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A NEW STRESS ROUTINE FOR THE

PROJECTILE DESIGN ANALYSIS SYSTEM

(PRODAS)

By

KURT G. GOTHE

EXCHANGE ENGINEER, WEST GERMANY

AERODYNAMICS BRANCH .

AIR FORCE ARMAMENT LABORATORY

EGLIN AFB, FLORIDA

Presented at the

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The purpose of this paper is to describe and discuss results obtained from a new stress routine, which is implemented in the Projectile Design Analysis System (PRODAS). This system is regularly used by the Aerodynamics Branch (FXA), Air Force Armament Laboratory (AFATL), Eglin Air Force Base, Florida, to design projectiles and to predict the aerodynamic behavior and performance of projectiles and rockets prior to testing in the Aeroballistic Research Facility (ARF). Due to the increasing variety of aerodynamic configurations that are being tested, a stress routine was desired which provides the model/sabot designer with a good estimation of the projectile stress during the launch phase.

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SECTION I

INTRODUCTION

The free flight ballistic range has been and still is an important tool in the testing and development of military weapon systems and ammunition as well as ballistic research. The design of the models used in ballistic research is a critical element in this process. As a result, a computer program entitled "The Projectile Design and Analysis System" (PRODAS) (Fig.1) was developed to predict the mass properties along with the aerodynamic parameters and the associated free flight behavior of various munitions. Another purpose of this program is to provide the design engineer with information about the expected acceleration loads during the launch cycle. For a complete description of the PRODAS program refer to Ref. 1.

PRODAS is basically a design tool which, as described herein, uses the interactive and display capabilities of the Eglin Air Force Base graphic computer system. The previously existing stress routine in PRODAS only performed a stress analysis for spin stabilized projectile configurations. The new stress routine discussed herein computes the projectile compressive loads in the outer shell at numerous locations along the projectile length. For certain projectile geometries containing a cylindrical body, the dynamic material properties are taken into consideration and the maximum allowable dynamic stresses are compared to the computed compressive stresses in the body. The basis for these dynamic properties are empirical results, taken from several sources (Ref. 2-4).

The purpose of this paper is to discuss this new stress routine and to present some typical results. However, it should be noted that the addition of this new stress routine does not represent the final solution to providing the engineer with design information. It is expected that this routine will be further improved and a finite element routine is already in development which will also be included in the near future. This finite element routine may be the subject of a future paper.

SECTION II

LAUNCH INDUCED STRESS

1. General Case

The maximum force (F), acting on a projectile-base during the acceleration phase is equal to the maximum base pressure (P) delivered by the propellant multiplied by the area (A) of the bore (Ref. 5, 6)

$$F = P \quad A \tag{1}$$

The setback force, caused by the acceleration (a) of the projectile is

$$F = a \quad W / g \tag{2}$$

where g is the acceleration of the gravity in ft/sec ^{2}, and W is the total weight of the projectile in pounds. Combining equations (1) and (2) leads to

$$a = P \, A \, g / W \tag{3}$$

The inertia of the mass of the parts of the projectile ahead of a transverse section will lead to a compression force (F_c) in that particular cross-section, assuming the projectile is acting as a rigid body.

$$F_c = W' \, a / g \tag{4}$$

W' is the weight of all projectile parts forward of the transverse section. The compressive stress is then defined as:

$$\sigma_c = F_c / A_i \tag{5}$$

Where A_i is the cross sectional area of the load carrying transverse section. This approach is applicable to projectile-models with a rigid body or with thick shell walls.

In the case of a rifled gun, a tangential force (F_t) also exists which is caused by the angular acceleration (a') imparted by the rotating band on the shell. This angular acceleration is a function of the rifling twist (n, in calibers per turn), the linear acceleration and the projectile diameter (d, in inches).

$$a' = 24 \pi a / (n d) \tag{6}$$

The torque applied to the projectile is

$$T = a' (I/g) \tag{7}$$

where I is the polar moment of inertia of the projectile (lb.in.^{2}) and T has the units of lb.in. The tangential force can be written as

$$F_t = T / (d/2) \tag{8}$$

Combining equations (3), (6), (7) and (8), we can obtain

$$F_t = \frac{48 \pi I P A}{n d^2 W} \tag{9}$$

Equation 9 shows that F_t is directly proportional to the propellant pressure acting on the base of the model and therefore F_t will be a maximum, when the base pressure is a maximum. The previously existing PRODAS program contained a simplified analysis which considered the shear stress caused by the tangential force (F_t) applied by the rotating band.

It should be noted that there are other forces which can contribute to the stress levels experienced during launch. For example, any projectile with internal cavities containing a filler material (i.e. a high explosive HEI round) can have longitudinal, tangential, and radial stresses resulting from the rotation, setback, or movement of filler material or any other internal components. As mentioned previously the purpose of the present work was to incorporate an additional routine in PRODAS where the compressive stresses acting on a rigid or semi-rigid projectile are caused by the setback forces encountered during launch (see equation 5). This is applicable for saboted projectiles fired from a smooth bore gun where the shear stresses resulting from rotation are negligible.

In order for the designer to determine whether or not the existing compressive stresses are high enough to possibly cause failure, they must be compared with the maximum allowable stress. Since the existing compressive loads are applied in a dynamic manner and exist only for a short period of time (i.e. Microseconds) the maximum allowable dynamic load (Q) can be significantly higher than the maximum allowable static load (see Ref. 6).

This maximum allowable dynamic load Q (lb.) can be calculated by the following secant formula:

$$\frac{Q}{A} = \frac{\sigma_y / m}{1 + .25 \sec\left(\frac{.75\ L}{2\ r}\sqrt{\frac{m\ Q}{E\ A}}\right)} \qquad (10)$$

Where m is normally set equal to 1.7 and L = length of column (in.), r = least radius of gyration of column section (in.), E = modulus of elasticity (psi), A = section area of column (in.^{2}), and σ_y is the static yield stress (psi).

Since this equation is nonlinear in Q, it can only be solved by trial and error or by the use of prepared charts (see Refs. 7,8 and the attached appendix).

Under certain circumstances the maximum load a body will sustain is not given by the strength of the material, but by the stiffness of the body. This behavior is known as "elastic stability" and arises when the load produces a bending or a twisting moment that is proportional to the corresponding deformation. An example of this is the Euler column, which is a straight column, axially loaded. It remains straight and suffers only axial compressive deformation under small loads. If while thus loaded it is slightly deflected by a transverse force, it will straighten after removal of this force. But there is some axial "critical load" that will hold the column in the deflected position, and since both the bending moment due to the load and the resisting moment due to the stress are directly proportional to the deflection, the load required to hold the column in the defected state is independent of the amount of the deflection. Any increase in the "critical load", leads immediately to a collapse of the column.

A very thorough discussion of the general problem, with detailed solutions of many cases are given in Ref. 6 and 7, from which many of the formulas presented in the Appendix were taken.

2. Special Case

A special model case, which is representative of many of the subscale models tested in the Aeroballistic Research Facility (ARF), was defined as follows: The model has a cylindrical body, with two concentric holes, drilled from the base of the projectile towards the tip. The model may consist of two different materials where the nose section and the body section is joined with a threaded stud. This threaded stud can be either part of the nose section or the body section. The nose section may also consist of various elements such as an ogive and conical elements capped with a hemispherical nose tip (see Fig. 2).

For the above defined projectile, the previously discussed stress analysis is performed and then compared with the maximum allowable dynamic stresses as calculated at both the base and joint. If the projectile does not fit the special case as defined above, only the general stress analysis will be computed and the design engineer will be left to his own means in determining whether or not the calculated stresses are critical.

SECTION III

RESULTS

When running the new stress routine in PRODAS, the program will automatically determine weather or not the conditions for the "special case" projectile exist. A projectile design will be treated as follows: The standard stress analysis corresponding to the previously discussed method will be computed for that projectile. The stress will be calculated at 200 transverse sections, beginning at the projectile tip and ending at the projectile base. The longitudinal distance from one transverse section to the next is equal. The information about the acceleration is taken from the PRODAS interior ballistic routine, and/or can be chosen by the designer. Results appear in the form of tables, as shown in Table 1, and plotted versus the projectile length, as shown in Figs. 3 and 4. These results provide the design engineer with the opportunity to redesign that specific model, for instance in the joint area to avoid inappropriate stress concentrations.

In addition to the above mentioned stress analysis, the maximum allowable dynamic stress will also be calculated if the conditions for the specially defined projectile exist. In order for this to be accomplished it is necessary for the designer to choose the materials used. This selection is made from the table as shown in Table 2. Depending on what materials are selected, subtables will appear on the screen for the designer to specify certain material properties (i.e. the maximum yeild point) of the selected material.

> "Enter the yield strength of the material (cylindrical part) in 10^3 psi. To keep the default value of 68 (hit 'return')"

The computed maximum allowable dynamic stresses are then displayed for both the base and joint cross sections of the specially defined projectile.

Projectile Base:

Stress: 9791 psi

Dynamic Allowed Stress: 39,217 psi

Safety Margin: 4.00

Joint Area:

Stress: 9106 psi

Dynamic Allowed Stress: 49,848 psi

Safety Margin: 5.47

SECTION IV

CONCLUSIONS

A Fortran V subroutine has been included in the Projectile Design and Analysis System (PRODAS) in order to analyze the compressive stress along a projectile body during launch. Also, the maximum allowable dynamic stresses are computed for a specially defined projectile. It is believed that this new stress routine will be of great assistance to the design engineers of the Aeroballistic Research Facility and will significantly reduce the risk of launch failures due to inadequately designed models. It is expected that this routine will be further improved in the future (i.e. by adding a sabot analysis) and that more advanced routines (i.e. finite element) will also be incorporated.

ACKNOWLEDGMENTS

The author wishes to acknowledge the technical guidance provided by Mr. G. L. Winchenbach who assisted in writing this paper and under whose supervision this work was performed. Also, to professor Wafa Yazigi of the Columbia Basin College, Pasco, Wa, who provided the theoretical analysis of the maximum allowable dynamic stresses and to Ms Cindi King for typing this paper.

REFERENCES

1. Armament System Department, General Electric Company, Burlington, VT, "PRODAS User Manual", Version 3.10, Sept. 1988.

2. Rinehart, J.S. and J. Pearson, "Behavior of Metals under impulsive loads", American Society for Metals, Cincinnati, Ohio, 1949.

3. Kornhauser, M. "Structural Effects of Impact", Sparta Books, Inc., Cleaner-Hume Press, London, 1964.

4. Editors, Shewmo N.P.G. and Zackay, V.F., "Response of Metals to High Velocity Deformation", Interscience Publishers, New York, 1960.

5. Alexander Blake, "Practical Stress Analysis in Engineering Design", Mariel Dekker, Inc. New York, 1982.

6. Roark, R.J. and Young, W.C., "Formulas for Stress and Strain", 5th Edition, McGraw-Hill, 1975.

7. Timoshenko, S. "Theory of Elastic Stability", Engineering Societies Monograph, MacGraw-Hill Book Company, 1936

8. Pflueger, A. "Stabilitaetsprobleme der Elastostatik, "Springer Verlag, 1964

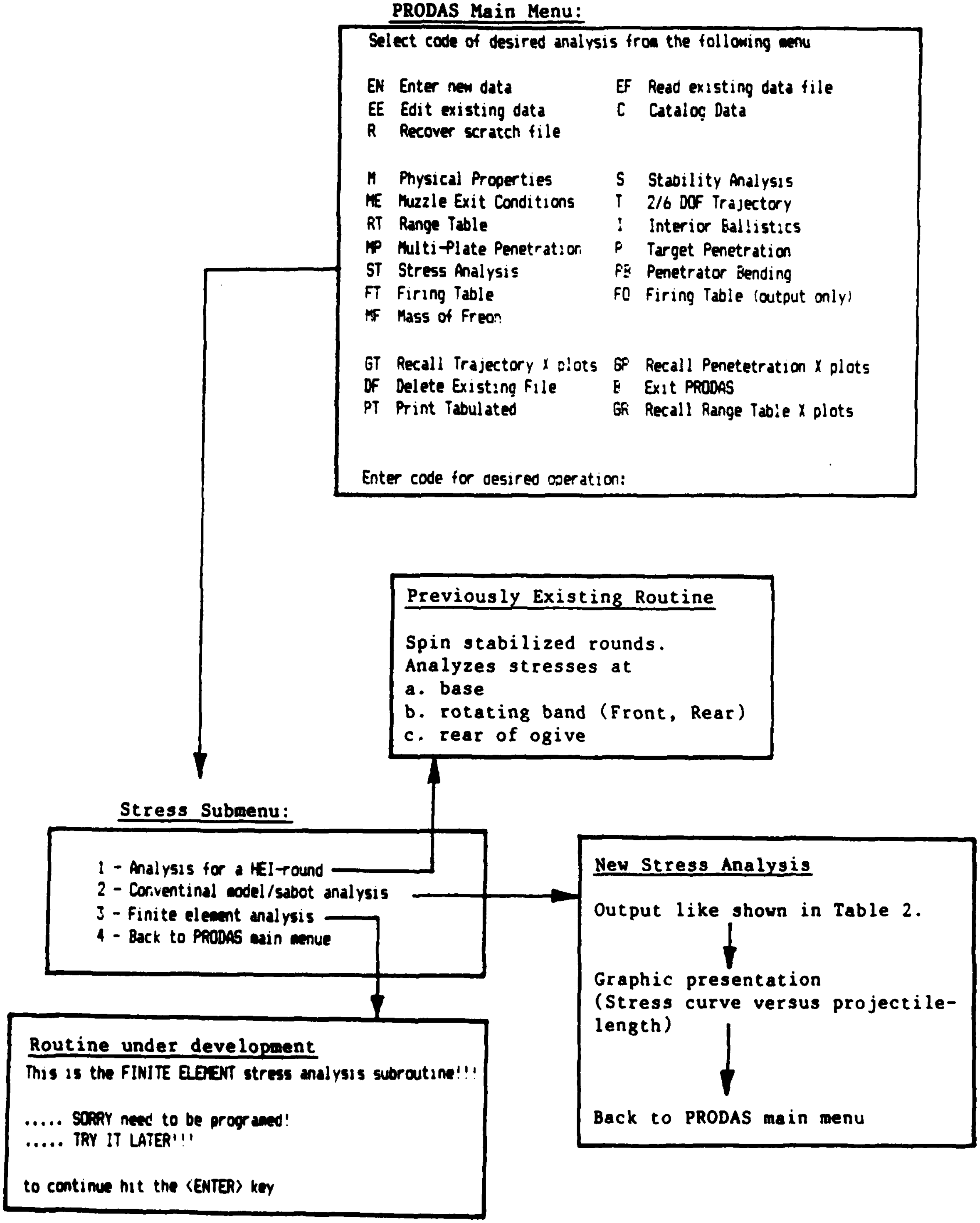

Figure 1: PRODAS STRESS MENU

ρ_1 = density of material for $L_5 < x < L_T$

ρ_2 = density of material for $L_0 < x < L_5$

D_0 = outside diameter of the cylinder. $L_0 < x < L_3$

D_1 = diameter of the first drill

D_2 = diameter of the second drill

D_h = diameter of the connection between the two materials

D_i = outer diameter of the cross section at $x = L_i$, $i = 3, 4, 5, 6, 7$

$L_8 = (D_0 L_7 - D_7 L_3) / (D_0 - D_7)$

L_T = total length of the projectile

a = acceleration (the maximum value of the acceleration obtained from interior ballistics)

E = modulus of elasticity of the material

M_i = mass of the projectile from $x = L_i$ to $x = L_T$, $i = 0, 1, \ldots, 7$

V_i = volume of the projectile from $x = L_i$ to $x = L_T$, $i = 0, \ldots, 7$

A_i = the cross sectional area at $X = L_i$, $i = 0, \ldots, 7$

R = radius of gyration of the cylinder $\sqrt{\frac{I}{A}}$

I = moment of inertia of the cylinder

σ_i = compressive stress at $x = L_i$. The stress at any cross section is

$\sigma_i = \frac{M_i a}{A_i}$, where $M_i = \rho\, V_i$

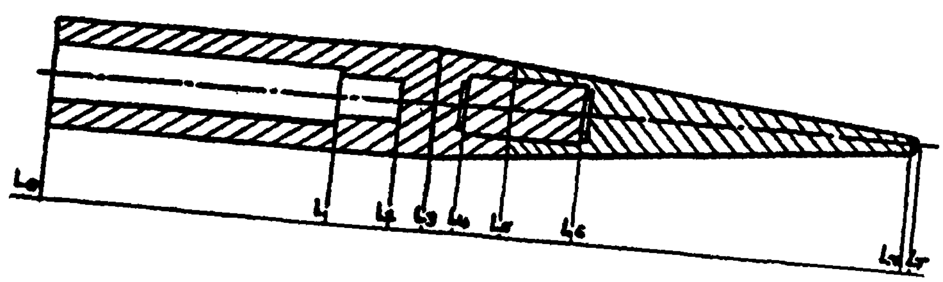

Figure 2: Inputs for Conventional Stress Analysis

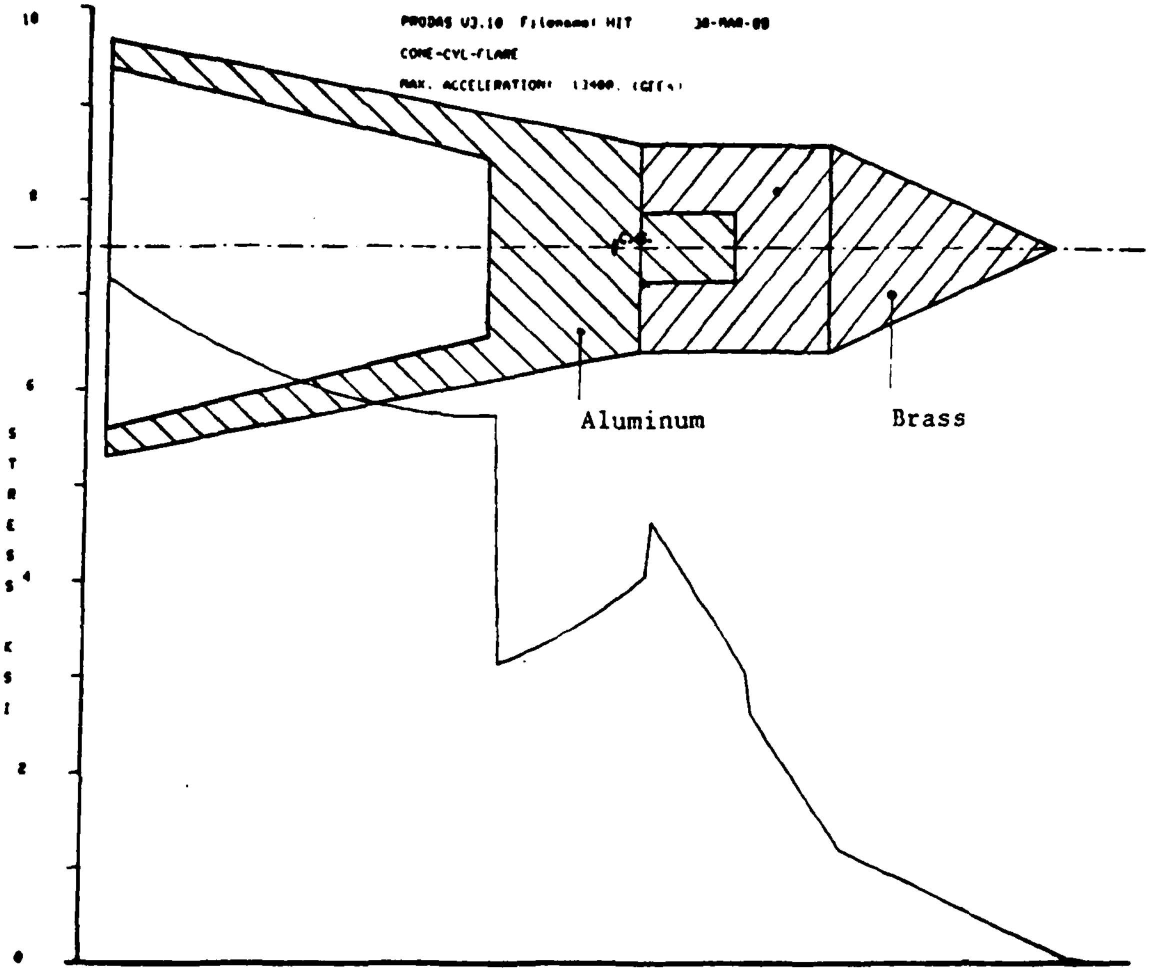

Figure 3: Stress versus projectile length for a general model.

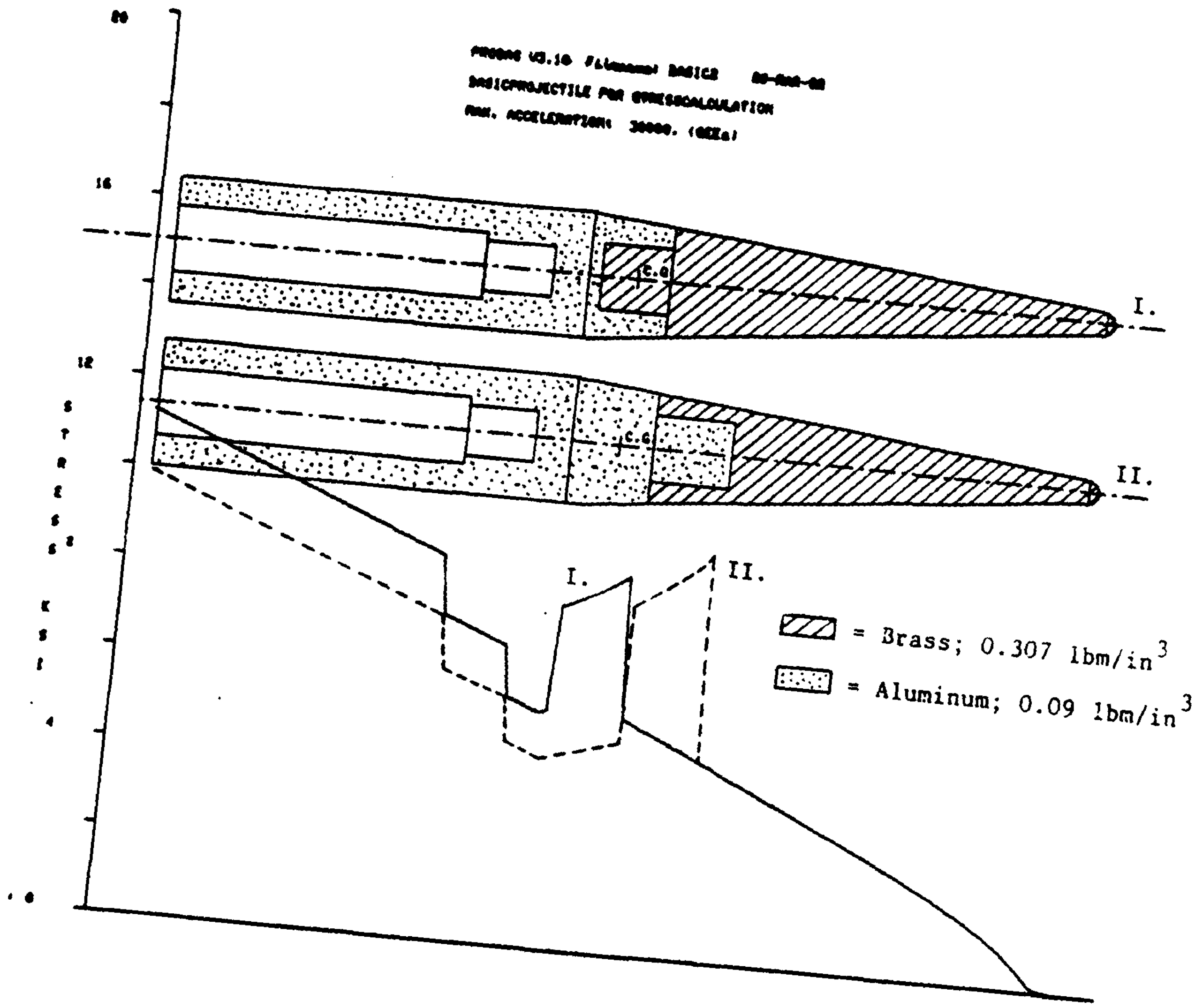

Figure 4: Stress versus projectile length for two joint designs

Table 1: Stress analysis results for the 'standard projectile'

Max. Acceleration (choosen by user)=11592000.IN/SEC**2
that is equivalent to 30000. GEEs
FILENAME: BASICSTR 20-MAR-89 :BASICPROJECTILE FOR STRESSCALCULATION (N
Stress in lb/in**2 in 200 cross-section aeras between
projectiletip (#:1) and projectilebottom (#:200)

#: 1	46.5	#: 51	3191.3	#:101	4532.8	#:151	7784.9
#: 2	152.2	#: 52	3331.0	#:102	4501.8	#:152	7825.9
#: 3	131.0	#: 53	3304.7	#:103	4495.2	#:153	7866.8
#: 4	270.6	#: 54	3283.6	#:104	4465.8	#:154	7907.8
#: 5	410.3	#: 55	3423.2	#:105	4437.4	#:155	7948.7
#: 6	481.2	#: 56	3401.3	#:106	4410.3	#:156	7989.6
#: 7	620.9	#: 57	3540.9	#:107	4384.1	#:157	8030.6
#: 8.	668.7	#: 58	3517.8	#:108	4359.2	#:158	8071.5
#: 9	808.4	#: 59	3657.4	#:109	4334.8	#:159	8112.5
#: 10	838.7	#: 60	3632.7	#:110	4311.6	#:160	8153.4
#: 11	978.3	#: 61	3772.4	#:111	4289.2	#:161	8194.3
#: 12	995.7	#: 62	3747.2	#:112	4266.4	#:162	8235.3
#: 13	1135.3	#: 63	3886.8	#:113	4307.3	#:163	8276.2
#: 14	1142.1	#: 64	3860.8	#:114	4348.3	#:164	8317.2
#: 15	1281.8	#: 65	4000.4	#:115	4389.2	#:165	8358.1
#: 16	1280.9	#: 66	3973.6	#:116	4430.2	#:166	8399.0
#: 17	1287.1	#: 67	3951.6	#:117	4471.1	#:167	8440.0
#: 18	1426.8	#: 68	4091.3	#:118	4512.0	#:168	8480.9
#: 19	1426.2	#: 69	4068.3	#:119	4553.0	#:169	8521.8
#: 20	1565.9	#: 70	4208.0	#:120	5465.3	#:170	8562.8
#: 21	1559.5	#: 71	4184.2	#:121	5506.3	#:171	8603.7
#: 22	1699.2	#: 72	4323.8	#:122	5547.2	#:172	8644.7
#: 23	1687.7	#: 73	4299.2	#:123	5588.2	#:173	8685.6
#: 24	1827.3	#: 74	4438.9	#:124	5629.1	#:174	8726.5
#: 25	1812.3	#: 75	4413.5	#:125	5670.0	#:175	8767.5
#: 26	1952.0	#: 76	4553.2	#:126	5711.0	#:176	8808.4
#: 27	1933.7	#: 77	4527.2	#:127	5751.9	#:177	8849.4
#: 28	2073.3	#: 78	4666.8	#:128	5792.8	#:178	8890.3
#: 29	2052.2	#: 79	9106.5	#:129	5833.8	#:179	8931.2
#: 30	2038.9	#: 80	8681.4	#:130	5874.7	#:180	8972.2
#: 31	2178.6	#: 81	8858.5	#:131	5915.7	#:181	9013.1
#: 32	2162.2	#: 82	8474.4	#:132	5956.6	#:182	9054.1
#: 33	2301.8	#: 83	8649.1	#:133	5997.6	#:183	9095.0
#: 34	2282.8	#: 84	8301.4	#:134	6038.5	#:184	9135.9
#: 35	2422.5	#: 85	8474.0	#:135	7129.9	#:185	9176.9
#: 36	2401.7	#: 86	8157.4	#:136	7170.9	#:186	9217.8
#: 37	2541.3	#: 87	8328.2	#:137	7211.8	#:187	9258.7
#: 38	2518.1	#: 88	8039.7	#:138	7252.7	#:188	9299.7
#: 39	2657.8	#: 89	8208.7	#:139	7293.7	#:189	9340.6
#: 40	2632.9	#: 90	7943.2	#:140	7334.6	#:190	9381.6
#: 41	2772.5	#: 91	8110.8	#:141	7375.6	#:191	9422.5
#: 42	2746.2	#: 92	7864.2	#:142	7416.5	#:192	9463.4
#: 43	2726.5	#: 93	7643.5	#:143	7457.4	#:193	9504.4
#: 44	2866.2	#: 94	7808.3	#:144	7498.4	#:194	9545.3
#: 45	2845.1	#: 95	4719.7	#:145	7539.3	#:195	9586.3
#: 46	2984.8	#: 96	4707.6	#:146	7580.3	#:196	9627.2
#: 47	2961.7	#: 97	4669.9	#:147	7621.2	#:197	9668.1
#: 48	3101.4	#: 98	4634.0	#:148	7662.1	#:198	9709.1
#: 49	3077.4	#: 99	4598.8	#:149	7703.1	#:199	9750.0
#: 50	3217.0	#:100	4565.2	#:150	7744.0	#:200	9790.9

1 - Do analysis again with other acceleration data
2 - Back to PRODAS main menue
<ENTER> to continue

Table 2: Table of possible materials

```
Please choose the material for the OGIVE part of the projectile!
------------------------------------------------------------------
1   Structural Steel
2   Carbon Steel  (yield strength = 33000 psi)
3   Silicon Steel (yield strength = 45000 psi)
4   Nickel Steel  (yield strength = 56000 psi)
5   High-Strength Steel
6   Low Carbon and Low Alloy Steel
7   Cast Iron
8   Structural Aluminum 6061-T6 or 6062-T6
9   Structural Aluminum 2014-T4
10  Structural Aluminum 2024-T3
11  Structural Aluminum 2024-T4
12  Structured Aluminum 7075-T6
13  Structured Magnesium Alloy AMC 58S-T51
14  Structured Magnesium Alloy AMC 58S
15  Structured Magnesium Alloy AMC 57S
16  Structured Magnesium Alloy AMC 52S
17  Structured Magnesium Alloy AM 3S

18  Other materials
99  Back to MAIN MENUE: 2
```

APPENDIX

EQUATIONS FOR MAXIMUM ALLOWABLE DYNAMIC STRESS

Material	$\frac{L}{R}$	$\sigma = \frac{Q}{A}$ = allowable unit load $\frac{Lb}{in^2}$
Structural Steel	$\frac{L}{R} < C_c$	$\frac{Q}{A} = \frac{1 - \frac{\left(\frac{L}{R}\right)^2}{2C_c^2}}{m} \cdot \sigma y$
	$C_c < \frac{L}{R} < 200$	$\frac{Q}{A} = \frac{149,000,000}{\left(\frac{L}{R}\right)^2}$

where $C_c = \sqrt{\frac{2\pi^2 E}{y}}$

$$m = \frac{5}{3} + \frac{3(L/r)}{8C_c} - \frac{\left(\frac{L}{R}\right)^3}{8C_c^3}$$

for σy =	33K	36K	42K	46K	50K
C_c =	131.7	126.1	116.7	111.6	107.0

Material	$\frac{L}{R}$	$\frac{Q}{A}$
Carbon Steel	$\frac{L}{R} \leq 140$	$\frac{Q}{A} = 15,000 - \frac{1}{4}\left(\frac{L}{R}\right)^2$
	$\frac{L}{R} > 140$	$\frac{Q}{A} = \frac{18,750}{1 + .25 \sec\left(\frac{.75L}{2R}\sqrt{\frac{1.76Q}{EA}}\right)}$
Silicon Steel	$\frac{L}{R} \leq 130$	$\frac{Q}{A} = 20,000 - .46\left(\frac{L}{R}\right)^2$
	$\frac{L}{R} > 130$	$\frac{Q}{A} = \frac{25,000}{1 + .25 \sec\left(\frac{.75L}{2R}\sqrt{\frac{1.8Q}{EA}}\right)}$
	$\frac{L}{R} \leq 120$	$\frac{Q}{A} = 24,000 - .66\left(\frac{L}{R}\right)^2$

Nickel Steel

$\frac{L}{R} > 120$

$$\frac{Q}{A} = \frac{30{,}000}{1 + .25 \sec\left(\frac{.75L}{2r}\sqrt{\frac{1.83Q}{EA}}\right)}$$

High-Strength Steel

$0 < \frac{L}{R} < 140$

$$\frac{Q}{A} = 15{,}000 - .325\left(\frac{L}{R}\right)^2 \qquad \text{for} \quad y = 33K$$

$140 < \frac{L}{R} < 200$

$$\frac{Q}{A} = \frac{15{,}000}{.5 + \frac{1}{15{,}860}\left(\frac{L}{R}\right)^2} \qquad \text{for} \quad y = 33K$$

$0 < \frac{L}{R} < 120$

$$\frac{Q}{A} = 20{,}500 - .605\left(\frac{L}{R}\right)^2 \qquad y = 45K$$

$120 < \frac{L}{R} < 200$

$$\frac{Q}{A} = \frac{20{,}500}{.5 + \frac{1}{11{,}630}\left(\frac{L}{R}\right)^2} \qquad y = 45K$$

$0 < \frac{L}{R} < 110$

$$\frac{Q}{A} = 22{,}500 - .738\left(\frac{L}{R}\right)^2 \qquad y = 50K$$

$110 < \frac{L}{R} < 200$

$$\frac{Q}{A} = \frac{22{,}500}{.5 + \frac{1}{10{,}460}\left(\frac{L}{R}\right)^2} \qquad y = 50K$$

$0 < \frac{L}{R} < 105$ $\qquad \frac{Q}{A} = 25{,}000 - .902\left(\frac{L}{R}\right)^2$ $\qquad \sigma y = 55K$

$105 < \frac{L}{R} < 200$ $\qquad \frac{Q}{A} = \dfrac{25{,}000}{.5 + \frac{1}{9{,}510}\left(\frac{L}{R}\right)^2}$ $\qquad \sigma y = 55K$

Low Carbon & Low Alloy Steel $\qquad \frac{L}{R} < 181$ $\qquad \frac{Q}{A} = 36{,}000 - 1.172\left(\frac{L}{1.5R}\right)^2$ for $\sigma y = 36K$

$\frac{L}{R} < 135$ $\qquad \frac{Q}{A} = 79{,}500 - 51.9\left(\frac{L}{1.5R}\right)^{1.5}$ for $\sigma y = 75K$

$\frac{L}{R} < 110$ $\qquad \frac{Q}{A} = 113{,}000 - 11.15\left(\frac{L}{1.5R}\right)^2$ for $\sigma y = 103K$

$\frac{L}{R} < 95$ $\qquad \frac{Q}{A} = 145{,}000 - 18.36\left(\frac{L}{1.5R}\right)^2$ for $\sigma y = 132K$

$\frac{L}{R} < 90$ $\qquad \frac{Q}{A} = 179{,}000 - 27.95\left(\frac{L}{1.5R}\right)^2$ for $\sigma y = 163K$

Cast Iron	$\frac{L}{R} < 100$	$\frac{Q}{A} = 12{,}000 - 60 \frac{L}{R}$
	$\frac{L}{R} < 70$	$\frac{Q}{A} = 9{,}000 - 40 \frac{L}{R}$
Structural Aluminum 6061-T6 6062-T6	$\frac{L}{R} < 10$	$\frac{Q}{A} = 19{,}000$
	$10 < \frac{L}{R} < 67$	$\frac{Q}{A} = 20{,}400 - 135 \frac{L}{R}$
	$\frac{L}{R} > 67$	$\frac{Q}{A} = \frac{51{,}000{,}000}{\left(\frac{L}{R}\right)^2}$
Structural Aluminum 2014-T4	$\frac{L}{R} < 1.732\pi\sqrt{\frac{1.5E}{F_{co}}}$	$\frac{Q}{A} = F_{co} \frac{1 - .385\left(\frac{L}{R}\right)}{\pi\sqrt{\frac{1.5E}{F_{co}}}}$
	$\frac{L}{R} > 1.732\pi\sqrt{\frac{1.5E}{F_{co}}}$	$\frac{Q}{A} = \frac{\pi^2 E(1.5)}{\left(\frac{L}{R}\right)^2}$

$$\text{where } F_{co} = F_{cy}\left(1 + \frac{F_{cy}}{200{,}000}\right)$$

and $F_{cy} = 35{,}000$ for 2014-T4

$F_{cy} = 42{,}000$ for 2024-T3

$F_{cy} = 40{,}000$ for 2024-T4

$F_{cy} = 35{,}000$ for 6061-T6

Structured Aluminum

$$\frac{L}{R} < 1.414\pi\sqrt{\frac{1.5E}{F_{co}}} \qquad \frac{Q}{A} = F_{co}\left[1 - \frac{F_{co}\left(\frac{L}{R}\right)^2}{6\pi^2 E}\right]$$

where $F_{co} = 1.075\ F_{cy}$ and $F_{cy} = 66{,}000$ for 7075-T6

Structured Magnesium Alloy

$$\frac{Q}{A} = \frac{\sigma}{1 + f\,K_1^2 \cdot \frac{L^2}{R^2}} \quad \text{not to exceed } \sigma^1$$

where:

ALLOY	σ	f	σ^1
AMC585-T51	160,900	.00249	36,000
AMC585	46,000	.00072	22,000
AMC575	34,300	.00053	19,000
AMC525	25,500	.00040	16,000
AM35	16,750	.00026	11,000

$K_1 = .5$

For other material use the following:

a. $\frac{L}{R} < 30$ Then the max. allowable stress is equal to the yield-point stress of material, δ_y.

b. $30 < \frac{L}{R} < 100$ Then max. allowable stress is given by:

$$(\sigma_{c_o})\ \text{static} = \frac{\delta_y}{1 + .25 \sec\left(.75\frac{L}{2R}\sqrt{\frac{\sigma_{co}}{AE}}\right)}$$

where σ_{c_o} = critical buckling load, lb.

I = least moment of inertia of cross sectional area, in^4

A = cross sectional area, in^2

R = least radius of gyration of cross sectional area($R = \sqrt{\frac{I}{A}}$, in

δ_y = yield-point stress of material, PSI

L = length of column, in

E = modulus of elasticity, PSI

c. if $\frac{L}{R} > 100$ then $(\sigma_{c_o})\text{static} = \frac{\pi^2 E}{\left(\frac{L}{R}\right)^2}$