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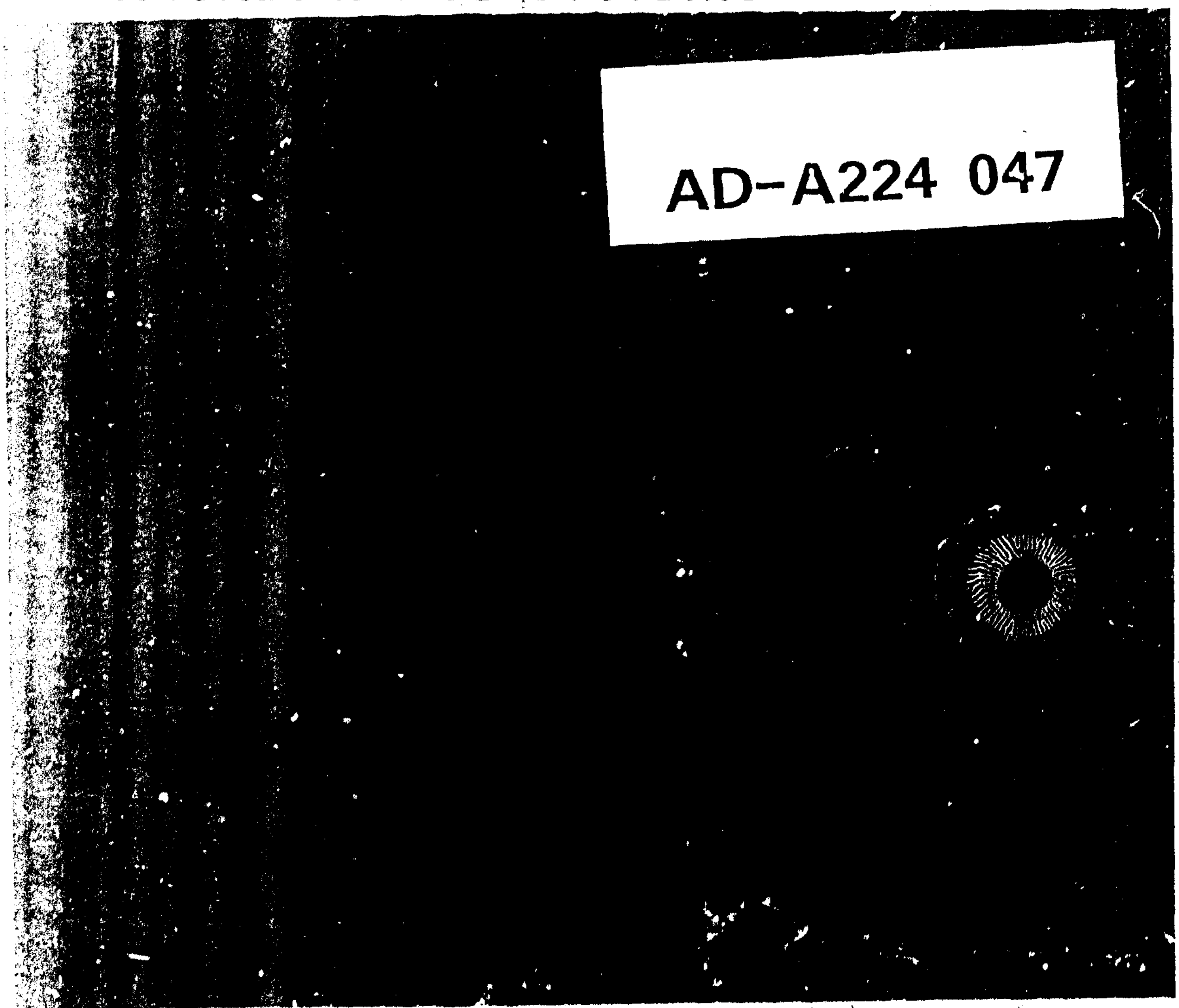
SEQUENCE SELECTION FOR SPREAD-SPECTRUM PACKET RADIO SYSTEMS



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SEQUENCE SELECTION FOR SPREAD-SPECTRUM PACKET RADIO SYSTEMS

by

Mark Steven Schmidt

This work was supported by the Defense Advanced Research Project Agency under Contract MDA 903-82-C-0026.

SEQUENCE SELECTION FOR SPREAD-SPECTRUM PACKET RADIO SYSTEMS

BY

MARK STEVEN SCHMIDT

B.S., University of Illinois, 1981

THESIS

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Electrical Engineering
in the Graduate College of the
University of Illinois at Urbana-Champaign, 1983

Thesis Advisor: Professor M. B. Pursley

Urbana, Illinois

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ABSTRACT

A direct-sequence spread-spectrum scheme is applied to multiple-access packet radio systems. Algorithms that find signature sequence phases of Gold sequences with low correlation parameter values are explored. The average probabilities of error for sets of Gold sequences are shown to be dependent on the phases of the sequences.

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CHAPTER 1

INTRODUCTION

The applications of direct-sequence spread-spectrum multiple-access (DS/SSMA) communications are varied. A contemporary employment of this scheme is in mobile packet radio communications. As a particular example, consider a packet radio network in which a number of mobile terminals are within range of a base station (See Fig. 1.1). The terminals communicate with each other. Assume this access is accomplished through the use of other terminals that serve as repeaters to relay packets between the terminals. Many different network protocols and multiplexing schemes are in use or have been proposed. The channel access may be accomplished by using receiver addressed waveforms [1]. A terminal that wishes to transmit to a particular receiver uses that receiver's unique code sequence.

In the packet radio application there may be a large number of mobile terminals. Thus, a large number of code sequences are required. For the application in which groups of mobile users are clustered (e.g., near base stations), the large set of codes may be divided into subsets, and a subset assigned to each group. It is easier to find several small subsets of sequences with the desired properties than to obtain a single large set.

A major concern of such a system is reliable communication between the terminals. There may be many terminals trying to transmit simultaneously, thus the multiple-access interference can be deleterious to communication if enough care is not given to signal set design. The system may be required to operate in an urban or hilly environment in which the interference due to

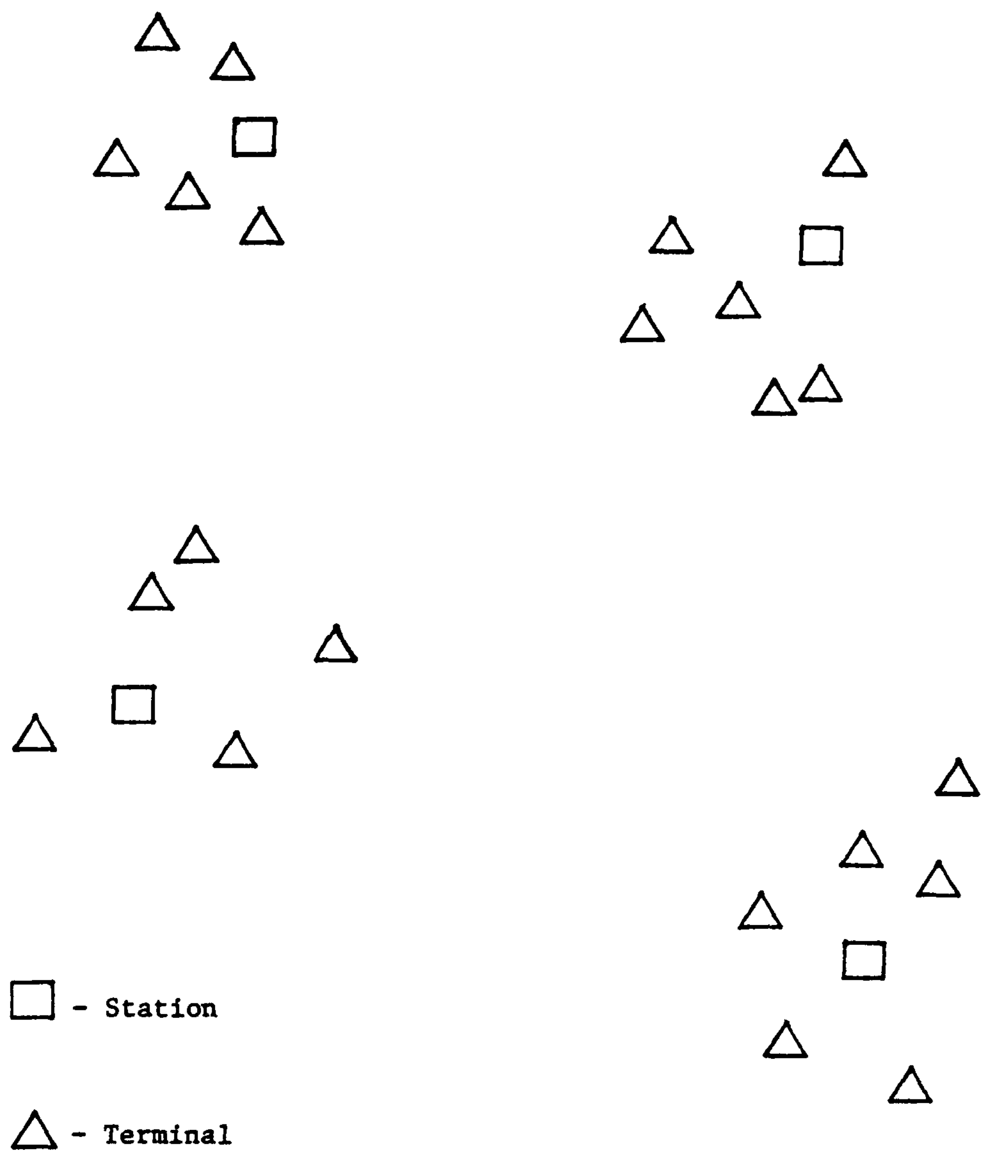

Figure 1.1. Mobile packet radio network.

multipath signal components becomes a serious consideration. In military applications it may be necessary to protect the system against narrowband interference (jamming) and eavesdropping. These and other factors can have detrimental effects on the performance of the system.

Direct-sequence spread-spectrum multiple-access systems provide good performance against these critical effects. The assumed DS/SSMA model is outlined in [6] in which there are K transmitters that are asynchronous in both time and phase. In this model the i-th user's data signal is modulated by a spectral-spreading signature sequence $\underline{x}_i$. In the application here, it will be assumed that the data pulse duration T is a multiple of the code sequence bit duration T_c and that the code sequence is periodic with period $N = T/T_c$. Thus, there is one code period $\underline{x}_i = (x_0^{(i)}, x_1^{(i)}, \ldots, x_{N-1}^{(i)})$ per data symbol. The relevant correlation parameters of these binary signature sequences are given in [3], where a detailed description is given for many binary sequence sets suitable for use as signature sequences in the packet radio system.

In [4] it is shown that the worst-case probability of error for the DS/SSMA system in the absence of channel effects such as multipath fading depends upon the maximum value of the magnitude of the multiple-access interference. Furthermore, it is seen that the maximum multiple-access interference is proportional to $\max\{\hat{M}(\underline{x}_i,\underline{x}_j), M(\underline{x}_i,\underline{x}_j)\}$ where

$$\hat{M}(\underline{x}_i,\underline{x}_j) = \max\{|\hat{\theta}(\underline{x}_i,\underline{x}_j)(\ell)|: 0 \leq \ell \leq N-1\}$$

and

$$M(\underline{x}_i,\underline{x}_j) = \max\{|\theta(\underline{x}_i,\underline{x}_j)(\ell)|: 0 \leq \ell \leq N-1\} \quad .$$

The odd crosscorrelation function $\hat{\theta}(\underline{x}_i,\underline{x}_j)(\ell)$ and the periodic crosscorrelation function $\theta(\underline{x}_i,\underline{x}_j)(\ell)$ are defined in [3]. For a single transmitter system used over a specular multipath channel the worst-case probability of error [2] depends upon $\max\{\hat{M}(\underline{x}),M(\underline{x})\}$ where

$$\hat{M}(\underline{x}) = \max\{|\hat{\theta}(\underline{x})(\ell)|: 1 \le \ell \le N-1\}$$

and

$$M(\underline{x}) = \max\{|\theta(\underline{x})(\ell)|: 1 \le \ell \le N-1\} \quad .$$

The odd autocorrelation function $\hat{\theta}(\underline{x})(\ell)$ and the periodic autocorrelation function $\theta(\underline{x})(\ell)$ are given in [3]. Coupling these results it can be seen that for a multi-user system over a specular multipath channel, the worst-case performance can be improved by selecting signature sequences that have smaller peak autocorrelation and crosscorrelation parameters.

Hopefully, in practice the worst-case error performance is rarely realized, therefore other parameters relating to the average performance of the DS/SSMA system are sought. In the multiple-access environment, the average signal-to-noise ratio at the output of the i-th receiver for a K-user system is defined by Pursley [6] as

$$SNR_i = \{(6N^3)^{-1} \sum_{\substack{k=1 \\ k\neq i}}^{K} r(\underline{x}_k,\underline{x}_i) + N_0/2E_b\} \quad ,$$

where E_b is the energy per data bit and $N_0/2$ is the two-sided channel noise density. The mean-squared interference $r(\underline{x}_k,\underline{x}_i)$ is easily found [7] from the aperiodic autocorrelation functions $C(\underline{x}_k)(\ell)$ and $C(\underline{x}_i)(\ell)$ [3] of $\underline{x}_k$ and $\underline{x}_i$ as

$$r(\underline{x}_k,\underline{x}_i) = 2N^2 + 4 \sum_{\ell=1}^{N-1} C(\underline{x}_k)(\ell)C(\underline{x}_i)(\ell)$$

$$+ \sum_{\ell=0}^{N-2} \{C(\underline{x}_k)(\ell)C(\underline{x}_i)(\ell+1) + C(\underline{x}_k)(\ell+1)C(\underline{x}_i)(\ell)\} .$$

Pursley claims that a very good approximation to the average probability of error for most cases of interest is given by $P_{e,i} = Q(SNR_i)$ where $Q(x) = 1-\Phi(x)$ and $\Phi(x)$ is the standard Gaussian cumulative distribution function. Indeed, evidence ([8], [11]) has shown that, except for small N and K, this approximation is good. Thus, another criterion in signature sequence selection is to choose sequences that give a large value of SNR_i.

Sets of binary sequences with small correlation parameters are desired for use in the DS/SSMA mobile packet radio example. A class of sequences that provides large sets of sequences with good periodic correlation parameters is the Gold sequence family. Gold sequences are generated by binary shift registers with tap connections specified by the binary product of a preferred pair of primitive polynomials [3]. A class of Gold sequences is equivalently formed by adding (modulo two) the distinct cyclic shifts of one maximal-length sequence to a fixed shift of another maximal-length sequence where the pair of m-sequences have a preferred three-valued crosscorrelation function [3]. For a signature sequence length $N = 2^n-1$ there will be $N + 2$ sequences in any given class of Gold sequences and any pair of sequences $\underline{x}_i$ and $\underline{x}_j$ in that class satisfies [5]

$$M(\underline{x}_k) \leq M(\underline{x}_i,\underline{x}_j) = 1 + 2^{\lfloor (n+2)/2 \rfloor} \; ; \; k = i,j$$

where $\lfloor y \rfloor$ is the integer part of the real number y. Thus, it is desired to choose Gold sequences that have small odd correlation values.

Chapter 2 describes some algorithms that find sets of Gold sequences with good parameter values. The AO/LSE algorithm may be used to find a large set of sequences with small odd autocorrelation values. The other algorithms are intended for use with smaller sequence sets (e.g., $10 \leq K \leq 15$ for length 127 sequences), and they find sets that have small peak correlation and mean-squared interference values. Chapter 3 presents data from the application of these algorithms to Gold sequence sets of various sizes and lengths. Chapter 4 gives the average probability of error performance for some of the sequence sets found by the algorithms. Algorithms and data for some very small sets of short-length sequences are presented to illustrate the benefits attainable through careful signature sequence selection.

CHAPTER 2

ALGORITHMS FOR FINDING SUBOPTIMAL SEQUENCE PHASES

The class of Gold sequences described in [3] provides large sets of sequences with good periodic autocorrelation and periodic crosscorrelation properties. As discussed in Chapter 1, it is desired to employ signature sequences with small odd autocorrelation values to combat the effects of multipath interference and aid in signal acquisition. Small odd cross-correlation values are required to ensure small multiple-access interference [4]. These odd correlation parameters are dependent on the cyclic shift or phase of the sequence [3], whereas the periodic correlation parameters are not. A search through all phases for those that yield the best odd correlation values is desired but the amount of computation is prohibitive for most sets of interest. Thus, suboptimal algorithms are derived to find phases of the Gold sequences that yield good odd correlation values. Suboptimal procedures to find sets of phases with small mean-squared interference values are also explored.

2.1 Auto-Optimal/Least-Sidelobe-Energy Algorithm

A simple algorithm that has a small computational requirement for many interesting signature sequence lengths is the Auto-Optimal/Least-Sidelobe-Energy (AO/LSE) sieve detailed in [5]. The phase of a sequence that gives a minimum value for the peak odd autocorrelation, a minimum number of occurrences of this peak value, and, in the case of ties, a minimum value of the sidelobe energy,

$$S(\underline{x}) = \sum_{\ell=1}^{N-1} \{C(\underline{x})(\ell)\}^2$$

is chosen as the AO/LSE phase by this program. This algorithm can be made efficient by the use of the identity [3]

$$\hat{\theta}(T\underline{x})(\ell) = \hat{\theta}(\underline{x})(\ell) - 2x_0x_\ell + 2x_{N-\ell}x_0,$$

where $\underline{x} = (x_0, x_1, \ldots, x_{N-1})$ and $T\underline{x} = (x_1, x_2, \ldots, x_{N-1}, x_0)$.

2.2 Algorithm for Finding Phases of Gold Sequences with Small Peak Odd Autocorrelation and Small Peak Odd Crosscorrelation

As mentioned, the computation involved in minimizing crosscorrelation parameters over the set of all possible phases is typically prohibitive. Indeed, for a scheme with K users each employing sequences of length N there are N^K different sets of phases for which the crosscorrelation parameters must be found. Clearly, this is a formidable task even for small N and K. Thus, a suboptimal algorithm that examines a subset of the N^K sets of phases is developed.

In the first step, a subset $\{\underline{x}_i : 1 \le i \le K\}$ of K sequences is arbitrarily chosen from a family of N+2 Gold sequences of length N. Secondly, a threshold value $\hat{\theta}_T$ for the maximum odd autocorrelation for the set is chosen and all phases $T^n\underline{x}_i$ of each of the K sequences that have $\hat{M}(T^n\underline{x}_i) \le \hat{\theta}_T$ are found, where

$$\hat{M}(T^n\underline{x}_i) = \max\{|\hat{\theta}(T^n\underline{x}_i)(\ell)| : 1 \le \ell \le N\}.$$

Let

$$S_i = \{T^n\underline{x}_i : \hat{M}(T^n\underline{x}_i) \le \hat{\theta}_T \; ; \quad 0 \le n \le N-1\}$$

for $1 \le i \le K$ and $N_i = \| S_i \|$. S_i is the set of all phases of $\underline{x}_i$ with odd autocorrelation sidelobes less than $\hat{\theta}_T$. Notice that $\hat{\theta}_T$ should be chosen such that $\hat{\theta}_T \ge \max\{\hat{\theta}_{AO}(\underline{x}_i) ; 1 \le i \le K\}$ where

$$\hat{\theta}_{AO}(\underline{x}_i) = \min\{\hat{M}(T^n\underline{x}_i) \ ; \ \ 0 \leq n \leq N-1\}$$

but not much larger to ensure small peak out-of-phase odd autocorrelation

$$\hat{\theta}_a = \max\{|\hat{\theta}(\underline{x}_i)(\ell)|: 1 \leq \ell \leq N-1, \ \ 1 \leq i \leq K\}$$

for the set. Next, the first sequence from each S_i becomes a member of the first set of phases to be evaluated. In the fourth step, the peak odd crosscorrelation

$$\hat{\theta}_c = \max\{|\hat{\theta}(\underline{x}_i,\underline{x}_j)(\ell)|: \ \ 0 \leq \ell \leq N-1, \ \ 1 \leq i < j \leq K\}$$

and the coordinates (i,j) of each sequence pair achieving $\hat{\theta}_c$ are found. Let $P = \{i: \max_{0 \leq \ell \leq N-1} |\hat{\theta}(\underline{x}_i,\underline{x}_j)(\ell)| = \hat{\theta}_c\}$. Fifth, for each $i \in P$ choose the next sequence phase from the set S_i, find $\hat{M}(\underline{x}_i,\underline{x}_j)$ for $1 \leq j \leq K$ and $j \neq i$ and compare to $\hat{\theta}_c$. If for any j this quantity is larger than or equal to $\hat{\theta}_c$ choose the next phase from S_i and reevaluate it. Do this until a phase for each $i \in P$ is found such that the peak crosscorrelation for the set is less than $\hat{\theta}_c$. Finally, return to step four, find this new $\hat{\theta}_c$ and repeat the process. This loop is continued until the algorithm fails. Failure occurs when for some $i \in P$, no sequence phase from S_i can be found such that

$$\max\{|\hat{\theta}(\underline{x}_i,\underline{x}_j)(\ell)|: \ \ 0 \leq \ell \leq N-1; \ \ 1 \leq j \leq k; \ \ j \neq i\}$$

is less than the current $\hat{\theta}_c$.

The worth of the algorithm rests on a few factors. First, a judicious choice of δ_T ensures that the possible number of different sets of sequences phases given by $\prod_{i=1}^{K} N_i$ is large enough to provide a good cross section for the search. Secondly, in step five the number of peak crosscorrelation values to be evaluated for each $i \in P$ is at most K-1 for each new set of phases rather than K(K-1). Finally, empirical verification has been obtained. For example, application of the algorithm to length 127 Gold sequences with K = 12 gives good results (see Section 3.2).

2.3 Algorithm for Finding Phases of Gold Sequences with Small Peak Odd Autocorrelation and Small Normalized Maximum Mean-Squared Multiple-Access Interference

The algorithm developed here suboptimally searches sets of phases of Gold sequences for those that give smaller values of the normalized maximum mean-squared multiple-access interference I_{max} defined in [9] by

$$I_{max} = \max\{(6N^3)^{-1} \sum_{\substack{k=1 \\ k \neq i}}^{K} r(\underline{x}_k, \underline{x}_i): \; 1 \leq i \leq K\} .$$

The first three steps of this algorithm proceed exactly as those of the previous algorithm to guarantee that any set of phases will have a small peak odd autocorrelation. The fourth step consists of selecting one sequence, say the first, from each S_i, $1 \leq i \leq K$. The sets S_i are defined in Section 2.2. In step five I_{max} is found for each of the sets formed by letting a different sequence phase from S_1 replace the first sequence in the set of K sequences while the sequences $\underline{x}_i$, $2 \leq i \leq K$ remain fixed. The phase from S_1 that gives the minimum value of I_{max} when used as the first sequence in the set of K sequences is kept as the first member of the desired set. Subsequently, I_{max} is found for each of the sets formed by letting a different sequence from S_i replace the i-th sequence while the other K-1 sequences remain fixed. The phase from S_i that gives the smallest value of I_{max} then becomes the i-th member of the set. This is done for each of the K sequences. Finally, step five is repeated so that the set just found is further optimized. This algorithm is applied to a set of twelve length 127 Gold sequences and the data is detailed in Section 3.3.

2.4 Algorithms for Finding Phases of Short-Length Gold Sequences with Small Normalized Maximum Mean-Squared Multiple-Access Interference and Small Correlation Parameters

In the application where N and K are small (e.g., N = 31, K = 3) all possible shifts of the K sequences may be examined and a set can be chosen that is optimal with respect to a desired parameter. This was done in [9] for three maximal-length sequences of length 31 with the optimization being performed on I_{max} defined in Section 2.3. From the definition of SNR_i in the introduction, the set of phases with smallest I_{max} are called maximum signal-to-noise ratio (max-SNR) phases. The algorithm proposed here is to apply the method of [9] to K Gold sequences of length N. There exists an extra degree of freedom since there are N + 2 different Gold sequences of length N. This implies that there are $\binom{N+2}{K}$ different sets of K sequences over which to look for the smallest I_{max} possible. Although N and K are assumed to be small, the quantity $\binom{N+2}{K}$ is typically large enough to make an exhaustive search infeasible. Thus, a linear search is applied to a small fraction of this total number of sets and, as shown in Section 3.3, this search yielded a significantly smaller I_{max} value than obtained for the m-sequences in [9].

Further improvement of the set found by the above algorithm may be desired. As seen in Section 3.4, sets with small I_{max} do not necessarily have the best attainable crosscorrelation parameters. Suboptimal shifts of the max-SNR phases may be found that yield better peak correlation parameters than the max-SNR phases while still having a small I_{max} parameter. Find $K' < K$ sets of shifts with small I_{max} and search these sets for the one set with smallest peak crosscorrelation, smallest peak odd autocorrelation and smallest numbers of occurrences of these peak values. K' may be determined by establishing a threshold value for I_{max} and choosing only those sets of phases that have I_{max} smaller than the threshold.

CHAPTER 3

NUMERICAL RESULTS

3.1 AO/LSE Phases for Two Families of Length 127 Gold Sequences

In this section we present data on the AO/LSE phases for two different families of Gold sequences of period 127. The first family of sequences is generated by the preferred pair of polynomials that are represented in octal [5, p. 1599] by 211 and 217. The sequences could also be generated by the product of these two polynomials over GF(2), which is given by 41567 in octal representation. The second family is generated by the pair of polynomials 211 and 235 whose product is 45365 in octal notation. The AO/LSE phase data is given in Tables A.1 and A.2 of Appendix A for these two sequences families. The AO/LSE phase of the sequence $\underline{x}_i$ is given in octal to be the initial loading of a 14-stage shift register with tap connections given by the product of one of the preferred pairs of polynomials above. The parameter $\hat{M}(\underline{x}_i)$ is the auto-optimal value of the peak odd autocorrelation for the sequence $\underline{x}_i$, and $\hat{L}(\underline{x}_i)$ is the number of occurrences of this peak value. $S(\underline{x}_i)$ is the sidelobe energy for the given phase. Sequences $\underline{x}_1$ and $\underline{x}_2$ in both tables are the m-sequences generated by preferred polynomials.

Table 3.1 gives the distribution of the sequences with respect to $\hat{M}(\underline{x}_i)$ for both families of Gold sequences. From this, it is evident that the family of sequences generated by the polynomial 41567 has a slightly higher density oi sequences with lower $\hat{M}(\underline{x}_i)$ values than the family generated by 45365. It can also be observed from Tables A.1 and A.2 that AO/LSE phases of the m-sequences of a set typically have low values of $\hat{M}(\underline{x}_i)$ with sidelobe energies significantly lower than the other sequences of the set.

Table 3.1

Distribution of the AO/LSE Phases of Length 127 Gold Sequences

Generating Polynomial:	41567	45365
	Number of Sequences	Number of Sequences
$\hat{M}(\underline{x}_i)$	Achieving $\hat{M}(\underline{x}_i)$	Achieving $\hat{M}(\underline{x}_i)$
13	1	0
15	1	2
17	17	17
19	68	56
21	40	49
23	2	4
25	0	1

3.2 Phases of Sets of Twelve Gold Sequences of Length 127 with Small Peak Odd Autocorrelation and Small Peak Odd Crosscorrelation

For an application of the algorithm outlined in Section 2.2, subsets of the Gold sequence family generated by the polynomial 41567 are used. Specifically, the sequences numbered 3 through 122 of Table A.1 are divided into ten sets of K = 12 sequences each. No attempt is made to choose sequences that will together produce optimal results; instead, the sets have the following arbitrary composition: set 1 consists of sequences 3-14, set 2 consists of sequences 15-26, ..., set 10 consists of sequences 111-122. The algorithm is performed on each of these ten sets.

It can be seen from Table A.1 that the largest value of the auto-optimal odd autocorrelation for sequences 3-122 is 23. $\hat{\theta}_T$ is consequently chosen to be 23 for each run of the algorithm. This ensures that all of the sets, S_i; $3 \leq i \leq 122$, are nonempty. Tables B.1 through B.10 of Appendix B give the phases (initial loadings in octal) and $N_i = \| S_i \|$, the number of shifts of each sequence that had peak odd autocorrelation less than $\hat{\theta}_T$ for the ten sets of phases generated by the program. Tables B.11 through B.20 give the peak correlation parameters for each set. In these tables the parameters $\hat{M}(\underline{x}_i)$, $\hat{M}(\underline{x}_i,\underline{x}_j)$, and $M(\underline{x}_i,\underline{x}_j)$ are given on, above, and below the diagonal, respectively, consistent with the data presentation of [5]. For purposes of comparison, the peak correlation parameters for each set in their AO/LSE phases are also given.

Table 3.2 gives the peak odd autocorrelation $\hat{\theta}_a$, the number of times $\hat{L}_a$ that $\hat{\theta}_a$ occurred for the set, the peak odd crosscorrelation $\hat{\theta}_c$, and the number of times $\hat{L}_c$ that $\hat{\theta}_c$ occurred for the set for each set of phases found by this sieve. Table 3.3 lists these parameters for the respective sets

Table 3.2

Peak Correlation Parameters for Ten Sets of Twelve Gold Sequences of Length 127 Shifted Into Small Crosscorrelation Phases

Set*	$\hat{\theta}_a$	$\hat{L}_a$	$\hat{\theta}_c$	$\hat{L}_c$
1	23	9	35	6
2	23	9	37	4
3	23	10	37	2
4	23	7	35	9
5	23	10	35	10
6	23	7	35	8
7	23	7	35	5
8	23	11	37	4
9	23	9	35	2
10	23	7	35	2

*The initial loadings for the sequences in each set are given in Tables B.1 through B.10.

Table 3.3

Peak Correlation Parameters for Ten Sets of Twelve Gold Sequences of Length 127 Shifted Into AO/LSE Phases

Set*	$\hat{\theta}_a$	$\hat{L}_a$	$\hat{\theta}_c$	$\hat{L}_c$
1	21	3	39	3
2	23	1	43	1
3	21	2	43	1
4	21	6	43	3
5	23	1	39	2
6	21	1	39	1
7	21	3	43	1
8	21	4	43	1
9	21	3	43	1
10	21	7	41	4

* The initial loadings for the sequences in each set are given in Table A.1.

shifted into their AO/LSE phases. From these tables and the tables of Appendix B some tradeoffs between the sets of sequences in low crosscorrelation phases and the sets in AO/LSE phases can be observed. The AO/LSE phases generally have smaller values of the odd autocorrelation parameter than the low crosscorrelation phases. Many of the low crosscorrelation sequences in each set achieve the threshold value of 23 established for $\hat{\theta}_a$ while many of the corresponding AO/LSE sequences attain values of 17, 19, or 21 for this parameter. The low crosscorrelation phases, however, perform better with respect to the peak odd crosscorrelation $\hat{\theta}_c$ than the corresponding AO/LSE phases; for some sets these phases give $\hat{\theta}_c = 35$ where the AO/LSE shifts give $\hat{\theta}_c$ as high as 43.

3.3 Phases of a Set of Twelve Gold Sequences of Length 127 with Small Peak Odd Autocorrelation and Small Normalized Maximum Mean-Squared Multiple-Access Interference

The set of twelve Gold sequences numbered 99-110 in Table A.1 is shifted into phases that yield a smaller value of I_{max} than the corresponding AO/LSE phases. $\hat{\theta}_T$ is again chosen to be 23. The initial loadings for this set are given in Table 3.4. For these phases the value of I_{max} is found to be 0.029012. The corresponding value of this parameter for the AO/LSE phases is 0.030730. This reflects only a 0.25 dB reduction in the I_{max} parameter. Nonetheless, the set of sequence phases produced by this algorithm give a smaller average probability of error than the AO/LSE shifts of these sequences (see Fig. 4.1).

Table 3.4

Phases of Sequences 99-110 of Table A.1 that have Small I_{max}

i	Initial Loadings
99	15370
100	34524
101	06747
102	37321
103	07627
104	36621
105	37762
106	04170
107	32077
108	34735
109	20607
110	25764

3.4 Phases of a Set of Three Gold Sequences of Length 31 with Small Normalized Maximum Mean-Squared Multiple-Access Interference and Small Correlation Parameters

The algorithm described in Section 2.4 is applied to subsets of the Gold sequence family generated by the preferred pair of polynomials 45 and 75 whose product is 3551 in octal. There are 33 sequences of length 31 in this Gold sequence family and a total of $\binom{33}{3}$ different sets of three sequences. A small fraction (less than one percent) of these sets is examined and a set of three sequences is found that when optimally shifted has a value of 0.014479 for the minimum I_{max}. This value compares to 0.017768 for the set of three m-sequences found in [9]. The initial loadings that give the max-SNR phases for the Gold sequences found above are listed in octal in Table 3.5.

Next, twenty sets of shifts of this set of sequences are found that had the smallest value of I_{max}. These twenty sets are examined to find the one that has correlation parameters meeting the criteria of Section 2.4. The selected phases give a value of 0.015217 for I_{max}. The initial loadings for this set of phases are given in Table 3.6. The peak correlation parameters for the max-SNR phases of m-sequences found in [9], max-SNR phases of Gold sequences of Table 3.5 and phases of Gold sequences of Table 3.6 are given in Table 3.7.

Table 3.5

Max SNR Phases for a Set of Three Gold Sequences of Length 31 Generated by 3551

	Initial Loadings
1	0417
2	1050
3	0115

Table 3.6

Phases for a Set of Three Gold Sequences of Length 31 Generated by 3551 that have Small I_{max} and Small Correlation Parameters

	Initial Loadings
1	0176
2	1230
3	1023

Table 3.7

Peak Correlation Parameters for Sets of Three Binary
Sequences of Length 31

Peak Correlation Parameters for a Set of Max-SNR Phases of m-Sequences
of Length 31 Generated by 45, 67, and 75

1	11	13	13
2	9	11	11
3	9	9	11

Peak Correlation Parameters for a Set of Max-SNR Phases of Gold Sequences
of Length 31 Generated by 3551

1	9	11	7
2	9	11	11
3	9	9	19

Peak Correlation Parameters for a Set of Phases of Gold Sequences of Length
31 Generated by 3551 that Have Small I_{max} and Small Correlation Parameters

1	11	9	7
2	9	9	9
3	9	9	15

CHAPTER 4

AVERAGE PROBABILITY OF ERROR EVALUATIONS

4.1 Characteristic-Function Method for Average Probability of Error Approximations

An evaluation of the average probability of error for various sets of Gold sequences applied to a direct-sequence spread-spectrum multiple-access system is carried out using the characteristic-function method first applied to DS/SSMA systems by Geraniotis [10]. It can be seen from the similarities between the multiple-access interference and multipath interference [2] that this type of analysis can be applied to a system used on a specular multipath channel.

The characteristic-function method involves integrating the characteristic function of the multiple-access interference component of the output of a correlation receiver. Two numerical integrations are performed to obtain the approximation of the probability of error. The first is part of the expression for the characteristic function of the multiple-access interference component and is accomplished using an n_ϕ point Simpson's rule integral approximation. The second integration is an nL point Simpson's rule approximation of a truncation of a semi-infinite integral. The integrand of this integral is the product of the characteristic function of a scaled channel noise random variable (Gaussian with zero mean and variance $N_0/2E_b$) and the characteristic function of the multiple-access interference random variable. The truncation of the second integral is at $L\pi$ and the actual evaluation is done by summing the n-point Simpson's rule approximations to the integrals from $\lambda\pi$ to $(\lambda+1)\pi$ for $\lambda = 0,1,\ldots,L-1$. Geraniotis and Pursley have shown [12] that the convergence of this method to the actual probability of error $P_{e,i}$

for the i-th receiver is insensitive to the value of n_ϕ as long $n_\phi \geq 10$. For a fixed sequence length N the convergence with respect to the value n is faster for a larger number of users K. It is shown in [11] that the error due to truncation ε of the second integral above is bounded by

$$\varepsilon \leq (8/\pi)^{\frac{1}{2}} \alpha Q(L\pi/\alpha)$$

where $\alpha = (2E_b/N_0)$ and $Q(x) = 1 - \Phi(x)$ and Φ is the standard Gaussian distribution function. For L = 20 and E_b/N_0 = 16 dB this guarantees that $\varepsilon < 2 \times 10^{-11}$.

4.2 Average Probability of Error for Sets of Twelve Gold Sequences of Length 127

The set composed of Gold sequences number 99-110 in Table A.1 is evaluated by the characteristic function method for various sets of phases. The value of n_ϕ is 10 and since K = 12 is large by the standards of analysis in [11], n is chosen to be 10 also. A value of L = 20 is used to assure truncation error bounds as given earlier.

The average probability of error at the output of the first correlation receiver $\bar{P}_{e,1}$ which by symmetry is the same for any receiver, is found for four different sets of phases of these sequences. The first three sets of phases were found earlier and are the AO/LSE phases of Table A.1, the low crosscorrelation phases of Table B.9, and the low I_{max} phases of Table 3.4. The fourth set of phases is a set of "random" shifts of the sequences; that is, an arbitrary phase for each sequence is chosen from the set of all cyclic shifts of the sequences numbered 99-110 of Table A.1. Figure 4.1 gives $\bar{P}_{e,1}$ versus the bit signal energy-to-noise ratio E_b/N_0. For further aid in comparing these sets of phases Table 4.1 gives the peak odd correlation parameters and

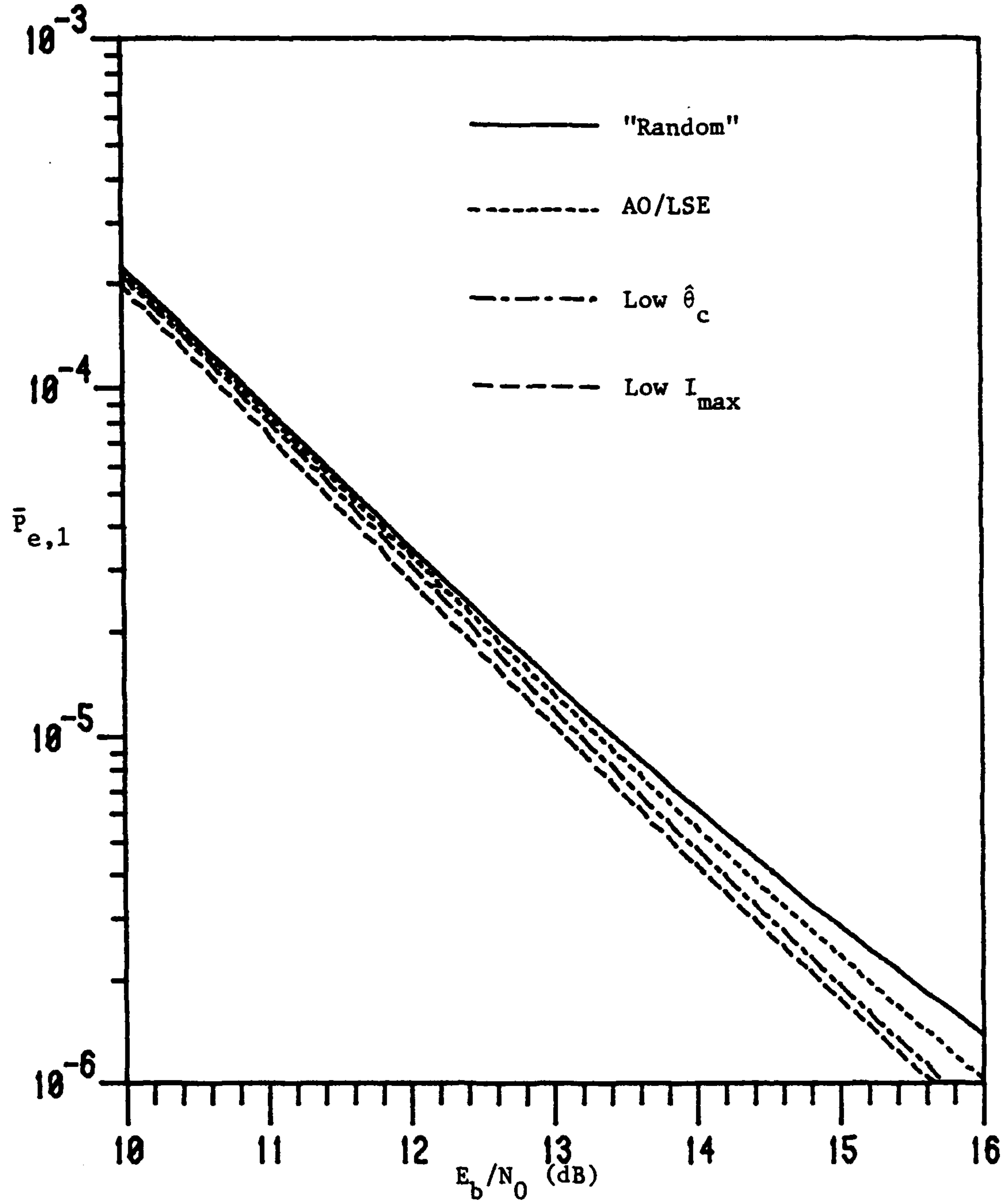

Figure 4.1. Average probability of error for four sets of twelve Gold sequences of length 127.

Table 4.1

Performance Parameters for Four Sets of Twelve Gold Sequences of Length 127 Generated by 41567

Phase*	$\hat{\theta}_c$	$\hat{\theta}_a$	I_{max}
Low I_{max}	39	23	0.029012
Low $\hat{\theta}_c$	35	23	0.030108
AO/LSE	43	21	0.030730
"Random"	43	43	0.030249

*The initial loadings for the low I_{max} phases are given in Table 3.4.

The initial loadings for the low $\hat{\theta}_c$ phases are given in Table B.9.

The initial loadings for the AO/LSE phases of sequences 99-110 are given in Table A.1.

I_{max} for each set. It can be seen from this table and Fig. 4.1 that although I_{max} is generally used as a predictor of average probability of error it cannot be solely considered. The I_{max} parameter for the "random" phases is smaller than that of the AO/LSE phases but its probability of error performance is worse at high signal-to-noise ratio values.

4.3 Average Probability of Error for Sets of Three Gold Sequences of Length 31

The average probability of error is found for the max-SNR phases of length 31 Gold sequences of Table 3.5 and for the suboptimal shifts of these sequences given by the loadings of Table 3.6. For these evaluations n_ϕ is chosen to be 10. Geraniotis and Pursley [11] claim that $n_\phi = 10$ and $n = 20$ are sufficient to obtain a very good approximation to the integral (even for $K = 3$), so $n = 20$ is used here in the evaluation of the Gold sequences. As before $L = 20$ is chosen to provide the necessary guard against large truncation errors that can occur at high values of E_b/N_0.

The error probabilities $\bar{P}_{e,1}$ for these two sets of phases are shown in Fig. 4.2. It can be seen that at high values of E_b/N_0 the shifts of the Gold sequences that have low odd correlation values and small I_{max} give better error performance than even the max-SNR shifts of the set. This suggests the need to employ algorithms that give consideration to more than one sequences parameter in the search for better shifts.

As a final note on sequence selection, a set of three Gold sequences of length 31 generated by 3551 is chosen such that shifts of these sequences yield a particularly large value of I_{max}. The chosen set of phases give $I_{max} = 0.037170$ and will be referred to as a min-SNR set of Gold sequences of length 31. The error probabilities $\bar{P}_{e,1}$ for these min-SNR Gold sequences

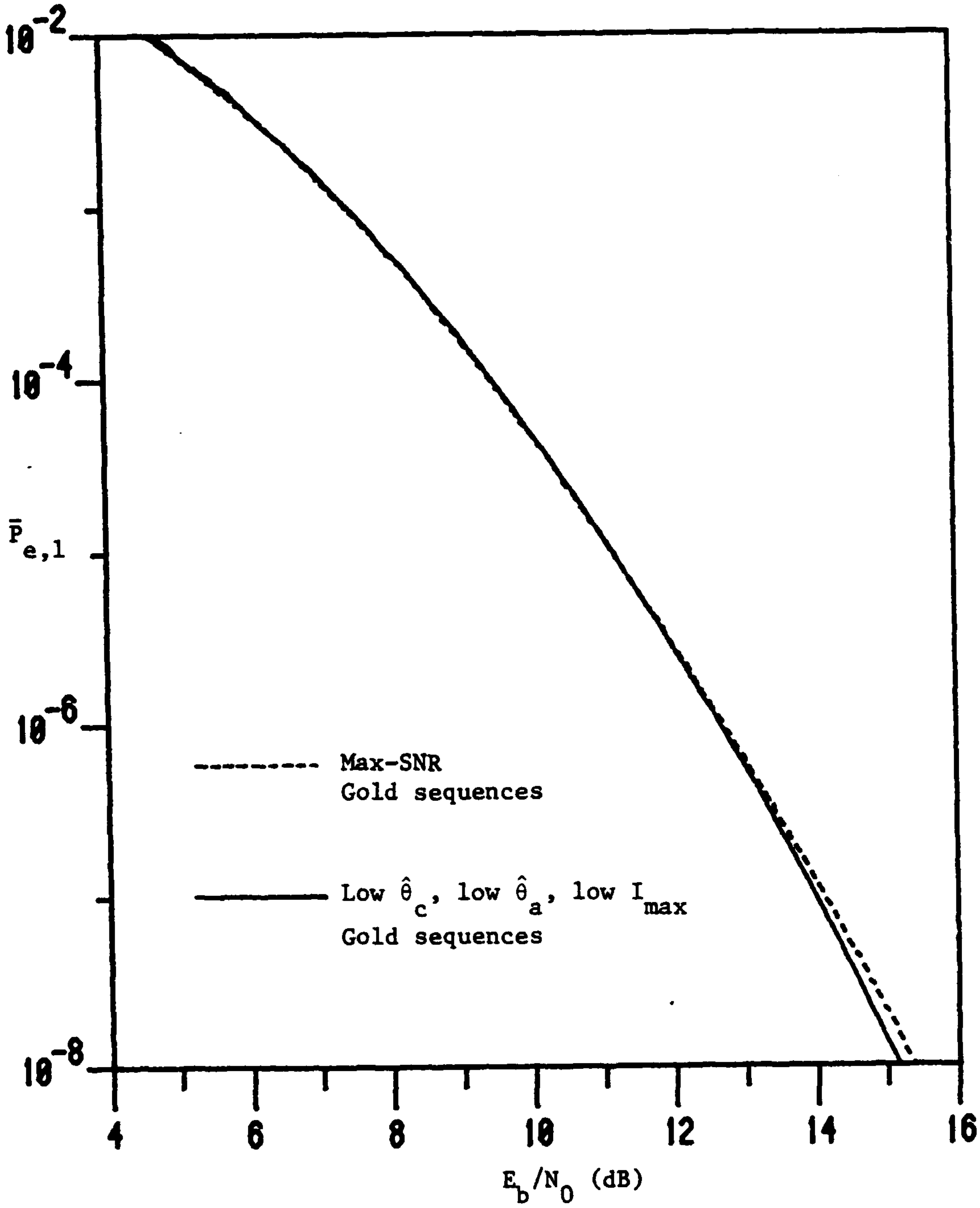

Figure 4.2. Average probability of error for two sets of three Gold sequences of length 31.

and the max-SNR Gold sequences of Table 3.5 are given in Fig. 4.3. Also included is $\bar{P}_{e,1}$ for the max-SNR and min-SNR phases of the three m-sequences generated by 45, 67, and 75 found in [9]. It can be seen from this figure that at an error rate of 10^{-5} one min-SNR phases of Gold sequences require a 2-db increase in E_b/N_0 over the max-SNR phases of Gold sequences. Also, at the same error rate, the max-SNR m-sequences need nearly a 1 dB increase in E_b/N_0 over the max-SNR phases of Gold sequences. Thus, careful signature sequence selection is necessary in order to obtain good average probability of error.

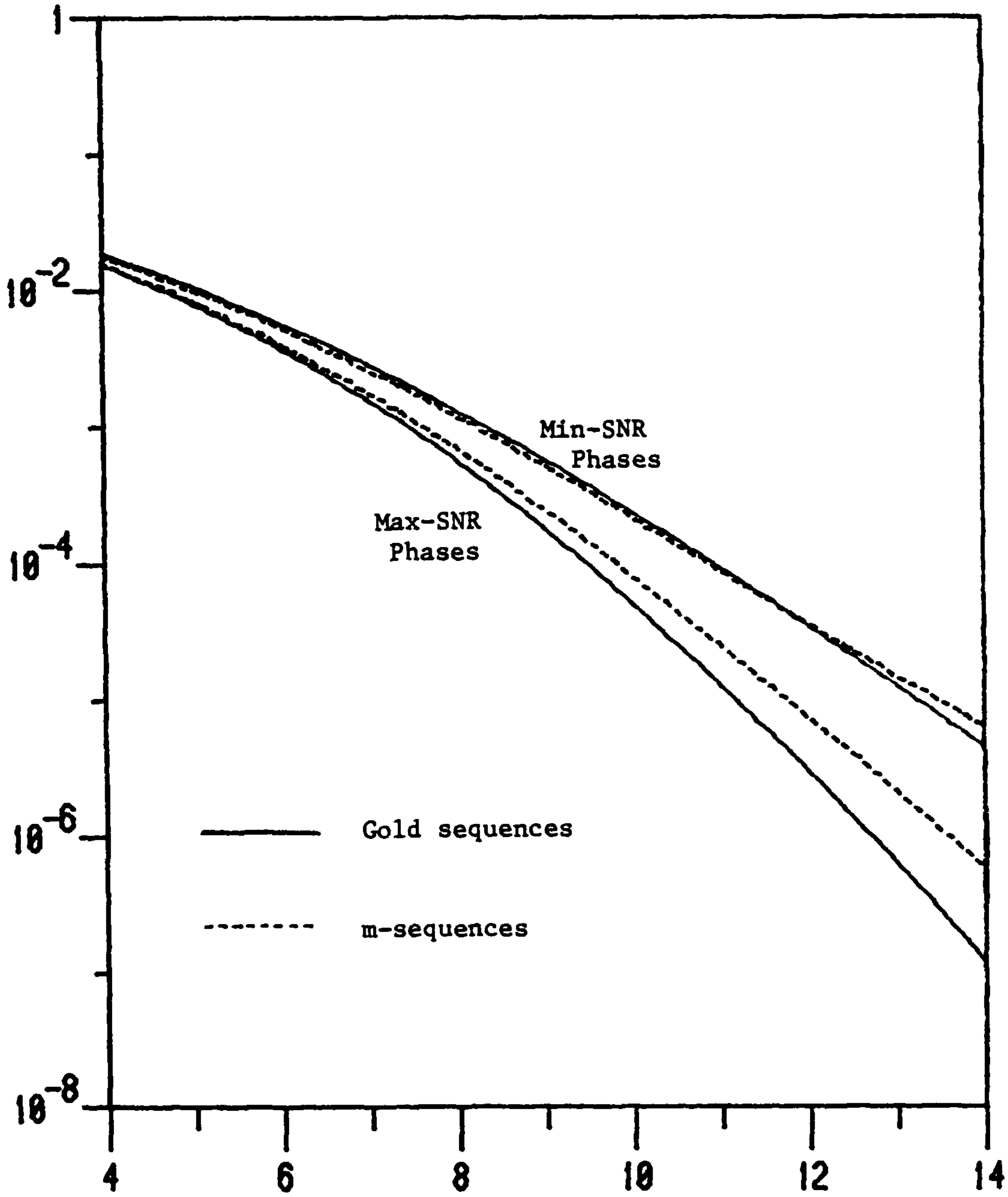

Figure 4.3. Average probabilities of error for sets of three Gold sequences and m-sequences of length 31 for two different sets of phases.

CHAPTER 5

CONCLUSIONS

The performance of a mobile packet radio system using a DS/SSMA scheme for channel access depends on the signal waveforms of the DS/SSMA system. To see how signature sequence selection affects a mobile packet radio system a specific example is considered. Suppose groups of eleven mobile terminals are clustered near the base stations as in Fig. 1.1. Let the mobile terminals in each group be constrained to travel within a geographical perimeter, called a cell, around the group's base station. In order to simplify the discussion, we assume the cells are located such that the interference in one cell due to an adjacent cell is small compared with the intra-cell interference. A coherent DS/SSMA system provides the channel access between terminals. Each terminal's receiver, including that of the base station, is assigned its own unique signature sequence. For example, each of the length 127 Gold sequences generated by the initial loadings given in Table B.9 may be assigned to the terminals in a given cell. A terminal may transmit to a receiver in his own cell by using the receiver's given code sequence. A terminal wishing to communicate with another terminal in a different cell must transmit to his own base station which in turn transmits to the desired terminal's base station. Thus, mobile terminals in different cells do not directly transmit to one another.

A prudent choice of the signature sequence sets assigned to the cells in this packet radio network helps offset undesirable effects such as multiple-access interference and multipath interference. If the multiple-access channel has a high throughput (i.e., many simultaneous transmitters), the worst-case probability of error due to multiple-access interference may be a critical design parameter. If so, sequence sets with low peak cross-correlation values, such as those of Appendix B, should be implemented in the mobile packet radio system. On the other hand, if the channel is not as actively used, perhaps sequence sets that give lower average probability of error values would be desired. In this case, sequence sets such as those found by the algorithm of Section 2.4 (e.g., the set specified by the initial loadings of Table 3.4) could be used in the network to give small average probability of error values.

For specular multipath channels small out-of-phase autocorrelation parameters are required to produce low probability of error. All of the algorithms of Chapter 2 find sequence sets with small peak autocorrelation parameters and, therefore, produce sets suitable for use in mobile packet radio communication over a specular multipath channel. Tables B.1 through B.10 each contain one set of twelve length 127 Gold sequence phases that can be assigned to each cell in a network of ten cells. This would assure out-of-phase autocorrelation peaks of at most 23 at the terminals' receivers. From Table 4.1 it can be seen that a random selection of shifts could give out-of-phase autocorrelation values as high as 43. Thus, the detrimental effects of multipath interference can be combatted by careful signature sequence selection.

The algorithms developed here have various applications for a mobile packet radio system. An easy method to find a large set of sequences with small out-of-phase autocorrelation peaks is to apply the AO/LSE algorithm to a family of Gold sequences. If low crosscorrelation magnitudes as well as low autocorrelation peaks are desired, small sets of sequence phases may be found by the program of Section 2.2. If small average probability of error values are required the algorithm of Section 2.3 can be applied to small sets of Gold sequences. It is shown that sequence phases produced by the algorithms proposed here have smaller average probability of error values than randomly selected phases. For example, it has been seen that at an error rate of 10^{-6} a randomly selected set of twelve length 127 Gold sequence phases requires nearly a 1 dB increase in E_b/N_0 over sets of phases found by the algorithms (See Fig. 4.1). At this same error rate Fig. 4.3 shows two sets of three length 31 Gold sequences that have a difference of almost 2 dB in E_b/N_0. Thus, the choice of signature sequences in a DS/SSMA mobile packet radio network affects the system performance.

APPENDIX A

DATA ON AO/LSE PHASES FOR TWO FAMILIES OF GOLD SEQUENCES OF LENGTH 127

(The initial loadings for these phases are given in octal as described in [5, p. 1599])

Table A.1

AO/LSE Phases of Gold Sequences of Length 127 Generated by 41567

i	Initial Loading	$\hat{M}(\underline{x}_i)$	$\hat{L}(\underline{x}_i)$	$S(\underline{x}_i)$
1	04021	17	6	2183
2	01261	15	12	2015
3	36576	17	12	6727
4	36606	19	2	6311
5	26172	17	6	5839
6	32715	19	4	6839
7	30371	19	2	7527
8	10214	17	8	5863
9	15167	19	2	5563
10	20743	21	2	7303
11	04312	19	6	7479
12	05355	21	2	7775
13	30074	21	2	7391
14	30005	19	2	6291
15	13750	21	2	6347
16	17160	19	2	5463
17	31216	19	2	6415
18	12142	21	4	6347
19	30400	21	6	7875
20	27525	21	4	8067
21	22356	23	2	6931
22	04376	21	4	8327
23	36522	19	12	7351
24	35104	21	2	7095
25	14045	19	4	6331
26	12513	19	4	6235
27	37160	17	10	6559
28	20151	19	4	6443
29	05263	19	8	7267
30	33112	19	2	5571
31	34745	19	2	7275
32	36762	17	10	7131
33	35154	19	6	6859
34	21565	21	2	6995
35	35754	19	4	7823
36	24710	19	6	7027
37	01756	21	2	6647
38	34424	19	2	6883
39	35117	19	6	7155
40	00506	19	4	6323
41	00203	21	2	7867
42	27761	17	8	6987

Table A.1 (Continued)

$\underline{i}$	Initial Loading	$\hat{M}(\underline{x}_i)$	$\hat{L}(\underline{x}_i)$	$S(\underline{x}_i)$
43	01344	21	2	7175
44	35671	21	2	7055
45	16246	19	6	7807
46	16311	21	2	6411
47	22476	21	2	6631
48	24272	19	6	6255
49	34031	21	2	7351
50	02104	19	2	5695
51	23212	19	8	6719
52	03637	21	2	6955
53	30351	17	6	6779
54	21255	21	4	6603
55	35234	21	4	7671
56	15015	21	2	6963
57	34112	19	2	6143
58	35576	19	2	5651
59	35332	23	2	7339
60	32037	19	4	6647
61	26453	19	2	5931
62	36256	17	4	6723
63	27463	21	2	7591
64	04754	19	4	6359
65	37727	19	4	6335
66	37347	19	10	6947
67	04726	19	4	6739
68	30621	19	4	7279
69	15665	13	8	4815
70	30152	19	6	7335
71	03337	19	4	6703
72	05253	19	4	6459
73	27445	19	4	6219
74	06175	17	4	6563
75	07161	19	8	6895
76	32613	17	12	6915
77	11506	21	2	6411
78	03354	19	6	6499
79	33717	19	6	6775
80	16757	21	2	6995
81	31204	19	6	6919
82	35664	19	2	7747
83	30130	19	4	6567
84	31167	19	8	7411
85	21172	21	6	7619
86	22207	19	4	6663

Table A.1 (Continued)

i	Initial Loading	$\hat{M}(\underline{x}_i)$	$\hat{L}(\underline{x}_i)$	$S(\underline{x}_i)$
87	15336	19	2	6879
88	36023	19	2	7383
89	32577	19	8	7331
90	15624	21	6	7579
91	04426	17	16	7083
92	02600	19	2	7115
93	37436	19	2	6207
94	35272	19	4	6855
95	16642	21	2	6267
96	30431	21	2	6719
97	05223	19	2	6915
98	24430	21	2	6407
99	03665	19	6	6307
100	34524	19	2	7271
101	25653	19	6	6899
102	02535	19	6	7227
103	17613	17	4	5895
104	36621	19	2	6495
105	30202	19	6	7363
106	12160	21	2	6331
107	24176	21	2	7539
108	01577	21	2	7195
109	31243	19	6	7243
110	07455	19	4	6911
111	16777	19	2	5911
112	23152	21	2	6751
113	13762	19	14	7983
114	05573	21	4	6535
115	06477	21	2	7243
116	27020	21	4	6719
117	10673	17	6	6703
118	17216	21	2	6943
119	27111	21	2	7759
120	30250	17	8	6067
121	12726	21	2	6999
122	13146	19	10	7291
123	25114	17	8	6671
124	37152	19	6	7211
125	30317	17	6	6443
126	27726	19	2	6271
127	25356	19	6	7343
128	22574	19	2	6223
129	01450	21	2	5447

Table A.2

AO/LSE Phases of Gold Sequences of Length 127 Generated by 45365

i	Initial Loading	$\hat{M}(\underline{x}_i)$	$\hat{L}(\underline{x}_i)$	$S(\underline{x}_i)$
1	04021	17	6	2183
2	03036	17	10	2283
3	24105	21	2	6015
4	13656	15	8	5803
5	27610	21	2	8315
6	35366	21	2	6371
7	31147	21	2	7307
8	10365	21	2	7887
9	30313	21	2	6739
10	31333	19	4	6747
11	07170	19	4	6255
12	23307	19	4	7131
13	10040	19	2	5787
14	20016	21	4	7759
15	04376	19	8	6355
16	21264	21	4	7495
17	21075	19	2	6451
18	30460	19	2	6183
19	05313	19	2	6183
20	37700	15	4	5655
21	16451	21	2	7259
22	26367	19	2	7067
23	11242	21	6	8519
24	33446	19	2	6083
25	14570	17	8	6775
26	21200	17	8	7399
27	25302	17	4	5779
28	37706	19	2	5983
29	15304	21	2	6235
30	23731	21	2	6575
31	37703	21	2	6535
32	11516	19	2	6735
33	11025	19	6	6487
34	13067	19	2	6435
35	02000	21	2	6803
36	00421	21	4	5967
37	01366	23	2	5575
38	13405	19	4	6015
39	01103	17	4	6595
40	20441	21	2	6643
41	35246	17	4	6143
42	35563	19	10	7999

Table A.2 (Continued)

i	Initial Loading	$\hat{M}(\underline{x}_i)$	$\hat{L}(\underline{x}_i)$	$S(\underline{x}_i)$
43	32411	21	2	7387
44	07500	21	2	7391
45	07164	19	6	7619
46	15537	19	2	5887
47	17466	19	2	6407
48	00422	21	2	7007
49	01157	21	2	7075
50	05201	19	4	7167
51	35360	19	4	6619
52	24700	17	4	6267
53	35465	21	2	7187
54	06152	19	2	6347
55	34336	21	2	7999
56	32131	19	8	7043
57	07714	21	2	7899
58	33735	17	2	5843
59	00621	21	2	7375
60	06616	19	4	6443
61	00753	25	2	8467
62	07440	19	2	5527
63	07453	21	2	7431
64	30057	21	4	6415
65	05636	21	2	6663
66	20713	19	6	6127
67	17010	19	2	7231
68	01642	19	6	7243
69	23355	19	2	6571
70	13101	19	8	7051
71	20215	21	2	6615
72	27734	21	2	7115
73	35312	19	2	6619
74	16006	21	2	6175
75	20274	21	2	6463
76	11000	19	2	6887
77	11356	19	4	6791
78	23132	21	2	6955
79	06543	17	10	6523
80	07713	17	10	6471
81	36774	19	2	6759
82	30203	17	8	6975
83	15637	21	4	6691
84	05277	21	2	7747
85	13044	19	4	6939
86	07657	19	6	7627

Table A.2 (Continued)

i	Initial Loading	$\hat{M}(\underline{x}_i)$	$\hat{L}(\underline{x}_i)$	$S(\underline{x}_i)$
87	37731	19	6	7315
88	30205	23	2	7327
89	36667	19	4	7123
90	00346	19	12	6807
91	33241	21	2	6955
92	07736	21	2	7103
93	21170	19	8	7335
94	24765	21	8	8023
95	11446	21	2	7871
96	12702	19	6	7127
97	13573	21	6	7951
98	12414	21	2	7755
99	37765	17	2	6483
100	14060	19	2	6015
101	12002	19	4	7575
102	02454	19	6	6799
103	33361	19	4	6767
104	14435	19	4	6351
105	24253	19	2	6503
106	17535	19	4	6911
107	36660	21	2	6911
108	35171	21	2	7471
109	24316	19	4	6927
110	11776	17	12	6867
111	12110	19	2	7303
112	25127	19	4	7659
113	21350	21	2	7855
114	04266	19	8	6919
115	33745	17	4	5663
116	04137	21	2	6875
117	20115	17	12	6483
118	04215	19	10	7179
119	26275	21	4	7779
120	35117	23	2	8763
121	02051	19	6	7043
122	22236	21	2	6995
123	27715	21	2	6591
124	30524	23	2	7627
125	01604	21	2	7855
126	27246	19	6	7115
127	25255	19	6	7055
128	04775	21	2	7175
129	13121	17	10	5347

APPENDIX B

PHASES AND PEAK CORRELATION PARAMETERS FOR TEN SETS OF TWELVE GOLD SEQUENCES OF LENGTH 127 WITH SMALL PEAK ODD AUTOCORRELATION AND SMALL PEAK ODD CROSSCORRELATION

(The sequences are shifts of the sequences generated by the polynomial 41567 recorded in Table A.1 and are identified by the numbers i, $3 \leq i \leq 122$ corresponding to those sequences. The initial loadings for these phases are given in octal as described in [5, p. 1599])

Table B.1

Phases for Sequences 3-14 That Give Low Peak Crosscorrelation

i	Initial Loading	N_i
3	21365	29
4	12232	21
5	00067	33
6	22471	11
7	32560	20
8	14615	44
9	36514	63
10	06374	31
11	37054	18
12	05704	27
13	26256	39
14	06321	37

Table B.2

Phases for Sequences 15-26 That Give Low Peak Crosscorrelation

i	Initial Loading	N_i
15	32026	22
16	23113	24
17	11320	36
18	17245	23
19	34700	32
20	04155	32
21	11326	10
22	22154	30
23	17636	15
24	34722	30
25	24301	16
26	10363	20

Table B.3

Phases for Sequences 27-38 That Give Low Peak Crosscorrelation

i	Initial Loading	N_i
27	06124	40
28	10675	27
29	14526	28
30	34574	47
31	01771	37
32	14757	20
33	36604	12
34	06643	24
35	24052	45
36	13077	18
37	20503	34
38	24441	57

Table B.4

Phases for Sequences 39-50 That Give Low Peak Crosscorrelation

i	Initial Loading	N_i
39	21356	31
40	20024	21
41	36515	23
42	03677	25
43	24134	14
44	33562	6
45	23007	34
46	01631	9
47	34512	35
48	17770	38
49	12555	7
50	24061	62

Table B.5

Phases for Sequences 51-62 That Give Low Peak Crosscorrelation

i	Initial Loading	N_i
51	14672	21
52	02010	18
53	35255	32
54	24130	8
55	27375	26
56	16640	10
57	25217	25
58	21553	50
59	16555	4
60	13613	20
61	12733	27
62	13625	22

Table B.6

Phases for Sequences 63-74 That Give Low Peak Crosscorrelation

i	Initial Loading	N_i
63	14040	33
64	33042	28
65	37727	11
66	23452	30
67	21216	17
68	14310	19
69	37205	31
70	30152	27
71	04315	48
72	15204	14
73	14333	16
74	13044	36

Table B.7

Phases for Sequences 75-86 That Give Low Peak Crosscorrelation

i	Initial Loading	N_i
75	34045	25
76	05665	34
77	10470	23
78	05616	36
79	26114	13
80	34125	23
81	16241	15
82	00003	37
83	06564	26
84	03116	13
85	21172	10
86	22071	34

Table B.8

Phases for Sequences 87-98 That Give Low Peak Crosscorrelation

i	Initial Loading	N_i
87	25326	38
88	06154	30
89	16553	21
90	37501	12
91	27703	16
92	16446	35
93	15735	51
94	01154	35
95	15054	22
96	23677	19
97	07461	25
98	05106	10

Table B.9

Phases for Sequences 99-110 That Give Low Peak Crosscorrelation

i	Initial Loading	N_i
99	27773	29
100	22523	28
101	35262	19
102	10150	15
103	04371	29
104	25332	39
105	22717	15
106	00631	23
107	21743	21
108	36356	19
109	07452	27
110	04717	21

Table B.10

Phases for Sequences 111-122 That Give Low Peak Crosscorrelation

i	Initial Loading	N_i
111	00163	54
112	23146	13
113	01377	11
114	02466	8
115	24026	29
116	35604	8
117	35002	31
118	30735	10
119	15674	25
120	23605	25
121	23600	22
122	07246	9

Table B.11

Peak Correlation Parameters for Sequences 3-14

Peak Correlation Parameters for Sequences in Low Crosscorrelation Phases

3	21	33	25	31	33	31	31	35	29	35	29	31
4	17	23	33	25	29	29	33	27	35	35	33	27
5	17	17	23	23	31	29	33	29	29	31	31	31
6	17	17	17	23	31	29	33	31	29	29	33	23
7	17	17	17	17	23	27	31	27	33	35	25	31
8	17	17	17	17	17	23	27	33	31	31	27	33
9	17	17	17	17	17	17	23	33	31	29	35	31
10	17	17	17	17	17	17	17	21	29	33	31	33
11	17	17	17	17	17	17	17	17	23	27	31	27
12	17	17	17	17	17	17	17	17	17	21	31	27
13	17	17	17	17	17	17	17	17	17	17	23	27
14	17	17	17	17	17	17	17	17	17	17	17	23

Peak Correlation Parameters for Sequences in AO/LSE Phases

3	17	29	35	29	27	29	29	31	27	39	29	31
4	17	19	27	31	29	31	29	33	33	31	37	29
5	17	17	17	27	33	27	35	25	31	29	31	29
6	17	17	17	19	39	31	35	35	37	35	33	27
7	17	17	17	17	19	27	33	31	27	33	33	33
8	17	17	17	17	17	17	37	29	31	33	33	29
9	17	17	17	17	17	17	19	29	35	29	27	35
10	17	17	17	17	17	17	17	21	29	33	29	29
11	17	17	17	17	17	17	17	17	19	23	25	31
12	17	17	17	17	17	17	17	17	17	21	35	39
13	17	17	17	17	17	17	17	17	17	17	21	27
14	17	17	17	17	17	17	17	17	17	17	17	19

Table B.12

Peak Correlation Parameters for Sequences 15-26

Peak Correlation Parameters for Sequences in Low Crosscorrelation Phases

15	23	27	31	31	31	27	25	35	29	27	27	27
16	17	23	29	27	37	29	35	35	33	27	25	31
17	17	17	23	31	27	33	29	25	31	35	37	33
18	17	17	17	21	33	25	31	31	29	29	31	31
19	17	17	17	17	23	25	29	31	33	27	35	37
20	17	17	17	17	17	21	33	27	23	31	31	25
21	17	17	17	17	17	17	23	29	29	23	33	27
22	17	17	17	17	17	17	17	23	29	37	27	27
23	17	17	17	17	17	17	17	17	23	27	27	33
24	17	17	17	17	17	17	17	17	17	23	31	25
25	17	17	17	17	17	17	17	17	17	17	23	33
26	17	17	17	17	17	17	17	17	17	17	17	21

Peak Correlation Parameters for Sequences in AO/LSE Phases

15	21	31	29	25	29	29	29	35	29	27	33	31
16	17	19	35	35	33	29	33	37	37	33	25	37
17	17	17	19	29	29	29	29	29	25	29	29	25
18	17	17	17	21	33	43	33	39	29	27	35	31
19	17	17	17	17	21	33	27	27	37	25	33	31
20	17	17	17	17	17	21	37	27	33	35	29	35
21	17	17	17	17	17	17	23	39	35	29	31	29
22	17	17	17	17	17	17	17	21	31	29	31	31
23	17	17	17	17	17	17	17	17	19	37	27	33
24	17	17	17	17	17	17	17	17	17	21	33	37
25	17	17	17	17	17	17	17	17	17	17	19	33
26	17	17	17	17	17	17	17	17	17	17	17	19

Table B.13

Peak Correlation Parameters for Sequences 27-38

Peak Correlation Parameters for Sequences in Low Crosscorrelation Phases

27	23	35	31	23	33	29	31	27	29	25	31	31
28	17	23	31	35	25	35	27	35	33	29	25	35
29	17	17	23	33	29	27	35	29	29	27	27	25
30	17	17	17	23	29	29	27	31	33	31	29	31
31	17	17	17	17	23	27	33	27	27	29	25	33
32	17	17	17	17	17	21	25	27	35	31	25	33
33	17	17	17	17	17	17	23	35	29	33	31	37
34	17	17	17	17	17	17	17	23	25	31	31	33
35	17	17	17	17	17	17	17	17	23	27	37	27
36	17	17	17	17	17	17	17	17	17	23	31	31
37	17	17	17	17	17	17	17	17	17	17	23	33
38	17	17	17	17	17	17	17	17	17	17	17	21

Peak Correlation Parameters for Sequences in AO/LSE Phases

27	17	35	29	35	27	41	31	29	29	33	27	31
28	17	19	29	25	29	35	31	43	25	25	29	31
29	17	17	19	31	29	25	35	31	29	27	29	29
30	17	17	17	19	31	27	33	27	27	41	29	31
31	17	17	17	17	19	33	33	31	29	29	29	39
32	17	17	17	17	17	17	29	25	33	33	27	31
33	17	17	17	17	17	17	19	29	29	37	27	29
34	17	17	17	17	17	17	17	21	27	33	29	33
35	17	17	17	17	17	17	17	17	19	25	33	31
36	17	17	17	17	17	17	17	17	17	19	31	29
37	17	17	17	17	17	17	17	17	17	17	21	37
38	17	17	17	17	17	17	17	17	17	17	17	19

Table B.14

Peak Correlation Parameters for Sequences 39-50

Peak Correlation Parameters for Sequences in Low Crosscorrelation Phases

39	21	33	29	29	29	29	33	31	27	33	29	29
40	17	23	25	31	33	33	29	35	27	33	25	35
41	17	17	23	33	35	35	27	29	33	25	29	29
42	17	17	17	23	27	27	25	27	27	31	33	27
43	17	17	17	17	21	29	31	35	35	35	35	33
44	17	17	17	17	17	21	29	25	35	25	31	31
45	17	17	17	17	17	17	23	31	29	29	33	27
46	17	17	17	17	17	17	17	21	31	29	27	31
47	17	17	17	17	17	17	17	17	21	27	33	29
48	17	17	17	17	17	17	17	17	17	23	33	29
49	17	17	17	17	17	17	17	17	17	17	23	33
50	17	17	17	17	17	17	17	17	17	17	17	23

Peak Correlation Parameters for Sequences in AO/LSE Phases

39	19	27	27	35	27	43	35	29	27	31	29	33
40	17	19	35	27	31	27	29	33	29	29	33	29
41	17	17	21	27	29	33	23	33	31	31	43	31
42	17	17	17	17	37	31	35	35	31	27	27	29
43	17	17	17	17	21	27	29	35	39	29	31	35
44	17	17	17	17	17	21	35	29	39	31	35	31
45	17	17	17	17	17	17	19	23	27	35	35	29
46	17	17	17	17	17	17	17	21	29	35	29	43
47	17	17	17	17	17	17	17	17	21	27	29	41
48	17	17	17	17	17	17	17	17	17	19	31	31
49	17	17	17	17	17	17	17	17	17	17	21	35
50	17	17	17	17	17	17	17	17	17	17	17	19

Table B.15

Peak Correlation Parameters for Sequences 51-62

Peak Correlation Parameters for Sequences in Low Crosscorrelation Phases

51	23	33	29	33	23	35	29	35	35	31	27	27
52	17	21	27	27	35	29	29	35	33	33	35	31
53	17	17	23	33	33	29	31	27	33	31	25	31
54	17	17	17	21	29	31	31	29	31	29	33	23
55	17	17	17	17	23	31	31	29	31	33	31	33
56	17	17	17	17	17	23	33	35	35	29	35	33
57	17	17	17	17	17	17	23	33	27	27	29	27
58	17	17	17	17	17	17	17	23	31	27	31	25
59	17	17	17	17	17	17	17	17	23	33	31	29
60	17	17	17	17	17	17	17	17	17	23	31	35
61	17	17	17	17	17	17	17	17	17	17	23	33
62	17	17	17	17	17	17	17	17	17	17	17	23

Peak Correlation Parameters for Sequences in AO/LSE Phases

51	19	35	29	33	37	25	35	39	33	31	31	31
52	17	21	31	27	37	27	33	29	33	27	37	33
53	17	17	17	27	29	29	29	31	29	31	29	25
54	17	17	17	21	31	33	29	37	25	33	29	25
55	17	17	17	17	21	29	31	27	29	31	27	35
56	17	17	17	17	17	21	33	29	29	27	29	35
57	17	17	17	17	17	17	19	27	39	35	29	31
58	17	17	17	17	17	17	17	19	37	27	27	25
59	17	17	17	17	17	17	17	17	23	39	29	33
60	17	17	17	17	17	17	17	17	17	19	31	25
61	17	17	17	17	17	17	17	17	17	17	19	29
62	17	17	17	17	17	17	17	17	17	17	17	17

Table B.16

Peak Correlation Parameters for Sequences 63-74

Peak Correlation Parameters for Sequences in Low Crosscorrelation Phases

63	23	33	25	27	31	29	33	33	29	27	29	25
64	17	23	29	27	35	35	27	27	27	27	33	29
65	17	17	19	29	27	29	25	27	33	31	31	31
66	17	17	17	21	35	31	35	29	31	27	35	25
67	17	17	17	17	23	27	31	35	29	29	27	33
68	17	17	17	17	17	21	27	31	31	35	27	31
69	17	17	17	17	17	17	21	31	31	25	25	27
70	17	17	17	17	17	17	17	19	31	27	29	35
71	17	17	17	17	17	17	17	17	23	35	29	31
72	17	17	17	17	17	17	17	17	17	23	33	29
73	17	17	17	17	17	17	17	17	17	17	23	33
74	17	17	17	17	17	17	17	17	17	17	17	23

Peak Correlation Parameters for Sequences in AO/LSE Phases

63	21	27	29	31	29	27	25	31	31	23	27	29
64	1[illegible]	19	31	33	29	27	33	29	27	37	37	27
65	1[illegible]	17	19	25	31	31	29	27	35	37	35	29
66	17	17	17	19	27	33	39	27	25	31	27	31
67	17	17	17	17	19	29	31	31	29	35	25	33
68	17	17	17	17	17	19	33	33	27	29	33	27
69	17	17	17	17	17	17	13	31	27	27	27	35
70	17	17	17	17	17	17	17	19	29	25	27	29
71	17	17	17	17	17	17	17	17	19	33	33	35
72	17	17	17	17	17	17	17	17	17	19	29	31
73	17	17	17	17	17	17	17	17	17	17	19	37
74	17	17	17	17	17	17	17	17	17	17	17	17

Table B.17

Peak Correlation Parameters for Sequences 75-86

Peak Correlation Parameters for Sequences in Low Crosscorrelation Phases

75	21	29	29	33	31	31	33	33	29	33	27	29
76	17	21	31	29	27	29	33	31	33	33	29	33
77	17	17	23	23	31	29	31	33	31	33	27	31
78	17	17	17	21	29	27	31	33	31	27	29	29
79	17	17	17	17	23	29	33	35	29	31	33	25
80	17	17	17	17	17	23	31	31	27	23	25	35
81	17	17	17	17	17	17	23	29	25	31	27	27
82	17	17	17	17	17	17	17	23	27	35	35	29
83	17	17	17	17	17	17	17	17	23	31	35	35
84	17	17	17	17	17	17	17	17	17	21	31	29
85	17	17	17	17	17	17	17	17	17	17	21	29
86	17	17	17	17	17	17	17	17	17	17	17	23

Peak Correlation Parameters for Sequences in AO/LSE Phases

75	19	33	27	29	31	41	31	29	33	23	25	31
76	17	17	31	25	29	35	27	27	29	29	27	33
77	17	17	21	29	35	31	27	31	31	39	27	27
78	17	17	17	19	33	29	37	29	25	35	41	31
79	17	17	17	17	19	35	25	29	33	27	43	27
80	17	17	17	17	17	21	31	37	31	35	31	27
81	17	17	17	17	17	17	19	37	29	37	27	31
82	17	17	17	17	17	17	17	19	29	27	31	25
83	17	17	17	17	17	17	17	17	19	29	35	31
84	17	17	17	17	17	17	17	17	17	19	33	33
85	17	17	17	17	17	17	17	17	17	17	21	27
86	17	17	17	17	17	17	17	17	17	17	17	19

Table B.18

Peak Correlation Parameters for Sequences 87-98

Peak Correlation Parameters for Sequences in Low Crosscorrelation Phases

87	23	35	31	33	31	27	25	25	33	29	33	27
88	17	21	29	25	31	27	27	35	31	29	33	31
89	17	17	23	29	31	31	29	33	27	31	29	31
90	17	17	17	23	29	25	27	27	35	31	25	27
91	17	17	17	17	23	35	37	35	29	35	33	37
92	17	17	17	17	17	23	27	29	27	33	37	31
93	17	17	17	17	17	17	23	37	31	27	35	35
94	17	17	17	17	17	17	17	23	33	29	27	29
95	17	17	17	17	17	17	17	17	23	31	31	27
96	17	17	17	17	17	17	17	17	17	23	31	29
97	17	17	17	17	17	17	17	17	17	17	23	29
98	17	17	17	17	17	17	17	17	17	17	17	23

Peak Correlation Parameters for Sequences in AO/LSE Phases

87	19	31	29	29	25	27	31	25	41	29	29	29
88	17	19	23	25	23	29	31	29	33	31	29	35
89	17	17	19	31	27	27	29	31	35	27	31	37
90	17	17	17	21	35	31	27	31	39	27	27	33
91	17	17	17	17	17	43	31	41	27	37	31	27
92	17	17	17	17	17	19	25	31	29	33	33	35
93	17	17	17	17	17	17	19	35	35	33	25	39
94	17	17	17	17	17	17	17	19	37	27	29	31
95	17	17	17	17	17	17	17	17	21	27	27	31
96	17	17	17	17	17	17	17	17	17	21	27	41
97	17	17	17	17	17	17	17	17	17	17	19	35
98	17	17	17	17	17	17	17	17	17	17	17	21

Table B.19

Peak Correlation Parameters for Sequences 99-110

Peak Correlation Parameters for Sequences in Low Crosscorrelation Phases

99	23	27	25	29	25	27	33	27	31	31	27	31
100	17	23	31	33	29	31	27	29	31	33	27	25
101	17	17	23	25	31	29	33	29	29	33	27	33
102	17	17	17	21	29	27	29	27	35	29	27	31
103	17	17	17	17	21	29	27	29	29	31	29	31
104	17	17	17	17	17	23	23	25	29	31	33	23
105	17	17	17	17	17	17	23	27	31	31	33	27
106	17	17	17	17	17	17	17	23	33	27	35	29
107	17	17	17	17	17	17	17	17	23	27	31	29
108	17	17	17	17	17	17	17	17	17	23	29	29
109	17	17	17	17	17	17	17	17	17	17	21	31
110	17	17	17	17	17	17	17	17	17	17	17	23

Peak Correlation Parameters for Sequences in AO/LSE Phases

99	19	27	27	25	33	27	37	31	37	27	41	31
100	17	19	35	27	35	31	25	27	25	33	39	29
101	17	17	19	31	31	31	27	29	25	33	27	31
102	17	17	17	19	37	29	31	33	33	25	35	29
103	17	17	17	17	17	37	33	37	29	27	27	29
104	17	17	17	17	17	19	35	33	27	37	37	23
105	17	17	17	17	17	17	19	33	41	31	29	27
106	17	17	17	17	17	17	17	21	31	35	39	31
107	17	17	17	17	17	17	17	17	21	31	43	37
108	17	17	17	17	17	17	17	17	17	21	29	31
109	17	17	17	17	17	17	17	17	17	17	19	35
110	17	17	17	17	17	17	17	17	17	17	17	19

Table B.20

Peak Correlation Parameters for Sequences 111-122

Peak Correlation Parameters for Sequences in Low Crosscorrelation Phases

111	21	25	29	27	29	29	29	31	29	31	33	29
112	17	21	31	29	29	27	27	29	33	31	31	31
113	17	17	21	29	31	33	27	31	27	29	27	27
114	17	17	17	23	29	35	31	31	31	29	31	31
115	17	17	17	17	23	27	25	23	29	29	25	25
116	17	17	17	17	17	21	29	31	29	33	29	33
117	17	17	17	17	17	17	21	29	35	25	27	27
118	17	17	17	17	17	17	17	23	23	29	27	31
119	17	17	17	17	17	17	17	17	23	29	29	29
120	17	17	17	17	17	17	17	17	17	23	31	29
121	17	17	17	17	17	17	17	17	17	17	23	29
122	17	17	17	17	17	17	17	17	17	17	17	23

Peak Correlation Parameters for Sequences in AO/LSE Phases

111	19	25	33	35	37	37	33	29	39	35	33	37
112	17	21	31	33	33	31	31	23	29	27	31	33
113	17	17	19	25	29	35	33	29	31	29	33	33
114	17	17	17	21	25	35	33	41	41	31	23	33
115	17	17	17	17	21	29	31	29	27	35	25	29
116	17	17	17	17	17	21	31	31	31	35	27	31
117	17	17	17	17	17	17	17	35	33	37	31	25
118	17	17	17	17	17	17	17	21	29	37	29	25
119	17	17	17	17	17	17	17	17	21	33	41	41
120	17	17	17	17	17	17	17	17	17	17	33	29
121	17	17	17	17	17	17	17	17	17	17	21	25
122	17	17	17	17	17	17	17	17	17	17	17	19

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