SEC

REPORT DOCUMENTATION PAGE

(2)

1a. AD-A222 993	1b. RESTRICTIVE MARKINGS
2a.	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S) ARO 26987.3-GS
6a. NAME OF PERFORMING ORGANIZATION Duke University	6b. OFFICE SYMBOL (If applicable)
7a. NAME OF MONITORING ORGANIZATION U. S. Army Research Office	
6c. ADDRESS (City, State, and ZIP Code) Durham, NC 27706	7b. ADDRESS (City, State, and ZIP Code) P. O. Box 12211 Research Triangle Park, NC 27709-2211
8a. NAME OF FUNDING/SPONSORING ORGANIZATION U. S. Army Research Office	8b. OFFICE SYMBOL (If applicable)
9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER DAAL03-89-K-0089	
8c. ADDRESS (City, State, and ZIP Code) P. O. Box 12211 Research Triangle Park, NC 27709-2211	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO

ELECTE JUN 15 1990 S D

11. TITLE (Include Security Classification)
Saltation and Suspension of Particulate Matter in Air

12. PERSONAL AUTHOR(S)
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13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Final Technical	FROM 4/16/89 TO 12/15/89	April 1, 1990	37

16. SUPPLEMENTARY NOTATION
The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Sand, Saltation, Aeolian, Suspension, Turbulence, Friction Velocity, Geology, Particulate Matter in Air, Simulations (JG)

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Abstract

A model of eolian sediment transport has been constructed, a special case of which is that corresponding to sand-sized mineral grains subjected to moderate winds: saltation. The model consists of four compartments corresponding to (1) aerodynamic entrainment, (2) grain trajectories, (3) grain-bed impacts, and (4) momentum extraction from the wind. Each sub-model encapsulates the physics of the process, and is constrained where necessary by experimental data. When combined, the full model allows simulation of eolian saltation from inception by aerodynamic entrainment to steady state.

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20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
☐ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	Unclassified

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE (Include Area Code)	22c. OFFICE SYMBOL

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

decay with height above the bed, representing the preponderance of short, low-energy trajectories in the saltation population. Yet the role of less abundant, longer, higher-energy trajectories is a strong one: at steady state the entire population of saltating grains is controlled by bed impacts rather than aerodynamic entrainment. Because the nature of the grain splash process is such that high-energy impacts are much more efficient at ejecting other grains from the bed, the response time of the system to changes in wind velocity is determined by the hop time of these long trajectories. Several hop times, or roughly 1-2 seconds, are required.

Varying wind velocity among the simulation runs allows mapping of the relation between steady state mass flux and wind velocity -- the mass flux "law" -- which may be expressed as a power law of the excess shear velocity. The hysteresis that led Bagnold to define both fluid and impact thresholds for saltation is apparent, reinforcing our conclusion that it is the impacts of saltating grains that support the large population of saltating grains at steady state.

These characteristics point to a serious need to collect high frequency near-bed wind velocity data simultaneously with mass flux measurements in natural environments. Ideally this data should be collected at a rate of several Hz, in order to capture the relevant spikes in wind velocity that contribute most strongly to the total flux over any period of time long relative to the response time of the saltation system Sample calculated time series of both wind speed (constrained by empirically gathered wind spectra) and the resulting saltation flux (taking into account both the response time and the mass flux law) confirm the importance of high frequency variations in wind speed.

Although such models lack the detail to treat all the complexity of the natural world they serve both to develop our intuition about the natural setting, and to guide future experimental efforts.

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Introduction

Considerable progress has been made in the understanding of the process by which sand sized grains are transported by the earth's atmosphere. Although ideas initially espoused by early workers, especially Bagnold (1941) and Chepil (1945), have enjoyed broad support over the decades through empirical findings, only recently has the theory of sand transport developed to provide the firm underpinnings to such empirical work. Theoretical work has now reached an approximate par with the empirical, and new questions raised by this theoretical work will require new field and wind tunnel efforts in the future.

The broad motivations for the study of the simple system of sand plus air range widely, but may be classed in four camps: environmental, geological, planetary and physical. The first motivated the work of Chepil and others, who lived in and through the Dust Bowl years of the US. Research founded by Chepil has been carried out since in extensive field and wind tunnel work centered at the Kansas research station, with the major emphasis being placed on the effects of wind on crops of the Great Plains and how to minimize these effects.

The geological motivations for performance of research on eolian sand systems include the need to understand the mechanisms by which eolian sandstone bodies of high porosity are produced, and how the grain packing geometry and other intrinsic variables of importance to the oil industry might differ according to the important variables in the problem: wind velocity, grain size distribution, etc.

The strong motivation to understand the settings into which we were sending planetary landing craft, made the 1970's and early 1980's ripe for a resurgence of work on both the theoretical and empirical sides of the problem. Wind tunnels were constructed capable of very high and very low atmospheric pressures, in which both threshold velocities and mass flux relations were measured; high-speed filming techniques were used to track individual trajectories in saltation; interest was renewed in terrestrial settings in which analogues to Martian terrains were suggested by surface features; and theoretical work was renewed on both grain trajectories and threshold velocities for a wide range of atmospheric conditions.

The fourth motivation may be termed physical in that the research has been on the physics of the problem, rather than on its planetary, geological, or environmental applications. The several components of the problem have been taken apart and studied in detail: e.g., the grain trajectory, the grain-bed impact, the wind velocity feedback, and the aerodynamic threshold. Only recently have computational capabilities allowed putting all of these components back together again. In addition, interest was spurred by the degree to which the sand-air system was self-organizational, resulting in sand ripples of a well-defined wavelength.

Below we give an overview of the current state of knowledge of the saltation problem, emphasizing results obtained as a result of Army Research Office support.

The physical picture

It is useful to picture eolian saltation in its simplest manifestation as being comprised of four linked processes: aerodynamic entrainment, grain trajectories, grain-bed impacts (the granular splash problem), and the wind field modification. Clearly, no sand is transported at the very lowest wind velocities. Sand transport begins with increasing wind velocity as the most easily moved grains are displaced. The aerodynamics of the threshold condition must therefore be known, and quantified appropriately. Once airborne, grains travel trajectories that are determined by the balance between grain weight, drag and lift forces on the grain. The smaller the grains, the more likely they are to respond effectively to the turbulence of the boundary layer. We will consider in this paper only those grains whose size is large enough to filter out short time scale variations in wind velocity, meaning that they respond effectively only to the mean wind velocity profile: we therefore limit the discussion to saltation, rather than suspension (see Anderson, 1987). Importantly, this reduces the grain trajectory problem to a deterministic one: once the liftoff conditions are specified, the entire grain trajectory is specified, including among other things the hop height, length, and hop time. Due to gravitational acceleration, all grains that are ejected from the bed ultimately return to the bed, upon which they interact energetically with a collection of other nearby particles. We must know in a quantitative sense the nature of this splash process, including what happens to the impacting grain, and to nearby grains. Because of the complicated microtopography of even a "flat" bed and because of the complexity of the packing geometry, this splash process is stochastic: we may know the results of any particular impact, characterized by the size of the impacting grain and its angle and speed, and the nature of the local bed (grain size distribution, angle with respect to horizontal), in only a statistical sense. All of the above-mentioned aspects of the saltation problem are dependent only upon the behavior of individual grains. The fourth facet to the saltation problem, that of the modification of the wind velocity, reflects the extraction of momentum from the wind by all of the particles in the air, and is an integral quantity that requires knowing the behavior of all of the airborne particles. In turn this modification will affect all of the other three processes, and may therefore be thought of as a negative feedback mechanism: aerodynamic entrainment will be curtailed to some extent, grain trajectories will be shortened by the fact that the entire wind velocity profile is slowed, and because the grain trajectories are slowed the impact velocities are diminished. Calculations such as we present here, which extend the work presented elsewhere (Anderson and Haff, 1988) allow determination of the relative importance of these two feedback effects.

We will first introduce each of the four processes mentioned, with attention being paid in inverse proportion to attention in the present literature. We then present the results of calculations performed with all processes linked.

Aerodynamic entrainment

Grains may be entrained by direct aerodynamic forces of lift and drag. Aerodynamic entrainment is required for the initiation of saltation in the absence of external mechanical disturbances. As the simplest possible model for aerodynamic entrainment, we take the number of entrained grains per unit area of bed per unit time (the aerodynamic ejection rate) to be proportional to the excess shear stress

$$N_a = \alpha\,(\tau_a - \tau_c)$$

where τ_a is the short term mean shear stress at the bed, τ_c is the critical fluid shear stress for entrainment, for which an expression has been determined from numerous wind tunnel experiments, and α is a constant. All such aerodynamically entrained grains are assumed to leave the bed with a velocity corresponding to that needed to reach a height of about one grain diameter, D, above the bed. This is a conservative assumption that effectively limits the role of the aerodynamic grains in the eolian saltation process.

As will be demonstrated below, this somewhat ad hoc entrainment rule does not play a critical role in a well-developed saltation curtain.

Grain trajectories

Once launched into the airstream by either ballistic impact forces or aerodynamic forces, grains are acted upon by the surface tractions of drag and lift forces imposed by the wind, and by the body force of the grain's weight. Many studies have addressed the details of the forces acting upon airborne grains, beginning with Bagnold (1941). They may be simply stated as follows. Consider a coordinate system in which positive x is in the downwind direction and positive z is upward. The particle weight is $\mathbf{F_w}$, written

$$\mathbf{F_w} = M_p\, \mathbf{g}$$

where M_p is the particle mass and $\mathbf{g}$ the acceleration due to gravity. The drag force, $\mathbf{F_d}$, operates in the direction of the relative velocity of the grain and the local wind velocity, $\mathbf{U_{rel}}$, and may be written

$$F_d = \frac{1}{2}\rho_a C_d A |U_{rel}| U_{rel}$$

where ρ_a is the density of air ($\simeq$1.22kg/m^3), A is the cross-sectional area of the grain presented to the flow ($=\pi D^2/4$ for spheres, where D is grain diameter), and C_d is the drag coefficient, which is dependent upon the local Reynold's number, $Re=U_{rel}D/\nu$, where ν is the kinematic viscosity of air ($=1.5\times10^{-5}$ m^2/s at 20°C).

As the shear-imposed lift force on a spherical grain is essentially negligible above a few grain diameters above the bed (Anderson and Hallet, 1986), we may neglect it in the first order sketch of the trajectory problem presented here. More elaborate and rigorous treatments will incorporate this effect as well as others, among them the Magnus effect (White and Schulz, 1977) and the virtual mass effect. The essence of the trajectory problem is, however, contained in the weight and drag forces. The downstream and vertical components of these two forces may be added to yield the resultant forces in the x and z directions, from which accelerations, velocities and displacements may be calculated for a given time step, dt, from Newton's second law:

$$a = \frac{dU}{dt} = \frac{F_w + F_d}{M_p} \;;\; U = \frac{dx}{dt} = \frac{dU}{dt}dt \;;\; x = \frac{dx}{dt}dt$$

Given the initial conditions of liftoff velocity and speed, these equations can be integrated forward in time until the particle reimpacts the bed. We find that very accurate calculations of trajectories may be accomplished with 100 time steps per trajectory, requiring that the time step, be chosen such that $dt=\tau_{hop}/100$, where τ_{hop} is the expected hop time in the absence of vertical drag, or $2w_o/g$, w_o being the initial vertical velocity of the grain. [It is the happy case in the eolian trajectory problem that the accelerations in the vertical are due primarily not to fluid drag but to gravity, meaning that hop times and hop heights are well approximated (less about 10-20%) by their values in a vacuum (see Anderson and Hallet, 1986).]

The splash process

Our knowledge of the splash process has been augmented greatly in the last decade by both physical and numerical experiments. The sophistication of these experiments has advanced considerably, and has resulted in placing us in a position to characterize in considerable detail the complexity of the process. Saltation is a stochastic process that must take into account the reaction not only of the grain impacting the bed, but the resulting rearrangement of the grains in the bed, including the possibility of ejecting some of these grains. The impact process is expected to depend upon the mass and velocity of the impacting grain, including its speed and approach angle with respect to the local bed slope, and the masses and elastic parameters describing the grains comprising the bed near the impact point.

Early numerical experiments were crude in the sense that they took into account only the geometrical complexity of the problem, while ignoring the dynamical nature of the problem. Grains in the bed were not allowed to be ejected, and the splash process was reduced to finding the distribution of rebound angles to be expected from beds of particular micro-topographies. Simple geometrical models such as these predict that if the bed were perfectly rigid the rebounding particle would most likely emerge from the collision at roughly 50° from the horizontal, having impacted the bed at angles typical of eolian saltation, i.e. 10-15°. This matches closely the measured averages of saltation angles in full saltation experiments, as first reported by White and Schulz (1977), and more recently in numerous publications by Willetts and Rice (e.g. 1986a,b ,1988, 1989). This was also the first work to question seriously the conclusions of Bagnold (1941) that the preponderance of grains were ejected from the bed vertically, conclusions that had played an important role in shaping later theoretical work (e.g. Owen, 1964).

The utilization of high-speed films by Willetts and Rice at the University of Aberdeen has allowed the characterization of the splash problem during saltation in wind tunnel settings. By elevating a small strip of the bed out of the high-concentration portion of the saltation layer, they were able to film individual impacts from which a statistical description of the impact process could be gathered. Typical saltation impacts strike the bed at angles of 10-15° from the horizontal. The impact results in two phenomena: the rebound of the impacting grain, and the emergence of several grains from among the bed grains. The rebound speed is reduced by roughly half from the pre-impact value, and the mean rebound angle is roughly 50° from the vertical. The number of splashed grains rises roughly linearly with the impact speed, being of the order of 3 grains per impact for impact speeds on the high end of those expected in eolian saltation. The ejection speeds of the splashed grains are of the order of 10 percent of the impact speed, and the ejection angles are canted more steeply toward the vertical than the rebound, within a broad range.

These measurements have been used to check the results of numerical simulations that attempt to take into account all the forces between all the grains in the problem. This was first addressed in detail by Werner (e.g., 1987; 1990) in his dynamical simulations of the impact process. Working with well-packed two dimensional beds, impactor grains could be fired into the beds at any chosen angle and speed. The statistics from many such impacts were collected to describe the outcome from any set of impact parameters. Werner reported distributions of rebound and ejection angles that paint a picture similar to that obtained from the high speed films. Most of the grains ejected from the bed were found to emerge from within the top one to two grain layers, and displayed a high ejection angle. Werner noted that this was probably due to the fact that this was the direction of the free face, and therefore the direction in which there was least resistance to motion. He further noted that when the bed was given some micro-relief, that grains were preferentially ejected from micro-discontinuities he termed steps. This allowed a greater range of possible ejection angles, and therefore resulted in a greater spread of the resulting ejection angle distribution. Simultaneously, the Caltech group (e.g., Mitha et al., 1986) developed a means of producing and documenting the results of individual grain impacts using real particles. These experiments started using spherical steel pellets (bb's) which were fired at a given angle into a wide bed of similarly sized pellets. Subsequent development of a "sand gun" allowed similar experiments using individual sand grains fired into sand beds (Werner, 1987).

Subsequent to Werner's simulations, Haff and Anderson (1988) reported further dynamical simulations that attempt to map out the full splash function. The well-packed nature of the simulated bed is abandoned in these calculations. Several two-dimensional simulated beds are created by dropping grains from a given height with a random distribution of initial velocities. The particles are allowed to settle in a "box" that is constructed with wrap-around boundary conditions that effectively extend the bed laterally. The resulting beds manifest more realistic micro-topography that importantly affect the splash outcome. These beds are then each subjected to impacts with identical impact parameters of speed and angle, many impacts being performed on each of three beds, each impact impinging on the bed at a different point in order to collect sufficient statistics. The resulting splash descriptions therefore have embedded in them the stochastic nature of the bed, as well as the possibility of hitting any particular grain in the bed at any of many different points.

Impact model

The splash function used in these calculations is generated from direct computer simulation of the motion of the impacting and impacted grains. The simulation is performed in two-dimensions, with each grain assumed to be circular. The position and velocity of each particle in the simulation is followed explicitly through time via integration of the Newtonian equations of motion

$$\mathbf{F}_i = M_i \, \mathbf{a}_i$$

where $\mathbf{F}_i$ is the force applied to particle i, M_i is the particle mass, and $\mathbf{a}_i$ is its acceleration. Since friction between particles is included in the model, torques may be applied leading to particle rotation. To the three degrees of freedom attaching to the motion of a two-dimensional extended particle, namely translational motion in the x and z directions, plus angular motion about the center of mass, there correspond the two components of Newton's second law, $F_x=Ma_x$ and $F_z=Ma_z$ plus a torque equation,

$$\tau = I\alpha$$

where τ is the applied torque, I the moment of inertia about the center of mass, and α the particle's angular acceleration. Following the motion of each particle therefore requires the integration of 3 second order or 6 first order differential equations, for a total of 6N equations for a model impact system with N grains. Typical impact scenarios have involved 80 to 100 grains, corresponding to ~ 500 - 600 simultaneous equations.

There are two basic types of forces which we must consider, body forces and contact forces. Contact forces can be categorized as normal (or compressive), and tangential (or frictional). The relevant body force here is just the weight of the grain, Mg (we drop the subscript i for now).

The normal force is modeled as a very stiff linear spring. This force is not activated until two particles "contact" one another, as determined by the spacing of their centers becoming less than the sum of their radii. The stiffness of the spring is chosen great enough that particle "overlap" is always less than a predetermined fraction of the particle diameter, a consideration which depends upon the maximum relative particle velocities expected to occur in the simulation. In the saltation problem, this is always determined by the speed of the impacting grain. Higher speeds require stiffer springs. Since the natural period of a linear spring decreases as the spring constant increases,

$$T = \sqrt{M/k}$$

an increase in k implies an increase in the number of incremental time steps Δt which are needed to effect the actual integration of the equations of motion, (since we must have $\Delta t \ll T$). This is one limitation on the number of independent splash simulations which can in practice be run, i.e., T must be small compared to the time required for the bed to respond sufficiently that final grain trajectories can be determined. And Δt is generally taken to be at least an order of magnitude smaller in order to ensure adequate accuracy of integration. (The integration itself is carried out using a predictor-corrector scheme for which preset integration tolerances can be specified.)

This calculation is made slightly more complicated by the fact that the grain collision is be damped, i.e., the grains are slightly inelastic. Damping is modeled by a velocity-dependent force which is proportional to the instantaneous normal relative particle velocity and which always opposes relative particle motion. In such a damped oscillator, there are two time scales, one is the effective or modified spring frequency, which is less than the natural (undamped) frequency, and the other is the damping time scale which gives the e-folding time for degradation of the oscillator amplitude. For an accurate integration of the equations of motion, the integration time step $\Delta t << T$ must be less than both of the system physical time scales.

For such an oscillator there are three characteristic response modes, namely, underdamping, critical damping and overdamping, depending upon whether the damping time-scale is greater than, equal to, or less than the natural frequency of the system. Critical and overdamped behavior correspond to collisions in which the two colliding partners lose all their normal velocity in a head-on collision (i.e., the coefficient of restitution, ε, *given by the ratio* of outgoing to incoming (normal) relative velocity, is zero). For the collision of resilient quartz grains the coefficient of restitution is greater than zero (it would be unity for a perfectly elastic collision) and consequently the normal forces in the impact simulation are modeled by underdamped springs. The choice of spring constant and damping strength is therefore made to satisfy the overlap requirements, and to produce a desired coefficient of restitution. The precise value of ε to be used in a given simulation is somewhat problematical, because, at the collision speeds characterizing typical impact events (a few ms^{-1}), the actual grain contact is not at all elastic, but rather is characterized by plastic deformation, spalling and chipping off of small pieces of the parent grains. Simulation runs on identical systems with reasonable variation of ε show that the main details of the splash function are not very sensitive to the exact value of ε. In our previous simulations we have taken $\varepsilon = 0.7$.

Besides the normal compressive and the corresponding damping forces, which act in a direction along the line of centers of two contacting particles, there is also a tangential force due to friction at the point of contact. Contact friction is modeled in one of two different ways, depending upon whether, at any instant, there is slippage at the contact or not. If the contact is slipping, i.e., if there is relative tangential motion of the material just on either side of the contact point, we invoke a rule corresponding to Coulomb friction, i.e., that the tangential frictional force is related to the (instantaneous) normal force F_n via a constant of proportionality, the friction coefficient μ. If the contact is not slipping, then we imagine that a tangential, damped, linear spring acts to oppose any slippage. Particles may still roll freely because the friction spring is a function of the total arc length separating the anchoring points of the spring minus the amount of rolling. If slippage starts to occur, the very stiff tangential spring effectively limits rim displacements unless or until the spring force exceeds μF_n . If this occurs, then the tangential spring is severed, and the maximum tangential force remains at μF_n. The tangential spring is damped in order to suppress low-amplitude, high-frequency (unphysical) tangential oscillations of the contacting particles.

The choice of a friction coefficient is somewhat problematical also, because, while perfectly smooth quartz grains exhibit low friction, real sand grains are pitted, faceted and generally irregular in shape. Thus, in nature, torques can be exerted across a particle contact even in the absence of true friction (Clayhold effect). The friction coefficient μ, for the disk model described here, is clearly an effective friction coefficient which incorporates not only the effects of true surface friction, but also effects of particle shape and surface morphology. In our saltation calculations, we have used a generic value of 0.5 for the friction coefficient. As with the coefficient of restitution, the splash function is only modestly sensitive to μ, as long as it is not too small.

Each saltation impact calculation is carried forward in time far enough that the nature of the bed-response is well-established. Depending upon bed conditions this might be 10 -20 T. If the simulation is followed too far in time, complications may arise from the presence of stress waves reflected off the substratum upon which the simulated collection of bed grains rests. These waves, which are artifacts of the calculation, can actually throw grains off the surface and lead to an overestimation of the number of reptating grains. This is one of several computational issues which arise as a result of the necessarily small number of grains which can be handled in each impact event.

In our simulation studies the "target" bed was prepared by dropping between 80 and 100 grains onto a fixed and immobile surface. The grains were allowed to fall under the influence of gravity and to rebound back again, colliding with one another however they might, until friction and the inelasticity of collisions reduced the kinetic energy of the grain mass to essentially zero. Because of the relatively small number of grains, periodic boundary conditions were applied at the edges of the region of calculation. At these lateral boundaries, particles and their contacts are "wrapped" around to the other side of the cell, so that if a particle moves too far to the right and crosses the right-hand dashed line, it is inserted again on the left, at precisely the same elevation it had on the right. While periodic boundary conditions can introduce spurious behavior in certain situations (especially in particle flow studies), they seem to be relatively innocuous here because the calculation of every impact is wisely terminated before the disturbance initiated by the impact can propagate out one side of the test region and back in the other.

Once a bed has been constructed in this way, we impinge an identical projectile of given velocity and impact angle upon it. Ten impact positions were chosen for each velocity and impact angle combination, corresponding to equally spaced impact points for a reference flat bed. Since the bed has its own unique topography, the actual impact points were not uniformly distributed. Thus the "upwind" side of any clumping of grains at the surface will tend to receive somewhat more impacts than the "downwind" side of such a clump, as would be the case in nature. (This is the physical origin of wind-ripple instabilties.)

One technical issue of importance in grain simulation revolves around contact detection. Two grains are in contact whenever the separation between their centers of mass is less than the sum of their radii. Since all particles in the simulation may potentially move, we must have a method for periodically detecting the generation of new contacts and the breaking of old ones. One way to do this is, at every integration step, to check the separation of the centers of mass of every pair of particles. If there are N particles in the system, then there are N(N-1)/2 pairs, so the number of pairs to be checked increases like N^2 for large N. Implementing contact detection this way leads to a rapid saturation of computing power when the number of particles becomes large. Various techniques exist to cut down the fraction of computer time spent in contact detection. Most of them involve some sort of fine- or cross-graining and take advantage of the fact that a grain's neighbors (contacting or not) at time t will still be the grain's (only) neighbors at time $t + \Delta t_{check}$, where t_{check} is large compared with the integration time step Δt, but small compared with the total elapsed time in the simulated system, t_{total} .Thus, during an interval of time Δt_{check} it is necessary to check only $n \ll N$ contacts within a small neighborhood of each particle. Only occasionally (at t_{check} intervals) need all pairs be checked.

The eolian saltation impact program implements a variation of this method, in that each particle carries attached to it a "neighbor list", which contains an identifying particle number for every (other) particle that was within a specified radius at time t. At each Δt, the collision detection code looks only at the neighbor list. The list is updated from time to time, using a rule based upon the speed of the fastest particle in the system, as old neighbors move away and new ones move in.

Numerical simulations of single grain impacts into a granular bed were performed with the aim of providing a quantitative picture of the splash process. The impact simulations were performed in two dimensions with identical slightly inelastic 1 mm diameter grains (disks) whose interactions were characterized by a coefficient of restitution equal to 0.7, an inter-grain friction coefficient equal to 0.5, and an elastic modulus equal to 10^7 dy cm^{-1}. Beds were generated by dropping 87 particles with random initial velocities into a box with periodic boundaries, its width chosen such that the resulting bed was about 10 grains deep. Three such granular beds, distinguished by different packing configurations, were each impacted 20 times to develop the splash statistics for each impact angle and speed. These computations soften assumptions made in earlier numerical work on impact dynamics (Werner 1987).

The picture of the grain splash resulting from our numerical simulations is qualitatively similar to that derived from earlier numerical (Werner 1987, 1990) and physical experiments (Mitha 1986; Willetts and Rice 1989) and fills in gaps in these latter results, especially in the low-impact velocity range. In our simulations the impacting grains were found to rebound from the surface with nearly unit (0.95) probability, a result postulated earlier by Rumpel (1985). For the chosen grain parameters, the mean rebound speed is approximately 50-60% of the impact speed, and the mean rebound angle is 30-40^{o} from the horizontal. The impact forces result in ejection of a number of grains from within a few grain diameters of the impact site. The mean number of grains ejected increases roughly linearly with impact speed, and compares well with data obtained from physical impact experiments with coarse sand. The mean speed of the ejected particles appears to saturate at roughly 10% of the speed of the impacting grain, and the mean ejection angle tends to be oriented downwind at 60-70^{o} from the horizontal. All results for impacts between 8° and 15°, which cover the range of expected impact angles in eolian saltation, display only slight dependence on the impact angle. These results are expressed in terms of the splash function (Ungar and Haff, 1987), an analytic expression for the number density of grains ejected from the bed and their distribution of initial velocities, for a given impact velocity.

Rebounds

The incident grain rebounds with probability close to unity for the impact speeds simulated. In the rare cases where the incident grain fails to rebound, the micro-geometry of the collision pocket is such that the momentum of the impact is efficiently delivered to one or more nearby grains, while the colliding particle is geometrically trapped. The mean speed of the rebound is 60% of that prior to collision and is distributed with a standard deviation of roughly 0.1 of the impact speed. The ejection angle is significantly steeper than the impact angle, as suggested by earlier work of Rumpel (1985), Werner and Haff (1988), and others.

The splash

The number of splashed grains resulting from an impact increases monotonically with the impact speed, as do the speed and angle of these ejections. The mean number of ejections increases slightly more than linearly with impact speed; for the chosen grain characteristics, impacts of 6m/s result in 10 ejecta. The mean ejection speed for the splashed grains increases less than linearly with impact speed, becoming approximately constant (at ~0.7m/s) for impacts greater than 5m/s. The mean ejection angle varies little, for all but the lowest speed impacts, being canted downwind at roughly 60-65^{o} form the horizontal. Low velocity impacts result in either no ejecta, or ejecta that appear very similar in speed and angle with that of a rebounding particle.

Discussion of the single impact simulations

The picture of the grain splash emerging from the numerical experiments is both qualitatively and quantitatively similar to that arising from physical experiments (e.g., Mitha et al., 1986; Willetts and Rice, 1986a,b, 1988), and from earlier numerical simulations (e.g., Werner and Haff, 1986; 1988a). A most impressive correspondence comes with the data on impacts collected with high speed motion photography taken during saltation in a wind tunnel (Willetts and Rice, 1986a, 1988).

Although considerable caution is warranted in relying upon these numerical experiments, which involve two-dimensional, perfect circles, we are encouraged by the correspondence with physical experiments using real sand grains in three-dimensional pockets. We are most skeptical of the low impact velocity results, where experimental confirmation is difficult due to problems associated with photography very near the bed. At these low impact speed conditions, the three-dimensional nature of the real beds should lead to more efficient capture of low velocity grains, leading to a reduction in the probability of rebound.

For reasons of computational efficiency, the simulated beds used in the impact experiments were only 10 grain diameters wide; they tended therefore to possess microtopography of the order of one grain diameter. This contrasts with Bagnold's observation that saltation impacts create craters with rim heights on the order of two grain diameters. In addition, preliminary measurements of microtopographic profiles with lengths of many thousand grain diameters on actual sand beds confirm that, even when artificially smoothed, such surfaces have micro-roughness at wavelengths much greater than 10D with amplitudes of up to 4D.

Impacts with the upwind sides of these bumps should reduce the number of rebounds to be expected, and may even result in occasional backwards-directed ejecta (or rebounds) that we will have missed in our simulations, but that have been observed in physical experiments (B.B. Willetts, personal communication, 1988). In many cases, impacts of grains with the granular surface result in rebounds that appear much as if the surface was rigid (Werner and Haff, 1988a), the angle of rebound being determined largely by the surface geometry, specifically the orientation of the surface normal. A knowledge of the microtopographic profiles of the real surfaces was used to calculate such surface normals for a range of possible impacts with known grain size and impact angle. The resulting reflections from the surface show distributions similar to those resulting from our calculated rebounds, and show the expected dependence of the distributions on impact angle and impacting particle diameter. Although our calculations are restricted to single grain size beds and to similarly sized impacting grains, these profiles allow estimation of the effects of variable impacting particle diameters. Larger grains impacting a finer surface will perceive a smoother surface, and should result in a lower angle rebound, while finer grains encountering a coarser surface ought to rebound at greater angles. The geometrical argument presented here should be altered by the fact that larger particles will create larger disturbances of the surface; simulations of these events are needed to clarify the relative roles of the geometrical and dynamical effects.

The statistics of the rebounds and splashed grains are effectively mingled for low impact velocity events, and become more distinguishable at higher velocities. In all cases, however, an event may be most easily characterized by the sum of these two separate populations, one representing the rebound, the other the splashed grains. We follow this approach in condensing the results of these simulations into a useful analytical representation of the grain-impact process to be used in the saltation simulations to follow.

Splash function

The results of the single impact simulations are condensed into analytical expressions for the number of ejecta and the probability distribution of their ejection velocities, as functions of impact velocity (Anderson and Haff, 1988). As we found little dependence of the properties of the grain splash over the relevant range of expected angles of impact in eolian saltation, we have ignored any dependence on impact angle. A reasonable expression for the number of rebounding particles to be expected from a single impact of velocity V_{imi} in each of j ejection velocity "bins", labelled V_{oj}, is a gaussian:

$$Nr(Voj) = a \exp(-(Voi-bVimi)/cVimi)2) \, dVo$$

where a, b and c are constants.
The distribution for the expected splashed grains is an exponential, this time scaled by the expected number of grains ejected per impact, fVimi

$$Ne(Voj) = [fVimi] \quad \exp(-Voi/hVim) \, dVo$$

In each case dVo is the width of the ejection velocity bin. The total number of grains leaving the bed in each velocity bin subsequent to a single impact of velocity Vimi, then, is the sum of these two expressions: Ns = Nr+Ne.

Modification of the wind velocity profile

As grains are accelerated by the force of the wind, they impose an equal and opposite force on the wind. The profile of this force per unit volume on the wind was calculated and used to alter the effective stress available to shear the air at all levels in the flow. Prediction of particle concentration, mass flux, and kinetic energy flux caused by wind requires accurate calculation of particle trajectories, which in turn requires a detailed knowledge of the wind velocity profile. A theoretical model for the magnitude and distribution of the drag imposed upon the wind by saltating grains is necessary to establish the relative magnitude of the three contributions to the overall drag over a mobile sand bed: saltating grains, stationary grains on the bed, and form drag due to ripples. In the eolian sediment transport system, the vertical region within which the velocity profile is modified by sediment transport is of the order of centimeters, making detailed measurement of the velocity structure possible.

In the lowest 10 m of the atmospheric boundary layer the shear stress, τ_b, is approximately constant. Within this region, but well above the heights of the roughness elements, a logarithmic velocity profile is expected to exist, dependent upon a single velocity scale, u_*, and a single length scale, z_0. The velocity scale is the shear velocity, u_*, defined as $\sqrt{\tau_b/\rho_b}$, where ρ_a is the air density. The shear stress, in turn, is governed by larger scale atmospheric circulation, driven by pressure gradients imposed largely by differential solar heating of the atmosphere. In the absence of large bedforms, the length scale setting the logarithmic velocity profile is proportional to the diameter of the grains in the bed: $z_0 = D/30$ for uniform grains.

Sediment transport alters the effective roughness of the bed in two ways: first, the horizontal acceleration of transported grains extracts momentum from the wind, and second, the formation of ripples in the bed imposes a form drag on the wind. The resulting sediment transport roughness, z_{0st}, measured by extrapolating the velocity profile outside the sediment transport region to the U=0 axis, has been shown to depend on the shear velocity [after Owen (1964)]

$$z_{0st} = a\frac{u_*^2}{2g}$$

with a~ .02.

Because we expect the mean vertical velocity of particles upon ejection from the bed to increase with the shear velocity, and, as argued above, in the absence of non-gravitational forces, a grain leaving the bed with velocity u_* reaches a height of $u_*^2/2g$, the success of Owen's formulation implies that the roughness is no longer proportional to the size of the stationary roughness elements in the bed, but rather to some typical hop height of a saltating grain.

Wind velocity measurements within the saltation region show significant deviations from the logarithmic profile. Velocity gradients nearest the bed are reduced from those expected by extrapolating the outer flow toward the bed, and result in ln(z) - U plots of wind profiles that are convex upward. This curvature corresponds to Bagnold's (1941) "kink" in the profile, which occurs typically on the order of one to a few centimeters above the bed. Such near-bed deviations from the logarithmic velocity profile during sediment transport have led to incorrect assessment of the shear velocity during experimental work in wind tunnels. All too often, the slope of the least-squares fit to the ln(z)-U plot, from which the shear velocity is calculated, incorporates at least in part the region in which saltating grains are expected to impose a systematic departure from the simple logarithmic profile. Correct assessment of the shear velocity is essential, for instance, in the development of the correct functional dependence of the total mass flux on the shear velocity (e.g., Bagnold, 1941 and compiled in Greeley and Iversen, 1985).

A full model of the modification of the wind profile by saltating grains must therefore be able to predict both this curvature of the profile, and the altered effective roughness of the bed. As emphasized by Ungar and Haff (1987), a complete steady state saltation model must include iteration through a wind-velocity profile feedback loop, as the altered wind profile will change the suite of particle trajectories, which in turn result in a new wind profile, etc.

Although it has long been recognized that the modification of the wind profile results from the extraction of momentum from the wind by the saltating grains, previous attempts to incorporate this effect have suffered from oversimplification of the saltation process. Most notably, Owen (1964) assumed that all particles trace identical trajectories. It is now clear, however, that the probabilistic nature of the grain-bed interaction leads to a broad distribution of initial conditions for particle trajectories, and to large gradients in particle concentration, mass flux, and kinetic energy flux with height above the bed (e.g., Anderson and Hallet, 1986; Anderson, 1986a,b; Jensen and Sorenson, 1985; Sorenson, 1986; Willetts and Rice, 1986a,b). This structure is expected to be reflected in the profile of the force imposed on the wind by the horizontal acceleration of transported grains.

Two recent attempts have been made to calculate wind profiles during sediment transport (Ungar and Haff, 1987; Sorensen, 1986). Ungar and Haff, in their analysis of the saltation problem, confine their calculations to the simplest possible case that retains all the important elements of the problem. At any shear velocity, they force their solution to yield only one particle trajectory. As the grain-bed interaction, characterized by their "splash function", is independent of wind speed, i.e. the same liftoff velocity is retained for a particular grain impact velocity no matter what the wind structure is, the single trajectory allowed at each shear velocity must have the same impact velocity. This requires that each such trajectory experience the same net acceleration by the wind. They argue, therefore, that in accord with Bagnold (1941) "no matter how hard the wind is made to blow ... the wind velocity at a height of about 3 cm remains almost the same. Moreover, at levels still closer to the ground the wind velocity actually falls as the wind above is made stronger." Ungar and Haff's computed profiles show this behavior. It remains to be seen whether similar results can be obtained for a more realistic range of particle trajectories, arising from a more realistic grain-bed interaction, or "splash function."

Sorenson's treatment of wind velocities during sediment transport allows a realistic range of particle trajectories. The principal contrast with the formalism presented below lies in the nature of the postulated "closure" relation between the fluid stress and the shear rate, discussed further below.

Momentum equation for the air

We seek an expression for the momentum of the air during saltation that both takes account of the momentum extracted by the grains during saltation, and reduces properly to the "law of the wall" appropriate for the planetary boundary layer in the absence of saltation. Following the work of Ungar and Haff, we first formalize the effect of saltating grains on the air, which results in the establishment of an extra body force term in the Navier-Stokes equation for the force balance on a parcel of air. We then seek an appropriate form for the distribution of this body force with height above the bed. Finally, we suggest a possible closure relating the turbulent stresses to the local mean wind shear that remains consistent with the law of the wall in the absence of saltation.

Within the saltating curtain, the assumption of constant shear stress used to construct the law of the wall breaks down, as the acceleration of massive grains imposes an additional force on the wind. Following the formulation of Ungar and Haff, we identify a horizontal body force on the wind, $F_x(z)$, acting in the upwind (negative-x) direction due to the acceleration of the grains by the air. This appears explicitly in the turbulent Navier-Stokes equations as an additional force term operating in the negative-x direction:

$$\rho_a \frac{\partial \mathbf{u}}{\partial t} + \rho_a \mathbf{u} \cdot \nabla \mathbf{u} = - \nabla p + \nabla \cdot \tau_t - \rho_a \, \mathbf{g} - \mathbf{F_x}$$

where ρ_a is the air density, u is the mean horizontal wind velocity, g is the acceleration due to gravity, and τ_t is the turbulent (Reynolds) shear stress, representing the flux of momentum via correlations in the velocity fluctuations. Given steady ($\partial/\partial t \sim 0$), horizontally uniform flow ($\mathbf{u} \cdot \nabla \mathbf{u} \approx 0$), and making boundary layer approximations ($\partial/\partial z >> \partial/\partial x$, $\partial/\partial y$), the equation for momentum in the downwind (positive-x) direction collapses to:

$$\frac{\partial (\tau_t)_{zx}}{\partial z} = F_x(z)$$

In the absence of saltating grains the right hand side vanishes, and the first integration yields τ_t = constant. After identifying this constant as the shear stress imposed by the exterior flow, $\tau_b = \rho_a u_*^2$, making the common "closure hypothesis" that the turbulent stresses may be identified as the product of an eddy diffusivity, K, with the strain rate, $\partial u/\partial z$, and making the further assumption that the eddy diffusivity varies linearly with height, $K=ku_*z$, where k is von Karman's constant (=0.40), a second integration yields the well known law of the wall, or logarithmic profile: $u=(u_*/k)\ln(z/z_o)$, where z_o is the effective roughness of the bed. Such conditions should apply throughout the profile in the absence of sediment transport, and above the region within which grains are being appreciably accelerated by the wind during sediment transport, the difference between them being in the value of the effective roughness, z_o.

Within the saltation region, however, the stress on the wind must vary in the vertical direction as the force on the wind due to the extraction of momentum by saltating grains varies. Assuming a constant total stress available for transporting momentum of either grains or fluid across any level z, the stresses may be partitioned according to

$$\tau_b = \tau_t(z) + \int_z^{z_{max}} F_x(z)\, dz$$

where z_{max} is the maximum height to which a saltating grain travels in the given transport conditions.

The first term on the right hand side represents the stress available to shear the air at the level z , or the flux of fluid momentum across that level; the second , also called the "grain stress", τ_g (Sorenson, 1986), represents the change in horizontal momentum of grains between their upward and downward crossings of the level z , or the net flux of horizontal grain momentum across that level. We see that as the bed is approached from above, the grain stress increases, leaving less shear stress available to shear the fluid.

This equation may be rewritten as

$$\tau_t(z) = \int_0^z F_x(z)dz + \tau_{sf}$$

where τ_{sf} is the fluid stress at the bed, or the skin friction, part of which may be taken up by form drag due to ripples.

We see then that the skin friction, or the shear stress exerted on the bed by the wind, is simply the far-field shear stress, τ_b, minus the total change in horizontal momentum of all grains ejected from a unit area of bed in a unit time

$$\tau_{sf} = \tau_b - \int_0^{z_{max}} F_x(z)dz$$

For a given exterior wind condition, characterized by τ_b, as the body force diminishes in magnitude, more fluid stress is made available at the bed for aerodynamic initiation of saltation; and conversely, as body force increases, the shear at the bed decreases. Owen (1964) hypothesized that the self-regulatory nature of eolian saltation arose from such a feedback, with skin friction held near the threshold shear stress necessary to entrain particles aerodynamically. It remains to test this hypothesis quantitatively.

We now seek an equation for the velocity gradient as a function of height, which when integrated will yield a velocity profile in equilibrium with sediment tranport. Given the above relations between far field shear stress, skin friction, and grain stress, we require both a constitutive relation between the turbulent stresses and the mean velocity gradient, and the body force profile, $F_x(z)$. For simplicity, we again postulate an eddy diffusivity closure: $\tau_t = \rho_a K \partial u/\partial z$, and retain the linear dependence of the eddy viscosity with height, $K = ku_* z$, that gives rise to the logarithmic velocity profile in the absence of sediment transport. However, referencing u_* to the total stress, τ_b, is no longer appropriate. We define a *local*, or effective shear velocity, referenced to the local turbulent stress, τ_t, as $u_{*eff}(z)=(\tau_t/\rho_a)^{1/2}$. Then, from $\tau_t = \rho_a k z u_{*eff} \partial u/\partial z$, we have

$$\frac{\partial u}{\partial z} = \frac{1}{kz}\left[\frac{\tau_b - \int_z^{z_{max}} F_x(z)dz}{\rho_a}\right]^{\frac{1}{2}}$$

Note that above the region within which grains accelerate, i.e when $z > z_{max}$, or in the absence of sediment transport altogether, i.e. when $F_x=0$ for all z, the numerator becomes $(\tau_b)^{1/2}$, and the rate of shear becomes $\partial u/\partial z = u_*/kz$, which again yields the logarithmic profile, as required. Given the form of the force profile at any time in the evolution of the saltation population, derived in the next section, this equation may be numerically integrated to yield the wind velocity profile throughout the saltation region.

Note that irrespective of the actual form of the force profile, the fluid stress, τ_t, and hence the effective shear velocity, will increase monotonically with height, giving rise to *convex upward* u-ln(z) plots. Those profiles showing inflections in semi-log space, as some of Bagnold's (1941) early profiles do, are suspect; the stress available to shear the fluid, and hence the effective shear velocity, should increase monotonically away from the bed, yielding convex-upward wind velocity profiles in semi-log space. Inflections resulting from an initial decrease in effective shear velocity, followed by an increase in shear velocity, should not exist.

Force on the wind due to saltating particles

Following the formalism established in previous work (Anderson and Hallet, 1986; Anderson, 1986), the force due to identical trajectories with unit ejection rate is first calculated, i.e., we compute numerically the ejection Greens function; the distribution of initial conditions and the actual ejection rate at any time in the evolution of the saltation population are then incorporated to yield a total instantaneous force profile.

The highest instantaneous force is attained early in the ascending limb of the trajectory, before the particle has been appreciably accelerated by the wind, and where the relative velocity between the particle and the air is therefore the greatest. The force on the particle becomes negative shortly before impacting the bed, indicating that the particle there is travelling faster than the wind.

Summing over the ascending and descending limbs of the trajectory, and assuming a single particle is ejected per unit area of bed per unit time, i.e. $N_1=1$, with initial vertical velocity w_o, results in the "identical trajectory" force on the wind:

$$F_x(z|w_0,N_1) = N_1 M\left[\frac{a_x}{|\langle w(w_0)\rangle|_+} + \frac{a_x}{|\langle w(w_0)\rangle|_-}\right]$$

where <w> is the mean vertical particle velocity in crossing the height element (z-dz, z), and the + and - denote upward and downward limbs of the trajectory, respectively (Ungar and Haff, 1987; Anderson, 1986b). The upwind direction of the force is left implicit. The "identical trajectory" force profile displays a distinct maximum at the top of the trajectory, similar to the maxima in particle concentration, mass flux, and kinetic energy flux profiles reported earlier (Anderson and Hallet, 1986; Anderson, 1986a). Although the instantaneous horizontal force on the particle peaks approximately one third of the way up the ascending limb of the trajectory, most of the particle's horizontal acceleration occurs near the top of the hop, where its vertical velocity is very low, and therefore where it spends the most time. A similar argument explains the maximum at the top of the particle path in the concentration, mass flux, and kinetic energy flux profiles for "identical trajectory" models (Anderson and Hallet, 1986; Anderson, 1987).

The probability density of the vertical liftoff velocity, $p(w_o)dw_o$, and the actual number of particles ejected per unit area of bed per unit time, N, are now introduced to yield an integral equation for the total horizontal body force per unit volume on the wind due to the presence of saltating particles:

$$F_x(z) = \frac{N}{N_1} \int_0^\infty p(w_0)\, F_x(z|w_0,N_1)\ dw_0$$

In earlier work (Anderson, 1986b), force profiles and the resulting wind velocity profiles were calculated using reported total mass fluxes and assumed forms for the probability distribution, $p(w_o)$, to constrain the values of the total ejection rate, N, in the above equation. These calculations revealed that the effect of the form of the probability distribution of liftoff velocities dominates over the shape of the "identical trajectory" force profile in determining the result of the integration. Force profiles decline sharply above the bed, with a scale height that is on the order of 1-3cm. The effect of transported grains on the wind profile, for moderate winds, should essentially vanish by 5cm above the bed, as is observed. As scale height is a reflection largely of the probability distribution of initial velocities, one expects this will be strongly influenced by grain size, and by the external forcing characterized by u_*.

The choice of a linearly increasing eddy diffusivity within the saltation region follows more closely the treatments of Ungar and Haff (1988) and of Sorenson (1986), (and is identical to the mixing length algorithm used by Werner, 1990) and diverges from that of Owen (1964), who argued that a constant eddy diffusivity would be appropriate. Owen made the plausibility argument that the intensity of turbulence and the mixing lengths of the turbulence ought both to be dominated by wakes cast by saltating particles, and should therefore be roughly constant within the saltation curtain. However, the concentration of particles in the flow is on the order of 10^{-2} to 10^{-4} near the bed (Anderson and Hallet, 1986; Sorenson, 1986). It may therefore be expected that the nearby presence of a continuous rough bed will dominate over the wakes cast by these sparsely distributed particles in setting the length scale of the turbulence. The use of a linearly varying eddy diffusivity also allows a much simpler feathering of the saltation region with the constant stress region above, rather than requiring the abrupt change in the nature of the turbulence suggested by Owen. This explicitly recognizes that the saltation region is neither physically isolated, nor even easily identified in the real world. The character of the saltation curtain is such that all profile quantities, including the force on the wind imposed by the accelerating particles, fall off rapidly away from the bed, as a consequence primarily of the heavily skewed nature of the probability density of liftoff conditions resulting from the complex grain-bed interaction. It is a sediment transport boundary layer.

That the shear velocity be tied to the *local* fluid stress, τ_t, rather than the total or far-field stress, τ_b, makes the present treatment different from those of both Ungar and Haff (1987), and Sorenson (1986). Such an hypothesis has proven fruitful in the analysis of aqueous systems, where the form drag due to multiple sets of bedforms is modelled with a corresponding number of matched logarithmic profiles, each characterized by a shear velocity referenced to the spatially averaged form drag associated with that particular bedform scale. The present expression for the wind shear identifies both the magnitude of the stress due to saltating grains, and its profile, allowing a smoothly varying wind profile rather than a matched logarithmic profile.

A further difficulty with Owen's formulation of the wind profile within the saltation region is that the no-slip condition is left unsatisfied. Using his formula, wind velocities at the bed remain of the order 7-8 u_*. Although this has prompted earlier workers to justify use of a constant wind velocity (=8.5 u_*) in the saltation layer, thereby simplifying trajectory calculations, the present formulation is considerably more realistic, is not computationally taxing, and provides a profile valid through the entire region of interest in sediment transport mechanics.

Saltation model

The actual particle motion through the air is handled as follows. Because of computational limitation, particles are allowed to leave the bed with one of only ten distinct velocities; i.e., the distribution of initial conditions is discretized. These are chosen such that the lowest liftoff velocity trajectory reaches a height δ equal to approximately one grain diameter, while that with the highest liftoff velocity reaches a height above which little saltation flux is expected. This latter limit was chosen to be $w_o=5u_*$, meaning that typical trajectory heights for these grains is $\delta=25u_*^2/2g$, or roughly 1.2m for $u_*=1.0$m/s. The intervening liftoff velocity "bins" are distributed logarithmically in order to maximize the profile information in the lower portion of the saltation curtain, where most of the grains travel. At any instant in time, each liftoff velocity bin $(V_o)_i$, contains N_i particles.

Collision list

Participation in the splash process requires impact with the surface, which occurs with decreasing frequency as the initial velocity of the trajectory increases. (As noted previously, trajectory times are well estimated by $t_{hop}=b(2w_o/g)$, where w_o is the vertical component of the liftoff velocity, and b is a coefficient of value close to 1, only weakly dependent upon the initial velocity of the grain.) To account for the variable trajectory durations, a "collision list" is established. For each trajectory class a list is maintained of the number of grains expected to be impacting the surface at each time increment in the future. At each time step, then, the collision list for each trajectory class is interrogated to reveal the number of impacts in each trajectory bin. Their impact velocities are then assessed by calculating the trajectories using the (changing) wind velocity profile averaged over the hop time of the trajectory; the number of splashed particles in each of the liftoff velocity bins is then calculated from the splash function. Finally, the collision list is updated in accord with this new set of ejecta, and the entire collision list is moved forward in time one step to reveal the new numbers of impacts in each bin to be expected upon the next time step.

In the calculation, time is discretized at two levels. The time step associated with the reassessments of the wind profile and the grain splash is determined by the shortest trajectory time, or roughly $2(2D/g)^{1/2}$. A second, much shorter time step is needed to assure that each trajectory is calculated with sufficient precision to account correctly for the impact velocity and the various profile quantities of concern. Were we to have to calculate the actual grain dynamics of the bed upon each and every impact with the bed, a third, much shorter time step would be involved corresponding to the elastic material properties of the bed grains; this would be of the order of 10^{-5} seconds. The use of the splash function to represent the response of the bed to impacts removes this necessity.

Full simulations

Inputs required for the full simulation of the eolian saltation system are (i) a grain diameter and density, (ii) an initial wind velocity profile, the constants determining the splash function, and the coefficient of proportionality between the excess shear stress and the aerodynamic entrainment rate, α. The initial wind velocity profile is taken to be logarithmic. For a flat bed, the roughness height is related to the size and packing of the stationary sand grains in the bed (=D/30). In his experiments on the effect of sand movement on the surface wind, Bagnold (1941) produced wind profiles over a wetted sand bed that was previously "not only pitted with tiny bombardment craters a few grain diameters in size, but was made to undulate in the usual flat transverse ripples". The resulting u- ln(z) profiles for u_*=.20-.62m/s show little or no curvature between 2mm and 10cm above the bed, and all yield roughness heights closely approximated by z_o=2D/30, where 2D is roughly the mean height of the impact craters (Bagnold, 1941). Accordingly, the initial roughness height was chosen to be 2D/30. The shear velocity, u_*, in turn sets the initial shear stress at the bed, $(\tau_a)_o=\tau_b=\rho_a u_*^2$, to be used in subsequent recalculations of the stress and wind profiles. Simulations are run until a steady state is achieved, a state characterized by little or no change in the mass flux, the wind velocity profile, or in the collision list from one time step to the next.

In a typical simulated evolution of the saltation population and of associated quantities, initial entrainment is entirely aerodynamic. These grains gain horizontal momentum from the wind, and impact with velocities such that a small proportion rebound with greater initial velocities than the initial trajectories; few subsequent grains are splashed at this stage. An initial delta function probability distribution of liftoff velocities therefore evolves to a broader probability distribution, filling out into the higher velocities. The grains with higher liftoff velocities are airborne for longer periods of time before impacting and contribute more strongly to the splash ejection of other grains. The full range of possible ejection velocities is populated only after many tens of short-trajectory times. At this point the total number of grains in transport begins to grow rapidly, the highest impact velocity grains being most efficient in splashing grains into the airstream. The resulting roughly exponential growth is curtailed only when the extraction of momentum from the wind is sufficient to alter significantly the wind velocity profile, which in turn alters the impact velocities of the longer grain trajectories.

The system eventually reaches a steady state characterized by a specific total mass flux, an equal number of impacting and ejected grains, and a stationary wind velocity profile. The overshoot of the steady state appears to be due to the time lag associated with the 0.2 to 0.3 second hop times of the most energetic trajectories responsible for the majority of the ejections from thc bed. The steady state mass flux is well within the range of fluxes measured in wind tunnel studies for the same combination of shear velocity and grain size. Whereas the initial saltation population is entirely aerodynamic, the steady state saltation population for most imposed shear velocities contains no aerodynamically entrained grains. As the population of splashed and rebounding grains increases, the shear stress at the bed is reduced, which produces a corresponding decrease in the rate of aerodynamic entrainment. The steady state shear stress at the bed is reduced to slightly below the critical shear stress for aerodynamic entrainment. The effective roughness of the bed is simultaneously greatly increased, as reflected in the rise of the U=0 intercept (from z_0 to z_0'). This change in bed roughness is in accord with numerous measured wind velocity profiles during saltation experiments, as summarized by Owen. The resulting steady state profile of mass flux displays the characteristic rapid decrease above the bed, implying that the system has evolved to produce a realistic probability distribution for the initial trajectory velocities.

By varying the initial shear velocity, several such simulations were performed (Anderson and Haff, 1988) to produce a mass flux relation that is broadly similar to those derived empirically: above a threshold shear velocity ($u_{*c} = (\tau_c/\rho_a)^{1/2}$), the flux increases as a power of ($u_* - u_{*c}$).

Further calculations demonstrated yet another long-recognized feature of eolian saltation: hysteresis. Once steady state had been established using a particular shear velocity, the externally imposed shear stress (and hence u_*) was diminished (in stepped changes) to below that necessary to entrain grains aerodynamically. For choices of this shear velocity down to approximately 0.75 of the aerodynamic threshold shear velocity, u_{*c}, a low but steady and finite flux was achieved. This reflects the hysteresis in the system that forced Bagnold to define two threshold velocities: a fluid threshold, and an impact threshold, the latter being on the order of 0.8 of the former.

The response time of the saltation system appears to be on the order of 1 to 2 seconds, or several long-trajectory times (Anderson and Haff, 1988; Anderson, 1988). For the cases run to date, it appears that the response time is a weak function of the shear velocity, being longer for lower shear velocities. A knowledge of the response time, which is difficult to obtain from wind tunnel experiments, will allow us to treat the problem of predicting total mass fluxes in variable winds.

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