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ENHANCEMENT OF THE OMEGA SYSTEM AVAILABILITY ALGORITHM

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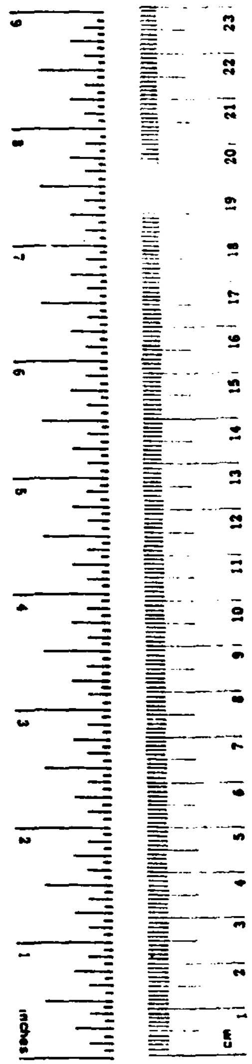
METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

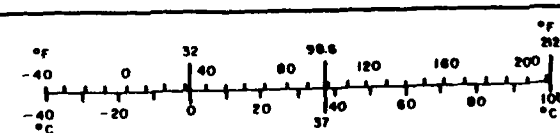
Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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16. ABSTRACT

Enhancements of the Omega System Availability Model are presented which: (1) widen the range of application of the model, (2) increase the efficiency of the model algorithm, and (3) broaden the model to include more realistic characterization of the signal coverage parameters. The system availability model/algorithm is modified to apply at the geographic area unit (cell) and multi-cell level. Further modifications include: a pre-computation method to speed calculation and extension of the coverage database (interpolation) from four-months to twelve months. The system availability model is extended to include the effects of random signal and noise variations. The augmented model structure is consistent with probabilistic definition of the Omega System Availability Index (P_{SA}).

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1. INTRODUCTION

1.1 BACKGROUND: SYSTEM AVAILABILITY MODEL

The System Availability Model (Ref. 1) was developed to provide an overall measure, or index, of Omega system performance by combining currently used, although disparate, measures of overall system performance:

- Omega receiver system reliability/availability
- Omega station reliability/availability
- Omega signal coverage (spatial/temporal)
- Omega user regional priority

The model structure consists of a probabilistic definition of the system availability index (P_{SA}) in terms of probabilistic/deterministic structures, or sub-models, for the above four performance elements. The initial development of this model (Ref. 1) treated the first two elements (above) probabilistically, while the third and fourth elements were addressed using deterministic sub-models. The model is flexible, allowing sub-models to be "turned on and off" as desired. For example, in the first sub-model, the probability that the receiver system is reliable/available can be set equal to one, thus eliminating any influence of receiver system reliability on P_{SA}. Similarly, the contribution of the fourth element above to P_{SA} can be eliminated by setting all regional weightings equal. Computed on a monthly basis, P_{SA} can be monitored as a continuing measure of system performance or to compare the effects of system options, e.g., reductions in radiated power at one or more stations.

The system availability index, P_{SA}, is the probability that, at any time and at any point on the earth's surface, an Omega user's receiver is properly functioning and three or more usable Omega signals are available to permit successful navigation, position-fixing, or other use of the system. Since receiver reliability/availability is independent of station signal access, P_{SA} may be expressed as:

$$P_{SA} = P_R \, P_A$$

where P_R is the probability that the receiver is reliable/available and P_A is the probability that three or more usable signals (in space) are available.

The phrases "any time" and "any point" in the definition of P_{SA} may be interpreted to mean that spatial and temporal averages should be taken in the calculation of P_{SA}. In practice, spatial averaging is done when computing P_{SA} over more than one (two-dimensional) spatial unit, i.e., cell. Temporal averaging is normally limited by the times at which signal coverage is available, e.g., two or 24 hours and four months. Since coverage at a fixed UT hour is assumed constant over the days in a month, P_{SA} may be specified by hour and month. P_{SA} can be averaged over the hours available from the coverage data base, or, alternatively, maximum/minimum values of P_{SA} over these hours may be specified. Because signal coverage changes less from month to month than from hour to hour, interpolation of signal coverage parameters over months is permissible, thus allowing specification of P_{SA} for all 12 months. This also permits averaging of P_{SA} over the months in a year.

The access probability, P_A, is specified for three or more usable Omega signals. This specification stems from conventional Omega usage and is not a limitation of the model which permits an arbitrary minimum usable number of signals. In developing the model (Ref. 1), P_A is written as a sum of two-factor terms -- the first factor being called the coverage element and the second factor the network reliability factor. Most of P_{SA}'s spatial and temporal dependence is contained in the coverage elements (spatial dependence may also be found in the user regional priority weightings). A month and year dependence for P_{SA} arises from the network reliability factors. Coverage elements define coverage in terms of the following signal properties:

- Signal-to-noise (SNR) ratio in a receiver's "front-end" bandwidth
- Relative strength and phasing of signal modes comprising the total signal
- Ratio of long-path to short-path signal strength
- Path/terminator crossing angle.

Criteria imposed on the above signal properties to determine signal coverage are presented in Reference 1. Network reliability factors define the probability that each station of the network is in a specific on-air/off-air condition. The station reliability sub-model includes three types of off-air conditions:

- Unscheduled off-air
- Scheduled off-air (excluding annual maintenance)
- Scheduled annual maintenance.

Recent-year station reliability data was used to determine average durations for the above off-air conditions. From these data, network reliability factors are derived using operational constraints governing concurrent off-air conditions.

1.2 OBJECTIVE

The objective of this memorandum is to describe two refinements of the System Availability Model which extend the model's applicability and serve to enhance its implementation via the Performance Assessment and Coverage Evaluation (PACE) workstation/tool. These refinements include:

(1) Extension of the System Availability (P_{SA}) algorithm to apply at the cell/region level

(2) Incorporating the random behavior of propagated signals and local atmospheric noise into the P_{SA} algorithm

The analytical basis for these refinements is to be developed using the probabilistic formalism of the the P_{SA} model. Specific assumptions and definitions of refined model parameters required by PACE should also be addressed. Recommendations should be made concerning how the refinements are to be integrated into PACE, e.g., as a replacement or alternative processing mode.

1.3 APPROACH

The System Availability Model is very general and applies at any level of spatial definition. The original development (Ref. 1), however, stressed global averaging, since only global coverage elements were available as a database to test hypothetical system options. In developing a refined algorithm for spatial calculations of P_{SA}, the approach used herein focuses on calculation of P_{SA} at the cell/multi-cell level. This algorithm is designed to minimize processing time, since it is to be frequently called within the overall PACE operation.

As noted in Section 1.1, the original system availability model treated coverage data as deterministic, i.e., signal and noise parameters (at a given time) are always as predicted. In fact, it is well-known that signals and, especially, noise vary randomly when observed in a narrow bandwidth over similar temporal periods. As a result, signal and noise levels as detected by an Omega receiver are considered in Chapter 3 of this report as random variables in treating SNR coverage data. This random variation is incorporated into the probabilistic structure of the system availability model. Distribution functions with appropriate parameters are selected to represent the random variations of signal and noise levels. The signal and noise distribution parameters are determined as a function of space and time. Although intrinsically more complex than the deterministic case, the modified algorithm provides much more accurate and realistic figures for system availability.

1.4 REPORT OVERVIEW

Refinement of the system availability algorithm to calculate P_{SA} at the cell/multi-cell level is described in Chapter 2. Specific assumptions concerning the model/algorithm necessary for PACE implementation are also provided in this chapter. A procedure for efficiently computing P_{SA} at the cell level is presented along with an interpolation method for computing P_{SA} at any given month.

Chapter 3 describes the P_{SA} model refinements required to incorporate random levels of signal and noise into the probabilistic model structure. From a very simple model of Omega receiver signal processing in the presence of noise, a theoretical description of the local coverage elements is obtained in terms of SNR probability distributions. Probability density functions describing signal and noise levels are presented together with the time- and path-dependent distribution parameters. Recommended methods of implementing the enhanced algorithm are discussed.

Appendix A presents the detailed mathematical structure of the original system availability model. Appendix B briefly explains the method for computing VLF signal amplitude standard deviation. Approximations to the enhanced P_{SA} algorithm (incorporating random signal and noise level variations) are contained in Appendix C.

2. CALCULATION OF P_{SA} AT THE CELL/MULTI-CELL LEVEL

This chapter explains how the original system availability algorithm is modified to compute P_{SA} at the cell and multi-cell level. Section 2.1 discusses specific assumptions regarding the receiver and station reliability/availability sub-models. In Section 2.2, types of time averaging are described which will be used when the algorithm is integrated into the Performance Assessment and Coverage Evaluation (PACE) software. Section 2.3 describes an efficient method for computing P_{SA} at the cell level and Section 2.4 develops a technique to compute P_{SA} for any month using interpolated coverage data. Finally, the chapter is summarized in Section 2.5.

2.1 RECEIVER AND STATION RELIABILITY/AVAILABILITY SUB-MODELS

2.1.1 Receiver Reliability/Availability Sub-model

In its most general form, the system availability index, P_{SA}, is given by

$$P_{SA} = \frac{1}{N} \sum_{i=1}^{n_c} n_i \, P_{R_i} \, P_{A_i} \qquad (2.1\text{-}1)$$

where:

n_i = number of receivers of class i (i=1,2,...,n)

n_c = number of receiver classes

N = total number of receivers = $\sum_{i=1}^{n_c} n_i$

P_{R_i} = reliability/availability for a class i receiver

P_{A_i} = probability that three or more usable signals are accessible by a class receiver.

The probability that a receiver is functioning properly, P_{R_i}, can be expressed in terms of the MTBF (mean time between failure) and the MTTR (mean time to repair) as explained in Ref. 1. It is clear that the reliability/availability probability depends on the receiver class but the dependence of P_{A_i} on receiver class is not so obvious. The relationship is linked through the coverage elements (on which P_{A_i} depends) which depend on the specific signal access criteria employed for assigning/defining coverage. One of the signal access criteria specifies a threshold for SNR which is based on the receiver sensitivity and other parameters. Since these receiver parameters are roughly the same for a given receiver class, the SNR threshold criterion may be keyed to receiver class.

Because PACE applications will not be concerned with multiple receiver classes, P_{R_i} is constant, and therefore independent of receiver class i. Thus, P_{A_i} is also independent of i and Eq. (2.1-1) becomes

$$P_{SA} = P_R P_A$$

Moreover, since there is no meaningful "average" reliability/availability figure which is valid for all receiver classes, P_R will be set to 1. Hence

$$P_{SA} = P_A$$

2.1.2 Station Reliability/Availability Sub-model

In the original system availability model (Ref. 1), station off-air events are treated as random variables, both in terms of event onset time and duration. The station reliability/availability sub-model defined two types of off-airs: unscheduled and scheduled. Unscheduled off-airs occur as a result of unforeseen circumstances — usually equipment failures. At the beginning of a month, the occurrence probability of an unscheduled off-air is essentially uniform over the month although it differs for each station. Consequently, compilations of monthly total off-air statistics are available as a function of station. Scheduled off-airs are planned conditions under which signal generation temporarily ceases. An important class of these events is the annual maintenance off-air for each station. A station's annual maintenance occurs in a specific month, unique to that station, and generally includes maintenance/repair work which is not urgent. Since users are usually notified of these annual maintenance periods well in advance, the occurrence time and duration of these events may be considered deterministic. Advance notice of other types of scheduled off-airs (a few days to two weeks) is such that these events may be considered random to a user at the beginning of a month (basic time interval over which off-air probabilities are defined). Although these types of scheduled off-airs and unscheduled off-airs are both random, they differ in an important way, as noted below.

The station reliability sub-model defines certain relationships between unscheduled and scheduled off-air events at the same or different stations based on intrinsic definitions and operational practice. The occurrence of an unscheduled off-air at a station is independent of the occurrence of an unscheduled or scheduled event at any other station. However, unscheduled and scheduled off-air events at the same station are exclusive, i.e., they cannot occur at the same time. Because of Omega system operational/management policy, scheduled off-air events at the same or different stations are not independent, i.e., they are excluded from simultaneous occurrence.

For PACE, it is assumed that the unscheduled off-air probability is 0.001 for all stations and months. Using the notation introduced in developing the system availability model (Ref. 1), this requirement is expressed

$$P\left(T_i^U\right) = 0.001 \; ; \; i = 1,2,\ldots,8$$

The scheduled off-air (excluding annual maintenance) event probabilities are assumed to be station-specific but are the same for each month of the year (all off-air probabilities are assumed independent of year). These values are obtained by averaging observed scheduled off-air times (excluding annual maintenance) over three recent years (data obtained from Ref. 2) for each station. Scheduled off-air probabilities for annual maintenance are computed by averaging the off-air times for each station's maintenance month over three recent years (Ref. 2). The resulting data is shown in Table 2.1-1. In this table, the first entry is the fixed unscheduled off-air event probability, the second is the scheduled off-air (excluding annual maintenance) event probability, and the third is the scheduled annual maintenance off-air event probability. Note that the default scheduled off-air (excluding annual maintenance) event probability is specified even for the months corresponding to a station's annual maintenance. This is because a scheduled off-air event (with a few days advance notice) may occur during the month, before or after the annual maintenance period with approximately the same relative probability as during the other months. Unscheduled off-air events at one or more stations may also occur, but scheduled events differ probabilistically in that they are *never* concurrent.

Table 2.1-1 Station Reliability/Availability Parameters for PACE

	A	B	C	D	E	F	G	H
JAN	.00100	.00100	.00100	.00100	.00100	.00100	.00100	.00100
	.00269	.00037	.03604	.00024	.00163	.00030	.00061	.00005
	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
FEB	.00100	.00100	.00100	.00100	.00100	.00100	.00100	.00100
	.00269	.00037	.03604	.00024	.00163	.00030	.00061	.00005
	.00000	.34569	.00000	.00000	.00000	.00000	.00000	.00000
MAR	.00100	.00100	.00100	.00100	.00100	.00100	.00100	.00100
	.00269	.00037	.03604	.00024	.00163	.00030	.00061	.00005
	.00000	.00000	.00000	.00000	.00000	.20511	.00000	.00000
APR	.00100	.00100	.00100	.00100	.00100	.00100	.00100	.00100
	.00269	.00037	.03604	.00024	.00163	.00030	.00061	.00005
	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
MAY	.00100	.00100	.00100	.00100	.00100	.00100	.00100	.00100
	.00269	.00037	.03604	.00024	.00163	.00030	.00061	.00005
	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
JUN	.00100	.00100	.00100	.00100	.00100	.00100	.00100	.00100
	.00269	.00037	.03604	.00024	.00163	.00030	.00061	.00005
	.00000	.00000	.28628	.00000	.00000	.00000	.00000	.00000
JUL	.00100	.00100	.00100	.00100	.00100	.00100	.00100	.00100
	.00269	.00037	.03604	.00024	.00163	.00030	.00061	.00005
	.00000	.00000	.00000	.07895	.00000	.00000	.00000	.00000
AUG	.00100	.00100	.00100	.00100	.00100	.00100	.00100	.00100
	.00269	.00037	.03604	.00024	.00163	.00030	.00061	.00005
	.11057	.00000	.00000	.00000	.00000	.00000	.00000	.00000
SEP	.00100	.00100	.00100	.00100	.00100	.00100	.00100	.00100
	.00269	.00037	.03604	.00024	.00163	.00030	.00061	.00005
	.00000	.00000	.00000	.00000	.61490	.00000	.00000	.00000
OCT	.00100	.00100	.00100	.00100	.00100	.00100	.00100	.00100
	.00269	.00037	.03604	.00024	.00163	.00030	.00061	.00005
	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.32515
NOV	.00100	.00100	.00100	.00100	.00100	.00100	.00100	.00100
	.00269	.00037	.03604	.00024	.00163	.00030	.00061	.00005
	.00000	.00000	.00000	.00000	.00000	.00000	.01726	.00000
DEC	.00100	.00100	.00100	.00100	.00100	.00100	.00100	.00100
	.00269	.00037	.03604	.00024	.00163	.00030	.00061	.00005
	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000

2.2 TIME AVERAGING IN P_{SA} CALCULATIONS

The assumption that $P_R = 1$ for PACE and thus $P_{SA} = P_A$ is addressed in Section 2.1. By definition, $P_A = P(X_3)$ where X_3 is the event that three or more station signals are accessible at any given location (cell)/time. It is shown in Appendix A that

$$\begin{aligned} P(X_3) &= P(X_3/B_0)P(B_0) + P(X_3/B_i)P(B_i) + P(X_3/B_{ij})P(B_{ij}) \\ &+ P(X_3/B_{ijk})P(B_{ijk}) + P(X_3/B_{ijkl})P(B_{ijkl}) \\ &+ P(X_3/B_{ijklm})P(B_{ijklm}) \end{aligned} \quad (2.2\text{-}1)$$

where B_{ijk}... is the event that stations i,j,k,... are off-air and the summation convention* is used to simplify the notation. Each term in Eq. 2.2-1 contains two factors: the first, which may be written Q_{ij}..., is called the coverage element, and the second, written as R_{ij}..., is known as the network reliability factor. Symbolically, Eq. 2.2-1 may be written

$$P(X_3) = Q_o\ R_o + Q_i\ R_i + Q_{ij}\ R_{ij} + \ldots$$

Thus, each term, in the expression for P_A is separated into a factor (Q) which depends only on signal coverage and a factor (R) which depends only on station reliability.

Since the signal coverage database specifies coverage in terms of hour and month (Ref. 1, Appendix C), Q depends on time through these two parameters. In passing, it should be noted that signal coverage databases do not specify coverage at all hours/months. PACE, however, will employ a database which provides coverage at 24 hours and four months (February, May, August, and November). Extension of the coverage information to twelve months is addressed in Section 2.4. Since station reliability is provided on a monthly basis, R depends on time through the numerical equivalent of the month. In general, R also depends on year, but PACE assumes year-independent station reliability statistics, so that R is also year-independent. Hence, in terms of PACE implementation, P_{SA} depends on time through:

- Month (01 - 12, independent of year)
- Hour (01 - 24, for any given day of the specified month)

As noted in Chapter 1, P_{SA} is a probabilistic measure, and thus cannot be specified on time scales smaller than a month (for one or more hours). For example, Q_0(FEB,0100) is the

*This convention states that a repeated index in any of the factors making up a given term indicates summation over understood limits. In Eq. 2.2-1, the B's are invariant under all interchanges of indices, all indices are distinct, and only the unique index combinations are included in the sum i.e. if B_{123} is included in the sum, B_{132} is excluded. All indices are summed from one to eight, subject to the stated conditions.

probability that three or more stations cover a specified cell on any given day in February at 0100 UT, given that no stations are off-air. For a fixed month, Q_0 or P_{SA} could be averaged over all (or a sub-group of) 24 hours, or other statistics could be computed, e.g.,

$$\overline{P_{SA}}(FEB) = (1/24) \sum_{h=1}^{24} P_{SA}(FEB,h) \qquad (2.2\text{-}2a)$$

$$\overline{P_{SA}}'(FEB) = MAX_h \, [P_{SA}(FEB,h)] \qquad (2.2\text{-}2b)$$

$$\overline{P_{SA}}''(FEB) = MIN_h \, [P_{SA}(FEB,h)] \qquad (2.2\text{-}2c)$$

Assuming that P_{SA} can be computed for any month (see Section 2.4),a year-averaged P_{SA} may be specified in several ways. For example, the hour could be fixed and averages could be taken over twelve months. If this procedure is followed for each hour, 24 numbers result which can be treated as indicated above (average, maximum, minimum). The average is clearly the same as if an expression like Eq. 2.2-2a were averaged over twelve months, since averaging is a linear operation. However, maximum and minimum operations do not commute with the averaging operation, so that, for example, the maximum of the 24 month-averaged hourly values is not the same as the average value of an expression like Eq. 2.2-2b over all months. However, the maximum of an expression like Eq. 2.2-2b over all months gives the "best" case (maximum P_{SA}) and the corresponding minimum of an expression like Eq. 2.2-2c over all months gives the "worst" case. In any case, the year-averaged value of P_{SA}, averaged over both hour and month is

$$(\overline{P_{SA}})_{year} = (1/12) \sum_{m=1}^{12} \overline{P_{SA}}(m)$$

where, in general, $\overline{P_{SA}}$ (m) is given by

$$\overline{P_{SA}}(m) = (1/N_h) \sum_{i=1}^{N_h} P_A(h_i)$$

and N_h is the number of hours to be included in the average (default N_h = 24).

2.3 EFFICIENT COMPUTATIONAL PROCEDURE FOR P_A

2.3.1 Direct Calculation of P_A

As discussed above, for the specific receiver reliability/availability sub-model considered by PACE,

$$P_{SA} = P_A = P(X_3) = Q_0R_0 + Q_iR_i + Q_{ij}R_{ij} + Q_{ijk}R_{ijk} + Q_{ijkl}R_{ijkl} + Q_{ijklm}R_{ijklm} \quad (2.3\text{-}1)$$

where, again, the repeated index summation convention is used, the indices are distinct and only unique combinations of indices are used in the sums. The factor R_{ijk}... is the network reliability factor for the specific case in which stations i, j, k, ...are off-air and all other stations are on-air (the subscript 0 means that no stations are off-air).

For a single cell, the Q's are known as *local* coverage elements, which depend only on the signal coverage for the given cell. For deterministic coverage elements, the Q's are binary-valued, i.e., 0 or 1, depending on the station signals accessible to the cell and the stations on-air. This can be written

$$Q_{ijk}\ldots = 1 \text{ if } \vec{V}_c \cdot \vec{V}_o \geq 3$$
$$= 0 \text{ otherwise}$$

where the eight elements of the coverage vector $\vec{V}_c$ are 0 or 1 depending on whether or not a station signal covers the cell and the dot indicates inner product; on-air vector $\vec{V}_o$ has elements

$$(V_o)_a = (1 - \delta_{ai})\,(1 - \delta_{aj})\,(1 - \delta_{ak})\ldots$$

where δ is the Kronecker delta function.

As an example, suppose a cell is covered at a given time by signals from stations A (Norway), C (Hawaii), F (Argentina), G (Australia), and H (Japan). Thus, the coverage vector may be written, (10100111). From the definition of Q_{ijk}... as the probability that three or more station signals are accessible given that stations i, j, k... are off-air, several local coverage elements are evaluated below to illustrate this example:

$$Q_0(10100111) = 1\ ;\ Q_3(10100111) = 1\ ;\ Q_{67}(10100111)=1$$
$$Q_{245}(10100111) = 1\ ;\ Q_{138}(10100111) = 0\ ;\ Q_{12345}(10100111) = 1$$

The total number of possible coverage elements/network reliability factors is simply the number of terms in Eq. 2.3-1. Clearly, there is one $Q_o\,R_o$ term and eight $Q_i\,R_i$ terms (i = 1,2,3,...8). The number of Q_{ij} R_{ij} terms is just the number of ways that 8 station signals can be chosen two at a time (28). In this way, the total number of terms in Eq. 2.3-1 is the sum of the combinatorials,

$$\begin{bmatrix}8\\0\end{bmatrix} + \begin{bmatrix}8\\1\end{bmatrix} + \begin{bmatrix}8\\2\end{bmatrix} + \begin{bmatrix}8\\3\end{bmatrix} + \begin{bmatrix}8\\4\end{bmatrix} + \begin{bmatrix}8\\5\end{bmatrix} = 219$$

Note that, for local coverage elements at the cell level, the cell weight does not enter the calculation for P_A, since the weighting is normalized.

For multi-celled regions (which may include the entire globe), the calculation of P_A for each cell comprising the region is the same as above, except that a weighted average is taken. Thus,

$$P_{SA} = P_A = P(X_3) = < w_i P_{A_i} >$$

where w_i is the relative (normalized) weight assigned to cell i, P_{A_i} is the quantity $P(X_3)$ for cell i, and the angle brackets, <>, indicate averaging over the cells (indexed by i) in the multi-cell region. These weights (which are conveniently specified between 0 and 10) indicate the relative importance of a cell to Omega usage. This could be measured by the number of users transiting, or located within, the cell region in a given time period, or by the strategic importance of the geographic region. For the purposes of PACE, the cell is a region of approximately 10 deg. (latitude) X 10 deg. (longitude, near the equator) in size. Since longitude intervals shrink in size as one progresses poleward, the cell definition correspondingly changes to maintain constant cell size (see Table 2.3-1). A sample weighting structure for U.S. civil users in a portion of the North Atlantic region is shown in Fig. 2.3-1.

As noted above, calculation of P_{SA} for a single cell involves an evaluation of 219 terms, each term a product of Q and R factors. Given a coverage vector, as defined above, calculation of the Q-factor is almost trivial. Calculation of the corresponding R-factor, however, is much more complex (see Appendix A of Ref. 1). Thus, computations of P_{SA} for regions containing more than a few cells may require an inconveniently long processing time on the machines targeted for PACE utilization. As a result, a method has been developed which considerably shortens the processing time for computing P_{SA} during PACE operation.

2.3.2 Pre-computation of P_A

In the direct procedure for a single cell described above, the Q-factors are computed as a function of the input coverage vector/set. In a pre-computational procedure, the Q's are

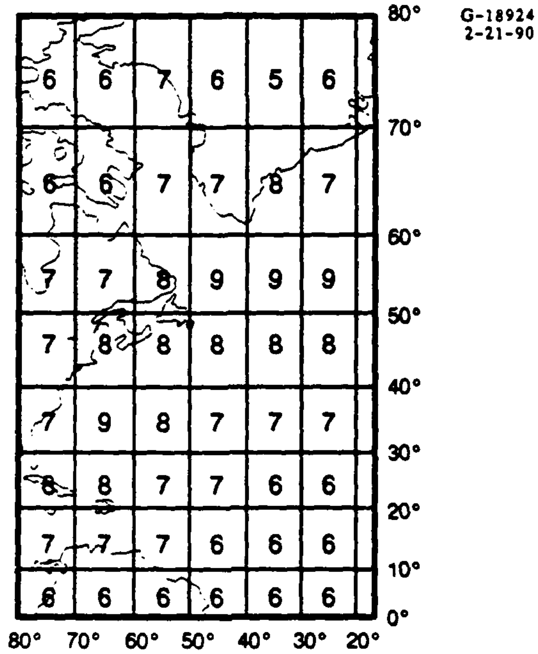

Figure 2.3-1 Example of Regional Weighting: Omega Civil Use

Table 2.3-1 Latitude/Longitude Dimensions of Cells in Grid Structure for Signal Coverage Database (Matrix Format)

LATITUDE RANGE*	LATITUDE DIMENSION OF CELL	LONGITUDE DIMENSION OF CELL	NUMBER OF CELLS IN BOTH HEMISPHERES
0° to 40°	10°	10°	288
40° to 60°	10°	15°	96
60° to 75°	15°	15°	48
75° to 90°	15°	60°	12
TOTAL NUMBER OF CELLS = 444			

*Same for northern and southern hemisphere

computed for *all possible* coverage sets. Since they do not depend on coverage, the corresponding R-factors are the same for each coverage set. As shown above, calculation of P_A for a given coverage set requires the evaluation of 219 terms.The total number of possible coverage sets/vectors with three or more station signals is just the number of combinations of 8 station signals taken three at a time, four at a time, etc. Thus, the total number of possible coverage sets is

$$\binom{8}{3} + \binom{8}{4} + \binom{8}{5} + \binom{8}{6} + \binom{8}{7} + \binom{8}{8} = 219$$

This is the same as the number of coverage elements because of the symmetry of the combinatorials,

$$\binom{N}{n} = \binom{N}{N-n}$$

Since there are 219 possible coverage sets (with a minimum 3-station coverage), pre-computation of 219 X 219 terms is required. The result is a vector of 219 P_A values corresponding to the 219 coverage sets. Since all possible coverage sets are included, no time input is required for the coverage part of the P_A calculation. However, since the network reliability factors, R, depend on month, the P_A vector is also a function of month. Hence, pre-computation and storage of 12 P_A vectors of length 219 is required for the P_{SA} computation in PACE. In the operational mode for a given cell, the month input determines the appropriate P_A vector and the hour/month input together determines the coverage set which indexes the corresponding element of the vector. Thus, determination of P_A is reduced from a lengthy calculation to a short search.

Because the coverage database specifies coverage for only four specific months (FEB, MAY, AUG, NOV), whereas R is defined for all 12 months, a problem arises in computing P_{SA} for any of the eight months for which coverage is not specified. This problem is addressed in Section 2.4.

2.4 CALCULATION OF P_A OVER 12 MONTHS

Omega signal propagation is well-known to be a sensitive function of time — especially over a diurnal period. Consequently, the new coverage database specifies signal coverage parameters at each of the 24 hours. Although the day-to-day time dependence of signal propagation is small enough to be ignored, month-to-month variations must be included in coverage calculations. However, these monthly variations are still smaller than hour-to-hour changes, so that coverage calculations are made over the full 24-hour diurnal scale but only over four months on the monthly scale. As a result, the coverage database includes coverage parameters for only four months of the year. The coverage months are chosen so as not to include equinoxes or solstices and are spaced so that each lies between two "non-coverage" months.

Because rapid variations in coverage are not expected between coverage months, linear interpolation of coverage parameters between these months should introduce negligible error. As an example, consider the interpolation of SNR at hour 0100 UT in March:

$$SNR(01,MAR) = (2 \cdot SNR(01,FEB) + SNR(01,MAY))/3$$

This linear interpolation should be carried out with SNR in logarithmic (dB) units since SNR is approximately lognormally distributed (see Chapter 3). Two signal coverage parameters are needed to determine whether or not a signal is modal (not dominated by Mode 1): signal phase deviation (from the dominant mode) and dominant mode number. Signal phase deviation is linearly interpolated in a similar fashion to SNR (except that logarithmic quantities are not used). Dominant mode number (not susceptible to linear interpolation) is properly interpolated using the following (conservative) voting scheme:

$$D_{ncm} = MAX(D_{pcm}, D_{fcm})$$

where D_{ncm} is the dominant mode number for the non-coverage month, and D_{pcm}, D_{fcm} are the dominant mode numbers for the preceding and following coverage months, respectively. With this information, the signal for the non-coverage month (at a given hour) is specified as modal if:

Interpolated phase deviation > threshold (e.g., 20 cecs)
OR
$D_{ncm} > 1$

Long-path coverage parameters are determined as ratios of long-path and short-path SNR values which are.interpolated as discussed above. Interpolation of other coverage parameters is not required.

2.5 SUMMARY

Calculation of P_{SA} at the cell and multi-cell level is reviewed in this chapter. Specific assumptions required for PACE implementation are outlined, including Omega receiver/station reliability/availability sub-model parameters and time averaging procedures. A method for computing P_{SA} is developed which uses pre-computed data to minimize the processing time required for PACE operation. Finally, a procedure is outlined for computing P_{SA} over 12 months by properly interpolating signal coverage parameters.

3. REVISION OF THE SYSTEM AVAILABILITY MODEL TO INCLUDE RANDOM SIGNAL/NOISE BEHAVIOR

This chapter traces the modification of the system availability model required for the treatment of signal and noise levels as random variables. Section 3.1 presents a simple model of Omega receiver signal processing in the presence of noise to provide a basis for determining the probability of coverage based on SNR. The SNR coverage data is reinterpreted on a statistical basis in Section 3.2. Section 3.3 develops the theoretical dependence of the local coverage elements on the probability distributions of signal and noise deviations. Section 3.4 describes the form and parameters of the distributions selected to describe the signal and noise level variations. The final expression for P_{SA} in terms of the selected distribution parameters is given in Section 3.5. Finally, the chapter is summarized in Section 3.6.

3.1 OMEGA RECEIVER SIGNAL PROCESSING IN THE PRESENCE OF NOISE

Once synchronized, a typical Omega receiver tracks (or attempts to track) all eight Omega station signals at one or more frequencies. At any given time, many of these signals are not usable, due to dominance by higher-order modes, dominance by long-path signals, low SNR, etc.

In conventional usage, the SNR is defined as either the ratio of the signal power to the noise power or the square root of this ratio. In the latter case, both signal and noise have the dimensions of electric field strength. For a harmonically varying signal with no dispersion, the electric field strength (at a given time) is expressed in terms of its amplitude (and phase). Narrowband noise, however, consists of a group of electric fields (wave packet) with slightly different frequencies, amplitudes, and phases. A single "amplitude" is not an appropriate characterization of the noise. Consequently, an "envelope" amplitude is defined (Ref. 3) for a narrowband VLF noise process which, in the limit of "zero" bandwidth, reverts to the customary definition of signal amplitude. **In the following development, the terms "noise level" or "noise amplitude" will be used as shorthand notation for the noise envelope amplitude.**

Ideally, the receiver's navigation filter will only process those signals which are dominated by the short-path Mode 1 component and which exceed a certain minimum phase stability/SNR. In most receivers, the SNR is derived from the stochastic fluctuations of the received signal phase (perturbed by noise), whereas the modal and long-path/short-path information are obtained from external sources (e.g., coverage diagrams). The signals from different stations are generally received on widely separated paths, and their random variations are assumed independent. The "natural" time unit over which received signal and noise level variations are defined is the receiver time constant (approximately 1 - 5 minutes). This signal independence assumption is supported by the following points:

- The signal propagation environments on widely separated paths are sufficiently different to produce independent signal level variations
- Paths with small azimuthal separation are generally rejected by the navigation filter because they can produce large position errors
- Paths with small azimuthal separation which are accepted by the navigation filter (e.g., because no other signals are available) have common propagation environments only between the receiver and the closer transmitting station; the remaining portion of the path (to the farther transmitting station) increases the independence of the two paths.

Although the propagated signal level variations are independent, the noise level accompanying each station signal processed is approximately the same because:

- All signals are processed over approximately the same time period (time constant)
- VLF electromagnetic noise is primarily the result of propagated VLF electromagnetic energy from lightning discharges. Although individually impulsive, when the noise levels from these events are aggregated and time-averaged over 1-5 minutes (typical receiver time constants), the "observed" noise level is found to vary little over an hour (Ref. 3).

Thus, for the purposes of characterizing random signal and noise variations for the probabilistic system availability model, an Omega receiver processes independent station signals corrupted by a common noise level/variation.

3.2 RANDOM SIGNAL/NOISE VARIATIONS AND SIGNAL COVERAGE DATA

In Section 3.1, a simple model is described for reception and processing of Omega signals subject to random variations in the presence of randomly varying noise. In terms of coverage parameters, it is clear that this model applies directly to the SNR of a given station signal at a specified location (cell)/time. One of the coverage parameters governing modal interference, signal phase deviation, deals strictly with the propagated signal (exclusive of noise), and although it probably has some random variation, its variation is neglected in this application because:

- From its definition, phase deviation must have a random variation which is the difference between the random variation of the Mode 1 phase and the random variation in the phasor sum of all modes; this difference in random variations is probably a second-order quantity which can be neglected
- No known reliable data exists for the distribution parameters of phase deviation due to the difficulty in isolating the uncertainties of Mode 1 phase prediction (usually due to uncertainties in the path ground/ionosphere environment) from the random variations in the higher-mode phase.

Also describing modal interference is the dominant mode number, a noise-independent parameter whose statistical distribution is unknown. The long-path coverage parameter is a ratio of long-path to short-path SNR, or, equivalently, a ratio of the corresponding amplitudes (since the noise is common). Thus, the statistics governing this parameter arise from the difference in the signal variations over the short- and long-paths which is generally expected to be no larger than the variation over a single (short) path.

As a result of the above discussion, only the SNR coverage parameter has statistical variations which are sufficiently large and well-defined to be incorporated into the P_{SA} algorithm. Thus, only the random variations of signal and noise levels as characterized by the SNR are considered herein.

When treated as a deterministic quantity, SNR is computed simply as the difference (in dB) between the signal amplitude and noise level provided by the coverage database (for a given hour/month). When signal and noise levels are modeled as random quantities, the "deterministic" SNR referred to above is interpreted as a difference between the mean value of the signal amplitude ($\bar{s}$) and the median value of the noise level ($\bar{n}$). Here, the statistics are taken over the ensemble of days at a fixed hour/month. If the lower-bound SNR threshold is -20 dB, then a deterministic treatment would define a signal with SNR ($\bar{s} - \bar{n}$) of -19 dB (and all other coverage parameters acceptable) as "accessible to" or "covering" a cell, whereas a signal with a SNR of -21 dB would not cover the cell. When the signal and noise levels are statistically defined (over a sufficiently wide range of values), there is always a finite probability that the SNR will exceed the threshold value of -20 dB.

These concepts are best illustrated by an example. Suppose the coverage information given in Table 3.2-1 is provided for a given hour/month/cell. The table lists the three principal coverage parameters (others, e.g., path-terminator crossing angle, are assumed acceptable) and the corresponding criteria/thresholds. The table shows that the signal from: station B is modal (due to both large phase deviation and higher-mode dominance), station D is modal (due to both large phase deviation and higher-mode dominance in the near-field), station E is long-path, and station F is modal (due to large phase deviation only). This leaves station signals A, C, G, and H as "potentially" covering signals since they contain no intrinsic "self-interference" effect which would exclude them from coverage. For these reasons, signals A, C, G,.H (corresponding to $\vec{V}_c$ (10100011)) make up what is known as the *maximal coverage set*.

Table 3.2-1 Cell Coverage Example

STATION SIGNALS	COVERAGE PARAMETERS CRITERIA/THRESHOLD		
	SNR $\bar{S} - \bar{N}$	MODAL CONDITION (Phs. Dev./Dom. Mode)	LONG-PATH/SHORT-PATH (SNR RATIO IN dB)
	> -20 dB	< 20 cecs/Mode 1	< -3 dB
A	−25 dB	15 cecs/Mode 1	−85 dB
B	−11 dB	24 cecs/Mode 2	−09 dB
C	+10 dB	03 cecs/Mode 1	−122 dB
D	+45 dB	61 cecs/Mode 3	−156 dB
E	−16 dB	13 cecs/Mode 1	+22 dB
F	−22 dB	35 cecs/Mode 1	−93 dB
G	−10 dB	08 cecs/Mode 1	−88 dB
H	−15 dB	11 cecs/Mode 1	−52 dB

3.3 DEPENDENCE OF LOCAL COVERAGE ELEMENTS ON SIGNAL/NOISE DISTRIBUTIONS

As noted above, the local coverage elements (Q's) for the deterministic coverage case are just 0 or 1, depending on the coverage data/criteria. For the case of random signal and noise levels, the Q's, which are conditional probabilities lie between 0 and 1 depending on the parameters of the signal and noise distributions.

To illustrate how this calculation proceeds, it is convenient to use the example given in Table 3.2-1. In that case, the maximal coverage set is {1,3,7,8}. The coverage element Q_0 always includes the maximal coverage set, so calculation of this quantity will be considered first. Define the events A_i and $\overline{A}_i$ as follows:

$$A_i \equiv \text{event that } S_i/N > -20\text{dB} \; ; \; \overline{A}_i \equiv \text{event that } S_i/N < -20\text{dB}$$

where S_i is the signal level from station i and N is the noise level in a 100 Hz BW about the signal frequency. With these definitions, Q_0 is written

$$Q_0 = P[A_1A_3A_7\overline{A}_8 + A_1A_3\overline{A}_7A_8 + A_1\overline{A}_3A_7A_8 + \overline{A}_1A_3A_7A_8 + A_1A_3A_7A_8] \qquad (3.3\text{-}1)$$

In this expression (and all succeeding development), a product of events implies set intersection and a sum means set union. Thus, Q_0 is the probability of the union of five events (terms), each of which is the intersection of four individual events. It is important to note that each of the event terms in brackets above are *mutually exclusive*. Because of this property, Eq. 3.3-1 can be written

$$\begin{aligned} Q_0 = \; & P(A_1A_3A_7\overline{A}_8) + P(A_1A_3\overline{A}_7A8) + P(A_1\overline{A}_3A_7A_8) \\ & + P(\overline{A}_1A_3A_7A_8) + P(A_1A_3A_7A_8) \end{aligned} \qquad (3.3\text{-}2)$$

The five probability terms in this expression cannot be further simplified because the A-events are *not* independent. The mutual dependence is due to the common noise processed with each measurement of signal (plus noise).

Calculation of the probability terms in Eq. 3.3-2 is best understood by first considering the probability of the single event A_i. From the definition of the event A_i, $P(A_i)$ is a distribution function which is defined over a probability density function $p_{R_i}(x)$, which is the probability that the station i SNR (denoted by R_i) lies between x and x+dx, i.e.

$$P(A_i) = \int_{-20\text{dB}}^{\infty} dx \, p_{R_i}(x) \qquad (3.3\text{-}3)$$

As explained in Section 3.4, it is convenient to define S_i and N in logarithmic units so that the SNR (R_i) for station signal i is S_i–N. The probability density function for R_i is expressed in terms of the joint probability density function of S_i and N subject to the constraint that $R_i = S_i - N$.

Moreover, the two-dimensional density function of s_i and n must be integrated over the space in which $s_i - n = x_i$, i.e., all possible s_i and n which yield a particular SNR value x_i. Finally, since S_i and N are independent random variables, the SNR density function may be written

$$p_{R_i}(x_i) = \int_{-\infty}^{+\infty} dn\ p_{s_i}(n + x_i)\ p_N(n) \qquad (3.3\text{-}4)$$

In this relation, p_{S_i} is the probability density function for the signal from station i and p_N is the probability density function for the common noise level. It must be emphasized that the random variable N here is a sample of the envelope amplitude of the noise in a narrow post-processing bandwidth (where $S_i > N$) and not a sample of the noise process in a pre-detection bandwidth of typically 100 Hz (see Section 3.4). Thus, although SNR thresholds are quoted in terms of a 100 Hz bandwidth for reference, the model actually applies to much narrower bandwidths.

Substituting Eq. 3.3-4b into Eq. 3.3-3 yields the expression for $P(A_i)$ in terms of the individual signal and noise level probability density functions, i.e.,

$$P(A_i) = \int_{-20}^{\infty} dx_i \int_{-\infty}^{+\infty} dn\ p_{s_i}(n + x_i)\ p_N(n) \qquad (3.3\text{-}5)$$

The probability of the complementary event, $P(\overline{A_i})$, is the same as the expression above, except that the integration over x_i is from minus infinity to −20 (dB). The joint distribution function for two received signals is expressed in a similar way, i.e.

$$P(A_iA_j) = \int_{-20}^{\infty} dx_i \int_{-20}^{\infty} dx_j \int_{-\infty}^{+\infty} dn\ p_{s_i}(n + x_i)\ p_{s_j}(n + x_j)\ p_N(n)$$

As mentioned above, substitution of the complementary event (e.g., $\overline{A_i}$) for the primary event (e.g., A_i) requires that the limits of integration are changed from −20 to plus infinity to minus infinity to −20. From the above, one easily extrapolates to the general case for the probability that the SNR is above threshold (−20dB) for each of m signals is

$$P(A_iA_j\ldots A_m) = \int_{-20}^{\infty} dx_i \int_{-20}^{\infty} dx_j \ldots \int_{-20}^{\infty} dx_m \int_{-\infty}^{+\infty} dn\ p_{s_i}(n + x_i)\ p_{s_j}(n + x_j)\ldots \qquad (3.3\text{-}6)$$

$$\ldots p_{s_m}(n + x_m)\ p_N(n)$$

Note that this form shows explicitly the mutual independence of the random variations in the amplitudes of signals i, j,..., m as well as the independence of the variations in the signal and noise levels.

The general form (Eq.3.3-6) is used for computing the individual terms for Q_0 in Eq. 3.3-2. The higher-order coverage elements (which appear in the expression for $P(X_3)$) are more numerous but generally involve fewer station signals. For illustration, some higher-order Q's for the example in Table 3.2-1 are given as:

$$Q_1 = P(A_3A_7A_8) = Q_{12} = Q_{12456}$$
$$Q_3 = P(A_1A_7A_8) \; ; \; Q_7 = P(A_1A_3A_8)$$
$$Q_2 = Q_4 = Q_5 = Q_6 = Q_{24} = Q_{56} = Q_{2456} = Q_0$$
$$Q_{13} = Q_{37} = Q_{78} = Q_{137} = Q_{378} = 0$$

3.4 SELECTED SIGNAL AND NOISE LEVEL DISTRIBUTIONS

3.4.1 Probability Density Functions for Signal and Noise Level Variations

In Section 3.1, atmospheric VLF noise level measurements integrated over a typical Omega receiver time constant (1-5 minutes) are characterized by small changes over the period of an hour (Ref. 3). Because the noise at a given location is primarily due to propagated VLF energy from lightning discharges, its level depends mostly on hour of the day (characteristic of VLF propagation) and season (governs location and source of thunderstorms). Atmospheric noise data compiled by CCIR (Ref. 3) specifies noise levels as median values of hourly data taken over the approximately 90 four-hour time blocks in each 3-month (seasonal) period of the year. In this work, deviations of the measurements about the median values are expressed as upper and lower deciles. An alternative approach to computing noise levels (Ref. 4) results in noise data which is averaged over the days in a month for a fixed hour. Corresponding upper and lower deciles are also derived in this approach to furnish a measure of the variation in the day-to-day noise levels. Year-to-year variations in the month-averaged (or 3-month-averaged) noise levels are provided in both types of approaches.

Narrowband electromagnetic noise from thermal sources can be characterized by an envelope amplitude which is Rayleigh distributed over an ensemble of short time-scale measurements (Ref. 5). Atmospheric noise at VLF, however, is only partly due to thermal sources; the largest contributor is the lightning discharge which radiates a substantial fraction of its spectral energy in the VLF band. This means that over short time scales, VLF noise is highly impulsive. Over longer time scales (~ several minutes), the impulsive component is smoothed considerably and the envelope amplitudes are generally found to be lognormally distributed (Refs. 3-6). Some models (Ref. 6) use two lognormal distributions — one below the median noise envelope amplitude and one above the median. For the purposes of the system availability model, however, only a single lognormal distribution for VLF noise envelope amplitude will be used. The mean value of this distribution will be assumed to coincide with the median day-to-day variation (over a month) of the noise level at a given hour and the standard deviation includes both the day-to-day variation (for a fixed hour) over a month and the year-to-year variation for the given month. The lognormal probability density function for the envelope amplitude of the noise is written

$$p_N(n) = \frac{1}{\sigma_N\sqrt{2\pi}} e^{-(n-\bar{n})^2/2\sigma_N^2} \qquad (3.4\text{-}1)$$

where σ_N and $\bar{n}$ are the standard deviation and mean value, respectively, of the noise envelope amplitude. The term "envelope" in this context means the narrowband VLF (centered on an Omega frequency) component of the relatively wideband noise sensed by the Omega receiver antenna. For example, the composite signal

$$A(t) = A_o \cos(\omega_1 t + \phi_1) \cos(\omega_2 t + \phi_2)$$

(where $\omega_2 >> \omega_1$) has an envelope given by $A_o\cos(\omega_1 t + \phi_1)$ which defines, in a sense, the upper and lower limits of the actual signal A(t). Eq. 3.4-1 differs from a normal probability density function in that the noise argument, n, is expressed logarithmically (in dB). This means that the distribution parameters (moments), σ_N and $\bar{n}$, are also expressed logarithmically.

The VLF signal amplitude also varies randomly due to random variations in the signal propagation environment, e.g., effective ionospheric reflection height. Measured over intervals of time comparable to a receiver time constant (1-5 minutes), these variations are, like noise, very small over a given hour but have a definite variation from day-to-day (over a month) at a fixed hour. Measurements over a range of VLF frequencies indicate a lognormal distribution for signal amplitude (Ref. 5), similar in form to the distribution of noise envelope amplitude discussed above, i.e.

$$p_{S_i} = \frac{1}{\sigma_{S_i}\sqrt{2\pi}} e^{-(s_i-\bar{s}_i)^2/2\sigma_{S_i}^2} \qquad (3.4\text{-}2)$$

where σ_{S_i} and $\bar{S}_i$ are the standard deviation and mean value, respectively, of the amplitude of the signal from station i.

3.4.2 Signal and Noise Distribution Parameters

The mean values of the noise envelope amplitude are available as a function of frequency, geographic location, and time using either of the two approaches to VLF noise field prediction mentioned in Section 3.4.1. In the second approach (Ref. 4) which is based on propagation of VLF energy from lightning discharges associated with thunderstorms, the mean noise is computed in terms of frequency, location (latitude/longitude), and time (hour/month) for a fixed noise bandwidth. Typical values range from 25-30 dB (relative to one microvolt/meter) under polar winter conditions (far from thunderstorm activity) to 55-60 dB in certain equatorial regions (within centers of thunderstorm activity). The standard deviation of noise level also depends on the above quantities. Typical values of the standard deviation range from 2-4 dB (polar regions) to 8-10 dB (equatorial regions).

The mean values of signal amplitude are assumed to be those signal amplitude predictions supplied by the 24-hour/4-month/2-frequency database (Appendix C of Ref. 1). This assumption is based on the premise that the signal coverage database contains no prediction bias error (for signal amplitude); the validity of this assumption is not known. Some research (Ref. 5) suggests that the standard deviation of the signal amplitude varies as the fourth root of path length. Other studies (Refs. 7 and 8) indicate primary dependence of the standard deviation on path illumination, season, and latitude. The algorithm associated with this latter work (see Appendix B) will be used to determine the signal amplitude standard deviations needed to compute the local coverage elements for the random signal/noise model.

3.5 Calculation of P_{SA} for Selected Signal and Noise Distributions

Using the selected signal and noise distributions described in Section 3.4 (Eqs. 3.4-1, 3.4-2), the local coverage elements may be computed with repeated use of Eq. 3.3-6. This equation represents an (m+1)-fold integral which cannot, in general, be expressed in closed form (finite series of elementary functions). Numerical integration is possible but not practical for an operational algorithm to compute P_{SA}. By changing the order of integration in Eq. 3.3-6 and integrating over each of the SNR random variables, x_i, the calculation is reduced to a single integral over noise. The result is

$$P(A_iA_j \ldots A_m) = \int_{-\infty}^{+\infty} dn\ (\mathrm{erfc}(b_i n - c_i)/2)(\mathrm{erfc}(b_j n - c_j)/2) \ldots \tag{3.5-1}$$

$$\ldots(\mathrm{erfc}(b_m n - c_m)/2)p_N(n)$$

where

$$b_i = \frac{1}{\sigma_{S_i}\sqrt{2}} \quad ; \ i = 1,2,3 \ldots 8$$

$$c_i = b_i(\bar{s}_i - a) \quad ; \ i = 1,2,3 \ldots 8$$

and $p_N(n)$ is given by Eq. 3.4-1. In Eq. 3.5-1, the constant "a" is the SNR threshold (e.g., –20 dB) and the function "erfc" is the complementary error function. Note that the mean amplitude for signal i enters Eq. 3.5-1 through the variable c_i and the standard deviation enters through both b_i and c_i. The noise parameters enter through the expression for $p_N(n)$ (see Eq. 3.4-1).

Except for certain special cases, the integration indicated in Eq. 3.5-1 must be numerically performed. Thus, in the example above, Q_0, as expressed by Eq. 3.3-2, is the sum of five terms, each of which is evaluated using Eq. 3.5-1 with m = 4. The higher-order Q's generally involve fewer terms since fewer station signals are available. If the execution time required for this algorithm (converted to code) is excessive, an approximate model may be used. In this model (described further in Appendix C), the standard deviation of the signal amplitude (which is nearly always smaller than the noise level standard deviation) shrinks to zero so that the signal level becomes deterministic while the noise level remains random. The deterministic signal approximation results in a closed form expression for the distribution function, $P(A_iA_j \ldots A_m)$, thus greatly decreasing the expected execution time for the implemented algorithm.

3.6 SUMMARY

The system availability model/algorithm is extended to include random signal and noise level variations which more realistically characterize the physical process of signal acquisition/utilization. A simple model of Omega receiver signal processing in the presence of noise is used as a basis for relating the coverage parameters to P_{SA}. The model considers a process in which multiple independent signals are processed together with common noise variations. The SNR is found to be the coverage parameter which best represents (in a statistical fashion) the random variations in the signal and noise levels. A maximal coverage set is defined as those signals which are (for a given time/location) short-path/Mode 1-dominated and whose path/terminator angle is greater than a selected threshold. In other words, a maximal coverage set is the largest set of signals whose accessibility is determined only by their SNR. With the basic receiver processing model and the maximal coverage set, the local coverage elements are expressed as a linear combination of joint probability distribution functions of SNR. These joint distribution functions are then written in terms of the independent signal and noise probability density functions. Measurements and studies by various researchers indicate that random variations of both signal and noise levels are lognormally distributed. The mean values of the signal amplitude distribution are assumed to be given by the signal coverage database (no bias error). The signal amplitude standard deviations depend on the temporal and spatial characteristics of the path and are provided by an algorithm based on historical measurements of VLF signal variations. Mean values and standard deviations of the noise envelope amplitude distributions are available from an algorithm which computes the noise level (at a given location/time) as propagated VLF energy from lightning discharges in thunderstorm centers throughout the world to the given location. With the particular form of the signal and noise distributions selected, the joint SNR probability distributions are computed as a single integral over noise, which must be evaluated numerically. The local coverage elements (Q's) are calculated from the joint SNR probability distributions and the Q's, together with the network reliability factors, determine P_{SA}.

If the calculations specified by this algorithm require an execution time considered excessive for operational software (PACE) on the target machine, an approximate model/algorithm, in which the signal is deterministic and the noise is random (see Appendix C), may be considered.

4. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

4.1 Summary

This memorandum presents modifications to the System Availability Model/Algorithm (Ref. 1) which are intended to:

- Adapt the model to a matrix coverage display representation
- Streamline the algorithm for real-time operation on a suitable workstation
- Extend the model's range of application through appropriate assumptions
- Increase the model realism by introducing uncertainty in the expected signal and noise levels.

The modified Psa algorithm will be the nucleus of a software package/workstation known as Performance Assessment and Coverage Evaluation (PACE). PACE will access a matrix/cell-based 24-hour/4-month/2-frequency signal coverage database in addition to a database containing median noise levels and uncertainties in both noise and signal levels. A much smaller database of station reliability figures (unscheduled and scheduled) based on averages of recent-year statistics will also be available to PACE.

Since PACE will access coverage data in a matrix/cell format, the Psa algorithm is tailored for application at the cell/multi-cell level. Model assumptions and data defaults are specified for PACE implementation. An algorithm is developed for pre-computation of Psa at the cell level, which will substantially improve the speed and efficiency of PACE operation. To expand the coverage database to 12 months, an interpolation method is developed for each of the coverage parameters.

The system availability model is modified to incorporate the observed random behavior of signal and noise levels into the signal coverage portion of the model. A simple model of signal processing by a "standard" Omega receiver in the presence of noise is formulated to relate the known signal and noise variations external to the receiver to the parameters of coverage contained in the database. The SNR is found to be the most appropriate coverage parameter to capture the random variation of signal amplitude and noise envelope amplitude. A theoretical treatment is developed to relate the local coverage elements to arbitrary signal and noise distributions. A search of the available VLF data/literature indicates that signal and noise variations are reasonably well described by a log normal distribution. A basic assumption is made that the predicted signal amplitude (given by the database) is equal to the mean of the randomly varying (true) amplitude. The veracity of this assumption (of negligible prediction bias error) is unknown. The noise level median values and standard deviations are obtained from a comprehensive model of VLF electromagnetic energy propagated from lightning discharges occurring in thunderstorm centers. Values of the standard deviation of signal amplitude are computed from an approximate semi-empirical model/algorithm based on limited data samples. The analytical form for Psa is reduced to a single integral over the random noise variable, thus requiring numerical integration for evaluation of the local coverage elements and Psa.

4.2 Conclusions

The methods developed here extend the range of application of the original system availability algorithm (e.g., interpolating coverage data over 12 months) and suggest new ways to interpret/compute system performance (e.g., through judicious choice of Psa time-averaging method). The result is a highly efficient procedure for real-time computation of Psa at the cell/multi-cell level.

The modifications of the system availability model required for inclusion of random signal and noise level variations maintain the probabilistic structure of the model and are rigorous within the context of a simple receiver processing model. The modified system availability model is structured to calculate Psa in terms of parameters of the assumed signal and noise level distributions and indirectly in terms of other coverage data. The assumed log normal form of the distributions is reasonable, based on reported data from several sources. However, the accuracy of the distribution parameters, in particular, the signal amplitude mean and standard deviation. is not known. In any case, the structural integrity of the model is independent of the uncertainty in the distribution parameters. Thus, predictions of Psa based on additional high-quality data will require only a modification of the database, i.e., no model changes will be necessary.

4.3 Recommendations

Since the results of this work indicate that a rapid, efficient algorithm for computing Psa can be readily implemented, it is recommended that the Psa algorithm for deterministic signal coverage parameters outlined in this memorandum be implemented as the basis for the PACE software package/workstation. Specifically, the original system availability algorithm (Ref. 1) should be modified to include the Psa pre-computation technique, as well as the 12-month coverage interpolation scheme and time-averaging options given in Chapter 1. Certain assumed data required for PACE, e.g., station reliability tables, should also be included in the algorithm.

It is also recommended that the algorithm for calculating Psa with random signal and noise level variations, as described in this memorandum, be implemented so that numerical results can be obtained and compared. As part of the current effort, the algorithm should then be tested for sensitivity of Psa to the magnitude of the signal and noise level standard deviations over the range indicated by available standard deviation data. If the test indicates a significant dependence of Psa on the magnitude of the signal/noise standard deviations, then the full random signal/noise model/algorithm described in Chapter 3 should be retained.

If the sensitivity test described above shows no significant Psa dependence on the size of the standard deviations, then three additional tests are suggested for future consideration:

(1) Execute the full random signal/noise algorithm for a fixed signal level standard deviation and a fixed noise level standard deviation using a set of baseline scenarios

(2) Implement and execute the deterministic signal/random noise approximation of the full random signal/noise model described in Chapter 3 for the same scenarios used in test (1)

(3) Execute the deterministic signal/noise algorithm described in Chapter 2 for the same scenarios used in tests (1) and (2).

If all three tests yield similar results for Psa, then only the deterministic signal/noise model should be implemented in the final version of PACE. If only tests (1) and (2) yield similar results, then the deterministic signal/random noise approximation introduced in Chapter 3 and in Appendix C should be implemented in place of the full random signal/noise computation option in PACE. Otherwise, fixed standard deviations for signal/noise should be used in the full random signal/noise option in PACE.

If the three tests listed above do indicate that the full random signal and noise model algorithm (with or without fixed standard deviations) must be used, then it is suggested that the implemented algorithm be tested for execution speed. If execution times (for a range of scenarios) prove excessive for operational use of PACE, then additional approximation techniques (e.g., selective elimination of higher-order coverage elements) should be explored.

APPENDIX A

ANALYTICAL DEVELOPMENT OF THE SYSTEM AVAILABILITY MODEL

A.1 DEVELOPMENT OF THE SYSTEM AVAILABILITY INDEX

A.1.1 Derivation of Expression for P_{SA}

The system availability index, P_{SA}, is the probability that, for any location on the earth at any time/time interval, an Omega user's receiver will be properly functioning and three or more Omega signals can be effectively used for navigation. Expressed analytically, P_{SA} is given very generally as

$$P_{SA} \equiv \frac{1}{N} \sum_{i=1}^{n_c} n_i P_{R_i} P_{A_i} \qquad \text{(A.1-1)}$$

where P_{R_i} ≡ probability that a receiver of class i is functioning normally and being operated correctly; also termed "receiver reliability"

P_{A_i} ≡ probability that three or more usable signals are accessible by a receiver of class i at any point on the earth's surface at any time/time interval

n_i ≡ number of receivers of class i currently in operation

$N \equiv \sum_{i=1}^{n_c} n_i$ = total number of receivers in the classes assumed

n_c ≡ number of receiver classes assumed.

Using a uniform failure interval and repair time model, it can be shown that the reliability for a receiver of class i is

$$P_{R_i} = 1 - \frac{MTTR_i}{MTBF_i}$$

where $MTTR_i$ = mean time to repair figure for receivers of class i

$MTBF_i$ = mean time between failure figure for receivers of class i

Since only very rough approximations to n_i are known, P_{SA} for a single receiver class (i.e., n_c = 1) will generally be considered.

Let X_3 be the event that three or more usable singles are available at a given point in time and space, i.e.,

$X_3(\theta, \phi, t) \equiv$ event that three or more usable signals are accessible to a point (θ, ϕ) on the earth's surface at time t

Event X_3 depends on

- Signal coverage
- Transmitting station reliability.

A signal's coverage is not only a function of space and time but also depends on the signal access *criteria* which define the usability of a signal. Thus, $P(X_3)$, the probability measure on event X_3, depends on receiver class *i* through the signal coverage/access criteria which, in general, change with different *i*.

P_A is just the weighted average of $P(X_3(\theta, \phi, t))$ over time and space, i.e.,

$$P_A = \frac{1}{N_w T} \int_{t_1}^{t_2} \int_0^{2\pi} \int_0^{\pi} P(X_3(\theta, \phi, t)) w(\theta, \phi) R_E^2 \sin\theta \, d\theta \, d\phi \, dt$$

where $N_w = \int_0^{2\pi} \int_0^{\pi} w(\theta, \phi,) R_E^2 \sin\theta \, d\theta \, d\phi$

$T = t_2 - t_1$

$w(\theta, \phi)$ = weight assigned to location (θ, ϕ) based on user's geographic priority

R_E = radius of earth

(θ, ϕ) = conventional angular spherical coordinates.

If time is partitioned into two dimensions (e.g., hour and day), then the integration will usually be carried out over day with hour fixed. In this case, T ~ 30 days, since longer time intervals would involve signal coverage changes.

$X_3(\theta, \phi, t)$ depends on which stations are on-air, in addition to the signal coverage dependence on space and time. The following notation is introduced:

$T_i \equiv$ event that station i is on-air (i=1,2,...8)

$\bar{T}_i \equiv$ event that station i is off-air (i=1,2,...8)

$B_{ijk\ldots} \equiv$ event that only stations i,j,k.,,, are concurrently off-air; i,j,k,... = 1,2,...8 and all indices distinct.

The universe (all possible events) can then be formed as

$$U = B_o + \sum_{i=1}^{8} B_i + \sum_{i=1}^{8} \sum_{j=i+1}^{8} B_{ij} + \ldots + \sum_{i=1}^{8} \sum_{j=i+1}^{8} \sum_{k=j+1}^{8} \sum_{l=k+1}^{8} \sum_{m=l+1}^{8} B_{ijklm} + \ldots + B_{12345678}$$

Notice that the sums are over the possible *combinations* of indices, not permutations, since the $B_{ijk\ldots}$ are symmetric under all possible interchanges of indices (i.e., it only matters *which* stations are off-air, not the order). This particular decomposition of the universe is used because the events $B_{ijk\ldots}$ are *mutually exclusive*. Now, by definition of the universe* (in the following, the θ, ϕ, t dependence of X_3 is suppressed)

$$X_3 U = X_3$$

and

$$X_3 B_{i_1 i_2 \cdots i_m} = 0$$

for m>5 since three or more signals cannot be available if more than five stations are concurrently off-air. Thus,

$$P(X_3) = P(X_3 U)$$

$$= P(X_3 B_o + \sum_{i=1}^{8} X_3 B_i + \ldots + \sum_{i=1}^{8} \sum_{j=i+1}^{8} \sum_{k=j+1}^{8} \sum_{l=k+1}^{8} \sum_{m=l+1}^{8} X_3 B_{ijklm})$$

$$= P(X_3 B_o) + \sum_{i=1}^{8} P(X_3 B_i) + \ldots + \sum_{i=1}^{8} \sum_{j=i+1}^{8} \sum_{k=j+1}^{8} \sum_{l=k+1}^{8} \sum_{m=l+1}^{8} P(X_3 B_{ijklm})$$

*In operations with events, product implies set intersection and sum implies set union. Thus, in set theory, the universe is equivalent to the identity operator.

where the last step follows because the $B_{ijk...}$ are all mutually exclusive.* Writing each of the above terms in terms of conditional probabilities, i.e.,

$$P(X_3\ B_{ijk...}) = P(X_3/B_{ijk...})\ P(B_{ijk...})$$

yields

$$P(X_3) = P(X_3/B_o)\ P(B_o) + \sum_{i=1}^{8} P(X_3/B_i)\ P(B_i) + \sum_{i=1}^{8} \sum_{j=i+1}^{8} P(X_3/B_{ij})\ P(B_{ij})$$

$$+ \ldots + \sum_{i=1}^{8} \sum_{j=i+1}^{8} \sum_{k=j+1}^{8} \sum_{l=k+1}^{8} \sum_{m=l+1}^{8} P(X_3/B_{ijklm})\ P(B_{ijklm}) \ . \qquad \text{(A.1-2)}$$

Since $X_3 = X_3(\theta, \phi, t)$, Eq. A.1-2 expresses a *local* definition (in space and time) of P_A. Thus, the factor

$$P(X_3(\theta, \phi, t)/B_{ijk...})$$

is a local coverage element (LCE). The second factor in each term in Eq. A.1-2, i.e.,

$$P(B_{ijk...})$$

is assumed approximately independent of time (up to a period of one month; see Section A.2) and is called the network reliability factor (NRF). Space and time integration of Eq. A.1-1 gives a relation of the same form but written as

$$P_A = Q_o\ R_o + \sum_{i=1}^{8} Q_i\ R_i + \ldots + \sum_{i=1}^{8} \sum_{j=i+1}^{8} \sum_{k=j+1}^{8} \sum_{l=k+1}^{8} \sum_{m=l+1}^{8} Q_{ijklm}\ R_{ijklm}$$

where

$$Q_{ijk...} = \frac{1}{N_W T} \int_{t_1}^{t_2} \int_0^{2\pi} \int_0^{\pi} P(X_3(\theta, \phi, t)/B_{ijk...})\ w(\theta, \phi)\ R_E^2 \sin\theta\ d\theta\ d\phi\ dt$$

*Events produced by intersections of one or more events with a set of mutually exclusive events are also mutually exclusive.

are called the global coverage elements (GCEs) and

$$R_{ijk...} = P(B_{ijk...})$$

are the NRFs.

P_{SA} is then given by Eq. A.1-1 in which the dependence on receiver class i is reflected through the GCEs.

A.1.2 Derivation of Expressions for the NRFs

The scheduled and unscheduled off-air probabilities are defined by

$$\bar{T}_i \equiv \bar{T}_i^u + \bar{T}_i^s \qquad i = 1,2,....8$$

where

$\bar{T}_i^u \equiv$ unscheduled off-air event for the i^{th} station

$\bar{T}_i^s \equiv$ scheduled off-air event for the i^{th} station

and, by definition,

$$\bar{T}_i^u \bar{T}_i^s = 0 \qquad i = 1,2,....8 \; . \qquad \text{(A.1-3a)}$$

Omega Navigation System operational doctrine bars the occurrence of concurrent scheduled off-airs at two or more stations. Thus

$$\bar{T}_i^s \bar{T}_j^s = 0 \qquad i,j = 1,2,....8 \qquad i \neq j \; . \qquad \text{(A.1-3b)}$$

Finally, the independence of an unscheduled off-air event at a given station from unscheduled/scheduled off-air events at other stations is expressed as

$$P(\bar{T}_i^u \bar{T}_j^u) = P(\bar{T}_i^u)\, P(\bar{T}_j^u) \qquad i,j = 1,2,...8 \; , \; i \neq j \qquad \text{(A.1-4a)}$$

$$P(\bar{T}_i^u \bar{T}_j^s) = P(\bar{T}_i^u)\, P(\bar{T}_j^s) \qquad i,j = 1,2,...8 \; , \; i \neq j \qquad \text{(A.1-4b)}$$

Before proceeding, it is necessary to establish the independence of unscheduled off-air events at a given station from on-air events at other stations, a property which can be derived from

Eqs. A.1-4a and A.1-4b. An indirect approach is employed which is used several times in this Appendix. The procedure begins by expanding $P(\bar{T}_i^u)$ and recalling $U \equiv$ universe is equivalent to the identity operator. Thus

$$P(\bar{T}_i^u) = P(\bar{T}_i^u U) = P(\bar{T}_i^u(T_j + \bar{T}_j)) = P(\bar{T}_i^u T_j + \bar{T}_i^u \bar{T}_j)$$

$$= P(\bar{T}_i^u T_j) + P(\bar{T}_i^u \bar{T}_j)$$

where the last step follows because events $\bar{T}_i^u T_j$ and $\bar{T}_i^u \bar{T}_j$ are mutually exclusive.* Rearranging the above relation gives

$$P(\bar{T}_i^u T_j) = P(\bar{T}_i^u) - P(\bar{T}_i^u \bar{T}_j)$$

$$= P(\bar{T}_i^u) - P(\bar{T}_i^u(\bar{T}_j^u + \bar{T}_j^s))$$

$$= P(\bar{T}_i^u) - P(\bar{T}_i^u \bar{T}_j^u + \bar{T}_i^u \bar{T}_j^s)$$

$$= P(\bar{T}_i^u) - P(\bar{T}_i^u \bar{T}_j^u) - P(\bar{T}_i^u \bar{T}_j^s)$$

where the last step follows because events $\bar{T}_i^u \bar{T}_j^u$ and $\bar{T}_i^u \bar{T}_j^s$ are mutually exclusive.* Now applying Eqs. A.1-4a and A.1-4b to the second and third terms on the RHS of the above relation yields

$$P(\bar{T}_i^u T_j) = P(\bar{T}_i^u) - P(\bar{T}_i^u)P(\bar{T}_j^u) - P(\bar{T}_i^u)P(\bar{T}_j^s)$$

$$= P(\bar{T}_i^u)[1 - P(\bar{T}_j^u) - P(\bar{T}_j^s)] = P(\bar{T}_i^u)[1 - P(\bar{T}_j^u + \bar{T}_j^s)]$$

$$= P(\bar{T}_i^u)[1 - P(\bar{T}_j)]$$

where the mutually exclusive property of $\bar{T}_i^u$ and $\bar{T}_j^u$ (Eq. A.1-3a) and the definition of $\bar{T}_j$ are used. Thus, by definition, the above relation gives

$$P(\bar{T}_i^u T_j) = P(\bar{T}_i^u)P(T_j) \qquad \text{(A.1-4c)}$$

Based on the above assumptions, the NDFs may now be computed in terms of the individual station off-air probabilities. Before considering the general case, a sample NRF calculation will be performed to illustrate the required component calculations. Consider R_{12}, i.e.,

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*If events A and B are mutually exclusive and C and D are two other unrestricted events, then AC and BD are also mutually exclusive.

$$P(B_{12}) = P\left[(\bar{T}_1^u + \bar{T}_1^s)(\bar{T}_2^u + \bar{T}_2^s)\ T_3\ T_4\ T_5\ T_6\ T_7\ T_8\right]$$

$$= P\left[\bar{T}_1^u\bar{T}_2^u V + \bar{T}_1^u\ \bar{T}_2^s V + \bar{T}_1^s\ \bar{T}_2^u V + \bar{T}_1^s\ \bar{T}_2^s V\right] \qquad \text{(A.1-5)}$$

where $V = T_3T_4T_5T_6T_7T_8$.

The last term/event inside the bracket in Eq. A.1-5 vanishes because of the concurrent scheduled off-air exclusion (Eq. A.1-3b). The remaining terms/events are mutually exclusive as expressed by Eq. A.1-3a. With these results and the repeated use of Eqs. A.1-4a, A.1-4b, and A.1-4c, then Eq. A.1-5 becomes

$$P(B_{12}) = P(\bar{T}_1^u)\ P(\bar{T}_2^u)\ P(V) + P(\bar{T}_1^u)\ P(\bar{T}_2^s V) + P(\bar{T}_2^u)\ P(\bar{T}_1^s V) \qquad \text{(A.1-6)}$$

Thus, the NRF has been reduced to an expression involving single station off-air probabilities, except for $P(V)$, $P(\bar{T}_2^s V)$, and $P(\bar{T}_1^s V)$ (similar results hold for reduction of other NRFs). Since $V = T_3T_4T_5T_6T_7T_8$, it is clear that $P(\bar{T}_1^s V)$ and $P(\bar{T}_2^s V)$ represent identical calculations, with $1 \rightarrow 2$.

To compute $P(V)$, first calculate $P(T_1T_2)$ and extend the result to higher-order products. As before, an indirect approach is used, in which $P(T_1)$ is expanded as

$$P(T_1) = P(T_1U) = P(T_1(T_2 + \bar{T}_2)) = P(T_1\ T_2 + T_1\ \bar{T}_2)$$

$$= P(T_1T_2) + P(T_1\bar{T}_2)$$

where the last step follows since T_1T_2 and $T_1\bar{T}_2$ are mutually exclusive events. Rearranging the above relation and using the definitions yield

$$P(T_1\ T_2) = P(T_1) - P(T_1\ \bar{T}_2) = P(T_1) - P(T_1\ (\bar{T}_2^u + \bar{T}_2^s))$$

$$= P(T_1) - P(T_1\ \bar{T}_2^u + T_1\ \bar{T}_2^s)$$

$$= P(T_1)\ (1 - P(\bar{T}_2^u)) - P(T_1\ \bar{T}_2^s) \qquad \text{(A.1-7)}$$

where the last step followed from Eq. A.1-4c and the fact that events $T_1\bar{T}_2^u$ and $T_1\bar{T}_2^s$ are mutually exclusive. Now, the quantity $P(T_1\bar{T}_2^s)$ is calculated using an approach similar to that used for $P(T_1T_2)$, but now expanding $P(\bar{T}_2^s)$, i.e.,

$$P(\bar{T}_2^s) = P(\bar{T}_2^s U) = P(\bar{T}_2^s\ (T_1 + \bar{T}_1)) = P(\bar{T}_2^s\ T_1 + \bar{T}_2^s\ \bar{T}_1)$$

$$= P(\bar{T}_2^s\ T_1) + P(\bar{T}_2^s\ \bar{T}_1)$$

because $\bar{T}_2^s T_1$ and $\bar{T}_2^s \bar{T}_1$ are mutually exclusive. Thus

$$\begin{aligned} P(T_1 \bar{T}_2^s) &= P(\bar{T}_2^s) - P(\bar{T}_2^s\, \bar{T}_1) \\ &= P(\bar{T}_2^s) - P(\bar{T}_2^s\, (\bar{T}_1^u + \bar{T}_1^s)) \\ &= P(\bar{T}_2^s) - P(\bar{T}_2^s \bar{T}_1^u) \end{aligned}$$

where the definition of $\bar{T}_1$ and the concurrent scheduled off-air exclusion (Eq. A.1-3a) was used. With the use of Eq. A.1-4b, the above expression yields

$$P(T_1 \bar{T}_2^s) = P(\bar{T}_2^s)\,(1 - P(\bar{T}_1^u)) \qquad \text{(A.1-8)}$$

Substituting this result into Eq. A.1-7 gives

$$P(T_1 T_2) = P(T_1)\,(1 - P(\bar{T}_2^u)) - P(\bar{T}_2^s)\,(1 - P(\bar{T}_1^u)) \qquad \text{(A.1-9)}$$

Now, by definition,

$$P(T_1) = 1 - P(\bar{T}_1) = 1 - P(\bar{T}_1^u + \bar{T}_1^s) = 1 - P(\bar{T}_1^u) - P(\bar{T}_1^s)$$

Thus

$$1 - P(\bar{T}_1^u) = P(T_1) + P(\bar{T}_1^s)$$

and, similarly, for $P(T_2)$,

$$1 - P(\bar{T}_2^u) = P(T_2) + P(\bar{T}_2^s)$$

Substituting these last two equations into Eq. A.1-9 yields

$$P(T_1 T_2) = P(T_1)P(T_2) - P(\bar{T}_1^s)P(\bar{T}_2^s) \qquad \text{(A.1-10)}$$

This result, which is properly symmetric in stations 1 and 2 shows explicitly the error in assuming on-air probabilities T_1 and T_2 independent.

To compute $P(\bar{T}_1^s V)$, it is again convenient to start with $P(\bar{T}_1^s T_2)$ and extrapolate the result for additional factors. Equation A.1-8 gives, with indices 1 and 2 interchanged,

$$P(\bar{T}_1^s T_2) = P(\bar{T}_1^s)(1 - P(\bar{T}_2^u))$$

Using a procedure identical to that used to derive Eq. A.1-8, it can be shown that

$$P(\bar{T}_1^s T_2 T_3) = P(\bar{T}_1^s)(1 - P(\bar{T}_2^u))(1 - P(\bar{T}_3^u))$$

and, in general,

$$P(\bar{T}_{i_1}^s T_{i_2} T_{i_3} \ldots T_{i_n}) = P(\bar{T}_{i_1}^s) \prod_{j=2}^{n} (1 - P(\bar{T}_{i_j}^u)) \qquad \text{(A.1-11)}$$

This general result can be used to decompose both $P(\bar{T}_1^s V)$ and $P(\bar{T}_2^s V)$.

To calculate the general form $P(T_{i_1}T_{i_2}..T_{i_n})$, a procedure similar to that used above is employed. Thus,

$$\begin{aligned} P(T_{i_1}T_{i_2}\ldots T_{i_{n-1}}) &= P(T_{i_1}T_{i_2}\ldots T_{i_{n-1}}U) \\ &= P(T_{i_1}T_{i_2}\ldots T_{i_{n-1}}(T_{i_n} + \bar{T}_{i_n})) \\ &= P(T_{i_1}T_{i_2}\ldots T_{i_n}) + P(T_{i_1}T_{i_2}\ldots T_{i_{n-1}}\bar{T}_{i_n}) \end{aligned}$$

because complementary sets are mutually exclusive. Thus,

$$\begin{aligned} P(T_{i_1}T_{i_2}\ldots T_{i_n}) &= P(T_{i_1}T_{i_2}\ldots T_{i_{n-1}}) - P(T_{i_1}T_{i_2}\ldots T_{i_{n-1}}(\bar{T}_{i_n}^u + \bar{T}_{i_n}^s)) \\ &= P(T_{i_1}T_{i_2}\ldots T_{i_{n-1}}) - P(T_{i_1}T_{i_2}\ldots T_{i_{n-1}}\bar{T}_{i_n}^u) \\ &\quad - P(T_{i_1}T_{i_2}\ldots T_{i_{n-1}}\bar{T}_{i_n}^s) \end{aligned}$$

Now, Eq. A.1-4c is applied to the second term and Eq. A.1-11 is used to reduce the third term. Hence,

$$\begin{aligned} P(T_{i_1}T_{i_2}\ldots T_{i_n}) &= P(T_{i_1}T_{i_2}\ldots T_{i_{n-1}})\,(1 - P(\bar{T}_{i_n}^u)) \\ &\quad - P(\bar{T}_{i_n}^s) \prod_{j=1}^{n-1} (1 - P(\bar{T}_{i_j}^u)) \end{aligned} \qquad \text{(A.1-12)}$$

Equation A.1-12 is in the form of a recursion relation for $P(T_{i_1}T_{i_2}\ldots T_{i_n})$. Although the symmetry on interchange of indices is not evident in this form, it is immediately adaptable to programming on a computer. For $n = 3$, Eq. A.1-12 can be manipulated to yield the expression

$$P(T_1T_2T_3) = P(T_1)P(T_2)P(T_3) - P(T_1^s)P(T_2^s)P(T_3^s) - \Big[P(T_1)PT_2^s)P(T_3^s)$$
$$+ P(T_1^s)P(T_2)P(T_3^s) + P(T_1^s)P(T_2^s)P(T_3) + P(T_1^s)P(T_2^s)P(T_3^s)\Big]$$

This relation is expressly written to exhibit the complete symmetry on interchange of indices 1, 2, and 3.

Thus, all terms and factors in the expression for $P(B_{12})$ (Eq. A.1-6) can be computed with the aid of Eqs. A.1-11 and A.1-12. For example, P(V) is computed from Eq. A.1-12 with $i_1 = 3$, $i_2 = 4$, $i_3 = 5$, $i_4 = 6$, $i_5 = 7$, $i_6 = 8$, and $n = 6$. Similarly, $P(T_1^s V)$ is computed from Eq. A.1-11 with $i_1 = 1$, $i_2 = 3$, $i_3 = 4$, $i_4 = 5$, $i_5 = 6$, $i_6 = 7, i_7 = 8$, and $n = 7$. $P(T_2^s V)$ is computed the same way except that $i_1 = 2$.

The general NRF, $R_{i_1 i_2 \ldots i_n}$, may be written as

$$P(B_{i_1 i_2 \ldots i_n}) = P\Big[(T_{i_1}^u + T_{i_1}^s)\,(T_{i_2}^u + T_{i_2}^s)\,\ldots\,(T_{i_n}^u + T_{i_n}^s)T_{i_{n+1}}\,T_{i_{n+2}}\,\ldots\,T_{i_8}\Big]$$

Although the above expression may appear formidable, the exclusion rule, Eq. A.1-3b, reduces the number of terms inside the brackets to just $n + 1$. Expanding the indicated product in brackets and using Eqs. A.1-2,3,4 yields the following general expression for the NRF:

$$R_{i_1 i_2 \ldots i_n} = P(W)\prod_{j=1}^{n} P(T_{i_j}^u) + \sum_{j=1}^{n}\prod_{k=1}^{n}\Big[P(T_{i_k}^u)(1 - \delta_{jk}) + P(T_{i_k}^s W)\delta_{jk}\Big]$$

where

$$W = T_{i_{n+1}}\,T_{i_{n+2}} \ldots T_{i_8}$$

$$\delta_{jk} = 1 \quad j = k \text{ (Kronecker } \delta)$$

$$= 0 \quad j \neq k$$

In this expression, P(W) is determined by means of Eq. A.1-12 and $P(T_{i_k}^s W)$ is computed with the use of Eq. A.1-11. Note that $n \leq 5$ since

$$P(X_3/B_{i_1 i_2 \ldots i_n}) = 0 \text{ for } n > 5$$

i.e., no more than 5 stations can be concurrently off-air.

It should be mentioned that the independence assumption regarding unscheduled off-air events is only approximate and is most accurate when the scheduled and unscheduled off-air probabilities are small. This is consistent with the definition of the off-air probabilities as ratios of total off-air duration to total time in a month as shown in Section A.2.

A.2 OFF-AIR PROBABILITY FUNCTIONS

A.2.1 Off-air Occurrence Probability Functions

A reasonable description of unscheduled (random) off-air occurrence is given by the probability density function shown in Fig. A.2-1(a). The probability density describes the situation in which an off-air occurs at time $t = 0$ and the probability per unit time of the next off-air occurrence is indicated by the plot. The probability density following the off-air is zero and gradually increases to a peak at time $1/\lambda$ which represents the average interval between off-air occurrences (based on empirical data). The probability density then gradually decreases to permit normalization. The normalized probability density function may be expressed as

$$p_{OAO}(t) = \lambda^2 t e^{-\lambda t} \qquad \text{(A.2-1)}$$

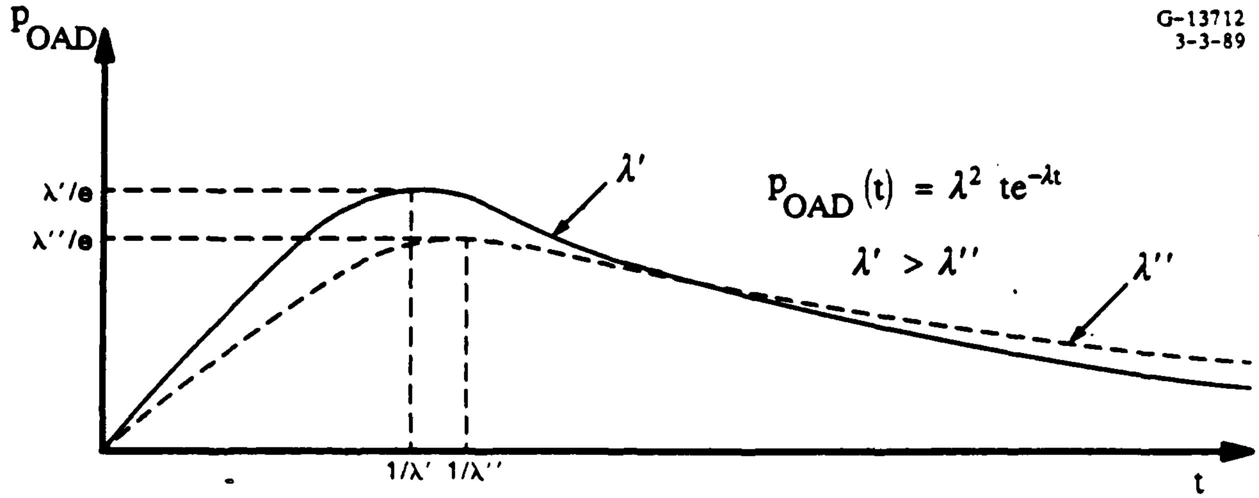
For scheduled off-airs the process is entirely deterministic so that the probability density function is given by the Dirac-delta function $\delta\,(t-T)$ where T is the known time of off-air occurrence referenced to a convenient initial point (such as the beginning of a month). This distribution is illustrated in Fig. A.2-1(b).

A.2.2 Off-air Duration Probability Functions

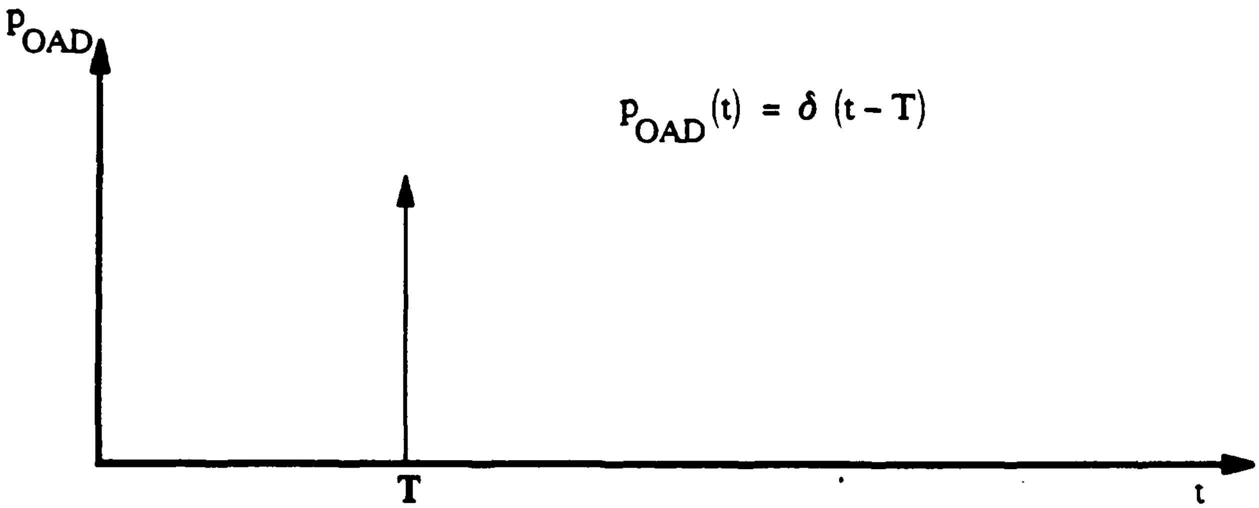
In the case of unscheduled off-airs, the off-air duration may be described by a simple exponential probability density function, which, in its normalized form is given by

$$p_{OAD}(t) = \mu e^{-\mu t} \qquad \text{(A.2-2)}$$

where $1/\mu$ is the average off-air duration, obtained from empirical data. Figure A.2-2(a) shows a plot of this function and Fig. A.2-2(b) illustrates the corresponding distribution function (integral of the density function) which describes the probability that the off-air duration is less than some value, T.

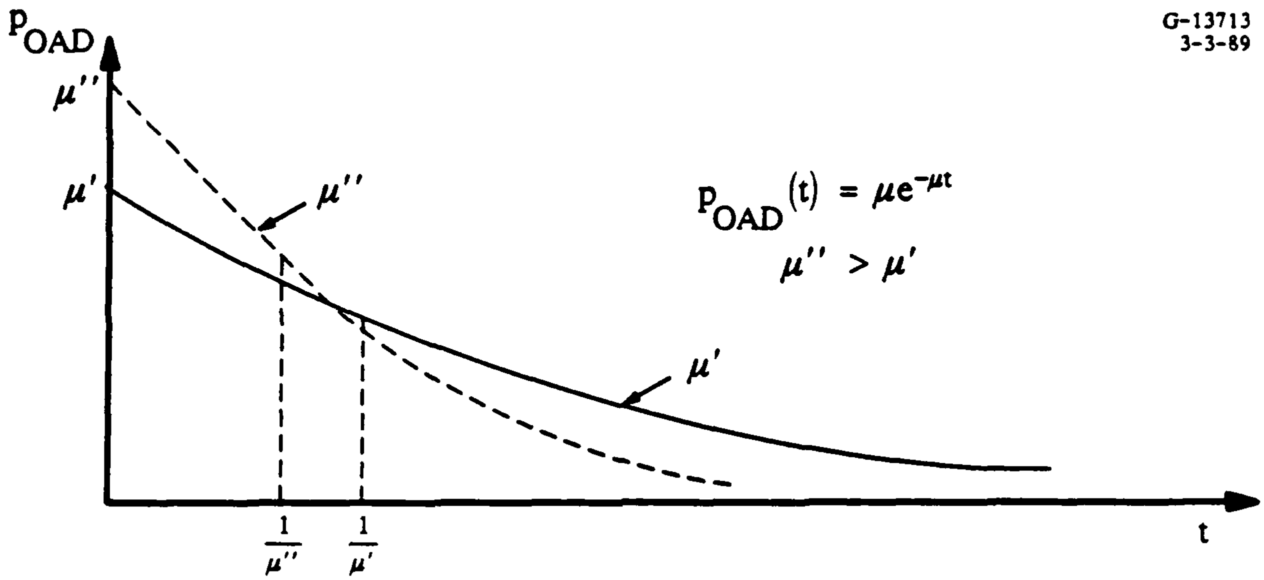


a) Unscheduled (random) Off-air Occurrence Probability Density Function for Two Values of the Average Time between Off-airs (1/λ)

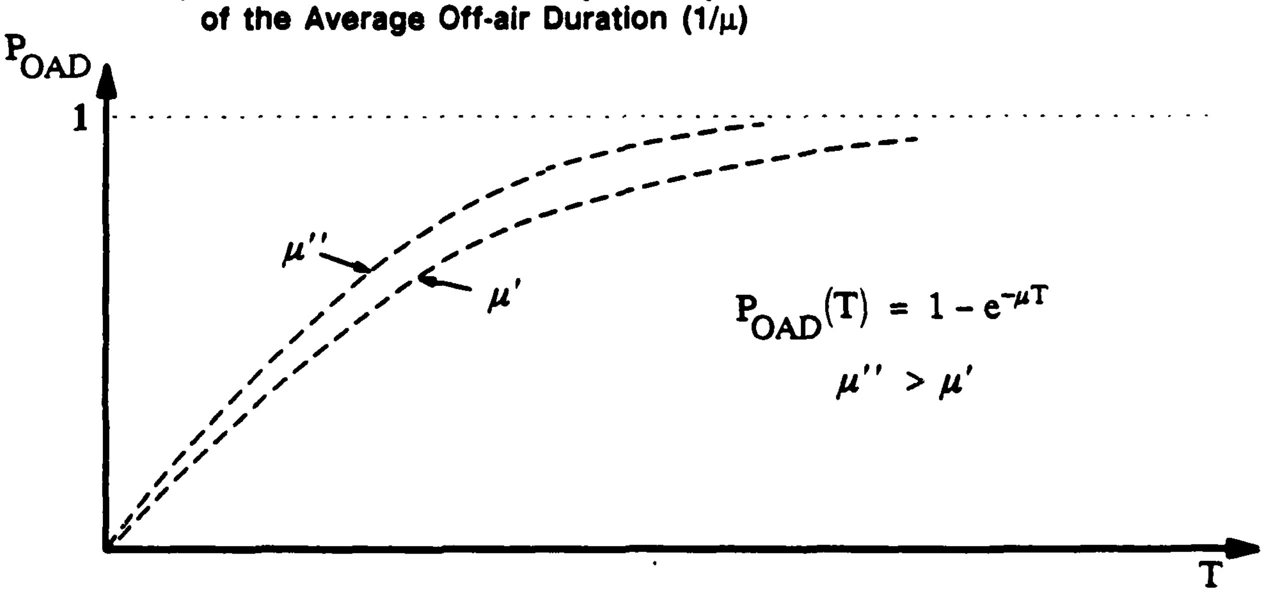


b) Scheduled (Deterministic) Off-air Occurrence Probability Density Function

Figure A.2-1 Off-air Occurrence Probability Density Functions for Unscheduled (random) and Scheduled (deterministic) Off-air Conditions



a) Off-air Duration Probability Density Function for Two Values of the Average Off-air Duration (1/μ)



b) Off-air Duration Probability Distribution Function for Two Values of the Average Off-air Duration (1/μ)

Figure A.2-2 Off-air Duration Probability Density and Distribution Functions for Unscheduled (random) Off-air Conditions

This density function differs from the off-air occurrence density function (aside from normalization constants) by a factor of t. This factor occurs in the expression for $p_{OAO}(t)$ to explicitly exclude very short intervals between off-air occurrences (e.g., before the station achieves an on-air condition). An unscheduled off-air condition may be indefinitely short, however, since immediate action is always taken to restore the on-air condition. Thus, the exponential factor appears alone (leading to a monotonically decreasing density function) in the expression for $p_{OAD}(t)$.

Since the duration of scheduled off-airs is a deterministic quantity, the probability density function has the same form as for off-air occurrences, i.e., $\delta\ (t-\Delta T)$ where ΔT is the known off-air duration. This density function is similar to the one shown in Fig. A.2-1(b).

A.2.3 Probability that a Station is Off-air at an Arbitrary Time

Assuming that the time of off-air occurrence is independent of the duration of the corresponding off-air period, the probability that a station is off-air at some arbitrary time *t* is

$$P_{OA}(t) = \int_{t'<t} dt' p_{OAO}(t') \int_{t-t'<t''} dt''\ p_{OAD}(t'')$$

In words this says that in order that a station be off-air at time *t*, the off-air (beginning at *t'*) must begin before *t and* the off-air duration (*t''*) must be longer than the current elapsed time since the off-air occurrence (*t*-*t'*). This reasoning is illustrated in Fig. A.2-3. With an arbitrary zero-time reference, the above may be written

$$P_{OA}(t) = \int_{0}^{t} dt'\ p_{OAO}(t') \int_{t-t'}^{\infty} dt''\ p_{OAD}(t'') \qquad \text{(A.2-3)}$$

For the case of unscheduled off-airs, Eqs. A.2-1 and A.2-2 are used for the off-air occurrence and off-air duration probability density functions, respectively. When inserted in Eq. A.2-3, the off-air probability becomes

$$P_{OA}(t) = \mu\lambda^2 \int_{0}^{t} dt'\ t'e^{-\lambda t'} \int_{t-t'}^{\infty} dt''e^{-\mu t''}$$

This integral is easily evaluated to give

$$P_{OA}(t) = \frac{\lambda^2}{\mu - \lambda}\left[\frac{e^{-\mu t}}{\mu - \lambda} + te^{-\lambda t} - \frac{e^{-\lambda t}}{\mu - \lambda}\right] \quad \text{(A.2-4)}$$

To evaluate this quantity, the assumption is made that the average time interval between successive off-airs is much larger than the average off-air duration, i.e.,

$$\frac{1}{\lambda} >> \frac{1}{\mu} \quad \text{or} \quad \lambda << \mu$$

Thus, for $t \neq 0, e^{-\lambda t} >> e^{-\mu t}$ and the first term in brackets in Eq. A.2-4 can be neglected in comparison to the second and third terms. Since $\mu >> \lambda$, the exponential in the third term in brackets in Eq. A.2-4 is essentially multiplied by $1/\mu$. Thus, for $t >> 1/\mu$ (i.e., for times large compared to an off-air duration), the third term in brackets in Eq. A.2-4 may be neglected in comparison to the second term. Thus, with t large compared to the average off-air duration, the off-air probability at time t may be written

$$P_{OA}(t) = \frac{\lambda^2}{\mu - \lambda}\, te^{-\lambda t} \cong \frac{\lambda^2}{\mu}\, t\, e^{-\lambda t}$$

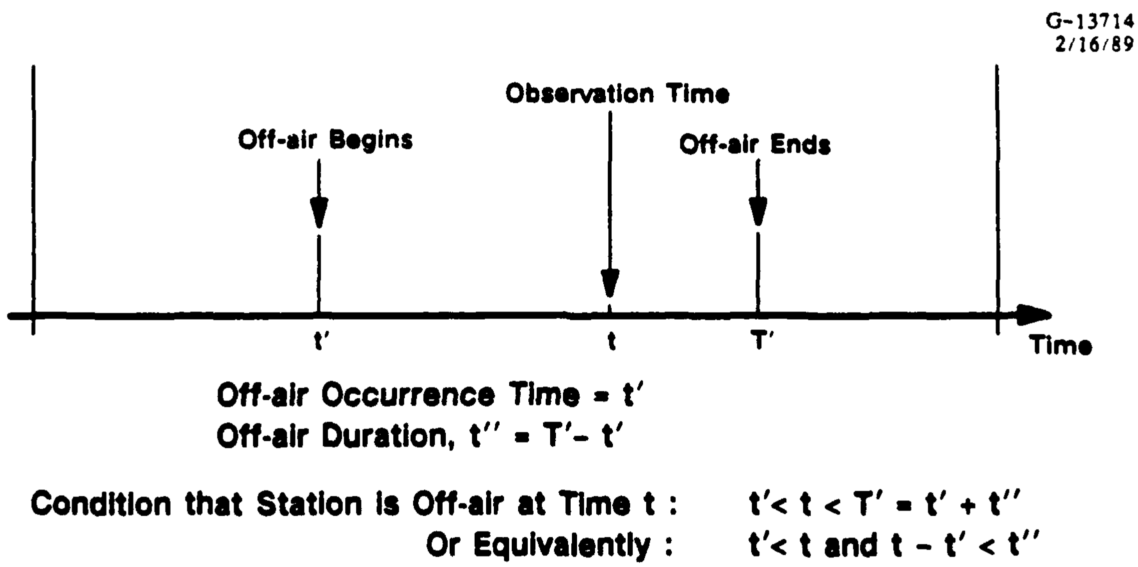


Figure A.2-3 Conditions Under Which a Station is Off-air at Time, *t* since $\mu >> \lambda$. Defining the month to begin at $t = 0$ and end at $t = T$, the average value of $P_{OA}(t)$ over the month may be computed as follows:

$$< P_{OA}(t) > \equiv \frac{1}{T} \int_0^T P_{OA}(t)\, dt$$

$$= \frac{\lambda^2}{\mu T} \left[\frac{1}{\lambda^2} (1 - e^{-\lambda T}) - \frac{T}{\lambda} e^{-\lambda T} \right] \qquad \text{(A.2-5)}$$

Now, assuming an average of about 3 off-air occurrences per month (i.e., $3/\lambda \approx T$)*, the exponential terms occurring inside the brackets in Eq. A.2-5 may be dropped in comparison to the non-exponential term. Thus,

$$< P_{OA}(t) > \cong \frac{\lambda^2}{\mu T} \left(\frac{1}{\lambda^2} \right) = \frac{1}{\mu T} = \frac{1/\mu}{T} = \frac{T_{OA}}{T} \qquad \text{(A.2-6)}$$

where T_{OA} is the average off-air duration and T is the total time in the month.

"Scheduled" off-airs which are not planned until after the beginning of the month can be modeled using the *a priori* probability functions (occurrence/duration) treated above with λ, μ given by historical reliability figures† for each station. Once the scheduled off-air is planned/announced, the randomness vanishes (for that particular kind of off-air) and the problem becomes deterministic. Equation A.2-6 may still be used as an approximation to the off-air probability, however, since it is valid except for those intervals during which advance information is known. For the completely deterministic cases/intervals, Eq. A.2-6 simply becomes a fractional off-air figure subject to the exclusion of concurrent scheduled off-airs from different stations (see Eq. A.1-3).

*This assumption is based on a sampling of off-airs (>1 min) in four separate months during 1988.

†Excluding scheduled off-airs for annual maintenance which are known well before the month begins and are thus completely deterministic.

APPENDIX B

ALGORITHM FOR COMPUTING VLF SIGNAL AMPLITUDE STANDARD DEVIATION

Based on VLF signal measurements and compilations by several research organizations, a semi-empirical algorithm has been developed to compute the standard deviation of VLF signal amplitude (Ref. 7). The algorithm is essentially a quantitative summary of the observations in terms of the following variables:

- Frequency
- Path illumination
- Fraction of path in equatorial band
- Local season (receiver and transmitter)
- Geomagnetic hemisphere (north/south) of transmitter.

The algorithm has been revised (Ref. 8) since the original development, presumably to reflect new/additional data.

The algorithm specifies the standard deviation of VLF signal amplitude, based on the following semi-empirical relationship:

$$\sigma_s = c_1 + c_2 (c_3 + c_4)$$

where the parameters c_1, c_2, c_3, and c_4 are defined as follows (all units in dB):

c_1 = 2.0

c_2 = $0.25 + 0.15 (f_{kHz} - 10)$; where f_{kHz} is the frequency in kHz, $10 \text{ kHz} \le f_{kHz} \le 30 \text{ kHz}$

c_3 = 0 if the path is fully illuminated (all-day path)
= 0.6 otherwise (night/transition paths)

c_4 = 0.5 if any portion of the path lies within the tropical belt (± 22.5° of the geographic equator)
= 0.5 if the local season at all points on the path is spring or autumn
= 1.0 if the path lies entirely in the non-tropical northern hemisphere (>22.5°) and the season is winter

= 1.0 if the path lies entirely in the non-tropical southern hemisphere (<22.5°) and the season is summer
= 0 if the path lies entirely in the non-tropical northern hemisphere (>22.5°) and the season is summer
= 0 if the path lies entirely in the non-tropical southern hemisphere (<22.5°) and the season is winter

The above algorithm for σ_S is specified (Ref. 7) for paths greater than 2 megameters (Mm) in length. A separate algorithm is presented for paths less than 2 Mm in length, but that algorithm is not used in the Omega System Availability Model for the following reasons:

- Since the daytime near-field (modal) region extends to a range of 2 Mm from the transmitter and the nighttime near-field range is even greater, very few station signals in the maximal coverage set will have a range less than 2 Mm
- The algorithm description (Ref. 7) states that the predicted signal amplitude standard deviations for paths less than 2 Mm are "largely speculative."

Because of the infrequent need and prediction uncertainty associated with signal amplitude standard deviations on paths less than 2 Mm in length, a default value of 3 dB (upper bound of the algorithm) for σ_S will be used if such paths are ever encountered.

APPENDIX C

APPROXIMATIONS TO THE SYSTEM AVAILABILITY MODEL FOR RANDOM SIGNAL AND NOISE LEVELS

Regarding the treatment of signal and noise levels, two types of system availability models are currently defined:

1) The "original" model which considers signal and noise levels (and therefore the coverage elements) as deterministic quantities.
2) The enhanced model which treats signal and noise levels as random variables.

In this Appendix, an approximation to Model (2) above (which may also be considered as a third, or alternative model) is described. This model treats noise envelope amplitude (averaged over several minutes) as a random quantity (same as Model (2)) but the signal amplitude from any given station/frequency is considered deterministic. This is consistent with the general observation that the signal amplitude standard deviation is smaller than the standard deviation of the noise envelope amplitude.

Using this model, signal-to-noise ratio (SNR) distribution functions are derived for a given threshold criterion. These distribution functions are used to compute the coverage elements needed in the calculation of P_{SA}. To facilitate understanding of the development, the SNR distribution functions are derived for one, two, and three station signals in sequence before presenting the general case.

A deterministic signal model means that the probability density function for amplitude of the i^{th} signal assumes the following form:

$$p_{S_i}(s_i) = \delta(s_i - \bar{s}_i) \tag{C-1}$$

where $\delta(\ )$ is the Dirac delta-function. In other words, the signal amplitude from station i is fixed at the predicted value, $\bar{s}_i$, for a given location/time. The noise envelope amplitude is assumed to be a random variable having a lognormal probability density function given by

$$p_N(n) = \frac{1}{\sigma_N \sqrt{2\pi}} e^{-(n-\bar{n})^2/2\sigma_N^2}$$

C.1 SNR DISTRIBUTION FUNCTIONS FOR ONE STATION SIGNAL

For a single station signal, the SNR probability density function (for logarithmically defined variables) is

$$p_R(x) = p_N(\bar{s} - x) = \frac{1}{\sigma_N \sqrt{2\pi}} e^{-(\bar{s}-x-\bar{n})^2/2\sigma_N^2} \qquad \text{(C-2)}$$

This equation states that the probability (per unit SNR interval) that the SNR is x is the probability (per unit noise interval) that the noise is $\bar{s}$-x. If event A is defined as $x \geq a$, where a is a minimum threshold of SNR (e.g. -20 dB in a 100 Hz bandwidth), then P(A) is a distribution function for SNR which is the integral of Eq. C-2 over the appropriate interval. Thus,

$$P(A) = P(x \geq a) = \int_a^\infty dx\, p_N(\bar{s} - x) \qquad \text{(C-3)}$$

By inserting the expression for the noise density function and performing suitable manipulation, it is seen that

$$P(A) = \frac{1}{\sqrt{\pi}} \int_{\frac{a-(\bar{s}-\bar{n})}{\sigma_N\sqrt{2}}}^{\infty} e^{-w^2} dw = \frac{1}{2} \mathrm{erfc}\left(\frac{a - (\bar{s} - \bar{n})}{\sigma_N \sqrt{2}}\right) \qquad \text{(C-4)}$$

where the complementary error function is defined by

$$\mathrm{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty e^{-y^2} dy \qquad \text{(C-5)}$$

To considerably simplify further development, a specific ranking is chosen for the deterministic signal amplitudes, i.e. for m signals,

$$\bar{s}_1 \geq \bar{s}_2 \geq \bar{s}_3 \ldots \geq \bar{s}_m \qquad \text{(C-6)}$$

It is important to differentiate this convention from the usual Omega convention in which subscripts indicate station number.

C.2 SNR DISTRIBUTION FUNCTIONS FOR TWO STATION SIGNALS

For two station signals of amplitudes $\bar{s}_1$ and $\bar{s}_2$, the joint SNR probability density function is

$$P_{R_1R_2}(x_1,x_2) = p_N(\bar{s}_1 - x_1)\delta(x_1 - x_2 - (\bar{s}_1 - \bar{s}_2)) \qquad \text{(C-7)}$$

The Dirac delta-function appearing in the expression for the density function limits the difference in SNR for station signals 1 and 2 to $\bar{s}_1 - \bar{s}_2$ due to the cancellation of the common noise.

The joint SNR distribution functions are defined in terms of the following events:

$A_i \equiv$ event that $\bar{s}_i - n \geq a$, where $\bar{s}_i$ is the deterministic signal amplitude from station i and n is the random noise level; both $\bar{s}_i$ and n have units of dB (relative to 1μ V/m)

$\bar{A}_i \equiv$ event that $\bar{s}_i - n < a$

Thus, using Eq. C-7,

$$P(A_1A_2) = \int_a^\infty dx_2 \int_a^\infty dx_1\, p_N(\bar{s}_1 - x_1)\, \delta(x_1 - x_2 - (\bar{s}_1 - \bar{s}_2)) \qquad \text{(C-8)}$$

Although this integral appears to be two-dimensional, the Dirac delta function in the integrand limits the integration to the line $x_1 - x_2 = \bar{s}_1 - \bar{s}_2$ shown in Fig. C.1-1. The figure shows that integration over the quarter-space $x_1 \geq a$, $x_2 \geq a$ is equivalent to integrating x_1 from $a + \bar{s}_1 - \bar{s}_2$ to infinity and x_2 from a to infinity. Thus, eq. (C-8) becomes

$$P(A_1A_2) = \int_a^\infty dx_2\, p_N(\bar{s}_2 - x_2) \qquad \text{(C-9)}$$

This expression has the same form as Eq. C-3 and thus, using Eq. C-4, Eq. C-9 becomes

$$P(A_1A_2) = \frac{1}{2}\, \text{erfc}\left(\frac{a - (\bar{s}_2 - \bar{n})}{\sigma_N \sqrt{2}}\right) \qquad \text{(C-10)}$$

Again, from Eq. (C-7), the joint distribution $P(A_1\bar{A}_2)$ is

$$P(A_1\bar{A}_2) = \int_{-\infty}^a dx_2 \int_a^\infty dx_1\, p_N(\bar{s}_1 - x_1)\, \delta(x_1 - x_2 - (\bar{s}_1 - \bar{s}_2))$$

Figure C.1-1 shows that integration over the quarter space $x_1 \geq a$, $x_2 \leq a$ is equivalent to integrating x_1 from a to $a + \bar{s}_1 - \bar{s}_2$ and x_2 from $a - (\bar{s}_1 - \bar{s}_2)$ to a. Thus,

$$P(A_1\bar{A}_2) = \int_{a-(\bar{s}_1-\bar{s}_2)}^a dx_2\, p_N(\bar{s}_2 - x_2) = \int_{a-(\bar{s}_1-\bar{s}_2)}^\infty dx_2\, p_N(\bar{s}_2 - x_2) - \int_a^\infty dx_2 p_N(\bar{s}_2 - x_2)$$

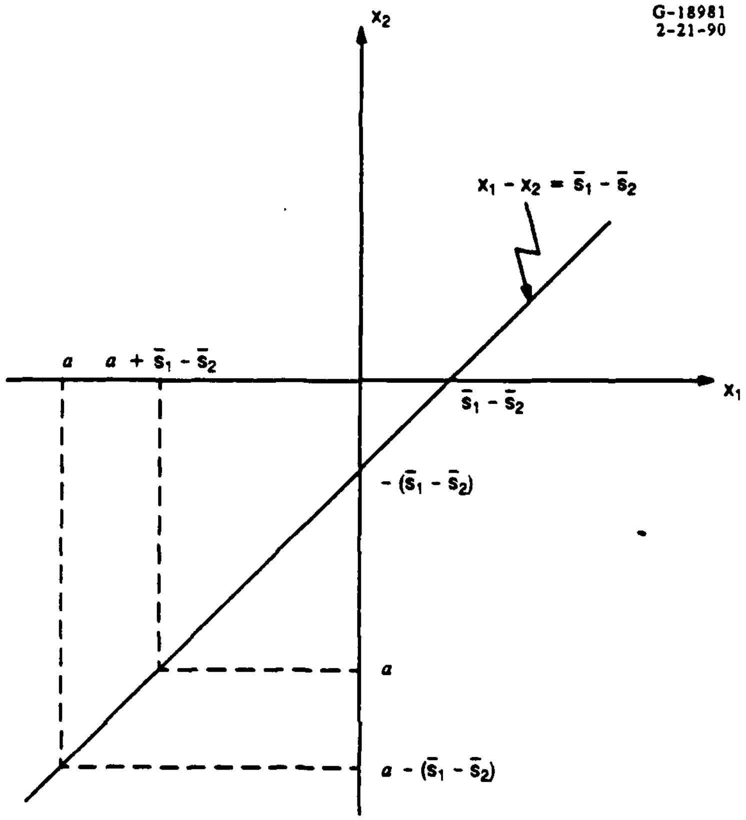


Figure C.1-1 Integration Regions for Joint Two-Signal SNR Distribution Function ($\bar{s}_1 > \bar{s}_2$)

Thus, $P(A_1\overline{A}_2)$ is expressed as the difference of two integrals, each of which may be compared with Eqs. C-3, C-4 to yield

$$P(A_1\overline{A}_2) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{a-(\bar{s}_1-\bar{s}_2)-(\bar{s}_2-\bar{n})}{\sigma_N\sqrt{2}}\right) - \frac{1}{2}\,\mathrm{erfc}\left(\frac{a-(\bar{s}_2-\bar{n})}{\sigma_N\sqrt{2}}\right)$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\frac{a-(\bar{s}_1-\bar{n})}{\sigma_N\sqrt{2}}\right) - \frac{1}{2}\,\mathrm{erfc}\left(\frac{a-(\bar{s}_2-\bar{n})}{\sigma_N\sqrt{2}}\right) \qquad \text{(C-11)}$$

In a similar way, the joint distribution $P(\overline{A}_1A_2)$ is computed by integrating over the quarter space $x_1 \le a$, $x_2 \ge a$. However, from Fig. C.1-1, it is seen that the line $x_1 - x_2 = \bar{s}_1 - \bar{s}_2$ does not intersect that quarter space so that

$$P(\overline{A}_1A_2) = 0 \qquad \text{(C-12)}$$

The joint distribution for the fourth and final event combination is $P(\bar{A}_1\bar{A}_2)$. This quantity is obtained by normalization, i.e.,

$$P(A_1A_2) + P(A_1\bar{A}_2) + P(\bar{A}_1A_2) + P(\bar{A}_1\bar{A}_2) = 1$$

Thus, using Eqs. C-10, C-11, and C-12 for $P(A_1A_2)$, $P(A_1\bar{A}_2)$, and $P(\bar{A}_1A_2)$, respectively, Eq. C-12 yields

$$P(\bar{A}_1\bar{A}_2) = 1 - \frac{1}{2}\ \mathrm{erfc}\left(\frac{a - (\bar{s}_1 - \bar{n})}{\sigma_N\sqrt{2}}\right)$$

C.3 SNR DISTRIBUTION FUNCTIONS FOR THREE STATION SIGNALS

For three station signals, the convention (Eq. C-6) is again invoked:

$$\bar{s}_1 \geq \bar{s}_2 \geq \bar{s}_3$$

The SNR probability density function for three station signals is analogous to that for two-station signals (Eq. C-7), i.e.,

$$p_{R_1R_2R_3}(x_1, x_2, x_3) = p_N(\bar{s}_1 - x_1)\delta(x_1 - x_2 - (\bar{s}_1 - \bar{s}_2))\delta(x_2 - x_3 - (\bar{s}_2 - \bar{s}_3))$$

The Dirac delta functions are again present to limit the SNR differences between the three signals to the differences in the respective deterministic signal amplitudes (only two independent differences/delta functions for three station signals).

For three station signals, eight SNR event combinations are defined: $A_1A_2A_3$, $A_1A_2\bar{A}_3$, $A_1\bar{A}_2A_3$, $\bar{A}_1A_2A_3$, $A_1\bar{A}_2\bar{A}_3$, $\bar{A}_1A_2\bar{A}_3$, $\bar{A}_1\bar{A}_2A_3$, and $\bar{A}_1\bar{A}_2\bar{A}_3$. Joint distribution functions, which give the probability of these event intersections are calculated using methods similar to those described in Section C.2 for the two station signal case. The integration limits are obtained from two plots similar to Fig. C.1-1: one for the pair of SNR variables x_1,x_2 and another for the pair x_2,x_3. Omitting the details of the calculations, it can be shown that

$$P(A_1A_2A_3) = \frac{1}{2}\ \mathrm{erfc}\left(\frac{a - (\bar{s}_3 - \bar{n})}{\sigma_N\sqrt{2}}\right)$$

$$P(A_1A_2\bar{A}_3) = \frac{1}{2}\ \mathrm{erfc}\left(\frac{a - (\bar{s}_2 - \bar{n})}{\sigma_N\sqrt{2}}\right) - \frac{1}{2}\ \mathrm{erfc}\left(\frac{a - (\bar{s}_3 - \bar{n})}{\sigma_N\sqrt{2}}\right)$$

$$P(A_1\overline{A}_2A_3) = 0 = P(\overline{A}_1A_2A_3)$$

$$P(A_1\overline{A}_2\overline{A}_3) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{a-(\bar{s}_1-\bar{n})}{\sigma_N\sqrt{2}}\right) - \frac{1}{2}\,\mathrm{erfc}\left(\frac{a-(\bar{s}_2-\bar{n})}{\sigma_N\sqrt{2}}\right)$$

$$P(\overline{A}_1A_2\overline{A}_3) = 0 = P(\overline{A}_1\overline{A}_2A_3)$$

$$P(\overline{A}_1\overline{A}_2\overline{A}_3) = 1 - \frac{1}{2}\,\mathrm{erfc}\left(\frac{a-(\bar{s}_1-\bar{n})}{\sigma_N\sqrt{2}}\right)$$

C.4 SNR DISTRIBUTION FUNCTIONS FOR THE GENERAL CASE

From the results given in Section C.3, the general case for m station signals may be inferred. With the basic ordering convention $\bar{s}_1 \geq \bar{s}_2 \geq \bar{s}_3 \geq \cdots \geq \bar{s}_m$, the general joint distribution function for the intersection of the m A-events is

$$P(A_1A_2\ldots A_m) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{a-(\bar{s}_m-\bar{n})}{\sigma_N\sqrt{2}}\right)$$

$$P(A_1A_2\ldots\overline{A}_m) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{a-(\bar{s}_{m-1}-\bar{n})}{\sigma_N\sqrt{2}}\right) - \frac{1}{2}\,\mathrm{erfc}\left(\frac{a-(\bar{s}_m-\bar{n})}{\sigma_N\sqrt{2}}\right)$$

$$\vdots$$

$$P(A_1A_2\ldots A_{r-1}\overline{A}_r\ldots\overline{A}_m) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{a-(\bar{s}_{r-1}-\bar{n})}{\sigma_N\sqrt{2}}\right) - \frac{1}{2}\,\mathrm{erfc}\left(\frac{a-(\bar{s}_r-\bar{n})}{\sigma_N\sqrt{2}}\right)$$

$$\vdots$$

$$P(A_1\overline{A}_2\ldots\overline{A}_m) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{a-(\bar{s}_1-\bar{n})}{\sigma_N\sqrt{2}}\right) - \frac{1}{2}\,\mathrm{erfc}\left(\frac{a-(\bar{s}_2-\bar{n})}{\sigma_N\sqrt{2}}\right)$$

$$P(\overline{A}_1\overline{A}_2\ldots\overline{A}_m) = 1 - \frac{1}{2}\,\mathrm{erfc}\left(\frac{a-(\bar{s}_1-\bar{n})}{\sigma_N\sqrt{2}}\right)$$

P (event combinations other than those listed above) = 0

The above expressions are used to calculate the local coverage elements in the expression for the system availability index, P_{SA}. The inputs required for calculation are the signal amplitudes (for a given time/location) for the signals in the maximal coverage set, the median noise level/ standard deviation (for the given location/time), and the minimum SNR threshold level.

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