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PHASE CHANGE MATERIAL FOR SPACECRAFT
THERMAL MANAGEMENT

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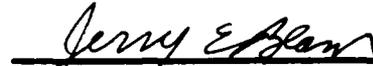
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Processes related to phase change encompass a wide range of engineering and scientific disciplines and occur in many applications. Owing to the release or absorption of latent heat, these phase change problems are nonlinear, and exact solutions are limited to a small class of problems involving pure substances in one-dimensional infinite or semi-infinite domains. Based on an enthalpy formulation, several models have been studied for solving the phase change heat transfer problem of melting. In this report, a continuum model is presented and utilized in a finite-difference numerical method. Thermophysical properties of the PCM (Phase Change Material) such as mass density, specific heat, thermal conductivity, etc. change while the PCM undergoes the solid-liquid phase change. Accommodation methods for these variations have been developed and are presented. The mathematical and numerical formulations of the conservation of mass, momentum and energy equations and thermodyanmic property equations are presented in detail. Limited preliminary results for P116 wax show velocity vector fields and the liquidus lines during melting. (U, words)

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TABLE OF CONTENTS

iii

LIST OF ILLUSTRATIONS

v

FOREWORD

The information presented in this report was generated during the performance of the Task No. 4 of AF Contract F33615-86-C-2721. The technical work was carried out at the University of Missouri-Rolla, Department of Mechanical and Aerospace Engineering and Engineering Mechanics.

Dr. John W. Sheffield served as the Principal Investigator of the program. Chudong Wen was the graduate student supported by the contract. This is the final report which presents the results generated during the period of 1 September 1988 to 31 March 1989. The program was sponsored by the Aeronautical Systems Division of the Air Force with Dr. Won Chang of the Aero Propulsion and Power Laboratory, WRDC/POOS, Wright-Patterson AFB, Ohio 45433-6563, serving as the Technical Monitor.

NOMENCLATURE

$A,\ B,\ C$	constants
a	combined convection-diffusion coefficient
b	source term in the discretization equation
c	specific heat
d	coefficient
f	mass fraction
g	acceleration due to gravity
$g_s,\ g_l$	volume fraction of the solid phase and liquid phase, respectively
h	enthalpy
h_1	solidus enthalpy
h_2	liquidus enthalpy
h_f	heat of fusion
K	permeability
K_o	constant for permeability
k	thermal conductivity
p	pressure
T	temperature
T_l	liquidus temperature
T_s	solidus temperature
t	time
$u,\ v$	velocity components
$u^*,\ v^*$	guessed velocity components
$u',\ v'$	velocity corrections

$\hat{u}, \hat{v}$	pseudo-velocity components
V	velocity vector
x, y	Cartesian coordinates

Greek Symbols

β	thermal expansion coefficient
μ	dynamic viscosity
ρ	density
$\bar{\rho}$	partial density

Subscripts

E, W, N, S	east, west, north, south neighbors of the control volume center
e, w, n, s	east, west, north, and south faces of a control volume
l	liquid phase
P	pertaining to the grid point, P
p	pressure
s	solid phase
u, v	pertaining to the velocity components, u and v
x, y	pertaining to the Cartesian coordinates

I. INTRODUCTION

Processes related to phase change encompass a wide range of engineering and scientific disciplines and occur in many applications such as welding, casting and energy storage. Owing to the absorption or release of latent thermal energy, phase change problems are nonlinear, and exact solutions are limited to a small class of problems involving pure substances in one-dimensional infinite or semi-infinite domains [1,2]. Unfortunately, most practical phase change problems are multi-dimensional; the thermophysical properties such as thermal conductivity, density, and specific heat for the phase change material (PCM) are changing with the changing of phase, and free convection occurs in melting liquid PCM. These have focused attention on development of suitable numerical procedures.

Generally, the numerical techniques can be divided into two groups. The first group uses the front tracking method, which utilizes two independent conservation equations for each phase and couples them with appropriate boundary conditions at the phase interface. Front tracking methods require the existence of discrete interfaces between phases in the domain and are generally limited to pure substances. The primary difficulty associated with this method centers on tracking the phase interface, which is generally an unknown function of space and time. The need for moving numerical grids and/or coordinate mapping procedures also complicates the application of this technique.

The second group uses the enthalpy method which is sometimes called the fixed grid method. The enthalpy method does not track the phase interface, instead, simply calculates the enthalpy of the PCM at each numerical grid.

Only one set of conservation equations are needed for both solid and liquid PCM domains. This eliminates the complications of tracking phase interface and moving numerical grids. Moreover, the enthalpy method allows a mushy region (solid-liquid phases coexist region). This enables the enthalpy method to be implemented for multiconstituent systems, which do not exhibit a sharp interface between the solid and liquid phases. The enthalpy method is generally easier to be implemented for solidification of alloys, melting of impure substances.

Based on enthalpy formulation, several models were developed to solve phase change problems. The first one is the conduction model [3,4]. This is the easiest model, where free convection effects of the liquid PCM are not considered. With the exception of microgravity applications, it is frequently this free-convective motion which is the dominant mode of heat transfer. To include free convection influences, Schneider [5] proposed a numerical model to solve phase change problems for pure substances. By assuming the behavior of fluid flows in the mushy region to be similar to that of fluid flows in porous media, Voller et al. [6,7] proposed the Enthalpy-Porosity model. This model was developed by utilizing Darcy's law to modify the pressure gradients in the momentum equations. A continuum model for analyzing the solid-liquid phase change behavior in a binary system was developed by Bennon and Incropera [8 – 10]. Semi-empirical laws, as well as microscopic descriptions of the transport behavior, have been integrated with the principles of classical mixture theory in obtaining this model.

In the present research, a continuum model is utilized and a finite-difference numerical method is used to solve the conservation equations.

A. CHANGING OF PHYSICAL PROPERTIES

Physical properties of the PCM such as density, specific heat, thermal conductivity, etc. change while undergoing solid-liquid phase change. In this research, a numerical method will be developed to accommodate the changing of these physical properties.

B. SLUMPING FLOATING PHENOMENON

The variation of solid and liquid densities can cause slumping/floating phenomenon during the melting processes. Floating of ice in the water is an example. Slumping or floating of solid PCMs in enclosures can significantly enhance the melting rate [11,12]. Simple analyses were developed by presuming the shapes of solid PCM [13,14], or by considering that conduction is the only energy transport mechanism [15]. However, no numerical studies for melting processes including both free convection and slumping have been reported. The difficulties are due to the fact that moving of the solid domain is unknown and time dependent. The utilizing of a continuum model enables the prediction of the velocity for each phase, and provides the possibility of solving the melting heat transfer problem having a slumping/floating phenomena.

3

C. DENSITY VARIATION INDUCED FLOW

During a melting process, the variation of phase densities can cause motion of the liquid PCM. If the density of the liquid PCM is smaller than that of the solid PCM, an expansion-induced flow would occur. Variation of phase densities is common, and for many materials, the variations are large. For example, the liquid density of P116 wax is 760 kg/m^3, and the solid density is 818 kg/m^3. However, the importance of density variation induced flow during melting has not yet been reported. In this research, the effect of density variation induced flow will be studied.

Experimental results will be used to verify the reliability of the numerical solutions. P116 wax with a melting temperature of 46.7 C (116 F) and a mushy temperature range of 10 C [16] has been selected for the experiments. Physical properties of this wax will be used in the numerical calculations.

II. MATHEMATICAL FORMULATION

A. CONSERVATION OF MASS

The principle of conservation of mass is that the time-rate of change of mass inside a control volume equals the net integral of the mass flux over the control volume. The two-dimensional, solid-liquid system, according to the principle of conservation of mass, yields

$$\frac{\partial[g_s\rho_s + g_l\rho_l]}{\partial t} + \frac{\partial[g_s\rho_s u_s + g_l\rho_l u_l]}{\partial x} + \frac{\partial[g_s\rho_s v_s + g_l\rho_l v_l]}{\partial y} = 0 \quad (1)$$

This can be seen from Fig. 1. Let the mixture density, ρ, be defined by

$$\rho = g_s\rho_s + g_l\rho_l$$

$$= \bar{\rho}_s + \bar{\rho}_l \quad (2)$$

and the mass averaged velocity, V, be defined by

$$V = \frac{\bar{\rho}_s}{\rho} V_s + \frac{\bar{\rho}_l}{\rho} V_l$$

$$= f_s V_s + f_l V_l \quad (3)$$

then substituting Eqs.(2) and (3) into Eq.(1), it follows that

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad (4)$$

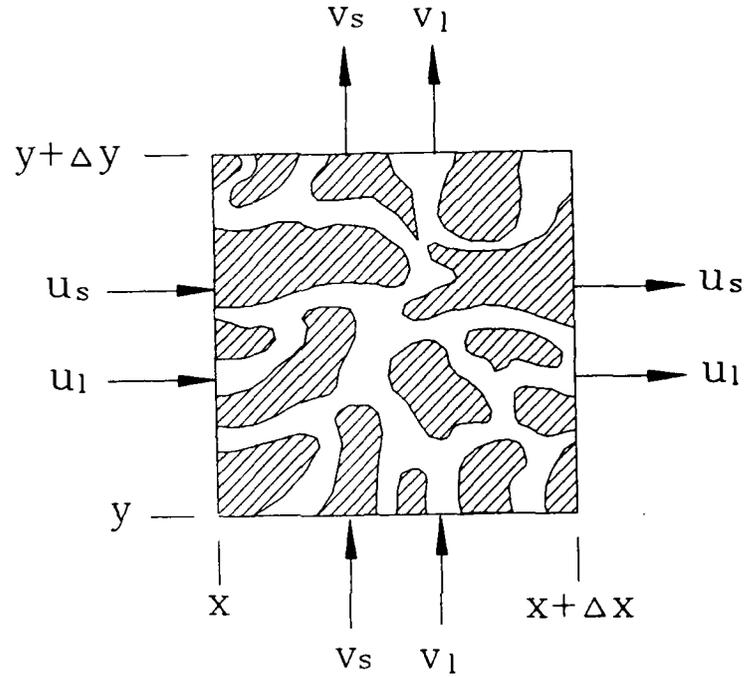This is the equation describing the conservation of mass for the two-dimensional, solid-liquid system.

Figure 1. A control volume in mushy region.

B. CONSERVATION OF MOMENTUM

The equations describing the conservation of momentum for the two-dimensional system are

$$\frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho V u) = \nabla \cdot (\mu_l \frac{\rho}{\rho_l} \nabla u) - \frac{\mu_l}{K} \frac{\rho}{\rho_l} (u - u_s) - \frac{\partial p}{\partial x} \qquad (5)$$

$$\frac{\partial(\rho v)}{\partial t} + \nabla \cdot (\rho V v) = \nabla \cdot (\mu_l \frac{\rho}{\rho_l} \nabla v) - \frac{\mu_l}{K} \frac{\rho}{\rho_l} (v - v_s) - \frac{\partial p}{\partial y} + \rho g \beta (T - T_s) \qquad (6)$$

6

where K is the permeability. Details of derivation of these equations are shown in the Appendix. In the present analysis, the permeability is assumed to vary with liquid volume fraction according to the Kozeny-Carman equation [17]

$$K = K_o [\frac{g_l^3}{(1 - g_l)^2}] \tag{7}$$

where K_o is a constant which depends on the specific multiphase region morphology.

C. CONSERVATION OF ENERGY

The equation describing the conservation of energy is

$$\frac{\partial(\rho h)}{\partial t} + \nabla \cdot (\rho \vec{V} h) = \nabla \cdot (\frac{k}{c_s} \nabla h) + \nabla \cdot [\frac{k}{c_s} \nabla(h_s - h)] \\ - \nabla \cdot [\rho(h_l - h)(V - V_s)] \tag{8}$$

where the mixture enthalpy and thermal conductivity are

$$h = f_s h_s + f_l h_l \tag{9}$$

$$k = g_s k_s + g_l k_l \tag{10}$$

Phase enthalpies are

$$h_s = c_s T \tag{11}$$

$$h_l = c_l T + [(c_s - c_l)T_s + h_f] \tag{12}$$

where it is presumed that $h_s|_{T=0} = 0$ and that $(h_l - h_s)|_{T=T_s} = h_f$. Details of derivation of the energy equation are shown in the Appendix.

D. THERMODYNAMIC RELATIONS

By assuming that the phase fractions are linear functions of temperature in the mushy range and in the saturated condition, we can obtain

$$f_l = \frac{T - T_s}{T_l - T_s} \tag{13}$$

$$f_s = 1 - f_l \tag{14}$$

From Eq.(2), we can find the phase volume fractions,

$$g_s = \frac{\rho - \rho_l}{\rho_s - \rho_l} \tag{15}$$

$$g_l = 1 - g_s \tag{16}$$

Let h_1 and h_2 represent the enthalpies of the PCM at the temperatures, T_s and T_l, respectively. By definitions,

$$h_1 = c_s T_s \tag{17}$$

and

$$h_2 = c_s T_s + c_l(T_l - T_s) + h_f \tag{18}$$

8

For $h < h_1$, we can determine the temperature of the PCM by using the following:

$$T = \frac{h}{c_s} \qquad (19)$$

For $h_1 \leq h \leq h_2$, from Eqs.(9), (11-14), we can determine the temperature of the PCM by using the following:

$$AT^2 + BT + C = 0 \qquad (20)$$

where

$$A = \frac{c_s - c_l}{T_l - T_s}$$

$$B = -\left[A(T_l + T_s) + \frac{h_f}{T_l - T_s} + c_l\right]$$

$$C = \frac{A}{T_l - T_s}T_s + h$$

For $h > h_2$, from Eq.(12), we can determine the temperature of the PCM by using the following:

$$T = \frac{h - (c_s - c_l)T_s - h_f}{c_l} \qquad (21)$$

9

III. NUMERICAL METHOD

A. STAGGERED GRID

Figure 2 shows staggered locations for u, v, and p. A staggered control volume for the x-momentum equation is shown in Fig 3. Figure 4 shows the control volume for the y-momentum equation.
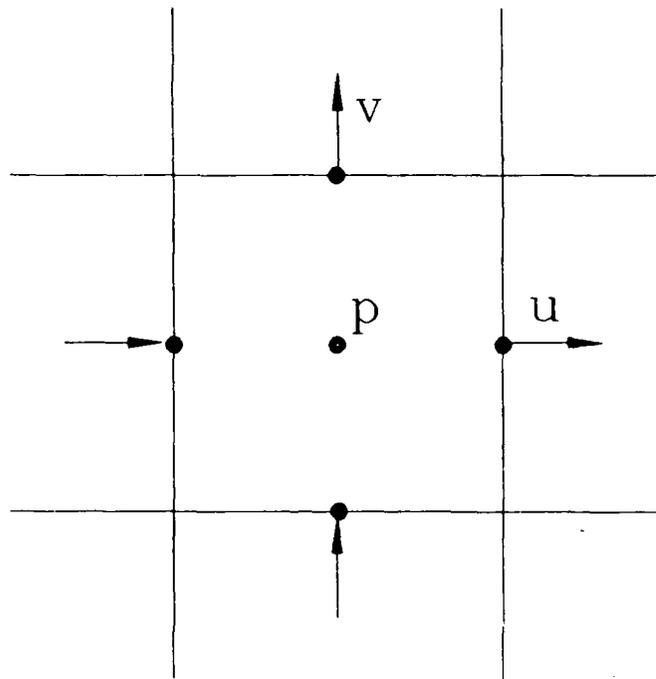

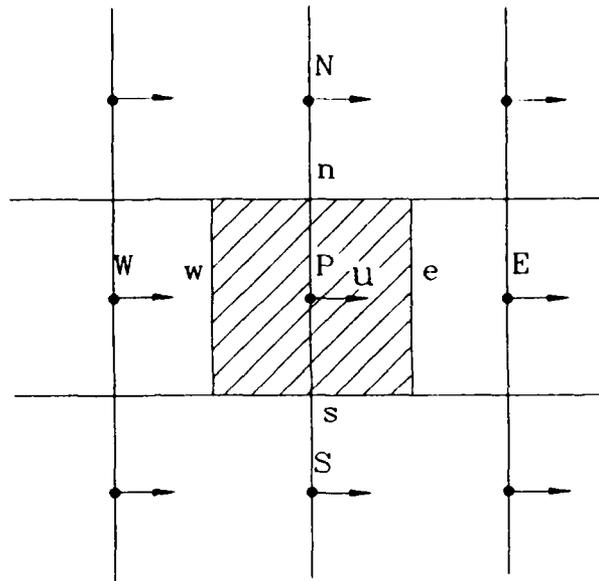Figure 2. Staggered locations for computational variables.

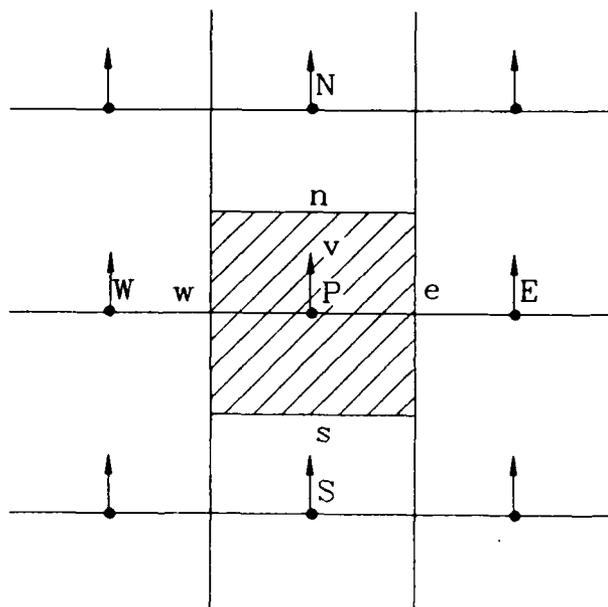Figure 3. Control volume for x-momentum equation

Figure 4. Control volume for y-momentum equation

B. NUMERICAL FORMULATION

The discretized equations for the continuity, x-momentum, and y-momentum equations are

$$\frac{(\rho_P - \rho_P^0)\Delta x \Delta y}{\Delta t} + \Delta y[\rho_e u_P - \rho_w u_W) + \Delta x(\rho_n v_P - \rho_s v_S) = 0 \tag{22}$$

$$a_{uP}u_P = a_{uE}u_E + a_{uW}u_W + a_{uN}u_N + a_{uS}u_S + b_x + \Delta y(p_P - p_E)$$
$$= \sum a_{unb}u_{nb} + b_x + \Delta y(p_P - p_E) \tag{23}$$

$$a_{vP}v_P = a_{vE}v_E + a_{vW}v_W + a_{vN}v_N + a_{vS}v_S + b_y + \Delta x(p_P - p_N)$$
$$= \sum a_{vnb}v_{nb} + b_y + \Delta x(p_P - p_N) \tag{24}$$

Let the pseudo-velocities $\hat{u}$ and $\hat{v}$ be defined by the following:

$$\hat{u}_P = \frac{\sum a_{unb}u_{nb} + b_x}{a_{uP}} \tag{25}$$

$$\hat{v}_P = \frac{\sum a_{vnb}v_{nb} + b_y}{a_{vP}} \tag{26}$$

and substituting into Eqs.(23) and (24), it follows that

$$u_P = \hat{u}_p + d_{uP}(p_P - p_E) \tag{27}$$

$$v_P = \hat{v}_p + d_{vP}(p_P - p_N) \tag{28}$$

12

where

$$d_{uP} = \Delta y / a_{uP}$$

$$d_{vP} = \Delta x / a_{vP}.$$

Substituting Eqs.(27) and (28) into Eq.(22), it follows that

$$a_{pP}p_P = \sum a_{pnb}p_{nb} + \hat{b}_P \qquad (29)$$

where

$$\hat{b}_P = -\left[\frac{(\rho_P - \rho_P^o)\Delta x \Delta y}{\Delta t} + \Delta y(\rho_e \hat{u}_P - \rho_w \hat{u}_W) + \Delta x(\rho_n \hat{v}_P - \rho_s \hat{v}_S)\right] \qquad (30)$$

Let u^*, v^* and p^* be the guessed values and

$$a_{uP}u^*_P = \sum a_{unb}u^*_{nb} + b_x + \Delta y(p^*_P - p^*_E) \qquad (31)$$

$$a_{vP}v^*_P = \sum a_{vnb}v^*_{nb} + b_y + \Delta x(p^*_P - p^*_N) \qquad (32)$$

It is proposed that

$$u'_p = d_{uP}(p'_P - p'_E) \qquad (33)$$

$$v'_p = d_{vP}(p'_P - p'_N) \qquad (34)$$

13

where u', v', and p' are corrections for velocity and pressure, respectively, and the correct velocities are obtained from

$$u = u^* + u'$$ (35)

$$v = v^* + v'$$ (36)

Substituting Eqs.(33) and (34) into Eqs.(35) and (36), respectively, it follows that

$$u_P = u^*_P + d_{uP}(p'_P - p'_E)$$ (37)

$$v_P = v^*_P + d_{vP}(p'_P - p'_N)$$ (38)

Substituting Eqs.(37) and (38) into Eq.(22), it follows that

$$a_{pP}p'_P = \sum a_{pnb}p'_{nb} + b_P$$ (39)

where

$$b_P = -\left[\frac{(\rho_P - \rho_P^o)\Delta x\Delta y}{\Delta t} + \Delta y(\rho_e u^*_P - \rho_w u^*_W) + \Delta x(\rho_n v^*_P - \rho_s v^*_S) \right]$$ (40)

C. NUMERICAL PROCEDURES

The SIMPLER (Semi-Implicit Method for Pressure-Linked Equations - Revised) [18] scheme is proposed to solve the conservation equations. The numerical procedures are

1.   Given initial and boundary conditions.

2.   Advance a time step.

3.   Given initial calculation values for velocities.

4.   Calculate pseudo-velocities $\hat{u}$ and $\hat{v}$.

5.   Calculate pressure field.

6.   Use the newest pressure as guessed pressure to find u^* and v^*.

7.   Calculate pressure corrections p'.

8.   Obtain velocities u and v.

9.   Find solutions for energy equation.

10.   Return to step 4 and repeat until convergence.

11.   Return to step 2 for next time step solutions or stop.

IV. RESULTS

Figure 5 is an example of the numerical results showing the velocity fields and the liquidus lines. P116 wax is the PCM constrained in a 2.5-cm x 2.5-cm square enclosure. The initial temperature is 35 C, and faces are subjected to constant temperature (50 C) heating.
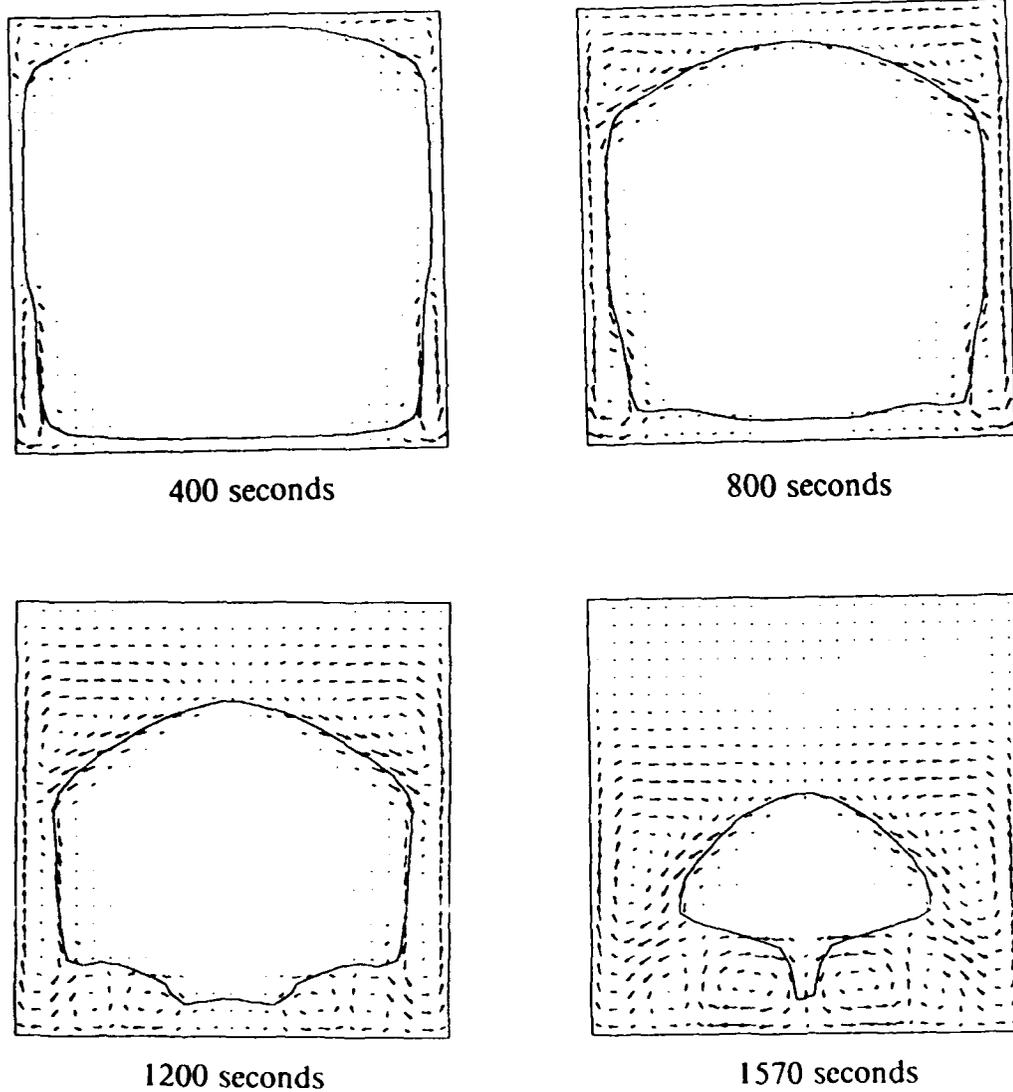

400 seconds

800 seconds

1200 seconds

1570 seconds

Figure 5. Velocity vectors and liquidus lines during melting.

V. SUMMARY

The research is summarized below:

1. This research is aimed at numerical examinations of melting heat transfer in enclosures.

2. A continuum model is used to describe the conservation equations for the PCM during melting.

3. SIMPLER, a control-volume-based numerical scheme, is utilized to solve the coupled conservation equations.

4. Influence of the density variation between solid and liquid phases on the melting process will be studied.

5. Influence of the slumping/floating phenomena on the melting process will be investigated.

6. Solid-liquid interfacial motion experiments will be performed.

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APPENDIX

DERIVATION OF CONSERVATION EQUATIONS

A. CONSERVATION OF MASS

The principle of conservation of mass states that the time rate of change of mass for a control volume equals the net mass flux in the control volume. For a two-dimensional, solid-liquid system, the conservation of mass equation is

$$\frac{\partial[g_s \rho_s + g_l \rho_l]}{\partial t} + \frac{\partial[g_s \rho_s u_s + g_l \rho_l u_l]}{\partial x} + \frac{\partial[g_s \rho_s v_s + g_l \rho_l v_l]}{\partial y} = 0 \quad (1)$$

The mass density of the mixture, ρ, is defined by

$$\rho = g_s \rho_s + g_l \rho_l$$

$$= \bar{\rho}_s + \bar{\rho}_l \quad (2)$$

and the mass averaged velocity, V, is defined by

$$V = \frac{\bar{\rho}_s}{\rho} V_s + \frac{\bar{\rho}_l}{\rho} V_l$$

$$= f_s V_s + f_l V_l \quad (3)$$

Substituting Eqs.(2) and (3) into Eq.(1), we obtain

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \qquad (4)$$

Thus, this equation describes the conservation of mass for the two-dimensional, solid-liquid system. In the liquid region, the form of this equation is the same as the traditional continuity equation. If phase densities are constants, the term, $\partial \rho / \partial t$, would be nonzero only in the mushy zone. However, in the mushy zone, if the solid density and the liquid density are different and if the solid and liquid fractions are time dependent, then the mass density of the mixture is time dependent. The flow from the density variation in the solid-liquid phase change is due mainly to the variation of solid and liquid densities. For example, during the solidification of many metals, shrinkage induced flow can result.

B. CONSERVATION OF MOMENTUM

The momentum equations are derived from Newton's Second Law, which states that the product of mass and acceleration is equal to the sum of the external forces acting on the body. Thus, the x-momentum equation of the two-dimensional, solid-liquid system is

$$\frac{D}{Dt} (\bar{\rho}_l u_l + \bar{\rho}_s u_s) = \nabla \cdot (g_l \sigma_{lx} + g_s \sigma_{sx}) + (\bar{\rho}_l B_{lx} + \bar{\rho}_s B_{sx})$$

$$+ (g_l \dot{G}_{lx} + g_s \dot{G}_{sx}) \qquad (5)$$

22

The flux vector, σ_x, represents the component of the general material stress tensor which influences the x-direction momentum, while B_x represents the x-component body force and $\dot{G}_x$ is the momentum production owing to phase interactions.

The left-hand side of Eq.(5) represents the product of mass and acceleration. It can be decomposed as follows:

$$\frac{D}{Dt}(\bar{\rho}_l u_l + \bar{\rho}_s u_s) = \frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\bar{\rho}_l V_l u_l + \bar{\rho}_s V_s u_s) \qquad (6)$$

Substituting Eq.(6) into Eq.(5), it follows that

$$\frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\bar{\rho}_l V_l u_l + \bar{\rho}_s V_s u_s) = \nabla \cdot (g_l \sigma_{lx} + g_s \sigma_{sx}) + \rho B_x + F_x \qquad (7)$$

where

$$B_x = \frac{\bar{\rho}_l}{\rho} B_{lx} + \frac{\bar{\rho}_s}{\rho} B_{sx} = f_l B_{lx} + f_s B_{sx} \qquad (8)$$

and

$$F_x = g_l \dot{G}_{lx} + g_s \dot{G}_{sx} \qquad (9)$$

The advective momentum flux can be decomposed as follows:

$$(\bar{\rho}_l V_l u_l + \bar{\rho}_s V_s u_s) = \rho V u + \bar{\rho}_s (V_s - V)(u_s - u)$$

$$+ \bar{\rho}_l (V_l - V)(u_l - u) \qquad (10)$$

Substituting Eq.(10) into Eq.(7), it follows that

23

$$\frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho V u) = \nabla \cdot (g_l \sigma_{lx} + g_s \sigma_{sx})$$

$$- \nabla \cdot [\bar{\rho}_s(V_s - V)(u_s - u) + \bar{\rho}_l(V_l - V)(u_l - u)] + \rho B_x + F_x \qquad (11)$$

The x-component of the material stress tensor can be separated into isotropic and deviatoric components,

$$\sigma_x = -p\hat{i} + \tau_x \qquad (12)$$

It is important to recognize that τ_x includes only stresses resulting from interaction of a single phase with itself. The effect of interactions between phases is accommodated by the quality, F_x. Specification of τ_x requires *a priori* assessment of the continuity of each mixture phase. A phase is considered to be continuous if any two points within the phase can be joined by a continuous curve which lies solely within the phase. If each phase mixture is considered to be continuous, the constitutive relationships are available to describe τ_x. In the present formulation, each phase is assumed to be continuous and Newtonian. In compact tensorial form, the average stress vector for each phase is

$$g_k \tau_k = \mu_k \left[\frac{\partial(g_k u_k)_i}{\partial x_j} + \frac{\partial(g_k u_k)_j}{\partial x_i} \right] - \frac{2}{3} \mu_k \delta_{ij} \frac{\partial(g_k u_k)_n}{\partial x_n}$$

$$(i, j, n = 1, 2) \qquad (13)$$

In x-direction, the average stress vector can be expressed as

24

$$g_k \tau_{kx} = [\mu_k \frac{\partial(g_k u_k)}{\partial x} - \frac{2}{3} \mu_k (\frac{\partial g_k u_k}{\partial x} + \frac{\partial g_k v_k}{\partial y})] \hat{i}$$

$$+ [\mu_k \frac{\partial(g_k u_k)}{\partial y}] \hat{j} + \nabla (\mu_k g_k u_k)$$

$$= \tau_{kxo} + \nabla(\mu_k g_k u_k) \qquad (14)$$

If μ_k is assumed to be constant, it can be shown that

$$\nabla \cdot \tau_{kxo} = \frac{1}{3} \mu_k \frac{\partial}{\partial x} [\nabla \cdot (g_k V_k)] \qquad (15)$$

Since the solid phase is assumed nondeformable, it is free of internal shear stress ($\tau_s = 0$, $\nabla(g_s u_s) = 0$) and translates at a prescribed velocity, V_s. With this assumption and substituting Eqs. (12), (14), and (15) into Eq. (11), it follows that

$$\frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho V u) = \nabla \cdot [\mu_l \nabla(g_l u_l)]$$

$$- \nabla \cdot [\bar{\rho}_s (V_s - V)(u_s - u) + \bar{\rho}_l (V_l - V)(u_l - u)] - \frac{\partial p}{\partial x} + \rho B_x + F_x$$

$$(16)$$

where

$$p = g_s p_s + g_l p_l - \frac{1}{3} \mu_l \nabla \cdot (g_l V_l) \qquad (17)$$

From Eq. (3), it can be shown that

$$g_l u_l = \frac{\rho}{\rho_l} u - \frac{\rho_s}{\rho_l} g_s u_s \qquad (18)$$

If it is assumed that the phase densities are constants and the viscous stresses resulting from local density gradients are negligible ($\nabla(\rho/\rho_l) = 0$), then from Eq. (18), Eq. (16) becomes

$$\frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho V u) = \nabla \cdot (\mu_l \frac{\rho}{\rho_l} \nabla u)$$

$$- \nabla \cdot [\bar{\rho}_s (V_s - V)(u_s - u) + \bar{\rho}_l (V_l - V)(u_l - u)] - \frac{\partial p}{\partial x} + \rho B_x + F_x$$

$$(19)$$

Invoking the following identities

$$V - V_s = f_l V_r$$

and

$$V_l - V = f_s V_r,$$

it can be shown that

$$\bar{\rho}_s (V_s - V)(u_s - u) + \bar{\rho}_l (V_l - V)(u_l - u) = \rho f_s f_l V_r u_r \qquad (20)$$

where

$$V_r = V_l - V_s$$

represents the relative phase velocity. Substituting Eq. (20) into Eq. (19), it follows that

$$\frac{\partial(\rho\,u)}{\partial t} + \nabla\cdot(\rho\,V\,u) = \nabla\cdot\left(\mu_l\,\frac{\rho}{\rho_l}\nabla\,u\right) - \rho\,f_s f_l\,V_r\,u_r - \frac{\partial p}{\partial x} + \rho\,B_x + F_x$$

$$(21)$$

To define the phase interaction force, F_x, it is necessary to consider the multiphase region morphology. For a wide region of multi-constitute solid-liquid phase change system, the multiphase region is characterized by a fine permeable solid matrix. The solid matrix is stationary or undergoes free body translation. Thus the liquid phase flow through the mushy region is analogy to flow through a porous media. Therefore, implementation of Darcy's law to prescribe the phase interaction force, F_x, is appropriate. Thus

$$F_x = -\frac{\mu_l}{K_x}\,(g_l u_r)$$

$$(22)$$

where K_x represents the component of anisotropic permeability which influences x-direction momentum transport. Since

$$g_l = \frac{\rho}{\rho_l}f_l$$

it can be shown that

$$F_x = -\frac{\mu_l}{K_x}\frac{\rho}{\rho_l}\,(u - u_s)$$

$$(23)$$

The second term on the right-hand side of Eq. (21) represents inertial forces induced as a result of variations in phase velocities. This inertial force only appears in the multiphase region, where permeabilities are extremely small and the

27

inertial contribution is negligible compared to the Darcian damping force. Hence the x-momentum reduces to

$$\frac{\partial(\rho\, u)}{\partial t} + \nabla \cdot (\rho\, V\, u) = \nabla \cdot (\mu_l\, \frac{\rho}{\rho_l}\, \nabla\, u) - \frac{\mu_l}{K_x}\, \frac{\rho}{\rho_l}\, (u \quad u_s)$$

$$-\frac{\partial p}{\partial x} + \rho\, B_x \qquad\qquad (24)$$

This final form of the equation represents conservation of momentum in the x-direction for the solid-liquid phase change system. In the solid region, the permeability is zero, thus the velocity is the solid phase velocity. In the liquid region, the permeability is infinite, thus the Darcian damping force will disappear. The form of the equation is the same as the normal momentum equation for a single phase fluid.

C. CONSERVATION OF ENERGY

Conservation of energy for a two-dimensional, solid-liquid phase change system can be expressed by the following equation:

$$\frac{\partial}{\partial t}\, (\bar{\rho}_s\, h_s + \bar{\rho}_l\, h_l) + \nabla \cdot (\bar{\rho}_l\, V_l\, h_l + \bar{\rho}_s\, V_s\, h_s) = \nabla \cdot (k\nabla T) \qquad (25)$$

where local thermodynamic equilibrium has been assumed ($T_k = T$) and the mixture conductivity is defined by

$$k = g_l\, k_l + g_s\, k_s \qquad\qquad (26)$$

28

The advective term may be decomposed into contributions owing to the mean mixture motion and the relative phase motion. Thus

$$\bar{\rho}_l \, V_l \, h_l \; + \; \bar{\rho}_s \, V_s \, h_s \; = \; \rho V h \; + \; \bar{\rho}_l \, (V_l - V)(h_l - h) \; + \; \bar{\rho}_s \, (V_s - V)(h_s - h)$$

(27)

Where the mixture enthalpy is

$$h \; = \; \frac{\bar{\rho}_s}{\rho} \, h_s \; + \; \frac{\bar{\rho}_l}{\rho} \, h_l \qquad\qquad (28)$$

It is noted that the flux owing to the relative phase motion only has a contribution in the mushy region. Substituting Eqs.(27) and (28) into Eq.(25), it follows that

$$\frac{\partial(\rho h)}{\partial t} \; + \; \nabla \bullet (\rho V h) \; = \; \nabla \bullet (k\nabla T)$$

$$- \; \nabla \bullet [\bar{\rho}_l(V_l - V)(h_l - h) \; + \; \bar{\rho}_s(V_s - V)(h_s - h)] \qquad (29)$$

Simplifying the term

$$\bar{\rho}_l(V_l - V)(h_l - h) \; + \; \bar{\rho}_s(V_s - V)(h_s - h) \; = \; \rho f_s \, (V - V_s)(h_l - h_s) \qquad (30)$$

it follows that

$$\frac{\partial(\rho h)}{\partial t} \; + \; \nabla \bullet (\rho V h) \; = \; \nabla \bullet (k\nabla T) \; - \; \nabla \bullet [\rho f_s \, (V - V_s)(h_l - h_s)] \qquad (31)$$

In the present formulation, the enthalpy of phase k is defined as

$$h_k = \int_0^T c_k dT + h_k^0 \tag{32}$$

where c_k represents an effective specific heat of phase k. Substituting the identity

$$\nabla T = \frac{1}{c_k} \nabla h + \frac{1}{c_k} \nabla(h_k - h) \tag{33}$$

into Eq.(31), it can be shown that

$$\frac{\partial(\rho h)}{\partial t} + \nabla \cdot (\rho V h) = \nabla \cdot \left(\frac{k}{c_s} \nabla h \right) + \nabla \cdot \left[\frac{k}{c_s} \nabla(h_s - h) \right] \\ - \nabla \cdot [\rho(h_l - h)(V - V_s)] \tag{34}$$

This is the final form of the equation which represents the conservation of energy in the x-momentum for the solid-liquid phase change system.