ONR/RR-90-2

PREDICTIONS OF RELIABILITY COEFFICIENTS AND STANDARD ERRORS OF MEASUREMENT USING THE TEST INFORMATION FUNCTION AND ITS MODIFICATIONS

FUMIKO SAMEJIMA

UNIVERSITY OF TENNESSEE

KNOXVILLE, TENN. 37996-0900

JULY, 1990

Prepared under the contract number N00014-87-K-0320,
4421-549 with the
Cognitive Science Research Program
Cognitive and Neural Sciences Division
Office of Naval Research

R01-1069-11-003-91

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No 0704-0188

1a REPORT SECURITY CLASSIFICATION: Unclassified	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; Distribution unlimited
2b DECLASSIFICATION/DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S)	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION: Fumiko Samejima, Ph.D. Psychology Department; 6b OFFICE SYMBOL (If applicable)	7a NAME OF MONITORING ORGANIZATION: Cognitive Science 1142 CS
6c ADDRESS (City, State, and ZIP Code): 310B Austin Peay Building, The University of Tennessee, Knoxville, TN 37996-0900	7b ADDRESS (City, State, and ZIP Code): Office of Naval Research, 800 N. Quincy Street, Arlington, VA 22217
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: Cognitive Science Research Program; 8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: N00014-87-K-0320
8c ADDRESS (City, State, and ZIP Code): Office of Naval Research, 800 N. Quincy Street, Arlington, VA [illegible]217	10 SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO	PROJECT NO	TASK NO	WORK UNIT ACCESSION NO
61153N	RR-042-04	042-04-01	4421-549

11 TITLE (Include Security Cla[illegible]ication)
Predictions of reliability coefficients and standard errors of measurement using the test information function and its modifications

12 PERSONAL AUTHOR(S)
Fumiko Samejima, Ph.D.

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
technical report	FROM 1987 TO 1990	1990, June, 30	28

16 SUPPLEMENTARY NOTATION

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Latent Trait Models, Mental Test Theory, Test Reliability, Standard Error of Measurement

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

Since we have more useful and informative measures like the test information function and its two modified formulae, the reliability coefficient of a test is no longer necessary in modern mental test theory. And yet it is interesting to know how to predict the coefficient using these functions, which are *tailored for each separate population of examinees.* In this process, it will become more obvious that the traditional concept of test reliability *is misleading*, for without changing the test the coefficient can be drastically different if we change the population of examinees.

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT: ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION
22a. NAME OF RESPONSIBLE INDIVIDUAL: Dr. Charles E. Davis	22b TELEPHONE (Include Area Code): 202-696-4046; 22c OFFICE SYMBOL: ONR-1142-CS

DD Form 1473, JUN 86 Previous editions are obsolete. SECURITY CLASSIFICATION OF THIS PAGE

S/N 0102-LF-014-6603

TABLE OF CONTENTS

The research was conducted at the principal investigator's laboratory, 405 Austin Peay Bldg., Department of Psychology, University of Tennessee, Knoxville, Tennessee. Those who worked as assistants for this research include Christine A. Golik, Barbara A. Livingston, Lee Hai Gan and Nancy H. Domm.

I Introduction

There seems to be a concensus that two main measures in classical mental test theory are the reliability and validity coefficients of a test. Although these measures have widely been accepted by psychologists and test users in the past decades, they are actually the attributes of a specified group of examinees as well as of a given test, since the correlation coefficient is used in either case. In addition, representation of these measures by single numbers results in over-simplification and the lack of useful information for both theorists and actual users of tests. The same applies for the standard error of measurement also.

In latent trait models, the item and test information functions provide us with abundant information about the *local* accuracy of estimation, a concept which is totally missing in classical mental test theory. These functions do not depend upon any specific group of examinees as the reliability coefficient does, or we can say that they are population-free. By virtue of this characteristic, adding further information about the MLE bias function of the test and the ability distribution of the examinee group, we can provide the *tailored* reliability coefficient and standard error of measurement in the classical mental test theory's sense for each and every specified group of examinees who have taken the same test! (cf. Samejima, 1977b, 1987).

This progressive desolution of the reliability coefficient and of the standard error of measurement in classical mental test theory and their replacement by the test information function in latent trait models is further facilitated by the recent proposal of the modifications of the test information function, using the MLE bias function (cf. Samejima, 1987, 1990). In the present paper, it will be shown how we can predict the *so-called* reliability coefficient and standard error of measurement *of a test* in the sense of classical mental test theory, taking advantage of the new developments in latent trait models.

II Test Information Function and Its Modifications

Let θ be ability, or latent trait, which takes on any real number. We assume that there is a set of n test items measuring θ whose characteristics are known. Let g denote such an item, k_g be a discrete item response to item g, and $P_{k_g}(\theta)$ denote the operating characteristic of k_g, or the conditional probability assigned to k_g, given θ, i.e.,

$$(2.1) \qquad P_{k_g}(\theta) = Prob.[k_g \mid \theta] .$$

We assume that $P_{k_g}(\theta)$ is three-times differentiable with respect to θ. We have for the *item response information function* (Samejima, 1972)

$$(2.2) \qquad I_{k_g}(\theta) = -\frac{\partial^2}{\partial\theta^2}\log P_{k_g}(\theta) ,$$

and the *item information function*, $I_g(\theta)$, is defined as the conditional expectation of $I_{k_g}(\theta)$, given θ, such that

$$(2.3) \qquad I_g(\theta) = E[I_{k_g}(\theta) \mid \theta] = \sum_{k_g} I_{k_g}(\theta)P_{k_g}(\theta) .$$

In the special case where the item g is scored dichotomously, this item information function is simplified to become

$$(2.4)\qquad I_g(\theta) = [\frac{\partial}{\partial\theta} P_g(\theta)]^2 [\{P_g(\theta)\}\{1-P_g(\theta)\}]^{-1} ,$$

where $P_g(\theta)$ is the operating characteristic of the correct answer to item g . Let V be a *response pattern* such that

$$(2.5)\qquad V = \{ k_g \}' \qquad g = 1,2,...,n .$$

The operating characteristic, $P_V(\theta)$, of the response patten V is defined as the conditional probability of V , given θ , and by virtue of *local independence* we can write

$$(2.6)\qquad P_V(\theta) = \prod_{k_g \epsilon V} P_{k_g}(\theta) .$$

The *response pattern information function* (Samejima, 1972), $I_V(\theta)$, is given by

$$(2.7)\qquad I_V(\theta) = -\frac{\partial^2}{\partial\theta^2} \log P_V(\theta) = \sum_{k_g \epsilon V} I_{k_g}(\theta) ,$$

and the *test information function*, $I(\theta)$, is defined as the conditional expectation of $I_V(\theta)$, given θ , and we obtain from (2.2), (2.3), (2.5), (2.6) and (2.7)

$$(2.8)\qquad I(\theta) = E[I_V(\theta) \mid \theta] = \sum_V I_V(\theta) P_V(\theta) = \sum_{g=1}^{n} I_g(\theta) .$$

A big advantage of modern mental test theory is that the standard error of estimation can *locally* be defined by using $[I(\theta)]^{-1/2}$. Unlike its counterpart in classical mental test theory, this function *does not depend upon the population of examinees,* but is solely a property of the test itself, which should be the way if we call it the standard error, or the reliability, *of a test.* It is well known that this function provides us with the asymptotic standard deviation of the conditional distribution of the maximum likelihood estimate of θ , given its true value.

Lord has proposed a bias function for the maximum likelihood estimate of θ in the three-parameter logistic model whose operating characteristic of the correct answer, $P_g(\theta)$, is given by

$$(2.9)\qquad P_g(\theta) = c_g + (1-c_g)[1+\exp\{-Da_g(\theta-b_g)\}]^{-1} ,$$

where a_g , b_g , and c_g are the item discrimination, difficulty, and guessing parameters, and D is a scaling factor, which is set equal to 1.7 when the logistic model is used as a substitute for the normal ogive model. Lord's bias function, which is denoted by $B(\hat{\theta}_V \mid \theta)$ in this paper, can be written as

$$(2.10)\qquad B(\hat{\theta}_V \mid \theta) = D[I(\theta)]^{-2} \sum_{g=1}^{n} a_g I_g(\theta) [\psi_g(\theta) - \frac{1}{2}] ,$$

where

$$(2.11)\qquad \psi_g(\theta) = [1+\exp\{-Da_g(\theta-b_g)\}]^{-1}$$

(cf. Lord, 1983). We can see in the above formula of Lord's MLE bias function that the bias should be negative when $\psi_g(\theta)$ is less than 0.5 for all the items, which is necessarily the case for some interval of θ, $(-\infty, \theta_L)$, and should be positive when $\psi_g(\theta)$ is greater than 0.5 for all the items, which also necessarily happens for some interval, (θ_H, ∞), and in between the bias tends to be close to sero, for the last factor in this formula assumes negative values for some items and positive for some others, and, therefore, they cancel themselves out, provided that the difficulty parameter b_g distributes widely. Lord has applied this MLE bias function to an 85-item SAT Verbal test (Lord, 1984), and the result shows a fairly wide range of θ in which the bias is practically nil.

In the general case of discrete item responses, we obtain for the bias function of the maximum likelihood estimate (cf. Samejima, 1987)

$$(2.12) \qquad B(\hat{\theta}_V \mid \theta) = E[\hat{\theta}_V - \theta \mid \theta] = -(1/2)[I(\theta)]^{-2} \sum_{g=1}^{n} \sum_{k_g} A_{k_g}(\theta) P''_{k_g}(\theta)$$
$$= -(1/2)[I(\theta)]^{-2} \sum_{g=1}^{n} \sum_{k_g} P'_{k_g}(\theta) P''_{k_g}(\theta) [P_{k_g}(\theta)]^{-1} ,$$

where $A_{k_g}(\theta)$ is the basic function for the discrete item response k_g, and $P'_{k_g}(\theta)$ and $P''_{k_g}(\theta)$ denote the first and second partical derivatives of $P_{k_g}(\theta)$ with respect to θ, respectively. On the graded response level where item score x_g assumes successive integers, 0 through m_g, each k_g in the above formula must be replaced by the graded item score x_g. On the dichotomous response level, it can be reduced to the form

$$(2.13) \qquad B(\hat{\theta}_V \mid \theta) = E[\hat{\theta}_V - \theta \mid \theta] = (-1/2)[I(\theta)]^{-2} \sum_{g=1}^{n} I_g(\theta) P''_g(\theta) [P'_g(\theta)]^{-1} ,$$

with $P'_g(\theta)$ and $P''_g(\theta)$ indicating the first and second partial derivatives of $P_g(\theta)$ with respect to θ, respectively. This formula includes Lord's bias function in the three-parameter logistic model as a special case.

Using this MLE bias function and taking the reciprocal of an approximate minimum variance bound of the maximum likelihood estimator, a modified test information function, $\Upsilon(\theta)$, has been defined by

$$(2.14) \qquad \Upsilon(\theta) = I(\theta) \, [1 + \frac{\partial}{\partial\theta} B(\hat{\theta}_V \mid \theta)]^{-2} ,$$

which is a reciprocal of an approximate minimum bound of the maximum likelihood estimator (cf. Samejima, 1990). From this formula, we can see that the relationship between this new function and the original test information function depends upon the first derivative of the MLE bias function. To be more precise, if the derivative is positive, then the new function will assume a lesser value than the original test information function. If it is negative, then this relationship will be reversed. If it is sero, i.e., if the MLE is conditionally unbiased, then these two functions will assume the same value.

The second modified test information function, $\Xi(\theta)$, is defined by

$$(2.15) \qquad \Xi(\theta) = I(\theta) \, \{[1 + \frac{\partial}{\partial\theta} B(\hat{\theta}_V \mid \theta)]^2 + I(\theta) \, [B(\hat{\theta}_V \mid \theta)]^2\}^{-1} ,$$

which is the reciprocal of an approximate minimum bound of the *mean squared error* of the maximum likelihood estimator (cf. Samejima, 1990). We can see that the difference between the two modified test information functions, $\Upsilon(\theta)$ and $\Xi(\theta)$, is the second and last term in the braces of the right hand side of formula (2.15). Since this term is nonnegative, we have

$$\Xi(\theta) \leq \Upsilon(\theta) \tag{2.16}$$

throughout the whole range of θ , regardless of the slope of the MLE bias function.

When the MLE bias function of the test is monotone increasing, as is the case with many existing tests, it is obvious from (2.14) that $\Upsilon(\theta)$ will assume no greater values than those of the original test information function $I(\theta)$. The same applies to $\Xi(\theta)$, and we have the relationship,

$$\Xi(\theta) \leq \Upsilon(\theta) \leq I(\theta) \ , \tag{2.17}$$

throughout the whole range of θ .

III Reliability Coefficient of a Test in the Sense of Classical Mental Test Theory

Although we can handle the concept of *reliability* much better in modern mental test theory by using the test information function, $I(\theta)$, or one of its modification formulae, $\Upsilon(\theta)$ or $\Xi(\theta)$, it has also been observed (Samejima, 1977b) that, if we wish, the reliability coefficient of a test in the sense of classical mental test theory can be obtained easily from the observed data and the test information function under a general condition. Since we have two modification formulae of the test information function now, we are in a position that can handle the prediction of the reliability coefficient *tailored* for a specified population of examinees even better.

[III.1] General Case

Let θ_V^* be any estimator of ability θ . We can write

$$\theta_V^* = \theta + \varepsilon \ , \tag{3.1}$$

where ε denotes the error variable. In the test-retest situation, we have

$$\begin{cases} \theta_{V1}^* = \theta + \varepsilon_1 \\ \theta_{V2}^* = \theta + \varepsilon_2 \ , \end{cases} \tag{3.2}$$

where the subscripts, 1 and 2 , indicate the test and retest situations, respectively. If we can reasonably assume that in the test and retest situations:

$$Cov.(\varepsilon_1, \varepsilon_2) = 0 \ , \tag{3.3}$$

$$Var.(\varepsilon_1) = Var.(\varepsilon_2) \tag{3.4}$$

and

(3.5) $$Cov.(\theta, \varepsilon_1) = Cov.(\theta, \varepsilon_2) = 0 ,$$

then we will have

(3.6) $$Corr.(\theta^*_{V1}, \theta^*_{V2}) = [Var.(\theta^*_{V1}) - Var.(\varepsilon_1)][Var.(\theta^*_{V1})]^{-1} .$$

Note that if we replace ability θ by one of its transformed forms, true test score T, and use the observed test score X as the estimator of T and E as its error of estimation, then (3.1) can be rewritten in the form

(3.7) $$T = X + E ,$$

which represents the fundamental assumption in classical mental test theory, and (3.6) becomes a familiar formula for the reliability coefficient $r_{X_1X_2}$,

(3.8) $$r_{X_1X_2} = Var.(T)[Var.(X)]^{-1} .$$

In classical mental test theory, however, researchers seldom check if these assumptions are acceptable. In fact, in many cases (3.5) is violated if we replace θ by T, and ε_1 and ε_2 by E_1 and E_2, respectively, unless the test has been constructed in such a way that most individuals from the target population have mediocre true scores.

We can write in general

(3.9) $$\begin{aligned} Var.(\varepsilon) &= E[\varepsilon - E(\varepsilon)]^2 \\ &= E[\varepsilon - E(\varepsilon \mid \theta)]^2 + E[E(\varepsilon \mid \theta) - E(\varepsilon)]^2 \\ &\quad + 2E[(\varepsilon - E(\varepsilon \mid \theta))(E(\varepsilon \mid \theta) - E(\varepsilon))] . \end{aligned}$$

This indicates that, if the error variable ε is conditionally unbiased for the interval of θ of interest, then (3.9) will be reduced to the form

(3.10) $$Var.(\varepsilon) = E[\varepsilon^2] .$$

[III.2] Maximum Likelihood Estimator

Let $\hat{\theta}_V$ or $\hat{\theta}$ denote the maximum likelihood estimator of θ based upon the response pattern V. If 1) $\hat{\theta}$ is conditionally unbiased for the interval of θ of interest and 2) the test information function $I(\theta)$ assumes reasonably high values for that interval, then we will be able to approximate the conditional distribution of $\hat{\theta}$, given θ, by the normal distribution $N(\theta, [I(\theta)]^{-1/2})$ for the interval of θ within which the examinees' ability practically distributes. Thus we have from (3.10)

$$(3.11) \qquad Var.(\varepsilon) \doteq E[\{I(\theta)\}^{-1}] \ .$$

When this is the case, from (3.6) we can write

$$(3.12) \qquad Corr.(\hat{\theta}_1, \hat{\theta}_2) = [Var.(\hat{\theta}_1) - E[\{I(\theta)\}^{-1}]][Var.(\hat{\theta}_1)]^{-1} \ .$$

Thus the reliability coefficient in the sense of classical mental test theory can be predicted by a single administration of the test, given the test information function $I(\theta)$ and the ability distribution of the examinees.

It has also been observed that in computerised adaptive testing we can predict the reliability coefficient if a specified amount of test information is used for the stopping rule for a given level of ability in each of the test and retest situations, provided that the above two conditions 1) and 2) are met. In such a case, we can write

$$(3.13) \qquad Corr.(\hat{\theta}_1, \hat{\theta}_2) = [Var.(\hat{\theta}_1) - E[\{I_{(1)}(\theta)\}^{-1}]][Var.(\hat{\theta}_1)\{Var.(\hat{\theta}_1) - E[\{I_{(1)}(\theta)\}^{-1}] + E[\{I_{(2)}(\theta)\}^{-1}]\}]^{-1/2} \ ,$$

where $I_{(1)}(\theta)$ and $I_{(2)}(\theta)$ are the *preset* criterion test information functions in the test and retest situations, respectively, which are adopted as the stopping rules for the two separate situations. Note that these two criterion test information functions need not be the same, and also that the reliability coefficient is obtainable from a single administration. In a simplified case where, in each situation, the same amount of test information is used as the criterion for terminating the presentation of new items for every examinee, we can rewrite the above formula into the form

$$(3.14) \qquad Corr.(\hat{\theta}_1, \hat{\theta}_2) = [Var.(\hat{\theta}_1) - \sigma_1^2][Var.(\hat{\theta}_1)\{Var.(\hat{\theta}_1) - \sigma_1^2 + \sigma_2^2\}]^{-1/2} \ ,$$

where σ_1^2 and σ_2^2 are the reciprocals of the constant amounts of criterion test information in the two separate situations, respectively. If we use the same constant amount of test information as the stopping rule in both the test and retest situations, then the reliability coefficient takes the simplest form

$$(3.15) \qquad Corr.(\hat{\theta}_1, \hat{\theta}_2) = [Var.(\hat{\theta}_1) - \sigma^2][Var.(\hat{\theta}_1)]^{-1} \ ,$$

where σ^2 denotes the reciprocal of this common constant amount of test information.

The appropriateness of the above normal approximation of the conditional distribution of $\hat{\theta}$, given θ, can be examined by the Monte Carlo method (cf. Samejima, 1977a). We also notice that a necessary condition for this approximation is that $\hat{\theta}$ is conditionally unbiased for the interval of θ of interest. Thus we can use the MLE bias function, which was introduced in Section 2, for a test for the support of the approximation. Note that the MLE bias function together with the ability distribution of the target population also determines whether the assumption described by (3.5) should be accepted.

If the conditional unbiasedness is *not* supported, i.e., if $B(\hat{\theta}_V \mid \theta)$ does not approximately equal zero for all values of θ in the interval of interest, however, then we shall be able to adopt one of the modified test information functions, $\Upsilon(\theta)$ or $\Xi(\theta)$. Thus we can rewrite (3.12) into the forms

$$(3.16) \qquad Corr.(\hat{\theta}_1, \hat{\theta}_2) = [Var.(\hat{\theta}_1) - E[\{\Upsilon(\theta)\}^{-1}]][Var.(\hat{\theta}_1)]^{-1}$$

and

$$(3.17) \qquad Corr.(\hat{\theta}_1, \hat{\theta}_2) = [Var.(\hat{\theta}_1) - E[\{\Xi(\theta)\}^{-1}]][Var.(\hat{\theta}_1)]^{-1} .$$

We can decide which of the modified formulae, (3.16) or (3.17), is more appropriate to use in a specified situation. Also in computerized adaptive testing, either $\Upsilon(\theta)$ or $\Xi(\theta)$ can be used as the stopping rule in place of the test information function $I(\theta)$, and we can revise (3.13) into the forms

$$(3.18) \qquad Corr.(\hat{\theta}_1, \hat{\theta}_2) = [Var.(\hat{\theta}_1) - E[\{\Upsilon_{(1)}(\theta)\}^{-1}]][Var.(\hat{\theta}_1)\{Var.(\hat{\theta}_1) - E[\{\Upsilon_{(1)}(\theta)\}^{-1}] + E[\{\Upsilon_{(2)}(\theta)\}^{-1}]\}]^{-1/2} ,$$

and

$$(3.19) \qquad Corr.(\hat{\theta}_1, \hat{\theta}_2) = [Var.(\hat{\theta}_1) - E[\{\Xi_{(1)}(\theta)\}^{-1}]][Var.(\hat{\theta}_1)\{Var.(\hat{\theta}_1) - E[\{\Xi_{(1)}(\theta)\}^{-1}] + E[\{\Xi_{(2)}(\theta)\}^{-1}]\}]^{-1/2} ,$$

where the subscripts (1) and (2) represent the test and retest situations, respectively.

IV Standard Error of Measurement of a Test in the Sense of Classical Mental Test Theory

In classical mental test theory, the standard error of estimation of ability is represented by a single number, which is heavily affected by the degree of heterogeneity of the group of examinees tested, as is the case with the reliability coefficient. In contrast, in latent trait models, the standard error of estimation is *locally* defined, i.e., as a function of ability, which is the reciprocal of the square root of test information function. Since the test information function does not depend upon any specific group of examinees, but is a *sole* property of the test itself, this locally defined standard error is much more appropriate than the standard error of estimation in classical mental test theory. Also this function indicates that no test is efficient in ability measurement for the entire range of ability, and each test provides us with large amounts of information *only locally*, which makes a perfect sense to our knowledge.

The standard error of measurement of a test *tailored for a specific ability distribution* is given by

$$(4.1) \qquad S.E. = E[\{I(\theta)\}^{-1/2}]$$

when the conditions 1) and 2) described in the preceding section are met, and by

(4.2) $$S.E.1 = E[\{\Upsilon(\theta)\}^{-1/2}]$$

or

(4.3) $$S.E.2 = E[\{\Xi(\theta)\}^{-1/2}]$$

otherwise.

V Examples

For the purpose of illustration, six ability distributions are hypothesized, and for a single test predictions are made for their *tailored* reliability coefficients and *tailored* standard errors of measurement in the sense of classical mental test theory, using (3.12), (3.16), (3.17), (4.1), (4.2) and (4.3). These six hypothetical ability distributions are normal distributions, i.e., $N(0.0, 1.0)$, $N(-0.8, 1.0)$, $N(0.0, 0.5)$, $N(-0.8, 0.5)$, $N(-1.6, 0.5)$ and $N(-2.4, 0.5)$. Figure 5-1 presents the density functions of these six distributions. The hypothetical test consists of thirty equivalent dichotomous items, which follow the logistic model represented by (2.9) with $c_g = 0.0$, and the common parameter values $a_g = 1.0$ and $b_g = 0.0$, respectively, with the scaling factor D set equal to 1.7. Figure 5-2 presents the MLE bias function of this hypothetical test. We can see in this figure that outside the interval of θ, $(-1.0,\ 1.0)$, the amount of bias is substantially large. The square roots of the test information function $I(\theta)$ and of its two modification formulae $\Upsilon(\theta)$ and $\Xi(\theta)$ of this test are shown in Figure 5-3.

Tables 5-1 and 5-2 present the resulting predicted reliability coefficients and standard errors of measurement for the six different ability distributions, respectively. In each table, the mean and the variance of θ of each of the six distributions are also given. We can see that these variances are slightly different from the squares of the second parameters of the normal distributions, i.e., 0.98322 vs. 1.00000 for the populations 1 and 2, and 0.25155 vs. 0.25000 for the populations 3, 4, 5 and 6, respectively, whereas all of the means are the same as the first parameters of the normal distributions. These discrepancies in variance come from the fact that we used frequencies for the equally spaced points of θ with the step width 0.05, which are given as integers, in order to approximate the normal distributions, instead of using the density functions themselves.

As you can see in the first table, the predicted reliability coefficient obtained by (3.12) distributes widely, i.e., it varies from 0.200 to 0.896 ! The the coefficient reduces as the main part of the distribution shifts from a range of θ where the amount of test information is greater to another range where it is lesser. The reduction is more conspicuous when the standard deviation of the normal distribution is smaller. The predicted reliability coefficient obtained by (3.16) using $\Upsilon(\theta)$ instead of $I(\theta)$ indicates a substantial reduction from the one obtained by (3.12) for each of the six ability distributions. The reduction is especially conspicuous for the populations 2, 5, and 6, whose ability distributes on lower levels of θ where the discrepancies between $I(\theta)$ and $\Upsilon(\theta)$ are large. Among the six populations the predicted reliability coefficient obtained by means of (3.16) varies from 0.012 to 0.781, showing even a larger range than that obtained by (3.12). Similar results were obtained for the predicted reliability coefficient given by (3.17), using $\Xi(\theta)$ instead of $I(\theta)$. The reliability coefficient varies from 0.011 to 0.766, and within each population the reduction in the value of the reliability coefficient from the one obtained by (3.16) is relatively small, as is expected from Figure 5-3.

As for the standard error of measurement, we can see in Table 5-2 that similar results were obtained, only in reversed order, of course. In classical mental test theory, the standard error of measurement σ_E is given by

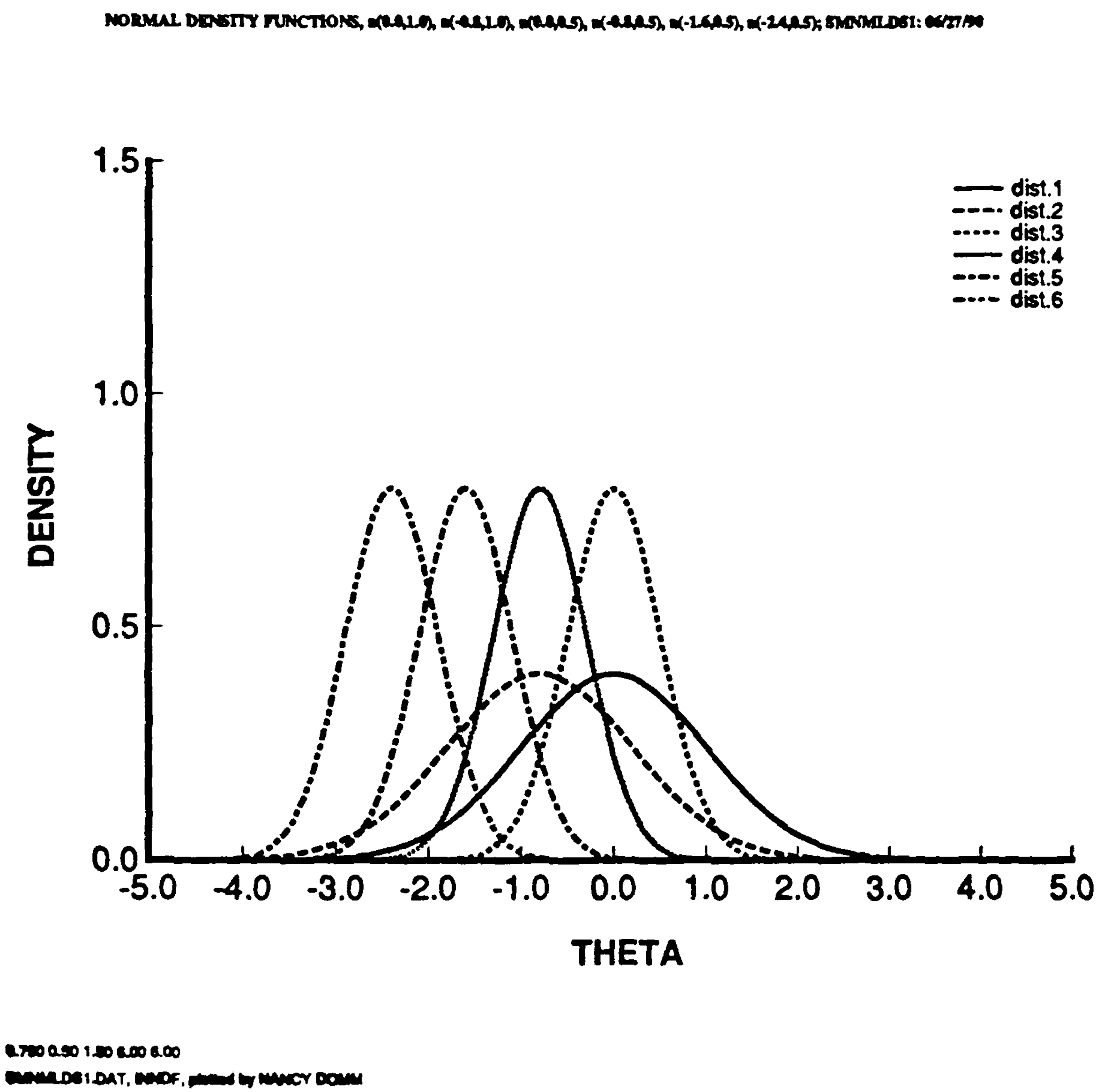


FIGURE 5-1

Density Functions of Six Hypothetical Ability Distributions: n(0.0, 1.0), n(-0.8, 1.0), n(0.0, 0.5), n(-0.8, 0.5), n(-1.6, 0.5) and n(-2.4, 0.5).

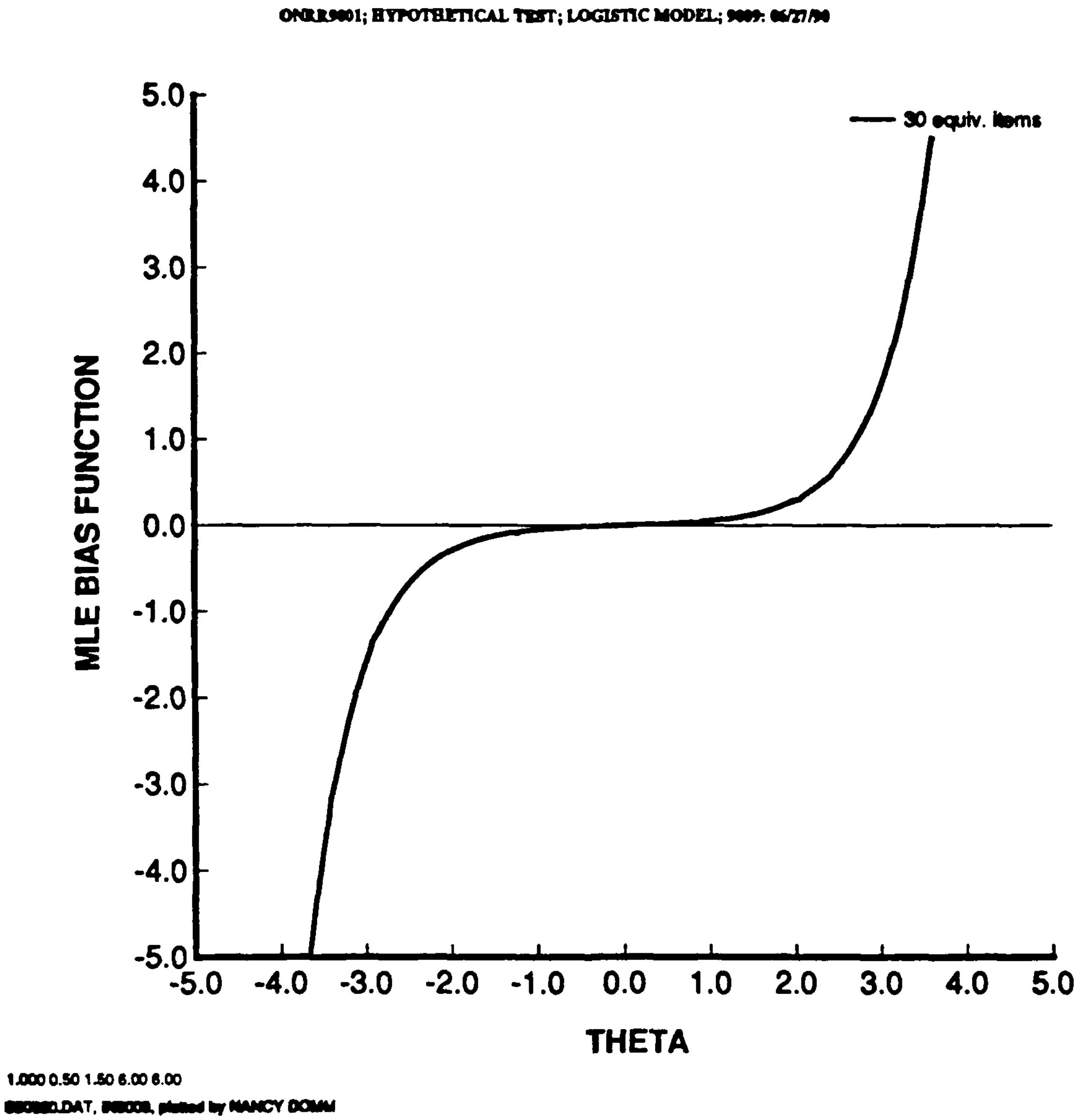
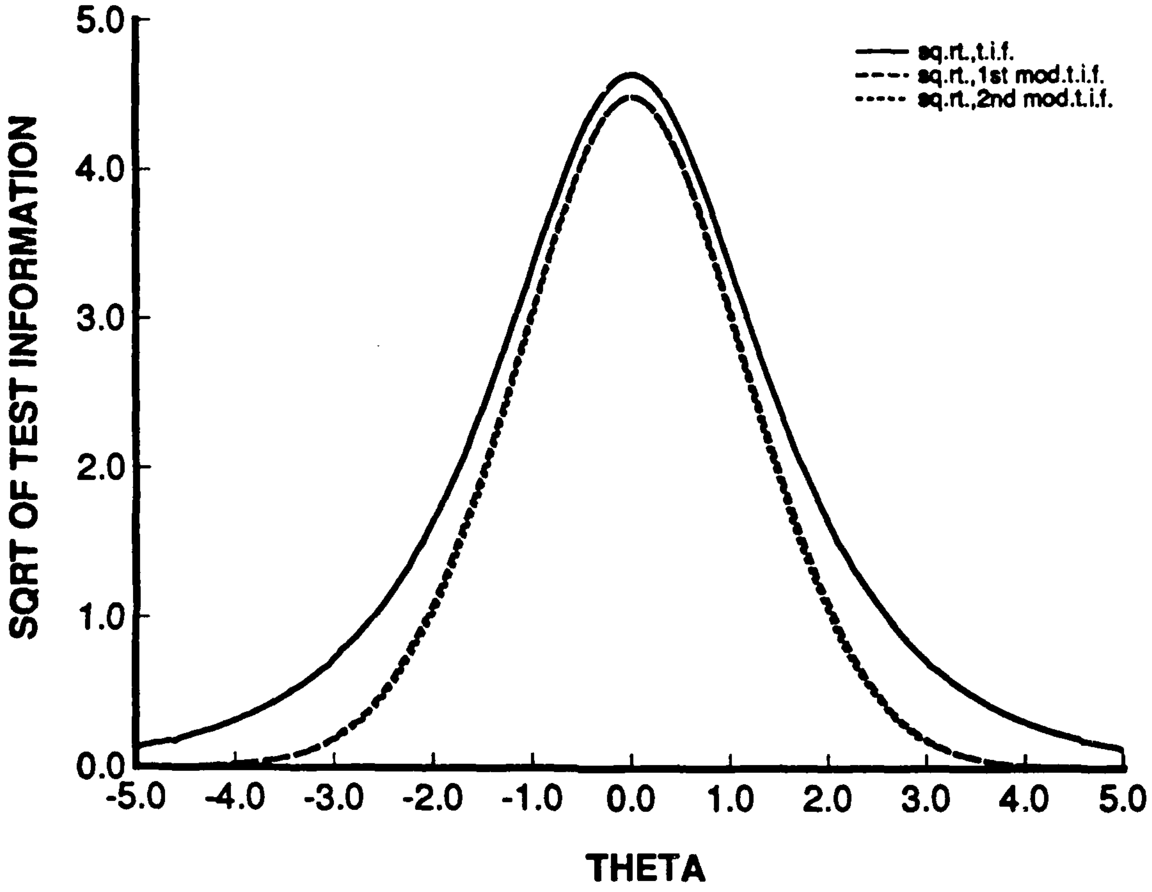


FIGURE 5-2

MLE Bias Function of the Hypothetical Test of Thirty Equivalent Test Items Following the Logistic Model with $a_g = 1.0$ and $b_g = 0.0$ As the Common Parameters.

ONRR9001; HYPOTHETICAL TEST; LOGISTIC MODEL; 9009: 06/27/90



0.750 0.50 1.50 6.00 6.00

[illegible].DAT, [illegible], plotted by NANCY DOMM

FIGURE 5-3

Square Roots of the Original (Solid Line) and the Two Modified (Dashed and Dotted Lines) Test Information Functions of the Hypothetical Test of Thirty Equivalent Items Following the Logistic Model with $a_g = 1.0$ and $b_g = 0.0$ As the Common Parameters.

TABLE 5-1

Three Predicted Reliability Coefficients Tailored for Each of the Six Hypothetical Ability Distributions, Using the Original Test Information Function and Its Two Modification Formulae. The Indices, 1, 2 and 3, Represent the Original Test Information Function, Modification Formula No. 1 and Modification Formula No. 2, Respectively. The Mean and the Variance of θ for Each Population Are Also Given.

POPULATION	RELIABILITY 1	RELIABILITY 2	RELIABILITY 3	MEAN OF THETA	VARIANCE OF THETA
1	0.89641	0.78053	0.76629	0.00000	0.98322
2	0.82324	0.26479	0.25256	-0.80000	0.98322
3	0.81738	0.80074	0.79920	0.00000	0.25155
4	0.73250	0.66611	0.65589	-0.80000	0.25155
5	0.47715	0.21681	0.20093	-1.60000	0.25155
6	0.20049	0.01182	0.01109	-2.40000	0.25155

TABLE 5-2

Three Predicted Standard Errors of Measurement Tailored for Each of the Six Hypothetical Ability Distributions, Using the Original Test Information Function and Its Two Modification Formulae. The Indices, 1, 2 and 3, Represent the Original Test Information Function, Modification Formula No. 1 and Modification Formula No. 2, Respectively. The Mean and the Variance of θ for Each Population Are Also Given.

POPULATION	STAND.ERROR 1	STAND.ERROR 2	STAND.ERROR 3	MEAN OF THETA	VARIANCE OF THETA
1	0.30548	0.37648	0.38514	0.00000	0.98322
2	0.37887	0.64293	0.66397	-0.80000	0.98322
3	0.23521	0.24717	0.24811	0.00000	0.25155
4	0.29172	0.32802	0.33326	-0.80000	0.25155
5	0.48839	0.73440	0.76583	-1.60000	0.25155
6	0.91974	2.76394	2.88922	-2.40000	0.25155

TABLE 5-3

Three Theoretical Variances of the Maximum Likelihood Estimates of θ for Each of the Six Hypothetical Ability Distributions, Using the Original Test Information Function and Its Two Modification Formulae. The Indices, 1, 2 and 3, Represent the Original Test Information Function, Modification Formula No. 1 and Modification Formula No. 2, Respectively. The Mean and the Variance of θ for Each Population Are Also Given.

POPULATION	VARIANCE OF MLE 1	VARIANCE OF MLE 2	VARIANCE OF MLE 3	MEAN OF THETA	VARIANCE OF THETA
1	1.09684	1.25968	1.28308	0.00000	0.98322
2	1.19432	3.71324	3.89296	-0.80000	0.98322
3	0.30775	0.31414	0.31475	0.00000	0.25155
4	0.34341	0.37763	0.38352	-0.80000	0.25155
5	0.52718	1.16023	1.25189	-1.60000	0.25155
6	1.25469	21.28788	22.68190	-2.40000	[illegible]5155

TABLE 5-4

Three Theoretical Error Variances for Each of the Six Hypothetical Ability Distributions, Using the Original Test Information Function and Its Two Modification Formulae. The Indices, 1, 2 and 3, Represent the Original Test Information Function, Modification Formula No. 1 and Modification Formula No. 2, Respectively. The Mean and the Variance of θ for Each Population Are Also Given.

POPULATION	VARIANCE OF ERROR 1	VARIANCE OF ERROR 2	VARIANCE OF ERROR 3	MEAN OF THETA	VARIANCE OF THETA
1	0.11363	0.27646	0.29987	0.00000	0.98322
2	0.21111	2.73003	2.90974	-0.80000	0.98322
3	0.05620	0.06260	0.06320	0.00000	0.25155
4	0.09186	0.12609	0.13197	-0.80000	0.25155
5	0.27563	0.90868	1.00034	-1.60000	0.25155
6	1.00314	21.03633	22.43035	-2.40000	0.25155

TABLE 5-5

Reliability Coefficient Computed for Each of the Six Hypothetical Ability Distributions Based upon the Maximum Likelihood Estimates of the Examinees for Test-Retest Situations Using a Test of Thirty Equivalent Items Following the Logistic Model with $D = 1.7$, $a_g = 1.0$ and $b_g = 0.0$. The Means and Variances of the Two Sessions and the Covariance Are Also Presented.

POPULATION	RELIABILITY	MEAN 1	MEAN 2	VARIANCE 1	VARIANCE 2	COVARIANCE
1	0.90788	-0.00311	0.00106	1.19069	1.16769	1.07051
2	0.88812	-0.81435	-0.80971	1.07982	1.09703	0.96663
3	0.80724	0.00785	-0.00754	0.33578	0.33443	0.27051
4	0.72334	-0.85777	-0.84349	0.40504	0.39310	0.28863
5	0.55304	-1.68722	-1.67511	0.42299	0.40820	0.22980
6	0.32187	-2.28115	-2.25897	0.21639	0.23189	0.07210

$$\sigma_E = [Var.(X)]^{1/2}[1 - r_{X_1X_2}]^{1/2} \quad , \tag{5.1}$$

where, as before, $r_{X_1X_2}$ indicates the reliability coefficient. Comparison of Table 5-1 and Table 5-2 reveals that there are substantial discrepancies between the values of σ_E obtained by formula (5.1) using the *tailored* reliability coefficients in Table 5-1, which are based upon the maximum likelihood estimate $\hat{\theta}$, in place of $r_{X_1X_2}$ in (5.1) and the corresponding standard errors of measurement, which were obtained by formulae (4.1) through (4.3) and presented in Table 5-2. To give some examples, for Population No. 1 the results of (5.1) are: 0.319 , 0.465 and 0.479 , respectively; for Population No. 3 they are: 0.214 , 0.224 and 0.225 ; and for Population No. 6 they are: 0.448 , 0.499 and 0.499 . These results are understandable, for the degree of violation from the assumptions behind the classical mental test theory is different for the separate ability distributions.

The three theoretical variances of the maximum likelihood estimate of θ and the three theoretical error variances are presented in Tables 5-3 and 5-4, respectively, for each of the six hypothetical populations. The latter were obtained by (3.11) and by replacing $I(\theta)$ in (3.11) by $\Upsilon(\theta)$ and $\Xi(\theta)$, respectively, and the former are the sum of these separate error variances and the variance of θ .

In order to satisfy our curiosity, a simulation study has been made in such a way that, following each of the six ability distributions, a group of examinees is hypothesized, and using the Monte Carlo method a response pattern of each hypothetical subject is produced for each of the test and retest situations. Since our test consists of thirty equivalent dichotomous test items, the simple test score is a sufficient statistic for the response pattern, and the maximum likelihood estimate of θ can be obtained upon this sufficient statistic. The numbers of hypothetical subjects are 1,998 for Populations No. 1 and No. 2, and 2,004 for Populations No. 3, No. 4, No. 5 and No. 6. The correlation coefficient between the two sets of $\hat{\theta}$'s was computed, and the results are presented in Table 5-5. Comparison of each of these results with the corresponding three *tailored* reliability coefficients in Table 5-1 gives the impression that, overall, these correlation coefficients are higher than the predicted *tailored* reliability coefficients. This enhancement comes from the fact that, in each distribution there are certain number of subjects who obtained negative or positive infinity as $\hat{\theta}$, and we have replaced these negative and positive infinities by more or less arbitrary values, -2.65 and 2.65 , respectively, in computing the correlation coefficients. Since in Population No. 3 none of the 2,004 hypothetical subjects got negative or positive infinity for their maximum likelihood estimates of θ in the first session, and only three got negative infinity and none got positive infinity in the second session, this result, 0.807 , will be the most trustworthy value. We can see that this value, 0.807 , is less than 0.817 obtained by using the original test information function $I(\theta)$, and a little greater than 0.801 obtained upon the Modification Formula No. 1, $\Upsilon(\theta)$. The next trustworthy value may be 0.723 of Population No. 4, for which none of the 2,004 subjects obtained positive infinity as their $\hat{\theta}$'s in each of the two sessions, and 56 and 45 got negative infinity in the first and second sessions, respectively. This value of correlation coefficient, 0.723 , is a little less than the predicted reliability coefficient 0.733 obtained upon $I(\theta)$, but somewhat greater than 0.666 , which is based upon $\Upsilon(\theta)$, the Modification Formula No. 1—the artificial enhancement is already visible. The numbers of subjects who obtained negative and positive infinities in the first session and in the second session are: 56 , 47 , 43 and 49 for Population No. 1; 197 , 4 , 195 and 6 for Population No. 2; 437 , 0 , 399 and 0 for Population No. 5; and 1,143 , 0 , 1,118 and 0 for Population No. 6. We must say that, for these four distributions, the values of correlation coefficient in Table 5-5 should not be taken too seriously, for these values are enhanced because of the involvement of too many substitute values for negative and positive infinities.

VI Discussion and Conclusions

Test information function $I(\theta)$ and its two modification formulae, $\Upsilon(\theta)$ and $\Xi(\theta)$, are used to predict the reliability coefficient and the standard error of measurement which are *tailored* for each specific ability distribution. Examples are given and a simulation study has been conducted for comparison.

These examples have been rather intentionally chosen to make the differences among the separate ability distributions, and among the three predicted indices for each ability distribution, clearly visible, using equivalent test items.

Since we have more useful and informative measures like the test information function and its two modified formulae, the reliability coefficient of a test is no longer necessary in modern mental test theory. And yet it is interesting to know how to predict the coefficient using these functions, which are *tailored for each separate population of examinees*. In this process, it will become more obvious that the traditional concept of test reliability *is misleading*, for without changing the test the coefficient can be drastically different if we change the population of examinees.

References

[1] Lord, F. M. Unbiased estimators of ability parameters, of their variance, and of their parallel-forms reliability. *Psychometrika, 48*, 1983, 233-245.

[2] Lord, F. M. *Technical problems arising in parameter estimation*. Paper presented at the 1984 Annual Meeting of the American Educational Research Association, New Orleans, Louisiana, 1984.

[3] Samejima, F. Estimation of ability using a response pattern of graded scores. *Psychometrika Monograph, No. 17*, 1969.

[4] Samejima, F. A general model for free-response data. *Psychometrika Monograph, No. 18*, 1972.

[5] Samejima, F. Effects of individual optimization in setting boundaries of dichotomous items on accuracy of estimation. *Applied Psychological Measurement, 1*, 1977a, 77-94.

[6] Samejima, F. A use of the information function in tailored testing. *Applied Psychological Measurement, 1*, 1977b, 233-247.

[7] Samejima, F. Bias function of the maximum likelihood estimate of ability for discrete item responses. *ONR/RR-87-1*, 1987.

[8] Samejima, F. Modifications of the test information function. *ONR/RR-90-1*, 1990.

ONRR9002.TEX
July 5, 1990

Distribution List

Dr. Terry Ackerman
Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Dr. James Algina
1403 Norman Hall
University of Florida
Gainesville, FL 32605

Dr. Erling B. Andersen
Department of Statistics
Studiestraede 6
1455 Copenhagen
DENMARK

Dr. Ronald Armstrong
Rutgers University
Graduate School of Management
Newark, NJ 07102

Dr. Eva L. Baker
UCLA Center for the Study
of Evaluation
145 Moore Hall
University of California
Los Angeles, CA 90024

Dr. Laura L. Barnes
College of Education
University of Toledo
2801 W. Bancroft Street
Toledo, OH 43606

Dr. William M. Bart
University of Minnesota
Dept. of Educ. Psychology
330 Burton Hall
178 Pillsbury Dr., S.E.
Minneapolis, MN 55455

Dr. Isaac Bejar
Mail Stop: 10-R
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

5/1/90

Dr. Menucha Birenbaum
School of Education
Tel Aviv University
Ramat Aviv 69978
ISRAEL

Dr. Arthur S. Blaiwes
Code N712
Naval Training Systems Center
Orlando, FL 32813-7100

Dr. Bruce Bloxom
Defense Manpower Data Center
99 Pacific St.
Suite 155A
Monterey, CA 93943-3231

Cdt. Arnold Bohrer
Sectie Psychologisch Onderzoek
Rekruterings-En Selectiecentrum
Kwartier Koningen Astrid
Bruijnstraat
1120 Brussels, BELGIUM

Dr. Robert Breaux
Code 281
Naval Training Systems Center
Orlando, FL 32826-3224

Dr. Robert Brennan
American College Testing
Programs
P. O. Box 168
Iowa City, IA 52243

Dr. John B. Carroll
409 Elliott Rd., North
Chapel Hill, NC 27514

Dr. John M. Carroll
IBM Watson Research Center
User Interface Institute
P.O. Box 704
Yorktown Heights, NY 10598

Dr. Robert M. Carroll
Chief of Naval Operations
OP-01B2
Washington, DC 20350

Dr. Raymond E. Christal
UES LAMP Science Advisor
AFHRL/MOEL
Brooks AFB, TX 78235

Mr. Hua Hua Chung
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Dr. Norman Cliff
Department of Psychology
Univ. of So. California
Los Angeles, CA 90089-1061

Director, Manpower Program
Center for Naval Analyses
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Director,
Manpower Support and
Readiness Program
Center for Naval Analysis
2000 North Beauregard Street
Alexandria, VA 22311

Dr. Stanley Collyer
Office of Naval Technology
Code 222
800 N. Quincy Street
Arlington, VA 22217-5000

Dr. Hans F. Crombag
Faculty of Law
University of Limburg
P.O. Box 616
Maastricht
The NETHERLANDS 6200 MD

Ms. Carolyn R. Crone
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

Dr. Timothy Davey
American College Testing Program
P.O. Box 168
Iowa City, IA 52243

Dr. C. M. Dayton
Department of Measurement
Statistics & Evaluation
College of Education
University of Maryland
College Park, MD 20742

Dr. Ralph J. DeAyala
Measurement, Statistics,
and Evaluation
Benjamin Bldg., Rm. 4112
University of Maryland
College Park, MD 20742

Dr. Lou DiBello
CERL
University of Illinois
103 South Mathews Avenue
Urbana, IL 61801

Dr. Dattprasad Divgi
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. Hei-Ki Dong
Bell Communications Research
6 Corporate Place
PYA-1K226
Piscataway, NJ 08854

Dr. Fritz Drasgow
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Defense Technical
Information Center
Cameron Station, Bldg 5
Alexandria, VA 22314
(12 Copies)

Dr. Stephen Dunbar
224B Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. James A. Earles
Air Force Human Resources Lab
Brooks AFB, TX 78235

Dr. Susan Embretson
University of Kansas
Psychology Department
426 Fraser
Lawrence, KS 66045

Dr. George Englehard, Jr.
Division of Educational Studies
Emory University
210 Fishburne Bldg.
Atlanta, GA 30322

ERIC Facility-Acquisitions
2440 Research Blvd, Suite 550
Rockville, MD 20850-3238

Dr. Benjamin A. Fairbank
Operational Technologies Corp.
5825 Callaghan, Suite 225
San Antonio, TX 78228

Dr. Marshall J. Farr, Consultant
Cognitive & Instructional Sciences
2520 North Vernon Street
Arlington, VA 22207

Dr. P-A. Federico
Code 51
NPRDC
San Diego, CA 92152-6800

Dr. Leonard Feldt
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Richard L. Ferguson
American College Testing
P.O. Box 168
Iowa City, IA 52243

5/1/90

Dr. Gerhard Fischer
Liebiggasse 5/3
A 1010 Vienna
AUSTRIA

Dr. Myron Fischl
U.S. Army Headquarters
DAPE-MRR
The Pentagon
Washington, DC 20310-0300

Prof. Donald Fitzgerald
University of New England
Department of Psychology
Armidale, New South Wales 2351
AUSTRALIA

Mr. Paul Foley
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Alfred R. Fregly
AFOSR/NL, Bldg. 410
Bolling AFB, DC 20332-6448

Dr. Robert D. Gibbons
Illinois State Psychiatric Inst.
Rm 529W
1601 W. Taylor Street
Chicago, IL 60612

Dr. Janice Gifford
University of Massachusetts
School of Education
Amherst, MA 01003

Dr. Drew Gitomer
Educational Testing Service
Princeton, NJ 08541

Dr. Robert Glaser
Learning Research
& Development Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15260

Dr. Sherrie Gott
AFHRL/MOMJ
Brooks AFB, TX 78235-5601

Dr. Bert Green
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

Michael Habon
DORNIER GMBH
P.O. Box 1420
D-7990 Friedrichshafen 1
WEST GERMANY

Prof. Edward Haertel
School of Education
Stanford University
Stanford, CA 94305

Dr. Ronald K. Hambleton
University of Massachusetts
Laboratory of Psychometric
and Evaluative Research
Hills South, Room 152
Amherst, MA 01003

Dr. Delwyn Harnisch
University of Illinois
51 Gerty Drive
Champaign, IL 61820

Dr. Grant Henning
Senior Research Scientist
Division of Measurement
Research and Services
Educational Testing Service
Princeton, NJ 08541

Ms. Rebecca Hetter
Navy Personnel R&D Center
Code 63
San Diego, CA 92152-6800

Dr. Thomas M. Hirsch
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Paul W. Holland
Educational Testing Service, 21-T
Rosedale Road
Princeton, NJ 08541

Dr. Paul Horst
677 G Street, #184
Chula Vista, CA 92010

Ms. Julia S. Hough
Cambridge University Press
40 West 20th Street
New York, NY 10011

Dr. William Howell
Chief Scientist
AFHRL/CA
Brooks AFB, TX 78235-5601

Dr. Lloyd Humphreys
University of Illinois
Department of Psychology
603 East Daniel Street
Champaign, IL 61820

Dr. Steven Hunka
3-104 Educ. N.
University of Alberta
Edmonton, Alberta
CANADA T6G 2G5

Dr. Huynh Huynh
College of Education
Univ. of South Carolina
Columbia, SC 29208

Dr. Robert Jannarone
Elec. and Computer Eng. Dept.
University of South Carolina
Columbia, SC 29208

Dr. Kumar Joag-dev
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright Street
Champaign, IL 61820

Dr. Douglas H. Jones
1280 Woodfern Court
Toms River, NJ 08753

Dr. Brian Junker
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Dr. Michael Kaplan
Office of Basic Research
U.S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333-5600

Dr. Milton S. Katz
European Science Coordination Office
U.S. Army Research Institute
Box 65
FPO New York 09510-1500

Prof. John A. Keats
Department of Psychology
University of Newcastle
N.S.W. 2308
AUSTRALIA

Dr. Jwa-keun Kim
Department of Psychology
Middle Tennessee State University
P.O. Box 522
Murfreesboro, TN 37132

Mr. Soon-Hoon Kim
Computer-based Education Research Laboratory
University of Illinois
Urbana, IL 61801

Dr. G. Gage Kingsbury
Portland Public Schools
Research and Evaluation Department
501 North Dixon Street
P. O. Box 3107
Portland, OR 97209-3107

Dr. William Koch
Box 7246, Meas. and Eval. Ctr.
University of Texas-Austin
Austin, TX 78703

Dr. Richard J. Koubek
Department of Biomedical & Human Factors
139 Engineering & Math Bldg.
Wright State University
Dayton, OH 45435

Dr. Leonard Kroeker
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Dr Jerry Lehnus
Defense Manpower Data Center
Suite 400
1600 Wilson Blvd
Rosslyn, VA 22209

Dr. Thomas Leonard
University of Wisconsin
Department of Statistics
1210 West Dayton Street
Madison, WI 53705

Dr. Michael Levine
Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Dr. Charles Lewis
Educational Testing Service
Princeton, NJ 08541-0001

Mr. Rodney Lim
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Dr. Robert L. Linn
Campus Box 249
University of Colorado
Boulder, CO 80309-0249

Dr. Robert Lockman
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. Frederic M. Lord
Educational Testing Service
Princeton, NJ 08541

Dr. Richard Luecht
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. George B. Macready
Department of Measurement Statistics & Evaluation
College of Education
University of Maryland
College Park, MD 20742

Dr. Gary Marco
Stop 31-E
Educational Testing Service
Princeton, NJ 08451

Dr. Clessen J. Martin
Office of Chief of Naval Operations (OP 13 F)
Navy Annex, Room 2832
Washington, DC 20350

Dr. James R. McBride
The Psychological Corporation
1250 Sixth Avenue
San Diego, CA 92101

Dr. Clarence C. McCormick
HQ, USMEPCOM/MEPCT
2500 Green Bay Road
North Chicago, IL 60064

Mr. Christopher McCusker
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Dr. Robert McKinley
Educational Testing Service
Princeton, NJ 08541

Mr. Alan Mead
c/o Dr. Michael Levine
Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Dr. Timothy Miller
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Robert Mislevy
Educational Testing Service
Princeton, NJ 08541

Dr. William Montague
NPRDC Code 13
San Diego, CA 92152-6800

Ms. Kathleen Moreno
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Headquarters Marine Corps
Code MPI-20
Washington, DC 20380

Dr. Ratna Nandakumar
Educational Studies
Willard Hall, Room 213E
University of Delaware
Newark, DE 19716

Library, NPRDC
Code P201L
San Diego, CA 92152-6800

Librarian
Naval Center for Applied Research in Artificial Intelligence
Naval Research Laboratory
Code 5510
Washington, DC 20375-5000

5/1/90

Dr. Harold F. O'Neil, Jr.
School of Education - WPH 801
Department of Educational
Psychology & Technology
University of Southern
California
Los Angeles, CA 90089-0031

Dr. James B. Olsen
WICAT Systems
1875 South State Street
Orem, UT 84058

Office of Naval Research,
Code 1142CS
800 N. Quincy Street
Arlington, VA 22217-5000
(6 Copies)

Dr. Judith Orasanu
Basic Research Office
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Jesse Orlansky
Institute for Defense Analyses
1801 N. Beauregard St.
Alexandria, VA 22311

Dr. Peter J. Pashley
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Wayne M. Patience
American Council on Education
GED Testing Service, Suite 20
One Dupont Circle, NW
Washington, DC 20036

Dr. James Paulson
Department of Psychology
Portland State University
P.O. Box 751
Portland, OR 97207

Dept. of Administrative Sciences
Code 54
Naval Postgraduate School
Monterey, CA 93943-5026

Dr. Mark D. Reckase
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Malcolm Ree
AFHRL/MOA
Brooks AFB, TX 78235

Mr. Steve Reiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455-0344

Dr. Carl Ross
CNET-PDCD
Building 90
Great Lakes NTC, IL 60088

Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208

Dr. Fumiko Samejima
Department of Psychology
University of Tennessee
310B Austin Peay Bldg.
Knoxville, TN 37916-0900

Mr. Drew Sands
NPRDC Code 62
San Diego, CA 92152-6800

Lowell Schoer
Psychological & Quantitative
Foundations
College of Education
University of Iowa
Iowa City, IA 52242

Dr. Mary Schratz
905 Orchid Way
Carlsbad, CA 92009

Dr. Dan Segall
Navy Personnel R&D Center
San Diego, CA 92152

Dr. Robin Shealy
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Dr. Kazuo Shigemasu
7-9-24 Kugenuma-Kaigan
Fujisawa 251
JAPAN

Dr. Randall Shumaker
Naval Research Laboratory
Code 5510
4555 Overlook Avenue, S.W.
Washington, DC 20375-5000

Dr. Richard E. Snow
School of Education
Stanford University
Stanford, CA 94305

Dr. Richard C. Sorensen
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Judy Spray
ACT
P.O. Box 168
Iowa City, IA 52243

Dr. Martha Stocking
Educational Testing Service
Princeton, NJ 08541

Dr. Peter Stoloff
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. William Stout
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Dr. Hariharan Swaminathan
Laboratory of Psychometric and
Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

Mr. Brad Sympson
Navy Personnel R&D Center
Code-62
San Diego, CA 92152-6800

Dr. John Tangney
AFOSR/NL, Bldg. 410
Bolling AFB, DC 20332-6448

Dr. Kikumi Tatsuoka
Educational Testing Service
Mail Stop 03-T
Princeton, NJ 08541

Dr. Maurice Tatsuoka
220 Education Bldg
1310 S. Sixth St.
Champaign, IL 61820

Dr. David Thissen
Department of Psychology
University of Kansas
Lawrence, KS 66044

Mr. Thomas J. Thomas
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

Mr. Gary Thomasson
University of Illinois
Educational Psychology
Champaign, IL 61820

Dr. Robert Tsutakawa
University of Missouri
Department of Statistics
222 Math. Sciences Bldg.
Columbia, MO 65211

Dr. Ledyard Tucker
University of Illinois
Department of Psychology
603 E. Daniel Street
Champaign, IL 61820

Dr. David Vale
Assessment Systems Corp.
2233 University Avenue
Suite 440
St. Paul, MN 55114

Dr. Frank L. Vicino
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Howard Wainer
Educational Testing Service
Princeton, NJ 08541

Dr. Michael T. Waller
University of Wisconsin-Milwaukee
Educational Psychology Department
Box 413
Milwaukee, WI 53201

Dr. Ming-Mei Wang
Educational Testing Service
Mail Stop 03-T
Princeton, NJ 08541

Dr. Thomas A. Warm
FAA Academy AAC934D
P.O. Box 25082
Oklahoma City, OK 73125

Dr. Brian Waters
HumRRO
1100 S. Washington
Alexandria, VA 22314

Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455-0344

Dr. Ronald A. Weitzman
Box 146
Carmel, CA 93921

5/1/90

Major John Welsh
AFHRL/MOAN
Brooks AFB, TX 78223

Dr. Douglas Wetzel
Code 51
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Rand R. Wilcox
University of Southern California
Department of Psychology
Los Angeles, CA 90089-1061

German Military Representative
ATTN: Wolfgang Wildgrube
Streitkraefteamt
D-5300 Bonn 2
4000 Brandywine Street, NW
Washington, DC 20016

Dr. Bruce Williams
Department of Educational Psychology
University of Illinois
Urbana, IL 61801

Dr. Hilda Wing
Federal Aviation Administration
800 Independence Ave, SW
Washington, DC 20591

Mr. John H. Wolfe
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. George Wong
Biostatistics Laboratory
Memorial Sloan-Kettering Cancer Center
1275 York Avenue
New York, NY 10021

Dr. Wallace Wulfeck, III
Navy Personnel R&D Center
Code 51
San Diego, CA 92152-6800

Dr. Kentaro Yamamoto
02-T
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Dr. Wendy Yen
CTB/McGraw Hill
Del Monte Research Park
Monterey, CA 93940

Dr. Joseph L. Young
National Science Foundation
Room 320
1800 G Street, N.W.
Washington, DC 20550

Mr. Anthony R. Zara
National Council of State Boards of Nursing, Inc.
625 North Michigan Avenue
Suite 1544
Chicago, IL 60611