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9. List of manuscripts submitted or published under ONR Sponsorship dur-

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Brief Outline of Research Findings

We have developed new perturbation formulas for signal and orthogonal subspaces which are estimated from a noisy data matrix. These formulas are:

- based on a *finite* amount of data,

- derived under the assumption of high signal-to-noise ratio,

- applicable to arrays of arbitrary geometry,

and they provide a common foundation for all our analyses, [1,3]. We have analyzed a number of array processing algorithms which we classify as follows: [4]

1. Signal subspace algorithms: ESPRIT, State-space realization (including TAM), and Matrix Pencil,

2. Orthogonal subspace algorithms: MUSIC and Min-Norm.

We have developed analytical variance formulas for the case in which estimates are obtained by searching for the extrema of a function (used with arbitrary array geometry) [1], as well as the case in which estimates are obtained by rooting a polynomial or finding the eigenvalues of a matrix (used with a uniform line array geometry) [2,6].

In addition, we have developed improvements to a state-space algorithm for frequency-wavenumber (2-D) estimation. We give a procedure to pair individual frequency and wavenumber estimates, and we also show how a 2-D forward-backward data matrix can be used to improve the performance of the state-space approach [5].

References

1. "Analytical Performance Comparison of The Min-Norm and MUSIC Algorithms with Arbitrary Array Geometry" (F. Li and R.J.Vaccaro), to appear in *IEEE Trans. on Aerospace and Electronic Systems.*

2. "A Unified Performance Analysis of Subspace-Based Polynomial-Rooting Algorithms for DOA Estimation" (F. Li and R.J.Vaccaro), *Proc. 24nd Annual Conference on Information Sciences and Systems*, Princeton, March, 1990.

3. "Unified Performance Analysis of Subspace-Based Estimation Algorithms" (F. Li, R.J.Vaccaro, and D.W.Tufts), *Proc. ICASSP'90*, Vol.5, pp.2575-2578, Alburquerque, NM, April 1990.

4. "Analytical Performance Prediction of Subspace-Based Algorithms for DOA Estimation" (F. Li and R.J.Vaccaro), *Proc. of 2nd International Workshop on SVD and Signal Processing*, pp.161-168, Kingston, RI, June, 1990

5. "On Frequency-wavenumber Estimation by State-Space Realization," (F. Li and R.J. Vaccaro), accepted for publication in *IEEE Trans. on Circuits and Systems,* 1990.

6. "Performance Analysis of the State Space Realization (TAM) and ESPRIT Algorithms for DOA Estimation," (F. Li, R.J. Vaccaro, and D.W. Tufts), submitted to *IEEE Trans. on Antennas and Propagation,* January, 1990.

Abstracts of Papers Sent to the Office of Naval Research as Attachments for Semiannual Progress Report

Analytical Performance Comparison of The Min-Norm and MUSIC Algorithms with Arbitrary Array Geometry

Fu Li and Richard J. Vaccaro

This paper presents a new, non-asymptotic performance comparison between the Min-Norm and MUSIC algorithms for estimating the directions of arrival of narrow-band plane waves impinging on an array of sensors. The analysis is based on a finite amount of sensor data, unlike many previous asymptotic analyses. The analysis presented here makes the assumption of high signal-to-noise ratio, and it applies to arrays of arbitrary geometry. It is shown that Min-Norm can be expressed as a certain data-dependent weighted MUSIC algorithm, and this relationship allows a unified performance comparison. It is also shown that the variances of the estimated directions-of-arrival from the MUSIC algorithm are always smaller than those of the Min-Norm algorithm at high SNR when both algorithms employ a numerical search procedure to obtain the estimates.

A Unified Performance Analysis of Subspace-Based Polynomial-Rooting Algorithms for DOA Estimation

Fu Li and Richard J. Vaccaro

In this paper, a non-asymptotic statistical performance analysis using matrix approximation is applied to a common model for orthogonal-subspace based polynomial-rooting methods for direction-of-arrival (DOA) estimation. In particular, the Min-Norm and Root-MUSIC algorithms are analyzed. The major result is a formula for variance of the estimated DOA's. The formula shows that the estimates obtained by Root-MUSIC have a smaller variance than those obtained by Min-Norm at high-SNR. The analytical results are verified by simulations.

Unified Performance Analysis of Subspace-Based Estimation Algorithms

Fu Li, Richard J. Vaccaro, and Donald W. Tufts
Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing '90, April, 1990

This paper presents a unified performance analysis of subspace-based algorithms for directions-of-arrival (DOA) estimation involving multiple signal arrivals in array signal processing. Following our previous analyses of MUSIC and Min-Norm, an analytical expression of the variance of the DOA estimation-error is developed here for State-Space Realization (TAM) and ESPRIT in a greatly simplified fashion. The tractable formulas provide insight for design. Simulation results verify the analysis.

Analytical Performance Prediction of Subspace-Based Algorithms for DOA Estimation

Fu Li and Richard J. Vaccaro
Proceedings of the Second International Workshop on SVD and Signal Processing, June, 1990

In this paper, a non-asymptotic statistical performance analysis using matrix approximation is applied to subspace-based algorithms for direction-of-arrival (DOA) estimation. In particular, the MUSIC, Min-Norm, State-Space Realization (TAM and DDA) and ESPRIT algorithms are analyzed. An analytical expression of the variance of the DOA estimation-error is developed for theoretical comparison in a greatly simplified and self-contained fashion. The tractable formulas provide insight into the algorithms. Simulation results verify the analysis.

On Frequency-Wavenumber Estimation By State-Space Realization

Fu Li and Richard J. Vaccaro
To appear in IEEE Transactions on Circuits and Systems

Most existing techniques for frequency-wavenumber estimation require a computationally expensive numerical search over a two-dimensional (2-D) parameter space for the maximum or minimum of a function. A method has been proposed using state-space models which decomposes the 2-D problem into two 1-D problems and provides an analytical solution (i.e. without searching) in each dimension separately. In this paper, we show how the separate estimates obtained by the state-space approach can be grouped into pairs of estimates corresponding to each 2-D signal. In addition, we show how a 2-D forward-backward data matrix can be used to improve the performance of the state-space approach.

Performance Analysis of The State Space Realization (TAM) and ESPRIT Algorithms for DOA Estimation

Fu Li, Richard J. Vaccaro, and Donald W. Tufts
Submitted to IEEE Transactions on Antennas and Propagation

This paper presents a performance analysis of signal subspace-based algorithms for directions-of-arrival (DOA) estimation involving multiple signal arrivals in array signal processing. Following our previous analyses of orthogonal subspace based algorithms (MUSIC and Min-Norm), an analytical expression of the variance of the DOA estimation-error is developed here for signal subspace based algorithms (State-Space Realization (TAM) and ESPRIT) Simulation results verify the analysis.