that both the equilibrium equations and the compatibility equations are satisfied if the stress function satisfies the boundary conditions of the problem and also satisfies the biharmonic equation:

$$
\begin{equation*}
\nabla^{2} \nabla^{2} \varphi=0 \tag{34}
\end{equation*}
$$

The relationships for the displacement components are determined by the application of Hooke's law, and are:

$$
\begin{align*}
& u=-\frac{1+\mu}{E} \frac{\partial^{2} \varphi}{\partial r \partial z}  \tag{35}\\
& w=\frac{1+\mu}{E}\left[2(1-\mu) \nabla^{2} \varphi-\frac{\partial^{2} \varphi}{\partial z^{2}}\right] \tag{36}
\end{align*}
$$

where

$$
E=\text { the modulus of elasticity of the material }
$$

For the problem of interest, a basic stress function may be used which has the form:

$$
\begin{equation*}
\varphi_{m}=Z_{m}(\zeta) J_{0}(m \rho) \tag{37}
\end{equation*}
$$

w!?are

$$
\begin{aligned}
& \zeta=\text { dimensionless depth }=z / H \\
& \rho=\text { dimensionless radial distance }=r / H \\
& H=\text { depth to lowermost layer, in arbitrary units } \\
& m=a \text { dimensionless parameter } \\
& J_{0}=\text { Bessel function of the first kind and order zero }
\end{aligned}
$$

It can be verified by substitution that Equation 37 satisfies the biharmonic equation (Equation 34) if $Z_{m}$ satisfies the differential equation:

$$
\begin{equation*}
Z_{m}^{\cdots \cdot \prime}-2 m Z_{m}^{\prime \prime}+m^{4} Z_{m}=0 \tag{38}
\end{equation*}
$$

Using a star superscript to denote that the quantities correspond to the basic stress function given in Equation 37, the stress and displacements may be written in terms of $Z_{m}$ :

$$
\begin{equation*}
\sigma_{z}^{*}=\frac{1}{H^{3}}\left[m^{2}(2-\mu) Z_{m}^{\cdot}-(1-\mu) Z_{m}^{\cdots \cdot}\right] J_{v}(m \rho) \tag{39}
\end{equation*}
$$

$$
\begin{align*}
& \theta^{*}=\sigma_{r}^{*}+\sigma_{\theta}^{*}+\sigma_{z}^{*}=-\frac{(1+\mu)}{H^{3}}\left[m^{2} Z_{m}^{\prime}-Z_{m}^{\prime \cdot}\right] J_{0}(m \rho)  \tag{40}\\
& \tau_{r z}^{*}=\frac{m}{H^{3}}\left[m^{2}(1-\mu) Z_{m}+\mu Z_{m}^{\prime \cdot}\right] J_{1}(m \rho)  \tag{41}\\
& w^{*}=-\frac{1+\mu}{E H^{2}}\left[2 m^{2}(1-\mu) Z_{m}-(1-2 \mu) Z_{m}^{\cdot \cdot}\right] J_{0}(m \rho)  \tag{42}\\
& u^{*}=\frac{1+\mu}{E H^{2}} m Z_{m}^{\prime} J_{1}(m \rho)  \tag{43}\\
& \sigma_{\theta}^{*}=\frac{\mu}{1+\mu} \theta^{*}+\frac{E}{1+\mu} \frac{u^{*}}{r}  \tag{44}\\
& \sigma_{r}^{*}=\theta^{*}-\sigma_{z}^{*}-\sigma_{\theta}^{*} \tag{45}
\end{align*}
$$

where

$$
J_{1}=\text { Bessel function of the first kind and order one }
$$

By inspection of Equations 39 through 45, the vertical normal stress, $\sigma_{2}^{*}$, and the vertical deflection, $w^{*}$, are proportional to the Bessel function of zero order, $J_{0}$. Similarly, the shearing stress, $\tau_{r z}^{*}$, and the radial deflection, $u^{*}$, are proportional to the Bessel function of order one, $J_{1}$. Equations 39 through 45 represent the solution for a layer subjected to a vertical pressure proportional to the Bessel function of zero order and a shearing stress proportional to the Bessel function of order one. Therefore, the solution for any layered system subjected to an axially symmetric load at the surface can be obtained by proper application of the basic solution represented by the stress function of Equation 37. The basic load of interest is a vertical disk load, represented by:

$$
\begin{equation*}
p_{m}^{*}(\rho)=-p_{m} J_{0}(m \rho) \tag{46}
\end{equation*}
$$

where

$$
p_{m}=\text { the load intensity for a vertical normal load }
$$

For the case of a pure vertical normal load, the shearing load intensity is equal to zero:

$$
\begin{equation*}
\tau_{\mathrm{m}}^{*}(\rho)=\tau_{\mathrm{m}} \mathrm{~J}_{1}(\mathrm{~m} \rho)=0 \tag{47}
\end{equation*}
$$

where

$$
\tau_{\mathrm{m}}=\text { the shearing load intensity }
$$

For the basic loading of a vertical normal load disk, the basic stress function for any layer, $j$, may be written in a form similar to Equation 37 , that is:

$$
\begin{equation*}
\varphi_{m j}=Z_{m j}(\zeta) J_{0}(m \rho) \tag{48}
\end{equation*}
$$

As previously noted, the stress function is valid for any layer if $Z_{m j}$ satisfies the differential equation:

$$
\begin{equation*}
Z_{m j}^{\prime \cdots}-2 m^{2} Z_{m j}^{\prime \cdot}+m^{4} Z_{m j}=0 \tag{49}
\end{equation*}
$$

A solution to the differential equation may be chosen such that the constants of integration in the general solution are dimensionless. This solution may be written as:

$$
\begin{equation*}
Z_{m j}=\frac{H^{3} p_{m}}{m^{3}}\left[A_{m j} e^{\alpha}-B_{m j} e^{\beta}+C_{m j} m \zeta e^{\alpha}-D_{m j} m \zeta e^{\beta}\right] \tag{50}
\end{equation*}
$$

where

$$
\begin{aligned}
& \alpha=-m\left(\zeta_{j}-\zeta\right) \\
& \beta=-m\left(\zeta-\zeta_{j-1}\right) \\
& A_{m j}, B_{m j}, C_{m j}, D_{m j}=\text { dimensionless constants of integration } \\
& \zeta_{:}=\text {the dimensionless depth to the bottom of the } j^{t h} \text { layer }
\end{aligned}
$$

Substitution of Equation 50 into the expression for vertical displacement given in Equation 42 yields the solution of interest:

$$
\begin{equation*}
\left(w^{\star}\right)_{j}=-\frac{1+\mu_{j}}{E_{j}}\left[\left(A_{m j}-C_{m j} \vartheta\right) e \alpha-\left(B_{m j}+D_{m j} \Omega\right) e^{\beta}\right]\left(H p_{m} / m\right) J_{0}(m \rho) \tag{51}
\end{equation*}
$$

where

$$
\begin{aligned}
& \left(w^{*}\right)_{j}=\text { vertical displacement for layer } j \\
& \theta=2-4 \mu_{j}-m \zeta \\
& \Omega=2-4 \mu_{j}+m \zeta
\end{aligned}
$$

So the problem of vertical deflection may be solved if the dimensionless constants of integration $A_{m j}, B_{m j}, C_{m j}$, and $D_{m j}$ are known. While not presented here, similar expressions exist for the other stress and displacement functions, so that all the stresses and displacements may be defined if the dimensionless constants of integration are known.

At the surface, the boundary conditions require that the vertical normal stress and the shearing stress be equal to the applied external loadings. At a rough interface, the normal stress and vertical displacement is continuous across the interface as well as the shearing stress and radial displacement. At a smooth interface, the normal stress and vertical displacement is continuous across the interface, but the shearing stress at the interface is zero, i.e. slip is allowed. Since the stresses and displacements vanish for infinite depth ( $\zeta \rightarrow \infty)$, it can be concluded from Equation 50 that for the $n^{\text {th }}$ layer, the constants $A_{m n}$ and $C_{m n}$ are equal to zero (since $\alpha \rightarrow \infty$ as $\zeta \rightarrow \infty$ ). There remains
$4 n-2$ constants which are evaluated by the simultaneous solution of the equations representing the boundary conditions for the various layer interfaces.

To summarize, the boundary conditions at the surface for the case of interest (vertical disk load) are:

$$
\begin{align*}
& \left(\sigma_{z}^{*}\right)_{1}=p_{m}^{*}  \tag{52}\\
& \left(\tau_{r z}^{*}\right)_{1}=\tau_{m}^{*}=0 \tag{53}
\end{align*}
$$

The boundary conditions for a rough interface are:

$$
\begin{align*}
& \left(\sigma_{z}^{*}\right)_{j}=\left(\sigma_{z}^{*}\right)_{j+1}  \tag{54}\\
& \left(w^{*}\right)_{j}=\left(w^{*}\right)_{j+1}  \tag{55}\\
& \left(\tau_{r z}^{*}\right)_{j}=\left(\tau_{r z}^{*}\right)_{j+1}  \tag{56}\\
& \left(u^{*}\right)_{j}=\left(u^{*}\right)_{j+1} \tag{57}
\end{align*}
$$

The boundary conditions for a smooth interface are:

$$
\begin{align*}
& \left(\sigma_{z}^{*}\right)_{j}=\left(\sigma_{z}^{*}\right)_{j+1}  \tag{58}\\
& \left(w^{*}\right)_{j}=\left(w^{*}\right)_{j+1}  \tag{59}\\
& \left(\tau_{r z}^{*}\right)_{j}=\left(\tau_{r z}^{*}\right)_{j+1}=0 \tag{60}
\end{align*}
$$

The boundary conditions for the lowest (infinite depth) layer are:

$$
\begin{equation*}
A_{m n}=C_{m n}=0 \tag{61}
\end{equation*}
$$

To determine the constants of integration, the boundary conditions are applied at each interface. At the surface, for $j=1$ and $\zeta=\zeta_{0}=0$, the equations are:

$$
\begin{align*}
& A_{m 1}+B_{m 1}-\left(1-2 \mu_{1}\right) C_{m 1}+\left(1-2 \mu_{1}\right) D_{m 1}=1  \tag{62}\\
& A_{m 1}-B_{m 1}+2 \mu_{1} C_{m 1}+2 \mu_{1} D_{m 1}=\tau_{m} / p_{m} \tag{63}
\end{align*}
$$

For a rough interface, the equations may be written in matrix form as:

$$
\begin{equation*}
R_{j} X_{j}=0 \tag{64}
\end{equation*}
$$

For a smooth interface, the equations may be written in matrix form as:

$$
\begin{equation*}
s_{j} x_{j}=0 \tag{65}
\end{equation*}
$$

Where for either interface type, the vector $X_{j}$ contains the constants $A_{m j}, B_{m j}, C_{m j}, D_{m j}, A_{m, j+1}, B_{m, j+1}, C_{m, j+1}, D_{m, j+1}$, and the matrices $R_{j}$ and $S_{j}$ are coefficient matrices of order $4 \times 8$, and are given below. To simplify, the following dummy variables are introduced:

$$
\begin{align*}
& k_{j}=\left(E_{j} / E_{j+1}\right)\left(1+\mu_{j+1} / 1+\mu_{j}\right)  \tag{66}\\
& \Psi_{j}=\alpha^{\alpha+\beta}  \tag{67}\\
& T_{j}=1-2 \mu_{j}  \tag{68}\\
& \nu_{j}=k_{j} \Psi_{j+1}  \tag{69}\\
& \eta_{j}=m \zeta_{j} \tag{70}
\end{align*}
$$

The matrix $R_{j}$ for a rough interface is given by:

$$
\left[\begin{array}{cccccccc}
1 & \Psi_{j} & -\left(T_{j}-\eta_{j}\right) & \left(\Upsilon_{j}+\eta_{j}\right) \Psi_{j} & -\Psi_{j+1} & -1 & \left(T_{j+1}-\eta_{j}\right) \Psi_{j+1} & -\left(T_{j+1}+\eta_{j}\right)  \tag{71}\\
1 & -\Psi_{j} & \left(2 \mu_{j}+\eta_{j}\right) & \left(2 \mu_{j}-\eta_{j}\right) \Psi_{j} & -\Psi_{j+1} & 1 & -\left(2 \mu_{j+1}+\eta_{j}\right) \Psi_{j+1} & -\left(2 \mu_{j+1}-\eta_{j}\right) \\
1 & \Psi_{j} & \left(1+\eta_{j}\right) & -\left(1-\eta_{j}\right) \Psi_{j} & -\nu_{j} & -k_{j} & -\left(1+\eta_{j}\right) \nu_{j} & \left(1-\eta_{j}\right) k_{j} \\
1 & -\Psi_{j} & -\left(2 T_{j}-\eta_{j}\right) & -\left(2 \Upsilon_{j}+\eta_{j}\right) \Psi_{j} & -\nu_{j} & k_{j} & \left(2 T_{j+1}-\eta_{j}\right) v_{j} & \left(2 T_{j+1}+\eta_{j}\right) k_{j}
\end{array}\right]
$$

The matrix $S_{j}$ for a smooth interface is given by:

$$
\left[\begin{array}{cccccccc}
1 & \Psi_{j} & -\left(\Upsilon_{j}-\eta_{j}\right) & \left(T_{j}+\eta_{j}\right) \Psi_{j} & -\Psi_{j+1} & -1 & \left(\Upsilon_{j+1}-\eta_{j}\right) \Psi_{j+1} & -\left(T_{j+1}+\eta_{j}\right) \\
1 & -\Psi_{j} & \left(2 \mu_{j}+\eta_{j}\right) & \left(2 \mu_{j}-\eta_{j}\right) \Psi_{j} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \Psi_{j+1} & -1 & \left(2 \mu_{j+1}+\eta_{j}\right) \Psi_{j+1} & \left(2 \mu_{j+1}-\eta_{j}\right) \\
1 & -\Psi_{j} & -\left(2 T_{j}-\eta_{j}\right) & -\left(2 \tau_{j}+\eta_{j}\right) \Psi_{j} & -v_{j} & k_{j} & \left(2 T_{j+1}-\eta_{j}\right) \nu_{j} & \left(2 T_{j+1}+\eta_{j}\right) k_{j}
\end{array}\right]
$$

The typical method of analysis is to choose either a rough or smooth condition at each interface. The BISAR method applies weighting factors to the matrices $R_{j}$ and $S_{j}$ to simulate a partial slip condition. After the weighted combined matrix is formed, the solution proceeds in the same way as for the ideal rough or smooth condition. After the interface matrices are formed, the global matrices are formed. The global problem is of the form:

$$
\begin{equation*}
K C=L \tag{73}
\end{equation*}
$$

where

$$
\begin{aligned}
& K=\text { the global matrix of coefficients } \\
& C=\text { the vector containing the constants of integration } \\
& L=\text { the external load vector }
\end{aligned}
$$

The problem is reduced to the numerical solution of the matrix problem shown below, where the $x^{\prime} s$ indicate the interface matrices:

where

$$
\tau_{\mathrm{m}} / \mathrm{p}_{\mathrm{m}}=0
$$

## BACKCALCULATION OF COMPOSITE PAVEMENT LAYER MODULI

The deformed shape of a pavement when subjected to a surface loading is known as a deflection basin. In the case of a uniform circular load on a layered system, the theoretical deflection basin has the property of radial symmetry and is typically represented by a series of deflections along a radial axis. Figure 10 illustrates ty: cal deflection basins for common pavement types. One reason that composite pavement evaluation is so difficult is that deflection basin shapes of ali of the types shown in Figure 10 can occur in a composite feature. This is due to variations in the condition of the underlying Portland cement concrete layer, which occur in addition to normal variations in subgrade properties. Often, the deflected shape does not specifically indicate a rigid or flexible behavior. Special analysis is needed to identify the relative behavior of the principle layers.

For a layered system with known properties, the elastic response at specific locations may be calculated for a given load by multilayer elastic theory. The FORTRAN subroutine BISAR developed by the Shell 0il Company uses multilayer elastic theory to make such calculations. Researchers at the Waterways Experiment Station have written an iterative program called BISDEF which uses BISAR as a subroutine to calculate theoretical deflection bowls to compare with measured deflection bowls. The program attempts to exactly match the measured deflection basin for a system with unknown layer moduli of elasticity with the theoretical deflection basin for a system with known layer moduli of elasticity. This process is commonly referred to as backcalculation of layer moduli.


FIGURE 10. Typical Deflection Basiiis

A program called COMDEF has been written which is designed specifically for the backcalculation of layer moduli in composite pavements. Improved speed and accuracy were achieved by making the program application specific (see Appendix A, COMDEF User's Guide, for a complete description and list of assumptions).

## EXECUTION TIME AS A CONTROLLING FACTOR

The cost of using a computer program is related to the time required to execute the program. For microcomputer applications, the primary cost is the cost of operator time, since there is no direct cost (other than electricity) associated with central processor unit (CPU) time. For mainframe applications, the direct cost of central processor unit time can be a controlling factor. To reduce computing costs, Waterways Experiment Station researchers have adapted many of their pavement analysis programs (including BISDEF) for use on microcomputers. For many programs which are input/output intensive, the operator time may actually be reduced by using a microcomputer version, due to ease of use and software capability. However, programs which are processor intensive, such as BISDEF, require significant amounts of execution time. The amount of time required to execute the program can have a controlling influence on the way the program is utilized. For the purpose of this report, comparisons of computer costs were made on the basis of execution time on an "MS-DOS compatible" microcomputer. Although the costs associated with other systems will not be discussed, it can be assumed that a program which executes significantly faster will result in reduced computing costs on any computer system.

Figure 11 illustrates the time savings of using the DELTA subroutine of COMDEF instead of the BISAR structural response model. The time comparisons were made on an 80286 -based microcomputer manufactured by Dell Computers. The time required for BISAR was more than 400 times greater than the COMDEF subroutine DELTA for making similar response predictions.


FIGURE 11. Forward Response Model Time Comparison (DELTA vs. BISAR)

BISDEF searches for a "matching" deflection basin by iteration, using a relaxation technique which solves simultaneously for all layer moduli based on gradient matrices. For a system with three unknown layers, four separate calls to the BISAR subroutine are required for each BISDEF iteration. For conventional pavements with three unknown layers and seven deflections, a BISDEF iteration will require about 3 minutes on an 80286 -based microcomputer. A reasonable tolerance can usually be achieved with 1 or 2 iterations, so that the total execution time for conventional pavements is usually between 3 and 6 minutes. For composite pavements, the convergence time of the BISAR subroutine is increased significantly. In some extreme cases, a single BISDEF iteration for a composite pavement can require as much as 20 minutes to
execute. A typical composite pavement case with three unknown layers and seven deflections will require about 4 minutes per BISDEF iteration. A reasonable solution for a composite pavement may require the full three iterations. Therefore, execution time may be a controlling factor for backcalculation of composite pavements moduli with BISDEF. In contrast, COMDEF will produce a reasonable solution for all cases in less than 2 minutes. Figure 12 illustrates the actual execution times for BISDEF and COMDEF for typical theoretical data. For the case tested, BISDEF closed on a solution in one iteration, so the time for a general BISDEF solution could be up to three times higher. The times indicated in Figure 12 for COMDEF are near the upper bound on execution time. Execution time is machine dependent. The execution time comparisons reported herein were made on a Dell 80286-based microcomputer.


FIGURE 12. Comparison of Execution Times for COMDEF and BISDEF

A program called COMDEF was developed for the analysis of composite airfield pavements. The method used in the program was the result of several iterations in methodology in various attempts to develop an improved method for backcalculation.

After completion of the literature review, the author felt that the iterative gradient relaxation method used in the program BISDEF (Bush and Alexander 1985) was the best currently available backcalculation method. A sensitivity study was performed to determine the sensitivity of the program to changes in important variables. Complete results of this study are included in a subsequent section. The sensitivity study showed clearly that the backcalculated moduli predicted by BISDEF were sensitive to the assumed values of moduli used as the initial trial values, and that the program would not converge to a reasonable soiution in many cases.

Initial efforts by the author to improve the backcalculation method for composite pavements centered around developing a driver subroutine which would predict good trial values for use in the BISDEF program. A subroutine called SEED was developed and integrated with the BISDEF program. The SEED method used fitted polynomials which represented the change in deflection at each sensor location as a function of the layer moduli and layer thicknesses. Early testing with SEED indicated a good deal of potential for this method. Results were favorable enough to pursue a new program which would not require the use of BISDEF.

Improvements in the method led to a second version of SEED, which was renamed COMDEF. COMDEF stands for Composite (pavement) Optimized Moduli from Deflection Evaluation with the Ealling Weight Deflectometer. The original version of COMDEF used polynomial representations which were extremely complex, and provided a direct interpolation of the moduli values. The early version of COMDEF used two important findings for modulus estimation. The first finding was that the modulus of subgrade correlated very closely with the outside sensor deflection, $\Delta_{72^{\prime \prime}}$. After a good estimation of the subgrade was obtained, the second finding was used. The basis of the second finding was that the ratio of the modulus of the Portland cement concrete to the modulus of the asphaltic concrete correlated very well with a ratio of the differences in predicted deflection, $\left(\Delta_{0^{n}}-\Delta_{12^{\sim}}\right) /\left(\Delta_{12^{\sim}}-\Delta_{24^{\sim}}\right)$. This ratio of deflections was given the name BRATIO, which stood for backcalculation ratio. A plot which illustrates the BRATIO methodology is shown in Figure 13. The new method worked exceptionally well with theoretical data. However, the program did not work well with some sample field data supplied by researchers at the Waterways Experiment Station. The data was from an extreme case, but served to point out two major problems with the method. The airfield pavements from which the data was collected was extremely thick ( 27 inches ( 0.7 m ) of Portland cement concrete). The polynomial representations used in the program became unstable for extreme values of pavement thickness. The other problem was that the data from a very strong, very thick pavement represented a very shallow deflection basin. That is, the difference in deflection with radial distance was small. In this case, the BRATIO was not an accurate indicator, and was very sensitive to minor changes in deflection values.

The conclusion was that although the program worked very well in predicting layer moduli for typical cases, it was urreliable for extreme cases. This was not an acceptable solution, so that further development was needed.


FIGURE 13. BRATIO Method Used in SEED Program

Initial efforts at improving COMDEF centered around using more and more complex polynomials to relate the deflection behavior to the layer moduli for a wider target range of typical values. For each increased level of complexity, the polynomials did a good job of predicting behavior within the chosen target range. However, for airfield pavements, the extrema values of thicknesses and moduli which can occur cover a broad range. As the number of polynomial constants became prohibitive, it became apparent that an improved method could utilize a
stored matrix of solutions, so that exact data could be used instead of polynomial representations. This improvement, coupled with a new method fur prediciing tridi vaiues based on two sensor deflections, made COMDEF a success. The complete method used in COMDEF is described in the COMDEF User's Guide, which is included in Appendix A. A narrative description of the program is included in the following section.

COMDEF is an interactive, user friendly fORTRAN program which backcalculates layer moduli for composite pavements. A cursory discussion is include herein. The interested reader is referred to the COMDEF User's Guide in Appendix A. The implementation version of the program, as implied by the title, is based on deflections measured by a Falling Weight Deflectometer. The COMDEF method uses a matrix of precalculated solutions stored in database files to replace the forward structural response model which would typically be included in a backcalculation algorithm. The forward modeling method used in COMDEF is completely automated and numerically approximates the theoretical deflection basin which would normally be calculated by a structural response model. The structural response model used in the implementation version of COMDEF is multilayered linear elastic theory. The data compression technique and interpolation routines used by COMDEF allow predicted deflections to be calculated almost instantaneously from a relatively small database with a high degree of accuracy. A comparison was made between predicted deflections calculated by COMDEF and BISAR for 45 cases with a wide range of thicknesses and layer moduli. The differences in the approximate COMDEF deflections and the theoretical deflections calculated by BISAR are small, with an average error of $0.001 \mathrm{mils}(25 \mathrm{~nm})$. This data is presented in the sensitivity study results.

COMDEF is an elementary expert system, since it automatically applies some engineering judgment in comparing results from multiple calculations. COMDEF uses repeated application of two general techniques to solve for layer moduli based on Falling Weight

Deflectometer deflections. One technique was developed specifically for the COMDEF program and has been named the method of stepwise direct optimization. The other technique is an iteration method similar to that used in the BISDEF backcalculation program. This method is an iterative relaxation technique which uses gradient matrices to quickly converge to a solution.

COMDEF uses database files to numerically calculate the deflection basins which are compared to Falling Weight Deflectometer deflections. The deflections in the database files were calculated by multilayer elastic theory. Assumptions which apply to multilayer elastic theory therefore apply to COMDEF, i.e. layers are horizontal, homogeneous, and isotropic and the loading plate applies a uniform pressure distribution to the pavement surface. Since the assumption of a uniform pressure distribution implies that some plate bending will occur, the deflection of the plate is assumed to be equal to the geometric average of the deflection under the load plate for a parabolic distribution of deflection under the load plate. In addition to these assumptions, there are other assumptions which facilitate a solution. It is assumed that the peak load and peak deflections measured by the Falling Weight Deflectometer can be compared to an equivalent elastic system under a static load. A rigid boundary is assumed to exist at a depth of 20 feet (as previously discussed in the section "Multilayer Linear Elastic Models"). Typical assumptions are made for interface friction, that is: (1) a "rough" interface between asphaltic concrete and Portland cement concrete, (2) a "smooth" interface between Portland cement concrete and subgrade, and (3) a "rough" interface between subgrade and rigid boundary. It is assumed that the equivalent pavement system can be
modeled by a three-layer system. This assumption is reasonably valid for most airfield pavements, since current design procedures utilize a bulk value for the stiffness of the subgrade under a Portland cement concrete pavement. Assumption of a uniform subgrade modulus gives reasonable results for backcalculated pavement layer moduli except when there are significant changes in subgrade stiffness relatively near the upper pavement layers (e.g., shallow depth to bedrock).

For each test location, the following is completed. The program performs a "global" iteration, using the subroutine ITERAT. The technique used by the subroutine is iterative relaxation, and the solution is achieved by forming gradient matrices and searching for solutions which minimize the errors in the fitted basin. This method has been used in BISDEF and has the advantage of rapid convergence. COMDEF stores the results from the global iteration and begins stepwise calculations. The term stepwise indicates that small increments are used for asphaltic concrete modulus. The use of stepwise calculations minimizes the occurrence of unrealistic nonunique solutions. For each increment, a best solution is calculated by two methods, stepwise direct optimization and stepwise iteration.

After each stepwise calculation, the total basin error and the total basin percentage error are compared to the previous best solution. These error terms are defined as:

$$
\begin{align*}
& \text { TOTAL BASIN ERROR }=\sum_{j=1}^{m}\left|D_{j}-\Delta_{j}\right|  \tag{75}\\
& \text { TOTAL BASIN }  \tag{76}\\
& \text { PERCENTAGE ERROR }=\sum_{j=1}^{m} \frac{\left|D_{j}-\Delta_{j}\right|}{D_{j}} * 100 \%
\end{align*}
$$

where

$$
\begin{aligned}
& D_{j}=\text { measured deflection for sensor } i \\
& \Delta_{j}=\text { theoretical deflection for sensor } i \\
& m=\text { number of sensor locations ( } m=7 \text { for COMDEF) }
\end{aligned}
$$

Each new incremental solution is accepted as an improved solution if both the total basin error and the total basin percentage error are less than the previous best solution. The stepwise calculations continue until a single best solution is found for all of the stepwise calculations. The best solution from the stepwise calculations is used as input data for a local optimizing iteration with the ITERAT subroutine. This final optimizing iteration allows more iterations and seeks a higher tolerance than the stepwise iterations, as well as allowing a wider range of possible moduli values. The total error and total percentage error of the basin for the local optimizing iteration are compared to the values from the stepwise calculations. The better of these solutions is compared to the total error and total percentage error in the basin from the global iteration. The best of all these solutions has the lowest total error and total percentage error in the fitted basin and is accepted as the final solution. The solution is output to the microcomputer screen and to an output file. This process is repeated until the moduli for all the test locations are computed. The major advantage of the method used in COMDEF is the rapid calculation of accurate comparison deflections, which allows a large number of iterations compared to other methods. This allows a much more accurate solution than would normally be obtained.

## THE METHOD OF STEPWISE DIRECT OPTIMIZATION

Stepwise calculations minimize the occurrence of unrealistic nonunique solutions. The method of stepwise direct optimization was devised by the author and includes both the stepwise calculation of trial moduli values by the subroutine RANGES and the local optimizing of the trial moduli values. Trial values for each increment of asphaltic concrete modulus are calculated by the method illustrated in Figure 14.


FIGURE 14. Trial Value Estimation for Stepwise Direct Optimization

The method illustrated in Figure 14 assumes that the outside sensor deflection $\left(D_{7}\right)$ is a strong indicator of subgrade modulus and that the inside sensor deflection ( $D_{1}$ ) is strongly influenced by the Portland cement concrete modulus. For each stepwise increment of asphaltic concrete modulus, a set of Portland cement concrete moduli values are assumed. For a given combination of asphaltic concrete modulus and Portland cement concrete modulus, each of the sensor deflections may be considered as a function of subgrade modulus. Each of the assumed values of Portland cement concrete moduli are used to calculate predicted subgrade moduli which give exact solutions for the deflections corresponding to the inside and outside sensors. When predicted subgrade modulus based on the outside sensor deflection is plotted versus assumed Portland cement concrete modulus, the curve is relatively flat. When predicted subgrade modulus based on the inside sensor deflection is plotted versus assumed Portland cement concrete modulus, a distinct curve is formed. The intersection of the two curves produces trial values which represent the solution of exact fit for both the inside and outside sensor deflections for that increment of asphaltic concrete modulus. A matrix of trial values is determined by the RANGES subroutine, with trial values for Portland cement concrete and subgrade moduli at each stepwise increment of asphaltic concrete modulus.

The basis of the direct optimizing technique is described below. In a layered system, if the modulus of one layer is varied while the moduli of the other layers is held constant, the predicted relationship between the variation in modulus and the variation in deflection at a given sensor location is nearly linear when plotted on a $\log -\log$ graph. This effect has been noted by numerous authors and is typically used in
backcalculation by gradient relaxation to speed closure by providing more accurate extrapolations based on the gradient matrices. for each application of direct optimization, it is assumed that all of the layer moduli values except one are known exactly and that the unknown modulus value is known approximately (the trial value). It is therefore assumed that a set of relationships exist which define the variations in the unknown layer modulus as a function of the variation in the deflections. These relationships are defined by using the predicted deflections for the trial modulus value and also the predicted deflections for a modulus value slightly offset from the trial modulus value. For each of the sensor locations, these values are used to predict a log-log relationship of the following form:

$$
\begin{equation*}
\left(E_{\text {pred }}\right)_{i}=\left(m \Delta^{n}\right)_{i} \tag{77}
\end{equation*}
$$

where

$$
\begin{aligned}
& \left(E_{\text {pred }}\right)_{i}=\text { predicted modulus for sensor } i \\
& \Delta=\text { deflection for sensor } i \\
& m, n= \\
& \\
& \quad \begin{array}{l}
\text { deflections calculated for the trial modulus value }
\end{array} \\
& \quad \text { and for a small deviation in the trial modulus value }
\end{aligned}
$$

Each of the relationships give a value of $E_{\text {pred }}$ for the actual measured deflection. However, the parameter of interest is the optimum value of modulus, $E_{\text {optimum }}$ to be determined by a least-squares analysis. The deviation from the optimum modulus value, $\delta E_{\text {opt imum }}$, may be expressed as:

$$
\begin{equation*}
\delta E_{\text {opt imum }}=E_{\text {opt imum }}-\left(E_{\text {pred }}\right)_{i} \tag{78}
\end{equation*}
$$

Therefore the expression, $f$, for the sum of the squared deviations may be written as:

$$
\begin{equation*}
f=\sum\left(\delta E_{\text {opt imum }}\right)^{2}=\sum\left[E_{\text {opt imum }}-\left(m \Delta^{n}\right)_{i}\right]^{2} \tag{79}
\end{equation*}
$$

By setting the derivative of $f$ with respect to $E_{\text {opt imum }}$ equal to zero, it may be easily shown that the least-squares optimized solution for the modulus jeing considered is simply the average of the $\left(E_{p r e d}\right)_{i}$. That is:

$$
\begin{equation*}
E_{\text {opt imum }}=\left(m \Delta^{n}\right)_{\text {average }} \tag{80}
\end{equation*}
$$

The technique described above is applicable only when the problem has been reduced to a single unknown layer modulus. The implementation of this method for the multilayer case is based on research ty the author which indicated that for fixed values of thicknesses and asphaltic concrete modulus, the subgrade modulus may be accurately estimated from the deflection basin if a reasonable value of Portland cement concrete modulus is assumed. A reasonabie value of Portland cement concrete modulus is provided by the trial value of Portland cement concrete modulus from the RANGES subroutine. The direct optimizing process proceeds as follows: (1) the asphaltic concrete modulus for each step is assumed to be equal to the midrange value of that asphaltic concrete modulus step, (2) the Portland cement concrete modulus is assumed to be equal to the trial value from the RANGES subroutine for that asphaltic concrete modulus step, (3) the subgrade trial modulus value is optimized, (4) using the new value of subgrade modulus, the Portland cement concrete trial modulus value is optimized, and (5) using the new subgrade and rortland cement concrete moduli, the asphaltic concrete
modulus is optimized. The direct optimizing technique is used for each stepwise increment of asphaltic concrete modulus and the best solution is chosen from all of the direct optimizations.

BISDEF is a general purpose multilayer linear elastic pavement evaluation program which has been used successfully for airfield and highway pavement analysis on various military installations and also in the analysis of nondestructive testing data from different pavement types and loading devices in comparison studies (Bush and Alexander 1985, Bush, Alexander, and Hall 1985). COMDEF is a program written for the specific application of composite pavement evaluation utilizing the Falling Weight Deflectometer. In this report, performance comparisons are made between COMDEF and BISDEF. The comparisons show that for the evaluation of composite pavements with the Falling Weight Deflectometer, the specific application program COMDEF will outperform the general application program BISDEF. This is not intended to detract from the track record of the general application program BISDEF, but rather to illustrate the capability of the specific application program COMDEF.

## RELATIONSHIP OF CRACKING IN PCC LAYER TO BACKCALCULATED PCC MODULI

Past experience (Sharpe, et al. 1986, Sharpe, et al. 1987) with in-place recycling (break and seat) of rigid pavements has shown that backcalculated Portland cement concrete moduli for composite pavements represent effective moduli which are directly related to the degree of cracking in the Portland cement concrete layer. In many cases, the determination of condition of the underlying Portland cement concrete layer is important for a complete nondestructive evaluation. Degree of cracking is not determined directly by COMDEF, but the backcalculated modulus of Portland cement concrete from COMDEF is a good indicator of the relative degree of cracking. While exact correlations do not exist, the following criteria based on past research provides approximate relationships between degree of Portland cement concrete cracking and backcalculated Portland cement concrete modulus. In general, composite pavements with backcalculated Portland cement concrete moduli of less than 1 million psi ( 6.89 MPa ) may be considered as behaving as flexible pavements. If the backcalculated Portland cement concrete modulus is less than $100 \mathrm{ksi}(689 \mathrm{kPa})$, the Portland cement concrete layer may be considered as completely shattered and roughly equivalent to a high quality granular base. If the backcalculated Portland cement concrete modulus is greater than $100 \mathrm{ksi}(689 \mathrm{kPa})$, but less than 1 million psi (6.89 MPa), the overall pavement system may be considered flexible and the Portland cement concrete layer will act similarly to a stabilized granular base. If the backcalculated Portland cement concrete modulus is above 1 million psi ( 6.89 MPa ), the Portland cement concrete layer may be considered intact and behaving rigidly.

## TREATMENT OF HIGH VALUES OF BACKCALCULATED PCC MODULUS

In some cases, backcalculated Portland cement concrete moduli may be relatively high. It is not unusual for values of backcalculated Portland cement concrete moduli to go above 15 million psi (103 MPa) when testing composite pavements with relatively undamaged Portland cement concrete layers. The reason for the high Portland cement concrete moduli values is the assumed logarithmic relationship between deflection and layer moduli. For composite pavements which behave rigidly, the deflection basin is relatively shallow and of low magnitude (as shown in Figure 10). When obtaini..j the best possible fitted basin, there may be only a small difference in the deflection behavior when the Portland cement concrete moduli is varied from 10 to 20 million psi (69 MPa to 138 MPa ). In general, the Portland cement concrete may be considered as a high strength layer when backcalculated moduli go above 7 million psi ( 48 MPa ). Most overlay design procedures for rigid pavements are based on tensile failure at the bottom of Portland cement concrete. For this reason, slightly higher values of Portland cement concrete modulus typically result in more conservative solutions. However, use of extremely high values of Portland cement concrete moduli may result in unconservative predictions of allowable load. For this reason, COMDEF assigns for a given test location a maximum value for the Portland cement concrete modulus of 7 million psi ( 48 MPa ) when calculating the average Portland cement concrete modulus for the given feature.

## CHAPTER IV

## SENSITIVITY STUDIES

## INTRODUCTION

A sensitivity study is defined herein as a systematic test of a computer program to determine the sensitivity of the program output to changes in important variables. Sensitivity studies are also known as parametric studies. To test a backcalculation program, a series of theoretical deflection basins is used as input, so that the theoretical values of layer moduli are known exactly. The backcalculation method is valid if the known values of moduli are backcalculated within a reasonable tolerance.

A sensitivity study using the program BISDEF was conducted. BISDEF requires an initial estimate (trial value) for each unknown moduli. Past experience with pavements which had stiff bases (Sharpe et al. 1985) led to concern about the sensitivity of backcalculated moduli to the chosen values of trial moduli. Preliminary BISDEF outputs indicated that the program was sensitive to the choice of trial values. A total of 45 cases were included in the sensitivity study. The cases studied included a matrix of 3 thicknesses of asphaltic concrete, 3 thicknesses of Portland cement concrete, and 5 layer moduli combinations. For each case, ten combinations of trial moduli values were used, for a total of 450 BISDEF runs. Table 1 summarizes the trial value combinations. BISDEF solutions were calculated from the
theoretical deflection basins using default conditions for all parameters except trial values. The default conditions set upper and lower limits on the layer moduli and allowed 3 iterations.

Table 1. Trial Value Combinations for Sensitivity Studies

| Trial Value Combination Identifier | Analysis Program | Trial Values for Moduli of Principle Layers |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $\left(E_{a c}\right)_{\text {trial }}$ | $\left(E_{p c c}\right)_{t r i a l}$ | $\left(E_{\text {sub }}\right)_{\text {trial }}$ |
| COM | COMDEF | + | + | + |
| A | BISDEF | 0.67* ( $\left.E_{\text {ac }}\right)_{\text {exact }}$ | 0.67* ( $\left.\mathrm{E}_{\mathrm{pcc}}\right)_{\text {exact }}$ | $0.67 *\left(E_{\text {sub }}\right)_{\text {exact }}$ |
| B | BISDEF | 1.50* ( $\left.E_{\text {ac }}\right)_{\text {exact }}$ | 1.50* ( $\left.\mathrm{E}_{\mathrm{pcc}}\right)_{\text {exact }}$ | $1.50 *\left(E_{\text {sub }}\right)_{\text {exact }}$ |
| c | BISDEF | 0.67* ( $\left.\mathrm{E}_{\mathrm{ac}}\right)_{\text {exact }}$ | $1.50 *\left(\mathrm{E}_{\mathrm{pcc}}\right)_{\text {exact }}$ | $0.67 *\left(E_{\text {sub }}\right)_{\text {exact }}$ |
| D | BISDEF | 1.50* ( $\left.\mathrm{E}_{\mathrm{ac}}\right)_{\text {exact }}$ | $0.67 *\left(E_{\text {pcc }}\right)_{\text {exact }}$ | $1.50 *\left(E_{\text {sub }}\right)_{\text {exact }}$ |
| E | BISDEF | $0.50 *\left(E_{\text {ac }}\right)_{\text {exact }}$ | 0.50* ( Epcc$)_{\text {exact }}$ | $0.50 *\left(E_{\text {sub }}\right)_{\text {exact }}$ |
| F | BISDEF | 2.00* $\left.E_{\text {ac }}\right)_{\text {exact }}$ | $2.00 *\left(E_{p c c}\right)_{\text {exact }}$ | 2.00* ( $\left.\mathrm{E}_{\text {sub }}\right)_{\text {exact }}$ |
| G | BISDEF | 0.50* ( $\left.\mathrm{E}_{\mathrm{ac}}\right)_{\text {exact }}$ | $2.00 *\left(\mathrm{E}_{\mathrm{pcc}}\right)_{\text {exact }}$ | $0.50 *\left(E_{\text {sub }}\right)_{\text {exact }}$ |
| H | BISDEF | $2.00 *\left(\mathrm{E}_{\mathrm{ac}}\right)_{\text {exact }}$ | 0.50* ( Epcc$)_{\text {exact }}$ | 2.00* ( $\left.\mathrm{E}_{\text {sub }}\right)_{\text {exact }}$ |
| I | BISDEF | 51,000 psi | 510,000 psi | 2,600 psi |
| J | BISDEF | 990,000 psi | 9,900,000 psi | 99,000 psi |

+ No trial values required for COMDEF
NOTE: $\quad 1 \mathrm{psi}=6.89 \mathrm{kPa}$

A sensitivity study using the program COMDEF was conducted. The theoretical deflection basins used in the BISDEF sensitivity study were also used in the COMDEF sensitivity study. Since COMDEF does not
require trial values, the only variables were layer moduli and layer thicknesses. Therefore, a total of 45 cases were analyzed with COMDEF. As part of the sensitivity study, check solutions were generated with BISAR to determine the sensitivity of the approximate deflections predicted by COMDEF to changes in important variables.

Results of the sensitivity studies involve presentation of various errors and percentage errors. For the purpose of this report, the error and the percentage error in a predicted quantity, $X$, is defined as :

$$
\begin{align*}
& \text { ERROR }=\left|x_{\text {actual }}-x_{\text {predicted }}\right|  \tag{81}\\
& \text { PERCENTAGE ERROR }=\frac{\left|x_{\text {actual }}-x_{\text {predicted }}\right|}{x_{\text {actual }}} * 100 \% \tag{82}
\end{align*}
$$

## COMDEF DEFLECTION APPROXIMATIONS

COMDEF does not contain a theoretical structural response model. Structural responses (deflections) are approximated from a matrix of precalculated solutions. In this section, it might appear to the casual reader that there is an implicit assumption that the values calculated by BISAR are "correct." However, this is not the issue under consideration here. The intent of this section is to show that the numerica! approximations of deflection made by COMDEF are sufficiently close to the predicted values of deflection from BISAR so that errors due to the COMDEF approximation process may be ignored. By analogy, it is probable that a similar result would be found for any structural response model of interest.

COMDEF approximates deflections by consecutive application of Lagrangian interpolation. Lagrange's form of the interpolating polynomial is :

$$
\begin{equation*}
p_{n}(x)=\sum_{i=0}^{n}\left[y_{i} \prod_{j \neq i}\left[\frac{x-x_{j}}{x_{i}-x_{j}}\right]\right] \tag{83}
\end{equation*}
$$

where

$$
\begin{aligned}
& p_{n}=\text { interpolating polynomial function } \\
& n=\text { the order of the interpolating function } \\
& \left(x_{i}, y_{i}\right)=\text { data points used in the interpolation } \\
& j=0,1, \ldots, n
\end{aligned}
$$

An important property of the Lagrangian interpolating function is that the function is exact at each of the known data points, that is:

$$
\begin{equation*}
p_{n}\left(x_{i}\right)=y_{i} \text { for all }\left(x_{i}, y_{i}\right) \tag{84}
\end{equation*}
$$

Two types of files are used to approximate deflections. These files have been named asphalt database files and constants files. These names are not meant to be inherently descriptive, but rather they are used as identifications. Each asphalt database files contain deflections for a fixed thickness of asphaltic concrete. "Standard" asphalt database files are available for thicknesses of asphaltic concrete from 0 to 16 inches ( 0 to 406 mm ) in increments of 0.5 inches ( 13 mm ). Each database file contains deflections for seven sensor locations for a matrix of Portland cement concrete thicknesses and layer moduli. The matrix of thicknesses and moduli are summarized in Table 2. "Custom" asphalt database files for non-standard thicknesses of asphaltic concrete may be calculated by COMDEF. The custom database files are calculated by applying the Lagrangian interpolation formula to the 6 standard asphalt database files whose thicknesses are nearest the asphaltic concrete thickness chosen. Constants files are actually smaller database files, but are named differently to minimize confusion. The constants files are created by applying the Lagrangian interpolation formula to interpolate a matrix of deflections for a fixed thickness of Portland cement concrete. Therefore, each constants file contains deflections for the matrix of layer moduli described in Table 2 but for a specific combination of asphaltic concrete and Portland cement concrete thicknesses. A deflection basin may be calculated for a given set of layer
moduli by interpolating the deflections in the constants file. To accomplish this, the Lagrangian formula is applied consecutively for each of the given values of layer moduli.

TABLE 2. Variable Matrix ${ }^{\text {a }}$ for Asphalt Database Files ${ }^{\text {b }}$

| Variable | Units | Values |
| :--- | :--- | :--- |
| AC thickness | in. | Fixed $^{\text {c }}$ |
| PCC thickness | in. | $4,6,9,14,20,30$ |
| AC modulus | ksi | $33,82,205,512,1280,3200$ |
| PCC modulus | ksi | $82,205,512,1280,3200,8000,20000$ |
| Subgrade modulus | ksi | $2,6,18,54,162$ |

${ }^{a}$ Deflections are calculated for each combination of the variable matrix at seven sensor locations located on 1 foot centers along a line measured from the center of the FWD load Plate (centers at $0,12,24,36,48,60$, and 72 inches).
b Total number of deflections in each of 33 asphalt database files is equal to the product of 7 sensor locations times 6 thicknesses of PCC times 6 moduli of AC times 7 moduli of PCC times 5 moduli of subgrade for a total of 8820 deflections per file.
${ }^{c}$ Each standard database file contains deflections for a fixed thickness of $A C$. There are 33 standard database files in 0.5 in . increments of $A C$ thickness from 0 in. to 16 in . of $A C$.

NOTE: $\quad 1 \mathrm{in} .=25.4 \mathrm{~mm} \quad 1 \mathrm{ksi}=6.89 \mathrm{MPa} \quad 1 \mathrm{ft}=0.305 \mathrm{~m}$

The subroutine BISAR was used to generate theoretical deflections to check the COMDEF deflections. For each of the 45 cases used in the COMDEF sensitivity study, BISAR check solutions calculated using the layer moduli predicted by COMDEF were used as a direct comparison with
the COMDEF calculated deflection basins. The results of this comparison are listed in Tables B1-B4, located in Appendix B. Table Bl lists the COMDEF values of predicted layer moduli and approximate deflection basins. Table B2 lists the corresponding theoretical deflection basins calculated by BISAR. Table B3 lists the errors in the deflections, and Table 84 lists the percentage errors in the deflections. Figure 15 summarizes the comparison of COMDEF approximate deflections with the theoretical BISAR deflections. A regression line fit through the data indicated that the data were almost exact, with slope of 1.00 , computed intercept of 0.00 , and correlation coefficient approximately equal to 1. Figure 15 also summarizes the errors and percentage errors in the deflections. The average error was equal to $0.001 \mathrm{mils}(25 \mathrm{~nm})$ and the average percentage error was $0.02 \%$. Maximum error and maximum percentage error are listed. The maximum error occurred in a relatively large deflection value and corresponded to a percentage error of $0.06 \%$. The data listed in Tables B1-84 and summarized in Figure 15 indicate that the interpolation procedure used in the COMDEF program does not introduce significant errors in the predicted deflection basins.


FIGURE 15. Deflection Errors Due to the COMDEF Approximation Method

## BISDEF AND COMDEF SENSITIVITY STUDIES

Both BISDEF and COMDEF calculate a fitted basin of predicted deflections which are calculated using the predicted values of layer moduli. The terms total basin error (or basin error) and total basin percentage error (or basin percentage error) have been defined previously and are summations of errors and percentage errors, respectively, in deflection for all of the sensor locations.

Complete results of both sensitivity studies are included in Appendix 8. Table B5 i, a tabular listing of the COMDEF sensitivity stud. Tables $\mathrm{B} 6-\mathrm{Bl} 5$ list results of the BISDEF sensitivity study for each of the 10 trial value combinations listed in Table 1. Tables 85-Bl5 list the thicknesses and layer moduli used to generate the theoretical deflection basins, the layer moduli which were backcalculated from the theoretical basins, and the error and percentage error in the fitted deflection basins. Since COMDEF deflections are mathematical approximations of multilayer elastic deflections, the BISAR check solutions listed in Table $B 2$ were used the calculate the basin errors and percentage errors for the COMDEF solutions listed in Table B5.

The tabular listings in Tables B5-B15 can be difficult to interpret. To make comparisons less difficult, graphical summaries are presented in Figures Bl-B45. These figures present graphically the percentage error (PE) in predicted values for each of the cases studied. Each figure includes results from the COMDEF solution, and also the results for each of the ten BISDEF solutions (corresponding to the tria? value combinations listed in Table l). Percentage error is plotted for each backcalculated moduli value. In addition, the total percentage
error of the fitted basin is plotted. Total percentage error is a good indicator of the overall fit of the matching deflection basin. A graphical presentation of the averages of Figures B1-B45 is shown in Figure 16.

$\square$ Summation of basin percentage errors ( 7 deflections)
$\boxed{\otimes}$ Percentage error in backcalculated AC modulus
$\square$ Percentage error in backcalculated PCC modulus
$\boxed{\square}$ Percentage error in backcalculated subgrade modulus COM = Results from COMDEF sensitivity study $\mathrm{A}-\mathrm{J}=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE 16. Summary of Results from Sensitivity Studies

It should be noted that some of the overall average errors in the BISDEF combinations shown in Figure 16 are relatively high due to a few divergent solutions. The figure clearly shows the high degree of accuracy achieved by COMDEF. Average errors were $0.32 \%$ for the asphaltic concrete modulus, $0.30 \%$ for the Portland cement concrete modulus, and $0.07 \%$ for the subgrade modulus. Average total percentage error in the fitted basin was equal to $0.07 \%$. The maximum percentage error in any backcalculated moduli in the COMDEF sensitivity study was $3.67 \%$ and the maximum total percentage error in any fitted basin in the COMDEF sensitivity study was $0.41 \%$.

Figures 17 and 18 make performance comparisons between BISDEF and COMDEF based on the proportion of cases which meet a given level of tolerance. Figure 17 compares levels of tolerance for the total percentage errors in fitted basins. This figure shows that all COMDEF basins matched within $1 \%$. BISDEF basins matched within $1 \%$ in $20 \%$ of the cases tested. It should be noted that the default condition used in the BISDEF sensitivity study accepted as correct any solution which matched the deflection basin within $10 \%$. This condition was met for $83 \%$ of the cases tested. Similarly, Figure 18 illustrates that all layer moduli calculated by COMDEF were within $4 \%$ of the known moduli. This accuracy was achieved by BISDEF in $70 \%$ of the cases studied. The layer moduli backcalculated by BISDEF were within $10 \%$ of the known values in $83 \%$ of the cases studied. BISDEF provided adequate solutions in the majority of the cases studied, but provided solutions which were not within a reasonable tolerance in $17 \%$ of the cases studied. COMDEF provided excellent solutions for all cases studied.

COMDEF


BISDEF

图 $\mathrm{PE}<1 \%$ $\square$ $1 \%<\mathrm{PE}<10 \% \mathrm{PE}>10 \%$

FIGURE 17. Comparison of the Range of Errors in Fitted Basins


PE $<4 \% \square 4 \%<\mathrm{PE}<10 \%$ PE $>10 \%$

FIGURE 18. Comparison of the Range of Errors in Backcalculated Moduli

The data presented in Appendix $B$ and summarized in Figure 16 indicate that the moduli values for the two upper layers which were backcalculated by BISDEF are sensitive to the selection of trial values for composite pavements. This is an important result, because past research (Bush 1980) has indicated that moduli values backcalculated by BISDEF are not sensitive to choice of trial values for other pavement types. The BISDEF sensitivity study indicated that acceptable solutions were most often obtained when trial values were relatively close to actual values. Poor, non-unique, and/or divergent solutions occurred most often for trial value combinations which were not relatively close to the actual values. Poor solutions are solutions which improve with iteration, but do not meet the tolerance requirements of BISDEF within the specified number of iterations. Poor solutions can be eliminated by increasing the number of iterations and are therefore of much less concern than non-unique or divergent solutions. A divergent solution occurs when one (or more) of the layer moduli is assigned a boundary value (either a maximum or a minimum) which cannot be corrected with additional iterations. Non-uniqueness occurs when an incorrect solution meets the tolerance criteria of the program. BISDEF accepts a solution when the total percentage error of the basin is less than $10 \%$, or when moduli values are within $10 \%$ of each other on subsequent iterations. BISDEF uses a relaxation technique based on gradient matrices to search for the solution with the minimum percentage error of the fitted basin. The presence of a local minimum in the percentage error can cause the program to move toward an incorrect solution. The problem is magnified for composite pavements because all the deflections are relatively small and because both of the upper layers are relatively stiff. Non-unique
and/or divergent solutions most often occurred when at least one of the upper layers was thin. Figure Bl illustrates a typical thin pavement case with non-unique solutions. For this case, COMDEF and trial value combination $D$ produced excellent results. Trial value combinations $A$, $B, F$, and $H$ produced solutions which were both non-unique and divergent. For each of these cases, the total percentage error of the basin is small, but one layer has been assigned a boundary value. For these cases, additional iterations will not improve the solution.

## RANDOM ERRORS

A cursory study was completed to examine the effects of random errors in individual sensor measurements on the backcalculated moduli values. An exact deflection basin for a typical case was used as the initial run. Subsequent runs were made with a $0.1 \mathrm{mil}(2.5 \mu \mathrm{~m})$ deviation applied sequentially to each of the 7 sensor deflections. Figure 19 illustrates the results of this study for COMDEF and for BISDEF. The notation used on the X-axis in Figure 19 (e.g., 5+) indicates which sensor was varied and whether the deviation was added or subtracted. COMDEF provided consistently low errors, with the average percentage error for the three layers always less than $5 \%$. BISDEF had large average percentage errors for some cases. This cursory comparison does not completely address the reliability of the COMDEF method, and a full parametric study is needed to determine the expected variability in moduli values due to random deflection errors. However, for the limited study presented here, COMDEF performed well.


FIGURE 19. Effect of "Random" Deflection Errors on Backcalculation

## CHAPTER V

## VERIFICATION TESTING

## INTRODUCTION

A series of field and laboratory tests on composite pavements were performed in conjunction with routine nondestructive evaluation testing. The purpose of these tests was to independently evaluate layer properties to validate the backcalculation procedure. Sites selected for this purpose were Godman Army Airfield (GAAF) at Fort Knox, Kentucky, Sherman Army Airfield (SAAF) at Fort Leavenworth, Kansas, and the Bomb Damage Repair Site (BDRS) at the Waterways Experiment Station, Vicksburg, Mississippi. These sites provided a wide range of layer thicknesses. The range of thicknesses of the asphaltic concrete overlay was 2.0 inches to 6.5 inches ( 51 mm to 165 mm ) and the range of thicknesses of the underlying Portland cement concrete was 6.0 inches to 11.0 inches ( 152 mm to 279 mm ). The Army Airfield sites were particularly attractive because the field testing was included as a portion of an ongoing Army Airfield evaluation program and was completed at a minimal cost to this project.

Figure 20 is a site map of Godman Army Airfield which indicates the composite pavement features tested in this project. The composite pavement features at Godman Army Airfield include the Runway 18-36 extensions (Features RIE and R5E), Taxiway A (Feature TlE), Taxiway D (Features T5E and T6E), the original apron (Feature AlE), and the apron extensions (Features A2E and A3E). Figure 20 also identifies the stations used in the evaluation program.


FIGURE 20. Site Identification Map, Godman Army Airfield (GAAF) (NOTE: 1 foot $=0.305 \mathrm{~m}$ )

To minimize confusion, designations from the Army Airfield Evaluation Program (Horihan 1986a,b) are used herein to identify pavement features from the verification sites.

Figure 21 provides corresponding site information for Sherman Army Airfield (SAAF). The composite pavement features at Sherman Army Airfield include the Runway 15-33 extensions (Features R1E and R5E), the Taxiways (Features T1E and T2E), and the Aprons (Features A1E and A2E). The Waterways Experiment Station Bomb Damage Repair Site site and test location is shown in Figure 22. The Waterways Experiment Station Bomb Damage Repair Site was a good verification site because the site was well engineered with excellent quality control (Cooksey 1981) and because a section was available which had been subjected to only minimal traffic.

Construction histories by feature for the three sites are included in Table 3. Figures 23-25 show typical sections for all composite features tested at Godman Army Airfield, Sherman Army Airfield, and the Waterways Experiment Station Bomb Damage Repair Site, respectively. Pavement thicknesses are assumed to be constant within each feature. Layer thicknesses for the features were based on past evaluation reports (Horihan 1986), construction history (Table 3), and on field core thicknesses (Table 4).


FIGURE 21. Site Identification Map, Sherman Army Airfield (SAAF)


FIGURE 22. Site Identification Map, WES Bomb Damage Repair Site (BDRS) (NOTE: $1 \mathrm{ft}=12$ inches $=0.305 \mathrm{~m}$ )

TABLE 3. Construction Histories for Verification Sites

| Pavement Feature Identification | Pavement |  | Construction Date | Agency ${ }^{*}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | Thickness (inches) | Type |  |  |
| Godman Army Airfield Runway 18-36 |  |  |  |  |
| R1E, RSE | 6.0 | PCC | 1944 | CE |
| R5E | 2.5** | AC | 1965 | PE |
| RIE, RSE | 2.0** | $A C$ | 1974 | CE |
| R1E, R5E |  | Fiberglass Mat | 1982 | CE |
| R1E, R5E | 2.0** | AC | 1982 | CE |
| Godman Army Airfield Taxiway A |  |  |  |  |
| T1E | 6.0 | PCC | 1942 | CE |
| tie | 2.0** | AC | + |  |
| Tie | 2.0** | AC | 1974 | CE |
| T1E | 2.0** | $A C$ | 1981 | CE |
| T1E |  | Slurry Seal | 1983 | CE |
| Gocman Army Airfield Taxiway D |  |  |  |  |
| T5E | 6.0 | PCC | 1942 | CE |
| T6E | 6.0 | PCC | 1943 | CE |
| T5E, T6E | 2.0** | AC | 1974 | CE |
| TSE, T6E |  | Slurry Seal | 1983 | CE |
| Godman Army Airfield Aprons |  |  |  |  |
| A1E | 5.0 | PCC | 1941 | CE |
| A2E, A3E | 6.0 | PCC | 1942 | CE |
| A1E, A2E, A3E | 1.5** | AC | 1966 | CE |
| A1E, A2E |  | Fiberglass Mat | 1981 | CE |
| A1E, A2E, A3E | 2.0** | AC | 1981 | CE |
| A1E, A2E, A3E |  | Slurry Seal | 1983 | CE |
| Sherman Army Airfield Runway 15-33 |  |  |  |  |
| R1E, R5E | 6.0 | PCC | Prior to 1942 | CE |
| R1E, R5E | 2.0 | AC | 1960 | PE |
| R1E, R5E | 3.0** | AC | ++ | PE |
| Sherman Army Airfield Taxiways |  |  |  |  |
| T1E, T2E | 6.0 | PCC | Prior to 1942 | cE |
| tie | 0.5** | AC | 1956 | PE |
| T2E | 0.5** | AC | 1960 | PE |
| T1E, T2E | 3.0** | ${ }^{\text {a }}$ C | ++ | PE |
| Sherman Army Airfield Parking Apron |  |  |  |  |
| A1E | 6.0 | PCC | Prior to 1942 | CE |
| AIE | 2.0** | AC | 1956 | PE |
| A1E | 2.0** | AC | 1985 | PE |
| Sherman Army Airfield Operations Apron |  |  |  |  |
| A2E | 6.0 | PCC | Prior to 1942 | CE |
| A2E | 2.0** | ${ }^{\text {AC }}$ | 1958 | PE |
| A2E | 2.0** | AC | ++ | PE |
| Waterways Experiment Station Bomb Damage Repair Site |  |  |  |  |
| 80RS | 11.0 | PCC | 1978 | CE |
| BORS | 4.0** | AC | 1978 | CE |

* CE = Corps of Engineers; PE = Post Engineer + Construction date and agency unknown (1942-1974) MOTE: $\quad 1$ inch $=25.4 \mathrm{~mm}$
** Overlay pavement
++ Construction date unknown (1972-1980)


FEATURE RIE
RUNWAY 18-36


FEATURE RSE
RUNWAY 18-36


FEATURE T5E taxiway 0


FEATURE TGE taxiwayo

FEATURE TIE TAXIWAY A


FEATURE AIE ORIGINAL APRON


FEATURES A2E, A $3 E$ NORTH AND SOUTH APRONS

FIGURE 23. Typical Sections, Godman Army Airfield (GAAF)

$$
\text { (NOTE: } 1 \text { रिt }=12 \text { inches }=0.305 \mathrm{~m})
$$



FEATURES RIE, R5E
RUNWAY 15-33


3-1/2" ASPHALTIC CONCRETE
6. PORTLAND CEMENT CONCRETE

FEATURES T1E, T2E NORTH AND SOUTH TAXIWAYS


FEATURES A IE, A2E
PARKING AND OPERATIONS APRON
FIGURE 24. Typical Sections, Sherman Army Airfield (SAAF)


FEATURE BDRS
BOMB DAMAGE REPAIR SITE
FIGURE 25. Typical Section, WES Bomb Damage Repair Site (BDRS) (NOTE: $1 \mathrm{ft}=12$ inches $=0.305 \mathrm{~m}$ )

TABLE 4. Core Logs and Verification Testing Quick Reference

| Feature <br> Identification | Station | Thickness of AC (inches) | Thickness of PCC (inches) | Testing Codes* |
| :---: | :---: | :---: | :---: | :---: |
| GAAF RIE, RUNWAY 18-36 | 1+00 | 4 | 8-1/2 | T |
| GAAF RIE, RUNWAY 18-36 | 2+00 | 3-3/4 | 8 | T |
| GAAF RIE, RUNWAY 18-36 | 4+00 | 3-1/4 | 8-1/2 | T |
| GAAF RIE, RUNWAY 18-36 | 6+00 | 4-1/4 | 7-3/4 | TF |
| GAAF R5E, RUNWAY 18-36 | $51+50$ | 6 | 7-3/4 | TF |
| GAAF TIE, TAXIWAY A | 1+50 | 6 | 7 | TFCUSI |
| GAAF TSE, TAXIWAY D | $2+00$ | 2 | 6 | TCUSI |
| GAAF T5E, TAXIWAY D | 16+50 | 2 | 6-1/4 | TFA |
| GAAF T6E, TAXIWAY D | $30+00$ | 2-1/2 | 9-1/2 | T |
| GAAF T6E, TAXIWAY D | $30+50$ |  | ++ | $T$ |
| GAAF AIE, ORIGINAL APRON | $11+00 \mathrm{~A}$ | 4-1/4 | 7 | TCUSI |
| GAAF A2E, NORTH APRON | $20+50 \mathrm{~B}$ | 3-1/4 | + |  |
| GAAF A3E, SOUTH APRON | $0+50 \mathrm{~A}$ | 3-1/4 | 7 | T |
| GAAF A3E, SOUTH APRON | $3+50 \mathrm{~B}$ | 3-1/4 | 7-1/2 | T |
| GAAF A3E, SOUTH APRON | $4+50 \mathrm{~A}$ | 3-1/2 | 7-1/2 | TF |
| SAAF RIE, RUNWAY 15-33 | $3+00$ | 6 | 7 | TCUI |
| SAAF R5E, RUNWAY 15-33 | 57+00 | 5-1/2 | 8+++ | TUSI |
| SAAF TIE, NORTH TAXIWAY | 1+25 | 3-1/2 | + | T |
| SAAF TIE, NORTH TAXIWAY | $3+25$ | 3-1/2 | 6-1/2+++ | TUI |
| SAAF T2E, SOUTH TAXIWAY | 1+50 | 4 | 5-1/2 | TUSI |
| SAAF T2E, SOUTH TAXIWAY | $3+00$ | 3 | 7 |  |
| SAAF T2E, SOUTH TAXIWAY | $3+25$ | 3 | 6-1/2 | TFCUI |
| SAAF AlE, PARKING APRON | 1+50 | 3 | + | T |
| SAAF AlE, PARKING APRON | $3+00$ | 3 | 7 | T |
| SAAF AlE, PARKING APRON | $4+00$ | 4 | 7+++ | T |
| SAAF A2E, OPERATIONS APRON | 0+00B | 4 | $6+++$ | T |
| SAAF A2E, OPERATIONS APRON | $1+00 \mathrm{~A}$ | 4 | 6 | T |
| SAAF A2E, OPERATIONS APRON | $1+00 \mathrm{~B}$ | 3-1/2 | 7-1/4 | TF |
| SAAF A2E, OPERATIONS APRON | $2+50 \mathrm{~B}$ | 3-3/4 | 6 |  |
| WES BOMB DAMAGE REPAIR SITE | BDRS | 4 | 11-1/2 | TANW |

+ Core could not be recovered
++ No PCC layer found at this location
+++ PCC core was shattered, thickness measured by aligering
* Testing codes: $T=$ Coring for Thickness Verification
$F=P C C$ Flexural Strength Test
C = In-place Small Aperture CBR Tests
$U=$ Soil Sampling by Thin-walled, Shelby Tube
S = Laboratory Soil Resilient Modulus Test
A = Laboratory Asphaltic Concrete Modulus Tests
I = Soil Classification, Index Tests
$N=$ Laboratory PCC Modulus by Fundamental Frequency
W = FWD Waveform Recording
NOTE: $\quad 1$ inch $=25.4 \mathrm{~mm}$


## VERIFICATION TESTS

Testing to verify backcalculated moduli values included both field and laboratory components. Field nondestructive testing included dynamic stiffness modulus tests performed with the WES 16-kip vibrator, pavement condition index surveys, and Falling Weight Deflectometer tests. In addition, at the Waterways Experiment Station Bomb Damage Repair Site, a seven channel recorder was used to record the outputs of the Falling Weight Deflectometer load cell and the first six geophones. Destructive field testing included pavement coring and subsurface exploration. Coring served the dual purpose of verifying pavement thicknesses and also providing core specimens from the pavements for laboratory testing. Subsurface exploration was completed at selected locations by obtaining thin-walled (Shelby) tube specimens and by performing in-place California bearing ratio tests. Laboratory testing for the project included soil classification tests, soil resilient modulus tests, asphaltic concrete resilient modulus tesis, Portland cement concrete splitting tensile strength tests, and Portland cement concrete modulus tests by fundamental frequency. Some of the testing, such as laboratory modulus tests, provided direct verification data. Some of the testing was not useful for verification of the backcalculation but was included as site documentation.

Table 4 includes a "quick reference" of testing codes which summarize the verification testing for this project. The testing codes include both field tests run at the given locations and also laboratory tests run on specimens from those locations. The locations for verification testing were chosen in the field. Pavement coring
operations and subsurface exploration could potentially cause an interruption of normal airfield operations, due to the total time required for setup, testing, and breakdown. For this reason, coring and subsurface exploration testing was completed at locations which minimized the impact of the testing on normal airfield operations. Results of subsurface exploration are illustrated in Figures 26-27. Inspection of Figures 26-27 indicate that subsurface exploration data was available only to depths of 42 inches. The available subsurface data was from an ongoing airfield evaluation program and was not under the control of the author. Under ideal conditions, the depth of subsurface exploration would be such that the actual variation in subgrade modulus as a function of depth could be evaluated.

LEGEND
$\boxtimes$ undisturbed sample icl)
W undisturbed sample (ch)
LJ LEAN CLAY (CL)
Zat Clay (Ch)
Ez2 ASPHALTIC CONCRETE (AC)
boid Portland cement
CONCRETE (PCC)
FIGURE 26. Subsurface Exploration, Godman Army Airfield (GAAF)
(NOTE: 1 inch $=25.4 \mathrm{~mm}$ )


## NONDESTRUCTIVE TESTING

Falling Weight Deflectometer tests were completed on all features at each station ( 100 feet). Table 5 summarizes the Falling Weight Deflectometer testing for this project. Table 5 includes for each test the station, surface temperature, peak Falling Weight Deflectometer load, peak deflections at seven sensor locations, and backcalculated moduli from COMDEF. The moduli shown in Table 5 were calculated by COMDEF using the temperature option (see Appendix A). The asphaltic concrete moduli listed in Table 5 are corrected to a standard temperature of 70 degrees Fahrenheit and frequency of loading of 1 Hertz. These values are for comparison purposes and should not be confused with asphaltic concrete design moduli which must take other variables (such as seasonal variation) into account. The representative deflection basins which were computed by the program BASIN are indicated in Table 5.

TABLE 5. FWD Data and Backcalculation Analysis for Verification Sites

| Station Number | Temp.$\left({ }^{\circ} \mathrm{F}\right)$ | Load <br> (b) | Deflection in mils for Sensor Number |  |  |  |  |  |  | $\begin{gathered} \mathrm{E}_{\mathrm{ac}} \\ (\mathrm{ksi}) \end{gathered}$ | $E_{p c c}$ (ksi) | $\begin{aligned} & E_{\text {sub }} \\ & (k s i) \end{aligned}$ | RB* |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |  |  |  |  |
| Godman Army Airfield, Feature Identification R1E |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0+00 | 68 | 27248 | 12.4 | 11.2 | 9.8 | 8.4 | 7.1 | 5.8 | 4.6 | 621 | 5383 | 16 |  |
| 1+00 | 73 | 26944 | 13.8 | 13.1 | 10.7 | 8.6 | 6.7 | 5.0 | 3.5 | 495 | 2145 | 19 |  |
| 2+00 | 74 | 26568 | 10.0 | 8.6 | 7.1 | 5.5 | 4.0 | 2.9 | 1.9 | 564 | 1883 | 32 |  |
| 3+00 | 74 | 26536 | 11.0 | 8.8 | 7.2 | 5.6 | 4.4 | 3.3 | 2.4 | 378 | 2934 | 29 | $\leftarrow$ |
| $4+00$ | 74 | 26344 | 8.5 | 7.1 | 5.8 | 4.4 | 3.2 | 2.3 | 1.6 | 632 | 2217 | 39 |  |
| $5+00$ | 74 | 26136 | 12.7 | 11.7 | 10.2 | 8.7 | 7.1 | 5.5 | 4.1 | 632 | 3388 | 16 |  |
| $6+00$ | 76 | 25920 | 13.1 | 12.3 | 10.6 | 9.0 | 7.5 | 5.9 | 4.7 | 621 | 3896 | 15 |  |

Godman Army Airfield, feature Identification RSE

| $51+00$ | 85 | 26920 | 8.8 | 6.9 | 5.7 | 4.6 | 3.6 | 2.8 | 2.1 | 526 | 2459 | 34 | $\leftarrow$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $52+00$ | 79 | 27064 | 12.7 | 11.2 | 9.6 | 7.3 | 4.8 | 3.8 | 3.0 | 621 | 595 | 24 |  |

Godman Army Airfield, Feature Identification T1E

| $1+00$ | 72 | 26640 | 10.6 | 8.7 | 7.7 | 6.9 | 5.9 | 5.0 | 4.2 | 387 | 15330 | 17 |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $2+00$ | 75 | 26712 | 11.6 | 10.2 | 8.9 | 7.8 | 6.5 | 5.4 | 4.3 | 591 | 5725 | 16 |  |
| $3+00$ | 73 | 26296 | 10.6 | 9.3 | 8.1 | 7.2 | 5.9 | 4.9 | 4.0 | 630 | 6159 | 17 |  |
| $4+00$ | 71 | 25344 | 18.5 | 16.8 | 15.0 | 13.0 | 10.8 | 8.8 | 7.2 | 653 | 1714 | 9 |  |

Godman Army Airfield, feature Identification TSE

| $2+00$ | 92 | 23680 | 27.4 | 24.1 | 19.1 | 14.8 | 10.9 | 7.5 | 4.7 | 387 | 2438 | 11 |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $3+00$ | 92 | 24256 | 19.6 | 16.8 | 13.5 | 10.6 | 7.8 | 5.3 | 3.4 | 387 | 4014 | 16 |  |
| $4+00$ | 92 | 24448 | 17.4 | 14.6 | 12.0 | 9.2 | 6.8 | 4.8 | 3.2 | 387 | 4959 | 18 |  |
| $5+00$ | 92 | 24480 | 22.9 | 19.2 | 16.3 | 13.0 | 9.9 | 7.1 | 4.6 | 387 | 4193 | 13 |  |
| $6+00$ | 92 | 24640 | 18.1 | 16.1 | 13.2 | 10.4 | 7.4 | 5.4 | 3.5 | 621 | 4015 | 16 |  |
| $7+30$ | 107 | 25664 | 21.7 | 19.7 | 16.9 | 13.5 | 10.1 | 7.0 | 3.7 | 387 | 4321 | 14 |  |
| $8+00$ | 107 | 25360 | 26.9 | 24.4 | 21.2 | 17.7 | 14.1 | 10.4 | 7.1 | 621 | 4790 | 9 |  |
| $9+00$ | 110 | 24944 | 23.4 | 21.5 | 17.8 | 14.3 | 10.9 | 7.8 | 5.3 | 564 | 4203 | 11 |  |
| $10+00$ | 110 | 25192 | 24.7 | 23.2 | 19.6 | 15.6 | 11.9 | 8.3 | 5.5 | 624 | 3650 | 11 |  |
| $11+00$ | 111 | 24864 | 25.7 | 23.4 | 19.3 | 15.3 | 11.5 | 8.1 | 5.7 | 631 | 3389 | 11 |  |
| $12+00$ | 110 | 24512 | 27.4 | 26.2 | 19.9 | 15.1 | 11.3 | 8.0 | 5.9 | 621 | 2513 | 11 |  |
| $13+00$ | 94 | 24856 | 22.3 | 21.0 | 18.2 | 14.3 | 11.3 | 8.2 | 5.9 | 644 | 4892 | 11 |  |
| $14+50$ | 96 | 25176 | 19.2 | 18.2 | 15.4 | 12.6 | 10.0 | 7.8 | 6.1 | 621 | 8142 | 11 |  |
| $15+00$ | 96 | 23936 | 33.8 | 31.5 | 26.3 | 19.7 | 14.8 | 10.7 | 7.4 | 479 | 2226 | 8 |  |
| $16+00$ | 98 | 24280 | 34.7 | 33.4 | 27.5 | 21.6 | 16.0 | 10.6 | 7.2 | 423 | 1967 | 8 |  |
| $17+00$ | 95 | 25376 | 27.6 | 25.3 | 21.5 | 17.0 | 13.2 | 9.9 | 7.3 | 626 | 3858 | 9 |  |
| $18+00$ | 90 | 24512 | 18.7 | 17.5 | 15.0 | 12.3 | 9.6 | 7.2 | 5.0 | 515 | 6678 | 12 |  |
| $19+00$ | 90 | 24016 | 24.1 | 21.9 | 17.9 | 14.0 | 10.7 | 7.6 | 5.3 | 561 | 3165 | 11 |  |
| $20+00$ | 97 | 24808 | 23.2 | 23.8 | 18.4 | 14.9 | 11.5 | 8.7 | 5.5 | 621 | 3643 | 11 |  |
| $21+00$ | 97 | 25120 | 20.0 | 14.0 | 11.7 | 11.1 | 8.8 | 7.0 | 4.7 | 387 | 9489 | 14 |  |
| $22+00$ | 97 | 24624 | 28.3 | 27.2 | 23.2 | 19.4 | 14.9 | 11.7 | 8.7 | 621 | 4532 | 7 |  |
| $23+00$ | 103 | 24816 | 40.2 | 38.3 | 33.7 | 28.5 | 21.5 | 16.6 | 10.9 | 397 | 3233 | 6 |  |
| $24+00$ | 101 | 25168 | 21.7 | 20.7 | 18.0 | 15.0 | 11.8 | 9.2 | 6.6 | 621 | 6789 | 10 |  |
| $25+00$ | 105 | 24672 | 20.7 | 19.2 | 15.7 | 12.1 | 8.9 | 6.2 | 3.7 | 567 | 3235 | 14 |  |
| $26+00$ | 102 | 24424 | 23.4 | 21.5 | 18.6 | 14.2 | 10.6 | 7.3 | 4.5 | 387 | 3790 | 12 | + |

* RB = Representative Basin for Each Feature as Calculated by the BASIN Program, noted by $\boldsymbol{\leftarrow}$ symbol NOTE: $\quad 0_{F}=\left({ }^{\circ} \mathrm{C} \times 9 / 5\right)+32 \quad 1 \mathrm{lb}=4.448 \mathrm{~N} \quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m} \quad 1 \mathrm{ksi}=6.89 \mathrm{mpa}$

TABLE 5. (Continued)

| Station Number | Temp.$\left({ }^{\circ} F\right)$ | $\begin{aligned} & \text { Load } \\ & \text { (lb) } \end{aligned}$ | Deflection in mils for Sensor Number |  |  |  |  |  |  | $E_{a c}$$(k s i)$ | $E_{\text {pce }}$ (ksi) | $E_{\text {sub }}$ (ksi) | RB* |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |  |  |  |  |

Godman Army Airfield, Feature Identification T6E

| $29+00$ | 107 | 26280 | 13.5 | 11.8 | 10.7 | 9.3 | 7.5 | 6.4 | 5.2 | 489 | 22517 | 14 |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :--- | :--- |
| $30+00$ | 107 | 25592 | 19.5 | 15.5 | 14.0 | 12.4 | 9.8 | 8.8 | 7.0 | 387 | 15917 | 10 | + |
| $31+00$ | 107 | 22488 | 67.2 | 46.5 | 25.6 | 13.9 | 8.2 | 6.4 | 5.1 | 621 | 32 | 10 |  |

Gocman Army Airfield, Feature Identification A1E

| $5+00 \mathrm{~A}$ | 111 | 23392 | 29.5 | 24.8 | 17.1 | 11.4 | 7.3 | 4.4 | 2.8 | 629 | 145 | 16 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6+00A | 119 | 23740 | 26.9 | 24.3 | 18.1 | 12.2 | 7.9 | 5.1 | 3.2 | 629 | 242 | 14 |  |
| 7+00A | 111 | 23664 | 29.1 | 24.1 | 17.6 | 12.0 | 7.9 | 4.8 | 2.9 | 621 | 152 | 15 |  |
| $8+00 \mathrm{~A}$ | 111 | 22160 | 50.5 | 40.8 | 24.0 | 15.4 | 9.1 | 5.6 | 3.1 | 526 | 34 | 12 |  |
| 9+00A | 111 | 22368 | 39.6 | 31.3 | 20.5 | 12.9 | 7.6 | 4.1 | 2.3 | 621 | 43 | 14 | - |
| 10+00a | 111 | 22924 | 34.3 | 27.8 | 18.6 | 12.2 | 7.3 | 4.9 | 3.3 | 629 | 114 | 14 |  |
| $11+00 \mathrm{~A}$ | 111 | 22832 | 32.8 | 24.8 | 17.6 | 12.7 | 8.7 | 6.6 | 3.9 | 396 | 296 | 12 |  |
| 12+00A | 111 | 22236 | 41.8 | 36.1 | 24.0 | 15.0 | 8.9 | 5.7 | 3.5 | 621 | 56 | 11 |  |
| 13+00A | 114 | 21488 | 73.2 | 51.4 | 30.4 | 20.2 | 13.3 | 8.7 | 5.8 | 387 | 39 | 7 |  |
| 14+00A | 117 | 23124 | 39.0 | 33.9 | 27.5 | 20.6 | 14.9 | 11.0 | 7.4 | 480 | 404 | 7 |  |
| 15+00A | 120 | 22728 | 48.8 | 43.6 | 32.6 | 24.4 | 15.6 | 12.8 | 9.1 | 626 | 196 | 6 |  |
| 16+00A | 119 | 22436 | 42.1 | 33.6 | 23.6 | 17.7 | 12.9 | 9.1 | 6.0 | 619 | 171 | 9 |  |
| $5+008$ | 129 | 21592 | 53.4 | 37.4 | 22.0 | 13.8 | 8.5 | 5.9 | 3.9 | 557 | 45 | 11 |  |
| 6+008 | 131 | 21956 | 53.5 | 36.1 | 25.8 | 17.6 | 11.3 | 7.3 | 4.7 | 441 | 101 | 9 |  |
| 7+008 | 129 | 22992 | 34.5 | 23.6 | 16.9 | 12.1 | 8.2 | 5.2 | 3.1 | 387 | 298 | 14 |  |
| $8+008$ | 126 | 22096 | 39.2 | 26.9 | 19.0 | 12.1 | 6.9 | 4.0 | 2.6 | 621 | 70 | 15 |  |
| $9+008$ | 127 | 23560 | 22.9 | 11.4 | 7.9 | 5.4 | 3.6 | 2.4 | 1.7 | 381 | 349 | 31 |  |
| 10.008 | 127 | 23168 | 29.3 | 17.0 | 9.5 | 6.3 | 3.7 | 2.6 | 1.8 | 381 | 163 | 26 |  |
| 11+008 | 121 | 23284 | 34.9 | 16.6 | 10.9 | 7.5 | 4.9 | 3.2 | 2.2 | 380 | 114 | 23 |  |
| 12+008 | 129 | 23144 | 37.9 | 24.1 | 17.9 | 12.2 | 8.3 | 5.3 | 3.4 | 387 | 252 | 14 |  |
| 134008 | 128 | 21444 | 54.3 | 44.2 | 32.2 | 23.1 | 15.7 | 10.1 | 6.1 | 621 | 82 | 7 |  |
| $14+008$ | 128 | 22316 | 44.0 | 37.6 | 28.5 | 20.4 | 14.3 | 9.9 | 6.5 | 638 | 168 | 8 |  |
| 15+008 | 126 | 23364 | 29.8 | 25.0 | 20.2 | 15.8 | 12.0 | 8.8 | 6.3 | 507 | 962 | 9 |  |
| $16+00 \mathrm{~B}$ | 126 | 22448 | 38.2 | 32.2 | 25.8 | 20.2 | 15.2 | 11.6 | 8.4 | 607 | 437 | 7 |  |

Godman Army Airfield, feature Identification A2E

| $0+00 A$ | 103 | 25780 | 13.1 | 11.9 | 10.2 | 8.4 | 6.6 | 5.1 | 2.5 | 621 | 3399 | 21 |  |
| :--- | :--- | :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: | :--- | :--- | :--- | :--- |
| $1+00 A$ | 110 | 25012 | 18.1 | 15.4 | 13.4 | 10.6 | 7.8 | 5.8 | 4.1 | 621 | 2679 | 15 |  |
| $2+00 A$ | 110 | 24732 | 20.1 | 15.0 | 11.8 | 9.3 | 6.8 | 4.7 | 3.3 | 387 | 2151 | 18 |  |
| $3+00 A$ | 110 | 24204 | 23.4 | 17.1 | 12.4 | 8.7 | 5.6 | 3.9 | 2.6 | 393 | 720 | 20 |  |
| $4+00 A$ | 110 | 24496 | 17.6 | 12.3 | 10.2 | 8.0 | 6.1 | 4.4 | 3.3 | 387 | 3659 | 19 |  |
| $0+00 B$ | 131 | 23400 | 21.6 | 18.0 | 15.2 | 12.3 | 9.5 | 7.2 | 5.3 | 510 | 3361 | 12 |  |
| $1+008$ | 128 | 23504 | 24.3 | 17.6 | 14.3 | 11.4 | 8.7 | 6.9 | 4.9 | 387 | 3118 | 13 |  |
| $2+00 B$ | 129 | 23968 | 22.6 | 16.4 | 13.2 | 10.4 | 7.8 | 5.8 | 4.3 | 387 | 3073 | 15 |  |
| $3+008$ | 130 | 23644 | 21.0 | 15.7 | 13.4 | 10.8 | 8.4 | 6.5 | 4.9 | 387 | 4597 | 13 |  |
| $4+008$ | 129 | 23824 | 18.4 | 14.4 | 12.4 | 10.2 | 7.4 | 6.5 | 4.8 | 388 | 6045 | 14 | + |

* RB = Representative Basin for Each Feature as Calculated by the BASIN Program, noted by $\leftarrow$ symbol

NOTE: $\quad o_{F}=\left({ }^{\circ} \mathrm{C} \times 9 / 5\right)+32 \quad 1 \mathrm{~b}=4.448 \mathrm{~N} \quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m} \quad 1 \mathrm{ksi}=6.89 \mathrm{MPa}$

TABLE 5. (Continued)

| Station Number | $\begin{aligned} & \text { Temp. } \\ & \left(\mathrm{O}_{\mathrm{f}}\right) \end{aligned}$ | Load <br> ( $b$ ) | Deflection in mils for Sensor Number |  |  |  |  |  |  | $\left\{\begin{array}{c} \mathrm{E}_{\mathrm{ac}} \\ (\mathrm{ksi}) \end{array}\right.$ | $E_{p c c}$(ksi) | $E_{\text {sub }}$ (ksi) | RB* |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |  |  |  |  |
| Godman Army Airfield, Feature Identification A3E |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 17+00A | 121 | 24888 | 18.4 | 14.4 | 12.4 | 10.1 | 7.8 | 5.8 | 4.3 | 398 | 4784 | 15 |  |
| 18+00A | 120 | 24400 | 17.7 | 15.5 | 13.7 | 10.0 | 7.7 | 6.2 | 4.5 | 625 | 3433 | 14 |  |
| 19+00a | 119 | 24484 | 18.1 | 14.6 | 12.9 | 10.9 | 8.8 | 6.5 | 5.2 | 398 | 6400 | 13 |  |
| 20+00A | 120 | 24596 | 20.6 | 15.6 | 13.8 | 11.9 | 9.1 | 7.2 | 5.8 | 380 | 5420 | 12 |  |
| $21+00 \mathrm{~A}$ | 120 | 24856 | 18.8 | 13.8 | 11.5 | 8.8 | 6.3 | 4.6 | 3.5 | 380 | 3188 | 19 |  |
| 17+008 | 128 | 23384 | 25.6 | 20.9 | 17.4 | 13.6 | 10.1 | 7.3 | 5.2 | 554 | 1566 | 11 | $\leftarrow$ |
| $18+008$ | 128 | 23272 | 28.6 | 23.1 | 18.6 | 14.5 | 11.2 | 8.6 | 6.4 | 383 | 2324 | 10 |  |
| $19+008$ | 128 | 22796 | 31.5 | 27.0 | 22.6 | 17.9 | 13.5 | 9.5 | 6.5 | 494 | 1250 | 8 |  |
| 20+008 | 120 | 23148 | 32.4 | 26.2 | 20.8 | 16.2 | 12.0 | 8.9 | 6.3 | 380 | 1308 | 9 |  |
| $21+008$ | 120 | 22720 | 43.3 | 31.0 | 21.8 | 15.2 | 10.3 | 6.7 | 4.8 | 487 | 203 | 11 |  |


| $0+00$ | 95 | 22928 | 47.8 | 31.5 | 21.9 | 15.3 | 10.9 | 8.0 | 6.1 | 378 | 84 | 10 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1+00 | 95 | 23160 | 37.8 | 23.8 | 16.8 | 13.2 | 10.3 | 7.8 | 6.0 | 387 | 274 | 10 |  |
| 2+00 | 94 | 24024 | 31.8 | 21.8 | 17.1 | 13.3 | 10.3 | 8.0 | 6.2 | 386 | 385 | 11 | $\leftarrow$ |
| $3+00$ | 92 | 23424 | 35.3 | 25.4 | 19.1 | 15.1 | 11.9 | 8.7 | 6.5 | 387 | 320 | 9 |  |
| 4+00 | 92 | 23704 | 31.4 | 23.2 | 18.1 | 13.9 | 10.7 | 8.1 | 6.2 | 373 | 394 | 10 |  |
| 5+00 | 92 | 24024 | 31.1 | 21.6 | 17.4 | 13.8 | 10.8 | 8.3 | 6.2 | 387 | 538 | 10 |  |
| $6+00$ | 93 | 25504 | 20.8 | 11.3 | 9.1 | 8.0 | 6.7 | 5.6 | 4.6 | 387 | 4775 | 16 |  |

Sherman Army Airfield, Feature Identification R5E

| $57+00$ | 94 | 23976 | 31.1 | 23.7 | 18.3 | 14.5 | 11.4 | 8.8 | 6.6 | 387 | 553 | 10 |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $58+00$ | 94 | 22928 | 46.7 | 29.1 | 24.4 | 19.3 | 15.7 | 11.8 | 8.9 | 387 | 274 | 7 |  |
| $59+00$ | 94 | 24000 | 36.6 | 24.9 | 19.4 | 16.0 | 13.0 | 10.2 | 7.7 | 387 | 524 | 8 | $\leftarrow$ |
| $59+61$ | 94 | 23248 | 51.6 | 34.8 | 20.2 | 16.1 | 12.3 | 9.6 | 7.4 | 386 | 87 | 9 |  |

Sherman Army Airfield, feature Identification TIE

| $0+00$ | 93 | 13768 | 77.6 | 49.5 | 22.1 | 10.8 | 8.3 | 6.7 | 5.3 | 387 | 21 | 6 |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $1+00$ | 93 | 18544 | 76.0 | 55.2 | 33.8 | 20.6 | 13.0 | 9.2 | 7.1 | 498 | 26 | 6 |  |
| $2+00$ | 93 | 22592 | 46.1 | 34.4 | 24.3 | 18.1 | 13.6 | 10.3 | 7.6 | 627 | 306 | 8 |  |
| $3+00$ | 93 | 22704 | 45.7 | 37.0 | 27.7 | 19.3 | 13.8 | 10.1 | 7.6 | 605 | 306 | 8 | $\leftarrow$ |
| $4+00$ | 93 | 24192 | 26.3 | 21.2 | 17.7 | 14.5 | 11.5 | 8.8 | 6.6 | 387 | 4610 | 10 |  |

Sherman Army Airfield, Feature Identification T2E

| $0+00$ | 96 | 24432 | 29.9 | 24.9 | 19.2 | 14.1 | 10.4 | 7.6 | 5.9 | 621 | 1019 | 11 | $\leftarrow$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $1+00$ | 97 | 24752 | 25.4 | 20.6 | 15.3 | 11.2 | 8.3 | 6.6 | 5.3 | 378 | 3358 | 13 |  |
| $2+00$ | 94 | 23208 | 42.2 | 32.7 | 23.7 | 17.0 | 12.0 | 8.3 | 6.3 | 621 | 310 | 9 |  |
| $3+00$ | 94 | 24288 | 27.3 | 24.1 | 18.9 | 14.3 | 10.5 | 7.6 | 5.7 | 570 | 1406 | 11 |  |
| $3+46$ | 94 | 23352 | 39.9 | 31.8 | 23.9 | 17.5 | 12.6 | 9.1 | 6.8 | 634 | 425 | 9 |  |

* RB $=$ Representative Basin for Each feature as Calculated by the BASIN Program, noted by $\leftarrow$ symbol
MOTE: $\quad{ }^{O_{F}}=\left({ }^{\circ} \mathrm{C} \times 9 / 5\right)+32$
$11 \mathrm{~b}=4.448 \mathrm{~N}$
$1 \mathrm{mil}=25.4 \mu \mathrm{~m}$
$1 \mathrm{ksi}=6.89 \mathrm{MPa}$

TABLE 5. (Concluded)

| Station Number | $\begin{aligned} & \text { Temp. } \\ & \left({ }^{\circ} \mathrm{F}\right) \end{aligned}$ | Load <br> (b) | Deflection in mils for Sensor Number |  |  |  |  |  |  | $\begin{gathered} \mathrm{E}_{\mathrm{ac}} \\ (\mathrm{ksi}) \end{gathered}$ | $E_{p c c}$ (ksi) | $\begin{aligned} & E_{\text {sub }} \\ & (k s i) \end{aligned}$ | RB* |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |  |  |  |  |
| Sherman Army Airfield, feature Identification A1E |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0+00A | 78 | 22512 | 63.7 | 41.7 | 25.1 | 15.3 | 10.6 | 8.0 | 6.3 | 387 | 82 | 9 |  |
| $1+00 \mathrm{~A}$ | 79 | 22248 | 71.0 | 44.8 | 25.6 | 19.1 | 14.0 | 9.8 | 7.3 | 387 | 99 | 7 |  |
| $2+00 \mathrm{~A}$ | 79 | 22360 | 64.6 | 47.4 | 31.2 | 21.5 | 14.8 | 10.1 | 7.2 | 381 | 95 | 7 |  |
| $3+00 \mathrm{~A}$ | 80 | 22336 | 63.6 | 43.3 | 28.0 | 18.9 | 11.5 | 9.4 | 7.2 | 371 | 103 | 8 |  |
| $4+00 \mathrm{~A}$ | 82 | 22328 | 77.9 | 52.5 | 31.8 | 20.5 | 13.9 | 10.1 | 7.8 | 387 | 64 | 7 | $\leftarrow$ |
| $5+00 \mathrm{~A}$ | 80 | 8032 | 63.9 | 43.6 | 20.1 | 8.5 | 4.9 | 3.8 | 3.3 | 387 | 32 | 5 |  |
| 0+00B | 85 | 22312 | 68.6 | 42.5 | 28.1 | 20.5 | 14.8 | 11.1 | 8.4 | 375 | 111 | 7 |  |
| $1+00 \mathrm{~B}$ | 85 | 23216 | 48.2 | 34.0 | 23.5 | 17.0 | 12.1 | 8.9 | 6.6 | 375 | 249 | 9 |  |
| $2+008$ | 85 | 22240 | 68.8 | 43.9 | 26.7 | 16.8 | 10.5 | 7.6 | 6.1 | 421 | 30 | 9 |  |
| $3+008$ | 80 | 13848 | 69.1 | 48.7 | 26.1 | 14.0 | 8.3 | 6.1 | 5.3 | 387 | 32 | 6 |  |

Sherman Army Airfield, Feature Identification A2E

| 0+00C | 68 | 24776 | 31.5 | 25.9 | 19.2 | 14.9 | 11.4 | 8.5 | 6.4 | 583 | 604 | 10 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0+90C | 68 | 25168 | 26.3 | 20.1 | 16.0 | 12.7 | 9.9 | 7.8 | 6.0 | 644 | 981 | i2 |  |
| 2+00c | 68 | 25208 | 21.1 | 17.7 | 15.1 | 12.4 | 10.0 | 8.0 | 6.1 | 387 | 5477 | 11 |  |
| 3+00C | 68 | 23720 | 42.2 | 31.2 | 22.3 | 15.5 | 11.3 | 8.6 | 6.5 | 371 | 303 | 10 |  |
| 4+00C | 69 | 23512 | 49.0 | 35.9 | 23.9 | 16.9 | 12.3 | 9.1 | 6.7 | 379 | 190 | 9 |  |
| 5+00c | 70 | 23248 | 49.2 | 35.4 | 24.0 | 17.5 | 12.5 | 9.5 | 7.0 | 386 | 198 | 9 |  |
| 6+00C | 68 | 22616 | 67.0 | 48.4 | 31.3 | 20.1 | 13.5 | 9.2 | 6.2 | 387 | 58 | 8 |  |
| 7+00c | 68 | 23248 | 53.1 | 37.7 | 25.5 | 17.5 | 11.8 | 8.0 | 6.0 | 371 | 121 | 9 |  |
| 8+00c | 68 | 23616 | 46.7 | 34.1 | 24.0 | 17.5 | 12.8 | 10.1 | 7.8 | 371 | 294 | 8 |  |
| $8+50 \mathrm{C}$ | 68 | 22768 | 65.1 | 43.7 | 30.5 | 21.5 | 15.5 | 11.9 | 8.1 | 371 | 100 | 7 |  |
| 0+000 | 68 | 23192 | 50.8 | 39.0 | 23.3 | 17.2 | 13.0 | 97 | 7.2 | 371 | 187 | 8 |  |
| 1+000 | 69 | 23800 | 36.2 | 28.4 | 22.3 | 17.5 | 13.5 | 10.2 | 7.7 | 591 | 572 | 8 |  |
| $2+000$ | 69 | 24376 | 27.9 | 24.6 | 20.5 | 16.8 | 13.4 | 10.5 | 8.2 | 621 | 1577 | 8 |  |
| $3+000$ | 71 | 23728 | 39.8 | 30.5 | 23.5 | 18.3 | 14.0 | 10.4 | 7.6 | 387 | 608 | 8 | $\leftarrow$ |
| $4+000$ | 72 | 23840 | 37.6 | 30.6 | 24.1 | 18.3 | 13.5 | 9.7 | 6.9 | 387 | 545 | 9 |  |
| $5+000$ | 69 | 23056 | 56.9 | 40.4 | 28.1 | 20.2 | 13.5 | 9.5 | 7.3 | 379 | 132 | 8 |  |
| 6+000 | 72 | 24144 | 33.9 | 27.1 | 20.9 | 16.2 | 12.5 | 9.5 | 7.0 | 586 | 627 | 9 |  |
| 7+000 | 73 | 23488 | 43.6 | 28.7 | 21.8 | 17.0 | 13.1 | 9.8 | 7.4 | 428 | 478 | 8 |  |
| $8+000$ | 73 | 23832 | 34.6 | 26.7 | 21.2 | 17.3 | 13.1 | 10.6 | 8.0 | 594 | 653 | 8 |  |
| $9+000$ | 75 | 23184 | 57.7 | 35.9 | 26.1 | 19.1 | 13.7 | 9.7 | 7.4 | 386 | 140 | 8 |  |

Waterways Experiment Station Bomb Damage Repair Site, feature Identification BDRS

| A | 88 | 26032 | 8.3 | 7.3 | 6.3 | 5.6 | 4.9 | 4.4 | 3.9 | 621 | 9753 | 16 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| B | 88 | 25896 | 8.2 | 7.4 | 5.7 | 5.5 | 4.3 | 3.9 | 3.4 | 621 | 6950 | 20 | $\leftarrow$ |

* RB = Representative Basin for Each Feature as calculated by the BASIN Program, noted by * symbol NOTE: ${ }^{O_{F}}=\left({ }^{\circ} \mathrm{C} \times 9 / 5\right)+32 \quad 1 \mathrm{tb}=4.448 \mathrm{~N} \quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m} \quad 1 \mathrm{ksi}=6.89 \mathrm{MPa}$


## DYNAMIC STIFFNESS MODULUS (DSM) TESTS

Mean dynamic stiffness modulus values were calculated and corrected for temperature (Hall 1978) for each feature. Dynamic stiffness modulus is the ratio of the steady-state dynamic force applied by the WES $16-k i p$ vibrator divided by the steady-state deflection measured at the center of the loading area. The dynamic stiffness modulus is a good indicator of overall pavement stiffness and has been directly correlated with allowable single-wheel load. Mean dynamic stiffness modulus values for each feature are reported in Table 6. Since the calibrations of dynamic stiffness modulus with design properties are empirical, only the WES 16-kip vibrator can provide useful values of dynamic stiffness modulus. A cursory study was made to see if the Falling Weight Deflectometer could be correlated with dynamic stiffness modulus for composite pavements. A quantity similar to dynamic stiffness modulus has been developed (Bush 1987) for the Falling Weight Deflectometer. This quantity is called impulse stiffness modulus (ISM) and is defined as the ratio of peak impulse load to the peak deflection measured at the center of the loading area. Impulse stiffness modulus values were calculated and corrected for temperature by the program FWDTCF (Bush 1987). Mean impulse stiffness modulus values are also included in Table 6.

TABLE 6. Verification Site Documentation by Feature

| Feature <br> Identification | Station |  | Mean DSM Kip/in. | Mean ISM Kip/in. | Mean PCI | PCI Rating |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | From | To |  |  |  |  |
| GAAF RIE (RUNWAY 18-36) | 0+00 | 6+00 | 1953 | 2688 | 85 | Very Good |
| gaAF R5E <br> (RUNWAY 18-36) | $50+50$ | $52+30$ | 677 | 3326 | 83 | Very Good |
| GAAF TIE <br> (TAXIWAY A) | $0+00$ | $4+20$ | 1422 | 2712 | 91 | Excellent |
| GAAF T5E <br> (TAXIWAY D) | $0+00$ | $27+00$ | 826 | 1129 | 64 | Good |
| GAAF T6E <br> (TAXIWAY D) | $27+00$ | $31+37$ | 676 | 1329 | 40 | Poor |
| gAAF AIE (ORIGINAL APRON) | $4+50$ | $16+50$ | 669 | 711 | 80 | Very Good |
| GAAF A2E <br> (NORTH APRON) | $16+50$ | $21+12$ | 899 | 1405 | 79 | Very Good |
| gAAF A3E <br> (SOUTH APRON) | $0+00$ | $4+50$ | 1041 | 1152 | 78 | Very Good |
| SAAF RIE (RUNWAY 15-33) | $0+00$ | 5+85 | 1089 | 941 | 69 | Good |
| SAAF R5E <br> (RUNWAY 15-33) | $56+40$ | 59+60 | 595 | 1002 | 49 | Fair |
| SAAF TIE (NORTH TAXIWAY) | 0+00 | $4+00$ | 519 | $0<0$ | 55 | Fair |
| SAAF T2E (SOUTH TAXIWAY) | 0+00 | 3+46 | 648 | 862 | 70 | Good |
| SAAF Ale (PARKING APRON) | $\begin{aligned} & 0+00 C \\ & 0+00 D \end{aligned}$ | $\begin{aligned} & 5+00 \\ & 3+00 \end{aligned}$ | 467 | 357 | 100 | Excellent |
| SAAF A2E (OPERATIONS APRON) | $\begin{aligned} & 0+00 \mathrm{~A} \\ & 0+00 B \end{aligned}$ | $\begin{aligned} & 8+00 \\ & 9+00 \end{aligned}$ | 563 | 693 | 59 | Good |
| WES BOMB DAMAGE REPAIR SITE | BDRS | BDRS | 2000* | 3147 | 23 | Very Poor |

* Approximate historical value, included for comparison purposes

NOTE: $\quad 1 \mathrm{kip} / \mathrm{in} .=175 \mathrm{kN} / \mathrm{m}$

Figure 28 shows a regression fit of mean dynamic stiffness modulus versus mean impulse stiffness modulus for all features tested. A reasonable fit could not be obtained until one data point (indicated in Figure 28) was excluded from the regression. This point was from an unusual feature, because the feature included only a portion of the Runway 18-36 extension. By excluding the offending point, a correlation coefficient of 0.80 was obtained. It appears that a relationship does exist between dynamic stiffness modulus and impulse stiffness modulus, but that additional research is needed to adequately define that relationship.


FIGURE 28. Mean DSM (WES 16-kip Vibrator) Versus Mean ISM (FWD)

## PAVEMENT CONDITION INDEX (PCI) SURVEYS

Pavement condition index surveys were made in accordance with AFR 93-5 (Headquarters, Department of the Air Force 1981). The pavement condition index numerical values are determined by tabulating pavement surface distresses. Pavement condition index numerical values may range from 0 to 100 . The pavement condition index numerical value is averaged for a given feature, and the mean pavement condition index value is converted into a condition rating. Possible condition ratings include excellent, very good, good, fair, poor, very poor, and failed. Average pavement condition index values and condition ratings are included for each feature in Table 6. Pavement condition index results are included as site documentation, but the pavement condition index does not appear to be a good indicator of overall pavement capacity for composite pavements. Figure 29 illustrates data for mean impulse stiffness modulus versus pavement condition index. A regression line through this data would have negative slope, which is the opposite of the expected value. The data shown in Figure 29 appears to indicate that pavement condition index is strongly influenced by recent maintenance activity. A recent nonstructural maintenance application, such as a slurry seal coat, could lead to high values of pavement condition index. Alternately, maintenance might be postponed on a feature which was structurally sound due to a strong supporting layer, leading to low pavement condition index values on structurally capable features.


FIGURE 29. Mean ISM (FWD) Versus Pavement Condition Index

## falling weight deflectometer wave measurements

Time domain measurements were recorded at the Waterways Experiment Station Bomb Damage Repair Site site for the Falling Weight Deflectometer load cell and the first six sensors. The recorder was a Teac Model R-71, which records 7 tracks at $17 / 8$ inches per second, wide band, frequency modulated. Analog data were digitized at the Waterways Experiment Station Instrumentation Services Division at a sampling rate of 10 kHz . The purpose of these measurements was to determine if an unmodified Falling Weight Deflectometer could provide wave propagation data suitable for the determination of layer moduli, in a manner similar to the spectral analysis of surface waves (SASW) method (Heisey 1981). Figure 7 shows the time domain of the Falling Weight Deflectometer load cell for a single Falling Weight Deflectometer drop. Average base width of the Falling Weight Deflectometer pulse was about 31 msec , corresponding to a predominant frequency of loading of about 16 Hz . Figure 8 shows the frequency spectrum of the load from a Fast Fourier Transform (FFT) of the load pulse in Figure 7. Figure 8 illustrates a bandwidth of 0 to 500 Hz , although the computed spectrum included frequencies up to 5 kHz . No significant energy was measured at frequencies above 300 Hz . Most on the energy is concentrated in frequencies less than 60 Hz . Significantly higher frequencies are needed to adequately predict layer moduli of pavements. Based on the load cell data, it was concluded that the unmodified Falling Weight Deflectometer was not a suitable loading device for general pavement evaluation. However, since the Falling Weight Deflectometer provides a large energy pulse of low frequency content, it is possible that it could be used as a testing
device to determine depth to bedrock or subgrade modulus at depths. This would require some system modification or external data acquisition system, since the current geophone configuration would be inadequate for such tests.

## SMALL APERTURE TESTING

Nominal 6 inches ( 152 mm ) cores for thickness verification were obtained from all features in accordance with ASTM C 42-85 (American Society for Testing and Materials 1986) and were measured in accordance with ASTM C 174-82 (American Society for Testing and Materials 1986). A summary of all field cores which were taken from the composite pavement features is included in Table 4.

Small aperture in-place California bearing ratio (CBR) tests (Hall and Elsea 1974) were performed at five locations. The tests were performed in accordance with Army TM 5-530 (Headquarters, Departments of the Army, the Navy, and the Air Force 1971), except that the surcharge loading plate was not used for the small aperture tests. In-place California bearing ratio tests are indicative of the soil properties at the time of the test. For this reason, in-place California bearing ratio values are more useful for validation of nondestructive testing data than are laboratory California bearing ratio tests. At each California bearing ratio test location, a test was performed near the surface of the subgrade and at a depth of about 3 feet ( 914 mm ). The results of all in-place California bearing ratio tests are shown in Figures 26-27. The California bearing ratio values were averaged at each location for comparison with the backcalculated layer moduli. Figure 30 is a graphical comparison of average in-place California bearing ratio versus backcalculated subgrade moduli. The data showed considerable scatter and a reasonable fit could not be achieved unless the regression line was forced through the origin. The regression indicated that subgrade modulus is equal to 1279 times the California bearing
ratio. This correlation agrees well with published equations (Heukelom and Foster 1960, Green and Hall 1975), but the high degree of scatter makes other conclusions questionable.


FIGURE 30. In-place CBR Versus Backcalculated Subgrade Modulus

Eight soil specimens were obtained with thin-walled (Shelby) tubes at selected locations in accordance with ASTM D 1587-83 (American Society for Testing and Materials 1986). All eight soil specimens were tested for soil classification and natural moisture content. Results of the soil classification and moisture content tests are shown in Figures 26-27. Three of the soil specimens were obtained at Godman Army Airfield and five of the specimens were obtained at Sherman Army Airfield. All of the Godman Army Airfield specimens were suitable for soil resilient modulus tests. Four of the specimens obtained at Sherman Army Airfield were unsuitable for soil resilient modulus testing due to sample disturbance. The remaining specimen from Sherman Army Airfield and the three specimens from Godman Army Airfield were tested for resilient modulus by the procedure described in Appendix $C$.

Results of the soil resilient modulus tests are shown in Figures 31-34. A dashed line on each plot indicates the backcalculated subgrade moduli value from the nearest testing location. There is reasonable agreement between the laboratory soil resilient moduli and backcalculated subgrade moduli from COMDEF.

figure 31. Soil Resilient Modulus test, Feature gaaf t5e

figure 32. Soil Resilient Modulus Test, Feature gaAf tie


DEVIATOR STRESS, ksi
figure 33. Soil Resilient Modulus, feature gaaf ale


FIGURE 34. Soil Resilient Modulus Test, Feature SAAF T2E

## LABORATORY ASPHALTIC CONCRETE MODULUS TESTS

## Background

Laboratory values of asphaltic concrete modulus were determined by two methods, ASTM D 4123-82 and ASTM D 3497-85e (American Society for Testing and Materials 1986). Reported herein are three different representations of asphaltic concrete modulus which are determined from the two test methods. These are: (1) asphaltic concrete instantaneous resilient modulus (ASTM $D$ 4123-82), (2) asphaltic concrete total resilient modulus (ASTM D 4123-82), and (3) asphaltic concrete dynamic modulus (ASTM D 3497-85 ).

Resilient modulus is the ratio of the recoverable strain to a repeated stress. The instantaneous resilient modulus is based on the instantaneous recovery and the total resilient modulus is based on the recovery over a given rest period. Therefore, the instantaneous resilient modulus is an indicator of the elastic behavior under repeated loadings and the total resilient modulus is an indicator of the overall viscoelastic behavior under repeated loadings. A conceptual view of the indirect tensile resilient modulus test is shown in Figure 35. The asphaltic concrete resilient modulus should be the best indicator of the in situ asphaltic concrete behavior under service or nondestructive test loadings. However, the difficulty in obtaining consistent resilient modulus results from the indirect tensile method has led to more reliance on other test methods. In this study, a new apparatus was developed which provided consistent results.


FIGURE 35. Apparatus for AC Resilient Modulus Test (ASTM D 4123)

The dynamic modulus of asphaltic concrete, as described in ASTM D 3497-85 , is an unconfined uniaxial steady-state vibratory response test. The asphaltic concrete dynamic modulus is the magnitude of the complex modulus and is the ratio of the sinusoidally varying stress to the sinusoidally varying strain. The sinusoidal loading of the asphaltic concrete dynamic modulus test provides a much more consistent test than repeated load tests. Although the loading used in this test is not typical of service loads, asphaltic concrete relationships developed from this test have been correlated with numerous pavement performance parameters (Kingham and Kallas 1972).

## Testing

Laboratory modulus tests were performed on asphaltic concrete specimens from Godman Army Airfield Taxiway D (Feature T5E) and from the Waterways Experiment Station Bomb Damage Repair Site. All laboratory asphaltic concrete modulus tests were completed with an MTS Model 312.31 loading frame excited with waveforms generated by a Wavetek Model 175 arbitrary waveform generator. Modulus tests were completed in a controlled temperature environment using an Instron Model 3111 temperature environment chamber.

Resilient modulus tests were performed by the indirect tension method in accordance with ASTM D 4123-82 (American Society for Testing and Materials 1986). Available equipment proved inadequate for completing these tests. Past testing performed in the Waterways Experiment Station asphalt laboratory for resilient modulus by the indirect tensile method had been on relatively soft specimens and had used magnetic mounts to hold the linearly variable differential transducers (LVDTs).

In addition, since the purpose of the past testing was to compare mix designs, all testing had been done at room temperature and no provision had been made for testing within a controlled temperature environment. To complete the testing on the relatively stiff specimens from the Waterways Experiment Station Bomb Damage Repair Site, and to provide a stable temperature environment, a new test apparatus was designed and assembled at the Waterways Experiment Station asphalt laboratory. Figure 35 is an assembly drawing of the apparatus built for this project. Complete design plans for the various components are included in Appendix D as Figures D1-09. The apparatus was designed so that it provided sufficient rigidity for very accurate radial displacement measurements, provided easy access for quick calibration, and was small enough to fit inside the Waterways Experiment Station temperature environment chamber (Instron Model 3111). In addition, computer control software was developed for calibration, testing and data acquisition. Source codes are listed in Appendices E and F.

Dynamic modulus (longitudinal) tests were performed in accordance with ASTM D 3497-85e (American Society for Testing and Materials 1986), except that "stacked" specimens were used. Since length to diameter ratio was not adequate, each longitudinal modulus specimen was made up of three cores. The faces of each core were carefully sawed smooth, and a very thin layer of epoxy was applied to each interface. The epoxy was thin enough to have a minimal effect on longitudinal stiffness, and was used to minimize horizontal slippage during testing. The strain gages were installed in the center of the middle core on each specimen. The
thickness of the middle section was at least twice the gage length of the strain gages for all the stacked specimens. There were no apparent deleterious effects from using the stacked specimens.

## Results

Figure 36 shows a family of curves published by The Asphalt Institute (TAI) for asphaltic concrete dynamic modulus as a function of temperature and frequency of loading (Kingham and Kallas 1972).


FIGURE 36. TAI Curves for AC Modulus (Kingham and Kallas 1972)

Figure 37 compares some published relationships for asphaltic concrete dynamic modulus versus temperature at a loading frequency of 16 Hz . The approximate frequency of loading for the Falling Weight Deflectometer is 16 Hz , based on an average pulse width of 31 msec . Although the curves
shown in Figure 37 are for a frequency of loading of 16 Hz , each curve is calculated from an equation defining a nonlinear family of curves relating asphaltic concrete modulus to temperature and frequency. The curves shown are based on The Asphalt Institute curve families for asphaltic concrete surface, asphaltic concrete base, and sand asphalt base, as well as a curve family by the Kentucky Transportation Research Program (KTRP) for asphaltic concrete surface mixes (Deen, Southgate, and Sharpe 1983), which was based on data by Shook and Kallas (1969). The equations for the families of curves shown in Figure 37 are included in Table 7.


FIGURE 37. Published Curves for AC Modulus Versus Temperature

TABLE 7. AC Modulus as a Function of Temperature and Frequency*


* Curve families have the general form:

$$
L O G_{10}\left(E_{a c}\right)=A_{0}+A_{1} T^{b}+A_{2} L O G_{10}(f)+A_{3} T^{b} L O G_{10}(f)
$$

where
$E_{a c}=$ Asphaltic Concrete Modulus, psi
$\mathrm{T}=\mathrm{AC}$ Temperature, ${ }^{\circ} \mathrm{F}$
$f=$ Loading frequency, Hz
** KTRP replaces the $T^{\text {b }}$ term with two terms involving $T$ and $T^{2}$
NOTE: $\quad{ }^{\circ} \mathrm{F}=\left({ }^{\circ} \mathrm{C} \times 9 / 5\right)+32 \quad 1 \mathrm{psi}=6.89 \therefore \mathrm{~Pa}$

Figures 38-43 illustrate laboratory values of asphaltic concrete moduli versus temperature and frequency of loading. Each plot includes mean asphaltic concrete modulus at each temperature and frequency and also a family of curves which were fit by regression and are of the same form as The Asphalt Institute curves (Kingham and Kallas 1972). Although the data points shown are average values, all data were used in the regression fits. Regression equations are included in Table 7 for each of the families shown in Figures $38-43$. Figures $38-40$ show results from the Godman Army Airfield specimens for total resilient modulus, instantaneous resilient modulus, and dynamic modulus, respectively. Figures 42-43 give corresponding results for the Waterways Experiment Station Bomb Damage Repair Site specimens. There was very little difference in the values of total and instantaneous asphaltic concrete resilient modulus. This indicates that most of the specimen recovery occurred soon after the load pulse.


FIGURE 38. Instantaneous AC Modulus by ASTM D 4123, Feature GAAF T5E


FIGURE 39. Total AC Modulus by ASTM D 4123, Feature GAAF T5E


FIGURE 40. Dynamic AC Modulus by ASTM D 3497, Feature GAAF T5E


FIGURE 41. Instantaneous AC Modulus by ASTM D 4123, Feature HES BDRS


FIGURE 42. Total AC Modulus by ASTM D 4123, Feature WES BDRS


FIGURE 43. Dynamic AC Modulus by ASTM D 3497, Feature WES BDRS

Figure 44 shows the laboratory test results superimposed on the published curves of Figure 37. The intent of this plot is to show that all the curves are reasonably close together. This is a very important result, because of the dissimilarity between the specimens tested. The Asphalt Institute curves and the Kentucky Transportation Research Program curves are based on data from laboratory compacted specimens. The Godman Army Airfield specimens appeared to be a typical asphaltic concrete mix with definite signs of aging. Specimens from the Waterways Experiment Station Bomb Damage Repair Site were highly oxidized, showing discoloration and visible brittleness. Figure 44 supports the work of Kingham and Kallas by verifying that asphaltic concrete modulus may be more strongly influenced by temperature and frequency of loading than by minor differences in mix properties.


FIGURE 44. Laboratory AC Modulus Curves Overlaid on Published Curves

Figures 45 and 46 compare field and laboratory values of asphaltic concrete moduli from the Waterways Experiment Station Bomb Damage Repair Site and Godman Army Airfield Feature T5E, respectively. Field asphaltic concrete moduli values were backcalculated by COMDEF and by BISDEF based on data from Falling Weight Deflectometer tests. Laboratory asphaltic concrete moduli curves are from the families summarized in Table 7 at 16 Hz . Backcalculated asphaltic concrete moduli are plotted versus mean asphaltic concrete temperature, where mean asphaltic concrete temperature is determined from Figure 47 (Southgate and Deen 1969) at mid-depth based on surface temperature and 5-day mean air temperature. Although temperature data were used to compute the mean asphaltic concrete temperature, the backcalculated asphaltic concrete moduli were computed by COMDEF with the "no temperature" option, so that backcalculated moduli were determined based only on the measured deflection basin. Default parameters were used for the BISDEF calculations, except that asphaltic concrete modulus limiting values were not enforced.

Backcalculated asphaltic concrete moduli from COMDEF and BISDEF agreed almost exactly with each other and with the laboratory curves for the Waterways Experiment Station Bomb Damage Repair Site, as illustrated in Figure 45. This was not the case for Godman Army Airfield Feature T5E, where there was significant difference between backcalculated values from COMDEF and BISDEF. The values from BISDEF were unrealistic, as shown in Figure 46. In fairness, the thickness of asphaltic concrete for Godman Army Airfield Feature T5E is only 2 inches ( 51 mm ). Backcalculation of asphaltic concrete surface moduli is, in general, more difficult for a thin asphaltic concrete layer.


FIGURE 45. AC Modulus Verification, Feature WES BDRS


FIGURE 46. AC Modulus Verification, Feature GAAF T5E


FIGURE 47. Pavement Temperature Distribution (Southgate and Deen 1969)

An old "rule-of-thumb" says that you cannot backcalculate an asphalt layer moduli when the layer is less than 3 inches ( 76 mm ) thick. However, with the exception of one scatter point, the agreement between backcalculated moduli from COMDEF and laboratory curves for Godman Army Airfield (Figure 46) was reasonable. Also, the agreement may be even better than it would otherwise appear. The data presented in Figures 45 and 46 ignore the inherent variability of asphaltic concrete. This is particularly true of Godman Army Airfield Feature T5E, where the asphaltic concrete laboratory specimens were taken from a single location which had been recently trenched. Direct comparison at this location
was impossible, so data from the entire feature was used in the comparison. At least some of the variation in the backcalculated COMDEF values can be attributed to localized material variation.

## Development of the COMDEF Temperature Data Option

The COMDEF temperature data option was developed from the results of the laboratory modulus testing. Use of the temperature data option allows the user to fix a range of asphaltic concrete moduli bounded by the curves shown in Figure 48. The lower boundary curve for the COMDEF temperature data option is The Asphalt Institute curve for asphaltic concrete surface mixes (Kingham and Kallas 1972). In general, laboratory asphaltic concrete moduli tests on field cores gave values higher than The Asphalt Institute curves for asphaltic concrete surface mixes (as shown in Figure 44). This is not a surprising result, since the specimens used in The Asphalt Institute study were laboratory compacted. Natural oxidation of in-service asphaltic concrete surfaces contribute to increased moduli values when field cores are tested. The upper boundary curve for the COMDEF temperature data option was chosen as the Godman Army Airfield Feature T5E dynamic modulus curve. This curve was chosen as the upper limit for the COMDEF temperature data option even though the Waterways Experiment Station Bomb Damage Repair Site curves predicted slightly higher values of asphaltic concrete modulus. The semilog relationship of the Waterways Experiment Station Bomb Damage Repair Site curves are almost linear, and therefore predict relatively high values of asphaltic concrete modulus at temperature extrema. The Godman Army Airfield curve used as the upper boundary gives values similar to the Waterways Experiment Station Bomb Damage

Repair Site curves at normal temperatures, but gives a more reasonable variation with temperature extrema. The specimens from the Waterways Experiment Station Bomb Damage Repair Site were highly oxidized and would appear to represent an upper limit on asphaltic concrete modulus.

An additional option in COMDEF allows the user to fix the asphaltic concrete modulus based solely on The Asphalt Institute curve. This option will generally give conservative values of asphaltic concrete modulus. For this reason, Portland cement concrete moduli values calculated using this option may be slightly higher, but will tend to be more consistent. Use of the COMDEF temperature data option is the recommended option for routine evaluations, because scatter points (such as the one point in Figure 46) are eliminated, and also because the time required for completion of the program is greatly reduced.


FIGURE 48. Allowable AC Moduli Range in COMNEF Temperature Data Option

## LABORATORY PCC MODULUS TESTS

Comparisons between laboratory Portland cement concrete modulus tests on intact field specimens and backcalculated Portland cement concrete modulus values are valid only when the intact field specimen is representative of the Portland cement concrete layer in situ. That is, comparison must be made on a feature with uncracked Portland cement concrete slabs. The Waterways Experiment Station Bomb Damage Repair Site was considered the best site for verification of the Portland cement concrete modulus because of the condition of the site. The site is located on a ridge with good natural drainage. The subgrade was well compacted and and the 11 inch ( 279 mm ) thick concrete sections were formed and cured for 40 days prior to the placement of the 4 inch ( 102 mm ) thick asphaltic concrete surface. No dowel bars for load transfer or structural reinforcement were used in the Portland cement concrete layer, although steel temperature mesh was used near the base of the Portland cement concrete layer. In the area of the Falling Weight Deflectometer test (see Figure 22), the Portland cement concrete layer had no significant traffic after construction. Three different modulus tests were performed on the Portland cement concrete core from the Waterways Experiment Station Bomb Damage Repair Site. The first was a pulse velocity test performed with a James V-Meter. A User's Guide prepared by the author is included in Appendix $G$ which describes the device and specific methods used in the pulse velocity test. The test measures the time of propagation of a longitudinal wave in a rod with free-free end conditions. The time of propagation divided by the length of specimen gives the phase velocity, or longitudinal wave propagation
velocity, $V_{c}$. The value of modulus of Portland cement concrete was calculated as 7.8 million psi, and was calculated from the following formula (Richart, Hall, and Woods 1970):

$$
\begin{equation*}
E=\rho V_{c}^{2} \tag{85}
\end{equation*}
$$

where

$$
\begin{aligned}
& E=\text { Young's modulus of elasticity, psi } \\
& \rho=\text { mass density, } 1 b-\sec ^{2} / \text { in. }^{4} \\
& V_{c}=\text { phase velocity, in. } / \mathrm{sec}
\end{aligned}
$$

The second and third tests were tests for longitudinal and transverse fundamental frequency in accordance with ASTM C 215-85 (American Society for Testing and Materials 1986), except that random frequencies were used as follows. A PCB Piezotronics Model 482A accelerometer was used to monitor random frequencies generated by a very small tap hammer. The accelerometer output was displayed as a function of frequency by a Hewlett-Packard 5423A structural dynamics (spectrum) analyzer to find the fundamental frequency. Each of the tests was completed very quickly by a two step procedure. A close approximation of the fundamental frequency was determined by analyzing a wide bandwidth of frequencies. The value from the wide bandwidth test was then used as the center frequency of a very narrow bandwidth test to improve the accuracy of the measurement of the fundamental frequency. Each of the tests was completed almost instantaneously, and provided a very accurate measurement of the fundamental frequency for calculation of the elastic modulus as described in ASTM C 215-85 (American Society for Testing and Materials 1986). The modulus of Portland cement concrete was calculated as 8.0 million psi for the longitudinal test and 8.7 million psi for the
transverse test. As expected, the modulus measured by the longitudinal fundamental frequency test was very close to the modulus measured by the longitudinal pulse velocity test. Laboratory tests for modulus agreed closely with the backcalculated value of Portland cement concrete modulus of 8.3 million psi (average of two tests). Figure 49 shows a bar chart of Portland cement concrete modulus for the laboratory and backcalculated values.


FIGURE 49. Field Versus Laboratory PCC Modulus, Feature WES BDRS

Use of small-strain wave propagation tests such as fundamental frequency tests and pulse velocity tests appear to provide a valid method for obtaining comparison values of moduli from cores. However, there are limits to their applicability. Cores from thin layers, for
example, cannot be tested properly by these methods. In most cases, the use of these tests are recommended even when other tests are performed, due to the relative ease of testing and reasonableness of results. The use of wave propagation tests on cores was limited in this study because the cores were tested with destructive tests in a standard evaluation procedure. This work was part of an ongoing airfield evaluation study and was beyond the control of the author.

## VERIFICATION OF THE DEGREE OF CRACKING IN PCC LAYERS

Cracked or shattered Portland cement concrete layers were encountered on some core attempts. These locations are identified in Table 4. In some areas at Sherman Army Airfield, a flight auger was used to verify the thickness of the shattered concrete. In addition to the coring listed in Table 4, three cores were attempted on Sherman Army Airfield Feature A2E (Operations Apron) Line A, between Stations $7+00$ and $9+00$. These coring attempts wre not included in the core logs, but the Portland cement concrete layer was obviously shattered. On these three attempts, the asphaltic concrete specimens were badly damaged. It appeared that the loose condition of the Portland cement concrete layer created an abrasive condition at the interface of the asphaltic concrete and Portland cement concrete during the coring operation. The asphaltic concrete specimen could not be accurately measured for thickness, and the Portland cement concrete particles "feathered" upward during the coring, making thickness verification of the asphaltic concrete impossible without auguring. Due to time constraints, no auguring was done at these three locations.

There was one area where there was no Portland cement concrete layer. Unusually high Falling Weight Deflectometer deflection values were noted at Godman Army Airfield Feature T6E (Taxiway D), Station $31+00$. A core was attempted at Station $30+50$, and it was discovered that there was no Portland cement concrete under this section. A representative from the Base Civil Engineer's office explained that the section had been removed and replaced during a construction operation to improve airfield drainage. The replacement pavement was a 4 inch
( 102 mm ) asphaltic concrete surface layer over 10 inches ( 254 mm ) of crushed stone. The backcalculated base modulus from COMDEF of 32 ksi ( 220 GPa ) for this section was reasonable for a granular base material.

The backcalculated Portland cement concrete moduli listed in Table 5 verify that areas with shattered Portland cement concrete layers can be identified. Figures 50-58 are bar charts of backcalculated Portland cement concrete modulus versus station for all composite features at Godman Army Airfield and Sherman Army Airfield. Some of the plots combine two similar or adjacent features. Labels are included which identify locations where cores were obtained or attempted. Cores were grouped as either intact, cracked, or shattered. Cores identified as cracked were recovered within the core barrel, but fell apart when removed from the core barrel. Cores identified as shattered could not be recovered in the core barrel and the Portland cement concrete was weak enough that so that particles could be easily dislodged with a steel probe and/or the Portland cement concrete was augured. Figures 50-58 show that intact cores were obtained at all locations where backcalculated moduli were high. In all cases, the backcalculated Portland cement concrete moduli was less than 1 million psi ( 7 GPa ) for locations where cracked or shattered cores were encountered. There were a few cases where an intact core was obtained at a location with a relatively low backcalculated Portland cement concrete modulus. This is not surprising, since localized unbroken areas of greater than 6 inch ( 152 mm ) diameter can exist within a Portland cement concrete layer which is badly cracked.


FIGURE 50. Backcalculated PCC Modulus with Core Status, gAAF AIE


FIGURE 51. Backcalculated PCC Modulus with Core Status, GAAF A1E \& A2E


## LEGEND

- APPROXIMATE DIVIDING LINE BETWEEN RIGID AND FLEXIBLE PCC BEHAVIOR
$R$ INTACT PCC CORE WAS RECOVERED
$1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1$ foot $=0.305 \mathrm{~m}$

FIGURE 52. Backcalculated PCC Modulus with Core Status, GAAF R1E \& R5E


## LEGEND

- APPROXIMATE DIVIDING LINE BETWEEN RIGID AND FLEXIBLE PCC BEHAVIOR
8 券 BACK-CALCULATED PCC MODULUS GREATER THAN 7 MILLION PSI, NUMBER LABEL indicates the back-Calculated value
R INTACT PCC CORE WAS RECOVERED
$1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1$ foot $=0.305 \mathrm{~m}$

FIGURE 53. Backcalculated PCC Modulus with Core Status, GAAF TIE


## LEGEND

APPROXIMATE DIVIDING LINE BETWEEN RIGID AND FLEXIBLE PCC BEHAVIOR

8BACK-CALCULATED PCC MODULUS GREATER THAN 7 MILLION PSI, NUMBER LABEL indicates the back-Calculated value
R INTACT PCC CORE WAS RECOVERED N NO PCC UNDERLAYMENT AT THIS LOCATION
$1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1$ foot $=0.305 \mathrm{~m}$

FIGURE 54. Backcalculated PCC Modulus with Core Status, GAAF T5E \& T6E


LEGEND
$\square$ APRON LINE A
区 APRON LINE B

- APPROXIMATE DIVIDING LINE BETWEEN RIGID AND FLEXIBLE PCC BEHAVIOR
R INTACT CORE WAS RECOVERED
C CRACKED CORE WaS RECOVERED
S SHATTERED PCC PROHIBITED RECOVERY E FEATURE ENDS (NO DATA)
$1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1$ foot $=0.305 \mathrm{~m}$

FIGURE 55. Backcalculated PCC Modulus with Core Status, SAAF AIE


FIGURE 56. Backcalculated PCC Modulus with Core Status, SAAF A2E


## LEGEND

- APPROXIMATE DIVIDING LINE BETWEEN RIGID AND FLEXIBLE PCC BEHAVIOR
R INTACT PCC CORE WAS RECOVERED
S SHATTERED PCC PREVENTED CORE RECOVERY $1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1$ foot $=0.305 \mathrm{~m}$

FIGURE 57. Backcalculated PCC Modulus with Core Status, SAAF R1E \& R5E


## LEGEND

- APPROXIMATE DIVIDING LINE BETWEEN RIGID AND FLEXIBLE PCC BEHAVIOR
R INTACT PCC CORE WAS RECOVERED
$S$ SHATTERED PCC PREVENTED CORE RECOVERY
$1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1$ foot $=0.305 \mathrm{~m}$

FIGURE 58. Backcalculated PCC Modulus with Core Status, SAAF T1E \& T2E

Average backcalculated Portland cement concrete modulus for each site was calculated for locations where intact cores were recovered and for locations where cores could not be recovered due to cracking in the Portland cement concrete layer. Average backcalculated Portland cement concrete moduli for the areas where cores were recovered were 3.9, 1.2, and 8.4 million psi ( $27 \mathrm{GPa}, 8 \mathrm{GPa}$, and 58 GPa ) for Godman Army Air field, Sherman Army Airfield, and the Waterways Experiment Station Bomb Damage Repair Site, respectively. Average backcalculated Pcrtland cement concrete modulus for areas where cores could not be obtained due to cracking of the Portland cement concrete layers was 0.2 million psi (1 GPa) for both Godman Army Airfield and Sherman Army Airfield. These average values are illustrated in Figure 59. While sufficient data have not been compiled to completely quantify this relationship, the data indicates a subjective verification of the use of backcalculated effective Portland cement concrete modulus as an indicator of the degree of cracking in the Portland cement concrete layer. It appears that, in general, the backcalculated Portland cement concrete moduli for badly cracked features approaches that which would be expected for a granular base, and backcalculated Portland cement concrete moduli for uncracked features approaches the actual Portland cement concrete modulus. As the backcalculated Portland cement concrete modulus is related to the structural behavior of a relatively large area, it may be inferred that the backcalculated Portland cement concrete modulus is a better indicator of the slab condition than a core which represents a limited area of test.


FIGURE 59. Average Backcalculated PCC Modulus for Core Sites

## LABORATORY PCC SPLITTING TENSILE TESTS

Splitting tensile tests were performed on various specimens in accordance with ASTM C 496-85 (American Society for Testing and Materials 1986). Results of the splitting tensile tests are included in Table 8.

TABLE 8. Approximate PCC Moduli Values from Splitting Tensile Tests

| Feature <br> Identification | Core Size, inches |  | Tensile Strength (psi) | Flexural Strength (psi)* | PCC <br> Modulus (psi)** |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Height | Diameter |  |  |  |
| GAAF RIE <br> (Runway 18-36) | 7.75 | 5.92 | 611 | 822 | 5255019 |
| GAAF R5E <br> (Runway 18-36) | 7.75 | 5.90 | 564 | 775 | 4992898 |
| gAAF TIE <br> (Taxiway A) | 7.00 | 5.93 | 721 | 932 | 5861016 |
| GAAF T5E <br> (Taxiway D) | 6.25 | 5.94 | 515 | 726 | 4717383 |
| GAAF A2E (North Apron) | 7.50 | 5.94 | 693 | 904 | 5707701 |
| SAAF T2E <br> (South Taxiway) | 6.53 | 5.97 | 391 | 602 | 4008702 |
| SAAF A2E <br> (Operations Apron) | 7.25 | 5.97 | 390 | 601 | 4002913 |
| WES BOMB DAMAGE REPAIR SITE | 11.46 | 3.86 | 729 | 940 | 5904708 |

* Flexural strength based on correlation by l'ammitt (1971)
** PCC Modulus based on correlation by Packard (1981)
NOTE: $\quad 1$ inch $=25.4 \mathrm{~mm} \quad 1 \mathrm{psi}=6.89 \mathrm{kPa}$

Approximate flexural strengths and Portland cement concrete modulus of elasticity are calculated and also presented in Table 8. Flexural strength approximation is based on the Waterways Experiment Station correlation curve shown in Figure 60 (Hammitt 1971).


FIGURE 60. Correlation Between Splitting Tensile and Flexural Strength (Hammitt 1971)

Portland cement concrete modulus approximation is based on the correlation curve shown in Figure 61 (Packard 1981). Figure 62 compares the approximate values of Portland cement concrete moduli based on the splitting tensile tests with the backcalculated Portland cement concrete moduli from COMDEF. In general, the modulus estimated from the splitting tensile test is lower than the backcalculated moduli from COMDEF. This is not surprising, since the empirical correlations relate the splitting tensile strength to the static-chord modulus (ASTM C 469-65).


FIGURE 61. Correlation Between PCC Modulus and Flexural Strength (Packard 1981)


FIGURE 62. Backcalculated Modulus Versus Splitting Tension "Modulus"

The important result from Figure 62 is that there is not a constant relationshif betweer the iaboratory performance of a core specimen and the structural performance of the Portland cement concrete layer in the composite system. All of the moduli estimated from the splitting tensile test were similar in magnitude, but there were significant variations in backcalculated Portland cement concrete moduli. In general, the Waterways Experiment Station Bomb Damage Repair Site specimen came from an area which had no cracking (assumed because test location has never had any significant traffic loading), the Godman Army Airfield specimens came from areas which had some cracking of the Portland cement concrete layer (based on coring attempts), and the Sherman Army Airfield specimens came from areas which had extensive cracking of the Portland cement concrete layer (based on coring attempts). These descriptions are subjective and it is certainly possible, for example, that some areas of Sherman Army Airfield are intact or that there has been some environmental degradation of the Waterways Experiment Station Bomb Damage Repair Site site due to temperature effects. Nevertheless, these subjective descriptors serve an important role in the examination of the prediction of cracking in the Portland cement concrete layer.

It is clear from Figure 62 that the testing of Portland cement concrete cores may not be an adequate description of the Portland cement concrete behavior in situ. The regression fits in Figure 62 show a generally increasing slope for improved in situ condition. (NOTE: Regression fits were forced through the origin.) This subjective evidence indicates that the backcalculated Portland cement concrete moduli are better indicators of actual in situ performance than tests which are based on very small areas of test (cores).

As previously discussed, use of wave propagation tests on the cores for the prediction of modulus would have been preferred over the use of empirical correlations with splitting tensile strength. However, this phase of the testing was not under the control of the author.

## REPRESENTATIVE BASIN

Analysis of a pavement feature with BISDEF is typically achieved by the use of a representative basin. A representative basin is used to reduce the time needed to analyze a pavement feature by reducing the data to one deflection basin per feature. A representative basin is not the same as an average basin. When mathematical averages are used for each sensor, the basin often has a distorted shape which produces unreasonable results. The representative basin is the measured basin which is most near the average basin. Waterways Experiment Station researchers have developed a program called BASIN which calculates the representative basin for a feature. The representative basin concept has been been used in numerous pavement evaluations. There are, however, some obvious problems with this approach when the analysis involves composite sections. One important result of a nondestructive evaluation of a composite pavement involves the identification of areas where the Portland cement concrete layer is badly cracked. Use of a representative basin for composite pavements could cause potential problem areas to be ignored. More importantly, the use of the representative basin method includes an implicit assumption that the behavior of the feature is relatively consistent and can be represented by data at a single point. To evaluate this assumption for composite pavements, coefficient of variation for each layer was calculated for each of the 15 composite features tested with the Falling Weight Deflectometer. The results of this analysis are shown in Figure 63. Mean coefficient of variation for backcalculated $A C$ and subgrade moduli were less than $25 \%$. However, mean coefficient of variation for backcalculated PCC moduli was
more than $80 \%$. This variability has a significant effect on the validity of a representative basin for a composite feature, as such variability could cause potential problem areas to be ignored, as described below.


FIGURE 63. Mean Coefficient of Variation of Layer Moduli

Figures 64-66 compare backcalculated layer moduli from representative basins with mean backcalculated layer moduli for the 15 features tested. A regression line forced through the origin was calculated for each plot to make general comparisons easier. Therefore a regression slope of 1 would indicate that the mean backcalculated moduli was $100 \%$ of the modu:i backcalculated from the representative basin. Figure 64 compares backcalculated subyrade moduli. Data for subgrade moduli had relatively low scatter. The plot illustrates that the mean subgrade modulus is about 91\% of the modulus based on the representative basin (and would be
almost $100 \%$ if the upper two data points were ignored). Figure 65 compares backcalculated asphaltic concrete moduli. Data for asphaltic concrete moduli had more scatter than the data for subgrade moduli, but was still relatively low. The plot illustrates that the mean asphaltic concrete modulus is near $100 \%$ of the modulus based on the representative basin. Figure 66 compares backcalculated Portland cement concrete moduli. Data for Portland cement concrete moduli had relatively low scatter. The plot illustrates that the mean Portland cement concrete modulus is about $89 \%$ of the modulus based on the representative basin. Based solely on Figures 64-66, it would appear that use of a representative basin is adequate for composite pavements. However, the variability noted in Figure 63 does lead to problems in evaluating composite features with a representative basin.


FIGURE 64. Mean Subgrade Modulus Versus Representative Basin

figure 65. Mean ac Modulus Versus Representative Basin

figure 66. Mean PCC Modulus Versus Representative Basin

To better study the effects of the variability of Portland cement concrete modulus, each composite feature was split into sub-features of cracked and uncracked areas. Sub-features were defined subjectively based on the author's experience and used, as a rough guide, the "rule-of-thumb" that the Portland cement concrete is cracked when backcalculated Portland cement concrete moduli are less thin 1 million psi ( 7 GPa ), and that the Portland cement concrete is uncracked when backcalculated Portland cement concrete moduli are greater than 1 million psi (7 GPa). Figures 67 and 68 are similar to Figure 66 except that Figure 67 plots mean Portland cement concrete moduli from the uncracked sub-features, and Figure 68 plots mean Portland cement concrete moduli from cracked sub-features. Figure 67 shows that the mean backcalculated Portland cement concrete modulus from the uncracked sub-features was about $112 \%$ of the backcalculated Portland cement concrete modulus based on the representative basin. Figure 68 shows that the mean backcalculated Portland cement concrete modulus from the cracked sub-features was about $8 \%$ of the backcalculated Portland cement concrete modulus based on the representative basin from the combined features. The results from Figu 66-68 are summarized in Figure 6S. The regression lines shown in Figure 69 clearly indicate that Portland cement concrete moduli backcalculated from a representative basin are more indicative of the uncracked areas in a feature when both uncracked and cracked areas exist within that feature. For this reason, use of a representative basin is not generally recommended for composite pavement evaluations.


FIGURE 67. Mean "Uncracked" PCC Modulus Versus Representative Basin


FIGURE 68. Mean "Cracked" PCC Modulus Versus Representative Basin


FIGURE 69. Summary of PCC Modulus Versus Representative Basin

## CHAPTER VI

## dYNAMIC VERSION OF COMDEF

## BACKGROUND

The implementation version of COMDEF utilizes multilayer linear elastic (quasistatic) theory as its basis. However, the nondestructive testing of pavements is a dynamic problem. While some authors imply that matching dynamic deflection basins with quasistatic basins is theoretically based, this is not at all the case. Use of quasistatic models to match dynamic deflection basins is inherently empirical. The COMDEF program provides a good framework for backcalculation and was designed so that a more complicated structural response model could be substituted simply by substituting new database files. A theoretical method for making dynamic structural response predictions of the deflections under nondestructive loading is presented. The method uses Green's functions to predict frequency domain responses. This structural response model was developed by Kausel (1981) and implemented in the microcomputer program GREEN-MA. The source code for GREEN-MA is included in Appendix $H$, except that two subroutines not written by the author have been omitted. The omitted subroutines calculate Hankel and Bessel functions and were taken directly from a mainframe program supplied by Dr. Eduardo Kausel. The advantage of the Green's function structural response model is that it provides a closed form solution to the dynamic wave propagation problem in algebraic form. A complete discussion of the theory used to make structural response predictions is presented in the following section. A major disadvantage of the method
used in GREEN-MA is the time required to make structural response predictions. As illustrated in Figure 70, the time required for structural response predictions by GREEN-MA is much greater than for BISAR. The method developed by Kausel (1981) represents a single frequency solution. However, a method presented by Roesset and Shao (1985) may be used to predict dynamic Falling Weight Deflectometer responses. A single case is used to demonstrate the applicability of Roesset and Shao's method to dynamic analysis of Falling Weight Deflectometer data, because the time required to calculate the dynamic structural responses by this method is significant. To demonstrate the usefulness of dynamic theory in nondestructive testing, a true dynamic backcalculation algorithm was developed for a vibratory nondestructive testing device. The advantage of using a single frequency testing device is that the structural responses can be predicted by a single frequency domain solution.


FIGURE 70. Time Required to Compute Comparison Basins

## DYNAMIC STRUCTURAL RESPONSE PREDICTIONS BY GREEN'S FUNETIONS

An improved method for the use of Green's functions for dynamic structural response predictions was developed by Kausel (1981). A cursory description of the method is presented here. More complete descriptions of the methods described herein may be found in Kausel (1981) and Wass (1972).

Dynamic structural response predictions for layered media were pioneered by Haskell (1953) and Thomson (1950). The Haskell-Thomson method uses transfer matrices to solve the problem in the frequency domain. For arbitrary loadings, the time domain loading function is resolved into a series of harmonic vibrations, typically by the fast Fourier Transform. This is equivalent to the use of the method of separation of variables to solve the wave equation. Closed form solutions are possible for simple cases, but numeric techniques are required for arbitrarily layered systems. Kausel and Roesset (1981) developed an approach which uses a stiffness matrix to relate external loads to displacements, where the stiffness matrix contains functions of frequency and wave number. This formulation is inefficient, because the arguments become transcendental functions which can be solved in closed form only for very simple geometries. If the layer thicknesses are small compared to the wavelengths of interest, it is possible to reduce the transcendental equations to a series of algebraic expressions. This procedure was first proposed by Lysmer and Wass (1972) and later generalized by Wass (1972) and Kausel (1974). The improved method presented by Kausel (1981) allows explicit, closed form solution for dynamic loads acting on a layered medium.

For this study, the problem of interest is the case of a vertical disc load, and discussion of the method will be restricted to this special case. Similar formulations are available for other common loading situations. For a disc load, it is appropriate to express the problem with cylindrical coordinates. In cylindrical coordinates, the stress and displacement vectors are:

$$
\begin{align*}
& S=\left\{\begin{array}{c}
\tau_{\rho z} \\
\tau_{\theta z} \\
\sigma_{z}
\end{array}\right\}  \tag{86}\\
& U=\left\{\begin{array}{l}
u_{\rho} \\
u_{\theta} \\
u_{z}
\end{array}\right) \tag{87}
\end{align*}
$$

where

$$
\begin{aligned}
& S=\text { stress vector } \\
& U=\text { displacement vector } \\
& U=\text { displacement component } \\
& \tau=\text { tangential stress component } \\
& \sigma=\text { normal stress component }
\end{aligned}
$$

The interrelationship with the spatial domain is now given by:

$$
\begin{equation*}
U=a_{\mu} \int_{0}^{\infty} \rho C_{\mu} \int_{0}^{2 \pi} T_{\mu} U d \theta d \rho \tag{88}
\end{equation*}
$$

where

$$
T_{\mu}=\operatorname{diag}(\cos (\mu \theta),-\sin (\mu \theta), \cos (\mu \theta))
$$

$$
\begin{aligned}
C_{\mu} & =\left\{\begin{array}{ccc}
\frac{d J_{\mu}}{d(k \rho)} & \frac{\mu J_{\mu}}{k \rho} & 0 \\
\frac{\mu J_{\mu}}{k \rho} & \frac{d J_{\mu}}{d(k \rho)} & 0 \\
0 & 0 & -J_{\mu}
\end{array}\right] \\
J_{\mu} & =J_{\mu}(k \rho)=\text { Bessel function of first kind and order } \mu \\
a_{\mu} & =\text { orthogonalization factor } \\
& =1 / 2 \pi \text { if } \mu=0 \\
& =1 / \pi \text { if } \mu \neq 0
\end{aligned}
$$

Similar expressions may be written for $S$. If a specific layer is isolated, and equilibrium preserved, the relationship between external loads and displacements may be written as:

$$
\begin{equation*}
P=K_{m} U \tag{89}
\end{equation*}
$$

where

$$
\begin{aligned}
& P=\text { external load vector } \\
& K_{\mathrm{m}}=\text { stiffness matrix } \\
& U=\text { displacement vector }
\end{aligned}
$$

For a multilayered system, the global stiffness matrix, $K=\left\{K_{m}\right\}$, is constructed by superimposing the contribution of the layer matrices at each interface. For the case of thin layers (Kausel and Roesset 1981), the layer stiffness matrix may be obtained as:

$$
\begin{equation*}
K_{m}=A_{m} k^{2}+B_{m} k+G_{m}-\omega^{2} M_{m} \tag{90}
\end{equation*}
$$

where

$$
\begin{aligned}
& k=\text { wave number } \\
& \omega=\text { frequc..cy of excitation }
\end{aligned}
$$

$A_{m}, B_{m}, C_{m}, M_{m}$ are defined as follows:

$$
\begin{align*}
& A_{m}=h / 6\left[\begin{array}{cccccc}
2(\lambda+2 G) & 0 & 0 & \lambda+2 G & 0 & 0 \\
0 & 2 G & 0 & 0 & G & 0 \\
0 & 0 & 2 G & 0 & 0 & G \\
\lambda+2 G & 0 & 0 & 2(\lambda+2 G) & 0 & 0 \\
0 & G & 0 & 0 & 2 G & 0 \\
0 & 0 & G & 0 & 0 & 2 G
\end{array}\right]  \tag{9}\\
& B_{m}=1 / 2\left[\begin{array}{ccccccc} 
\\
0 & 0 & \lambda-G & 0 & 0 & -(\lambda+G) \\
0 & 0 & 0 & 0 & 0 & 0 \\
\lambda-G & 0 & 0 & \lambda+G & 0 & 0 \\
0 & 0 & \lambda+G & 0 & 0 & -(\lambda-G) \\
0 & 0 & 0 & 0 & 0 & 0 \\
-(\lambda+G) & 0 & 0 & -(\lambda-G) & 0 & 0
\end{array}\right]  \tag{92}\\
& G_{m}=1 / h\left[\begin{array}{ccccccc} 
\\
G & 0 & 0 & -G & 0 & 0 & 0 \\
0 & G & 0 & 0 & -G & 0 \\
0 & 0 & \lambda+2 G & 0 & 0 & -(\lambda+2 G) \\
0 & 0 & 0 & G & 0 & 0 \\
0 & 0 & -(\lambda+2 G) & 0 & 0 & \lambda+2 G
\end{array}\right] \tag{93}
\end{align*}
$$

$$
M_{m}=\rho h / 6\left[\begin{array}{llllll}
2 & 0 & 0 & 1 & 0 & 0  \tag{94}\\
0 & 2 & 0 & 0 & 1 & 0 \\
0 & 0 & 2 & 0 & 0 & 1 \\
1 & 0 & 0 & 2 & 0 & 0 \\
0 & 1 & 0 & 0 & 2 & 0 \\
0 & 0 & 1 & 0 & 0 & 2
\end{array}\right]
$$

where

$$
\begin{aligned}
& \lambda=\text { Lame's constant } \\
& G=\text { Shear modulus } \\
& \rho=\text { mass density } \\
& h=\text { layer thickness }
\end{aligned}
$$

For this formulation, the stiffness matrices are functions only of the layer material properties, and the relationships are algebraic. For the thin layer approximation, the displacements within the layer are obtained by linear interpolation between interfaces, that is:

$$
\begin{equation*}
U_{\xi}=\xi U_{1}+(1-\xi) U_{2} \tag{95}
\end{equation*}
$$

where

$$
\begin{aligned}
& U_{\xi}=\text { displacement within the layer } \\
& U_{1}=\text { displacement at the upper interface } \\
& U_{2}=\text { displacement at the lower interface } \\
& 0 \leq \xi \leq 1
\end{aligned}
$$

The global stiffness matrix is formed by superimposing the layer stiffness matrices, and the displacements may be obtained by the formal inversion of the stiffness matrix, that is:

$$
\begin{equation*}
U=K^{-1} P \tag{96}
\end{equation*}
$$

Formal inversion is not usually necessary and a spectral decomposition may be used for the problem of interest. The natural modes of wave propagation are obtained from the eigenvalue problem produced by setting the load vector equal to zero:

$$
\begin{equation*}
\left(A k_{j}^{2}+B k_{j}+C\right) \phi_{j}=0 \tag{97}
\end{equation*}
$$

where

$$
C=G-\omega^{2} M
$$

The notation for the displacement vector has been changed from $U$ to $\phi_{j}$ to identify it as an eigenvector. The problem yields 6 N eigenvalues, $k_{j}$, and eigenvectors, $\phi_{j}$, where $N$ is the total number of layers. Half of the solution set corresponds to $k_{j}, \phi_{j}$. The other half of the solution set corresponds to $-k_{j}$, $\phi_{j}^{*}$, with $\phi_{j}^{*}$ being obtained trivially from $\phi_{j}$ by reversing the sign of the vertical components. Following Wass (1972), we choose the $3 N$ modes that decay with distance from the source, or propagate away from the source. These correspond to eigenvalues $k_{j}$, whose imaginary part is negative if $k_{j}$ is complex, or whose real part is positive if $k_{j}$ is real. Solution of the quadratic eigenvalue problem as a linear double dimension problem is not necessary because of the spe-
cial structure of the matrices involved. The rows and columns may be rearranged by degrees of freedom rather than by interface. The resulting eigenvalue problem is of the form:

$$
\left[\begin{array}{ccc}
k_{j}^{2} A_{x}+C_{x} & k_{j} B_{x z} &  \tag{98}\\
k_{j} B_{x z}^{\top} & k_{j}^{2} A_{z}+C_{z} & \\
& & k_{j}^{2} A_{y}+C_{y}
\end{array}\right]\left\{\begin{array}{l}
\phi_{x j} \\
\phi_{z j} \\
\phi_{y j}
\end{array}\right]=\left\{\begin{array}{l}
0 \\
0 \\
0
\end{array}\right\}
$$

The antiplane mode $\phi_{y}$ is uncoupled, and the eigenvalue problem may be transformed into a linear, nonsymmetric eigenvalue problem in $k^{2}$ :

$$
\left[\begin{array}{ccc}
k_{j}^{2} A_{x}+C_{x} & B_{x z} &  \tag{99}\\
k_{j}^{2} B_{x z}^{\top} & k_{j}^{2} A_{z}+C_{z} & \\
& & k_{j}^{2} A_{y}+C_{y}
\end{array}\right]-\left[\begin{array}{c}
\phi_{x j} \\
k_{j} \phi_{z j} \\
\phi_{y j}
\end{array}\right]=\left\{\begin{array}{l}
0 \\
0 \\
0
\end{array}\right]
$$

An alternative linear eigenvalue problem is:

$$
\left[\begin{array}{ccc}
k_{j}^{2} A_{x}+C_{x} & k_{j}^{2} B_{x z} &  \tag{100}\\
& B_{x z}^{T} & k_{j}^{2} A_{z}+C_{z} \\
& & k_{j}^{2} A_{y}+C_{y}
\end{array}\right]\left\{\begin{array}{c}
k_{j} \phi_{x j} \\
\phi_{z j} \\
\phi_{y j}
\end{array}\right]=\left\{\begin{array}{l}
0 \\
0 \\
0
\end{array}\right]
$$

Both eigenvalue problems yield the same eigenvalues and have associated left and right eigenvectors which are mutually orthogonal with respect to the characteristic equation:

$$
Y_{j}=\left\{\begin{array}{c}
k_{j} \phi_{x j}  \tag{101}\\
\phi_{z j} \\
\phi_{y j}
\end{array}\right\}
$$

$$
Z_{j}=\left\{\begin{array}{c}
\phi_{x j}  \tag{102}\\
k_{j} \phi_{z j} \\
\phi_{y j}
\end{array}\right\}
$$

The eigenvalue problem may be rewritten as:

$$
\begin{equation*}
k_{j}^{2} A Z_{j}+C Z_{j}=0 \text { or } A Z K^{2}+C Z=0 \tag{103}
\end{equation*}
$$

and

$$
\begin{equation*}
k_{j}^{2} A^{\top} Y_{j}+C^{\top} Y_{j}=0 \quad \text { or } \quad A^{\top} Y K^{2}+C^{\top} Y=0 \tag{104}
\end{equation*}
$$

where

$$
\begin{aligned}
& A=\left[\begin{array}{lll}
A_{x} & & \\
B_{x z}^{\top} & A_{z} & \\
& & A_{y}
\end{array}\right] \\
& C=\left[\begin{array}{lll}
C_{x} & B_{x z} & \\
& C_{z} & \\
& & C_{y}
\end{array}\right] \\
& Y=\left\{Y_{j}\right\}, \quad Z=\left\{Z_{j}\right\}, \quad j=1,2, \ldots, 3 N
\end{aligned}
$$

$$
K=\operatorname{diag}\left\{K_{j}\right\}=\left[\begin{array}{ll}
K_{R} & \\
& K_{L}
\end{array}\right]=\left[\begin{array}{cc}
\begin{array}{cc}
\text { Rayleigh } \\
\text { modes }
\end{array} & \\
& \text { Love } \\
& \text { modes }
\end{array}\right]
$$

Normalization of the eigenvectors gives:

$$
Y^{\top} A Z=\left[\begin{array}{lll}
K_{R} &  \tag{105}\\
& I
\end{array}\right]=N
$$

Substitution into the eigenvalue problem gives:

$$
\begin{equation*}
Y^{\top} C Z=-N K^{2} \tag{106}
\end{equation*}
$$

Considering the equilibrium equation in the wave-number domain (Equation 89), and rearranging rows and columns by degree of freedom, as before:

$$
\begin{equation*}
\left(A k^{2}+C\right) U^{\star}=P^{\star} \tag{107}
\end{equation*}
$$

where

$$
\begin{aligned}
& U^{*}=\left\{\begin{array}{c}
U_{x} \\
\mathrm{k} U_{z} \\
U_{y}
\end{array}\right) \\
& P^{*}=\left\{\begin{array}{c}
P_{x} \\
k P_{z} \\
P_{y}
\end{array}\right)
\end{aligned}
$$

Premultiplying by $Y^{\top}$ and introducing $Z Z^{-1}=I$ gives:

$$
\begin{equation*}
Y^{\top}\left(A k^{2}+C\right) Z Z^{-1} U^{*}=Y^{\top} P^{*} \tag{108}
\end{equation*}
$$

Using Equations 102 and 103:

$$
\begin{equation*}
\left(N K^{2}-N K^{2}\right) Z^{-1} U^{*}=Y^{\top} P^{*} \tag{109}
\end{equation*}
$$

This allows solution for $U^{*}$ :

$$
\begin{equation*}
U^{\star}=Z N^{-1}\left(I k^{2}-K^{2}\right)^{-1} T^{\top} P^{\star} \tag{110}
\end{equation*}
$$

Since the in-plane eigenvalue problem is uncoupled from the antiplane problem, the in-plane problem of interest may be considered separately. Using matrix algebra, the solution for the flexibility matrix (inverted stiffness matrix) may be obtained as:

$$
\left\{\begin{array}{l}
U_{x}  \tag{111}\\
U_{z}
\end{array}\right\}=\left\{\begin{array}{ll}
\phi_{x} D_{R} \phi_{x}^{\top} & k \phi_{x} K_{R}^{-1} D_{R} \phi_{z}^{\top} \\
k \phi_{z} D_{R} K_{R}^{-1} \phi_{x}^{\top} & \phi_{z} D_{R} \phi_{z}^{\top}
\end{array}\right\}\left\{\begin{array}{l}
P_{x} \\
P_{z}
\end{array}\right\}
$$

where

$$
D_{R}=\left(k^{2} I-K_{R}^{2}\right)^{-1}
$$

For the special case of the uniform vertical disk load, the solution can be expressed as:

$$
\begin{equation*}
u_{z}=q R \sum_{l=1}^{2 N} \phi_{z}^{m l} \phi_{z}^{n \ell} I_{1 l}^{R} \tag{112}
\end{equation*}
$$

where

$$
\begin{aligned}
q & =\text { load intensity } \\
R & =\text { vertical disk radius } \\
m & =\text { interface for displacements ( } m=1 \text { for surface) } \\
n & =\text { interface for disk load }(1=1 \text { for surface }) \\
\ell & =\text { Rayleigh modes }=1,2, \ldots, 2 N \\
I_{1 \ell}^{R} & =\left(\pi / 2 i k_{\ell}\right) J_{0}\left(k_{\ell} \rho\right) H_{1}^{2}\left(k_{\ell} R\right)-\left(1 / R k_{\ell}\right) \text { for } 0 \leq \rho \leq R \\
& =\left(\pi / 2 i k_{\ell}\right) J_{1}\left(k_{\ell} R\right) H_{0}^{2}\left(k_{\ell} \rho\right) \quad \text { for } R \leq \rho \\
i & =(-1)^{1 / 2} \\
J_{\zeta} & =\text { Bessel function of the first kind and order } \zeta \\
H_{\zeta}^{\eta} & =\text { Hankel function of the } \eta^{\text {th }} \text { kind and order } \zeta
\end{aligned}
$$

The solution presented herein is for a normal loading from a massless vertical disk. The Falling Weight Deflectometer loading plate weighs about 35 pounds ( 156 N ). The dimensionless mass ratio (Richart, Hall, and Woods 1970), $b$, for the Falling Weight Deflectometer may be computed as:

$$
\begin{equation*}
b_{F W 0}=m /\left(\rho r_{0}^{3}\right)=W / \gamma r_{0}^{3} \approx 2 \tag{113}
\end{equation*}
$$

where

$$
\begin{aligned}
& m=\text { mass of FWD load plate } \\
& W=\text { weight of FWD lo:. } 1 \text { plate } \approx 35 \text { pounds }(156 \mathrm{~N}) \\
& r_{0}=\text { radius of FWD load plate } \approx 5.906 \text { inches }(150 \mathrm{~mm}) \\
& \rho=\text { mass density of system } \\
& \gamma=\text { unit weight of system } \approx 125 \mathrm{pcf}\left(20 \mathrm{kN} / \mathrm{m}^{3}\right)
\end{aligned}
$$

The mass ratio of 2 computed for the Falling Weight Deflectometer is relatively low. The load plate system of vibratory test devices, such as the Road Rater 2008, is much more massive. Based on the plate system dimensions, the weight of the dual load plate system of the Road Rater 2008 was estimated at about 200 pounds ( 890 N ). Using similar assumptions for system density, the dimensionless mass ratio, $b$, for the Road Rater 2008 may be computed as:

$$
\begin{equation*}
b_{R R 2008}=m /\left(\rho r_{0}^{3}\right)=W / \gamma r_{0}^{3} \approx 4 \tag{114}
\end{equation*}
$$

where

$$
\begin{aligned}
& m=\text { mass of plate } \\
& W=\text { weight of plate } \approx 200 \text { pounds }(890 \mathrm{~N}) \\
& r_{0}=\text { radius of plate } \approx 18 \text { inches }(229 \mathrm{~mm})
\end{aligned}
$$

The dimensionless mass ratio of 4 for the Road Rater 2008 is a relatively low, alinough significant, value for mass ratio. For the purpose of the anclysis presented herein the effects of the mass of the load plate will be ignored for both the Falling Weight Deflectometer and the Road Rater 2008.

## USE OF DYNAMIC THEORY TO PREDICT FALLING WEIGHT DEFLECTOMETER RESPONSES

The method presented by Roesset and Shao (1985) was used to predict dynamic responses to the Falling Weighi Deflectometer for the test at the Waterways Experiment Station Bomb Damage Repair Site. The method used consisted of the following:

1. The program GREEN-MA was used to predict the Green's functions of the pavement system. Green's functions relate complex responses to loads of unit magnitude. Numerous single frequency solutions were calculated to define the Green's function versus frequency spectra for each Falling Weight Deflectometer sensor location.
2. The recorded load pulse of the Falling Weight Deflectometer presented in Figure 7 was analyzed by Fast Fourier Transform (FFT) analysis to determine the complex load versus frequency spectrum.
3. The frequency spectra of load and Green's functions were multiplied together using complex multiplication to obtain the predicted dynamic deflection versus frequency spectrum for each Falling Weight Deflectometer sensor location.
4. Inverse Fast Fourier Transform (IFFT) analysis was used to predict time histories of the Falling Weight Deflectometer sensor responses from the predicted dynamic deflection versus frequency spectra.

The method described above is very time consuming due to the large number of dynamic solutions which must be calculated. A demonstration of the method was used for two purposes. The first purpose was to demonstrate the use of a theoretically based dynamic model to predict the dynamic structural responses of the Falling Weight Deflectometer. The second purpose was to examine the backcalculated moduli determined from the implementation (quasistatic) version of COMDEF for validity. To accomplish both of these purposes, the average backcalculated moduli values for the Waterways Experiment Station Bomb Damage Repair Site feature were used to predict dynamic time histories by the method previously described. For the dynamic modeling, the best possible estimate was made for the depth to rigid boundary (depth to rock). Data obtained from Waterways Experiment Station engineers indicated that the depth to rock was 133 feet for a test drilling at a nearby site. This value appeared consistent with geological maps of the Vicksburg area and was used in the dynamic analyses.

Figure 71 illustrates the predicted time histories for each Falling Weight Deflectometer sensor location and the measured peak values from the Falling Weight Deflectometer. Input values for the dynamic structural responses included the average backcalculated moduli values from the implementation version of COMDEF and the assumed value of depth to rock of 133 feet. Agreement between the magnitudes of the predicted dynamic deflections and the measured dynamic deflections was satisfactory. This means that, for this particular case, the moduli values predicted by the implementation version of COMDEF were good approximations of the actual dynamic moduli values.


FIGURE 71. Dynamic Time Histories Compared to Peak FWD Deflections

Although the quasistatic model utilized in the implementation version of COMDEF gave satisfactory moduli values for the Waterways Experiment Station Bomb Damage Repair Site, there are situations where the quasistatic method fails. The most obvious example is the case of shallow bedrock. The quasistatic solution is empirically correlated to the dynamic solution by assuming a depth to rigid boundary of 20 feet $(6.1 \mathrm{~m})$, and use of this correlation will not adequately predict layer moduli values when the actual depth to rigid boundary is small (e.g., less than 20 feet ( 6.1 m )). Proper treatment of anomalous data, such as for shallow bedrock, must utilize a more realistic model which will undoubtedly require a dynamic treatment.

## USE OF ARRIVAL TIME TO DETERMINE MODULI VALUES

The "ideal" backcalculation algorithm could automatically take into account any localized anomalies for a given test site. While development of such an "ideal" algorithm is beyond the scope of this effort, there are dynamic aspects of the Falling Weight Deflectometer problem which have potential application in the near future of backcalculation. One specific area of dynamics which could provide additional information for backcalculation, particularly in the analysis of anomolous data, is the use of arrival times. Richart, Hall, and Woods (1970) was used as the reference for all of the wave propagation analysis presented in this section.

Figure 72 shows velocity versus time signals for each of the inside six Falling Weight Deflectometer sensors.


FIGURE 72. Measured Velocity Time Histories from FWD Geophones

Approximate ranges are indicated in Figure 72 for the compression wave ( $P$-wave), the shear wave ( $S$-wave), and the Rayleigh wave ( $R$-wave). Figure 73 shows velocity versus time for Falling Weight Deflectometer sensors at 24 inches ( 0.6 m ) and 48 inches ( 1.2 m ) from the load plate center, referred to herein as Sensors 3 and 5, respectively. Sensors 3 and 5 were chosen for additional analysis because they appeared to have the smoothest signals (less signal noise).


FIGURE 73. Measured Velocity Time Histories from Selected FWD Geophones

Figures 74-76 show enlargements of the approximate arrivals of the compression wave, the shear wave, and the Rayleigh wave, respectively, at the two geophone locations previously selected. The time axes shown in Figures 74-76 correspond to the time axis shown in Figure 72. The difference in direct arrival times between the two sensors located 2 feet ( 0.6 m ) apart may be used to approximate the compression, shear, and Rayleigh wave velocities of the surface layer.


FIGURE 74. Difference in Direct Arrival Time for FWD Compression Wave


FIGURE 75. Difference in Direct Arrival Time for FWD Shear Wave


FIGURE 76. Difference in Direct Arrival Time for FWD Rayleigh Wave

Because the time histories shown in Figures 72-76 are digitized, the determination of difference in direct arrival times was made on the basis of the reference points marked $A, B, C$, and $D$. The reference points were chosen because they indicated corresponding "break points" on the digitized curves. The shear and Rayleigh wave arrivals were much more clearly defined and are indicated by the single reference points $C$ and $D$, respectively. The compression wave arrival was more difficult to distinguish, so two easily distinguishable reference points, $A$ and $B$, were chosen. The difference in arrival times between the reference points for each wave type was used to compute the direct arrival wave speed for each wave type by:

$$
\begin{equation*}
V=\Delta X / \Delta t \tag{115}
\end{equation*}
$$

where

$$
\begin{aligned}
& V=\text { Wave velocity } \\
& \Delta X=\text { Distance between sensors }=2 \text { feet }(0.6 \mathrm{~m}) \\
& \Delta t=\text { Difference in arrival times at the reference points }
\end{aligned}
$$

The time differences for reference points "A" and "B" in Figure 74 are 0.0003 seconds and 0.0004 seconds, respectively. For the average value of 0.00035 seconds, the corresponding value of compression wave velocity is $5714 \mathrm{ft} / \mathrm{s}(1742 \mathrm{~m} / \mathrm{s})$. Similarly, the time difference for reference points "C" in Figure 75 is 0.0006 seconds and the time difference for reference points "D" in Figure 76 is 0.0008 seconds. These arrival time differences correspond to shear and Rayleigh wave velocities of $3333 \mathrm{ft} / \mathrm{s}(1016 \mathrm{~m} / \mathrm{s})$ and $2500 \mathrm{ft} / \mathrm{s}(762 \mathrm{~m} / \mathrm{s})$, respectively.

The measured wave velocities were used to calculate values of elastic modulus from the following equations:

$$
\begin{align*}
& G=E /(2+2 \nu)  \tag{116}\\
& V_{p}=[(\lambda+2 G) / \rho]^{1 / 2}  \tag{117}\\
& V_{\mathrm{s}}=(G / \rho)^{1 / 2}  \tag{118}\\
& V_{r}^{2} / V_{s}^{2}=K^{2}  \tag{119}\\
& K^{6}-8 K^{4}+\left(24-16 \alpha^{2}\right) K^{2}+16\left(\alpha^{2}-1\right)=0 \tag{120}
\end{align*}
$$

where

$$
\begin{aligned}
& V_{p}=\text { compression wave velocity } \\
& V_{s}=\text { shear wave velocity } \\
& V_{r}=\text { Rayleigh wave velocity } \\
& G=\text { shear modulus }
\end{aligned}
$$

$$
\begin{aligned}
& E=\text { Young's (elastic) modulus } \\
& \nu=\text { Poisson's ratio } \\
& \rho=\text { mass density } \\
& \lambda=\nu E /[(1+\nu)(1-2 \nu)] \\
& \alpha^{2}=(1-2 \nu) /(2-2 \nu)
\end{aligned}
$$

Using Equations 116-120, the corresponding values of elastic moduli were calculated for each of the wave arrivals. In addition, comparison values of wave velocities were calculated using the elastic modulus predicted by the COMDEF backcalculation, $621 \mathrm{ksi}(4.3 \mathrm{GPa})$, and an assumed value of Poisson's ratio for asphaltic concrete of 0.35 . The results of the direct arrival analysis are reasonable and are summarized in Table 9. It is clear from the data in Table 9 that classical direct arrival survey methods can be applied to nondestructive test data. By analogy, it would appear that other classical techniques, such as reflection and refraction survey methods could also be applied to nondestructive test data.

TABLE 9. Use of Direct Arrival Survey Method for FWD Data

| Analysis <br> Method | Predicted Velocity* <br> Using COMDEF Values <br> (ft/sec) | Measured Velocity <br> Using Arrival Times <br> (ft/sec) | Calculated Modulus <br> of Elasticity* <br> $(\mathrm{psi})$ |
| :---: | :---: | :---: | :---: |
| COMDEF | $\ldots-$ | $\ldots$ | 620,513 |
| P-wave | 5,849 | 5,714 | 592,200 |
| S-wave | 2,810 | 3,333 | 872,989 |
| R-wave | 2,627 | 2,500 | 561,967 |
| Average |  |  | 675,719 |

* Assumed value of Poisson's ratio $=0.35$ for asphaltic concrete NOTE: $\quad 1 \mathrm{ft} / \mathrm{sec}=0.305 \mathrm{~m} / \mathrm{s} \quad \mathrm{psi}=6.89 \mathrm{kPa}$


## A TRUE DYNAMIC BACKCALCULATION ALGORITHM

Background
To demonstrate the importance of dynamic analysis, as well as to demonstrate the adaptability of the COMDEF program, a dynamic backcalculation program was developed. Due to the intensive computer time required for a dynamic treatment of Falling Weight Deflectometer data, as previously discussed, the dynamic version of COMDEF was developed for the case of a single frequency vibratory nondestructive testing device. The structural response model used in the dynamic version of COMDEF was the Green's function model developed by Kausel (1981) and implemented in the microcomputer program GREEN-MA.

A vibratory nondestructive device was selected for the dynamic portion of the study. Devices considered for this phase of the study included the Dynaflect, the WES l6-kip Vibrator, and the Road Rater 2008. The Dynaflect was ruled out because of the very low maximum load of $1 \mathrm{kip}(4.4 \mathrm{kN})$. A cursory analysis of responses (see discussion below) indicated that the Road Rater 2008 provided a smoother sinusoidal signal than the WES 16 -kip Vibrator. Typical waveforms from the WES 16-kip Vibrator were presented in a study by Drnevich (1985). The advantage of the WES 16-kip Vibrator is the large magnitude harmonic loads which can be applied to the pavement system. However, this advantage was offset by the manpower needed to use the equipment. Use of the WES 16-kip Vibrator typically requires a 4 person crew. In addition, use of the WES 16-kip Vibrator was discouraged, due to difficulties in scheduling during an ongoing US Army airfield evaluation study. The Road Rater 2008 can be operated by a single person and provides a reasonably high
load magnitude of up to 8 kips ( 36 kN ). This made the Road Rater 2008 the best alternative for the testing in this phase of the project. A Road Rater 2008 which was temporarily assigned to the Pavement Systems Division of the Waterways Experiment Station in September and October of 1987 was made available to the author for dynamic testing. The Road Rater used in the project was undergoing maintenance, calibration, and painting. The device was disassembled during most of the period it was at the Waterways Experiment Station. The device was used prior to disassembly on September 16,1987 to complete an initial test series at the Waterways Experiment Station Bomb Damage Repair Site. This initial series was used by the author to assess the suitability of the device for the project, to provide data for determining the best use of the spectrum analyzer when capturing the waveform data, and to plan the testing and analysis which is presented in this document in a later section. Following the repair and maintenance program performed at the Waterways Experiment Station, the Road Rater 2008 was made available to the author on October 28, 1987, as a "shake-down" test of the equipment prior to its return. The equipment was operated by Waterways Experiment Station personnel. One disadvantage of this particular device was that a switching device was used to obtain frequencies of 15,20 , and 25 Hz . In general, the Road Rater 2008 is capable of generating a frequency "sweep" by a variable frequency generator. Due to the short amount of time that the device was available, it was decided that only the preset frequencies would be used. Even with this disadvantage, the Road Rater 2008 was still the best alternative for the project, considering time and budgetary constraints. The Road Rater 2008 is illustrated in Figure 77 and a schematic illustration is shown in Figure 78.


FIGURE 77. Road Rater Model 2008


FIGURE 78. Schematic of the Road Rater Model 2008

## Verification Data For Dynamic Analysis

The Waterways Experiment Station Bomb Damage Repair Site (BDRS) previously described and shown in Figure 22 was used as the verification site for the dynamic backcalculation. Verification data were collected using the Road Rater 2008 in its standard configuration, except that the outputs from two of the geophones were run directly into a HewlettPackard 3562A Dynamic Signal Analyzer, which will be referred to herein as the Analyzer. Calibration factors for the Road Rater 2008 geophones at the frequencies of interest were obtained from the Waterways Experiment Station Instrumentation Systems Division. Time and frequency domain measurements were recorded by the Analyzer for subsequent analysis. Measurements were made at 1 foot ( 305 mm ) intervals from the center of the load plate using the second sensor location, 1 foot ( 305 mm ) from the plate center, as the reference location. Data of interest include the magnitudes of the sensor deflections at each location and the relative phase shift between each sensor location and the reference sensor.

The cross spectrum function of the Analyzer was used to record the relative phase shift between the 2 sensors, as illustrated in Figure 79. Fast Fourier Transform analysis was used to determine the magnitude of the deflection at each sensor location from time domain recordings. The Road Rater 2008 recording system was used to monitor the load. There was no way to externally monitor the load cell output without destructively modifying the Road Rater 2008. Although the load output was adjusted to 7 kips ( 31 kN ) before each recording, there was no way to precisely synchronize the measurement of the load output (by the internal Road Rater system) with the external measurement of the geophone
responses (by the Analyzer). Although the recorded waveforms were smooth and very near a pure sinusoid, the peak-to-peak magnitudes had some variation with time due to fluctuations in the hydraulic pressure system of the Road Rater. Since the load measurements might not have occurred at precisely the same instant as the pavement response measurements, some errors could have been created if the peak-to-peak fluctuations were ignored for the six response measurements. Although the load measurement was not precisely synchronized with the response measurements, the test response measurement for each location was, in fact, synchronized with the reference sensor response, so that the ratio of the test sensor response to the corresponding reference sensor response should be unaffected by load fluctuations.

Relative Phase Shift at $20 \mathrm{~Hz}=-2.539$ degrees
1 degree $=0.017$ radians


FIGURE 79. Relative Phase Shift from Cross Spectrum Measurement

To minimize the errors in deflection magnitudes due to load fluctuations for the six response measurements, the following procedure was used: (1) time domain waveforms of voltage versus time were recorded, as illustrated in Figures 80 and 81, (2) calibration factors were applied to obtain velocity versus time waveforms, as illustrated in Figures 82 and 83, (3) the Fast fourier Transform of each waveform was performed to obtain the frequency domain spectrum of velocity magnitude, as illustrated in Figures 84 and 85 , (4) the velocity was integrated to obtain deflection using division by $-j \omega$ (assumes a single degree of freedom system under sinusoidal loading), as illustrated in Figures 86 and 87 , (5) the deflection magnitude spectrum at each location was divided by the corresponding deflection magnitude spectrum of the reference sensor, obtaining a ratio analogous to a transfer function value, as illustrated in Figure 88 , (6) the average magnitude of the reference deflection for the six tests was assumed to be the value corresponding to a 7 kip loading, and (7) the magnitudes of deflection at each sensor location was obtained by multiplying each of the "transfer function" ratios by the average value of reference deflection. This procedure minimized the deleterious effect of load level fluctuations, resulting in measurements similar to that which would be measured if the data for seven channels were taken simultaneously. Figures 79-88 are provided for illustration and represent waveform analyses which were performed directly using Analyzer functions.


FIGURE 80. Time Domain of Reference Geophone Voltage


FIGURE 81. Time Domain of Comparison Geophone Voltage


FIGURE 82. Time Domain of Reference Geophone Velocity


FIGURE 83. Time Domain of Comparison Geophone Velocity


FIGURE 84. Frequency Domain of Reference Geophone Velocity


FIGURE 85. Frequency Domain of Comparison Geophone Velocity


FIGURE 86. Frequency Domain of Reference Geophone Deflection


FIGURE 87. Frequency Domain of Comparison Geophone Deflection


FIGURE 88. Deflection Ratio Analogous to Transfer Function

Table 10 summarizes the Road Rater 2008 raw data taken at the Waterways Experiment Station Bomb Damage Repair Site, including peak deflection magnitudes from the Fast Fourier Transform analyses and phase shift data from the cross spectrum analyses. Table 11 summarizes data calculated from the raw data presented in Table 10 . The average deflection magnitude of the reference sensor was calculated and assumed to be the value associated with an exact loading of 7 kips ( 31 kN ). This value was multiplied by the "transfer function" ratios to obtain the peak magnitude of sensor deflection at each test location which is assumed to correspond to an exact loading of $7 \mathrm{kips}(31 \mathrm{kN})$.

TABLE 10. Road Rater 2008 Raw Data

| Sensor Location | Peak Reference Deflection from FFT (mils) | Peak Sensor Deflection from FFT (mils) | Ratio of Peak Sensor to Peak Reference Deflection | Phase Shift Relative to Reference (degrees) | Phase Shift Relative to First Sensor (degrees) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 inches | 1.19715 | 1.53902 | 1.28556 | -7.7578 | 0.0000 |
| 12 inches | --- | --- | 1.00000 | 0.0000 | 7.7578 |
| 24 inches | 1.17047 | 1.09339 | 0.93415 | 2.5390 | 10.2968 |
| 36 inches | 1.18661 | 0.99009 | 0.83439 | 8.2780 | 16.0358 |
| 48 inches | 1.19536 | 0.92482 | 0.77367 | 14.2900 | 22.0478 |
| 60 inches | 1.14104 | 0.78680 | 0.68955 | 21.5100 | 29.2678 |
| 72 inches | 1.17927 | 0.77212 | 0.65474 | 30.4130 | 38.1708 |

NOTE: $\quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m} \quad 360$ degrees $=2 \pi$ radians

TABLE 11. Real Time (Dynamic) Deflection Basin

| Sensor Location | Mean Reference Deflection for 7 kip p-p Load (mils) | Ratio of Peak <br> Sensor to Peak Reference Deflection | Calculated <br> Peak Sensor Deflection (mils) | Phase Shift Relative to First Sensor (degrees) | Real rime Deflection Basin (mils) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 inches | 1.17832 | 1.28556 | 1.51480 | 0.0000 | 1.51480 |
| 12 inches | 1.17832 | 1.00000 | 1.17832 | 7.7578 | 1.16753 |
| 24 inches | 1.17832 | 0.93415 | 1.10072 | 10.2968 | 1.08299 |
| 36 inches | 1.17832 | 0.83439 | 0.98317 | 16.0358 | 0.94492 |
| 48 inches | 1.17832 | 0.77367 | 0.91163 | 22.0478 | 0.84496 |
| 60 inches | 1.17832 | 0.68955 | 0.81251 | 29.2678 | 0.70879 |
| 72 inches | 1.17832 | 0.65474 | 0.77149 | 38.1708 | 0.60653 |

NOTE: $\quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m} \quad 360$ degrees $=2 \pi$ radians

In addition to the traditional deflection data summarized in Tables 10 and 11, a "real time" deflection basin was calculated using the magnitude and phase information, and is also included in Table 11. The real time basin is the equivalent of a "photograph" of the dynamic deflection basin at a given instant of time. The concept of a real time deflection basin is clearly illustrated in Figure 89. The traditional "basin" used to compare to theoretical models is actually the envelope of peak deflections. As can be readily seen, the peak values of deflection do not occur at the same time. The data presented in Table 11 are for a real time basin at the instant that the first sensor has reached its peak. Both peak deflections and real time deflections will be used in backcalculation schema in following sections.


FIGURE 89. "Real Time" Deflection Basins

## Backcalculation Schema

The two main purposes of the dynamic backcalculation analyses presented herein are to demonstrate the importance of dynamic analysis and the adaptability of the COMDEF program. In addition, an attempt was made to demonstrate various levels of implementation of the dynamic methodology. To achieve this additional goal, several backcalculation schema were devised. These schema are summarized in Table 12.

TABLE 12. Backcalculation Schema for Dynamic Study

| Schema <br> ID | Sensor Locations Used | Phase Data Used | Forward <br> Model |
| :---: | :---: | :---: | :--- |
| STD | $0,12,24,36$ inches | None | Static |
| 4D-OP | $0,24,48,72$ inches | None | Dynamic |
| $40-3 P$ | $0,24,48,72$ inches | Relative Phase Shifts | Dynamic |
| 7D-PK | $0,12,24,36,48,60,72 \mathrm{in}$. | None | Dynamic |
| 7D-RT | $0,12,24,36,48,60,72 \mathrm{in}$. | Real Time Basin | Dynamic |
| 6D-TP* | $12,24,36,48,60,72$ inches | Total Phase Shift | Dynamic |

* Schema 6D-TP added to examine the effect of the load plate sensor NOTE: 1 inch $=25.4$ mm

The schema presented in Table 12 generally represent increased cost of implementation to the end user. The six backcalculation schema include:
(1) STD A "standard" method which represents the typical backcalculation method currently used in practice for nondestructive evaluations with the Road Rater 2008. This method measures peak deflection measurements from 4 sensors at 1 foot ( 0.3 m ) intervals. These
deflection measurements are analyzed by a backcalculation algorithm which utilizes multilayer linear elastic (quasistatic) theory as its structural response model.
(2) 4D-OP A dynamic method which represents the minimum investment by the user. In this method, the standard Road Rater 2008 sensor arrangement is utilized and the standard 4 peak deflections are measured. However, for this case a dynamic structural response model is substituted for the quasistatic model typically used in practice. Therefore the dynamic peak values measured by the Road Rater are compared to true dynamic peak values.
(3) 4D-3P A dynamic method representing relatively minor modifications of the Road Rater 2008 system. For this schema, the spacing of the 4 standard Road Rater 2008 sensors is increased to 2 foot ( 0.6 m ) intervals. In addition, a spectrum analyzer is incorporated to measure phase shifts. In this schema, the 4 peak deflections and the 3 relative phase shifts between sensors are used as 7 inputs to a true dynamic backcalculation algorithm.
(4) 7D-OP A dynamic method requiring significant modifications to the Road Rater 2008 but not requiring a spectrum analyzer. In this schema, the number of
sensors is increased to 7 and the peak deflections are used as 7 inputs to a true dynamic backcalculation algorithm.

7D-RT A dynamic method which represents the complete implementation by the end user. In this schema, a real time basin is measured and used as input to a true dynamic backcalculation algorithm.
(6) 6D-TP A dynamic method which was added to determine if better backcalculated moduli could be obtained by ignoring the sensor mounted on the load plate. In this schema, the outer 6 deflections and the total phase shift across the 6 sensors are used as the 7 inputs to a true dynamic backcalculation algorithm.

## Formulation of Database Files

A matrix of 210 precalculated solutions is required to form a constants file for use in the COMDEF program. Structural response predictions were made with GREEN-MA for the 210 cases of interest. The 210 cases represent the variable matrix of moduli values listed in Table 2, but for a fixed thickness of Portland cement concrete. A single set of 210 solutions provided the data to create the databases for all 5 of the dynamic backcalculation schema, since each of the 210 GREEN-MA runs provided the complex Green's functions for each of the 7 sensor locations. In addition to the dynamic structural response solutions, a set of 210 quasistatic solutions were used in the "standard" schema (STD).

One of the important aspects of this portion of the study was to show that the standard COMDEF program could be used with database files utilizing improved structural response models. Therefore, all backcalculations performed in this portion of the study used the standard COMDEF program without modification. That is, the "implementation version" of COMDEF and the "dynamic version" of COMDEF used the same executable file (COMDEF.EXE) and differed only in the database files utilized.

The COMDEF program expects 7 deflections and corrects the deflections to a standard load of $25 \mathrm{kips}(111 \mathrm{kN})$. Since some of the schema utilized data which did not meet this criteria, special manipulations were used in the creation of the database files and in the creation of the data files to insure that the backcalculation was completed properly without requiring changes to the COMDEF program. Schema 7D-PK and 7D-RT required no special manipulations. For these schema, the peak deflections and the real time deflections, respectively, for the 210 GREEN-MA solutions were stored in database files. GREEN-MA predicts deflections for a unit load, so that correction to a standard load is appropriate. Schema 4D-3P and 6D-TP replaced some of the deflection inputs with phase shift data. The database files for these schema therefore included both deflection data and phase shift data from the 210 GREEN-MA solutions. The theoretical values of phase shift are not dependent on load magnitude and errors would result if the standard load correction were made for a data file containing 7 kip deflections along with phase data. Therefore, for these schema, the load correction to 25 kips (111 kN) was calculated for the deflection data prior to running the COMDEF program. The data file therefore contained the values of equivalent 25 kip
(111 kN) deflections and the uncorrected phase shift data. For a 25 kip (111 kN) load, no correction is made by the COMDEF program, so that the phase shift values were handled properly by the program. For schema STD and 4D-OP. unit "dummy" deflections were input both to the database and the data files for the three unused sensor locations, where the databases for schema STD and 4D-OP contained deflections from the 210 quasistatic solutions and the 210 GREEN-MA solutions, respectively. As with the schema involving phase data, the load correction to 25 kips (111 kN) was made prior to running the deflection data so that the unit deflections in the data file were handled properly. In addition, all schema requiring special treatment (STD, 4D-OP, 4D-3P, and 6D-TP) were organized so that the first and seventh sensor locations contained actual deflection data, since these deflections are used in the RANGES subroutine of COMDEF to compute trial values for the stepwise direct optimization.

## Depth to Bedrock as a Controlling Factor

Assumption of a rigid layer at a depth of 20 feet ( 6.1 m ) is common for backcalculation using multilayer linear elastic theory (Bush 1980). This assumption has been used in the past for dynamic structural response predictions (Mamlouk 1985). However, this assumption cannot be used in a dynamic backcalculation method unless the depth to rigid material is actually 20 feet ( 6.1 m ). Just as system resonance is a function of frequency for a fixed pavement modulus profile, system resonance becomes a function of the pavement modulus profile for a fixed frequency of test. For the case of vibratory loading at 20 Hz , and for the assumption of a rigid layer at 20 feet ( 6.1 m ), the predicted system
resonance occurs when the modulus of subgrade is about 50 ksi ( 345 MPa ). This predicted resonance will bias the backcalculation so that reasonable moduli values, even those far from the predicted resonant condition, cannot be found routinely. Based on a cursory review of the resonance condition described above, it is concluded that an assumed depth to rigid boundary must be at least 60 feet ( 18.3 m ) to avoid the predicted resonant condition for a reasonable range of subgrade moduli. For the dynamic analyses included in this study, the best possible estimate was made for the depth to rigid boundary. As previously discussed, a rigid layer was assumed to occur at a depth of 133 feet based on data obtained from Waterways Experiment Station engineers.

Role of Material Damping Ratio in the Dynamic Calculations
Geometric, or radiation, damping due to the vibratory loading of a massless disc is taken into account automatically in the method developed by Kausel, as implemented in the program GREEN-MA. Dynamic effects due to the mass of the Road Rater plate system (mass ratio $\approx 4$ ) are ignored, as previously discussed. Material damping must be assumed for structural response calculations by this method. Numerous authors have shown that the upper limit for material damping in soils is about 10\% (Richart, Hall, and Woods 1970). It is common to assume a value of $5 \%$ for the material damping of all pavement layers for dynamic calculations (Mamlouk 1985). It has been reported that the choice of assumed material damping ratio has only a small effect on the predicted structural responses (Roesset and Shao 1985, Mamlouk 1985, Davies and Mamlouk 1985). A set of structural response predictions were used to evaluate this effect. Arbitrarily chosen values of layer moduli which
gave predicted values of deflection similar to the actual data were used with a series of assumed material damping ratios. The moduli values used were 1500,10000 , and $25 \mathrm{ksi}(10.3,68.9$, and 0.2 GPa ), for the asphaltic concrete, Portland cement concrete, and subgrade, respectively. The material damping ratios used were $0 \%$ (undamped), $2 \%, 5 \%$, $10 \%, 20 \%$, and $40 \%$. The theoretical total phase shift across the deflection basin is plotted in Figure 90.


FIGURE 90. Effect of Material Damping Ratio on Phase Shift

Theoretical deflections are plotted in Figures 91-97, for sensors 1-7, respectively. Measured data taken with the Analyzer are plotted on Figures 90-97 for illustration. Table 13 summarizes load-frequency sweeps of Road Rater data taken with the standard Road Rater recording system prior to recording with the Analyzer. Average values of the data in Table 13 are included in Figures 91-94 for illustration.


FIGURE 91. Effect of Material Damping Ratio on Sensor 1 Deflection


FIGURE 92. Effect of Material Damping Ratio on Sensor 2 Deflection


FIGURE 93. Effect of Material Damping Ratio on Sensor 3 Deflection


FIGURE 94. Effect of Material Damping Ratio on Sensor 4 Deflection


FIGURE 95. Effect of Material Damping Ratio on Sensor 5 Deflection


FIGURE 96. Effect of Material Damping Ratio on Sensor 6 Deflection


FIGURE 97. Effect of Material Damping Ratio on Sensor 7 Deflection

TABLE 13. Standard Road Rater 2008 Data from BDRS Site

| Load <br> (kips) | Sensor 1 <br> Deflection <br> (mils) | Sensor 2 <br> Deflection <br> (mils) | Sensor 3 <br> Deflection <br> (mils) | Sensor 4 <br> Deflection <br> (mils) | Frequency <br> (Hz) |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 1.03 | 0.5 | 0.4 | 0.3 | 0.3 | 15 |
| 1.03 | 0.4 | 0.3 | 0.3 | 0.2 | 20 |
| 1.02 | 0.3 | 0.2 | 0.2 | 0.2 | 25 |
| 4.98 | 2.3 | 1.9 | 1.6 | 1.5 | 15 |
| 4.97 | 1.9 | 1.5 | 1.3 | 1.2 | 20 |
| 4.75 | 1.6 | 1.2 | 1.0 | 0.9 | 25 |
| 6.77 | 3.1 | 2.6 | 2.3 | 2.2 | 15 |
| 6.94 | 2.7 | 2.2 | 1.9 | 1.8 | 20 |
| 7.05 | 2.3 | 1.8 | 1.6 | 1.4 | 25 |
| 1.09 | 0.5 | 0.4 | 0.4 | 0.3 | 15 |
| 1.08 | 0.4 | 0.3 | 0.3 | 0.2 | 20 |
| 1.04 | 0.4 | 0.2 | 0.2 | 0.2 | 25 |
| 4.71 | 2.2 | 1.6 | 1.5 | 1.4 | 15 |
| 4.98 | 1.9 | 1.5 | 1.4 | 1.3 | 20 |
| 4.79 | 1.5 | 1.2 | 1.1 | 1.0 | 25 |
| 6.80 | 4.1 | 2.5 | 2.2 | 2.1 | 15 |
| 6.95 | 2.8 | 2.1 | 1.9 | 1.7 | 20 |
| 6.75 | 2.3 | 1.7 | 1.5 | 1.3 | 25 |

NOTE: $\quad 1 \mathrm{kip}=4.448 \mathrm{kN} \quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m}$

The effects of assumed material damping ratio at the test frequency of 20 Hz are summarized in Table 14. At the limits of the realistic values of material damping ratio ( $0 \%-10 \%$ ) the differences are relatively small. All deflections agree within $0.07 \mathrm{mils}(2 \mu \mathrm{~m})$ and the total phase shift agrees within about 2 degrees. Even when an extreme range of values of material damping ratio is considered ( $0 \%-20 \%$ ), all the deflections agree
within $0.15 \mathrm{mils}(4 \mu \mathrm{~m})$ and the total phase shift differs by about 4 degrees. When expressed as a percentage of the measured values these differences are relatively small, particularly since it is almost certain that a material with a material damping ratio of $0 \%$ will not be encountered. This agrees with other authors conclusions that material damping ratio is a minor effect in comparison with the geometric damping associated with wave propagation in a continuum for nondestructive testing of pavements (Roesset and Shao 1985, Mamlouk 1985, Davies and Mamlouk 1985). Based on the literature and on the data presented in Figures 90-97 and in Table 14, it was concluded that the assumption of material damping of $5 \%$ for all layers was appropriate and reasonable. All subsequent calculations for dynamic deflection data used this assumption.

TABLE 14. Effects of Assumed Material Damping Ratio (DR) at $20 \mathbf{~ H z}$

| Parameter | Difference Between $D R=0 \%$ and $D R=10 \%$ | Measured Value | Difference <br> Expressed as <br> X of Measured | Difference Between $O R=0 \%$ and DR=20\% | Measured Value | Difference Expressed as x of Measured |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Phase shift (degrees) | 2.149325 | 38.1708 | 5.63 | 4.413946 | 38.1708 | 11.56 |
| Sensor 1 (mils) | 0.058243 | 1.514802 | 3.84 | 0.142233 | 1.514802 | 9.39 |
| Sensor 2 (mils) | 0.062818 | 1.178317 | 5.33 | 0.146132 | 1.178317 | 12.40 |
| Sensor 3 (mils) | 0.064480 | 1.100721 | 5.86 | 0.145344 | 1.100721 | 13.20 |
| Sensor 4 (mils) | 0.066011 | 0.983174 | 6.71 | 0.142210 | 0.983174 | 14.46 |
| Sensor 5 (mils) | 0.066893 | 0.911630 | 7.34 | 0.138440 | 0.911630 | 15.19 |
| Sensor 6 (mils) | 0.067167 | 0.812506 | 8.27 | 0.133575 | 0.812506 | 16.44 |
| Sensor 7 (mils) | 0.067483 | 0.771491 | 8.74 | 0.129206 | 0.771491 | 16.74 |
| NOTE: 360 | 360 degrees $=2 \pi$ radians $\quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m}$ |  |  |  |  |  |

## Results of the Dynamic Backcalculations

The results of the dynamic backcalculations are summarized in Table 15, and illustrated in Figures 98, 99, and 100 for the asphaltic concrete, Portland cement concrete, and subgrade, respectively. Backcalculated results for both the temperature and no temperature options are plotted in Figure 98 for the asphaltic concrete layer. Only the options which use temperature data are plotted in Figures 99 and 100 for the Portland cement concrete and subgrade layers, due to the extreme variability in results.

TABLE 15. Tabular Results of Dynamic Backcalculation

| Schema ID | Fixed AC Modulus Option |  |  | Temperature Data Option |  |  | No Temperature Data Option |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & E_{a c} \\ & (p s i) \end{aligned}$ | $\begin{aligned} & E_{p c c} \\ & (p s i) \end{aligned}$ | $\begin{aligned} & E_{\text {sub }} \\ & \text { (psi) } \end{aligned}$ | $\begin{aligned} & E_{a c} \\ & (p s i) \end{aligned}$ | $\begin{aligned} & E_{p c c} \\ & (p s i) \end{aligned}$ | $\begin{aligned} & E_{\text {sub }} \\ & \text { (psi) } \end{aligned}$ | $\begin{aligned} & E_{a c} \\ & (p s i) \end{aligned}$ | $\begin{aligned} & E_{p c c} \\ & (p s i) \end{aligned}$ | $\begin{aligned} & E_{\text {sub }} \\ & (p s i) \end{aligned}$ |
| STD | 1464000 | 76893 | 158489 | 799583 | 79749 | 158489 | 81518 | 5172561 | 15684 |
| 40-0p | 1464000 | 4370211 | 43715 | 799583 | 5979334 | 42544 | 202234 | 13702707 | 24619 |
| 40.3p | 1464000 | 7309035 | 23934 | 799583 | 9011286 | 27437 | 357419 | 10336315 | 26731 |
| 70-PK | 1464000 | 11572309 | 18833 | 799583 | 13423884 | 18108 | 168300 | 19952623 | 11366 |
| 70-RT | 1464000 | 9833925 | 25143 | 1353083 | 10025688 | 24799 | 181791 | 19952623 | 10391 |
| $60.1 p$ | 1464000 | 11864229 | 19590 | 1353083 | 12072143 | 19621 | 3162278 | 9292454 | 18837 |

NOTE: $\quad 1 \mathrm{psi}=6.89 \mathrm{kPa}$


## BACKCALCULATION SCHEMA

FIGURE 98. Backcalculated AC Modulus from Dynamic Analysis


BACKCALCULATION SCHEMA
FIGURE 99. Backcalculated PCC Modulus from Dynamic Analysis


FIGURE 100. Backcalculated Subgrade Modulus from Dynamic Analysis

Initially, only the first 5 backcaiculation schema listed in Table 12 were used. Backcalculation schema $6 D-T P$ was added to evaluate the effect of the load plate deflection on the backcalculated asphaltic concrete modulus. As shown in Figure 98 and Table 15 , the predicted modulus of the surface (asphaltic concrete) layer was typically underestimated. Mamlouk (1985) reported that the upper layer modulus tends to be underestimated with dynamic backcalculation and that there is a corresponding overestimation of the subgrade moduli. Figure 99 shows that the subgrade modulus is overestimated primarily when phase data is not considered. In backcalculation, the sensors near the load plate tend to define the upper pavement layers. All of the sensors are useful in predicting the subgrade modulus. Therefore, it was of interest to include
schema 6D-TP because the load plate sensor seemed high in comparison to the other sensor deflections. In this schema, the load plate deflection was omitted. In this case, the opposite phenomena was achieved, as the surface layer modulus was overestimated, compared with laboratory values. It is clear that not enough is known about the interaction of the load plate of the Road Rater 2008 and the pavement structure. Problems with load plate interaction could also affect testing with other nondestructive testing devices, including the Falling Weight Deflectometer. Future research in nondestructive evaluation of pavements should consider this phenomena.

## Interpretation of Dynamic Backcalculation Results

Figure 98 illustrates the backcalculated values of asphaltic concrete moduli for the available options. The dashed lines indicate the limits of the allowable data for the temperature option. For the fixed modulus of asphalt option, the laboratory values (corresponding to the upper limit of modulus, as previously described) were used. The data for this option were included for illustrative purposes only. When the temperature option was used, only for schema 7D-RT did the backcalculated value approach the laboratory value when the load plate sensor was considered. Schema 6D-TP, which did not consider the load plate deflection, also gave a high value of asphaltic concrete modulus. For the no temperature option, all of the backcalculated values of asphaltic concrete moduli were unrealistically low except for schema 60-TP. Of primary significance is that the values were improved when phase data were considered, both for the 4 sensor cases ( $40-3 P$ compared to 40-0P) and for the 7 sensor cases (7D-RT compared to 7D-PK). The most
unrealistic value came from the standard (STD) schema using the standard Road Rater 2008 deflection sensor locations and multilayer linear elastic theory. As previously described, schema 6D-1P gave higher than expected values for the asphaltic concrete.

Figure 99 illustrates the values of backcalculated Portland cement concrete moduli for the various schema. Schema STD proved inadequate in this case, backcalculating a very low modulus value. As previously shown for the asphaltic concrete moduli, the backcalculated values of Portland cement concrete moduli were improved compared to laboratory values when phase data were included.

Figure 100 illustrates the values of backcalculated subgrade moduli. Schema STD was inadequate in this case, predicting an unrealistically high value of subgrade modulus. Although an exact value of subgrade modulus at the time of the test was not known, discussions with engineers at the Waterways Experiment Station who had built the test section at the Waterways Experiment Station Bomb Damage Repair Site indicated that the expected subgrade modulus should be about 15 to 25 ksi , based on the season. If it is assumed that the subgrade should fall in this range, then all of the dynamic solutions except schema 4DOP gave reasonable values for subgrade moduli. This points out that 4 peak deflections, even with a dynamic solution, may not give good results for backcalculated subgrade unless more data are included either in the form of phase data or more deflection data.

## CHAPTER VII

CLOSURE

## SUMMARY

A literature review of the state-of-the-art of nondestructive testing of pavements was presented. Four general types of nondestructive testing devices were discussed, including quasistatic deflection devices, vibratory deflection devices, impulsive testing devices, and wave propagation devices. Five general types of structural response models were discussed, including equivalent thickness models, plate bending models, multilayer linear elastic models, time-independent continuum models, and time-dependent models. Three existing methods of backcalculation were discuss:d, including simplified methods, gradient relaxation methods, and direct interpolation methods.

A program called COMDEF was developed for the specific application of backcalculation of layer moduli of composite pavements. Improved speed and accuracy were achieved by making the program application specific and using a matrix of precalculated solutions to speed the structural response predictions for forward modeling. The implementation version of COMDEF uses the Falling Weight Deflectometer as the nondestructive loading device. The structural response model chosen for the precalculated solutions for the standard version was a multilayer linear elastic model. The method used to predict surface deflections based on multilayer linear elastic theory was presented. Backcalculation was discussed and the importance of execution time as a controlling factor in backcalculation analyses was emphasized. The evolution of the

COMDEF method was discussed, including early work with BISDEF, development of a program called SEED, and the eventual development of the complete COMDEF methodology. The COMDEF method is described and particular emphasis is given to a new method called the method of stepwise direct optimization. The relative performance of the specific application program COMDEF compared to the general application program BISDEF is discussed. It is emphasized that superior performance of COMDEF for the specific application of composite pavement backcalculation is not intended to detract from the capabilities and track record of BISDEF. The relationship of backcalculated Portland cement concrete moduli values to the degree of cracking and the treatment of high values of Portland cement concrete modulus was discussed.

Sensitivity studies using theoretical deflection basins were performed using BISDEF and COMDEF to determine the sensitivity of the backcalculated moduli to variations in input parameters. The BISDEF sensitivity study indicated clearly that backcalculated moduli for composite pavements were sensitive to the choice of trial values. BISDEF provided reasonable solutions $83 \%$ of the time, but very poor solutions $17 \%$ of the time. Solutions were improved when better choices of trial values were made. The COMDEF sensitivity study indicated that COMDEF will always predict "near-perfect" solutions when theoretical deflection basins are used as input. The numerically approximate COMDEF deflections predicted by the DELTA subroutine were compared to BISAR predicted deflections. Differences between the COMDEF approximate deflections and the theoretical BISAR deflections were relatively small. A cursory
study of the effects of random variations in the deflection data seemed to indicate that COMDEF will perform better than BISDEF when these random variations occur in composite pavements.

A number of verification data were presented. All available data were presented as site verification for the three test sites, but some of the data did not provide useful correlations with backcalculated moduli values. An approximate relationship between dynamic stiffness modulus measured by the WES 16 -kip vibrator and the impulse stiffness modulus measured by the Falling Weight Deflectometer was developed, but the degree of scatter in the data makes the relationship questionable. Pavement condition index surveys could not be correlated with backcalculated moduli values. Analysis of the Falling Weight Deflectometer load pulse indicated that the frequencies generated were too low to be useful in a methodology similar to the Spectral Analysis of Surface Waves (SASW) method, although the Falling Weight Deflectometer might be useful for generating high-energy low-frequency pulses to determine subgrade properties at depth. Cores were taken for thickness verification and small aperture in-place California bearing ratio (CBR) tests were performed. The relationship of in-place California bearing ratio values with backcalculated subgrade moduli appeared to agree with published data, but scatter in the data made such a correlation questionable. Laboratory resilient modulus tests were performed on four soil specimens. Agreement between backcalculated subgrade modulus and laboratory soil resilient modulus values was reasonable. Laboratory asphaltic concrete modulus tests were performed by two methods. A new apparatus, as well as machine control and data acquisition software were developed during this project for the dynamic testing of asphaltic
concrete for laboratory modulus values. Agreement between the laboratory asphaltic concrete modulus values and backcalculated asphaltic concrete moduli values was reasonable. Laboratory tests on a core from a test section with a relatively uncracked Portland cement concrete layer indicated that the backcalculated Portland cement concrete modulus agreed reasonably with dynamic modulus tests for Portland cement concrete modulus. Other results from sections where the Portland cement concrete layer was cracked or shattered indicated that the effective modulus of Portland cement concrete predicted by COMDEF is a good indicator of the degree of cracking in the Portland cement concrete layer. A test location where the Portland cement concrete had been removed was identified based on the backcalculated Portland cement concrete moduli and verified by coring and personal contact with representatives of the Base Civil Engineer. The high variability in the Portland cement concrete layer was presented as a reason for not using the representative basin approach for these pavement types.

The empirical nature of quasistatic modeling of nondestructive testing data was discussed. A dynamic structural response model was implemented in a microcomputer program called GREEN-MA and a dynamic approach was presented for the prediction of Falling Weight Deflectometer structural responses. A demonstration case was presented, and for the single case studied, the moduli values from the implementation version of COMDEF gave good results when put into the dynamic structural response model. Dynamic modeling is presented as a way of handing anomalous conditions, such as shallow bedrock. Direct arrival times are presented as a way to obtain additional data from Falling Weight

Deflectometer pulses. It is likely that other classical methods of seismic exploration, such as reflection and refraction surveys, could be applied.

A dynamic version of COMDEF for backcalculation of moduli using the Road Rater 2008 was presented. GREEN-MA was used to calculate dynamic solutions for various backcalculation schema. Verification data were collected and analyzed with the backcalculation schema. The dynamic backcalculations strongly indicated that phase shift data should be included in the backcalculation process for vibratory testing devices.

## CONCLUSIONS

The standard COMDEF system represents a significant improvement in the state-of-the-art of backcalculation of layer moduli values for composite pavements. The speed and accuracy of the program is unmatched at the present time for these pavement types. The linear elastic moduli values predicted by the COMDEF system are consistent with the linear elastic evaluation and overlay procedures currently in use. The COMDEF system is currently being used by both the Army and the Air Force in evaluations of these pavement types.

The COMDEF system can be used in all routine evaluations of composite pavements which meet the basic assumptions of the COMDEF system. The system was designed so that it could be run on any IBM-compatible computer equipped a 360K floppy disk. However, the COMDEF system should be installed on a hard-disk system for routine use.

The approximate deflections predicted by the DELTA subroutine of COMDEF closely approximate theoretical deflections. Similarly, backcalculated moduli using theoretical deflection data match the known moduli values almost perfectly. Backcalculated moduli from field data agree reasonably with laboratory and field validation data from a limited number of verification sites.

The method of stepwise direct optimization is a good alternative to traditional iteration techniques. When combined with traditional iteration methods, as in the COMDEF system, the occurrence of nonunique or unrealistic solutions is minimized.

The effective modulus of Portland cement concrete is a good indicator of the degree of cracking in that layer. Variability in the degree of cracking of the Portland cement concrete layer is a good reason not to use the representative basin approach for these pavement types.

The backcalculated moduli predicted by the standard COMDEF system using the Falling Weight Deflectometer and multilayer linear elastic theory appear to be more consistent and reasonable than backcalculated moduli predicted using dynamic analysis of vibratory test data. It is concluded, based on limited verification data, that if a vibratory testing device is used for nondestructive testing then phase shift data must be collected and used in the backcalculation schema to predict reasonable results for these pavement types.

A demonstration of dynamic structural response predictions for the Falling Weight Deflectometer gave reasonable responses. It is concluded that dynamic modeling is superior to quasistatic models because it does not require empirical correlation with structural responses. Arrival time data is a good possibility for additional data to determine anomalous conditions.

## FUTURE RESEARCH

The framework for a good backcalculation methodology has been developed. However, improvements in the COMDEF method are needed in the future. Examples of such improvements include options for varying the depth to rigid boundary, options for varying the load plate diameter, options for variation in the number and location of sensors, and addition of a fourth layer so that a granular base material can be considered underneath the Portland cement concrete layer. Additionally, it is anticipated that future research will lead to the routine use of more realistic dynamic structural response models both in the analysis and design of pavement structures.

A thorough study of the interaction of the load plate of vibratory pavement testing devices with the pavement structure is needed. The role of damping in the backcalculation process needs additional research, as well. More research is needed to develop the technology to capture phase data routinely so that it can be included in routine evaluation schema.

COMDEF provides a method whereby moduli values can be backcalculated for each test point. Future research in stochastic modeling is needed so that the additional moduli data can be used in a reliability based evaluation method.

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## VITA

The author, Mark Anderson, was born in Birmingham, Alabama on January 14, 1957. He attended Salyersville High School in Salyersville, Kentucky, where his role as student-athlete included selection as class salutatorian, co-captain of the basketball team, and captain of the baseball team. He also worked part-time for Mortimer Engineering.

He attended the University of Kentucky and was awarded a Bachelor of Science degree in Civil Engineering with High Distinction in May, 1980. Among his undergraduate achievements were being initiated into Chi Epsilon and Tau Beta Pi honor societies, earning Dean's List honors for six semesters, and being listed in the National Dean's List in 1980. He won the American Society for Testing and Materials Student Award for Excellence in Engineering Materials Research in 1979 for the paper "Plastic Necking in Beams of Differing Geometry." As an undergraduate, he worked part-time as a construction engineer for Carey \& Adams Construction Company and as a geotechnical engineer for Stokely \& Cheeks Engineering.

He was awarded a University of Kentucky Research Foundation fellowship for the 1980-81 academic year and a summer fellowship from the Department of Civil Engineering for the Summer of 1981 to attend graduate school at the University of Kentucky. He was awarded the degree of Master of Science in Civil Engineering in August, 1981.

He was awarded a College of Engineering fellowship for the 1981-82 academic year to pursue post-graduate study at the University of Kentucky. In the Fall of 1982, he was selected to teach a graduate level soil mechanics class, filling a temporary faculty vacancy.

In January, 1983, he joined the University of Kentucky Transportation Research Program, and in July, 1983, he was promoted to Senior Engineer. He became a Professional Engineer in July, 1985. In January, 1986, he began his doctoral research at the US Army Engineer Waterways Experiment Station in Vicksburg, Mississippi. In January, 1988, he accepted a position as a Senior Engineer in the newly formed Gulf Coast Division of Applied Research Associates, Inc., providing scientific and engineering technical assistance to the Air Force Engineering and Services Center at Tyndall Air Force Base, Florida. He currently resides in Panama City, Florida.

He has published numerous reports and papers on a wide range of civil engineering topics, including mine reclamation, slope stability, exotic paving materials, in-place recycling of pavements, composite pavement behavior, and nondestructive testing of pavements. He is active in professional societies, including the National Society of Professional Engineers and the Society of American Military Engineers, and is a committee member for the Transportation Research Board.

## APPENDIX A

COMDEF USER'S GUIDE

## INTRODUCTION

COMDEF, which stands for Composite (pavement) Optimized Moduli from Deflection Evaluation with the Ealling Weight Deflectometer, is an interactive, user friendly program which backcalculates layer moduli for composite pavements based on deflections measured by a Falling Weight Deflectometer (FWD). A compiled FORTRAN executable file is supplied with the standard COMDEF system which is suitable for use with an MS-DOS compatible microcomputer.

COMDEF uses two techniques to solve for layer moduli based on Falling Weight Deflectometer deflections. One technique was developed specifically for the COMDEF program and has been named the method of stepwise direct optimization. The other technique is an iteration technique similar to that used in the BISDEF backcalculation program developed by Waterways Experiment Station researchers. This method is an iterative relaxation technique using gradient matrices to quickly converge to an optimum solution. This technique will be referred herein as iteration.

COMDEF uses database files to numerically calculate the deflection basins which are compared to Falling Weight Deflectometer deflections. The deflections in the database files were calculated by multilayer linear elastic theory. Assumptions which apply to multilayer linear elastic theory therefore apply to COMDEF, i.e., layers are horizontal, homogeneous, and isotropic and the loading plate applies a uniform
pressure distribution to the pavement surface. It is assumed that the peak load and peak deflections measured by the Falling Weight Deflectometer can be compared to an equivalent elastic system under a quasistatic load. Agreement between the peak deflections and the idealized solution is improved by assuming that a rigid layer exists at a depth of 20 feet. It is further assumed that that the composite pavement system can be modeled by a three layer system of asphaltic concrete surface, Portland cement concrete base, and subgrade. This assumption is reasonably valid for most airfield composite pavements. Many composite airfield pavements were originally full-depth Portland cement concrete pavements, and many others had a relatively low quality base material under the Portland cement concrete which served primarily as a leveling course. Assumption of a uniform subgrade modulus gives reasonable results for pavement layer moduli except when there are significant changes in subgrade stiffness near the surface. COMDEF can be used successfully for most composite airfield pavements with speed and accuracy.

The information provided in this Appendix includes the following:
a. A description of the files contained in the COMDEF analysis system.
b. Input guide.
c. Example input data file.
d. Example of interactive screen display as onscreen during execution.
e. Example output data file.
f. Flowcharts and descriptions of the program and major subroutines.

## FILE DESCRIPTIONS

The complete COMDEF analysis system is contained on 2 high density 5-1/4-inch floppy disks (1.2-Mb disks). The complete analysis system includes the COMDEF executable driver program (COMDEF.EXE), 33 standard asphalt database files (*.ADB), an example data file (EXAMPLE.DTA), an example output file (EXAMPLE.OUT), and a dummy data file (DUMMY.DTA) which may be used to generate constants files (described below). The COMDEF program is described in following sections. The asphalt database files are so named because each file contains deflections for a fixed thickness of asphalt. The asphalt database files contain deflections for a matrix of Portland cement concrete thicknesses and layer moduli. The combinations are summarized in Table Al. Analysis of a data file requires that a "constants" file be calculated for the particular thicknesses of asphaltic concrete and Portland cement concrete. The constants file is actually a smaller database file which is interpolated from the solutions contained in the corresponding asphalt database file. The difference in names serves two purposes. The first purpose is to minimize general confusion between the two file types. The other purpose is to distinguish between the standard asphalt database files which contain theoretical deflections calculated by multilayer linear elastic theory and the constants files which contain approximate deflections calculated numerically from the asphalt database file. Custom database files are asphalt database files for nonstandard thicknesses of asphaltic concrete and also contain approximate deflections. The custom
database files are interpolated from the 6 standard asphalt database files which are closest in asphaltic concrete thickness to the chosen value of asphaltic concrete thickness.

TABLE Al. Variable Matrix ${ }^{\text {d }}$ for Asphalt Database Files ${ }^{\text {b }}$

| Variable | Units | Values |
| :--- | :--- | :--- |
| AC thickness | in. | Fixed $^{\text {c }}$ |
| PCC thickness | in. | $4,6,9,14,20,30$ |
| AC modulus | ksi | $33,82,205,512,1280,3200$ |
| PCC modulus | ksi | $82,205,512,1280,3200,8000,20000$ |
| Subgrade modulus | ksi | $2,6,18,54,162$ |

${ }^{a}$ Deflections are calculated for each combination of the variable matrix at seven sensor locations located on 1 foot centers along a line measured from the center of the FWD load Plate (centers at $0,12,24,36,48,60$, and 72 inches).

- Total number of deflections in each of 33 asphalt database files is equal to the product of 7 sensor locations times 6 thicknesses of PCC times 6 moduli of AC times 7 moduli of PCC times 5 moduli of subgrade for a total of 8820 deflections per file.
c Each standard database file contains deflections for a fixed thickness of AC. There are 33 standard database files in 0.5 in . increments of $A C$ thickness from 0 in. to 16 in. of $A C$.

NOTE: $\quad 1 \mathrm{in} .=25.4 \mathrm{~mm} \quad 1 \mathrm{ksi}=6.89 \mathrm{MPa} \quad 1 \mathrm{ft}=0.305 \mathrm{~m}$

Analysis of a feature with COMDEF is easiest when an MS-DOS compatible microcomputer with a hard disk drive is used. All the asphalt database files can then be put in a single directory, using a little more than $2-\mathrm{Mb}$ of hard disk storage. However, the program can be run with a single $360-\mathrm{Kb}$ floppy disk. The COMDEF program requires about

260-Kb of disk space, and each constants file requires about 11 -Kb of space. If a $360-\mathrm{Kb}$ floppy disk will be used in a field situation, as on a portable computer, it will probably be best to create the constants files in advance. Use of the dummy data file DUMMY.DTA, shown in Figure Al, will allow quick creation of constants files, because the COMDEF program will stop execution when it encounters a negative value of deflection.

DUMMY DATA FILE FOR GENERATING CONSTANTS FILES 1 DROP TESTED

1111111111111

FIGURE A1. Data File "DUMMY.DTA" Used to Generate Constants Files

Asphalt database files are named by the fixed thickness of asphalt the file represents and are given the extension ADB. Thicknesses are given to the 0.01 inches ( $254 \mu \mathrm{~m}$ ) and a dash is used to separate the fractional portion. For example, the standard asphalt database file for 11.50 inches ( 293 mm ) of asphaltic concrete is named 11-50.ADB. The same convention is used for custom asphalt database files. For example a custom file for 2.75 inches ( 70 mm ) of asphaltic concrete would be given the name 02-75.ADB. Files for asphalt thicknesses less than 10 inches ( 254 mm ) begin with a zero, so that the filename always has the generai form $X X-X X . A D B$, where $X X-X X$ is the asphalt thickness as previously described.

A naming convention similar to that of the asphalt database files is used to name constants files. Constants file names have the general form $X X-X X-Y Y . Y Y-$, where $X X-X X$ is the asphalt thickness as described above, and YY. YY is the thickness of Portland cement concrete. For example, a constants file for 9.5 inches ( 242 mm ) of asphaltic concrete and for 5.75 inches ( 146 mm ) of Portland cement concrete would be named 09-50-05.75-. The dashes following the thicknesses have no special significance, but must be included as placeholders. If constants files are created in advance, they will be named with the naming convention described. The user should not change these names. COMDEF automatically assembles the constants filename needed from the user input values of asphaltic concrete and Portland cement concrete thickness. If the constants filenames are changed, COMDEF will not recognize them and will recalculate the constants file.

## INPUT GUIDE

A typical data file, EXAMPLE.DTA, is shown in figure A2. This file is included in the COMDEF analysis system. A data file may be created with a word processor from hard copies of Falling Weight Deflectometer data, or may be created by the program CONVERT, which was developed by Waterways Experiment Station researchers to interpret Falling Weight Deflectometer data stored on a data tape by the Hewlett-Packard HP-85 computer used in the Falling Weight Deflectometer system.

```
PAVEMENT FACILITY OR FEATURE ID: EXAMPLE DATA
    I NDT LOADINGS PER TEST LOCATION
STATION TRACK DATE TEMP LOAD D1 D2 D3 D4 D5 D5 D6 D7
```



```
    1.0 1 870421 70.0 25025 20.9 18.1 16.2 14.1 12.0 10.0 8.2
```

FIGURE A2. Typical Data File "EXAMPLE.DTA" for COMDEF

The data illustrated in Figure $A 2$ is of the following form :
(1) LINE 1 contains title information up to 80 characters.
(2) LINE 2 contains the number of drops per location. The input is free format, but the first character(s) must be the number of drops in integer format. If analysis of all drops is desired, the number of drops must be set equal to 1.
(3) LINES 3 and 4 are dummy lines which are not used, but must be included. If the input data file is created by the program CONVERT, these lines contain column headings. If the input data file is created with a word processor, a character should be put on these lines as a placeholder, as shown in Figure Al.
(4) DATA LINES continue after the first four lines. The total number of data lines must be an even multiple of the number of drops given in LINE 2. Each line contains the test information for one drop. All places must be filled, but some may be filled with placeholders, as in Figure Al. The following information is contained on each line, with FORTRAN variable type indicated:
(a) Station (real)
(b) Track number (integer)
(c) Date (integer)
(d) Pavement surface temperature (real)
(e) Falling Weight Deflectometer peak load (real)
(f) Seven deflections in mils ( $1 \mathrm{mil}=25.4 \mu \mathrm{~m}$ ) on 1 foot ( 0.3 m ) centers along a line from the center of the Falling Weight Deflectometer load plate (real)

Mixed mode input is supported, so that real number fields may be filled with integers in the input file.

An additional option which is not documented on screen is that the user may type USER INPUT as the data file name, and the program will allow interactive input of Falling Weight Deflectometer data. This option is very useful for field use with a portable computer.

## EXAMPLE OF COMDEF EXECUTION AND OUTPUT FILE

Figure A3 illustrates a COMDEF execution run by listing the interactive screen display during execution. Examination of this figure will allow the user to quickly become familiar with the interactive input needed by the program. In the example given, data from the file EXAMPLE.DTA shown in Figure A2 is averaged with data input interactively using the "USER INPUT" option previously described. Keyboard input from the program user is emphasized to assist the reader. The output file EXAMPLE.OUT created by this example run is shown in Figure A4.

## c.COMDEF

ENTER INPUT DATA FILENAME

## EXAMPLE.DTA

READING INPUT FILE ...

THERE ARE 4 DROPS PER LOCATION. WHICH DROP DO YOU WISH TO ANALYSE ? ... 4

INPUT FILE example.dta SUCESSFULLY READ ...

INPUT FILE IS example.dta
WHAT IS OUTPUT FILENAME ?
EXAMPLE.OUT

OUTPUT FILE example.out SUCESSFULLY OPENED ..

FIGURE A3. Interactive Screen Display During CONDEF Execution (Sheet 1 of 11)

INPUT FOR TEMPERATURE OPTIONS FOR DATA FILE example.dta

COMDEF HAS 2 OPTIONS, TEMPERATURE AND NO TEMPERATURE.

If THE TEMPERATURE OPTION IS CHOSEN, A RANGE OF POSSIBLE ASPHALT MODULI WILL BE ASSIGNED.
*


*     * TEMPERATURE OPTION IS NORMALLY RECOMMENDED IF * *
*     * RELIABLE TEMPERATURE INFORMATION IS AVAILABLE. * *



DATA REQUIRED FOR THE TEMPERATURE OPTION IS :
(1) THE PAVEMENT SURFACE TEMPERATURE (SHOULD BE IN DATA FILE)
(2) THE 5-DAY MEAN AIR TEMPERATURE

DO YOU WANT THE TEMPERATURE OPTION (Y/N) ?

YOU HAVE CHOSEN THE TEMPERATURE OPTION !!

PAVEMENT SURFACE TEMPERATURE IS REQUIRED ...
IF DATA FILE TEMPERATURES ARE CORRECT, NO INPUT IS REQUIRED !!

WHAT WAS THE PAVEMENT SURFACE TEMPERATURE ?
NOTE : DEFAULT WILL ACCEPT DATA FILE TEMPERATURE VALUES

WHAT WAS THE 5-DAY MEAN AIR TEMPERATURE ?

$$
80 .
$$

TEMPERATURE DATA COMPLETE
MINIMUM AND MAXImUM VALUES OF AC MODULUS WILL BE SET

FIXED AC MOOULUS OPTION IS AVAILABLE.
IF YOU USE THIS OPTION, AC MODULUS MAY BE FIXED BASED ON 5-DAY MEAN AIR TEMPERATUTE AND MEASURED
SURFACE TEMPERATURE. MODULUS-TEMPERATURE RELATIONSHIP
IS BASED ON CORRELATION BY THE ASPHALT INSTITUTE (TAI).
IF YOU PREFER, YOU MAY INPUT A FIXED VALUE OF ASPHALT MODULUS.

DO YOU WANT TO USE EITHER OPTION TO FIX AC MODULUS (Y/N) ?


FIGURE A3. (Continued)
(Sheet 3 of 11)

INPUT PAVEMENT THICKNESSES FOR DATA FILE example.dta !!!

WHAT IS THE ASPHALT THICKNESS IN INCHES ?
6.

WHAT IS THE PCC THICKNESS IN INCHES ?
7.

WHAT DRIVE IS CONSTANTS FILE IN ? (DEFAULT TO CREATE NEW FILE)
A
the constants file for the selected thicknesses is not on the SELECTED DRIVE. FILE MAY NOT EXIST, OR IT MAY BE ON ANOTHER DRIVE. YOU MAY REENTER THE PAVEMENT THICKNESSES AND DRIVE SPECIFICATION, OR YOU MAY CREATE A NEW CONSTANTS FILE (WILL REQUIRE ADB FILE) !!

DRIVE. DO YOU WISH TO CREATE A NEW CONSTANTS FILE (Y/N) ? NOTE : DEFAULT ALLOWS YOU TO REENTER PARAMETERS

```
THIS IS SUBROUTINE CREATE ...
    A CONSTANTS FILE WILL BE CREATED BASED ON YOUR
    SELECTIONS OF ASPHALT AND PCC THICKNESSES.
A DATABASE FILE FOR AC THICKNESS OF 6.00 INCHES
    ASPHALT WILL BE REQUIRED. STANDARD FILES ARE BASED
    ON NOMINAL ASPHALT THICKNESS TO NEAREST 1/2 INCH.
DATABASE FILE FOUND ON DEFAULT DRIVE ...
ASPHALT DATABASE FILE SUCESSFULLY OPENED !!
CONSTANTS FILE REQUIRES 11 K-BYTES OF DRIVE SPACE !!
WHAT DRIVE TO SEND CONSTANTS FILE TO ? (MAY BE DEFAULTED)
    C
CONSTANTS FILE OPENED...
CALCULATIONS STARTING ...
CALCULATIONS WILL REQUIRE 2 TO 5 MINUTES (DEPENDS ON DRIVE SPEED)
READING CONSTANTS FILE, PLEASE WAIT ABOUT 1 MIN ...
CONSTANTS FILE SUCESSFULLY READ ...
```

FIGURE A3. (Continued)
(Sheet 5 of 11)

```
** COMDEF (MA-4.87.21) EXECUTING PROBLEM 1 OF 1 FOR example.out **
SOLUTION FOR PROBLEM 1 OF 1 FOR FILE example.out :
STATION NUMBER = 1.00
TRACK NUMBER =1
DATE OF TEST = 870421
SURFACE TEMPERATURE = 70.0 DEGREES F
MEAN 5-DAY AIR TEMPERATURE = 80.00 DEGREES F
MEAN PAVEMENT TEMPERATURE = 71.96 DEGREES F
THICKNESS OF AC = 6.00 INCHES
THICKNESS OF PCC = 7.00 INCHES
DYNAMIC LOAD = 25025. POUNDS
MODULUS OF AC = 546948. PSI
STANOARDIZED AC MODULUS (T=70 F, f=1 H)= 387365. PSI
MODULUS OF PCC = 4706035. PSI
MODULUS OF SUBGRADE = 8059. PSI
SUM OF ABSOLUTE VALUE OF ERRORS IN DEFLECTION = 1.03 MILS TOTAL PERCENTAGE ERROR IN DEFLECTION BASIN = \(5.91 \%\)
WOULD YOU LIKE TO AVERAGE MORE DATA INTO THIS FEATURE (Y/N) ? Y
enter input data filename
USER INPUT
```

FIGURE A3. (Continued)
(Sheet 6 of 11)

USER INPUT OPTION HAS BEEN CHOSEN INPUT TEST DATA NOW

ALL DATA IS FREE FORMAT
PROMPTS IDENTIFY REAL AND INTEGER INPUT

USER INPUT : STATION NUMBER (REAL) $=2$ 。
USER INPUT : TRACK NUMBER (INTEGER) $=2$
USER INPUT : DATE (INTEGER) $=870421$
USER INPUT : SURFACE TEMPERATURE (REAL) $=70$. USER INPUT : FWD FORCE READING (REAL) $=\mathbf{2 4 8 0 0}$. USER INPUT : 7 DEFLECTION READINGS (REAL) $=20.78$
17.97
16.1014 .0011 .889 .888 .12

OUTPUT FILE example.out SUCESSFULLY OPENED ...

FIGURE A3. (Continued)
(Sheet 7 of 11)

INPUT FOR TEMPERATURE OPTIONS FOR DATA FILE example.dta

COMDEF HAS 2 OPTIONS, TEMPERATURE AND NO TEMPERATURE.

IF THE TEMPERATURE OPTION IS CHOSEN, A RANGE OF POSSIBLE ASPHALT MODULI WILL BE ASSIGNED.


DATA REQUIRED FOR THE TEMPERATURE OPTION IS :
(1) THE PAVEMENT SURFACE TEMPERATURE (SHOULD BE IN DATA FILE)
(2) THE 5-DAY MEAN AIR TEMPERATURE

DO YOU WANT THE TEMPERATURE OPTION (Y/N) ?
Y

FIGURE A3. (Continued)
(Sheet 8 of 11)

YOU HAVE CHOSEN THE TEMPERATURE OPTION !!

PAVEMENT SURFACE TEMPERATURE IS REQUIRED ...
IF DATA FILE TEMPERATURES ARE CORRECT, NO INPUT IS REQUIRED !!

WHAT WAS THE PAVEMENT SURFACE TEMPERATURE ?
NOTE : DEFAULT WILL ACCEPT DATA FILE TEMPERATURE VALUES

WHAT WAS THE 5-DAY MEAN AIR TEMPERATURE ?

## 80.

TEMPERATURE DATA COMPLETE
MINIMUM AND MAXIMUM VALUES OF AC MODULUS WILL BE SET

FIXED AC MODULUS OPTION IS AVAILABLE.

IF YOU USE THIS OPTION, AC MODULUS MAY BE FIXED BASED ON 5-DAY MEAN AIR TEMPERATUTE AND MEASURED SURFACE TEMPERATURE. MODULUS-TEMPERATURE RELATIONSHIP IS BASED ON CORRELATION BY THE ASPHALT INSTITUTE (TAI).

IF YOU PREFER, YOU MAY INPUT A FIXED VALUE OF ASPHALT MODULUS.

DO YOU WANT TO USE EITHER OPTION TO FIX AC MODULUS (Y/N) ?
N

FIGURE A3. (Continued)
(Sheet 9 of 11)

INPUT PAVEMENT THICKNESSES FOR DATA FILE example.dta !!!

WHAT IS THE ASPHALT THICKNESS IN INCHES ?

$$
6 .
$$

WHAT IS THE PCC THICKNESS IN INCHES ?

$$
7 .
$$

READING CONSTANTS FILE, PLEASE WAIT ABOUT 1 MIN ...

CONSTANTS FILE SUCESSFULLY READ ...

FIGURE A3. (Continued)
(Sheet 10 of 11)

```
** COMDEF (MA-4.87.21) EXECUTING PROBLEM 1 OF l FOR example.out **
    SOLUTION FOR PROBLEM 1 OF 1 FOR FILE example.out :
    STATION NUMBER = 2.00
    TRACK NUMBER = 2
    DATE OF TEST = 870421
    SURFACE TEMPERATURE = 70.0 DEGREES F
    MEAN 5-DAY AIR TEMPERATURE = 80.00 DEGREES F
    MEAN PAVEMENT TEMPERATURE = 77.96 DEGREES F
    THICKNESS OF AC = 6.00 INCHES
    THICKNESS OF PCC = 7.00 INCHES
    DYNAMIC LOAD = 24800. POUNDS
    MODULUS OF AC = 546948. PSI
    STANDARDIZED AC MODULUS (T=70 F, f=1 H) = 387365. PSI
    MODULUS OF PCC = 4591860. PSI
    MODULUS OF SUBGRADE = 8087. PSI
    SUM OF ABSOLUTE VALUE OF ERRORS IN DEFLECTION = 1.02 MILS
TOTAL PERCENTAGE ERROR IN DEFLECTION BASIN = 5.77%
    WOULD YOU LIKE TO AVERAGE MORE DATA INTO THIS FEATURE (Y/N) ?
        N
    NOTE : MAXIMUM PCC MOOULUS OF 7 MILLION PSI IS ENFORCED
        AT EACH TEST LOCATION PRIOR TO AVERAGING.
    AVERAGE MODULI VALUES FOR FEATURE IN FILE example.out :
    AVERAGE AC MODULUS = 387365.
    AVERAGE SUBGRADE MODULUS = 8073.
    OVERALL AVERAGE PCC MODULUS FOR 2 POINTS = 4648947.
    AVERAGE PCC MODULUS FOR 2 RIGID TEST POINTS = 4648947.
    AVERAGE PCC MODULUS FOR 0 FLEXIBLE TEST POINTS = 0.
    Execution terminated : 0
    C>
```

```
PAVEMENT FACILITY OR FEATURE ID: EXAMPLE DATA
SOLUTION FOR PROBLEM 1 OF 1 FOR FILE example.out :
STATION NUMBER = 1.00
TRACK NUMBER = 1
DATE OF TEST = 870421
SURFACE TEMPERATURE = 70.0 DEGREES F
MEAN 5-DAY AIR TEMPERATURE = 80.00 DEGREES F
MEAN PAVEMENT TEMPERATURE = 77.96 DEGREES F
THICKNESS OF AC =6.00 INCHES
THICKNESS OF PCC = 7.00 INCHES
DYNAMIC LOAD = 25025. POUNDS
MODULUS OF AC = 546948. PSI
STANDARDIZED AC MODULUS (T=70 F, f=1 Hz) = 387365. PSI
MODULUS OF PCC = 4706035. PSI
MODULUS OF SUBGRADE = 8059. PSI
\begin{tabular}{cccc} 
& DISTANCE & ACTUAL & PREDICTED \\
SENSOR & FROM LOAD & DEFLECTION & DEFLECTION \\
NUMBER & (INCHES) & (MILS) & (MILS) \\
1 & 0. & 20.90 & 20.41 \\
2 & 12. & 18.10 & 18.32 \\
3 & 24. & 16.20 & 16.39 \\
4 & 36. & 14.10 & 14.18 \\
5 & 48. & 12.00 & 11.99 \\
6 & 60. & 10.00 & 9.97 \\
7 & 72. & 8.20 & 8.17
\end{tabular}
Sum OF ABSOLUTE value of errors in DEFLECTION = 1.03 MILS
total percentage error in deflection basin = 5.91%
```

FIGURE A4. Typical Output File "EXAMPLE.OUT"
(Sheet 1 of 3)

```
SOLUTION FOR PROBLEM l OF l FOR FILE example.out :
STATION NUMBER = 2.00
TRACK NUMBER =
    2
DATE OF TEST = 870421
SURFACE TEMPERATURE = 70.0 DEGREES F
MEAN 5-DAY AIR TEMPERATURE = 80.00 DEGREES F
MEAN PAVEMENT TEMPERATURE = 77.96 DEGREES F
THICKNESS OF AC = 6.00 INCHES
THICKNESS OF PCC = 7.00 INCHES
DYNAMIC LOAD =24800. POUNDS
MODULUS OF AC = 546948. PSI
STANDARDIZED AC MODULUS (T=70 F, f=1 Hz) = 387365. PSI
MODULUS OF PCC = 4591860. PSI
MODULUS OF SUBGRADE = 8087. PSI
    DISTANCE ACTUAL PREDICTED
SENSOR FROM LOAD DEFLECTION DEFLECTION
NUMBER (INCHES) (MILS) (MILS)
    1 0. 20.78 20.29
    2 12. 17.97 18.21
    3 24. 16.10 16.27
    4
    5
    6
    7
        36.
        48.
                            14.00
                            14.06
        60. 9.88 9.87
        8.88
        11.88
        72.
    8.12
    8.08
SUM OF ABSOLUTE VALUE OF ERRORS IN DEFLECTION = 1.02 MILS
TOTAL PERCENTAGE ERROR IN DEFLECTION BASIN = 5.77%
```

FIGURE A4. (Continued)
(Sheet 2 of 3)

NOTE : MAXIMUM PCC MODULUS OF 7 MILLION PSI IS ENFORCED at each test location prior to averaging.

AVERAGE MODULI VALUES FOR FEATURE IN FILE example.out :

AVERAGE AC MODULUS $=387365$.
AVERAGE SUBGRADE MODULUS $=8073$.
OVERALL AVERAGE PCC MODULUS FOR 2 POINTS $=4648947$.

AVERAGE PCC MODULUS FOR 2 RIGID TEST POINTS $=4648947$. AVERAGE PCC MODULUS FOR 0 FLEXIBLE TEST POINTS $=0$.

FIGURE A4. (Concluded) (Sheet 3 of 3)

Figure $A 5$ is a flow chart of the COMDEF analysis system. On execution, the program calls the subroutine INPUTF for interactive data input. After successful completion of data input, the subroutine E3EQUA is called. This subroutine calculates an approximate relationship between subgrade modulus and the predicted deflection at the outside sensor $\left(\Delta_{7}\right)$. This relationship is used to predict a trial value of subgrade modulus based on the measured value of the outside sensor deflection $\left(D_{7}\right)$ at each test location. The program then calls the subroutine EACHL to set asphaltic concrete modulus allowable ranges based on the temperature data option chosen (see INPUTF description for temperature data options). The program then begins a loop for all of the data lines (test locations) to be analyzed, with the following performed for each line of data:
(1) The program performs a "global" iteration, using the subroutine ITERAT. The global iteration is similar to the backcalculation method used in BISDEF, but sets a more rigorous tolerance on the basin fit to force a high number of iterations.
(2) The program stores the results from the global iteration and begins stepwise calculations. The term stepwise indicates that small increments are used for asphaltic concrete modulus. For each increment, a best solution is calculated by two methods, the method of stepwise direct optimization and stepwise iteration. The stepwise iteration is performed by calling ITERAT for each asphaltic concrete modulus increment. The method of stepwise direct optimization is begun by the subroutine RANGES and completed within the main COMDEF program.


FIGURE A5. The COMDEF Analysis System
(3) After each stepwise calculation, the total basin error and the total basin percentage error is compared to the previous best solution, where total basin error and total basin percentage error are defined as follows:

TOTAL BASIN ERROR $=\sum_{j=1}^{m}\left|D_{j}-\Delta_{j}\right|$
$\underset{\operatorname{TOTAL} \text { BASIN }}{\text { PERCENTAGE ERROR }}=\sum_{j=1}^{m} \frac{\left|D_{j}-\Delta_{j}\right|}{D_{j}} * 100 \%$
where
$D_{j}=$ measured deflection for sensor $i$
$\Delta_{j}=$ theoretical deflection for sensor $i$
$m$ * number of sensor locations ( $m=7$ for COMDEF)
The new incremental solution is accepted if both the total error and the total percentage error in the basin are less than the previous best solution.
(4) The stepwise calculations continue until a single best solution is found for the stepwise methods. This best solution is used as trial value input for a final optimizing iteration with the ITERAT subroutine. This final optimizing iteration allows more iterations and seeks a higher tolerance than the stepwise iterations, as well as giving a wider range of possible moduli values.
(5) The total error and total percentage error of the basin for the final optimizing iteration are compared to the values from the stepwise calculations. The better of these solutions is compared to the error and percentage error in the basin from the global iteration. The best of all these solutions has the lowest error and percentage error in the fitted basin and is accepted as the final solution. The solution is output to the screen and to the output file.

The analysis is repeated until the moduli for all the data lines (test locations) are computed. The average modulus for each of the three layers is computed. If the temperature data option was used, the average asphaltic concrete modulus is calculated from the corrected asphaltic concrete moduli values $\left(T=70^{\circ} \mathrm{F}, \mathrm{f}=1 \mathrm{~Hz}\right.$ ). If the temperature data option was not used, the average asphaltic concrete moduli is based on uncorrected asphaltic concrete moduli values. Three values of average Portland cement concrete modulus are calculated. An overall average Portland cement concrete modulus is calculated, as well as an average for the "rigid" test locations (backcalculated Portland cement concrete modulus of at least 1 million psi ( 7 GPa )) and an average for the flexible locations (backcalculated Portland cement concrete modulus less than 1 million psi ( 7 GPa$)$ ). For all test locations, a maximum value of Portland cement concrete modulus of 7 million psi ( 48 GPa ) is enforced to reduce bias in the average value.

## delta subroutine

All COMDEF deflection calculations are performed by the subroutine DELTA. A flow chart of subroutine DELTA is shown in Figure A6. DELTA calculates a deflection basin consisting of seven deflection locations at 1 foot ( 0.3 m ) intervals from a standard load. Layer moduli for the pavement system are supplied as subroutine arguments. The subroutine uses a common block which contains the data read from the particular constants file. The common block (constants file) contains data for a matrix of layer moduli, but for fixed values of thickness of asphaltic concrete and Portland cement concrete. Based on the moduli values supplied as subroutine arguments, DELTA performs a fifth order LaGrangian interpolation for asphaltic concrete modulus, a sixth order LaGrangian interpolation for Portland cement concrete modulus, and a fourth order Lagrangian interpolation for subgrade modulus. The result of these interpolations is a set of seven deflections which correspond to the given layer thicknesses and moduli. Deflections are calculated for a standard load of $25-k i p s$. Corrections are applied so that comparisons with measured deflections are made at equivalent load levels.


FIGURE A6. Flowchart of DELTA Subroutine

A flow chart of INPUTF is shown in Figure A7. When INPUTF is called, the user is asked for the name of the data file for input data. The input data file is described in the section labeled "Input Guide." If there was more than one drop per test location, the user chooses the drop number to analyze. The data file is read. After successfully reading the input data file, the user is asked for the output file name. After successfully opening the output file, the user chooses a temperature data option.

There are two temperature data options which determine how the asphaltic concrete modulus is calculated. The "no temperature data" option does not utilize pavement temperature measurements in the analysis. With this option, the solution is based only on the deflection basin. The user may input a fixed value of asphaltic concrete modulus with this option. The "no temperature data" option is not the recommended option. Use of the "temperature data" option forces a realistic solution for asphaltic concrete modulus which reduces scatter in the program output, and also greatly speeds up the program. Although the user may specify a fixed value of asphaltic concrete modulus with this option, there are two methods of interest which are unique to the "temperature data" option. One is to fix a possible range of asphaltic concrete moduli based on curves for minimum and maximum asphaltic concrete modulus, and the other is to fix a single value of asphaltic concrete modulus based on The Asphalt Institute (Kingham and Kallas 1972) surface mix curve.


FIGURE A7. Flowchart of the INPUTF Subroutine

The curves used in the temperature data option are shown in Figure A8. The curves used in the temperature data option relate asphaltic concrete modulus to mean pavement temperature at the predominant loading frequency ( 16 Hz ). The mean pavement temperature is calculated from an equation which interpolates the curves in Figure A9 (Southgate and Deen 1969). Data needed to calculate the mean pavement temperature is the sum of the pavement surface temperature and the 5 -day mean air temperature, as well as the thickness of asphaltic concrete. Pavement thicknesses and the 5 -day mean air temperature must be input by the user. If pavement surface temperatures were recorded properly in the data file they can be used as recorded. Values of surface temperature can be input interactively by the user during analysis.


FIGURE A8. Allowable AC Moduli Range in COMDEF Temperature Data Option


FIGURE A9. AC Pavement Temperature Distribution (Southgate \& Deen 1969)

Some options are available in the COMDEF analysis system which are not documented in the on-screen help. These options are intended for experienced users and should be used with caution:
(1) If the file combines data from different test periods, an easy way to include the proper temperature data is to calculate the sum of the pavement surface temperature and the 5-day mean air temperature at each test location. When prompted, a value of 0 may be input for the 5 -day mean air temperature, and then the sum of temperatures is input for each test location when prompted to input the pavement surface temperatures (the data file may be edited so that the sum of temperatures is automatically substituted for the surface temperature).
(2) If a value of -999 is given for the 5 -day mean air temperature, then the program assumes that the 5 -day mean air temperature is equal to the pavement surface temperature.
(3) If a value of -9999 is given for the 5 -day mean air temperature, then the mean pavement temperature is assumed to be equal to the pavement surface temperature. This option could be useful, for example, if temperature devices were embedded in the asphalt layer so that mean pavement temperatures were recorded instead of surface temperatures.

In all, there are 4 possible ways of calculating asphaltic concrete modulus by the program COMDEF. These are: (1) user input fixed asphaltic concrete modulus, (2) asphaltic concrete modulus fixed based on The Asphalt Institute curve, (3) asphaltic concrete modulus fixed within the range defined by the curves in Figure Al2, and (4) asphaltic concrete modulus calculated by the program based purely on the deflection basin.

After the asphaltic concrete modulus option is determined, the pavement layer thicknesses must be input. Based on the pavement layer thicknesses, a constants file is opened or created. The program first looks for the proper constants file on the default drive. If the proper constants file is not found on the default drive, the user is prompted to input the identifying letter of an alternative disk drive. If the constants file is found, it is read, and the subroutine returns to the main program. If the constants file is not found, the subroutine CREATE may be used to create a new constants file. This is accomplished fairly quickly if a standard option is used. Standard options include nominal thickness of asphaltic concrete to the nearest 0.5 inch ( 12 mm ) and less
than 16 inches ( 406 mm ). If the asphaltic concrete thickness does not meet this condition, the user is allowed to round the asphaltic concrete thickness to a standard value. If rounding is used, tne difference in asphaltic concrete thickness is corrected by adjusting the Portland cement concrete thickness, so that the total pavement thickness is constant. If a standard option is not used, the subroutine ASPADB is called to create a custom asphalt database file. This option takes much more time, and requires the presence of 6 standard asphalt database files (as previously discussed). After the custom asphalt database file is created, or when a standard database file is used, a constants file is calculated and the data is read into the proper common block.

## EACHL SUBROUTINE

A flow chart of suoroutine EACHL is shown in Figure AiO. The subroutine first determines if the temperature data option was chosen. If the no temperature data option was chosen, the subroutine sets dummy values for the upper and lower limits of asphaltic concrete modulus which will be ignored in the main program. The program then determines if there was a user input value of fixed asphaltic concrete modulus. If so, the ranges are collapsed to the user input value. If the temperature data option is chosen, the mean pavement temperature is calculated based on the asphaltic concrete thickness, the pavement surface temperature, and the 5-day mean air temperature. The upper and lower limits are calculated automatically for the curves shown in Figure A8, but the ranges are collapsed to a single value if the fixed asphaltic concrete modulus option was enabled. In this case, the subroutine determines if the fixed value was input by the user. If not, the fixed value is calculated from The Asphalt Institute curve (lower curve in Figure A8). The subroutine returns the asphaltic concrete moduli range to the program. The high and low values will either contain dummy values (no temperature data option) which will be ignored, a fixed value, or a range of values based on Figure A8.


FIGURE A10. Flowchart of the EACHL Subroutine

## ITERAT SUBROUTINE

A flow chart of the ITERAT subroutine is shown in Figure All. The ITERAT subroutine solves for the best solution of layer moduli using iterative relaxation. The solution is achieved by forming gradient matrices and solving for solutions which minimize the errors in the fitted basin. This method has been used in the programs CHEVDEF and BISDEF, developed by Waterways Experiment Station researchers, and has the advantage of rapid convergence. The specific method of solution is described below.

It is assumed that a relationship exists between deflection and layer moduli. The predicted deflection at a given sensor location $j$, $\Delta_{j}$, is assumed to be a function of the unknown layer moduli, that is:

$$
\begin{equation*}
\Delta_{j}=f\left(E_{1}, E_{2}, E_{3}\right) \tag{3}
\end{equation*}
$$

The deviation at sensor location $\mathbf{j}, \boldsymbol{\delta}_{\mathrm{j}}$, between the measured deflection, $D_{j}$, and the predicted deflection, $\Delta_{j}$, is given by:

$$
\begin{equation*}
\delta_{j}=D_{j}-\Delta_{j}=D_{j}-f\left(E_{1}, E_{2}, E_{3}\right) \tag{A4}
\end{equation*}
$$

The sum of the squares of the deviations for the seven sensor locations may be written as:

$$
\begin{equation*}
\sum_{j=1}^{7} \delta_{j}^{2}=\sum_{j=1}^{7}\left[D_{j}-f\left(E_{1}, E_{2}, E_{3}\right)\right]^{2} \tag{A5}
\end{equation*}
$$



FIGURE All. Flowchart of the ITERAT Subroutine

To minimize the error with respect to the unknown values of moduli, the fartial derivatives of the error function are taken with respect to the unknown moduli values. This gives a solution matrix of 3 equations involving the unknown moduli values. The solution is calculated numerically by forming gradient equations which approximate the partial derivative relationships. The gradient equations are formed by the following method. A initial set of modulus values, $E^{0}$, is assumed and corresponding deflections $\Delta^{0}$ are computed. A second set of moduli values, $E^{1}$, are assumed. A new set of deflections is calculated for each of the combinations of moduli variations. That is, combinations where all but one of the moduli have values as in $E^{0}$ and one of the moduli is varied to a new value $E^{1}$. The deflection at a given sensor location $j$ may then be given as a function of the gradient equation and the unknown modulus of layer $\mathfrak{i}$. The general equation is:

$$
\begin{equation*}
\Delta_{j}=A_{j i}+S_{j i} \log _{10}\left(E_{i}\right) \tag{A6}
\end{equation*}
$$

where
$\Delta_{j}=$ the predicted deflection at sensor location $j$
$E_{i}=$ the unknown layer moduli of layer $i$
$S_{j i}=\frac{\Delta_{i}^{0}-\Delta_{i}^{1}}{\log _{10} E_{i}^{0}-\log _{10} E_{i}^{1}}$
$A_{j i}=\Delta_{i}^{0}-S_{j i} E_{i}^{1}$
$E_{i}^{0}=$ first assumed value of modulus for layer $i$
$E_{i}^{1}=$ modulus for layer $i$ after the variation

$$
\begin{aligned}
& \Delta_{i}^{0}=\text { predicted deflection at sensor location } j \text { for } E_{i}^{0} \\
& \Delta_{i}^{1}=\text { predicted deflection at sensor location } j \text { for } E_{i}^{1}
\end{aligned}
$$

An expression can be written for the deflection at sensor location $j$, $\Delta_{j}$, as a function of all the unknown moduli values, $E_{i}$. The equation must relate the following:

$$
\begin{equation*}
\Delta_{j}=\Delta_{j}^{0}+\left(\Delta_{i}^{0} \text { change due to moduli variations }\right) \tag{A7}
\end{equation*}
$$

The general equation is:

$$
\begin{equation*}
\Delta_{j}=\Delta_{j}^{0}+\sum_{i=1}^{3} S_{j i}\left[\log _{10} E_{i}-\log _{10} E_{i}^{0}\right] \tag{A8}
\end{equation*}
$$

The value of $\Delta_{j}^{0}$ can be expressed in terms of one of the unknown moduli as:

$$
\begin{equation*}
\Delta_{j}^{0}=A_{j 3}+S_{j 3} \log _{10} E_{3}^{0} \tag{A9}
\end{equation*}
$$

Therefore the expression for $\Delta_{j}$ can be written as:

$$
\begin{equation*}
\Delta_{j}=A_{j 3}+S_{j 3} \log _{10} E_{3}^{0}+\sum_{i=1}^{3} S_{j i}\left(\log _{10} E_{i}-\log _{10} E_{i}\right) \tag{AlO}
\end{equation*}
$$

The expression for the summation of the squared deviations may be written as:

$$
\begin{equation*}
\sum_{j=1}^{n} \delta_{j}^{2}=\sum_{j=1}^{m}\left[D_{j}-A_{j 3}^{0}-S_{j 3} \log _{10} E_{3}^{0}-\sum_{i=1}^{n} S_{j i}\left[\log _{10} E_{i}-\log _{10} E_{i}^{0}\right)\right]^{2} \tag{All}
\end{equation*}
$$

The squared errors in deviation are minimized by taking the partial derivatives of the error expression with respect to each of the unknown moduli values. By setting the partial derivatives equal to zero, the following matrix equation may be obtained:

$$
\begin{equation*}
[B]\{E\}=\{C\} \tag{Al2}
\end{equation*}
$$

where, for $\mathfrak{i}$ and $k$ equal to the layer number, the matrix terms are:

$$
\begin{equation*}
C_{k}=\sum_{j=1}^{7} S_{j k}\left[D_{j}-A_{j 3}-S_{j 3} \log _{10} E_{3}-\sum_{i=1}^{3} S_{j i} \log _{10} E_{i}\right] \tag{Al3}
\end{equation*}
$$

$$
B_{k i}=\sum_{j=1}^{m} S_{j k} S_{j i}
$$

$$
\begin{equation*}
E \text {. = unknown layer moduli of layer } k \tag{Al5}
\end{equation*}
$$

Solution of the matrix equation formed from the gradient expressions noted above provides the solution vector of layer moduli values.

## RANGES SUBROUTINE

A flow chart of the RANGES subroutine is shown in Figure Al2. The subroutine RANGES has two main functions. The first is a search routine to determine practical boundaries for ranges of layer moduli and the second is to do the initial calculations for the method of stepwise direct optimization.

To accomplish the first function, RANGES starts with the minimum asphaltic concrete modulus set by the subroutine EACHL and steps through small increments of asphaltic concrete modulus. For each step, a set of deflections is calculated based on the given step value of asphaltic concrete modulus and extremely low values of Portland cement concrete modulus $\left(E_{p c c}=32 \mathrm{ksi}(220 \mathrm{MPa})\right.$ ) and subgrade modulus ( $E_{\text {sub }}=1 \mathrm{ksi}$ (7 MPa)). The deflections calcu?ated may be considered the "absolute" maximum deflections which can practically occur for the given value of asphaltic concrete modulus. If any of the measured deflections exceed the computed maximum values, the solution is unacceptable, and the minimum value of asphaltic concrete modulus is incremented. The process is repeated (asphaltic concrete modulus incremented) until a solution is found where all measured deflections are less than the absolute maximum deflections. This value is then set as the absolute minimum value of asphaltic concrete modulus.


FIGURE Al2. Flowchart of the RANGES Subroutine

A similar process is used to fix the absolute maximum value of asphaltic concrete modulus. Calculations start with the maximum asphaltic concrete modulus calculated by EACHL, and the asphaltic concrete modulus is decremented in steps. For each step, a set of deflections are calculated based on the given value of asphaltic concrete modulus and extremely high values of Portland cement concrete modulus ( $E_{p c c}=32$ million psi ( 220 GPa )) and subgrade modulus $\left(E_{\text {sub }}=200 \mathrm{ksi}\right.$ (1.4 GPa)). The calculated deflections can be considered the "absolute" minimum deflections for the given value of asphaltic concrete modulus. If any of the measured deflections are less than the absolute minimum deflections, then the solution is unacceptable, and the maximum value of asphaltic concrete modulus is decremented. The process is repeated (asphaltic concrete modulus decremented) until a solution is found such that all measured deflections are greater than the absolute minimum deflections. This value is then set as the absolute maximum value of asphaltic concrete modulus. The process described above is achieved very quickly and can often save a significant amount of computer time by eliminating impossible solutions. This feature is particularly important in cases where the no temperature data option is used. When the temperature data option is used, the maxima and minima set by EACHL usually meet the criteria for the absolute maxima and minima as described above.

The second function of the RANGES subroutine involves the initial calculations for the method of stepwise direct optimization. Only asphaltic concrete moduli values within the absolute limits previously set are considered. Starting with the absolute minimum value, the asphaltic concrete modulus is incremented (stepped) up to the absolute
maximum asphaltic concrete modulus. For each asphaltic concrete modulus step, an attempt is made to calculate trial values. If suitable trial values are found, they are stored. If suitable trial values are not found, this particular asphaltic concrete increment is eliminated from consideration in the direct optimization. The technique for calculating trial values uses two main assumptions. The first assumption is that the measured value of the outside sensor deflection $\left(D_{7}\right)$ is a strong indicator of subgrade strength. This assumption may be considered reasonably valid for all but very extreme cases. The second assumption is that the inside sensor deflection $\left(D_{1}\right)$ is an accurate indicator of the overall pavement structural capability. This is also a good assumption for non-extreme cases. Based on these assumptions, trial values are predicted for each asphaltic concrete modulus step very quickly by the method illustrated in Figure Al3. For the fixed value of asphaltic concrete modulus corresponding the the step being considered, the Portland cement concrete modulus is incremented through a range of reasonable values. or each set of fixed asphaltic concrete and Portland cement concrete moduli, a value of predicted subgrade modulus is calculated which corresponds to an exact solution for the inside sensor deflection and a second value of predicted subgrade modulus is calculated which corresponds to an exact solution for the outside sensor deflection. When predicted subgrade modulus based on the outside sensor deflection is plotted versus assumed Portland cement concrete modulus, the curve is relatively flat (verifying that the outer sensor strongly reflects the subgrade modulus). When predicted subgrade modulus based on the inside sensor deflection is plotted versus assumed Portland cement concrete modulus, a distinct curve is formed. The intersection
of the two curves represents the single solution which gives exact fit for both the inside and outside sensor deflections. If the curves do not intersect, then there exists no combination of Portland cement concrete and subgrade moduli which will give an exact fit for both the inside and outside sensor deflections. If no intersection is found, the given asphaltic concrete step is not included when the stepwise direct optimization is completed. When all the ranges have been tested, the trial values are returned for completion of the method of stepwise direct optimization.


FIGURE A13. Trial Value Estimation for Stepwise Direct Optimization

## THE METHOD OF STEPWISE DIRECT OPTIMIZATION

This method is accomplished in two parts. The first part is completed as previously described by the RANGES subroutine, and illustrated in the flowchart of Figure Al2. A flowchart showing the completion of the method of stepwise direct optimization is shown in Figure A14. The method is completed within the main COMDEF program, but a separate flowchart is included for additional clarity. The completion of the method uses the trial values which were calculated by the subroutine RANGES for each asphaltic concrete modulus increment. The program then does local optimizing separately on each layer moduli, starting with the subgrade moduli and working upward in the pavement structure. The optimizing calculations involve calculating a set of deflections for the initial solution, then offsetting the given modulus value slightly. The direct optimizing technique uses as its basis the assumption that that for small increments in layer moduli there is a loglinear relationship between incremental layer moduli and incremental deflections. That is:

$$
\begin{equation*}
\left(E_{\text {pred }}\right)_{i}=\left(m \Delta^{n}\right)_{i} \tag{A16}
\end{equation*}
$$

where

$$
\begin{aligned}
& \left(E_{\text {pred }}\right)_{i}=\text { predicted modulus for sensor } i \\
& \Delta=\text { deflection for sensor } i \\
& m, n= \\
& \quad \begin{array}{l}
\text { loglinear fit parameters for sensor } i, \text { based on }
\end{array} \\
& \quad \text { and for a small deviation in the trial modulus value }
\end{aligned}
$$



FIGURE A14. Flowchart of the Completion of the Method of Stepwise Direct Optimization

Therefore the expression, $f$, for the sum of the squared deviations may be written as:

$$
\begin{equation*}
f=\sum\left(\delta E_{\text {opt imum }}\right)^{2}=\sum\left[E_{\text {opt imum }}-\left(m \Delta^{n}\right)_{i}\right]^{2} \tag{A17}
\end{equation*}
$$

By setting the derivative of $f$ with respect to $E_{\text {optimum }}$ equai to zero, it may be easily shown that the least-squares optimized solution for the modulus being considered is simply the average of the $\left(E_{\text {pred }}\right)$. That is:

$$
\begin{equation*}
E_{\text {opt imum }}=\left(m \Delta^{\mathrm{N}}\right)_{\text {average }} \tag{A18}
\end{equation*}
$$

The process is continued for each layer. After each optimizing calculation, the subroutine ERRORS is called and the error and percentage error of the fitted basin are compared with previous minimums. When all the asphaltic concrete modulus ranges have been completed, the moduli values for the best solution are saved, and the error and percentage error of the fitted basin for the best solution are saved for comparison with solutions from the other methods.

## APPENDIX B

## COMPLETE RESULTS FROM SENSITIVITY STUDIES

Tables Bl-B4 list tabular results of the analysis of the errors created by the COMDEF deflection approximation method. Table Bl lists the deflections calculated in the COMDEF sensitivity study by the DELTA subroutine. Table $B 2$ gives "check" deflections calculated by multilayer linear elastic tneory, using the BISAR subroutine. Tables B3 and B4 summarize the errors and percentage errors, respectively, due to the COMDEF approximation method.

Tables $85-$ Bl5 list tabular results from all the sensitivity studies. Table $B 5$ summarizes the COMDEF sensitivity study and Tables B6-B15 summarize the BISDEF sensitivity study for each trial value combination.

Figures Bl-B45 provide graphical summaries of the sensitivity studies, with direct comparisons between trial value combinations for the 45 cases studied.

TABLE Bl. Fitted Basins from COMDEF Sensitivity Study (45 cases)

| Thickness (inches) |  | Backealculated Modulus (psi) |  |  | COMOEF Deflections in mils for Sensor Number |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AC | PCC | AC | PCC | subgr. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 3 | 6 | 244369 | 2029185 | 9990 | 20.516 | 16.987 | 13.288 | 9.897 | 7.198 | 5.185 | 3.739 |
| 3 | 6 | 246802 | 4025679 | 9994 | 17.061 | 14.415 | 11.845 | 9.289 | 7.094 | 5.327 | 3.960 |
| 3 | 6 | 247979 | 6017766 | 9999 | 15.160 | 12.897 | 10.880 | 8.783 | 6.905 | 5.330 | 4.058 |
| 3 | 6 | 481660 | 2055771 | 9992 | 18.507 | 15.851 | 12.643 | 9.635 | 7.165 | 5.258 | 3.842 |
| 3 | 6 | 490096 | 4045941 | 9995 | 15.653 | 13.710 | 11.387 | 9.050 | 7.007 | 5.332 | 4.010 |
| 3 | 12 | 249560 | 1998330 | 10005 | 11.006 | 9.069 | 8.038 | 6.911 | 5.817 | 4.815 | 3.927 |
| 3 | 12 | 249546 | 3991794 | 10011 | 8.694 | 7.122 | 6.499 | 5.774 | 5.035 | 4.324 | 3.664 |
| 3 | 12 | 249522 | 5985887 | 10014 | 7.539 | 6.103 | 5.648 | 5.101 | 4.529 | 3.966 | 3.430 |
| 3 | 12 | 500755 | 1996194 | 10004 | 10.029 | 8.696 | 7.736 | 6.692 | 5.671 | 4.728 | 3.887 |
| 3 | 12 | 500143 | 3989779 | 10012 | 7.936 | 6.939 | 6.340 | 5.649 | 4.942 | 4.260 | 3.624 |
| 3 | 18 | 249824 | 1996271 | 10009 | 7.405 | 5.738 | 5.281 | 4.778 | 4.257 | 3.745 | 3.257 |
| 3 | 18 | 249520 | 4001855 | 9997 | 5.725 | 4.318 | 4.061 | 3.759 | 3.434 | 3.104 | 2.777 |
| 3 | 18 | 249607 | 6004896 | 9995 | 4.932 | 3.616 | 3.436 | 3.216 | 2.974 | 2.724 | 2.473 |
| 3 | 18 | 500122 | 1996680 | 10008 | 6.661 | 5.569 | 5.130 | 4.652 | 4.157 | 3.669 | 3.203 |
| 3 | 18 | 499490 | 3997235 | 10004 | 5.088 | 4.245 | 3.991 | 3.697 | 3.383 | 3.061 | 2.744 |
| 9 | 6 | 249245 | 2012147 | 9997 | 15.816 | 11.642 | 9.874 | 8.102 | 6.500 | 5.130 | 3.996 |
| 9 | 6 | 249289 | 4014916 | 10000 | 14.498 | 10.571 | 9.169 | 7.683 | 6.286 | 5.054 | 4.006 |
| 9 | 6 | 249385 | 6010268 | 10003 | 13.688 | 9.884 | 8.687 | 7.377 | 6.114 | 4.978 | 3.993 |
| 9 | 6 | 499507 | 1095573 | 10005 | 12.290 | 9.967 | 8.659 | 7.316 | 6.056 | 4.933 | 3.965 |
| 9 | 6 | 499567 | 3995933 | 10005 | 11.231 | 9.084 | 8.030 | 6.896 | 5.801 | 4.801 | 3.919 |
| 9 | 12 | 249630 | 1996190 | 10003 | 10.889 | 7.273 | 6.587 | 5.837 | 5.065 | 4.325 | 3.643 |
| 9 | 12 | 249702 | 3995556 | 10006 | 9.536 | 6.065 | 5.603 | 5.073 | 4.503 | 3.936 | 3.397 |
| 9 | 12 | 249794 | 5994593 | 10008 | 8.769 | 5.357 | 4.999 | 4.581 | 4.119 | 3.651 | 3.197 |
| 9 | 12 | 499438 | 1994237 | 10007 | 8.389 | 6.446 | 5.885 | 5.273 | 4.642 | 4.028 | 3.453 |
| 9 | 12 | 499321 | 4000485 | 10004 | 7.331 | 5.506 | 5.113 | 4.661 | 4.175 | 3.689 | 3.221 |
| 9 | 18 | 249927 | 1995190 | 10009 | 8.392 | 4.915 | 4.555 | 4.187 | 3.785 | 3.378 | 2.980 |
| 9 | 18 | 250055 | 3988780 | 10017 | 7.245 | 3.872 | 3.650 | 3.415 | 3.147 | 2.866 | 2.584 |
| 9 | 18 | 250113 | 5986471 | 10018 | 6.646 | 3.313 | 3.146 | 2.972 | 2.766 | 2.548 | 2.325 |
| 9 | 18 | 499489 | 1996711 | 10007 | 6.315 | 4.478 | 4.166 | 3.846 | 3.500 | 3.149 | 2.804 |
| 9 | 18 | 499829 | 3994708 | 10012 | 5.368 | 3.623 | 3.424 | 3.210 | 2.969 | 2.716 | 2.462 |
| 15 | 6 | 249689 | 2001914 | 10004 | 12.783 | 8.475 | 7.288 | 6.290 | 5.340 | 4.474 | 3.705 |
| 15 | 6 | 249638 | 4010644 | 10004 | 12.088 | 7.866 | 6.836 | 5.975 | 5.128 | 4.342 | 3.634 |
| 15 | 6 | 249648 | 6011988 | 10004 | 11.694 | 7.516 | 6.573 | 5.789 | 5.002 | 4.262 | 3.589 |
| 15 | 6 | 499551 | 1993409 | 10010 | 9.178 | 6.939 | 6.160 | 5.447 | 4.747 | 4.087 | 3.479 |
| 15 | 6 | 499596 | 3992716 | 10007 | 8.527 | 6.349 | 5.685 | 5.080 | 4.473 | 3.892 | 3.351 |
| 15 | 12 | 249752 | 1995172 | 10005 | 10.005 | 5.924 | 5.231 | 4.738 | 4.214 | 3.696 | 3.204 |
| 15 | 12 | 249792 | 3996375 | 10004 | 9.203 | 5.183 | 4.613 | 4.245 | 3.834 | 3.414 | 3.005 |
| 15 | 12 | 249881 | 5996750 | 10003 | 8.720 | 4.728 | 4.219 | 3.918 | 3.572 | 3.219 | 2.854 |
| 15 | 12 | 499469 | 1996969 | 10009 | 7.019 | 4.926 | 4.461 | 4.084 | 3.687 | 3.289 | 2.906 |
| 15 | 12 | 499548 | 3999265 | 10006 | 6.401 | 4.351 | 3.976 | 3.685 | 3.365 | 3.037 | 2.715 |
| 15 | 18 | 249926 | 1995900 | 10004 | 8.326 | 4.320 | 3.814 | 3.550 | 3.253 | 2.942 | 2.634 |
| 15 | 18 | 250054 | 3990252 | 10014 | 7.552 | 3.591 | 3.172 | 2.997 | 2.788 | 2.562 | 2.331 |
| 15 | 18 | 250117 | 5983349 | 10020 | 7.116 | 3.173 | 2.793 | 2.661 | 2.496 | 2.314 | 2.126 |
| 15 | 18 | 499631 | 1997620 | 10004 | 5.710 | 3.665 | 3.332 | 3.113 | 2.874 | 2.626 | 2.378 |
| 15 | 18 | 499854 | 4001659 | 10005 | 5.114 | 3.106 | 2.840 | 2.687 | 2.511 | 2.322 | 2.129 |

NOTE: $\quad 1$ inch $=25.4 \mathrm{~mm} \quad 1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m}$

TABLE B2. BISAR Basins from COMDEF Backcalculated Moduli (45 cases)

| Thickness (inches) |  | Backealeulated Modulus (psi) |  |  | COMOEF Deflections in mils for Sensor Mumber |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AC | PCC | AC | PCC | Subgr. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 3 | 6 | 244369 | 2029185 | 9990 | 20.520 | 16.999 | 13.288 | 9.894 | 7.200 | 5.193 | 3.748 |
| 3 | 6 | 246802 | 4025679 | 9994 | 17.063 | 14.416 | 11.843 | 9.285 | 7.090 | 5.329 | 3.968 |
| 3 | 6 | 247979 | 60!776 | 9999 | 15.162 | 12.897 | 10.877 | 8.779 | 6.900 | 5.328 | 4.063 |
| 3 | 6 | 481660 | 2055771 | 9992 | 18.513 | 15.857 | 12.644 | 9.633 | 7.164 | 5.264 | 3.851 |
| 3 | 6 | 490096 | 4045941 | 9995 | 15.653 | 13.710 | 11.385 | 9.047 | 7.004 | 5.333 | 4.017 |
| 3 | 12 | 249560 | 1998330 | 10005 | 11.009 | 9.070 | 8.038 | 6.911 | 5.816 | 4.812 | 3.926 |
| 3 | 12 | 249546 | 3991794 | 10011 | 8.698 | 7.123 | 6.499 | 5.774 | 5.033 | 4.322 | 3.662 |
| 3 | 12 | 249522 | 5985887 | 10014 | 7.543 | 6.106 | 5.648 | 5.101 | 4.528 | 3.964 | 3.427 |
| 3 | 12 | 500755 | 1996194 | 10004 | 10.031 | 8.697 | 7.737 | 6.692 | 5.670 | 4.727 | 3.886 |
| 3 | 12 | 500143 | 3989779 | 10012 | 7.938 | 6.940 | 6.340 | 5.649 | 4.941 | 4.258 | 3.622 |
| 3 | 18 | 249824 | 1996271 | 10009 | 7.408 | 5.739 | 5.282 | 4.778 | 4.257 | 3.744 | 3.256 |
| 3 | 18 | 249520 | 4001855 | 9997 | 5.727 | 4.318 | 4.061 | 3.759 | 3.434 | 3.104 | 2.778 |
| 3 | 18 | 249607 | 6004894 | 9995 | 4.933 | 3.616 | 3.436 | 3.216 | 2.974 | 2.725 | 2.473 |
| 3 | 18 | 500122 | 1996680 | 10008 | 6.662 | 5.570 | 5.130 | 4.652 | 4.157 | 3.668 | 3.201 |
| 3 | 18 | 499490 | 3997235 | 10004 | 5.089 | 4.245 | 3.991 | 3.698 | 3.383 | 3.061 | 2.744 |
| 9 | 6 | 249245 | 2012147 | 9997 | 15.826 | 11.646 | 9.876 | 8.102 | 6.499 | 5.130 | 3.999 |
| 9 | 6 | 249289 | 4014916 | 10000 | 14.507 | 10.573 | 9.170 | 7.683 | 6.285 | 5.053 | 4.007 |
| 9 | 6 | 249385 | 6010268 | 10003 | 13.695 | 9.885 | 8.687 | 7.376 | 6.112 | 4.976 | 3.993 |
| 9 | 6 | 499507 | 1995573 | 10005 | 12.295 | 9.970 | 8.660 | 7.315 | 6.054 | 4.931 | 3.964 |
| 9 | 6 | 499567 | 3995933 | 10005 | 11.233 | 9.084 | 8.030 | 6.895 | 5.800 | 4.800 | 3.918 |
| 9 | 12 | 249630 | 1996190 | 10003 | 10.898 | 7.276 | 6.589 | 5.839 | 5.066 | 4.326 | 3.643 |
| 9 | 12 | 249702 | 3995556 | 10006 | 9.541 | 6.065 | 5.603 | 5.073 | 4.503 | 3.936 | 3.396 |
| 9 | 12 | 249794 | 5994593 | 10008 | 8.771 | 5.356 | 4.998 | 4.580 | 4.118 | 3.650 | 3.195 |
| 9 | 12 | 499438 | 1994237 | 10007 | 8.394 | 6.448 | 5.886 | 5.275 | 4.642 | 4.028 | 3.451 |
| 9 | 12 | 499321 | 4000485 | 10004 | 7.333 | 5.505 | 5.113 | 4.661 | 4.173 | 3.689 | 3.220 |
| 9 | 18 | 249927 | 1995190 | 10009 | 8.394 | 4.916 | 4.556 | 4.188 | 3.785 | 3.377 | 2.979 |
| 9 | 18 | 250055 | 3988780 | 10017 | 7.245 | 3.872 | 3.650 | 3.415 | 3.146 | 2.865 | 2.583 |
| 9 | 18 | 250113 | 5984471 | 10018 | 6.645 | 3.312 | 3.146 | 2.971 | 2.766 | 2.547 | 2.324 |
| 9 | 18 | 499489 | 1996711 | 10007 | 6.318 | 4.679 | 4.166 | 3.846 | 3.501 | 3.149 | 2.804 |
| 9 | 18 | 499829 | 3994708 | 10012 | 5.368 | 3.622 | 3.423 | 3.209 | 2.968 | 2.715 | 2.461 |
| 15 | 6 | 249689 | 2001914 | 10004 | 12.788 | 8.475 | 7.288 | 6.290 | 5.338 | 4.472 | 3.704 |
| 15 | 6 | 249638 | 4010644 | 10004 | 12.093 | 7.864 | 6.836 | 5.974 | 5.127 | 4.341 | 3.633 |
| 15 | 6 | 249648 | 6011988 | 10004 | 11.699 | 7.515 | 6.573 | 5.789 | 5.001 | 4.261 | 3.588 |
| 15 | 6 | 499551 | 1993409 | 10010 | 9.181 | 6.939 | 6.160 | 5.467 | 4.746 | 4.085 | 3.477 |
| 15 | 6 | 499596 | 3992716 | 10007 | 8.529 | 6.348 | 5.685 | 5.080 | 4.473 | 3.891 | 3.369 |
| 15 | 12 | 249752 | 1995172 | 10005 | 10.019 | 5.926 | 5.233 | 4.739 | 4.215 | 3.696 | 3.204 |
| 15 | 12 | 249792 | 3996375 | 10006 | 9.207 | 5.183 | 4.614 | 4.245 | 3.834 | 3.413 | 3.004 |
| 15 | 12 | 249881 | 5996750 | 10003 | 8.723 | 4.728 | 4.220 | 3.918 | 3.572 | 3.210 | 2.853 |
| 15 | 12 | 499469 | 1994969 | 10009 | 7.023 | 4.924 | 4.462 | 4.034 | 3.686 | 3.289 | 2.905 |
| 15 | 12 | 499548 | 3999265 | 10006 | 6.402 | 4.350 | 3.976 | 3.685 | 3.365 | 3.037 | 2.714 |
| 15 | 18 | 249926 | 1995900 | 10006 | 8.329 | 4.321 | 3.816 | 3.550 | 3.253 | 2.943 | 2.634 |
| 15 | 18 | 250054 | 3990252 | 10014 | 7.551 | 3.590 | 3.172 | 2.997 | 2.78 | 2.561 | 2.330 |
| 15 | 18 | 250117 | 5983349 | 10020 | 7.113 | 3.172 | 2.792 | 2.660 | 2.495 | 2.313 | 2.124 |
| 15 | 18 | 499631 | 1997620 | 10004 | 5.712 | 3.666 | 3.332 | 3.113 | 2.874 | 2.626 | 2.378 |
| 15 | 18 | 499854 | 4001659 | 10005 | 5.113 | 3.105 | 2.839 | 2.686 | 2.510 | 2.321 | 2.128 |

NOTE: $\quad 1$ inch $=25.4 \mathrm{~mm} \quad 1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m}$

TABLE B3. Deflection Errors Due to COMDEF Approximation (45 cases)

| Thickness (inches) |  | Backcalculated Modulus (psi) |  |  | Deflection Errors in mils for Sensor Number |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AC | PCC | AC | PCC | Subgr. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 3 | 6 | 244369 | 2029185 | 9990 | 0.004 | 0.003 | 0.000 | 0.002 | 0.002 | 0.008 | 0.009 |
| 3 | 6 | 246802 | 4025679 | 9994 | 0.003 | 0.001 | 0.002 | 0.004 | 0.003 | 0.001 | 0.008 |
| 3 | 6 | 247979 | 6017766 | 9999 | 0.002 | 0.000 | 0.002 | 0.005 | 0.005 | 0.002 | 0.005 |
| 3 | 6 | 481660 | 2055771 | 9992 | 0.005 | 0.005 | 0.001 | 0.002 | 0.001 | 0.005 | 0.009 |
| 3 | 6 | 490096 | 4045941 | 9995 | 0.000 | 0.000 | 0.001 | 0.003 | 0.003 | 0.001 | 0.007 |
| 3 | 12 | 249560 | 1998330 | 10005 | 0.003 | 0.001 | 0.000 | 0.001 | 0.002 | 0.002 | 0.001 |
| 3 | 12 | $249540^{\circ}$ | 3991794 | 10011 | 0.004 | 0.001 | 0.000 | 0.000 | 0.009 | 0.002 | 0.002 |
| 3 | 12 | 249522 | 5985887 | 10014 | 0.004 | 0.000 | 0.000 | 0.000 | 0.001 | 0.002 | 0.003 |
| 3 | 12 | 500755 | 1996194 | 10004 | 0.002 | 0.001 | 0.001 | 0.000 | 0.001 | 0.001 | 0.001 |
| 3 | 12 | 500143 | 3949779 | 10012 | 0.002 | 0.001 | 0.000 | 0.001 | 0.001 | 0.002 | 0.003 |
| 3 | 18 | 249824 | 1996271 | 10009 | 0.003 | 0.001 | 0.001 | 0.001 | 0.000 | 0.000 | 0.002 |
| 3 | 18 | 249520 | 4001855 | 9997 | 0.002 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 18 | 249607 | 6004894 | 9995 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 3 | 18 | 500122 | 1996680 | 10008 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.001 |
| 3 | 18 | 499490 | 3997235 | 10004 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 9 |  | 249245 | 2012147 | 9997 | 0.010 | 0.004 | 0.002 | 0.000 | 0.001 | 0.001 | 0.003 |
| 9 | 6 | 249289 | 4014916 | 10000 | 0.009 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| 9 | 6 | 249385 | 6010268 | 10003 | 0.007 | 0.000 | 0.000 | 0.001 | 0.002 | 0.002 | 0.000 |
| 9 | 6 | 499507 | 1995573 | 10005 | 0.005 | 0.003 | 0.001 | 0.001 | 0.002 | 0.002 | 0.001 |
| 9 | 6 | 499567 | 3995933 | 10005 | 0.002 | 0.000 | 0.000 | 0.001 | 0.002 | 0.002 | 0.001 |
| 9 | 12 | 249630 | 1996190 | 10003 | 0.009 | 0.003 | 0.003 | 0.002 | 0.001 | 0.000 | 0.000 |
| 9 | 12 | 249702 | 3995556 | 10006 | 0.005 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.001 |
| 9 | 12 | 249794 | 5994593 | 10008 | 0.002 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| 9 | 12 | 499438 | 1994237 | 10007 | 0.005 | 0.002 | 0.002 | 0.004 | 0.000 | 0.000 | 0.001 |
| 9 | 12 | 499321 | 4000485 | 10004 | 0.002 | 0.001 | 0.000 | 0.000 | 0.000 | 0.001 | 0.001 |
| 9 | 18 | 249927 | 1995190 | 10009 | 0.003 | 0.001 | 0.001 | 0.000 | 0.000 | 0.000 | 0.001 |
| 9 | 18 | 250055 | 3988780 | 10017 | 0.000 | 0.000 | 0.000 | 0.000 | 0.001 | 0.001 | 0.001 |
| 9 | 18 | 250113 | 5984471 | 10018 | 0.001 | 0.000 | 0.000 | 0.000 | 0.001 | 0.001 | 0.001 |
| 9 | 18 | 499489 | 1996711 | 10007 | 0.003 | 0.001 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
| 9 | 18 | 499829 | 3994708 | 10012 | 0.000 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| 15 | 6 | 249689 | 2001914 | 10004 | 0.005 | 0.000 | 0.000 | 0.001 | 0.001 | 0.001 | 0.001 |
| 15 | 6 | 249638 | 4010644 | 10004 | 0.005 | 0.001 | 0.000 | 0.001 | 0.001 | 0.001 | 0.001 |
| 15 | 6 | 269648 | 6011988 | 10004 | 0.005 | 0.001 | 0.000 | 0.000 | 0.001 | 0.004 | 0.001 |
| 15 | 6 | 499551 | 1993609 | 10010 | 0.003 | 0.000 | 0.000 | 0.000 | 0.009 | 0.002 | 0.002 |
| 15 | 6 | 499596 | 3992716 | 10007 | 0.002 | 0.000 | 0.000 | 0.000 | 0.009 | 0.001 | 0.001 |
| 15 | 12 | 249732 | 1905172 | 10005 | 0.007 | 0.001 | 0.002 | 0.001 | 0.001 | 0.000 | 0.001 |
| 15 | 12 | 249792 | 3996375 | 10004 | 0.004 | 0.000 | 0.001 | 0.001 | 0.000 | 0.000 | 0.001 |
| 15 | 12 | 249881 | 5996750 | 10003 | 0.002 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 15 | 12 | 499469 | 1994969 | 10009 | 0.004 | 0.001 | 0.001 | 0.000 | 0.000 | 0.001 | 0.001 |
| 15 | 12 | 499548 | 3999265 | 10006 | 0.002 | 0.001 | 0.000 | 0.000 | 0.000 | 0.001 | 0.001 |
| 15 | 18 | 269926 | 1995900 | 10004 | 0.003 | 0.001 | 0.001 | 0.001 | 0.001 | 0.000 | 0.000 |
| 15 | 18 | 250054 | 3990252 | 10014 | 0.001 | 0.000 | 0.000 | 0.001 | 0.001 | 0.001 | 0.001 |
| 15 | 18 | 250117 | 5983349 | 10020 | 0.002 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| 15 | 18 | 499631 | 1997620 | 10004 | 0.002 | 0.001 | 0.001 | 0.001 | 0.000 | 0.000 | 0.000 |
| 15 | 18 | 499854 | 4001659 | 10005 | 0.000 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.009 |
| Maximen Errors Average Errors |  |  |  |  | 0.010 | 0.005 | 0.003 | 0.005 | 0.005 | 0.008 | 0.009 |
|  |  |  |  |  | 0.003 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.002 |

Maximum deflection error found in 45 cases $=0.010 \mathrm{mils}$
Average deflection error found in 45 cases $=0.001 \mathrm{mils}$

NOTE:
1 inch $=25.4 \mathrm{~mm}$
$1 \mathrm{psi}=6.89 \mathrm{kPa}$
$1 \mathrm{mil}=25.4 \mu \mathrm{~m}$

TABLE B4. Percentage Errors Due to COMDEF Approximation (45 cases)

| Thickness (inches) |  | Backcalculated Modulus (psi) |  |  | Percentage Error in Deflection for Sensor Number |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AC | PCC | AC | PCC | Subgr . | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 3 | 6 | 244369 | 2029185 | 9990 | 0.017 | 0.020 | 0.002 | 0.023 | 0.026 | 0.151 | 0.251 |
| 3 | 6 | 246802 | 4025679 | 9994 | 0.016 | 0.007 | 0.015 | 0.045 | 0.049 | 0.025 | 0.207 |
| 3 | 6 | 247979 | 6017766 | 9999 | 0.013 | 0.000 | 0.022 | 0.052 | 0.071 | 0.042 | 0.111 |
| 3 | 6 | 481660 | 2055771 | 9992 | 0.028 | 0.034 | 0.007 | 0.023 | 0.009 | 0.097 | 0.228 |
| 3 | 6 | 490096 | 4045941 | 9995 | 0.001 | 0.003 | 0.011 | 0.038 | 0.046 | 0.018 | 0.171 |
| 3 | 12 | 249560 | 1998330 | 10005 | 0.027 | 0.008 | 0.001 | 0.011 | 0.028 | 0.051 | 0.019 |
| 3 | 12 | 249546 | 3991794 | 10011 | 0.044 | 0.009 | 0.004 | 0.006 | 0.026 | 0.048 | 0.063 |
| 3 | 12 | 249522 | 5985887 | 10014 | 0.048 | 0.006 | 0.001 | 0.006 | 0.024 | 0.044 | 0.075 |
| 3 | 12 | 500755 | 1996194 | 10004 | 0.020 | 0.016 | 0.017 | 0.006 | 0.011 | 0.024 | 0.024 |
| 3 | 12 | 500143 | 3989779 | 10012 | 0.023 | 0.010 | 0.005 | 0.010 | 0.025 | 0.047 | 0.072 |
| 3 | 18 | 249824 | 1996271 | 10009 | 0.041 | 0.012 | 0.014 | 0.011 | 0.003 | 0.011 | 0.051 |
| 3 | 18 | 249520 | 4001855 | 9997 | 0.036 | 0.001 | 0.002 | 0.005 | 0.006 | 0.013 | 0.009 |
| 3 | 18 | 249607 | 6004894 | 9995 | 0.027 | 0.009 | 0.005 | 0.001 | 0.003 | 0.012 | 0.007 |
| 3 | 18 | 500122 | 1996680 | 10008 | 0.021 | 0.005 | 0.008 | 0.002 | 0.001 | 0.015 | 0.046 |
| 3 | 18 | 499490 | 3997235 | 10004 | 0.023 | 0.006 | 0.006 | 0.008 | 0.006 | 0.004 | 0.013 |
| 9 | 6 | 249245 | 2012147 | 9997 | 0.066 | 0.030 | 0.020 | 0.003 | 0.012 | 0.012 | 0.085 |
| 9 | 6 | 249289 | 4014916 | 10000 | 0.061 | 0.013 | 0.008 | 0.008 | 0.021 | 0.012 | 0.037 |
| 9 | 6 | 249385 | 6010268 | 10003 | 0.054 | 0.009 | 0.003 | 0.016 | 0.031 | 0.033 | 0.003 |
| 9 | 6 | 499507 | 1995573 | 10005 | 0.039 | 0.027 | 0.014 | 0.008 | 0.031 | 0.041 | 0.023 |
| 9 | 6 | 499567 | 3995933 | 10005 | 0.019 | 0.003 | 0.000 | 0.013 | 0.029 | 0.038 | 0.030 |
| 9 | 12 | 249630 | 1996190 | 10003 | 0.082 | 0.041 | 0.043 | 0.036 | 0.023 | 0.007 | 0.008 |
| 9 | 12 | 269702 | 3995556 | 10006 | 0.049 | 0.005 | 0.006 | 0.000 | 0.007 | 0.018 | 0.039 |
| 9 | 12 | 249794 | 5994593 | 10008 | 0.026 | 0.016 | 0.012 | 0.018 | 0.025 | 0.034 | 0.046 |
| 9 | 12 | 499438 | 1994237 | 10007 | 0.061 | 0.034 | 0.030 | 0.023 | 0.009 | 0.009 | 0.032 |
| 9 | 12 | 499321 | 4000485 | 10004 | 0.027 | 0.012 | 0.004 | 0.005 | 0.008 | 0.016 | 0.027 |
| 9 | 18 | 249927 | 1995190 | 10009 | 0.034 | 0.022 | 0.021 | 0.011 | 0.000 | 0.013 | 0.030 |
| 9 | 18 | 250055 | 3988780 | 10017 | 0.001 | 0.003 | 0.000 | 0.010 | 0.022 | 0.038 | 0.058 |
| 9 | 18 | 250113 | 5984471 | 10018 | 0.017 | 0.004 | 0.003 | 0.011 | 0.022 | 0.037 | 0.056 |
| 9 | 18 | 499489 | 1996711 | 10007 | 0.048 | 0.021 | 0.017 | 0.011 | 0.004 | 0.005 | 0.017 |
| 9 | 18 | 499829 | 3994708 | 10012 | 0.001 | 0.023 | 0.020 | 0.027 | 0.034 | 0.043 | 0.054 |
| 15 | 6 | 249689 | 2001914 | 10004 | 0.041 | 0.006 | 0.007 | 0.010 | 0.025 | 0.031 | 0.021 |
| 15 | 6 | 249638 | 4010664 | 10004 | 0.041 | 0.019 | 0.002 | 0.012 | 0.024 | 0.031 | 0.026 |
| 15 | 6 | 249648 | 6011988 | 10004 | 0.043 | 0.018 | 0.001 | 0.007 | 0.018 | 0.027 | 0.027 |
| 15 | 6 | 499551 | 1993409 | 10010 | 0.035 | 0.006 | 0.006 | 0.008 | 0.025 | 0.043 | 0.057 |
| 15 | 6 | 499596 | 3992716 | 10007 | 0.026 | 0.003 | 0.006 | 0.003 | 0.015 | 0.028 | 0.041 |
| 15 | 12 | 249752 | 1995172 | 10005 | 0.066 | 0.025 | 0.038 | 0.030 | 0.017 | 0.001 | 0.017 |
| 15 | 12 | 249792 | 3996375 | 10006 | 0.047 | 0.002 | 0.016 | 0.012 | 0.005 | 0.004 | 0.018 |
| 15 | 12 | 249881 | 5996750 | 10003 | 0.027 | 0.005 | 0.010 | 0.004 | 0.000 | 0.005 | 0.015 |
| 15 | 12 | 499469 | 1994969 | 10009 | 0.050 | 0.015 | 0.018 | 0.009 | 0.003 | 0.016 | 0.033 |
| 15 | 12 | 499548 | 3099265 | 10006 | 0.025 | 0.016 | 0.004 | 0.007 | 0.012 | 0.021 | 0.033 |
| 15 | 18 | 249926 | 1995900 | 10006 | 0.035 | 0.032 | 0.033 | 0.026 | 0.019 | 0.012 | 0.004 |
| 15 | 18 | 250056 | 3990252 | 10014 | 0.012 | 0.011 | 0.006 | 0.017 | 0.028 | 0.039 | 0.048 |
| 15 | 18 | 250117 | 5983349 | 10020 | 0.032 | 0.026 | 0.020 | 0.032 | 0.045 | 0.057 | 0.068 |
| 15 | 18 | 499631 | 1997620 | 10004 | 0.043 | 0.021 | 0.021 | 0.016 | 0.012 | 0.007 | 0.001 |
| 15 | 18 | 499854 | 4001659 | 10005 | 0.007 | 0.035 | 0.028 | 0.031 | 0.033 | 0.036 | 0.038 |
| Maximan Percentage Error Average Percentage Error |  |  |  |  | 0.082 | 0.041 | 0.043 | 0.052 | 0.071 | 0.159 | 0.251 |
|  |  |  |  |  | 0.033 | 0.014 | 0.012 | 0.015 | 0.020 | 0.029 | 0.052 |

Maximum deflection percentage error found in 45 cases $=0.251 \%$ Average deflection percentage error found in 45 cases $=0.025 \%$

NOTE: $\quad 1$ inch $=25.4 \mathrm{~mm} \quad 1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m}$

TABLE B5. Results From COMDEF Sensitivity Study

| Analys is |  | Thickness (inches) |  | Exact Modulus (psi) |  |  | Calculated Modulus (psi) |  |  | $\begin{aligned} & \text { Error } \\ & \text { in } \\ & \text { Basin } \\ & \text { (mils) } \end{aligned}$ | Percent Error in Basin |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Program | 10* | AC | PCC | AC | PCC | Subgr. | AC | PCC | Subgr |  |  |
| comotf | com | 3 | 6 | 250000 | 2000000 | 10000 | 246369 | 2029185 | 9990 | 0.03 | 0.36 |
| COMOEF | COM | 3 | 12 | 250000 | 2000000 | 10000 | 249560 | 1998330 | 10005 | 0.00 | 0.06 |
| comot | COM | 3 | 18 | 250000 | 2000000 | 10000 | 249824 | 1996271 | 10009 | 0.00 | 0.06 |
| comot | COM | 9 | 6 | 250000 | 2000000 | 10000 | 249245 | 2012147 | 9997 | 0.01 | 0.17 |
| comot | COM | 9 | 12 | 250000 | 2000000 | 10000 | 249630 | 1996190 | 10003 | 0.00 | 0.03 |
| COMOEF | com | 9 | 18 | 250000 | 2000000 | 10000 | 249927 | 1995190 | 10009 | 0.00 | 0.02 |
| comot | COM | 15 | 6 | 250000 | 2000000 | 10000 | 249689 | 2001914 | 10004 | 0.00 | 0.05 |
| COMDEF | COM | 15 | 12 | 250000 | 2000000 | 10000 | 249752 | 1995172 | 10005 | 0.00 | 0.05 |
| COMDEF | COM | 15 | 18 | 250000 | 2000000 | 10000 | 249926 | 1995900 | 10004 | 0.00 | 0.01 |
| COMOEF | COM | 3 | 6 | 250000 | 4000000 | 10000 | 246802 | 4025679 | 9994 | 0.03 | 0.38 |
| COMOEF | COH | 3 | 12 | 250000 | 4000000 | 10000 | 249546 | 3991794 | 10011 | 0.00 | 0.02 |
| COMDEF | COM | 3 | 18 | 250000 | 4000000 | 10000 | 249520 | 4001855 | 9997 | 0.00 | 0.01 |
| COMCEF | COM | 9 | 6 | 250000 | 4000000 | 10000 | 249289 | 4014916 | 10000 | 0.01 | 0.11 |
| comota | COM | 9 | 12 | 250000 | 4000000 | 10000 | 249702 | 3995556 | 10006 | 0.00 | 0.02 |
| comotf | COM | 9 | 18 | 250000 | 4000000 | 10000 | 250055 | 3988780 | 10017 | 0.00 | 0.02 |
| COMOEF | COM | 15 | 6 | 250000 | 4000000 | 10000 | 249638 | 4010644 | 10004 | 0.00 | 0.04 |
| COMOEF | COM | 15 | 12 | 250000 | 4000000 | 10000 | 249792 | 3996375 | 10004 | 0.00 | 0.04 |
| COMOEF | COM | 15 | 18 | 250000 | 46.10000 | 10000 | 250054 | 3990252 | 10014 | 0.00 | 0.02 |
| COMOEF | COM | 3 | 6 | 250000 | 6000000 | 10000 | 247979 | 6017766 | 9999 | 0.02 | 0.30 |
| COMOEF | COM | 3 | 12 | 250000 | 6000000 | 10000 | 269522 | 5985887 | 10014 | 0.00 | 0.03 |
| COMDEF | COM | 3 | 18 | 250000 | 6000000 | 10000 | 249607 | 6004896 | 9995 | 0.00 | 0.01 |
| COMOEF | COM |  | 6 | 250000 | 6000000 | 10000 | 249385 | 6010268 | 10003 | 0.00 | 0.07 |
| COMOEF | COM |  | 12 | 250000 | 6000000 | 10000 | 249794 | 5994593 | 10008 | 0.00 | 0.02 |
| COMOEF | COM | 9 | 18 | 250000 | 6000000 | 10000 | 250113 | 5984471 | 10018 | 0.00 | 0.02 |
| COMDEF | Can | 15 | 6 | 250000 | 6000000 | 10000 | 269648 | 6011988 | 10004 | 0.00 | 0.04 |
| COMOEF | $\mathrm{COM}^{\text {a }}$ | 15 | 12 | 250000 | 6000000 | 10000 | 249881 | 5996750 | 10003 | 0.00 | 0.03 |
| COMDEF | COM | 15 | 18 | 250000 | 6000000 | 10000 | 250117 | 5983349 | 10020 | 0.00 | 0.02 |
| camof | COM | 3 | ${ }^{\circ}$ | 500000 | 2000000 | 10000 | 481660 | 2055771 | 9992 | 0.04 | 0.41 |
| COMDEF | COM | 3 | 12 | 500000 | 2000000 | 10000 | 500735 | 1996194 | 10004 | 0.00 | 0.03 |
| COMOEF | COM | 3 | 18 | 500000 | 2000000 | 10000 | 500122 | 1996680 | 10008 | 0.00 | 0.05 |
| COMDEF | COM | 9 | 6 | 500000 | 2000000 | 10000 | 499507 | 1995573 | 10005 | 0.00 | 0.06 |
| comotf | COM |  | 12 | 500000 | 2000000 | 10000 | 499438 | 1994237 | 10007 | 0.00 | 0.03 |
| COMOEF | COM | 9 | 18 | 500000 | 2000000 | 10000 | 499489 | 1996711 | 10007 | 0.00 | 0.09 |
| COMOEF | COM | 15 | 6 | 500000 | 2000000 | 10000 | 499551 | 1993409 | 10010 | 0.00 | 0.02 |
| COMOEF | COM | 15 | 12 | 500000 | 2000000 | 10000 | 499469 | 1994969 | 10009 | 0.00 | 0.03 |
| COMDEF | com | 15 | 18 | 500000 | 2000000 | 10000 | 499631 | 1997620 | 10004 | 0.00 | 0.01 |
| COMDEF | CCW | 3 | 6 | 500000 | 4000000 | 10000 | 490096 | 4045941 | 9995 | 0.02 | 0.32 |
| COMOEF | COM | 3 | 12 | 500000 | 4000000 | 10000 | 500143 | 3989779 | 10012 | 0.00 | 0.02 |
| COMDEF | COM | 3 | 18 | 500000 | 4000000 | 10000 | 499490 | 3997235 | 10004 | 0.00 | 0.02 |
| compef | COM | 9 | 6 | 500000 | 4000000 | 10000 | 499567 | 3995933 | 10005 | 0.00 | 0.03 |
| COMDEF | COM | 9 | 12 | 500000 | 4000000 | 10000 | 499321 | 4000485 | 10004 | 0.00 | 0.03 |
| COMDEF | COM | 9 | 18 | 500000 | 4000000 | 10000 | 499829 | 3994708 | 10012 | 0.00 | 0.02 |
| COMDEF | COM | 15 | 6 | 500000 | 4000000 | 10000 | 499596 | 3992716 | 10007 | 0.00 | 0.03 |
| COMDEF | COM | 15 | 12 | 500000 | 4000000 | 10000 | 499548 | 3999265 | 10006 | 0.00 | 0.04 |
| COMDEF | COM | 15 | 18 | 500000 | 4000000 | 10000 | 499854 | 4001659 | 10005 | 0.00 | 0.01 |

* ID is the trial value combination identifier, as defined in Table 1.

NOTE: 1 inch $=25.4 \mathrm{~mm} \quad 1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m}$

TABLE B6. Results from BISDEF Sensitivity Study (Case 1 of 10)

| Analysis |  | Thickness (inches) |  | Exact Modulus (psi) |  |  | Calculated Modulus (psi) |  |  | Error in Basin (mils) | Percent <br> Error <br> in <br> Basin |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Program | 10* | $A C$ | PCC | AC | PCC | subgr. | AC | PCC | Subgr. |  |  |
| 8!SOEF | A | 3 | 6 | 250000 | 2000000 | 10000 | 1000000 | 731876 | 10031 | 0.52 | 5.47 |
| BISDEF | A | 3 | 12 | 250000 | 2000000 | 10000 | 147181 | 2102345 | 10136 | 1.00 | 11.33 |
| BISDEF | A | 3 | 18 | 250000 | 2000000 | 10000 | 239387 | 2012839 | 10003 | 0.07 | 1.12 |
| BISDEF | A | 9 | 6 | 250000 | 2000000 | 10000 | 250761 | 1980704 | 10011 | 0.03 | 0.48 |
| BISOEF | $A$ | 9 | 12 | 250000 | 2000000 | 10000 | 231490 | 2347872 | 9859 | 0.42 | 6.15 |
| BISDEF | A | 9 | 18 | 250000 | 2000000 | 10000 | 247598 | 2010236 | 10013 | 0.05 | 1.01 |
| BISOEF | A | 15 | 6 | 250000 | 2000000 | 10000 | 250337 | 1997545 | 10005 | 0.03 | 0.38 |
| BISOEF | A | 15 | 12 | 250000 | 2000000 | 10000 | 250217 | 1996509 | 10008 | 0.02 | 0.35 |
| BISOEF | A | 15 | 18 | 250000 | 2000000 | 10000 | 242486 | 2084423 | 10058 | 0.24 | 5.36 |
| BISOEF | A | 3 | 6 | 250000 | 4000000 | 10000 | 184196 | 4329891 | 10141 | 0.89 | 9.33 |
| 8ISDEF | A | 3 | 12 | 250000 | 4000000 | 10000 | 239568 | 4026360 | 9994 | 0.06 | 0.77 |
| 8ISDEF | A | 3 | 18 | 250000 | 4000000 | 10000 | 235477 | 4070415 | 10042 | 0.20 | 5.21 |
| BISDEF | A | 9 | 6 | 250000 | 4000000 | 10000 | 249684 | 4075632 | 10028 | 0.21 | 2.64 |
| BISOEF | A | 9 | 12 | 250000 | 4000000 | 10000 | 267166 | 4030524 | 10007 | 0.06 | 0.98 |
| $8150 E F$ | A | 9 | 18 | 250000 | 4000000 | 10000 | 210809 | 4369323 | 10084 | 0.95 | 20.50 |
| BISDEF | A | 15 | 6 | 250000 | 4000000 | 10000 | 250290 | 3974875 | 10002 | 0.01 | 0.09 |
| BISOEF | A | 15 | 12 | 250000 | 4000000 | 10000 | 261786 | 4514451 | 9979 | 0.38 | 8.64 |
| 8ISDEF | $A$ | 15 | 18 | 250000 | 4000000 | 10000 | 248638 | 4018931 | 10006 | 0.03 | 0.70 |
| BISDEF | A | 3 | 6 | 250000 | 6000000 | 10000 | 218889 | 6202150 | 10017 | 0.04 | 2.39 |
| BISDEF | A | 3 | 12 | 250000 | 6000000 | 10000 | 245091 | 6014230 | 9998 | 0.03 | 0.45 |
| BISDEF | A | 3 | 18 | 250000 | 6000000 | 10000 | 239653 | 6053646 | 10065 | 0.15 | 4.50 |
| BISDEF | A | 9 | 6 | 250000 | 6000000 | 10000 | 244575 | 6297682 | 9976 | 0.10 | 1.20 |
| BISDEF | A | 9 | 12 | 250000 | 6000000 | 10000 | 247766 | 6022658 | 10015 | 0.06 | 1.07 |
| 8ISDEF | A | 9 | 18 | 250000 | 6000000 | 10000 | 268664 | 6007163 | 10012 | 0.03 | 0.75 |
| 8ISOEF | A | 15 | 6 | 250000 | 6000000 | 10000 | 250259 | 5987838 | 10002 | 0.01 | 0.20 |
| 8ISOEF | A | 15 | 12 | 250000 | 6000000 | 10000 | 248460 | 6037197 | 10008 | 0.04 | 0.76 |
| BISOEF | A | 15 | 18 | 250000 | 6000000 | 10000 | 248760 | 6010236 | 10014 | 0.03 | 0.78 |
| BISDEF | A | 3 | 6 | 500000 | 2000000 | 10000 | 394860 | 2686895 | 9999 | 1.64 | 12.07 |
| BJSDEF | 4 | 3 | 12 | 500000 | 2000000 | 10000 | 1000000 | 1565682 | 9770 | 0.54 | 8.40 |
| BISOEF | A | 3 | 18 | 500000 | 2000000 | 10000 | 466373 | 2048140 | 10015 | 0.11 | 2.22 |
| 8 SOEF | A | 9 | 6 | 500000 | 2000000 | 10000 | 500605 | 1986506 | 10010 | 0.02 | 0.27 |
| 8ISOEF | A | 9 | 12 | 500000 | 2000000 | 10000 | 480592 | 2079748 | 9909 | 0.28 | 5.05 |
| BISOEF | A | 9 | 18 | 500000 | 2000000 | 10000 | 492336 | 2065833 | 10037 | 0.19 | 4.77 |
| BlSDEF | A | 45 | 6 | 500000 | 2000000 | 10000 | 492933 | 1961277 | 9964 | 0.33 | 5.41 |
| BISOEF | A | 15 | 12 | 500000 | 2000000 | 10000 | 497056 | 1933656 | 9943 | 0.30 | 6.73 |
| BISOEF | A | 15 | 18 | 500000 | 2000000 | 10000 | 497910 | 2018935 | 9995 | 0.02 | 0.45 |
| 8ISOEF | A | 3 | 6 | 500000 | 4000000 | 10000 | 371301 | 5652823 | 9987 | 2.28 | 19.01 |
| BI SOEF | A | 3 | 12 | 500000 | 4000000 | 10000 | 655889 | 4085266 | $99 \%$ | 0.09 | 1.63 |
| 8150EF | A | 3 | 18 | 500000 | 4000000 | 10000 | 479228 | 4076730 | 10038 | 0.18 | 4.89 |
| Bisoef | A | 9 | 6 | 500000 | 4000000 | 10000 | 500570 | 3987052 | 10007 | 0.02 | 0.30 |
| BISOEF | A | 9 | 12 | 500000 | 4000000 | 10000 | 484089 | 4318022 | 10055 | 0.43 | 9.00 |
| BISDEF | A | 9 | 18 | 500000 | 4000000 | 10000 | 495959 | 410\%470 | 10096 | 0.26 | 8.08 |
| 8150EF | A | 15 | 6 | 500000 | 4000000 | 10000 | 500171 | 4001864 | 10002 | 0.01 | 0.19 |
| 81SOEF | A | 15 | 12 | 500000 | 4000000 | 10000 | 496366 | 3990.03 | 999\% | 0.07 | 1.61 |
| BISOEF | A | 15 | 18 | 500000 | 4000000 | 10000 | 490090 | 4105197 | 10066 | 0.16 | 5.71 |

* ID is the trial value combination identifier, as defined in Table 1.

NOTE: $\quad 1$ inch $=25.4 \mathrm{~mm} \quad 1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m}$

TABLE B7. Results from BISDEF Sensitivity Study (Case 2 of 10)

| Anatys is |  | Thickness (inches) |  | Exact Modulus (psi) |  |  | Calculated Modulus (psi) |  |  | Error in Basin (mils) | Percent <br> Error in Bes in |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Program | 10* | AC | PCC | AC | PCC | Subgr . | AC | PCC | Subgr. |  |  |
| bisoef | 8 | 3 | 6 | 250000 | 2000000 | 10000 | 893095 | 814483 | 9980 | 0.41 | 3.25 |
| BISDEF | 8 | 3 | 12 | 250000 | 2000000 | 10000 | 239988 | 2015150 | 10024 | 0.11 | 1.69 |
| bisoef | 8 | 3 | 18 | 250000 | 2000000 | 10000 | 257187 | 1984658 | 10067 | 0.19 | 2.27 |
| Bisdef | 8 | 9 | 6 | 250000 | 2000000 | 10000 | 259364 | 1680263 | 10140 | 0.28 | 4.57 |
| BISDEF | 8 | 9 | 12 | 250000 | 2000000 | 10000 | 255358 | 1997486 | 10137 | 0.54 | 8.63 |
| Bisoef | 8 | 9 | 18 | 250000 | 2000000 | 10000 | 256633 | 1974137 | 10082 | 0.21 | 3.94 |
| BISOEF | 8 | 15 | 6 | 250000 | 2000000 | 10000 | 258561 | 1662154 | 10096 | 0.19 | 2.55 |
| BISDEF | 8 | 15 | 12 | 250000 | 2000000 | 10000 | 256650 | 1940938 | 10053 | 0.25 | 4.03 |
| BISDEF | 8 | 15 | 18 | 250000 | 2000000 | 10000 | 256708 | 1970689 | 10065 | 0.24 | 4.81 |
| BISDEF | 8 | 3 | 6 | 250000 | 4000000 | 10000 | 181306 | 4265158 | 10077 | 0.66 | 6.02 |
| BISDEF | 8 | 3 | 12 | 250000 | 4000000 | 10000 | 256454 | 3961202 | 10089 | 0.17 | 3.11 |
| Bisdef | 8 | 3 | 18 | 250000 | 4000000 | 10000 | 257792 | 4033562 | 9957 | 0.06 | 1.30 |
| BISOEF | 8 | 9 | 6 | 250000 | 4000000 | 10000 | 254711 | 4004438 | 10133 | 0.74 | 9.02 |
| BJSDEF | B | 9 | 12 | 250000 | 4000000 | 10000 | 256274 | 3968726 | 10128 | 0.39 | 7.00 |
| BISDEF | 8 | 9 | 18 | 250000 | 4000000 | 10000 | 25697 | 3959580 | 10018 | 0.09 | 1.35 |
| BISDEF | 8 | 15 | 6 | 250000 | 4000000 | 10000 | 258288 | 3410368 | 10034 | 0.17 | 2.01 |
| bisdef | B | 15 | 12 | 250000 | 4000000 | 10000 | 256437 | 4015001 | 10076 | 0.40 | 7.66 |
| BISDEF | 8 | 45 | 18 | 250000 | 4000000 | 10000 | 256769 | 3947656 | 10031 | 0.15 | 2.67 |
| BISDEF | 8 | 3 | 6 | 250000 | 6000000 | 10000 | 210505 | 6181933 | 10038 | 0.34 | 3.31 |
| BISDEF | B | 3 | 12 | 250000 | 6000000 | 10000 | 257638 | 5999909 | 10016 | 0.09 | 1.54 |
| bisdef | 8 | 3 | 18 | 250000 | 6000000 | 10000 | 25759 | 6143002 | 9879 | 0.10 | 2.60 |
| BISDEF | B | 9 | 6 | 250000 | 6000000 | 10000 | 250605 | 5960915 | 10015 | 0.04 | 0.53 |
| BISDEF | 8 | 9 | 12 | 250000 | 6000000 | 10000 | 256631 | 5916836 | 10097 | 0.23 | 4.16 |
| bisoef | $B$ | 9 | 18 | 250000 | 6000000 | 10000 | 256982 | 6009009 | 9947 | 0.11 | 2.07 |
| 81SDEF | 8 | 15 |  | 250000 | 6000000 | 10000 | 257671 | 5560523 | 10007 | 0.15 | 1.58 |
| BISDEF | 8 | 15 | 12 | 250000 | 6000000 | 10000 | 256575 | 6008454 | 10086 | 0.37 | 7.63 |
| Bisoef | 8 | 15 | 18 | 250000 | 600000 | 10000 | 256858 | 5923860 | 9998 | 0.12 | 2.12 |
| BISOEF | B | 3 | 6 | 500000 | 2000000 | 10000 | 827308 | 1314392 | 10161 | 0.80 | 9.67 |
| 8ISOEF | 8 | 3 | 12 | 500000 | 2000000 | 10000 | 416118 | 2116426 | 10038 | 0.23 | 3.28 |
| sisoef | 8 | 3 | 18 | 500000 | 2000000 | 10000 | 556712 | 1924677 | 10164 | 0.19 | 4.10 |
| 81SDEF | 8 | 9 | 6 | 500000 | 2000000 | 10000 | 540037 | 1651813 | 10106 | 0.23 | 3.45 |
| BISDEF | 8 | 9 | 12 | 500000 | 2000000 | 10000 | 527916 | 1898856 | 10049 | 0.22 | 3.63 |
| 8ISOEF | 8 | 9 | 18 | 500000 | 2000000 | 10000 | 527693 | 1939839 | 10022 | 0.13 | 2.71 |
| BISDEF | B | 15 | 6 | 500000 | 2000000 | 10000 | 530341 | 1753342 | 10070 | 0.18 | 2.64 |
| GISDEF | 8 | 15 | 12 | 500000 | 2000000 | 10000 | 528190 | 1863898 | 9978 | 0.12 | 1.88 |
| 81SOEF | B | 15 | 18 | 500000 | 2000000 | 10000 | 527254 | 1925639 | 9967 | 0.14 | 3.03 |
| BISDEF | 8 | 3 | 6 | 500000 | 4000000 | 10000 | 1000000 | 2571543 | 10163 | 0.79 | 9.61 |
| BISDEF | 8 | 3 | 12 | 500000 | 4000000 | 10000 | 423124 | 4172390 | 10139 | 0.49 | 9.01 |
| bisoef | 8 |  | 18 | 500000 | 4000000 | 10000 | 533181 | 3981871 | 9976 | 0.05 | 1.27 |
| bisotf | 8 | 9 | 6 | 500000 | 4000000 | 10000 | 535589 | 3370258 | 10058 | 0.15 | 2.19 |
| BISOEF | 8 | 9 | 12 | 500000 | 4000000 | 10000 | 526341 | 3890345 | 10051 | 0.25 | 4.66 |
| Bisoef | 8 | 9 | 18 | 500000 | 4000000 | 10000 | 527888 | 3893072 | 9997 | 0.11 | 2.45 |
| 81SDEF | 8 | 15 | 6 | 500000 | 4000000 | 10000 | 530729 | 3337015 | 10006 | 0.16 | 2.42 |
| BISDEF | 8 | 15 | 12 | 500000 | 4000000 | 10000 | 527123 | 3810306 | 9963 | 0.16 | 3.06 |
| bisoef | B | 15 | 18 | 500000 | 4000000 | 10000 | 527052 | 3866746 | 9969 | 0.11 | 2.37 |

* ID is the trial value combination identifier, as defined in Table 1.

$$
\text { NOTE: } \quad 1 \text { inch }=25.4 \mathrm{~mm} \quad 1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m}
$$

TABLE B8. Results from BISDEF Sensitivity Study (Case 3 of 10)

| Analys is |  | Thickness (inches) |  | Exact Moculus (psi) |  |  | Calculated Modulus (p6i) |  |  | Error in Basin (mils) | Percent Error in Basin |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Program | 10* | AC | PCC | AC | PCC | Subgr . | ${ }^{\text {A }}$ | PCC | subgr. |  |  |
| bisotf | c | 3 | 6 | 250000 | 2000000 | 10000 | 161248 | 2396335 | 10460 | 2.67 | 27.18 |
| BISOEF | c | 3 | 12 | 250000 | 2000000 | 10000 | 160041 | 2137793 | 10202 | 1.13 | 14.79 |
| BISOEF | c | 3 | 18 | 250000 | 2000000 | 10000 | 207783 | 2054010 | 9994 | 0.28 | 4.45 |
| BISDEF | c | 9 | 6 | 250000 | 2000000 | 10000 | 250160 | 1997344 | 10023 | 0.10 | 1.27 |
| BISDEF | c | 9 | 12 | 250000 | 2000000 | 10000 | 269222 | 1992938 | 10960 | 0.39 | 6.87 |
| BISOEF | c | 9 | 18 | 250000 | 2000000 | 10000 | 234481 | 2409578 | 9415 | 0.34 | 7.31 |
| BISDEF | $c$ | 15 | 6 | 250000 | 2000000 | 10000 | 250300 | 1994576 | 10005 | 0.02 | 0.30 |
| BISDEF | c | 15 | 12 | 250000 | 2000000 | 10000 | 250132 | 1994865 | 10030 | 0.06 | 1.19 |
| Bisdef | c | 15 | 18 | 250000 | 2000000 | 10000 | 250168 | 1985363 | 10067 | 0.07 | 1.84 |
| BISDEF | c | 3 | 6 | 250000 | 4000000 | 10000 | 211797 | 4201053 | 10269 | 1.41 | 15.77 |
| BISDEF | c | 3 | 12 | 250000 | 4000000 | 10000 | 246292 | 3941918 | 10162 | 0.23 | 4.53 |
| BISDEF | c | 3 | 18 | 250000 | 4000000 | 10000 | 223373 | 4237355 | 9788 | 0.21 | 4.68 |
| bisoff | c | 9 | 6 | 250000 | 4000000 | 10000 | 249706 | 4048901 | 10034 | 0.19 | 2.41 |
| BISDEF | c | 9 | 12 | 250000 | 4000000 | 10000 | 238825 | 4751107 | 9440 | 0.35 | 7.39 |
| BISDEF | $c$ | 9 | 18 | 250000 | 4000000 | 10000 | 241211 | 4328049 | 9746 | 0.20 | 4.84 |
| Bisdef | c | 15 | 6 | 250000 | 4000000 | 10000 | 250243 | 3985448 | 10003 | 0.09 | 0.16 |
| BISOEF | c | 15 | 12 | 250000 | 4000000 | 10000 | 243175 | 4461627 | 9584 | 0.45 | 9.83 |
| BISDEF | $c$ | 15 | 18 | 250000 | 4000000 | 10000 | 243318 | 4326505 | 9718 | 0.14 | 2.98 |
| BISOEF | c | 3 | 6 | 250000 | 6000000 | 10000 | 235886 | 6093186 | 10135 | 0.67 | 7.94 |
| BISOEF | c | 3 | 12 | 250000 | 6000000 | 10000 | 226250 | 6458072 | 9520 | 0.44 | 9.25 |
| Bisdef | c | 3 | 18 | 250000 | 6000000 | 10000 | 237095 | 6072893 | 9809 | 0.22 | 6.24 |
| BISDEF | c | 9 | 6 | 250000 | 6000000 | 10000 | 249751 | 6068144 | 10038 | 0.21 | 2.70 |
| BISDEF | c | 9 | 12 | 250000 | 6000000 | 10000 | 261497 | 6603368 | 9601 | 0.26 | 5.16 |
| bisoef | c | 9 | 18 | 250000 | 6000000 | 10000 | 243158 | 6180319 | 9810 | 0.13 | 2.95 |
| BISOEF | c | 15 | 6 | 250000 | 6000000 | 10000 | 250208 | 6001417 | 10004 | 0.02 | 0.36 |
| BISOEF | c | 15 | 12 | 250000 | 6000000 | 10000 | 243540 | 6562433 | 9687 | 0.22 | 4.77 |
| Bisoef | c | 15 | 18 | 250000 | 6000000 | 10000 | 243899 | 6256269 | 9810 | 0.12 | 2.40 |
| 8150EF | c | 3 | 6 | 500000 | 2000000 | 10000 | 406371 | 2297935 | 10099 | 0.65 | 6.34 |
| BISOEF | c | 3 | 12 | 500000 | 2000000 | 10000 | 251673 | 2333352 | 10178 | 0.98 | 13.60 |
| B1SOEF | c | 3 | 18 | 500000 | 2000000 | 10000 | 357252 | 2131699 | 10010 | 0.31 | 5.70 |
| bisoef | c | 9 | 6 | 500000 | 2000000 | 10000 | 500229 | 1983058 | 10019 | 0.03 | 0.55 |
| 81 SOEF | c | 9 | 12 | 500000 | 2000000 | 10000 | 499107 | 1991353 | 10061 | 0.11 | 2.19 |
| Bisoef | c | 9 | 18 | 500000 | 2000000 | 10000 | 483578 | 2193187 | 9531 | 0.37 | 9.79 |
| B1SOEF | c | 15 | 6 | 500000 | 2000000 | 10000 | 494695 | 1939382 | 9965 | 0.34 | 5.61 |
| Bisoef | c | 15 | 12 | 500000 | 2000000 | 10000 | 500106 | 1998399 | 10008 | 0.01 | 0.30 |
| Bisoef | c | 15 | 18 | 500000 | 2000000 | 10000 | 499971 | 1996546 | 10017 | 0.01 | 0.46 |
| bisoef | $c$ | 3 | 6 | 500000 | 4000000 | 10000 | 406450 | 4435306 | 10205 | 1.24 | 13.40 |
| Bisdef | c | 3 | 12 | 500000 | 4000000 | 10000 | 360906 | 4916732 | 9322 | 0.48 | 9.59 |
| 81 SOEF | c | 3 | 18 | 500000 | 4000000 | 10000 | 436223 | 4264346 | 9746 | 0.13 | 3.16 |
| Bisoef | c | 9 | 6 | 500000 | 4000000 | 10000 | 500000 | 4000189 | 10017 | 0.06 | 0.90 |
| BISOEF | c | 9 | 12 | 500000 | 4000000 | 10000 | 499599 | 3991952 | 10045 | 0.07 | 1.66 |
| 81 SDEF | c | 9 | 18 | 500000 | 4000000 | 10000 | 493056 | 4206988 | 9700 | 0.13 | 4.04 |
| 8150EF | c | 15 | 6 | 500000 | 4000000 | 10000 | 500155 | 399661 | 10004 | 0.01 | 0.19 |
| Bisoef | c | 15 | 12 | 500000 | 4000000 | 10000 | 497948 | 6120650 | 9711 | 0.37 | 10.00 |
| BISOEF | c | 15 | 18 | 500000 | 4000000 | 10000 | 497982 | 4110128 | 9731 | 0.20 | 7.13 |

* ID is the trial value combination identifier, as defined in Table 1.
NOTE:
1 inch $=25.4 \mathrm{~mm}$
$1 \mathrm{psi}=6.89 \mathrm{kPa}$
$1 \mathrm{mil}=25.4 \mu \mathrm{~m}$

TABLE B9. Results from BISDEF Sensitivity Study (Case 4 of 10)

| Analysis |  | Thickness (inches) |  | Exact Modulus (psi) |  |  | Calculated Modulus (psi) |  |  | Error in Basin (mils) | Percent <br> Error in Basin |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Program | 10* | AC | PCC | AC | PCC | Subgr. | AC | PCC | Subgr. |  |  |
| Bisoef | D | 3 | 6 | 250000 | 2000000 | 10000 | 259062 | 1965344 | 10029 | 0.18 | 1.85 |
| BISOEF | 0 | 3 | 12 | 250000 | 2000000 | 10000 | 251117 | 1994865 | 10026 | 0.07 | 1.07 |
| bisoef | 0 | 3 | 18 | 250000 | 2000000 | 10000 | 255567 | 1986501 | 10226 | 0.46 | 9.32 |
| BISDEF | 0 | 9 | 6 | 250000 | 2000000 | 10000 | 259257 | 1708286 | 10148 | 0.33 | 5.18 |
| BISOEF | 0 | 9 | 12 | 250000 | 2000000 | 10000 | 251139 | 199017 | 10015 | 0.04 | 0.66 |
| Bisoef | 0 | 9 | 18 | 250000 | 2000000 | 10000 | 256486 | 1989977 | 10168 | 0.42 | 8.83 |
| bisoef | 0 | 15 | 6 | 250000 | 2000000 | 10000 | 258568 | 1657855 | 10114 | 0.19 | 2.75 |
| BISDEF | D | 15 | 12 | 250000 | 2000000 | 10000 | 256630 | 1933361 | 10084 | 0.30 | 5.07 |
| BISOEF | 0 | 15 | 18 | 250000 | 2000000 | 10000 | 256715 | 1990351 | 10107 | 0.35 | 7.80 |
| BISOEF | 0 | 3 | 6 | 250000 | 4000000 | 10000 | 250394 | 3988049 | 10028 | 0.12 | 1.47 |
| BISOEF | 0 | 3 | 12 | 250000 | 4000000 | 10000 | 251123 | 3991944 | 10027 | 0.06 | 1.05 |
| 8ISOEF | 0 | 3 | 18 | 250000 | 4000000 | 10000 | 251157 | 3993006 | 10032 | 0.04 | 1.07 |
| BISDEF | 0 | 9 | 6 | 250000 | 4000000 | 10000 | 251002 | 3966171 | 10017 | 0.06 | 0.81 |
| BISDEF | 0 | 9 | 12 | 250000 | 4000000 | 10000 | 251225 | 3985475 | 10017 | 0.04 | 0.71 |
| BisoEf | 0 | 9 | 18 | 250000 | 4000000 | 10000 | 256679 | 3971402 | 10220 | 0.38 | 9.56 |
| bisoff | 0 | 15 | 6 | 250000 | 4000000 | 10000 | 258369 | 3350531 | 10055 | 0.17 | 2.20 |
| BISDEF | 0 | 15 | 12 | 250000 | 4000000 | 10000 | 251191 | 3978920 | 10005 | 0.03 | 0.36 |
| BISOEF | 0 | 15 | 18 | 250000 | 4000000 | 10000 | 256669 | 4001199 | 10149 | 0.36 | 9.39 |
| BISOEF | 0 | 3 | 6 | 250000 | 6000000 | 10000 | 251156 | 5980508 | 10028 | 0.11 | 1.36 |
| BISOEF | 0 | 3 | 12 | 250000 | 6000000 | 10000 | 251198 | 5989496 | 10029 | 0.06 | 1.10 |
| BISOEF | 0 | 3 | 18 | 250000 | 6000000 | 10000 | 251129 | 5988695 | 10039 | 0.04 | 1.19 |
| BISOEF | 0 | 9 | 6 | 250000 | 6000000 | 10000 | 250713 | 5956153 | 10017 | 0.05 | 0.67 |
| BISOEF | 0 | 9 | 12 | 250000 | 6000000 | 10000 | 251171 | 5984237 | 10018 | 0.04 | 0.76 |
| BISOEF | 0 | 9 | 18 | 250000 | 6000000 | 10000 | 251198 | 5989790 | 10024 | 0.04 | 0.92 |
| 8isoEf | 0 | 15 | 6 | 250000 | 6000000 | 10000 | 257734 | 5482291 | 10026 | 0.15 | 1.75 |
| BISOEF | 0 | 15 | 12 | 250000 | 6000000 | 10000 | 251232 | 5974666 | 10005 | 0.03 | 0.43 |
| BISOEF | 0 | 15 | 18 | 250000 | 6000000 | 10000 | 256742 | 5976336 | 10188 | 0.34 | 9.69 |
| B1SOEF | 0 | 3 | 6 | 500000 | 2000000 | 10000 | 407681 | 2464038 | 10005 | 0.90 | 6.56 |
| Bisoef | 0 | 3 | 12 | 500000 | 2000000 | 10000 | 505029 | 1990394 | 10022 | 0.05 | 0.85 |
| 8isotf | 0 | 3 | 18 | 500000 | 2000000 | 10000 | 523158 | 1976571 | 10204 | 0.39 | 8.54 |
| BISDEF | 0 | 9 | 6 | 500000 | 2000000 | 10000 | 539517 | 1648274 | 10131 | 0.28 | 4.30 |
| BISOEF | 0 | 9 | 12 | 500000 | 2000000 | 10000 | 526836 | 1883982 | 10107 | 0.29 | 5.26 |
| BISOEF | D | 9 | 18 | 500000 | 2000000 | 10000 | 526646 | 1960772 | 10105 | 0.32 | 7.56 |
| BISOEF | 0 | 15 | 6 | 500000 | 2000000 | 10000 | 530419 | 1743021 | 10105 | 0.24 | 3.81 |
| B1SOEF | 0 | 15 | 12 | 500000 | 2000000 | 10000 | 528252 | 1831222 | 10024 | 0.13 | 2.16 |
| BISOEF | 0 | 15 | 18 | 500000 | 2000000 | 10000 | 527090 | 193964 | 10012 | 0.23 | 5.67 |
| 8ISOEF | D | 3 | 6 | 500000 | 4000000 | 10000 | 365279 | 5392805 | 10061 | 2.38 | 19.69 |
| BISDEF | 0 | 3 | 12 | 500000 | 4000000 | 10000 | 506160 | 3985240 | 10023 | 0.05 | 0.87 |
| bisoef | 0 | 3 | 18 | 500000 | 4000000 | 10000 | 522851 | 3954332 | 10266 | 0.36 | 9.76 |
| Bispef | 0 | 9 | 6 | 500000 | 4000000 | 10000 | 535160 | 3295503 | 10098 | 0.19 | 2.93 |
| BISDEF | 0 | 9 | 12 | 500000 | 4000000 | 10000 | 506327 | 3959337 | 10004 | 0.02 | 0.34 |
| 8ISOEF | D | 9 | 18 | 500000 | 4000000 | 10000 | 526287 | 3952167 | 10162 | 0.34 | 9.48 |
| BISOEF | 0 | 45 | 6 | 500000 | 4000000 | 10000 | 531170 | 3218562 | 10054 | 0.15 | 2.32 |
| BISOEF | 0 | 15 | 12 | 500000 | 4000000 | 10000 | 526641 | 3748265 | 10022 | 0.18 | 3.72 |
| BISOEF | D | 13 | 18 | 500000 | 4000000 | 10000 | 526676 | 3957921 | 10058 | 0.30 | 9.00 |

* ID is the trial value combination identifier, as defined in Table 1.

NOTE: $\quad 1$ inch $=25.4 \mathrm{~mm} \quad 1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m}$

TABLE B10. Results from BISDEF Sensitivity Study (Case 5 of 10)

| Analysis |  | Thickness (inches) |  | $\begin{aligned} & \text { Exact Modul us } \\ & \text { (psi) } \end{aligned}$ |  |  | Calculated Modulus ( p i i ) |  |  | Error in Basin (mils) | Percent Error in Basin |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Program | 10* | AC | PCC | AC | PCC | Subgr. | AC | PCC | Subgr. |  |  |
| BISDEF | $E$ | 3 | 6 | 250000 | 2000000 | 10000 | 133241 | 2504070 | 10628 | 3.30 | 35.13 |
| BISOEF | $E$ | 3 | 12 | 250000 | 2000000 | 10000 | 50000 | 5920516 | 10256 | 10.85 | 151.28 |
| BISDEF | E | 3 | 18 | 250000 | 2000000 | 10000 | 165185 | 2044621 | 10064 | 0.61 | 8.78 |
| BISDEF | E | 9 | 6 | 250000 | 2000000 | 10000 | 249705 | 2027634 | 10046 | 0.26 | 3.05 |
| BISDEF | E | 9 | 12 | 250000 | 2000000 | 10000 | 249289 | 2011356 | 10103 | 0.30 | 5.23 |
| BISDEF | $E$ | 9 | 18 | 250:30 | 2000000 | 10000 | 236279 | 2046910 | 10034 | 0.23 | 3.58 |
| BISDEF | E | 15 | 6 | 250000 | 2000000 | 10000 | 250539 | 1986973 | 10010 | 0.04 | 0.52 |
| bisdef | E | 15 | 12 | 250000 | 2000000 | 10000 | 25079 | 1987385 | 10004 | 0.01 | 0.17 |
| BISDEF | E | 15 | 18 | 250000 | 2000000 | 10000 | 231681 | 2099482 | 10127 | 0.41 | 6.98 |
| Bisdef | $E$ | 3 | 6 | 250000 | 4000000 | 10000 | 50000 | 10000000 | 11407 | 11.57 | 115.35 |
| bisoef | E | 3 | 12 | 250000 | 4000000 | 10000 | 159073 | 4017631 | 10125 | 0.72 | 9.14 |
| 8isoef | E | 5 | 18 | 250000 | 4000000 | 10000 | 237627 | 4010427 | 10030 | 0.09 | 2.09 |
| Bisoef | E | 9 | 6 | 250000 | 4000000 | 10000 | 250463 | 4041732 | 10029 | 0.20 | 2.35 |
| Bisoef | E | 9 | 12 | 250000 | 4000000 | 10000 | 240420 | 4118975 | 10014 | 0.21 | 3.32 |
| 8ISDEF | E |  | 18 | 250000 | 4000000 | 10000 | 245232 | 4021122 | 10029 | 0.10 | 2.06 |
| BISOEF | E | 15 | 6 | 250000 | 4000000 | 10000 | 250849 | 3906283 | 10008 | 0.02 | 0.32 |
| bisoef | E | 15 | 12 | 250000 | 4000000 | 10000 | 230366 | 4827785 | 9873 | 0.53 | 9.30 |
| BISDEF | $E$ | 15 | 18 | 250000 | 4000000 | 10000 | 246130 | 4043869 | 10031 | 0.10 | 2.26 |
| bisoef | $\varepsilon$ | 3 | 6 | 250000 | 6000000 | 10000 | 184879 | 6290070 | 10297 | 1.30 | 16.07 |
| gisoef | E | 3 | 12 | 250000 | 6000000 | 10000 | 224677 | 6037907 | 10016 | 0.16 | 2.39 |
| bisoef | E | 3 | 18 | 250000 | 6000000 | 10000 | 242877 | 6002143 | 10035 | 0.06 | 1.70 |
| bisdef | E | 9 | 6 | 250000 | 6000000 | 10000 | 248631 | 6376402 | 10107 | 0.76 | 9.47 |
| bisoef | E | 9 | 12 | 250000 | 6000000 | 10000 | 262215 | 6067810 | 10021 | 0.14 | 2.26 |
| bisoef | E | 9 | 18 | 250000 | 6000000 | 10000 | 175136 | 6988718 | 10196 | 2.06 | 48.06 |
| bisoef |  | 15 | 6 | 250000 | 6000000 | 10000 | 250819 | 5875107 | 10003 | 0.03 | 0.40 |
| bisoef | E | 15 | 12 | 250000 | 6000000 | 10000 | 245743 | 6129836 | 10007 | 0.11 | 2.00 |
| bisoef | E | 15 | 18 | 250000 | 6000000 | 10000 | 246425 | 6032066 | 10044 | 0.09 | 2.33 |
| bisoef | E | 3 | 6 | 500000 | 2000000 | 10000 | 302959 | 3306509 | 10122 | 3.18 | 22.80 |
| bisoef | E | 3 | 12 | 500000 | 2000000 | 10000 | 378105 | 2448220 | 9958 | 1.30 | 17.26 |
| B1S0EF | E | 3 | 18 | 500000 | 2000000 | 10000 | 320646 | 2128307 | 10064 | 0.40 | 7.02 |
| bisoef | E | 9 | 6 | 500000 | 2000000 | 10000 | 495845 | 2009776 | 10041 | 0.09 | 1.51 |
| bisoef | E | 9 | 12 | 500000 | 2000000 | 10000 | 500575 | 1993885 | 10013 | 0.02 | 0.45 |
| bisoef | E | 9 | 18 | 500000 | 2000000 | 10000 | 477073 | 2006085 | 10343 | 0.36 | 9.93 |
| bisoef | E | 15 | 6 | 500000 | 2000000 | 10000 | 487225 | 2279598 | 9956 | 0.17 | 2.87 |
| Bisdef | $E$ | 15 | 12 | 500000 | 2000000 | 10000 | 493760 | 1811284 | 10043 | 0.46 | 9.85 |
| BISDEF | E | 15 | 18 | 500000 | 2000000 | 10000 | 521830 | 1942643 | 9964 | 0.11 | 2.29 |
| gispef | E | 3 | 6 | 500000 | 4000000 | 10000 | 360910 | 5260935 | 10164 | 2.27 | 20.55 |
| Bisoef | E | 3 | 12 | 500000 | 4000000 | 10000 | 198282 | 4004845 | 10410 | 1.17 | 16.99 |
| B1SDEF | $E$ | 3 | 18 | 500000 | 4000000 | 10000 | 460939 | 4028513 | 10029 | 0.08 | 1.94 |
| bisdef | E | 9 | 6 | 500000 | 4000000 | 10000 | 500492 | 3971804 | 10016 | 0.03 | 0.53 |
| bisoef | E | 9 | 12 | 500000 | 4000000 | 10000 | 487189 | 409708 | 10012 | 0.08 | 1.47 |
| SISOEF | E | 9 | 18 | 500000 | 4000000 | 10000 | 496037 | 4009577 | 10035 | 0.04 | 1.29 |
| BISDEF | E | 15 | 6 | 500000 | 4000000 | 10000 | 489651 | 3335941 | 10139 | 0.54 | 8.74 |
| Bisoef | E | 15 | 12 | 500000 | 4000000 | 10000 | 492762 | 3731126 | 10085 | 0.26 | 5.63 |
| BISDEF | E | 15 | 18 | 500000 | 4000000 | 17000 | 49390 | 3990124 | 10049 | 0.03 | 1.28 |

* 10 is the trial value combination identifier, as defined in Table 1.

NOTE: $\quad 1$ inch $=25.4 \mathrm{~mm} \quad 1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m}$

TABLE B11. Results from BISDEF Sensitivity Study (Case 6 of 10)

| Anatysis |  | Thickness (inches) |  | Exact Modulus (psi) |  |  | Calculated Modulus (psi) |  |  | Error in Basin (mils) | Percent Error in Basin |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Program | 10* | AC | PCC | AC | PCC | Subgr. | AC | PCC | Subgr. |  |  |
| bisoef | F | 3 | 6 | 250000 | 2000000 | 10000 | 879935 | 827657 | 9977 | 0.42 | 3.26 |
| BISOEF | F | 3 | 12 | 250000 | 2000000 | 10000 | 151254 | 2098252 | 10091 | 0.85 | 9.10 |
| 81SOEF | F | 3 | 18 | 250000 | 2000000 | 10000 | 228702 | 2015781 | 10125 | 0.28 | 5.86 |
| BISDEF | F | 9 | 6 | 250000 | 2000000 | 10000 | 262131 | 1626267 | 10150 | 0.32 | 5.06 |
| BiSOEF | F | 9 | 12 | 250000 | 2000000 | 10000 | 249196 | 2001356 | 10026 | 0.05 | 0.97 |
| BISOEF | F | 9 | 18 | 250000 | 2000000 | 10000 | 250214 | 1999244 | 10013 | 0.03 | 0.56 |
| BISDEF | F | 15 | 6 | 250000 | 2000000 | 10000 | 258533 | 1615740 | 10080 | 0.22 | 2.99 |
| BISOEF | $F$ | 15 | 12 | 250000 | 2000000 | 10000 | 253462 | 1813697 | 10040 | 0.24 | 4.58 |
| Bisdef | F | 15 | 18 | 250000 | 2000000 | 10000 | 253038 | 2031291 | 10092 | 0.32 | 7.66 |
| 8150EF | F | 3 | 6 | 250000 | 4000000 | 10000 | 1000000 | 1813380 | 10042 | 0.73 | 6.68 |
| BISOEF | F | 3 | 12 | 250000 | 4000000 | 10000 | 234763 | 4017062 | 10185 | 0.42 | 7.90 |
| BISDEF | F | 3 | 18 | 250000 | 4000000 | 10000 | 250413 | 4044409 | 9930 | 0.03 | 0.71 |
| 8ISOEF | F | 9 | 6 | 250000 | 4000000 | 10000 | 249662 | 4030754 | 9999 | 0.03 | 0.27 |
| BISOEF | F | 9 | 12 | 250000 | 4000000 | 10000 | 249849 | 3996977 | 10019 | 0.03 | 0.70 |
| 81SOEF | $F$ | 9 | 18 | 250000 | 4000000 | 10000 | 253472 | 3983805 | 10031 | 0.08 | 1.61 |
| BISOEF | F | 15 | 6 | 250000 | 4000000 | 10000 | 249413 | 4035047 | 9997 | 0.01 | 0.15 |
| bisoff | F | 15 | 12 | 250000 | 4000000 | 10000 | 250398 | 3992510 | 10002 | 0.01 | 0.12 |
| 81SDEF | F | 15 | 18 | 250000 | 4000000 | 10000 | 253533 | 4080918 | 10069 | 0.29 | 7.95 |
| bisoef | F | 3 | 6 | 250000 | 6000000 | 10000 | 218951 | 6569209 | 10063 | 1.07 | 10.16 |
| bisdef | F | 3 | 12 | 250000 | 6000000 | 10000 | 246311 | 5950721 | 10141 | 0.20 | 4.33 |
| BISDEF | F | 3 | 18 | 250000 | 6000000 | 10000 | 252679 | 6110033 | 9907 | 0.06 | 1.55 |
| Bisoef | F | 9 | 6 | 250000 | 6000000 | 10000 | 248811 | 6014005 | 10025 | 0.07 | 1.05 |
| BisoEf | F | 9 | 12 | 250000 | 6000000 | 10000 | 250265 | 5991858 | 10021 | 0.04 | 0.84 |
| Bisoef | F | 9 | 18 | 250000 | 6000000 | 10000 | 253932 | 5969563 | 10020 | 0.05 | 0.91 |
| BISOEF | F | 15 | 6 | 250000 | 6000000 | 10000 | 249372 | 6093681 | 10000 | 0.03 | 0.43 |
| B1SDEF | F | 15 | 12 | 250000 | 6000000 | 10000 | 245706 | 6108663 | 10021 | 0.11 | 2.12 |
| Bisot | F | 15 | 18 | 250000 | 6000000 | 10000 | 253932 | 6000007 | 10090 | 0.19 | 5.42 |
| BISOEF | F | 3 | 6 | 500000 | 2000000 | 10000 | 67442 | 1588329 | 10022 | 0.14 | 1.79 |
| BISOEF | F | 3 | 12 | 500000 | 2000000 | 10000 | 827236 | 1609486 | 10238 | 0.54 | 8.83 |
| BISOEF | F | 3 | 18 | 500000 | 2000000 | 10000 | 387283 | 2088084 | 10050 | 0.25 | 4.75 |
| BisoEf | $F$ | 9 | 6 | 500000 | 2000000 | 10000 | 565446 | 1559577 | 10128 | 0.39 | 5.42 |
| Bisoef | F | 9 | 12 | 500000 | 2000000 | 10000 | 524058 | 1818226 | 10059 | 0.12 | 1.98 |
| BISDEF | F | 9 | 18 | 500000 | 2000000 | 10000 | 521140 | 1981504 | 10039 | 0.23 | 5.13 |
| BISOEF | F | 15 | 6 | 500000 | 2000000 | 10000 | 508755 | 1950299 | 10001 | 0.07 | 0.97 |
| SISOEF | F | 15 | 12 | 500000 | 2000000 | 10000 | 361747 | 3860852 | 10364 | 1.56 | 34.19 |
| bisoef | F | 15 | 18 | 500000 | 2000000 | 10000 | 525101 | 1926951 | 9914 | 0.13 | 2.82 |
| Bisot | F | 3 | 6 | 500000 | 4000000 | 10000 | 387754 | 5446661 | 10056 | 2.69 | 22.61 |
| 8ISDEF | F | 3 | 12 | 500000 | 4000000 | 10000 | 400947 | 4151131 | 10033 | 0.23 | 3.76 |
| BISDEF | F | 3 | 18 | 500000 | 4000000 | 10000 | 518270 | 3964795 | 9963 | 0.06 | 1.70 |
| BISDEF | F | 9 | 6 | 500000 | 4000000 | 10000 | 548004 | 2623268 | 10135 | 0.70 | 8.57 |
| BISOEF | F | 9 | 12 | 500000 | 4000000 | 10000 | 503781 | 3975467 | 10001 | 0.01 | 0.21 |
| BISOEF | F |  | 18 | 500000 | 4000000 | 10000 | 523506 | 3961266 | 9996 | 0.12 | 2.74 |
| BISOEF | $F$ | 15 | 6 | 500000 | \$000000 | 10000 | 534999 | 2660997 | 10059 | 0.35 | 6.68 |
| Bisoef | F | 15 | 12 | 500000 | 4000000 | 10000 | 526591 | 3779924 | 9870 | 0.20 | 4.73 |
| Bisotf | F | 15 | 18 | 500000 | 4000000 | 10000 | 524063 | $399 \% 11$ | 9955 | 0.21 | 5.86 |

* ID is the trial value combination identifier, as defined in Table 1.

NOTE: 1 inch $=25.4 \mathrm{~mm} \quad 1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m}$

TABLE B12. Results from BISDEF Sensitivity Study (Case 7 of 10)

| Analys is |  | Thickness (inches) |  | Exact Modulus (psi) |  |  | Calculated Modulus (psi) |  |  | Error in Basin (mils) | Percent <br> Error in <br> Basin |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Program | 10* | AC | PCC | AC | PCC | Subgr. | AC | PCC | Stogr . |  |  |
| 8ISDEF | G | 3 | 6 | 250000 | 2000000 | 10000 | 105563 | 3090122 | 11446 | 8.87 | 84.21 |
| bisdef | $G$ | 3 | 12 | 250000 | 2000000 | 10000 | 129858 | 1919528 | 17693 | 14.56 | 224.85 |
| bisdef | G | 3 | 18 | 250000 | 2000000 | 10000 | 136344 | 1918016 | 12636 | 3.74 | 81.99 |
| bisdef | G | 9 | 6 | 250000 | 2000000 | 10000 | 250555 | 1995277 | 10043 | 0.20 | 2.50 |
| BISDEF | G | 9 | 12 | 250000 | 2000000 | 10000 | 245521 | 2076907 | 10969 | 2.62 | 45.26 |
| bisdef | G | 9 | 18 | 250000 | 2000000 | 10000 | 247451 | 1871041 | 11707 | 2.24 | 53.95 |
| 8ISDEF | G | 15 | 6 | 250000 | 2000000 | 10000 | 250795 | 1977329 | 10008 | 0.03 | 0.34 |
| BISCEF | G | 15 | 12 | 250000 | 2000000 | 10000 | 250410 | 1999714 | 10087 | 0.20 | 4.07 |
| BISDEF | $G$ | 15 | 18 | 250000 | 2000000 | 10000 | 250284 | 1978574 | 10371 | 0.50 | 13.45 |
| BISOEF | G | 3 | 6 | 250000 | 4000000 | 10000 | 192590 | 4756573 | 10737 | 4.83 | 49.05 |
| BISOEF | G | 3 | 12 | 250000 | 4000000 | 10000 | 210021 | 3528192 | 12696 | 4.43 | 83.11 |
| BISDEF | 6 | 3 | 18 | 250000 | 4000000 | 10000 | 239514 | 3746862 | 11120 | 0.97 | 27.19 |
| BISOEF | $G$ | 9 | 6 | 250000 | 4000000 | 10000 | 249814 | 4091792 | 10069 | 0.40 | 4.92 |
| BISDEF | G | 9 | 12 | 250000 | 4000000 | 10000 | 249201 | 3971678 | 10697 | 1.45 | 28.86 |
| BISDEF | $G$ | 9 | 18 | 250000 | 4000000 | 10000 | 250207 | 3861399 | 10694 | 0.68 | 19.81 |
| BISDEF | $G$ | 15 | 6 | 250000 | 4000000 | 10000 | 250727 | 39674.t. | 10003 | 0.01 | 0.17 |
| BISDEF | G | 15 | 12 | 250000 | 4000000 | 10000 | 250433 | 40164: | 10129 | 0.28 | 6.20 |
| BISDEF | G | 15 | 18 | 250000 | 4000000 | 10000 | 250429 | 3961488 | 10307 | 0.33 | 10.26 |
| BISOEF | G | 3 | 6 | 250000 | 6000000 | 10000 | 231272 | 6365777 | 10348 | 2.14 | 23.62 |
| Bisdef | G | 3 | 12 | 250000 | 6000000 | 10000 | 245935 | 5900618 | 10439 | 0.76 | 15.73 |
| bisoef | $G$ | 3 | 18 | 250000 | 6000000 | 10000 | 250246 | 5945582 | 10183 | 0.16 | 4.94 |
| BISOEF | G |  | 6 | 250000 | 6000000 | 10000 | 249324 | 6227761 | 10113 | 0.64 | 8.29 |
| B1SDEF | G | 9 | 12 | 250000 | 6000000 | 10000 | 250285 | 5960264 | 10274 | 0.50 | 10.84 |
| BISOEF | 6 | 5 | 18 | 250000 | 6000000 | 10000 | 250585 | 5951286 | 10166 | 0.15 | 4.74 |
| bisoef | 6 | 15 | 6 | 250000 | 6000000 | 10000 | 250686 | 5971906 | 10003 | 0.03 | 0.39 |
| Bisdef | G | 15 | 12 | 250000 | 6000000 | 10000 | 250551 | 6008014 | 10109 | 0.20 | 4.74 |
| 8isdef | 6 | 15 | 18 | 250000 | 6000000 | 10000 | 250567 | 5964402 | 10122 | 0.19 | 3.79 |
| BISDEF | G | 3 | 6 | 500000 | 2000000 | 10000 | 134573 | 3642821 | 11000 | 5.66 | 58.01 |
| BISOEF | G | 3 | 12 | 500000 | 2000000 | 10000 | 155930 | 2244997 | 13008 | 6.19 | 103.53 |
| BiSOEF | G | 3 | 18 | 500000 | 2000000 | 10000 | 163627 | 1968355 | 12882 | 3.88 | 86.00 |
| Bisoef | $G$ |  | 6 | 500000 | 2000000 | 10000 | 499790 | 1972911 | 10059 | 0.14 | 2.24 |
| BISDEF | 6 | 9 | 12 | 500000 | 2000000 | 10000 | 494667 | 2026066 | 10305 | 0.72 | 13.91 |
| Bisoef | $G$ | 9 | 18 | 500000 | 2000000 | 10000 | 496017 | 1965246 | 10556 | 0.72 | 19.13 |
| BISOEF | G | 15 | 6 | 500000 | 2000000 | 10000 | 492813 | 2023165 | 9829 | 0.54 | 9.60 |
| BISOEF | 6 | 15 | 12 | 500000 | 2000000 | 10000 | 500098 | 198337 | 10048 | 0.06 | 1.55 |
| BISOEF | 6 | 15 | 18 | 500000 | 2000000 | 10000 | 499851 | 1981532 | 10125 | 0.12 | 3.76 |
| Bisoef | $G$ | 3 | 6 | 500000 | 4000000 | 10000 | 343449 | 5050879 | 10595 | 3.88 | 40.67 |
| BISDEF | $G$ | 3 | 12 | 500000 | 4000000 | 10000 | 377118 | 3784686 | 12116 | 3.69 | 70.24 |
| Bisoef | 6 | 3 | 18 | 500000 | 4000000 | 10000 | 460432 | 3828411 | 10961 | 0.89 | 25.41 |
| BISDEF | 6 | 9 | 6 | 500000 | 4000000 | 10000 | 499710 | 4015614 | 10055 | 0.20 | 2.99 |
| 8150 FF | G | 9 | 12 | 500000 | 4000000 | 10000 | 498145 | 4012798 | 10251 | 0.51 | 11.06 |
| Bisoef | G | 9 | 18 | 500000 | 4000000 | 10000 | 499430 | 3946127 | 10340 | 0.34 | 10.71 |
| Bisdef | 6 | 15 | 6 | 500000 | 4000000 | 10000 | 500217 | 398849 | 10011 | 0.02 | 0.39 |
| Bisoef | 6 | 15 | 12 | 500000 | 4000000 | 10000 | 499919 | 3992533 | 10050 | 0.07 | 1.82 |
| Bisdef | G | 15 | 18 | 500000 | 4000000 | 10000 | 49963 | 3980458 | 10117 | 0.10 | 3.66 |

* ID is the trial value combination identifier, as defined in Table 1.
NOTE:
1 inch $=25.4$ mm
$1 \mathrm{psi}=6.89 \mathrm{kPa}$
$1 \mathrm{mil}=25.4 \mu \mathrm{~m}$

TABLE B13. Results from BISDEF Sensitivity Study (Case 8 of 10)

| Analys is |  | Thickness (inches) |  | Exact Mooulus (psi) |  |  | Calculated Modulus (psi) |  |  | Error in Basin (mils) | Percent <br> Error in Basin |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Program | 10* | AC | PCC | $A C$ | PCC | Subgr . | $A C$ | PCC | Subgr. |  |  |
| bisdef | H | 3 | 6 | 250000 | 2000000 | 10000 | 889594 | 817377 | 9982 | 0.42 | 3.32 |
| BISDEF | H | 3 | 12 | 250000 | 2000000 | 10000 | 247971 | 1999886 | 10031 | 0.08 | 1.30 |
| BISDEF | H | 3 | 18 | 250000 | 2000000 | 10000 | 249997 | 1997721 | 10033 | 0.06 | 1.21 |
| BISDEF | H | 9 | 6 | 250000 | 2000000 | 10000 | 259627 | 1516644 | 10171 | 0.63 | 7.56 |
| Bisoef | ${ }^{H}$ | 9 | 12 | 250000 | 2000000 | 10000 | 249864 | 1997435 | 10020 | 0.04 | 0.74 |
| BISOEF | H | 9 | 18 | 250000 | 2000000 | 10000 | 250369 | 1997562 | 10022 | 0.04 | 0.88 |
| BISDEF | H | 15 | 6 | 250000 | 2000000 | - 900 | 257417 | 1476157 | 10189 | 0.36 | 5.12 |
| BISDEF | ${ }_{H}$ | 15 | 12 | 250000 | 2000000 | . 3000 | 253617 | 1886130 | 10068 | 0.08 | 1.39 |
| BISDEF | H | 15 | 18 | 250000 | 2000000 | 10000 | 250665 | 1994306 | 10012 | 0.02 | 0.49 |
| Bisoef | H | 3 | 6 | 250000 | 4000000 | 10000 | 222022 | 4133917 | 10037 | 0.27 | 2.68 |
| Bisdef | H | 3 | 12 | 250000 | 4000000 | 10000 | 250446 | 3992004 | 10033 | 0.07 | 1.27 |
| bisoef | H | 3 | 18 | 250000 | 4000000 | 10000 | 250591 | 3991540 | 10048 | 0.06 | 1.58 |
| Bisot | H | 9 | 6 | 250000 | 4000000 | 10000 | 250628 | 3367824 | 10186 | 0.56 | 6.93 |
| 8ISOEF | H | 9 | 12 | 250000 | 4000000 | 10000 | 250330 | 3989929 | 10022 | 0.04 | 0.75 |
| BisoEf | H | 9 | 18 | 250000 | 4000000 | 10000 | 250669 | 3989145 | 10035 | 0.04 | 1.08 |
| 8isoEf | H | 15 | 6 | 250000 | 4000000 | 10000 | 257703 | 2637815 | 10109 | 0.73 | 9.89 |
| BISDEF | H | 15 | 12 | 250000 | 4000000 | 10000 | 250464 | 3987940 | 10010 | 0.02 | 0.36 |
| BISDEF | H | 15 | 18 | 250000 | 4000000 | 10000 | 250624 | 3996227 | 10013 | 0.03 | 0.65 |
| BISOEF | H | 3 | 6 | 250000 | 6000000 | 10000 | 246431 | 6004513 | 10030 | 0.11 | 1.45 |
| BISDEF | H | 3 | 12 | 250000 | 6000000 | 10000 | 250764 | 5988325 | 10637 | 0.07 | 1.39 |
| Bisot | H | 3 | 18 | 250000 | 6000000 | 10000 | 250648 | 5984919 | 10062 | 0.06 | 1.90 |
| BISDEF | H | 9 | 6 | 250000 | 6000000 | 10000 | 249084 | 6004527 | 10018 | 0.05 | 0.69 |
| BISDEF | H | 9 | 12 | 250000 | 6000000 | 10000 | 250527 | 5986767 | 10024 | 0.04 | 0.88 |
| BISDEF | H | 9 | 18 | 250000 | 6000000 | 10000 | 250719 | 5989876 | 10037 | 0.04 | 1.28 |
| Bisoef | H | 15 | 6 | 250000 | 6000000 | 10000 | 249427 | 6080825 | 10000 | 0.02 | 0.35 |
| 81SOEF | ${ }^{\prime}$ | 15 | 12 | 250000 | 8000000 | 10000 | 250596 | 5980799 | 10006 | 0.01 | 0.20 |
| Bisdef | H | 15 | 18 | 250000 | 6000000 | 10000 | 250709 | 5983825 | 10025 | 0.03 | 0.83 |
| BISOEF | H | 3 | 6 | 500000 | 2000000 | 10000 | 643374 | 1663801 | 10009 | 0.11 | 1.20 |
| Bisot | H | 3 | 12 | 500000 | 2000000 | 10000 | 448769 | 2073080 | 10031 | 0.15 | 2.35 |
| Bisoef | H | 3 | 18 | 500000 | 2000000 | 10000 | 497366 | 1999220 | 10033 | 0.05 | 1.18 |
| Bisotf | H | 9 | 6 | 500000 | 2000000 | 10000 | 553025 | 1507549 | 10187 | 0.32 | 5.31 |
| 8isotf | H | 9 | 12 | 500000 | 2000000 | 10000 | 524632 | 1821200 | 10116 | 0.13 | 2.43 |
| bisoef | H | 9 | 18 | 500000 | 2000000 | 10000 | 505431 | 1985732 | 10009 | 0.03 | 0.53 |
| Bisoef | H | 15 | 6 | 500000 | 2000000 | 10000 | 534136 | 1792955 | 10149 | 0.55 | 9.06 |
| Bisoef | H | 15 | 12 | 500000 | 2000000 | 10000 | 529226 | 1748103 | 9992 | 0.22 | 4.97 |
| BISOEF | H | 15 | 18 | 500000 | 2000000 | 10000 | 525692 | 1922346 | 9974 | 0.13 | 2.60 |
| gisoef | $\stackrel{H}{H}$ | 3 | 6 | 500000 | 4000000 | 10000 | 347687 | 5898103 | 10047 | 3.24 | 27.12 |
| bisote | H | 3 | 12 | 500000 | 4000000 | 10000 | 499610 | 3995224 | 10030 | 0.07 | 1.23 |
| BISOEF | $\stackrel{H}{*}$ | 3 | 18 | 500000 | 4000000 | 10000 | 503500 | 3987155 | 10046 | 0.05 | 1.38 |
| bisoef | ${ }_{N}$ | 9 | 6 | 500000 | 4000000 | 10000 | 544914 | 2728681 | 10157 | 0.54 | 6.92 |
| Bisote | H | 9 | 12 | 500000 | 4000000 | 10000 | 504035 | 3970681 | 10006 | 0.02 | 0.24 |
| BISDEF | H | 9 | 18 | 500000 | 4000000 | 10000 | 505095 | 3974505 | 10020 | 0.03 | 0.64 |
| Bisotm | ${ }^{\text {H }}$ | 15 | 6 | 500000 | 4000000 | 10000 | 534470 | 2099937 | 10117 | 0.24 | 4.24 |
| O1SOEF | H | 15 | 12 | 500000 | 4000000 | 10000 | 525105 | 3593931 | $9 \% 6$ | 0.21 | 5.06 |
| Bisot | N | 15 | 18 | 500000 | 4000000 | 10000 | 505646 | 3965610 | 9996 | 0.03 | 0.62 |

* ID is the trial value combination identifier, as defined in Table 1.

NOTE: $\quad 1$ inch $=25.4 \mathrm{~mm} \quad 1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m}$

TABLE B14. Results from BISDEF Sensitivity Study (Case 9 of 10)

| Analysis |  | Thickness <br> (inches) |  | Exact Modulus (psi) |  |  | Calculated Modulua (psi) |  |  | Error in Basin (mils) | Percent <br> Error in Basin |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Program | 10* | AC | PCC | AC | PCC | Subgr. | ${ }^{\text {AC }}$ | PCC | Subgr . |  |  |
| BISDEF | 1 | 3 | 6 | 250000 | 2000000 | 10000 | 168808 | 2358043 | 10201 | 1.20 | 12.14 |
| BISDEF | 1 | 3 | 12 | 250000 | 2000000 | 10000 | 528257 | 1541424 | 10319 | 0.79 | 11.03 |
| BISDEF | 1 | 3 | 18 | 250000 | 2000000 | 10000 | 167035 | 2069460 | 10007 | 0.58 | 8.32 |
| Bisoef | 1 | 9 | 6 | 250000 | 2000000 | 10000 | 243033 | 2273141 | 10113 | 0.81 | 9.18 |
| bisoff | 1 | 9 | 12 | 250000 | 2000000 | 10000 | 245301 | 2016103 | 10069 | 0.17 | 2.85 |
| BiSOEF | 1 | 9 | 18 | 250000 | 2000000 | 10000 | 238084 | 2026756 | 10145 | 0.32 | 6.46 |
| BISOEF | 1 | 15 | 6 | 250000 | 2000000 | 10000 | 256658 | 1512770 | 10183 | 0.33 | 4.74 |
| bisoef | 1 | 15 | 12 | 250000 | 2000000 | 10000 | 251274 | 1985225 | 9987 | 0.04 | 0.71 |
| B1SOEF | 1 | 15 | 18 | 250000 | 2000000 | 10000 | 65495 | 4232873 | 9324 | 12.26 | 159.86 |
| BISOEF | 1 | 3 | 6 | 250000 | 4000000 | 10000 | 154482 | 4538729 | 10002 | 0.81 | 5.96 |
| BISDEF | 1 | 3 | 12 | 250000 | 4000000 | 10000 | 50000 | 8848968 | 11194 | 10.69 | 178.70 |
| BiSOEF | 1 | 3 | 18 | 250000 | 4000000 | 10000 | 155974 | 4105146 | 9732 | 1.00 | 20.69 |
| SISOEF | 1 | 9 | 6 | 250000 | 4000000 | 10000 | 238354 | 4429342 | 9974 | 0.19 | 1.99 |
| BISOEF | 1 | 9 | 12 | 250000 | 4000000 | 10000 | 237896 | 4035027 | 10217 | 0.45 | 8.57 |
| 81SOEF | 1 | 9 | 18 | 250000 | 4000000 | 10000 | 230950 | 3924100 | 10534 | 0.62 | 15.76 |
| BISOEF | 1 | 15 | 6 | 250000 | 4000000 | 10000 | 249506 | 4040417 | 9993 | 0.01 | 0.21 |
| BISOEF | 1 | 15 | 12 | 250000 | 4000000 | 10000 | 251590 | 3919017 | 10021 | 0.04 | 0.73 |
| Bisoef | 1 | 15 | 18 | 250000 | ¢. 200000 | 10000 | 230436 | 4114299 | 10083 | 0.39 | 6.25 |
| BISOEF | 1 | 3 | 6 | 250000 | 6000000 | 10000 | 158512 | 6444378 | 9969 | 0.68 | 5.19 |
| 81SOEF | 1 | 3 | 12 | 250000 | 6000000 | 10000 | 50000 | 10000000 | 12452 | 10.16 | 190.13 |
| alsoef | 1 | 3 | 18 | 250000 | 6000000 | 10000 | 156115 | 6063662 | 9251 | 1.71 | 45.75 |
| bisoef | 1 | 9 | 6 | 250000 | 6000000 | !0000 | 236769 | 6533122 | 9990 | 0.23 | 2.18 |
| bisoef | 1 | 9 | 12 | 250000 | 6000000 | 10000 | 232828 | 5984338 | 10389 | 0.66 | 13.75 |
| BISOEF | 1 | 9 | 18 | 250000 | 6000000 | 10000 | 209667 | 6297082 | 9876 | 0.64 | 10.30 |
| Bisoef | 1 | 15 | 6 | 250000 | 6000000 | 10000 | 249289 | 6164541 | 10004 | 0.06 | 0.96 |
| Bisotf | 1 | 15 | 12 | 250000 | 6000000 | 10000 | 251657 | 5828802 | 10094 | 0.05 | 1.18 |
| 8tsoef | 1 | 15 | 18 | 250000 | 6000000 | 10000 | 228263 | 6016634 | 10195 | 0.45 | 8.02 |
| bisoef | 1 | 3 | 6 | 500000 | 2000000 | 10000 | 185337 | 3202228 | 9971 | 0.93 | 7.28 |
| bisota | ! | 3 | 12 | 500000 | 2000000 | 10000 | 343468 | 2655563 | 10247 | 1.98 | 28.45 |
| BISDEF | ! | 3 | 18 | 500000 | 2000000 | 10000 | 448055 | 2014078 | 10263 | 0.37 | 8.68 |
| 31SOEF | 1 | 9 | 6 | 500000 | 2000000 | 10000 | 567021 | 1438168 | 10100 | 0.29 | 3.90 |
| Bisoef | 1 | 9 | 12 | 500000 | 2000000 | 10000 | 496290 | 2047836 | 9979 | 0.04 | 0.73 |
| 8isotf | 1 | 9 | 18 | 500000 | 2000000 | 10000 | 503646 | 1936063 | 10233 | 0.21 | 5.62 |
| bisoef | 1 | 15 | 6 | 500000 | 2000000 | 10000 | 502572 | 1932442 | 10083 | 0.11 | 2.19 |
| 81 SOEF | 1 | 15 | 12 | 500000 | 2000000 | 10000 | 500900 | 2001083 | 9998 | 0.01 | 0.30 |
| 81SOEF | 1 | 15 | 18 | 500000 | 2000000 | 10000 | 500929 | 198846 | 9997 | 0.02 | 0.51 |
| BISOEF | 1 | 3 | 6 | 500000 | 4000000 | 10000 | 178810 | 528489 | 9979 | 1.03 | 7.54 |
| 81SOEF | 1 | 3 | 12 | 500000 | 4000000 | 10000 | 227126 | 5629548 | 10748 | 4.14 | 74.20 |
| 81SOEF | 1 | 3 | 18 | 500000 | 4000000 | 10000 | 240255 | 411096 | 9917 | 0.89 | 19.63 |
| BISOEF | 1 | 9 | 6 | 500000 | 4000000 | 10000 | 490673 | 4133367 | 9993 | 0.06 | 0.76 |
| Bisote | 1 | 9 | 12 | 500000 | 4000000 | 10000 | 497506 | 4006433 | 10058 | 0.10 | 2.23 |
| BISOEF | 1 | 9 | 18 | 500000 | 4000000 | 10000 | 495867 | 3881822 | 10240 | 0.10 | 3.39 |
| 8isotf | ! | 15 | 6 | 500000 | 4000000 | 10000 | 503548 | 3576676 | 10151 | 0.10 | 2.14 |
| bisoff | 1 | 15 | 12 | 500000 | 4000000 | 10000 | 500235 | 3863915 | 10085 | 0.03 | 0.88 |
| aisoef | 1 | 45 | 18 | 500000 | 4000000 | 10000 | 499095 | 3802077 | 10213 | 0.06 | 1.48 |

* ID is the trial value combination identifier, as defined in Table 1.

NOTE: $\quad 1$ inch $=25.4 \mathrm{~mm} \quad 1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m}$

TABLE B15．Results from BISDEF Sensitivity Study（Case 10 of 10）

| Analysis |  | Thickness （inches） |  | Exact Modulus （psi） |  |  | Calculated Modulus （psi） |  |  | Error in Besin （mils） | Percent <br> Error in Basin |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Program | 10＊ | AC | PCC | AC | PCC | Subgr ． | AC | PCC | subgr． |  |  |
| 81SDEF | $J$ | 3 | 6 | 250000 | 2000000 | 10000 | 144114 | 2641739 | 10010 | 0.98 | 7.82 |
| BISOEF | $J$ | 3 | 12 | 250000 | 2000000 | 10000 | 591410 | 1461168 | 10258 | 0.75 | 9.77 |
| bisotf | J | 3 | 18 | 250000 | 2000000 | 10000 | 50000 | 4663487 | 10236 | 9.31 | 180.88 |
| BISDEF | $J$ | 9 | 6 | 250000 | 2000000 | 10000 | 257893 | 1908035 | 10132 | 0.70 | 8.20 |
| BISDEF | J | 9 | 12 | 250000 | 2000000 | 10000 | 236747 | 2142502 | 10119 | 0.51 | 8.64 |
| BISOEF | J | 9 | 18 | 250000 | 2000000 | 10000 | 191729 | 2271274 | 10001 | 0.96 | 14.70 |
| BISDEF | 」 | 15 | 6 | 250000 | 2000000 | 10000 | 246621 | 2105109 | 9956 | 0.13 | 1.79 |
| 8ISDEF | $\downarrow$ | 15 | 12 | 250000 | 2000000 | 10000 | 238570 | 2239413 | 10137 | 0.50 | 10.05 |
| B1SOEF | $J$ | 15 | 18 | 250000 | 2000000 | 10000 | 232907 | 2153475 | 10394 | 0.75 | 18.95 |
| bisdef | $J$ | 3 | 6 | 250000 | 4000000 | 10000 | 154668 | 4518102 | 10005 | 0.80 | 5.75 |
| bisoef | J | 3 | 12 | 250000 | 4000000 | 10000 | 50000 | 9041119 | 11100 | 10.70 | 179.00 |
| BiSOEF | $J$ | 3 | 18 | 250000 | 4000000 | 10000 | 154819 | 4058073 | 9653 | 1.26 | 27.42 |
| bisoef | 」 | 9 | 6 | 250000 | 4000000 | 10000 | 244353 | 3827716 | 10116 | 0.31 | 3.98 |
| BISOEF | 」 | 9 | 12 | 250000 | 4000000 | 10000 | 181324 | 4793917 | 9938 | 1.46 | 18.72 |
| Bisdef | $J$ | 9 | 18 | 250000 | 4000000 | 10000 | 50000 | 9466215 | 12697 | 18.16 | 361.85 |
| BISOEF | $J$ | 15 | 6 | 250000 | 4000000 | 10000 | 250547 | 3962260 | 9996 | 0.02 | 0.30 |
| Bisotf | J | 15 | 12 | 250000 | 4000000 | 10000 | 233578 | 4517397 | 10074 | 0.55 | 10.79 |
| 8ISDEF | $\checkmark$ | 15 | 18 | 250000 | 4000000 | 10000 | 191964 | 4697433 | 9993 | 1.38 | 22.91 |
| BISDEF | J | 3 |  | 250000 | 6000000 | 10000 | 158614 | 6427307 | 9970 | 0.68 | 5.17 |
| BiSOEF | J | 3 | 12 | 250000 | 6000000 | 10000 | 162149 | 5886694 | 9763 | 1.60 | 27.32 |
| BISOEF | $J$ | 3 | 18 | 250000 | 6000000 | 10000 | 215570 | 6018441 | 10089 | 0.25 | 6.26 |
| bisoef | $J$ | 9 | 6 | 250000 | 6000000 | 10000 | 228403 | 6276504 | 10143 | 0.50 | 5.92 |
| BISDEF | J | 9 | 12 | 250000 | 6000000 | 10000 | 225238 | 6122835 | 10251 | 0.66 | 11.99 |
| BISDEF | J | 9 | 18 | 250000 | 6000000 | 10000 | 236700 | 6062516 | 10018 | 0.21 | 3.84 |
| BISOEF | J | 15 | 6 | 250000 | 6000000 | 10000 | 250719 | 5916260 | 10002 | 0.02 | 0.25 |
| 8：50EF | J | 15 | 12 | 250000 | 6000000 | 10000 | 229563 | 673420 | 10437 | 0.71 | 14.95 |
| BISDEF | $J$ | 15 | 18 | 250000 | 6000000 | 10000 | 238528 | 6154374 | 9995 | 0.22 | 4.00 |
| BISDEF | J | 3 | 6 | 500000 | 2000000 | 10000 | 160312 | 3317711 | 10183 | 1.66 | 14.23 |
| BISOEF | $\checkmark$ | 3 | 12 | 500000 | 2000000 | 10000 | 1000000 | 1401646 | 10156 | 0.35 | 5.30 |
| 8ISOEF | J | 3 | 18 | 500000 | 2000000 | 10000 | 377649 | 2112767 | 10007 | 0.25 | 4.65 |
| BISDEF | $J$ | 9 | 6 | 500000 | 2000000 | 10000 | 501386 | 1990755 | 9094 | 0.02 | 0.30 |
| BISDEF | J | 9 | 12 | 500000 | 2000000 | 10000 | 500617 | 1995664 | 9998 | 0.01 | 0.14 |
| BISDEF | $J$ | 9 | 18 | 500000 | 2000000 | 10000 | 375766 | 2640857 | 10129 | 1.16 | 27.42 |
| BISOEF | J | 15 | 6 | 500000 | 2000000 | 10000 | 508529 | 1959031 | 10009 | 0.10 | 1.57 |
| BISOEF | $J$ | 15 | 12 | 500000 | 2000000 | 10000 | 500644 | 1996986 | 9983 | 0.03 | 0.65 |
| BiSOEF | $J$ | 15 | 18 | 500000 | 2000000 | 10000 | 500522 | 1990681 | 10045 | 0.04 | 1.39 |
| 81SOEF | $J$ | 3 | 6 | 500000 | 4000000 | 10000 | 178985 | 5264465 | 9981 | 1.01 | 7.39 |
| 8ISDEF | $J$ | 3 | 12 | 500000 | 4000000 | 10000 | 3554Cn | 4670219 | 10289 | 1.80 | 31.81 |
| BISDEF | $J$ | 3 | 18 | 500000 | 4000000 | 10000 | 405179 | 4080422 | 10052 | 0.20 | 4.77 |
| BISOEF | $J$ | 9 | 6 | 500000 | 4000000 | 10000 | 697178 | 4161222 | 9990 | 0.08 | 0.91 |
| BISOEF | J | 9 | 12 | 500000 | 4000000 | 10000 | 499226 | 3953545 | 10163 | 0.25 | 5.51 |
| BISOEF | $\downarrow$ | 9 | 18 | 500000 | 4000000 | 10000 | 501029 | 397734 | 10085 | 0.08 | 2.54 |
| BISOEF | J | 15 | 6 | 500000 | 4000000 | 10000 | 508648 | 3805432 | 10033 | 0.09 | 1.55 |
| BISOEF | 」 | 15 | 12 | 500000 | 4000000 | 10000 | 530057 | 2960726 | 10519 | 0.22 | 5.73 |
| BISDEF | J | 15 | 18 | 500000 | 4000000 | 10000 | 520329 | 3498239 | 10451 | 0.14 | 4.45 |

＊ID is the trial value combination identifier，as defined in Table 1.
NOTE： 1 inch $=25.4 \mathrm{~mm} \quad 1 \mathrm{psi}=6.89 \mathrm{kPa} \quad 1 \mathrm{mil}=25.4 \mu \mathrm{~m}$


## LEGEND

Thickness of $\mathrm{AC}=3$ inches
Thickness of $\mathrm{PCC}=6$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\mathbb{\otimes}$ Percentage error in back-calculated AC modulus
$\triangle \Delta$ Percentage error in back-calculated PCC modulus
$\boxed{5}$ Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
$A-J=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B1. Graphical Sumary of Sensitivity Study, Case 1 of 45


## LEGEND

Thickness of $\mathrm{AC}=3$ inches
Thickness of $\mathrm{PCC}=12$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10.000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
区 Percentage error in back-calculated AC modulus
$\mathbb{Z}$ Percentage error in back-calculated PCC modulus
$\int$ Percentage error in back-calculated subgrade modulus COM $=$ Results from COMDEF sensitivity study
$A-J=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B2. Graphical Sumary of Sensitivity Study, Case 2 of 45


Thickness of $\mathrm{AC}=3$ inches
Thickness of $\mathrm{PCC}=18$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\mathbb{\otimes}$ Percentage error in back-calculated AC modulus
$\square \square$ Percentage error in back-calculated PCC modulus
$\square$ Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
$A-J=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B3. Graphical Sumary of Sensitivity Study, Case 3 of 45


Thickness of $\mathrm{AC}=9$ inches
Thickness of PCC $=6$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
区 Percentage error in back-calculated AC modulusPercentage error in back-calculated PCC modulusPercentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
A-J = Results from BISDEF sensitivity study with trial value combinations as difined in Table 1

FIGURE B4. Graphical Sumary of Sensitivity Study, Case 4 of 45


LEGEND
Thickness of $\mathrm{AC}=9$ inches
Thickness of $\mathrm{PCC}=12$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
区 Percentage error in back-calculated AC modulusPercentage error in back-calculated PCC modulus
$\pi$ Percentage error in back-calculated subgrade modulus
COM = Results from COMDEF sensitivity study
$A-J=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B5. Graphical Sumary of Sensitivity Study, Case 5 of 45


FIGURE B6. Graphical Sumary of Sensitivity Study, Case 6 of 45


## LEGEND

Thickness of $\mathrm{AC}=15$ inches
Thickness of $\mathrm{PCC}=6$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\mathbb{X}$ Percentage error in back-calculated AC modulus
$\mathbb{Z}$ Percentage error in back-calculated PCC modulus
$\int$ Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
$\mathrm{A}-\mathrm{J}=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B7. Graphical Sumary of Sensitivity Study, Case 7 of 45


## LEGEND

Thickness of $\mathrm{AC}=15$ inches
Thickness of $\mathrm{PCC}=12$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
区 Percentage error in back-calculated AC modulus
$\boxed{Z}$ Percentage error in back-calculated PCC modulus
$\square$ Percentage error in back-calculated subgrade modulus COM = Results from COMDEF sensitivity study
$A-J=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B8. Graphical Sumary of Sensitivity Study, Case 8 of 45


Thickness of $\mathrm{AC}=15$ inches
Thickness of PCC $=18$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\triangle \otimes$ Percentage error in back-calculated AC modulus
$\boxed{Z}$ Percentage error in back-calculated PCC modulusPercentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
$\mathrm{A}-\mathrm{J}=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B9. Graphical Sumary of Sensitivity Study, Case 9 of 45


FIGURE B10. Graphical Sumary of Sensitivity Study, Case 10 of 45


LEGEND
Thickness of $\mathrm{AC}=3$ inches
Thickness of $\mathrm{PCC}=12$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=4,000,000 \mathrm{psi}$
Modulus of subgrade $=10.000$ psi
Summation of basin percentage errors ( 7 deflections)
$\mathbb{\otimes}$ Percentage error in back-calculated AC modulus
$\square \triangle$ Percentage error in back-calculated PCC modulus
$\triangle$ Percentage error in back-calculated subgrade modulus
COM = Results from COMDEF sensitivity study
A-J = Results from BISDEF sensitivity study with triai value combinations as defined in Table 1

FIGURE 811. Graphical Summary of Sensitivity Study, Case 11 of 45


Thickness of $\mathrm{AC}=3$ inches
Thickness of PCC $=18$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=4,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\otimes \otimes$ Percentage error in back-calculated AC modulus
$\square \square$ Percentage error in back-calculated PCC modulus
$\square$ Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
A-J = Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B12. Graphical Summary of Sensitivity Study, Case 12 of 45


Thickness of $\mathrm{AC}=9$ inches
Thickness of $\mathrm{PCC}=6$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=4,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
区 Percentage error in back-calculated AC modulus
$\triangle \triangle$ Percentage error in back-calculated PCC modulus
$\Phi$ Percentage error in back-calculated subgrade modulus
$C O M=$ Results from COMDEF sensitivity stud
$A-J=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1


## LEGEND

Thickness of $\mathrm{AC}=9$ inches
Thickness of $\operatorname{PCC}=12$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=4,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
区 Percentage error in back-calculated AC modulus


Percentage error in back-calculated PCC modulus
$\triangle$ Percentage error in back-calculated subgrade modulus
COM = Results from COMDEF sensitivity study
A-J $=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B14. Graphical Sumary of Sensitivity Study, Case 14 of 45


## LEGEND

Thickness of $\mathrm{AC}=9$ inches
Thickness of $\mathrm{PCC}=18$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=4,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
X Percentage error in back-calculated AC modulus
$\square$ Percentage error in back-calculated PCC modulus
$\triangle$ Percentage error in back-calculated subgrade modulus COM $=$ Results from COMDFF sensitivity study
A-J $=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B15. Graphical Summary of Sensitivity Study, Case 15 of 45


## LEGEND

Thickness of $\mathrm{AC}=15$ inches
Thickness of $\mathrm{PCC}=6$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=4,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\Delta>$ Percentage error in back-calculated AC modulus
$\boxed{7}$ Percentage error in back-calculated PCC modulus
$\triangle \Delta$ Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
A-J $=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B16. Graphical Summary of Sensitivity Study, Case 16 of 45


## LEGEND

Thickness of $\mathrm{AC}=15$ inches
Thickness of PCC $=12$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=4,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\mathbb{X}$ Percentage error in back-calculated AC modulusPercentage error in back-calculated PCC modulus
D] Percentage error in back-calculated subgrade modulus COM $=$ Results from COMDEF sensitivity study
A-J = Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE 817. Graphical Summary of Sensitivity Study, Case 17 of 45


## LEGEND

Thickness of $\mathrm{AC}=15$ inches
Thickness of PCC $=18$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=4,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\mathbb{\otimes}$ Percentage error in back-calculated AC modulus
$\square \triangle$ Percentage error in back-calculated PCC modulus
$\triangle \square$ Percentage error in back-calculated subgrade modulus
$\mathrm{COM}=$ Results from COMDEF sensitivity study
$A-J=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1


## LEGEND

Thickness of $\mathrm{AC}=3$ inches
Thickness of $\mathrm{PCC}=6$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Kiodulus of $\mathrm{PCC}=6,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
区 Percentage error in back-calculated AC modulus
$\square \triangle$ Percentage error in back-calculated PCC modulus
$\triangle$ Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
A-J $=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B19. Graphical Summary of Sensitivity Study, Case 19 of 45


## LEGEND

Thickness of $\mathrm{AC}=3$ inches
Thickness of $\mathrm{PCC}=12$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=6,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\boxed{\square}$ Percentage error in back-calculated AC modulus
$\square \square$ Percentage error in back-calculated PCC modulus
$\triangle$ Percentage error in back-calculated subgrade modulus COM $=$ Results from COMDEF sensitivity study $\mathrm{A}-\mathrm{J}=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B20. Graphical Summary of Sensitivity Study, Case 20 of 45


## LEGEND

Thickness of $\mathrm{AC}=3$ inches
Thickness of PCC $=18$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=6,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\mathbb{X}$ Percentage error in back-calculated AC modulus
Percentage error in back-calculated PCC modulus
$\triangle$
Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
$A-J=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1


LEGEND
Thickness of $\mathrm{AC}=9$ inches
Thickness of $\mathrm{PCC}=6$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=6,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\otimes$ Percentage error in back-calculated AC modulus
$Z \triangle$ Percentage error in back-calculated PCC modulus
$\triangle$ Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
A-J $=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE 822. Graphical Summary of Sensitivity Study, Case 22 of 45


## LEGEND

Thickness of $\mathrm{AC}=9$ inches
Thickness of $\mathrm{PCC}=12$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=6,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
区 Percentage error in back-calculated AC modulus
Z P Percentage error in back-calculated PCC modulus
$\triangle$ Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
$\mathrm{A}-\mathrm{J}=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1


LEGEND
Thickness of $\mathrm{AC}=9$ inches
Thickness of $\mathrm{PCC}=18$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=6,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
区 Percentage error in back-calculated AC modulusPercentage error in back-calculated PCC modulusPercentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
A-J $=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B24. Graphical Summary of Sensitivity Study, Case 24 of 45


Thickness of $\mathrm{AC}=15$ inches
Thickness of PCC $=6$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=6,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\mathbb{\otimes}$ Percentage error in back-calculated AC modulus
$\square \square$ Percentage error in back-calculated PCC modulus
Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
A-J $=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B25. Graphical Summary of Sensitivity Study, Case 25 of 45


## LEGEND

Thickness of $\mathrm{AC}=15$ inches
Thickness of PCC $=12$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=6,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
区 Percentage error in back-calculated AC modulus
Z $\triangle$ Percentage error in back-calculated PCC modulus
5 Percentage error in back-calculated subgrade modulus
$C O M=$ Results from COMDEF sensitivity study
$\mathrm{A}-\mathrm{J}=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1


## LEGEND

Thickness of $\mathrm{AC}=15$ inches
Thickness of $\mathrm{PCC}=18$ inches
Modulus of $\mathrm{AC}=250,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=6,000,000 \mathrm{psi}$
Modulus of subgrad $\epsilon=10,000 \mathrm{psi}$

- Summation of basin percentage errors ( 7 deflections)

区 Percentage error in back-calculated AC modulus
$\square$ Percentage error in back-calculated PCC modulus
$\triangle$ Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
A-J = Results from BISDEF sensitivity study with trial value combinations as defined in Table 1


Thickness of $\mathrm{AC}=3$ inches
Thickness of $\mathrm{PCC}=6$ inches
Modulus of $\mathrm{AC}=500.000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
区 Percentage error in back-calculated AC modulus
$\square \triangle$ Percentage error in back-calculated PCC modulus
$\triangle$ Percentage error in back-cr'^ulated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
$A-J=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B28. Graphical Summary of Sensitivity Study, Case 28 of $\mathbf{4 5}$


Thickness of $\mathrm{AC}=3$ inches
Thickness of PCC $=12$ inches
Modulus of $\mathrm{AC}=500,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
区 Percentage error in back-calculated AC modulus


Percentage error in back-calculated PCC modulusPercentage error in back-calculated subgrade modulus COM = Results from COMDEF sensitivity study
A-J = Resuits from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B29. Graphical Summary of Sensitivity Study, Case 29 of 45


## LEGEND

Thickness of $A C=3$ inches
Thickness of $\mathrm{PCC}=18$ inches
Modulus of $\mathrm{AC}=500,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
双 Percentage error in back-calculated AC modulus
$Z \triangle$ Percentage error in back-calculated PCC modulus
$\triangle$ Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
$\mathrm{A}-\mathrm{J}=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1


## LEGEND

Thickness of $\mathrm{AC}=9$ inches
Thickness of PCC = 6 inches
Modulus of $\mathrm{AC}=500,000 \mathrm{psi}$
Modulus of PCC $=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
Q Percentage error in back-calculated AC modulus
$\mathbb{Z}$ Percentage error in back-calculated PCC modulus
$\triangle$ Percentage error in back-calculated subgrade modulus COM = Results from COMDEF sensitivity study $A-J=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B31. Graphical Summary of Sensitivity Study, Case 31 of 45


## LEGEND

Thickness of $\mathrm{AC}=9$ inches
Thickness of PCC $=12$ inches
Modulus of $\mathrm{AC}=500,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\otimes \otimes$ Percentage error in back-calculated AC modulus
Percentage error in back-calculated PCC modulus
$\triangle$ Percentage error in back-calculated subgrade modulus
COM = Results from COMDEF sensitivity study
A-J $=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B32. Graphical Summary of Sensitivity Study, Case 32 of 45


## LEGEND

Thickness of $\mathrm{AC}=9$ inches
Thickness of $\mathrm{PCC}=18$ inches
Modulus of $\mathrm{AC}=500,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\mathbb{X}$ Percentage error in back-calculated AC modulus
Percentage error in back-calculated PCC modulus
Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
$A-J=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B33. Graphical Summary of Sensitivity Study, Case 33 of 45


## LEGEND

Thickness of $\mathrm{AC}=15$ inches
Thickness of $\mathrm{PCC}=6$ inches
Modulus of $\mathrm{AC}=500,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\otimes \otimes$ Percentage error in back-calculated AC modulus
$\boxed{Z}$ Percentage error in back-calculated PCC modulus
$\triangle \square$ Percentage error in back-calculated subgrade modulus
COM = Results from COMDEF sensitivity study
A-J = Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B34. Graphical Summary of Sensitivity Study, Case 34 of 45


## LEGEND

Thickness of $\mathrm{AC}=15$ inches
Thickness of PCC $=12$ inches
Modulus of $\mathrm{AC}=500,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors (7 deflections)
区 Percentage error in back-calculated AC modulus
$Z \triangle$ Percentage error in back-calculated PCC modulus
$\square$ Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
$A-J=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B35. Graphical Summary of Sensitivity Study, Case 35 of 45


## LEGEND

Thickness of $\mathrm{AC}=15$ inches
Thickness of PCC $=18$ inches
Modulus of $\mathrm{AC}=500,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=2,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
( $\mathbb{Z}$ Percentage error in back-calculated AC modulus
$\mathbb{Z}$ Percentage error in back-calculated PCC modulusPercentage error in back-calculated subgrade modulus
COM = Results from COMDEF sensitivity study
$\mathrm{A}-\mathrm{J}=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B36. Graphical Summary of Sensitivity Study, Case 36 of 45


## LEGEND

Thickness of $\mathrm{AC}=3$ inches
Thickness of PCC $=6$ inches
Modulus of $\mathrm{AC}=500,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=4,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\mathbb{\otimes}$ Percentage error in back-calculated AC modulus
$\nabla \square$ Percentage error in back-calculated PCC modulus
$\triangle$ Percentage error in back-calculated subgrade modulus COM $=$ Results from COMDEF sensitivity study
A-J $=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE 837. Graphical Summary of Sensitivity Study, Case 37 of 45



Thickness of $\mathrm{AC}=3$ inches
Thickness of PCC $=12$ inches
Modulus of $\mathrm{AC}=500,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=4,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
区 Percentage error in back-calculated AC modulus
$\square \square$ Percentage error in back-calculated PCC modulusPercentage error in back-calculated subgrade modulus
COM = Results from COMDEF sensitivity study
A-J $=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B38. Graphical Summary of Sensitivity Study, Case 38 of 45


## LEGEND

Thickness of $\mathrm{AC}=3$ inches
Thickness of PCC $=18$ inches
Modulus of $\mathrm{AC}=500,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=4,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
区 Percentage error in back-calculated AC modulus
$\square \square$ Percentage error in back-calculated PCC modulus
$\boxed{\square}$ Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
$\mathrm{A}-\mathrm{J}=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1


$$
\begin{aligned}
& \text { LEGEND } \\
& \text { Thickness of } \mathrm{AC}=9 \text { inches } \\
& \text { Thickness of } \mathrm{PCC}=6 \text { inches } \\
& \text { Modulus of } \mathrm{AC}=500,000 \mathrm{psi} \\
& \text { Modulus of } \mathrm{PCC}=4,000,000 \mathrm{psi} \\
& \text { Modulus of subgrade }=10,000 \mathrm{psi} \\
& \text { Summation of basin percentage errors ( } 7 \text { deflections) } \\
& \text { 区 Perce.. cage error in back-calculated AC modulus } \\
& \square \searrow \text { Percentage error in back-calculated PCC modulus } \\
& \square \text { Percentage error in back-calculated subgrade modulus } \\
& \text { COM = Results from COMDEF sensitivity study } \\
& \text { A-J = Results from BISDEF sensitivity study with trial } \\
& \text { value combinations as defined in Table } 1
\end{aligned}
$$

FIGURE B40. Graphical Summary of Sensitivity Study, Case 40 of 45


## LEGEND

Thickness of $\mathrm{AC}=9$ inches
Thickness of PCC $=12$ inches
Modulus of $\mathrm{AC}=500,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=4,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\mathbb{\otimes}$ Percentage error in back-calculated AC modulus
$\square$ Percentage error in back-calculated PCC modulus
$\triangle$ Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
A-J = Results from BISDEF sensitivity study with trial value combinations as defined in Table 1


## LEGEND

Thickness of $\mathrm{AC}=9$ inches
Thickness of $\mathrm{PCC}=18$ inches
Modulus of $\mathrm{AC}=500,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=4,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\mathbb{\otimes}$ Percentage error in back-calculated AC modulus
$\boxtimes$ Percentage error in back-calculated PCC modulus
$\Delta$ Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
$A-J=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B42. Graphical Summary of Sensitivity Study, Case 42 of 45


Thickness of $\mathrm{AC}=15$ inches
Thickness of $\mathrm{PCC}=6$ inches
Modulus of $\mathrm{AC}=500,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=4,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\mathbb{Z}$ Percentage error in back-calculated AC modulus
$Z \triangle$ Percentage error in back-calculated PLC modulus
$\triangle$ Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
$\mathrm{A}-\mathrm{J}=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B43. Graphical Summary of Sensitivity Study, Case 43 of 45


Thickness of $\mathrm{AC}=15$ inches
Thickness of PCC $=12$ inches
Modulus of $\mathrm{AC}=500,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=4,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$\boxtimes \otimes$ Percentage error in back-calculated AC modulus
$\boxed{\square}$ Percentage error in back-calculated PCC modulus
$\triangle$ Percentage error in back-calculated subgrade modulus
COM $=$ Results from COMDEF sensitivity study
$A-J=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1

FIGURE B44. Graphical Summary of Sensitivity Study, Case 44 of 45


## LEGEND

Thickness of $\mathrm{AC}=15$ inches
Thickness of PCC $=18$ inches
Modulus of $\mathrm{AC}=500,000 \mathrm{psi}$
Modulus of $\mathrm{PCC}=4,000,000 \mathrm{psi}$
Modulus of subgrade $=10,000 \mathrm{psi}$
Summation of basin percentage errors ( 7 deflections)
$区 \boxtimes$ Percentage error in back-calculated AC modulusPercentage error in back-calculated PCC modulusPercentage error in back-calculated subgrade modulus COM $=$ Results from COMDEF sensitivity study
A-J $=$ Results from BISDEF sensitivity study with trial value combinations as defined in Table 1
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## APPENDIX C

## PROCEDURE FOR SOIL RESILIENT MODULUS TESTS

## INTRODUCTION

The objective of this test procedure is to determine a modulus value for subgrade soils by means of resilient triaxial techniques. The test is similar to a standard triaxial compression test, except that the deviator stress is applied repetitively and at several stress levels. This procedure allows testing of soil specimens in a repetitive stress state similar to that encountered by a soil in a pavement under a moving wheel load, and also similar to the stress state produced by nondestructive testing with the Falling Weight Deflectometer.

The results of the soil resilience tests were computed in a spreadsheet. Values of resilient modulus, $M_{R}$, were plotted versus deviator stress, $\sigma_{d}$, for all confining pressures. The results of these tests are illustrated in Figures 31 to 34.

## DEFINITIONS

The following symbols and terms are used in the description of this procedure:

```
\sigma}=\mathrm{ total axial stress
\sigma}=\mathrm{ total radial stress
    = confining pressure in the chamber
\sigmad}=\mp@subsup{\sigma}{1}{}-\mp@subsup{\sigma}{3}{
    = deviator stress
    = repeated axial stress in this procedure
\epsilon}\mp@subsup{|}{R}{}=\mathrm{ total axial strain due to }\mp@subsup{\sigma}{d}{}
M
Load duration = time interval over which the
specimen is subjected to a
deviator stress
```

Cycle duration = time interval between successive applications of a deviator stress

## SPECIMENS

Various diameter soil specimens may be used for soil resilient modulus tests, but the recommended specimen diameter is 2.5 to 3.0 inches ( 64 mm to 76 mm ) or approximately four times the maximum aggregate size. The samples used for this project were untrimmed samples from nominal 3 inch ( 76 mm ) thin-walled tubes (actual diameters slightly less than 3 inches ( 76 mm )). Minimum length to diameter ratio of 2 was maintained for all samples tested. Since the main purpose of this testing was comparison with nondestructive testing backcalculated moduli values, back-pressure saturation was not used.

## EQUIPMENT

The Waterways Experiment Station triaxial cell used for resilience testing of soils is similar to a standard triaxial cell, with the exceptions of being somewhat larger to facilitate the internally mounted load and deformation measuring equipment and having additional outlets for the electrical leads from the measuring devices. Repeated axial stresses were applied pneumatically with the Waterways Experiment Station cyclic triaxial loading device with a nearly rectangular stresstime wave form for a load duration of $0.2-\mathrm{sec}$ at a cycle duration of 3 seconds. The load was monitored by a 500 pound ( 2.2 kN ) capacity miniature load cell mounted inside the confining chamber between the specimen cap and the loading piston. The axial strains were measured by two matching linearly variable differential transformers (LVDTs) calibrated to the nearest 0.0001 inches $(2.5 \mu \mathrm{~m})$ and held in position on the specimen by spring-loaded LVDT clamps. The clamps were positioned so that axial strains were measured over the central 4 inches ( 102 mm ) of the specimen. A small amount of Devcon 5-min epoxy was applied at the contact points to minimize slippage between the membrane and clamp. Frictionless end plates incorporating polished stainless steel surfaces, silicone grease, and Teflon inserts were utilized to minimize end effects. Use of this measurement configuration offers several advantages:
(1) It is not necessary to reference deformations to the equipment (which deforms during loading).
(2) The effect of end-cap restraint on soil response is virtually eliminated.
(3) Any effects of piston friction are eliminated by measuring loads inside the triaxial cell.

## ADDITIONAL EQUIPMENT

In addition to the equipment described above, the following items were also used:
a. A 10 ton ( 89 kN ) capacity loading machine.
b. Calipers, a micrometer gage, and a calibrated steel rule.
c. Rubber membranes, 0.010 to 0.025 inches (254 $\mu \mathrm{m}$ to $635 \mu \mathrm{~m})$ thick.
d. Rubber o-rings.
e. A vacuum source with a bubble chamber and regulator.
f. A back-pressure chamber with pressure transducers.
g. A membrane stretcher.
h. Porous stones.

## SAMPLE PREPARATION AND PLACEMENT IN THE TRIAXIAL CELL

The following procedure should be followed in preparing and placing specimens:
a. In accordance with procedures specified in Army Engineering Manual 1110-2-1906 (ASTM D 2850), prepare the specimen and place it on the base-plate complete with porous stones, cap, and base and equipped with a rubber membrane secured with 0 -rings. Check for leakage. The specimen is now ready to receive the LVDTs.
b. Extend the lower LVDT clamp and slide it carefully down over the specimen to approximately the lower quarter point of the specimen.
c. Repeat this step for the upper clamp, placing it at the upper quarter point. Insure that both clamps lie in horizontal planes.
d. Connect the LVDTs to the recording unit, and balance the recording bridges. This step will require recorder adjustments and adjustment of the LVDT stems. When a recording bridge balance has been obtained, determine (to the nearest 0.01 inches ( $254 \mu \mathrm{~m}$ )) the vertical spacing between the LVDT clamps and record this value.
e. Place the triaxial chamber in position. Set the load cell in place on the specimen.
f. Place the cover plate on the chamber. Insert the loading piston, and obtain a firm connection with the load cell.
g. Tighten the tie rods firmly.
h. Slide the assembled apparatus into position under the axial loading device. Bring the loading device to a position in which it nearly contacts the loading piston.

## RESILIENT TESTING OF COHESIVE SOILS

The resilient properties of cohesive soils are only slightly affected by the magnitude of the confining pressure $\sigma_{3}$. For most applications, this effect can be disregarded. A range of chamber pressures from 3 to 25 psi ( 21 to 172 kPa ) were used for testing. Resilient properties are highly dependent on the magnitude of the deviator stress $\sigma_{d}$. It is therefore necessary to conduct the tests for a range in deviator stress values. The following procedure was used:
a. Connect the chamber supply line and apply a chamber pressure of $5 \mathrm{psi}(34 \mathrm{kPa})$, i.e. $\sigma_{3}=5 \mathrm{psi}(34 \mathrm{kPa})$.
b. Rebalance the recording bridges for the LVDTs, and balance the load cell recording bridge.
c. Begin the test by applying 1000 repetitions of a deviator stress, $\sigma_{d}$ of not more than one-half the unconfined compressive strength.
d. Decrease confining pressure and deviator stress to the lowest values to be used.
e. Apply 200 repetitions of $\sigma_{d}$, recording the recovered axial deformation, $\epsilon_{R}$, at or near the last repetition.
f. Increase the deviator stress, recording deformations as in Step e. Repeat over a range of deviator stresses.
g. Repeat Steps e. and f. for other chamber pressures, or until the sample fails.
h. At the completion of the loading, reduce the chamber pressure to zero. Remove the chamber LVDTs and load cell. Use the entire specimen for the purpose of determining the moisture content.

## APPENDIX D <br> DESIGN PLANS FOR ASPHALTIC CONCRETE RESILIENT MODULUS APPARATUS

This appendix contains 9 design drawings for the apparatus shown as an assembly drawing in Figure 35. To aid the reader, a reduced version of the assembly drawing is included with each design drawing, with an arrow pointing to the corresponding part.


FIGURE D1. Load Platen Surface Mount


FIGURE D2. Base Plate Attachment


FIGURE D3. Apparatus Base Plate


FIGURE D4. Top Loading Plate


FIGURE D5. Front Vertical LVDT Bracket


FIGURE D6. Front Vertical LVDT Bracket Clamp (Lockdown)


FIGURE D7. Rear Vertical LVDT Bracket


FIGURE D8. Horizontal LVDT Bracket(s)


FIGURE D9. Load Application Strip

## APPENDIX E SOURCE CODE FOR RESMA.BAS

RESMA.BAS was developed at the Waterways Experiment Station by a design team which included Mark Anderson, Thomas V. McEwen, Mitchell S. Jones, Harold T. Carr, and Terry V. Jobe. The program collects, displays, processes, and plots data acquired for the $A C$ resilient modulus test by indirect tension (ASTM C 4123). A complete source code listing follows.

1 REM
10 REM 1 *************************** 2/13/87
11 REM
20 REM THIS IS A WES DEVELOPED PROGRAM TO COLLECT,DISPLAY,PROCESS AND PLOT DATA ACQUIRED FOR THE AC RESILIENT MODULUS TEST BY INDIRECT TENSION (ASTM C 4123)
22 REM
23 REM PROGRAM DEVELOPMENT BY MARK ANDERSON, THOMAS V. MCEWEN, MITCHELI. S. JONES, HAROLD T. CARR, AND TERRY V. JOBE
25 REM
30 REM THE TEST CONDUCTED ON THE ** MTS CLOSED LOOP SYSTEM ** PAVEMENT SYSTEMS DIVISION, GEOTECHNICAL LABORATORY, WATERWAYS EXPERIMENT STATION
50 REM

70 REM
100 REM DIMENSION SECTION AND LIST OF VARIABLES IN USE. LINES 100-799
101 REM
130 REM VARIABLE USED ARE

```
STS,N,N2$,DS,N$,P$,P3$,P4$,A$,A1$,L1$,R1$,LS,R2$
```

140 REM H\$,Z1S=(Y/N),Z,
$155 \mathrm{~N}=950$
160 DIM X1(N), X2(N), X3(N), X4(950)
170 DIM R1(10) REM ARRAY FOR INITIAL PRECONDITION READINGS
180 DIM R2(50) REM ARRAY FOR PRECONDITION DATA
200 REM
965 WAIT 1
1000 KILL ALL
1002 TASK 2,6900,5
1003 TASK 3,6380,5

```
1004 TASK 4,7500,2
1010 PNT }186\mathrm{ REM MENU LIST
1020 DISPLAY O PNT 195 PNT 180 CLS O PNT 10
1030 PNT 195 !" ";PNT 196 !"***** PAVEMENT & SOIL TRAFFICABILITY INFOR-
    MATION ANALYSIS CENTER (GL)
1050 IF STS<>"" THEN PRINT "
STS
1060 PNT 10 PNT 10 PNT 195
1070 PNT 195 !" ";PNT 193 !"** Select from the following
        queries ** ACTION DESIRED **"
1080 PNT 10 PNT 10
1085 PNT 195 !" ";PNT 197 !"
1090 PNT 195!"
        CODE "
1100 PNT 195 !"
FOR SERIES)
1105 PNT 195 !"
        INFO)
1110 PNT 195 !"
        PH "
1120 PNT 195 !"
        SCALE FACTOR)
1130 PNT 195 !" 
1130 PNT 195 !" 
1150 PNT 195 !"
1180 PNT 195 !" ";PNT 197 !" PLOT DATA
1182 PNT 195 !"
        PD "
1183 PNT 195 !"
        SD "
1184 PNT 195 !" ";PNT 197 !" READ DATA FROM DISKETTE
        RD "
1185 PNT 195 !"
        TION
    ";PNT 197!" EDIT VARIBLE HEADER INFORMA-
EV
1186 PNT 195 !"
HI
                                    ";PNT 197 !"
                                    ACTION DESIRED
                    ";PNT 197 !" HEADER INFORMATION (CONSTANT
    ";PNT 197 !" HEADER INFORMATION (VARIABLE
VI
```

ACTION DESIRED

```;PNT 197 !"
```

";PNT 194 !
1050 IF STS<>"" THEN PRINT "

```ST\$
```

```1070 PNT 195 !" ";PNT 193 !"** Select from the followingPNT 10 PNT 10
```

1085 PNT 195 !"

```";PNT 197 !
```

```1120 PNT 195 !"SCALE FACTOR)VI
```

";PNT 197!" HEADER INFORMATION (VARIABLE

```v
```

";PNT 197 !" PRINT HEADER INFORMATION
";PNT 197 !" CALIBRATION INPUT (ZERO \&

```CI
```

";PNT 197 !" BEGIN CHECK-OUT PROCEDURE
BC
";PNT 197 !" START TEST (TAKE DATA)

```ST "
```

";PNT 197 !" PLOT DATA

```PL "1182 PNT 195 !"PD "
```

1183 PNT 195 !" ..... SD "
";PNT 197 !" PROCESS DATA

```";PNT 197 !" SAVE DATA ON DISKETTE";PNT 197 !" READ DATA FROM DISKETTERD "
```

``` TIONEV1186 PNT 195 !"";PNT 197 !"
```

1190 PNT 10 PNT ..... 10
1200 PNT 195 !" ";PNT 192 INPUT " Please enter action code,

```then hit RETURN "N2S
```

1205 RESET SUB
1210 IF N2S="HI" LET N2S="" GOSUB 2000

```1215 IF N2S="VI" LET N2S="n GOSUB 2600
```

```
1216 IF N2$="EV" LET Z1$="N"
1217 IF N2S="EV" LET N2S="" GOSUB 2710
1220 IF N2S="PH" LET N2S="" GOSUB 3000
1230 IF N2$="CI" LET N2$="" GOSUB 4000
1240 IF N2$="BC" LET N2$="" GOSUB 5000
1260 IF N2S="ST" LET N2S="" GOSUB 6000
1290 IF N2$="PL" LET N2$="" GOTO 10000
1292 IF N2$="PD" KILL ALL NCHAIN "PROCESS"
1294 IF N2$="SD" LET N2S="" GOSUB 16000
1296 IF N2$="RD" LET N2$="" GOSUB 17000
1300 IF N2$="" THEN 1000
1310 PNT 176 PNT 194 ! "INVALID ACTION CODE PLEASE RE-ENTER!!" WAIT 2
GOTO 1000
2000 REM ******************(THIS ROUTINE ACCEPTS CONSTANT HEADER
INFORMATION)*
2010 PNT 195 CLS O PNT 10 PNT 192
2020 PRINT " THIS ROUTINE ACCEPTS CONSTANT HEADER INFORMA-
TION
2025 WAIT l PNT 10 PNT 10 PNT 10
2030 PNT 195 !"
"D$
2040 PNT 195 !"
TYPE ? "ACS
2050 PNT 195 !"
DURATION ?"LDS
2153 PNT 195 !" ";PNT 192 INPUT " 4) LOAD CELL
CAPACITY (LBS.) ? "LCS
2154 PNT 195 !" ";PNT 192 INPUT " 5) MTS LOAD
RANGE ? "LMS
2155 PNT 195 !" ";PNT 192 INPUT " 6) LVDT
(LAT.) RANGE (IN.) ? "LRS
2157 PNT 195 !"
(VERT.) RANGE (IN.) ? "LIS
2160 PNT 10 PNT 10 PNT 195 !"
                                    ";PNT 196 INPUT "
                                    ARE ALL ENTRIES CORRECT (Y/N)? "Zl$
2162 IF Z1$="Y" RETURN
2163 IF Z1$="N" THEN 2270
2164 PNT 176 PNT 195 !"
                                    ";PNT 194 !" PLEASE
ENTER Y OR N!!" GOTO 2160
2270 PNT 195 CLS O PNT 10 PNT 10 PNT 10
2280 PNT 195 !" 1) DATE : "OS
2300 PNT 195 !"
2) ASPHALT TYPE : "ACS
2310 PNT 195 !"
3) LOAD DURATION : "LOS
2350 PNT 195 !"
4) LOAD CELL CAPACITY (LBS.): "LCS
2355 PNT 195 !" 5) MTS LOAD RANGE: "LMS
```




```
2806 IF Z=5 PNT 193 !" ";PNT 197 INPUT " 5) LENGTH OF
    TOTAL PULSE (SEC.)"UW$
2807 IF Z=6 PNT 193 !"
        LOAD PULSE (SEC.)"LPS
2820 IF Z=7 PNT 193 !" ";PNT 197 INPUT " 7) DIAMETER ?
        "LOS
2830 IF Z=8 PNT 193 !" ";PNT 197 INPUT " 8) THICKNESS ?
        "DES
2831 IF Z=9 PNT 193 !"
        ?"VLS
2833 IF Z=10 PNT 193 !" ";PNT 197 INPUT " 10) TEST TEM-
        PERTURE ? "TS
2835 IF Z<l THEN PNT 176 PNT 193 !"
        ENTER NUMBERS 1 THRU 10 ONLY!! "
2838 IF Z>10 THEN PNT l76 PNT 193 !" ";PNT 194 !" PLEASE
        enter numbers 1 thru 10 ONLY!!"
2839 PNT 10
2840 GOTO 2690
3000 REM
3010 REM THIS ROUTINE PRINTS HEADER INFORMATION
3020 OPENW :2 "$QTO:1"
3025 PRINT:2 CHR$(24)
3030 PRINT:2 CHR$(27);CHR$(65);CHR$(27);CHR$(54);CHR$(29);CHR$(31);
3040 PRINT:2 TAB(12);
3050 A=1
3060 A=A+1
3070 IF A>2 THEN GOTO 3090
3080 PRINT:2 CHR$(151);
3090 PRINT:2 CHR$(131);
3100 IF A=>78 THEN PRINT:2 CHRS(149) GOTO 3120
3110 GOTO 3060
3120 PRINT:2 TAB(12);CHRS(149);TAB(33);"HEADER INFORMATION FOR TEST NO.
        "TNS;TAB(90);CHR$(149)
3125 PRINT:2 TAB(12);CHR$(149);TAB(15);"BLEND NO. = "BL$;TAB(60);"SAMPLE
        NO. = "SAS;TAB(90);CHRS(149)
3130 PRINT:2 TAB(12);CHRS(149);TAB(15);"DATE OF TEST =
        "D$;TAB(60);"TEMPERTURE = "TS;TAB(90);CHRS(149)
3140 PRINT:2 TAB(12);CHR$(149);TAB(15);"TYPE OF TEST =
        "BOS;TAB(60);"FREQUENCY = "UWS;TAB(90);CHRS(149)
3145 PRINT:2 TAB(12);CHR$(149);TAB(15);"ASPHALT TYPE =
        "ACS;TAB(60);"LOAD DURATION = "LDS;TAB(90);CHRS(149)
3147 PRINT:2 TAB(12);CHRS(149);TAB(15);"PERCENT AC =
        "PAS;TAB(90);CHRS(149)
3150 PRINT:2 TAB(12);CHRS(149);TAB(15);"OIAMETER = "LOS"
        IN.";TAB(60);"THICKNESS = "OES" IN.";TAB(90);CHRS(149)
```

```
3160 PRINT:2 TAB(12);CHR$(149);TAB(15);"LOAD CELL CAPACITY = "LCS"
    LBS.";TAB(60);"MTS LOAD RANGE = "LMS" %";TAB(90);CHR$(149)
3170 PRINT:2 TAB(12);CHR$(149);TAB(15);"LVDT (LAT.) FS RANGE = "LRS"
    IN.";TAB(60);"LVDT (VERT.) FS RANGE = "LIS" IN.";TAB(90);CHRS(149)
3180 A=1
3190 PRINT:2 TAB(12);
3200 A=A+1
3210 PRINT:2 CHRS(I31);
3220 IF A=79 THEN PRINT:2 CHR$(129) GOTO 3240
3230 GOTO 3200
3240 PRINT:2 CHR$(30)
3250 PNT :2 10
3280 CLOSE :2
3290 GOTO 1000
4000 REM ***************************(THIS ROUTINE GIVES VALUES IN EN-
    GINEERING UNITS)
4001 PNT 194 CLS O PNT 10 PNT 10 PNT 10
4 0 0 2 ~ F L A G M E = 1
4003 PNT 194 !" ";PNT 19; INPUT " DO YOU WANT TO
    CALIBRATE ? "Z1$
4005 IF Z1$="Y" THEN 4010
4 0 0 7 ~ I F ~ Z 1 \$ = " N " ~ T H E N ~ R E T U R N
4008 PNT 10 PNT 176 PNT 194 !" ";PNT 197 !" PLEASE TYPE Y
    OR N !! " WAIT l.5 GOTO 4000
4010 PNT 191 CLS 0 WAIT 1 PNT 10 PNT 197
4 0 2 0 ~ P R I N T ~ " ~ C A L I B R A T I O N ~ F O R ~ I E S T ~ C O N D U C T E D ~ O N ~ M T S
        SYSTEM
4025 ON ERROR 205,4056
4030 PNT 191 !" ";PNT 10 PNT 197 INPUT " CHANNEL
    NUMBER? " C' PNT 191
4035 OFF ERROR
4040 IF C'<1 THEN }405
4 0 4 3 ~ I F ~ C ' = 1 ~ T H E N ~ 4 0 6 0 ~
4045 IF C'>4 THEN }405
4 0 5 0 ~ I F ~ C ' = 2 ~ T H E N ~ 4 4 1 0 ~
4 0 5 3 ~ I F ~ C ' = 3 ~ T H E N ~ 4 7 5 0 ~
4 0 5 5 ~ I F ~ C ' = 4 ~ T H E N ~ 4 8 8 7 ~
4056 PNT 10 PNT 176 PNT 191 !" ";PNT 194 !" INCORRECT
    INPUT!! PLEASE TRY AGAIN."GOTO 4030
4060 !" ";PNT 197 INPUT " LOAD CELL CAPACITY? "L1$
    PNT 191
4070 PNT 191 !"
    100%-10% ? "R1S
4075 IF RlS>"100" THEN PNT 176 PNT 10 PNT 191 !" ";PNi 194
        !" INCORRECT INPUT!! NEEDS TO BE 100%-10%!!! " PNT 10 GOTO 4070
```

4076 IF R1\$<"10" THEN PNT 176 PNT 10 PNT 191 !" ";PNT 194
!" INCORRECT INPUT!! NEEOS TO BE 100\%-10\%!!!" PNT 10 GOTO 4070
4080 PNT 191 !" ";PNT 197 INPUT " ARE ZERO AND
SCALE FACTOR CORRECT? (Y,N) "Zl\$ PNT 191
4090 IF Z1\$="Y" THEN 4120
4095 IF Z1\$="N" THEN 4100
4096 PNT 176 GOTO 4080
4100 ON ERROR 122, 4112
4101 PNT 191 !" ";PNT 197 INPUT " ZERO OFFSET? "Al
PNT 191
4110 PNT 191 !" ";PNT 197 INPUT " SCALE FACTOR FOR
ENGINEERING UNITS= "B1 PNT 191
4111 OFF ERROR GOTO 4120
4112 PNT 176 PNT 191 !" ";PNT 194 !" INCORRECT INPUT!!
PLEASE TRY AGAIN. " GOTO 4101
4120 WAIT I CLS 0 PNT 10 PNT 10
4130 !" ";PNT 10 PNT 197 PRINT " LOAD CELL
CAPACITY = "L1S" LBS." PNT 191
4140 !" ";PNT 197 PRINT " MTS LOAD RANGE = "Rl\$
PNT 191
4150 !" ";PNT 197 PRINT " ZERO OFFSET = "A1 PNT
191
4160 !" ";PNT 197 PRINT " SCALE FACTOR FOR
ENGINEERING UNITS= "B1 PNT 191
4170 PNT $10 \mathrm{C}^{\prime}=1$
4180 Xl=AIN: $0\left(2, C^{\prime}-1\right)$
4190 FOR I=1 TO 5
4195 PNT 191
4200 L=VAL(L1\$)
4210 R=VAL(R1\$)
4220 IF $R=100$ THEN $L 1=L / 10$
4250 IF $R=10$ THEN $L 1=L / 100$
$4260 \mathrm{X}=(\operatorname{AIN}: 0(2,0)) *$ LI*Bl+Al
4270 WAIT 2
4280 !" ";PNT 193 PRINT USING "-\#\#\#.\#
LBS.",X PNT 191
4290 NEXT I
4295 PNT 10
4300 PNT 191 !" ";PNT 196 INPUT " IS MORE CAL DATA
REQURIED FOR CHANNEL \#1? (Y,N) "Z1S PNT 191
4310 IF Z1\$="Y" THEN GOTO 4120
4315 IF Z1S="N" THEN GOTO 4330
4320 PNT 176 GOTO 4300
4330 PNT 191 !" ";PNT 10 PNT 196 INPUT " ARE ZERO
AND SCALE FACTOR CORRECT? (Y,N) "Z1\$ PNT 191

```
4340 IF Z1S="N" THEN 4100
4 3 4 5 ~ I F ~ Z 1 S = " Y " ~ T H E N ~ 4 3 5 0
4 3 4 6 ~ P N T ~ 1 7 6 ~ G O T O ~ 4 3 3 0
4350 PNT 191 !" ";PNT 10 PNT 192 INPUT " DO YOU REQUIRE
    ADDITIONAL CHANNELS CALIBRATED? (Y,N) "Z1$
4 3 6 0 ~ I F ~ Z 1 \$ = " Y " ~ T H E N ~ 4 0 0 0 ~
4370 IF Z1$="N" THEN RETURN
4371 PNT 176 GOTO 4350
4400 PNT 191 CLS 0 PNT 10 PNT 10 PNT 10
4410 !" ";PNT 197 INPUT " ACT. LVDT RANGE FULL
SCALE?"L$ PNT 191
4430 PNT 191 !" ";PNT 197 INPUT " ARE ZERO AND
    SCALE FACTOR CORRECT? (Y,N) "Z1$ PNT 191
4440 IF Z1$="Y" THEN 4470
4441 IF ZI$="N" THEN GOTO 4450
4442 PNT 176 GOTO 4430
4450 ON ERROR 122,4465
4451 PNT 191 !" ";PNT I97 INPUT " ZERO OFFSET?"A2
    PNT 191
4460 PNT 191 !" ";PNT 197 INPUT " SCALE FACTOR FOR
ENGINEERING UNITS?"B2 PNT 191
4461 OFF ERROR GOTO 4470
4465 PNT 176 PNT 191 !" ";PNT 194 !" INCORRECT INPUT!!!
PLEASE TRY AGAIN. " GOTO 4451
4470 CLS O PNT 191 PNT 10 PNT 10 PNT 10 WAIT 1
4475 PNT 191
4480 PNT 191 !" ";PNT 197 PRINT " ACT. LVDT RANGE
    FS="L$" IN." PNT 191
4500 PNT 191 !"
";PNT 197 !" ZERO OFFSET= "A2
4510 PNT 191 !" ";PNT 197 PRINT " SCALE FACTOR FOR
ENGINEERING UNITS="B2 PNT 191
4 5 2 0 ~ P N T ~ 1 0 ~
4530 C'=2
4540 FOR I=1 TO 5
4600 X=(AIN:O(2,1)+A2)*M1*B2
4 6 1 0 ~ W A I T ~ 2 ~
4620 PNT 191 !" ";PNT 193 PRINT USING
"-##.### IN.",X
4 6 4 0 ~ N E X T ~ I ~
4660 PNT 191 !" ";PNT 10 PNT 196 INPUT " IS MORE CAL
DATA REQURIED FOR CHANNEL #2? "Z1$
4665 PNT 191
4670 IF Z1S="Y" THEN 4470
4671 IF Z1S="N" THEN 4680
4672 PNT 176 GOTO 4660
```

4680 PNT 191 !"
";PNT 10 PNT 196 INPUT " ARE ZERO
AND SCALE FACTOR CORRECT? "Z1S
4690 IF Z1S $=$ "N" THEN 4450
4691 IF $21 \$=$ "Y" THEN 4700
4692 PNT 176 GOTO 4680
4700 PNT 191 !" ";PNT 10 PNT 192 INPUT " DO YOU REQUIRE ADDITIONAL CHANNELS CALIBRATED? "Z1\$
4710 IF $21 \$=" Y$ " THEN 4000
4720 IF Z1\$="N" THEN RETURN
4740 PNT 176 GOTO 4700
4745 PNT 191 CLS 0 PNT 10 PNT 10 PNT 10
4750 !" ";PNT 197 INPUT " LAT. LVDT RANGE FULL SCALE?"L2\$ PNT 191
4780 PNT 191 !" ";PNT 197 INPUT " ARE ZERO AND SCALE FACTOR CORRECT? (Y,N) "Z1\$ PNT 191
4785 IF Z1\$="Y" THEN 4810
4790 IF Z1\$="N" THEN GOTO 4797
4795 PNT 176 GOTO 4785
4797 ON ERROR 122,4807
4800 PNT 191 !" ";PNT 197 INPUT " ZERO OFFSET?"A3
PNT 191
4803 PNT 191 !" ";PNT 197 INPUT " SCALE FACTOR FOR
ENGINEERING UNITS?"B3 PNT 19]
4805 OFF ERROR GOTO 4810
4807 PNT 176 PNT 191 !" ";PNT 194 !" INCORRECT INPUT!!!
PLEASE TRY AGAIN. " GOTO 4800
4810 CLS 0 PNT 191 PNT 10 PNT 10 PNT 10 WAIT 1
4813 PNT 191
4815 PNT 191 !"
FS="L2\$" IN." PNT 191
4820 PNT 191 !"
";PNT 197 !" ZERO OFFSET= "A3
4823 PNT 191 !" ";PNT 197 PRINT " SCALE FACTOR FOR
ENGINEERING UNITS="B3 PNT 191
4825 PNT 10
$4827 C^{\prime}=5$
4830 FOR $I=1$ TO 5
$4845 X=(\operatorname{AIN}: 0(3,1)+A 3) * B 3$
4847 WAIT 2
4850 PNT 191 !" ";PNT 193 PRINT USING
"-\#.\#\#\#\#\# IN.", X
4853 NEXT I
4855 PNT 191 !" ";PNT 10 PNT 196 INPUT " IS MORE CAL DATA REQURIED FOR CHANNEL "3? "Zls
4857 PNT 191
4860 IF Z1S="Y" THEN 4810

```
4 8 6 3 ~ I F ~ Z 1 S = " N " ~ T H E N ~ 4 8 6 7 ~
4 8 6 5 ~ P N T ~ 1 7 6 ~ G O T O ~ 4 8 5 5 ~
4867 PNT 191 !" ";PNT 10 PNT 196 INPUT " ARE ZERO
    AND SCALE FACTOR CORRECT? "Z1$
4870 IF 21$="N" THEN 4800
4 8 7 3 ~ I F ~ Z 1 S = " Y " ~ T H E N ~ 4 8 7 7 ~
4875 PNT 176 GOTO 4680
4877 PNT 191 !" ";PNT 10 PNT 192 INPUT " DO YOU REQUIRE
        ADDITIONAL CHANNELS CALIBRATED? "Z1S
4 8 8 0 ~ I F ~ Z 1 \$ = " Y " ~ T H E N ~ 4 0 0 0 ~
4 8 8 3 ~ I F ~ Z 1 \$ = " N " ~ T H E N ~ R E T U R N
4885 PNT 176 GOTO 4877
4 8 8 7 \text { PNT 191}
4890 !" ";PNT 197 INPUT " VERT. LVDT RANGE FULL
        SCALE?"L3$ PNT 191
4 8 9 3 ~ P N T ~ 1 9 1 ~ ! " ~ " ; P N T ~ 1 9 7 ~ I N P U T ~ " ~ A R E ~ Z E R O ~ A N D
        SCALE FACTOR CORRECT? (Y,N) "Z1$ PNT 191
4 8 9 5 ~ I F ~ Z 1 S = " Y " ~ T H E N ~ 4 9 1 5 ~
4897 IF Z1$="N" THEN GOTO 4903
4900 PNT 176 GOTO 4893
4903 ON ERROR 122,4913
4905 PNT 191 !" ";PNT 197 INPUT " ZERO OFFSET?"A4
        PNT 191
4907 PNT 191 !" ";PNT 197 INPUT " SCALE FACTOR FOR
        ENGINEERING UNITS?"B4 PNT 191
4910 OFF ERROR GOTO 4915
4913 PNT 176 PNT 191 !" ";PNT 194 !" INCORRECT INPUT!!!
    PLEASE TRY AGAIN. " GOTO 4903
4915 CLS 0 PNT 191 PNT 10 PNT 10 PNT 10 WAIT 1
4917 PNT 191
4920 PNT 191 !" ";PNT 197 PRINT " VERT. LVDT RANGE
    (FS)="L3S" IN." PNT 191
4923 PNT 191 !"
";PNT 197 !" ZERO OFFSET= "A4
4925 PNT 191 !" ";PNT 197 PRINT " SCALE FACTOR FOR
    ENGINEERING UNITS="B4 PNT 191
4 9 2 7 ~ P N T ~ 1 0 ~
4930 C'=5
4 9 3 3 ~ F O R ~ I = 1 ~ T O ~ 5 ,
4 9 3 5 X = ( A I N : O ( 3 , 0 ) + A 4 ) * B 4
4 9 3 7 \text { WAIT 2}
4 9 4 0 ~ P N T ~ 1 9 1 ~ ! " ~ " ; P N T ~ 1 9 3 ~ P R I N T ~ U S I N G ~
        "-#.##### IN.",X
4 9 4 3 ~ N E X T ~ I ~
4 9 4 5 ~ P N T ~ 1 9 1 ~ ! " ~ " ; P N T ~ 1 0 ~ P N T ~ 1 9 6 ~ I N P U T ~ " ~ I S ~ M O R E ~ C A L ~
        DATA REQURIED FOR CHANNEL #4? "ZlS
```

```
4 9 4 7 ~ P N T ~ 1 9 1 ~
4 9 5 0 ~ I F ~ Z l \$ = " Y " ~ T H E N ~ 4 9 1 5 ~
4 9 5 3 ~ I F ~ 2 1 \$ = " N " ~ T H E N ~ 4 9 5 7 ~
4 9 5 5 ~ P N T ~ 1 7 6 ~ G O T O ~ 4 9 4 5
4957 PNT 191 !" ";PNT 10 PNT 196 INPUT " ARE ZERO
    AND SCALE FACTOR CORRECT? "Z1$
4960 IF Zl$="N" THEN 4905
4 9 6 3 ~ I F ~ Z 1 S = " Y " ~ T H E N ~ 4 9 6 7 ~
4 9 6 5 ~ P N T ~ 1 7 6 ~ G O T O ~ 4 9 5 7 ~
4967 PNT 191 !" ";PNT 10 PNT 192 INPUT " DO YOU REQUIRE
    ADDITIONAL CHANNELS CALIBRATED? "Z1$
4970 IF Z1$="Y" THEN 4000
4 9 7 3 ~ I F ~ Z 1 \$ = " N " ~ T H E N ~ 1 0 0 0 ~
4 9 7 5 ~ P N T ~ 1 7 6 ~ G O T O ~ 4 9 6 7 ~
5000 REM *************************THIS ROUTINE PROMPTS CHECK-OUT
PROCEDURE PRIOR TO TEST*************************
5010 PNT 192 CLS 0 PNT 197 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10 PNT
        10 PNT 10
5012 IF FLAGME=O PNT 192!" ";PNT 197 !"ONLY FEABLE
    MINDS FORGET TO CALIBRATE!" WAIT 5 N2$="CI" GOTO 1230
5020 PNT 192 !" ";PNT 197 INPUT " IS THIS A NEW
        TEST (Y/N) ? "Z1$
5030 IF Z1S="N" GOTO 5055
5040 IF Z1S="Y" GOTO 5220
5050 IF Zl$<>"Y" PNT 176 GOTO 5020
5 0 5 5 \text { PNT 195 CLS O PNT 197 PNT 10 PNT 10 PNT 10}
5060 PNT 195 !" ";PNT 197 !"
5070 PNT 195 !" ";PNT 197 !"
COMPLETE ?
5080 PNT 195 !" ";PNT 197 !"
5085 PNT 195 !"
    AND READY ?
5090 PNT 195 !"
5100 PNT 195 !"
    CONTROLS SET ?
5105 PNT 195 !"
        #
5110 PNT 195 !"
        FUNCTION (8) !!
5120 PNT 195 !"
    ";PNT 197 !"
                                    IS THE CHART SET
";PNT 197 !"
";PNT 197 !" MTS GAIN & DITHER
";PNT 197 !"
    ";PNT 197 !" SELECT WAVETEK
";PNT 197 !"
```

```
5130 PNT 195 !" ";PNT 197 !" MTS SPAN CONTROL SET TO
5140 PNT 195 !" ";PNT 197 !"
5150 PNT 195 !" ";PNT 197 !" TEST SPECIMEN &
    APPARATUS READY ?
5155 PNT 195 !" ";PNT 197 !"
5160 PNT 195 !" ";PNT 197 !" PROJECT
        ENGINEER/TECHNICIAN READY ?
5165 PNT 195 !" ";PNT 197 !"
5170 PNT 195 !" ";PNT 197 !" ALL CONTROLS & VALVES IN
        CORRECT POSITION FOR TEST ?
5175 PNT 195 !" ";PNT 197 !"
5180 PNT 10 PNT 195 ! " ";PNT 192 INPUT " SYSTEM IS NOW READY TO
        START TEST << TYPE (Y) TO RETURN TO MENU >> "Z1$
5185 22$="GO"
5190 IF Z1$="Y" WAIT 1 GOTO 1000
5200 PNT 176 GOTO 5180
5220 REM THIS ROUTINE SETS TIME & DATA ARRAYS TO ZERO
5230 Z2S="0"
5240 PNT 27 PNT 12 WAIT l
5250 PNT 10 PNT 192 !" ";PNT 196 !" *** TO ZERO TIME & DATA
        ARRAYS TYPE (Y) & PRESS RETURN *** "
5255 PNT 10 PNT 192 !" ";PNT !`.2 INPUT" ** OR TO ABORT
        TYPE (N) & PRESS RETURN ** "Z1S
5260 IF Z1S="N" THEN 1000
5270 IF Z1$<>"Y" ! "INCORRECT INPUT" WAIT 1 GOTO 5250
5360 GOTO 5055
5370 END
6000 REM ****************************************************************
6001 REM ****************** ROUTINE FOR PRECONDITIONING SAMPLE
```



```
6002 F7=0
6005 PNT 185 PNT 192 CLS O PNT 196 PNT 10 PNT 10 PNT 10
6 0 0 6 ~ I F ~ F L A G M E = 0 ~ P N T ~ 1 9 2 ! " ~ " ; P N T ~ 1 9 7 ~ ! " O N L Y ~ F E A B L E ~
        MINDS FORGET TO CALIBRATE!" WAIT 5 N2S="CI" GOTO 1230
6110 PNT 194 CLS O FOR X=1 TO 6 PNT 10 NEXT X
6111 PNT 194 !" ";PNT 193 !" PLEASE WAIT WHILE WAVETEK IS
        BEING PROGRAMMED. "
6112 GPIB:4 0,13
6113 CONFIG 0,13
6114 PRINT:4 "C901P1B1A1000F";1/VAL(UWS)
```

```
6115 U=VAL(LP$)/VAL(UW$)
6117 W=INT(255*U)
6119 AA=(180/57.29577951)/W
6121 XX=0
6 1 2 2 ~ P R I N T : 4 ~ " X O Y O " ~
6123 FOR Z=1 TO W
6125 XX=XX +AA
6127 YY=SIN(XX)*127
6130 PRINT:4 "X";Z;"Y";YY
6 1 3 2 ~ N E X T ~ Z ~
6134 FOR T= W TO 255
6 1 3 6 ~ P R I N T : 4 ~ " X " ; T ; " Y O " '
6 1 3 8 \text { NEXT T}
6267 WAIT 1.3 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10
6270 P9=0
6340 PNT 192 CLS 0 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10
6345 PNT 192 !" ";PNT 193 !"
6346 PNT 192 !" ";PNT 193 !" Please enter no. of cycles
PRECONDITION is set to run. "
6347 PNT 192 !" ";PNT 193 !"
    ";INPUT P8
6348 PRINT:4 "L";P8;"Il"
6 3 4 9 ~ L O C A L
6350 CLOSE:4
6 3 5 1 ~ C L S ~ 4 ~ G R A P H I C S ~ 4 ~ C O L O R ~ 4 , 1 ~ F O N T ~ 4 ~ D I S P L A Y ~ 4 ~ W I N D O W ~ - 1 0 , 1 0 , - 1 0 , 1 0
6352 HPRINT -2.2,5,"TEST"
6 3 5 3 ~ H P R I N T ~ - 1 , . 5 , " I N " '
6354 HPRINT -4.5,-4,"PROGRESS"
6 3 5 5 \text { FONT 1 COLOR } 6
6 3 5 6 ~ H P R I N T ~ - 6 , - 9 . 5 , " P R E S S ~ R E T U R N ~ T O ~ T A K E ~ D A T A " ~
6358 R4=((AIN:0(3,0)+A4)*B4)+.001 F7=0 ACT 4 WAIT . }2\mathrm{ DOT:0(4,0)=1
        DOT:0(4,0)=0
6 3 5 9 ~ C O U N T = 0 ~ T = . 1
6360 V1=AIN:O(2,3) IF V1>8 THEN COUNT=COUNT+1 WAIT . }
        R3=((AIN:O(3,0)+A4)*B4) ACT 3
6 3 6 5 ~ I F ~ C O U N T = 2 ~ T H E N ~ T = 1 ~
6370 IF F7=1 THEN KILL 2 KILL 3 GOTO 7000
6371 IF F7=2 THEN 6630
6372 GOTO 6360
6 3 8 0 \text { GOTO } 6 4 4 0
6381 FOR I=1 TO 49
6390 R3=R2(I)
6400 IF R3>R2(I+1) THEN EXIT FOR GOTO }644
6 4 1 0 ~ N E X T ~ I ~
```

```
6440 P9=P9+1 IF PG=>P8-5 THEN P8=0 F7=1
6442 REM IF P9=1 THEN R4=(R3+.001)
6445 IF P9>1 THEN IF R3>R4 THEN F7=2
6450 SUSPEND 3
6455 GOTO 6380
6 4 5 6 ~ E N D
6630 KILL 3 KILL 2
6 6 4 0 ~ F O R ~ I = 1 ~ T O ~ 1 2 ~ P N T ~ 1 0 ~ N E X T ~ I ~
6650 CLS 4 DISPLAY 0 PNT 7 PNT }7\mathrm{ PNT }7\mathrm{ PNT }194\mathrm{ CLS 0 PNT 10 PNT 10 PNT
    10 PNT 10
6655 PNT 194 !" ";PNT 193 !"
6 6 6 0 ~ P N T ~ 1 9 4 ~ ! " '
        deformation has
6 6 7 0 ~ P N T ~ 1 9 4 ~ ! " '
        . }001\mathrm{ inches!
6 6 7 3 \text { PNT 194 !"}
6 6 7 5 \text { WAIT 2 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10}
6677 PNT 194 !" ";PNT 193 !"
6 6 8 0 ~ P N T ~ 1 9 4 ~ ! " '
";PNT 193 !" THE LAST THREE
    PULSES ARE NOW
6 6 8 5 \text { PNT 194 !"}
6 6 9 0 ~ P N T ~ 1 9 4 ~ ! " '
        "
6 7 0 0 \text { GOTO 7000}
6900 REM *****************<TASK 2 STARTS HERE>******************
6910 FOR I=1 TO 50
6920 R2(I)= (AIN:0(3,0)+A4)*B4
6 9 3 0 ~ N E X T ~ I ~
6 9 4 0 ~ A C T ~ 3 ~
6 9 5 0 ~ S U S P E N D ~ 2 ~ G O T O ~ 6 9 1 0 ~
6 9 6 0 \text { END}
7000 REM
7001 REM **** THIS ROUTINE STARTS TEST ****
7005 IF VAL(UWS)=2 DDD=.5
7006 IF VAL(UW$)=3 DDD=.25
7007 IF VAL(UW$)=1 DDD=.5
7010 IF AIN:O(2,3)>1 THEN WAIT VAL(UWS)-DDD*VAL(LP$) GOTO }702
7020 GOTO 7010
7021 T6=TIMER
7022 FOR I=1 TO 950
7024 X1(I)=AIN:0(3,1)
```

```
7026
    X2(I)=AIN:0(3,0)
7027 X3(I)=AIN:O(2,0)
7 0 3 0 ~ N E X T ~ I ~
7036 T7=TIMER T7=T7-T6
7040 IF AIN:O(2,3)<.2 THEN DOT:O(4,1)=1 DOT:0(4,1)=0 GOTO 7046
7 0 4 5 \text { GOTO } 7 0 4 0
7046 FOR I=1 TO 950
7047 XI(I) =(XI(I)+A3)*83
7048 X2(I) =(X2(I)+A4)*B4
7049 X3(I)=X3(I)*L1*B1+A1
7 0 5 0 ~ N E X T ~ I ~
7051 CLS }4\mathrm{ GRAPHICS 4 COLOR 2,1 FONT 2 DISPLAY 4 WINDOW -10,10,-10,10
7060 HPRINT -7,5,"TESTING" HPRINT -1,1,"IS" HPRINT 1,-3,"COMPLETE"
7070 FONT 1 COLOR 6 HPRINT -2,-8,"PRESS RETURN"
7 0 8 0 ~ I N P U T ~ G O T O ~ 7 2 2 0 ~
7090 FOR I=601 TO 900
7095 X5=X2(I)
7100 X3=X3(601)
7110 IF X5>X2(I+1) AND X5>X2(I+25) EXIT FOR GOTO }713
7120 NEXT I
7130 FOR X=I TO 900
7140 IF X3(X)<=X3(601) THEN X9=X2(X) EXIT FOR GOTO 7151
7150 NEXT X
7151 FOR P=601 TO 900
7152 IF X3(P)<20 GOTO 7155
7153 X10=X3(P)
7154 IF X10>X3(P+1) AND X10>X3(P+15) EXIT FOR GOTO }715
7155 NEXT P
7156 X10=X10-5
7160 DV = X5-XJ
7170 MR=(3.59*X10)/(VAL(DES)*DV)
780 CLS 4 DISPLAY O CLS O PNT 191 PNT 174 PNT 180 PNT }19
7190 PNT 10 PNT 10 PNT 10 PNT 10
7200 FOR Y=1 TO 20 PNT 171 NEXT Y !" THE RESILIENT MODULUS = "MR PNT 185
PNT 180 PNT 191
7210 INPUT
7220 GOTO 16000
7500 INPUT
7510 F7=1
7 5 2 0 ~ S U S P E N D ~ 4 ~
7530 GOTO 7500
7540 END
10000 |************************************ THIS ROUTINE DRAWS ALL
    PLOTS ********************************************
10010 PNT 180 PNT 190 CLS 0 PNT 10 PNT 10
```

```
10020 PNT 190 !"
10030 PNT 190 !"
10040 PNT 190 !"
    "
10050 PNT 10 PNT 10 PNT 10 PNT 10
10060 PNT 190 !" ";PNT 196 !"
10070 PNT 190 !" ";PNT 196 !" 1) PLOT DATA ON
    SCREEN "
10080 PNT 190 !" ";PNT 196 !" 2) RETURN TO
    MAIN MENU
10090 PNT 190 !"
10100 PNT 10 PNT 10 PNT 10 PNT 10 PNT 190 !" ";PNT
    192 !" "
10110 PNT 181 PNT 190 !"
    ENTER NO. OF CHOICE
10120 PNT 190 !" ";PNT 192 INPUT"
    "Z1S PNT 183
10130 IF Z1$="1" GOTO 10170
10140 IF Z1$="2" GOTO 1000
10150 PNT 176 CLS O GOTO 10000
10160 REM
10170 PNT 190 CLS O PNT 183 PNT 180 PNT 185 DISPLAY O
10180 PNT 10 PNT 10 PNT 190 !" ";PNT 193 ! "
10190 PNT 190 !" ";PNT 193 ! " MENU FOR SCREEN
    PLOTTING "
10200 PNT 190 !" ";PNT 193 ! "
10210 PNT 10 PNT 10 PNT 10 PNT 10
10220 PNT 190 !"
    "
10230 PNT 190 !"
    "
10240 PNT 190 !"
    DEFORMATION "
10250 PNT 190 !"
    DEFORMATION "
10260 PNT 190 !" ";PNT 196 !" 4) RETURN TO PLOT
    MENU
10270 PNT 190 !"
    "
```

```
10280 PNT 10 PNT 190 !"
    "
10290 PNT 181 PNT 190 !"
    ENTER NO. OF CHOICE
10300 PNT 190 !"
    "Z1$ PNT 183
10310 IF Z1$="1" GOTO 10360
10320 IF Z1$="2" GOTO 10654
10330 IF Z1$="3" GOTO 10895
10340 IF Z1$="4" GOTO 10000
10350 PNT 176 GOTO 10170
10360 REM PLOT LOAD ROUTINE
10370 PNT 180 PNT 10 PNT 190 !"";PNT 193 INPUT " PLEASE ENTER NO. OF
    PULSE TO PLOT (1,2,3 OR 4 FOR ALL PULSES) "Z1
10376 BB=0
10379 IF Z1=4 THEN PT=1
10380 GRAPHICS 4 COLOR 0,7 Yl=VAL(VL$)+20 Y5=-5*(Yl/l00)
10387 IF PT=1 THEN FOR Zl=1 TO 3
10389 MOVE 0,0
10390 IF Z1=1 THEN S1'=1 S2'=300 WINDOW -4,300,Y5,Y1 V=0 0=1 P=300
10400 IF Z1=2 THEN S1'=300 S2'=600 WINDOW 296,600,Y5,Y1 V=300 0=300
    P=600
10410 IF Zl=3 THEN S1'=600 S2'=900 WINDOW 596,900,Y5,Yl V=600 0=600
    P=900
10412 IF Z1=2 AND PT=1 THEN COLOR 2 MOVE 0,0
10414 IF Zl=3 AND PT=1 THEN COLOR 5 MOVE O,0
10420 DISPLAY 4
10430 FOR I=S1' TO S2' X4(I)=X3(I) NEXT I
10440 IF Z1=1 AND PT=1 THEN VAXISP V,(Yl/10) HAXIS 0,30
10444 IF PT=0 THEN VAXISP V,(Y1/10) HAXIS 0,30
10445 IF Z1=2 COLOR 2
10446 IF Zl=3 COLOR 5
10448 MOVE S1',X4(S1')
10449 FOR I=S1' TO S2'
10450 PLOT I,X4(I)
10455 NEXT I
10458 IF Z1=3 THEN PT=0
10459 IF PT=0 THEN EXIT FOR GOTO 10570
10460 NEXT Z1
10570 FOR I=0 TO P
10575 IF X3(I)<20 THEN GOTO 10600
10580 X5=X3(I)
10590 IF X5>X3(I+1) AND X5>X3(I+15) THEN EXIT FOR GOTO 10620
10600 NEXT I
10610 GOTO 10340
```

```
10620 LET L1S = STR$(X5) LET L2$="PEAK LOAD =" LET L3$=" LBS."
10625 IF BB=0 THEN COLOR l MOVE I,X5 IPLOT 50,0 HPRINT I+50,X5,L2$L1$L3$
10631 IF BB=O THEN GOTO 10650
10650 INPUT GOTO 10160
10654 PNT 183 REM ***** ROUTINE FOR VERTICAL DEF *****
10655 FLAG=0
10660 FLAG=FLAG+1
10670 PNT 190 CLS O PNT 180 PNT 185 DISPLAY 0
10680 PNT 10 PNT 10 PNT 190 !" ";PNT 193 !"
10690 PNT 190 !" ";PNT 193 !" PLOT VERT
    DEFORMATION ROUTINE "
10700 PNT 190 !" ";PNT 193 !"
10710 PNT 10 PNT 10 PNT 10 PNT 190 !" ";PNT 196
    INPUT" DO YOU WISH TO RETURN TO MENU ? (Y/N) "N1$
10720 IF N1$="Y" THEN 10170
10730 PNT 10 PNT 10 PNT 190 !"";PNT 197 INPUT" PLEASE ENTER NO. OF
    PULSE TO SEE. (1 THRU 3 OR 4 FOR ALL PULSES) "Z1
10736 BB=0
10739 [F Z1=4 THEN PT=1
10740 GRAPHICS 4 COLOR 0,7 CLS 4 DISPLAY }
10750 WINDOW -4,300,-.0001,.0025
10760 VAXISP 0,.00005 HAXIS 0,30
10765 IF PT=1 THEN FOR Zl=1 TO 3
10770 MOVE 0,0
10780 IF Z1=1 THEN S1'=1 S2'=300
10790 IF Zl=2 THEN S1'=300 S2'=600 WINDOW 296,600,-.0002,.0025 COLOR 2
10800 IF Zl=3 THEN S1'=600 S2'=900 WINDOW 596,900,-.0002,.0025 COLOR 5
10810 DISPLAY 4
10815 MOVE S1',X2(S1')
10820 FOR I=S1' TO S2'
10830 Y=X2(I)
10840 PLOT I,Y
10850 NEXT I
10852 IF Z1=3 THEN PT=0
10855 IF PT=0 THEN EXIT FOR GOTO 10860
10856 NEXT Z1
10860 IF BB=0 THEN GOTO 10892
10892 INPUT GOTO 10660
10895 PNT 183 REM ***** ROUTINE FOR HORIZONTAL DEF. *****
10900 FLAG=0
10910 FLAG=FLAG+1
10920 PNT 190 CLS O PNT 180 PNT 185 DISPLAY O
```

```
10930 PNT 10 PNT 10 PNT 190 !"
    "
10940 PNT 190 !" ";PNT 193 !" PLOT HORIZONTAL
    DEFORMATION "
10950 PNT 190 !" ";PNT 193 !"
10960 PNT 10 PNT 10 PNT 10 PNT 190 !" ";PNT 196
    INPUT" DO YOU WISH TO RETURN TO MENU ? (Y/N) "NIS
10970 IF N1S="Y" THEN }1017
10980 PNT 10 PNT 10 PNT 190 !"";PNT 197 INPUT" PLEASE ENTER NO. OF
    PULSE TO SEE. (1 THRU 3 OR 4 FOR ALL PULSES) "Z1
10986 BB=0
10989 IF Z1=4 THEN PT=1
10990 GRAPHICS 4 COLOR 0,7 CLS 4 DISPLAY 4
11000 WINDOW -40,300,-.0003,.0003
11010 VAXIS 0,.000010 HAXIS -.00027,30
11015 IF PT=1 THEN FOR Zl=1 TO 3
11020 MOVE 0,0
11030 IF 21=1 THEN S1'=1 S2'=300
11040 IF 21=2 THEN S1'=300 S2'=600 WINDOW 260,600,-.0003,.0003 COLOR 2
11050 IF Z1=3 THEN S1'=600 S2'=900 WINDOW 560,900,-.0003,.0003 COLOR 5
11060 DISPLAY 4
11065 MOVE S1',X1(S1')
11070 FOR I=S1' TO S2'
11080 Y=(X1(I))
11090 PLOT I,Y
11100 NEXT I
11110 IF 2l=3 THEN PT=0
11112 IF PT=0 THEN EXIT FOR GOTO 11115
11114 NEXT Z1
11115 IF BB=0 THEN GOTO 11150
11150 INPUT GOTO 10910
11160 PLOT I,X4(1)
15999 REM
16000 CLS 4 DISPLAY O CLS 0 PNT 191 PNT 174 PNT 180 PNT 192
16005 PNT 10 PNT 10 PNT 10
16007 PNT 191 !" ";PNT 192 !"
16010 PNT 185 PNT 191 !" ";PNT 192 !" This routine stores
    header information, and also the
16020 PNT 191 !" ";PNT 192 !" the data on a disk in DRIVE
    A for future reference.
16025 PNT 191 !" ";PNT 192 !"
16030 PNT 10 PNT 10 WAIT 2
```

```
16070 PNT 191 !" ";PNT 192 !"
16076 PNT 191 !" ";PNT l92 !" Please insert disk to cantain
    data in DRIVE A and close door.
16080 PNT 191 !" ";PNT 192 !" Press RETURN when
    this is done "
16085 PNT 191 !" ";PNT 192 !"
    "; INPUT
16090 DSKRESET PNT 191 PNT 10 PNT 10 PNT 10
16100 PNT 191 !" ";PNT 192 !"
16105 PNT 191 !" ";PNT 192 !" Please enter a name for the data
    to be filed under. The name
16110 PNT 191 !" ";PNT 192 !" may be 8 characters in length, no
    spaces are allowed, so
16120 PNT 191 !" ";PNT 192 !" words may be separated by the
    underline mark.
16121 PNT 191 !" ";PNT 192 !"
    ";INPUT N7$
16125 PNT 186 DSKRESET
16130 OPENW:3 ""N7$".TES"
16135 PRINT:3 BLS
16137 PRINT:3 SAS
16139 PRINT:3 PAS
16140 PRINT:3 DS
16141 PRINT:3 ACS
16142 PRINT:3 LDS
16143 PRINT:3 LCS
16144 PRINT:3 LRS
16145 PRINT:3 LIS
16146 PRINT:3 TNS
16147 PRINT:3 BDS
16148 PRINT:3 UWS
16149 PRINT:3 LOS
16150 PRINT:3 DES
16151 PRINT:3 VLS
16152 PRINT:3 T$
16153 PRINT:3 LMS
16154 PRINT:3 LPS
16155 PRINT:3 A3
16156 PRINT:3 A4 !:3 B3 !:3 B4 !:3 Ll !:3 B1 !:3 Al
16160 SAVE ARRAY:3 X1(1)
16165 SAVE ARRAY:3 X2(1)
16170 SAVE ARRAY:3 X3(1)
16185 PNT 191 CLS 0 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10
```

```
16190 PNT 191 !"
    NAME "N7$"."
16200 WAIT 3
16210 CLOSE:3 RETURN
16235 INPUT:3 BLS
16237 INPUT:3 SA$
16239 INPUT:3 PA$
16999 REM
17000 PNT 195 PNT 174 PNT 10 PNT 10
17010 PNT 195 !" ";PNT 197 PRINT "
17020 PNT 195 !" ";PNT 197 PRINT " THIS ROUTINE READS DATA
    FROM A DISKETTE AND STORES IT IN "
17030 PNT 195 !" ";PNT 197 PRINT " PROGRAM MEMORY
    FOR PLOTTING AND ANALYSIS
17040 PNT 195 !" ";PNT 197 PRINT "
    "
17050 FOR I=1 TO 900
17060 X1(I)=0 X2(I)=0 X3(I)=0
17070 NEXT I
17080 WAIT 1.5 PNT 195 PNT 10 PNT 10 PNT 10 PNT 10
17090 PNT 195 !" ";PNT 196 !"
17100 PNT 195 !" ";PNT 196 !" I AM NOW READY TO READ AN
    EXISTING DATA FILE.
17110 PNT 195 !" ";PNT 196 !"
    FILE TO BE READ.
17120 PNT 195 !" ";PNT 196 !"
17130 PNT 195 !" ";PNT 196 !" ";INPUT N7$
17140 PNT 10 PNT 10 PNT 10
17150 PNT 195 !" ";PNT 197 !"
17160 PNT 195 !" ";FNT 197 !" PLEASE INSERT DISKETTE CONTAINING
    FILE TO BE READ IN DRIVE A.
17170 PNT 195 !" ";PNT 197 !" WHEN READY
    PRESS RETURN.
17180 PNT 195 !" ";PNT 197 !"
    ";INPUT
17190 DSKRESET
17200 WAIT 2
17210 ON ERROR 17,17430
17220 OPENR:3 ""N7S".TES"
17230 CLS 0 PNT 181 PNT 10 PNT 10 PNT 10 PRINT "
    ";PNT 193 !" FILE IS BEING READ. ";PNT 183 PNT 180
```

```
17235 INPUT:3 BL$
17237 INPUT:3 SA$
17239 INPUT:3 PA$
17240 INPUT:3 DS
17250 INPUT:3 ACS
17260 INPUT:3 LD$
17270 INPUT:3 LC$
17280 INPUT:3 LRS
17290 INPUT:3 LI$
17300 INPUT:3 TN&
17310 INPUT:3 BDS
17320 INPUT:3 UWS
17330 INPUT:3 LO$
17340 INPUT:3 DES
17350 INPUT:3 VLS
17360 INPUT:3 TS
17370 INPUT:3 LM$
17375 INPUT:3 LP$
17380 INPUT:3 A3
17390 INPUT:3 A4 INPUT:3 B3 INPUT:3 B4 INPUT:3 Ll INPUT:3 Bl INPUT:3 Al
17400 LOAD ARRAY:3 Xl(1)
17410 LOAD ARRAY:3 X2(1)
17420 LOAD ARRAY:3 X3(1)
17425 CLOSE:3 RETURN
17430 PNT 194 PRINT " UNABLE TO FIND FILE! " WA!T 2 OFF
    ERROR PNT 195 CLS O GOTO 17080
20000 END
```


## APPENDIX F

## SOURCE CODE FOR LONGMA.BAS

LONGMA.BAS was developed at the Waterways Experiment Station by a design team which included Mark Anderson, Thomas V. McEwen, Mitchell S. Jones, Harold T. Carr, and Terry V. Jobe. The program collects, displays, processes, and plots data acquired for the $A C$ dynamic modulus test (ASTM C 3497). A complete source code listing follows.

2 REM
10 REM*************************** 2/13/87
11 REM
20 REM THIS IS A WES DEVELOPED PROGRAM TO COLLECT,DISPLAY,PROCESS AND PLOT DATA ACQUIRED FOR THE AC DYNAMIC MODULUS TEST (ASTM C 3497)
22 REM
23 REM PROGRAM DEVELOPMENT BY MARK ANDERSON, THOMAS V. MCEWEN, MITCHELL S. JONES, HAROLD T. CARR, AND TERRY V. JOBE
25 REM
30 REM THE TEST CONDUCTED ON THE ** MTS CLOSED LOOP SYSTEM ** PAVEMENT SYSTEMS DIVISION, GEOTECHNICAL LABORATORY, WATERWAYS EXPERIMENT STATION
50 REM
60 REM************************************************************************) 70 REM
100 REM DIMENSION SECTION AND LIST OF VARIABLES IN USE. LINES 100-799
101 REM
130 REM VARIABLES USED ARE
ST\$,N,N2\$, DS,NS, P\$, P3\$, P4\$,AS,A1\$,L1\$,R1\$,L\$,R2\$
140 REM H\$,Z1\$=(Y/N),Z
$155 \mathrm{~N}=1426$
160 DIM X2(N), X3(N), X4(N)
170 DIM R1(10) REM ARRAY FOR INITIAL PRECONDITION READINGS
180 DIM R2(50) REM ARRAY FOR PRECONDITION DATA
965 WAIT 1
1000 KILL ALL
1002 TASK 2,6900,5
1003 TASK 3,6380,5
1004 TASK 4,7500,2
1010 PNT 186 REM MENU LIST

```
1020 DISPLAY O PNT 195 PNT 180 CLS O PNT 10
1030 PNT 195 !" ";PNT 196 !"***** PAVEMENT & SOIL TRAFFICABILITY
        INFORMATION ANALYSIS CENTER (GL) *****"
1050 IF ST$<>"" THEN PRINT " ";PNT 194 !
    STS
1060 PNT 10 PNT 10 PNT 195
1070 PNT 195 !" ";PNT 193 !"** Select from the following
    queries ** ACTION DESIRED **"
1080 PNT 10 PNT 10
1085 PNT 195 !" ";PNT 197 !"
1090 PNT 195 !" ";PNT 197 !" ACTION DESIRED
        CODE
1100 PNT 195 !" ";PNT 197 !" HEADER INFORMATION (CONSTANT
        FOR SERIES) HI
1105 PNT 195 !"
        INFO)
1110 PNT 195 !"
        PH "
1120 PNT 195 !"
        SCALE FACTOR)
1I30 PNT 195 !"
        PRIOR TO TEST
1150 PNT 195 !:
        ST "
1180 PNT 195 !"
        PL "
1182 PNT 195 !"
        PD "
1183 PNT 195 !"
        SD "
1184 PNT 195 !"
        RD "
1185 PNT 195 !"
    ";PNT 197 !"
1190 PNT 10 PNT 10
1200 PNT 195 !" ";PNT 192 INPUT " Please enter action code,
        then hit RETURN "N2S
1205 UU=0 00=0 II=0
1210 IF N2S="HI" LET N2S="" GOSUB 2000
1215 IF N2S="VI" LET N2S="" GOSUB 2600
1220 IF N2S="PH" LET N2S="" GOSUB 3000
1230 IF N2S="CI" LET N2S="" GOSUB 4000
1240 IF N2S="BC" LET N2S="" GOSUB 5000
1260 if N2S="ST" LET N2S="" GOSUB }600
```

```
1290 IF N2$="PL" LET N2$="" GOTO 10000
1292 IF N2$="PD" KILL ALL NCHAIN "PROCESS"
1294 IF N2$="SD" LET N2$="" GOSUB 16000
1296 IF N2$="RD" LET N2$="" GOSUB 17000
1300 IF N2$="" THEN }100
1310 PNT 176 PNT 194 ! "INVALID ACTION CODE PLEASE RE-ENTER!!" WAIT 2
    GOTO 1000
2000 REM *******************(THIS ROUTINE ACCEPTS CONSTANT HEADER
        INFORMATION)
2010 PNT 195 CLS O FNT 10 PNT 192
2020 PRINT " THIS ROUTINE ACCEPTS CONSTANT HEADER
        INFORMATION
2025 WAIT 1 PNT 10 PNT 10 PNT 10
2030 PNT 195 !" ";PNT 192 INPUT " 1) DATE ?
        "D$
2040 PNT 195 !"
        TYPE ? "ACS
2050 PNT 195 !"
        DURATION ? "LDS
2153 PNT 195 !"
        CAPACITY (LBS.) ? "LCS
2l54 PNT 195 !"
        RANGE ? "LM$
2155 PNT 195 !" ";PNT 192 INPUT " 6) LVDT
        (LAT.) RANGE (IN.) ? "LRS
2157 PNT 195 !"
        (VERT.) RANGE (IN.) ? "LIS
2160 PNT 10 PNT 10 PNT 195 !" ";PNT 196 INPUT "
        ARE ALL ENTRIES CORRECT (Y/N)? "Z1$
2162 IF ZlS="Y" RETURN
2163 IF Z1S="N" THEN 2270
2164 PNT 176 PNT 195 !"
                                    ";PNT 194 !" PLEASE
        ENTER Y OR N!!" GOTO 2160
2270 PNT 195 CLS 0 PNT 10 PNT 10 PNT 10
2280 PNT 195 !" 1) DATE : "DS
2300 PNT 195 !" 2) ASPHALT TYPE : "AC$
2310 PNT 195 !" 3) LOAD DURATION : "LDS
2350 PNT 195 !" 4) LOAD CELL CAPACITY (LBS.): "LCS
2355 PNT 195 !" 5) MTS LOAD RANGE: "LMS
2360 PNT 195 !" 6) LVDT (LAT.) RANGE (IN.): "LRS
2365 PNT 195 !" 7) LVDT (VERT.) RANGE (IN.): "LIS
2370 ON ERROR 122,2371
2371 PNT 10 PNT 10 PNT 195 !" ";PNT 193 INPUT "
    PLEASE ENTER NUMBER OF WRONG ENTRY. "Z PNT 10
2372 OFF ERROR
```

```
2380 IF Z=1 PNT 195 !"
    DATE ?"DS
2410 IF Z=2 PNT 195 !"
        ASPHALT TYPE ? "ACS
2420 IF Z=3 PNT 195 !"
    LOAD DURATION ?"LDS
2493 IF Z=4 PNT 195 !"
        LOAD CELL CAPACITY (LBS.) ? "LCS
2494 IF Z=5 PNT 195 !"
        MTS LOAD RANGE ? "LMS
2495 IF Z=6 PNT 195 !" ";PNT 192 INPUT " 6)
        LVDT (LAT.) RANGE (IN.) ? "LR$
2496 IF Z=7 PNT 195 !"
        LVDT (VERT.) RANGE (IN.) ? "LIS
2500 IF Z<1 THEN PNT 176 PNT 195 !" ";PNT 194 !" PLEASE ENTER
        NUMBERS 1 THRU }7\mathrm{ ONLY!! " WAIT 2
2505 REM
2510 IF Z>7 THEN PNT 176 PNT 195 !" ";PNT 194 !" PLEASE ENTER
        NUMBERS 1 THRU }7\mathrm{ ONLY!! " WAIT 2
2520 GOTO 2160
2600 REM ************************(THIS ROUTINE ACCEPTS VARIABLE HEADER
        INFO)
2610 PNT 193 CLS O PNT 10 PNT }19
2620 PRINT " THIS ROUTINE ACCEPTS VARIABLE HEADER
        INFORMATION
2630 WAIT 1 PNT 10
2640 PNT 193 !" ";PNT 196 INPUT " 1) TEST NO. ?"TN$
2650 PNT 193 !" ";PNT 196 INPUT " 2) TYPE OF TEST ?"BOS
2653 PNT 193 !" ";PNT 196 INPUT " 3) BLEND NO.? "BLS
2655 PNT 193 !" ";PNT 196 INPUT " 4) SAMPLE NO.? "SAS
2657 PNT 193 !" ";PNT 196 INPUT " 5) LENGTH OF TOTAL
        PULSE (SEC.)"UWS
2660 PNT 193 !"
        PULSE (SEC.) "LPS
2670 PNT 193 !"
2680 PNT 193 !"
2682 PNT 193 !"
        "VLS
2684 PNT 193 !"
        "TS
2685 PNT 10
2686 REM
2690 PNT 193 !" ";PNT 192 INPUT " ARE ALL ENTRIES
        CORRECT? "Z1S
2700 IF Z15="Y" FLAG1=1 GOTO 1000
```

```
2705 IF Z1S="N" THEN 2710
2707 PNT 10 PNT 10 PNT 176 PNT 193 !" ";PNT 194 !" PLEASE
    ENTER Y OR N!!! " PNT 10 GOTO 2690
2710 PNT 193 CLS O PNT 10 PNT 10
2715 PNT 193 !" ";PNT 196 !" THIS ROUTINE ACCEPTS VARIABLE
    HEADER INFORMATION " PNT 10
2720 PNT 193 !" ";PNT l92 !" 1) TEST NO. ? "TNS
2725 PNT 193 !" ";PNT 192 !" 2) TYPE OF TEST ? "BDS
2726 PNT 193 !" ";PNT 192 !" 3) BLEND NO.? "BLS
2727 PNT 193 !" ";PNT 192 !" 4) SAMPLE NO.? "SAS
2729 PNT 193 !" ";PNT 192 !" 5) LENGTH OF TOTAL PULSE
    (SEC.)"UWS
2730 PNT 193 !"
    (SEC.)"UWS
2735 PNT 193 !" ";PNT 192 !" 7) DIAMETER ? "LOS
2740 PNT 193 !" ";PNT 192 !" 8) THICKNESS ? "DES
2750 PNT 193 !" ";PNT 192 !" 9) VERTICAL LOAD ? "VLS
2760 PNT 193 !" ";PNT 192 !" 10) TEST TEMPERTURE ? "T$
2770 PNT 10
2775 ON ERROR 122,2780
2780 IF Z1S="N" THEN PNT 193 !" ";PNT 196 INPUT " PLEASE
    ENTER NO. OF WRONG ENTRY. "Z
2783 OFF ERROR
2785 FLAGl=1 PNT 10
2790 IF Z=1 PNT 193 !"
    "TNS
2800 IF Z=2 PNT 193 !"
    ? "BDS
2802 IF Z=3 PNT 193 !"
        "BL$
2804 IF Z=4 PNT 193 !"
        "SAS
2806 IF Z=5 PNT 193 !"
    TOTAL PULSE (SEC.)"UWS
2810 IF Z=6 PNT 193 !"
    LOAD PULSE (SEC.)"LPS
2820 IF Z=7 PNT 193 !"
        "LOS
2830 IF Z=8 PNT 193 !"
        "DES
2831 IF Z=9 PNT 193 !"
        ? "VLS
2833 IF Z=10 PNT 193 !"
        TEMPERTURE ? "TS
```

```
2835 IF Z<1 THEN PNT 176 PNT 193 !"
    ENTER NUMBERS 1 THRU 10 ONLY!!
2838 IF Z>10 THEN PNT 176 PNT 193 !" ";PNT 194 !" PLEASE
        ENTER NUMBERS 1 THRU 10 ONLY!! "
2839 PNT 10
2840 GOTO 2690
3000 REM
3010 REM THIS ROUTINE PRINTS HEADER INFORMATION
3020 OPENW :2 "$QTO:1"
3025 PRINT:2 CHR$(24)
3030 PRINT:2 CHRS(27);CHR$(65);CHR$(27);CHRS(54);CHRS(29);CHRS(31);
3040 PRINT:2 TAB(12);
3050 A=1
3060 A=A+1
3070 IF A>2 THEN GOTO 3090
3080 PRINT:2 CHRS(151);
3090 PRINT:2 CHR$(131);
3100 IF A=>78 THEN PRINT:2 CHR$(149) GOTO 3120
3110 GOTO 3060
3120 PRINT:2 TAB(12);CHR$(149);TAB(33);"HEADER INFORMATION FOR TEST NO.
    "TNS;TAB(90);CHR$(149)
3125 PRINT:2 TAB(12);CHR$(149);TAB(15);"BLEND NO. = "BL$;TAB(60);"SAMPLE
    NO. = "SAS;TAB(90);CHRS(149)
3130 PRINT:2 TAB(12);CHR$(149);TAB(15);"DATE OF TEST =
        "D$;TAB(60);"TEMPERTURE = "T$;TAB(90);CHR$(149)
3140 PRINT:2 TAB(12);CHRS(149);TAB(15);"TYPE OF TEST =
        "BD$;TAB(60);"FREQUENCY = "UW$;TAB(90);CHRS(149)
3145 PRINT:2 TAB(12);CHR$(149);TAB(15);"ASPHALT TYPE =
        "AC$;TAB(60);"LOAD DURATION = "LD$;TAB(90);CHRS(149)
3147 PRINT:2 TAB(12);CHR$(149);TAB(15);"PERCENT AC =
        "PAS;TAB(90);CHRS(149)
3150 PRINT:2 TAB(12);CHRS(149);TAB(15);"DIAMETER = "LOS"
        IN.";TAB(60);"THICKNESS = "DES" IN.";TAB(90);CHRS(149)
3160 PRINT:2 TAB(12);CHRS(149);TAB(15);"LOAD CELL CAPACITY = "LC$"
        LBS.";TAB(60);"MTS LOAD RANGE = "LM$" %";TAB(90);CHR$(149)
3170 PRINT:2 TAB(12);CHRS(149);TAB(15);"LVDT (LAT.) FS RANGE = "LRS"
        IN.";TAB(60);"LVDT (VERT.) FS RANGE = "LIS" IN.";TAB(90);CHRS(149)
3180 A=1
3190 PRINT:2 TAB(12);
3200 A=A+1
3210 PRINT:2 CHRS(131);
3220 IF A=79 THEN PRINT:2 CHRS(129) GOTO 3240
3230 GOTO 3200
3240 PRINT:2 CHRS(30)
3250 PNT :2 10
```

```
3280 CLOSE :2
3290 GOTO 1000
4000 REM ***************************(THIS ROUTINE GIVES VALUES IN
ENGINEERING UNITS)****************************
4001 PNT 194 CLS 0 PNT 10 PNT 10 PNT 10
4 0 0 2 ~ F L A G M E = 1
4003 PNT 194 !" ";PNT 193 INPUT " DO YOU WANT TO
    CALIBRATE ? "Z1S
4 0 0 5 ~ I F ~ Z 1 \$ = " Y " ~ T H E N ~ 4 0 1 0 ~
4 0 0 7 ~ I F ~ Z 1 \$ = " N " ~ T H E N ~ R E T U R N
4008 PNT 10 PNT 176 PNT 194 !" ";PNT 197 !" PLEASE TYPE Y
    OR N !! " WAIT 1.5 GOTO 4000
4010 PNT 191 CLS O WAIT 1 PNT 10 PNT 197
4 0 2 0 ~ P R I N T ~ " ~ C A L I B R A T I O N ~ F O R ~ T E S T ~ C O N D U C T E D ~ O N ~ M T S
        SYSTEM
        "
4025 ON ERROR 205,4056
4030 PNT 191 !" ";PNT 10 PNT 197 INPUT " CHANNEL
    NUMBER? " C' PNT 191
4035 OFF ERROR
4040 IF C'<l THEN }405
4 0 4 3 ~ I F ~ C ' = 1 ~ T H E N ~ 4 0 6 0 ~
4045 IF C'>4 THEN }405
4 0 5 0 ~ I F ~ C ' = 2 ~ T H E N ~ 4 4 1 0 ~
4 0 5 3 ~ I F ~ C ' = 3 ~ T H E N ~ 4 7 5 0 ~
4 0 5 5 ~ I F ~ C ' = 4 ~ T H E N ~ 4 8 8 7 ~
4 0 5 6 ~ P N T ~ 1 0 ~ P N T ~ 1 7 6 ~ P N T ~ 1 9 1 ~ ! " ~ " ; P N T ~ 1 9 4 ~ ! " ~ I N C O R R E C T
    INPUT!! PLEASE TRY AGAIN."GOTO 4030
4060 !" ";PNT 197 INPUT " LOAD CELL CAPACITY? "Ll$
        PNT 191
4070 PNT 191 !"
        100%-10% ? "R1$
4075 IF RlS>"100" THEN PNT 176 PNT 10 PNT 191 !" ";PNT 194
        !" INCORRECT INPUT!! NEEDS TO BE 100%-10%!!! " PNT 10 GOTO 4070
4076 IF RlS<"10" THEN PNT 176 PNT 10 PNT 191 !" ";PNT 194
        !" INCORRECT INPUT!! NEEDS TO BE 100%-10%!!!" PNT 10 GOTO 4070
4080 PNT 191 !" ";PNT 197 INPUT " ARE ZERO AND
        SCALE FACTOR CORRECT? (Y,N) "Z1S PNT 191
4090 IF ZlS="Y" THEN 4120
4 0 9 5 ~ I F ~ Z 1 S = " N " ~ T H E N ~ 4 1 0 0 ~
4096 PNT 176 GOTO 4080
4100 ON ERROR 122, 4112
4101 PNT 191 !" ";PNT 197 INPUT " ZERO ÖFFSET? "Al
    PNT 191
4110 PNT 191 !" ";PNT 197 INPUT " SCALE FACTOR FOR
ENGINEERING UNITS= "B1 PNT 191
```

4112 PNT 176 PNT 191 !" ";PNT 194 !" INCORRECT INPUT!!
PLEASE TRY AGAIN. " GOTO 4101
4120 WAIT 1 CLS 0 PNT 10 PNT 10
4130 !" ";PNT 10 PNT 197 PRINT " LOAD CELL CAPACITY = "Ll\$" LBS." PNT 191
4140 !" ";PNT 197 PRINT " MTS LOAD RANGE = "R1S
PNT 191
4150 !" ";PNT 197 PRINT " ZERO OFFSET = "Al PNT
191
4160 !" ";PNT 197 PRINT " SCALE FACr.R FOR
ENGINEERING UNITS= "Bl PNT 191
4170 PNT $10 \mathrm{C}^{\prime}=1$
4180 X1=AIN: O(2, C'-1)
4190 FOR I=1 TO 5
4195 PNT 191
4200 L=VAL(LI\$)
4210 R=VAL (R1\$)
4220 IF $R=100$ THEN $L 1=L / 10$
4250 IF $R=10$ THEN $\mathrm{L}=\mathrm{L} / 100$
$4260 \mathrm{X}=(\mathrm{AIN}: 0(2,0)) * \mathrm{~L} 1 * \mathrm{Bl}+\mathrm{Al}$
4270 WAIT 2
4280 !" ";PNT 193 PRINT USING "-\#\#\#\#.\#
LBS.", X PNT 191
4290 NEXT I
4295 PNT 10
4300 PNT 191 !" ";PNT 196 INPUT " IS MORE CAL DATA
REQURIED FOR CHANNEL \#I? (Y,N) "ZIS PNT 191
4310 IF Z1S="Y" THEN GOTO 4120
4315 IF Z1\$="N" THEN GOTO 4330
4320 PNT 176 GOTO 4300
4330 PNT 191 !" ";PNT 10 PNT 196 INPUT " ARE ZERO
AND SCALE FACTOR CORRECT? (Y,N) "ZIS PNT 191
4340 IF Z1S="N" THEN 4100
4345 IF Z1S="Y" THEN 4350
4346 PNT 176 GOTO 4330
4350 PNT 191 !" ";PNT 10 PNT 192 INPUT " DO YOU REQUIRE
ADOITIONAL CHANNELS CALIBRATED? (Y,N) "Z1\$
4360 IF Z1S="Y" THEN 4000
4370 IF 21S="N" THEN RETURN
4371 PNT 176 GOTO 4350
4400 PNT 191 CLS 0 PNT 10 PNT 10 PNT 10
4410 !" ";PNT 197 INPUT " ACT. LVDT RANGE FULL SCALE?"L\$ PNT 191

4430 PNT 191 !" ";PNT 197 INPUT " ARE ZERO AND SCALE FACTOR CORRECT? (Y,N) "Zl\$ PNT 191
4440 IF Z1S="Y" THEN 4470
4441 IF Z1S="N" THEN GOTO 4450
4442 PNT 176 GOTO 4430
4450 ON ERROR 122,4465
4451 PNT 191 !" ";PNT 197 INPUT " ZERO OFFSET?"A2 PNT 191
4460 PNT 191 !" ";PNT 197 INPUT " SCALE FACTOR FOR ENGINEERING UNITS?"B2 PNT 191
4461 OFF ERROR GOTO 4470
4465 PNT 176 PNT 191 !" ";PNT 194 !" INCORRECT INPUT!!!
PLEASE TRY AGAIN. " GOTO 4451
4470 CLS 0 PNT 191 PNT 10 PNT 10 PNT 10 WAIT 1
4475 PNT 191
4480 PNT 191 !"
FS="LS" IN." PNT 191
4500 PNT 191 !" ";PNT 197 !" ZERO OFFSET= "A2
4510 PNT 191 !" ";PNT 197 PRINT " SCALE FACTOR FOR
ENGINEERING UNITS="B2 PNT 191
4520 PNT 10
$4530 C^{\prime}=2$
4540 FOR I=1 TO 5
4600 X $=(\operatorname{AIN}: 0(2,1)+A 2) * M 1 * B 2$
4610 WAIT 2
4620 PNT 191 !" ";PNT 193 PRINT USING
"-\#\#.\#\#\# IN.", X
4640 NEXT I
4660 PNT 191 !" ";PNT 10 PNT 196 INPUT " IS MORE CAL DATA REQURIED FOR CHANNEL \#2? "Z1\$
4665 PNT 191
4670 IF Z1S="Y" THEN 4470
4671 IF Z1\$="N" THEN 4680
4672 PNT 176 GOTO 4660
4680 PNT 191 !" ";PNT 10 PNT 196 INPUT " ARE ZERO
AND SCALE FACTOR CORRECT? "Z1\$
4690 IF Z1\$="N" THEN 4450
4691 IF Z1S="Y" THEN 4700
4692 PNT 176 GOTO 4680
4700 PNT 191 !" ";PNT 10 PNT 192 INPUT " DO YOU REQUIRE ADOITIONAL CHANNELS CALIBRATED? "Z1S
4710 IF Z1S="Y" THEN 4000
4720 IF Z1s="N" THEN RETURN
4740 PNT 176 GOTO 4700
4745 PNT 191 CLS 0 PNT 10 PNT 10 PNT 10

```
4750 !"
                                    ";PNT 197 INPUT " LAT. LVDT RANGE FULL
        SCALE?"L2$ PNT 191
4780 PNT 191 !" ";PNT 197 INPUT " ARE ZERO AND
    SCALE FACTOR CORRECT? (Y,N) "Z1$ PNT 191
4 7 8 5 ~ I F ~ Z 1 \$ = " Y " ~ T H E N ~ 4 8 1 0 ~
4790 IF Z1S="N" THEN GOTO 4797
4 7 9 5 ~ P N T ~ 1 7 6 ~ G O T O ~ 4 7 8 5 ~
4797 ON ERROR 122,4807
4800 PNT 191 !" ";PNT 197 INPUT " ZERO OFFSET?"A3
    PNT 191
4803 PNT 191 !" ";PNT 197 INPUT " SCALE FACTOR FOR
    ENGINEERING UNITS?"B3 PNT 191
4805 OFF ERROR GOTO 4810
4807 PNT 176 PNT 191 !" ";PNT 194 !" INCORRECT INPUT!!!
    PLEASE TRY AGAIN. " GOTO 4800
4810 CLS 0 PNT 191 PNT 10 PNT 10 PNT 10 WAIT 1
4 8 1 3 ~ P N T ~ 1 9 1 ~
4815 PNT 191 !" ";PNT 197 PRINT " LAT. LVDT RANGE
    FS="L2$" IN." PNT 191
4820 PNT 191 !"
";PNT 197 !" ZERO OFFSET= "A3
4823 PNT 191 !" ";PNT 197 PRINT " SCALE FACTOR FOR
    ENGINEERING UNITS="B3 PNT 191
4825 PNT 10
4 8 2 7 ~ C ' = 5
4 8 3 0 ~ F O R ~ I = 1 ~ T O ~ 5 ~
4845 X=(AIN:O(3,1)+A3)*B3
4 8 4 7 \text { WAIT 2}
4850 PNT 191 !" ";PNT 193 PRINT USING
    "-#.##### IN.",X
4 8 5 3 ~ N E X T ~ I ~
4855 PNT 191 !" ";PNT 10 PNT 196 INPUT " IS MORE CAL
    DATA REQURIED FOR CHANNEL #3? "ZIS
4857 PNT 191
4 8 6 0 ~ I F ~ Z 1 S = " Y " ~ T H E N ~ 4 8 1 0 ~
4 8 6 3 ~ I F ~ Z 1 S = " N " ~ T H E N ~ 4 8 6 7 ~
4865 PNT 176 GOTO 4855
4867 PNT 191 !" ";PNT 10 PNT 196 INPUT " ARE ZERO
    AND SCALE FACTOR CORRECT? "Zl$
4870 IF Z1$="N" THEN 4800
4 8 7 3 ~ I F ~ Z 1 \$ = " Y " ~ T H E N ~ 4 8 7 7 ~
4875 PNT 176 GOTO 4680
4877 PNT 191 !" ";PNT 10 PNT 192 INPUT " DO YOU REQUIRE
        AODITIONAL CHANNELS CALIBRATED? "Z1S
4 8 8 0 ~ I F ~ Z 1 S = " Y " ~ T H E N ~ 4 0 0 0 ~
4883 IF 21S="N" THEN RETURN
```

```
4 8 8 5 \text { PNT 176 GOTO 4877}
4 8 8 7 \text { PNT 191}
4890 !" ";PNT 197 INPUT " VERT. LVDT RANGE FULL
    SCALE?"L3S PNT 191
4893 PNT 191 !" ";PNT 197 INPUT " ARE ZERO AND
    SCALE FACTOR CORRECT? (Y,N) "Z1$ PNT 191
4 8 9 5 ~ I F ~ Z 1 S = " Y " ~ T H E N ~ 4 9 1 5 ~
4 8 9 7 ~ I F ~ Z 1 \$ = " N " ~ T H E N ~ G O T O ~ 4 9 0 3 ~
4900 PNT 176 GOTO 4893
4903 ON ERROR 122,4913
4905 PNT 191 !" ";PNT 197 INPUT " ZERO OFFSET?"A4
    PNT 19]
4907 PNT 191 !" ";PNT 197 INPUT " SCALE FACTOR FOR
    ENGINEERING UNITS?"B4 PNT 191
4910 OFF ERROR GOTO 4915
4913 PNT 176 PNT 191 !" ";PNT 194 !" INCORRECT INPUT!!!
    PLEASE TRY AGAIN. " GOTO 4903
4915 CLS O PNT 191 PNT 10 PNT 10 PNT 10 WAIT I
4917 PNT 191
4920 PNT 191 !" ";PNT 197 PRINT " VERT. LVDT RANGE
        (FS)="L3$" IN." PNT 191
4 9 2 3 ~ P N T ~ 1 9 1 ~ ! " ~ " ; P N T ~ 1 9 7 ~ ! " ~ Z E R O ~ O F F S E T = ~ " A 4 ,
4925 PNT 191 !" ";PNT 197 PRINT " SCALE FACTOR FOR
        ENGINEERING UNITS="B4 PNT 191
4 9 2 7 ~ P N T ~ 1 0 ~
4 9 3 0 ~ C ' = 5
4 9 3 3 ~ F O R ~ I = 1 ~ T O ~ 5 , ~
4 9 3 5 X = ( A I N : O ( 3 , 0 ) + A 4 ) * B 4
4 9 3 7 \text { WAIT 2}
4940 PNT 191 !" ";PNT 193 PRINT USING
    "-####.##### MICRO IN/IN.",X
4 9 4 3 ~ N E X T ~ I ~
4945 PNT 191 !" ";PNT 10 PNT 196 INPUT " IS MORE CAL
        DATA REQURIED FOR CHANNEL #4? "Z1S
4947 PNT 191
4 9 5 0 ~ I F ~ Z 1 S = " Y " ~ T H E N ~ 4 9 1 5 ~
4 9 5 3 ~ I F ~ Z 1 S = " N " ~ T H E N ~ 4 9 5 7 ~
4 9 5 5 ~ P N T ~ 1 7 6 ~ G O T O ~ 4 9 4 5 ~
4957 PNT 191 !" ";PNT 10 PNT 196 INPUT " ARE ZERO
    AND SCALE FACTOR CORRECT? "Z1$
4960 IF Z1S="N" THEN 4905
4 9 6 3 ~ I F ~ Z 1 S = " Y " ~ T H E N ~ 4 9 6 7 ~
4 9 6 5 ~ P N T ~ 1 7 6 ~ G O T O ~ 4 9 5 7 ~
4967 PNT 191 !" ";PNT 10 PNT 192 INPUT " DO YOU REQUIRE
        ADDITIONAL CHANNELS CALIBRATED? "Z1$
```

```
4 9 7 0 ~ I F ~ Z 1 \$ = " Y " ~ T H E N ~ 4 0 0 0 ~
4 9 7 3 ~ I F ~ Z 1 S = " N " ~ T H E N ~ 1 0 0 0 ~
4 9 7 5 \text { PNT 176 GOTO 4967}
5000 REM *************************THIS ROUTINE PROMPTS CHECK-OUT
PROCEDURE PRIOR TO TEST*************************
5010 PNT 192 CLS 0 PNT 197 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10 PNT
    10 PNT 10
5012 IF FLAGME=0 PNT 192!" ";PNT 197 !"ONLY FEABLE
MINDS FORGET TO CALIBRATE!" WAIT 5 N2$="CI" GOTO 1230
5020 PNT 192 !" ";PNT 197 INPUT " IS THIS A NEW
    TEST (Y/N) ? "Zl$
5030 IF Z1S="N" GOTO 5055
5040 IF Z1$="Y" GOTO 5220
5050 IF Z1$<>"Y" PNT 176 GOTO 5020
5055 PNT 195 CLS O PNT 197 PNT 10 PNT 10 PNT 10
5060 PNT 195 !" ";PNT 197 !"
5070 PNT 195 !" ";PNT 197 !"
    CALIBRATIONS
    COMPLETE ?
5080 PNT 195 !" ";PNT 197 !"
5085 PNT 195 !" ";PNT 197 !" IS THE CHART SET
    AND READY ?
5090 PNT 195 !" ";PNT 197 !"
5100 PNT 195 !" ";PNT 197 !" MTS GAIN & DITHER
    CONTROLS SET ?
5105 PNT 195 !" ";PNT 197 !"
5110 PNT 195 !" ";PNT 197 !"
    FUNCTION (8) !!
5120 PNT 195 !" ";PNT 197 !"
5130 PNT 195 !" ";PNT 197 !"
DESIRE VALUE ?
5140 PNT 195 !" ";PNT 197 !"
5150 PNT 195 !" ";PNT 197 !"
    APPARATUS READY ?
5155 PNT 195 !" ";PNT 197 !"
5160 PNT 195 !" ";PNT 197 !" PROJECT
        ENGINEER/TECHNICIAN READY ?
5165 PNT 195 !" ";PNT 197 !"
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5170 PNT 195 !" ";PNT 197 !" ALL CONTROLS & VALVES IN
    CORRECT POSITION FOR TEST ? "
5175 PNT 195 !" ":PNT 197 !"
5180 PNT 10 PNT 195 ! " ";PNT 192 INPUT " SYSTEM IS NOW READY TO
        START TEST << TYPE (Y) TO RETURN TO MENU >> "Zl$
5185 Z2S="GO"
5190 IF Z1S="Y" WAIT 1 GOTO 1000
5200 PNT 176 GOTO 5180
5210 REM
5220 REM THIS ROUTINE SETS TIME & DATA ARRAYS TO ZERO
5230 Z2S="O"
52!0 PNT 27 PNT 12 WAIT 1
5250 PNT 10 PNT 192 !" ";PNT 196 !" *** TO ZERO TIME & DATA
    ARRAYS TYPE (Y) & PRESS RETURN *** "
5255 PNT 10 PNT 192 !" ";PNT 193 INPUT" ** OR TO ABORT
TYPE (N) & PRESS RETURN ** "Zl$
5260 IF Z1S="N" THEN 1000
5270 IF Z1S<>"Y" ! "INCORRECT INPUT" WAIT 1 GOTO 5250
5360 GOTO 5055
5370 END
6000 REM
6001 REM ****************** ROUTINE FOR PRECONDITIONING SAMPLE
        ****************
6002 F7=0
6 0 0 5 \text { PNT 185 PNT 192 CLS 0 PNT 196 PNT 10 PNT 10 PNT 10}
6006 IF FLAGME=0 PNT 192!" ";PNT 197 !"ONLY FEABLE
    MINDS FORGET TO CALIBRATE!" WAIT 5 N2$="CI" GOTO 1230
6007 PNT 192 !" ";PNT 196 !"
6010 PNT 192 !" ";PNT 196 !" This routine preconditions
        aspahlt samples for
6020 PNT 192 !" ";PNT 196 !" RESILIENT MODULUS OR
        INDIRECT TENSILE TESTING. "
6021 PNT 192 !" ":PNT i96 !" If waveform stableizes
        before limits are exceeded,
6022 PNT 192 !" ";PNT 196 !" data will be taken and
    test will end.
6023 PNT 192 !" ";PNT 196 !"
6 0 2 5 \text { WAIT 1 PNT 193 PNT 10 PNT 10 PNT 10}
6027 PNT 192 !" ";PNT 193 !"
6030 PNT 192 !" ";PNT 193 !" Verticle deformation will
        be monitered closely.
```

```
6040 PNT 192 !" ";PNT 193 !" If deformation exceeds 0.001
    inches, precondition
6050 PNT 192 !" ";PNT 193 !" will halt and main menu
    will return to screen. "
6053 PNT 192 !" ";PNT 193 !"
6 0 5 4 \text { WAIT 1 PNT 10 PNT 10 PNT 10 PNT 192 !" ";PNT 197}
    !" PRESS RETURN TO CONTINUE ";INPUT
6055 PNT 193 CLS 0 PNT 196 PNT 10 PNT 10
6060 PNT 192 !" SAMPLE NEEDS TO BE READY FOR TESTING
6065 WAIT 1.2 PNT 10 PNT 10 PNT 193 !"
                                    ";PNT 196 !"
6066 PNT 193 !" ";PNT 196 !"
6070 PNT 193 !" ";PNT 196 !" (Has a preload been
        applied to the sample ?)
6075 PNT 193 !" ";PNT 196 !"
6076 PNT 193 !" ";PNT 196 !" (Place MTS program switch
    in RUN position !)
6 0 7 8 \text { PNT 193 !"}
6080 PNT 193 !"
    LVDTS been zeroed ?)
6085 PNT 193 !"
    "
6095 PNT 193 !"
        INITIALIZED ?)
6098 PNT 193 !"
6100 WAIT 2 PNT 10 PNT 10 PNT 10 PNT 10 PNT 193 !" ";PNT 197 !"
        Please do this, and then press RETURN. ";INPUT
6 1 1 0 ~ P N T ~ 1 9 4 ~ C L S ~ O ~ F O R ~ X = 1 ~ T O ~ 6 ~ P N T ~ 1 0 ~ N E X T ~ X ~
6111 PNT 194 !" ";PNT 193 !" PLEASE WAIT WHILE WAVETEK IS
        BEING PROGRAMMED.
6112 GPIB:4 0,13
6 1 1 3 ~ C O N F I G ~ 0 , 1 3
6114 PRINT:4 "CO01P1BIA5D2.5F16"
6267 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10
6270 P9=0
6340 PNT 192 CLS 0 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10
6345 PNT 192 !" ";PNT 193 !"
```

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6346 PNT 192 !" ";PNT 193 !" Please enter no. of cycles
    PRECONDITION is set, to run.
6347 PNT 192 !" ";PNT 193 !"
    ";INPUT P8
6348 IF II=2 PRINT:4 "Fl"
6 3 4 9 ~ I F ~ I I = 1 ~ P R I N T : 4 ~ " F 4 " '
6350 PRINT:4 "L";P8 PRINT:4 "I"
6351 CLS 4 GRAPHICS 4 COLOR 4,1 FONT 3 DISPLAY 4 WINDOW -10,10,-10,10
6 3 5 2 ~ H P R I N T ~ - 2 , 4 , " T E S T " '
6353 HPRINT -1,-1,"IN"
6354 HPRINT -4,-6,"PROGRESS"
6355 R4={(AIN:O(3,0)+A4)*B4)+.0025 F7=0 ACT 4 WAIT . 2 DOT:O(4,0)=1
        DOT:O(4,0)=0
6 3 5 6 \text { COUNT=0 T=.1}
6360 Vl=AIN:0(2,3) IF V1>8 THEN COUNT=COUNT+1 WAIT . }
    R3=((AIN:O(3,0)+A4)*B4) ACT 3
6 3 6 5 ~ I F ~ C O U N T = 2 ~ T H E N ~ T = 1 ~
6370 IF F7=1 THEN SUSP 2 SUSP 3 GOTO 7000
6371 IF F7=2 THEN PRINT:4 "A5"
6372 GOTO 6360
6380 GOTO 6440
6381 FOR I=1 TO 49
6390 R3=R2(1)
6400 IF R?`R2(I+1) THEN EXIT FOR GOTO 6440
6 4 1 0 ~ N E X T ~ I ~ I ~
6440 P9=P9+1 IF P9=>P8-5 THEN P8=0 F7=1 PNT }7\mathrm{ PNT }
6442 REM IF PG=1 THEN R4=(R3+.001)
6445 IF P9>1 THEN IF R3>R4 THEN F7=2
6450 SUSPEND 3
6455 GOTO 6380
6 4 5 6 ~ E N D
6 6 3 0 \text { SUSPEND } 3 \text { SUSPEND 2}
6640 FOR I=1 TO 12 PNT 10 NEXT I
6 6 5 0 \text { CLS } 4 \text { DISPLAY O PNT 194 CLS O PNT 10 PNT 10 PNT 10 PNT 10}
6655 PNT 194 !" ";PNT 193 !"
6660 PNT 194 !" ";PNT 193 !" The overall verticle
    deformation has
6670 PNT 194 !" ";PNT 193 !" EXCEEDED the limit of
        . }001\mathrm{ inches!
6673 PNT 194 !" ";PNT 193 !"
6 6 7 5 \text { WAIT 2 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10}
6677 PNT 194 !" ";PNT 193 !"
```

```
6 6 8 0 ~ P N T ~ 1 9 4 ~ ! " '
    PULSES ARE NOW
6685 PNT 194 !"
";PNT 193 !" BEING RECORDED
6 6 9 0 ~ P N T ~ 1 9 4 ~ ! " ~ " ; P N T ~ 1 9 3 ~ ! " ~
6700 GOTO 7000
6900 REM *****************<TASK 2 STARTS HERE>******************
6 9 1 0 ~ F O R ~ I = 1 ~ T O ~ 5 0 ~
6920 R2(I)= (AIN:O(3,0)+A4)*B4
6 9 3 0 ~ N E X T ~ I ~
6 9 4 0 ~ A C T ~ 3 ~
6 9 5 0 ~ S U S P E N D ~ 2 ~ G O T O ~ 6 9 1 0 ~
6 9 6 0 \text { END}
7 0 0 0 ~ R E M
7001 REM **** THIS ROUTINE STARTS TEST ****
7010 IF AIN:O(2,3)>1 THEN WAIT VAL(UW$)-.08 GOTO }701
7012 GOTO 7010
7014 T6=TIMER
7018 IF II=0 00=1 UU=112
7019 IF II=1 00=113 UU=450
7020 IF II=2 00=451 UU=1425
7021 REM
7022 FOR I=00 TO UU
7026 X2(I)=AIN:0(3,0)
7027 X3(I)=AIN:0(2,0)
7 0 3 0 ~ N E X T ~ I ~
7 0 3 2 ~ P N T ~ 7 ~ W A I T ~ . ~ 5 ~ P N T ~ 7 ~
7 0 3 4 ~ I I = 1 I + 1
7036 T7=TIMER T7=T7-T6
7046 FOR I=00 TO UU
7048 X2(I)=(X2(I)+A4)*B4
7049 X3(I)=X3(I)*L1*Bl+A1
7 0 5 0 ~ N E X T ~ I ~
7051 INPUT
7 0 5 3 ~ I F ~ I I < 3 ~ D I S P L A Y ~ O ~ G O T O ~ 6 3 4 0
7054 PRINT:4 "G" WAIT 10
7 0 5 5 \text { CLS } 4 \text { GRAPHICS 4 COLOR 4,1 FONT 2 DISPLAY 4 WINDOW -10,10,-10,10}
7060 HPRINT -7,5,"TESTING" HPRINT -1,1,"IS" HPRINT 1,-3,"COMPLETE"
7070 FONT 1 HPRINT -2,-8,"PRESS RETURN"DISPLAY 4 WINDOW -10,10,-10,10
7075 LOCAL CLOSE:4
7080 INPUT
7220 GOTO 16000
7450 REM
7500 INPUT
```

```
7510 F7=1
7520 SUSPEND 4
7530 GOTO 7500
7540 END
7545 REM
10000 @ ************************************ THIS ROUTINE DRAWS ALL
    PLOTS ********************************************
10010 PNT 180 PNT 190 CLS O PNT 10 PNT 10
10020 PNT 190 !" ";PNT 193 !"
10030 PNT 190 !" ";PNT 193 !" PLOT MENU
10040 PNT 190 !" ";PNT 193 !"
10050 PNT 10 PNT 10 PNT 10 PNT 10
10060 PNT 190 !" ";PNT 196 !"
10070 PNT 190 !"
    SCREEN "
10080 PNT 190 !" ";PNT 196 !" 2) RETURN TO
    MAIN MENU "
10090 PNT 190 !" ";PNT 196 !"
10100 PNT 10 PNT 10 PNT 10 PNT 10 PNT 190 !" ";PNT
    192 !"
10110 PNT 181 PNT 190 !" ";PNT 192 !" PLEASE
    ENTER NO. OF CHOICE
";PNT 192 INPUT"
    "Z1S PNT }18
10130 IF Z1S="1" GOTO 10170
10140 IF Z1$="2" GOTO 1000
10150 PNT 176 CLS O GOTO 10000
10160 REM
10170 PNT 190 CLS O PNT 183 PNT 180 PNT 185 DISPLAY O
10180 PNT 10 PNT 10 PNT 190 !" ";PNT 193 ! "
    H
10190 PNT 190 !" ";PNT 193 ! " MENU FOR SCREEN
    PLOTTING
10200 PNT 190 !"
    "
10210 PNT 10 PNT 10 PNT 10 PNT 10
10220 PNT 190 !" ";PNT 196 !"
10230 PNT 190 !"
";PNT 196 !"
    1) PLOT LOAD
```

```
10240 PNT 190 !"
    DEFORMATION
10250 PNT 190 !"
    DEFORMATION "
10260 PNT 190 !"
    MENU
10270 PNT 190 !"
10280 PNT 10 PNT 190 !"
    "
10290 PNT 181 PNT 190 !"
    ENTER NO. OF CHOICE
10300 PNT 190 !"
    "ZlS PNT }18
10310 IF Z1$="1" GOTO 10360
10320 IF Z1$="2" GOTO 10654
10330 IF Z1$="3" GOTO 10895
10340 IF Z1$="4" GOTO 10000
10350 PNT 176 GOTO 10170
10360 REM PLOT LOAD ROUTINE
10370 PNT 180 PNT 10 PNT 190 !"";PNT 193 INPUT " PLEASE ENTER NO. OF
    PULSE TO PLOT (1,2,3 OR 4 FOR ALL PULSES) "Z1
10376 BB=0
10379 IF 2l=4 THEN PT=1
10380 GRAPHICS 4 COLOR 0,7 Y1=VAL(VL$)+20 Y5 =-5*(Yl/l00)
10387 IF PT=1 THEN FOR Z1=1 TO 3
10389 MOVE 0,0
10390 IF Z1=1 THEN S1'=1 S2'=1425 WINDOW -4,1425,Y5,YI V=0 0=1 P=1425
10400 IF Z1=2 THEN S1'=300 S2'=600 WINDOW 296,600,Y5,Y1 V=300 0=300
    P=600
10410 IF 21=3 THEN S1'=600 S2'=900 WINDOW 596,900,Y5,Y1 V=600 0=600
    P=900
10412 IF Zl=2 AND PT=1 THEN COLOR 2 MOVE 0,0
10414 IF Z1=3 AND PT=1 THEN COLOR 5 MOVE 0,0
10420 DISPLAY 4
10425 PEM
10430 FOR I=S1' TO S2' X4(I)=X3(I) NEXT I
10440 IF Zl=1 AND PT=1 THEN VAXISP V,(Y1/10) HAXIS 0,30
10444 IF PT=0 THEN VAXISP V,(Yl/10) HAXIS 0,30
10445 IF Z1=2 COLOR 2
10446 IF Z1=3 COLOR 5
10447 REM
10448 MOVE Sl',X4(S1')
10449 FOR I=S1' TO S2'
10450 PLOT I,X4(I)
```

```
10455 NEXT I
10458 IF Z1=3 THEN PT=0
10459 IF PT=0 THEN EXIT FOR GOTO 10570
10460 NEXT Z1
10570 FOR I=0 TO P
10575 IF X3(I)<20 THEN GOTO 10600
10580 X5=X3(I)
10590 IF X5>X3(I+1) AND X5>X3(I+15) THEN EXIT FOR GOTO 10620
10600 NEXT I
10610 GOTO 10340
10620 LET L1S = STRS(X5) LET L2$="PEAK LOAD *"
10625 IF BB=0 THEN COLOR 1 MOVE I,X5 IPLOT 50,0 HPRINT I+50,X5,L2SL1S
10631 IF BB=0 THEN GOTO 10650
10650 INPUT GOTO 10160
10654 PNT 183 REM ***** ROUTINE FOR VERTICAL DEF *****
10655 FLAG=0
10660 FLAG=FLAG+1
10670 PNT 190 CLS 0 PNT 180 PNT 185 DISPLAY 0
10680 PNT 10 PNT 10 PNT 190 !" ";PNT 193 !"
10690 PNT 190 !" ";PNT 193 !" PLOT VERT
    DEFORMATION ROUTINE
10700 PNT 190 !" ";PNT 193 !"
    |
10710 PNT 10 PNT 10 PNT 10 PNT 190 !" ";PNT 196
    INPUT" DO YOU WISH TO RETURN TO MENU ? (Y/N) "N1S
10720 IF N1$="Y" THEN }1017
10730 PNT 10 PNT 10 PNT 190 !"";PNT 197 INPUT" PLEASE ENTER NO. OF
    PULSE TO SEE. (1 THRU 3 OR 4 FOR ALL PULSES) "Z1
10736 BB=0
10739 If Z1=4 THEN PT=1
10740 GRAPHICS 4 COLOR 0,7 CLS 4 DISPLAY 4
10750 WINDOW -4,1425,-1,3000
10760 VAXISP 0,100 HAXIS 0,30
10765 IF PT=1 THEN FOR Z1=1 TO 3
10770 MOVE 0,0
10780 IF Zl=1 THEN S1'=1 S2'=1425
10790 IF 21=2 THEN S1'=112 S2'=450 WINDOW 110,450,-1,1000 COLOR 2
10800 IF Z1=3 THEN S1'=450 S2'=1425 WINDOW 446,1425,-1,1000 COLOR 5
10810 DISPLAY 4
10815 MOVE S1',X2(S1')
10820 FOR I=S1' TO S2'
10830 Y=x2(I)
10840 PLOT I,Y
10850 NEXT I
```

```
10851 REM
10852 IF Z1=3 THEN PT=0
10855 IF PT=O THEN EXIT FOR GOTO 10860
10856 NEXT Z1
10860 IF BB=0 THEN GOTO 10892
10892 INPUT GOTO 10660
10895 PNT 183 REM ***** ROUTINE FOR HORIZONTAL DEF. *****
10900 FLAG=0
10910 FLAG=FLAG+1
10920 PNT 190 CLS O PNT 180 PNT 185 DISPLAY O
10930 PNT 10 PNT 10 PNT 190 !" ";PNT 193 !"
10940 PNT 190 !" ";PNT 193 !" PLOT HORIZONTAL
    DEFORMATION "
10950 PNT 190 !" ";PNT 193 !"
10960 PNT 10 PNT 10 PNT 10 PNT 190 !"
                                    ";PNT }19
    INPUT" DO YOU WISH TO RETURN TO MENU ? (Y/N) "NIS
10970 IF N1$="Y" THEN 10170
10980 PNT 10 PNT 10 PNT 190 !"";PNT 197 INPUT" PLEASE ENTER NO. OF
    PULSE TO SEE. (1 THRU 3 OR 4 FOR ALL PULSES) "Z1
10986 BB=0
10989 IF Z1=4 THEN PT=1
10990 GRAPHICS 4 COLOR 0,7 CLS 4 DISPLAY }
11000 WINDOW -40,300,-.00001,.0005
11010 VAXISP 0,.000010 HAXIS 0,30
11015 IF PT=1 THEN FOR Z1=1 TO 3
11020 MOVE 0,0
11030 IF Z1=1 THEN S1'=1 S2'=300
11040 IF Z1=2 THEN S1'=300 S2'=600 WINDOW 260,600,-.00001,.0005 COLOR 2
11050 IF Z1=3 THEN S1'=600 S2'=900 WINDOW 560,900,-.00001,.0005 COLOR 5
11060 DISPLAY 4
11065 MOVE S1',Xl(S1')
11070 FOR I=S1' TO S2'
11080 Y=(X1(I))
11090 PLOT I,Y
11100 NEXT I
11110 IF Zl=3 THEN PT=0
11112 IF PT=0 THEN EXIT FOR GOTO 11115
11114 NEXT Z1
11115 IF BB=O THEN GOTO 11150
11150 INPUT GOTO 10910
11160 PLOT I,X4(I)
15999 REM
16000 CLS 4 DISPLAY O CLS O PNT 191 PNT 174 PNT 180 PNT }19
```

```
16005 PNT 10 PNT 10 PNT 10
16007 PNT 191 !" ";PNT 192 !"
16010 PNT 185 PNT 191 !" ";PNT 192 !" This routine stores
header information, and also the "
16020 PNT 191 !" ";PNT 192 !" the data on a disk in DRIVE
    B for future reference.
16025 PNT 191 !" ";PNT 192 !"
    "
16030 PNT 10 PNT 10 WAIT 2
16070 PNT 191 !" ";PNT 192 !"
    "
16076 PNT 191 !" ";PNT 192 !" Please insert disk to cantain
    data in DRIVE B and close door.
16080 PNT 191 !" ";PNT 192 !" Press RETURN when
        this is done "
16085 PNT 191 !" ";PNT 192 !"
        ";INPUT
16090 DSKRESET PNT 191 PNT 10 PNT 10 PNT 10
16100 PNT 191 !" ";PNT 192 !"
16105 PNT 191 !" ";PNT 192 !" Please enter a name for the data
        to be filed under. The name "
16110 PNT 191 !" ";PNT 192 !" may be 8 characters in length, no
    spaces are allowed, so
16120 PNT 191 !" ";PNT 192 !" words may be separated by the
    underline mark.
16121 PNT 191 !" ";PNT 192 !"
        ";INPUT N7$
16125 PNT 186 DSKRESET
16130 OPENW:3 "SQTO:O"
16135 PRINT:3 BLS
16137 PRINT:3 SAS
16139 PRINT:3 PA$
16140 PRINT:3 DS
16141 PRINT:3 ACS
16142 PRINT:3 LDS
16143 PRINT:3 LCS
16144 PRINT:3 LRS
16145 PRINT:3 LIS
16146 PRINT:3 TNS
16147 PRINT:3 BD$
16148 PRINT:3 UW$
16149 PRINT:3 LO$
16150 PRINT:3 DE$
```

```
16151 PRINT:3 VL$
16152 PRINT:3 T$
16153 PRINT:3 LMS
16155 PRINT:3 A3
16156 PRINT:3 A4 !:3 B3 !:3 B4 !:3 Ll !:3 Bl !:3 A1
16165 FOR I=1 TO 1425
16170 PRINT USING:3 "-######.##### -######.#####", X2(I),X3(I)
16172 WAIT . }05\mathrm{ NEXT I
16185 PNT 191 CLS 0 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10 PNT 10
16190 PNT 191 !" ";PNT 194 PRINT " FILE HAS BEED CREATED WITH
    NAME "N7$"."
16200 WAIT 3
i6210 CLOSE:3 RETURN
16235 INPUT:3 BL$
16237 INPUT:3 SA$
16239 INPUT:3 PA$
16999 REM
17000 PNT 195 PNT 174 PNT 10 PNT 10
17000 PNT 195 PNT 174 PNT 10 PNT 10
17010 PNT 195 !" ";PNT 197 PRINT "
17020 PNT 195 !" ";PNT 197 PRINT " THIS ROUTINE READS DATA
    FROM A DISKETTE AND STORES IT IN "
17030 PNT 195 !" ";PNT 197 PRINT " PROGRAM MEMORY
    FOR PLOTTING AND ANALYSIS
17040 PNT 195 !" ";PNT 197 PRINT "
    "
17050 FOR I=1 TO 1425
17060 X2(I)=0 X3(1)=0
17070 NEXT I
17080 WAIT 1.5 PNT 195 PNT 10 PNT 10 PNT 10 PNT 10
17090 PNT 195 !" ";PNT 196 !"
    "
17100 PNT 195 !" ";PNT 196 !" I AM NOW READY TO READ AN
    EXISTING DATA FILE.
17110 PNT 195 !" ";PNT 196 !" PLEASE ENTER NAME OF
    FILE TO BE READ.
17120 PNT 195 !" ";PNT 196 !"
17130 PNT 195 !" ";PNT 196 !" ";INPUT N7S
17140 PNT 10 PNT 10 PNT 10
17150 PNT 195 !" ";PNT 197 !"
17160 PNT 195 !" n;PNT 197 !" PLEASE INSERT DISKETTE CONTAINING
        FILE TO BE READ IN DRIVE B. "
```

```
17170 PNT 195 !" ";PNT 197 !" WHEN READY
    PRESS RETURN.
17180 PNT 195 !" ";PNT 197 !"
    ";INPUT
17190 DSKRESET
17200 WAIT 2
17210 ON ERROR 17,17430
17220 OPENR:3 ""N7$".TES"
17230 CLS O PNT l81 PNT 10 PNT 10 PNT 10 PRINT "
    ";PNT 193 !" FILE IS BEING READ. ";PNT 183 PNT 180
17235 INPUT:3 BL$
17237 INPUT:3 SAS
17239 INPUT:3 PA$
17240 INPUT:3 DS
17250 INPUT:3 AC$
17260 INPUT:3 LD$
17270 INPUT:3 LC$
17280 INPUT:3 LR$
17290 INPUT:3 LI$
17300 INPUT:3 TNS
17310 INPUT:3 BDS
17320 INPUT:3 UWS
17330 INPUT:3 LOS
17340 INPUT:3 DE$
17350 INPUT:3 VL$
17360 INPUT:3 T$
17370 INPUT:3 LMS
17380 INPUT:3 A3
1 7 3 8 9 \text { REM}
17390 INPUT:3 A4 INPUT:3 B3 INPUT:3 B4 INPUT:3 Ll INPUT:3 B1 INPUT:3 Al
17410 LOAD ARRAY:3 X2(1)
17420 LOAD ARRAY:3 X3(1)
17425 CLOSE:3 RETURN
17430 PNT 194 PRINT " UNABLE TO FIND FILE! " WAIT 2 OFF
    ERROR PNT 195 CLS O GOTO 17080
20000 REM
20009 REM OPENW:2 "SQTO:1"
20010 FOR I=1 TO 300
20015 REM
20016 PRINT X3(I)
20017 REM NX$=STRS(X3(I))
20018 REM YS=STRS(X2(I))
20019 REM Z$=STRS(Xl(I))
20020 REM PRINT:2 X$,Y$,Z$
20021 REM PRINT:2 X3(I),X2(I),X1(I)
```

20022 WAIT . 1
20030 NEXT I
20040 END

## APPENDIX G <br> A GUIDE FOR THE USE OF THE JAMES V-METER

This appendix contains a guide prepared oy the author during this study. The guide was delivered to HQ AFESC/RDCP, Tyndall AFB, Florida, in July :987. The guide is presented herein in its original form. The following metric conversion factors apply:

$$
\begin{aligned}
& 1 \text { foot }=0.305 \mathrm{~m} \\
& 1 \text { inch }=25.4 \mathrm{~mm} \\
& 1 \text { pound }=4.448 \mathrm{~N} \\
& 1 \mathrm{psi}=6.89 \mathrm{kPa}
\end{aligned}
$$

# A GUIDE FOR THE USE OF THE JAMES V-METER TO DETERMINE THE MODULUS OF ELASTICITY OF CONCRETE CORES 

by
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JULY 1987

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## Introduction

The James $V$-Meter is a device which generates low-frequency ultrasonic pulses and measures the time of propagation between 2 transducers through a material of interest. This user's guide is intended to be a quick reference for the testing of concrete core samples with the $V$-Meter for the determination of modulus of elasticity. The principle reference for this guide is the V-Meter Instruction Manual published by James Instruments, Inc.

## Equipment

Figure Gl illustrates the front panel display of the V-Meter and Figure G2 illustrates the back panel display. Connections and switches mentioned below are indicated in the figures.

The complete $V$-Meter test system includes :
a) C-4902 V-Meter
b) AC power cord
c) Connector for external battery
d) Two C-4898 transducers
e) Two transducer leads ( $10-\mathrm{ft}$ )
f) Leather case for $V$-Meter
g) C-4897 reference bar for checking zero
h) Couplant material
i) $V$-Meter manual


FIGURE G1. Front View of the James V-Meter (Model C-4902)


FIGURE 62. Back View of the James V-Meter (Model C-4902)

## Test Procedure for Concrete Cores

The following procedure may be used to test concrete cores :
(1) Prepare all cores to be tested (see discussion on sample preparation).
(2) Choose type of power supply and attach cord if necessary (see discussion on power supply).
(3) Set power selector switch to OFF.
(4) Use transducer leads to connect the C-4898 transducers to the V-Meter. One transducer acts as transmitter (connected to TRAN) and the other acts as receiver (connected to REC).
(5) Set voltage selector switc to 500 V .
(6) Set range switch to 0.1.
(7) Set power selector switch to enable the appropriate power supply (see discussion on power supply).
(8) Depress the push-on switch until a clicking noise is heard from the transducers.
(9) Apply a small amount of couplant material (see discussion on couplant material) to the face of each transducer.
(10) Press the transducers tightly to the ends of the calibrating rod.
(11) Observe the reading on the transit time display. The display should read 26.0 for the $26 \mu \mathrm{sec}$ calibration rod (or be equal to the calibration transit time engraved on the rod).
(12) If the display reading does not match the calibration value, adjust the reading with the set reference dial (this is much easier with two people, but one person can do it by trial and error).
(13) Remove the transducers from the calibrating rod.
(14) If necessary, apply an additional small amount of couplant material to the transducers.
(15) Press the transducers tightly against the ends of the cores.
(16) Read and record the transit time from the transit time display.
(17) Repeat Steps (14) through (16) for each core to be tested.
(18) For each core calculate the uncorrected pulse velocity which is the length of travel (core length) divided by the transit time.
(19) Use Figure G3 or Figure G4 to obtain correction factor if core contains reinforcing steel (correction factor equals 1 for no steel).
(20) Multiply the uncorrected pulse velocity by the correction fastor to obtain the corrected pulse velocity.
(21) Choose the appropriate equation and calculate the modulus of elasticity (see discussion on equations for modulus of elasticity).


FIGURE G3. Correction Factor for Steel Perpendicular to Travel Path


FIGURE G4. Correction Factor for Steel Parallel to Travel Path

## Sample Preparation

Core length should be at least 4 inches when maximum aggregate size is 0.75 inches or less. Core length should be at least 6 inches for larger aggregate sizes. Accuracy is improved if the core faces are sawed smooth. Special precautions may be needed to insure that the resulting faces are "square" (perpendicular to the longitudinal axis of symmetry). Properly faced core samples can be accurately measured for length.

Use calipers to measure the length of travel, $L$, through the core. Make sure that the calipers are positioned on the same spots that the transducers will be placed.

Measure average length, $\ell$, and average diameter, D. Use sufficient readings at equidistant spacing to insure that the average dimensions are representative of the sample. Weigh the sample. Calculate the mass density of the sample with the following formula :

$$
\begin{equation*}
\rho=\frac{0.003295133 \mathrm{~W}}{D^{2} \ell} \tag{G1}
\end{equation*}
$$

where

$$
\begin{aligned}
& \rho=\text { mass density }, 1 b-\sec ^{2} / i n^{4} . \\
& W=\text { weight }, 1 b \\
& D=\text { average diameter, inches } \\
& \ell=\text { average length, inches }
\end{aligned}
$$

## Power Supply

The power may be supplied by three sources, AC power, internal battery, or external battery. AC power is recommended for laboratory testing. Internal battery power is recommended for field testing. An external battery may be used as power supply if the internal battery fails.

An $A C$ connecting cable is provided with the $V$-Meter system. The AC connecting cable attaches to the AC Main Line Socket. The V-Meter is internally connected to run from a standard $A C$ connection, and will operate properly for voltage ranges of 110 V to 130 V supplied at either 50 Hz or 60 Hz . The AC power supply option is enabled when the power selection switch is set to MAINS.

The internal battery is enabled when the power selection switch is set to BATT. The battery condition indicator gives the approximate battery condition. The battery will operate for about 9 hours when the needle is at the left hand side of the black line. The battery will operate for about 4 hours when the needle is at the right hand side of the black line. Overcharging can damage the internal battery. Charging should be done as described in the next section which describes internal battery charging.

An external battery capable of supplying 12 V DC at 140 mA may be used. The V-Meter system includes an external battery connecting cable which is color coded (red = positive, black = negative) for the external battery connection on the back of the V-Meter. The external battery is enabled when the power selection switch is set to EXT.

## Charging the Internal Battery

Charging the internal battery requires connection of the $A C$ power supply. The battery may be damaged by overcharging. If the battery is fully discharged, the battery may be recharged by setting the power selector switch to CH for 12 hours. If the battery has been used for less than 9 hours, recharge should be 1.3 times the discharge time. If the discharge time is unknown, let the $V$-Meter run with power selection switch set to BATT until it switches itself off, then charge for 12 hours by turning the power selection switch to CH .

If the discharge time is unknown, or if extended charging is necessary, use the trickle charge feature by setting the power selector switch to TC. When set to trickle charge, at least 30 hours is required to charge a fully discharged battery. Trickle charging will recover at least $70 \%$ of the battery capacity.

It is essential on non-metallic material to use a coupling medium between the faces of the transducers and the material being tested. A can of water pump grease is supplied with the $V$-Meter system for this purpose. However, silicon grease, medium bearing grease, liquid soap, or petroleum jelly may be used as couplant material.

The amount of couplant material to be used varies with the surface roughness of the face of the core. Use just enough couplant material so that all the surface voids underneath the transducers are filled and a good acoustic coupling is provided when the transducers are pressed tightly against the faces. Use of additional couplant material should be avoided.

## Correction of Pulse Velocity for Steel Reinforcing Bars

Use Figure G3 or Figure G4 to obtain a correction factor which is multiplied by the uncorrected phase velocity to obtain the corrected phase velocity. If no steel is present, the correction factor is equal to 1 and no correction is required.

## Calculation of Modulus of Elasticity

There are two formulas for the calculation of modulus of elasticity based on the transit time measured by the $V$-Meter. One formula is correct for small diameter cores and one formula is correct for large diameter cores.

For small diameter cores, the proper equation is the equation for propagation of a compression wave in a rod. This formula is :

$$
\begin{equation*}
E=\rho V^{2} \tag{G2}
\end{equation*}
$$

where

$$
\begin{aligned}
E & =\text { Young's modulus of elasticity, psi } \\
\rho & =\text { mass density, } 1 \mathrm{~b}-\mathrm{sec}^{2} / \mathrm{in}^{4} \\
V & =\text { pulse velocity, inches } / \text { sec } \\
& =\text { transit time/length of travel }
\end{aligned}
$$

For large diameter cores, the proper equation is the equation for propagation of a compression wave in an infinite solid. This formula is:

$$
\begin{equation*}
E=\frac{\rho V^{2}(1+\nu)(1-2 \nu)}{(1-\nu)} \tag{G3}
\end{equation*}
$$

where

$$
\begin{aligned}
E & =\text { Young's modulus of elasticity, psi } \\
\rho & =\text { mass density, } 1 b-\sec ^{2} / \mathrm{in}^{4} \\
V & =\text { pulse velocity, inches/sec } \\
& =\text { transit time/length of travel } \\
\nu & =\text { Poisson's ratio }
\end{aligned}
$$

Use of the small-diameter (rod) equation is considered valid if the radius of the core is less than $l$ wavelength of the pulse wave. The pulse wave is generated at 54 kHz . Therefore, the maximum diameter for the small-diameter equation is equal to :

$$
\begin{equation*}
D_{\max }=V / 27000 \tag{G4}
\end{equation*}
$$

where

$$
\begin{aligned}
& D_{\text {max }}=\text { maximum diameter for rod equation, inches } \\
& V=\text { pulse velocity, inches } / \mathrm{sec}
\end{aligned}
$$

For cases where the core radius is greater than 1 wavelength, the large-diameter (infinite solid) equation should be used. For the normal range of concrete mixtures, a 2 inch diameter core can be considered a small-diameter sample and a 6 inch diameter core can be considered a large-diameter sample. Core samples with diameters between 2 inches and 6 inches may be either small-diameter or large-diameter samples, depending on the pulse velocity (see equation above).

Use of 2 inch diameter samples is recommended for this test. If 2 inch diameter cores are used, the small-diameter (rod) equation may be used, and Poisson's ratio is not needed. If the large-diameter (infinite solid) equation is used, an assumption must be made for Poisson's ratio. Figure $G 5$ plots the ratio of moduli from each of the equations versus Poisson's ratio. For materials with low Poisson's ratio, the two equations produce reasonably similar results. For Poisson's ratio of 0.15 (typical assumed value for $P C C$ ), the difference in the two equations is less than 6\%. However, the difference becomes very significant for higher values of Poisson's ratio.


FIGURE G5. Influence of Poisson's Ratio on Modulus Equations

## Selected References

James Instruments Inc, Instruction Manual for Model C-4092 and Model C-4901 V-Meter, James Instruments Inc, Chicago, Illinois, January 1986.

Prakash, Shamsher, Soil Dynamics, McGraw-Hill, New York, New York, 1981.
Richart, F. E., Jr., Hall, J. R., Jr., and Woods, R. D., Vibrations of Soils and Foundations, Prentice-Hall, Englewood Cliffs, New Jersey, 1970.

APPENDIX H

## SOURCE CODE FOR GREEN-MA.FOR

## C $X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X ~ D R I V E . M A ~ X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X$

 \$LARGEIMPLICIT REAL*8(A-H,0-Z)
COMMON AR1,AR2,BR1,BR2,CR1,CR2,GR1,GR2,ML1,ML2,FR,KR,NLAY
COMPLEX*16 AR1(62), AR2(62), GR1(62), GR2(62)
COMPLEX*16 BR1(62),BR2(62),CR1(62),CR2(62)
COMPLEX*16 KR(60), $\operatorname{FR}(60,60)$
REAL*8 ML1(31), ML2(31)
REAL*8 HH(30), $\operatorname{WGT}(30), \mathrm{G}(30), \operatorname{DAMP}(30), \operatorname{LAME}(30)$
COMPLEX*16 CON,CON1
COMPLEX*16 BETA
REAL*8 FQ,RR,R2(8)
DIMENSION FREQ(100)
CHARACTER*14 DUMMY
CHARACTER*1 CHECKC(7)
DATA CHECKC/' ','d','D','R','r','c','C'/
WRITE(*,*)
WRITE(*,*)'WELCOME TO GREEN-MA, A PROGRAM FOR CALCULATION OF' WRITE(*,*)'GREENS FUNCTIONS FOR SURFACE DISPLACEMENTS CAUSED' WRITE(*,*)'BY A VERTICAL DISK LOAD AT THE SURFACE OF A MULTI-' WRITE(*,*)'LAYERED SYSTEM. GREEN-MA IS BASED ON A PROGRAM BY'
WRITE(*,*)'EDUARDO KAUSEL AND ADAPTED BY MARK ANDERSON.'
WRITE(*,*)
WRITE(*,*)'THE USER MUST CHOOSE THE RADIUS CONFIGURATION.'
WRITE(*,*)
WRITE(*,*)'OPTIONS INCLUDE:'
WRITE(*,*)
WRITE(*,*)' DEFAULT (RETURN) = FWD WITH 7 SENSORS'
WRITE (*,*)' D = DYNAFLECT WITH 6 SENSORS
WRITE(*,*)' R = ROAD RATER WITH 4 SENSORS'
WRITE(*,*)' C = CHOOSE CONFIGURATION'
WRITE (*,*)
WRITE(*,*)'WHICH OPTION WOULD YOU LIKE ?'
WRITE(*,*)
READ(*, 4199)CHECKC(1)
4199 FORMAT(AI)
IF (CHECKC(1).EQ.CHECKC(2).OR.CHECKC(1).EQ.CHECKC(3))GO TO 4198
IF (CHECKC(1).EQ.CHECKC (4).OR.CHECKC(1).EQ.CHECKC(5))GO TO 4197

IF (CHECKC (1).EQ.CHECKC(6).OR.CHECKC(1).EQ.CHECKC(7))GO TO 4196
$R 2(1)=0$.
R2 (2) $=0.5$
$R 2(3)=1$.
$R 2(4)=2$.
$R 2(5)=3$.
$R 2(6)=4$.
$R 2(7)=5$.
$\mathrm{R} 2(8)=6$.
NRR=2
NR=8
GO TO 4195
4198 R2(1) $=0.08333333$
$R 2(2)=0.8333335 .3$
R2 (3) $=1.30$
R2(4) $=2.16666667$
R2(5) $=3.11666667$
$R 2(6)=4.08333333$
NRR=1
$N R=6$
GO TO 4195
4197 R2(1) $=0.25$
$R 2(2)=0.4375$
$R 2(3)=1.09166667$
$R 2(4)=2.0475$
$R 2(5)=3.03166667$
NRR $=1$
$N R=5$
GO TO 4195
4196 WRITE(*,*)
WRITE(*,*)'INPUT NUMBER OF RADIUS VALUES, INCLUDING LOAD, MAX $=10^{\prime}$
WRITE(*,*)
READ(*,*)NR
WRITE(*,*)
WRITE(*,*)'INPUT RADIUS VALUES, FREE FORMAT'
WRITE(*,*)
READ(*,*)(R2(IIIIII), IIIIII=1,NR)
WRITE (*,*)
WRITE(*,*)'INPUT RADIUS NUMBER FOR LOAD RADIUS'
WRITE(*,*)
READ(*,*)NRR
4195 CONTINUE
RR=R2 (NRR)
WRITE(*,*)
WRITE(*,*)'INPUT FILENAME FOR INPUT'

```
    WRITE(*,*)
    READ(*,2118)DUMMY
2118 FORMAT(A14)
OPEN(8,FILE=DUMMY)
WRITE(*,*)
WRITE(*,*)'INPUT FILENAME FOR OUTPUT'
WRITE(*,*)
READ(*,2218)DUMMY
2218 FORMAT(A14)
    OPEN(6,FILE=DUMMY,STATUS='NEW')
    WRITE(*,*)
    WRITE(*,*)'END INPUT PHASE - START CALCULATIONS'
    WRITE(*,*)
    READ(8,*)NLAY
    DO 3716 I=1,NLAY
    READ(8,*)HH(I),WWWWWW,EE,POIS,DAMP(I)
    WGT(I)=WWWWW/32.2
    G(I)=0.5*EE/(1.+POIS)
    CLA=1.-2.*POIS+.00000000000000000001
    LAME(I)=2.*POIS*G(I)/CLA
3716 CONTINUE
    LAY2=NLAY +NLAY
    AR1(1)=0.
    AR1(2)=0.
    BR1(1)=0.
    BR1(2)=0.
    ML1(1)=0.
    GR1(1)=0.
    GR1(2)=0.
    DO 1107 I=1,NLAY
        K=2*I
        IF(DAMP(I).LT.1.D-12)BETA=DCMPLX(1.DO)
        IF(DAMP(1).GE.1.D-12)BETA=DCMPLX(1.D0,2.D0*DAMP(I))
        CON=HH(I)*G(I)*BETA/3.
        CONl=HH(I)*(LAME(I)+2.*G(I))*BETA/3.
        ARI(K-1)=ARI(K-1)+CON1
        ARI (K)=AR1 (K)+CON
        AR1(K+1)=CON1
        AR1(K+2)=CON
        AR2(K-1)=0.5*CON1
        AR2(K)=0.5*CON
        CON=BETA*G(I)/HH(I)
        CONl=BETA*(LAME(I)+2.*G(I))/HH(I)
        GRI(K-1)=GRI(K-1)+CON
        GRI(K)=GR1(K)+CON1
```

```
                GRI (K+1)=CON
                GR1(K+2)=CON1
                GR2(K-1)=-CON
                GR2(K)=-CON1
                CONS=WGT(I)*HH(I)/3.
                ML1(I)=ML1(I)+CONS
                MLI(I+1)=CONS
                ML2(I)=0.5*CONS
                CON=0.5*(G(I)-LAME(I))*BETA
                CONl=0.5*(G(I)+LAME(I))*BETA
                BRI(K-1)=CON-BRI(K-1)
                BR1(K)=-CON1
                    BR1(k+1)=CON
            BR2(K-1)=CON1
            BR2(K)=0.
1107 CONTINUE
    ML2(NLAY)=0.
    AR2(LAY2-1)=0.
    AR2(LAY2)=0.
        BR1(LAY2)=0.
        BR2(LAY2-2)=0.
        BR2(LAY2-1)=0.
        BR2(LAY2)=0.
        GR2(LAY2-1)=0.
        GR2(LAY2)=0.
c
c READ FREQUENCY INTERVALS
C
    READ(8,*)NFR
        NOF=0
        TDF=-1.D-20
        DO 3 1=1,NFR
            READ(8,*)NOM,DOM,OM
            J=NOF+1
            IF(NOM.EQ.0)NOM=1
            NOF=NOF+NOM
            TDF=OM+(NOM-1)*DOM
            DO 333 K=J,NOF
                FREQ(K)=OM
                    OM=OM+DOM
333 CONTINUE
CONTINUE
c
C LOOP OVER FREQUENCIES
C
```

```
        DO 11 JJJJ=1,NOF
        WRITE(*,*)
    WRITE(*,*)'WORKING ON FREQUENCY NUMBER ',JJJJ,' OF ',NOF
    WRITE(*,*)
    FQ=FREQ(JJJJ)
    OM2=39.478417604357DO*FQ*FQ
C
C SOLVE EIGENVALUE PROBLEM
C
    DO 2 I=1,LAY2
        K=(I+1)/2
        CR1(I)=GR1(I)-OM2*ML1(K)
        CR2(I)=GR2(I)-0M2*ML2(K)
    CONTINUE
    CALL RAYLGH(*33)
    CALL GREEN(R2,FQ,RR,NR,NRR)
    GO TO 11
33 WRITE (6,210) FREQ(JJJJ)
11 CONTINUE
    STOP
210 FORMAT(1HO,'FREQUENCY F=',F10.4,' CAUSED EIGENVALUE ERROR')
        END
C
C
C
C XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX RAYLGH XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    SUBROUTINE RAYLGH(*)
    IMPLICIT REAL*8(A-H,0-Z)
    COMMON A1,A3,B2,B4,C1,C3,GR1,GR2,ML1,ML2,V,E,NLAY
    COMPLEX*16 Al(62),A3(62),GR1(62),GR2(62)
    COMPLEX*16 B2(62),B4(62),C1(62),C3(62)
    COMPLEX*16 E(60),V(60,60)
    REAL*8 MLl(31),ML2(31)
    COMPLEX*16 A(248),V1(62),V2(62),U1(62),U2(62)
    COMPLEX*16 U3(62),R1(62),R2(62),S1(62),S2(62)
    DIMENSION MVB(62)
    COMPLEX*16 CF
    COMPLEX*16 EV,EVS,CE,CD,DEV,C,D
    DATA EPS1/1.0D-5/,EPS2/1.00-10/
    NN=NLAY+NLAY
    Nl=NN-1
    N2=NN-2
    DO 10 J=1,N2,2
        MVB(J)=J+3*NN
        I=J+1
```

$\operatorname{MVB}(\mathrm{I})=\mathrm{I}+2 * \mathrm{NN}$
10 CONTINUE
$\operatorname{MVB}(\mathrm{N} 1)=\mathrm{Nl}+\mathrm{NN}$
$X N=0$.
DO $15 \mathrm{~J}=1, \mathrm{NN}$
$X=\mathrm{Cl}(\mathrm{J}) / \mathrm{Al}(\mathrm{J})$
$X N=X N+D A B S(X)$
15 CONTINUE
XN=DSQRT (XN/NN)
$C=D C M P L X(X N, X N+X N)$
DO $20 \mathrm{~J}=1, \mathrm{NN}$
Sl(J) $=1.00$
S2 (J) = C
$\mathrm{V} 1(\mathrm{~J})=\mathrm{Sl}(\mathrm{J})$
$V 2(\mathrm{~J})=\mathrm{S} 2(\mathrm{~J})$
R1 (J) $=0.00$
R2 $(\mathrm{J})=0 . \mathrm{DO}$
20 CONTINUE$E V=X N / N N$
$I S W=0$
$K C=0$$M C=0$
400
DO $50 \mathrm{~N}=1, \mathrm{NN}$
$\mathrm{Ul}(\mathrm{N})=\mathrm{Cl}(\mathrm{N}) * \mathrm{Vl}(\mathrm{N})$ $\mathrm{U} 2(\mathrm{~N})=\mathrm{Al}(\mathrm{N}) * \mathrm{~V} 2(\mathrm{~N})$
50 CONTINUE
DO $55 \mathrm{~N}=3$, NN
$L=N-2$
$\mathrm{Ul}(\mathrm{L})=\mathrm{Ul}(\mathrm{L})+\mathrm{C} 3(\mathrm{~L}) * \mathrm{~V} 1(\mathrm{~N})$$\mathrm{Ul}(\mathrm{N})=\mathrm{Ul}(\mathrm{N})+\mathrm{C} 3(\mathrm{~L}) * \mathrm{~V} 1(\mathrm{~L})$$\mathrm{U} 2(\mathrm{~L})=\mathrm{U} 2(\mathrm{~L})+\mathrm{A} 3(\mathrm{~L}) * \mathrm{~V} 2(\mathrm{~N})$$\mathrm{U} 2(\mathrm{~N})=\mathrm{U} 2(\mathrm{~N})+\mathrm{A} 3(\mathrm{~L}) * \mathrm{~V} 2(\mathrm{~L})$
55 CONTINUE
IF (ISW.EQ.l) GO TO 85
ISW=0
CD=1.D0
$X=C D A B S(E V) / 2 . D 0$
$E V=D C M P L X(X, X+X)$
GO TO 87
85 EV=DCONJG(EV)
ISW=2
$C D=1.00$
87 IK=0
DO 200 IT=1,100

```
EVS=EV*EV
DO 90 N=1,NN
    A(N)=EVS*Al(N)+Cl(N)
    A(N+NN)=EV *B2(N)
    A(N+2*NN)=EVS*A3(N)+C3(N)
    A(N+3*NN)=EV *B4(N)
    V2(N)=(U1 (N)-U2(N)*EV)*CD
90 CONTINUE
    DO 150 N=1,N1
        I=N
        J=N+NN
        M=MVB(N)
    DO 120 L=J,M,NN
        C=A(L)/A(N)
        I=I+1
        JI = I
        DO 110 K=L,M,NN
                A(JI) = A(JI) - C*A(K)
                JI = JI + NN
                CONTINUE
                A(L)=C
                V2(I)=V2(I)-C*V2(N)
    CONTINUE
    V2(N)=V2(N)/A(N)
150 CONTINUE
    N=NN
    IF(CDABS(A(N)).NE.O.DO)GO TO 151
    DEV=0.5DO*DEV
    EV=EV-DEV
    GO TO 200
151 V2(N)=V2(N)/A(N)
    DO 160 K=1,N1
        N=N-1
        I=N
        J=N+NN
        M=MVB(N)
        DO 1600 L=J,M,NN
            I=I+1
                V2(N)=V2(N)-A(L)*V2(I)
            CONTINUE
160 CONTINUE
    CE=0.00
    DO 170 N=1,NN
        V1(N)=(V2(N)-V1(N)*CD)/EV
        CE=CE-U1(N)*V1(N)+U2(N)*V2(N)
```

```
        Ul(N)=Cl(N)*Vl(N)
        U2(N)=Al(N)*V2(N)
    170 CONTINUE
    CE=CE*CD
    DO 180 N=3,NN
        L=N-2
        U1(L)=U1(L)+C3(L)*V1(N)
        Ul(N)=U1(N)+C3(L)*V1(L)
        U2(L)=U2(L)+A3(L)*V2(N)
        U2(N)=U2(N)+A3(L)*V2(L)
180 CONTINUE
    CD=0.DO
    DO 190 N=1,NN
        CD=CD-U1 (N)*Vl(N)+U2(N)*V2(N)
    190 CONTINUE
    DEV=CE/CD
    IF (IT.GT.15) DEV=DEV*0.5DO
    EV=EV+DEV
    CF=CD
    CD=CDSQRT (2.DO/CD)
    C=DEV/EV
    X=DABS(DREAL(C))+DABS(DIMAG(C))
    IF (IK.EQ.1.AND.X.LT.EPS2) GO TO 300
    IF (X.LT.EPSI) IK=1
    2OO CONTINUE
    WRITE (6,2020)
    RETURN 1
    300 E(KC)=EV
    ICR=0
    X=DABS(DREAL(EV))
    Y=DABS(DIMAG(EV))
    Z=X+Y
    IF (Y/Z.LT.EPS2) ICR=1
    IF (X/Z.LT.EPS2) ICR=2
    IF(ICR.EQ.1) E(KC)=DREAL(EV)
    IF(ICR.EQ.2)E(KC)=DIMAG(EV)*(0.DO,1.DO)
    DO 310 N=1,NN
    310 V(N,KC)=V2(N)*CD
C
C IF THE EIGENVALUE IS REAL CHOOSE THE SIGN SUCH THAT THE GROUP VELOCIT
C BECOMES POSITIVE
C
    IF (ICR.NE.1) GO TO 316
    X=(V1(1)*V1(1)+V1(2)*V1(2))*EV
    IF(X.GT.O.) GOTO 316
```

```
    E(KC)=-E(KC)
            DO 315 N=2,NN,2
    315 V(N,KC)=-V(N,KC)
C CHECK THE SPECIAL ORTHOGONALITY OF THE NEWLY FOUND EIGENVECTOR AND
C THE SUM OF THE PREVIOUSLY FOUND EIGENVECTORS
    316 CE=0.00
            DO 330 N=1,NN
                CE=CE-Rl(N)*Ul(N)+R2(N)*U2(N)
    330 CONTINUE
            CE=CE*CD
            X=DABS(DREAL(CE))+DABS(DIMAG(CE))
            IF (X.LT.EPS1) GO TO 335
            WRITE (6,2021)
            RETURN 1
335 C=1.DO/E(KC)
            DO 340 N=1,NN
                R1(N)=R1(N)+V(N,KC)*C
                R2(N)=R2(N)+V(N,KC)
    340 CONTINUE
C FIND A STARTING VECTOR ORTHOGONAL TO ALL EIGENVECTORS FOUND FOR THE
C ITERATION TOWARD THE NEXT EIGENVECTOR
    C=O.DO
    D=0.DO
    DO 342 N=2,NN,2
        M=N-1
        C=C-U1(M)*S1(M)+U2(N)*S2(N)
        D=D-U1(N)*S1(N)+U2(M)*S2(M)
    342 CONTINUE
    CE=2.DO/CF
    C=C*CE
    D=D*CE
    DO 343 N=2,NN,2
        M=N-i
        Sl(M)=Sl(M)-C*V1(M)
        Sl(N)=Sl(N)-D*V1(N)
        S2(M)=S2(M)-D*V2(M)
        S2(N)=S2(N)-C*V2(N)
        Vl(M)=S1(M)
        Vl(N)=S1(N)
        V2(M)=S2(M)
        V2(N)=S2(N)
    343 CONTINUE
    IF (ICR+ISW.EQ.0) ISW=1
    MC=MC+1
    IF (MC.GE.NN ) GO TO 350
```

```
        GOTO 400
C FORM VECTORS FOR DISPLACEMENT EXPANSION
    350 DO 610 N=1,NN
                C=-1.DO
                X =DIMAG(E(N))
                    IF (X.LE.O.) GO TO }60
                    E(N)=-E(N)
                    C=-C
                    DO 6100 J=2,NN,2
                                    V(J,N)=V (J,N)*C
            CONTINUE
    6 1 0 ~ c o n t i n u e ~
        RETURN
C
    2020 FORMAT(IX,'FAIL TO CONVERGE 100 ITERATIONS IN QUAD. EIGEN. PROB.')
    2021 FORMAT(1X,'FAIL FIND EIGENVECTOR ORTHOGONAL TO QUAD EIGEN PROB')
        END
C
C
C
C
C XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX GREEN XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    SUBROUTINE GREEN(R,FQ,RR,NR,NRR)
    IMPLICIT REAL*8(A-H,O-Z)
    COMMON AR1,AR2,BR1,BR2,CR1,CR2,GR1,GR2,ML1,ML2,FR.KR,LAY
    COMPLEX*16 AR1(62),AR2(62),GR1(62),GR2(62)
    COMPLEX*16 BR1(62),BR2(62),CR1(62),CR2(62)
    COMPLEX*16 KR(60),FR(60,60)
    REAL*8 ML1(31),ML2(31),R(8)
        COMPLEX*16 ILI(30,8),IL2,IL3(30,8),IL4
        COMPLEX*16 IRI (60,8),IR2 (60,8),IR3(60,8),IR4 (60,8)
        COMPLEX*16 Wl(60,60),W2(60,60),W3(30,30)
        COMPLEX*16 U,V,W,Ul
        LAY2=LAY+LAY
C
C COMPUTE INTEGRALS
C
        DO 3 J=1,NR
        DO 1 I=1,LAY2
            CALL EVINT(IR1(I,J),IR2(I,J),IR3(I,J),IR4(I,J),KR(I),R(J),RR)
    1 CONTINUE
    3 CONTINUE
C
C FORM PRODUCT OF NODAL MATRIX BY ROW CORRESPONDING TO ELEVATION
C WHERE LOAD IS APPLIED. LOOP OVER ALL LOADS
```

```
C
    PIR=3.14159265*RR
        PIR=1.D0/PIR
            DO 4 I=1,LAY2
                DO 4431 J=1,LAY2
                    W1(I,J)=FR(I,J)*FR(1,J)
                    W2(I,J)=FR(I,J)*FR(2,J)
                            CONTINUE
        CONTINUE
C
C COMPUTATION OF AVERAGE DISPLACEMENTS UNDER LOAD
C
        W=0.
        WRITE (6,104) FQ
        DO 6 L=1,LAY2
                        W=W+W2(2,L)*IR3(L,NRR)
        CONTINUE
        W=2.D0*W*PIR/RR
        WRITE(6,101) W
C
C COMPUTE DISPLACEMENTS DUE TO VERTICAL DISK LOAD
C
    WRITE(6,102)
    DO 17 J=1,NR
        U=0
        V=0.
        W=0.
        IF(R(J).EQ.O.DO) GO TO 15
        DO 14 L=1,LAY2
            U=U+W2(1,L)*IR2(L,J)/KR(L)
        CONTINUE
        U=U*PIR
        DO 16 L=1,LAY2
            W=W+W2(2,L)*IR1(L,J)
        CONTINUE
        W=W*PIR
                                WRITE(6,103) R(J),U,W
                            CONTINUE
101 FORMAT(' AVERAGE VERTICAL DISPLACEMENT OF VERTICAL LOAD DISK',/,
    *' ',2E15.5,/,/)
102 FORMAT(' SURFACE DISPLACEMENTS DUE TO VERTICAL SURFACE LOAD DISK'
    *,/,/,' RADIUS',7X,'RADIAL DISPLACEMENT',
    * 12X,'VERTICAL DISPLACEMENT')
103 FORMAT(F10.4,2E15.7,2X,2E15.7)
104 FORMAT(1H1,30X,'FREQUENCY=',F10.4,' CPS',//)
```

```
        WRITE(6,*)
        WRITE(6,*)
        WRITE(6,*)
        RETURN
    END
C
C
C
C
C XXXXXXXXXXXXXXXXXXXXXXXXXX EVINT XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
    SUBROUTINE EVINT (I1,I2,I3,I4,Z,R,RR)
    IMPLICIT REAL*8 (A-H,O-Z)
    COMPLEX*16 I1,I2,13,I4,Z,Z1,Z2,J0,J1,HO,H1,C,P,ZZ
    DATA P/(00.D0,-1.570796D0)/
    Zl=Z*R
    Z2=Z*RR
    ZZ=Z*Z
    J0=1.D0
    JI=0.DO
    IF(R.GT.RR) GOTO I
C
C FOR R.LT.RR INSIDE LOAD
    IF(R.NE.0.DO) CALL BESSEL(Zl,J0,J1,C)
    CALL HANKEL(Z2,HO,Hl,C,2)
    I2=P*J1*H1
    Il = (P*JO*H1-1.DO/Z2)/Z
    I3=(P*Jl*H1-0.5DO*R/RR)/ZZ
    I4=P*JO*HO
    RETURN
C
C FOR R.GE.RR OUTSIDE LOAD
C
    1 CONTINUE
    CALL BESSEL(Z2,JO,J1,C)
    CALL HANKEL(Z1,HO,H1,C,2)
    I2=P*Jl*H1
    [1=P*J1*HO/Z
    I3=(P*J1*H1-0.5D0*RR/R)/ZZ
    I4=P*JO*HO
    RETURN
    END
C
C
C
```


## $C$

C $X X X X X X X X X X X X X X X X X X X X X X X X X X X ~ B E S S E L ~ X X X X X X X X X X X X X X X X X X X X X X X X X X X ~$ SUBROUTINE BESSEL(ZZ,JO,Jl,C)
C COMPUTATION OF THE BESSEL FUNCTIONS OF ZERO AND FIRST ORDER
C $Z Z=\operatorname{COMPLEX} \operatorname{ARGUMENT}=(X, Y) \quad$-180. LT ARG(ZZ) LT 180. DEGREES
C $C=J 0 / J 1$
C
C WRITTEN BY E.KAUSEL
C
C
C
C
C
C
C
C
C $X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X ~ H A N K E L ~ X X X X X X X X X X X X X X X X X X X X X X X X X X ~$
SUBROUTINE HANKEL ( $2 Z, H 0, H 1, C, I N D)$
c
C COMPUTATION OF HANKEL FUNCTION
C $Z$ - COMPLEX ARGUMENT, -3.1415...LE.ARG(Z) .LE. 3.1415...
C HO- HANKEL FUNCTION OF IND'TH KIND AND ZERO ORDER
C Hl - HANKEL FUNCTION OF IND'TH KIND AND FIRST ORDER
C IND $=1,2$
C C $-\mathrm{HO} / \mathrm{Hl}$
C
C WRITTEN BY G.WAAS/E.KAUSEL
C
C
C
C AUTHOR'S NOTE : THIS SUBROUTINE WAS OMITTED FROM THE C C C

