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Combining Zero Doppler Filter Calculations with MTI Filter Calculations to Increase Computational Speed

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The implementation of a Zero Doppler Filter in Radar applications can be simplified to cause a substantial saving of computational time when an associated Moving Target Indicator (MTI) filter is computed. Because the zero doppler filter is actually the MTI Filter shifted 180 degrees in the phase domain, the Zero Doppler output of the filter can be derived from the MTI output.

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COMBINING ZERO DOPPLER FILTER CALCULATIONS WITH MTI FILTER CALCULATIONS TO INCREASE COMPUTATIONAL SPEED

INTRODUCTION

In implementing an MTI filter and a related zero doppler filter, calculation time can be reduced by using the fact that the two filters are identical except for a 180 degree phase shift. The MTI can be calculated and then the zero doppler can be obtained from the MTI outputs without doing the full zero doppler calculation. This report will explain one method for reducing these calculations.

DERIVATION OF COEFFICIENTS

In designing an MTI filter the general equation of a FIR filter with complex coefficients can be used to derive the transfer function. The complex coefficients of the transfer function can be determined to place the null of the filter at any preselected frequency. In terms of the Z-transform the transfer function of a FIR filter is

$$H(z)=\sum_{k=0}^{N-1} a_k z^{-k} \tag{1}$$

Letting $z=e^{j2\pi\theta}$ to evaluate $H(z)$ on the unit circle, one obtains the frequency response of the filter

$$H(e^{j2\pi\theta})=\sum_{k=0}^{N-1} a_k e^{-j2\pi\theta k} \tag{2}$$

The coefficients are in general complex and will be represented here in polar form. This form makes the derivation more apparent. Substituting $a_k=r_k e^{j2\pi\theta_k}$ into Eq. 2 gives

$$H(e^{j2\pi\theta})=\sum_{k=0}^{N-1} r_k e^{j2\pi\theta_k} e^{-j2\pi k\theta}=\sum_{k=0}^{N-1} r_k e^{j2\pi(\theta_k-k\theta)} \tag{3}$$

To design an MTI filter we want to take the magnitude squared of the FIR filter frequency response and set it equal to zero at a preselected null frequency (often the null is set to zero). The magnitude squared is

$$|H(e^{j2\pi\theta})|^2=H(e^{j2\pi\theta})H^*(e^{j2\pi\theta})=\left[\sum_{m=0}^{N-1} r_m e^{j2\pi(\theta_m-m\theta)}\right]\left[\sum_{n=0}^{N-1} r_n e^{-j2\pi(\theta_n-n\theta)}\right] \tag{4}$$

Simplifying, we obtain

$$|H(e^{j2\pi\theta})|^2=\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} r_m r_n e^{j2\pi[(\theta_m-\theta_n)+(n-m)\theta]} \tag{5}$$

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Eq. 5 can be written as

$$|H(e^{j2\pi\theta})|^2=\sum_{k=0}^{N-1} r_k^2+\sum_{m=n+1}^{N-1}\sum_{n=0}^{N-2} r_m r_n e^{j2\pi[(\theta_m-\theta_n)+(n-m)\theta]}+r_n r_m e^{-j2\pi[(\theta_m-\theta_n)+(n-m)\theta]} \qquad (6)$$

Noticing that the double summation is a sum of complex conjugates we can combine the two exponentials into twice the real part

$$|H(e^{j2\pi\theta})|^2=\sum_{k=0}^{N-1} r_k^2+\sum_{m=n+1}^{N-1}\sum_{n=0}^{N-2} r_m r_n 2\cos\left\{2\pi[(\theta_m-\theta_n)+(n-m)\theta)]\right\} \qquad (7)$$

This can be recombined into

$$|H(e^{j2\pi\theta})|=\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} r_m r_n \cos\left\{2\pi[(\theta_m-\theta_n)+(n-m)\theta]\right\} \qquad (8)$$

Now pick $\frac{\alpha}{f_r}$ to be the angle where the null occurs, substitute this for theta in eq. 8 and set the expression equal to zero

$$\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} r_m r_n \cos\left\{2\pi[(\theta_m-\theta_n+(n-m)\frac{\alpha}{f_r}]\right\}=0 \qquad (9)$$

One method of solving this equation is to set the cosine term equal to one. This changes eq. 9 to

$$\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} r_m r_n=0 \qquad (10)$$

This last relation is equivalent to

$$(\sum_{i=0}^{N-1} r_i)^2=0 \qquad (11)$$

or,

$$\sum_{i=0}^{N-1} r_i=0 \qquad (12)$$

This last equation means that the signed magnitude of the complex coefficients must be equal to zero. (In the real case this would mean that the sum of the coefficients must be equal to zero as do the binomial coefficients when accompanied with alternating signs).

Since we set $\cos\left\{2\pi[(\theta_m-\theta_n)+(n-m)\frac{\alpha}{f_r}]\right\}=1$ for all θ_k's, given $\frac{\alpha}{f_r}$, this implies that the next equation is valid

$$2\pi[(\theta_m-\theta_n)+(n-m)\frac{\alpha}{f_r}]=2k\pi \quad (13)$$

where k is any integer. Solving for θ_m we obtain

$$\theta_m=k+\theta_n+(m-n)\frac{\alpha}{f_r} \quad (14)$$

We pick a value of zero for k because this equation is valid for all integers and zero simplifies the equation

$$\theta_m=\theta_n+(m-n)\frac{\alpha}{f_r} \quad (15)$$

This formula can be used to generate the phase angles of the complex coefficients for a given null at $\frac{\alpha}{f_r}$ provided of course that the sum of the signed magnitudes of the coefficients is equal to zero.

The zero doppler coefficients can be derived from the MTI coefficients via the phase shift formula, eq. 15, because a zero doppler filter is none other than an MTI filter shifted 180 degrees. To accomplish this begin with an MTI filter wiuh a null at $\frac{\alpha}{f_r}$, where f_r is the prf. The coefficients can be shown to be in general

$$a_k=r_k e^{j2\pi(\theta_0+k\frac{\alpha}{f_r})} \quad (16)$$

with $0\leq k \leq$ the size of the filter minus one. For ease of notation write this as

$$a_k=r_k e^{j2\pi\theta_k} \quad (17)$$

We will construct a zero doppler filter from the above MTI coefficients. We need to take the above coefficients and shift the null 180 degrees. This sets the null at $\frac{(\alpha+.5)f_r}{f_r}$. The coefficients turn out to be

$$b_k=r_k e^{j2\pi[\theta_0+k(\frac{\alpha}{f_r}+.5)]} \quad (18)$$

or,

$$b_k=r_k e^{j2\pi(\theta_0+k\frac{\alpha}{f_r})} e^{j2\pi.5k} \quad (19)$$

or,

$$b_k=r_k e^{j2\pi\theta_k} e^{jk\pi} \quad (20)$$

Noticing that $e^{jk\pi}=\left[e^{j\pi}\right]^k=-1^k$, we obtain

$$b_k=(-1)^k r_k e^{j2\pi\theta_k}=(-1)^k a_k \tag{21}$$

This last equation can save large quantities of computational time.

IMPLEMENTATION

Consider a 4 pulse MTI filter, and a corresponding 4 pulse zero doppler filter. The MTI output can be obtained by the sum

$$y(n)=r_0e^{j2\pi\theta_0}x(n)+r_1e^{j2\pi\theta_1}x(n-1)+r_2e^{j2\pi\theta_2}x(n-2)+r_3e^{j2\pi\theta_3}x(n-3) \tag{22}$$

The related zero doppler output can be calculated from the above MTI output as follows

$$z(n)=y(n)-2r_1e^{j2\pi\theta_1}x(n-1)-2r_3e^{j2\pi\theta_3}x(n-3) \tag{23}$$

The two products $r_1e^{j2\pi\theta_1}x(n-1)$ and $r_3e^{j2\pi\theta_3}x(n-3)$ need only to be calculated once for both filters and then stored in the registers of the processor. The zero doppler filter would then only require 2 bit shifts and 2 subtractions, resulting in a tremendous saving of time when calculated with the MTI filter instead of calculating each separately.

Six graphs of different filters are contained in figures 1 though 6. The graphs are of 3 pulse, 4 pulse, and 5 pulse MTI filters and their related Zero Doppler filters. Notice that the Zero Doppler coefficients in each case are identical to their MTI counterparts except for a sign change consistent with equation 21. Also note that the Zero Doppler graphs are exactly 180 degrees phase shifted from their MTI counterparts.

CONCLUSION

It was shown that the coefficients of a Zero Doppler filter can be derived from its associated MTI filter coefficients. Furthermore, it was shown that the relationship between the coefficients is a simple one that can be exploited to reduce the computational time of a Zero Doppler filter when the associated MTI filter is also calculated. An example of a 4 pulse Zero Doppler derived from a 4 pulse MTI filter was included to show the reduction in calculations to obtain the Zero Doppler output. Six graphs of filters are also included to illuminate the relationship between the two types of filters.

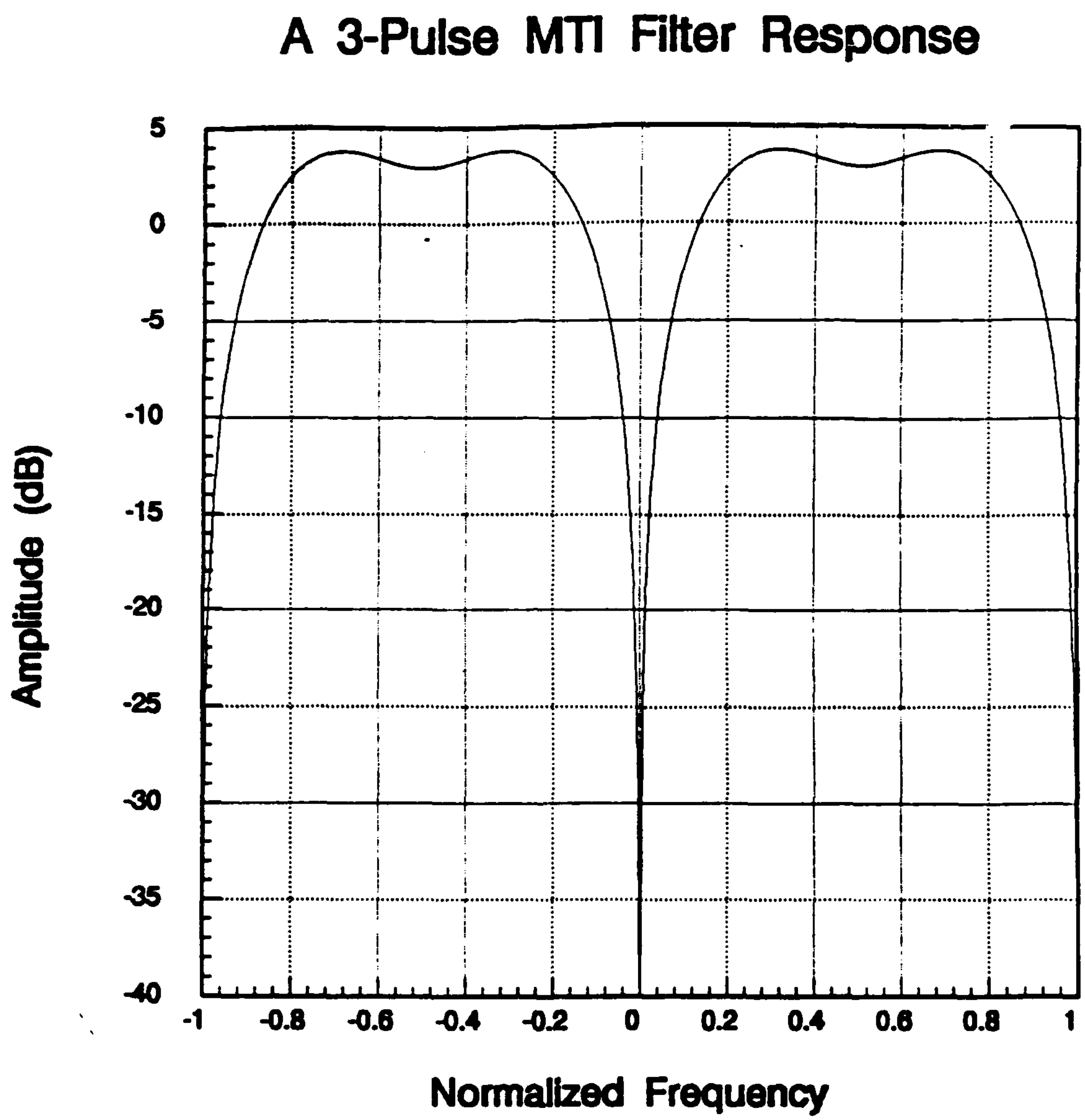

Figure 1. The coefficients for this filter are 1, -.7, and -.3.

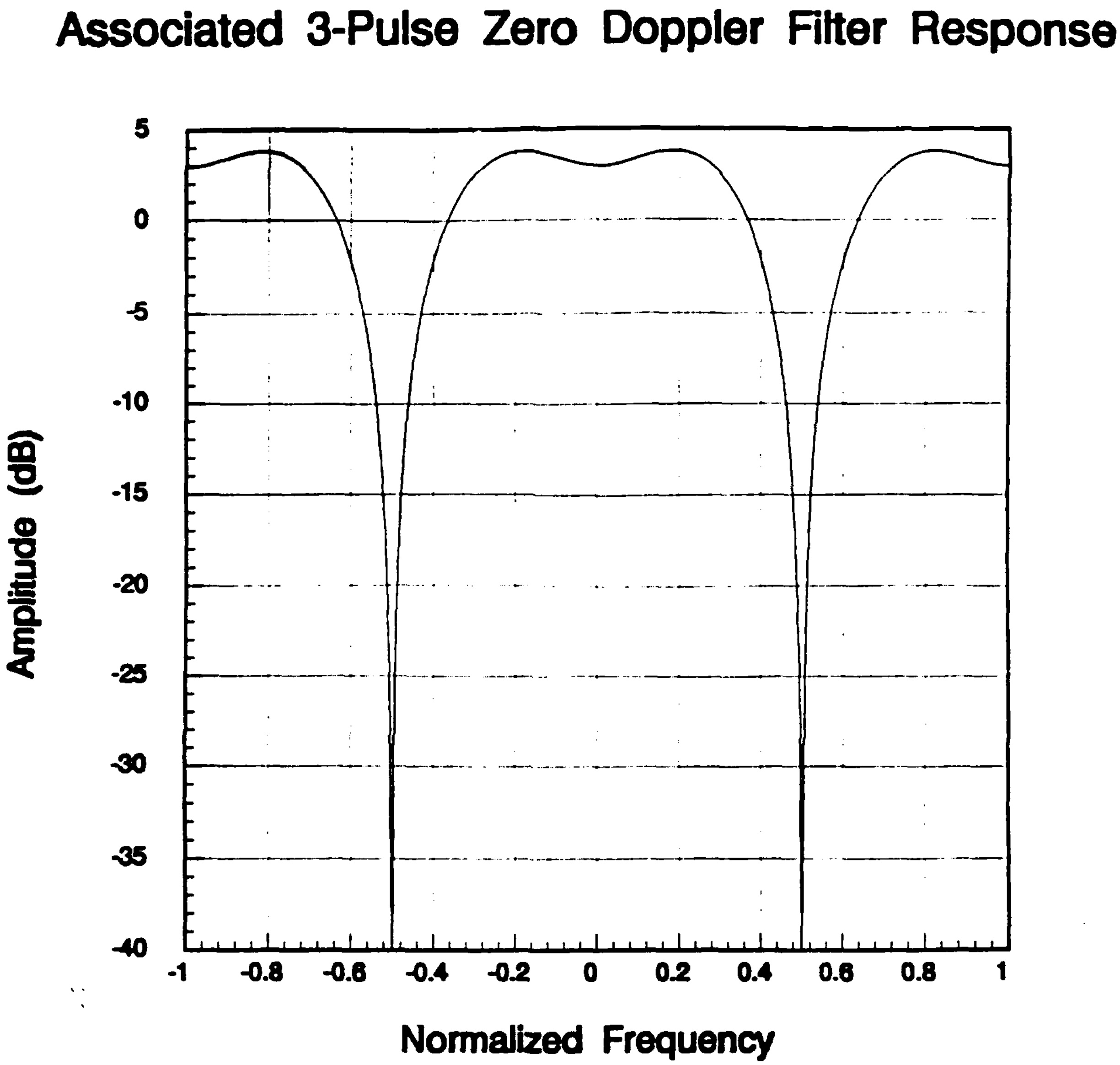

Figure 2. The coefficients for this filter are 1, .7, and -.3.

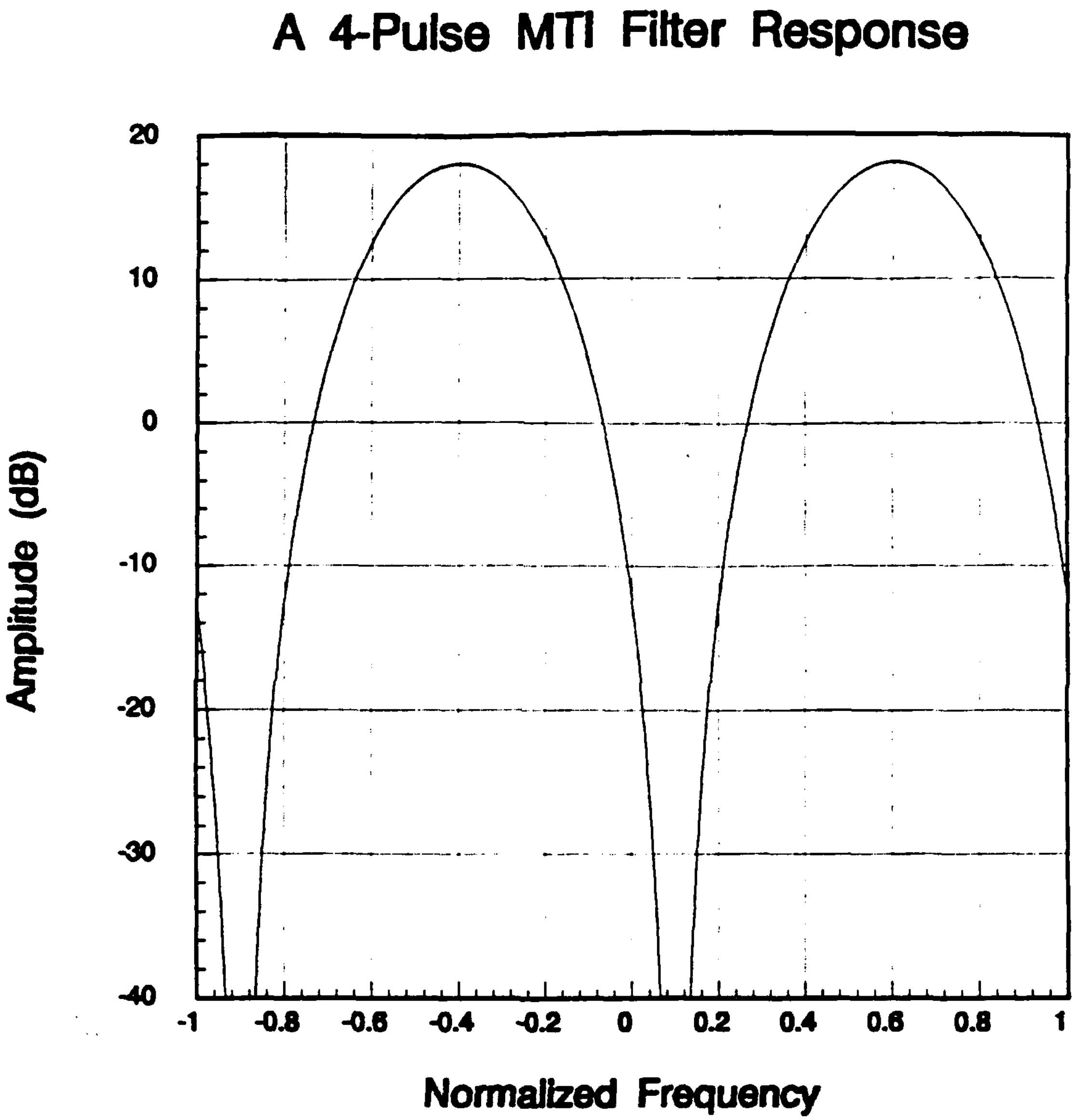

Figure 3. The complex coefficients for this filter are $1e^{j2\pi 0}$, $-3e^{j2\pi 1}$, $3e^{j2\pi 2}$, and $-1e^{j2\pi 3}$.

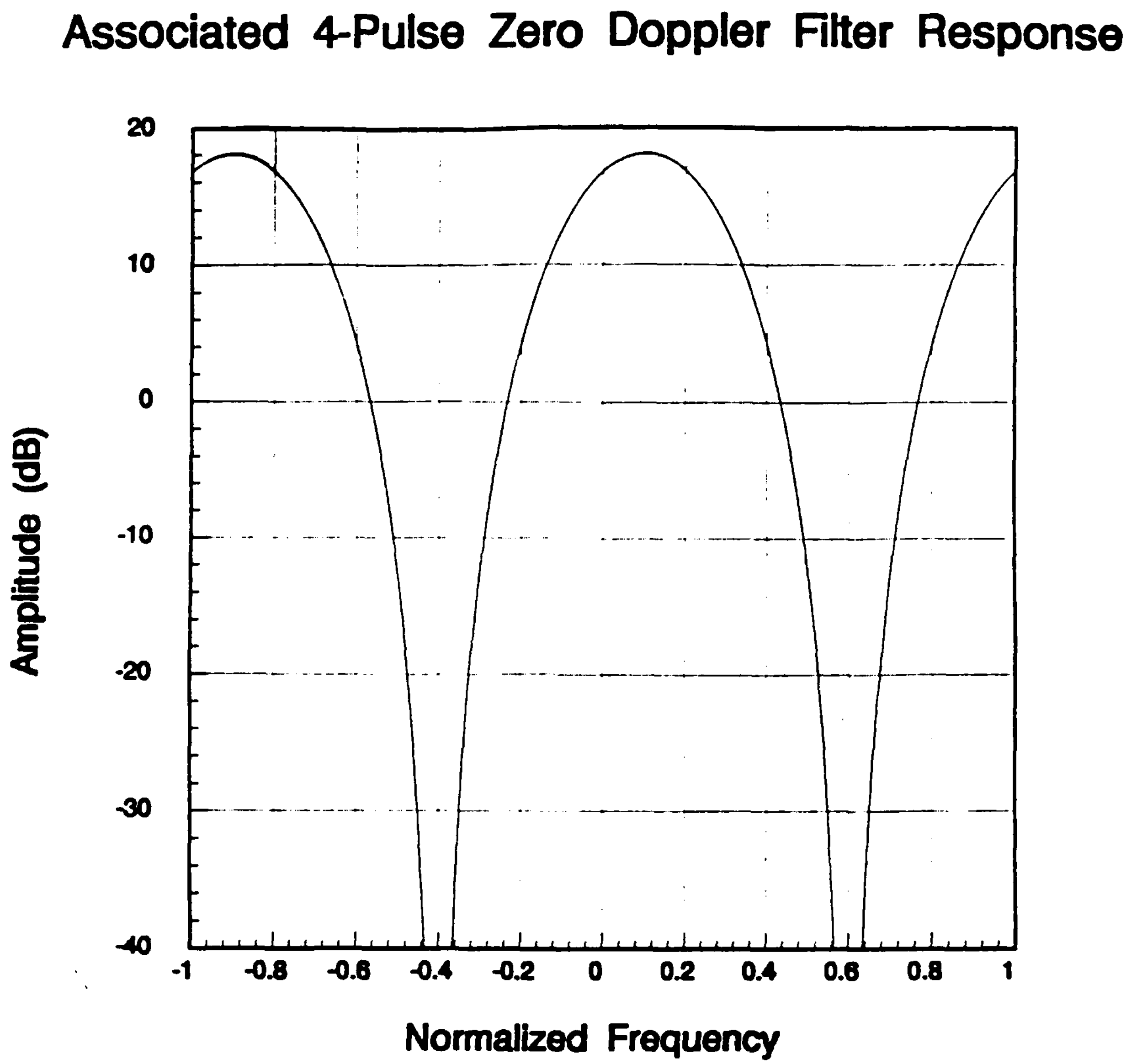

Figure 4. The complex coefficients for this filter are $1e^{j2\pi 0}$, $3e^{j2\pi .1}$, $3e^{j2\pi .2}$, and $1e^{j2\pi .3}$.

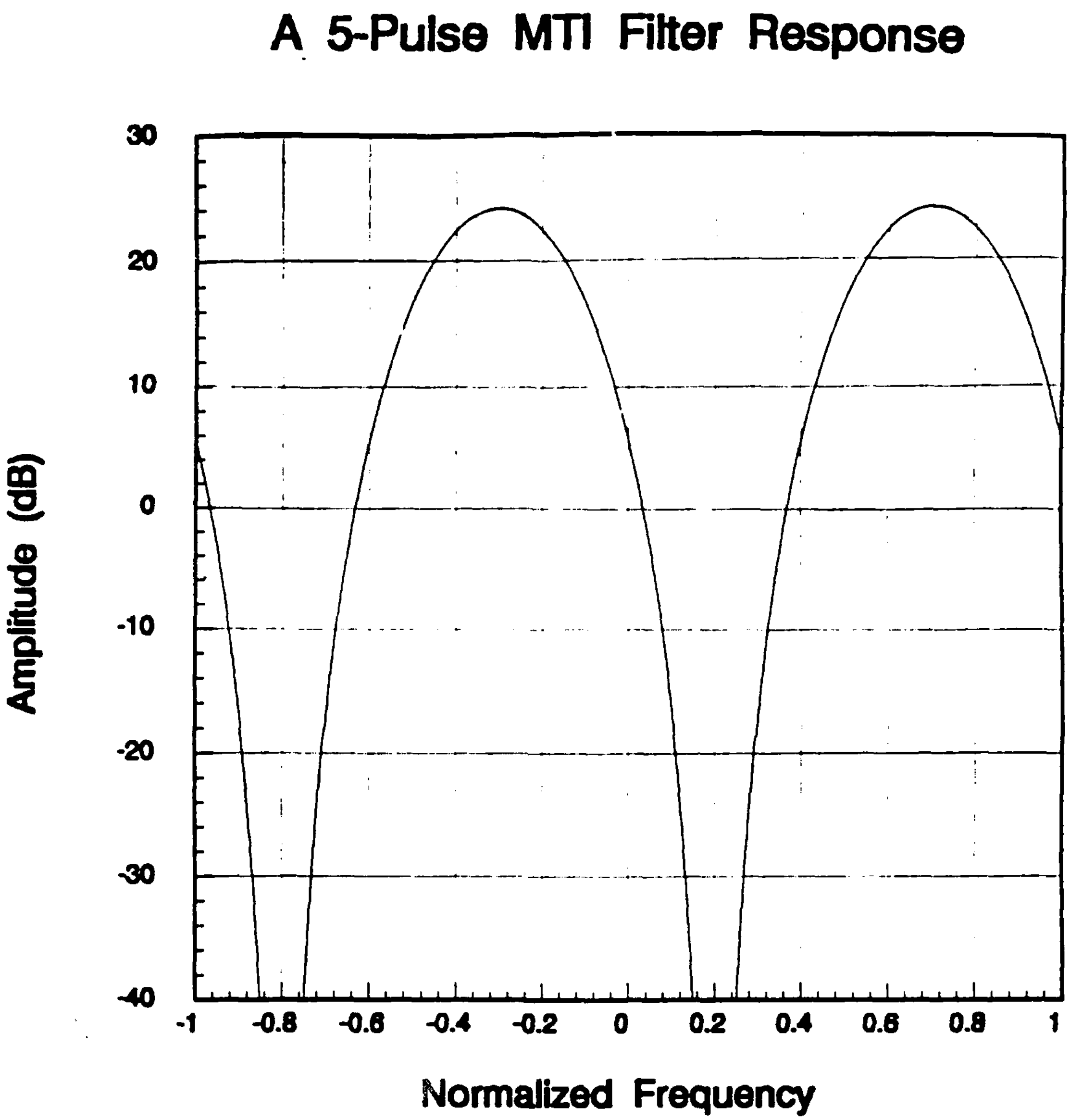

Figure 5. The complex coefficients for this filter are $1e^{j2\pi 0}$, $-4e^{j2\pi .2}$, $6e^{j2\pi .4}$, $-4e^{j2\pi .6}$, and $1e^{j2\pi .8}$.

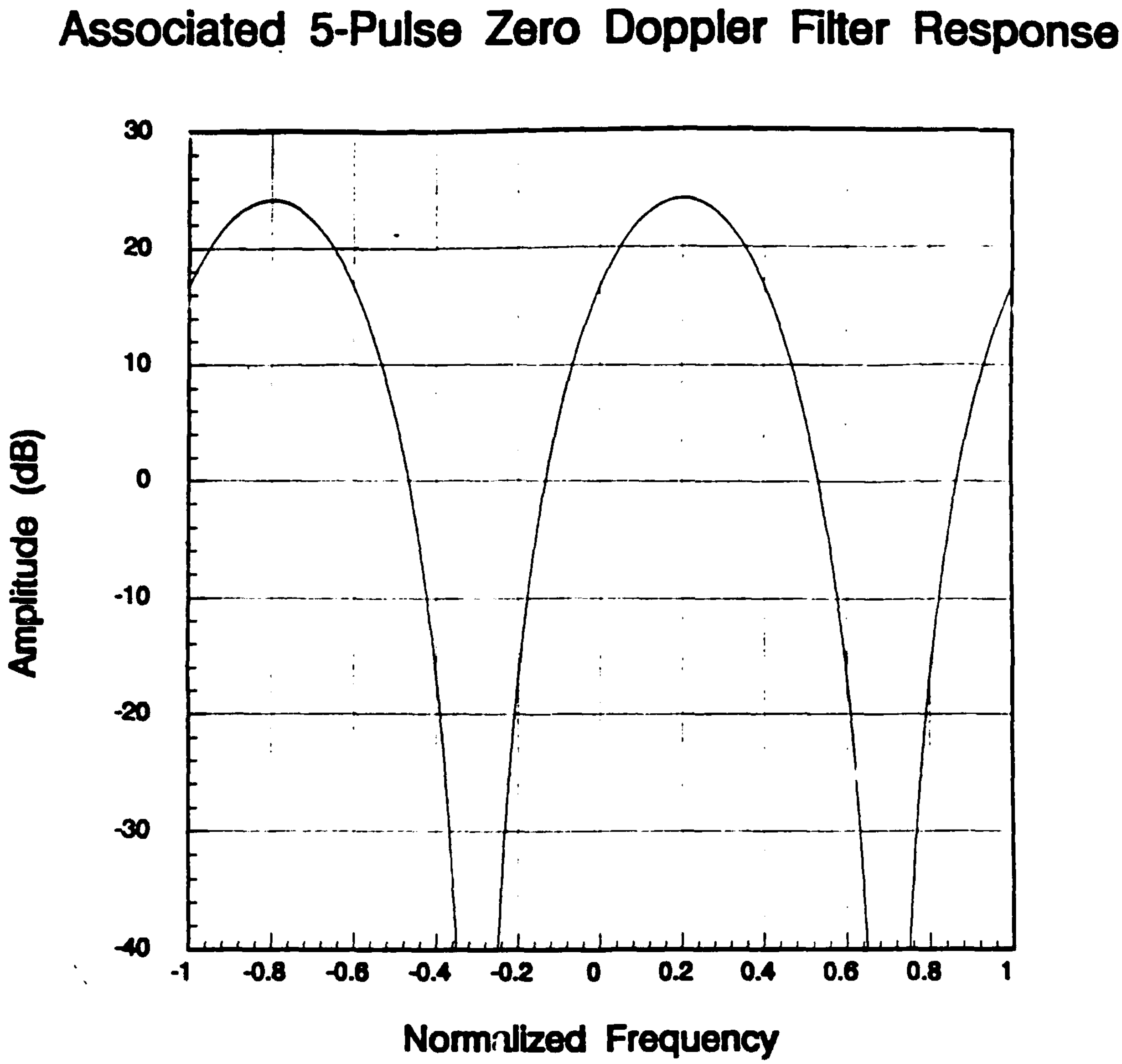

Figure 6. The complex coefficients for this filter are $1e^{j2\pi 0}$, $4e^{j2\pi .2}$, $6e^{j2\pi .4}$, $4e^{j2\pi .6}$, and $1e^{j2\pi .8}$.