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Effect of Worst-Case Multiple Jammers on Coded FH/SSMA Systems

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In this report ~~we~~ characterize and evaluate the effect of simultaneous multiple partial-band noise or tone jammers and other-user interference on a single communication link employing frequency-hopped spread-spectrum (FH/SS) signaling, *M*-ary frequency-shift-keying (FSK) modulation with noncoherent demodulation, and Reed-Solomon coding. For the symbol error probability of these systems, we derive exact expressions in the absence of multiple-access (MA) interference and tight upper bounds in the presence of other-user interference. Although our analytical methods are valid for any number of multiple jammers, we restrict our numerical study to the cases of two and three partial-band noise and tone jammers.

For fixed values of the spectral densities of noise jammers, or the energies per symbol of tone jammers, we evaluate the worst-case fraction of the band that each jammer should use in order to maximize the error probability of the FH/SS or FH/SSMA system. For the range of the signal-to-jammer power ratios examined, multiple noise or tone jammers appear to have no advantage over a single noise or tone jammer of equivalent spectral density or energy per symbol but achieve aproximately the same worst-case performance by jamming smaller fractions of the band.

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EFFECT OF WORST-CASE MULTIPLE JAMMERS ON CODED FH/SSMA SYSTEMS

1. INTRODUCTION

The effect of worst-case partial-band noise and tone jamming on frequency-hopped spread-spectrum (FH/SS) has been thoroughly studied over the years. References [1]-[6] constitute a representative selection of works that describe FH/SS systems operating in the presence of a single partial-band noise or tone jammer. Moreover, worst-case interference has been identified and several error-control coding schemes have been proposed for enabling the FH/SS systems to combat the interference.

More recently, the combined effects of partial-band noise jamming, other-user interference--also termed multiple-access (MA) interference, Rician non-selective fading, and additive white Gaussian noise (AWGN) were studied in [8]. A common characteristic of the work described in [1]-[6] and [8] is that hostile interference consists either of a single unmodulated signal which is hopped around the targeted frequency band, or of white noise generated in different sub-bands of the targeted frequency band. A single jamming device generates these signals.

In this report we characterize and evaluate the effect of simultaneous multiple partial-band noise or tone jammers and other-user interference on a single communication link employing frequency-hopped spread-spectrum (FH/SS) signaling, M-ary frequency-shift-keying (FSK) modulation with noncoherent demodulation, and Reed-Solomon coding. We develop techniques for the evaluation of the symbol error probability--also termed symbol error rate (SER)--of these systems. In particular, we derive (i) exact expressions and tight upper bounds for SER when multiple partial-band noise or tone jammers but no other-user interference are present, and (ii) tight upper bounds on SER when both multiple noise or tone jammers and

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other-user interference are present. In the case of frequency-hopped spread-spectrum multiple-access (FH/SSMA) systems we can analyze accurately the effect of other-user interference with different power levels by extending the results of [9]. The expressions for SER can also serve as upper bounds for the bit error rate (BER).

Although our analytical methods are valid for an arbitrary number of jammers and interfering users, we restrict our numerical study to the cases of two and three multiple noise or tone jammers; the numerical study of FH/SSMA systems disturbed by a larger number of simultaneous jammers was prohibited by the excessive computations needed to evaluate the worst-case allocation of fractions of the band jammed. There is no restriction on the number of interfering users. Numerical results are generated for both uncoded and Reed-Solomon coded FH/SS and FH/SSMA systems with errors-only decoding. When the spectral densities of the noise jammers or the energies per symbol of the tone jammers are fixed, we evaluate the optimal fraction of the band that each jammer should use in order to maximize the error probability of the FH/SSMA system; this corresponds to a worst-case scenario from the communicator's standpoint. We also compare the performance of multiple noise (or tone) jammers with that of a single noise (or tone) jammer, whose spectral density (or energy per symbol) equals the sum of the spectral densities (or energies per symbol) of the multiple jammers. Finally, we compare the performance of noise and tone jammers and assess the effects of other-user interference on the worst-case scenario.

The report is organized as follows: multiple partial-band noise jammers are treated in Section 2, where error probabilities are computed exactly (or bounded) for the single-user and multi-user cases in subsections 2.1 and 2.2, respectively; this is repeated in Section 3 for multiple partial-band tone jammers. In both sections the cases of two and three simultaneous jammers are treated in detail. In Section 4, error-control coding considerations are discussed; in

Section 5, numerical results are presented, while Section 6 contains the conclusions.

2. EFFECT OF MULTIPLE PARTIAL-BAND NOISE JAMMERS ON FH/SSMA SYSTEMS

2.1 Multiple Partial-Band Noise Jammers and AWGN

The model for the scenario of multiple partial-band noise jammers considered in this report is the following. There are L distinct jammers, each jamming a fraction ρ_i $(0 \le \rho_i \le 1)$ of the total bandwidth W of the FH/SS system. This bandwidth is $W = qMR/log_2M$ (in Hz), where q is the number of distinct frequency sub-bands used for frequency hopping (a sub-band being the hop bandwidth of the FH/SS system around a particular hopping frequency), M the number of frequency tones used in the MFSK data modulation scheme (orthogonal tone spacing is assumed), and R (in bits/sec) is the data (information) rate of the uncoded (no FEC) FH/SS system. The term RM/log_2M denotes the bandwidth of each sub-band. The hopping is slow, that is more than one M-ary symbol is transmitted per hop; the corresponding hopping rate is $R_h = R/N_b$ (hops per sec), where R is the data rate defined above and N_b is the number of bits per hop.

Each jammer visits the q different sub-bands randomly with equal probability. This means that the i-th jammer selects randomly and jams $\rho_i q$ of the q hopping frequencies. The different jammers select independently of each other (with no cooperation) which sub-bands to jam. Thus, with non-zero probability the same sub-band is jammed by more than two jammers. The i-th jammer's effective power spectral density is denoted by N_{J_i}. This is the power spectral density of a broadband jammer that jams the entire bandwidth W with the same total power as the partial-band jammer of interest. The total jammer power is $\overline{P}_{J_i} = W \cdot N_{J_i}$. The statistics of the noise jammers are assumed to be Gaussian. AWGN is also present and has spectral density N_0. It is assumed that the jamming power remains constant throughout the

MFSK symbol duration. With probability ρ_i the i-th jammer is present in any sub-band and has spectral density N_{J_i}/ρ_i; with probability $1-\rho_i$ it is absent from that sub-band or equivalently has spectral density 0.

The variables ρ_i are chosen in an optimal way for the jammers (this corresponds to a worst-case scenario for the communicator), whereas N_{J_i} are assumed to be known constants determined by the specifications of the device that each jammer uses and its distance from the receiver. In our model, the different noise jammers cooperate only for the common optimization of the ρ_i's; this, of course, presumes that knowledge of the N_{J_i}s is shared by all the jammers, $i = 1, 2, ..., L$.

The average symbol error probability for a FH/SS MFSK system with noncoherent demodulation operating in the presence of L partial-band noise jammers is easily obtained be enumerating all possible cases as follows:

$$\begin{aligned}
P_e^{(L)} = & \left[\prod_i (1-\rho_i)\right] P_{e,M}(N_0) \\
& + \sum_{i=1}^{L} \rho_i \left[\prod_{j\neq i}(1-\rho_j)\right] P_{e,M}\left[\frac{N_{J_i}}{\rho_i} + N_0\right] \\
& + \sum_{i<j} \rho_i \rho_j \left[\prod_{k\neq(i,j)}(1-\rho_k)\right] P_{e,M}\left[\frac{N_{J_i}}{\rho_i} + \frac{N_{J_j}}{\rho_j} + N_0\right] \\
& + \cdots \\
& + \sum_{l}(1-\rho_l)\left[\prod_{i\neq l}\rho_i\right] P_{e,M}\left[\sum_{i\neq l}\frac{N_{J_i}}{\rho_i} + N_0\right] \\
& + \left[\prod_k \rho_k\right] P_{e,M}\left[\sum_k \frac{N_{J_k}}{\rho_k} + N_0\right]
\end{aligned} \tag{1}$$

where $P_{e,M}(\eta)$, the probability of a symbol error for an MFSK system with energy per symbol E_s ($E_s = E_b \log_2 M$) operating in AWGN of spectral density η, is given by

$$P_{e,M}(\eta) = \sum_{m=1}^{M-1} \binom{M-1}{m} \frac{(-1)^{m+1}}{m+1} \exp\left[-\frac{m}{m+1} \cdot \frac{E_s}{\eta}\right]. \tag{2}$$

Since the jammers, like the background noise, are Gaussian, the total interference process for a given set of jammers is AWGN with spectral density equal to the sum of the spectral densities of each noise source. The enumeration process involved in the derivation of (1) is better understood with an example. Consider the case of three jammers ($L = 3$), the error probability of the uncoded FH/SS system takes the form

$$\begin{aligned} P_e^{(3)} = {} & (1-\rho_1)(1-\rho_2)(1-\rho_3)P_{e,M}(N_0) + \rho_1(1-\rho_2)(1-\rho_3)P_{e,M}\left[\frac{N_1}{\rho_1}+N_0\right] \\ & + \rho_2(1-\rho_3)(1-\rho_1)P_{e,M}\left[\frac{N_2}{\rho_2}+N_0\right] + \rho_3(1-\rho_1)(1-\rho_2)P_{e,M}\left[\frac{N_3}{\rho_3}+N_0\right] \\ & + \rho_1\rho_2(1-\rho_3)P_{e,M}\left[\frac{N_1}{\rho_1}+\frac{N_2}{\rho_2}+N_0\right] + \rho_2\rho_3(1-\rho_1)P_{e,M}\left[\frac{N_2}{\rho_2}+\frac{N_3}{\rho_3}+N_0\right] \\ & + \rho_3\rho_1(1-\rho_2)P_{e,M}\left[\frac{N_3}{\rho_3}+\frac{N_1}{\rho_1}+N_0\right] + \rho_1\rho_2\rho_3P_{e,M}\left[\frac{N_1}{\rho_1}+\frac{N_2}{\rho_2}+\frac{N_3}{\rho_3}+N_0\right] \end{aligned} \tag{3}$$

Consider the first line of this expression. The first term represents the case in which a symbol is affected only by background noise (i.e., no jammers are transmitting in the same frequency sub-band as the desired signal). The second term represents the case in which jammer 1 (but not the two others) is transmitting in the same frequency sub-band as the desired signal. The other terms represent the remaining possibilities. The enumeration process covers all cases of only one jammer (out of the three) being present in the same sub-band with the desired signal, of two jammers being present, and of all three jammers being present. In each case the power spectral densities of the noise sources present add together as noted in the comment following (2).

2.2 Multiple Partial-Band Noise Jammers and Other-User Interference

The model of the FH/SS multiple-access system is that of [7]. Since other-user interference is present, we cannot use (2) to compute the probability of error for the MFSK system in the absence of jamming, which is necessary for the computations in (1) and (3). Instead, we can use an upper bound on the total error probability using the technique of [7] and [8], or the approximation techniques developed in [9].

Using the approach found in [8] for FH/SSMA systems with a single jammer, we upper-bound the uncoded symbol error probability by

$$p_s^{(L)} \leq P_e^{(L)} \cdot (1-P_h)^{\bar{K}} + \frac{M-1}{M}\left[1-(1-P_h)^{\bar{K}}\right]. \tag{4}$$

In (4) $P_e^{(L)}$ is given by (1) for general $L > 1$ and by (3) for $L = 3$ in particular; P_h is the probability of being hit--i.e., interfered with by another user in the sense that both users use the same hopping frequency for part or all of the duration of the particular hop; this probability is given for memoryless random hopping patterns and asynchronous FH/SSMA systems (see [7]) by

$$P_h = \left[1 + \frac{1}{N_s}\left[1 - \frac{1}{q}\right]\right]\frac{1}{q}, \tag{5}$$

where N_s is the number of M-ary symbols per dwell-time (hop duration) and q has already been defined as the number of frequencies available for hopping. Clearly, N_s and the number of bits per hop N_b are related as $N_s = N_b / \log_2 M$. $\bar{K}$ is the total number of other users who share the FH channel together with the user under consideration; thus the total number of users transmitting simultaneously is $\bar{K} + 1$. Eq.(5) is valid when the interfering users are assumed to be asynchronous with respect to the user of interest. The corresponding expression for synchronous systems is $P_h = 1/q$, which is independent of the number of symbols per hop (N_s).

For the asynchronous case, when $N_s = 1$ (one M-ary symbol per hop) P_h takes on its maximum (worst-case) value of $2/q$; this asynchronous FH/SSMA system in the absence of other forms of interference can support almost half as many users as the corresponding synchronous FH/SSMA system. For $N_s > 1$, $1/q < P_h < 2/q$ and the asynchronous FH/SSMA system can support more users than in the asynchronous case with $N_s = 1$ but fewer than in the synchronous case.

Equation (4) provides a pessimistic upper bound on the performance of the FH/SSMA system. The recent results of [9] indicate that, if several of the relative power levels of the other interfering (not jamming) signals with respect to that of the desired signals are smaller than 1, then the hard bound of [8] that we used in (4) is one or two orders of magnitude away from the actual result. By contrast, tight upper bounds were derived in [9] for the symbol error probability of M-ary FSK FH/SSMA systems based on the union bound and the characteristic function method. These bounds appear to have satisfactory tightness (accuracy in the case of approximations) over a large range of the relative power levels of other-user interference as shown in [9]. For binary FSK FH/SSMA systems exact expressions for BER were also derived.

We can actually use a more general model for the FH/SS multiple-access system than that of [7] or [8]. Instead of having users with equal power levels as assumed there, we have n qroups of users altogether, with the $\overline{K}_i$ ones in the i-th group ($1 \le i \le n$) having relative power $\overline{P}_i = E_{s_i}/E_s$ with respect to the user under consideration (see [9]). This model allows for distinct groups of users with different received power levels at a particular receiver. This is a realistic model for situations in which the interfering users are distributed over a wide geographical area and/or their distance from the receiver of interest varies with time; some of these users may have power levels which are approximately the same, these users are grouped together in the above model. Therefore, this model allows for the investigation of the resistance (or immunity) of the FH/SS system to the near-far problem which may be present in terrestrial mobile radio networks or even in satellite networks.

The analysis in this report (as well as that of [9]) can handle the general case of unequal power levels, thus all the expressions involving other-user interference and cited below are valid for the general model of n groups of users with the i-th group having parameters $(\bar{K}_i, \bar{P}_i)$. However, in Section 5, where the numerical results are presented, the power levels of the interfering users were taken to be equal for the sake of simplicity in the presentation and for emphasizing the effect of the multiple jammers on the FH/SS system.

A tight upper bound on the symbol error probability can be obtained by using the results of [9] (for the M-ary case) to handle the multiple-access interference in the presence of multiple noise jammers. The union bound of [9] takes the form

$$\bar{P}_{e,M}(\bar{K}_1, \bar{K}_2, \ldots, \bar{K}_n; N_0) \le (1 - P_h)^{\sum_{i=1}^{n} \bar{K}_i} P_{e,M}(N_0) + (M-1) \cdot \hat{P}^{\,0}_{e,M}(\bar{K}_1, \bar{K}_2, \ldots, \bar{K}_n; N_0) \tag{6}$$

where $\hat{P}^{\,0}_{e,M}(\bar{K}_1, \bar{K}_2, \ldots, \bar{K}_n; N_0)$ denotes the probability of deciding in favor of any particular M-ary FSK symbol other than the one transmitted by the user under consideration, when there is at least one interfering user [i.e., the event $(K_1=0, K_2=0, \ldots, K_n=0)$ is excluded from the computation of the average]. The expression in (6) can be manipulated further to give

$$\bar{P}_{e,M}(\bar{K}_1, \bar{K}_2, \ldots, \bar{K}_n; N_0) \le (1 - P_h)^{\sum_{i=1}^{n} \bar{K}_i} \left[P_{e,M}(N_0) - \frac{M-1}{2} \exp\left[-\frac{E_s}{2N_0} \right] \right]$$

$$+ (M-1) \int_0^{\infty} \exp\left[-\frac{u^2}{2E_s/N_0} \right] J_0(u) \bar{\Phi}_I(u)\, du \tag{7}$$

where $P_{e,M}(\cdot)$ is given by (2) and the n-th order ($n = 0, 1, \cdots$) Bessel function J_n is

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x\sin\theta - n\theta)d\theta.$$

We first consider a pessimistic model in which all hits are assumed to be full hits, i.e., all hits result in interference that is present for the entire duration of the symbol. In this case, $\overline{\Phi}_I(u)$ is given by [9]:

$$\overline{\Phi}_I(u) = \prod_{i=1}^{n}\left[1-P_h+P_h\left[1-\frac{2}{M}+\frac{2}{M}J_0(\overline{a}_i u)\right]\right]^{\overline{K}_i}\left[\sum_{i=1}^{n}\frac{\overline{K}_i\overline{a}_i P_h J_1(\overline{a}_i u)}{M\left[1-P_h+P_h\left[1-\frac{2}{M}+\frac{2}{M}J_0(\overline{a}_i u)\right]\right]}+\frac{u}{2E_s/N_0}\right], \tag{8}$$

where $\overline{a}_i = \sqrt{\overline{P}_i} = \sqrt{E_{s_i}/E_s}$. A more accurate model can be developed by taking into account the effect of partial hits into the analysis. This model incorporates the fact that partial hits result in interference that is present for only a portion of the symbol duration. We can use the following expression of [9] for $\overline{\Phi}_I(u)$:

$$\overline{\Phi}_I(u) = \prod_{i=1}^{n} A(u;P_f,P_p,R_\psi,\hat{R}_\psi,M;\overline{a}_i)^{\overline{K}_i}$$

$$\left[\sum_{i=1}^{n}\overline{K}_i\overline{a}_i\frac{P_f\left[\frac{1}{M^2}J_1(\overline{a}_i u)+\frac{2}{M^2}E_\tau\left\{J_0\left[\overline{a}_i R_\psi(\tau)u\right]J_1\left[\overline{a}_i\hat{R}_\psi(\tau)u\right]\right\}\right]+P_p\frac{1}{M}E_\tau\left\{J_1\left[\overline{a}_i R_\psi(\tau)u\right]\right\}}{A(u;P_f,P_p,R_\psi,\hat{R}_\psi,M;\overline{a}_i)}\right.$$

$$\left.+\frac{u}{2E_s/N_0}\right] \tag{9}$$

where

$$A(u;P_f,P_p,R_\psi,\hat{R}_\psi,M;\overline{a}_i) = 1-P_h$$
$$+P_f\left[1-\frac{4}{M^2}+\frac{2}{M^2}J_0(\overline{a}_i u)+\frac{2}{M^2}E_\tau\left\{J_0\left[\overline{a}_i R_\psi(\tau)u\right]J_0\left[\overline{a}_i\hat{R}_\psi(\tau)u\right]\right\}\right]$$

$$+P_p\left[1-\frac{2}{M}+\frac{2}{M}E_\tau\left\{J_0\left[\overline{a}_i R_\psi(\tau)u\right]\right\}\right] \tag{10}$$

In (9) and (10), $E_\tau\{\cdot\}$ denotes expectation with respect to τ being uniformly distributed in the interval [0,1]--τ (the subscript k has been dropped) represents the normalized delay of the k-th user τ_k/T_s; the model of [7]-[9] assumes that the delays of the asynchronous interfering users τ_k are uniformly distributed in $[0,T_s]$. $R_\psi(\tau)$ and $\hat{R}_\psi(\tau)$ are the partial continuous autocorrelation functions of the shaping waveform $\psi(t)$ used, and $P_f=[1-(1-1/q)/N_s]/q$ and $P_p=2(1-1/q)/(N_s q)$ are the the probabilities of full hits and partial hits (see [10]), respectively, for random memoryless hopping patterns. Suitable choice of the shaping waveform $\psi(t)$ can reduce the other-user interference in SSMA systems; for example a sine waveform performs better than the rectangular waveform as established by Table II of [10] where such a comparison was carried out for hybrid frequency-hopped/direct-sequence SSMA systems.

In order to obtain the error probability in the presence of multiple noise jammers, we use $\overline{P}_{e,M}(\overline{K}_1,\overline{K}_2,\ldots,\overline{K}_n;\cdot)$ from either (7) and (8) or (7) and (9)-(10), whenever $P_{e,M}(\cdot)$ is called by equation (1) (for the case of L jammers) or by equation (3) (for the example of three jammers). The free variable $(\cdot)$ is the effective spectral density of the total noise, which, for example, takes the value N_1/ρ_1+N_0 when only the first jammer is present, and the value $N_1/\rho_1+N_2/\rho_2+N_0$ when two jammers are present. So modified, equations (1) and (3) provide the uncoded symbol error probability of the system under consideration.

3. EFFECT OF MULTIPLE PARTIAL-BAND TONE JAMMERS ON FH/SSMA SYSTEMS

3.1 Multiple Partial-Band Tone Jamming and AWGN

Our model for the scenario of multiple partial-band tone jammers is the following. As for the case of multiple partial-band noise jammers, there are L jammers each jamming a fraction ρ_i $(0 \le \rho_i \le 1,\ 1 \le i \le L)$ of the total bandwidth W (evaluated in Section 2.1). The effective energy per bit E_{J_i} of the i-th jammer remains constant over the duration of a symbol. The total power available to the i-th tone jammer is $\overline{P}_{J_i} = W(log_2 ME_{J_i})$. The see this notice that qM is the total number of distinct frequencies visited by the jammer, whereas the power the jammer places in each frequency is its power per symbol given by $(log_2 ME_{J_i})/(log_2 MT_b)$, where the numerator is the energy per symbol and the denominator is the symbol duration; since the data bit duration is given by $T_b = 1/R$ we obtain that the power per frequency is RE_{J_i}, and thus $qM \cdot RE_{j_i} = W(\log_2 ME_{J_i})$.

As in Section 2.1, each jammer randomly selects which $\rho_i q$ out of the total q sub-bands he jams. Thus, with probability ρ_i the i-th tone jammer is present in a particular sub-band and has energy per bit E_{J_i}/ρ_i, and with probability $1-\rho_i$ it is absent from that sub-band or effectively has energy per bit 0. When a jammer is present in a sub-band, it jams **only one of the** M tones of the MFSK system, it actually transmits an unmodulated carrier of the form $\sqrt{2P_{J_i}/\rho_i}\cos[2\pi(\nu_l+f_m)t+\phi_{l,m}]$, where $P_{J_i} = E_{j_i}/T_b$ denotes the jammer power per bit (or symbol), ν_l the hopping frequency during the l-th hop, f_m the m-th tone of the MFSK FH/SS system which is visited by the jammer, and $\phi_{l,m}$ the phase angle introduced by the jammer's local oscillator. All M tones have the same probability $\frac{1}{M}$ of being jammed. Therefore, it is possible that any number of jammers between 0 and L jam the same MFSK tone or that each jammer jams a different MFSK tone.

For this case it is very difficult, although not impossible, to obtain a general expression for the symbol error probability. We prefer to illustrate the situation for the cases $L = 2$ and $L = 3$ distinct jammers. We first present the *exact expression* for the symbol error probability of the MFSK FH/SS system when two tone jammers ($L = 2$) are present:

$$
\begin{aligned}
P_e^{(2)} = {} & \rho_1(1-\rho_2)\cdot\left[P_{e,J}^{(1)}\left[\frac{E_{J_1}}{\rho_1}\right]\frac{1}{M} + P_{e,J}^{(2)}\left[\frac{E_{J_1}}{\rho_1}\right]\frac{M-1}{M}\right] \\
& + \rho_2(1-\rho_1)\cdot\left[P_{e,J}^{(1)}\left[\frac{E_{J_2}}{\rho_2}\right]\frac{1}{M} + P_{e,J}^{(2)}\left[\frac{E_{J_2}}{\rho_2}\right]\frac{M-1}{M}\right] \\
& + \rho_1\rho_2\cdot\left[P_{e,J}^{(3)}\left[\frac{E_{J_1}}{\rho_1},\frac{E_{J_2}}{\rho_2}\right]\frac{M-1}{M^2} + P_{e,J}^{(3)}\left[\frac{E_{J_2}}{\rho_2},\frac{E_{J_1}}{\rho_1}\right]\frac{M-1}{M^2}\right. \\
& \quad + P_{e,J}^{(4)}\left[\frac{E_{J_1}}{\rho_1},\frac{E_{J_2}}{\rho_2}\right]\frac{M-1}{M^2} + P_{e,J}^{(5)}\left[\frac{E_{J_1}}{\rho_1},\frac{E_{J_2}}{\rho_2}\right]\frac{(M-1)(M-2)}{M^2} \\
& \quad \left. + P_{e,J}^{(6)}\left[\frac{E_{J_1}}{\rho_1},\frac{E_{J_2}}{\rho_2}\right]\frac{1}{M^2}\right] \\
& + (1-\rho_1)(1-\rho_2)\cdot P_{e,M}(N_0), \qquad (11)
\end{aligned}
$$

where $P_{e,J}^{(1)}\left[E_J\right]$ is the probability of error for an MFSK system when the jammer and the signal occupy the same frequency tone; $P_{e,J}^{(2)}\left[E_J\right]$ is the probability of error when the jammer and the signal occupy different frequency tones; $P_{e,J}^{(3)}\left[E_{J_1},E_{J_2}\right]$ is the probability of error when jammer 1 and the signal occupy the same frequency tone and jammer 2 occupies a different frequency tone; $P_{e,J}^{(4)}\left[E_{J_1},E_{J_2}\right]$ is the probability of error when jammers 1 and 2 occupy the same frequency tone and the signal occupies a different frequency tone; $P_{e,J}^{(5)}\left[E_{J_1},E_{J_2}\right]$ is the probability of error when jammer 1, jammer 2, and the signal occupy different frequency tones; and $P_{e,J}^{(6)}\left[E_{J_1},E_{J_2}\right]$ is the probability of error when jammer 1, jammer 2, and the signal occupy the same frequency tone.

The above probabilities can be all evaluated using the methods of [11]. For the first we have

$$
P_{e,J}^{(1)}\left[E_J\right] = 1 - \int_0^\infty \left[R\int_0^\infty uJ_0(Ru)\exp(-\frac{1}{2}u^2)J_0(\alpha_s u)J_0(\alpha_J u)du\right]\left[1-e^{-R^2/2}\right]^{M-1} dR
$$

(12)

where $\alpha_s = \sqrt{2E_s/N_0}$ and $\alpha_J = \sqrt{2E_J/N_0}$. In deriving this equation we used the fact that the outputs of $M-1$ branches of the MFSK demodulator have (normalized) pdfs $re^{-r^2/2}$ (and thus $\int_0^R re^{-r^2/2}dr = 1-e^{-R^2/2}$), whereas the output of the branch in which the signal and the jammer are both present has pdf

$$g_1(R) = R\int_0^\infty uJ_0(Ru)\Phi_1(u)du$$

where the characteristic function $\Phi_1(u)$ is given by

$$\Phi_1(u) = \exp(-\frac{1}{2}u^2)J_0(\alpha_s u)J_0(\alpha_J u)du.$$

Both formulas come from the theory of circularly symmetric distributions (see [11]).

For $P_{eJ}^{(2)}(E_J)$ we can write

$$P_{eJ}^{(2)}\left[E_J\right] = 1 - \int_0^\infty Re^{-\frac{1}{2}(R^2+\alpha_s^2)} I_0(R\alpha_s)\left[\int_0^R re^{-\frac{1}{2}(r^2+\alpha_J^2)} I_0(r\alpha_J)dr\right]\left[1-e^{-R^2/2}\right]^{M-2} dR \tag{13}$$

where α_s and α_J are as defined above. In deriving this equation we used the fact that the pdfs of the outputs of the branches, in which the signal and the jammer are present, are Rician r.v.s, whereas for the other $M-2$ branches they are Rayleigh r.v.s.

Similarly for $P_{eJ}^{(3)}(E_J)$ we can write

$$P_{eJ}^{(3)}\left[E_{J_1},E_{J_2}\right] = 1 - \int_0^\infty \left[R\int_0^\infty uJ_0(Ru)\exp(-\frac{1}{2}u^2)J_0(\alpha_s u)J_0(\alpha_{J_1}u)du\right]$$
$$\left[\int_0^R re^{-\frac{1}{2}(r^2+\alpha_{J_2}^2)} I_0(r\alpha_{J_2})dr\right]\left[1-e^{-R^2/2}\right]^{M-2} dr. \tag{14}$$

For $P_{eJ}^{(4)}\left[E_{J_1},E_{J_2}\right]$ we can write,

$$P_e^{(4)}\left[E_{J_1},E_{J_2}\right] = 1 - \int_0^{\infty} Re^{-\frac{1}{2}(R^2+\alpha_s^2)} I_0(R\alpha_s)\left[1-e^{-R^2/2}\right]^{M-2}$$

$$\left\{\int_0^R \left[r\int_0^{\infty} uJ_0(ru)\exp\left[-\frac{1}{2}u^2\right]J_0(\alpha_{J_1}u)J_0(\alpha_{J_2}u)du\right]dr\right\}dR. \tag{15}$$

For $P_e^{(5)}\left[E_{J_1},E_{J_2}\right]$ we can write,

$$P_e^{(5)}\left[E_{J_1},E_{J_2}\right] = 1 - \int_0^{\infty} Re^{-\frac{1}{2}(R^2+\alpha_s^2)} I_0(R\alpha_s)\left[1-e^{-R^2/2}\right]^{M-3}$$

$$\left[\int_0^R re^{-\frac{1}{2}(r^2+\alpha_{J_1}^2)} I_0(r\alpha_{J_1})dr\right]\left[\int_0^R re^{-\frac{1}{2}(r^2+\alpha_{J_2}^2)} I_0(r\alpha_{J_2})dr\right]dR \tag{16}$$

and, finally, for $P_e^{(6)}\left[E_{J_1},E_{J_2}\right]$ we can write

$$P_e^{(6)}\left[E_{J_1},E_{J_2}\right] = 1- \int_0^{\infty} \left[R\int_0^{\infty} uJ_0(Ru)\exp\left[-\frac{1}{2}u^2\right]J_0(\alpha_s u)J_0(\alpha_{J_1}u)J_0(\alpha_{J_2}u)du\right]$$

$$\left[1-e^{-R^2/2}\right]^{M-1} dR \tag{17}$$

An expression similar to (11) can be obtained for three tone jammers. However, it involves terms $P_e^{(i)}$ for $i = 1,2,...,13$, the first six of which are cited in (12)-(17) and the other seven involve even more complicated integrations. Although the expression for the symbol error probability of the MFSK FH/SS system in (11) is exact, it has two disadvantages: First, it involves multiple integrals, some of which are computed from 0 to ∞ and are thus very demanding computationally; this computational complexity is exacerbated by the need to call $P_e^{(2)}$ and $P_e^{(3)}$ several times by the optimization subroutines, when the optimal jamming fractions are being searched for. Second, it cannot be easily extended to the case of combined multiple tone jammer and other-user interference.

A *tight upper bound* on the symbol error probability can be obtained by applying the results of [9] (for the M-ary case) to the case of multiple tone jammers. This bound has computational complexity which is linear in the number of jammers L and can be used for the case of combined mutiple-tone jammer and other-user interference. The union bound of [9] is now modified to give

$$\bar{P}_{e,M}^{(L)}\left[E_{J_1},E_{J_2},\ldots,E_{J_L}\right] \leq (M-1)\cdot\hat{P}_{e,M}^{(L)}\left[E_{J_1},E_{J_2},\ldots,E_{J_L}\right] \tag{18}$$

where $\hat{P}_{e,M}^{(L)}\left[E_{J_1},E_{J_2},\ldots,E_{J_L}\right]$ denotes the probability of deciding in favor of any particular M-ary FSK symbol other than the one transmitted by the user under consideration; for the case of multiple-tone jammers, which is equivalent to other-user interferers causing only full hits, it is given by

$$\hat{P}_{e,M}^{(L)}\left[E_{J_1},E_{J_2},\ldots,E_{J_L}\right] = \int_0^{\infty}\exp\left[-\frac{u^2}{2E_s/N_0}\right]J_0(u)\bar{\Phi}_J(u)du \tag{19}$$

where

$$\bar{\Phi}_J(u) = \prod_{l=1}^{L}\left[1-\rho_l+\rho_l\left[1-\frac{2}{M}+\frac{2}{M}J_0(\bar{a}_{J_l}u/\sqrt{\rho_l})\right]\right]$$
$$\cdot\left[\sum_{l=1}^{L}\frac{\bar{a}_l\sqrt{\rho_l}J_1(\bar{a}_{J_l}u/\sqrt{\rho_l})}{M\left[1-\rho_l+\rho_l\left[1-\frac{2}{M}+\frac{2}{M}J_0(\bar{a}_{J_l}u/\sqrt{\rho_l})\right]\right]}+\frac{u}{2E_s/N_0}\right]. \tag{20}$$

and $\bar{a}_{J_l} = \sqrt{\bar{P}_{J_l}} = \sqrt{E_{J_l}/E_s}$.

3.2 Multiple Partial-Band Tone Jammers and Other-User Interference

A tight upper bound on the symbol error probability can be obtained by extending the results of [9] (for the M-ary case) to the case of simultaneous multiple-access interference and multiple tone jammers. The union bound of [9] is now extended to

$$\overline{P}_{e,M}^{(L)}\left[\overline{K}_1,\overline{K}_2,\ldots,\overline{K}_n;E_{J_1},E_{J_2},\ldots,E_{J_L}\right] \le (M-1)\cdot\hat{P}_{e,M}^{(L)}\left[\overline{K}_1,\overline{K}_2,\ldots,\overline{K}_n;E_{J_1},E_{J_2},\ldots,E_{J_L}\right] \tag{21}$$

where $\hat{P}_{e,M}^{(L)}\left[\overline{K}_1,\overline{K}_2,\ldots,\overline{K}_n;E_{J_1},E_{J_2},\ldots,E_{J_L}\right]$ denotes the probability of deciding in favor of a any particular M-ary FSK symbol other than the one transmitted by the user under consideration; for the case of other-user interference causing only full hits it is given by

$$\hat{P}_{e,M}^{(L)}\left[\overline{K}_1,\overline{K}_2,\ldots,\overline{K}_n;E_{J_1},E_{J_2},\ldots,E_{J_L}\right] = \int_0^\infty \exp\left[-\frac{u^2}{2E_s/N_0}\right] J_0(u)\overline{\Phi}_{I,J}(u)du \tag{22}$$

where

$$\overline{\Phi}_{I,J}(u) = \prod_{i=1}^{n}\left[1-P_h+P_h\left[1-\frac{2}{M}+\frac{2}{M}J_0(\overline{a}_i u)\right]\right]^{\overline{K}_i}\prod_{l=1}^{L}\left[1-\rho_l+\rho_l\left[1-\frac{2}{M}+\frac{2}{M}J_0(\overline{a}_{J_l}u/\sqrt{\rho_l})\right]\right]$$

$$\cdot\left[\sum_{i=1}^{n}\frac{\overline{K}_i\overline{a}_i P_h J_1(\overline{a}_i u)}{M\left[1-P_h+P_h\left[1-\frac{2}{M}+\frac{2}{M}J_0(\overline{a}_i u)\right]\right]}\right.$$

$$\left.+\sum_{l=1}^{L}\frac{\overline{a}_l\sqrt{\rho_l}J_1(\overline{a}_{J_l}u/\sqrt{\rho_l})}{M\left[1-\rho_l+\rho_l\left[1-\frac{2}{M}+\frac{2}{M}J_0(\overline{a}_{J_l}u/\sqrt{\rho_l})\right]\right]}+\frac{u}{2E_s/N_0}\right]. \tag{23}$$

$\overline{a}_i$, $i = 1, 2,\ldots, n$, was defined after (8), and $\overline{a}_{J_l}$, $l = 1, 2,\ldots, L$, was defined after eq. (20). If, instead of considering only the full hits, we take into account both the full and partial hits that the other-user interference causes, we can obtain

$$\overline{\Phi}_I(u) = \prod_{i=1}^{n}A(u;P_f,P_p,R_\psi,\hat{R}_\psi,M;\overline{a}_i)^{\overline{K}_i}\prod_{l=1}^{L}\left[1-\rho_l+\rho_l\left[1-\frac{2}{M}+\frac{2}{M}J_0(\overline{a}_{J_l}u/\sqrt{\rho_l})\right]\right]\cdot$$

$$\left[\sum_{i=1}^{n}\overline{K}_i\overline{a}_i\frac{P_f\left[\frac{1}{M^2}J_1(\overline{a}_i u)+\frac{2}{M^2}E_\tau\left\{J_0\left[\overline{a}_i R_\psi(\tau)u\right]J_1\left[\overline{a}_i\hat{R}_\psi(\tau)u\right]\right\}\right]+P_p\frac{1}{M}E_\tau\left\{J_1\left[\overline{a}_i R_\psi(\tau)u\right]\right\}}{A(u;P_f,P_p,R_\psi,\hat{R}_\psi,M;\overline{a}_i)}\right.$$

$$+\sum_{l=1}^{L}\frac{\bar{a}_{J_l}\sqrt{\rho_l}J_1(\bar{a}_{J_l}u/\sqrt{\rho_l})}{M\left[1-\rho_l+\rho_l\left[1-\frac{2}{M}+\frac{2}{M}J_0(\bar{a}_{J_l}u/\sqrt{\rho_l})\right]\right]}+\frac{u}{2E_s/N_0}\Bigg] \tag{24}$$

where $A(u;P_f,P_p,R_\psi,\hat{R}_\psi,M;\bar{a}_i)$ was defined in (10).

4. ERROR-CONTROL CODING CONSIDERATIONS

We consider coded MFSK FH/SSMA systems that employ Reed-Solomon (n,k) codes. One M-ary symbol is put on each RS code symbol ($n = M$) and this is transmitted during one dwell-time (hop) (i.e., $N_s = 1$). Three decoding schemes are discussed here. The simplest decoding algorithm of interest is that of **errors-only decoding**. For this scheme, the probability of a symbol error (or symbol error rate: SER) of the coded system is an increasing function of the SER of the uncoded system. However, the fractions of the band jammed that maximize the SER may or may not be the same as those of the uncoded case, depending on what the relationship between the data rate, the coding rate, and the total bandwidth is. Recall that the total bandwidth of the uncoded FH/SS system W was computed in section 2.1 as $W = qMR/log_2M$, where R is the data rate of the uncoded system. Similarly, the total bandwidth of the coded system is $W' = qMR'/(r\log_2M)$, where $r = k/n < 1$ is the code rate and R' the data rate of the coded system.

We consider two distinct coded situations, in both of which q and M are the same as in the uncoded case. In the first we assume that the total system bandwidth is fixed, that is, $W' = W$ for the coded and uncoded systems, and, consequently, $R' = Rr < R$, which implies that a lower data rate should be used in the coded case. Thus, the number of channel symbols per sec remains the same ($R'/(r\log_2M) = R/log_2M$) for the coded and uncoded systems, and, since the total energy of the FH/SS signal of interest is fixed, so does the energy per channel symbol. Since the total jammer power is kept fixed and the bandwidth is the same for the coded and uncoded systems, the spectral densities of the noise jammers and the energies per symbol of the tone jammers of the coded systems remain the same as those of the uncoded system. This implies that the same ratios E_s/N_0, E_s/N_{J_i}, and E_s/E_{J_i}, for $i = 1, 2, ..., L$, are involved in the expressions about the SERs of the coded and uncoded systems; for the uncoded systems they represent symbol signal-to-jammer energy ratios, for the coded systems they represent channel symbol signal-to-jammer energy ratios. Since the SER of the coded system is an increasing function of the SER of the uncoded system, the same combination of ρ_is achieves the worst-case performance for the uncoded and coded systems. Of course, the value of the coded SER is much smaller than that of the uncoded SER.

In the second case, the data rate of the FH/SS system remains fixed, that is $R' = R$. Since the number of channel symbols per sec increases to $R'/(r\log_2 M) > R/\log_2 M$ (because $1/r$ code symbols are transmitted through the channel for each information symbol), the bandwidth of each frequency sub-band also increases by a factor of $1/r$ (because orthogonal tone spacing is maintained). The hopping bandwidth is therefore increased to $W' = W/r$ (assuming q is the same in the coded and uncoded systems). Since the spectral density of the AWGN is fixed, AWGN of larger power is now affecting the system performance and thus the channel symbol signal-to-noise (AWGN) ratio becomes $(r\log_2 M)E_b/N_0$ where rE_b is the energy per channel bit for the coded system. By contrast, for both the noise and tone jammers, since the bandwidth increases to $W' = W/r > W$ but the total power of these jammers is kept fixed, the new spectral densities and energies per channel bit will be $N'_{J_i} = rN_{J_i}$ and $E'_{J_i} = rE_{J_i}$, respectively; to see this we express the jammer power as $\overline{P}_{J_i} = W \cdot N_{J_i} = W' \cdot N'_{J_i}$ for the noise jammers (see Section 2.1) and as $\overline{P}_{J_i} = W \cdot (log_2 M)E_{J_i} = W' \cdot (log_2 M)E'_{J_i}$ for the tone jammers (see Section 3.1). Therefore, the channel symbol signal-to-jammer ratios for both the noise jammers and the tone jammers remain the same as in the uncoded case (i.e., independent of r). Consequently, the expressions for the uncoded symbol error probability provided by

(7)-(10) and (21)-(24) now depend on the code rate r through the signal-to-noise (AWGN). This implies that the combination of ρ_is that achieve the worst-case performance for the coded system is different from that of the uncoded system.

The second scheme for a decoding strategy is **erasures/errors decoding** (refer to [8] for the case of combined single-jammer and multiple-access interference). This algorithm erases symbols with detected errors and attempts to correct symbols with undetected errors. However, if broadband jamming ($\rho_i = 1$ for all jammers) was allowable, it would result in the erasure of all symbols, thus causing failure of the erasures-decoding part of the algorithm. This decoding algorithm is not sufficiently robust and is not analyzed here.

The third scheme is the **parallel erasures/errors decoding** algorithm introduced in [3] (see also [8] for the case of combined single-jammer and multiple-access interference). According to this algorithm, the decoder executes erasures-only decoding, as long as the number of erasures is smaller than $n-k$, and errors-only decoding, when the number of erasures exceeds $n-k$. This algorithm is especially suited for our problem and should be preferred over all others in a realistic implementation. Unfortunately, the optimization of the fractions of the band jammed by the multiple jammers is extremely demanding computationally for this decoding scheme due to the extra complexity in the evaluation of the SER of the coded system from that of the uncoded system. Therefore, we did not generate any numerical results for this case, although we have developed, together with the work in [8], all necessary equations for analyzing this algorithm. For a detailed description of the expressions involved here refer to equations (8)-(11) in Section III.A of [8]. These equations need to be modified to reflect the fact that only jammers cause erasures. This is because, in this report, errors caused by multiple-access interference (MAI) are quantified more accurately than in [8] and are elaborated in Section 2.2. Consequently, we need not treat errors due to MAI as erasures.

5. NUMERICAL RESULTS

We first present results for two partial-band noise or tone jammers. In Tables 1 and 2, E_b/N_0 denotes the bit signal-to-noise ratio, where $E_b = E_s/log_2 m$; E_b/N_{J_l} and E_b/E_{J_l} the bit signal-to-jammer ratios for the noise and tone jammers, respectively, for the l-th jammer ($l = 1, 2$); ρ_l the worst-case (optimal for the jammer) fraction of the band jammed; $P_e^{(2)}$ the symbol error probability of the uncoded MFSK FH/SSMA system when two jammers are present; E_b/N_J and E_b/E_J the equivalent bit signal-to-jammer ratio of a single noise or tone jammer with power equal to the sum of the powers of the two noise or tone jammers, that is,

$$\frac{E_b}{N_J} = \frac{E_b}{N_{J_1}+N_{J_2}} = \left[\left[\frac{E_b}{N_{J_1}}\right]^{-1} + \left[\frac{E_b}{N_{J_2}}\right]^{-1}\right]^{-1}$$

and

$$\frac{E_b}{E_J} = \frac{E_b}{E_{J_1}+E_{J_2}} = \left[\left[\frac{E_b}{E_{J_1}}\right]^{-1} + \left[\frac{E_b}{E_{J_2}}\right]^{-1}\right]^{-1}; \qquad (25)$$

ρ is the worst-case fraction of the band jammed by the single jammer; $P_e^{(1)}$ the symbol error probability of the uncoded MFSK FH/SSMA system, when only the single jammer is present; and K the total number of users in the system (including the desired signal), under the assumption that all users have equal power levels. As discussed in Section 2.2, this assumption was made for the sake of simplicity in the presentation of numerical results and is not required by the analysis.

For all numerical results presented in this section, the symbol error probabilitiies (SERs) are computed from (1) and (7) - (8) for noise jammers and (21) - (23) for tone jammers, as these expressions represent our most accurate analytical results. Moreover, all results in this section pertain to a 32-ary FSK FH/SSMA system. The number of frequencies available for frequency-hopping is q = 100 for Subtables b and c and q = 1000 for Subtables d and e of

Tables 1 and 2. For Subtables 1a and 2a, the value of q is immaterial, since both the choice of the worst-case fractions of the band jammed and the performance of the FH/SS system in the absence of other-user interference and in the presence of partial-band noise or tone jammers are independent of q. This is true because the signal-to-jammer ratio is given in terms of the bit energy to noise spectral density ratio E_b/N_{J_i}. For a given E_b/N_{J_i}, the total power of the jammer is proportional to q. The frequency-hopping patterns employed are modeled as random memoryless patterns [7] for all performance results presented. In all cases, the number of symbols per hop N_s is 1 (slow hopping).

Tables 1 and 2 show that two partial-band noise or tone jammers result in an SER slightly lower than the one caused by a single noise or tone jammer with power (spectral density) equal to the sum of powers (spectral densities) of the two jammers. However, for the two-jammer case, a value of the SER almost identical to that of the single-jammer case is achieved by smaller fractions of the band jammed. These observations on the relative performance of single and multiple jammers, which are among the most important conclusions of this study, could not have been predicted intuitively before the generation of these numerical results. Notice that equal signal-to-jammer power ratios imply that the worst-case fractions of the band jammed by the two noise or tone jammers are equal, as one would intuitively expect.

For small values of the signal-to-jammer power ratios, tone jammers result in a larger SER than noise jammers of the same power; this is reversed for moderate to large values of the signal-to-jammer power ratios. This is justified by the fact that the tone jammers considered here are of a particular type, according to which a jammer is not allowed to jam more than one of the MFSK tones simultaneously. By contrast, multi-tone (comb) jammers, which are not considered here, outperform noise jammers for all values of signal-to-jammer power ratios.

From our exhaustive search for the worst-case fractions of the band corresponding to the cases presented in Tables 1 and 2, we found out that, for each non-symmetric or completely symmetric allocation of the jammers' energies-per-bit or spectral densities, the worst-case combination of fractions of the band is unique. For partially symmetric allocations, the worst-case combination is unique within permutations of components, that is, three jammers with vector of signal-to-jammer power ratios (5,5,10) (in dB) have the same performance as the three jammers with vectors (5,10,5) or (10,5,5).

Finally, the performance degrades gracefully as the number of users increases from $K = 1$ to 6 and then to 11. As expected, $q = 1000$ provides a better performance than $q = 100$. Notice that, as the level of other-user interference K increases, the worst-case fractions of the band jammed decrease.

Tables 3 and 4 present results for three simultaneous noise and tone jammers, respectively. In these tables, $P_e^{(3)}$ denotes the SER for the three-jammer configuration and $P_e^{(1)}$ denotes the SER for the equivalent single-jammer configuration. The equivalent E_b/N_J or E_b/E_J of the single jammer are given by expressions similar to (25) of the two-jammer case; three terms ($l = 1, 2, 3$) instead of two are now involved in the sums of the right hand side of each of the two equations in (25). Tables 3 and 4 reveal similar trends as those of Tables 1 and 2. Also notice that three jammers (noise or tone) result in a larger SER than two jammers.

Tables 5 to 8 present results for the SER of Reed-Solomon coded 32-ary FSK FH/SSMA systems operating in the presence of two or three simultaneous noise or tone jammers. It is assumed that the data rate is fixed (this corresponds to the second case described in Section 4). The RS code used in generating the numerical results is a (32,16) code with errors-only decoding. Each 32-ary symbol is placed on one RS symbol; the number of symbols per hop N_s is 1. The number of frequencies used for hopping is $q = 100$. In these tables, we considered higher signal-to-AWGN and signal-to-jammer power ratios than in Tables 1 to 4. This is necessary in order to guarantee that the SER for the uncoded systems is smaller than .5, so that error-control coding is effective (higher SERs can be tolerated if lower code rates are used). Notice that for the range of lower signal-to-jammer power ratios resulting from the multiplication of the original SNRs by r as discussed in the third paragraph of Section 4, the SER of uncoded systems in Tables 1 to 4 is larger than .5. Besides the SERs $P_e^{(2)}$, $P_e^{(3)}$, and $P_e^{(1)}$, the corresponding codeword error probabilities ($P_E^{(2)}$ and $P_E^{(3)}$) of the coded systems (or, equivalently, the packet error probabilities, if one codeword per packet is transmitted) are provided. However, we note that the worst-case jamming fractions shown in Tables 5 - 8 have been chosen to maximize SER, and that these fractions are not necessarily worst case in terms of maximizing codeword error probability. All trends observed in the previous tables (1 to 4) for the uncoded systems are also observed here. Since in these tables we consider higher signal-to-jammer power ratios than in Tables 1 - 4, the tone jammers always result in a smaller uncoded SER than that of the noise jammers and this gets amplified for the coded SER as observed by comparing Tables 5 and 6, as well as Tables.7 and 8.

6. CONCLUSIONS

In this report, we characterize and evaluate the effect of simultaneous multiple partial-band noise or tone jammers and other-user interference on a single communication link employing frequency-hopped spread-spectrum (FH/SS) signaling, M-ary FSK modulation with noncoherent demodulation, and Reed-Solomon coding. We develop techniques for the evaluation of the symbol error probability (SER) of these systems; these include exact expressions and tight upper bounds for SER, when multiple partial-band noise or tone jammers but no other-user interference are present, and tight upper bounds on SER when both multiple noise or tone jammers and other-user interference are present. Our analytical results are valid for an arbitrary number of simultaneous noise or tone jammers and an arbitrary number of interfering users with different power levels.

Our numerical study of the cases of two and three multiple noise or tone jammers and of a varying number of interfering users established the following facts about uncoded and Reed-Solomon coded FH/SSMA systems in multiple partial-band noise or tone jamming:

(1) Two or three partial-band noise or tone jammers result in a symbol error probability slightly lower than the one caused by a single noise or tone jammer with power (or spectral density) equal to the sum of powers (spectral densities) of the two or three jammers; however, an almost identical value of the SER is achieved by smaller fractions of the band jammed for the two-jammer or three-jammer case than for the single-jammer case.

(2) For symmetric allocations of the jammers' energies per bit or spectral densities (i.e., when these jammer-to-signal energies are the same for all jammers), the resulting worst-case fractions of the band jammed by each jammer are equal for all cases considered in this report.

(3) For each non-symmetric or completely symmetric allocation of the jammers' energies-per-bit or spectral densities, the worst-case combination of fractions of the band is unique. For partially symmetric power allocations, the worst-case combination is unique within permutations of the components of the vector of the power ratios.

(4) For small values of the signal-to-jammer power ratios, tone jammers result in a larger SER than noise jammers of the same power; this is reversed for moderate to large values of the signal-to-jammer power ratios and is further amplified for coded systems.

(5) The performance degrades gracefully as the number of interfering users increases.

(6) As the level of other-user interference increases, the worst-case fractions of the band jammed decrease.

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Uncoded 32-ary FSK FH/SSMA Systems in Multiple Partial-Band Noise Jamming

Worst-Case Error Probability and Optimal Jamming Fractions of Band
for Two Simultaneous Partial-Band Noise Jammers
(All Signal-to-Noise and Signal-to-Jammer Ratios are in dB)

Table 1a

No Other-User Interference ($K = 1$)

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	ρ_1	ρ_2	$P_e^{(2)}$	E_b/N_J	ρ	$P_e^{(1)}$
10.	0.	0.	1.	1.	.822186	-3.01	1.	.822186
10.	0.	10.	1.	1.	.689945	-.41	1.	.689945
10.	5.	5.	1.	1.	.498023	1.99	1.	.498023
10.	5.	10.	1.	1.	.327839	3.81	.815	.335213
10.	10.	10.	.210	.210	.155455	6.99	.392	.161099

Table 1b

$K = 6$ Simultaneous Transmissions and $q = 100$

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	ρ_1	ρ_2	$P_e^{(2)}$	E_b/N_J	ρ	$P_e^{(1)}$
10.	0.	0.	1.	1.	.881234	-3.01	1.	.894512
10.	0.	10.	1.	1.	.854392	-.41	1.	.854392
10.	5.	5.	1.	1.	.551427	1.99	1.	.551427
10.	5.	10.	.548	.208	.372706	3.81	.667	.380374
10.	10.	10.	.169	.169	.206632	6.99	.320	.209295

Table 1c

$K = 11$ Simultaneous Transmissions and $q = 100$

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	ρ_1	ρ_2	$P_e^{(2)}$	E_b/N_J	ρ	$P_e^{(1)}$
10.	0.	0.	1.	1.	.931862	-3.01	1.	.933218
10.	0.	10.	1.	1.	.880732	-.41	1.	.886317
10.	5.	5.	1.	1.	.609893	1.99	.636	.613816
10.	5.	10.	.340	.125	.441143	3.81	.419	.445377
10.	10.	10.	.106	.106	.265479	6.99	.201	.266938

Table 1d

$K = 6$ Simultaneous Transmissions and $q = 1000$

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	ρ_1	ρ_2	$P_e^{(2)}$	E_b/N_J	ρ	$P_e^{(1)}$
10.	0.	0.	1.	1.	.862827	-3.01	1.	.862827
10.	0.	10.	1.	1.	.706137	-.41	1.	.706137
10.	5.	5.	1.	1.	.502952	1.99	1.	.502952
10.	5.	10.	1.	1.	.330853	3.81	.804	.338989
10.	10.	10.	.198	.198	.160124	6.99	.386	.165537

Table 1e

$K = 11$ Simultaneous Transmissions and $q = 1000$

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	ρ_1	ρ_2	$P_e^{(2)}$	E_b/N_J	ρ	$P_e^{(1)}$
10.	0.	0.	1.	1.	.904437	-3.01	1.	.904437
10.	0.	10.	1.	1.	.722979	-.41	1.	.722979
10.	5.	5.	1.	1.	.508349	1.99	1.	.508349
10.	5.	10.	1.	1.	.334217	3.81	.792	.343178
10.	10.	10.	.198	.198	.165182	6.99	.381	.170206

Uncoded 32-ary FSK FH/SSMA Systems in Multiple Partial-Band Tone Jamming

Worst-Case Error Probability and Optimal Jamming Fractions of Band
for Two Simultaneous Partial-Band Tone Jammers
(All Signal-to-Noise and Signal-to-Jammer Ratios are in dB)

Table 2a

No Other-User Interference ($K = 1$)

E_b/N_0	E_b/E_{J_1}	E_b/E_{J_2}	ρ_1	ρ_2	$P_e^{(2)}$	E_b/E_J	ρ	$P_e^{(1)}$
10.	0.	0.	.757	.757	.922783	-3.01	1.	.928334
10.	0.	10.	.758	.076	.697365	-.41	.834	.700918
10.	5.	5.	.240	.240	.399421	1.99	.480	.402972
10.	5.	10.	.240	.076	.264055	3.81	.316	.265179
10.	10.	10.	.076	.076	.127030	6.99	.152	.127386

Table 2b

$K = 6$ Simultaneous Transmissions and $q = 100$

E_b/N_0	E_b/E_{J_1}	E_b/E_{J_2}	ρ_1	ρ_2	$P_e^{(2)}$	E_b/E_J	ρ	$P_e^{(1)}$
10.	0.	0.	.757	.757	.939562	-3.01	1.	.945371
10.	0.	10.	.758	.076	.744939	-.41	.834	.748471
10.	5.	5.	.240	.240	.448491	1.99	.480	.452021
10.	5.	10.	.240	.076	.313803	3.81	.316	.314920
10.	10.	10.	.076	.076	.177466	6.99	.152	.177819

Table 2c

$K = 11$ Simultaneous Transmissions and $q = 100$

E_b/N_0	E_b/E_{J_1}	E_b/E_{J_2}	ρ_1	ρ_2	$P_e^{(2)}$	E_b/E_J	ρ	$P_e^{(1)}$
10.	0.	0.	.757	.757	.956161	-3.01	1.	.963251
10.	0.	10.	.759	.076	.792299	-.41	.835	.795810
10.	5.	5.	.240	.240	.497339	1.99	.480	.500849
10.	5.	10.	.240	.076	.363325	3.81	.316	.364436
10.	10.	10.	.076	.076	.227671	6.99	.152	.228022

Table 2d

$K = 6$ Simultaneous Transmissions and $q = 1000$

E_b/N_0	E_b/E_{J_1}	E_b/E_{J_2}	ρ_1	ρ_2	$P_e^{(2)}$	E_b/E_J	ρ	$P_e^{(1)}$
10.	0.	0.	.757	.757	.877182	-3.01	1.	.891772
10.	0.	10.	.759	.100	.689924	-.41	.834	.705681
10.	5.	5.	.240	.240	.404335	1.99	.480	.407885
10.	5.	10.	.240	.100	.256597	3.81	.316	.270113
10.	10.	10.	.076	.076	.132082	6.99	.152	.132438

Table 2e

$K = 11$ Simultaneous Transmissions and $q = 1000$

E_b/N_0	E_b/E_{J_1}	E_b/E_{J_2}	ρ_1	ρ_2	$P_e^{(2)}$	E_b/E_J	ρ	$P_e^{(1)}$
10.	0.	0.	.757	.757	.926897	-3.01	1.	.945432
10.	0.	10.	.759	.100	.694695	-.41	.834	.710441
10.	5.	5.	.184	.184	.411299	1.99	.376	.412453
10.	5.	10.	.240	.100	.261586	3.81	.316	.275141
10.	10.	10.	.076	.076	.137132	6.99	.152	.137487

Uncoded 32-ary FSK FH/SSMA Systems in Multiple Partial-Band Noise Jamming

Worst-Case Error Probability and Optimal Jamming Fractions of Band
for Three Simultaneous Partial-Band Noise Jammers
(All Signal-to-Noise and Signal-to-Jammer Ratios are in dB)

Table 3a

No Other-User Interference ($K = 1$)

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	E_b/N_{J_3}	ρ_1	ρ_2	ρ_3	$P_e^{(3)}$	E_b/N_J	ρ	$P_e^{(1)}$
10.	0.	0.	0.	1.	1.	1.	.875196	-4.77	1.	.875197
10.	0.	0.	10.	1.	1.	.900	.829834	-3.22	1.	.829875
10.	5.	5.	5.	1.	1.	1.	.644932	.23	1.	.644932
10.	5.	5.	10.	1.	1.	1.	.554691	1.35	1.	.554691
10.	10.	10.	10.	.233	.233	.233	.225581	5.23	.588	.241622

Table 3b

$K = 6$ Simultaneous Transmissions and $q = 100$

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	E_b/N_{J_3}	ρ_1	ρ_2	ρ_3	$P_e^{(3)}$	E_b/N_J	ρ	$P_e^{(1)}$
10.	0.	0.	0.	1.	1.	1.	.9288816	-4.77	1.	.928931
10.	0.	0.	10.	1.	1.	1.	.851542	-3.22	1.	.851612
10.	5.	5.	5.	1.	1.	1.	.728838	.23	1.	.728838
10.	5.	5.	10.	1.	1.	1.	.586046	1.35	1.	.586046
10.	10.	10.	10.	.182	.182	.182	.237246	5.23	.483	.245077

Table 3c

$K = 11$ Simultaneous Transmissions and $q = 100$

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	E_b/N_{J_3}	ρ_1	ρ_2	ρ_3	$P_e^{(3)}$	E_b/N_J	ρ	$P_e^{(1)}$
10.	0.	0.	0.	1.	1.	1.	.940719	-4.77	1.	.940823
10.	0.	0.	10.	1.	1.	1.	.893176	-3.22	1.	.893248
10.	5.	5.	5.	1.	1.	1.	.806069	.23	.967	.806230
10.	5.	5.	10.	1.	1.	1.	.615831	1.35	.746	.624938
10.	10.	10.	10.	.115	.115	.115	.258097	5.23	.306	.262352

Table 3d

$K = 6$ Simultaneous Transmissions and $q = 1000$

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	E_b/N_{J_3}	ρ_1	ρ_2	ρ_3	$P_e^{(3)}$	E_b/N_J	ρ	$P_e^{(1)}$
10.	0.	0.	0.	1.	1.	1.	.9013787	-4.77	1.	.901378
10.	0.	0.	10.	1.	1.	1.	.829178	-3.22	1.	.831542
10.	5.	5.	5.	1.	1.	1.	.652993	.23	1.	.652993
10.	5.	5.	10.	1.	1.	1.	.557423	1.35	1.	.557423
10.	10.	10.	10.	.228	.228	.228	.226273	5.23	.580	.241428

Table 3e

$K = 11$ Simultaneous Transmissions and $q = 1000$

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	E_b/N_{J_3}	ρ_1	ρ_2	ρ_3	$P_e^{(3)}$	E_b/N_J	ρ	$P_e^{(1)}$
10.	0.	0.	0.	1.	1.	1.	.917285	-4.77	1.	.917302
10.	0.	0.	10.	1.	1.	1.	.836127	-3.22	1.	.837229
10.	5.	5.	5.	1.	1.	1.	.661653	.23	1.	.661653
10.	5.	5.	10.	1.	1.	1.	.569122	1.35	.872	.570256
10.	10.	10.	10.	.223	.223	.223	.227268	5.23	.571	.241547

Uncoded 32-ary FSK FH/SSMA Systems in Multiple Partial-Band Tone Jamming

Worst-Case Error Probability and Optimal Jamming Fractions of Band
for Three Simultaneous Partial-Band Tone Jammers
(All Signal-to-Noise and Signal-to-Jammer Ratios are in dB)

Table 4a

No Other-User Interference ($K = 1$)

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	E_b/N_{J_3}	ρ_1	ρ_2	ρ_3	$P_e^{(3)}$	E_b/N_J	ρ	$P_e^{(1)}$
10.	0.	0.	0.	.755	.755	.755	.921735	-4.77	1.	.921892
10.	0.	0.	10.	.757	.757	.100	.830357	-3.22	1.	.830425
10.	5.	5.	5.	.239	.239	.239	.593903	.23	.720	.604490
10.	5.	5.	10.	.240	.240	.100	.448579	1.35	.556	.466697
10.	10.	10.	10.	.076	.076	.076	.190047	5.23	.228	.191111

Table 4b

$K = 6$ Simultaneous Transmissions and $q = 100$

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	E_b/N_{J_3}	ρ_1	ρ_2	ρ_3	$P_e^{(3)}$	E_b/N_J	ρ	$P_e^{(1)}$
10.	0.	0.	0.	.755	.755	.755	.934158	-4.77	1.	.934169
10.	0.	0.	10.	.757	.757	.100	.850813	-3.22	1.	.850911
10.	5.	5.	5.	.240	.240	.240	.597114	.23	.720	.607643
10.	5.	5.	10.	.240	.240	.100	.452380	1.35	.556	.470375
10.	10.	10.	10.	.076	.076	.076	.194779	5.23	.228	.195837

Table 4c

$K = 11$ Simultaneous Transmissions and $q = 100$

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	E_b/N_{J_3}	ρ_1	ρ_2	ρ_3	$P_e^{(3)}$	E_b/N_J	ρ	$P_e^{(1)}$
10.	0.	0.	0.	.755	.755	.755	.962143	-4.77	1.	.962185
10.	0.	0.	10.	.757	.757	.100	.895132	-3.22	1.	.895177
10.	5.	5.	5.	.240	.240	.240	.600275	.23	.720	.610747
10.	5.	5.	10.	.240	.240	.100	.456127	1.35	.556	.474000
10.	10.	10.	10.	.076	.076	.076	.199452	5.23	.228	.200505

Table 4d

$K = 6$ Simultaneous Transmissions and $q = 1000$

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	E_b/N_{J_3}	ρ_1	ρ_2	ρ_3	$P_e^{(3)}$	E_b/N_J	ρ	$P_e^{(1)}$
10.	0.	0.	0.	.755	.755	.755	.914287	-4.77	1.	.919523
10.	0.	0.	10.	.757	.757	.100	.837122	-3.22	1.	.838517
10.	5.	5.	5.	.240	.240	.240	.578236	.23	.720	.580172
10.	5.	5.	10.	.240	.240	.100	.431872	1.35	.556	.436918
10.	10.	10.	10.	.076	.076	.076	.172083	5.23	.228	.174682

Table 4e

$K = 11$ Simultaneous Transmissions and $q = 1000$

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	E_b/N_{J_3}	ρ_1	ρ_2	ρ_3	$P_e^{(3)}$	E_b/N_J	ρ	$P_e^{(1)}$
10.	0.	0.	0.	.755	.755	.755	.938562	-4.77	1.	.939106
10.	0.	0.	10.	.757	.757	.100	.872159	-3.22	1.	.876218
10.	5.	5.	5.	.240	.240	.240	.581327	.23	.720	.593261
10.	5.	5.	10.	.240	.240	.100	.449343	1.35	.556	.467436
10.	10.	10.	10.	.076	.076	.076	.187236	5.23	.228	.198327

Coded 32-ary FSK FH/SSMA Systems in Multiple Partial-Band Noise Jamming
[Reed-Solomon (32,16) Codes with Errors-Only Decoding]

Worst-Case Error Probability and Optimal Jamming Fractions of Band
for Two Simultaneous Partial-Band Noise Jammers
(All Signal-to-Noise and Signal-to-Jammer Ratios are in dB)

Table 5a

No Other-User Interference ($K = 1$)

E_b/N_0	E_b/E_{J_1}	E_b/E_{J_2}	ρ_1	ρ_2	$P_e^{(2)}$	$P_E^{(2)}$	E_b/E_J	ρ	$P_e^{(1)}$
15.	10.	10.	.392	.392	.201435	.594018	6.99	.689	.228685
15.	10.	15.	.357	.123	.044520	.144891	8.81	.454	.051789
15.	15.	15.	.112	.112	.000587	.002040	11.99	.218	.000684
15.	15.	20.	.110	.035	.000029	.000101	13.81	.143	.000032
15.	20.	20.	.035	.035	.0000003	.000001	16.99	.069	.0000003

Table 5b

$K = 6$ Simultaneous Transmissions

E_b/N_0	E_b/E_{J_1}	E_b/E_{J_2}	ρ_1	ρ_2	$P_e^{(2)}$	$P_E^{(2)}$	E_b/E_J	ρ	$P_e^{(1)}$
15.	10.	10.	.351	.351	.309391	.832119	6.99	.557	.315324
15.	10.	15.	.287	.097	.122684	.380412	8.81	.367	.127953
15.	15.	15.	.090	.090	.009892	.033235	11.99	.176	.010280
15.	15.	20.	.089	.029	.002120	.007244	13.81	.116	.002120
15.	20.	20.	.028	.028	.000205	.000710	16.99	.056	.000207

Table 5c

$K = 11$ Simultaneous Transmissions

E_b/N_0	E_b/E_{J_1}	E_b/E_{J_2}	ρ_1	ρ_2	$P_e^{(2)}$	$P_E^{(2)}$	E_b/E_J	ρ	$P_e^{(1)}$
15.	10.	10.	.198	.198	.405087	.959123	6.99	.357	.411723
15.	10.	15.	.184	.062	.243503	.695912	8.81	.235	.246832
15.	15.	15.	.058	.058	.055377	.178896	11.99	.113	.056108
15.	15.	20.	.057	.018	.022237	.073658	13.81	.074	.022369
15.	20.	20.	.018	.018	.005972	.020197	16.99	.036	.005988

Coded 32-ary FSK FH/SSMA Systems in Multiple Partial-Band Tone Jamming
[Reed-Solomon (32,16) Codes with Errors-Only Decoding]

Worst-Case Error Probability and Optimal Jamming Fractions of Band
for Two Simultaneous Partial-Band Tone Jammers
(All Signal-to-Noise and Signal-to-Jammer Ratios are in dB)

Table 6a

No Other-User Interference ($K = 1$)

E_b/N_0	E_b/E_{J_1}	E_b/E_{J_2}	ρ_1	ρ_2	$P_e^{(2)}$	$P_E^{(2)}$	E_b/E_J	ρ	$P_e^{(1)}$
15.	10.	10.	.097	.097	.027952	.092126	6.99	.432	.028085
15.	10.	15.	.071	.028	.001120	.003844	8.81	.211	.001219
15.	15.	15.	.025	.025	.000025	.000087	11.99	.134	.000027
15.	15.	20.	.024	.011	.000003	.000001	13.81	.081	.000003
15.	20.	20.	.940	.940	.0000001	.0000003	16.99	.852	.0000001

Table 6b

$K = 6$ Simultaneous Transmissions

E_b/N_0	E_b/E_{J_1}	E_b/E_{J_2}	ρ_1	ρ_2	$P_e^{(2)}$	$P_E^{(2)}$	E_b/E_J	ρ	$P_e^{(1)}$
15.	10.	10.	.082	.082	.037816	.123668	6.99	.389	.038364
15.	10.	15.	.065	.024	.001374	.004711	8.81	.191	.001401
15.	15.	15.	.021	.021	.000038	.000132	11.99	.122	.000039
15.	15.	20.	.019	.009	.000005	.000017	13.81	.067	.000005
15.	20.	20.	1.	1.	.0000002	.0000007	16.99	1.	.0000002

Table 6c

$K = 11$ Simultaneous Transmissions

E_b/N_0	E_b/E_{J_1}	E_b/E_{J_2}	ρ_1	ρ_2	$P_e^{(2)}$	$P_E^{(2)}$	E_b/E_J	ρ	$P_e^{(1)}$
15.	10.	10.	.073	.073	.047313	.153669	6.99	.357	.047923
15.	10.	15.	.057	.018	.001937	.006622	8.81	.156	.002038
15.	15.	15.	.018	.018	.000049	.000170	11.99	.098	.000050
15.	15.	20.	1.	1.	.000008	.000028	13.81	.062	.000008
15.	20.	20.	1.	1.	.0000003	.000001	16.99	1.	.0000003

Coded 32-ary FSK FH/SSMA Systems in Multiple Partial-Band Noise Jamming
[Reed-Solomon (32,16) Codes with Errors-Only Decoding]

Worst-Case Error Probability and Optimal Jamming Fractions of Band
for Three Simultaneous Partial-Band Noise Jammers
(All Signal-to-Noise and Signal-to-Jammer Ratios are in dB)

Table 7a

No Other-User Interference ($K = 1$)

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	E_b/N_{J_3}	ρ_1	ρ_2	ρ_3	$P_e^{(3)}$	$P_E^{(3)}$	E_b/N_J	ρ	$P_e^{(1)}$
15.	10.	10.	10.	1.	1.	1.	.447750	.984378	5.23	1.	.447749
15.	10.	15.	15.	.373	.129	.129	.099672	.313338	7.87	.563	.124236
15.	15.	15.	15.	.116	.116	.116	.007186	.024256	10.23	.327	.009299
15.	15.	15.	20.	.113	.113	.037	.001507	.005163	11.35	.252	.001850
15.	20.	20.	20.	.035	.035	.035	.000002	.000007	15.23	.103	.000002

Table 7b

$K = 6$ Simultaneous Transmissions

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	E_b/N_{J_3}	ρ_1	ρ_2	ρ_3	$P_e^{(3)}$	$P_E^{(3)}$	E_b/N_J	ρ	$P_e^{(1)}$
15.	10.	10.	10.	1.	1.	1.	.450602	.985493	5.23	.841	.455930
15.	10.	15.	15.	.298	.101	.101	.132605	.408766	7.87	.457	.134838
15.	15.	15.	15.	.093	.093	.093	.010222	.034334	10.23	.266	.011442
15.	15.	15.	20.	.092	.092	.030	.002262	.007728	11.35	.205	.002473
15.	20.	20.	20.	.028	.028	.028	.000223	.000772	15.23	.084	.000223

Table 7c

$K = 11$ Simultaneous Transmissions

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	E_b/N_{J_3}	ρ_1	ρ_2	ρ_3	$P_e^{(3)}$	$P_E^{(3)}$	E_b/N_J	ρ	$P_e^{(1)}$
15.	10.	10.	10.	.237	.237	.237	.478772	.993374	5.23	.543	.494538
15.	10.	15.	15.	.193	.065	.065	.298062	.811142	7.87	.296	.295947
15.	15.	15.	15.	.061	.061	.061	.068148	.218371	10.23	.172	.069245
15.	15.	15.	20.	.059	.059	.019	.031325	.102959	11.35	.133	.031465
15.	20.	20.	20.	.018	.018	.018	.006532	.022076	15.23	.054	.006549

Coded 32-ary FSK FH/SSMA Systems in Multiple Partial-Band Tone Jamming [Reed-Solomon (32,16) Codes with Errors-Only Decoding]

Worst-Case Error Probability and Optimal Jamming Fractions of Band for Three Simultaneous Partial-Band Tone Jammers (All Signal-to-Noise and Signal-to-Jammer Ratios are in dB)

Table 8a

No Other-User Interference ($K = 1$)

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	E_b/N_{J_3}	ρ_1	ρ_2	ρ_3	$P_e^{(3)}$	$P_E^{(3)}$	E_b/N_J	ρ	$P_e^{(1)}$
15.	10.	10.	10.	.077	.077	.077	.054788	.177074	5.23	.232	.056266
15.	10.	15.	15.	.077	.024	.024	.001762	.006032	7.87	.126	.001799
15.	15.	15.	15.	.024	.024	.024	.000036	.000125	10.23	.073	.000036
15.	15.	15.	20.	.024	.024	.008	.000005	.000017	11.35	.057	.000005
15.	20.	20.	20.	.940	.975	.824	.0000003	.000001	15.23	.790	.0000003

Table 8b

$K = 6$ Simultaneous Transmissions

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	E_b/N_{J_3}	ρ_1	ρ_2	ρ_3	$P_e^{(3)}$	$P_E^{(3)}$	E_b/N_J	ρ	$P_e^{(1)}$
15.	10.	10.	10.	.077	.077	.077	.059279	.099991	5.23	.232	.060814
15.	10.	15.	15.	.077	.024	.024	.002219	.007582	7.87	.126	.002263
15.	15.	15.	15.	.024	.024	.024	.000057	.000198	10.23	.073	.000058
15.	15.	15.	20.	.053	.053	.018	.000008	.000028	11.35	.057	.000009
15.	20.	20.	20.	1.	1.	1.	.0000004	.000001	15.23	1.	.0000004

Table 8c

$K = 11$ Simultaneous Transmissions

E_b/N_0	E_b/N_{J_1}	E_b/N_{J_2}	E_b/N_{J_3}	ρ_1	ρ_2	ρ_3	$P_e^{(3)}$	$P_E^{(3)}$	E_b/N_J	ρ	$P_e^{(1)}$
15.	10.	10.	10.	.077	.077	.077	.063899	.205319	5.23	.232	.065488
15.	10.	15.	15.	.077	.024	.024	.002758	.009403	7.87	.126	.002809
15.	15.	15.	15.	.024	.024	.024	.000089	.000309	10.23	.073	.000090
15.	15.	15.	20.	1.	1.	1.	.000015	.000052	11.35	.057	.000016
15.	20.	20.	20.	1.	1.	1.	.0000005	.000002	15.23	1.	.0000005