Presented in this letter are least-squares regression relationships between velocity ratio (R; the ratio between surficial sediment sound speed and bottom water sound speed) and sediment mean grain size (Mz, phi units), porosity (φ, %), and density (ρ, kg/m³). The relationships are: R = 1.296 - 6.01 x 10⁻⁴ Mz + 6.23 x 10⁻⁷ Mz²; R = 1.675 - 1.639 x 10⁻² + 9.762 x 10⁻⁶ η²; and R = 1.513 - 8.24 x 10⁻⁵ ρ + 3.2249 x 10⁻⁷ ρ². These equations, respectively, explain 91.6%, 88.0%, and 86.4% of the variation observed in velocity ratio. Velocity ratio relationships are more convenient than those previously available because they do not require temperature and pressure correction to in situ conditions.
Estimating velocity ratio in marine sediment

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Presented in this letter are least-squares regression relationships between velocity ratio ($R$: the ratio between surficial sediment sound speed and bottom water sound speed) and sediment mean grain size (Mz, phi units), porosity ($\eta$, %), and density ($\rho$, kg/m$^3$). The relationships are:

- $R = 1.296 - 5.01 \times 10^{-2} Mz + 2.83 \times 10^{-3} Mz^2 - 1.675 \times 10^{-7} \eta - 9.762 \times 10^{-5} \rho - 3.2249 \times 10^{-7} \rho^2$

These equations, respectively, explain 91.6%, 88.0%, and 86.4% of the variation observed in velocity ratio. Velocity ratio relationships are more convenient than those previously available because they do not require temperature and pressure correction to in situ conditions.

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INTRODUCTION

For more than a decade, workers have used Hamilton’s equations (Hamilton, 1970, 1974; Hamilton and Bachman, 1982; Bachman, 1985) to estimate the speed of sound in seafloor sediment when measurements were not available. These empirical equations relate sound speed at laboratory conditions to mean grain size, density, and porosity (the volume percent of voids in the sediment). Hamilton’s equations are useful because sediment physical property measurements are frequently available when acoustic measurements are not. However, to be used in acoustic propagation studies, laboratory sound speed (23°C, 1-atm pressure) must be corrected to in situ conditions.

A more convenient method is to construct regressions relating the ratio $\left[ \text{sound speed in sediment}/\text{sound speed in seawater} \right]$ to appropriate physical properties. This ratio
I. LABORATORY MEASUREMENTS

The measurements used in this letter are those listed and discussed in Hamilton and Bachman (1982) and Bachman (1985). The information presented here supplements the relationships between sediment properties discussed in these and earlier papers. Sound velocity was determined by measuring temperature and sound speed in a sediment sample. These velocities were corrected to 23°C and 1-atm pressure using tables for the speed of sound in seawater (Bialek, 1966). Velocity ratio was determined by dividing sediment sound speed at 23°C by the speed of sound in seawater at 23°C, 1-atm pressure, and of the same salinity as the bottom water at the site.

Grain size distribution was determined using the pipet technique for the silt and clay sample fraction (Krumbein and Pettijohn, 1938) and a settling column for the sand fraction (Emery, 1938). Mean grain size was obtained from the cumulative grain size curves using one of the following equations:

- $M_z = \phi_{50} \times 64\%,$ \hspace{2cm} (1)
- $M_z = (\phi_{30} + \phi_{50} + \phi_{70})/3 \times 82\%,$ \hspace{2cm} (2)
- $M_z = (\phi_{25} + \phi_{50} + \phi_{75})/3 \times 86\%,$ \hspace{2cm} (3)
- $M_z = (\phi_{20} + \phi_{50} + \phi_{80})/3 \times 88\%,$ \hspace{2cm} (4)
- $M_z = (\phi_{16} + \phi_{50} + \phi_{84})/3 \times 88\%,$ \hspace{2cm} (5)
- $M_z = (\phi_{10} + \phi_{30} + \phi_{50} + \phi_{70} + \phi_{90})/5 \times 93\%.$ \hspace{2cm} (6)

The notation $\phi_n$ means the grain size in phi units corresponding to the $n$th percentile [$\phi = -\log_2 (\text{grain size in mm})$]. The percentage following each equation is the efficiency of the equation at estimating the moment mean (McCannom, 1962). The equation used for a particular size depended on availability of the appropriate cumulative percentages. Equations (1), (5), and (6) are reviewed by Folk (1966, p. 81). Equation (4) was suggested by McCannom (1962). Equations (2) and (3) were introduced to permit computation of $M_z$ when the measured size distribution did not permit the use of Eqs. (4), (5), or (6).

Sediment density was determined by weighing the known volume of sediment obtained using a stainless steel tube. Porosity was measured by drying the density sample and determining the void volume from the weight of water lost. Porosities were corrected for the salt content of the pore water.

II. REGRESSION ANALYSIS

Weighted regression analysis produced the following equation relating mean grain size in phi units to velocity ratio ($R$):

$$R = 1.296 - 6.01 \times 10^{-2} M_z + 2.83 \times 10^{-3} M_z^2.$$ \hspace{2cm} (7)

Mean grain size measurement efficiencies were expressed as decimal fractions and used as weights. This equation explains 91.6% of the observed ratio variation. The equation for porosity is

$$R = 1.675 - 1.639 \times 10^{-2} \eta + 9.762 \times 10^{-5} \eta^2,$$ \hspace{2cm} (8)

which explains 88.0% of the observed ratio variation. The equation for density is

$$R = 1.513 - 8.24 \times 10^{-4} \rho + 3.2249 \times 10^{-7} \rho^2.$$ \hspace{2cm} (9)

Equation (9) explains 86.4% of the variation observed in velocity ratio. The data for these regressions and the resulting equations are illustrated in Fig. 1.

Table I shows statistics for each regression equation. This table also contains the elements of a matrix ($C$) useful in calculating the variance of velocity ratio estimated for a particular value of the independent variable (Draper and Smith, 1981, p. 210):

$$\text{Var}(R) = s^2 [1, X_0, X_0^2] C [X_0, X_0^2].$$ \hspace{2cm} (10)

where $X_0$ is the value of the independent variable, and $s^2$ is residual mean-square error (from Table I). The square root of the variance is the standard error of the estimate of the true value of velocity ratio.

III. DISCUSSION

It is instructive to compare the use and results of the equations presented here with Hamilton's. For example, consider an abyssal hill silt clay of 8.76 phi mean grain size, a porosity of 81.2%, and a density of 1.344 g/cm$^3$ (the averages for this sediment type; Hamilton and Bachman, 1982, Tables III and IV). Assume that this sediment was recovered from 6000-m depth where the bottom water salinity is 34.69 ppt, and the temperature is 1.5°C. The bottom water sound speed is 1559.8 m/s in situ, and 1529.7 m/s at 23°C and 1-atm pressure (Hamilton, 1971, p. 272).
Equations (7)–(9) predict sound-speed ratios of 0.987, 0.988, and 0.988, which, when multiplied by 1559.8 m/s, indicate in situ sediment sound speeds of 1539.5 and 1541.1 m/s. The variance of the predicted ratios is between 1 and 2×10⁻³, which equates to a standard error of 0.001, or 2 m/s in situ.

Hamilton’s equations (Hamilton and Bachman, 1982, p. 1902) for abyssal hill sediment predict laboratory sound speeds of 1504.9, 1506.3, and 1506.4 m/s from mean grain size, density, and porosity, respectively. The standard errors for these predictions are 12, 13, and 13 m/s. The sound-speed ratios are 0.984 and 0.985, with a standard error of 0.008. These ratios indicate in situ sound speeds of 1534.8 and 1536.4 m/s, with a standard error of 12 m/s.

Thus the equations presented in this letter produce results that are close to Hamilton’s and offer an improvement in the standard error of the estimates. This discussion also illustrates the savings in computation afforded by directly predicting sound velocity ratio.

IV. CONCLUSIONS

Because it eliminates the need to correct for temperature and pressure differences, velocity ratio is a more convenient measure of surficial sediment sound speed than is sound speed in the laboratory. Equations (7)–(9) provide a means to estimate easily velocity ratio from sediment density, porosity, and grain size distribution. Frequently, one of these is available when acoustic measurements are not.

It must be emphasized that these regression relationships are for surficial, unconsolidated sediment only (i.e., within a few meters of the seafloor for silts and clays, and within a few tens of centimeters of the seafloor for sands). It must also be emphasized that these relationships are for sediments composed dominantly of solid particles: Hollow biogenic grains behave differently (e.g., Hamilton et al., 1982; Bachman, 1984).

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