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THESIS

Simulation and Analysis of a MFQPSK Signal
Transmitted Through an Acoustic Medium

by

Anita S. Daniel

December 1989

Thesis Advisor: Paul H. Moose

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A multi-frequency quadrature phase shift keyed (MFQPSK) signal has been developed at NPS to be used in computer-to-computer communications. This report discusses the simulation of a MFQPSK signal transmitted from a moving transmitter platform through a near vertical acoustic channel as seen by a moored receiver. The simulated signal is tested against MFQPSK signal theory. The simulation was developed to be an experimental tool for testing various Doppler, synchronization, and coding algorithms/techniques for a MFQPSK communication signal. The degradation of output signal-to-noise ratio due to Doppler shifts caused by the moving transmitter is analyzed. An algorithm for estimating Doppler compression/expansion in the received signal is evaluated.

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Anita S. Daniel
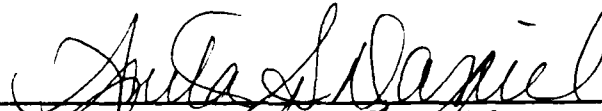B.S., Northern Arizona University, 1983

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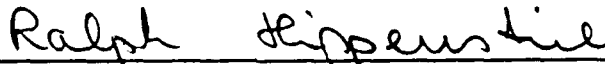
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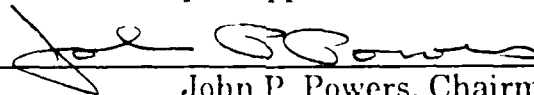December 1989

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ABSTRACT

A multi-frequency quadrature phase shift keyed (MFQPSK) signal has been developed at NPS to be used in computer-to-computer communications.

This report discusses the simulation of a MFQPSK signal transmitted from a moving transmitter platform through a near vertical acoustic channel as seen by a moored receiver. The simulated received signal is tested against MFQPSK signal theory. The simulation was developed to be an experimental tool for testing various Doppler, synchronization, and coding algorithms/ techniques for a MFQPSK communication signal. The degradation of output signal-to-noise ratio due to Doppler shifts caused by the moving transmitter is analyzed. An algorithm for estimating Doppler compression/expansion in the received signal is evaluated.

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I. INTRODUCTION

Data links used in many modern military communication systems re-
quire computer-to-computer communications which transmit bits from one
terminal to another with a low error rate. The communication channel may
be a lowpass channel, if the transmission medium is wire or optical fiber, or a
bandpass channel, if the transmission medium is radio frequency (RF) link or
acoustic channel. The signal used to carry the data bits must be a properly
modulated signal with sufficient energy. It must also be positioned in the fre-
quency spectrum to propagate effectively through the intended medium.
Multi-Frequency Modulation (MFM) is a signal modulation format that is
readily adaptable to a variety of data link scenarios. MFM does not require
special purpose MODEMS to translate between the digital and analog do-
mains and can emulate most existing signal modulation formats as well as
generate entirely new formats. MFM's descriptive language is that of Digital
Signal Processing (DSP) [Ref. 1]. Simulation and analysis of a Naval Post-
graduate School (NPS)-developed MFM signal received through a bandpass
channel is the subject of this thesis.

A. BACKGROUND

Classical modulation methods for bandpass channels use amplitude and/
or phase to carry signal information on a carrier wave in the channel. When
the information source is a finite alphabet, as with data or quantized analog
sources such as digitized speech or video, then only a finite number of signal

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states are needed to represent or code the source. Phase shift keying (PSK) is an efficient method for coding these signal states or message [Ref 2].

MFM is a modulation technique that is ideally suited for computer-to-computer data transmission and reception because its basic structure is one of time and frequency slots. In MFM, the signals are directly encoded, modulated, decoded, and demodulated using DSP techniques within the host computer. In MFM, signal "packets" located in a given frequency spectrum and time are created as shown in Figure 1. Figure 1 shows a single signal packet. Each packet consists of L bauds of K tones. The length of a baud is ΔT seconds during which K discrete tones are transmitted. The phase of each tone is the coded information. These LK subsignals form an orthogonal signal set. Each subsignal may be independently modulated with phase information.

Dr. P. H. Moose at NPS, in conjunction with the Naval Ocean Systems Center (NOSC) in San Diego, has developed Multi-Frequency Quadrature Phase Shift Keying (MFQPSK) signals to be used in high data-rate acoustic burst communications from moving platforms using medium frequency, relatively wideband (25 percent bandwidths) links [Ref. 1, 3]. A simulation of the MFQPSK signal from a moving platform as seen through a bandpass channel was needed to analyze and test various Doppler, synchronization, and coding techniques and/or algorithms before final implementation of the modulation scheme is actually realized. Thus, the subject of this thesis is the simulation and analysis of a MFQPSK signal from a moving platform as seen through a bandpass channel.
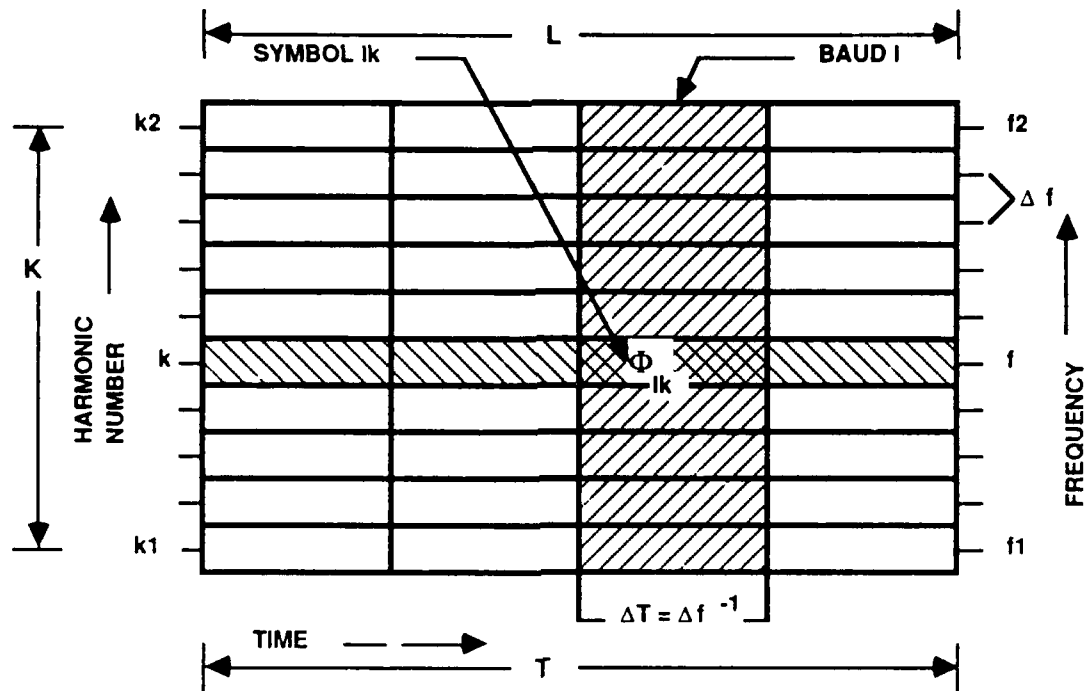
Figure 1. MFM Signal Packet (after Ref. 1: p. 3.)

B. OVERVIEW

The NPS-designed MFQPSK signal will be transmitted from a moving platform. In Chapter II a simple model of the moving transmitter platform's dynamics is developed; its performance is compared to actual track data. This model is implemented in the simulation.

A mathematical description of the transmitted signal and the design parameters of the MFQPSK signal are introduced in Chapter III. The channel parameters which delay and compress/expand this transmitted signal and which characterize the received signal, the additive Gaussian noise which is

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added to the received signal after sampling, and the final simulated sampled received signal are derived and presented in Chapter III.

Once the simulation was complete, testing and analysis was performed. First, the simulation was tested against MFQPSK signal theory to insure that the simulated received MFQPSK signal is consistent with theory. The simulated received signal should be consistent with theory so that the simulation may be used as an experimental tool for testing various Doppler, synchronization, and coding algorithms. Chapter IV consists of this analysis and simulation results.

Second, the amount of residual Doppler mismatch which the signal can tolerate due to the moving transmitter was analyzed. Chapter V provides the analysis of the output signal-to-noise ratio degradation due to the Doppler mismatch. Then, in Chapter VI, an estimation of the Doppler compression factor within a baud is derived using the Discrete Fourier Transform of a received signal's baud. The simulated outcome of this estimated Doppler versus the Doppler due to the moving transmitter within a baud is also illustrated in Chapter VI.

In the simulation, the actual Doppler compression/expansion factor due to the dynamics of the moving transmitter is computed; therefore, it is known. The start time of the signal reception is also known; thus, the signal is always perfectly synchronized in the simulation.

II. MOVING TRANSMITTER PLATFORM DYNAMICS AND BEAM PATTERN

A. BACKGROUND

Since it is assumed that the MFQPSK signal will be transmitted from a moving platform, provision must be made at the receiver to compensate for Doppler shifts. The Doppler shift factor is computed based on the geometry in the simulation using the radial velocity of the moving transmitter relative to the stationary receiver. The complete set of equations for the Doppler shift factor is presented in the next chapter. The radial velocity is the derivative with respect to time of the slant range to the receiver. Realistic slant range values are necessary to compute realistic Doppler shift factors. Slant range is computed using the time-varying x-position, y-position, and z-position of the transmitter relative to the receiver.

B. ACTUAL TRACK DATA VS. SIMULATED TRACK DATA

To realistically model the dynamics of the moving transmitter, actual track data was provided by NOSC. The parameters that were given in the actual track data were x-position, y-position, z-position, and slant range relative to the receiver. A simple random walk model was used to model the track data. This model assumes the transmitter is moving along a straight line with random fluctuations in the x, y, and z velocity components. Figures 2 through 7 illustrate how well the random walk model corresponds to the actual track data on six different runs. The z-position seems to consistently have the largest error. The largest error in the z-position is 25 feet, (see

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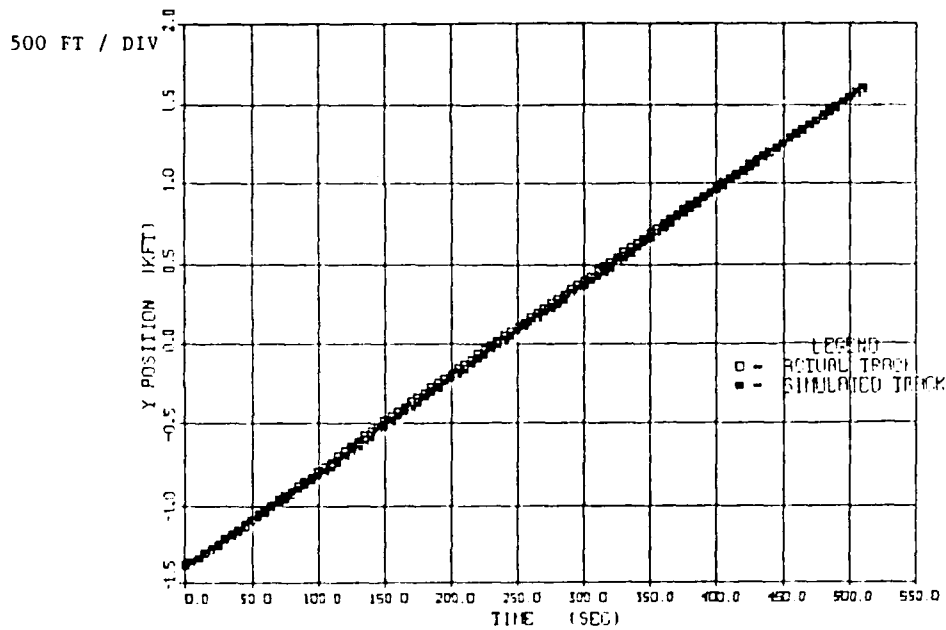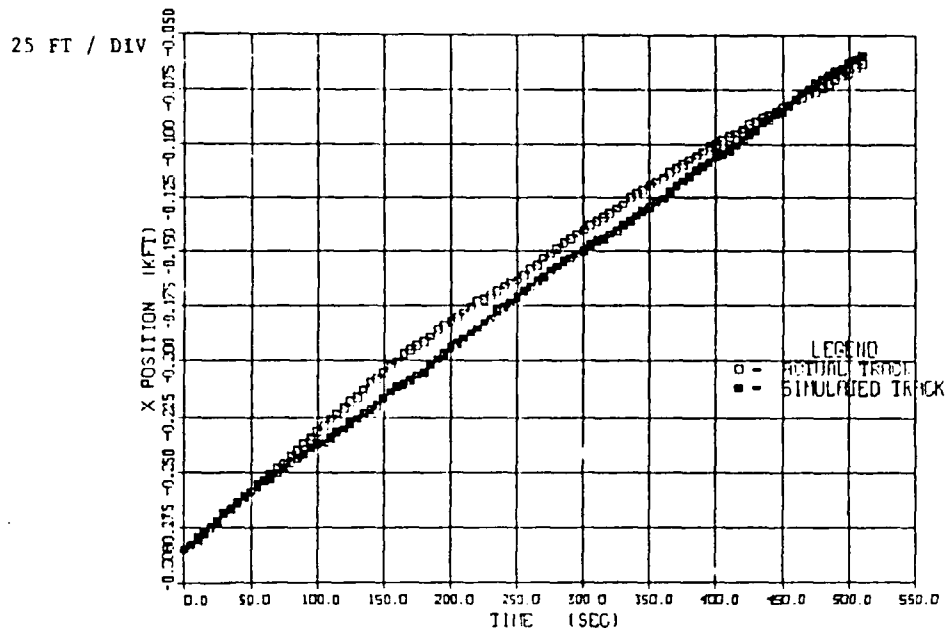
Figure 2a. Run 1: Actual and Simulated Track Data
X–Position and Y–Position vs. Time

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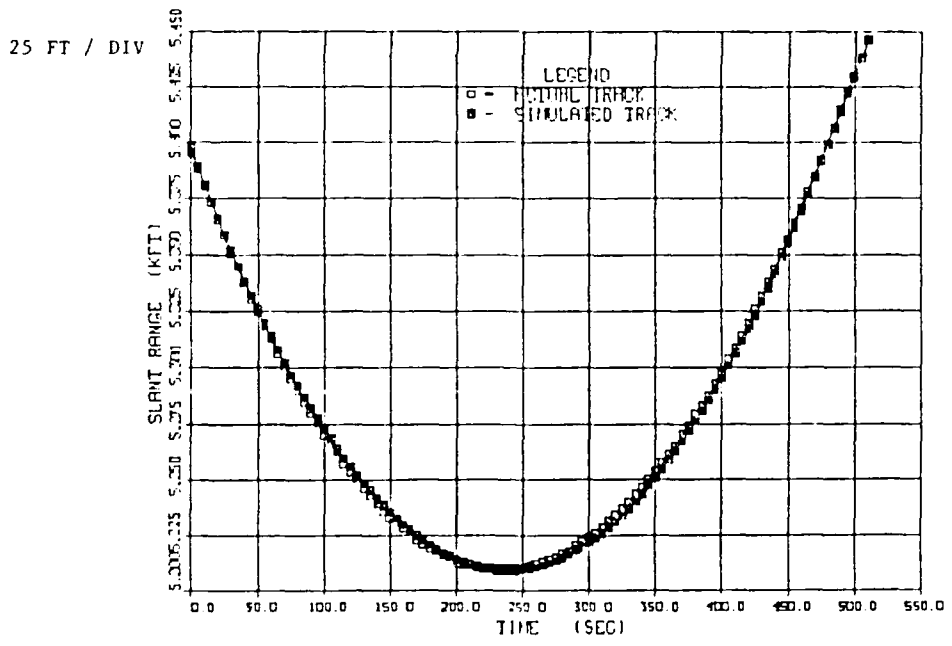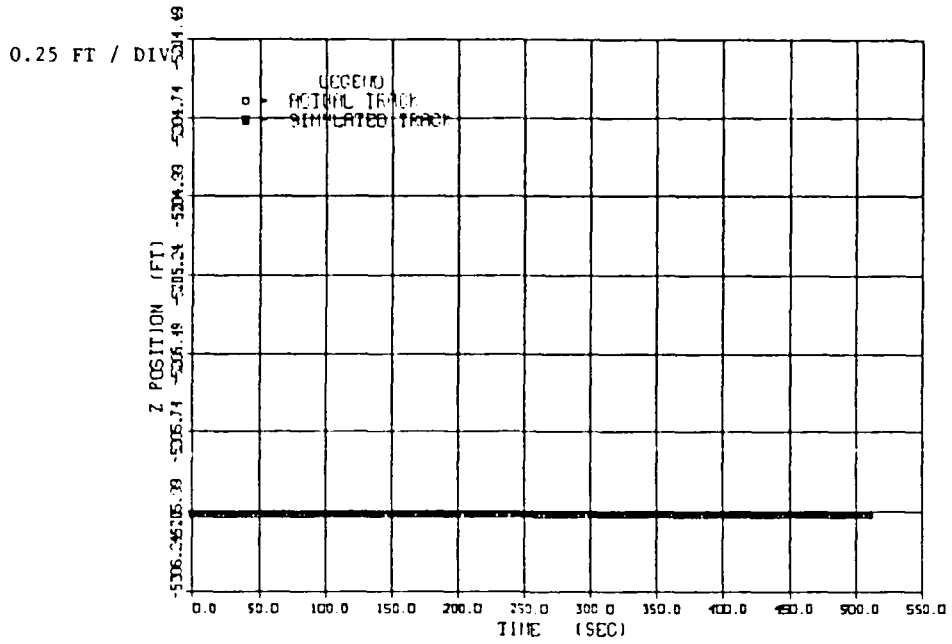
Figure 2b.   Run 1: Actual and Simulated Track Data
Z–Position and Slant Range vs. Time

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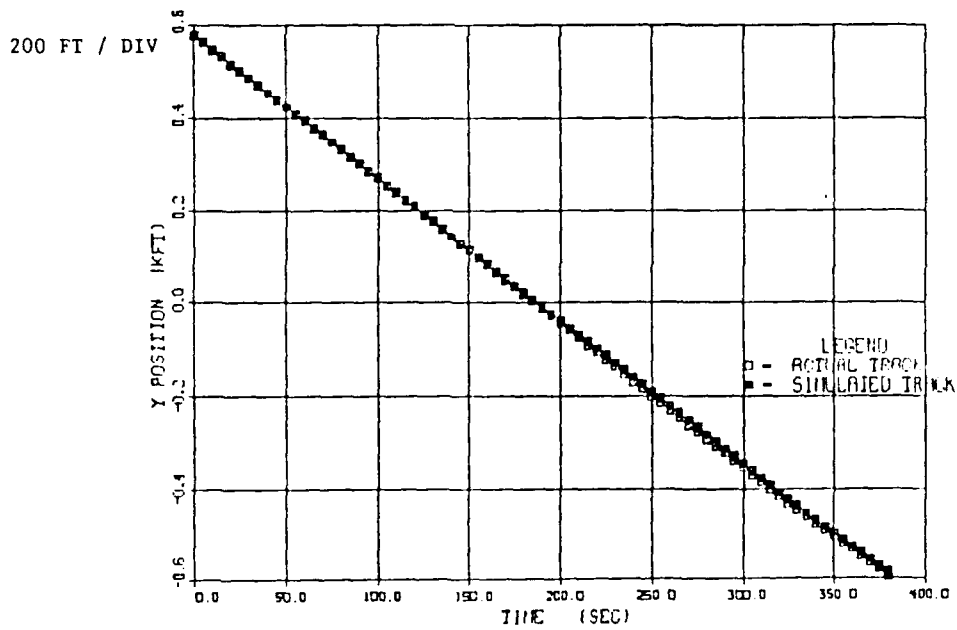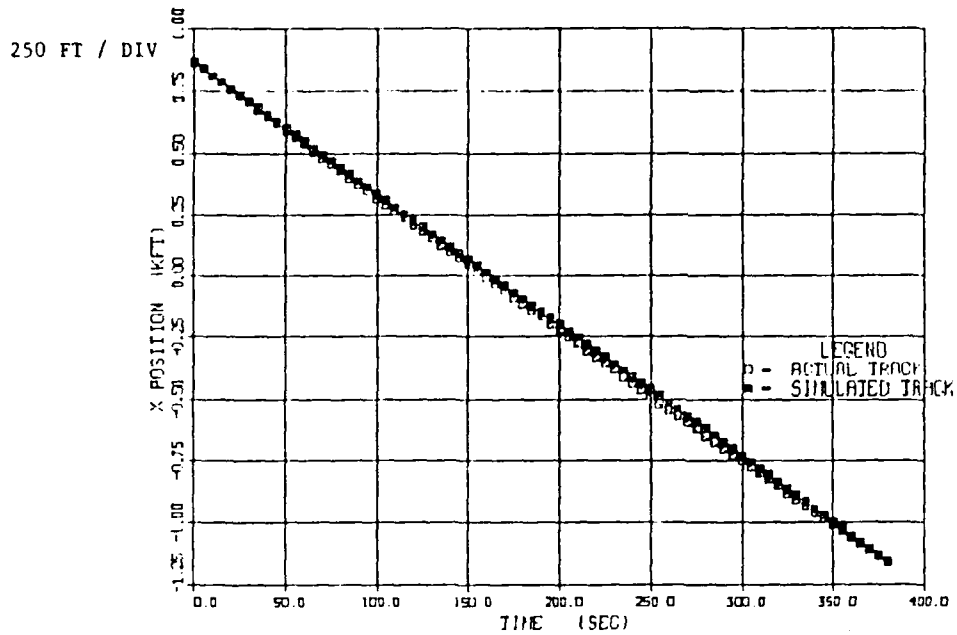
Figure 3a. Run 2: Actual and Simulated Track Data
X-Position and Y-Position vs. Time

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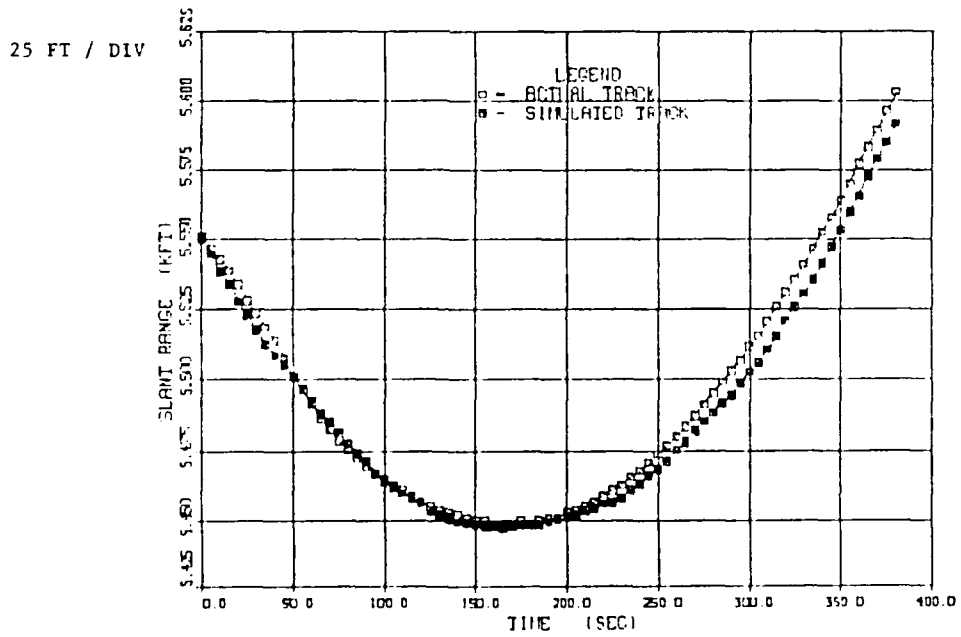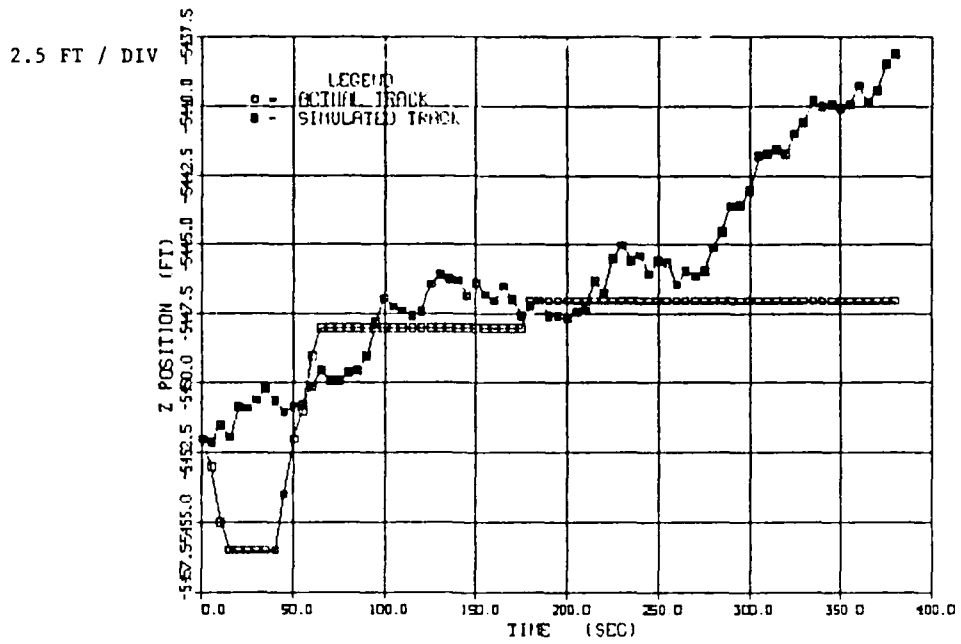
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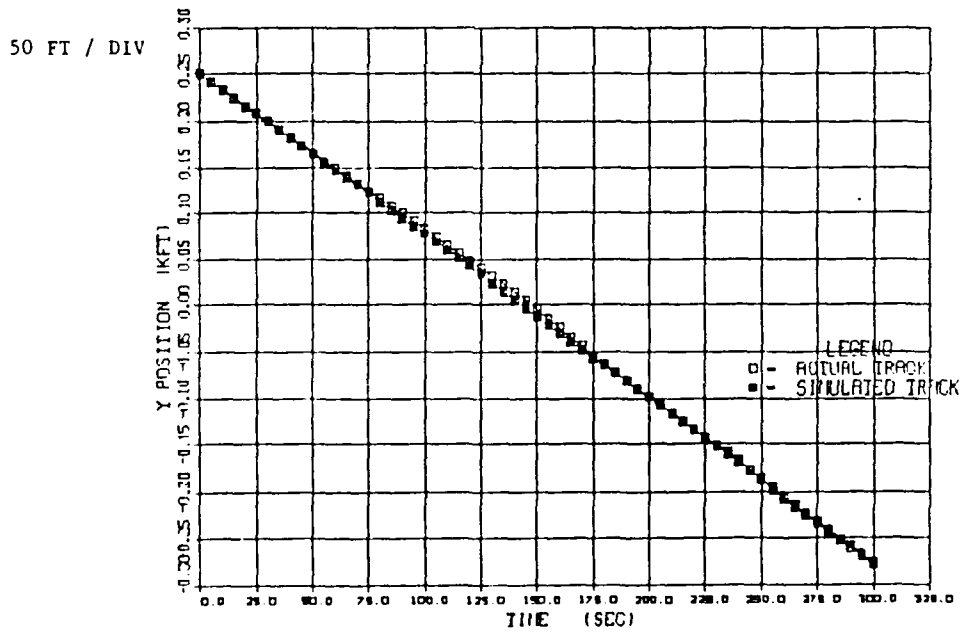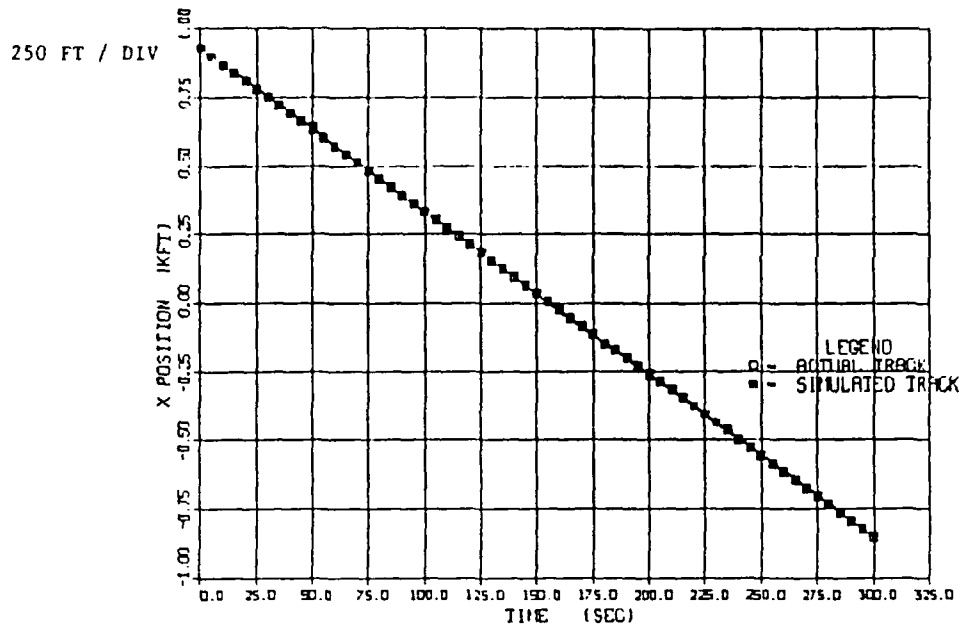
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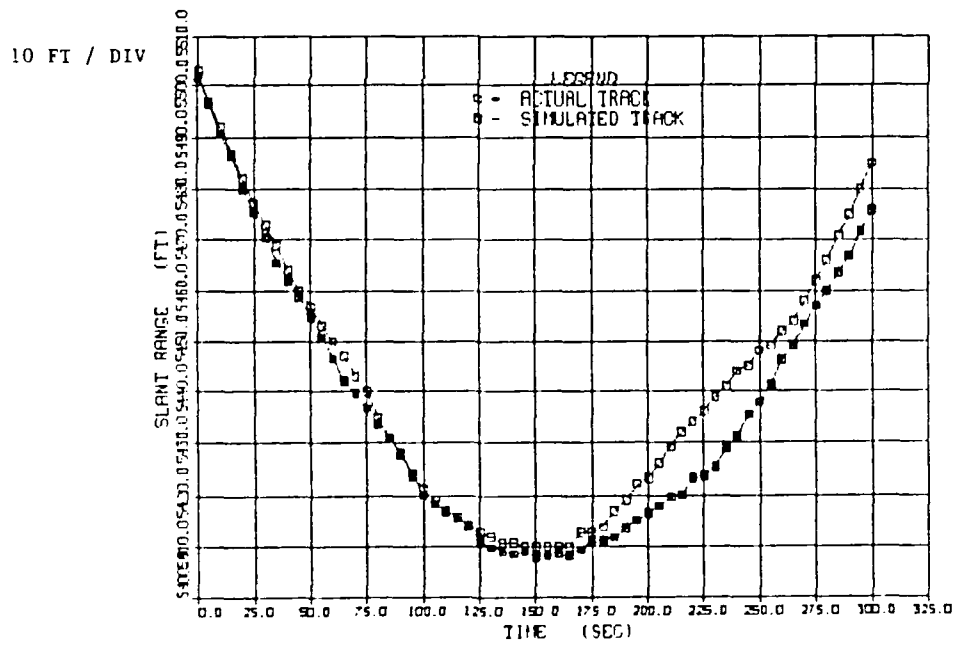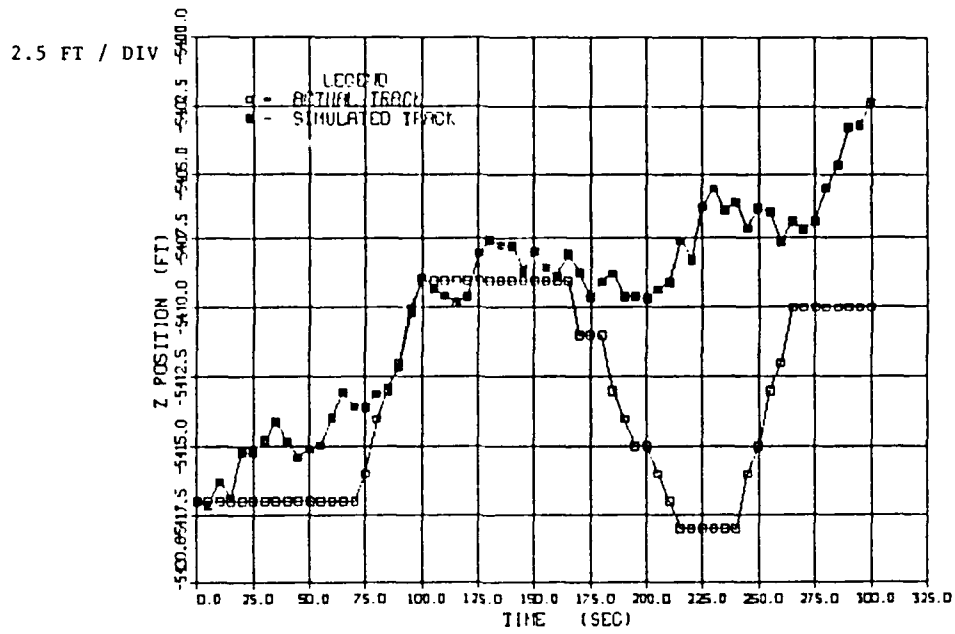
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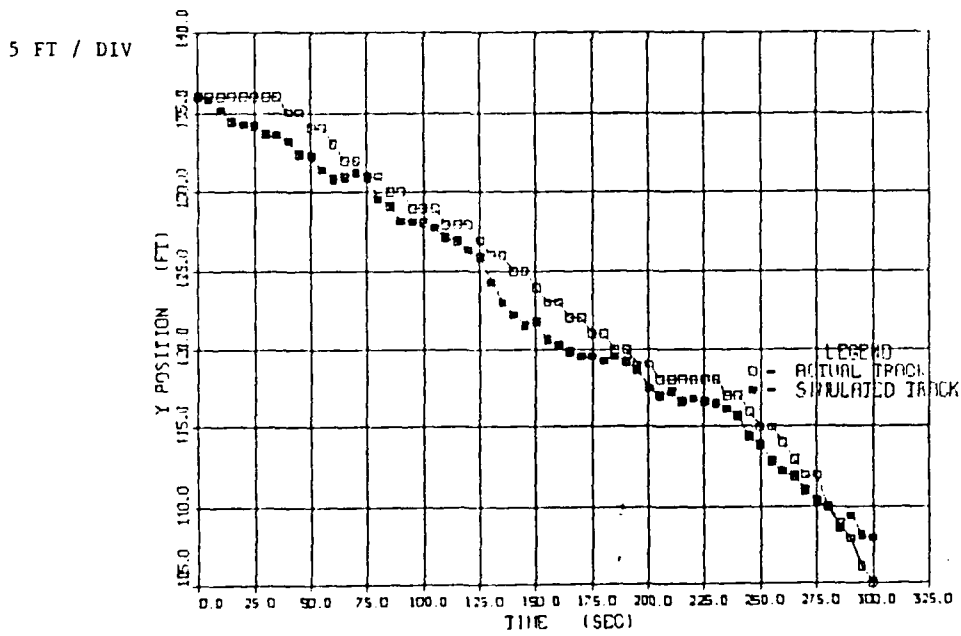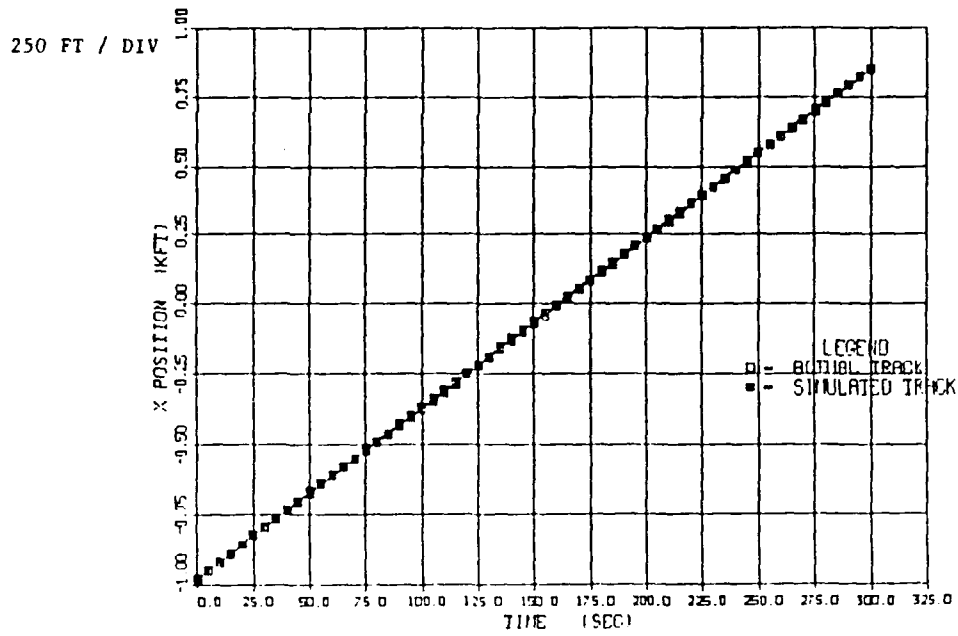
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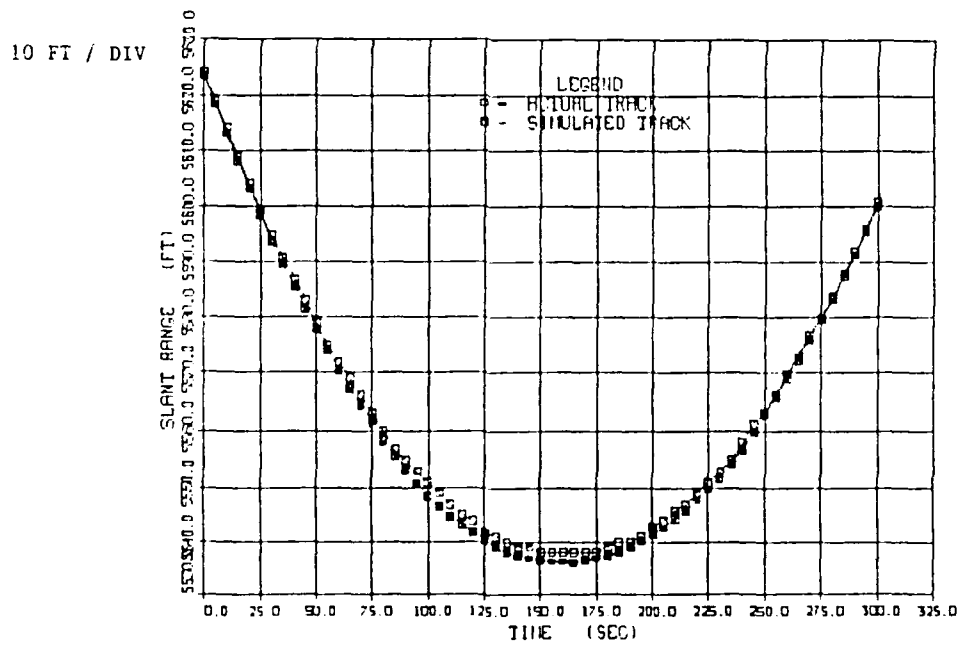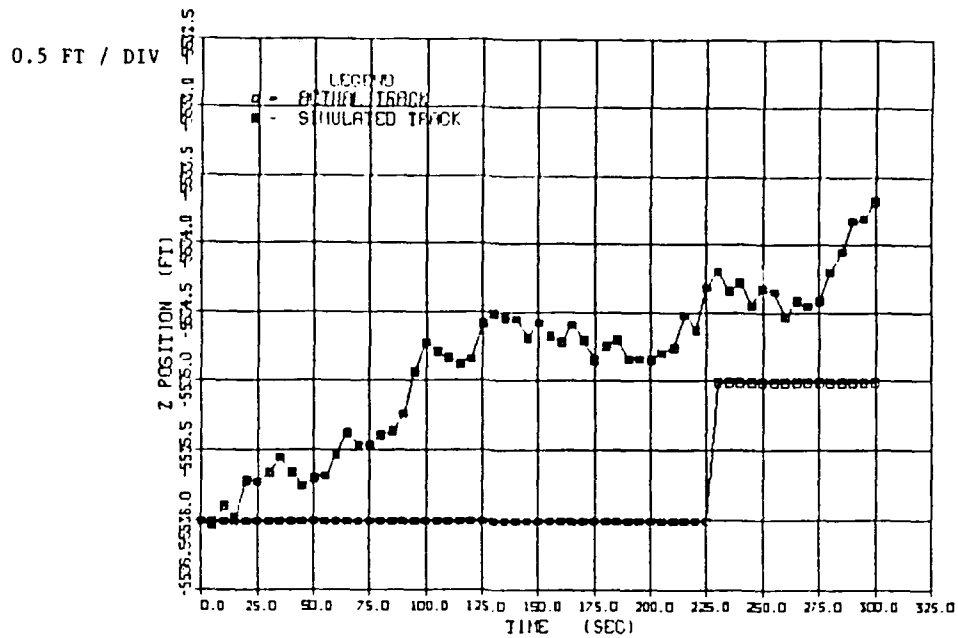
Figure 5b. Run 4: Actual and Simulated Track Data
Z–Position and Slant Range vs. Time

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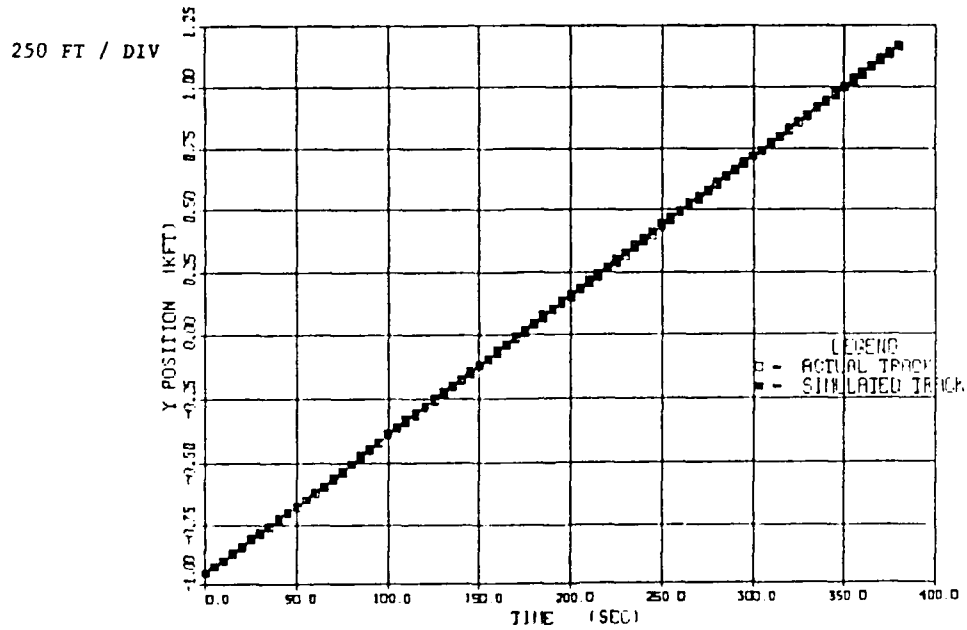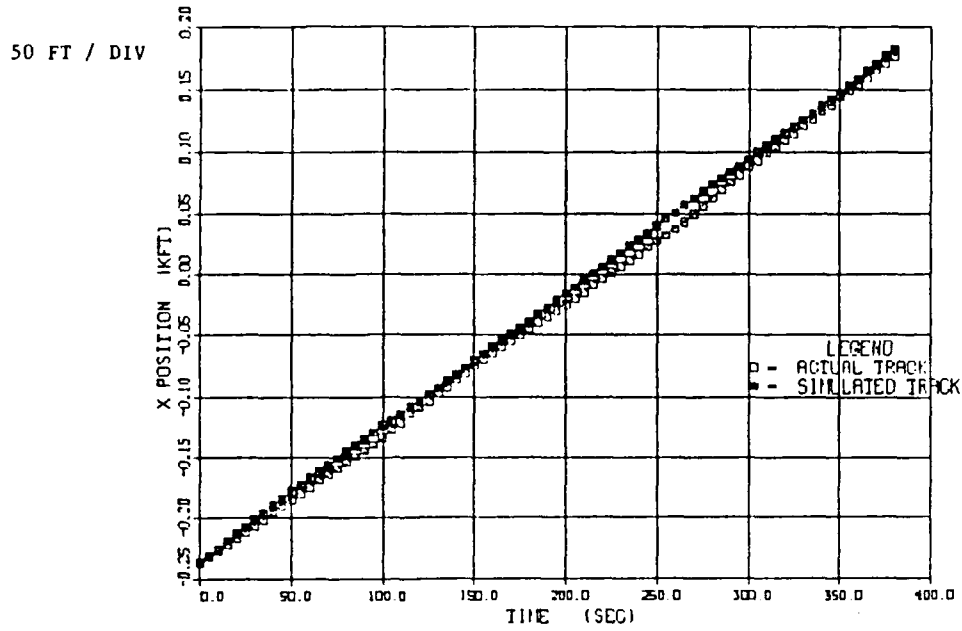
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Figure 6a. Run 5: Actual and Simulated Track Data
X-Position and Y-Position vs. Time

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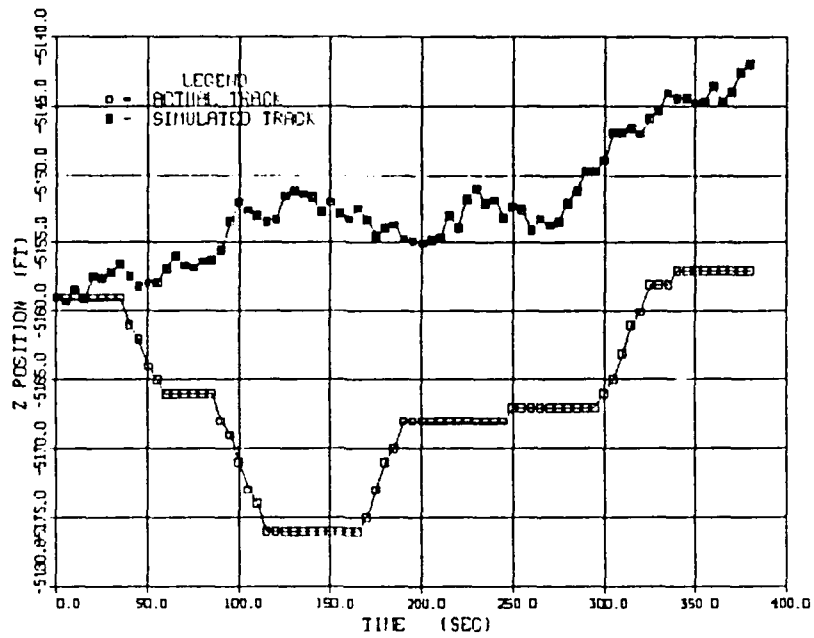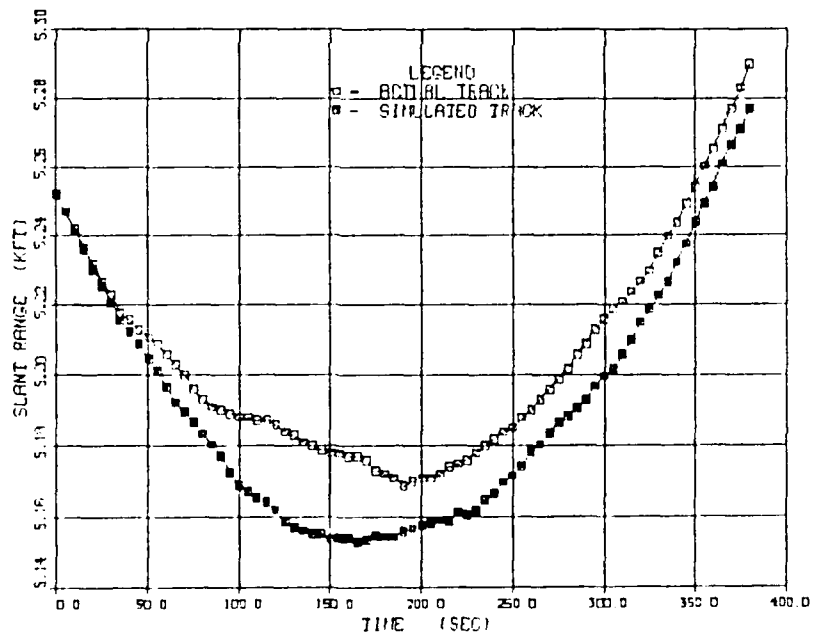
Figure 6b.   Run 5: Actual and Simulated Track Data
Z–Position and Slant Range vs. Time

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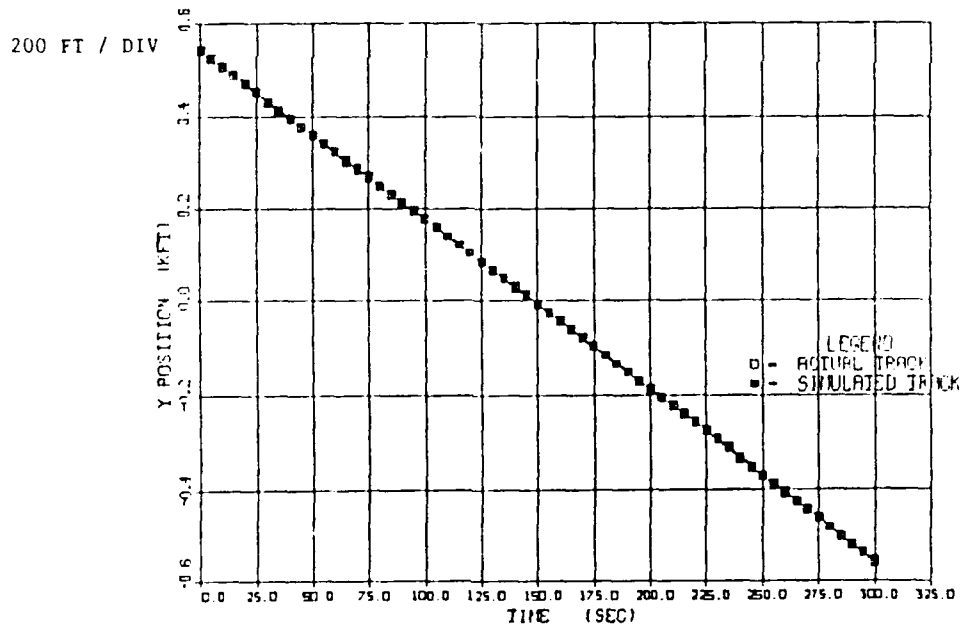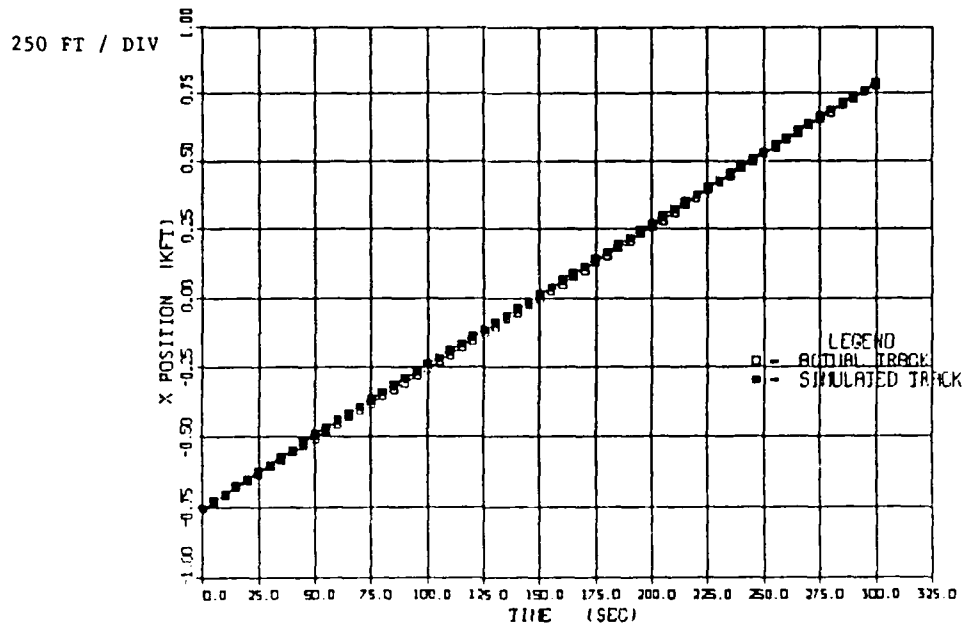
Figure 7a.   Run 6: Actual and Simulated Track Data
X–Position and Y–Position vs. Time

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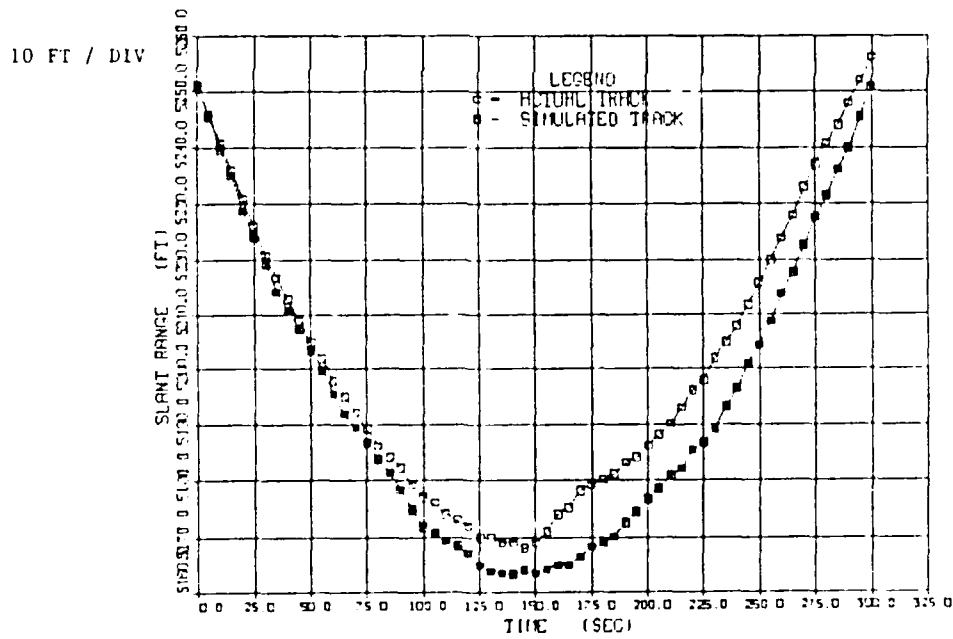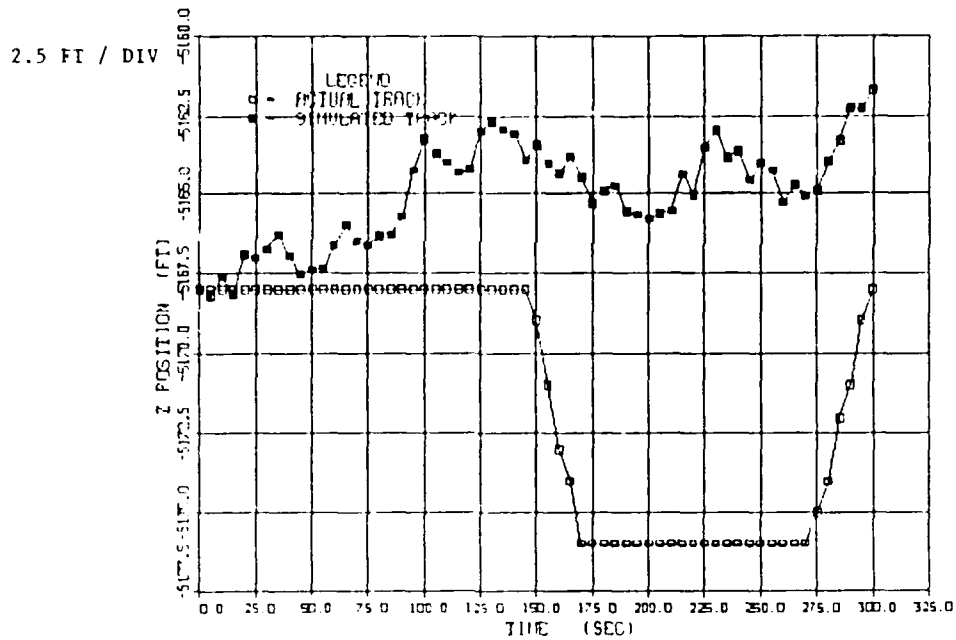
Figure 7b.   Run 6: Actual and Simulated Track Data
Z–Position and Slant Range vs. Time

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Figure 6b). Note that the z-position graphs have a smaller vertical scale than the x- and y-position graphs. Ideally, since the transmitter is attempting to maintain a constant depth, the location should not fluctuate much in the z direction as was the case in Run 5 shown in Figure 6b. However, overall there is little error for these six runs in the model output for the slant range; therefore, the random walk model was considered to be an adequate model of the transmitter's dynamics for the purpose of simulating Doppier shifts, time delays, and propagation losses of the MFM acoustic communication signal.

C.   TRANSMITTER PLATFORM DYNAMICS MODEL

The following three equations are the random walk model used to simulate the transmitter's dynamics. The names of the variables used in the simulation are the same as those used in the equations below. When applicable, the parameters from Figure 1, which are printed in boldface, are equated to their simulation variables, printed in capital letters. The dynamics model is described by:

$$X(LL) = X(LL-1) + (VX(LL) * DELT(LL)) \qquad (1)$$
$$Y(LL) = Y(LL-1) + (VY(LL) * DELT(LL)) \qquad (2)$$
$$Z(LL) = Z(LL-1) + (VZ(LL) * DELT(LL)) \ . \qquad (3)$$

The slant range to the receiver is computed using the following equation;

$$R(LL) = (X(LL)^2 + Y(LL)^2 + Z(LL)^2)^{0.5} \qquad (4)$$

where,

X(LL):   The position of the transmitter relative to the receiver in the x direction during the LLth baud

Y(LL):   The position of the transmitter relative to the receiver in the y direction during the LLth baud

Z(LL): The position of the transmitter relative to the receiver in the z direction during the LLth baud

R(LL): Slant range of the transmitter relative to the receiver during the LLth baud

LL = l: The baud number of the transmitted signal

DELT(LL) = ΔT : The LLth baud length in seconds

VX(LL): The velocity in the x direction during time DELT(LL). VX(LL) is Gaussian distributed with mean = VXAVG and variance = VXVAR. VXAVG and VXVAR are input by the user.

VY(LL): The velocity in the y direction during time DELT(LL). VY(LL) is Gaussian distributed with mean = VYAVG and variance = VYVAR. VYAVG and VYVAR are input by the user.

VZ(LL): The velocity in the z direction during time DELT(LL). VZ(LL) is Gaussian distributed with mean = VZAVG and variance = VZVAR. VXAVG and VXVAR are input by the user.

Note: LL and l are used interchangably throughout.

The rectangular coordinate system of the transmitter's motion is relative to the receiver. The origin (0, 0, 0) is perpendicular to the receiver in the plane of the transmitter (the x-y plane) as shown in Figure 8.

D. TRANSMITTER BEAM PATTERN

If the receiver is not within the transmitter's transmission beam, then the message will not be received. The simulation does not use the actual beam pattern of the transmitter; therefore, it does not know if the receiver is within the beam pattern. The simulation assumes the receiver is within the transmitter's beam the entire time of transmission (i.e., from the beginning of the first signal packet to the end of the last signal packet).

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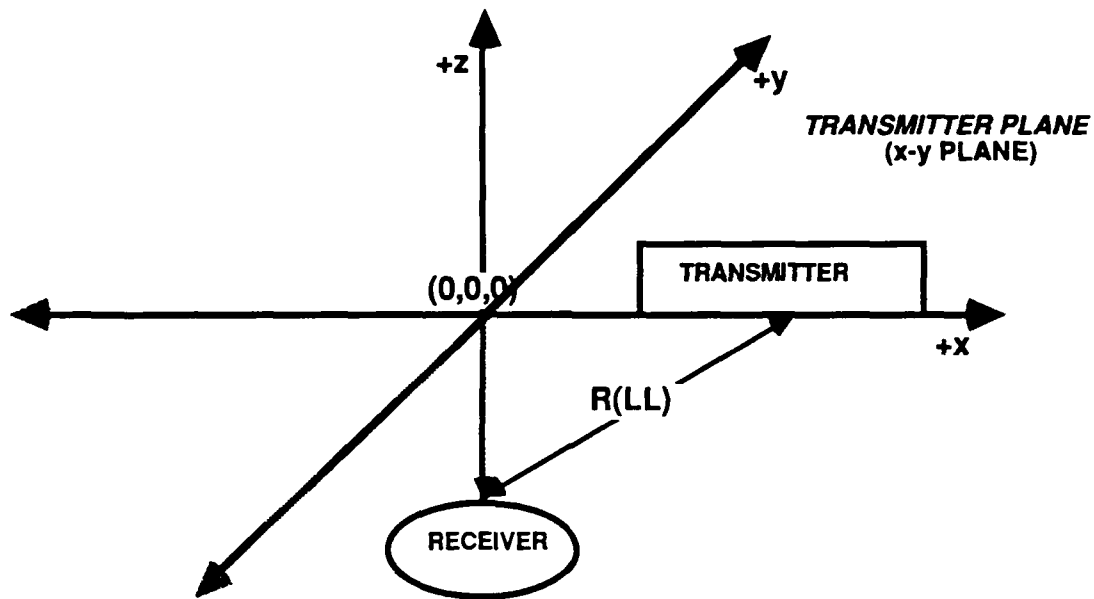
Figure 8. Coordinate System of the Transmitter and Receiver

The angle between the transmitter's slant range and z-position relative to the receiver is computed and output graphically by the simulation. This angle, THETAD(LL) in the simulation, is

$$\text{THETAD(LL)} = \text{arccos}(\ \text{abs(Z(LL))} / \text{R(LL)}\ ) * (180.0 / \text{PI}) \qquad (5)$$

where PI = 3.141592654. Figure 9 illustrates the geometry of the line of sight angle, THETAD(LL) = θ, and THETA0 = θ_0, the half beam width of the transmission beam.

The simulation user can graphically observe the value of THETAD(LL) for each baud and determine for a given THETA0 if the receiver would actually be able to receive the transmitted signal. The simulation does not

stop if THETAD(LL) is greater than the input half beam width of the transmitter, THETA0, but a warning message is output to the screen. The initial position (X0, Y0, Z0) and THETA0 are inputs to the simulation.
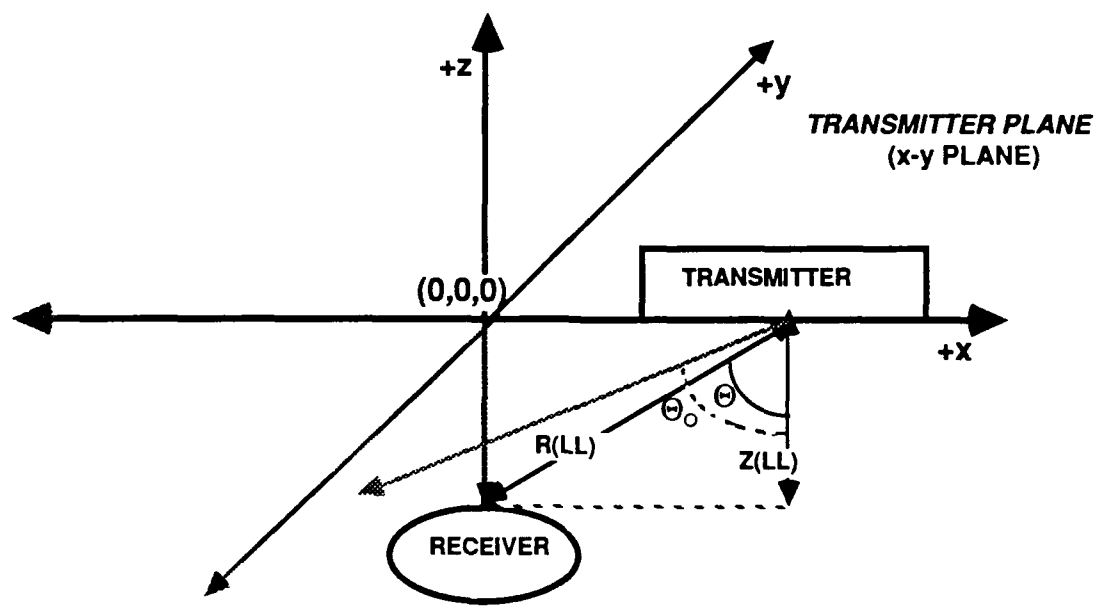
Figure 9. Geometry of the Transmission Beam

III. DERIVATION OF THE SIMULATED RECEIVED MFQPSK SIGNAL

A. DESCRIPTION OF THE TRANSMITTED MFQPSK SIGNAL

Referring to Figure 1, the following definitions are used in MFQPSK [Ref. 1]:

T :   Packet length in seconds

ΔT : Baud length in seconds

k_x : Baud length in number of samples (not in Figure 1)

L :   Number of bauds per signal packet

Δt : Time between samples in seconds

$f_x = 1 / \Delta t$: Sampling frequency in Hz

$\Delta f = 1 / \Delta T$: Minimum frequency spacing between MFM tones in Hz

K :   Number of MFM tones in a baud

l :   Baud number

k :   Harmonic number of the MFM tone

Φ_{lk} : Symbol or phase on the kth tone of the lth baud

Since $\Delta t = \Delta T / k_x$, the sampling frequency is $f_x = k_x * \Delta f$. Consequently there are a maximum of $k_x/2$ tones spaced Δf Hz apart in frequency covering the range from 0 Hz to $f_x/2$ Hz. Here $f_x/2$ is the Nyquist frequency [Ref. 2]. The K tones carry the phase information during each baud. The relationship between f_i, the ith frequency, and k_i, the ith harmonic, is $k_i = f_i / \Delta f$. Some of the tones may not be used (i.e., their amplitudes are set equal to zero) during any or all bauds of the packet. To generate the transmitted bandpass signals

between frequencies f_1 and f_2, only tones between harmonics $k_1 = f_1 / \Delta f$, the minimum harmonic number or tone, and $k_2 = f_2 / \Delta f$, the maximum tone, will be given non-zero amplitudes. The harmonic numbers less than k_1 and greater than k_2 are given zero amplitudes. The number of tones in a baud is $K = k_2 - k_1 + 1$. These K contiguous tones are transmitted with non-zero amplitudes. The signal bandwidth is $W = K * \Delta f$. Thus, the time bandwidth product of the entire signal packet is $TW = L * \Delta T * \Delta f * K = LK$, which is the total number of symbols that can be sent in one signal packet.

A mathematical description of the transmitted MFQPSK signal is necessary to understand some of the parameters used to simulate the received MFQPSK signal. The lth baud of the transmitted signal is described by:

$$x_l(u) = \sum_{k=k_1}^{k_2} x_{lk}(u) \quad ; \quad 0 \le u \le \Delta T \qquad (6)$$

where $x_{lk}(u) = A_{lk} \cos(2\pi k \, \Delta f \, u + \Phi_{lk})$ [Ref. 1]. Here, u is time referenced to the beginning of the baud. Actual real time is $t = t_o + (l * \Delta T) + u$ where t_o is the time at the beginning of the 0th baud (i.e., the beginning time of the first signal packet).

The discrete time signal corresponding to the lth baud is generated by sampling (6) at the sampling intervals $\Delta t = 1/f_x$. Thus, the discrete time signal is given by:

$$x_l(n) = \sum_{k=k_1}^{k_2} x_{lk}(n) \quad ; \quad 0 \le n \le (k_x - 1) \qquad (7)$$

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where $x_{lk}(n) = A_{lk} \cos((2\pi k n) / k_x + \Phi_{lk})$. Here, n is discrete time referenced to the beginning of the baud.

Note that a baud interval of time ΔT seconds contains *exactly* k cycles of tone k. Therefore, adjacent tones differ by one in the number of cycles they generate during a baud.

The phase, Φ_{lk}, may be given one of the four values $\pi/4$, $3\pi/4$, $-3\pi/4$, or $-\pi/4$, which are in quadrants 1 through 4 respectively. These four values make up the symbol set used to code the information or message on the signal. The simulation variable for Φ_{lk} is PHI(LL,K).

The design parameters used in the MFQPSK signal designed and developed by Dr. P. H. Moose at NPS are listed in Table I. These parameters are for a signal packet in a 16 to 20 KHz bandpass channel. These parameters are also used in the simulation to uniquely characterize a chosen baud type (i.e., baud types 1 through 5). The simulation variables in capital letters are equated to the parameters they represent. Recall that LL = 1, the baud number.

B. TIME DELAY AND COMPRESSION/EXPANSION OF THE TRANSMITTED SIGNAL

When the MFQPSK signal is transmitted, there are various factors which can affect the signal such as a moving transmitter, the channel or the medium, and the receiver. In the simulation, these parameters are computed and applied to the transmitted signal, thus producing a model of the received signal baud. By delaying and compressing/expanding the signal in time, the frequencies of the transmittedsignal are shifted in the received signal. Some

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TABLE I

DESIGN PARAMETERS FOR SIGNAL PACKET IN A 16-20KHZ BANDPASS CHANNEL

BAUD TYPE:	1	2	3	4	5
ΔT = DELT(LL) (sec)	1/240	1/120	1/60	1/30	1/15
Δf = DELF(LL) (Hz)	240	120	60	30	15
k_1 = KMIN(LL)	68	135	269	537	1073
f_1 (Hz)	16320	16200	16140	16110	16095
k_2 = KMAX(LL)	83	166	332	664	1328
f_2 (Hz)	19920	19920	19920	19920	19920
k_x = KX(LL)	256	512	1024	2048	4096
f_x (Hz)	61440	61440	61440	61440	61440

of the parameters or variables, which delay and compress or expand the transmitted signal in time, are illustrated in Figure 10 for the lth baud.

Recall $x_l(u)$ is the lth baud of the transmitted signal, which is described by (6). In Figure 10, $x_l(u)$ and $y_l(u)$ are represented with a rectangle for illustration purposes only. Recall $x_l(u)$ is the superposition of $x_{lk}(u)$, for $k_1 \leq k \leq k_2$, over a total of K tones. The frequency of $x_{lk}(u)$ is

$$\omega_k = \omega_x * (k / k_x) \qquad (8)$$
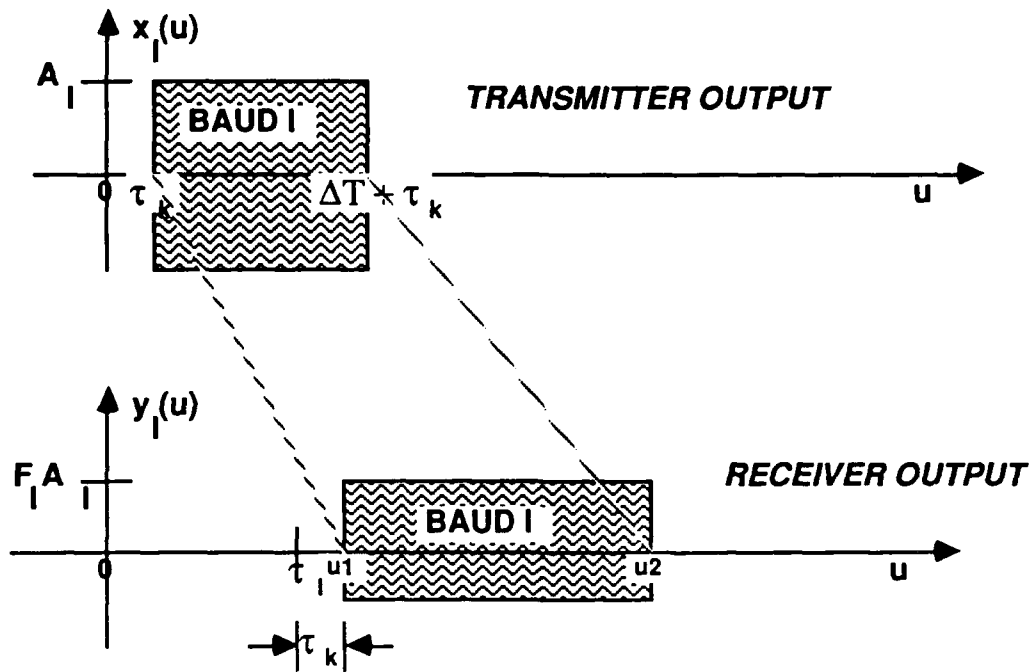
Figure 10.   The lth Baud of the Transmitted Signal Delayed and
Dilated in Time

where $\omega_x = 2\pi f_x$ and f_x is the sampling frequency of the transmitted signal.
Table I lists the values of these parameters and the applicable variables
which are used in the simulation.

When $x_{lk}(u)$ propagates through the transmitter system, it may be
delayed by the transmitter system electronics. In the simulation, this delay is
denoted by τ_k = TAUK(K), and is a constant for all k. (In the simulation,
TAUK(K) may be set to a constant or to any known function of k.) Acoustic
transmission of $x_l(u)$ begins at $u = \tau_k$ and ends at $u = \Delta T + \tau_k$.

When the signal, $x_l(u - \tau_k)$, leaves the moving transmitter, it is delayed
and compressed or expanded in time due to the movement of the transmitter
and the distance it travels through the channel to the receiver. (The time

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delay introduced by the electronics of the receiver has not been included in the simulation.) This time delay and compression/expansion factor will be referred to as £(u). Derivation of £(u) is presented in Appendix A. The time delay and compression/expansion factor due to the medium is described by

$$£(u) = (\tau_l + \alpha_l u) / (1 + \alpha_l) \tag{9}$$

where

$\tau_l =$ The time for the lth baud of the transmitted signal to travel through the channel and arrive at the output of the receiver.

$\alpha_l =$ The Doppler compression factor due to the moving transmitter platform.

The parameters, τ_l and α_l, depend on the slant range to the receiver, R(LL), and the speed of sound of the acoustic channel, C(LL), for the lth baud. Recall R(LL) is computed using equation (4). The speed C(LL) is Gaussian distributed with mean = C0 and variance = CVAR. The parameters C0 and CVAR are input by the simulation user. The parameters, τ_l and α_l, are related to R(LL) and C(LL) in the following manner:

$$\tau_l = TAUL(LL) = R(LL) / C(LL) \tag{10}$$

and

$$\alpha_l = ALPHA(LL) = \dot{R}(LL) / C(LL) \tag{11}$$

where

$$\dot{R}(LL) = \frac{[(X(LL) * VXAVG) + (Y(LL) * VYAVG) + (Z(LL) * VZAVG)]}{R(LL)} \tag{12}$$

Recall X(LL), VXAVG, Y(LL), VYAVG, Z(LL), and VZAVG were described in the previous chapter with equations (1) through (3). TAUL(LL) and

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ALPHA(LL) are the simulation variables for τ_l and α_l, respectively. When the transmitted signal, $x_l(u - \tau_k)$, arrives at the receiver output, it is compressed/expanded in time by $\pounds(u)$.

The received signal is

$$y_l(u) = \sum_{k=k_1}^{k_2} y_{lk}(u) \qquad ; u_1 \le u \le u_2$$

(13)

where

$$y_{lk}(u) = F_{lk} \, A_{lk} \, \cos(2\pi \, k \, \Delta f \, (u - \tau_k - \pounds(u)) + \Phi_{lk}),$$

(14)

and F_{lk} is the attenuation of tone k due to the propagation through the channel. The signal, $y_l(u)$, will begin at time u_1 and end at time u_2. The signal, $x_l(0)$, transmitted at time $u = 0$ is the same signal, $y_l(u_1)$, that arrives at the receiver at time u_1; and likewise, the signal, $x_l(\Delta T)$, transmitted at time $u = \Delta T$ is the same signal, $y_l(u_2)$, which arrives at the receiver at time u_2 (see Figure 10). Since it is the same signal, the times that this signal arrives at the output of the receiver may be equated to the corresponding times that this signal was sent to yield

$$u_1 = (1 + \alpha_l) \, \tau_k + \tau_l$$

(15)

and

$$u_2 = (1 + \alpha_l) \, \Delta T + (1 + \alpha_l) \, \tau_k + \tau_l \ .$$

(16)

It is obvious that $y_l(u)$ is also a superposition of signals, $y_{lk}(u)$. However, $y_{lk}(u)$ is at frequency ω_k', where

$$\omega_k' = \omega_k / (1 + \alpha_l)$$

(17)

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due to the moving transmitter. At the receiver, $y_l(u)$ will be sampled at

$$\omega_y = \omega_x / (1 + \alpha_m) \tag{18}$$

where α_m is the Doppler factor associated with the mth Doppler channel. If $\alpha_l = \alpha_m$, then the frequencies of the received signal are exactly in the center of Doppler channel m. However, in general, α_l will fall between Doppler channels and there will be some residual Doppler mismatch $\alpha_l - \alpha_m$. If the channels are spaced at intervals $\Delta\alpha$, then the maximum mismatch will be $\pm$ $\Delta\alpha/2$. In the following section, a derivation for the spacing of the Doppler channels $\Delta\alpha$ is presented.

C.   DOPPLER ESTIMATION AND DOPPLER CHANNELS

Several processing approaches were considered for estimating the channel model parameters. However, the ultimate goal is to send a signal through a bandpass channel using MFQPSK modulation on a succession of bauds which constitute a signal "packet". In the presence of white noise, the optimum receivers for these signals are filters matched to each of the tone/phase combinations. Recall the four phases are $\{\pi/4, 3\pi/4, -3\pi/4, -\pi/4\}$ for a MFQPSK signal. Therefore, a filter matched to the transmitted signal with $\Phi_{lk} = \pi/4$ will have a positive output when $\pi/4$ is sent, a negative output when $-3\pi/4$ is sent, and an output of zero when $3\pi/4$ or $-\pi/4$ is sent, provided that the receiver is synchronized with the signal. Similarly, the output of a filter matched to the transmitted signal with phase equal to $3\pi/4$ is positive when $3\pi/4$ is sent, negative when $-\pi/4$ is sent, and zero otherwise. Also filter pairs matched to the ith frequency of the transmitted signal, $f_i = k_i\Delta f$, produce zero outputs for all phases at the frequency $f_j = k_j\Delta f$ when $k_i \neq k_j$.

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To summarize, a matched filter system for MFQPSK signals with K tones and baud length $\Delta T = 1/\Delta f$ consists of K pairs of filters, one filter of each pair matched to phase $\pi/4$ and the other filter of the pair matched to phase $3\pi/4$. Each filter pair output demodulates the phase information encoded on a particular tone.

Since the maximum Doppler shift will be present on the highest frequency or tone in the baud, the response of the filter pair matched to this tone will be analyzed to show how the Doppler channel spacing, $\Delta\alpha$, is derived and how α_m is computed in the simulation.

Let $h_i(u)$ and $h_q(u)$ be the filters matched to the signals:

$$x_{lk_2,i}(u) = A_{lk_2}\cos(2\pi\, k_2\, \Delta f\, u + \pi/4)\,;\, 0 \le u \le \Delta T \qquad (19a)$$

and

$$x_{lk_2,q}(u) = A_{lk_2}\cos(2\pi\, k_2\, \Delta f\, u + 3\pi/4)\,;\, 0 \le u \le \Delta T. \qquad (19b)$$

where the indices i and q denote in-phase and quadrature phase, respectively. Thus, $h_i(u)$ and $h_q(u)$ are described as follows:

$$h_i(u) = 2\,\cos(2\pi\, k_2\, \Delta f\, (\Delta T - u) + \pi/4)\,;\, 0 \le u \le \Delta T \qquad (20a)$$

and

$$h_q(u) = 2\,\cos(2\pi\, k_2\, \Delta f\, (\Delta T - u) + 3\pi/4)\,;\, 0 \le u \le \Delta T. \qquad (20b)$$

Now the received signal from equation (14), given $\pi/4$ is the phase of the tone k_2, is

$$y_{lk_2}(u) = F_{lk_2}A_{lk_2}\cos(2\pi\, k_2\, \Delta f\, (u - \tau_{k_2} - \pounds(u)) + \pi/4)\,;\, u_1 \le u \le u_2\ . \qquad (21)$$

The received signal, given $3\pi/4$ is the phase of the tone k_2, is similar to (21) with $\Phi_{lk} = 3\pi/4$ instead of $\pi/4$ as above.

As illustrated in Figure 11, the output of the matched filter pair, in general, is the convolution of $h(\tau)$ and $y_{lk}(\tau)$ given by

$$z_{lk,i}(u) = \int_0^{\tau_{max}} h_i(\tau) \, y_{lk}(u - \tau) \, d\tau$$

(22a)

and

$$z_{lk,q}(u) = \int_0^{\tau_{max}} h_q(\tau) \, y_{lk}(u - \tau) \, d\tau \, ,$$

(22b)

where $\tau_{max} = u_2 - u_1$. Evaluating (22a) and (22b), the following expressions are obtained for the output of the matched filter pairs due to tone k_2:

$$z_{lk_2,i}(u) = F_{lk_2} A_{lk_2} \int_0^{\tau_{max}} \cos\,[2\pi k_2 \Delta f(u - \pounds(u - \tau) - \tau_{k_2} - \Delta T)] \, d\tau$$

(23a)

and

$$z_{lk_2,q}(u) = F_{lk_2} A_{lk_2} \int_0^{\tau_{max}} \sin\,[2\pi k_2 \Delta f(u - \pounds(u - \tau) - \tau_{k_2} - \Delta T)] \, d\tau$$

(23b)
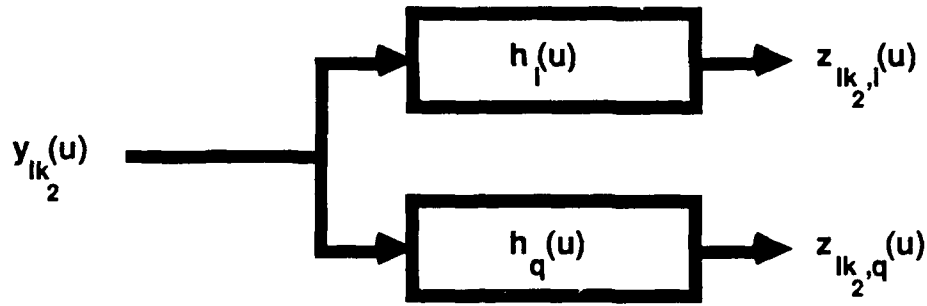
Figure 11. Matched Filter Pair for Tone k_2

Inserting equation (9) for £(u) in the above two equations yields

$$z_{lk_2,i}(u) = F_{lk_2} A_{lk_2} \int_0^{\tau_{max}} \cos\left[(2\pi k_2 \Delta f)\,(u\text{-}t_1 + \alpha_1 \tau - (1+\alpha_1)\,(\tau_{k_2} + \Delta T))\,(1+\alpha_1)^{-1}\right] d\tau$$

(24a)

and

$$z_{lk_2,q}(u) = F_{lk_2} A_{lk_2} \int_0^{\tau_{max}} \sin\left[(2\pi k_2 \Delta f)\,(u\text{-}t_1 + \alpha_1 \tau - (1+\alpha_1)\,(\tau_{k_2} + \Delta T))\,(1+\alpha_1)^{-1}\right] d\tau$$

(24b)

Recall that the received signal arrives when $u_1 \leq u \leq u_2$ with

$$u_1 = [(1 + \alpha_1)\,\tau_k + \tau_1] \quad \text{and} \quad u_2 = [(1 + \alpha_1)\,\Delta T + (1 + \alpha_1)\,\tau_k + \tau_1] \quad .$$

Also recall from the previous section that α_1 is the Doppler compression/ expansion factor due to the moving transmitter. Note that for $\alpha_1 \leq 0$, $\tau_{max} = u_2 - u_1 \leq \Delta T$ and for $\alpha_1 > 0$, $\tau_{max} = u_2 - u_1 > \Delta T$. Using $\tau_{max} = \Delta T / (1+\alpha_1)$ for the upper limit in the integration in the convolution integrals (24a) and (24b), will not affect the integrals significantly because α_1 is of the order 10^{-3}; therefore, dividing by $(1 + \alpha_1)$ is very close to dividing by 1.

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Sampling $z_{lk_2,i}(u)$ and $z_{lk_2,q}(u)$ at $u = u_2$ and evaluating their integrals in (24a) and (24b) from 0 to τ_{max} yields

$$z_{lk_2,i}(u) \Big|_{u=u_2} = F_{lk_2} A_{lk_2} \Delta T (1+\alpha_1) \frac{\sin [2\pi k_2 \alpha_1]}{2\pi k_2 \alpha_1} = Z_{lk_2,i}(\alpha_1) \tag{25a}$$

and

$$z_{lk_2,q}(u) \Big|_{u=u_2} = F_{lk_2} A_{lk_2} \Delta T (1+\alpha_1) \frac{(1-\cos [2\pi k_2 \alpha_1])}{2\pi k_2 \alpha_1} = Z_{lk_2,q}(\alpha_1) . \tag{25b}$$

The total power, P_{lk}, in the kth tone of the lth baud is one-half the amplitude, $F_{lk} A_{lk}$, of the received signal squared. That is

$$F_{lk} A_{lk} = (2 P_{lk})^{0.5}. \tag{26}$$

Also assume $1+\alpha_1 = 1$; therefore, $\Delta T (1+\alpha_1)$ is approximately equal to ΔT. With these substitutions, equations (25a) and (25b) become

$$Z_{lk_2,i}(\alpha_1) = (2P_{lk_2})^{0.5} \Delta T \frac{(\sin [2\pi k_2 \alpha_1])}{2\pi k_2 \alpha_1} \tag{27a}$$

and

$$Z_{lk_2,q}(\alpha_1) = (2P_{lk_2})^{0.5} \Delta T \frac{(1-\cos [2\pi k_2 \alpha_1])}{2\pi k_2 \alpha_1} . \tag{27b}$$

A graph of $Z_{lk_2,i}(\alpha_1)$ and $Z_{lk_2,q}(\alpha_1)$ from (27a) and (27b), respectively, is shown in Figure 12.

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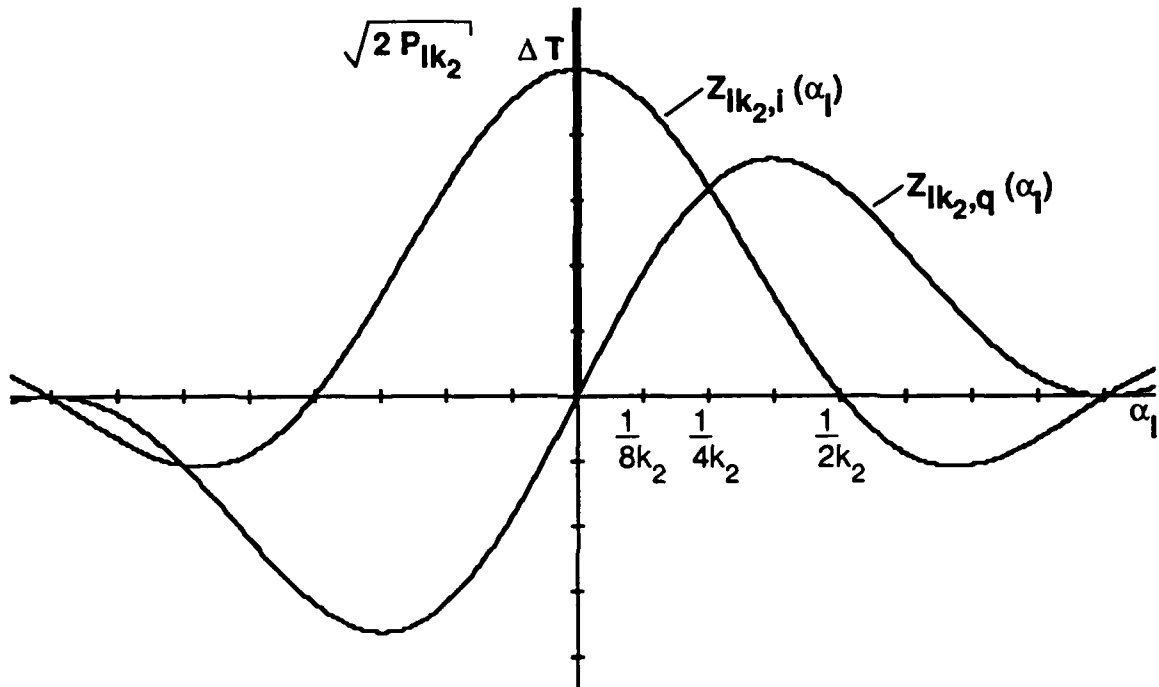
Figure 12. Matched Filter Outputs Versus Doppler Shift

Note that at $\alpha_l = 1/(4 k_2)$, $| Z_{lk_2,i} | = | Z_{lk_2,q} |$; this is illustrated in Figure 12. This means that the response of the in-phase and quadrature channels are equal even though an in-phase $\pi/4$ symbol was sent; therefore, a significant decoding error would be made if $\alpha_l = 1/(4 k_2)$. It is obvious that, at $\alpha_l = 0$, $Z_{lk_2,i}(0)$ is maximum and $Z_{lk_2,q}(0)$ is zero; in which case, no decoding errors would be made. The ratio of $Z_{lk_2,i}$ to $Z_{lk_2,q}$ below, evaluated for various α_l's less than $1/(4 k_2)$, produces the maximum width of the Doppler channels, $\Delta\alpha$.

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The ratio is described by:

$$\frac{Z_{1k_2,i}}{Z_{1k_2,q}} = \frac{\sin(2\pi k_2 \alpha_1)}{1 - \cos(2\pi k_2 \alpha_1)} \; .$$

(28)

For $\alpha_1 = 1/(16\,k_2)$, the ratio above is ≥ 5.03 which is 14 dB. This is called inter-symbol separation.

The value of 14 dB was considered the minimum amount of inter-symbol separation to be tolerated. Recall that the maximum amount of Doppler mismatch is $\alpha_1 - \alpha_m = \pm\Delta\alpha/2$. Therefore, $\Delta\alpha$, the Doppler channel spacing, is equal to $1/(8\,k_2)$ and the magnitude of the maximum amount of residual Doppler mismatch is $\Delta\alpha/2 = 1/(16\,k_2)$, where α_m is the Doppler factor associated with the mth Doppler channel. In the simulation, $\Delta\alpha = \text{DELALF(LL)} = 1/(8*\text{KMAX(LL)})$.

Solving $\alpha_1 - \alpha_m = \pm\,\Delta\alpha/2$ for α_m, yields

$$\alpha_m = \alpha_1 \pm \Delta\alpha/2 \; ,$$

(29)

where α_m, the Doppler factor due to the mth Doppler channel, is equal to $m\,\Delta\alpha$. Substituting $\alpha_m = m\,\Delta\alpha$ into (29) yields the following expressions for the lower bound, m_1, and upper bound, m_2, of m, the Doppler channel number as

$$m_1 = (\alpha_1 / \Delta\alpha) - 0.5$$

(30a)

and

$$m_2 = (\alpha_1 / \Delta\alpha) + 0.5 \; .$$

(30b)

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Since m is an integer between m_1 and m_2, it is computed as

$$m = \text{int}[(m_1 + m_2)/2].\qquad(31)$$

In the simulation, m = M(LL) and is computed using (31); and

$$\alpha_m = \text{ALPHAM(LL)} = \text{M(LL)} * \text{DELALF(LL)}.$$

D. THE ATTENUATION FACTOR DUE TO AN ACOUSTIC CHANNEL

When the transmitted signal, $x_{lk}(u - \tau_k)$, travels through the medium or channel, its amplitude will be attenuated by a factor F_{lk} due to the channel. In the simulation, the user has the input option to have this attenuation applied to the received signal or to let all the received signal tones have equal amplitudes. The quantity AA(LL,K) is the amplitude of the lth baud and k^{th} tone of the simulation's received signal. If the user chooses to have equal amplitudes, then AA(LL,K) is set equal to 1 for all LL and all K. If the user desires the attenuation to be put on the received signal, then $\text{AA(LL,K)} = F_{lk}$, which is computed below.

In general, the transmission loss depends on frequency, depth, pressure, and temperature. However, when specific propagation conditions are of no interest and only a rough approximation of the transmission loss is adequate; the spherical-spreading law plus an added loss due to absorption is equal to the transmission loss [Ref. 4]. Therefore, the transmission loss in dB is

$$\text{TL} = (20 \log_{10} R) + \beta R \qquad(32)$$

where

R is the depth of the receiver from the surface in feet and

β is the attenuation coefficient due to absorption in dB per foot.

Thus, the following expression for F_{lk} is used in the simulation:

$$F_{lk} = 10^{-(TL/20)}. \tag{33}$$

The attenuation coefficient, β, is described by [Ref. 4]

$$\beta = \left(\frac{0.1\,f^2}{1 + f^2} + \frac{40\,f^2}{4.100 + f^2} + (0.000275)\,f^2 + 0.003 \right) (3000)^{-1} \tag{34}$$

where

$$f = (k\,\Delta f) / 1000 \text{ in kilohertz.}$$

This expression applies for a temperature of 39°F (4°C), which is an average seawater temperature, and a depth of about 3000 feet. The constant 0.003 is added to take care of the attenuation at very low frequencies [Ref. 4]. In the simulation, ABSORP = β.

The absorption of seawater decreases by about 2 percent for every increase of 1000 feet in depth [Ref. 4]. Note in the simulation, the transmitter is at a depth of 1000 feet below the ocean's surface. This depth is set with the simulation variable TXDEP. Thus, the receiver is R = TXDEP + Z(LL) below the ocean's surface. TXDEP is not an input of the simulation; therefore, to change TXDEP from 1000, one must change TXDEP in the simulation code. The simulation computes β using equation (34), then increases β by 2 percent for every 1000 feet that the receiver's depth exceeds 3000 feet from the

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ocean's surface or decreases β by 2 percent for every 1000 feet that the receiver's depth is less than 3000 feet from the ocean's surface.

E. THE SAMPLED RECEIVED SIGNAL WITHOUT NOISE

The received signal, $y_l(u)$, is sampled at an interval

$$\Delta u = [\Delta T (1 + \alpha_m)] / k_x . \tag{35}$$

(Recall $\omega_y = 2\pi k_x / [(1 + \alpha_m) \Delta T]$, (18), is the sampling frequency.) The parameters ΔT and k_x are defined in section A. The Doppler factor, $\alpha_m = m \Delta\alpha$, associated with the mth Doppler channel was derived in the previous section. The first sample will be taken at $\hat{u}_1$, which is the best estimate of u_1, the beginning or synchronization point of the lth baud at the receiver. This is illustrated in Figure 13. The synchronization error of the lth baud is $\Delta u_1 = u_1 - \hat{u}_1$. The simulation variable names for the above parameters are $u_1 = U1$, $\hat{u}_1 = UHAT1$, and $\Delta u_1 = DELU1$.

The sampled received signal for the lth baud is (13) with $u = n\Delta u + \hat{u}_1$. When $\hat{u}_1 = u_1$, the sampled received signal for the lth baud is (13),

$$y_l(n) = y_l (n\Delta u + u_1) = \sum_{k=k_1}^{k_2} y_{lk}(n\Delta u + u_1) \quad ; \quad 0 \le n \le (k_x - 1) \tag{36}$$

where

$$y_{lk}(n\Delta u + u_1) = F_{lk}A_{lk}\cos[2\pi k \, \Delta f (n \, \Delta u - (\pounds(n \, \Delta u + u_1) - \pounds(u_1))) + \Phi_{lk}]. \tag{37}$$

Substituting (9) for $\pounds(u)$ and computing $\pounds(n \, \Delta u + u_1) - \pounds(u_1)$ yields

$$\pounds(n \, \Delta u + u_1) - \pounds(u_1) = (\alpha_l \, n \, \Delta u) / (1 + \alpha_l). \tag{38}$$

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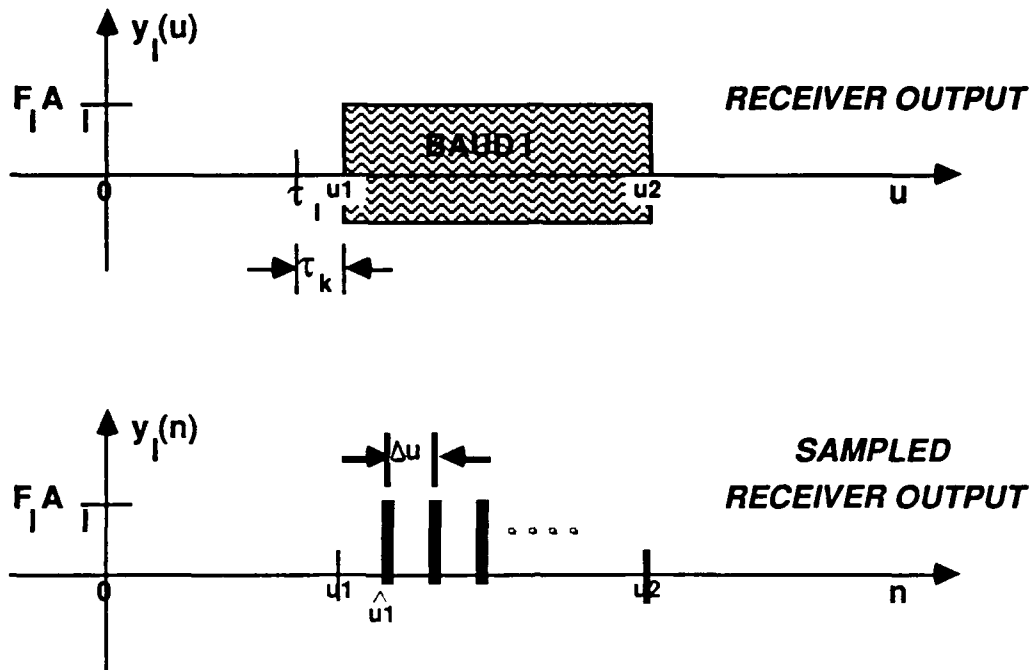
Figure 13. The Sampled Receiver Output

Now substituting $\Delta u = [\Delta T (1 + \alpha_m)] / k_x$ into (38) produces

$$\pounds(n \, \Delta u + u_l) - \pounds(u_l) = [\alpha_l \, n \, \Delta T \, (1 + \alpha_m)] / k_x \, (1 + \alpha_l). \tag{39}$$

Collecting the results from (39) and $n\Delta u = [n \, \Delta T \, (1 + \alpha_m)] / k_x$ together and substituting them into (37) yields the following sampled received signal for the lth baud and k^{th} tone:

$$y_{lk}(n) = F_{lk} A_{lk} \, \cos[(2\pi k/k_x) \, ((1+\alpha_m) / (1+\alpha_l)) \, n + \Phi_{lk}]. \tag{40}$$

If a factor v_{lk} is added to n to account for random timing jitters of the lth baud and k^{th} tone due to synchronization error, then (40) becomes

$$y_{lk}(n) = F_{lk} A_{lk} \, \cos[(2\pi k/k_x) \, ((1+\alpha_m) / (1+\alpha_l)) \, (n - v_{lk}) + \Phi_{lk}] \tag{41}$$

$$\text{for } 0 \le n \le (k_x - 1) ,$$

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where

$$v_{lk} = \frac{\Delta u_1 \, k_x}{\Delta T \, (1+\alpha_m)} \cdot \qquad (42)$$

Finally summing (41) over all the tones in the lth baud gives

$$y_l(n) = \sum_{k=k_1}^{k_2} y_{lk}(n) \qquad ; \quad 0 \le n \le (k_x-1) \; . \qquad (43)$$

Summarizing, (41) is the final computational form of the received signal which is denoted by YY in the simulation. The received signal, YY, is then summed over all the k's for each baud from KMIN(LL) to KMAX(LL) to yield YRX in the simulation, which is $y_l(n)$ above. In the simulation, all the packets are contiguous. The continuous sampled received signal is YYRX(II) where II ranges from 0, which is the first sample of the first baud (LL=1) in the first packet, to NPTS, which is the last sample of the last baud (LL = BDTOTL = the total number of bauds) in the last packet. YYRX(II) is equal to YRX plus NOISE with all the packets contiguous and the bauds within each packet contiguous, where NOISE is white Gaussian noise added to the received signal. NOISE, which depends on the input signal-to-noise ratio (SNR), is described in the next section.

F. SIGNAL PLUS ADDITIVE NOISE

Inevitably noise will be added to the signal, either from the environment, the electronics, or both. This noise is assumed to be additive white Gaussian noise. Since it is assumed that the received signal, $y_l(u)$, has been ideally

bandlimited to one-half the sampling frequency, there is no power in frequencies greater than or equal to $f_x/2$. Recall that the signal is sampled at the transmitter at f_x. If $w(u)$ is white noise and has the white power spectral density equal to $N_0/2$, then

$$E[w(u)] = E[w(n)] = 0 \tag{44}$$

and

$$\sigma^2 = VAR[w(u)] = VAR[w(n)] = N_0 f_x/2, \tag{45}$$

where $w(n)$ is the white noise sequence [Ref. 1].

Let the receiver input SNR be defined as the signal power in bandwidth W divided by the noise power in bandwidth W. The average tone signal power in the lth baud is defined as

$$PAVG_l = (1/K) \sum_{k=k_1}^{k_2} P_{lk} \tag{46}$$

where P_{lk} is the total power in the kth tone of the lth baud. (Recall that K is the total number of MFM tones in the lth baud.) Therefore, the wideband input SNR for the lth baud [Ref. 1] is

$$SNRWB_l = (K\,PAVG_l)/(W\,N_0) = PAVG_l/(\Delta f\,N_0) = PAVG_l\,k_x/(2\,\sigma_l^2), \tag{47}$$

because $\Delta f = f_x / k_x$ and $N_0 = (2\,\sigma_l^2)/f_x$. The narrowband input SNR for the lth baud is

$$SNRNB_l = P_{lk}/(\Delta f\,N_0) = P_{lk}\,k_x/(2\,\sigma_l^2). \tag{48}$$

Note that if all the tones have equal amplitudes (i.e., AA(LL,K) = 1), then $PAVG_l$ is equal to P_{lk}; in which case, $SNRWB_l = SNRNB_l$.

Solving (47) for σ_l^2 yields

$$\sigma_l^2 = [(1/K) \sum_{k=k_1}^{k_2} P_{lk}] \, k_x \, / \, (2 \, SNRWB_l) \quad .$$

(49)

Recall $P_{lk} = (AA_{lk}^2) / 2$, where AA_{lk} is the amplitude of the kth tone and l^{th} baud of the received signal. In the simulation, NOISE is added to the received signal, $y_l(n) = YRX$, which is white Gaussian noise with zero mean and variance $= \sigma_l^2$ computed in (49). A single input wideband SNR is input by the simulation user for all bauds, l. The following simulation variables are used for the above parameters:

$\sigma_l^2 = NOSVAR(LL)$,

$K = KPTS(LL)$,

$AA_{lk} = AA(LL,K)$,

$k_x = KX(LL)$, and

$SNRWB_l = SNRIN$.

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IV. TESTING THE SIMULATION AGAINST MFQPSK THEORY

A. BACKGROUND

The purpose of developing this simulation was to create an experimental tool to test and analyze various Doppler, synchronization, and coding techniques and/or algorithms to support the implementation and testing of an MFQPSK acoustic link. Before the simulation could be used in this capacity, it had to be tested against MFQPSK signal theory. There was no actual test data to use for comparison; therefore, tests were performed to verify that the simulation output, the received signal, agrees with the theory of MFQPSK signals. It is anticipated that the simulation will be tested further when actual test data is available.

It has been shown that, in theory, the output SNR (SNR_{OUT}) equals the input narrowband SNR ($SNRNB_{IN}$) for MFQPSK signals in additive white Gaussian noise memoryless channels that are demodulated coherently with a Discrete Fourier Transform (DFT) [Ref. 1]. The results of testing the simulation's ability to reproduce this result are described and presented below.

B. THE TESTING METHODOLOGY

The following approach was taken to verify that the simulation performs at an acceptable level (i.e., $SNR_{OUT} = SNRNB_{IN}$). A set of 5 signal packets were generated with the simulation with a given input wideband SNR ($SNRWB_{IN}$). Each packet contained a single unique baud type; therefore, each simulation run consisted of the 5 different baud types. Referring to Table I, the 5 different baud types are

Baud Type 1: 256 sample points,

Baud Type 2: 512 sample points,

Baud Type 3: 1024 sample points,

Baud Type 4: 2048 sample points, and

Baud Type 5: 4096 sample points.

Recall from Chapter III that if all the tones have the same power within a baud (i.e., the amplitudes of tones within a baud are all equal), then the input wideband SNR is equal to the input narrowband SNR for that baud. All the runs used for this analysis were produced with normalized amplitudes within each baud. The phases that were transmitted were generated randomly by the computer code.

It is known that the minimum input SNR required to accurately decode MFQPSK is approximately 15 dB. This analysis considered input narrowband SNRs from 0 to 20 dB. The initial conditions for the transmitter's position and velocity were chosen from the NOSC track data. These initial conditions are

X0 = -5.0 feet, VXAVG = 1.0 ft/sec, VXVAR = 0.0025 (ft/sec)^2,

Y0 = 5.0 feet, VYAVG = 5.4 ft/sec, VYVAR = 0.0025 (ft/sec)^2,

Z0 = -5400.0 feet, VZAVG = 0.0 ft/sec, and VZVAR = 0.0025 (ft/sec)^2.

The initial conditions for the speed of sound was C0=4900 ft/sec and CVAR=0. Note the transmitter was placed almost directly above the receiver to reduce the amount of Doppler shift in the received signal. A description and examples of the simulation inputs and outputs are presented in Appendix B.

An average output SNR was estimated for each baud using the real and imaginary parts of the received signal's DFT. A k_x-point DFT was performed on each baud of the received signal to decode the phase of the received signal. (The real and imaginary parts of the received signal's DFT contain the phase information sent on each of the tones between harmonic numbers k_1 and k_2 in each baud.) If $\Phi_{lk} = \pi/4$ was transmitted (which is in the first quadrant), then the received signal DFT's real and imaginary parts should be located in the first quadrant (i.e., the real and imaginary parts should be positive). Likewise, for a transmitted phase angle of $3\pi/4$, $-3\pi/4$, or $-\pi/4$ the received signal's DFT phase angle (i.e., real and imaginary parts) should lie in the second, third, or fourth quadrant, respectively. The conditional distributions within each baud were produced by categorizing the DFT's real and imaginary parts on a tone-by-tone basis given the phase that was transmitted on each tone between harmonic numbers k_1 and k_2. There are two distributions of the received signal DFT in each of the four quadrants, a distribution of the real part and one of the imaginary part. The real and imaginary parts are statistically independent [Ref. 1].

For example, in baud type 5 there are 4096 samples of the 256 tones encoded with phase information between harmonic numbers k_1=1073 and k_2=1328. There should be approximately 64 (i.e., one fourth of 256) tones transmitted and received in each quadrant because the four transmitted phases were generated randomly from a uniform distribution. The distribution of each DFT component should be close to a Gaussian distribution.

Once the DFT of K tones in each baud were conditionally partitioned by knowing the transmitted phase (i.e., quadrant) into the four quadrants, then

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the sample mean and variance were computed on each of 4 real part distributions and each of the 4 imaginary part distributions. The sample means are, $\bar{R}_i$ and $\bar{I}_i$, where the subscript i denotes the quadrant number. The sample means for each quadrant in a baud are

$$\bar{R}_i = (1/N_i) \sum_{j=1}^{N_i} R_{ij} \quad \text{for i = 1, 2, 3, or 4}$$

(50)

and

$$\bar{I}_i = (1/N_i) \sum_{j=1}^{N_i} I_{ij} \quad \text{for i = 1, 2, 3, or 4}$$

(51)

where

$N_i =$ the number of times the phase corresponding to quadrant i was sent in that baud

$R_{ij} =$ the jth real part of the received signal's DFT corresponding to a harmonic number that was transmitted with phase in quadrant i

$I_{ij} =$ the jth imaginary part of the received signal's DFT corresponding to a harmonic number that was transmitted with phase in quadrant i.

The sample variances of a given quadrant in a baud are

$$S_{R,i}^2 = [1/(N_i-1)] \sum_{j=1}^{N_i} (R_{ij} - \bar{R}_i)^2$$

(52)

and

$$S_{I,i}^2 = [1/(N_i-1)] \sum_{j=1}^{N_i} (I_{ij} - \bar{I}_i)^2 .$$

(53)

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The output SNRs for the ith quadrant within a baud were estimated by the sample mean squared divided by the sample variance as follows [Ref.1]

$$\widehat{SNROUT}_{R,i} = (\bar{R}_i)^2 / S_{R,i}^2 \tag{54}$$

and

$$\widehat{SNROUT}_{I,i} = (\bar{I}_i)^2 / S_{I,i}^2 . \tag{55}$$

The mean squared value of the real and imaginary parts of the DFT is a measure of the signal power received in that quadrant. The variance of the real and imaginary parts is a measure of the noise received in that quadrant. These values form an estimate of the output SNR. Appendix C contains the statistics for the real and imaginary distributions of each of the four quadrants and the overall statistics for each baud.

To derive an estimate of the output SNR for each baud, the real and imaginary distributions of each quadrant were averaged over all four quadrants. There were 2K points in the overall mean and in the overall variance due to the K real parts and K imaginary parts. $\widehat{SNROUT}$ is the overall mean squared divided by the overall variance. Figure 14 is a plot of $\widehat{SNROUT}$ versus the input narrowband SNR, where $\widehat{SNROUT}$ is described by

$$\widehat{SNROUT} = \frac{\left(\sum_{i=1}^{4} \left[N_i \left| \bar{R}_i \right| + N_i \left| \bar{I}_i \right| \right] \right) / \left(2 \sum_{i-1}^{4} N_i \right)}{\left(\sum_{i=1}^{4} \left[\left(N_i - 1 \right) S_{R,i}^2 + \left(N_i - 1 \right) S_{I,i}^2 \right] \right) / \left(\left(2 \sum_{i=1}^{4} N_i \right) - 1 \right)} \tag{56}$$

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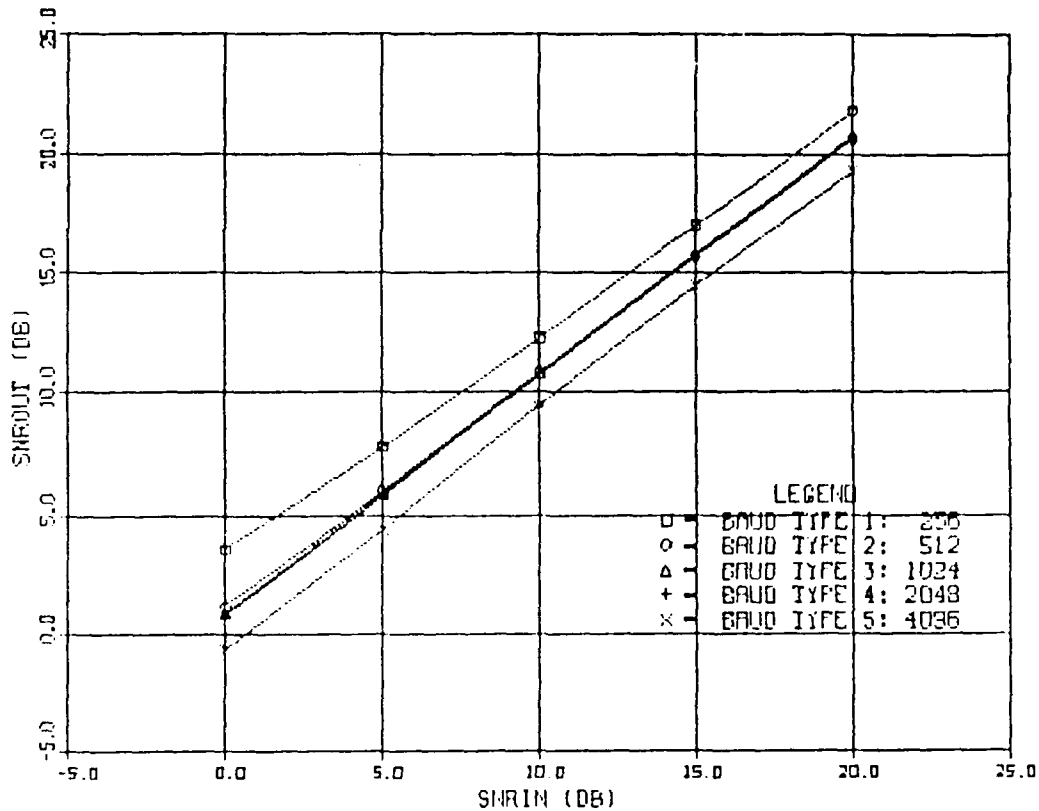
Figure 14. SNR_{OUT} vs. SNR_{IN} for Five Different Bauds

where

$$\sum_{i=1}^{4} N_i = K$$

is the total number of MFM tones in each baud (see Chapter III, Section A).

Note that the absolute value of the sample means must be used in the estimate (56), because, if they are not used, the overall mean will always be close to zero. This is due to the fact that the eight sample means are approximately equal in magnitude, but four have opposite signs due to their quadrant location.

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C. RESULTS

If the simulation results agree with the MFQPSK theory, then the input narrowband SNR should equal the output SNR. As can be seen in Figure 14, baud types 2, 3, and 4 agree within one dB with the MFQPSK theory. The longest baud, baud type 5, and the shortest baud, baud type 1, produce estimated output SNRs that are within two dB of the input SNR. As the input SNR increases, it can be observed from Figure 14 that the output SNRs for baud types 1 and 5 converge towards the value of the input SNR.

In conclusion, the output SNR is approximately equal to the input narrowband SNR; therefore, the simulation reproduces what the MFQPSK theory predicts.

V. OUTPUT SNR DEGRADATION DUE TO DOPPLER

A. BACKGROUND

When the MFQPSK signal is transmitted from a moving platform, Doppler will shift the received signal's frequencies. The received signal (at frequency ω_k', (17),) is sampled at ω_y, (18), baud by baud. In general, α_l, the time Doppler compression/expansion factor, does not equal α_m, the Doppler factor set for the mth Doppler channel; hence, there will be some Doppler mismatch. This mismatch will cause degradation in the output SNR.

The output SNR degradation versus the Doppler mismatch was simulated and compared to theory. If inter-symbol interference (ISI), which is energy leaking from one tone to the next tone, is negligible, then the output SNR should degrade due to the Doppler mismatch as sinc($\pi\epsilon/4$) from (27a) where $\epsilon = \alpha_l/\Delta\alpha$. Derivation of the theoretical degradation and the comparison to the simulation results follows.

B. ANALYSIS APPROACH

The simulation was run several times. Each run generated one signal packet with an input narrowband SNR = 15 dB. (The values of 15 dB is considered to be the minimum input SNR required to decode the MFQPSK signal.) A packet consisted of five bauds of baud type 3 (i.e., $k_x = 1024$ points and K = 64 tones). Baud type 3 was chosen based on the results of the simulation versus MFQPSK theory in the previous chapter. Recall that baud type 3 was in close agreement with the theory of MFQPSK signals (i.e., $SNR_{OUT} = SNRNB_{IN}$). The Doppler channel, m, was set to zero for all the

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packets. If $m = 0$, then $\alpha_l - \alpha_m = \alpha_l - m\,\Delta\alpha = \alpha_l$. Thus, by increasing α_l, the amount of Doppler mismatch increases.

Each packet was generated with a different fixed value of $\alpha_l = \varepsilon\,\Delta\alpha$, where ε was fixed at 0, 0.25, 0.5, 0.75, 1.0, ..., 2.5, 2.75 and $\Delta\alpha = 1/(8k_2)$. There were 12 packets generated, one for each value of ε. The phases in the passband of each baud were selected randomly in the simulation. The initial conditions on the transmitter position and velocity and the speed of sound were the same as used in the earlier simulation analysis of the previous chapter.

For each baud, the conditional statistics based on knowing the transmitted phase (i.e., quadrant) and an estimate of the output SNR (56) were computed with the method described in the previous chapter. A packet's estimated output SNR, $\overline{\widehat{\mathrm{SNROUT}}}(\varepsilon)$, is simply the average of the estimated output SNR's from each of the five bauds within that packet. $\overline{\widehat{\mathrm{SNROUT}}}(\varepsilon)$ is

$$\overline{\widehat{\mathrm{SNROUT}}}(\varepsilon) = (1/5)\sum_{i=1}^{5}\widehat{\mathrm{SNROUT}}(\varepsilon)_i \quad .$$

(57)

Theoretically, if ISI is neglected, the output SNR, $\mathrm{SNR_{OUT}}$ should degrade due to Doppler, α_l, from (26a) as $(\mathrm{SNR_{IN}}\ \mathrm{sinc}^2(2\pi\,k_2\,\alpha_l))$. Substituting $\alpha = \varepsilon\,\Delta\alpha = \varepsilon/(8\,k_2)$ yields

$$\mathrm{SNR_{OUT}} = \mathrm{SNR_{IN}} + 20\,\log_{10}[\sin(\pi\varepsilon/4)/(\pi\varepsilon/4)]$$

(58)

where $\mathrm{SNR_{OUT}}$ is in dB. For this analysis, the input SNR, $\mathrm{SNR_{IN}}$, was equal to 15 dB. $\overline{\widehat{\mathrm{SNROUT}}}(\varepsilon)$ and $\mathrm{SNR_{OUT}}$ were plotted versus ε to compare the simulation results with the theoretical degradation for an input SNR of 15 dB (see Figure 15).
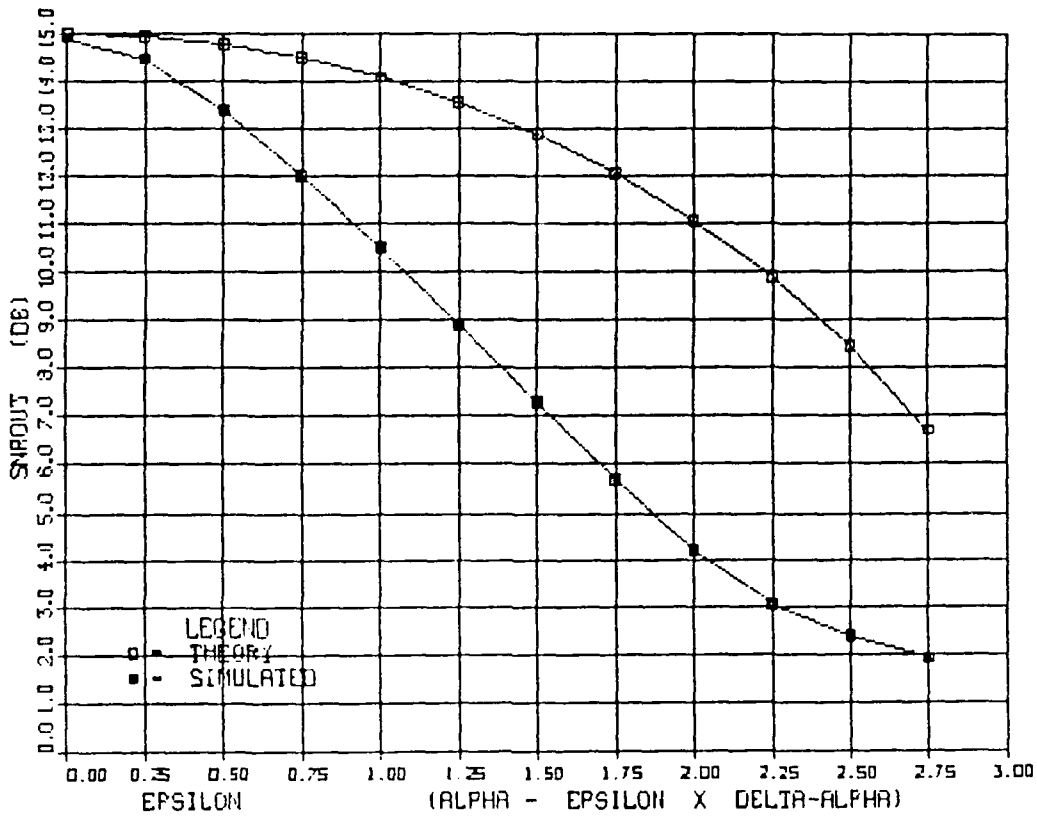
Figure 15. SNR_{OUT} vs. ALPHA for SNR_{IN} = 15 dB

C. RESULTS AND CONCLUSIONS

There is a significant difference between the simulation results and theory as shown in Figure 15. It was speculated that this difference is due to ISI. As α_1 increases, the tones "smear" into adjacent tones (i.e., ISI). To prove whether or not the difference was due to ISI, the analysis was repeated with an input SNR of 5 db. By lowering the input SNR to 5 dB, the signal level is not much higher than the noise level within the band; therefore, symbol interference from adjacent tones is at about the same level as the noise level of

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that tone. The noise level of each tone, at $SNR_{IN} = 5$ dB, did not increase as much relative to increases in α_1 as when $SNR_{IN} = 15$ dB; thus, the output SNR at a given tone should not degrade as rapidly as α_1 increases. Figure 16 is a plot of the theoretical degradation and the simulation results for both input SNR's (5 and 15 dB). By examining Figure 16 one can see that, indeed, the output SNR at 5 dB degrades at a much slower rate than the output SNR at 15 dB. The statistics of each packet for both 5 and 15 dB input SNR on a baud by baud basis are presented in Appendix D.

The analysis of the simulation results for 15 and 5 dB seemed to substantiate that ISI can not be neglected. An estimate and correction of Doppler shift may be used to reduce the ISI and lessen the output SNR degradation at higher SNR_{IN}'s. An estimate of the Doppler shift can be fed back into the receiver sampling system to compensate for the Doppler shift.
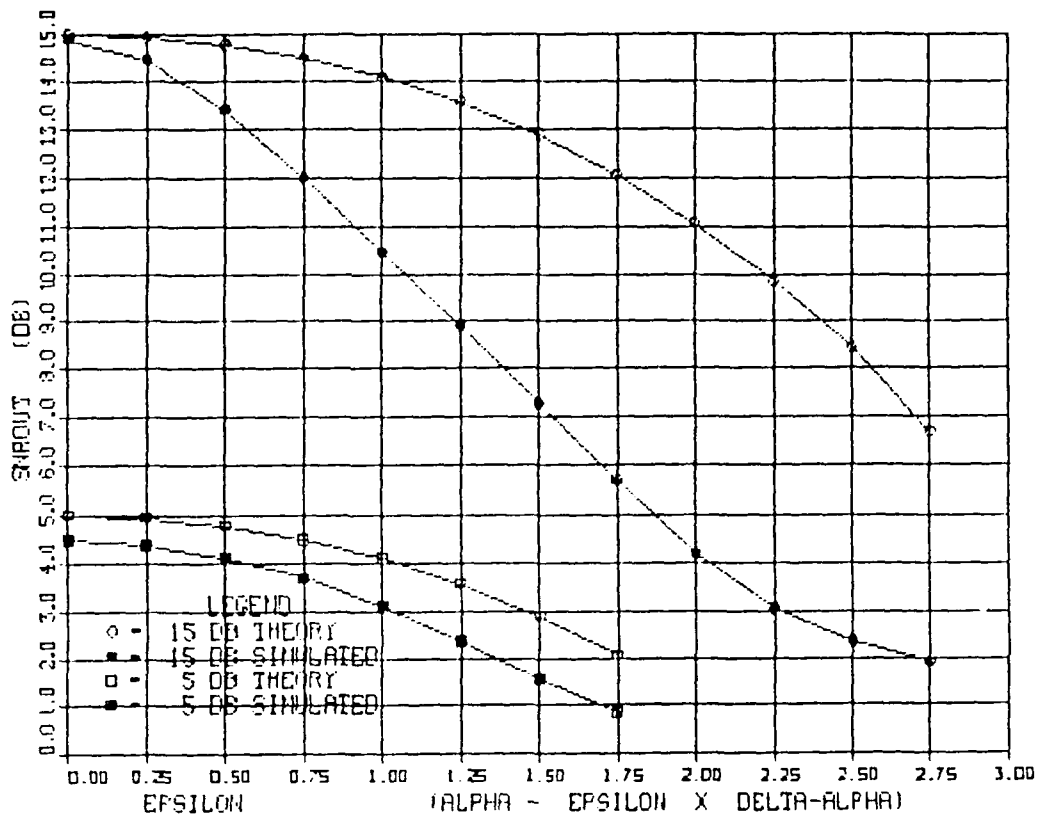
Figure 16. SNR_{OUT} vs. ALPHA for SNR_{IN} = 15 dB and 5 dB

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VI. A DOPPLER ESTIMATE FOR FINE CORRECTIONS

A. BACKGROUND

The Doppler shifts due to the moving transmitter should be estimated so that they can be removed from the received signal in the receiver system to reduce the errors in decoding the MFQPSK signal. One technique for estimating the Doppler compression/expansion factor was examined. An adaptation of a Doppler estimation method recently proposed for ordinary QPSK [Ref. 5] was developed for use with MFQPSK and is the subject of this chapter.

B. IMPLEMENTATION OF THE DOPPLER ESTIMATION

This Doppler estimation method approximates the Doppler time compression/expansion factor within a single baud. To implement this estimator, several simulation runs were produced. A simulation run consisted of a single packet. In each packet, five bauds of baud type 3 (i.e., $k_x = 1024$ points and $K = 64$ tones between harmonics 269 and 332) were generated with randomly selected phases. The initial conditions of the transmitter's position and velocity are the same as used in the simulation versus MFQPSK theory in Chapter IV. Each packet was generated with m, the Doppler channel number, equal to zero (i.e., $\alpha_m = 0$, which means the residual Doppler mismatch, $\alpha_l - \alpha_m$, is equal to α_l) and a fixed $\alpha = \alpha_l = \varepsilon \Delta\alpha$ where ε is between 0 and 1.5 and $\Delta\alpha = 1/(8k_2)$. For this analysis (using baud type 3), k_2 is 332; therefore, $\Delta\alpha = 1/2656$. Recall $\alpha = \alpha_l$ is the Doppler compression/expansion factor. A set of runs or packets were produced for four different input

narrowband SNR's (10, 15, 20, and 40 dB). The amplitudes for all the packets were normalized (i.e., AA(LL,K) = 1 for all LL and K).

Comparison of the phase for a given tone in the first half of a baud to the phase of the same tone in the second half of the baud yields an estimate of the Doppler compression/expansion factor within that baud. If there is no Doppler shift and no noise on the signal, the corresponding phases from the first and last half of the baud should be equal. The following method was used to compute $\hat{\alpha}_l$ which is an estimate of the Doppler compression/expansion factor within the lth baud. To obtain the phases of the first and second half of the received signal's baud, a 512 (i.e., $k_x/2$) point DFT was taken on the first 512 points of the baud and another 512 point DFT was taken on the last 512 points of the baud. Note that, when the bauds were generated with the simulation, the odd tones within the passband (harmonics between 269 and 332) were zeroed out by setting their amplitudes to zero (all the harmonics outside the passband are always zero). If the odd harmonics in the passband are not zero, then the output of the DFT's of the first and last halves of the baud will have interference between harmonic numbers because 1/2 of an odd harmonic does not produce an integer harmonic, causing an extra half cycle of signal between the harmonics, k', of the 512 point DFT's. With an ideal received signal (i.e., no noise and no Doppler), if the odd harmonics in the passband are nonzero, the corresponding phases of the harmonics of the first and last half of the baud will not be equal due to the interference of the signals between each harmonic in the 512 point DFT's.

A Doppler estimate, $\hat{\alpha}_l$, was then computed for each baud by averaging the differences between the received signal's phases from the first half and

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the received signal's phases from the second half of the baud. Thus, the Doppler compression/expansion factor estimate for a baud is

$$\hat{\alpha}_1 = [1/(K/2)] \sum_{k'} [\text{PHASE of } (S2^*(k') \, S1(k'))] / (2\pi k')$$

(59)

where,

S2*(k') = Conjugate of the DFT of the second half of the baud

S1(k') = The DFT of the first half of the baud

k' = A harmonic number of the first and second half DFT's
(Note k' = k/2, where k are the harmonics of the original k_x point DFT of the baud.)

K = The number of MFM tones in the original baud.

An average Doppler estimate, $\hat{\alpha}$, was calculated for each packet by averaging the five Doppler estimates, $\hat{\alpha}_1$, within each packet and was plotted versus α for each of the four input SNRs (see Figure 17). Appendix E contains the estimates and statistics for each packet at each input SNR.

C. RESULTS AND CONCLUSIONS

From the analysis for $\alpha = \alpha_1$ between 0 and 1.5 $\Delta\alpha$, $\hat{\alpha}$ does not estimate α accurately for input SNRs less than 15 dB. Recall that an input SNR = 15 db is the minimum for successfully decoding this MFQPSK signal. At an input SNR of 15 dB, $\hat{\alpha}$ estimates α well for α between 0 and 0.8 $\Delta\alpha$; and for input SNRs of 20 dB or greater, $\hat{\alpha}$ estimates α for α between 0 and $\Delta\alpha$. These results are shown clearly in Figure 17.

The estimate $\hat{\alpha}$ is not acceptable for large Doppler shifts (i.e., Doppler shifts > $\Delta\alpha$); therefore, this Doppler estimation/correction method may only be used for fine corrections of Doppler within a received signal baud and Doppler channel, where the maximum Doppler mismatch is $\pm \Delta\alpha/2$. Note from Figure 17 that the estimation is perfect for 0.5 $\Delta\alpha$.

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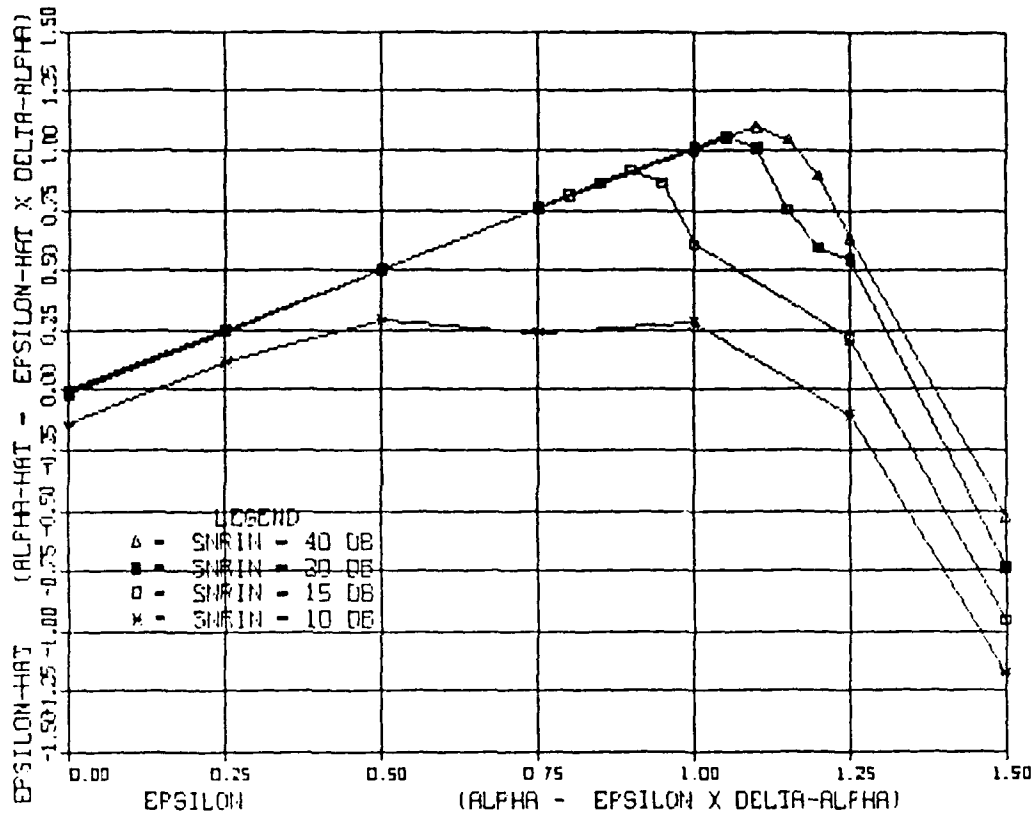
Figure 17. ALPHA-HAT vs. ALPHA for SNR_{IN} = 10, 15, 20, and 40 dB

VII. CONCLUSIONS AND RECOMMENDATIONS

A numerical code to simulate a MFQPSK signal received from a moving transmitter through a bandpass channel has been developed. It resides presently on the NPS IBM mainframe computer. The simulation is written in FORTRAN 77 and the code is included in Appendix F.

The model of the transmitter platform dynamics in the simulation is limited to straight line motion with random fluctuations. Actual transmitter platforms have motion in the pitch, yaw, and roll planes too and thus a six-degree-of-freedom model may be more realistic for future studies of platform motion effects on MFQPSK signalling.

The simulation should be tested further with actual test data to ensure that it produces a realistic received signal. Validation with actual test data may show that other channel characteristics or parameters which have not been considered in the simulation are important and must be included.

An important discovery of the simulation was that the output SNR degradation due to Doppler shift was primarily due to ISI. ISI cannot be neglected; therefore a theoretical analysis of ISI should be developed to realistically analyze MFQPSK in the presence of Doppler.

The Doppler estimation/correction method analyzed and presented in chapter VI only appears useful to remove small Doppler shifts (i.e., for fine adjustments only). Analysis of methods to remove larger Doppler shifts (i.e., coarse adjustments) across several bauds or a packet should be performed for MFQPSK signals.

Synchronization error estimation and correction has not yet been addressed with this simulation, but should be in the future.

This NPS developed MFQPSK signal has been implemented in hardware by T. Gantenbein and Dr. P. H. Moose using differential phase coding [Ref. 6]. It is conjectured that with differential coding the Doppler shifts, ISI, and synchronization errors will not affect the received signal as significantly as they affect the individually phase coded signal which is the coding technique presently implemented in the simulation. Therefore,the differential phase coding should be coded in the simulation. Doppler, ISI, and synchronization estimation/correction methods should be studied that are compatible with differential phase coding.

APPENDIX A

DERIVATION OF £(u)

Given the model of the channel's time delays below in Figure 18, assume x(u) is a superposition of N impulses;

$$x(u) = \sum_{i=1}^{N} A_i \ \delta(u-u_i) \ .$$

$$(60)$$
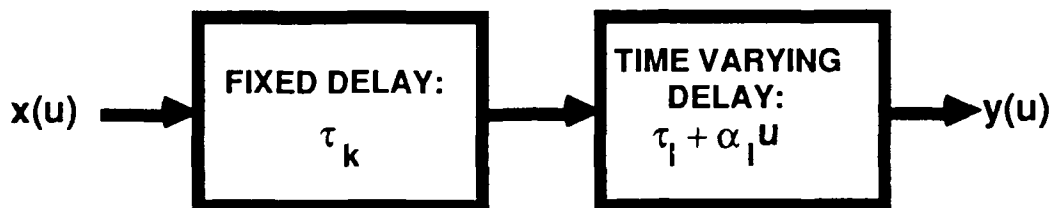
Figure 18. Time Delay Model of Channel

An impulse sent at time $u = 0$ arrives at y at time $\tau_k + \tau_l + \alpha_l\tau_k$. Therefore, the output signal from the channel $y(\tau_k + \tau_l + \alpha_l\tau_k)$ is the impulse $x(0)$; that is, $y(\tau_k + \tau_l + \alpha_l\tau_k) = x(0)$. An impulse sent at time ΔT arrives at the output of the channel at time

$$\Delta T + \tau_k + \tau_l + \alpha_l(\Delta T + \tau_k).$$

$$(61)$$

Therefore, the output signal from the channel

$$y(\Delta T + \tau_k + \tau_l + \alpha_l(\Delta T + \tau_k))$$

$$(62)$$

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which is the impulse $x(\Delta T)$; is

$$y(\Delta T + \tau_k + \tau_l + \alpha_l(\Delta T + \tau_k)) = x(\Delta T) \ . \tag{63}$$

Now letting

$$u = \Delta T + \tau_k + \tau_l + \alpha_l(\Delta T + \tau_k) \tag{64}$$

which can be rewritten as

$$u = \Delta T(1 + \alpha_l) + \tau_k(1 + \alpha_l) + \tau_l \ , \tag{65}$$

then solving for ΔT yields

$$\Delta T = (\ (u - \tau_l) / (1 + \alpha_l)\ ) - \tau_k = u - \tau_k - ((\tau_l + \alpha_l u) / (1 + \alpha_l)). \tag{66}$$

Thus,

$$y(u) = x(\ \Delta T\ ) = x(\ u - \tau_k - ((\tau_l + \alpha_l u) / (1 + \alpha_l))\ ) \tag{67}$$

or

$$y(u) = x(\ u - \tau_k - \pounds(u)) \tag{68}$$

where

$$\pounds(u) = ((\tau_l + \alpha_l u) / (1 + \alpha_l)). \tag{69}$$

APPENDIX B. AN EXAMPLE OF A SIMULATION RUN

A.   INPUT FROM THE SCREEN

```
PLEASE ENTER THE INITIAL POSITION  X0,Y0,Z0 (FT)
  OF THE RECEIVER RELATIVE TO THE TRANSMITTER ...
X0 =  -10.0000   Y0 =   20.0000   Z0 =  -6000.000


ENTER THE TRANSMITTER"S VELOCITY AVERAGE AND VARIANCE
  IN THE X, Y, Z-DIRECTIONS (FT/SEC) AND (FT/SEC)**2 ...
  (I.E.  VXAVG, VXVAR, VYAVG, VYVAR, VZAVG, VZVAR)
VXAVG =   2.00000000       VXVAR =  0.500000007E-01
VYAVG =   4.00000000       VYVAR =  0.499999896E-02
VZAVG =  0.000000000E+00   VZVAR =  0.249999994E-02


ENTER THE AVERAGE SPEED OF SOUND IN FT/SEC
  AND THE VARIANCE IN (FT/SEC)**2   ...
C0 =   5000.00000      CVAR =  0.000000000E+00


ENTER THE TRANSMITTER"S DOWN LINK TRANSMITTED
  HALF BEAM WIDTH ANGLE (DEG)   ...
THETA0 =   4.00000000


DO YOU WANT NORMALIZED AMPLITUDE FOR THE RECEIVED SIGNAL?
     PLEASE ENTER 1:YES OR 0:NO ...
IAMP =            1

ENTER THE # OF PACKETS IN THE TRANSMITTED SIGNAL ...
NPAKS =           2
```

```
THIS PROGRAM ENCODES A QPSK MULTIFREQUENCY SIGNAL.
 THE PHASES ARE SHOWN BELOW FOR ONE FREQUENCY ...


         *                    *
         2                    1



         _____  _____


         *                    *
         3                    4




SELECT ONE OF THE FOLLOWING METHODS FOR ENCODING
  THE PHASES FOR EACH FREQUENCY FOR EVERY BAUD
  WITHIN EACH PACKET ...

    ENTER ...
      1: THE PROGRAM RANDOMLY SELECTS ALL THE PHASES
            FOR ALL     2 PACKETS
      2: YOU INDIVIDUALLY SELECT THE PHASES
METHOD =               1


ENTER THE BAUD TYPE # CORRESPONDING TO THE FOLLOWING
  BAUD LENGTH FOR PACKET NUMBER:     1 ...

        1 : BAUD LENGTH (DELT) = 1/240 SECONDS
        2 : BAUD LENGTH (DELT) = 1/120 SECONDS
        3 : BAUD LENGTH (DELT) = 1/60 SECONDS
        4 : BAUD LENGTH (DELT) = 1/30 SECONDS
        5 : BAUD LENGTH (DELT) = 1/15 SECONDS
BDTYPE(1) =              1


ENTER THE NUMBER OF BAUDS IN PACKET NUMBER:     1 ...
NBAUDS(1) =              1


ENTER THE BAUD TYPE # CORRESPONDING TO THE FOLLOWING
  BAUD LENGTH FOR PACKET NUMBER:     2 ...

        1 : BAUD LENGTH (DELT) = 1/240 SECONDS
        2 : BAUD LENGTH (DELT) = 1/120 SECONDS
        3 : BAUD LENGTH (DELT) = 1/60 SECONDS
        4 : BAUD LENGTH (DELT) = 1/30 SECONDS
        5 : BAUD LENGTH (DELT) = 1/15 SECONDS
BDTYPE(2) =              2


ENTER THE NUMBER OF BAUDS IN PACKET NUMBER:     2 ...
NBAUDS(2) =              1
```

```
WOULD YOU LIKE THE DISCETE FOURIER TRANSFORM
  OF THE OUTPUT SIGNAL ?  (ENTER  1:YES OR 0:NO) ...
IDFT =             1


WOULD YOU LIKE THE DFT OUTPUT WINDOWED ?
  ENTER  1:YES OR 0:NO ...
IWNDOW =           1


PLEASE ENTER THE DESIRED INPUT WIDE BAND
  SIGNAL-TO-NOISE RATIO IN DB ....
SNRDB =    15.0000
```

B. LIST OUTPUT TO DISK FILE 30

```
48
LL        K      PHI(LL,K)    IPHI
 1       68        -0.785       4
 1       69         0.785       1
 1       70         0.785       1
 1       71         2.356       2
 1       72        -2.356       3
 1       73        -2.356       3
 1       74        -2.356       3
 1       75        -0.785       4
 1       76         2.356       2
 1       77        -0.785       4
 1       78         2.356       2
 1       79         0.785       1
 1       80        -0.785       4
 1       81         0.785       1
 1       82         0.785       1
 1       83        -2.356       3
 2      135        -0.785       4
 2      136        -2.356       3
 2      137        -2.356       3
 2      138         2.356       2
 2      139        -2.356       3
 2      140         2.356       2
 2      141        -0.785       4
 2      142        -0.785       4
 2      143         0.785       1
 2      144        -0.785       4
 2      145         2.356       2
 2      146        -0.785       4
 2      147         2.356       2
 2      148         0.785       1
 2      149         0.785       1
 2      150         0.785       1
 2      151        -2.356       3
 2      152        -0.785       4
 2      153        -0.785       4
 2      154         2.356       2
 2      155        -2.356       3
 2      156         2.356       2
 2      157         2.356       2
 2      158        -2.356       3
 2      159         2.356       2
 2      160         2.356       2
 2      161         0.785       1
 2      162         2.356       2
 2      163         2.356       2
 2      164        -2.356       3
 2      165        -2.356       3
 2      166         2.356       2
```

```
DFT INPUT DATA FOR BAUD #    1
  N      REAL PART        IMAG PART
  0    0.200484E+01     0.000000E+00
  1   -.291531E+01      0.000000E+00
  2   -.122278E+01      0.000000E+00
  3    0.225909E+01     0.000000E+00
  4    0.176306E+01     0.000000E+00
  5   -.118927E+01      0.000000E+00
  6   -.218088E-01      0.000000E+00
  7   -.235837E+00      0.000000E+00
  8   -.613874E+00      0.000000E+00
  9    0.194238E+01     0.000000E+00
 10   -.130624E+01      0.000000E+00
 11   -.342880E+00      0.000000E+00
 12    0.207660E+01     0.000000E+00
 13   -.329895E+00      0.000000E+00
 14   -.369812E+01      0.000000E+00
 15    0.348756E+01     0.000000E+00
 16    0.125413E+01     0.000000E+00
 17   -.485057E+01      0.000000E+00
 18    0.209897E+01     0.000000E+00
 19    0.725176E+01     0.000000E+00
 20   -.267107E+01      0.000000E+00
 21   -.327209E+01      0.000000E+00
 22    0.502687E+01     0.000000E+00
 23   -.287833E+01      0.000000E+00
 24   -.696552E+01      0.000000E+00
 25    0.376308E+01     0.000000E+00
 26    0.269107E+01     0.000000E+00
 27   -.425671E+01      0.000000E+00
 28   -.142092E+01      0.000000E+00
 29    0.417586E+01     0.000000E+00
 30   -.685801E+00      0.000000E+00
 31   -.910082E-01      0.000000E+00
 32    0.237153E+01     0.000000E+00
 33    0.686129E+00     0.000000E+00
 34   -.128562E+01      0.000000E+00
 35    0.271385E+01     0.000000E+00
 36    0.143204E+01     0.000000E+00
 37    0.122637E+01     0.000000E+00
 38    0.156954E+01     0.000000E+00
 39   -.215876E+01      0.000000E+00
 40   -.159364E+00      0.000000E+00
 41    0.406544E+01     0.000000E+00
 42    0.115762E+01     0.000000E+00
 43   -.275345E+01      0.000000E+00
 44    0.252170E+01     0.000000E+00
 45   -.110620E+01      0.000000E+00
 46   -.378736E+00      0.000000E+00
 47    0.253587E+01     0.000000E+00
 48   -.494904E+00      0.000000E+00
 49   -.336829E+01      0.000000E+00
 50    0.299053E+01     0.000000E+00
 51   -.839522E+00      0.000000E+00
 52   -.154595E+01      0.000000E+00
 53   -.102779E+01      0.000000E+00
 54    0.128035E+01     0.000000E+00
 55   -.365980E+00      0.000000E+00
 56   -.550354E+01      0.000000E+00
 57   -.281839E+00      0.000000E+00
 58    0.125277E+01     0.000000E+00
 59   -.697582E+00      0.000000E+00
 60    0.705833E+00     0.000000E+00
 61    0.133619E+01     0.000000E+00
 62    0.229186E+00     0.000000E+00
 63   -.356922E+00      0.000000E+00
 64    0.444179E+00     0.000000E+00
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 65   0.413470E+01   0.000000E+00
 66  -.175258E+01   0.000000E+00
 67  -.939483E+00   0.000000E+00
 68   0.279167E+00   0.000000E+00
 69  -.211374E+00   0.000000E+00
 70   0.749620E+00   0.000000E+00
 71   0.227079E+01   0.000000E+00
 72   0.169964E+01   0.000000E+00
 73  -.360312E+01   0.000000E+00
 74   0.181549E+01   0.000000E+00
 75   0.192265E+01   0.000000E+00
 76  -.243610E+01   0.000000E+00
 77  -.571788E+00   0.000000E+00
 78   0.171358E+01   0.000000E+00
 79  -.227510E+01   0.000000E+00
 80   0.154184E+01   0.000000E+00
 81   0.240438E+01   0.000000E+00
 82  -.142191E+01   0.000000E+00
 83  -.171134E+01   0.000000E+00·
 84   0.349656E+01   0.000000E+00
 85  -.214113E+01   0.000000E+00
 86   0.219070E+01   0.000000E+00
 87   0.363189E+01   0.000000E+00
 88   0.713565E+00   0.000000E+00
 89  -.240271E+01   0.000000E+00
 90   0.625637E+00   0.000000E+00
 91   0.177416E+01   0.000000E+00
 92  -.548617E+01   0.000000E+00
 93   0.517214E+01   0.000000E+00
 94   0.379215E+01   0.000000E+00
 95  -.540784E+01   0.000000E+00
 96   0.428047E+01   0.000000E+00
 97   0.534627E+01   0.000000E+00
 98  -.433480E+01   0.000000E+00
 99  -.320334E+00   0.000000E+00
100   0.504450E+01   0.000000E+00
101  -.394868E+01   0.000000E+00
102  -.180450E+01   0.000000E+00
103   0.731280E+01   0.000000E+00
104  -.358992E+01   0.000000E+00
105  -.684641E+01   0.000000E+00
106   0.478245E+01   0.000000E+00
107   0.353709E+01   0.000000E+00
108  -.738240E+01   0.000000E+00
109   0.855238E+00   0.000000E+00
110   0.565053E+01   0.000000E+00
111  -.466334E+01   0.000000E+00
112  -.202646E+01   0.000000E+00
113   0.416927E+01   0.000000E+00
114   0.943901E+00   0.000000E+00
115  -.129343E+01   0.000000E+00
116  -.182671E+00   0.000000E+00
117   0.314196E+01   0.000000E+00
118  -.641427E+00   0.000000E+00
119  -.229205E+01   0.000000E+00
120   0.894737E-02   0.000000E+00
121  -.208729E+01   0.000000E+00
122   0.127476E+01   0.000000E+00
123   0.189270E+01   0.000000E+00
124   0.827031E+00   0.000000E+00
125  -.180424E+01   0.000000E+00
126   0.181878E-01   0.000000E+00
127   0.738333E+00   0.000000E+00
128   0.130366E+01   0.000000E+00
129   0.426684E+00   0.000000E+00
130   0.200011E+01   0.000000E+00
131  -.176676E+01   0.000000E+00
132  -.184777E+01   0.000000E+00
133   0.305751E+01   0.000000E+00
134  -.415754E+00   0.000000E+00
135  -.356674E+01   0.000000E+00
136   0.429929E+01   0.000000E+00
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137   0.331470E+01   0.000000E+00
138  -.484753E+01    0.000000E+00
139  -.127909E+01    0.000000E+00
140   0.602902E+01   0.000000E+00
141   0.847187E-01   0.000000E+00
142  -.353355E+01    0.000000E+0U
143   0.508767E+01   0.000000E+00
144   0.341302E+01   0.000000E+00
145  -.246052E+01    0.000000E+00
146   0.850205E+00   0.000000E+00
147   0.319615E+01   0.000000E+00
148  -.407688E+01    0.000000E+00
149  -.112038E+00    0.000000E+00
150   0.436206E+01   0.000000E+00
151   0.205910E+01   0.000000E+00
152  -.242958E+01    0.000000E+00
153   0.360312E+01   0.000000E+00
154   0.238491E+01   0.000000E+00
155   0.885219E+00   0.000000E+00
156  -.307407E+01    0.000000E+00
157   0.230542E+01   0.000000E+00
158   0.173215E+01   0.000000E+00
159  -.932735E+00    0.000000E+00
160   0.963823E+00   0.000000E+00
161   0.193234E+01   0.000000E+00
162   0.574894E-01   0.000000E+00
163  -.168030E+01    0.000000E+00
164   0.330895E+01   0.000000E+00
165  -.280785E+01    0.000000E+00
166  -.376674E+01    0.000000E+00
167   0.258334E+01   0.000000E+00
168   0.592477E+00   0.000000E+00
169  -.419450E+01    0.000000E+00
170   0.111468E+01   0.000000E+00
171   0.271280E+01   0.000000E+00
172  -.502696E+01    0.000000E+00
173  -.321339E+01    0.000000E+00
174   0.732952E+01   0.000000E+00
175  -.217458E+01    0.000000E+00
176  -.501133E+01    0.000000E+00
177   0.301020E+01   0.000000E+00
178   0.179601E+01   0.000000E+00
179  -.189239E+01    0.000000E+00
180   0.346009E+01   0.000000E+00
181  -.109039E+01    0.000000E+00
182  -.391839E+00    0.000000E+00
183   0.417718E+00   0.000000E+00
184  -.272576E+01    0.000000E+00
185  -.153335E+01    0.000000E+00
186   0.340410E+01   0.000000E+00
187  -.250186E+00    0.000000E+00
188  -.513401E+01    0.000000E+00
189   0.468323E+01   0.000000E+00
190   0.310399E+01   0.000000E+00
191  -.444465E+01    0.000000E+00
192  -.127312E+01    0.000000E+00
193   0.250906E+01   0.000000E+00
194  -.666324E+00    0.000000E+00
195  -.134897E+01    0.000000E+00
196   0.570884E+01   0.000000E+00
197  -.896835E-01    0.000000E+00
198  -.293430E+01    0.000000E+00
199  -.239388E+01    0.000000E+00
200  -.312660E+00    0.000000E+00
201   0.871717E+00   0.000000E+00
202  -.115026E+01    0.000000E+00
203  -.344144E+01    0.000000E+00
204   0.286601E+01   0.000000E+00
205   0.451813E+01   0.000000E+00
206  -.516409E+01    0.000000E+00
207  -.216182E+01    0.000000E+00
208   0.682289E+01   0.000000E+00
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              209   0.316336E+00    0.000000E+00
              210  -.910015E+01     0.000000E+00
              211   0.373189E+01    0.000000E+00
              212   0.459230E+01    0.000000E+00
              213  -.709038E+01     0.000000E+00
              214  -.790167E-01     0.000000E+00
              215   0.640498E+01    0.000000E+00
              216   0.885365E+00    0.000000E+00
              217  -.101914E+02     0.000000E+00
              218   0.376423E+01    0.000000E+00
              219  -.109551E+00     0.000000E+00
              220  -.386085E+01     0.000000E+00
              221  -.380749E+01     0.000000E+00
              222   0.589190E+01    0.000000E+00
              223   0.234406E+00    0.000000E+00
              224  -.489929E+01     0.000000E+00
              225  -.257231E+01     0.000000E+00
              226   0.668187E+01    0.000000E+00
              227   0.256053E+00    0.000000E+00
              228  -.820556E+01     0.000000E+00
              229   0.317285E+01    0.000000E+00
              230   0.728581E+01    0.000000E+00
              231  -.353177E+01     0.000000E+00
              232  -.473028E+01     0.000000E+00
              233   0.651311E+01    0.000000E+00
              234  -.125509E+01     0.000000E+00
              235  -.923538E+01     0.000000E+00
              236   0.317662E+01    0.000000E+00
              237   0.596793E+01    0.000000E+00
              238  -.415065E+01     0.000000E+00
              239   0.927513E+00    0.000000E+00
              240   0.638412E+01    0.000000E+00
              241   0.116869E+01    0.000000E+00
              242  -.316552E+01     0.000000E+00
              243   0.217262E+01    0.000000E+00
              244   0.134112E+01    0.000000E+00
              245  -.508769E+01     0.000000E+00
              246   0.219943E+01    0.000000E+00
              247   0.279915E+01    0.000000E+00
              248  -.269927E+01     0.000000E+00
              249   0.254480E+00    0.000000E+00
              250   0.117177E+01    0.000000E+00
              251  -.115213E+01     0.000000E+00
              252  -.553449E+00     0.000000E+00
              253   0.370947E+01    0.000000E+00
              254   0.146894E+01    0.000000E+00
              255   0.176158E+01    0.000000E+00
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DFT OUTPUT DATA FOR BAUD #    1
  K      REAL PART        IMAG PART        MAGNITUDE              PHASE
                                                            (DEG)           (RAD)
  68   0.90637344E+02   -.90041351E+02   0.12775980E+03   -.44811005E+02   -.78209955E+00
  69   0.12512837E+03   0.87728928E+02   0.15281844E+03   0.35034729E+02   0.61147141E+00
  70   0.85056396E+02   0.10223833E+03   0.13299347E+03   0.50241470E+02   0.87687898E+00
  71  -.95022675E+02   0.10510611E+03   0.14169193E+03   0.13211565E+03   0.23058529E+01
  72  -.80296448E+02   -.10472539E+03   0.13196561E+03   -.12747855E+03   -.22249212E+01
  73  -.11772476E+03   -.10389015E+03   0.15701044E+03   -.13857214E+03   -.24185400E+01
  74  -.10582492E+03   -.87098938E+02   0.13705887E+03   -.14054395E+03   -.24529552E+01
  75   0.95074265E+02   -.11210881E+03   0.14699489E+03   -.49700256E+02   -.86743307E+00
  76  -.73995346E+02   0.71338562E+02   0.10278374E+03   0.13604727E+03   0.23744726E+01
  77   0.95286469E+02   -.91973923E+02   0.13243378E+03   -.43986572E+02   -.76771045E+00
  78  -.75572342E+02   0.88835632E+02   0.11663167E+03   0.13038779E+03   0.22756958E+01
  79   0.84246826E+02   0.97707855E+02   0.12901299E+03   0.49231018E+02   0.85924339E+00
  80   0.11124681E+03   -.82108078E+02   0.13826636E+03   -.36429962E+02   -.63582295E+00
  81   0.10516679E+03   0.83761368E+02   0.13444707E+03   0.38535995E+02   0.67257994E+00
  82   0.11486154E+03   0.10973633E+03   0.15885600E+03   0.43692764E+02   0.76258266E+00
  83  -.89434708E+02   -.95062175E+02   0.13051968E+03   -.13325296E+03   -.23257027E+01

NOTE:  The DFT is windowed (IWNDOW = 1)
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DFT INPUT DATA FOR BAUD #    2
 N      REAL PART        IMAG PART
 0  -.526725E+01      0.000000E+00
 1  -.394182E+00      0.000000E+00
 2   0.696870E+01     0.000000E+00
 3  -.290471E+01      0.000000E+00
 4  -.384541E+01      0.000000E+00
 5   0.468083E+01     0.000000E+00
 6  -.439102E+01      0.000000E+00
 7  -.115464E+01      0.000000E+00
 8   0.533558E+01     0.000000E+00
 9  -.240291E+01      0.000000E+00
10  -.667564E+01      0.000000E+00
11   0.728867E+01     0.000000E+00
12   0.474582E+00     0.000000E+00
13  -.380783E+01      0.000000E+00
14   0.311805E+01     0.000000E+00
15   0.450043E+01     0.000000E+00
16  -.446419E+01      0.000000E+00
17   0.199608E+01     0.000000E+00
18   0.185680E+01     0.000000E+00
19  -.970748E+01      0.000000E+00
20   0.440136E+01     0.000000E+00
21   0.601283E+01     0.000000E+00
22  -.750463E+01      0.000000E+00
23  -.344410E+01      0.000000E+00
24   0.110174E+02     0.000000E+00
25   0.249212E+01     0.000000E+00
26  -.811214E+01      0.000000E+00
27   0.863213E+01     0.000000E+00
28   0.576661E+01     0.000000E+00
29  -.834345E+01      0.000000E+00
30  -.330458E+01      0.000000E+00
31   0.113159E+02     0.000000E+00
32   0.149053E+01     0.000000E+00
33  -.194054E+01      0.000000E+00
34   0.498989E+01     0.000000E+00
35   0.492170E+01     0.000000E+00
36  -.811450E+01      0.000000E+00
37   0.138805E+01     0.000000E+00
38   0.412452E+01     0.000000E+00
39  -.386714E+01      0.000000E+00
40  -.588585E+01      0.000000E+00
41   0.650927E+01     0.000000E+00
42   0.396164E+00     0.000000E+00
43   0.101974E+00     0.000000E+00
44   0.422124E+01     0.000000E+00
45  -.261774E+01      0.000000E+00
46  -.652925E+01      0.000000E+00
47   0.692594E+01     0.000000E+00
48   0.912038E+00     0.000000E+00
49  -.929524E+01      0.000000E+00
50   0.202819E+01     0.000000E+00
51   0.106854E+02     0.000000E+00
52  -.710419E+01      0.000000E+00
53  -.457107E+01      0.000000E+00
54   0.104375E+02     0.000000E+00
55  -.104101E+01      0.000000E+00
56  -.712630E+01      0.000000E+00
57   0.622043E+01     0.000000E+00
58   0.965949E+01     0.000000E+00
59  -.112449E+02      0.000000E+00
60  -.477618E+01      0.000000E+00
61   0.101622E+02     0.000000E+00
62   0.207549E+01     0.000000E+00
63  -.460409E+01      0.000000E+00
64   0.214426E+01     0.000000E+00
65   0.406143E+01     0.000000E+00
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66   -.242523E+01   0.000000E+00
67   -.426298E+01   0.000000E+00
68   -.407959E+00   0.000000E+00
69   0.548756E+01   0.000000E+00
70   -.242346E+01   0.000000E+00
71   -.285746E+01   0.000000E+00
72   0.931297E+01   0.000000E+00
73   0.285819E+01   0.000000E+00
74   -.899141E+01   0.000000E+00
75   0.387245E+00   0.000000E+00
76   0.578286E+01   0.000000E+00
77   -.490578E+01   0.000000E+00
78   -.144855E+00   0.000000E+00
79   0.642747E+01   0.000000E+00
80   0.213817E+00   0.000000E+00
81   -.223994E+01   0.000000E+00
82   0.170248E+01   0.000000E+00
83   -.325185E+00   0.000000E+00
84   -.174552E+01   0.000000E+00 .
85   0.526715E+00   0.000000E+00
86   -.176461E+01   0.000000E+00
87   -.392745E+01   0.000000E+00
88   0.614061E+01   0.000000E+00
89   -.354380E+00   0.000000E+00
90   -.750051E+01   0.000000E+00
91   0.551929E+01   0.000000E+00
92   0.380251E+01   0.000000E+00
93   -.484067E+01   0.000000E+00
94   -.182470E+01   0.000000E+00
95   0.584987E+01   0.000000E+00
96   -.323802E+01   0.000000E+00
97   -.358901E+01   0.000000E+00
98   0.548582E+01   0.000000E+00
99   0.277453E+01   0.000000E+00
100  -.177448E+01   0.000000E+00
101  -.301150E+00   0.000000E+00
102  0.494978E+00   0.000000E+00
103  0.216534E+01   0.000000E+00
104  -.216204E+01   0.000000E+00
105  -.329386E+01   0.000000E+00
106  -.238950E+01   0.000000E+00
107  0.349077E+01   0.000000E+00
108  -.454049E+01   0.000000E+00
109  -.237934E+00   0.000000E+00
110  0.778620E+01   0.000000E+00
111  0.125057E+01   0.000000E+00
112  -.132939E+01   0.000000E+00
113  -.121500E+01   0.000000E+00
114  0.302725E+01   0.000000E+00
115  -.207522E+01   0.000000E+00
116  -.233488E+01   0.000000E+00
117  0.142058E+01   0.000000E+00
118  -.349241E+00   0.000000E+00
119  -.429151E+01   0.000000E+00
120  0.323071E+01   0.000000E+00
121  0.584821E+00   0.000000E+00
122  -.693258E+00   0.000000E+00
123  -.275253E+01   0.000000E+00
124  0.208057E+01   0.000000E+00
125  0.542997E+00   0.000000E+00
126  -.197652E+01   0.000000E+00
127  -.154319E+01   0.000000E+00
128  -.420868E+01   0.000000E+00
129  -.113642E+01   0.000000E+00
130  -.201206E+01   0.000000E+00
131  0.105195E+01   0.000000E+00
132  -.264909E+01   0.000000E+00
133  -.401588E+00   0.000000E+00
134  0.531781E+01   0.000000E+00
135  -.470942E+01   0.000000E+00
136  -.477054E+01   0.000000E+00
137  0.643067E+01   0.000000E+00
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138   -.110350E+01    0.000000E+00
139   -.466090E+01    0.000000E+00
140   0.382527E+01    0.000000E+00
141   0.389133E+01    0.000000E+00
142   -.508665E+01    0.000000E+00
143   0.366893E+01    0.000000E+00
144   0.604237E+01    0.000000E+00
145   -.266780E+01    0.000000E+00
146   -.120114E+01    0.000000E+00
147   0.394714E+01    0.000000E+00
148   -.545985E-01    0.000000E+00
149   -.313556E+01    0.000000E+00
150   -.733842E+00    0.000000E+00
151   -.177183E-01    0.000000E+00
152   -.281948E+00    0.000000E+00
153   0.643579E+0'    0.000000E+00
154   -.231300E+01    0.000000E+00
155   -.248661E+01    0.000000E+00
156   0.441114E+01    0.000000E+00
157   -.283533E+01    0.000000E+00
158   -.626540E+01    0.000000E+00
159   0.582976E+01    0.000000E+00
160   0.176179E+00    0.000000E+00
161   -.254206E+01    0.000000E+00
162   0.552283E+00    0.000000E+00
163   0.443619E+01    0.000000E+00
164   -.811100E+00    0.000000E+00
165   0.614064E+00    0.000000E+00
166   0.348493E+01    0.000000E+00
167   -.542510E+01    0.000000E+00
168   -.285739E+00    0.000000E+00
169   -.908447E+00    0.000000E+00
170   -.758899E+01    0.000000E+00
171   0.872746E+00    0.000000E+00
172   0.510780E+01    0.000000E+00
173   -.642361E+01    0.000000E+00
174   -.196627E+01    0.000000E+00
175   0.604545E+01    0.000000E+00
176   -.406701E+01    0.000000E+00
177   -.907386E+01    0.000000E+00
178   0.522828E+01    0.000000E+00
179   0.274564E+01    0.000000E+00
180   -.836539E+01    0.000000E+00
181   0.758526E+00    0.000000E+00
182   0.555079E+01    0.000000E+00
183   -.524344E+00    0.000000E+00
184   -.604758E+01    0.000000E+00
185   0.451275E+01    0.000000E+00
186   0.691295E+00    0.000000E+00
187   -.403368E+01    0.000000E+00
188   -.421285E+01    0.000000E+00
189   0.297227E+01    0.000000E+00
190   0.138190E+01    0.000000E+00
191   -.497323E+01    0.000000E+00
192   0.272180E+01    0.000000E+00
193   0.999976E+00    0.000000E+00
194   -.198997E+01    0.000000E+00
195   -.321780E+01    0.000000E+00
196   0.117637E+01    0.000000E+00
197   0.322415E+01    0.000000E+00
198   -.458584E+01    0.000000E+00
199   -.421095E+01    0.000000E+00
200   0.435145E+01    0.000000E+00
201   -.580684E+00    0.000000E+00
202   -.501254E+01    0.000000E+00
203   0.115667E+01    0.000000E+00
204   -.203047E+00    0.000000E+00
205   -.292859E+01    0.000000E+00
206   -.315172E+01    0.000000E+00
207   0.280452E+01    0.000000E+00
208   0.373484E+01    0.000000E+00
209   -.424158E+01    0.000000E+00
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210  -.262466E+01   0.000000E+00
211  0.494710E+01   0.000000E+00
212  -.142531E+01   0.000000E+00
213  -.941097E+01   0.000000E+00
214  0.318101E+01   0.000000E+00
215  0.424388E+01   0.000000E+00
216  -.650536E+01   0.000000E+00
217  -.124455E+01   0.000000E+00
218  0.738448E+01   0.000000E+00
219  -.604091E+01   0.000000E+00
220  -.656762E+01   0.000000E+00
221  0.350399E+01   0.000000E+00
222  0.608059E+01   0.000000E+00
223  -.523418E+01   0.000000E+00
224  -.253131E+01   0.000000E+00
225  0.883400E+01   0.000000E+00
226  -.569328E+01   0.000000E+00
227  -.112117E+02   0.000000E+00
228  0.645991E+01   0.000000E+00·
229  0.547792E+01   0.000000E+00
230  -.782587E+01   0.000000E+00
231  -.427295E+01   0.000000E+00
232  0.980781E+01   0.000000E+00
233  -.190006E+01   0.000000E+00
234  · 471819E+01   0.000000E+00
235   `28085E+01   0.000000E+00
236  0.314933E+01   0.000000E+00
237  -.106632E+02   0.000000E+00
238  0.194443E+01   0.000000E+00
239  0.765045E+01   0.000000E+00
240  -.357001E+01   0.000000E+00
241  -.640628E+00   0.000000E+00
242  0.354356E+01   0.000000E+00
243  0.182806E+01   0.000000E+00
244  -.669535E+00   0.000000E+00
245  -.275834E+01   0.000000E+00
246  0.240171E+01   0.000000E+00
247  -.151564E+01   0.000000E+00
248  -.253017E+01   0.000000E+00
249  -.379693E+00   0.000000E+00
250  0.388394E+01   0.000000E+00
251  -.289334E+01   0.000000E+00
252  0.214223E+00   0.000000E+00
253  -.294030E+01   0.000000E+00
254  0.413015E+00   0.000000E+00
255  0.353020E+01   0.000000E+00
256  0.116558E+01   0.000000E+00
257  -.362228E+00   0.000000E+00
258  -.38607 E+01   0.000000E+00
259  -.129466E+01   0.000000E+00
260  0.532838E+00   0.000000E+00
261  0.584236E+00   0.000000E+00
262  -.339053E+01   0.000000E+00
263  0.256464E+01   0.000000E+00
264  0.152053E+01   0.000000E+00
265  -.475879E+01   0.000000E+00
266  0.368118E+01   0.000000E+00
267  -.267406E+00   0.000000E+00
268  -.466596E+01   0.000000E+00
269  -.184211E+01   0.000000E+00
270  0.490506E+01   0.000000E+00
271  -.452730E+01   0.000000E+00
272  -.526406E+01   0.000000E+00
273  0.578228E+01   0.000000E+00
274  0.348354E+01   0.000000E+00
275  -.493828E+01   0.000000E+00
276  -.308919E+01   0.000000E+00
277  0.871899E+01   0.000000E+00
278  0.399700E+00   0.000000E+00
279  -.53887 E+01   0.000000E+00
280  0.1565 0E+01   0.000000E+00
281  0.289917E+01   0.000000E+00
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282	0.650645E-01	0.000000E+00
283	-.374218E+01	0.000000E+00
284	0.392436E+01	0.000000E+00
285	0.753166E+01	0.000000E+00
286	-.239218E+01	0.000000E+00
287	-.625107E+01	0.000000E+00
288	0.464018E+01	0.000000E+00
289	0.803769E+01	0.000000E+00
290	-.486152E+01	0.000000E+00
291	-.450123E+01	0.000000E+00
292	0.736135E+01	0.000000E+00
293	-.191086E+01	0.000000E+00
294	-.418490E+01	0.000000E+00
295	0.810541E+01	0.000000E+00
296	0.481734E+01	0.000000E+00
297	-.821385E+01	0.000000E+00
293	-.678597E+00	0.000000E+00
299	0.493847E+01	0.000000E+00
300	-.376643E+01	0.000000E+00
301	-.355329E+01	0.000000E+00
302	0.366560E+01	0.000000E+00
303	0.115503E+01	0.000000E+00
304	-.399879E+01	0.000000E+00
305	0.126453E+01	0.000000E+00
306	0.735960E+01	0.000000E+00
307	-.381905E+01	0.000000E+00
308	-.238577E+01	0.000000E+00
309	0.122439E+01	0.000000E+00
310	0.642341E+00	0.000000E+00
311	-.11559SE+01	0.000000E+00
312	0.284103E+01	0.000000E+00
313	0.501446E+00	0.000000E+05
314	0.177095E+01	0.000000E+00
315	0.128382E+01	0.000000E+00
316	-.779019E+01	0.000000E+00
317	0.361360E+01	0.000000E+00
318	0.235632E+01	0.000000E+00
319	-.544813E+00	0.000000E+00
320	-.177875E+01	0.000000E+00
321	0.902049E+00	0.000000E+00
322	0.172862E+01	0.000000E+00
323	-.228558E+01	0.000000E+00
324	0.150589E+01	0.000000E+00
325	0.379671E+01	0.000000E+00
326	-.367524E+01	0.000000E+00
327	0.321874E+01	0.000000E+00
328	0.288017E+01	0.000000E+00
329	0.122940E+01	0.000000E+00
330	0.290588E+01	0.000000E+00
331	0.193080E+01	0.000000E+00
332	0.253495E-02	0.000000E+00
333	-.331905E+01	0.000000E+00
334	0.732367E+01	0.000000E+00
335	-.264367E+01	0.000000E+00
336	-.805418E+01	0.000000E+00
337	0.289561E+01	0.000000E+00
338	0.549653E+01	0.000000E+00
339	-.565418E+01	0.000000E+00
340	-.205987E+01	0.000000E+00
341	0.556493E+01	0.000000E+00
342	-.594744E+01	0.000000E+00
343	-.371291E+01	0.000000E+00
344	0.430186E+01	0.000000E+00
345	0.409844E+00	0.000000E+00
346	-.677720E+01	0.000000E+00
347	0.455239E+01	0.000000E+00
348	0.546079E+01	0.000000E+00
349	-.594345E+01	0.000000E+00
350	-.872314E+00	0.000000E+00
351	0.585900E+01	0.000000E+00
352	-.262263E+01	0.000000E+00
353	-.573845E+01	0.000000E+00

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354   0.433496E+01   0.000000E+00
355   -.593394E+00   0.000000E+00
356   -.681707E+01   0.000000E+00
357   0.652774E+01   0.000000E+00
358   0.150886E+01   0.000000E+00
359   -.614447E+01   0.000000E+00
360   0.239488E+01   0.000000E+00
361   0.226228E+01   0.000000E+00
362   -.481258E+01   0.000000E+00
363   -.354793E+01   0.000000E+00
364   0.451782E+01   0.000000E+00
365   -.756871E+00   0.000000E+00
366   -.148307E+01   0.000000E+00
367   0.335441E+01   0.000000E+00
368   0.871370E+00   0.000000E+00
369   0.123139E+01   0.000000E+00
370   -.170679E+01   0.000000E+00
371   0.252001E+01   0.000000E+00
372   -.361835E+00   0.000000E+00 ·
373   -.495687E+01   0.000000E+00
374   0.162119E+01   0.000000E+00
375   0.185459E+01   0.000000E+00
376   -.309277E+01   0.000000E+00
377   -.320235E+01   0.000000E+00
378   0.138036E+01   0.000000E+00
379   -.292664E+01   0.000000E+00
380   -.307330E+01   0.000000E+00
381   0.156635E+01   0.000000E+00
382   0.748866E+01   0.000000E+00
383   -.562336E+01   0.000000E+00
384   0.100312E+00   0.000000E+00
385   0.132461E+01   0.000000E+00
386   -.355028E+01   0.000000E+00
387   -.232426E+01   0.000000E+00
388   0.451112E+01   0.000000E+00
389   -.531282E+00   0.000000E+00
390   -.288922E+01   0.000000E+00
391   0.354156E+01   0.000000E+00
392   -.109373E+01   0.000000E+00
393   -.283108E+01   0.000000E+00
394   0.396496E+01   0.000000E+00
395   -.698274E+00   0.000000E+00
396   -.531781E+01   0.000000E+00
397   0.843346E+00   0.000000E+00
398   0.356507E+01   0.000000E+00
399   -.822237E+00   0.000000E+00
400   -.128205E+01   0.000000E+00
401   0.401342E+01   0.000000E+00
402   -.109053E+00   0.000000E+00
403   -.560845E+01   0.000000E+00
404   0.179668E+01   0.000000E+00
405   -.377383E+01   0.000000E+00
406   -.398152E+01   0.000000E+00
407   -.198354E+01   0.000000E+00
408   0.325370E+01   0.000000E+00
409   -.542649E+01   0.000000E+00
410   -.373510E+01   0.000000E+00
411   0.330313E+01   0.000000E+00
412   0.116460E+01   0.000000E+00
413   -.330263E+01   0.000000E+00
414   -.428371E+01   0.000000E+00
415   0.166203E+01   0.000000E+00
416   0.294798E+01   0.000000E+00
417   -.260710E+01   0.000000E+00
418   0.193795E+01   0.000000E+00
419   0.609513E+01   0.000000E+00
420   -.514264E+01   0.000000E+00
421   -.528953E+01   0.000000E+00
422   0.720675E+01   0.000000E+00
423   -.123952E+01   0.000000E+00
424   -.733272E+01   0.000000E+00
425   0.468533E+01   0.000000E+00
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426   0.573544E+01    0.000000E+00
427  -.845249E+01     0.000000E+00
428   0.516992E+00    0.000000E+00
429   0.119343E+02    0.000000E+00
430  -.442223E+01     0.000000E+00
431  -.389617E+01     0.000000E+00
432   0.306069E+01    0.000000E+00
433  -.741859E+00     0.000000E+00
434  -.601171E+01     0.000000E+00
435   0.915210E+00    0.000000E+00
436  -.662840E+00     0.000000E+00
437  -.279501E+01     0.000000E+00
438   0.102183E+01    0.000000E+00
439   0.129399E+01    0.000000E+00
440   0.942253E+00    0.000000E+00
441  -.105543E+00     0.000000E+00
442   0.422624E+01    0.000000E+00
443  -.604331E+00     0.000000E+00
444  -.160753E+01     0.000000E+00 .
445   0.489188E+00    0.000000E+00
446  -.570710E+01     0.000000E+00
447  -.202896E+01     0.000000E+00
448   0.476189E+01    0.000000E+00
449  -.340101E+01     0.000000E+00
450  -.432951E+01     0.000000E+00
451   0.849652E+01    0.000000E+00
452   0.419645E+01    0.000000E+00
453  -.520679E+01     0.000000E+00
454   0.318501E+01    0.000000E+00
455   0.661741E+01    0.000000E+00
456  -.293308E+01     0.000000E+00
457  -.549945E+00     0.000000E+00
458   0.337600E+01    0.000000E+00
459  -.188377E+01     0.000000E+00
460  -.153699E+01     0.000000E+00
461   0.728273E+00    0.000000E+00
462   0.173535E+01    0.000000E+00
463   0.842127E-01    0.000000E+00
464  -.321595E+01     0.000000E+00
465   0.280456E+01    0.000000E+00
466   0.515843E+01    0.000000E+00
467   0.127567E+01    0.000000E+00
468  -.271868E+01     0.000000E+00
469   0.314677E+01    0.000000E+00
470  -.684675E+00     0.000000E+00
471  -.370720E+01     0.000000E+00
472   0.485160E+00    0.000000E+00
473   0.458457E+01    0.000000E+00
474  -.628002E+01     0.000000E+00
475  -.598999E+01     0.000000E+00
476   0.536761E+01    0.000000E+00
477  -.155600E+01     0.000000E+00
478  -.832894E+01     0.000000E+00
479   0.648226E+00    0.000000E+00
480   0.308144E+01    0.000000E+00
481  -.259356E+01     0.000000E+00
482   0.101954E+01    0.000000E+00
483  -.183804E+00     0.000000E+00
484   0.914496E+00    0.000000E+00
485   0.103868E+00    0.000000E+00
486  -.548742E+01     0.000000E+00
487  -.193591E+01     0.000000E+00
488   0.333655E+01    0.000000E+00
489  -.407597E+01     0.000000E+00
490  -.881921E+01     0.000000E+00
491   0.956705E+01    0.000000E+00
492   0.924545E+00    0.000000E+00
493  -.905340E+01     0.000000E+00
494   0.402092E+01    0.000000E+00
495   0.527295E+01    0.000000E+00
496  -.906265E+01     0.000000E+00
497  -.233179E+01     0.000000E+00
```

```
498   0.880906E+01   0.000000E+00
499  -.545897E+01    0.000000E+00
500  -.745451E+01    0.000000E+00
501   0.784850E+01   0.000000E+00
502   0.331564E+01   0.000000E+00
503  -.686388E+01    0.000000E+00
504   0.161638E+01   0.000000E+00
505   0.279832E+01   0.000000E+00
506  -.107526E+02    0.000000E+00
507   0.452893E+01   0.000000E+00
508   0.396753E+01   0.000000E+00
509  -.574705E+01    0.000000E+00
510   0.397997E+01   0.000000E+00
511   0.242562E+01   0.000000E+00
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DFT OUTPUT DATA FOR BAUD #   2

K	REAL PART	IMAG PART	MAGNITUDE	PHASE (DEG)	(RAD)
135	0.21869614E+03	-.13437286E+03	0.25667896E+03	-.31567688E+02	-.55096024E+00
136	-.20494591E+03	-.20220082E+03	0.28790259E+03	-.13538632E+03	-.23629370E+01
137	-.14575168E+03	-.15294678E+03	0.21127296E+03	-.13362015E+03	-.23321114E+01
138	-.21255940E+03	0.19105444E+03	0.28580273E+03	0.13804984E+03	0.24094248E+01
139	-.15577037E+03	-.13739233E+03	0.20770425E+03	-.13858707E+03	-.24188013E+01
140	-.13250822E+03	0.14138666E+03	0.19377464E+03	0.13314339E+03	0.23237906E+01
141	0.17392702E+03	-.22279741E+03	0.28264697E+03	-.52022568E+02	-.90796512E+00
142	0.13896306E+03	-.13225531E+03	0.19183899E+03	-.43583252E+02	-.76067126E+00
143	0.16901073E+03	0.14665094E+03	0.22376578E+03	0.40948242E+02	0.71468163E+00
144	0.19780035E+03	-.15818835E+03	0.25327560E+03	-.38650635E+02	-.67458087E+00
145	-.20230388E+03	0.27242383E+03	0.33932495E+03	0.12659782E+03	0.22095499E+01
146	0.20827090E+03	-.18137456E+03	0.27617651E+03	-.41051254E+02	-.71647972E+00
147	-.19098985E+03	0.22640436E+03	0.29620264E+03	0.13015030E+03	0.22715511E+01
148	0.15571500E+03	0.14765985E+03	0.21459401E+03	0.43479050E+02	0.75885254E+00
149	0.22490648E+03	0.18650417E+03	0.29217578E+03	0.39667282E+02	0.69232482E+00
150	0.18717580E+03	0.16107140E+03	0.24695880E+03	0.40713165E+02	0.71057868E+00
151	-.13020734E+03	-.21927161E+03	0.25501762E+03	-.12070264E+03	-.21066589E+01
152	0.21125522E+03	-.22968059E+03	0.31206079E+03	-.47392807E+02	-.82716048E+00
153	0.16326271E+03	-.17587959E+03	0.23997571E+03	-.47130524E+02	-.82258284E+00
154	-.18001019E+03	0.15380461E+03	0.23676892E+03	0.13948868E+03	0.24345369E+01
155	-.19258081E+03	-.13529289E+03	0.23535405E+03	-.14491098E+03	-.25291739E+01
156	-.19027963E+03	0.16097487E+03	0.24923732E+03	0.13976907E+03	0.24394302E+01
157	-.21412935E+03	0.18872530E+03	0.28542749E+03	0.13860835E+03	0.24191723E+01
158	-.20541913E+03	-.22383812E+03	0.30381006E+03	-.13254298E+03	-.23133116E+01
159	-.19955086E+03	0.20583072E+03	0.28668237E+03	0.13411246E+03	0.23407049E+01
160	-.21075299E+03	0.22362857E+03	0.30728906E+03	0.13330223E+03	0.23265629E+01
161	0.17198572E+03	0.17881973E+03	0.24810397E+03	0.46116013E+02	0.80487633E+00
162	-.17657683E+03	0.15956850E+03	0.23799471E+03	0.13789659E+03	0.24067497E+01
163	-.18814537E+03	0.14442966E+03	0.23718896E+03	0.14248830E+03	0.24868908E+01
164	-.17734966E+03	-.17915202E+03	0.25208797E+03	-.13471030E+03	-.23511391E+01
165	-.18872591E+03	-.18354022E+03	0.26325732E+03	-.13579803E+03	-.23701239E+01
166	-.18190575E+03	0.12815350E+03	0.22251521E+03	0.14483507E+03	0.25278492E+01

NOTE: The DFT is windowed (IWNDOW = 1)

C. GRAPHICAL OUTPUT USING DISSPLA

1. DFT Windowed Between k_1 and k_2 (IWNDOW=1)

DFT OUTPUT OF THE RECEIVED SIGNAL
FOR BAUD NUMBER   1
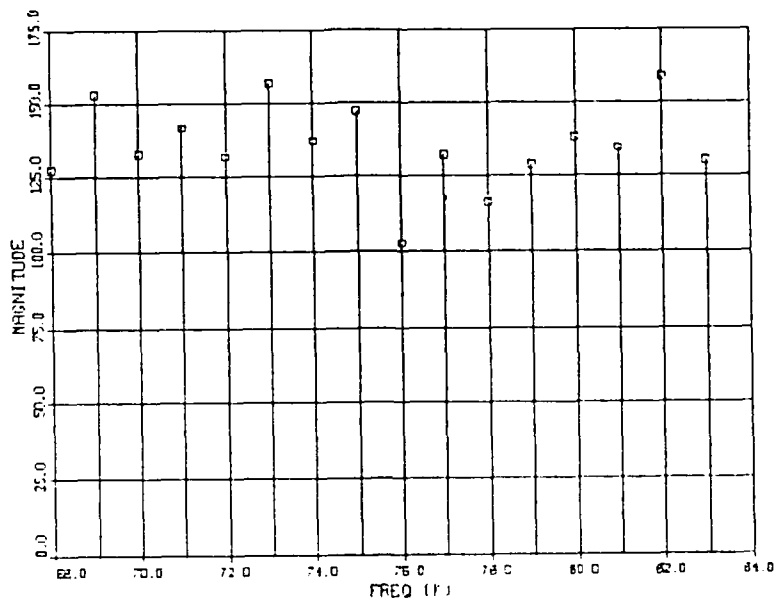
DFT OUTPUT OF THE RECEIVED SIGNAL
FOR BAUD NUMBER   1

79

DFT OUTPUT OF THE RECEIVED SIGNAL
FOR BAUD NUMBER 2

DFT OUTPUT OF THE RECEIVED SIGNAL
FOR BAUD NUMBER 2

80
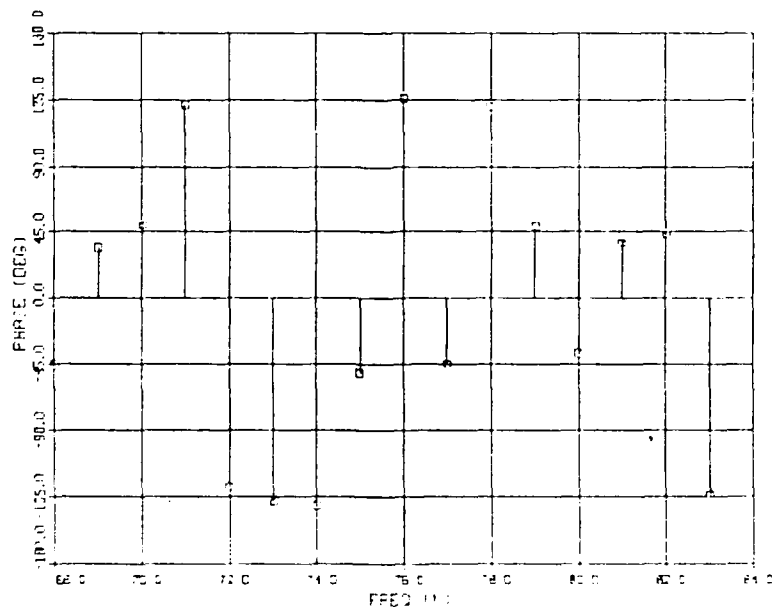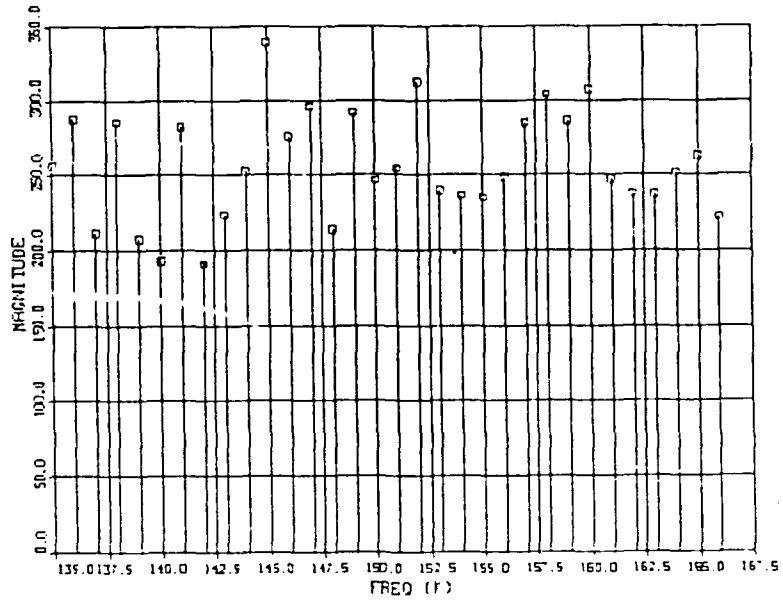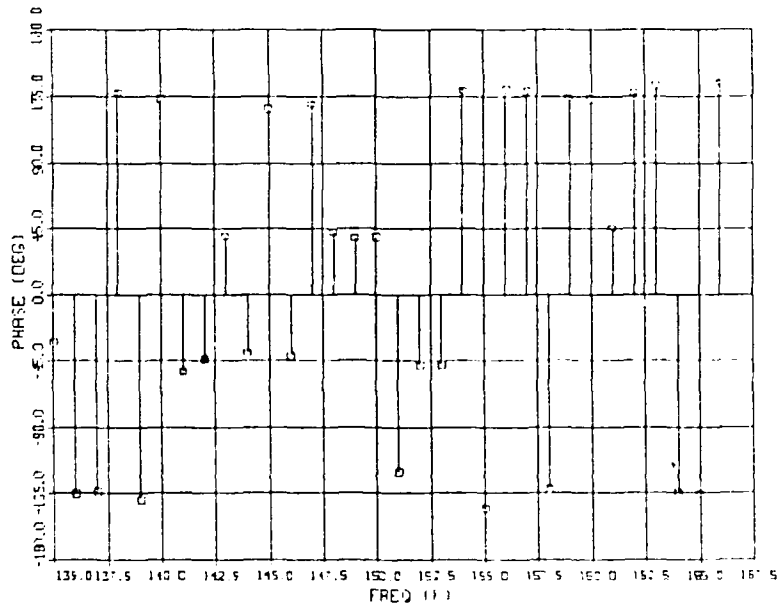
RECEIVED SIGNAL

X-POSITION

81

Y-POSITION

Z-POSITION

82
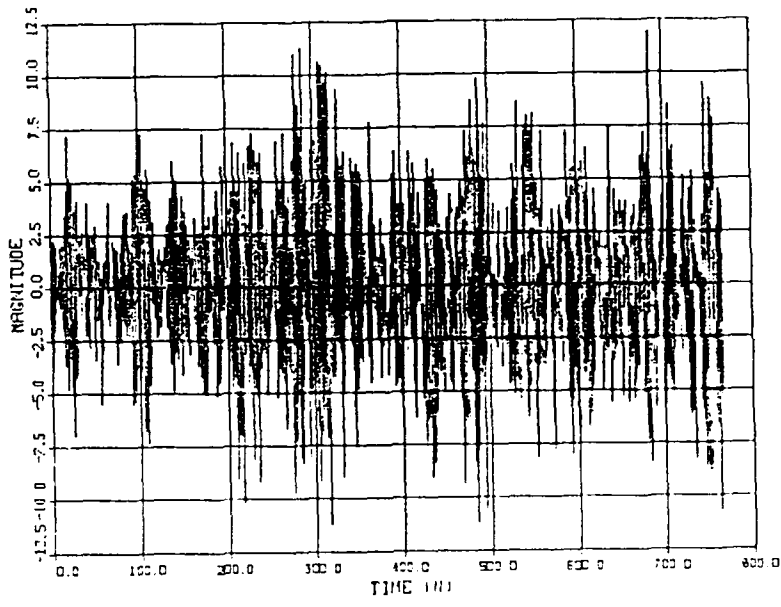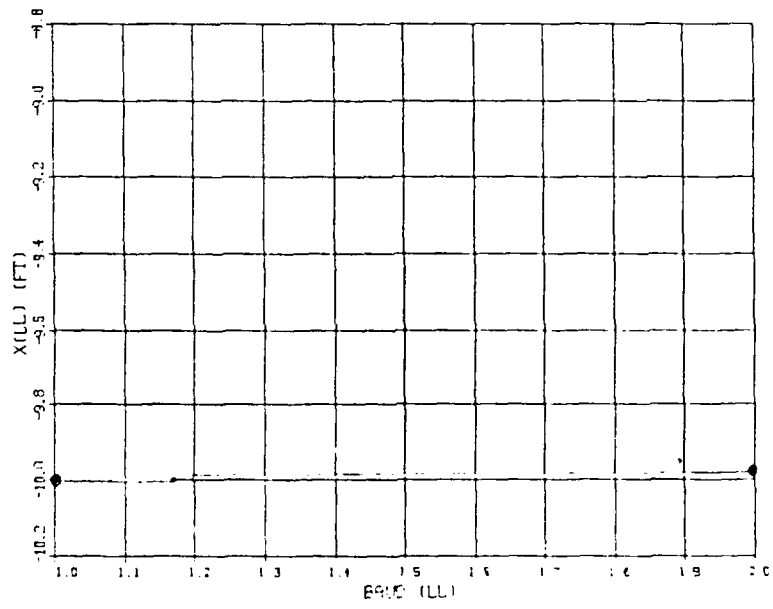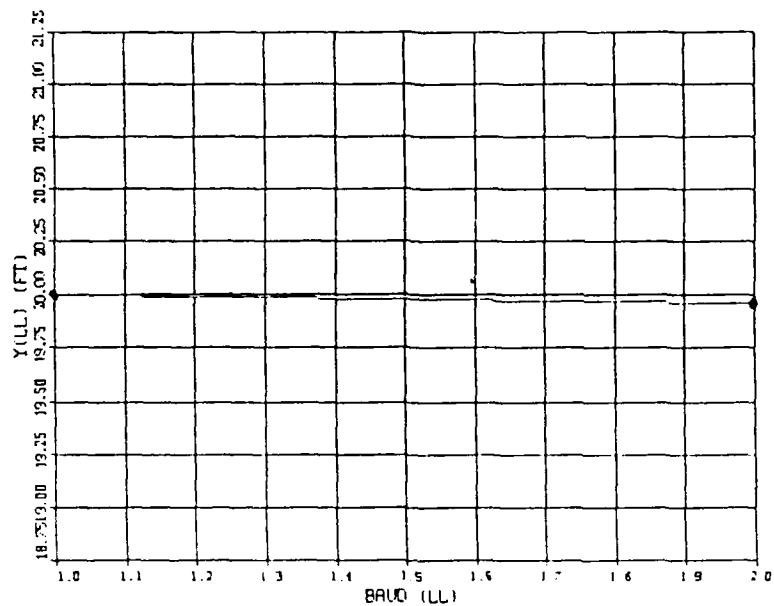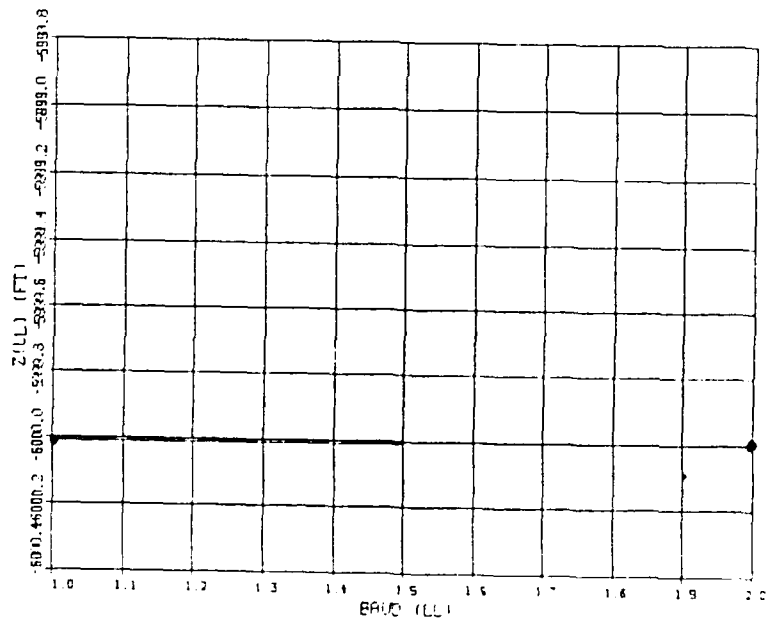
SLANT RANGE TO RECEIVER

SPEED OF SOUND

83
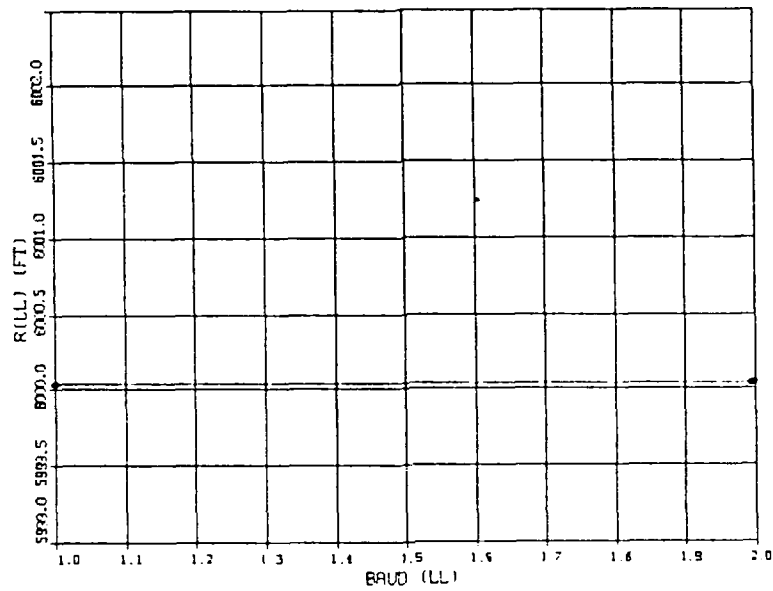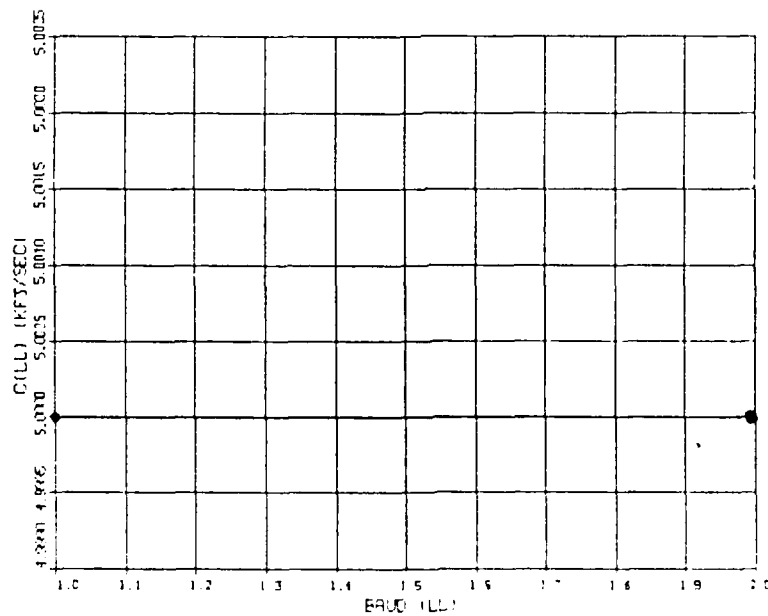
COMPRESSION FACTOR DUE TO THE MOVING TX

ANGLE BETWEEN R(LL) AND ZO

84
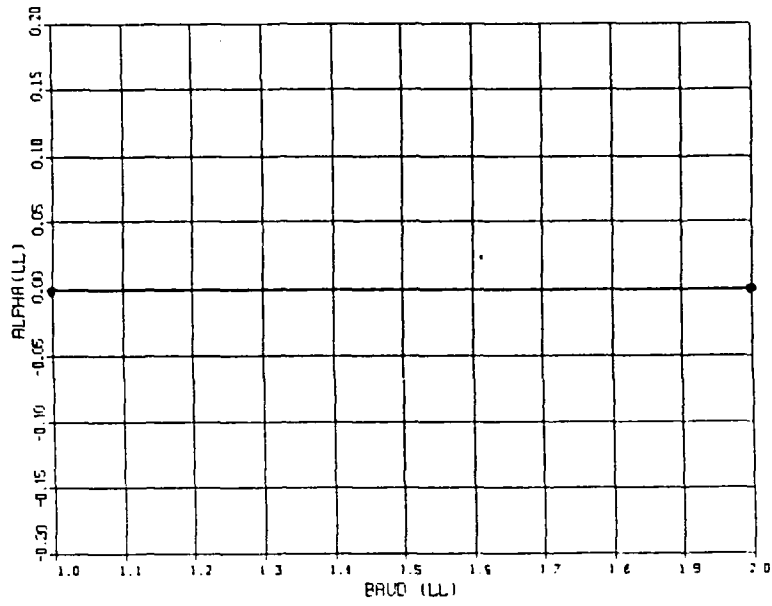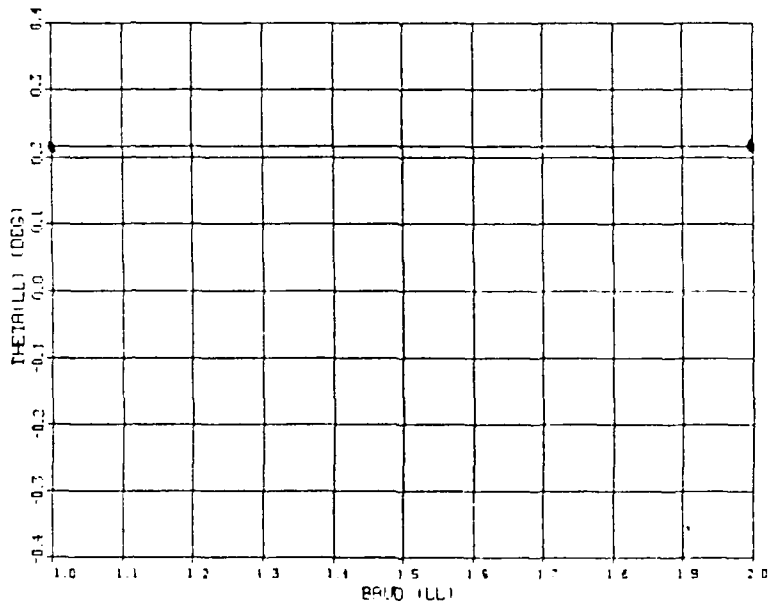
2. DFT Not Windowed (IWNDOW=0)

DFT OUTPUT OF THE RECEIVED SIGNAL
FOR BAUD NUMBER   1
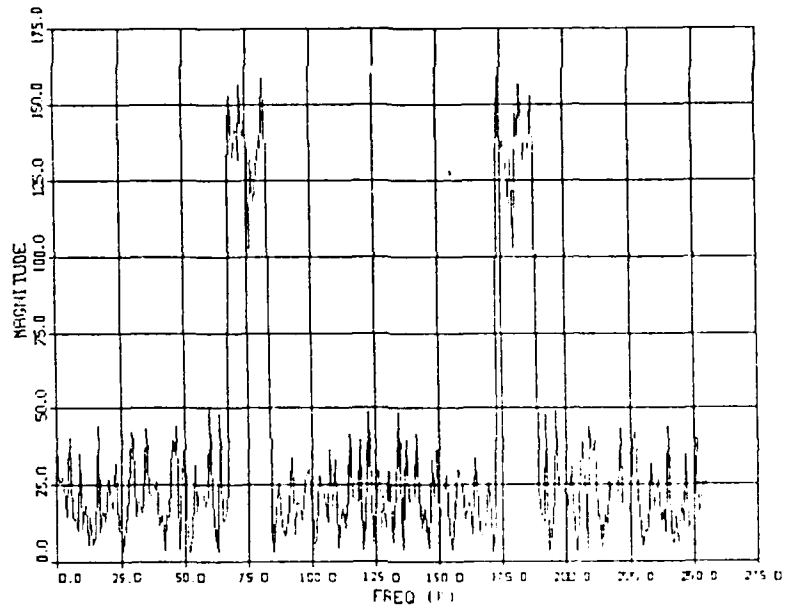
DFT OUTPUT OF THE RECEIVED SIGNAL
FOR BAUD NUMBER   1

85

DFT OUTPUT OF THE RECEIVED SIGNAL
FOR BAUD NUMBER   2

DFT OUTPUT OF THE RECEIVED SIGNAL
FOR BAUD NUMBER   2

86
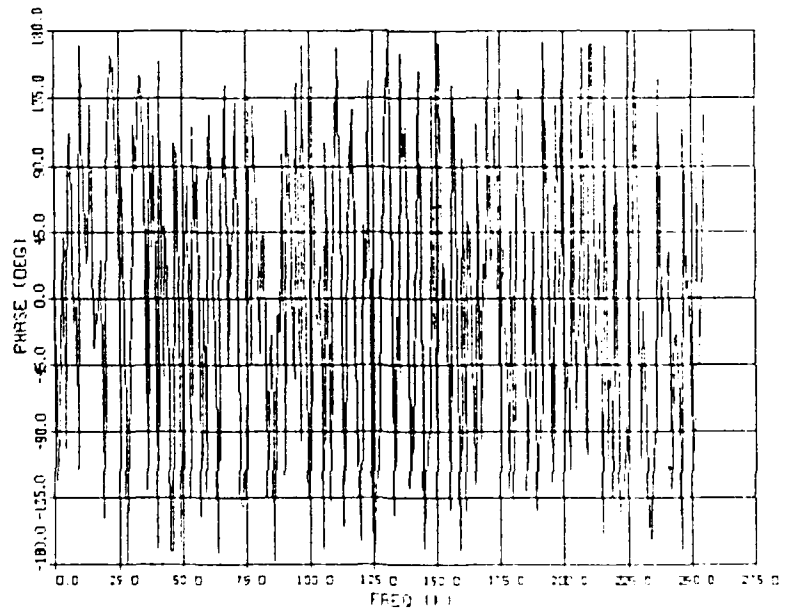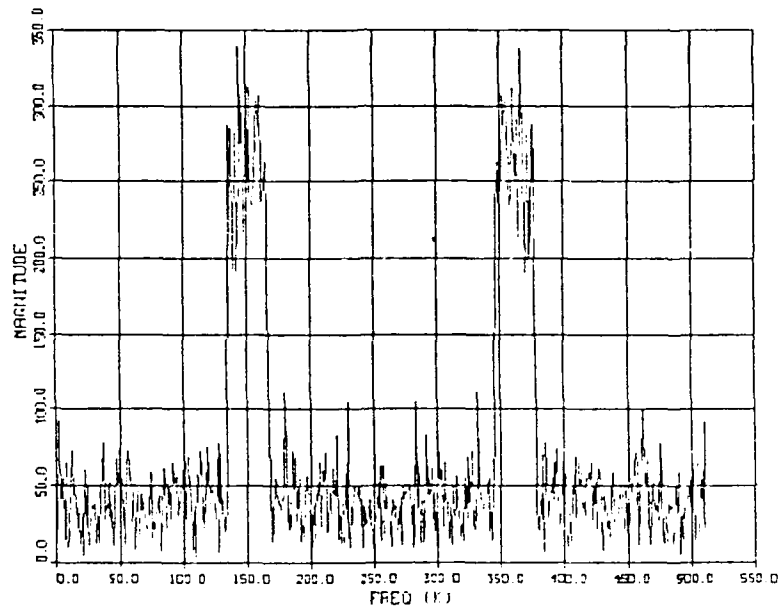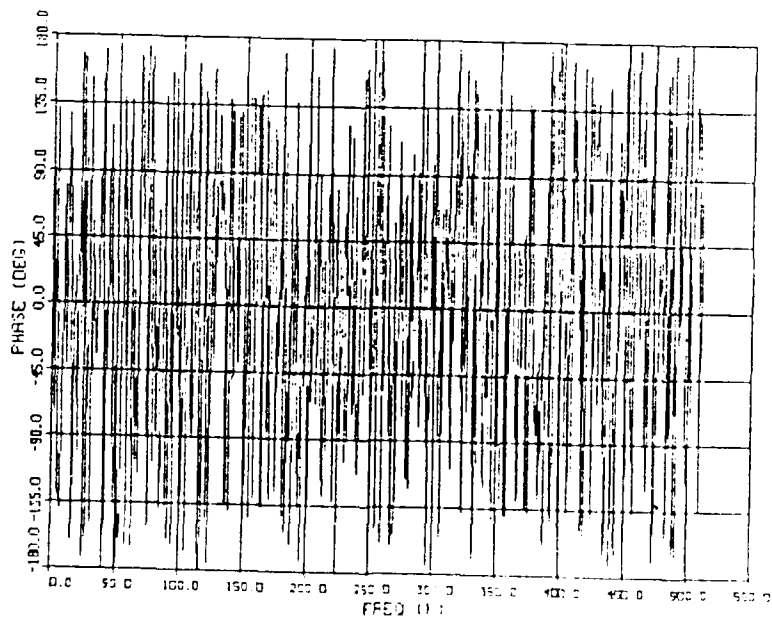
APPENDIX C. STATISTICS OF SNR$_{OUT}$ VS. SNR$_{IN}$ ANALYSIS

A. INPUT SNR = 0 dB

```
INPUT SNRNB =   1.000 =   0.000 DB

BAUD TYPE 1:  KX =  256 SAMPLE POINTS; K =  16 TONES


** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **
                       NUMBER OF TONES =    4
             MEAN OF DFT REAL PART =    97.73993
         VARIANCE OF DFT REAL PART =     8419.37
QUADRANT SNROUT OF DFT REAL PART =     1.1347 =    0.5486 DB

             MEAN OF DFT IMAG PART =   129.16241
         VARIANCE OF DFT IMAG PART =     6640.81
QUADRANT SNROUT OF DFT IMAG PART =     2.5122 =    4.0005 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **
                       NUMBER OF TONES =    2
             MEAN OF DFT REAL PART =   -85.21500
         VARIANCE OF DFT REAL PART =      429.54
QUADRANT SNROUT OF DFT REAL PART =    16.9056 =   12.2803 DB

             MEAN OF DFT IMAG PART =    82.06499
         VARIANCE OF DFT IMAG PART =     2787.80
QUADRANT SNROUT OF DFT IMAG PART =     2.4158 =    3.8305 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **
                       NUMBER OF TONES =    6
             MEAN OF DFT REAL PART =   -61.63330
         VARIANCE OF DFT REAL PART =    12897.87
QUADRANT SNROUT OF DFT REAL PART =     0.2945 =   -5.3089 DB

             MEAN OF DFT IMAG PART =  -157.14986
         VARIANCE OF DFT IMAG PART =     1129.39
QUADRANT SNROUT OF DFT IMAG PART =    21.8667 =   13.3978 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **
                       NUMBER OF TONES =    4
             MEAN OF DFT REAL PART =   135.55243
         VARIANCE OF DFT REAL PART =     7555.41
QUADRANT SNROUT OF DFT REAL PART =     2.4320 =    3.8596 DB

             MEAN OF DFT IMAG PART =  -134.99750
         VARIANCE OF DFT IMAG PART =    11628.12
QUADRANT SNROUT OF DFT IMAG PART =     1.5673 =    1.9514 DB


*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 1 ***

    TOTAL NUMBER OF POINTS, 2K =     32
                   BAUD MEAN =   113.65836
               BAUD VARIANCE =     5680.15
                 BAUD SNROUT =     2.2743 =    3.5684 DB
```

```
INPUT SNRNB =    1.000 =    0.000 DB

BAUD TYPE 2:  KX =   512 SAMPLE POINTS; K =   32 TONES


** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **
                      NUMBER OF TONES =      5
            MEAN OF DFT REAL PART =    137.35571
        VARIANCE OF DFT REAL PART =     14934.41
QUADRANT SNROUT OF DFT REAL PART =      1.2633 =    1.0151 DB

            MEAN OF DFT IMAG PART =    263.66382
        VARIANCE OF DFT IMAG PART =     54654.25
QUADRANT SNROUT OF DFT IMAG PART =      1.2720 =    1.0448 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

                      NUMBER OF TONES =      8
            MEAN OF DFT REAL PART =   -126.14920
        VARIANCE OF DFT REAL PART =     60110.95
QUADRANT SNROUT OF DFT REAL PART =      0.2647 =   -5.7718 DB

            MEAN OF DFT IMAG PART =    137.62115
        VARIANCE OF DFT IMAG PART =     19688.26
QUADRANT SNROUT OF DFT IMAG PART =      0.9620 =   -0.1684 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

                      NUMBER OF TONES =     10
            MEAN OF DFT REAL PART =   -191.87024
        VARIANCE OF DFT REAL PART =     19641.78
QUADRANT SNROUT OF DFT REAL PART =      1.8743 =    2.7283 DB

            MEAN OF DFT IMAG PART =   -288.79492
        VARIANCE OF DFT IMAG PART =     30312.55
QUADRANT SNROUT OF DFT IMAG PART =      2.7514 =    4.3956 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

                      NUMBER OF TONES =      9
            MEAN OF DFT REAL PART =    154.75328
        VARIANCE OF DFT REAL PART =     40444.66
QUADRANT SNROUT OF DFT REAL PART =      0.5921 =   -2.2758 DB

            MEAN OF DFT IMAG PART =   -215.35548
        VARIANCE OF DFT IMAG PART =     21217.86
QUADRANT SNROUT OF DFT IMAG PART =      2.1858 =    3.3961 DB



*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 2 ***

    TOTAL NUMBER OF POINTS, 2K =      64
                     BAUD MEAN =    191.45129
                 BAUD VARIANCE =     28251.36
                   BAUD SNROUT =      1.2974 =    1.1308 DB
```

```
INPUT SNRNB =    1.000 =    0.000 DB

BAUD TYPE 3:   KX = 1024 SAMPLE POINTS; K =   64 TONES


** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **
                     NUMBER OF TONES =     13
            MEAN OF DFT REAL PART =    295.59131
        VARIANCE OF DFT REAL PART =    171068.06
QUADRANT SNROUT OF DFT REAL PART =      0.5108 =  -2.9179 DB

            MEAN OF DFT IMAG PART =    272.87988
        VARIANCE OF DFT IMAG PART =     75231.81
QUADRANT SNROUT OF DFT IMAG PART =      0.9898 =  -0.0446 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

                     NUMBER OF TONES =     15
            MEAN OF DFT REAL PART =   -502.71387
        VARIANCE OF DFT REAL PART =    107979.00
QUADRANT SNROUT OF DFT REAL PART =      2.3405 =   3.6930 DB

            MEAN OF DFT IMAG PART =    393.86084
        VARIANCE OF DFT IMAG PART =     91919.19
QUADRANT SNROUT OF DFT IMAG PART =      1.6876 =   2.2728 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

                     NUMBER OF TONES =     20
            MEAN OF DFT REAL PART =   -365.57617
        VARIANCE OF DFT REAL PART =    158013.50
QUADRANT SNROUT OF DFT REAL PART =      0.8458 =  -0.7274 DB

            MEAN OF DFT IMAG PART =   -287.26196
        VARIANCE OF DFT IMAG PART =    110806.31
QUADRANT SNROUT OF DFT IMAG PART =      0.7447 =  -1.2801 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

                     NUMBER OF TONES =     16
            MEAN OF DFT REAL PART =    328.53247
        VARIANCE OF DFT REAL PART =     97164.81
QUADRANT SNROUT OF DFT REAL PART =      1.1108 =   0.4565 DB

            MEAN OF DFT IMAG PART =   -504.95386
        VARIANCE OF DFT IMAG PART =    125734.50
QUADRANT SNROUT OF DFT IMAG PART =      2.0279 =   3.0705 DB


*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 3 ***

    TOTAL NUMBER OF POINTS, 2K =     128
                    BAUD MEAN =    368.99438
                BAUD VARIANCE =    111852.19
                  BAUD SNROUT =      1.2173 =    0.8540 DB
```

```
INPUT SNRNB =    1.000 =    0.000 DB

BAUD TYPE 4:   KX = 2048 SAMPLE POINTS; K = 128 TONES


** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

                    NUMBER OF TONES =    38
            MEAN OF DFT REAL PART =    765.55249
        VARIANCE OF DFT REAL PART =    480832.37
QUADRANT SNROUT OF DFT REAL PART =    1.2189 =    0.8596 DB

            MEAN OF DFT IMAG PART =    487.95532
        VARIANCE OF DFT IMAG PART =    436912.56
QUADRANT SNROUT OF DFT IMAG PART =    0.5450 =   -2.6363 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

                    NUMBER OF TONES =    34
            MEAN OF DFT REAL PART =   -919.41406
        VARIANCE OF DFT REAL PART =    473379.37
QUADRANT SNROUT OF DFT REAL PART =    1.7857 =    2.5181 DB

            MEAN OF DFT IMAG PART =    909.34058
        VARIANCE OF DFT IMAG PART =    403385.00
QUADRANT SNROUT OF DFT IMAG PART =    2.0499 =    3.1173 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

                    NUMBER OF TONES =    35
            MEAN OF DFT REAL PART =   -744.60986
        VARIANCE OF DFT REAL PART =    535541.62
QUADRANT SNROUT OF DFT REAL PART =    1.0353 =    0.1506 DB

            MEAN OF DFT IMAG PART =   -645.88306
        VARIANCE OF DFT IMAG PART =    493911.50
QUADRANT SNROUT OF DFT IMAG PART =    0.8446 =   -0.7334 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

                    NUMBER OF TONES =    21
            MEAN OF DFT REAL PART =    845.74316
        VARIANCE OF DFT REAL PART =    506747.94
QUADRANT SNROUT OF DFT REAL PART =    1.4115 =    1.4968 DB

            MEAN OF DFT IMAG PART =   -607.39453
        VARIANCE OF DFT IMAG PART =    472505.12
QUADRANT SNROUT OF DFT IMAG PART =    0.7808 =   -1.0746 DB


*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 4 ***

TOTAL NUMBER OF POINTS, 2K =    256
                 BAUD MEAN =    738.25732
             BAUD VARIANCE =    460690.81
                BAUD SNROUT =    1.1831 =    0.7301 DB
```

```
INPUT SNRNB =   1.000 =   0.000 DB

BAUD TYPE 5:   KX = 4096 SAMPLE POINTS; K = 256 TONES


** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

                    NUMBER OF TONES =   64
            MEAN OF DFT REAL PART =  1507.82715
        VARIANCE OF DFT REAL PART =  2880635.00
QUADRANT SNROUT OF DFT REAL PART =     0.7893 =  -1.0279 DB

            MEAN OF DFT IMAG PART =  1674.69238
        VARIANCE OF DFT IMAG PART =  2577858.00
QUADRANT SNROUT OF DFT IMAG PART =     1.0880 =   0.3661 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

                    NUMBER OF TONES =   61
            MEAN OF DFT REAL PART = -1636.89526
        VARIANCE OF DFT REAL PART =  2406180.00
QUADRANT SNROUT OF DFT REAL PART =     1.1136 =   0.4671 DB

            MEAN OF DFT IMAG PART =  1345.84106
        VARIANCE OF DFT IMAG PART =  3274481.00
QUADRANT SNROUT OF DFT IMAG PART =     0.5532 =  -2.5715 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

                    NUMBER OF TONES =   59
            MEAN OF DFT REAL PART = -1447.54858
        VARIANCE OF DFT REAL PART =  1710154.00
QUADRANT SNROUT OF DFT REAL PART =     1.2253 =   0.8823 DB

            MEAN OF DFT IMAG PART = -1208.50415
        VARIANCE OF DFT IMAG PART =  2686264.00
QUADRANT SNROUT OF DFT IMAG PART =     0.5437 =  -2.6465 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

                    NUMBER OF TONES =   72
            MEAN OF DFT REAL PART =  1315.15088
        VARIANCE OF DFT REAL PART =  1992730.00
QUADRANT SNROUT OF DFT REAL PART =     0.8680 =  -0.6150 DB

            MEAN OF DFT IMAG PART = -1541.86792
        VARIANCE OF DFT IMAG PART =  2939624.00
QUADRANT SNROUT OF DFT IMAG PART =     0.8087 =  -0.9220 DB


*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 5 ***

    TOTAL NUMBER OF POINTS, 2K =     512
                    BAUD MEAN =  1461.01611
                BAUD VARIANCE =  2524291.00
                  BAUD SNROUT =     0.8456 =  -0.7283 DB
```

B. INPUT SNR = 5 dB

```
INPUT SNRNB =   3.162 =   5.000 DB

BAUD TYPE 1:  KX =  256 SAMPLE POINTS; K =  16 TONES


** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

                     NUMBER OF TONES =     4
              MEAN OF DFT REAL PART =    94.57245
          VARIANCE OF DFT REAL PART =     2661.94
QUADRANT SNROUT OF DFT REAL PART =     3.3599 =    5.2633 DB

              MEAN OF DFT IMAG PART =   112.22491
          VARIANCE OF DFT IMAG PART =     2098.79
QUADRANT SNROUT OF DFT IMAG PART =     6.0008 =    7.7821 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

                     NUMBER OF TONES =     2
              MEAN OF DFT REAL PART =   -87.53000
          VARIANCE OF DFT REAL PART =      135.80
QUADRANT SNROUT OF DFT REAL PART =    56.4194 =   17.5143 DB

              MEAN OF DFT IMAG PART =    85.78000
          VARIANCE OF DFT IMAG PART =      883.68
QUADRANT SNROUT OF DFT IMAG PART =     8.3268 =    9.2048 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

                     NUMBER OF TONES =     6
              MEAN OF DFT REAL PART =   -74.27998
          VARIANCE OF DFT REAL PART =     4080.46
QUADRANT SNROUT OF DFT REAL PART =     1.3522 =    1.3103 DB

              MEAN OF DFT IMAG PART =  -127.96658
          VARIANCE OF DFT IMAG PART =      357.45
QUADRANT SNROUT OF DFT IMAG PART =    45.8118 =   16.6098 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

                     NUMBER OF TONES =     4
              MEAN OF DFT REAL PART =   115.84491
          VARIANCE OF DFT REAL PART =     2387.74
QUADRANT SNROUT OF DFT REAL PART =     5.6204 =    7.4977 DB

              MEAN OF DFT IMAG PART =  -115.54242
          VARIANCE OF DFT IMAG PART =     3679.88
QUADRANT SNROUT OF DFT IMAG PART =     3.6278 =    5.5965 DB


*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 1 ***

     TOTAL NUMBER OF POINTS, 2K =     32
                     BAUD MEAN =   103.52617
                 BAUD VARIANCE =     1796.58
                   BAUD SNROUT =     5.9656 =    7.7565 DB
```

92

```
INPUT SNRNB =   3.162 =   5.000 DB

BAUD TYPE 2:  KX =  512 SAMPLE POINTS; K =  32 TONES


** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

                    NUMBER OF TONES =    5
           MEAN OF DFT REAL PART =   156.34995
       VARIANCE OF DFT REAL PART =     4717.56
QUADRANT SNROUT OF DFT REAL PART =     5.1818 =   7.1448 DB

           MEAN OF DFT IMAG PART =   227.40594
       VARIANCE OF DFT IMAG PART =    17291.69
QUADRANT SNROUT OF DFT IMAG PART =     2.9907 =   4.7577 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

                    NUMBER OF TONES =    8
           MEAN OF DFT REAL PART =  -150.08240
       VARIANCE OF DFT REAL PART =    19011.11
QUADRANT SNROUT OF DFT REAL PART =     1.1848 =   0.7365 DB

           MEAN OF DFT IMAG PART =   156.51974
       VARIANCE OF DFT IMAG PART =     6230.11
QUADRANT SNROUT OF DFT IMAG PART =     3.9323 =   5.9464 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

                    NUMBER OF TONES =   10
           MEAN OF DFT REAL PART =  -186.99594
       VARIANCE OF DFT REAL PART =     6212.98
QUADRANT SNROUT OF DFT REAL PART =     5.6281 =   7.5036 DB

           MEAN OF DFT IMAG PART =  -241.53493
       VARIANCE OF DFT IMAG PART =     9587.26
QUADRANT SNROUT OF DFT IMAG PART =     6.0851 =   7.8427 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

                    NUMBER OF TONES =    9
           MEAN OF DFT REAL PART =   166.15880
       VARIANCE OF DFT REAL PART =    12788.79
QUADRANT SNROUT OF DFT REAL PART =     2.1588 =   3.3422 DB

           MEAN OF DFT IMAG PART =  -200.21880
       VARIANCE OF DFT IMAG PART =     6707.95
QUADRANT SNROUT OF DFT IMAG PART =     5.9761 =   7.7642 DB


*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 2 ***

       TOTAL NUMBER OF POINTS, 2K =    64
                      BAUD MEAN =   186.78583 ·
                  BAUD VARIANCE =     8934.94
                    BAUD SNROUT =     3.9048 =   5.9160 DB
```

INPUT SNRNB =   3.162 =   5.000 DB

BAUD TYPE 3:   KX = 1024 SAMPLE POINTS; K =   64 TONES

** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

                      NUMBER OF TONES =   13
             MEAN OF DFT REAL PART =   324.77417
         VARIANCE OF DFT REAL PART =   54069.19
QUADRANT SNROUT OF DFT REAL PART =   1.9508 =   2.9021 DB

              MEAN OF DFT IMAG PART =   312.15674
          VARIANCE OF DFT IMAG PART =   23785.73
QUADRANT SNROUT OF DFT IMAG PART =   4.0967 =   6.1243 DB

** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

                      NUMBER OF TONES =   15
             MEAN OF DFT REAL PART =   -441.42603
         VARIANCE OF DFT REAL PART =   34121.99
QUADRANT SNROUT OF DFT REAL PART =   5.7106 =   7.5668 DB

              MEAN OF DFT IMAG PART =   380.12280
          VARIANCE OF DFT IMAG PART =   29067.91
QUADRANT SNROUT OF DFT IMAG PART =   4.9709 =   6.9643 DB

** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

                      NUMBER OF TONES =   20
             MEAN OF DFT REAL PART =   -364.12720
         VARIANCE OF DFT REAL PART =   49952.47
QUADRANT SNROUT OF DFT REAL PART =   2.6543 =   4.2395 DB

              MEAN OF DFT IMAG PART =   -320.32812
          VARIANCE OF DFT IMAG PART =   35015.78
QUADRANT SNROUT OF DFT IMAG PART =   2.9304 =   4.6693 DB

** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

                      NUMBER OF TONES =   16
             MEAN OF DFT REAL PART =   343.55005
         VARIANCE OF DFT REAL PART =   30738.97
QUADRANT SNROUT OF DFT REAL PART =   3.8396 =   5.8429 DB

              MEAN OF DFT IMAG PART =   -442.46045
          VARIANCE OF DFT IMAG PART =   39734.58
QUADRANT SNROUT OF DFT IMAG PART =   4.9270 =   6.9258 DB

*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 3 ***

        TOTAL NUMBER OF POINTS, 2K =   128
                       BAUD MEAN =   366.16089
                   BAUD VARIANCE =   35357.58
                     BAUD SNROUT =   3.7919 =   5.7886 DB

94

INPUT SNRNB =   3.162 =   5.000 DB

BAUD TYPE 4:   KX = 2048 SAMPLE POINTS; K = 128 TONES

** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

```
                   NUMBER OF TONES =    38
             MEAN OF DFT REAL PART =    754.15430
         VARIANCE OF DFT REAL PART =    151360.81
QUADRANT SNROUT OF DFT REAL PART =    3.7576 =    5.7491 DB

             MEAN OF DFT IMAG PART =    598.70117
         VARIANCE OF DFT IMAG PART =    138265.00
QUADRANT SNROUT OF DFT IMAG PART =    2.5924 =    4.1371 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

```
                   NUMBER OF TONES =    34
             MEAN OF DFT REAL PART =    -842.04663
         VARIANCE OF DFT REAL PART =    150349.12
QUADRANT SNROUT OF DFT REAL PART =    4.7160 =    6.7357 DB

             MEAN OF DFT IMAG PART =    834.70776
         VARIANCE OF DFT IMAG PART =    127103.50
QUADRANT SNROUT OF DFT IMAG PART =    5.4817 =    7.3891 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

```
                   NUMBER OF TONES =    35
             MEAN OF DFT REAL PART =    -741.81006
         VARIANCE OF DFT REAL PART =    168815.31
QUADRANT SNROUT OF DFT REAL PART =    3.2597 =    5.1317 DB

             MEAN OF DFT IMAG PART =    -688.02417
         VARIANCE OF DFT IMAG PART =    155789.25
QUADRANT SNROUT OF DFT IMAG PART =    3.0386 =    4.8267 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

```
                   NUMBER OF TONES =    21
             MEAN OF DFT REAL PART =    799.47021
         VARIANCE OF DFT REAL PART =    159680.31
QUADRANT SNROUT OF DFT REAL PART =    4.0027 =    6.0235 DB

             MEAN OF DFT IMAG PART =    -665.15771
         VARIANCE OF DFT IMAG PART =    149754.06
QUADRANT SNROUT OF DFT IMAG PART =    2.9544 =    4.7047 DB
```

*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 4 ***

```
    TOTAL NUMBER OF POINTS, 2K =    256
                    BAUD MEAN =    739.13794
                BAUD VARIANCE =    145479.56
                  BAUD SNROUT =    3.7553 =    5.7465 DB
```

```
INPUT SNRNB =   3.162 =   5.000 DB

BAUD TYPE 5:   KX = 4096 SAMPLE POINTS; K = 256 TONES


** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

                    NUMBER OF TONES =    64
             MEAN OF DFT REAL PART =  1508.21289
         VARIANCE OF DFT REAL PART =   915649.00
 QUADRANT SNROUT OF DFT REAL PART =     2.4843 =   3.9520 DB

             MEAN OF DFT IMAG PART =  1605.33105
         VARIANCE OF DFT IMAG PART =   823249.25
 QUADRANT SNROUT OF DFT IMAG PART =     3.1304 =   4.9560 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

                    NUMBER OF TONES =    61
             MEAN OF DFT REAL PART = -1585.36865
         VARIANCE OF DFT REAL PART =   762098.37
 QUADRANT SNROUT OF DFT REAL PART =     3.2980 =   5.1825 DB

             MEAN OF DFT IMAG PART =  1415.77759
         VARIANCE OF DFT IMAG PART =  1035021.56
 QUADRANT SNROUT OF DFT IMAG PART =     1.9366 =   2.8704 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

                    NUMBER OF TONES =    59
             MEAN OF DFT REAL PART = -1473.39185
         VARIANCE OF DFT REAL PART =   538915.81
 QUADRANT SNROUT OF DFT REAL PART =     4.0282 =   6.0512 DB

             MEAN OF DFT IMAG PART = -1342.45337
         VARIANCE OF DFT IMAG PART =   845222.31
 QUADRANT SNROUT OF DFT IMAG PART =     2.1322 =   3.2883 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

                    NUMBER OF TONES =    72
             MEAN OF DFT REAL PART =  1401.73340
         VARIANCE OF DFT REAL PART =   632763.94
 QUADRANT SNROUT OF DFT REAL PART =     3.1052 =   4.9209 DB

             MEAN OF DFT IMAG PART = -1526.53638
         VARIANCE OF DFT IMAG PART =   927957.37
 QUADRANT SNROUT OF DFT IMAG PART =     2.5112 =   3.9989 DB



*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 5 ***

    TOTAL NUMBER OF POINTS, 2K =    512
                     BAUD MEAN =  1433.02075 ·
                 BAUD VARIANCE =   799351.69
                   BAUD SNROUT =     2.7514 =   4.3956 DB
```

C. INPUT SNR = 10 dB

```
INPUT SNRNB =  10.000 =  10.000 DB

BAUD TYPE 1:  KX =  256 SAMPLE POINTS; K =  16 TONES


** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

                    NUMBER OF TONES =    4
            MEAN OF DFT REAL PART =    92.78491
        VARIANCE OF DFT REAL PART =      840.84
QUADRANT SNROUT OF DFT REAL PART =    10.2386 =   10.1024 DB

            MEAN OF DFT IMAG PART =   102.71246
        VARIANCE OF DFT IMAG PART =      663.56
QUADRANT SNROUT OF DFT IMAG PART =    15.8989 =   12.0137 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

                    NUMBER OF TONES =    2
            MEAN OF DFT REAL PART =   -88.83499
        VARIANCE OF DFT REAL PART =       42.97
QUADRANT SNROUT OF DFT REAL PART =  183.6700 =   22.6404 DB

            MEAN OF DFT IMAG PART =    87.84000
        VARIANCE OF DFT IMAG PART =      278.95
QUADRANT SNROUT OF DFT IMAG PART =    27.6602 =   14.4185 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

                    NUMBER OF TONES =    6
            MEAN OF DFT REAL PART =   -81.36990
        VARIANCE OF DFT REAL PART =     1289.56
QUADRANT SNROUT OF DFT REAL PART =     5.1344 =    7.1049 DB

            MEAN OF DFT IMAG PART =  -111.58658
        VARIANCE OF DFT IMAG PART =      113.19
QUADRANT SNROUT OF DFT IMAG PART =  110.0076 =   20.4142 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

                    NUMBER OF TONES =    4
            MEAN OF DFT REAL PART =   104.76740
        VARIANCE OF DFT REAL PART =      756.27
QUADRANT SNROUT OF DFT REAL PART =    14.5137 =   11.6178 DB

            MEAN OF DFT IMAG PART =  -104.59491
        VARIANCE OF DFT IMAG PART =     1164.08
QUADRANT SNROUT OF DFT IMAG PART =     9.3981 =    9.7304 DB


*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 1  ***

    TOTAL NUMBER OF POINTS, 2K =       32
                  BAUD MEAN =    97.82896
              BAUD VARIANCE =      568.06
                BAUD SNROUT =    16.8477 =   12.2654 DB
```

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INPUT SNRNB = 10.000 = 10.000 DB

BAUD TYPE 2:   KX =   512 SAMPLE POINTS; K =   32 TONES

** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

```
                   NUMBER OF TONES =      5
             MEAN OF DFT REAL PART =    167.03993
         VARIANCE OF DFT REAL PART =      1490.65
QUADRANT SNROUT OF DFT REAL PART =     18.7182 =   12.7226 DB

             MEAN OF DFT IMAG PART =    207.00189
         VARIANCE OF DFT IMAG PART =      5471.27
QUADRANT SNROUT OF DFT IMAG PART =      7.8318 =    8.9386 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

```
                   NUMBER OF TONES =      8
             MEAN OF DFT REAL PART =   -163.52493
         VARIANCE OF DFT REAL PART =      6010.42
QUADRANT SNROUT OF DFT REAL PART =      4.4490 =    6.4826 DB

             MEAN OF DFT IMAG PART =    167.12369
         VARIANCE OF DFT IMAG PART =      1969.51
QUADRANT SNROUT OF DFT IMAG PART =     14.1813 =   11.5172 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

```
                   NUMBER OF TONES =     10
             MEAN OF DFT REAL PART =   -184.25990
         VARIANCE OF DFT REAL PART =      1965.78
QUADRANT SNROUT OF DFT REAL PART =     17.2714 =   12.3733 DB

             MEAN OF DFT IMAG PART =   -214.98996
         VARIANCE OF DFT IMAG PART =      3034.56
QUADRANT SNROUT OF DFT IMAG PART =     15.2314 =   11.8274 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

```
                   NUMBER OF TONES =      9
             MEAN OF DFT REAL PART =    172.58655
         VARIANCE OF DFT REAL PART =      4046.22
QUADRANT SNROUT OF DFT REAL PART =      7.3615 =    8.6696 DB

             MEAN OF DFT IMAG PART =   -191.69989
         VARIANCE OF DFT IMAG PART =      2121.56
QUADRANT SNROUT OF DFT IMAG PART =     17.3216 =   12.3859 DB
```

*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 2 ***

```
    TOTAL NUMBER OF POINTS, 2K =      64
                     BAUD MEAN =    184.16351 ·
                 BAUD VARIANCE =      2826.23
                   BAUD SNROUT =    12.0005 =   10.7920 DB
```

```
INPUT SNRNB =  10.000 =  10.000 DB

BAUD TYPE 3:  KX = 1024 SAMPLE POINTS; K =  64 TONES


** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

                    NUMBER OF TONES =   13
          MEAN OF DFT REAL PART =   341.20361
      VARIANCE OF DFT REAL PART =    17078.35
QUADRANT SNROUT OF DFT REAL PART =    6.8168 =   8.3358 DB

          MEAN OF DFT IMAG PART =   334.22290
      VARIANCE OF DFT IMAG PART =     7515.77
QUADRANT SNROUT OF DFT IMAG PART =   14.8627 =  11.7210 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

                    NUMBER OF TONES =   15
          MEAN OF DFT REAL PART =  -406.96606
      VARIANCE OF DFT REAL PART =    10785.23
QUADRANT SNROUT OF DFT REAL PART =   15.3563 =  11.8629 DB

          MEAN OF DFT IMAG PART =   372.41919
      VARIANCE OF DFT IMAG PART =     9200.02
QUADRANT SNROUT OF DFT IMAG PART =   15.0756 =  11.7828 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

                    NUMBER OF TONES =   20
          MEAN OF DFT REAL PART =  -363.32935
      VARIANCE OF DFT REAL PART =    15794.66
QUADRANT SNROUT OF DFT REAL PART =    8.3578 =   9.2209 DB

          MEAN OF DFT IMAG PART =  -338.93921
      VARIANCE OF DFT IMAG PART =    11063.66
QUADRANT SNROUT OF DFT IMAG PART =   10.3835 =  10.1634 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

                    NUMBER OF TONES =   16
          MEAN OF DFT REAL PART =   351.99341
      VARIANCE OF DFT REAL PART =     9729.22
QUADRANT SNROUT OF DFT REAL PART =   12.7348 =  11.0499 DB

          MEAN OF DFT IMAG PART =  -407.30591
      VARIANCE OF DFT IMAG PART =    12552.48
QUADRANT SNROUT OF DFT IMAG PART =   13.2164 =  11.2111 DB


*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 3  ***

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   364.57397
                BAUD VARIANCE =    11176.81
                  BAUD SNROUT =   11.8920 =  10.7525 DB
```

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```
INPUT SNRNB =  10.000 =  10.000 DB

BAUD TYPE 4:   KX = 2048 SAMPLE POINTS; K = 128 TONES


** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

                       NUMBER OF TONES =    38
              MEAN OF DFT REAL PART =   747.69604
          VARIANCE OF DFT REAL PART =    47469.11
QUADRANT SNROUT OF DFT REAL PART =   11.7771 =   10.7104 DB

              MEAN OF DFT IMAG PART =   660.96704
          VARIANCE OF DFT IMAG PART =    43777.00
QUADRANT SNROUT OF DFT IMAG PART =    9.9796 =    9.9911 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

                       NUMBER OF TONES =    34
              MEAN OF DFT REAL PART =  -798.57202
          VARIANCE OF DFT REAL PART =    47932.87
QUADRANT SNROUT OF DFT REAL PART =   13.3044 =   11.2399 DB

              MEAN OF DFT IMAG PART =   792.75439
          VARIANCE OF DFT IMAG PART =    39958.12
QUADRANT SNROUT OF DFT IMAG PART =   15.7280 =   11.9667 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

                       NUMBER OF TONES =    35
              MEAN OF DFT REAL PART =  -740.16992
          VARIANCE OF DFT REAL PART =    53078.29
QUADRANT SNROUT OF DFT REAL PART =   10.3216 =   10.1375 DB

              MEAN OF DFT IMAG PART =  -711.70435
          VARIANCE OF DFT IMAG PART =    49063.26
QUADRANT SNROUT OF DFT IMAG PART =   10.3239 =   10.1384 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

                       NUMBER OF TONES =    21
              MEAN OF DFT REAL PART =   773.37500
          VARIANCE OF DFT REAL PART =    50136.43
QUADRANT SNROUT OF DFT REAL PART =   11.9296 =   10.7663 DB

              MEAN OF DFT IMAG PART =  -697.57031
          VARIANCE OF DFT IMAG PART =    47513.27
QUADRANT SNROUT OF DFT IMAG PART =   10.2414 =   10.1036 DB


*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 4  ***

    TOTAL NUMBER OF POINTS, 2K =    256
                   BAUD MEAN =   739.60791 .
               BAUD VARIANCE =    45891.41
                 BAUD SNROUT =   11.9199 =   10.7627 DB
```

INPUT SNRNB =   10.000 =   10.000 DB

BAUD TYPE 5:   KX = 4096 SAMPLE POINTS; K = 256 TONES

** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

```
                    NUMBER OF TONES =    64
           MEAN OF DFT REAL PART =   1508.31250
       VARIANCE OF DFT REAL PART =    292452.31
QUADRANT SNROUT OF DFT REAL PART =      7.7791 =    8.9093 DB

              MEAN OF DFT IMAG PART =   1566.18359
          VARIANCE OF DFT IMAG PART =    265130.50
QUADRANT SNROUT OF DFT IMAG PART =      9.2518 =    9.6623 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

```
                    NUMBER OF TONES =    61
           MEAN OF DFT REAL PART =  -1556.26514
       VARIANCE OF DFT REAL PART =    242012.25
QUADRANT SNROUT OF DFT REAL PART =     10.0076 =   10.0033 DB

              MEAN OF DFT IMAG PART =   1455.11865
          VARIANCE OF DFT IMAG PART =    327317.06
QUADRANT SNROUT OF DFT IMAG PART =      6.4689 =    8.1083 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

```
                    NUMBER OF TONES =    59
           MEAN OF DFT REAL PART =  -1487.87915
       VARIANCE OF DFT REAL PART =    169614.31
QUADRANT SNROUT OF DFT REAL PART =     13.0519 =   11.1567 DB

              MEAN OF DFT IMAG PART =  -1417.67261
          VARIANCE OF DFT IMAG PART =    265248.00
QUADRANT SNROUT OF DFT IMAG PART =      7.5770 =    8.7950 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

```
                    NUMBER OF TONES =    72
           MEAN OF DFT REAL PART =   1450.37061
       VARIANCE OF DFT REAL PART =    202104.12
QUADRANT SNROUT OF DFT REAL PART =     10.4084 =   10.1738 DB

              MEAN OF DFT IMAG PART =  -1517.82886
          VARIANCE OF DFT IMAG PART =    292801.12
QUADRANT SNROUT OF DFT IMAG PART =      7.8682 =    8.9587 DB
```

*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 5   ***

```
TOTAL NUMBER OF POINTS, 2K =     512
                  BAUD MEAN =   1495.31201
              BAUD VARIANCE =    253713.62
                BAUD SNROUT =      8.8129 =    9.4512 DB
```

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D. INPUT SNR = 15 dB

INPUT SNRNB = 31.623 = 15.000 DB

BAUD TYPE 1: KX = 256 SAMPLE POINTS; K = 16 TONES

** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

                 NUMBER OF TONES =   4
                 MEAN OF DFT REAL PART =   91.78998
     VARIANCE OF DFT REAL PART =   265.70
QUADRANT SNROUT OF DFT REAL PART = 31.7097 = 15.0119 DB

                 MEAN OF DFT IMAG. PART =   97.37494
     VARIANCE OF DFT IMAG PART =   209.96
QUADRANT SNROUT OF DFT IMAG PART = 45.1599 = 16.5475 DB

** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

                 NUMBER OF TONES =   2
                 MEAN OF DFT REAL PART =   -89.56999
     VARIANCE OF DFT REAL PART =   13.62
QUADRANT SNROUT OF DFT REAL PART = 588.8657 = 27.7002 DB

                 MEAN OF DFT IMAG PART =   89.00999
     VARIANCE OF DFT IMAG PART =   88.18
QUADRANT SNROUT OF DFT IMAG PART = 89.8486 = 19.5351 DB

** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

                 NUMBER OF TONES =   6
                 MEAN OF DFT REAL PART =   -85.36328
     VARIANCE OF DFT REAL PART =   407.69
QUADRANT SNROUT OF DFT REAL PART = 17.8737 = 12.5221 DB

                 MEAN OF DFT IMAG PART = -102.36160
     VARIANCE OF DFT IMAG PART =   35.63
QUADRANT SNROUT OF DFT IMAG PART = 294.1055 = 24.6850 DB

** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

                 NUMBER OF TONES =   4
                 MEAN OF DFT REAL PART =   98.51746
     VARIANCE OF DFT REAL PART =   238.91
QUADRANT SNROUT OF DFT REAL PART = 40.6241 = 16.0878 DB

                 MEAN OF DFT IMAG PART =   -98.39490
     VARIANCE OF DFT IMAG PART =   366.81
QUADRANT SNROUT OF DFT IMAG PART = 26.3941 = 14.2151 DB

*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 1 ***

     TOTAL NUMBER OF POINTS, 2K =   32
                    BAUD MEAN =   94.61931
                 BAUD VARIANCE =   179.44
                  BAUD SNROUT = 49.8941 = 16.9805 DB

INPUT SNRNB = 31.623 = 15.000 DB

BAUD TYPE 2: KX = 512 SAMPLE POINTS; K = 32 TONES

** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

```
                   NUMBER OF TONES =      5
          MEAN OF DFT REAL PART =    173.05991
      VARIANCE OF DFT REAL PART =       469.50
QUADRANT SNROUT OF DFT REAL PART =    63.7903 =   18.0475 DB

          MEAN OF DFT IMAG PART =    195.51991
      VARIANCE OF DFT IMAG PART =      1735.51
QUADRANT SNROUT OF DFT IMAG PART =    22.0270 =   13.4295 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

```
                   NUMBER OF TONES =      8
          MEAN OF DFT REAL PART =   -171.09995
      VARIANCE OF DFT REAL PART =      1899.12
QUADRANT SNROUT OF DFT REAL PART =    15.4151 =   11.8795 DB

          MEAN OF DFT IMAG PART =    173.09988
      VARIANCE OF DFT IMAG PART =       623.56
QUADRANT SNROUT OF DFT IMAG PART =    48.0524 =   16.8171 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

```
                   NUMBER OF TONES =     10
          MEAN OF DFT REAL PART =   -182.69992
      VARIANCE OF DFT REAL PART =       622.43
QUADRANT SNROUT OF DFT REAL PART =    53.6277 =   17.2939 DB

          MEAN OF DFT IMAG PART =   -200.01991
      VARIANCE OF DFT IMAG PART =       961.04
QUADRANT SNROUT OF DFT IMAG PART =    41.6297 =   16.1940 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

```
                   NUMBER OF TONES =      9
          MEAN OF DFT REAL PART =    176.17767
      VARIANCE OF DFT REAL PART =      1279.21
QUADRANT SNROUT OF DFT REAL PART =    24.2639 =   13.8496 DB

          MEAN OF DFT IMAG PART =   -186.88879
      VARIANCE OF DFT IMAG PART =       670.12
QUADRANT SNROUT OF DFT IMAG PART =    52.1213 =   17.1702 DB
```

*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 2 ***

```
TOTAL NUMBER OF POINTS, 2K =      64
                 BAUD MEAN =    182.67627
             BAUD VARIANCE =       894.04
               BAUD SNROUT =    37.3256 =   15.7201 DB
```

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```
INPUT SNRNB =  31.623 =  15.000 DB

BAUD TYPE 3:  KX = 1024 SAMPLE POINTS; K =  64 TONES


** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

                      NUMBER OF TONES =    13
            MEAN OF DFT REAL PART =   350.43823
        VARIANCE OF DFT REAL PART =     5392.23
QUADRANT SNROUT OF DFT REAL PART =  22.7748 =  13.5745 DB

            MEAN OF DFT IMAG PART =   346.65332
        VARIANCE OF DFT IMAG PART =     2375.41
QUADRANT SNROUT OF DFT IMAG PART =  50.5885 =  17.0405 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

                      NUMBER OF TONES =    15
            MEAN OF DFT REAL PART =  -387.58618
        VARIANCE OF DFT REAL PART =     3403.85
QUADRANT SNROUT OF DFT REAL PART =  44.1332 =  16.4476 DB

            MEAN OF DFT IMAG PART =   368.07251
        VARIANCE OF DFT IMAG PART =     2911.24
QUADRANT SNROUT OF DFT IMAG PART =  46.5359 =  16.6779 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

                      NUMBER OF TONES =    20
            MEAN OF DFT REAL PART =  -362.88428
        VARIANCE OF DFT REAL PART =     4993.27
QUADRANT SNROUT OF DFT REAL PART =  26.3725 =  14.2115 DB

            MEAN OF DFT IMAG PART =  -349.39429
        VARIANCE OF DFT IMAG PART =     3493.32
QUADRANT SNROUT OF DFT IMAG PART =  34.9457 =  15.4339 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

                      NUMBER OF TONES =    16
            MEAN OF DFT REAL PART =   356.73682
        VARIANCE OF DFT REAL PART =     3080.93
QUADRANT SNROUT OF DFT REAL PART =  41.3060 =  16.1601 DB

            MEAN OF DFT IMAG PART =  -387.54321
        VARIANCE OF DFT IMAG PART =     3962.38
QUADRANT SNROUT OF DFT IMAG PART =  37.9039 =  15.7868 DB


*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 3 ***

    TOTAL NUMBER OF POINTS, 2K =     128
                   BAUD MEAN =   363.68042
               BAUD VARIANCE =     3531.64
                 BAUD SNROUT =  37.4510 =  15.7346 DB
```

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INPUT SNRNB = 31.623 = 15.000 DB

BAUD TYPE 4: KX = 2048 SAMPLE POINTS; K = 128 TONES

** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

```
                   NUMBER OF TONES =    38
          MEAN OF DFT REAL PART =   744.14038
      VARIANCE OF DFT REAL PART =    14843.46
QUADRANT SNROUT OF DFT REAL PART =    37.3056 =  15.7177 DB

          MEAN OF DFT IMAG PART =   696.01440
      VARIANCE OF DFT IMAG PART =    13916.01
QUADRANT SNROUT OF DFT IMAG PART =    34.8114 =  15.4172 DB
```


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

```
                   NUMBER OF TONES =    34
          MEAN OF DFT REAL PART =  -774.12231
      VARIANCE OF DFT REAL PART =    15403.46
QUADRANT SNROUT OF DFT REAL PART =    38.9046 =  15.9000 DB

          MEAN OF DFT IMAG PART =   769.15747
      VARIANCE OF DFT IMAG PART =    12525.41
QUADRANT SNROUT OF DFT IMAG PART =    47.2322 =  16.7424 DB
```


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

```
                   NUMBER OF TONES =    35
          MEAN OF DFT REAL PART =  -739.27295
      VARIANCE OF DFT REAL PART =    16651.30
QUADRANT SNROUT OF DFT REAL PART =    32.8217 =  15.1616 DB

          MEAN OF DFT IMAG PART =  -725.06152
      VARIANCE OF DFT IMAG PART =    15437.08
QUADRANT SNROUT OF DFT IMAG PART =    34.0553 =  15.3218 DB
```


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

```
                   NUMBER OF TONES =    21
          MEAN OF DFT REAL PART =   758.77905
      VARIANCE OF DFT REAL PART =    15687.69
QUADRANT SNROUT OF DFT REAL PART =    36.7005 =  15.6467 DB

          MEAN OF DFT IMAG PART =  -715.81812
      VARIANCE OF DFT IMAG PART =    15135.91
QUADRANT SNROUT OF DFT IMAG PART =    33.8530 =  15.2960 DB
```


*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 4 ***

```
     TOTAL NUMBER OF POINTS, 2K =    256
                   BAUD MEAN =   739.90381
               BAUD VARIANCE =    14483.24
                 BAUD SNROUT =    37.7994 =  15.7748 DB
```


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INPUT SNRNB = 31.623 = 15.000 DB

BAUD TYPE 5: KX = 4096 SAMPLE POINTS; K = 256 TONES

** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

```
                      NUMBER OF TONES =    64
              MEAN OF DFT REAL PART =  1508.39551
          VARIANCE OF DFT REAL PART =    94514.62
QUADRANT SNROUT OF DFT REAL PART =    24.0731 =   13.8153 DB

              MEAN OF DFT IMAG PART =  1544.17187
          VARIANCE OF DFT IMAG PART =    87073.12
QUADRANT SNROUT OF DFT IMAG PART =    27.3846 =   14.3751 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

```
                      NUMBER OF TONES =    61
              MEAN OF DFT REAL PART = -1539.88403
          VARIANCE OF DFT REAL PART =    77501.44
QUADRANT SNROUT OF DFT REAL PART =    30.5961 =   14.8567 DB

              MEAN OF DFT IMAG PART =  1477.20898
          VARIANCE OF DFT IMAG PART =   103902.56
QUADRANT SNROUT OF DFT IMAG PART =    21.0018 =   13.2226 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

```
                      NUMBER OF TONES =    59
              MEAN OF DFT REAL PART = -1496.03369
          VARIANCE OF DFT REAL PART =    53483.48
QUADRANT SNROUT OF DFT REAL PART =    41.8469 =   16.2166 DB

              MEAN OF DFT IMAG PART = -1459.93311
          VARIANCE OF DFT IMAG PART =    83136.06
QUADRANT SNROUT OF DFT IMAG PART =    25.6375 =   14.0888 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

```
                      NUMBER OF TONES =    72
              MEAN OF DFT REAL PART =  1477.83325
          VARIANCE OF DFT REAL PART =    65498.62
QUADRANT SNROUT OF DFT REAL PART =    33.3441 =   15.2302 DB

              MEAN OF DFT IMAG PART = -1512.91821
          VARIANCE OF DFT IMAG PART =    92605.37
QUADRANT SNROUT OF DFT IMAG PART =    24.7169 =   13.9299 DB
```

*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 5 ***

```
TOTAL NUMBER OF POINTS, 2K =    512
                   BAUD MEAN =  1502.23193
               BAUD VARIANCE =    81161.50
                 BAUD SNROUT =    27.8051 =   14.4412 DB
```

E.   INPUT SNR = 20 dB

```
INPUT SNRNB = 100.000 =  20.000 DB

BAUD TYPE 1:  KX =  256 SAMPLE POINTS; K =  16 TONES


** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

                      NUMBER OF TONES =    4
             MEAN OF DFT REAL PART =    91.23743
         VARIANCE OF DFT REAL PART =       84.15
QUADRANT SNROUT OF DFT REAL PART =  98.9206 =  19.9529 DB

             MEAN OF DFT IMAG PART =    94.37244
         VARIANCE OF DFT IMAG PART =       66.41
QUADRANT SNROUT OF DFT IMAG PART = 134.1027 =  21.2744 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

                      NUMBER OF TONES =    2
             MEAN OF DFT REAL PART =   -89.98000
         VARIANCE OF DFT REAL PART =        4.32
QUADRANT SNROUT OF DFT REAL PART = ****** =  32.7263 DB

             MEAN OF DFT IMAG PART =    89.66499
         VARIANCE OF DFT IMAG PART =       27.90
QUADRANT SNROUT OF DFT IMAG PART = 288.1604 =  24.5963 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

                      NUMBER OF TONES =    6
             MEAN OF DFT REAL PART =   -87.61157
         VARIANCE OF DFT REAL PART =      129.06
QUADRANT SNROUT OF DFT REAL PART =  59.4730 =  17.7432 DB

             MEAN OF DFT IMAG PART =   -97.17159
         VARIANCE OF DFT IMAG PART =       11.23
QUADRANT SNROUT OF DFT IMAG PART = 840.5669 =  29.2457 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

                      NUMBER OF TONES =    4
             MEAN OF DFT REAL PART =    95.00745
         VARIANCE OF DFT REAL PART =       75.29
QUADRANT SNROUT OF DFT REAL PART = 119.8836 =  20.7876 DB

             MEAN OF DFT IMAG PART =   -94.95740
         VARIANCE OF DFT IMAG PART =      116.38
QUADRANT SNROUT OF DFT IMAG PART =  77.4774 =  18.8917 DB


*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 1 ***

      TOTAL NUMBER OF POINTS, 2K =       32
                      BAUD MEAN =    92.82147
                  BAUD VARIANCE =       56.79
                    BAUD SNROUT = 151.7199 =  21.8104 DB
```

INPUT SNRNB = 100.000 = 20.000 DB

BAUD TYPE 2:   KX = 512 SAMPLE POINTS; K = 32 TONES

** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

```
                NUMBER OF TONES =      5
        MEAN OF DFT REAL PART =   176.45985
    VARIANCE OF DFT REAL PART =      147.68
QUADRANT SNROUT OF DFT REAL PART = 210.8442 =  23.2396 DB

        MEAN OF DFT IMAG PART =   189.09990
    VARIANCE OF DFT IMAG PART =      549.10
QUADRANT SNROUT OF DFT IMAG PART =  65.1219 =  18.1373 DB
```


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

```
                NUMBER OF TONES =      8
        MEAN OF DFT REAL PART =  -175.34995
    VARIANCE OF DFT REAL PART =      599.80
QUADRANT SNROUT OF DFT REAL PART =  51.2631 =  17.0980 DB

        MEAN OF DFT IMAG PART =   176.44992
    VARIANCE OF DFT IMAG PART =      197.26
QUADRANT SNROUT OF DFT IMAG PART = 157.8330 =  21.9820 DB
```


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

```
                NUMBER OF TONES =     10
        MEAN OF DFT REAL PART =  -181.83990
    VARIANCE OF DFT REAL PART =      197.03
QUADRANT SNROUT OF DFT REAL PART = 167.8235 =  22.2485 DB

        MEAN OF DFT IMAG PART =  -191.60988
    VARIANCE OF DFT IMAG PART =      304.44
QUADRANT SNROUT OF DFT IMAG PART = 120.5977 =  20.8134 DB
```


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

```
                NUMBER OF TONES =      9
        MEAN OF DFT REAL PART =   178.21101
    VARIANCE OF DFT REAL PART =      405.12
QUADRANT SNROUT OF DFT REAL PART =  78.3938 =  18.9428 DB

        MEAN OF DFT IMAG PART =  -184.21101
    VARIANCE OF DFT IMAG PART =      211.55
QUADRANT SNROUT OF DFT IMAG PART = 160.4063 =  22.0522 DB
```


*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 2 ***

```
    TOTAL NUMBER OF POINTS, 2K =      64
                    BAUD MEAN =   181.85138
                BAUD VARIANCE =      282.75
                  BAUD SNROUT = 116.9591 =  20.6803 DB
```


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INPUT SNRNB = 100.000 = 20.000 DB

BAUD TYPE 3: KX = 1024 SAMPLE POINTS; K = 64 TONES

** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

```
                      NUMBER OF TONES =   13
               MEAN OF DFT REAL PART =   355.62256
           VARIANCE OF DFT REAL PART =     1701.75
QUADRANT SNROUT OF DFT REAL PART = 74.3160 =  18.7108 DB

               MEAN OF DFT IMAG PART =   353.64551
           VARIANCE OF DFT IMAG PART =      749.14
QUADRANT SNROUT OF DFT IMAG PART = 166.9445 =  22.2257 DB
```


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

```
                      NUMBER OF TONES =   15
               MEAN OF DFT REAL PART =  -376.68604
           VARIANCE OF DFT REAL PART =     1074.68
QUADRANT SNROUT OF DFT REAL PART = 132.0319 =  21.2068 DB

               MEAN OF DFT IMAG PART =   365.62622
           VARIANCE OF DFT IMAG PART =      922.01
QUADRANT SNROUT OF DFT IMAG PART = 144.9895 =  21.6134 DB
```


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

```
                      NUMBER OF TONES =   20
               MEAN OF DFT REAL PART =  -362.62378
           VARIANCE OF DFT REAL PART =     1577.91
QUADRANT SNROUT OF DFT REAL PART = 83.3354 =  19.2083 DB

               MEAN OF DFT IMAG PART =  -355.27930
           VARIANCE OF DFT IMAG PART =     1101.76
QUADRANT SNROUT OF DFT IMAG PART = 114.5647 =  20.5905 DB
```


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

```
                      NUMBER OF TONES =   16
               MEAN OF DFT REAL PART =   359.39941
           VARIANCE OF DFT REAL PART =      977.57
QUADRANT SNROUT OF DFT REAL PART = 132.1315 =  21.2101 DB

               MEAN OF DFT IMAG PART =  -376.43091
           VARIANCE OF DFT IMAG PART =     1248.45
QUADRANT SNROUT OF DFT IMAG PART = 113.5008 =  20.5500 DB
```


*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 3 ***

```
       TOTAL NUMBER OF POINTS, 2K =    128
                       BAUD MEAN =    363.17578
                   BAUD VARIANCE =     1115.50
                    BAUD SNROUT = 118.2399 =  20.7276 DB
```

INPUT SNRNB = 100.000 = 20.000 DB

BAUD TYPE 4: KX = 2048 SAMPLE POINTS; K = 128 TONES

** GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

```
                    NUMBER OF TONES =    38
           MEAN OF DFT REAL PART =   742.11963
       VARIANCE OF DFT REAL PART =     4625.64
QUADRANT SNROUT OF DFT REAL PART = 119.0627 =   20.7578 DB

           MEAN OF DFT IMAG PART =   715.70630
       VARIANCE OF DFT IMAG PART =     4457.19
QUADRANT SNROUT OF DFT IMAG PART = 114.9235 =   20.6041 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

```
                    NUMBER OF TONES =    34
           MEAN OF DFT REAL PART =  -760.36914
       VARIANCE OF DFT REAL PART =     5032.17
QUADRANT SNROUT OF DFT REAL PART = 114.8930 =   20.6029 DB

           MEAN OF DFT IMAG PART =   755.90723
       VARIANCE OF DFT IMAG PART =     3920.06
QUADRANT SNROUT OF DFT IMAG PART = 145.7620 =   21.6364 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

```
                    NUMBER OF TONES =    35
           MEAN OF DFT REAL PART =  -738.78418
       VARIANCE OF DFT REAL PART =     5230.00
QUADRANT SNROUT OF DFT REAL PART = 104.3598 =   20.1853 DB

           MEAN OF DFT IMAG PART =  -732.57861
       VARIANCE OF DFT IMAG PART =     4854.96
QUADRANT SNROUT OF DFT IMAG PART = 110.5407 =   20.4352 DB
```

** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

```
                    NUMBER OF TONES =    21
           MEAN OF DFT REAL PART =   750.57007
       VARIANCE OF DFT REAL PART =     4882.30
QUADRANT SNROUT OF DFT REAL PART = 115.3872 =   20.6216 DB

           MEAN OF DFT IMAG PART =  -726.06567
       VARIANCE OF DFT IMAG PART =     4864.54
QUADRANT SNROUT OF DFT IMAG PART = 108.3701 =   20.3491 DB
```

*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 4 ***

```
TOTAL NUMBER OF POINTS, 2K =    256
                BAUD MEAN =   740.06909
            BAUD VARIANCE =     4585.54
              BAUD SNROUT = 119.4413 =   20.7715 DB
```

```
INPUT SNRNB = 100.000 =  20.000 DB

BAUD TYPE 5:   KX = 4096 SAMPLE POINTS; K = 256 TONES


**  GIVEN THE TRANSMITTED PHASE IS IN THE 1ST QUADRANT **

                   NUMBER OF TONES =    64
            MEAN OF DFT REAL PART =  1508.39062
        VARIANCE OF DFT REAL PART =    31407.17
QUADRANT SNROUT OF DFT REAL PART =  72.4434 =   18.6000 DB

            MEAN OF DFT IMAG PART =  1531.78125
        VARIANCE OF DFT IMAG PART =    29779.73
QUADRANT SNROUT OF DFT IMAG PART =  78.7903 =   18.9647 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 2ND QUADRANT **

                   NUMBER OF TONES =    61
            MEAN OF DFT REAL PART = -1530.73755
        VARIANCE OF DFT REAL PART =    25465.21
QUADRANT SNROUT OF DFT REAL PART =  92.0140 =   19.6385 DB

            MEAN OF DFT IMAG PART =  1489.81445
        VARIANCE OF DFT IMAG PART =    33534.58
QUADRANT SNROUT OF DFT IMAG PART =  66.1868 =   18.2077 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 3RD QUADRANT **

                   NUMBER OF TONES =    59
            MEAN OF DFT REAL PART = -1500.62695
        VARIANCE OF DFT REAL PART =    17125.93
QUADRANT SNROUT OF DFT REAL PART = 131.4895 =   21.1889 DB

            MEAN OF DFT IMAG PART = -1483.72876
        VARIANCE OF DFT IMAG PART =    26270.10
QUADRANT SNROUT OF DFT IMAG PART =  83.8006 =   19.2325 DB


** GIVEN THE TRANSMITTED PHASE IS IN THE 4TH QUADRANT **

                   NUMBER OF TONES =    72
            MEAN OF DFT REAL PART =  1493.18042
        VARIANCE OF DFT REAL PART =    22084.69
QUADRANT SNROUT OF DFT REAL PART = 100.9562 =   20.0413 DB

            MEAN OF DFT IMAG PART = -1510.18042
        VARIANCE OF DFT IMAG PART =    29661.86
QUADRANT SNROUT OF DFT IMAG PART =  76.8881 =   18.8586 DB


*** OVERALL (REAL + IMAG) STATISTICS FOR BAUD TYPE 5 ***

     TOTAL NUMBER OF POINTS, 2K =    512
                    BAUD MEAN =  1506.13916
                BAUD VARIANCE =    26586.55
                  BAUD SNROUT =  85.3234 =   19.3107 DB
```

APPENDIX D. STATISTICS OF SNR_{OUT} VS. DOPPLER ANALYSIS

A. INPUT SNR = 15 dB

```
SNROUT VS. DOPPLER TEST -- EPSILON =  0.00

INPUT SNRNB =   31.623 =   15.000 DB

BAUD TYPE 3:  KX = 1024 SAMPLE POINTS; K =  64 TONES


** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

    TOTAL NUMBER OF POINTS, 2K =    128
                 BAUD MEAN =   358.19678
             BAUD VARIANCE =    3614.89
               BAUD SNROUT =    35.4935 =    15.5015 DB


** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

    TOTAL NUMBER OF POINTS, 2K =    128
                 BAUD MEAN =   359.38672
             BAUD VARIANCE =    4296.25
               BAUD SNROUT =    30.0631 =    14.7803 DB


** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

    TOTAL NUMBER OF POINTS, 2K =    128
                 BAUD MEAN =   353.96387
             BAUD VARIANCE =    3632.05
               BAUD SNROUT =    34.4958 =    15.3777 DB


** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =    128
                 BAUD MEAN =   360.04858
             BAUD VARIANCE =    3868.04
               BAUD SNROUT =    33.5144 =    15.2523 DB


** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =    128
                 BAUD MEAN =   344.44604
             BAUD VARIANCE =    5204.43
               BAUD SNROUT =    22.7965 =    13.5787 DB



*** TOTAL OVER ALL 5 BAUDS WITH EPSILON =   0.00 ***

            TOTAL NUMBER OF BAUDS =     5
                     MEAN SNROUT =    14.8981 DB
    STANDARD DEVIATION OF SNROUT =     0.7865 DB
```

SNROUT VS. DOPPLER TEST -- EPSILON = 0.25

INPUT SNRNB =   31.623 =   15.000 DB

BAUD TYPE 3:   KX = 1024 SAMPLE POINTS; K =   64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                     BAUD MEAN =   356.60303
                 BAUD VARIANCE =   4178.20
                   BAUD SNROUT =   30.4355 =   14.8338 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                     BAUD MEAN =   357.83740
                 BAUD VARIANCE =   4579.19
                   BAUD SNROUT =   27.9630 =   14.4658 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                     BAUD MEAN =   352.04834
                 BAUD VARIANCE =   3697.63
                   BAUD SNROUT =   33.5182 =   15.2528 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                     BAUD MEAN =   358.16406
                 BAUD VARIANCE =   4301.95
                   BAUD SNROUT =   29.8194 =   14.7450 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

    TOTAL NUMBER OF. POINTS, 2K =   128
                     BAUD MEAN =   342.34058
                 BAUD VARIANCE =   5904.97
                   BAUD SNROUT =   19.8472 =   12.9770 DB

*** TOTAL OVER ALL 5 BAUDS WITH EPSILON =   0.25 ***

               TOTAL NUMBER OF BAUDS =   5
                         MEAN SNROUT =   14.4549 DB
         STANDARD DEVIATION OF SNROUT =   0.8730 DB

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SNROUT VS. DOPPLER TEST -- EPSILON = 0.50

INPUT SNRNB =   31.623 =   15.000 DB

BAUD TYPE 3: KX = 1024 SAMPLE POINTS; K = 64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   351.68213
                 BAUD VARIANCE =   5379.54
                  BAUD SNROUT =   22.9909 =   13.6156 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   353.07666
                 BAUD VARIANCE =   5645.37
                  BAUD SNROUT =   22.0823 =   13.4404 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   346.26807
                 BAUD VARIANCE =   4283.94
                  BAUD SNROUT =   27.9886 =   14.4698 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   352.39697
                 BAUD VARIANCE =   5489.51
                  BAUD SNROUT =   22.6220 =   13.5453 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   335.90625
                 BAUD VARIANCE =   7204.71
                  BAUD SNROUT =   15.6610 =   11.9482 DB

*** TOTAL OVER ALL 5 BAUDS WITH EPSILON =   0.50 ,***

            TOTAL NUMBER OF BAUDS =   5
                      MEAN SNROUT =   13.4039 DB
        STANDARD DEVIATION OF SNROUT =   0.9112 DB

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SNROUT VS. DOPPLER TEST -- EPSILON = 0.75

INPUT SNRNB = 31.623 = 15.000 DB

BAUD TYPE 3: KX = 1024 SAMPLE POINTS; K = 64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

```
    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   343.56445
                BAUD VARIANCE =     7197.11
                  BAUD SNROUT =   16.4005 =   12.1486 DB
```

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

```
    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   345.20166
                BAUD VARIANCE =     7479.94
                  BAUD SNROUT =   15.9312 =   12.0225 DB
```

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

```
    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   336.72485
                BAUD VARIANCE =     5392.54
                  BAUD SNROUT =   21.0260 =   13.2276 DB
```

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

```
    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   342.88135
                BAUD VARIANCE =     7409.56
                  BAUD SNROUT =   15.8670 =   12.0049 DB
```

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

```
    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   325.28076
                BAUD VARIANCE =     9073.28
                  BAUD SNROUT =   11.6614 =   10.6675 DB
```


*** TOTAL OVER ALL 5 BAUDS WITH EPSILON = 0.75 ***

```
            TOTAL NUMBER OF BAUDS =      5
                      MEAN SNROUT =   12.0142 DB
    STANDARD DEVIATION OF SNROUT =    0.9088 DB
```

SNROUT VS. DOPPLER TEST -- EPSILON = 1.00

INPUT SNRNB = 31.623 = 15.000 DB

BAUD TYPE 3: KX = 1024 SAMPLE POINTS; K = 64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

```
    TOTAL NUMBER OF POINTS, 2K =    128
                  BAUD MEAN =    332.46948
              BAUD VARIANCE =      9581.14
                BAUD SNROUT =     11.5368 =    10.6209 DB
```

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

```
    TOTAL NUMBER OF POINTS, 2K =    128
                  BAUD MEAN =    334.46362
              BAUD VARIANCE =     10027.16
                BAUD SNROUT =     11.1563 =    10.4752 DB
```

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

```
    TOTAL NUMBER OF POINTS, 2K =    128
                  BAUD MEAN =    323.73706
              BAUD VARIANCE =      7001.29
                BAUD SNROUT =     14.9695 =    11.7521 DB
```

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

```
    TOTAL NUMBER OF POINTS, 2K =    128
                  BAUD MEAN =    329.95630
              BAUD VARIANCE =     10013.46
                BAUD SNROUT =     10.8725 =    10.3633 DB
```

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

```
    TOTAL NUMBER OF POINTS, 2K =    128
                  BAUD MEAN =    310.86450
              BAUD VARIANCE =     11450.02
                BAUD SNROUT =      8.4399 =     9.2634 DB
```


*** TOTAL OVER ALL 5 BAUDS WITH EPSILON = 1.00 ***

```
            TOTAL NUMBER OF BAUDS =        5
                   MEAN SNROUT =    10.4949 DB
    STANDARD DEVIATION OF SNROUT =     0.8847 DB
```

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SNROUT VS. DOPPLER TEST -- EPSILON = 1.25

INPUT SNRNB =   31.623 =   15.000 DB

BAUD TYPE 3:   KX = 1024 SAMPLE POINTS; K =   64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

       TOTAL NUMBER OF POINTS, 2K =   128
                      BAUD MEAN =   318.38794
                  BAUD VARIANCE =   12548.15
                    BAUD SNROUT =   8.0786 =   9.0733 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

       TOTAL NUMBER OF POINTS, 2K =   128
                      BAUD MEAN =   320.88208
                  BAUD VARIANCE =   13305.20
                    BAUD SNROUT =   7.7387 =   8.8867 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

       TOTAL NUMBER OF POINTS, 2K =   128
                      BAUD MEAN =   307.26807
                  BAUD VARIANCE =   9126.79
                    BAUD SNROUT =   10.3447 =   10.1472 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

       TOTAL NUMBER OF POINTS, 2K =   128
                      BAUD MEAN =   313.63037
                  BAUD VARIANCE =   13315.13
                    BAUD SNROUT =   7.3874 =   8.6849 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

       TOTAL NUMBER OF POINTS, 2K =   128
                      BAUD MEAN =   292.66162
                  BAUD VARIANCE =   14353.81
                    BAUD SNROUT =   5.9671 =   7.7576 DB

*** TOTAL OVER ALL 5 BAUDS WITH EPSILON =   1.25 ***

                TOTAL NUMBER OF BAUDS =   5
                        MEAN SNROUT =   8.9099 DB
       STANDARD DEVIATION OF SNROUT =   0.8568 DB

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SNROUT VS. DOPPLER TEST -- EPSILON = 1.50

INPUT SNRNB = 31.623 = 15.000 DB

BAUD TYPE 3: KX = 1024 SAMPLE POINTS; K = 64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

```
    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =    301.64111
                BAUD VARIANCE =    16039.55
                   BAUD SNROUT =    5.6727 =    7.5379 DB
```

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

```
    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =    304.77563
                BAUD VARIANCE =    17242.49
                   BAUD SNROUT =    5.3872 =    7.3136 DB
```

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

```
    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =    287.72412
                BAUD VARIANCE =    11751.02
                   BAUD SNROUT =    7.0449 =    8.4788 DB
```

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

```
    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =    294.33276
                BAUD VARIANCE =    17252.69
                   BAUD SNROUT =    5.0213 =    7.0082 DB
```

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

```
    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =    271.16650
                BAUD VARIANCE =    17716.52
                   BAUD SNROUT =    4.1504 =    6.1809 DB
```


*** TOTAL OVER ALL 5 BAUDS WITH EPSILON = 1.50 ***

```
            TOTAL NUMBER OF BAUDS =      5
                     MEAN SNROUT =    7.3039 DB
    STANDARD DEVIATION OF SNROUT =    0.8342 DB
```


118

SNROUT VS. DOPPLER TEST -- EPSILON = 1.75

INPUT SNRNB =   31.623 =   15.000 DB

BAUD TYPE 3:   KX = 1024 SAMPLE POINTS; K = 64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                  BAUD MEAN =   282.71875
              BAUD VARIANCE =   19969.26
                BAUD SNROUT =   4.0026 =   6.0235 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                  BAUD MEAN =   286.64087
              BAUD VARIANCE =   21735.70
                BAUD SNROUT =   3.7801 =   5.7750 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                  BAUD MEAN =   265.68335
              BAUD VARIANCE =   14806.48
                BAUD SNROUT =   4.7673 =   6.7828 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                  BAUD MEAN =   272.69653
              BAUD VARIANCE =   21720.69
                BAUD SNROUT =   3.4236 =   5.3449 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                  BAUD MEAN =   247.10403
              BAUD VARIANCE =   21451.55
                BAUD SNROUT =   2.8464 =   4.5430 DB

*** TOTAL OVER ALL 5 BAUDS WITH EPSILON = 1.75 ***

              TOTAL NUMBER OF BAUDS =   5
                      MEAN SNROUT =   5.6938 DB
      STANDARD DEVIATION OF SNROUT =   0.8287 DB

119

SNROUT VS. DOPPLER TEST -- EPSILON = 2.00

INPUT SNRNB = 31.623 = 15.000 DB

BAUD TYPE 3: KX = 1024 SAMPLE POINTS; K = 64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   261.53735
                BAUD VARIANCE =   24362.90
                  BAUD SNROUT =   2.8076 =   4.4834 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   266.46338
                BAUD VARIANCE =   26814.79
                  BAUD SNROUT =   2.6479 =   4.2290 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   242.98599
                BAUD VARIANCE =   18343.12
                  BAUD SNROUT =   3.2188 =   5.0769 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   248.73459
                BAUD VARIANCE =   26755.45
                  BAUD SNROUT =   2.3124 =   3.6406 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   239.01096
                BAUD VARIANCE =   25585.32
                  BAUD SNROUT =   2.2328 =   3.4884 DB

*** TOTAL OVER ALL 5 BAUDS WITH EPSILON = 2.00 ***

            TOTAL NUMBER OF BAUDS =   5
                     MEAN SNROUT =   4.1837 DB
    STANDARD DEVIATION OF SNROUT =   0.6458 DB

120

SNROUT VS. DOPPLER TEST -- EPSILON = 2.25

INPUT SNRNB = 31.623 = 15.000 DB

BAUD TYPE 3: KX = 1024 SAMPLE POINTS; K = 64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

```
TOTAL NUMBER OF POINTS, 2K =    128
                 BAUD MEAN =    243.64471
             BAUD VARIANCE =     29136.31
                BAUD SNROUT =    2.0374 =    3.0908 DB
```

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

```
TOTAL NUMBER OF POINTS, 2K =    128
                 BAUD MEAN =    248.93130
             BAUD VARIANCE =     32359.91
                BAUD SNROUT =    1.9149 =    2.8215 DB
```

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

```
TOTAL NUMBER OF POINTS, 2K =    128
                 BAUD MEAN =    240.99722
             BAUD VARIANCE =     22283.86
                BAUD SNROUT =    2.6064 =    4.1603 DB
```

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

```
TOTAL NUMBER OF POINTS, 2K =    128
                 BAUD MEAN =    230.72388
             BAUD VARIANCE =     32238.76
                BAUD SNROUT =    1.6512 =    2.1781 DB
```

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

```
TOTAL NUMBER OF POINTS, 2K =    128
                 BAUD MEAN =    244.09976
             BAUD VARIANCE =     30041.91
                BAUD SNROUT =    1.9834 =    2.9741 DB
```


*** TOTAL OVER ALL 5 BAUDS WITH EPSILON = 2.25 ***

```
      TOTAL NUMBER OF BAUDS =      5
               MEAN SNROUT =    3.0450 DB
STANDARD DEVIATION OF SNROUT =    0.7163 DB
```

121

SNROUT VS. DOPPLER TEST -- EPSILON = 2.50

INPUT SNRNB =   31.623 =   15.000 DB

BAUD TYPE 3: KX = 1024 SAMPLE POINTS; K = 64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                   BAUD MEAN =   240.04703
               BAUD VARIANCE =   34182.69
                  BAUD SNROUT =   1.6857 =   2.2679 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                   BAUD MEAN =   234.67413
               BAUD VARIANCE =   38249.63
                  BAUD SNROUT =   1.4398 =   1.5830 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                   BAUD MEAN -   254.86404
               BAUD VARIANCE =   26564.81
                  BAUD SNROUT =   2.4452 =   3.8831 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                   BAUD MEAN =   228.89236
               BAUD VARIANCE =   38044.78
                  BAUD SNROUT =   1.3771 =   1.3897 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                   BAUD MEAN =   255.39224
               BAUD VARIANCE =   34706.46
                  BAUD SNROUT =   1.8793 =   2.7400 DB

*** TOTAL OVER ALL 5 BAUDS WITH EPSILON = 2.50 ***

              TOTAL NUMBER OF BAUDS =   5
                      MEAN SNROUT =   2.3727 DB
    STANDARD DEVIATION OF SNROUT =   1.0021 DB

122

SNROUT VS. DOPPLER TEST -- EPSILON = 2.75

INPUT SNRNB = 31.623 = 15.000 DB

BAUD TYPE 3: KX = 1024 SAMPLE POINTS; K = 64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

       TOTAL NUMBER OF POINTS, 2K =   128
                     BAUD MEAN =   250.07709
                 BAUD VARIANCE =   39538.09
                   BAUD SNROUT =   1.5817 =   1.9913 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

       TOTAL NUMBER OF POINTS, 2K =   128
                     BAUD MEAN =   225.95108
                 BAUD VARIANCE =   44515.95
                   BAUD SNROUT =   1.1469 =   0.5951 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

       TOTAL NUMBER OF POINTS, 2K =   128
                     BAUD MEAN =   265.65649
                 BAUD VARIANCE =   31226.82
                   BAUD SNROUT =   2.2600 =   3.5411 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

       TOTAL NUMBER OF POINTS, 2K =   128
                     BAUD MEAN =   236.69147
                 BAUD VARIANCE =   44222.02
                   BAUD SNROUT =   1.2669 =   1.0273 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

       TOTAL NUMBER OF POINTS, 2K =   128
                     BAUD MEAN =   262.79248
                 BAUD VARIANCE =   39631.13
                   BAUD SNROUT =   1.7426 =   2.4119 DB

*** TOTAL OVER ALL 5 BAUDS WITH EPSILON = 2.75 ***

              TOTAL NUMBER OF BAUDS =   5
                       MEAN SNROUT =   1.9133 DB
        STANDARD DEVIATION OF SNROUT =   1.1648 DB

B. INPUT SNR = 5 dB

```
SNROUT VS. DOPPLER TEST -- EPSILON =  0.00

INPUT SNRNB =     3.162 =     5.000 DB

BAUD TYPE 3:  KX = 1024 SAMPLE POINTS; K =  64 TONES


** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

     TOTAL NUMBER OF POINTS, 2K =     128
                     BAUD MEAN =     348.85937
                  BAUD VARIANCE =      36188.43
                   BAUD SNROUT =      3.3630 =     5.2673 DB


** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

     TOTAL NUMBER OF POINTS, 2K =     128
                     BAUD MEAN =     352.66479
                  BAUD VARIANCE =      43009.19
                   BAUD SNROUT =      2.8918 =     4.6116 DB


** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

     TOTAL NUMBER OF POINTS, 2K =     128
                     BAUD MEAN =     335.33081
                  BAUD VARIANCE =      36318.57
                   BAUD SNROUT =      3.0961 =     4.9082 DB


** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

     TOTAL NUMBER OF POINTS, 2K =     128
                     BAUD MEAN =     354.62695
                  BAUD VARIANCE =      38677.65
                   BAUD SNROUT =      3.2515 =     5.1208 DB


** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

     TOTAL NUMBER OF POINTS, 2K =     128
                     BAUD MEAN =     305.20776
                  BAUD VARIANCE =      52039.11
                   BAUD SNROUT =      1.7900 =     2.5286 DB



*** TOTAL OVER ALL 5 BAUDS WITH EPSILON =  0.00 ***

               TOTAL NUMBER OF BAUDS =       5
                       MEAN SNROUT =     4.4873 DB
        STANDARD DEVIATION OF SNROUT =     1.1224 DB
```

SNROUT VS. DOPPLER TEST -- EPSILON = 0.25

INPUT SNRNB =   3.162 =   5.000 DB

BAUD TYPE 3: KX = 1024 SAMPLE POINTS; K = 64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                   BAUD MEAN =   347.25366
               BAUD VARIANCE =   37339.82
                 BAUD SNROUT =   3.2294 =   5.0912 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                   BAUD MEAN =   351.10767
               BAUD VARIANCE =   43143.09
                 BAUD SNROUT =   2.8574 =   4.5597 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                   BAUD MEAN =   333.41895
               BAUD VARIANCE =   35979.64
                 BAUD SNROUT =   3.0898 =   4.8992 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                   BAUD MEAN =   352.74438
               BAUD VARIANCE =   39246.14
                 BAUD SNROUT =   3.1705 =   5.0112 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                   BAUD MEAN =   303.09644
               BAUD VARIANCE =   53554.99
                 BAUD SNROUT =   1.7154 =   2.3436 DB

*** TOTAL OVER ALL 5 BAUDS WITH EPSILON =   0.25 ***

              TOTAL NUMBER OF BAUDS =   5
                      MEAN SNROUT =   4.3810 DB
        STANDARD DEVIATION OF SNROUT =   1.1568 DB

SNROUT VS. DOPPLER TEST -- EPSILON = 0.50

INPUT SNRNB =   3.162 =   5.000 DB

BAUD TYPE 3: KX = 1024 SAMPLE POINTS; K = 64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                  BAUD MEAN =   342.34521
              BAUD VARIANCE =   39210.31
                BAUD SNROUT =   2.9890 =   4.7553 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                  BAUD MEAN =   346.34448
              BAUD VARIANCE =   44183.36
                BAUD SNROUT =   2.7149 =   4.3376 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                  BAUD MEAN =   327.63818
              BAUD VARIANCE =   36156.82
                BAUD SNROUT =   2.9689 =   4.7260 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                  BAUD MEAN =   346.97962
              BAUD VARIANCE =   40537.41
                BAUD SNROUT =   2.9700 =   4.7275 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                  BAUD MEAN =   296.65332
              BAUD VARIANCE =   55526.02
                BAUD SNROUT =   1.5849 =   2.0000 DB

*** TOTAL OVER ALL 5 BAUDS WITH EPSILON = 0.50 ***

              TOTAL NUMBER OF BAUDS =   5
                        MEAN SNROUT =   4.1093 DB
        STANDARD DEVIATION OF SNROUT =   1.1917 DB

126

SNROUT VS. DOPPLER TEST -- EPSILON = 0.75

INPUT SNRNB =   3.162 =   5.000 DB

BAUD TYPE 3:   KX = 1024 SAMPLE POINTS; K = 64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   334.21777
                BAUD VARIANCE =   41755.36
                 BAUD SNROUT =   2.6751 =   4.2735 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   338.46997
                BAUD VARIANCE =   46133.04
                 BAUD SNROUT =   2.4833 =   3.9503 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   318.09814
                BAUD VARIANCE =   36878.21
                 BAUD SNROUT =   2.7438 =   4.3835 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   337.46533
                BAUD VARIANCE =   42530.41
                 BAUD SNROUT =   2.6777 =   4.2776 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   286.04077
                BAUD VARIANCE =   57909.20
                 BAUD SNROUT =   1.4129 =   1.5011 DB

*** TOTAL OVER ALL 5 BAUDS WITH EPSILON = 0.75 ***

              TOTAL NUMBER OF BAUDS =   5
                      MEAN SNROUT =   3.6772 DB
        STANDARD DEVIATION OF SNROUT =   1.2273 DB

127

SNROUT VS. DOPPLER TEST -- EPSILON = 1.00

INPUT SNRNB =   3.162 =   5.000 DB

BAUD TYPE 3: KX = 1024 SAMPLE POINTS; K = 64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                   BAUD MEAN =   323.12573
               BAUD VARIANCE =   44902.68
                 BAUD SNROUT =   2.3253 =   3.6647 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                   BAUD MEAN =   327.73608
               BAUD VARIANCE =   48904.04
                 BAUD SNROUT =   2.1964 =   3.4170 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                   BAUD MEAN =   305.10498
               BAUD VARIANCE =   38135.35
                 BAUD SNROUT =   2.4410 =   3.8757 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                   BAUD MEAN =   324.53516
               BAUD VARIANCE =   45177.82
                 BAUD SNROUT =   2.3313 =   3.6760 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                   BAUD MEAN =   271.62476
               BAUD VARIANCE =   60591.47
                 BAUD SNROUT =   1.2177 =   0.8553 DB

*** TOTAL OVER ALL 5 BAUDS WITH EPSILON = 1.00 ***

            TOTAL NUMBER OF BAUDS =   5
                    MEAN SNROUT =   3.0977 DB
    STANDARD DEVIATION OF SNROUT =   1.2641 DB

128

SNROUT VS. DOPPLER TEST -- EPSILON = 1.25

INPUT SNRNB =   3.162 =   5.000 DB

BAUD TYPE 3:   KX = 1024 SAMPLE POINTS; K = 64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                 BAUD MEAN =   309.05103
              BAUD VARIANCE =   48664.34
                BAUD SNROUT =   1.9627 =   2.9285 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                 BAUD MEAN =   314.15283
              BAUD VARIANCE =   52524.69
                BAUD SNROUT =   1.8790 =   2.7392 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                 BAUD MEAN =   288.63892
              BAUD VARIANCE =   39937.87
                BAUD SNROUT =   2.0861 =   3.1932 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                 BAUD MEAN =   308.20581
              BAUD VARIANCE =   48482.35
                BAUD SNROUT =   1.9593 =   2.9210 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                 BAUD MEAN =   253.42087
              BAUD VARIANCE =   63620.02
                BAUD SNROUT =   1.0095 =   0.0409 DB

*** TOTAL OVER ALL 5 BAUDS WITH EPSILON = 1.25 ***

              TOTAL NUMBER OF BAUDS =   5
                      MEAN SNROUT =   2.3646 DB
        STANDARD DEVIATION OF SNROUT =   1.3090 DB

129

SNROUT VS. DOPPLER TEST -- EPSILON = 1.50

INPUT SNRNB =   3.162 =   5.000 DB

BAUD TYPE 3:   KX = 1024 SAMPLE POINTS; K = 64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   292.29761
                BAUD VARIANCE =   52952.92
                  BAUD SNROUT =   1.6135 =   2.0776 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   298.04663
                BAUD VARIANCE =   56898.22
                  BAUD SNROUT =   1.5612 =   1.9347 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   269.08667
                BAUD VARIANCE =   42304.40
                  BAUD SNROUT =   1.7116 =   2.3340 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   288.91211
                BAUD VARIANCE =   52405.19
                  BAUD SNROUT =   1.5928 =   2.0216 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

    TOTAL NUMBER OF POINTS, 2K =   128
                    BAUD MEAN =   237.78751
                BAUD VARIANCE =   66898.56
                  BAUD SNROUT =   0.8452 =   -0.7304 DB

*** TOTAL OVER ALL 5 BAUDS WITH EPSILON = 1.50 ***

            TOTAL NUMBER OF BAUDS =   5
                      MEAN SNROUT =   1.5275 DB
    STANDARD DEVIATION OF SNROUT =   1.2709 DB

130

SNROUT VS. DOPPLER TEST -- EPSILON = 1.75

INPUT SNRNB =   3.162 =   5.000 DB

BAUD TYPE 3:   KX = 1024 SAMPLE POINTS; K = 64 TONES

** OVERALL (REAL + IMAG) STATISTICS FOR THE 1ST BAUD **

        TOTAL NUMBER OF POINTS, 2K =   128
                        BAUD MEAN =   278.15869
                    BAUD VARIANCE =   57652.27
                      BAUD SNROUT =   1.3420 =   1.2777 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 2ND BAUD **

        TOTAL NUMBER OF POINTS, 2K =   128
                        BAUD MEAN =   288.96216
                    BAUD VARIANCE =   61909.75
                      BAUD SNROUT =   1.3487 =   1.2992 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 3RD BAUD **

        TOTAL NUMBER OF POINTS, 2K =   128
                        BAUD MEAN =   247.04657
                    BAUD VARIANCE =   45172.92
                      BAUD SNROUT =   1.3511 =   1.3068 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 4TH BAUD **

        TOTAL NUMBER OF POINTS, 2K =   128
                        BAUD MEAN =   267.27612
                    BAUD VARIANCE =   56809.34
                      BAUD SNROUT =   1.2575 =   0.9950 DB

** OVERALL (REAL + IMAG) STATISTICS FOR THE 5TH BAUD **

        TOTAL NUMBER OF POINTS, 2K =   128
                        BAUD MEAN =   241.30634
                    BAUD VARIANCE =   70345.50
                      BAUD SNROUT =   0.8278 =   -0.8210 DB

*** TOTAL OVER ALL 5 BAUDS WITH EPSILON = 1.75 ***

                TOTAL NUMBER OF BAUDS =   5
                        MEAN SNROUT =   0.8115 DB
        STANDARD DEVIATION OF SNROUT =   0.9218 DB

APPENDIX E. STATISTICS OF THE DOPPLER ESTIMATION ALGORITHM

A. INPUT SNR = 10 dB

```
ALPHA-HAT VS. ALPHA -- INPUT SNR = 10 DB
   BAUD TYPE 3:  KX = 1024 SAMPLE POINTS;  K = 64 TONES


**  EPSILON = 0.0      DELTA-ALPHA = 0.000376506

    LL   ALPHA         ALPHA-HAT       EPSILON-HAT

    1    0.000          0.00008162      0.21678060
    2    0.000         -0.00005518     -0.14657104
    3    0.000          0.00000018      0.00046881
    4    0.000         -0.00029124     -0.77353448
    5    0.000         -0.00000678     -0.01801626


             NUMBER OF BAUDS =           5
         MEAN OF EPSILON-HAT = -0.144174457
     VARIANCE OF EPSILON-HAT =  0.140806437



**  EPSILON = 0.25     DELTA-ALPHA = 0.000376506

    LL   ALPHA         ALPHA-HAT       EPSILON-HAT

    1    0.000094128    0.00017936      0.47636849
    2    0.000094128    0.00003786      0.10056025
    3    0.000094128    0.00011023      0.29278034
    4    0.000094128   -0.00020184     -0.53608972
    5    0.000094128    0.00009492      0.25209653


             NUMBER OF BAUDS =           5
         MEAN OF EPSILON-HAT =  0.117143154
     VARIANCE OF EPSILON-HAT =  0.151272893



**  EPSILON = 0.50     DELTA-ALPHA = 0.000376506

    LL   ALPHA         ALPHA-HAT       EPSILON-HAT

    1    0.000188255   -0.00011252     -0.29886585
    2    0.000188255    0.00013242      0.35169941
    3    0.000188255    0.00021961      0.58327287
    4    0.000188255    0.00011711      0.31104386
    5    0.000188255    0.00019830      0.52669251


             NUMBER OF BAUDS =           5
         MEAN OF EPSILON-HAT =  0.294768512
     VARIANCE OF EPSILON-HAT =  0.123232782
```

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ALPHA-HAT VS. ALPHA -- INPUT SNR = 10 DB
   BAUD TYPE 3:   KX = 1024 SAMPLE POINTS;   K = 64 TONES

** EPSILON = 0.75   DELTA-ALPHA = 0.000376506

    LL   ALPHA   ALPHA-HAT   EPSILON-HAT

    1   0.000282383   -0.00024254   -0.64418107
    2   0.000282383   0.00003668   0.09741002
    3   0.000282383   0.00013307   0.35344487
    4   0.000282383   0.00021555   0.57250100
    5   0.000282383   0.00030271   0.80399001

              NUMBER OF BAUDS =   5
         MEAN OF EPSILON-HAT =   0.236632884
     VARIANCE OF EPSILON-HAT =   0.310890436

** EPSILON = 1.00   DELTA-ALPHA = 0.000376506

    LL   ALPHA   ALPHA-HAT   EPSILON-HAT

    1   0.000376506   -0.00034564   -0.91803193
    2   0.000376506   0.00013396   0.35578859
    3   0.000376506   0.00023758   0.63100237
    4   0.000376506   0.00031679   0.84140372
    5   0.000376506   0.00019155   0.50875187

              NUMBER OF BAUDS =   5
         MEAN OF EPSILON-HAT =   0.283782721
     VARIANCE OF EPSILON-HAT =   0.482913494

** EPSILON = 1.25   DELTA-ALPHA = 0.000376506

    LL   ALPHA   ALPHA-HAT   EPSILON-HAT

    1   0.000470638   -0.00088145   -2.34113503
    2   0.000470638   0.00023501   0.62418330
    3   0.000470638   0.00034109   0.90593851
    4   0.000470638   0.00023151   0.61489588
    5   0.000470638   -0.00013302   -0.35329860

              NUMBER OF BAUDS =   5
         MEAN OF EPSILON-HAT = -0.109883070
     VARIANCE OF EPSILON-HAT =   1.78344536

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```
ALPHA-HAT VS. ALPHA -- INPUT SNR = 10 DB
  BAUD TYPE 3:   KX = 1024 SAMPLE POINTS;   K = 64 TONES


**  EPSILON = 1.50      DELTA-ALPHA = 0.000376506

    LL    ALPHA          ALPHA-HAT          EPSILON-HAT

    1    0.000564765    -0.00100695      -2.67446518
    2    0.000564765    -0.00051077      -1.35661030
    3    0.000564765    -0.00061805      -1.64155102
    4    0.000564765    -0.00006197      -0.16459131
    5    0.000564765    -0.00002642      -0.07016611


            NUMBER OF BAUDS =              5
        MEAN OF EPSILON-HAT =   -1.18147659
    VARIANCE OF EPSILON-HAT =    1.18510342
```

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B. INPUT SNR = 15 dB

```
ALPHA-HAT VS. ALPHA -- INPUT SNR = 15 DB
    BAUD TYPE 3:  KX   1024 SAMPLE POINTS;  K = 64 TONES


**  EPSILON = 0.0        DELTA-ALPHA = 0.000376506

    LL   ALPHA          ALPHA-HAT        EPSILON-HAT

    1    0.000          0.00004005       0.10637712
    2    0.000         -0.00002999      -0.07964146
    3    0.000         -0.00000758      -0.02012977
    4    0.000         -0.00003488      -0.09264153
    5    0.000          0.00000226       0.00600086


             NUMBER OF BAUDS =              5
        MEAN OF EPSILON-HAT = -0.160069466E-01
    VARIANCE OF EPSILON-HAT =  0.635034963E-02


**  EPSILON = 0.25       DELTA-ALPHA = 0.000376506

    LL   ALPHA          ALPHA-HAT        EPSILON-HAT

    1    0.000094128    0.00013718       0.36434567
    2    0.000094128    0.00006437       0.17096901
    3    0.000094128    0.00009453       0.25108421
    4    0.000094128    0.00005696       0.15127569
    5    0.000094128    0.00010090       0.26797980


             NUMBER OF BAUDS =              5
        MEAN OF EPSILON-HAT =  0.241130829
    VARIANCE OF EPSILON-HAT =  0.724961236E-02


**  EPSILON = 0.50       DELTA-ALPHA = 0.000376506

    LL   ALPHA          ALPHA-HAT        EPSILON-HAT

    1    0.000188255    0.00023408       0.62171990
    2    0.000188255    0.00016005       0.42509770
    3    0.000188255    0.00019705       0.52335626
    4    0.000188255    0.00015115       0.40144742
    5    0.000188255    0.00020043       0.53233200


             NUMBER OF BAUDS =              5
        MEAN OF EPSILON-HAT =  0.500790179
    VARIANCE OF EPSILON-HAT =  0.793160871E-02
```

```
ALPHA-HAT VS. ALPHA -- INPUT SNR = 15 DB
   BAUD TYPE 3:   KX = 1024 SAMPLE POINTS;   K = 64 TONES


**  EPSILON = 0.75      DELTA-ALPHA = 0.000376506

     LL    ALPHA          ALPHA-HAT        EPSILON-HAT

     1     0.000282383    0.00033001       0.87650335
     2     0.000282383    0.00025643       0.68107283
     3     0.000282383    0.00029901       0.79417539
     4     0.000282383    0.00024693       0.65585577
     5     0.000282383    0.00029999       0.79678440

                NUMBER OF BAUDS =            5
            MEAN OF EPSILON-HAT =   0.760878146
        VARIANCE OF EPSILON-HAT =   0.829143077E-02


**  EPSILON = 0.80      DELTA-ALPHA = 0.000376506

     LL    ALPHA          ALPHA-HAT        EPSILON-HAT

     1     0.000301205    0.00034829       0.92506492
     2     0.000301205    0.00027502       0.73044205
     3     0.000301205    0.00031849       0.84590751
     4     0.000301205    0.00026549       0.70512855
     5     0.000301205    0.00031912       0.84758091

                NUMBER OF BAUDS =            5
            MEAN OF EPSILON-HAT =   0.810824394
        VARIANCE OF EPSILON-HAT =   0.831642002E-02


**  EPSILON = 0.85      DELTA-ALPHA = 0.000376506

     LL    ALPHA          ALPHA-HAT        EPSILON-HAT

     1     0.000320030    0.00036756       0.97623181
     2     0.000320030    0.00029466       0.78261632
     3     0.000320030    0.00033904       0.90049165
     4     0.000320030    0.00028515       0.75734794
     5     0.000320030    0.00033927       0.90109152

                NUMBER OF BAUDS =            5
            MEAN OF EPSILON-HAT =   0.863555133
        VARIANCE OF EPSILON-HAT =   0.832509249E-02
```

```
ALPHA-HAT VS. ALPHA -- INPUT SNR = 15 DB
  BAUD TYPE 3:  KX = 1024 SAMPLE POINTS;  K = 64 TONES


**  EPSILON = 0.90      DELTA-ALPHA = 0.000376506

    LL    ALPHA          ALPHA-HAT      EPSILON-HAT

    1    0.000338855    0.00038678     1.02729702
    2    0.000338855    0.00031433     0.83485985
    3    0.000338855    0.00035951     0.95485568
    4    0.000338855    0.00030481     0.80957907
    5    0.000338855    0.00035939     0.95453286


             NUMBER OF BAUDS =             5
         MEAN OF EPSILON-HAT =  0.91224658
     VARIANCE OF EPSILON-HAT =  0.832261145E-02


**  EPSILON = 0.95      DELTA-ALPHA = 0.000376506

    LL    ALPHA          ALPHA-HAT      EPSILON-HAT

    1    0.000357681    0.00020686     0.54943258
    2    0.000357681    0.00033398     0.88705266
    3    0.000357681    0.00037992     1.00906944
    4    0.000357681    0.00032449     0.86183619
    5    0.000357681    0.00037949     1.00792885


             NUMBER OF BAUDS =             5
         MEAN OF EPSILON-HAT =  0.863063574
     VARIANCE OF EPSILON-HAT =  0.353112072E-01


**  EPSILON = 1.00      DELTA-ALPHA = 0.000376506

    LL    ALPHA          ALPHA-HAT      EPSILON-HAT

    1    0.000376506    0.00003566     0.09470624
    2    0.000376506    0.00016103     0.42768973
    3    0.000376506    0.00020522     0.54507101
    4    0.000376506    0.00034329     0.91177434
    5    0.000376506    0.00039861     1.05871677


             NUMBER OF BAUDS =             5
         MEAN OF EPSILON-HAT =  0.607591391
     VARIANCE OF EPSILON-HAT =  0.148841441
```

```
ALPHA-HAT VS. ALPHA -- INPUT SNR = 15 DB
   BAUD TYPE 3:   KX = 1024 SAMPLE POINTS;   K = 64 TONES


**   EPSILON = 1.25        DELTA-ALPHA = 0.000376506

     LL    ALPHA           ALPHA-HAT        EPSILON-HAT

     1     0.000470638     -0.00049591      -1.31713200
     2     0.000470638      0.00025919       0.68840146
     3     0.000470638      0.00030580       0.81219983
     4     0.000470638      0.00005479       0.14553374
     5     0.000470638      0.00028259       0.75056285


                NUMBER OF BAUDS =            5
           MEAN OF EPSILON-HAT =   0.215912998
       VARIANCE OF EPSILON-HAT =   0.804957628



**   EPSILON = 1.50        DELTA-ALPHA = 0.000376506

     LL    ALPHA           ALPHA-HAT        EPSILON-HAT

     1     0.000564765     -0.00082413      -2.18889141
     2     0.000564765     -0.00046605      -1.23783207
     3     0.000564765     -0.00020983      -0.55731559
     4     0.000564765     -0.00024733      -0.65690029
     5     0.000564765     -0.00004868      -0.12930280


                NUMBER OF BAUDS =            5
           MEAN OF EPSILON-HAT = -0.954047918
       VARIANCE OF EPSILON-HAT =   0.632816792
```

C.   INPUT SNR = 20 dB

```
ALPHA-HAT VS. ALPHA -- INPUT SNR = 20 DB
   BAUD TYPE 3:   KX = 1024 SAMPLE POINTS;   K = 64 TONES


**   EPSILON = 0.0        DELTA-ALPHA = 0.000376506

     LL    ALPHA          ALPHA-HAT        EPSILON-HAT

     1     0.000          0.00002055       0.05457596
     2     0.000         -0.00001621      -0.04305860
     3     0.000         -0.00000631      -0.01675819
     4     0.000         -0.00001915      -0.05085765
     5     0.000          0.00000311       0.00826014


              NUMBER OF BAUDS =              5
        MEAN OF EPSILON-HAT = -0.956766680E-02
    VARIANCE OF EPSILON-HAT =  0.182760879E-02


**   EPSILON = 0.25       DELTA-ALPHA = 0.000376506

     LL    ALPHA          ALPHA-HAT        EPSILON-HAT

     1     0.000094128    0.00011643       0.30924392
     2     0.000094128    0.00007815       0.20756477
     3     0.000094128    0.00009204       0.24444658
     4     0.000094128    0.00007348       0.19517177
     5     0.000094128    0.00009959       0.26451528


              NUMBER OF BAUDS =              5
        MEAN OF EPSILON-HAT =  0.244188309
    VARIANCE OF EPSILON-HAT =  0.209734496E-02


**   EPSILON = 0.50       DELTA-ALPHA = 0.000376506

     LL    ALPHA          ALPHA-HAT        EPSILON-HAT

     1     0.000188255    0.00021262       0.56471968
     2     0.000188255    0.00017358       0.46102297
     3     0.000188255    0.00019098       0.50724143
     4     0.000188255    0.00016762       0.44520897
     5     0.000188255    0.00019673       0.52252698


              NUMBER OF BAUDS =              5
        MEAN OF EPSILON-HAT =  0.500143766
    VARIANCE OF EPSILON-HAT =  0.231742300E-02
```

```
ALPHA-HAT VS. ALPHA -- INPUT SNR = 20 DB
  BAUD TYPE 3:   KX = 1024 SAMPLE POINTS;   K = 64 TONES


**  EPSILON = 0.75      DELTA-ALPHA = 0.000376506

    LL    ALPHA           ALPHA-HAT        EPSILON-HAT

    1    0.000282383      0.00030833       0.81892306
    2    0.000282383      0.00026935       0.71539086
    3    0.000282383      0.00028965       0.76931149
    4    0.000282383      0.00026252       0.69726127
    5    0.000282383      0.00029374       0.78016865


              NUMBER OF BAUDS =            5
         MEAN OF EPSILON-HAT =   0.756210685
     VARIANCE OF EPSILON-HAT =   0.245493464E-02


**  EPSILON = 1.00      DELTA-ALPHA = 0.000376506

    LL    ALPHA           ALPHA-HAT        EPSILON-HAT

    1    0.000376506      0.00040266       1.06947517
    2    0.000376506      0.00036461       0.96840972
    3    0.000376506      0.00038703       1.02794170
    4    0.000376506      0.00035726       0.94887573
    5    0.000376506      0.00038970       1.03504181


              NUMBER OF BAUDS =            5
         MEAN OF EPSILON-HAT =     1.00994873
     VARIANCE OF EPSILON-HAT =   0.248805061E-02


**  EPSILON = 1.05      DELTA-ALPHA = 0.000376506

    LL    ALPHA           ALPHA-HAT        EPSILON-HAT

    1    0.000395336      0.00042184       1.12041664
    2    0.000395336      0.00038408       1.02010632
    3    0.000395336      0.00040684       1.08056164
    4    0.000395336      0.00037666       1.00041580
    5    0.000395336      0.00040925       1.08697891


              NUMBER OF BAUDS =            5
         MEAN OF EPSILON-HAT =     1.06169510
     VARIANCE OF EPSILON-HAT =   0.248205289E-02
```

```
ALPHA-HAT VS. ALPHA -- INPUT SNR = 20 DB
  BAUD TYPE 3:  KX = 1024 SAMPLE POINTS;  K = 64 TONES


**  EPSILON = 1.10      DELTA-ALPHA = 0.000376506

    LL   ALPHA          ALPHA-HAT        EPSILON-HAT

    1    0.000414161    0.00044105       1.17141533
    2    0.000414161    0.00021188       0.56274819
    3    0.000414161    0.00042667       1.13323879
    4    0.000414161    0.00039613       1.05211163
    5    0.000414161    0.00042883       1.13897419


              NUMBER OF BAUDS =           5
         MEAN OF EPSILON-HAT =     1.01169682
     VARIANCE OF EPSILON-HAT =  0.649175048E-01


**  EPSILON = 1.15      DELTA-ALPHA = 0.000376506

    LL   ALPHA          ALPHA-HAT        EPSILON-HAT

    1    0.000432987    0.00026110       0.69347852
    2    0.000432987    0.00023134       0.61445159
    3    0.000432987    0.00025232       0.67015254
    4    0.000432987    0.00021900       0.58167678
    5    0.000432987    0.00044832       1.19074154


              NUMBER OF BAUDS =           5
         MEAN OF EPSILON-HAT =   0.750099897
     VARIANCE OF EPSILON-HAT =   0.626323223E-01


**  EPSILON = 1.20      DELTA-ALPHA = 0.000376506

    LL   ALPHA          ALPHA-HAT        EPSILON-HAT

    1    0.000451812    0.00008400       0.22311223
    2    0.000451812    0.00024993       0.66380346
    3    0.000451812    0.00027113       0.72011048
    4    0.000451812    0.00004699       0.12480402
    5    0.000451812    0.00046693       1.24015808


              NUMBER OF BAUDS =           5
         MEAN OF EPSILON-HAT =   0.594397545
     VARIANCE OF EPSILON-HAT =   0.198999524
```

```
ALPHA-HAT VS. ALPHA -- INPUT SNR = 20 DB
   BAUD TYPE 3:  KX = 1024 SAMPLE POINTS;   K = 64 TONES


**  EPSILON = 1.25      DELTA-ALPHA = 0.000376506

     LL   ALPHA         ALPHA-HAT       EPSILON-HAT

     1    0.000470638   0.00010312      0.27389467
     2    0.000470638   0.00026947      0.71572357
     3    0.000470638   0.00010148      0.26952112
     4    0.000470638   0.00006651      0.17663908
     5    0.000470638   0.00048646      1.29204941


            NUMBER OF BAUDS =            5
         MEAN OF EPSILON-HAT =   0.545565367
     VARIANCE OF EPSILON-HAT =   0.218075991


**  EPSILON = 1.50      DELTA-ALPHA = 0.000376506

     LL   ALPHA         ALPHA-HAT       EPSILON-HAT

     1    0.000564765   -0.00062383     -1.65688896
     2    0.000564765   -0.00024093     -0.63990915
     3    0.000564765   -0.00022707     -0.60310274
     4    0.000564765   -0.00023840     -0.63318038
     5    0.000564765   -0.00006303     -0.16741729


            NUMBER OF BAUDS =            5
         MEAN OF EPSILON-HAT =  -0.740099311
     VARIANCE OF EPSILON-HAT =   0.302176237
```

D. INPUT SNR = 40 dB

```
ALPHA-HAT VS. ALPHA -- INPUT SNR = 40 DB
   BAUD TYPE 3:  KX = 1024 SAMPLE POINTS;  K = 64 TONES


**  EPSILON = 0.0        DELTA-ALPHA = 0.000376506

    LL   ALPHA         ALPHA-HAT       EPSILON-HAT

    1    0.000          0.00000226       0.00601395
    2    0.000         -0.00000109      -0.00289558
    3    0.000         -0.00000038      -0.00100686
    4    0.000         -0.00000142      -0.00376268
    5    0.000          0.00000092       0.00243571


              NUMBER OF BAUDS =             5
         MEAN OF EPSILON-HAT =   0.156908704E-03
     VARIANCE OF EPSILON-HAT =   0.163832447E-04


**  EPSILON = 0.25       DELTA-ALPHA = 0.000376506

    LL   ALPHA         ALPHA-HAT       EPSILON-HAT

    1    0.000094128    0.00009623       0.25558215
    2    0.000094128    0.00009274       0.24631453
    3    0.000094128    0.00009378       0.24908329
    4    0.000094128    0.00009219       0.24485534
    5    0.000094128    0.00009487       0.25196874


              NUMBER OF BAUDS =             5
         MEAN OF EPSILON-HAT =   0.249561667
     VARIANCE OF EPSILON-HAT =   0.187394326E-04


**  EPSILON = 0.50       DELTA-ALPHA = 0.000376506

    LL   ALPHA         ALPHA-HAT       EPSILON-HAT

    1    0.000188255    0.00019089       0.50700271
    2    0.000188255    0.00018733       0.49755794
    3    0.000188255    0.00018857       0.50083911
    4    0.000188255    0.00018658       0.49555242
    5    0.000188255    0.00018942       0.50309473


              NUMBER OF BAUDS =             5
         MEAN OF EPSILON-HAT =   0.500809073
     VARIANCE OF EPSILON-HAT =   0.204471289E-04
```

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```
ALPHA-HAT VS. ALPHA -- INPUT SNR = 40 DB
  BAUD TYPE 3:  KX = 1024 SAMPLE POINTS;  K = 64 TONES


**  EPSILON = 0.75      DELTA-ALPHA = 0.000376506

    LL   ALPHA          ALPHA-HAT        EPSILON-HAT

    1    0.000282383    0.00028547       0.75821120
    2    0.000282383    0.00028194       0.74884433
    3    0.000282383    0.00028319       0.75216144
    4    0.000282383    0.00028101       0.74635839
    5    0.000282383    0.00028383       0.75386328


               NUMBER OF BAUDS =              5
          MEAN OF EPSILON-HAT =    0.751887321
      VARIANCE OF EPSILON-HAT =    0.209499267E-04


**  EPSILON = 1.00      DELTA-ALPHA = 0.000376506

    LL   ALPHA          ALPHA-HAT        EPSILON-HAT

    1    0.000376506    0.00037910       1.00688934
    2    0.000376506    0.00037569       0.99783307
    3    0.000376506    0.00037675       1.00063419
    4    0.000376506    0.00037461       0.99496496
    5    0.000376506    0.00037728       1.00205612


               NUMBER OF BAUDS =              5
          MEAN OF EPSILON-HAT =    1.00047493
      VARIANCE OF EPSILON-HAT =    0.202523515E-04


**  EPSILON = 1.05      DELTA-ALPHA = 0.000376506

    LL   ALPHA          ALPHA-HAT        EPSILON-HAT

    1    0.000395336    0.00039818       1.05757332
    2    0.000395336    0.00039480       1.04860115
    3    0.000395336    0.00039580       1.05124378
    4    0.000395336    0.00039370       1.04568005
    5    0.000395336    0.00039634       1.05267429


               NUMBER OF BAUDS =              5
          MEAN OF EPSILON-HAT =    1.05115414
      VARIANCE OF EPSILON-HAT =    0.200020440E-04
```

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```
ALPHA-HAT VS. ALPHA -- INPUT SNR = 40 DB
   BAUD TYPE 3:   KX = 1024 SAMPLE POINTS;   K = 64 TONES


**  EPSILON = 1.10      DELTA-ALPHA = 0.000376506

     LL    ALPHA         ALPHA-HAT      EPSILON-HAT

     1    0.000414161    0.00041730     1.10836124
     2    0.000414161    0.00041396     1.09947109
     3    0.000414161    0.00041488     1.10192490
     4    0.000414161    0.00041284     1.09649086
     5    0.000414161    0.00041541     1.10333538

              NUMBER OF BAUDS =           5
           MEAN OF EPSILON-HAT =    1.10191631
     VARIANCE OF EPSILON-HAT =   0.197413901E-04


**  EPSILON = 1.15      DELTA-ALPHA = 0.000376506

     LL    ALPHA         ALPHA-HAT      EPSILON-HAT

     1    0.000432987    0.00043634     1.15891647
     2    0.000432987    0.00043304     1.15016556
     3    0.000432987    0.00043389     1.15240288
     4    0.000432987    0.00023536     0.62512481
     5    0.000432987    0.00043442     1.15381145

              NUMBER OF BAUDS =           5
           MEAN OF EPSILON-HAT =    1.04808331
     VARIANCE OF EPSILON-HAT =   0.559149086E-01


**  EPSILON = 1.20      DELTA-ALPHA = 0.000376506

     LL    ALPHA         ALPHA-HAT      EPSILON-HAT

     1    0.000451812    0.00045451     1.20717335
     2    0.000451812    0.00045124     1.19849777
     3    0.000451812    0.00026262     0.69750923
     4    0.000451812    0.00006299     0.16730273
     5    0.000451812    0.00045256     1.20199013

              NUMBER OF BAUDS =           5
           MEAN OF EPSILON-HAT =   0.894494414
     VARIANCE OF EPSILON-HAT =   0.213087618
```

```
ALPHA-HAT VS. ALPHA -- INPUT SNR = 40 DB
   BAUD TYPE 3:  KX = 1024 SAMPLE POINTS;  K = 64 TONES


**  EPSILON = 1.25      DELTA-ALPHA = 0.000376506

     LL    ALPHA          ALPHA-HAT        EPSILON-HAT

     1     0.000470638    0.00047359       1.25785255
     2     0.000470638    0.00008576       0.22777945
     3     0.000470638    0.00028165       0.74806517
     4     0.000470638   -0.00011694      -0.31059778
     5     0.000470638    0.00047162       1.25261021


              NUMBER OF BAUDS =          5
          MEAN OF EPSILON-HAT =  0.635141730
      VARIANCE OF EPSILON-HAT =  0.460538387


**  EPSILON = 1.50      DELTA-ALPHA = 0.000376506

     LL    ALPHA          ALPHA-HAT        EPSILON-HAT

     1     0.000564765   -0.00044042      -1.16975403
     2     0.000564765   -0.00001852      -0.04919485
     3     0.000564765   -0.00024438      -0.64908123
     4     0.000564765   -0.00022513      -0.59793478
     5     0.000564765   -0.00006345      -0.16852963


              NUMBER OF BAUDS =          5
          MEAN OF EPSILON-HAT = -0.526898742
      VARIANCE OF EPSILON-HAT =  0.197466671
```

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APPENDIX F. THE SIMULATION CODE: RXSIM

```
C  THIS SIMULATION PRODUCES A PREDICTION OF THE RECEIVED SIGNAL THROUGH
C  AN ACOUSTIC CHANNEL.  THE TRANSMITTED SIGNAL IS MFQPSK SIGNAL.
C
        REAL NU(100.1350), PHI(100,1350), U1, DELU1
        REAL FREQ, FF, ABSORP, AAUNDB, TL, AA(100,1350)
        REAL YRX, YY, TIME(410000), YYRX(410000)
        REAL RXKM(0:4096), RXKP(0:4096), RXKPD(0:4096), KFREQ(0:4096)
        REAL DELT(100), DELF(100)
        REAL VX(100), X(100)
        REAL VY(100), Y(100)
        REAL VZ(100), Z(100)
        REAL C(100), R(100), THETAD(100)
        REAL TAUL(100), ALPHA(100), ALPHAM(100), DELALF(100)
        REAL UHAT1(100), LBAUD(100), PTOT(100), NOSVAR(100)
        REAL TAUK(1350), NOSAVG, NOISE

        INTEGER BDTYPE(100), BDTOTL, NBAUDS(50), IDFT
        INTEGER KMIN(100), KMAX(100), KX(100), M(100), KPTS(100)
        INTEGER IIPHI(100,1350), IPHI

        COMPLEX RXIN(0:4096), RXOUT(0:4096)

        DOUBLE PRECISION DSEED

        CHARACTER*40 PLABEL

C
C    USER INPUTS FOR THE SIMULATION
C
        WRITE(6,1000)
        READ(5,*) X0, Y0, Z0
C
        WRITE(30,1000)
        WRITE(30,*) ' X0 = ',X0,' Y0 = ',Y0,' Z0 = ',Z0
        WRITE(30,*) ' '
C
        WRITE(6,1001)
        READ(5,*) VXAVG, VXVAR, VYAVG, VYVAR, VZAVG, VZVAR
C
        WRITE(30,1001)
        WRITE(30,*) ' VXAVG = ',VXAVG,'  VXVAR = ',VXVAR
        WRITE(30,*) ' VYAVG = ',VYAVG,'  VYVAR = ',VYVAR
        WRITE(30,*) ' VZAVG = ',VZAVG,'  VZVAR = ',VZVAR
        WRITE(30,*) ' '
C
        WRITE(6,1002)
        READ(5,*) C0, CVAR
C
        WRITE(30,1002)
        WRITE(30,*) ' C0 = ',C0,' CVAR = ',CVAR
        WRITE(30,*) ' '
C
        WRITE(6,1003)
        READ(5,*) THETA0
C
        WRITE(30,1003)
        WRITE(30,*) ' THETA0 = ',THETA0
        WRITE(30,*) ' '
C
        WRITE(6,1004)
        READ(5,*) IAMP
C
C
        WRITE(30,1004)
        WRITE(30,*) ' IAMP = ',IAMP
        WRITE(30,*) ' '
C
C    INITIALIZE VARIABLES
C
C    BDTOTL = THE TOTAL NUMBER OF BAUDS IN THE SIGNAL
C    R0 = CLOSEST POINT OF APPROACH
C
```

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```
          BDTOTL = 0
          R0 = 10**15.0
          X(1) = X0
          Y(1) = Y0
          Z(1) = Z0
C
          IF (X0 .GT. 0)  VXAVG = -ABS(VXAVG)
          IF (X0 .LE. 0)  VXAVG = ABS(VXAVG)
          IF (Y0 .GT. 0)  VYAVG = -ABS(VYAVG)
          IF (Y0 .LE. 0)  VYAVG = ABS(VYAVG)
          IF (Z0 .GT. 0)  VZAVG = -ABS(VZAVG)
          IF (Z0 .LE. 0)  VZAVG = ABS(VZAVG)
C
          WRITE(6,1010)
          READ(5,*) NPAKS
C
          WRITE(30,1010)
          WRITE(30,*) ' NPAKS = ',NPAKS
          WRITE(30,*) ' '
C
   10 WRITE(6,1100)
          WRITE(6,1101)
          WRITE(6,1102)
          WRITE(6,1103) NPAKS
          READ(5,*) METHOD
C
          WRITE(30,1100)
          WRITE(30,1101)
          WRITE(30,1102)
          WRITE(30,1103) NPAKS
          WRITE(30,*) ' METHOD = ',METHOD
          WRITE(30,*) ' '
C
          IF (METHOD .LE. 0) GO TO 10
          IF (METHOD .GE. 3) GO TO 10
          DO 100  I = 1, NPAKS
   50     WRITE(6,1020)
          WRITE(6,1021) I
          WRITE(6,1022)
          WRITE(6,1023)
          WRITE(6,1024)
          WRITE(6,1025)
          WRITE(6,1026)
          READ(5,*) BDTYPE(I)
C
          WRITE(30,1020)
          WRITE(30,1021) I
          WRITE(30,1022)
          WRITE(30,1023)
          WRITE(30,1024)
          WRITE(30,1025)
          WRITE(30,1026)
          WRITE(30,*) ' BDTYPE(',I,') = ',BDTYPE(I)
          WRITE(30,*) ' '
C
          IF (BDTYPE(I) .LE. 0) GO TO 50
          IF (BDTYPE(I) .GT. 5) GO TO 50
   60     WRITE(6,1027) I
          READ(5,*) NBAUDS(I)
C
          WRITE(30,1027) I
          WRITE(30,*) ' NBAUDS(',I,') = ',NBAUDS(I)
          WRITE(30,*) ' '
C
          IF (NBAUDS(I) .LE. 0) GO TO 60
  100 CONTINUE
          WRITE(6,1030)
          READ(5,*) IDFT
C
          WRITE(30,1030)
          WRITE(30,*) ' IDFT = ',IDFT
          WRITE(30,*) ' '
```

```
C
        IF (IDFT .EQ. 1)   THEN
          WRITE(6,1040)
          READ(5,*) IWNDOW
C
          WRITE(30,1040)
          WRITE(30,*) ' IWNDOW = ',IWNDOW
          WRITE(30,*) '  '
C
        ENDIF
        WRITE(6,1050)
        READ(5,*) SNRDB
C
        WRITE(30,1050)
        WRITE(30,'(A10,F10.4)') ' SNRDB = ',SNRDB
        WRITE(30,*) '  '
C
        SNRIN = 10.0**(SNRDB/10.0)
C
        LL = 0
        DO 300   I = 1, NPAKS
          DO 200   J = 1, NBAUDS(I)
            LL = LL + 1
            IF (BDTYPE(I) .EQ. 1)   THEN
              DELT(LL) = 1.0 / 240.0
              DELF(LL) = 240.0
              KMIN(LL) = 68
              KMAX(LL) = 83
              KX(LL) = 256
            ENDIF
            IF (BDTYPE(I) .EQ. 2)   THEN
              DELT(LL) = 1.0 / 120.0
              DELF(LL) = 120.0
              KMIN(LL) = 135
              KMAX(LL) = 166
              KX(LL) = 512
            ENDIF
            IF (BDTYPE(I) .EQ. 3)   THEN
              DELT(LL) = 1.0 / 60.0
              DELF(LL) = 60.0
              KMIN(LL) = 269
              KMAX(LL) = 332
              KX(LL) = 1024
            ENDIF
            IF (BDTYPE(I) .EQ. 4)   THEN
              DELT(LL) = 1.0 / 30.0
              DELF(LL) = 30.0
              KMIN(LL) = 537
              KMAX(LL) = 664
              KX(LL) = 2048
            ENDIF
            IF (BDTYPE(I) .EQ. 5)   THEN
              DELT(LL) = 1.0 / 15.0
              DELF(LL) = 15.0
              KMIN(LL) = 1073
              KMAX(LL) = 1328
              KX(LL) = 4096
            ENDIF
  200     CONTINUE
  300 CONTINUE
C
C   INITIALIZE VARIABLES
C
      PI = 4.0*ATAN(1.0)
      BDTOTL = LL
      IF (BDTOTL .GT. 100)   THEN
        WRITE(6,*)' ***  ERROR:   TOTAL NUMBER OF BAUDS EXCEEDS 100  '
        WRITE(30,*)' ***  ERROR:   TOTAL NUMBER OF BAUDS EXCEEDS 100  '
        GO TO 9999
      ENDIF
C
C   ALLOW THE USER TO ENCODE THE PHASES
```

```fortran
C
      IF (METHOD .EQ. 2)  THEN
        LL = 0
        DO 490  I = 1, NPAKS
          WRITE(6,1110) I
          WRITE(6,1111) I, I
  410     READ(5,*) MTHDPK
C
          WRITE(30,1110) I
          WRITE(30,1111) I, I
          WRITE(30,*) ' MTHDPK = ',MTHDPK
          WRITE(30,*) '  '
C
          IF (MTHDPK .LE. 0)  GO TO 410
          IF (MTHDPK .GE. 3)  GO TO 410
          IF (MTHDPK .EQ. 1)  THEN
            DO 430  J = 1, NBAUDS(I)
              LL = LL + 1
              DO 420  K = KMIN(LL), KMAX(LL)
                DSEED = (K * 3.5729) + DSEED
                CALL PHASE(DSEED,RNDPHI,IPHI)
                PHI(LL,K) = RNDPHI
                IIPHI(LL,K) = IPHI
  420         CONTINUE
  430       CONTINUE
          ENDIF
          IF (MTHDPK .EQ. 2)  THEN
            DO 480  J = 1, NBAUDS(I)
              LL = LL + 1
              WRITE(6,1120) I, J
              WRITE(6,1121) I, J, I, J
  440         READ(5,*) MTHDBD
C
              WRITE(30,1120) I, J
              WRITE(30,1121) I, J, I, J
              WRITE(30,*) ' MTHDBD = ',MTHDBD
              WRITE(30,*) '  '
C
              IF (MTHDBD .LE. 0)  GO TO 440
              IF (MTHDBD .GE. 3)  GO TO 440
              IF (MTHDBD .EQ. 1)  THEN
                DO 450  K = KMIN(LL), KMAX(LL)
                  DSEED = (K * 5.7317) + DSEED
                  CALL PHASE(DSEED,RNDPHI,IPHI)
                  PHI(LL,K) = RNDPHI
                  IIPHI(LL,K) = IPHI
  450           CONTINUE
              ENDIF
              IF (MTHDBD .EQ. 2)  THEN
                WRITE(6,1100)
                WRITE(6,1101)
C
                WRITE(30,1100)
                WRITE(30,1101)
C
                DO 470  K = KMIN(LL), KMAX(LL)
  460             WRITE(6,1130) I, J, K
                  READ(5,*) IPHI
                  IIPHI(LL,K) = IPHI
C
                  WRITE(30,1130) I, J, K
                  WRITE(30,*) ' IPHI = ',IPHI
                  WRITE(30,*) '  '
C
                  IF (IPHI .LE. 0)  GO TO 460
                  IF (IPHI .GE. 5)  GO TO 460
                  IF (IPHI .EQ. 1) PHI(LL,K) = (45.0 * PI) / 180.0
                  IF (IPHI .EQ. 2) PHI(LL,K) = (135.0 * PI) / 180.0
                  IF (IPHI .EQ. 3) PHI(LL,K) = (-135.0 * PI) / 180.0
                  IF (IPHI .EQ. 4) PHI(LL,K) = (-45.0 * PI) / 180.0
  470           CONTINUE
              ENDIF
```

```
   480        CONTINUE
           ENDIF
    490   CONTINUE
        ENDIF
C
        DO 550  LL = 1, BDTOTL
C
C   INITIALIZE VARIABLES
C
        DSEED = (LL * 15.5987) + DSEED
        LBAUD(LL) = LL
C
C   COMPUTE:
C        X(LL) = DISTANCE THE TRANSMITTER TRAVELED IN THE
C               X-DIRECTION DURING THE LLTH BAUD
C
        CALL GAUSS(DSEED,VXAVG,VXVAR,ZRND)
        VX(LL) = ZRND
        IF (LL .GT. 1) X(LL) = X(LL-1) + (VX(LL) * DELT(LL))
C
C
C   COMPUTE:
C        Y(LL) = DISTANCE THE TRANSMITTER TRAVELED IN THE
C               Y-DIRECTION DURING THE LLTH BAUD
C
        CALL GAUSS(DSEED,VYAVG,VYVAR,ZRND)
        VY(LL) = ZRND
        IF (LL .GT. 1) Y(LL) = Y(LL-1) + (VY(LL) * DELT(LL))
C
C
C   COMPUTE:
C        Z(LL) = DISTANCE THE TRANSMITTER TRAVELED IN THE
C               Z-DIRECTION DURING THE LLTH BAUD
C
        CALL GAUSS(DSEED,VZAVG,VZVAR,ZRND)
        VZ(LL) = ZRND
        IF (LL .GT. 1) Z(LL) = Z(LL-1) + (VZ(LL) * DELT(LL))
C
C
C   COMPUTE:
C        C(LL) = THE SPEED OF SOUND DURING THE LTH BAUD
C
        CALL GAUSS(DSEED,C0,CVAR,ZRND)
        C(LL) = ZRND
C
C   COMPUTE:
C        R(LL) = SLANT RANGE TO THE RECEIVER
C
        XXX = ABS(X(LL))
        YYY = ABS(Y(LL))
        ZZZ = ABS(Z(LL))
        R(LL) = ( XXX**2.0 + YYY**2.0 + ZZZ**2.0 )**0.5
        IF (R(LL) .LE. R0)  R0 = R(LL)
C
C   COMPUTE:
C        THETAD(LL) = ANGLE BETWEEN THE SLANT RANGE AND Z0
C
        ARG = ABS(Z(LL)) / R(LL)
        THETAD(LL) = ACOS(ARG) * (180.0/PI)
        IF (THETAD(LL) .GT. THETA0)  THEN
          WRITE(6,*)
     %    ' ** WARNING:   THETA IS GREATER THAN ',THETA0,' DEGREES ** '
          WRITE(6,*)
     %    ' **********   THE RECEIVER CAN NOT RECEIVE THE SIGNAL  ** '
          WRITE(6,*)' '
          WRITE(30,*)
     %    ' ** WARNING:   THETA IS GREATER THAN ',THETA0,' DEGREES ** '
          WRITE(30,*)
     %    ' **********   THE RECEIVER CAN NOT RECEIVE THE SIGNAL  ** '
          WRITE(30,*)' '
        ENDIF
C
```

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```
C     INITIALIZE LIMITS FOR GRAPHS
C
      IF (LL .EQ. 1)  THEN
         XXMIN = INT(X(1))
         XXMAX = INT(X(1)) + 1.0
         YYMIN = INT(Y(1))
         YYMAX = INT(Y(1)) + 1.0
         ZZMIN = INT(Z(1))
         ZZMAX = INT(Z(1)) + 1.0
         CCMIN = INT(C(1))
         CCMAX = INT(C(1)) + 1.0
         RRMIN = INT(R(1))
         RRMAX = INT(R(1)) + 1.0
         THTMIN = INT(THETAD(1))
         THTMAX = INT(THETAD(1)) + 0.5
      ENDIF
C
C     SET LIMITS FOR GRAPHS                   .
C
      IF (X(LL) .LT. XXMIN) XXMIN = X(LL)
      IF (X(LL) .GT. XXMAX) XXMAX = X(LL)
      IF (Y(LL) .LT. YYMIN) YYMIN = Y(LL)
      IF (Y(LL) .GT. YYMAX) YYMAX = Y(LL)
      IF (Z(LL) .LT. ZZMIN) ZZMIN = Z(LL)
      IF (Z(LL) .GT. ZZMAX) ZZMAX = Z(LL)
      IF (C(LL) .LT. CCMIN) CCMIN = C(LL)
      IF (C(LL) .GT. CCMAX) CCMAX = C(LL)
      IF (R(LL) .LT. RRMIN) RRMIN = R(LL)
      IF (R(LL) .GT. RRMAX) RRMAX = R(LL)
      IF (THETAD(LL) .LT. THTMIN) THTMIN = THETAD(LL)
      IF (THETAD(LL) .GT. THTMAX) THTMAX = THETAD(LL)
C
C     ASSIGN ALL THE PHASES RANDOMLY FOR EVERY PACKET
C
      DO 500  K = KMIN(LL), KMAX(LL)
        IF (METHOD .EQ. 1)  THEN
           DSEED = (K * 2.767653) + DSEED
           CALL PHASE(DSEED,RNDPHI,IPHI)
           PHI(LL,K) = RNDPHI
           IIPHI(LL,K) = IPHI
        ENDIF
C
  500   CONTINUE
  550 CONTINUE
C
      DO 660  LL = 1, BDTOTL
C
C     COMPUTE:
C           ALPHA(LL) = THE DOPPLER COMPRESSION FACTOR
C                       DUE TO THE MOVING TRANSMITTER
C
      AVX = VXAVG * X(LL)
      AVY = VYAVG * Y(LL)
      AVZ = VZAVG * Z(LL)
      ALPHA(LL) = (AVX + AVY + AVZ) / (R(LL)*C(LL))
C
C     COMPUTE:
C           DELALF(LL) = THE MAXIMUM CHANGE IN ALPHA FOR THE LTH BAUD
C           M(LL) = THE DOPPLER CHANNEL # NEEDED FOR THE LTH BAUD
C           ALPHAM(LL) = THE DOPPLER CHANNEL FACTOR FOR THE LTH BAUD
C                       NOTE:
C                           ALPHAM(LL) IS USED TO ESTIMATE THE SAMPLING
C                           FREQUENCY OF THE RECEIVED SIGNAL
C
      DELALF(LL) = 1 / (8.0 * KMAX(LL))
      AMIN = (ALPHA(LL) / DELALF(LL)) - 0.5
      AMAX = (ALPHA(LL) / DELALF(LL)) + 0.5
      M(LL) = INT((AMIN + AMAX) / 2.0)
      ALPHAM(LL) = M(LL) * DELALF(LL)
C
C     COMPUTE:
C           TAUL(LL) = THE TIME DELAY DUE TO THE SIGNAL TRAVELING
```

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```
C                          THROUGH THE MEDIUM OR CHANNEL AND THE RECEIVER
C                          FOR THE LTH BAUD
C
        TAUL(LL) = R(LL) / C(LL)
C
        IF (LL .EQ. 1)  THEN
          ALFMIN = INT(ALPHA(1))
          ALFMAX = INT(ALPHA(1)) + 0.1
        ENDIF
        IF (ALPHA(LL) .LT. ALFMIN) ALFMIN = ALPHA(LL)
        IF (ALPHA(LL) .GT. ALFMAX) ALFMAX = ALPHA(LL)
        PTOT(LL) = 0.0
        NOSVAR(LL) = 0.0
        DO 640  K = KMIN(LL), KMAX(LL)
C
C    COMPUTE:
C          U1 = THE RECEIVE SIGNAL SAMPLE START TIME
C          DELU1 = THE SYNCHRONIZATION ERROR
C
C        WRITE(6,1140) LL, K
C1140 FORMAT(/,2X,'ENTER THE ESTIMATED RECEIVE SIGNAL START TIME ',/,2X,
C     %  '  FOR BAUD NUMBER ',I4,/,4X,
C     %  '  AND THE TIME DELAY THROUGH THE TRANSMITTER ',/,4X,
C     %  '    ELECTRONICS FOR TONE NUMBER ',I4,' ... ')
C        READ(5,*)  UHAT1(LL), TAUK(K)
         TAUK(K) = 0.005
         U1 = TAUL(LL) + (TAUK(K) * (1 + ALPHA(LL)))
         UHAT1(LL) = U1
         DELU1 = U1 - UHAT1(LL)
C
C    COMPUTE:
C          NU(LL,K) = A RANDOM SEQUENCE OF TIMING JITTERS
C
         NU(LL,K) = (DELU1/DELT(LL)) * (KX(LL)/(1+ALPHAM(LL)))
C
C    COMPUTE:
C          AA(LL,K) = THE AMPLITUDE OF THE RECEIVED SIGNAL, WHICH IS
C                     1 FOR ALL LL AND K IF NORMALIZED AMPLITUDES ARE
C                     DESIRED (I.E., IAMP=1), OR ATTENUATION FACTOR
C                     DUE TO THE TRANSMISSION LOSS IF NORMALIZED
C                     AMPLITUDES ARE NOT DESIRED (I.E., IAMP=0)
C
         IF (IAMP .EQ. 0) THEN
C
C    COMPUTE:
C          ABSORP = THE AMOUNT OF ABSORPTION IN DB/FT AT 4 DEGREES C
C                   AT A DEPTH OF APPROXIMATELY 3000 FT
C
           FREQ = (K * DELF(LL)) / 1000.0
           FF = FREQ**2.0
           ABSORP = (0.003 + ( (0.1*FF) / (1+FF) )
     %                     + ( (40.0*FF) / (4.100+FF) )
     %                     + ( 0.000275 * FF )) / 3000
C
C    TXDEP = THE DEPTH THE TRANSMITTER IS BELOW THE SURFACE IN FEET
C
           TXDEP = 1000.0
           ZTOTAL = TXDEP + ABS(Z(LL))
C
           AAUNDB = 10 ** (ABSORP / 20.0)
           IFAC = INT((ZTOTAL - 3000.0) / 1000.0)
           IF (IFAC .LT. 0) FACTOR = 1.02
           IF (IFAC .GT. 0) FACTOR = 0.98
           IIFAC = ABS(IFAC)
           IF (IIFAC .NE. 0)  THEN
             DO 620  I = 1, IIFAC
               AAUNDB = AAUNDB * FACTOR
 620         CONTINUE
           ENDIF
           ABSORP = 20.0 * ALOG10(AAUNDB)
C
C    COMPUTE:
```

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```
C              TL = THE TRANSMISSION LOSS IN DB DUE TO SPHERICAL-SPREADING
C                   AND ABSORPTION OF THE ACOUSTIC CHANNEL
C
               TL = (20.0 * ALOG10(ZTOTAL)) + (ABSORP * ZTOTAL)
C
               AA(LL,K) = 10.0**(-(TL/20.0))
            ELSE
               AA(LL,K) = 1.0
            ENDIF
C
C    COMPUTE THE NOISE VARIANCE FOR THE DESIRED
C      WIDE BAND SIGNAL-TO-NOISE RATIO
C
C         PTOT(LL) = THE TOTAL POWER OF THE LLTH BAUD
C         NOSVAR(LL) = THE NOISE VARIANCE OF THE LLTH BAUD
C
            PTOT(LL) = PTOT(LL) + ( (AA(LL,K)**2/2.0 ) )
  640    CONTINUE
            KPTS(LL) = (KMAX(LL) - KMIN(LL)) + 1
            NOSVAR(LL) = (PTOT(LL) * KX(LL)) / (2.0 * SNRIN * KPTS(LL))
  660 CONTINUE
C
      II = 0
      RXMAX = 0.0
      RXMIN = 0.0
      NOSAVG = 0.0
      NOISE = 0.0
      DO 700  LL = 1, BDTOTL
        DO 690  N = 1, KX(LL)
          DSEED = (LL * 32.7673) + DSEED
          CALL GAUSS(DSEED,NOSAVG,NOSVAR(LL),NOISE)
          YRX = 0.0
          NN = N - 1
          II = II + 1
          TIME(II) = II - 1
          DO 680  K = KMIN(LL), KMAX(LL)
C
C
C    COMPUTE:
C         YRX = THE RECEIVED SIGNAL WITH ALL OF THE ABOVE
C               PARAMETERS COMBINED FOR THE LTH BAUD
C               AT TIME = NN OVER ALL KMIN TO KMAX FREQUENCIES
C
          YY = AA(LL,K) * COS( (((2*PI*K)/KX(LL)) *
     %          ((1+ALPHAM(LL)) / (1+ALPHA(LL))) *
     %          (NN - NU(LL,K))) + PHI(LL,K) )
          YRX = YRX + YY
  680     CONTINUE
          YYRX(II) = YRX + NOISE
          IF (YYRX(II) .GT. RXMAX) RXMAX = YYRX(II)
          IF (YYRX(II) .LT. RXMIN) RXMIN = YYRX(II)
  690   CONTINUE
  700 CONTINUE
C
      NPTS = II
      KLL = 0
      DO 710  LL = 1, BDTOTL
        KLL = KLL + KPTS(LL)
  710 CONTINUE
      WRITE(30,2000) KLL
      WRITE(30,2001)'       LL       K     PHI(LL,K)   IPHI '
      DO 740  LL = 1, BDTOTL
        DO 720  K = KMIN(LL), KMAX(LL)
          WRITE(30,2010) LL, K, PHI(LL,K), IIPHI(LL,K)
  720   CONTINUE
  740 CONTINUE
C
C    COMPUTE THE DFT OF THE OUTPUT (RECEIVED) SIGNAL
C
      IF (IDFT .EQ. 1)  THEN
        RXKMIN = 0.0
        RXKMAX = 0.0
```

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```
          DO 800  LL = 1, BDTOTL
            II = 0
            IF (LL .EQ. 1)   THEN
              JJ = 1
            ELSE
              JJ = JJJ + 1
            ENDIF
            KXM1 = KX(LL) - 1
            JJJ = JJ + KXM1
            DO 760   J = JJ, JJJ
              RXIN(II) = CMPLX(YYRX(J),0.0)
              II = II + 1
760         CONTINUE
            CALL DFT(KXM1,RXIN,RXOUT)
C
C   CONVERT OUTPUT DATA TO EXPONENTIAL FORM,   I.E., RXKM()*EXP(J*RXKP())
C
            DO 770   I = 0, KXM1
              RXKM(I) = SQRT( (REAL(RXOUT(I)))**2 + (AIMAG(RXOUT(I)))**2 )
              IF (ABS(REAL(RXOUT(I))) .GE. 1.E-15)   THEN
                RXKP(I) = ATAN2(AIMAG(RXOUT(I)), REAL(RXOUT(I)))
              ELSE
                IF (ABS(AIMAG(RXOUT(I))) .LE. 1.E-15)   RXKP(I) = 0.0
                IF (AIMAG(RXOUT(I)) .GT. 1.E-15)   RXKP(I) = PI/2.0
                IF (AIMAG(RXOUT(I)) .LT. -1.E-15)   RXKP(I) = -PI/2.0
              ENDIF
              RXKPD(I) = (RXKP(I) * 180.0) / PI
770         CONTINUE
C
C   WRITE DFT INPUT AND OUTPUT TO A FILE
C
            WRITE(30,2200) LL
            DO 780   I = 0, KXM1
              WRITE(30,2210) I, REAL(RXIN(I)), AIMAG(RXIN(I))
780         CONTINUE
            WRITE(30,2220) LL
            IF (IWNDOW .EQ. 0)   THEN
              IMIN = 0
              IMAX = KXM1
              WMIN = 0
              WMAX = KXM1
            ELSE
              IMIN = KMIN(LL)
              IMAX = KMAX(LL)
              WMIN = KMIN(LL)
              WMAX = KMAX(LL)
            ENDIF
            KXX = (IMAX - IMIN) + 1
            DO 790   I = IMIN, IMAX
              WRITE(30,2230) I, REAL(RXOUT(I)), AIMAG(RXOUT(I)),
     %                          RXKM(I), RXKPD(I), RXKP(I)
              IF (RXKM(I) .LT. RXKMIN) RXKMIN = RXKM(I)
              IF (RXKM(I) .GT. RXKMAX) RXKMAX = RXKM(I)
              KFREQ(I) = I
790         CONTINUE
C
C   PLOT THE DFT OUTPUT
C
            WRITE(PLABEL,2300) LL
            IF (LL .EQ. 1) CALL COMPRS
            CALL PAGE(11,8.5)
            CALL NOBRDR
            CALL AREA2D(8,6)
            CALL XNAME('   FREQ (K) $',100)
            CALL YNAME('   MAGNITUDE $',100)
            CALL HEADIN(' DFT OUTPUT OF THE RECEIVED SIGNAL $',100,4,2)
            CALL HEADIN(PLABEL,100,3,2)
            CALL GRAF(WMIN,'SCALE',WMAX,RXKMIN,'SCALE',RXKMAX)
            CALL GRID(1,1)
            CALL SETCLR('MAGENTA')
            IF (KXX .LE. 100)   THEN
              CALL CURVE(KFREQ(IMIN),RXKM(IMIN),KXX,-1)
```

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```
                CALL VLINE(IMIN,IMAX,KFREQ,RXKM)
            ELSE
                CALL CURVE(KFREQ(IMIN),RXKM(IMIN),KXX,0)
            ENDIF
            CALL ENDGR(0)
            CALL ENDPL(0)
C
            CALL PAGE(11,8.5)
            CALL NOBRDR
            CALL AREA2D(8,6)
            CALL XNAME('  FREQ (K) $',100)
            CALL YNAME('  PHASE (DEG) $',100)
            CALL HEADIN(' DFT OUTPUT OF THE RECEIVED SIGNAL $',100,4,2)
            CALL HEADIN(PLABEL,100,3,2)
            CALL GRAF(WMIN,'SCALE',WMAX,-180.0,45.0,180.0)
            CALL GRID(1,1)
            CALL SETCLR('MAGENTA')
            IF (KXX .LE. 100)  THEN
                CALL CURVE(KFREQ(IMIN),RXKPD(IMIN),KXX,-1)
                CALL VLINE(IMIN,IMAX,KFREQ,RXKPD)
            ELSE
                CALL CURVE(KFREQ(IMIN),RXKPD(IMIN),KXX,0)
            ENDIF
            CALL ENDGR(0)
            CALL ENDPL(0)
  800   CONTINUE
        ENDIF
C
C   FORMATS
C
 1000 FORMAT(/,2X,
      % 'PLEASE ENTER THE INITIAL POSITION  X0,Y0,Z0 (FT) ',/,2X,
      % '  OF THE RECEIVER RELATIVE TO THE TRANSMITTER ...')
 1001 FORMAT(/,2X,
      % 'ENTER THE TRANSMITTER"S VELOCITY AVERAGE AND VARIANCE',/,2X,
      % '  IN THE X, Y, Z-DIRECTIONS (FT/SEC) AND (FT/SEC)**2   ...',/,2X,
      % '  (I.E. VXAVG, VXVAR, VYAVG, VYVAR, VZAVG, VZVAR) ')
 1002 FORMAT(/,2X,
      % 'ENTER THE AVERAGE SPEED OF SOUND IN FT/SEC ',/,2X,
      % '  AND THE VARIANCE IN (FT/SEC)**2   ...')
 1003 FORMAT(/,2X,
      % 'ENTER THE TRANSMITTER"S DOWN LINK TRANSMITTED ',/,2X,
      % '  HALF BEAM WIDTH ANGLE (DEG)   ...')
 1004 FORMAT(/,2X,'DO YOU WANT NORMALIZED AMPLITUDE ',
      % 'FOR THE RECEIVED SIGNAL?',/,2X,
      % '      PLEASE ENTER 1:YES OR 0:NO ...')
 1010 FORMAT(2X,'ENTER THE # OF PACKETS IN THE TRANSMITTED SIGNAL ...')
 1020 FORMAT(/,2X,
      % 'ENTER THE BAUD TYPE # CORRESPONDING TO THE FOLLOWING ')
 1021 FORMAT(2X,'  BAUD LENGTH FOR PACKET NUMBER: ',I4,' ...')
 1022 FORMAT(/,10X,' 1 : BAUD LENGTH (DELT) = 1/240 SECONDS ')
 1023 FORMAT(10X,' 2 : BAUD LENGTH (DELT) = 1/120 SECONDS ')
 1024 FORMAT(10X,' 3 : BAUD LENGTH (DELT) = 1/60 SECONDS ')
 1025 FORMAT(10X,' 4 : BAUD LENGTH (DELT) = 1/30 SECONDS ')
 1026 FORMAT(10X,' 5 : BAUD LENGTH (DELT) = 1/15 SECONDS ')
 1027 FORMAT(/,2X,
      % 'ENTER THE NUMBER OF BAUDS IN PACKET NUMBER: ',I4,' ...')
 1030 FORMAT(//,2X,'WOULD YOU LIKE THE DISCETE FOURIER TRANSFORM ',/,
      % 2X,'  OF THE OUTPUT SIGNAL ?  (ENTER  1:YES OR 0:NO) ... ')
 1040 FORMAT(/,2X,
      % 'WOULD YOU LIKE THE DFT OUTPUT WINDOWED ? ',/,2X,
      % '  ENTER  1:YES OR 0:NO ... ')
 1050 FORMAT(/,2X,
      % 'PLEASE ENTER THE DESIRED INPUT WIDE BAND ',/,2X,
      % '  SIGNAL-TO-NOISE RATIO IN DB ... ')
 1100 FORMAT(/,2X,
      % 'THIS PROGRAM ENCODES A QPSK MULTIFREQUENCY SIGNAL.',/,2X,
      % '  THE PHASES ARE SHOWN BELOW FOR ONE FREQUENCY ...')
 1101 FORMAT(//,
      % 6X,'                   ',' ',' ',' '            '              ',/,
      % 6X,'        *          ',' ',' ',' '        *     ',/,
```
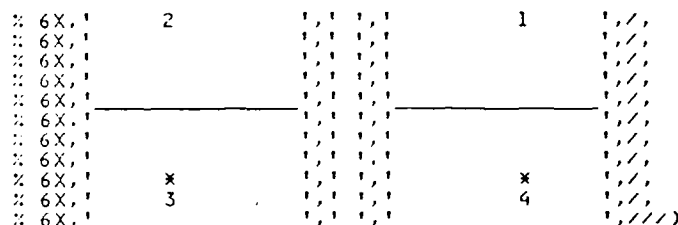
```
      % 6X,'         2            ',' ',' '         1        ',/,
      % 6X,'                      ',' ',' '                  ',/,
      % 6X,'                      ',' ',' '                  ',/,
      % 6X,'                      ',' ',' '                  ',/,
      % 6X,'---------------       ',' ',' '---------------   ',/,
      % 6X,'                      ',' ',' '                  ',/,
      % 6X,'                      ',' ',' '                  ',/,
      % 6X,'                      ',' ',' '                  ',/,
      % 6X,'                      ',' ',' '                  ',/,
      % 6X,'         *            ',' ',' '         *        ',/,
      % 6X,'         3            ',' ',' '         4        ',/,
      % 6X,'                      ',' ',' '                  ',/,///)
 1102 FORMAT(//,2X,
      % 'SELECT ONE OF THE FOLLOWING METHODS FOR ENCODING ',/,4X,
      % 'THE PHASES FOR EACH FREQUENCY FOR EVERY BAUD ',/,4X,
      % 'WITHIN EACH PACKET ...')
 1103 FORMAT(/,5X,'ENTER ...',/,7X,
      % '1: THE PROGRAM RANDOMLY SELECTS ALL THE PHASES ',/,7X,
      % '        FOR ALL ',I4,' PACKETS ',/,7X,
      % '2: YOU INDIVIDUALLY SELECT THE PHASES ')
 1110 FORMAT(//,2X,
      % 'SELECT ONE OF THE FOLLOWING METHODS FOR ENCODING ',/,4X,
      % 'THE PHASES FOR PACKET NUMBER: ',I4,' ...')
 1111 FORMAT(/,5X,'ENTER ...',/,7X,
      % '1: THE PROGRAM RANDOMLY SELECTS ALL THE PHASES ',/,7X,
      % '        FOR PACKET NUMBER: ',I4,/,7X,
      % '2: YOU INDIVIDUALLY SELECT THE PHASES ',/,7X,
      % '        FOR PACKET NUMBER: ',I4)
 1120 FORMAT(//,2X,
      % 'SELECT ONE OF THE FOLLOWING METHODS FOR ENCODING ',/,4X,
      % 'THE PHASES FOR PACKET NUMBER: ',I4,' BAUD NUMBER: ',I4,' ...')
 1121 FORMAT(/,5X,'ENTER ...',/,7X,
      % '1: THE PROGRAM RANDOMLY SELECTS ALL THE PHASES ',/,7X,
      % '        FOR PACKET NUMBER: ',I4,' BAUD NUMBER: ',I4,/,7X,
      % '2: YOU INDIVIDUALLY SELECT THE PHASES ',/,7X,
      % '        FOR PACKET NUMBER: ',I4,' BAUD NUMBER: ',I4)
 1130 FORMAT(//,2X,
      % 'PLEASE ENTER THE DESIRED QUADRANT FOR THE PHASE ',/,4X,
      % 'OF PACKET NUMBER: ',I4,' BAUD NUMBER: ',I4,/,4X,
      % 'FREQUENCY NUMBER (K): ',I4,' ...')
C
 2000 FORMAT(I8)
 2001 FORMAT(A38)
 2010 FORMAT(2X,I6,2X,I6,2X,F10.3,2X,I4)
 2020 FORMAT(//,2X,I8)
 2100 FORMAT(2X,I6,4X,F14.4)
 2200 FORMAT(//,2X,' DFT INPUT DATA FOR BAUD # ',I4,/,5X,
      % 'N',5X,'REAL PART',5X,'IMAG PART')
 2210 FORMAT(2X,I4,2X,E12.6,2X,E12.6)
 2220 FORMAT(//,2X,' DFT OUTPUT DATA FOR BAUD # ',I4,/,5X,
      %'K',7X,'REAL PART',7X,'IMAG PART',7X,'MAGNITUDE',16X,'PHASE',/,
      % 64X,'(DEG)',11X,'(RAD)')
 2230 FORMAT(2X,I4,2X,E14.8,2X,E14.8,2X,E14.8,4X,E14.8,2X,E14.8)
 2300 FORMAT(' FOR BAUD NUMBER ',I4,' $')
C
C    PLOTTING
C
          XXMIN = INT(XXMIN) - 0.2
          XXMAX = INT(XXMAX) + 0.2
          YYMIN = INT(YYMIN) - 0.2
          YYMAX = INT(YYMAX) + 0.2
          ZZMIN = INT(ZZMIN) - 0.2
          ZZMAX = INT(ZZMAX) + 0.2
          CCMIN = INT(CCMIN) - 1.0
          CCMAX = INT(CCMAX) + 1.0
          RRMIN = INT(RRMIN) - 1.0
          RRMAX = INT(RRMAX) + 1.0
          ALFMIN = INT(ALFMIN) - 0.2
          ALFMAX = INT(ALFMAX) + 0.2
          THIMIN = INT(THIMIN) - 0.4
          THIMAX = INT(THIMAX) + 0.4
C
      IF (IDFT .EQ. 0)  CALL COMPRS
```

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```
        CALL PAGE(11,8.5)
        CALL NOBRDR
        CALL AREA2D(8,6)
        CALL XNAME('  TIME (N) $',100)
        CALL YNAME('  MAGNITUDE $',100)
        CALL HEADIN(' RECEIVED SIGNAL $',100,3,1)
        CALL GRAF(0,'SCALE',NPTS,RXMIN,'SCALE',RXMAX)
        CALL GRID(1,1)
        CALL SETCLR('GREEN')
        CALL CURVE(TIME,YYRX,NPTS,0)
        CALL ENDGR(0)
        CALL ENDPL(0)
        IF (BDTOTL .GT. 1)  THEN
          CALL PAGE(11,8.5)
          CALL NOBRDR
          CALL AREA2D(8,6)
          CALL XNAME('  BAUD (LL) $',100)
          XXM = XXMAX - XXMIN
          IF (XXM .GE. 100.0)  THEN
            XXMIN = XXMIN / 1000.0
            XXMAX = XXMAX / 1000.0
            CALL YNAME('  X(LL) (KFT)      $',100)
            DO 900  I = 1, BDTOTL
              X(I) = X(I) / 1000.0
900         CONTINUE
          ELSE
            CALL YNAME('  X(LL) (FT)      $',100)
          ENDIF
          CALL HEADIN(' X-POSITION         $',100,3,1)
          CALL GRAF(1,'SCALE',BDTOTL,XXMIN,'SCALE',XXMAX)
          CALL GRID(1,1)
          CALL SETCLR('CYAN')
          CALL CURVE(LBAUD,X,BDTOTL,0)
          CALL ENDGR(0)
          CALL ENDPL(0)
          CALL PAGE(11,8.5)
          CALL NOBRDR
          CALL AREA2D(8,6)
          CALL XNAME('  BAUD (LL) $',100)
          YYM = YYMAX - YYMIN
          IF (YYM .GE. 100.0)  THEN
            YYMIN = YYMIN / 1000.0
            YYMAX = YYMAX / 1000.0
            CALL YNAME('  Y(LL) (KFT)      $',100)
            DO 905  I = 1, BDTOTL
              Y(I) = Y(I) / 1000.0
905         CONTINUE
          ELSE
            CALL YNAME('  Y(LL) (FT)      $',100)
          ENDIF
          CALL HEADIN(' Y-POSITION         $',100,3,1)
          CALL GRAF(1,'SCALE',BDTOTL,YYMIN,'SCALE',YYMAX)
          CALL GRID(1,1)
          CALL SETCLR('MAGENTA')
          CALL CURVE(LBAUD,Y,BDTOTL,0)
          CALL ENDGR(0)
          CALL ENDPL(0)
          CALL PAGE(11,8.5)
          CALL NOBRDR
          CALL AREA2D(8,6)
          CALL XNAME('  BAUD (LL) $',100)
          ZZM = ZZMAX - ZZMIN
          IF (ZZM .GE. 100.0)  THEN
            ZZMIN = ZZMIN / 1000.0
            ZZMAX = ZZMAX / 1000.0
            CALL YNAME('  Z(LL) (KFT)      $',100)
            DO 910  I = 1, BDTOTL
              Z(I) = Z(I) / 1000.0
910         CONTINUE
          ELSE
            CALL YNAME('  Z(LL) (FT)      $',100)
          ENDIF
```

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```
          CALL HEADIN(' Z-POSITION        $',100,3,1)
          CALL GRAF(1,'SCALE',BDTOTL,ZZMIN,'SCALE',ZZMAX)
          CALL GRID(1,1)
          CALL SETCLR('BLUE')
          CALL CURVE(LBAUD,Z,BDTOTL,0)
          CALL ENDGR(0)
          CALL ENDPL(0)
          CALL PAGE(11,8.5)
          CALL NOBRDR
          CALL AREA2D(8,6)
          CALL XNAME('  BAUD (LL) $',100)
          CCM = CCMAX - CCMIN
          IF (CCMAX .GE. 100.0)  THEN
            CCMIN = CCMIN / 1000.0
            CCMAX = CCMAX / 1000.0
            CALL YNAME('  C(LL) (KFT/SEC)     $',100)
            DO 915  I = 1, BDTOTL
              C(I) = C(I) / 1000.0
915         CONTINUE                       .
          ELSE
            CALL YNAME('  C(LL) (FT/SEC)     $',100)
          ENDIF
          CALL HEADIN(' SPEED OF SOUND  $',100,3,1)
          CALL GRAF(1,'SCALE',BDTOTL,CCMIN,'SCALE',CCMAX)
          CALL GRID(1,1)
          CALL SETCLR('RED')
          CALL CURVE(LBAUD,C,BDTOTL,0)
          CALL ENDGR(0)
          CALL ENDPL(0)
          CALL PAGE(11,8.5)
          CALL NOBRDR
          CALL AREA2D(8,6)
          CALL XNAME('  BAUD (LL) $',100)
          RRM = RRMAX - RRMIN
          IF (RRM .GE. 100.0)  THEN
            RRMIN = RRMIN / 1000.0
            RRMAX = RRMAX / 1000.0
            CALL YNAME('  R(LL) (KFT)     $',100)
            DO 920  I = 1, BDTOTL
              R(I) = R(I) / 1000.0
920         CONTINUE
          ELSE
            CALL YNAME('  R(LL) (FT)     $',100)
          ENDIF
          CALL HEADIN(' SLANT RANGE TO RECEIVER $',100,3,1)
          CALL GRAF(1,'SCALE',BDTOTL,RRMIN,'SCALE',RRMAX)
          CALL GRID(1,1)
          CALL SETCLR('GREEN')
          CALL CURVE(LBAUD,R,BDTOTL,0)
          CALL ENDGR(0)
          CALL ENDPL(0)
          CALL PAGE(11,8.5)
          CALL NOBRDR
          CALL AREA2D(8,6)
          CALL XNAME('  BAUD (LL) $',100)
          CALL YNAME('  ALPHA(LL)  $',100)
          CALL HEADIN('COMPRESSION FACTOR DUE TO THE MOVING TX $',100,3,1)
          CALL GRAF(1,'SCALE',BDTOTL,ALFMIN,'SCALE',ALFMAX)
          CALL GRID(1,1)
          CALL SETCLR('RED')
          CALL CURVE(LBAUD,ALPHA,BDTOTL,0)
          CALL ENDGR(0)
          CALL ENDPL(0)
          CALL PAGE(11,8.5)
          CALL NOBRDR
          CALL AREA2D(8,6)
          CALL XNAME('  BAUD (LL) $',100)
          CALL YNAME('  THETA(LL) (DEG) $',100)
          CALL HEADIN('ANGLE BETWEEN R(LL) AND Z0 $',100,3,1)
          CALL GRAF(1,'SCALE',BDTOTL,THTMIN,'SCALE',THTMAX)
          CALL GRID(1,1)
          CALL SETCLR('BLUE')
```

```
            CALL CURVE(LBAUD,THETAD,BDTOTL,0)
            CALL ENDGR(0)
            CALL ENDPL(0)
         ENDIF
         CALL DONEPL

 9999 STOP
      END
C
      SUBROUTINE GAUSS(DSEED,AVG,VAR,ZRND)
C
C    THIS SUBROUTINE GENERATES A GAUSSIAN RANDOM NUMBER WITH
C    MEAN = AVG AND VARIANCE = VAR
C
C       INPUTS:   DSEED = A DOUBLE PRECISION SEED THAT MUST BE A VARIABLE
C                 AVG   = THE MEAN OF THE GAUSSIAN RANDOM VARIABLE
C                 VAR   = THE VARIANCE OF THE GAUSSIAN RANDOM VARIABLE
C
C       OUTPUTS:  DSEED = THE SEED IS CHANGED DURING EXECUTION WHICH IS
C                         REQUIRED FOR THE NEXT SUBROUTINE CALL
C                 ZRND  = A GAUSSIAN RANDOM NUMBER WITH MEAN = AVG AND
C                         VARIANCE = VAR
C
      REAL AVG, VAR, ZRND
      DOUBLE PRECISION ZZ, DSEED, URND
      ZZ = 0.0
      DO 3000  I = 1, 12
         DSEED = DSEED + 13579345.1397537
         URND = GGUBFS(DSEED)
         ZZ = (URND - 0.5) + ZZ
 3000 CONTINUE
      ZRND = (ZZ * (VAR**0.5)) + AVG
      RETURN
      END
C
      SUBROUTINE PHASE(DSEED,RNDPHI,IPHI)
C
C    THIS SUBROUTINE SELECTS A PHASE RANDOMLY FROM QUADRANTS 1 TO 4
C
C       INPUTS:   DSEED = A DOUBLE PRECISION SEED THAT MUST BE A VARIABLE
C
C       OUTPUTS:  DSEED = THE SEED IS CHANGED DURING EXECUTION WHICH IS
C                         REQUIRED FOR THE NEXT SUBROUTINE CALL
C                 RNDPHI = A RANDOM PHASE FROM ONE OF THE FOUR QUADRANTS
C                          IN RADIANS
C                 IPHI = THE QUADRANT NUMBER OF THE RANDOM PHASE
C
      REAL PHIRND, RNDPHI
      DOUBLE PRECISION DSEED
      INTEGER IPHI
C
      PI = 4.0*ATAN(1.0)
      PHIRND = 0.0
      IPHI = 0
      PHIRND = GGUBFS(DSEED)
      DSEED = DSEED + 127.453
      PHIRND = (PHIRND * 4.0)
      IPHI = INT(PHIRND) + 1
      IF (IPHI .EQ. 1) RNDPHI = (45.0 * PI) / 180.0
      IF (IPHI .EQ. 2) RNDPHI = (135.0 * PI) / 180.0
      IF (IPHI .EQ. 3) RNDPHI = (-135.0 * PI) / 180.0
      IF (IPHI .EQ. 4) RNDPHI = (-45.0 * PI) / 180.0
      RETURN
      END
C
      SUBROUTINE DFT(NM1,XIN,XOUT)
C
C    THIS SUBROUTINE COMPUTES THE DISCRETE FOURIER TRANSFORM OF
C    A COMPLEX DATA SET OF N (= NM1+1) POINTS STORED IN THE ARRAY XIN
C    THE RESULT IS STORED IN THE COMPLEX ARRAY XOUT
C
      COMPLEX XIN(0:NM1), XOUT(0:NM1), W, WM
```

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```fortran
          N = NM1 + 1
          PI = 4.0*ATAN(1.0)
          EN = N
          NM2 = NM1 - 1
          IF (NM1 .EQ. 0)  THEN
            XOUT(0) = XIN(0)
          ELSE
            BETA = 2.0 * PI / EN
            W = CMPLX(COS(BETA), -SIN(BETA))
            DO 3110  K = 0, NM1
              WM = W**K
              XOUT(K) = XIN(NM1)
              DO 3100  L = NM2, 0, -1
                XOUT(K) = XOUT(K)*WM + XIN(L)
 3100       CONTINUE
 3110     CONTINUE
C
          ENDIF
          RETURN
          END
C
          SUBROUTINE VLINE(J1,JN,XARRAY,B)
C
          REAL XARRAY(0:4096), B(0:4096)
C
C     DRAW VERTICAL LINES
C
          DO 3200  JLINE = J1, JN
            XFROM = XARRAY(JLINE)
            XTO = XFROM
            YFROM = 0.0
            YTO = B(JLINE)
            CALL RLVEC(XFROM,YFROM,XTO,YTO,0000)
 3200   CONTINUE
          RETURN
          END
```

LIST OF REFERENCES

1. Paul H. Moose, "Theory of Multi-Frequency Modulation (MFM) Digital Communications", Technical Report No. NPS 62-89-019, Naval Postgraduate School, Monterey, California, May 1989

2. Leon W. Couch II, *Digital and Analog Communication Systems,* 2nd Edition, Macmillian Publishing Company, New York, 1987

3. Robert D. Childs, "High Speed Output Interface for a Multifrequency Quaternary Phase Shift Keyed Signal Generated on an Industry Standard Computer", MSEE Thesis, Naval Postgraduate School, Monterey, California, December 1988

4. Robert J. Urich, *Principles of Underwater Sound,* 3rd Edition, McGraw Hill Publishing Company, New York, 1983

5. Marvin K. Simon and Dariush Divsalar, "Doppler-Corrected Differential Detection of MPSK", *IEEE Transactions of Communications,* Vol. 37, No. 2, Feb 1989, pp. 99-109

6. Terry K. Gantenbein, "Implementation of Multi-Frequency Modulation on an Industry Standard Computer", MSEE Thesis, Naval Postgraduate School, Monterey, California, September 1989.

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