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Part I of this report indicated a novel solution to the basic diffusion equation of Physics where the field boundary extends from zero to positive infinity. The nodal points of the field net are identified as terminating polynomials with the numerators of the coefficients found first by deduction--for the lower orders--and then by extrapolation. ~~Part II~~ considers the numerical analysis employed to complete the entire set of tables.

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INTRODUCTION

Part I of this report indicated a novel solution to the basic diffusion equation of Physics where the field boundary extends from zero to positive infinity. The nodal points of the field net are identified as terminating polynomials with the numerators of the coefficients found first by deduction - for the lower orders - and then by extrapolation. Part II considers the numerical analysis employed to complete the entire set of tables.

PROCEDURE

A. Direct Differences

Formulation of the polynomial form of the discrete solutions, Eq. (1), of the diffusion equation from the Schmidt plot geometry is described in References (1) and (2) and reflects a progressive trigonometric construction where the degree and term extension increases with time and decreases with distance, time and distance referring to the unsteady heat flow application.

$$T\ (N,P) = \frac{\varphi}{2^n\ 2^j}\left(A - A_1 m + A_2 m^2 - A_3 m^3 + A_4 m^4 - \cdots \mp A_k m^k \right) \quad (1)$$

where m is the independent variable,

N is a distance index,

P is a time index,

$$h = \frac{(P+N)-2-\left|\sin\ (P+N)\pi/2\right|}{2},$$

$j = (k$ - term exponent of $m)$, the individual term denominator exponent, and $\varphi = m\ (T_0 - T_1 0)$, with

$A, A_1, A_2, \cdots\cdots A_k$ the numerical coefficients of the interior terms of the equation.

The numerators of each term of the "nodal" equations, are uniquely related to adjacent time and distance term coefficients of the same degrees. This relation, originally found accidentally, is correlated by the table of differences shown in Table 1 & 1-A where the boxed vertical sequence; 17548, 25147, 35401, 49024 and 66868, is established by the reduction of the Schmidt plot through the trigonometric analysis. Step-wise right moving subtraction generates a column of residual zeroes, an adjoining column of ones, and a digital sequence identified as "IV" in Table 1. Reducing this column "IV" to zero vertically then allows a corresponding determination of the particular values of the entire matrix from inspection of the biased rows I, II and III.

Table 1. Difference Progression for Degree Zero Numerators

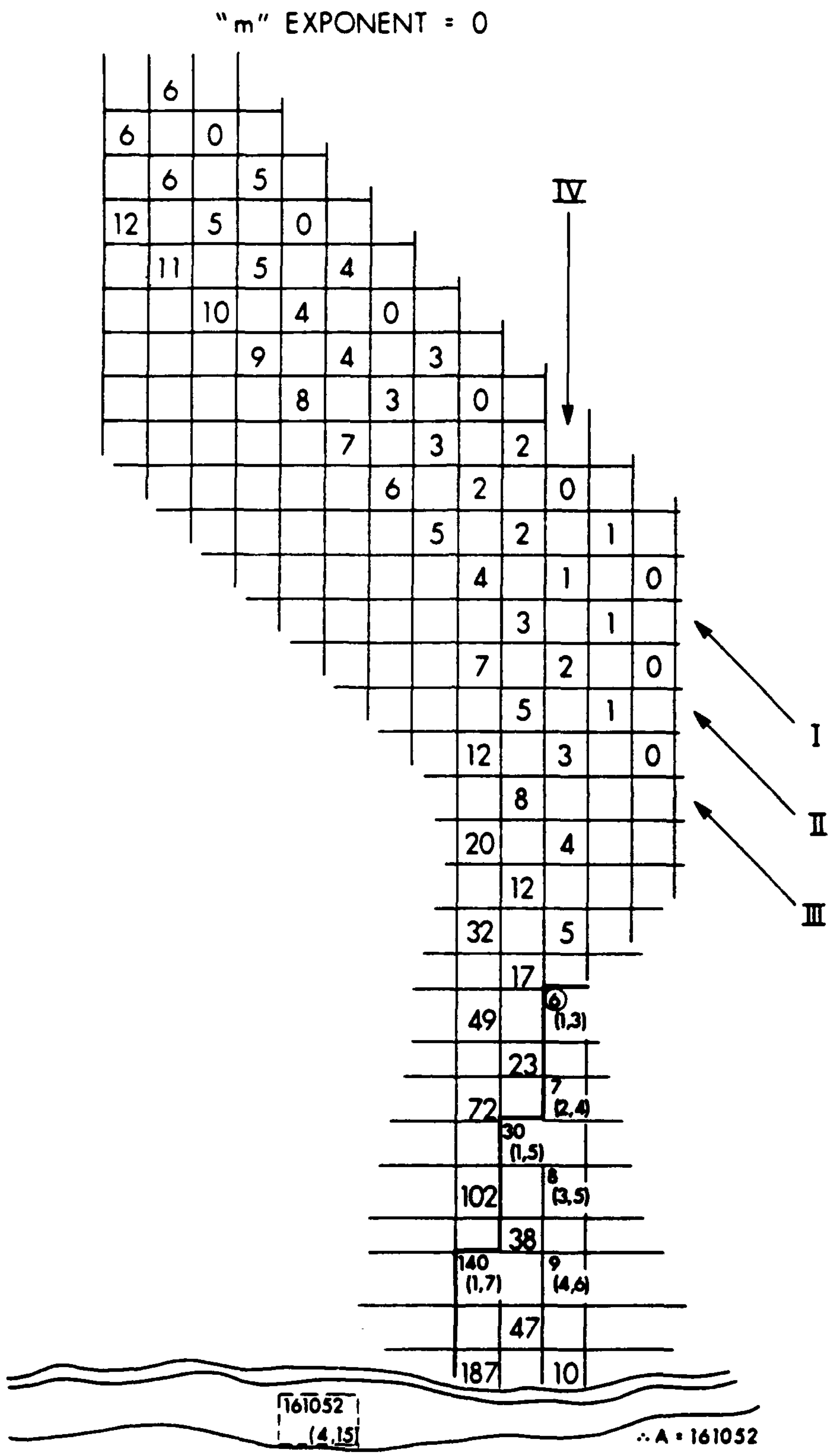

Table 1-a (continuation of Table 1)

							630 (1,9)		57 (4,8)			
						1898		244 (3,9)		11 (6,8)		0
					5282		874 (2,10)		68 (5,9)		1 (8,8)	
				13866		2772 (1,11)		312 (4,10)		12 (7,9)		0
					8054		1186 (3,11)		80 (6,10)		1 (9,9)	
				21920		3958 (2,12)		392 (5,11)		13 (8,10)		0
					12012 (1,13)		1578 (4,12)		93 (7,11)		1 (10,10)	
				33932		5536 (3,13)		485 (6,12)		14 (9,11)		0
			90683		17548 (2,14)		2063 (5,13)		107 (8,12)		1 (11,11)	
		232009		51480 (1,15)		7599 (4,14)		592 (7,13)		15 (10,12)		0
	572312		142163		25147 (3,15)		2655 (6,14)		122 (9,13)		1 (12,12)	
		374172		76627 (2,16)		10254 (5,15)		714 (8,14)		16 (11,13)		0
	946484		218790 (1,17)		35401 (4,16)		3369 (7,15)		138 (10,14)		1 (13,13)	
		592962		112028 (3,17)		13623 (6,16)		852 (9,15)		17 (12,14)		0
	1539446		330818 (2,18)		49024 (5,17)		4221 (8,16)		155 (11,15)		1 (14,14)	
		923780 (1,19)		161052 (4,18)		17644 (7,17)		1007 (10,16)		18 (13,15)		0
	2463226		491870 (3,19)		66868 (6,18)		5228 (9,17)		173 (12,16)		1 (15,15)	
								1180 (11,17)		19 (14,16)		0
									192 (13,17)			

B. Summing Progression

Rewriting Table 1 as Table 2, where the biased rows are horizontal and the zero column is left-justified instead of right-justified, shows that the sum of any two adjacent column values of any row gives the value of the next row entry directly under the right-wise addendum. The sequence of Row I, regardless of the degree of the term represented by the matrix, always starts with zero and then maintains alternate zeroes to infinity. Each of the difference tables corresponding to a given "m" exponent (Eq. (1)) can be resolved similarly, except that each first row is carried in a unique progression. This progression is repeated without the interspersed zeroes within the matrix.

C. Pascal Triangles

The classical Pascal triangle, Table 3, can be formed by simple addition where each term is the sum of the two previous superior terms and individual entries are represented by the binomial coefficient, $\binom{z}{w} = \frac{z!}{w!(z-w)!}$, where z and w represent the row and column of a particular coefficient of a binomial expansion. A modification of the Pascal triangle is found by writing the diagonals as rows which then generates the "arithmetic square", Table 4, also known historically.[3]

It is precisely these progressions alternating with zeroes, which comprise the first rows of the individual "summing progressions" shown as Table 2. In this case the digital enumeration of the rows indicates the degree of the "m" term.

D. Pockhammer's Symbol

Each row of Table 4 can be examined by finite differencing to establish, via Gregory-Newton,[4] a definitive polynomial expansion extending to infinity. Furthermore, the algebraic equation can be simplified to a factorial form known as Pockhammer's Symbol or as a π factorial. Appendix A presents an example of such a development for "m" degrees zero through three. The complete arithmetic square, Table 2, can then be written as

$$f(v) = \frac{1}{(r+1)!} (v)_r \qquad (2)$$

where

r = degree of m,

v = column value

and f(v) = row value of Table 2. From Table 2, a complete construction of Table 1 follows.

Table 2. Summing Progression for Degree zero Numerators

"m" EXPONENT = 0

ROW↓ \ COLUMN →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	1	0	2	0	3	0	4	0	5	0	6	0	7	0	8	0
2	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8
3	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
4	0	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31
5	0	1	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
6	0	1	5	12	20	28	36	44	52	60	68	76	84	92	100	108	116
7	0	1	6	17	32	48	64	80	96	112	128	144	160	176	192	208	224
8	0	1	7	23	49	80	112	144	176	208	240	272	304	336	368	400	432
9	0	1	8	30	72	129	192	256	320	384	448	512	576	640	704	768	832
10	0	1	9	38	102	201	321	448	576	704	832	960	1084	1216	1344	1472	1600

Table 3. Pascal Triangle

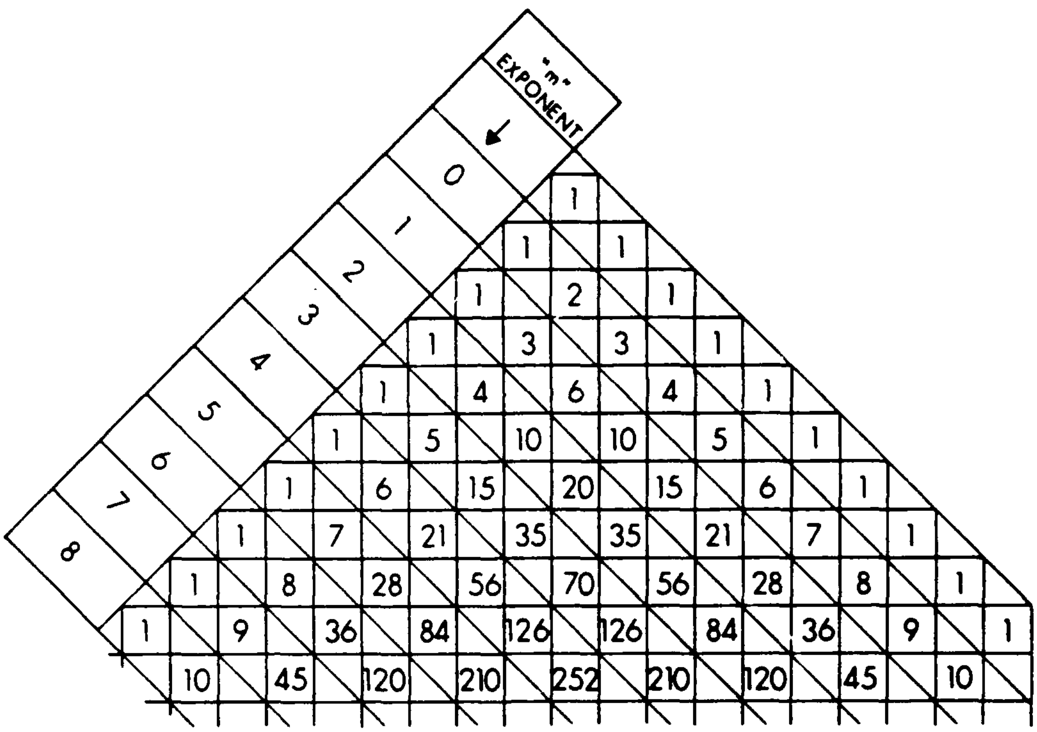

Table 4. Arithmetic Square

"m" EXPONENT ↓	BASELINE SEQUENCE →										
	1	1	1	1	1	1	1	1	1	1	1
0	1	2	3	4	5	6	7	8	9	10	11
1	1	3	6	10	15	21	28	36	45	55	66
2	1	4	10	20	35	56	84	120	165	220	286
3	1	5	15	35	70	126	210	330	495	715	1001
4	1	6	21	56	126	252	462	792	1287	2002	3003
5	1	7	28	84	210	462	924	1716	3003	5005	8008
6	1	8	36	120	330	792	1716	3432	6435	10010	18018
7	1	9	45	165	495	1287	3003	6435	12870	22880	40898

Summary

A straight-forward method (arithmetic squares) is described to permit the numerical construction of the differencing tables of Part I of this report. The derivation through the Pascal triangle and correspondence to Pockhammer's Symbol notation is demonstrated.

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1. William F. Donovan, "Determination of Heat Transfer Coefficient in a Gun Barrel from Experimental Data," Memorandum Report BRL-MR-3428, January 1985.

2. William F. Donovan, "Polynomial Definition of Discrete Field Points of Map Diffusion Equation, Part I," Memorandum Report BRL-MR-3649, March 1988.

3. N.Ya. Vilenkin, <u>Combinatorics</u>, Academic Press, New York and Long, 1971, p.p. 90-94.

4. Spiegel, M.R., "<u>Theory and Problems of Finite Differences and Finite Difference Equations</u>, Schaum's Outline Series in Mathematics, McGraw-Hill Book Company, N.Y., etc., 1971, p.p. 36-44.

APPENDIX A

Determination of Pockhammer's Notation

A discussion of the Gregory-Newton analysis is presented in Reference 4. It consists of determining a polynomial expression to represent a progressive sequence of numbers. In the present application, it is used to examine the base row development of Table 4.

Given a unit stepping difference in a counting reference, v, and a matched sequence f(y);

$$f(y) = f(v) + \frac{\Delta f(v)}{1!} y^{(1)} + \frac{\Delta^2 f(v)}{2!} y^{(2)} + \frac{\Delta^3 f(v)}{3!} y^{(3)} + \cdots\cdots$$

where

v is the step level,

f(y) is the dependent variable,

$\Delta^m f(v)$ are the diagonal values of the difference table, and

$y^{(0)} = 1$

$y^{(1)} = y$

$y^{(2)} = y(y-1)$

$y^{(3)} = y(y-1)\ (y-2) \cdots$

etc.

For the case of "m" degree zero where f(y) is from Table 4:

v	f(y)	f(v)	Δf(v)
0	1		
		1	
1	2		0
		1	
2	3		0
		1	
3	4		0
		1	
4	5		0
		1	
5	6		0
		1	
6	7		0
		1	
7	8		0
		1	
8	9		0
		1	
9	10		0
		1	
10	11		0
		1	
11	12		0

$$f(y) = f(v) + \frac{\Delta f(v)}{1!} \quad y^{(1)} \cdots\cdots$$

$$= 1 + v + 0 \quad \cdots\cdots$$

$$= (v + 1)$$

For m degree 1:

v	f(y)	f(v)	Δf(v)	$\Delta^2 f(v)$
0	1			
		2		
1	3		1	
		3		0
2	6		1	
		4		0
3	10		1	
		5		0
4	15		1	

$$f(y) = f(v) + \frac{\Delta f(v)}{1!} \, y^{(1)} + \frac{\Delta^2 f(v)}{2!} \, y^{(2)} + \cdots\cdots\cdots$$

$$= 1 + 2v + \frac{1}{2}\, v(v-1) + 0 + \cdots\cdots$$

$$= \frac{1}{2}\,(v^2 + 3v + 2)$$

$$= \frac{1}{2}\,(v + 1)\,(v + 2)$$

For m degree 2

v	f(y)	f(v)	$\Delta f(v)$	$\Delta^2 f(v)$	$\Delta^3 f(v)$
0	1				
		3			
1	4		3		
		6		1	
2	10		4		0
		10		1	
3	20		5		0
		15		1	
4	35		6		
		21			
5	56				

$$= 1 + 3v + \frac{3}{2}\,(v)(v-1) + \frac{1}{6}\,(v)\,(v-1)\,(v-2) + 0 \cdots\cdots\cdots$$

$$= 1 + 3v + \frac{3}{2}\,(v^2 - v) + \frac{v}{6}\,(v^2 - 3v + 2)$$

$$= \frac{1}{6}\,(v^3 + 6v^2 + 11v + 6)$$

$$= \frac{1}{6}\,(v + 1)\,(v + 2)\,(v + 3)$$

The pattern continues so that:

Degree of "m" = r	f(y)
0	$(v + 1)$
1	$\frac{1}{2}\,(v + 1)\,(v + 2)$
2	$\frac{1}{6}\,(v + 1)\,(v + 2)\,(v + 3)$
3	$\frac{1}{24}\,(v + 1)\,(v + 2)\,(v + 3)\,(v + 4)$
4	$\frac{1}{120}\,(v + 1)\,(v + 2)\,(v + 3)\,(v + 4)\,(v + 5)$

5 $\quad \frac{1}{720} (v + 1)(v + 2)(v + 3)(v + 4)(v + 5)(v + 6)$

and the general expression is

$$f(y) = \frac{1}{(r + 1)!} (v)_{r+1}$$

where $(v)_{r+1} = (v + 1)(v + 2)(v + 3) \ldots \ldots (v + r + 1)$.

With respect to the original time index, P, of the diffusion equation polynomial; $v = P-1$ by Table 3, and

$$(v)_{r+1} = (P - 1 + 1)(P - 1 + 2) \ldots . (P + r)$$

$$= P (P + 1)(P + 2)(P + 3) \ldots . (P + r)$$

so that

$$f(y) = \frac{1}{(r + 1)!} (P)_r$$

which is known as Pockhammer's Symbol.*

---

*Gravio A. Korn, Mathematical Handbook for Scientists and Engineers, McGraw-Hill Book Co., Inc., New York, etc., 1961.

List of Symbols

h	exponent of 2 in external denominator
j	exponent of 2 in each term denominator
k	exponent of "m" in final term
m	independent variable
r	degree of "m"
v	column value of stepping sequence
$f(v)$	row value of stepping sequence
w	inferior component of binomial coefficient
z	superior component of binomial coefficient
$A_1, A_2, A_3, \ldots.$	numerical coefficients
N	distance index
P	time index
T	dependent variable
φ	external numerator

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