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Mode Competition in the Quasioptical Gyrotron

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13. ABSTRACT (<i>Maximum 200 words</i>) A set of equations describing the nonlinear multimode dynamics in the Quasioptical Gyrotron is derived. These equations, involving the slow amplitude and phase variation for each mode, result from an expansion of the nonlinear induced current up to fifth order in the wave amplitude. The interaction among various modes is mediated by coupling coefficients, of known analytic dependence on the normalized current I, the interaction length μ , and the frequency detunings ω_1 corresponding to the competing frequencies ω_1 . The particular case when the modes form triads with frequencies $\omega_1 + \omega_3 - 2\omega_3 \equiv 0$ is examined in more detail. The equations are quite general and can be used to study mode competition, the existence of a final steady state, its stability, as well as its accessibility from given initial conditions. It is shown that when $(\mu/\beta_1) >> 1$, μ can be eliminated as an independent parameter. The control space is then reduced to a new normalized current I and the desyn- chronism parameters $\nu_1 = \Delta_1 \omega$ for the interacting frequencies. Each coupling coefficient G_{ij}^- is written as $G_{ij} = I S_{ij}^- G_{ij}(\nu_1, \nu_j)$, where the nonlinear filling factor S_{ij} , carrying the information of the beam current spatial profile, can be computed independently. Therefore, it suffices to compute table of G_{ij}^- as functions of ν_1 , ν_2 and ν_3 once to cover the parameter space. Results for a cold beam are presented here. 14. SUBJECT TERMS Quasioptical Gyrotron Multismode Dup amisc				
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CONTENTS

I.	INTRODUCTION	1
П.	GENERAL FORMALISM	4
Ш.	NONLINEAR ELECTRON DISTRIBUTION	10
IV.	MODE COUPLING EQUATIONS	13
V.	THREE MODE INTERACTION	15
VI.	RESULTS AND CONCLUSIONS	21
ACKN	OWLEDGEMENT	24
REFE	RENCES	25
APPE	NDIX A — Computation of the Nonlinear Current	27
APPE	NDIX B — Computation of the Three-Mode Coupling Coefficients	33
DISTR	UBUTION LIST	55



MODE COMPETITION IN THE QUASI-OPTICAL GYROTRON

I. INTRODUCTION

The quasi-optical gyrotron¹⁻¹⁰ (QOG) is a promising millimeter and submillimeter radiation source that has already demonstrated⁹ power levels \leq 150 kW with pulse durations up to 13 µs at electronic efficiencies \leq 14%. Potential applications, ranging from heating of fusion plasmas to short wavelength radars, require stable CW operation at high power levels within a narrow frequency band. Because of the strong fields developed inside the cavity, the high density of longitudinal resonator modes, and the relatively long time of operation, nonlinear interaction among many modes will inevitably occur.^{8,9} In fact, it is the nonlinear coupling among cavity modes¹¹ that will determine the existence of a final steady state, its accessibility from the initial small signal phase, and whether it is a single or a multimode state.

In short, the evolution of the cavity fields can be described as follows. The injected electron beam initially excites all the cavity eigenmodes characterized by a start-up current below the beam current. At small amplitude each mode grows exponentially in time with its linear growth rate, unaffected by the presence of other modes. Soon one or more modes reach finite amplitude and start interacting through the induced modifications in the distribution of the Abetron beam. When this happens a single mode may nonlinearly suppress all other unstable modes and eventually dominate. Equally well, it may destabilize modes that are linearly stable. In some cases the fastest linearly growing mode may be overtaken by another, linearly slower mode, causing mode switching. The final steady state, if one exists, may involve more than one large amplitude modes. The point to be made here is that the existence of a final steady state, be it single or multimode, is determined nonlinearly Manuscript approved February 6, 1990.

and cannot be predicted from the linear behavior.

Finding the regimes in parameter space associated with the desired type of final equilibrium proves too costly for direct numerical simulation, given that the performance of the QOG depends on four independent parameters. It should be stressed that the final equilibrium cannot always be obtained by examining the linear stability of every possible single mode at saturation.¹⁰ Even if a saturated single mode is found stable to small perturbations by other modes, there is no guarantee that the system will eventually evolve to this state. In general, more than one stable equilibrium exists for a given set of modes, however, only one of them is accessible from the appropriate initial conditions. In the case that the final steady state is dominated by a single mode, it is not necessarily the mode with the largest linear growth rate. Consequently, all participating modes must be treated on equal basis Mode selection and accessibility are until (and if) one dominates. inherently nonlinear processes.

In this work an analytic model of the multimode dynamics is developed by expanding the nonlinear current in powers of the radiation amplitude. A set of equations is then obtained for the slow time scale evolution of the wave amplitudes and phases. The strength of the nonlinear interactions enters through coupling coefficients of known analytic dependence on the gyrotron parameters. Numerical simulations of particle trajectories are then replaced by a set of first order ordinary differential equations, one for each participating mode. Not only is the computation time reduced by orders of magnitude but, equally important, the final equilibria can be predicted analytically from the coupling coefficients. In this paper we will mainly discuss cases with up to three interacting modes, coupled up to fifth order in the wave amplitude.

The coupled equations obtained here can be applied to the description of phase-locked¹² operation of the QOG through the injection of a small external signal. They can also describe phase locking through the use of a prebunching resonator,¹³ via certain modifications resulting from the use of a prebunched injected electron distribution in place of a uniform one.

II. GENERAL FORMALISM

The configuration for the QOG is shown in Fig. 1. The cavity fields are expressed through the vector potential A(r,t),

$$\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} , \qquad \mathbf{B} = \nabla \times \mathbf{A} , \qquad (1)$$

where A is a superposition of eigenmodes of the Fabry-Perot resonator,

$$\mathbf{A}(\mathbf{r},t) = \frac{1}{2} \sum_{m} \tilde{\mathbf{A}}_{m}(t) U_{m}(\mathbf{r}) e^{-i\omega_{m}t} e_{\mathbf{y}} + cc, \qquad (2a)$$

$$U_{m}(\mathbf{r}) = u_{p_{m}q_{m}}(\mathbf{r}) \cos k_{m} \mathbf{x} .$$
^(2b)

In the orthogonal coordinates (x,y,z) with x along the resonator axis and z along the electron beam axis, the transverse profiles $u_{pq}(r)$ are given by the Hermite-Gaussian functions

$$u_{p_m q_m}(r) = h_{p_m}(y) h_{q_m}(z) , \qquad h_{p_m}(z) = H_{p_m}\left(\frac{\sqrt{2}z}{w(x)}\right) \exp\left(-\frac{z^2}{w^2(x)}\right) ,$$
(3)

 $w(x) = W (1+x^2/b_m^2)^{1/2}$ is the radiation spot size, W is the radiation waist, $b_m = k_m W^2/2$ is the Rayleigh length and $H_p(z)$ are the Hermite polynomials. The wavenumber k_m is equal to ω_m/c , where the cavity frequencies ω_m take the values

$$\omega_{\rm m} = \frac{\pi c}{L} \left[m - 1 + \frac{2(p_{\rm m} + q_{\rm m} + 1)}{\pi} \tan^{-1} \left(\frac{L}{2b} \right) \right] \simeq \frac{\pi c}{L} (m + p_{\rm m} + q_{\rm m}) , \quad (4)$$

where L is the cavity length and p_m and q_m are integers; the notation p_m , q_m is used for labeling frequencies, $\omega_m = \omega_{mpq}$. According to the weak coupling approximation, the slowly varying amplitudes $A_m(t)$ are complex quantities with magnitude $A_m(t)$ and phase $\phi_m(t)$,

$$\tilde{A}_{m}(t) = A_{m}(t) e^{i\phi_{m}(t)}$$
(5)

The superposition (2), involving various frequencies ω_m , may also contain various modal structures $p_m q_m$ for a given frequency (i.e. degeneracy).

(a) Particle Equations

We use the guiding center description for the transverse particle coordinates

$$x = x_{g} + \rho \cos \theta, \qquad y = y_{g} + \rho \sin \theta,$$

$$p_{x} = p_{gx} - p_{\perp} \sin \theta, \qquad p_{y} = p_{gy} + p_{\perp} \cos \theta,$$
(6)

where (x_g, y_g) and (p_{gx}, p_{gy}) denote the guiding center position and momentum, p_{\perp} is the magnitude of the transverse momentum, θ is the gyroangle, $\rho = p_{\perp}/\Omega_c$ is the Larmor radius, $\Omega_c = \Omega_o/\gamma$ is the relativistic cyclotron frequency, where $\Omega_o = |e|B_o/mc$, and the relativistic factor $\gamma =$ $[1 + (p_{\perp}/mc)^2 + (p_z/mc)^2]^{1/2}$. By averaging the exact Lorentz force equations in the vector potential representation over the fast (cyclotron) time scale, the slow-time-scale nonlinear relativistic equations of motion are cast in the form

$$\frac{du_{\perp}}{dt} = \sum_{n} \omega_{n} a_{n} J_{1}'(k_{n} \rho) \sin (\psi_{n} + \phi_{n}) \cos k_{n} x_{g}, \qquad (7a)$$

$$\frac{d\theta}{dt} = \frac{Q_0}{\gamma} + \sum_n \frac{\omega_n}{u_\perp} a_n \frac{J_1(k_n \rho)}{k_n \rho} \cos(\psi_n + \phi_n) \cos k_n x_g.$$
(7b)

In Eqs. (7) time has been normalized to $\omega_0^{-1} = (\Omega_0 / \gamma_0)^{-1}$, length to $k_0^{-1} = c/\omega_0$, $a_n = |e|A_n/mc^2$ is the normalized radiation amplitude, u is the normalized momentum $u = p/mc = \gamma v/c$ and the relativistic factor $\gamma = (1 + u_1^2 + u_2^2)^{1/2}$. The angle ψ_m is the relative phase between the m-th mode and the particle, $\psi_m = \theta - \omega_m t$, evolving in time as

$$\frac{d\Psi_n}{dt} = -\delta\omega_n + \sum_n \frac{\omega_n}{u_\perp} a_n \frac{J_1(k_n \rho)}{k_n \rho} \cos(\Psi_n + \Phi_n) \cos k_n x_g, \qquad (8)$$

where $\delta \omega_n = (\omega_n - \Omega_0/\gamma_0) \omega_0^{-1}$ is the zero-order frequency detuning. The prime (') signifies the Bessel function derivative in respect to the argument. The evolution of γ is found combining Eqs. (7a) to (7b), to obtain

$$\frac{d\gamma}{dt} = \sum_{n} \omega_{n} \frac{u_{\perp}}{\gamma} a_{n} J_{1}'(k_{n}\rho) \sin (\psi_{n} + \phi_{n}) \cos k_{n}x_{g} . \qquad (9)$$

Two approximations are implicit in Eqs. (7)-(9). The guiding center drift has been ignored and the axial relativistic momentum u_z is taken as constant. They are both justified for $a_n \ll 1$, kW >> 1, since $dx_g/dt \sim dy_g/dt \sim 0(a_n^2)$ and $du_z/dt \sim 0(a_n/kW)$, which are much smaller than $du/dt \sim d\theta/dt \sim 0(a_n)$.

(b) Field Equations

Substituting Eq. (2) into Maxwell's equations and ignoring second order derivatives $\partial^2 \tilde{A}_n / \partial t^2 \ll \omega_n^2 \tilde{A}_n$, the slow-time evolution of the vector potentials is given by

$$\sum_{n} \left\{ i\omega_{n} \frac{\partial \widetilde{A}_{n}}{\partial t} U_{n} e^{-i\omega_{n}t} + cc \right\} = \frac{4\pi}{c} j_{NL} - \sum_{n} \left\{ \widetilde{A}_{n} \left(\nabla^{2} - \frac{1}{c^{2}} \frac{\partial^{2}}{\partial t^{2}} \right) U_{n} e^{-i\omega_{n}t} + cc \right\}.$$
(10)

The second term in the right-hand side of Eq. (10) is zero by definition for the vacuum eigenmodes defined in Eq. (2b), thus, all the information for the slow evolution of the fields is carried in the nonlinear transverse current j_{NL} . Equation (10) can also be derived from the general expression governing coherent wave interaction,¹⁴

$$\left(\frac{\partial \varepsilon}{\partial \omega}\right)_{n} \left[\frac{\partial}{\partial t} - v_{g} \frac{\partial}{\partial x} \right] \tilde{A}_{n} = \frac{4\pi i}{c} j_{NL} , \qquad (11)$$

in the limit of standing waves inside the resonator (zero group velocity, $v_g = 0$) and near vacuum dielectric constant ε (weak beam limit: $\omega_b^2 / \omega_o^2 \ll 1$), $(d\varepsilon/d\omega)_n \simeq 2\omega_n$.

Multiplying both sides of Eq. (10) by $\exp(i\omega_n t)$ and taking the fast time average over $\tau_0 \sim 2\pi/\omega_0$ isolates the nth mode on the left-hand side. Introducing the normalized variables used in Eqs. (7), and letting $\tilde{a}_n = |\tilde{A}_n/mc = a_n \exp(i\phi_n)$, we have

$$U_{n}(r) \frac{\tilde{\partial a_{n}}}{\partial t} = -\frac{i 4\pi |e|}{2\omega_{0}^{2} m c^{3}} j_{NL}(r;\omega_{n}) , \qquad (12a)$$

where

$$j_{NL}(r;\omega_n) = \frac{1}{\tau_o} \int_0^{\tau_o} dt \ j_{NL}(r,t) \ e^{i\omega_n t} .$$
(12b)

Multiplying both sides by the mode profile U_n and integrating over the cavity volume lead to

$$\left(\frac{\partial}{\partial t} - \frac{\omega_n}{2Q_n}\right)\tilde{a}_n = -\frac{i 2\pi |e|}{\omega_0^2 m c^3 V_n} \int d^3 r U_n(r) j_{NL}(r;\omega_n) . \qquad (13)$$

Above $V_n = \int d^3r U_n^2$ is the weighted cavity volume, and the phenomenological term ω_n/Q_n was added to account for resonator losses due to boundary effects, where the cavity Q_n is defined by

$$\frac{d}{dt}a_n^2 = -\frac{\omega_n}{Q_n}a_n^2.$$
(14)

The current is expressed in terms of the nonlinear distribution function $f_{\rm NL}$,

$$\mathbf{j}_{\mathrm{NL}}(\mathbf{r},t) = -|\mathbf{e}| \mathbf{n}_{\mathrm{b}}(\mathbf{r}) \int_{0}^{2\pi} \mathrm{d}\theta \int_{0}^{\infty} \mathrm{d}u_{\mathrm{z}} \int_{0}^{\infty} \mathrm{d}u_{\mathrm{\perp}} \mathbf{u}_{\mathrm{\perp}} \frac{\mathbf{u}_{\mathrm{\perp}}}{\gamma} \cos\theta \mathbf{f}_{\mathrm{NL}}(\mathbf{r},\mathbf{u},t) \cdot (15)$$

Letting $f_{NL}(\mathbf{r}, \mathbf{u}, t) = n_0 \quad n_b(\mathbf{r}_{\perp}) \quad f_{NL}(\mathbf{u}_{\perp}, \mathbf{u}_z, z; t)$, substituting (15) in Eq. (13), and expressing $\mathbf{r}_{\perp} = (x, y)$ and $\mathbf{u}_{\perp} = (\mathbf{u}_x, \mathbf{u}_y)$ in the guiding center representation Eq. (6), the right-hand side of Eq. (15) becomes

$$-i \frac{2\pi c |\mathbf{e}| \mathbf{n}_{0}}{\omega_{0}^{2} \mathbf{m}c^{3} \mathbf{V}_{m}} \int_{0}^{L} d\mathbf{x}_{g} \int_{-\infty}^{\infty} d\mathbf{y}_{g} \int_{-\infty}^{\infty} d\mathbf{z} \mathbf{n}_{b}(\mathbf{x}_{g}, \mathbf{y}_{g}) \mathbf{h}_{p_{n}}(\mathbf{y}_{g}) \mathbf{h}_{q_{n}}(\mathbf{z}) \cos \mathbf{k}_{n} \mathbf{x}_{g}$$
$$\int_{0}^{2\pi} d\theta \int_{0}^{\infty} d\mathbf{u}_{z} \int_{0}^{\infty} d\mathbf{u}_{\perp} \mathbf{u}_{\perp} \frac{\mathbf{u}_{\perp}}{\gamma} \cos \theta \mathbf{J}_{1}(\mathbf{k}_{n} \rho) \mathbf{f}_{NL}(\mathbf{u}_{\perp}, \mathbf{u}_{z}, \theta, z; \omega_{n}) , \qquad (16)$$

where again $f_{NL}(r;\omega_n) = (1/\tau_0) \int_0^{\tau} dt f_{NL}(r,t) \exp(i\omega_n t)$. Expression (16) is finally recast in terms of the injected beam current

$$\mathbf{I}_{\mathbf{b}} = |\mathbf{e}| c\beta_{\mathbf{z}0} n_{\mathbf{0}} \int d\mathbf{x}_{\mathbf{g}} \int d\mathbf{y}_{\mathbf{g}} \hat{\mathbf{n}}_{\mathbf{b}}(\mathbf{x}_{\mathbf{g}}, \mathbf{y}_{\mathbf{g}}) , \qquad (17)$$

as

$$\left(\frac{\partial}{\partial t} - \frac{\omega_n}{2Q_n}\right) \tilde{a}_n = -2\pi i < j_{NL}(\omega_n) > .$$
(18)

The dimensionless, volume averaged current on the right-hand side of Eq. (18),

$$\langle j_{NL}(\omega_{n}) \rangle = \frac{\gamma}{u_{z}} \frac{I_{o}}{\sigma_{b} V_{n}} \frac{1}{\tau_{o}} \int_{0}^{\tau_{o}} \frac{i\omega_{n}t}{\int_{V}} \int_{V} d\mathbf{r} \ U(\mathbf{r}) \ j_{NL}(\mathbf{r},t)$$

$$= \frac{\gamma}{u_{z}} \frac{I_{o}}{\sigma_{b} V_{n}} \int_{V} d\mathbf{r} \ U_{n}(\mathbf{r}) \ j_{NL}(\mathbf{r};\omega_{n})$$
(19a)

is given, in terms of $f_{\rm NL}$ from Eq. (16), by

$$\langle \mathbf{j}_{\mathrm{NL}}(\boldsymbol{\omega}_{\mathrm{n}}) \rangle = \frac{\mathbf{I}_{0}}{\sigma_{\mathrm{b}} \mathbf{V}_{\mathrm{n}}} \frac{\gamma}{\mathbf{u}_{z}} \int_{0}^{\mathbf{L}} d\mathbf{x}_{g} \int_{-\infty}^{\infty} d\mathbf{y}_{g} \mathbf{n}_{\mathrm{b}}(\mathbf{x}_{g}, \mathbf{y}_{g}) \mathbf{h}_{\mathrm{p}_{\mathrm{n}}}(\mathbf{y}_{g}) \cos \mathbf{k}_{\mathrm{n}} \mathbf{x}_{g}$$

$$\int_{-\infty}^{\infty} dz \mathbf{h}_{\mathrm{q}_{\mathrm{n}}}(z) \int_{0}^{2\pi} d\theta \int_{0}^{\infty} du_{z} \int_{0}^{\infty} du_{\perp} \mathbf{u}_{\perp} \mathbf{j}_{1}'(\mathbf{k}_{\mathrm{n}}\boldsymbol{\rho}) \frac{\mathbf{u}_{\perp}}{\gamma} \cos\theta \mathbf{f}_{\mathrm{NL}}(\mathbf{u}_{\perp}, \mathbf{u}_{z}, \theta, z; \boldsymbol{\omega}_{\mathrm{n}}) ,$$

$$(19b)$$

where $I_o = |e|I_b/mc^3$ is the dimensionless beam current (Budker parameter), and $\sigma_b = \iint dx_g dy_g n_b(x_g, y_g)$ is the effective beam cross section.

III. NONLINEAR ELECTRON DISTRIBUTION

The nonlinear electron distribution in phase space $f_{NL}(u_{\perp}, u_{z}, \theta, z, t)$ will be obtained in the form of an asymptotic expansion in powers of the small amplitudes $a_n \ll 1$. Once the form of f_{NL} in the multimode field (2) is known, the nonlinear transverse current j_{NL} is obtained from Eq. (19). The evolution of f_{NL} along the particle trajectories Eqs. (7) is given by the collisionless kinetic equation

$$\frac{\partial f_{NL}}{\partial t} + u_z \frac{\partial f_{NL}}{\partial z} + \frac{d\theta}{dt} \frac{\partial f_{NL}}{\partial \theta} + \frac{du_z}{dt} \frac{\partial f_{NL}}{\partial u_z} + \frac{du_\perp}{dt} \frac{\partial f_{NL}}{\partial u_1} = 0.$$
(20)

By formally expanding f_{NL} and ordering in powers of a^j , $a \sim 0(a_n) << 1$

$$f_{NL} = f^{(0)} + a f^{(1)} + \dots + a^j f^{(j)}$$

one obtains a hierarchy of equations

$$L_{o} f^{(j)} = L_{1} f^{(j-1)} = Df^{(j-1)},$$
 (21)

where

$$L_{0} = \frac{\partial}{\partial t} + u_{z} \frac{\partial}{\partial z} + Q_{c} \frac{\partial}{\partial \theta} , \qquad (22a)$$

$$L_{1} = \sum_{n} \tilde{a}_{n} \left\{ \frac{1}{2i} J_{1}(k_{n}\rho) \cos k_{n}x_{g} e^{i\psi_{n}} \frac{\partial}{\partial u_{\perp}} + \frac{1}{2u_{\perp}} \frac{J_{1}(k_{n}\rho)}{k_{n}\rho} \cos k_{n}x_{g} e^{i\psi_{n}} \frac{\partial}{\partial \theta} \right\}.$$
(22b)

Operator L_0 is the total time derivative $L_0 \equiv d/dt$ along the unperturbed orbits of electrons gyrating in the external magnetic field B_0

$$\theta' = \theta - \Omega_c \tau$$
, $u_{\perp}' = u_{\perp}$, $z' = z - \frac{u_z}{\gamma} \tau$, $\psi_n' = \psi_n - \delta \omega_n \tau$, (23)

(the characteristic curves of $L_0 f = 0$ in the theory of partial differential equations) where $\theta' \equiv \theta(t')$, $\theta \equiv \theta(t)$ and $\tau = t - t'$. Replacing the exact trajectories in the right-hand side of (21) by the unperturbed orbits (23), (integration along the characteristics), it follows that

$$f^{(j)} = \int_{0}^{\infty} d\tau \ Df^{(j-1)} \left[u'(\tau), \ \theta'(\tau), \ z'(\tau) \right], \qquad (24)$$

We exploit the fact that the interaction time of an electron inside the cavity, of the order of the transit time $t_i \approx W/\beta_z$, is much shorter than the characteristic growth time $t_c^{-1} = d(\ln a_m)/dt$ for the fields. Keeping \tilde{a}_n constant during the integration (24) yields

$$f^{(j)} = \sum_{n} \tilde{a}_{n} \sin k_{n} x_{g} e^{(j)}$$

$$(25)$$

$$\int_{0}^{d\tau_{j}} h_{n}\left(z - \frac{u_{z}\tau_{j}}{\gamma}\right) e^{i\delta\omega_{n}\tau_{j}} \left(\frac{J_{1}(k_{n}\rho)}{2i} \frac{\partial}{\partial u_{\perp}} + \frac{J_{1}(k_{n}\rho)}{(k_{n}\rho)} \frac{1}{2u_{\perp}} \frac{\partial}{\partial \theta}\right) f^{(j-1)} + cc,$$

where $\tau_j = t_j - t_{j-1}$ and we used

$$\Psi_{n}(t_{j-1}) = \Theta(t_{j-1}) - \omega_{n}t_{j-1} = \Theta_{j} - \omega_{n}t_{j} + \delta\omega_{n}\tau_{j}.$$

It is now convenient to change variables, introducing the set $\zeta_0,\ \zeta_1,\ \ldots$, $\zeta_i,$ defined by

$$\zeta_{0} = \frac{z}{W}, \quad \zeta_{1} = \zeta_{0} - \frac{u_{z}}{\gamma W} \tau_{1}, \quad \dots, \quad \zeta_{j} = \zeta_{j-1} - \frac{u_{z}}{\gamma W} \tau_{j} = \frac{z}{W} - \frac{u_{z}}{\gamma W} (\tau_{1} + \tau_{2} + \dots + \tau_{j})$$
(26)

where $d\zeta_j = -(u_z/\gamma W) d\tau_j$ and $\zeta_j(\tau_j = 0) = \zeta_{j-1}$. Successive iterations of Eq. (25) yield

$$f^{(j)} = \int_{0}^{\infty} d\tau_{1}L_{1}(\tau_{1}) \int_{0}^{\infty} d\tau_{2}L_{1}(\tau_{2}) \int_{0}^{\infty} \dots \int_{0}^{\infty} d\tau_{j} L_{1}(\tau_{j}) f^{(0)}$$

$$= (-1)^{j} \left(\frac{\gamma W}{u_{z}}\right)^{j} \int_{-\infty}^{\zeta_{0}} d\zeta_{1}L_{1}(\zeta_{1}) \int_{-\infty}^{\zeta_{1}} d\zeta_{2}L_{1}(\zeta_{2}) \int_{-\infty}^{\zeta_{2}} \dots \int_{-\infty}^{\zeta_{j-1}} d\zeta_{j} L_{1}(\zeta_{j}) f^{(0)} ,$$
(27)

where $f^{(0)}$ is the injected beam distribution, and L_1 is written as

$$\int_{-\infty}^{\zeta_{j-1}} d\zeta_{j} L_{1}(\zeta_{j}) \equiv \sum_{n} a_{n} e^{i(\theta - \omega_{n}t_{j} + \nu_{n}t_{j})} \cos k_{n}x_{g}$$
(28)

$$\int_{\infty}^{\zeta_{j-1}} d\zeta_{j} h_{n}(\zeta_{j}) e^{i\nu_{n}\zeta} \left(\frac{J_{1}(k_{n}\rho)}{2i} \frac{\partial}{\partial u_{\perp}} + \frac{J_{1}(k_{n}\rho)}{(k_{n}\rho)} \frac{1}{2u_{\perp}} \frac{\partial}{\partial \theta} \right) + cc.$$

The quantity $v_n = (W_Y/u_z) \, \delta \omega_n$, the product of the frequency detuning times the transit time through the cavity is the most important parameter for the mode coupling coefficients. The integrations, Eqs. (27) and (28), are carried out in Appendix A. The expressions for $f^{(j)}$, j=1,2,3 are given by (A1)-(A3). For $j \ge 4$ the full expressions are getting rather complicated. However, one may exploit the existing ordering inside $f^{(j)}$ in powers of the large parameter $\eta = (kW/\gamma)(u_1/u_2) >> 1$. It is then easier to find an approximation for $f^{(j)}$ to any order in j, Eq. (A6), by keeping the dominant contribution. Additional simplifications occur during the gyroangle averaging in obtaining the nonlinear current, given by Eqs. (A7) and (A8).

IV. MODE COUPLING EQUATIONS

Substituting expression (A7) for the nonlinear current inside the right hand side of Eq. (A8) and keeping terms up to fifth order in complex amplitudes $\tilde{a}_m = a_m \exp(i\phi_m)$, the general mode coupling equations assume the form

$$\frac{da_{n}}{dt} = -\frac{2\pi i \gamma_{0}I_{0}}{u_{z0} \sigma_{b} V_{n}} \sum_{\{i,j,\ldots,m\}}^{\Delta} \left(C_{m;n}^{(1)} \tilde{a}_{m} + C_{klm;n}^{(3)} \tilde{a}_{k}\tilde{a}_{l}\tilde{a}_{m} + C_{ijklm;n}^{(5)} \tilde{a}_{i}\tilde{a}_{j}\tilde{a}_{k}\tilde{a}_{l}\tilde{a}_{m} \right).$$
(29)

A mode of given frequency ω_n interacts through all possible frequency combinations satisfying the resonant condition

 $\Delta \omega_{ij...m;n} \equiv (\omega_i \pm \omega_j \pm ... \pm \omega_m) - \omega_n \approx 0, \qquad (30)$ denoted by $\sum_{i=1}^{\Delta}$; the notation {i,j,...,m} implies summation over all permutations among i,j,...,m (except n) inside Eq. (29), for a given set of resonant frequencies (30). A negative index -m will imply $-\omega_m$, $-\phi_m$, $-\nu_m$ and \tilde{a}_n^* in place of ω_m , ϕ_m , ν_m and \tilde{a}_n respectively.

The dominant contribution to the coupling coefficients $C^{(j)}$ is given by

$$C_{m;n}^{(1)} = \left(\frac{i}{2}\right) \mathbb{V}^{2} S_{m;n} \int_{0}^{\infty} du_{\perp} \frac{u_{\perp}^{2}}{\gamma} \left(\frac{\gamma}{u_{zo}}\right) \int_{-\infty}^{\infty} dz v_{n}(z) \int_{-\infty}^{z} d\zeta_{1} v_{m}(\zeta_{1}) \frac{df^{(0)}}{du_{\perp}},$$
(31a)

$$C_{klm;n}^{(3)} = \left(\frac{i}{2}\right)^{3} V^{4} S_{klm;n} \exp \left[i\left(\Delta\omega_{klm;n}t + \Phi_{klm;n}\right)\right] \int_{0}^{\infty} du_{\perp} \frac{u_{\perp}^{2}}{\gamma} \left(\frac{\gamma}{u_{zo}}\right)^{3} \\ \times \int_{-\infty}^{\infty} dz v_{n}(z) \int_{-\infty}^{z} d\zeta_{1} v_{k}(\zeta_{1}) \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_{1}} d\zeta_{2} v_{1}(\zeta_{2}) \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_{2}} d\zeta_{3} v_{m}(\zeta_{3}) \frac{df^{(0)}}{du_{\perp}},$$
(31b)

$$c_{ijklm;n}^{(5)} = \left(\frac{i}{2}\right)^{5} w^{6} s_{ijklm;n}^{exp} \left[i\left(\Delta \omega_{ijklm;n}^{t} + \Phi_{ijklm;n}\right)\right] \int_{0}^{\infty} du_{\perp} \frac{u_{\perp}^{2}}{\gamma} \left(\frac{\gamma}{u_{zo}}\right)^{5}$$

$$\int_{-\infty}^{\infty} dz v_{n}(z) \int_{-\infty}^{z} d\zeta_{1} v_{i}(\zeta_{1}) \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_{1}} d\zeta_{2} v_{j}(\zeta_{2}) \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_{2}} \dots \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_{4}} d\zeta_{5} v_{m}(\zeta_{5}) \frac{df^{(0)}}{du_{\perp}}$$

$$(31c)$$

where

$$v_{m}(\zeta) \equiv J_{1}'(k_{m}\rho) h_{m}(\zeta) e^{-iv_{m}\zeta}$$
, (32)

 $\Phi_{ij...m;n} = (\phi_i \pm \phi_j \pm ... \pm \phi_m) - \phi_n$ is the combined slow phase, and the nonlinear filling factor is given by

$$s_{ij...m;n} = 2 \int_{0}^{\frac{L}{2}} dx_{g} \int_{\infty}^{\infty} dy_{g} \hat{n}_{b}(x_{g}, y_{g}) \left(1 + \frac{x_{g}^{2}}{b_{i}^{2}}\right)^{-\frac{1}{2}} \left(1 + \frac{x_{g}^{2}}{b_{j}^{2}}\right)^{-\frac{1}{2}} \dots \left(1 + \frac{x_{g}^{2}}{b_{n}^{2}}\right)^{-\frac{1}{2}}$$

$$h_{i}(y_{g}) h_{j}(y_{g}) \dots h_{n}(y_{g}) \cos k_{i} x_{g} \cos k_{j} x_{g} \dots \cos k_{n} x_{g}.$$
(33)

The results appearing in Eqs. (29)-(33) apply for any spatial profile and velocity distribution of the electron beam, as well as any radiation modal structure in the open resonator.

V. THREE MODE INTERACTION

Equations (29) are quite general, involving all possible resonant combinations of modes. In many cases it has been observed that only a small number of modes participates in the final equilibrium with finite amplitude. In this section the three mode synchronous interaction, satisfying the condition

$$v_k + v_1 \approx 2v_m$$
, or $\omega_k + \omega_1 \approx 2\omega_m$, (34)

will be considered in more detail. This is a special case of the four mode interaction, with $\omega_m = \omega_n$ in Eq. (30). Hence the frequencies ω_k , ω_l are symmetric around ω_m , $\omega_k = \omega_m - d\omega$, $\omega_l = \omega_m + d\omega$, where d ω is any frequency interval allowed by the mode spacing in the cavity. The possible resonant frequency triads entering C⁽³⁾ are

$$\omega_{\rm m} - \omega_{\rm l} - \omega_{\rm k} ,$$

$$- \omega_{\rm m} - \omega_{\rm l} + \omega_{\rm l} ,$$

$$- \omega_{\rm m} - \omega_{\rm k} + \omega_{\rm k} ,$$

$$- \omega_{\rm m} - \omega_{\rm m} + \omega_{\rm m} ,$$
(35)

and all 3! permutations of each arrangement, while the possible arrangements for $C^{(5)}$ are given, setting i = k and j = 1, by

$$- \omega_{m} - \omega_{1} + \omega_{1} - \omega_{k} + \omega_{k} ,$$

$$- \omega_{m} - \omega_{k} + \omega_{k} - \omega_{m} + \omega_{m} ,$$

$$- \omega_{m} - \omega_{m} + \omega_{m} - \omega_{m} + \omega_{m} ,$$

$$\omega_{m} - \omega_{1} - \omega_{k} + \omega_{k} - \omega_{k} ,$$

$$\omega_{m} - \omega_{1} - \omega_{k} + \omega_{1} - \omega_{1} ,$$

$$\omega_{m} - \omega_{1} - \omega_{k} + \omega_{m} - \omega_{m} ,$$
(36)

with all the 5! permutations of each.

There has been a tendency recently to employ the following renormalized parameters in gyrotron theory: amplitude $F_n = a_n / \gamma_0 \beta_{\perp 0}^3 = E_n / B_0 \beta_{0\perp}^3$, detuning $\Delta_n = (2/\beta_{\perp 0}^2) \delta \omega_n$, interaction length $\mu = k_0 V(\beta_{\perp 0}^2/2\beta_{z0})$ and current $I = I_0 (8\pi/V_0)(Q/\gamma_0 \beta_{\perp 0}^3)$ where $V_0 = (\pi/2)k_0^3LV^2$ and $I_0 = I_b(Ams) |e|/m_e c^3 \times 3 10^9$. Setting $\omega_k = \omega_1$, $\omega_m = \omega_2$, $\omega_1 = \omega_3$ and separating real and imaginary parts, Eqs. (29) are recast in the form

$$\frac{dF_{1}}{dt} + \frac{\omega_{1}}{2Q_{1}}F_{1} = Re\Gamma_{1}F_{1} + \sum_{m=1}^{3}F_{1}F_{m}^{2}ReG_{1m} + F_{2}^{2}F_{3}Re\left(G_{123}e^{-i\Phi}\right) + \sum_{l=1}^{3}\sum_{m=1}^{3}F_{1}F_{1}^{2}F_{m}^{2}ReD_{1lm} + \sum_{m=1}^{3}F_{m}^{2}F_{2}^{2}F_{3}Re\left(D_{m123}e^{-i\Phi}\right), \quad (37a)$$

$$\frac{dF_2}{dt} + \frac{\omega_2}{2Q_2}F_2 = Re\Gamma_2F_2 + \sum_{m=1}^{3}F_2F_m^2 ReG_{2m} + F_1F_2F_3 Re(G_{231}e^{i\Phi}) + \sum_{l=1}^{3}\sum_{m=1}^{3}F_2F_1^2F_m^2 ReD_{2lm} + \sum_{m=1}^{3}F_m^2F_1F_2F_3 Re(D_{m231}e^{i\Phi}), (37b)$$

$$\frac{dF_{3}}{dt} + \frac{\omega_{3}}{2Q_{3}}F_{3} = Re\Gamma_{3}F_{3} + \sum_{m=1}^{3}F_{3}F_{m}^{2}ReG_{1m} + F_{2}^{2}F_{1}Re(G_{312}e^{-i\Phi}) + \sum_{l=1}^{3}\sum_{m=1}^{3}F_{3}F_{1}^{2}F_{m}^{2}ReD_{31m} + \sum_{m=1}^{3}F_{m}^{2}F_{2}^{2}F_{1}Re(D_{m312}e^{-i\Phi}), \quad (37c)$$

where Γ is the complex linear growth rate. The evolution of the slow phase $\Phi = \phi_1 + \phi_3 - 2\phi_2$ is given by

$$\frac{d\Phi}{dt} = \delta \Omega + Im \left(\Gamma_{1} + \Gamma_{3} - 2 \Gamma_{2} \right)$$

$$- F_{1}^{2} Im \left(G_{11}^{+} G_{31}^{-} 2G_{21}^{-} \right) - F_{3}^{2} Im \left(G_{13}^{+} G_{33}^{-} 2G_{23}^{-} \right) + F_{2}^{2} Im \left(G_{22}^{-} G_{12}^{-} G_{32}^{-} \right)$$

$$- \frac{F_{3}}{F_{1}} \frac{F_{2}^{2}}{F_{1}} Im \left(G_{123}^{-} e^{-i\Phi} \right) - \frac{F_{1}}{F_{3}} \frac{F_{2}^{2}}{F_{3}} Im \left(G_{312}^{-} e^{-i\Phi} \right) + F_{1}^{-} F_{3}^{-} Im \left(G_{231}^{-} e^{--i\Phi} \right)$$

$$- \frac{3}{\Gamma_{1}} \sum_{m=1}^{3} Im \left(D_{11m}^{-} + D_{31m}^{-} - 2D_{21m}^{-} \right) F_{1}^{2} F_{m}^{-2}$$

$$- \frac{3}{\Gamma_{m}} F_{m}^{-2} \left\{ \frac{F_{3}}{F_{1}} \frac{F_{2}^{-2}}{F_{1}} Im \left(D_{m123}^{-} e^{-i\Phi} \right) + \frac{F_{1}}{F_{3}} Im \left(D_{m312}^{-} e^{-i\Phi} \right) - F_{1}^{-} F_{3}^{-} Im \left(D_{m231}^{-} e^{-i\Phi} \right) \right\},$$

$$(37d)$$

where $\delta \Omega = (\omega_1 + \omega_3 - 2\omega_2)/\omega_0$. Note that only one combined slow phase appears for each triad.

The dependence on the gyrotron parameters I, μ and Δ is contained in the coupling coefficients. It will be shown now that when $\mu >>$ $\gamma_0 \beta_{\perp 0}/2$, it can be eliminated as an independent parameter by proper scaling. The computation of Γ , G, and D, carried out in Appendix B, yields

$$\Gamma_{n} = I_{s}S_{1} \hat{\Gamma}(\nu_{n}) ,$$

$$G_{nm} = I_{s}S_{3} \xi^{2} \hat{G}(\nu_{n}, \nu_{m}) ,$$

$$D_{nm1} = I_{s}S_{5} \xi^{4} \hat{D}(\nu_{n}, \nu_{m}, \nu_{1}) ,$$
(38)

where $\xi = (k_0 W)^2 (\beta_{\perp 0}^4 / 2\beta_{z0}^2) J_1'(k_0 \rho_0)$ and $I_s = I(\beta_{\perp 0}^2 / 80) (k_0 W)^3 (\beta_{\perp 0} / \beta_{z0})^3 [J_1'(k_0 \rho_0)]^2$. The nonlinear filling factors S_1 , S_3 , and S_5 are given in (B11). The quantities Γ , G, D, given for a cold beam by the integrals (B16)-(B18), depend on the desynchronism parameter

 v_m alone, the product of the interaction length μ with the detuning Δ_m ,

$$v_{\rm m} = \mu \Delta_{\rm m} \,. \tag{39}$$

This suggests the following scaling transformation, valid when $\omega_n\simeq\omega_o$ and $Q_n\simeq Q_,$

$$I = 2 I_{s}Q$$

$$\hat{F} = \xi F, \qquad (40)$$

$$\tau = t/2Q,$$

putting the mode coupling equations in the final reduced form

$$\frac{d\tilde{F}_{1}}{d\tau} + \tilde{F}_{1} = i\left\{S_{1}Re\tilde{\Gamma}_{1}\tilde{F}_{1} + S_{3}\sum_{m=1}^{3}\tilde{F}_{1}\tilde{F}_{m}^{2}Re\tilde{G}_{1m} + S_{3}\tilde{F}_{2}^{2}\tilde{F}_{3}Re\left(\tilde{G}_{123}e^{-i\Phi}\right)\right\}, \quad (41a)$$

$$+ S_{5}\sum_{l=1}^{3}\sum_{m=1}^{3}\tilde{F}_{1}\tilde{F}_{1}^{2}\tilde{F}_{m}^{2}Re\tilde{D}_{11m} + S_{5}\sum_{m=1}^{3}\tilde{F}_{m}^{2}\tilde{F}_{2}^{2}\tilde{F}_{3}Re\left(\tilde{D}_{m123}e^{-i\Phi}\right)\right\}, \quad (41a)$$

$$\frac{d\tilde{F}_{2}}{d\tau} + \tilde{F}_{2} = i\left\{S_{1}Re\tilde{\Gamma}_{2}\tilde{F}_{2} + S_{3}\sum_{m=1}^{3}\tilde{F}_{2}\tilde{F}_{m}^{2}Re\tilde{G}_{2m} + S_{3}\tilde{F}_{1}\tilde{F}_{2}\tilde{F}_{3}Re\left(\tilde{G}_{123}e^{-i\Phi}\right)\right\}, \quad (41a)$$

$$+ S_{5}\sum_{l=1}^{3}\sum_{m=1}^{3}\tilde{F}_{2}\tilde{F}_{1}^{2}\tilde{F}_{m}^{2}Re\tilde{D}_{21m} + S_{5}\sum_{m=1}^{3}\tilde{F}_{m}^{2}\tilde{F}_{1}\tilde{F}_{2}\tilde{F}_{3}Re\left(\tilde{D}_{m231}e^{-i\Phi}\right)\right\}, \quad (41b)$$

$$\frac{d\tilde{F}_{3}}{d\tau} + \tilde{F}_{3} = i\left\{S_{1}Re\tilde{\Gamma}_{3}\tilde{F}_{3} + S_{3}\sum_{m=1}^{3}\tilde{F}_{3}\tilde{F}_{m}^{2}Re\tilde{G}_{3m} + S_{3}\tilde{F}_{2}^{2}\tilde{F}_{1}Re\left(\tilde{G}_{312}e^{-i\Phi}\right)\right\}, \quad (41b)$$

$$+ S_{5}\sum_{l=1}^{3}\sum_{m=1}^{3}\tilde{F}_{3}\tilde{F}_{1}^{2}\tilde{F}_{m}^{2}Re\tilde{D}_{31m} + S_{5}\sum_{m=1}^{3}\tilde{F}_{m}^{2}\tilde{F}_{2}^{2}\tilde{F}_{1}Re\left(\tilde{D}_{m312}e^{-i\Phi}\right)\right\}. \quad (41c)$$

It follows, from (41), that the normalized start-up current is $I_n = 1/s_1 \hat{\Gamma}_n$.

The beam parameters I, β_{zo} , β_{jo} the interaction length μ , the cavity Q and the three frequencies combine into only 3 control parameters: \hat{I} , v_1 and v_2 (v_3 is related to the other two frequencies through the resonant condition). The interaction length μ does not appear explicitly, but only implicitly through \hat{I} and v. For simple density profiles, such as a pencil or a thin annular beam, the filling factors may also be expressed as powers of a single factor s,

$$S_{j} = s^{j+1}$$
, $s = \cos k_{0} \bar{x}_{g} h(\bar{y}_{g})$, (42)

with \bar{x}_g and \bar{y}_g the effective values of x_g and y_g . Thus, they could also be absorbed in the scaling factor, redefining

$$\xi' = s \xi,$$

$$\hat{I}' = s^2 \hat{I}.$$
 (43)

Since I is given analytically in terms of the gyrotron parameters, it suffices to compute tables of Γ , G and D as functions of v_m once, to completely know the coupling coefficients for any combination of the parameters I, μ , β_{zo} , β_{lo} and frequencies ω_m . Note that the beam spatial profile enters only in the nonlinear filling factor S_j ; in case of complicated beam profiles S_j can be computed independently from Eq. (33). The discussion so far has been limited to a cold beam. In case of thermal effects the general expressions (A7) and (A8) are applicable and the control space will increase by two parameters, the pitch angle spread and the energy spread.

Equations (37) and their reduced form, Eqs. (41), are the basic result in this paper, and describe three mode interaction up to fifth

order in magnitude in the QOG. The diagonal terms G_{nn} describe the effects of nonlinear saturation (or self excitation) for each mode. The cross-coupling terms G_{nm} describe a phase-independe. nonlinear damping (positive or negative) among different modes. The interaction through the G_{nml} terms does involve the slow phase and describes mode locking effects. The fifth order terms D_{nml} are necessary to stabilize the system in regimes where most of the third order interactions are positive and mutually destabilizing. They are also required to account for potential amplitude bistability,¹⁵ i.e., the existence of two equilibria of the same frequency but of different amplitude.

Similar results, to third order in amplitude, have been obtained for the conventional gyrotron¹¹, using a somewhat different approach. The present derivation of the coefficients is considerably simpler, since it requires a (j+1)-fold integration for the j-th order, compared to the (2j+1)-fold integration required in the single particle approach.¹¹ Mode coupling equations, identical in structure, first appeared in the treatment of LASER cavities¹⁶.

VI. RESULTS AND CONCLUSIONS

The normalized linear growth rate $\operatorname{ReF}(v_n)$ is given in Fig. 2. Tables of the third order coupling coefficients $\widehat{G}(v_n, v_m)$ for Gaussian profile modes and a cold pencil electron beam have been obtained by numerical evaluation of the expressions (B5)-(B7). The contour plots for the real and imaginary parts of \widehat{G}_{nm} , \widehat{G}_{123} and \widehat{G}_{231} within the regime $0 < v_n$, $v_m < 3$ are shown in Figs. 3a, 3b, and 3c respectively (the coefficients \widehat{G}_{312} are given in terms of \widehat{G}_{123} , $\widehat{G}_{312}(v_1, v_2) = \widehat{G}_{123}(v_3, v_2)$). Dotted lines represent regions of negative values for \widehat{G} , corresponding to mutually stabilizing influence among interacting modes in that region. The mode coupling is destabilizing in the regions with solid lines, corresponding to positive \widehat{G} . Note the absence of symmetry in the coupling coefficients, where, in general, $G_{nm} \neq G_{mn}$. Contour plots for the fifth order coupling coefficients $\widehat{D}(v_n, v_m, v_1)$ as functions of v_m , v_1 for selected values of v_n appear in Figs. 4(a)-4(d).

According to the final number of participating equilibrium modes, Eqs. (41) demonstrate three types of equilibria: (a) single mode, where one mode dominates the other two (b) two mode equilibria, where one mode is of negligible amplitude and (c) equilibria among three modes of comparable amplitudes. The complete set of equilibria among a given set of frequencies is given by the zeros of the right-hand side of (42). Since F_n are defined positive, only solutions with F_1 , F_2 , $F_3 > 0$ are acceptable. It is well known that every stable equilibrium of a nonlinear dissipative system, such as the system represented by Eqs. (41), is associated with a region of initial conditions in amplitude space (basin of attraction) that eventually fall into this equilibrium. Which equilibrium the system will choose to settle in cannot be analytically

predicted, in the absence of invariants of the motion. This can only be done by integration of the set of ordinary differential equations (41).

The simplest case, a single mode equilibrium for the n-th mode, is found from

$$\frac{dF_n}{dt} = F_n \left(Re\Gamma_n + ReG_{nn} F_n^2 + ReD_{nnn} F_n^4 \right) = 0 , \qquad (44a)$$

$$\frac{d\phi_n}{dt} = Im\Gamma_n - F_n^2 \left(ImG_{nn} + ImD_{nnn} F_n^2 \right).$$
(44b)

Equations (44) are obtained by dropping the cross-coupling terms from (37); the combined phase Φ is irrelevant in that case. Acceptable, non-trivial solutions $F_n > 0$ for the steady-state amplitude should always exist, since a single mode saturates through particle trapping. The fifth order saturation term D_{nnn} is therefore important on two counts. First, it is necessary for saturation in regimes where the third order G_{nn} is positive and causes self-excitation instead of suppression. Second, it is required to account for amplitude bistability, when two acceptable equilibrium values F_{n1} and F_{n2} exist for a given mode, since the third order does nonzero solutions, $F_n = \pm (\Gamma_n/G_{nn})^{1/2}$, can provide at most one acceptable solution F > 0. At steady-state the slow phase ϕ_n varies at a constant rate, Eq. (44b); $d\phi_n/dt$ expresses the nonlinear frequency shift from the linear frequency ω_n .

Equations (37) are used to provide some examples of mode competition in a QOG driven by a cold pencil beam in a typical parameter regime, with beam current $I_b = 13$ A, $\gamma_o = 1.146$, $\beta_{\perp o}/\beta_{zo} = 1$, $W/\lambda = 5$, cavity Q = 40,000 and cavity length L = 48cm, corresponding to a frequency separation $d\omega/\omega_o = 0.003$. In Figs. 4(a)-4(c) we examine mode interaction

among the first three frequencies $\omega_1 < \omega_2 < \omega_3$ above ω_0 , counting only the odd resonator modes ω_n separated by $d\omega/\omega_n = 0.006$. In Fig. 5(a) initially $F_{20} >> F_{10}$, F_{30} . The mode F_2 quickly reaches a quasi-steady state, which, however, is unstable. The mode F_3 , which initially grows much slowly than its uncoupled linear rate due to nonlinear suppression from F_2 , eventually overtakes and suppresses F₂ (mode switching in the cavity). In contrast, the steady state reached from $F_{10} >> F_{20}$, F_{30} , shown in Fig. 5(b), is stable since a large amplitude F_1 suppresses F_2 and F_3 to negligible amplitude. However, the single mode equilibrium of F_1 is not accessible from the usual start-up initial conditions, where all three modes have small comparable amplitudes $F_{10} \approx F_{20} \approx F_{30} \ll 1$. Figure 5(c) shows that the final state in this case has F_3 as the dominant mode. Note that F_2 has been completely suppressed in Fig. 5(c), although its linear growth rate differs from F_2 by a few percent. Since F_3 wins the competition with F_1 and F_2 , we next examine the coupling of F_3 with the next two frequencies F_4 and F_5 . It is seen from Fig. 6 that the final steady state involves three modes with comparable amplitudes. Note that F5 has been excited despite being linearly stable. Figure 7 shows the mode competition among the first three modes in a cavity of reduced length, L = 24cm, and increased frequency separation $d\omega/\omega_0 = 0.012$ among odd modes.

In conclusion, the general analytic formalism for multimode interaction in the QOG has been developed in this paper. Applications of the coupling equations to interactions involving three main modes have demonstrated the ability to model nonlinear effects, such as nonlinear destabilization, suppression, and mode switching. A new scaling has been introduced, during the derivation of the coupling equations, that reduces the number of the control parameters. The ratio of operation to start-up current \hat{I} , and the desynchronism parameters v_i for the participating modes

become the only free parameters. The inclusion of coupling terms up to fifth order in amplitude was shown to stabilize the system in the hard excitation regime (where the third order coupling coefficients are positive.) The coupling coefficients up to fifth order have been tabulated for a cold pencil beam. These results will be generalized in a future paper to include thermal spreads in velocity, and arbitrary beam current profiles. The set of equations will also be expanded to include interactions among a large number of modes, by forming all the possible resonant frequency combinations.

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Appendix A: Computation of the Nonlinear Current

Substituting operator (28) inside (27) and performing the successive iterations yields

$$f^{(1)}(u_{\perp},\theta,\zeta;t) = \frac{kW}{2i} \sum_{n} \tilde{a}_{n} e^{i(\psi_{n} + v_{n}\zeta)} \int_{-\infty}^{\zeta} d\zeta_{1} \chi_{-n}(\zeta_{1},u_{\perp}) \frac{\partial f^{(0)}}{\partial u_{\perp}} + cc, \quad (A1)$$

$$f^{(2)}(u_{\perp},\theta,\zeta;t) = \left(\frac{kV}{2i}\right)^{2} \sum_{n} \sum_{m} \tilde{a}_{n} \tilde{a}_{m}^{*} \left\{ e^{i(\Psi_{n}-\Psi_{m})} e^{i(\Psi_{n}-\Psi_{m})\zeta} \int_{-\infty}^{\zeta} d\zeta_{1} \chi_{-n} \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_{1}} d\zeta_{2} \chi_{m} \right.$$

$$\left. + \left[n, m \right] \right]$$

$$\left. + e^{i(\Psi_{n}-\Psi_{m})} e^{i(\Psi_{n}-\Psi_{m})\zeta} \int_{-\infty}^{\zeta} d\zeta_{1} \chi_{-n} \frac{1}{u_{\perp}} \int_{-\infty}^{\zeta_{1}} d\zeta_{2} \tilde{\chi}_{m} \right.$$

$$\left. + \left[n, m \right] \right\} \frac{\partial f^{(0)}}{\partial u_{\perp}} + cc , \qquad (A2)$$

and

$$\begin{aligned} f^{(3)}(u_{1},\theta,\zeta,t) &= \left(\frac{kv}{2i}\right)^{3} \sum_{n} \sum_{m} \sum_{m} \sum_{n} \sum_{m} \sum_{n} a_{n} a_{m}^{*} a_{1}^{*} \right\{ \\ e^{i(\psi_{n}^{-}\psi_{m}^{-}\psi_{1}^{-})} e^{i(\psi_{n}^{-}\psi_{m}^{-}\psi_{1}^{-})\zeta} \int_{-\infty}^{\zeta} d\zeta_{1} \chi_{-n} \frac{\partial}{\partial u_{1}} \int_{-\infty}^{\zeta_{1}} d\zeta_{2} \chi_{m} \frac{\partial}{\partial u_{1}} \int_{-\infty}^{\zeta_{2}} d\zeta_{3} \chi_{1} \\ &+ \left[-n, m, -1 \right] + \left[-n, -m, 1 \right] + \left[-n, -m, -1 \right] + \\ e^{i(\psi_{n}^{-}\psi_{m}^{-}\psi_{1}^{-})} e^{i(\psi_{n}^{-}\psi_{m}^{-}\psi_{1}^{-})\zeta} \int_{-\infty}^{\zeta} d\zeta_{1} \chi_{-n} \frac{\partial}{\partial u_{1}} \int_{-\infty}^{\zeta_{1}} d\zeta_{2} \chi_{-m} \frac{1}{u_{1}} \int_{-\infty}^{\zeta_{2}} d\zeta_{3} \dot{\chi}_{1} \\ &+ \left[-n, m, -1 \right] + \left[-n, -m, 1 \right] + \left[-n, -m, -1 \right] + \\ e^{i(\psi_{n}^{-}\psi_{m}^{-}\psi_{1}^{-})} e^{i(\psi_{n}^{-}\psi_{m}^{-}\psi_{1}^{-})\zeta} \int_{-\infty}^{\zeta} d\zeta_{1} \chi_{-n} \frac{1}{u_{1}} \int_{-\infty}^{\zeta_{1}} d\zeta_{2} \dot{\chi}_{m} \frac{\partial}{\partial u_{1}} \int_{-\infty}^{\zeta_{2}} d\zeta_{3} \chi_{1} \\ &+ \left[-n, -m, -1 \right] \end{aligned}$$

+
$$\left[-n, -m, -1\right]$$
 $\left\{\frac{\partial f^{(0)}}{\partial u_{\perp}} + cc, \right\}$ (A3)

where the explicit $\theta,$ t dependence is contained in ψ_n = θ - $\omega_n t$,

$$\begin{split} \chi_{n}(\zeta,u_{\perp}) &= \frac{\Upsilon}{u_{zo}} J_{1}' \left(\frac{k_{n}u_{\perp}}{\Omega_{o}} \right) \cos k_{n} x_{g} h_{n}(\zeta) e^{iv_{n}\zeta} , \qquad \chi_{-n}(\zeta,u_{\perp}) \equiv \chi_{n}(\zeta,u_{\perp})^{\star}, , \\ \vdots \\ \chi_{n}(\zeta,u_{\perp}) &= \chi_{n}(\zeta,u_{\perp}) J_{1} \left(\frac{k_{n}u_{\perp}}{\Omega_{o}} \right) / J_{1}' \left(\frac{k_{n}u_{\perp}}{\Omega_{o}} \right) \left(\frac{k_{n}u_{\perp}}{\Omega_{o}} \right) , \qquad \chi_{-n}(\zeta,u_{\perp}) \equiv \chi_{n}(\zeta,u_{\perp})^{\star}. \end{split}$$

In obtaining (A1)-(A3) we used the invariance: $\psi_n + \upsilon_n \zeta = \text{const.}$, along the unperturbed trajectories. The notation [-n, m, -1] inside (A2)-(A3)

means an integral similar to the preceding fully written term except that a negative index -n in place of n implies $-\psi_n$, $-\nu_n$, $\chi_n \stackrel{\sim}{a_n}^{\star}$ in place of ψ_n , ν_n , χ_{-n} , $\stackrel{\sim}{a}_n$, etc.

The dominant contribution in each integral comes from the derivative $\partial/\partial u_1$ acting on v_m ,

$$\frac{\partial v_{m}}{\partial u_{\perp}} = \frac{\omega_{m}}{\omega_{o}} \eta(u_{\perp}), \text{ where } \eta(u_{\perp}) = \frac{k_{o}Wu_{\perp}}{\gamma u_{z}} = \frac{2\pi W}{\lambda_{\gamma}} \frac{u_{\perp}}{u_{z}} >> 1. \quad (A4)$$

Therefore, the most important terms inside $f^{(j)}$ are those with the $(\partial/\partial u_{\downarrow})^{j}$ derivative. Neglecting, for the same reason, terms like $\partial \gamma/\partial u_{\downarrow} \sim \partial u_{\downarrow}^{k}/\partial u_{\downarrow} \sim 1 \ll \partial v_{m}/\partial u_{\downarrow}$ one obtains from (A3)

$$f^{(3)}(u_{\perp},\theta,\zeta,t) = \left(\frac{k\Psi}{2i}\right)^{3} \left(\frac{\gamma}{u_{zo}}\right)^{3} \sum_{n} \sum_{m} \sum_{m} \sum_{n} s_{n} s_{n} s_{n} \sum_{n} a_{n} a_{m}^{*} a_{1}^{*}$$

$$e^{i(\Psi_{n} - \Psi_{m} - \Psi_{1})} e^{i(\Psi_{n} - \Psi_{m} - \Psi_{1})\zeta} \int_{-\infty}^{\zeta} d\zeta_{1} \Psi_{-n} \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_{1}} d\zeta_{2} \Psi_{m} \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_{2}} d\zeta_{3} \Psi_{1} \frac{\partial f^{(0)}}{\partial u_{\perp}}$$

$$+ \left[-n, m, -1\right] + \left[-n, -m, 1\right] + \left[-n, -m, -1\right] + cc, \quad (A5)$$
where $\Psi_{m}(\zeta) = J_{1}'\left(\frac{k_{n}u_{\perp}}{\varphi_{0}}\right) h_{m}(\zeta) e^{i\Psi_{m}\zeta}, \quad s_{m} = \cos k_{m}x_{g} h_{m}(\Psi_{g}).$

One can show by induction, proceeding along the same lines, that

$$f^{(j)}(u_{\perp},\theta,\zeta,t) = \left(\frac{kW}{2i}\right)^{j} \left(\frac{\gamma}{u_{zo}}\right)^{j} \sum_{m_{1}} \sum_{m_{2}} \cdots \sum_{m_{j}} s_{m_{1}} s_{m_{2}} \cdots s_{m_{j}} \tilde{a}_{m_{1}} \tilde{a}_{m_{2}}^{\star} \tilde{a}_{m_{3}} \cdots \tilde{a}_{m_{j}}^{\star}$$

$$e^{i(\psi_{m_{1}} - \psi_{m_{2}} + \psi_{m_{3}} - \cdots - \psi_{m_{j}})} e^{i(\psi_{m_{1}} - \psi_{m_{2}} + \psi_{m_{3}} - \cdots - \psi_{m_{j}})\zeta}$$

$$\int_{-\infty}^{\zeta} d\zeta_{1} v_{-m_{1}} \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_{1}} d\zeta_{2} v_{m_{2}} \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_{2}} d\zeta_{3} v_{-m_{3}} \cdots \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_{j-1}} d\zeta_{j} v_{m_{j}} \frac{\partial f^{(0)}}{\partial u_{\perp}}$$

$$+ \left\{ m_{1}, -m_{2}, m_{3}, \cdots, -m_{j} \right\} + cc , \qquad (A6)$$

where $\{ \dots \}$ implies all the permutations among the m_j' s.

When the j-th nonlinear piece $f^{(j)}$ of f_{NL} is substituted inside (15) and combined with $\cos\theta = (1/2)(e^{i\theta} + e^{-i\theta})$, the resulting expression contains even θ -harmonics, 1, $e^{\pm 2i\theta}$, ..., $e^{\pm (j+1)i\theta}$, for j odd, and odd θ harmonics, $e^{\pm i\theta}$, $e^{\pm 3i\theta}$, ..., $e^{\pm (j+1)i\theta}$, for j even. Consequently, coupling interactions involving even number of modes vanish completely during the θ integration. The first nonlinear correction is of third order in amplitude, the second of fifth, and so on. It should be emphasized that this is a consequence of assuming a uniform in θ initial distribution $f^{(0)}(u)$; however, for a prebunched distribution of the form $f^{(0)}(u,\theta) = \tilde{f}_0(u) + \tilde{f}_1(u)$ $e^{i\theta} + \ldots + cc$, both even and odd terms survive and the nonlinear effects enter to second order in amplitude. Splitting the slowly varying $f_{NL}(\omega_n)$ component off the nonlinear distribution, by multiplying each $f^{(j)}$ with $\exp(i\omega_n t)$ and averaging over the fast time, substituting $f_{NL}(\omega_n)$ inside (19), and finally averaging over the gyroangle θ and over the resonator volume, one obtains

$$\langle j_{NL}(\omega_{n}) \rangle = \frac{I_{o}}{\sigma_{b} V_{n}} \frac{v_{o}}{u_{zo}} \{m_{1}, m_{2}, m_{3}, \dots, m_{j}; n\}$$

$$\left(C_{m_{1};n}^{(1)} \tilde{a}_{m_{j}}^{+} C_{m_{1},m_{2},m_{3};n}^{(3)} \tilde{a}_{m_{1}}^{-} \tilde{a}_{m_{2}}^{-} \tilde{a}_{m_{3}}^{-} + \dots + C_{m_{1},m_{2},m_{3},\dots,m_{j};n}^{(j)} \tilde{a}_{m_{1}}^{-} \tilde{a}_{m_{2}}^{-} \tilde{a}_{m_{j}}^{-} \right),$$

$$(A7)$$

$$C_{m_{1}\dots m_{j};n}^{(j)} = \left(\frac{i}{2} \right)^{j} (kW)^{j+1} S_{m_{1}\dots m_{j};n} \exp \left[i \left(\Delta \omega_{m_{1}\dots m_{j};n}^{-} t + \Phi_{m_{1}\dots m_{j};n}^{-} t \right) \right] \int_{0}^{\infty} du_{\perp} \frac{u_{\perp}^{2}}{v} \left(\frac{v}{u_{zo}} \right)^{j}$$

$$\int_{-\infty}^{\infty} dz \, v_{n}(z) \int_{-\infty}^{z} d\zeta_{1} \, v_{m}(\zeta_{1}) \, \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_{1}} d\zeta_{2} \, v_{m}(\zeta_{2}) \, \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_{2}} \cdots \, \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_{j-1}} d\zeta_{j} v_{m}(\zeta_{j}) \, \frac{df^{(0)}}{du_{\perp}}$$
(A8)

The quantities $\Delta \omega_{m_1 m_2 \cdots m_j;n}$, v_{m_j} , $m_{m_1 m_2 \cdots m_j;n}$ and $S_{m_1 m_2 \cdots m_j;n}$ are defined in a similar manner as in equations (30), (32) and (33) respectively. Of all the possible combinations $[m_1 \pm m_2 \pm m_3 \pm \cdots \pm m_j]$ only those with (j-1)/2 positive and (j+1)/2 negative signs have survived the Θ -integration and contribute to $C^{(j)}$; for example, the term [-1, -m, -n] in $f^{(3)}$, Eq. (A5), has vanished from $C^{(3)}$. Also note that the complex conjugate terms in (A1)-(A6) drop out during the fast time averaging; they contribute to the $j_{NL}(-\omega_n)$ component. Appendix B: Computation of the Three-Mode Coupling Coefficients

The three-mode coupling coefficients in Eqs. (37) are computed from the general expressions in (31), keeping the frequency combinations (35) and (36), and yielding,

$$\Gamma_{n} = K C_{-n;n}^{(1)}, \quad G_{nm} = K \sum_{\{-m,m,-n\}} C_{-mm-n;n}^{(3)},$$

$$G_{123} = K \sum_{\{-2,-2,3\}} C_{-2-23;1}^{(3)}, \quad G_{312} = K \sum_{\{-2,-2,1\}} C_{-2-21;3}^{(3)}, \quad G_{231} = K \sum_{\{2,-3,-1\}} C_{2-3-1;2}^{(3)},$$

$$D_{m123} = K \sum_{\{-m,m,-2,-2,3\}} C_{-mm-2-23;1}^{(5)}, \quad D_{nlm} = K \sum_{\{-m,m,-1,1,-n\}} C_{-mm-1l-n;n}^{(5)},$$
where $K = 2\pi i \frac{\gamma_{o}}{u_{zo}} \frac{I}{\sigma_{b} V}$. (B1)

A cold pencil beam is employed for simplicity, where the velocity distribution $f^{(0)}$ and the density profile $n(x_g, y_g)$ are given by

$$f^{(0)} = \frac{1}{2\pi u_{\perp 0}} \, \delta(u_{\perp} - u_{\perp 0}) \, \delta(u_{z} - u_{z0}) \, , \qquad \hat{n}(x_{g}, y_{g}) = \, \delta(x_{g} - \frac{L}{2}) \, \delta(y_{g}) \, .$$

The calculations are simplified by the following observation involving the derivatives of $\delta(u_{\perp}-u_{\perp 0})$. If A(u), B(u) are any functions of u satisfying $d^{n}A/du^{n} \ll d^{n}B/du^{n}$ for every n, then

$$\int du A B \frac{d\delta}{du} = - \int du \left(\frac{dA}{du} B + \frac{dB}{du} A \right) \delta \simeq - A \frac{dB}{du} \Big|_{U_0} = A(U_0) \int du B \frac{d\delta}{du} ,$$

and by induction

$$\int du \ A \ B \ \frac{d^n \delta}{du^n} \simeq A(u_0) \ \int du \ B \ \frac{d^n \delta}{du^n}$$

One can see that
$$\frac{d^n}{du_{\perp}^n} \left(\gamma, u_{\perp}^k, J(k_n \rho) \right) << \left| \frac{d^n}{du_{\perp}^n} e^{i\nu_n \zeta} \right| \sim \eta_0^n$$
, where

$$\eta_{o} = \eta(u_{\perp o}) = \frac{k_{o}^{W}}{\gamma_{o}} \frac{\beta_{\perp o}}{\beta_{zo}} = \frac{2 \mu}{\gamma_{o} \beta_{\perp o}} >> 1 .$$
(B2)

Therefore, moving everything but the exponentials $\exp(i\nu_n\zeta)$ in front of the velocity integrals, one obtains

$$\Gamma_{n} = \frac{1}{2} \frac{2\pi I_{o}}{\sigma_{b} V_{n}} \frac{u_{\perp o}}{u_{zo}} \left(\frac{\gamma_{o}}{u_{zo}}\right) \left(k_{o} W\right)^{2} J_{no}^{2} s_{1} \tilde{\Gamma}_{n} , \qquad (B3)$$

$$S_{nm} = -\left(\frac{1}{2}\right)^{3} \frac{2\pi I_{o}}{\sigma_{b} V_{n}} \frac{u_{10}}{u_{20}} \left(\frac{\gamma_{o}}{u_{20}}\right)^{3} \left(k_{o} V\right)^{4} (J_{no} J_{mo})^{2} S_{3} \tilde{G}_{nm}$$
(B4)

$$D_{nlm} = \left(\frac{1}{2}\right)^{5} \frac{2\pi I_{o}}{\sigma_{b} V_{n}} \frac{u_{\perp o}}{u_{zo}} \left(\frac{\gamma_{o}}{u_{zo}}\right)^{5} \left(k_{o} V\right)^{6} \left(J_{no} J_{1o} J_{mo}\right)^{2} S_{5} \tilde{D}_{nlm} , \qquad (B5)$$

where $J_{no} = J_1'(k_n \rho_0)$, $\rho_0 = u_{\perp 0}/\Omega_0$ and

$$\tilde{\Gamma}_{n} = \int_{0}^{\infty} du_{\perp} \int_{-\infty}^{\infty} d\zeta \, v_{n} \int_{-\infty}^{\zeta} d\zeta_{1} \, v_{-n} \, \frac{\partial}{\partial u_{\perp}} \delta\left(u_{\perp} - u_{\perp 0}\right) , \qquad (B6)$$

$$\tilde{G}_{nm} = \sum_{\{-n,m,-m\}} \int_{0}^{\infty} du_{\perp} \int_{-\infty}^{\infty} d\zeta \, \tilde{v}_{n} \int_{-\infty}^{\zeta} d\zeta_{1} \, \tilde{v}_{-n} \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_{1}} d\zeta_{2} \, \tilde{v}_{m} \, \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_{2}} d\zeta_{3} \, \tilde{v}_{-m}$$

$$\times \frac{\partial}{\partial u_{1}} \delta \left(u_{\perp} - u_{\perp 0} \right) , \qquad (B7)$$

$$\tilde{D}_{nlm} = \sum_{\{n,-1,1,-m,m\}} \int_{-\infty}^{\infty} d\zeta \, \hat{v}_n \int_{-\infty}^{\zeta} d\zeta_1 \, \hat{v}_{-n} \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_1} d\zeta_2 \, \hat{v}_1 \, \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_2} d\zeta_3 \, \hat{v}_{-1}$$

$$\frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_3} d\zeta_4 \, \hat{v}_n \, \frac{\partial}{\partial u_{\perp}} \int_{-\infty}^{\zeta_4} d\zeta_5 \, \hat{v}_{-m} \, \frac{\partial}{\partial u_{\perp}} \delta\left(u_{\perp} - u_{\perp 0}\right) , \qquad (B8)$$

where

$$v_n(\zeta) = e^{iv_n\zeta} h_n(\zeta)$$

We limit the mode coupling among fundamental Gaussian modes where the transverse profiles u_{00} are

$$u_{00}(y_g,z) = h_0(y_g) h_0(z)$$
, $h_0(z) = \exp\left(-\frac{z^2}{y^2}\right)$, (B9)

for all frequencies ω_n . The normalized volume in that case is given by

$$V_{n} = 2 \int_{0}^{\frac{L}{2}} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dz \frac{\cos^{2} k_{n} x}{\left(1 + \frac{x^{2}}{b_{n}^{2}}\right)^{1/2}} \exp\left(-2\frac{y^{2} + z^{2}}{w(x)^{2}}\right) = \frac{\pi}{2} k_{n} k_{0}^{2} L W^{2}, \quad (B10)$$

$$\sigma_{b} = 1 \quad \text{and} \quad \dot{v}_{n} = e^{-\zeta^{2}} e^{iv_{n}\zeta}.$$

For the pencil beam at $x_g = L/2$, and according to Eq. (4), $\cos k_n x_g = 0$ for n even and $\cos k_n x_g = 1$ for n odd. In other words, the electron beam passes through a null of the electric field for modes with an even number of half wavelengths between the mirrors, or through a maximum, for an odd number of half wavelengths. The j-th order nonlinear filling factor S_j, Eq. (33), is then reduced to

$$S_{j} = S_{m_{1}m_{2}\cdots m_{j}}; n^{=2} \int_{0}^{L/2} dx_{g} \int_{-\infty}^{\infty} dy_{g} \hat{n}(x_{g}, y_{g}) \left(1 + \frac{x_{g}^{2}}{b_{o}^{2}}\right)^{-\frac{j+1}{2}} e^{-\frac{(j+1)y_{g}^{2}}{y^{2}}}. (B11)$$

Since the frequency separation among the coupled frequencies ω_n is very small, it is important only when the detuning $\delta\omega_n$, ν_n appears explicitly. Thus, one may set $k_n \simeq k_0$, letting $J_1'(k_n \rho) \simeq J_1'(k_0 \rho) \equiv J_0$, $V_n \simeq V = (\pi/2)k_0^3 LW^2$ and $Q_n = Q$ for all modes. To further simplify (B6)-(B10), one may notice that, according to (A4) and for $\omega_n \simeq \omega_0$,

$$\frac{dv_n}{du_\perp} \simeq \eta(u_\perp) = \frac{k\Psi}{\gamma} \frac{u_\perp}{u_z}$$

is independent of frequency. Since the δ -function causes every derivative to be taken at $u_{\perp} = u_{\perp 0}$, one may employ v_n , v_m , v_{\perp} as independent variables and use

$$\frac{\partial}{\partial u_{\perp}} = \eta_0 \left(\frac{\partial}{\partial v_n} + \frac{\partial}{\partial v_m} + \frac{\partial}{\partial v_{\perp}} \right) \equiv \hat{M} , \qquad (B12)$$

to show that

$$\tilde{\Gamma}_{n} = \eta_{0} \hat{\Gamma}(\nu_{n}) , \qquad (B13)$$

$$\tilde{G}_{nm} = \eta_0^3 \tilde{G}(\nu_n, \nu_m) , \qquad (B14)$$

$$\tilde{\tilde{D}}_{nlm} = \eta_0^5 \tilde{D}(\nu_n, \nu_l, \nu_m) , \qquad (B15)$$

where

$$\hat{\Gamma}(\nu_{n}) = \int_{-\infty}^{\infty} d\nu_{n} \int_{-\infty}^{\infty} d\zeta \, \hat{\nu}_{n} \int_{-\infty}^{\zeta} d\zeta_{1} \, \hat{\nu}_{-n} \, \hat{H} \, \delta\left(\nu_{n} - \nu_{no}\right) = \frac{\partial}{\partial \nu_{no}} \frac{\pi}{2} e^{-\frac{\nu_{no}^{2}}{2}}$$

$$= \frac{\pi}{2} \nu_{no} e^{-\frac{\nu_{no}^{2}}{2}}, \qquad (B16)$$

$$\hat{G}(\nu_{n},\nu_{m}) = \sum_{\{-n,m,-m\}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\nu_{n} d\nu_{m} \int_{-\infty}^{\infty} d\zeta \, \hat{\nu}_{n} \int_{-\infty}^{\zeta} d\zeta_{1} \, \hat{\nu}_{-n} \, \hat{H} \int_{-\infty}^{\zeta_{1}} d\zeta_{2} \, \hat{\nu}_{m} \, \hat{H} \int_{-\infty}^{\zeta_{2}} d\zeta_{3} \, \hat{\nu}_{-m}$$

$$\hat{H} \, \delta(\nu_{n} - \nu_{no}) \, \delta(\nu_{m} - \nu_{mo}) , \qquad (B17)$$

$$\hat{D}(v_{n}, v_{1}, v_{m}) =$$

$$\sum_{\{n, -1, 1, -m, m\}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dv_{n} dv_{1} dv_{m} \int_{-\infty}^{\infty} d\zeta \, \hat{v}_{n} \int_{-\infty}^{\zeta} d\zeta_{1} \, \hat{v}_{-n} \hat{M} \int_{-\infty}^{\zeta_{1}} d\zeta_{2} \, \hat{v}_{1} \, \hat{M} \int_{-\infty}^{\zeta_{2}} d\zeta_{3} \, \hat{v}_{-1}$$

$$\hat{\mathbf{M}} \int_{-\infty}^{\zeta_3} d\zeta_4 \, \hat{\mathbf{v}}_m \, \hat{\mathbf{M}} \, \int_{-\infty}^{\zeta_4} d\zeta_5 \, \hat{\mathbf{v}}_{-m} \, \hat{\mathbf{M}} \, \delta\left(\nu_1 - \nu_{10}\right) \, \delta\left(\nu_n - \nu_{n0}\right) \, \delta\left(\nu_m - \nu_{m0}\right). \tag{B18}$$

It is clear from (B16)-(B18) that $\hat{\Gamma}$, \hat{G} and \hat{D} depend only on the initial values for the detuning parameters ν_{no} . Combining (B13)-(B15) with (B16)-(B18), converting the amplitudes a_n into the normalized F_n , $a_n = F_n (\gamma_0 \beta_{\perp 0}^{-3})$, and absorbing the factors $(\gamma_0 \beta_{\perp 0}^{-3})$ in the coupling coefficients, finally yields

$$\Gamma_{n} = \frac{2\pi I_{o}}{\sigma_{b} V_{n}} \frac{\beta_{\perp o}}{\beta_{zo}} \left(\frac{1}{2\beta_{zo}}\right) \eta_{o} \left(k_{o} W\right)^{2} J_{o}^{2} S_{1} \hat{\Gamma}(v_{n})$$
(B19)

$$G_{nm} = \frac{2\pi I_{o}}{\sigma_{b} V_{n}} \frac{\beta_{\perp o}}{\beta_{zo}} \left(\frac{1}{2\beta_{zo}}\right)^{3} \left(\gamma_{o} \beta_{\perp o}^{3}\right)^{2} \eta_{o}^{3} \left(k_{o} W\right)^{4} J_{o}^{4} S_{3} \hat{G}(\nu_{n}, \nu_{m}) , \quad (B20)$$

$$D_{nlm} = \frac{2\pi I_{o}}{\sigma_{b} V_{n}} \frac{\beta_{lo}}{\beta_{zo}} \left(\frac{1}{2\beta_{zo}}\right)^{5} \left(\gamma_{o} \beta_{lo}^{3}\right)^{4} \eta_{o}^{5} \left(k_{o} W\right)^{6} J_{o}^{6} S_{5} \hat{D}(\nu_{n}, \nu_{l}, \nu_{m}) ,$$
(B21)

and, in general, for the j-th order coupling coefficient,

$$D_{m_{1}\cdots m_{j}} = \frac{2\pi I_{o}}{\sigma_{b} V_{n}} \frac{\beta_{1o}}{\beta_{zo}} \left(\frac{1}{2\beta_{zo}}\right)^{j} \left(\gamma_{o} \beta_{1o}^{3}\right)^{j-1} \eta_{o}^{j} (k_{o} V)^{j+1} J_{o}^{j+1} S_{j} \hat{D}(\nu_{m_{1}}, \dots, \nu_{m_{j}})$$
(B22)

Defining the scaling factor

$$\xi = \frac{\gamma_0 \beta_{\perp 0}}{2\beta_{z_0}} k_0 V J_1'(k_0 \rho_0) \eta_0 , \qquad (B23)$$

and the new normalized current

$$I_{s} = \frac{\pi I_{o}}{V_{o}} \frac{\beta_{10}^{2}}{\gamma_{o}\beta_{z0}^{3}} (k_{o}W)^{3} [J_{1}'(k_{o}\rho_{o})]^{2} = \frac{I}{8 Q_{o}} \frac{\beta_{10}^{5}}{\beta_{z0}^{3}} [J_{1}'(k_{o}\rho_{o})]^{2} (k_{o}W)^{3},$$
(B24)

puts Eqs. (B19)-(B21) in the form given in Eq. (38).







Fig. 2 – Plot of the linear growth rate Γ_n against the mismatch $\delta \omega_n / \omega_o$.



(a) Fig. 3 — Contour plots for the real parts of third order coupling coefficients (a) \hat{G}_{am} (b) \hat{G}_{123} and (c) \hat{G}_{231} . Dashed lines signify negative values.



(c) \hat{G}_{231} . Dashed lines signify negative values.



(c) Fig. 3 (Continued) — Contour plots for the real parts of third order coupling coefficients (a) \hat{G}_{am} (b) \hat{G}_{123} and (c) \hat{G}_{231} . Dashed lines signify negative values.



(a) Fig. 4 — Contour plots for the real parts of the fifth order coupling coefficients \hat{D}_{nlm} on the $\nu_n = \text{const. plane}$ for (a) $\nu_n = 0.8$ (b) $\nu_n = 1.9$ (c) $\nu_n = 2.9$. The contour plots in (d) are drawn for $\nu_m = \nu_1$.



(b) Fig. 4 (Continued) — Contour plots for the real parts of the fifth order coupling coefficients \hat{D}_{alm} on the $\nu_a = const.$ plane for (a) $\nu_n = 0.8$ (b) $\nu_n = 1.9$ (c) $\nu_n = 2.9$. The contour plots in (d) are drawn for $\nu_m = \nu_1$.



Fig. 4 (Continued) — Contour plots for the real parts of the fifth order coupling coefficients \hat{D}_{nlm} on the $\nu_n = const.$ plane for (a) $\nu_n = 0.8$ (b) $\nu_n = 1.9$ (c) $\nu_n = 2.9$. The contour plots in (d) are drawn for $\nu_m = \nu_1$.



(d) Fig. 4 (Continued) — Contour plots for the real parts of the fifth order coupling coefficients \hat{D}_{nlm} on the $\nu_n = const.$ plane for (a) $\nu_n = 0.8$ (b) $\nu_n = 1.9$ (c) $\nu_n = 2.9$. The contour plots in (d) are drawn for $\nu_m = \nu_1$.



Fig. 5 — Mode competition in the QOG for the first three modes above ω_o . The time evolution of the amplitudes F in (a)-(c) corresponds to different initial conditions.



the amplitudes F in (a)-(c) corresponds to different initial conditions.



(c) Fig. 5 (Continued) — Mode competition in the QOG for the first three modes above ω_o . The time evolution of the amplitudes F in (a)-(c) corresponds to different initial conditions.



Fig. 6 — Mode competition among the third, fourth and fifth modes for same parameters as in Fig. 5.



Fig. 7 — Mode competition among the first three modes in a cavity of half the length and same other parameters as in Figs. 5 and 6

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