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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A PROPOSED ANALYTIC FILTER MODEL FOR USE IN FAADS FORCE MIX ANALYSIS AT THE U. S. ARMY AIR DEFENSE CENTER

by

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September 1989

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Unclassified
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No 0704-0188

1a REPORT SECURITY CLASSIFICATION Unclassified
1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY
2b DECLASSIFICATION/DOWNGRADING SCHEDULE
3 DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; Distribution is unlimited
4 PERFORMING ORGANIZATION REPORT NUMBER(S)
5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION Naval Postgraduate School
6b OFFICE SYMBOL (If applicable) 55
7a NAME OF MONITORING ORGANIZATION Naval Postgraduate School
6c ADDRESS (City, State, and ZIP Code) Monterey, California 93943-5000
7b ADDRESS (City, State, and ZIP Code) Monterey, California 93943-5000
8a NAME OF FUNDING/SPONSORING ORGANIZATION
8b OFFICE SYMBOL (If applicable)
9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c ADDRESS (City, State, and ZIP Code)
10 SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO	PROJECT NO	TASK NO	WORK UNIT ACCESSION NO

11 TITLE (Include Security Classification) A PROPOSED ANALYTIC FILTER MODEL FOR USE IN FAADS FORCE MIX ANALYSIS AT THE U.S. ARMY AIR DEFENSE CENTER
12 PERSONAL AUTHOR(S) COLE, Stephen A.
13a TYPE OF REPORT Master's thesis
13b TIME COVERED FROM ________ TO ______
14 DATE OF REPORT (Year, Month, Day) 1989 September
15 PAGE COUNT 85
16 SUPPLEMENTARY NOTATION The views expressed in this thesis are those of the author and do not reflect the official policy or positon of the Department of Defense or the U.S. Government

17 COSATI CODES

FIELD	GROUP	SUB-GROUP

18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number) FAADS, CARMO-FAAD, ANALYTIC FILTER MODEL, AIR DEFENSE,

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

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20 DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS
21 ABSTRACT SECURITY CLASSIFICATION Unclassified
22a NAME OF RESPONSIBLE INDIVIDUAL Samuel H. Parry
22b TELEPHONE (Include Area Code) 408-646-2779
22c OFFICE SYMBOL 55py

DD Form 1473, JUN 86
Previous editions are obsolete
SECURITY CLASSIFICATION OF THIS PAGE Unclassified
S/N 0102-LF-014-6603

A Proposed Analytic Filter Model for use in FAADS Force Mix Analysis at the U.S. Army Air Defense Center

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September 1989

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ABSTRACT

This thesis proposes an analytic filter model to support the Forward Area Air Defense System (FAADS) force mix analysis studies ongoing at the U.S. Army Air Defense Center. The FAADS Force Mix Analysis Model (FFMAM) focuses on the air defense versus combat aviation battle in the maneuver brigade's forward and rear areas. Particular attention is given to the representation of the FAADS attrition cycle as a Semi-Markov renewal process. Additional emphasis is placed on combat aviation tactics and the impact of terrain on system employment. Model output is presented to demonstrate FFMAM's reaction to selected input changes and to identify attrition related trends.

THESIS DISCLAIMER

The reader is cautioned that the computer program developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the program is free of computational and logic errors, it cannot be considered validated. Any application of this program without additional verification is at the risk of the user.

TABLE OF CONTENTS

I. INTRODUCTION 1

A. BACKGROUND 1

B. OVERVIEW OF CARMO-FAAD 3

C. RESEARCH OBJECTIVE 7

II. ATTRITION RATE COEFFICIENT METHODOLOGIES 9

A. GENERAL 9

B. KEY PARAMETERS USED IN ATTRITION RATE COEFFICIENT DEVELOPMENT 12

1. Example 1: Key Parameters Equation . . . 14

C. BONDER/FARRELL APPROACH 16

D. ALGEBRAIC METHOD FOR CALCULATING EXPECTED TIMES TO ATTRITION FOR A SEMI-MARKOV PROCESS 19

1. Example 2: Bonder/Farrell Fire Dependent Model 22

E. SUMMARY 24

III. PROPOSED FAADS ATTRITION RATE COEFFICIENT ESTIMATION METHODOLOGY 25

A. GENERAL 25

B. STATE SPACE DEFINITION FOR A FAAD WEAPON SYSTEM 26

1. Example 3. Approximation for **PRL** 32

C. SUMMARY 36

IV. FAADS FORCE MIX ANALYSIS MODEL (FFMAM) 37

A. GENERAL 37

B. BASIC MODEL 37

1. Model description 37

2. Model resolution 38

3. User interface 39

4. Attrition 40

5. FFMAM input 47

C. SUMMARY 49

V. FFMAM OUTPUT AND ANALYSIS 50

A. GENERAL 50

B. SCENARIO 50

C. DEMONSTRATION OF MODEL TRENDS 51

D. SUMMARY 63

VI. FUTURE ENHANCEMENTS 65

A. SUMMARY 65

B. RECOMMENDATIONS 65

APPENDIX A. MODEL VARIABLES 67

APPENDIX B. THE FAADS FORCE MIX ANALYSIS MODEL . . . 71

APPENDIX C. SCENARIO INPUTS 74

LIST OF REFERENCES 77

INITIAL DISTRIBUTION LIST 78

I. INTRODUCTION

A. BACKGROUND

The high cost of U.S. Army force modernization demands a critical look at all phases of the procurement process. In the case of weapon system acquisition, projected force structure is especially important due to the escalating costs of individual fire units and their logistical and training bases. The battlefield of the future presents numerous technological and tactical advances that have driven the development and fielding of numerous systems currently in the Army inventory. Many other systems, still in the test and evaluations phase, require detailed analysis to determine the "optimal" force structure to accomplish their projected mission(s). The Army's Forward Area Air Defense System (FAADS) is one such system.

The FAAD system concept was developed to counter the air threat to divisional ground maneuver forces into and beyond the year 2000. The FAAD system consists of three separate weapon systems, each with specific capabilities designed to counter the threat to their respective areas on the battlefield. Each of the three types of fire units comprising the FAAD system are described as follows:

Line-of-Sight-Forward-Heavy (LOS-F-H):

A gun/missile mix system with 8 ready-to-fire missiles mounted on a tracked vehicle. Upon acquisition, the gunner activates a laser beam guidance system which is designed to track/engage fixed and rotary wing targets in the brigade forward area.

Line-of-Sight-Rear (LOS-R):

A stinger based missile system with eight read-to-fire missiles and a gun surrogate mounted on a high mobility multipurpose wheeled vehicle. Employs direct view optics, a forward looking infrared device (FLIR), and a laser range finder. Primary system deployment is in the brigade and division rear areas.

Non-Line-of-Sight (NLOS):

Track mounted system deployed in defilade near the forward line of troops where it can be masked from detection. Its primary munition is a fiber optic guided missile system which uses an image seeker for target acquisition. Often referred to as a "television guided" munition. Primary targets include attack helicopters in defilade positions. Does not engage fixed wing aircraft.

The current baseline force mix for the divisional FAAD battalion consists of 36 LOS-F-H, 18 NLOS, and 36 LOS-R. While prototype testing for each of the FAAD weapon systems is complete, the problem of the "optimal" force mix structure of the FAAD battalion remains unanswered. On 3 Sep 1988, the Defense Acquisition Board issued an Acquisition Defense Memorandum outlining the requirement for detailed force mix analysis of the FAAD system. On 28 Sep 1988, the Directorate of Concepts and Studies, U.S. Army Air Defense School, released an update study plan for the conduct of its FAADS force mix analysis. The study plan calls for a myriad of simulation and non-simulation analyses to supplement follow-on cost effectiveness studies. The plan identifies a total of 36 FAADS force mix alternatives for consideration. The initial objective is to screen each of the 36 alternatives and through various analytical methodologies, filter out those that are least effective. Analysis of the remaining alternatives, ideally less than five, would be accomplished through the application of high resolution simulation, such as CASTFOREM and JANUS, of tactical scenarios and low resolution modeling of logistical scenarios such as VIC. [Ref. 1]

Initial simulation screening of the FAADS force mix alternatives was to be accomplished through use of a low resolution combat model called CARMO-FAAD.

B. OVERVIEW OF CARMO-FAAD

CARMO-FAAD is an adaptation of the Combined Arms Model (CARMO) developed by CACI; Inc.. It was developed for use as a decision support tool by the U.S. Army Training and Doctrine Command (TRADOC) and the U.S. Army Air Defense Artillery Center employing analytic simulations to study the effects of specific changes in FAAD system characteristics and war fighting capability. CARMO-FAAD is an aggregated deterministic force-on-force model written in LOTUS 123 requiring 640K RAM. Its structure is a heterogeneous, time-stepped simulation of combined arms engagement between two opposing forces through the use of a system of difference equation to calculate losses by weapon system type. [Ref. 2]

As currently configured, CARMO-FAAD inadequately represents numerous aspects of modern combat. Given its original purpose as a low resolution analytic decision aid, CARMO-FAAD nevertheless fails as a credible model of the dynamics of air-to-ground and ground-to-air tactical engagements. Despite numerous equations to calculate probabilities of line of sight, acquisition, and subsequent attrition, CARMO-FAAD lacks algorithms to accommodate any tactic beyond a linear battle in which a static defending force is closed upon by an attacking force at constant rate. Such a scenario might be feasible for two dimensional ground force battles, but when the third dimension of aviation is added it is hardly acceptable. Although an in depth review of CARMO-FAAD would seem in order, it is not the focus of this research effort. It is, however, necessary to point out a few of CARMO-FAAD's significant shortcomings as a FAAD analytic filter model.

Several problems with CARMO-FAAD stem from the "tactical perspective. The most notable of these is the static defender. Combat aviation's most important characteristic as a weapon system is its mobility, yet CARMO-FAAD models attack helicopters in the defense as stationary systems. Furthermore, in the offense, they are mobile but close on the static defender at a constant rate with a constant level of exposure. This representation eliminates the mobility advantage of combat aviation while subjecting it to the same rules as the ground systems fighting the two dimensional battle.

Another problem with CARMO-FAAD is the fixed rate of movement of air defense systems in the attack. CARMO-FAAD does not allow for static overwatch attacking weapon systems. Detailed terrain analysis will often identify key enemy air avenues of approach and in turn "optimal" air defense positioning to protect freedom of maneuver from the air threat.

A third consideration not adequately modeled in CARMO-FAAD is the impact of terrain on the three dimensional battle. CARMO-FAAD relies on a constant value for "terrain range" which applies to all systems on both sides for the duration of the battle. The value represents the terrain's "openness" and ultimately impacts on line-of-sight calculations. Again, the dynamics of the three dimensional battle would certainly demand that "terrain range" be a variable, or a set of variables, not a single fixed value for each model run.

Each of the aforementioned shortcomings are potentially correctable through embellishment of CARMO-FAAD. Having worked exclusively with CARMO-FAAD for approximately six weeks, the author gained an intimate appreciation of another model shortcoming: CARMO-FAAD is extremely cumbersome to embellish. Keeping an audit trail on the impact of individual changes on program algorithms proved to be tedious and very time consuming. Relating input and output files proved a problem when attempting to provide updated input with each time-step or when consolidating output. Additionally, limited availability of memory quickly placed an upper bound on embellishments.

Finally, due to the structure of the program, the analyst will not be able to model more than one attacking or defending force per model run. For example, a brigade in the offense is modeled either as a completely aggregated force (with up to eight system types modeled heterogeneously), or as two or more battalion sized forces. When attempting to set up the model to fight the brigade in the offense scenario designated by the FAADS force mix study, a minimum of four runs were required to simulate the battle (including the brigade rear area battle). When attempting to screen 36 alternative force mixes for the brigade in the offense, one quickly realizes the cumbersome nature of the model and Lotus 123 as the programming language.

In summary, CARMO-FAAD has been identified as the low resolution analytic decision aid to act as a filter model in support of the ongoing FAADS force mix analysis. However, CARMO-FAAD does not adequately simulate the forward area air defense versus combat aviation battle. CARMO-FAAD could be embellished to add some credibility as a FAAD model, but the time and effort involved in such an endeavor would outweigh the limited utility of those embellishments that are possible. The best solution to the CARMO-FAAD problem is to create a low resolution model that better simulates the dynamics of the three dimensional battle in the forward area.

C. RESEARCH OBJECTIVE

The objective of this research is to initiate the development of an alternative model for use as a decision aid in the conduct of FAADS force mix analysis. The proposed model is a low resolution, heterogeneous, lanchester time-stepped simulation of the air defense versus combat aviation engagements in the division forward area. The model will be structured to simulate a brigade area of operations, to include the brigade rear area.

The model emphasizes the impact of terrain masking and combat aviation mobility on the air defense battle. Detailed discussion of model assumptions, structure, and determination of input is presented in Chapter IV. Similarly, in an effort to provide a more accurate representation of FAAD weapon system capabilities, a methodology for the estimation of FAAD attrition rate coefficients is presented in Chapter III. A basis for the development of the FAAD attrition rate methodology is provided in Chapter II.

Finally, due to the non-availability of classified performance data for FAAD and projected threat systems, input values used to demonstrate the model do not represent actual or projected system capabilities. However, a concerted effort was made to provide credible values for input parameters. Model output for selected FAADS force mix alternatives and changes of input parameters is presented in Chapter V.

II. ATTRITION RATE COEFFICIENT METHODOLOGIES

A. GENERAL

Lanchester-type attrition models consist of a set of differential equations that describe the combat process. Given an aggregated force of size **x**, an opposing aggregated force of size **y**, and the initial conditions of modern warfare (aimed fire), Lanchester describes the attrition rate of **x** as a function of how many **y**'s are shooting at him. That is:

$$dx/dt = -ay \qquad (1)$$

where **a** is the attrition rate coefficient and **t** is time. When breaking down the **x** and **y** forces and desegregating them by weapon system type, equation (1) becomes the heterogeneous Lanchester equation given by equation (2).

$$dx_j/dt = -a_{ij}y_i \qquad (2)$$

The **y** firer is of weapon system type i, the **x** target is of weapon system type j and the attrition rate coefficient $\mathbf{a}_{ij}$ is for firer i against target j. The challenge to the combat modeler is how to determine the attrition rate coefficient that best suits his model.

A key element in Lanchester type attrition models is time. When conducting analytical studies such as the FAADS force mix analysis, the ability to assess weapon system effectiveness at different points in the battle is essential. Accordingly, the number and size of the time steps in which an aggregated model allows opposing forces to attrite each other is critical. Models in which entire battles are fought in one time step certainly provide less information than those that fight the same battle in 30 time steps. The increased resolution of information derived from the multiple time step approach enables the analyst to more accurately simulate the capabilities and/or limitations of the weapon systems to be modeled.

Because the rate at which opposing systems attrite each other ultimately determines battle duration, it is important that attrition rate coefficients be as realistic as possible. Inaccurate coefficients can result in biased output which leads to faulty analysis. This is especially true for analytic studies with measures of effectiveness (MOE) that are based on killer-victim scoreboards. Indeed, every MOE listed for the FAADS force mix analysis in FAADS Update Study Plan is attrition based (Ref. 2;p. 17]. Two methodologies are currently in use for the estimation of attrition rate coefficients. As described in Hartman:

> The COMAN approach, developed by G. Clark,is a fitted parameter model which takes a time series of casualty times and computes the maximum likelihood estimates of the mean time between casualties. The Bonder/Farrell technique is used in independent analytical models (they do not depend on outside models for input). In this methodology, a stochastic process model of a single $\mathbf{Y}_j$ firing at type $\mathbf{X}_i$ targets is built and then $\mathbf{E}[\mathbf{T}_{ij}]$ values are determined. [Ref. 3:p. 49]

Both COMAN and Bonder/Farrell assume that a Lanchester attrition process is occurring. Because COMAN typically obtains its data from high resolution, small unit combat models, it assumptions are whatever are implicit in the data source model(s). This creates an inherent problem in that the data source models usually contain numerous complex assumptions which are not readily apparent in the output. Conversely, Bonder/Farrell takes on whatever explicit assumptions are made in the i-j independent engagement model. Consequently, when comparing the two methodologies:

> ...we see that the Bonder technique is generally more restrictive since the in-depth engagement is analytic and in turn suppresses detail. The assumptions are explicit and up-front which makes it easy to criticize and finally, there typically is no possibility for synergystic effects to occur in the Bonder approach. [Ref. 3:p. 49]

Both COMAN and Bonder/Farrell are data intensive. While Bonder/Farrell requires extensive engineering data one each weapon system modeled, COMAN relies on large libraries of $\mathbf{a}_{ij}$'s and selects the particular value that corresponds to the current situation. Consequently, the hardest thing in the COMAN methodology is to be sure that scenarios are consistent between data source models and the current aggregated model being used.

Considering the need to utilize the first principles approach and eliminate the possibility of synergystic effects in the development of the proposed FAADS force mix filter model, the attrition estimation technique of choice is Bonder/Farrell. Using the basic principles underlying Bonder/Farrell, it is possible to develop an analytic model "tailor made" for the task at hand. The remainder of this chapter will focus on parameters key to the attrition process, a discussion of the basics of Bonder/Farrell technique, and an algebraic method to solve the inherent stochastic process.

B. KEY PARAMETERS USED IN ATTRITION RATE COEFFICIENT DEVELOPMENT

The ability of weapon system type i to attrite weapon system type j is a function of numerous parameters. Both high and low resolution small unit combat models tend to emphasize five conditions.

1) probability of target acquisition
2) probability of hitting a target
3) probability of killing a target given a hit
4) weapon system rate of fire

5) allocation of weapon fires

The current range to target can have a direct impact on these parameters depending on explicit model assumptions and model structure. A functional form containing all five parameters for a heterogeneous Lanchester attrition model might be:

$$A_{ij} = \alpha_{ij} \times \Psi_{ij} \times P_{ij} \times \nu_i \times (1 - R/MAXR_i)^{\beta i} \qquad (3)$$

where

A_{ij} = rate at which weapon system type i attrites targets of type j

α_{ij} = probability that system type i acquires target type j during the current time-step

Ψ_{ij} = percent of system type i fires allocated to target type j

P_{ij} = probability of i killing j given a single round hit

ν_i = rate of fire for weapon system i

R = current range between opposing systems

$MAXR_i$ = maximum effective range of weapon system type i

β_i = single round accuracy parameter of weapon system i

and the value $(1 - R/MAXR_i)^{\beta i}$ equates to the probability of hitting a target over the effective range of the firing system. In the more realistic models the probability of acquisition (α_{ij}) is also a function of range to target and the effective range of a firer's acquisition system. The following example reflects typical data entries used in the generation of attrition rate coefficients using equation (3).

1. Example 1: Key Parameters Equation

An MX tank with a main gun maximum effective range of 3km has a 75% probability of acquiring targets over a one minute time interval at a range of 2km. The MX fires at a rate of 6 rounds per minute. Priority of fires require that 60% of MX fires be targeted against enemy tanks. The weapon accuracy parameter for the MX is 0.3. The probability of killing an enemy tank given a hit is 0.89. The attrition rate coefficient then becomes:

$$A_{ij} = (.75)x(.60)x(.89)x(6)x(1 - 2/3)^{.3} = 1.73$$

This implies that one MX will kill 1.73 enemy tanks per minute at a range of 2km. The reader should note that as the range to target decreases, the attrition rate increases. When the MX system is modeled heterogeously per equation (2), the number of enemy tanks attrited in a single 1 minute time-step becomes a linear function of the number of MX tanks. For example, 10 MX tanks would destroy 17.3 enemy tanks at 2 km in one minute. Although such success on the modern battlefield is certainly desireable, it is unlikely to occur at the pace derived using equation (3).

An argument for the use of equation (3) might be that it is applied to all systems on both sides of the battlefield and therefore provides no advantage to either force; it simply expedites the pace of battle. This is faulty logic for a number of reasons, the primary one being terrain. Use of equation (3) assumes that a system will be able to engage targets inside its maximum effective range. This gives a distinct advantage to a tank with a 3km maximum range versus one that has a 2.5 km range. The results of such representation would have tacticians ever increasing the range of their combat systems (oddly enough, they are). the tank commander on the ground quickly realizes the chance of engaging a target at maximum effective range is virtually nonexistent due to terrain masking. Terrain has a direct impact on target acquisition and target exposure. There is some validity to the argument that hilly terrain with dense foliage does more to defeat the advanced capabilities of modern combat systems than does the opposing force.

Another problem with equation (3) is the assessment of system rates of fire. In ideal conditions with multiple targets and a highly efficient crew, a tank might get off 6 rounds in 1 minute. Engineers would argue that indeed it is possible, but in the "fog of war" the time required to acquire may alone be in excess of 1 minute.

Despite accounting for the combined effects of those key parameters, equation (3) will always produce an inflated attrition rate. When employed in a model that does not make specific adjustments for terrain, it is biased towards weapon systems with greater maximum effective ranges. But the major problem with equation (3) is its failure to adequately represent the dimension of time and its ultimate impact on the "rate" of attrition.

C. BONDER/FARRELL APPROACH

Another approach to generating attrition rate coefficients is to calculate the reciprocal of the expected time between target kills given by equation (4).

$$a_{ij} = 1/E[T_{ij}] \tag{4}$$

where $E[T_{ij}]$ is the expected time required for weapon system i to kill target system j. Bonder/Farrell developed a Markov dependent fire model which represents the time between kills as a Semi-Markov renewal process. By splitting the renewal state into an initial state and an absorbing state, the Bonder/Farrell approach can be represented as a Semi-Markov process with an absorbing state as depicted in Figure 1. It is important to note that the Bonder/Farrell approach accounts for various activities inherent to the attrition process. In doing so, activity time requirements are factored into attrition rate determination.

The four states shown in the model are defined as follows:

State 0 = new engagement; First round about to be fired.

State 1 = hit: previous round resulted in a sensed hit.

State 2 = miss: previous round resulted in a sensed miss.

State 3 = kill: previous round resulted in a kill.

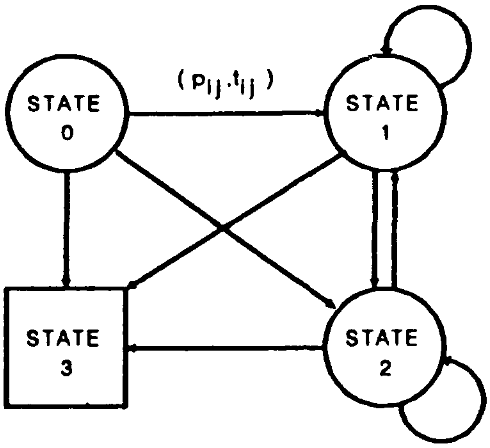

Figure 1. Bonder/Farrell Dependent Fire Model

Transition probabilities between states are a function of the following discrete probabilities:

P_h = probability of first round hit

$P_{k|h}$ = probability of a kill given a hit

$P_{h|h}$ = probability of a hit on the current shot given a hit on the previous shot

$P_{h|m}$ = probability of a hit on the current shot given a miss on the previous shot

$P_{m|m}$ = probability of a miss on the current shot given a miss on the previous shot

Assuming critical event times are deterministic, the transition time between states is the summation of two or more of the following event times:

t_a = time to acquire a target from a battle ready position

t_e = time to fire the first round after target acquisition

t_f = round time of flight

$t_{h|h}$ = time to fire a round following a hit

$t_{m|m}$ = time to fire a round following a miss

For the purpose of this model, State 0 is always the initial state with State 3 the absorbing state. The expected time between target kills $E[T_{ij}]$ is expressed stochastically as the expected time to absorption, that is the expected time to reach State 3 given the process starts in State 0. The reader should note that as defined the Bonder/Farrell approach accounts for four of the five attrition parameters identified in the previous section. The probability of acquisition is used to derive an expected time to acquisition t_a; transitions between states are a function of P(hit) and P(kill|hit); and the rate of fire is determined by the number of transitions between states prior to absorption. The resulting $E[T_{ij}]$ need only be multiplied by the percent of i fires allocated to target j to account for all five parameters. A method for solving for the expected time to absorption is described in

Section D of this chapter, as is an example using the Bonder/Farrell model shown above.

The Bonder/Farrell fire dependent model makes several assumption about the attrition process. The first assumption is that the probability of a hit after the first round is dependent on the outcome of the previous shot. The second assumption is that more than one hit may be required to kill a target. These assumptions reflect a "Shoot-Look-Shoot" firing doctrine. Although these assumptions are valid, they are by no means generic to every weapon system on the battlefield. However, the flexibility of the Semi-Markov process allows the analyst to modify the Bonder/Farrell approach to accommodate any firing doctrine currently in use. Modification of the Bonder/Farrell approach is the basis for the FAAD attrition model presented in Chapter III.

D. ALGEBRAIC METHOD FOR CALCULATING EXPECTED TIMES TO ATTRITION FOR A SEMI-MARKOV PROCESS

The remainder of this chapter will focus on solving for the expected time to absorption (attrition) for a Semi-Markov process. One approach to finding the expected time to absorption of a Semi-Markov process is through the application of matrix algebra as presented by Taylor.

> Consider a Markov chain whose states are labeled $0,1,\ldots,N$. States $0,1,\ldots,r-1$ are transient in that $p_{ij}^{(n)} \to 0$ as $n \to \infty$ for $0 \le i, j < r$, while states $r,\ldots,N$ are absorbing, or trap, and here $p_{ii} = 1$ for $r \le i \le N$. The transition matrix has the form

$$\mathbf{P} = \begin{bmatrix} \mathbf{Q} & \mathbf{R} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{5}$$

where $\mathbf{0}$ is an (N-r+1) matrix all of whose components are zero, $\mathbf{I}$ is an (N-r+1)x(N-r+1) identity matrix and $q_{ij} = p_{ij}$ for $0<i$, $j<r$. [Ref. 4;p. 116]

The matrix $\mathbf{Q}$ is an (r-1)x(r-1) transition matrix whose entries q_{ij} are the transition probabilities from transient state i to transient state j. The matrix $\mathbf{R}$ is an (r-1)x(N-r+1) transition matrix whose entries r_{ij} are the transition probabilities from transient state i to absorbing state j. An intermediate matrix $\mathbf{W}$ can then be defined by:

$$\mathbf{W} = (\mathbf{I}-\mathbf{Q})^{-1} \tag{6}$$

where $\mathbf{I}$ is an (r-1)x(r-1) identity matrix with the same dimension as $\mathbf{Q}$. The matrix $\mathbf{W}$ is known as the fundamental matrix and can be used to determine the expected number of visits to a state prior to absorption. The fundamental entry w_{ij} is the expected number of visits to state j prior to absorption given an initial state i.

The probability of absorption in each of the absorbing states can be obtained from:

$$\mathbf{U} = \mathbf{W}\mathbf{R} \tag{7}$$

where **U** is an (r-1)x(N-r+1) matrix whose entries u_{ij} are the probabilities of absorption in state j given the process started at state i.

Having determined the expected number of visits to each state prior to absorption and the probability of absorption for each absorbing state, the next step is to determine the expected time to absorption. It is first necessary to construct **T** which is an (r-1)x(N) matrix of transition times whose individual entries, t_{ij}, represent the transition times from state i to state j. The next step is to calculate the mean sojourn time, μ_i, in state i prior to transition to the next state, given by:

$$\mu_i = \sum_{j=0}^{N} (p_{ij} \times t_{ij}) \tag{8}$$

where p_{ij} are values from the transition probability matrix P. The expected time to absorption can then be obtained from:

$$\mathbf{T}_{abs} = \mathbf{W}\boldsymbol{\mu} \tag{9}$$

where $\boldsymbol{\mu}$ is a column vector of the expected sojourn times for each transient state, **W** is the fundamental matrix, and $\mathbf{T}_{abs}$ is a vector of the expected times to absorption in state i given the initial state j. The following example applies the algebraic method to the Bonder/Farrell approach for calculating attrition rate coefficients.

1. Example 2: Bonder/Farrell Fire Dependent Model

Given the conditions outlined in example 1 and adjusting for the conditional probabilities intrinsic to the Bonder/Farrell fire dependent model, the resulting P matrix might be:

$$\mathbf{P} = \begin{bmatrix} 0 & .08 & .28 & .64 \\ 0 & .09 & .19 & .72 \\ 0 & .08 & .26 & .66 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

with the resulting **Q** and **R** matrices:

$$\mathbf{Q} = \begin{bmatrix} 0 & .08 & .29 \\ 0 & .09 & .19 \\ 0 & .08 & .28 \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} .64 \\ .72 \\ .66 \end{bmatrix}$$

Given the following transition times matrix T (in seconds):

$$\mathbf{T} = \begin{bmatrix} 0 & 37 & 37 & 37 \\ 0 & 12 & 12 & 12 \\ 0 & 17 & 17 & 17 \end{bmatrix}$$

equation (6) yields the fundamental matrix:

$$\mathbf{W} = \begin{bmatrix} 1 & .12 & .41 \\ 0 & 1.12 & .29 \\ 0 & .12 & 1.38 \end{bmatrix}$$

Because there is only one absorbing (tactical kill) state, we know the probability of absorption is 1. Equation(8) yields the following vector of mean sojourn times:

$$\mu = [\ 37\ 12\ 17] \qquad \text{(transposed)}$$

The expected time to absorption can then be obtained using equation (9):

$$\mathbf{T}_{abs} = [45.5 \quad 18.4 \quad 25] \qquad \text{(transposed)}$$

Assuming the attrition process always starts in state 0 (new engagement), the resulting attrition rate coefficient would be:

$$a_{ij}=1/E[T_{abs(1,4)}]=(1/45.5\text{ sec.})\text{x}(60\text{ sec./min.})=1.32\text{ per min.}$$

When accounting for allocation fires the revised attrition coefficient is:

$$A_{ij} = \Psi_{ij}\ \text{x}\ a_{ij} = (.60)\text{x}(1.32) = .79$$

By making a more formal accounting of the time required to execute specific events, Bonder/Farrell slows the attrition process significantly over the key parameters of equation (3).

E. SUMMARY

The attrition process is an integral part of combat modeling. For heterogeneous Lanchester type attrition models, the ability of one system to attrite another is a function of several key parameters. Because the values of the key parameters are fixed for each model time-step, selection of the appropriate time-step size is important because the values of various key parameters change with the flow of the battle. Similarly, a time-step size that allows for few iterations prior to battle capitulation results in attrition data which may prove useless to the analyst.

The rate at which systems attrite is the inverse of the time between kills, therefore accurate determination of the time between kills is essential. For analytic type models, the attrition rate estimation methodology of choice is the Bonder/Farrell approach. The Bonder/Farrell approach utilizes a Semi-Markov renewal process to determine the time between kills. The process can be modified to represent a myriad of weapon system types and their associated firing doctrines. The process is modified in Chapter III to obtain FAADS attrition rates.

III. PROPOSED FAADS ATTRITION RATE COEFFICIENT ESTIMATION METHODOLOGY

A. GENERAL

The Bonder/Farrell approach showed that the engagement process can be described as a set of specific states and in turn modeled as a Semi-Markov process. It follows that a more "realistic" expected time to kill would be obtained if the series of events leading up to and including Bonder/Farrell's engagement process were modeled as a Semi-Markov process.

A basic "combat" attrition process can be defined for all weapon systems on the modern battlefield. This basic process is comprised of several specific events such as target search, target acquisition, target engagement, weapon system repositioning, weapon system reload, etc.. Each of these activities vary in the amount of time required to completion and the probability of inception and/or successful completion. This variation is often a function of weapon system characteristics as in reload time or munition flight time. However, variation may also be caused by other independent variables such as target type, range to target, or target exposure. This section will model the "combat attrition process for the LOS-F-H and the LOS-R FAAD weapon systems and present a methodology for incorporating key independent variables into the attrition process.

B. STATE SPACE DEFINITION FOR A FAAD WEAPON SYSTEM

Whether in a static defensive position providing air defense for a fixed asset or on the move in support of a maneuver task force, a FAAD weapon system's "combat" attrition process can be described as a collection of distinct events. Accordingly, these events can be represented as a Semi-Markov process. Several assumptions were made in conjunction with the FAADS attrition model design:

1. The primary munition for the FAAD weapon system is a single missile.
2. An individual FAAD system will expend no more than two missiles on any given target.
3. Allocation of air defense fires to aviation targets is 100%, therefore no time is spent searching for or engaging ground targets.
4. The probability of a second round hitting the target is independent of the outcome of the previous round.
5. Weapon system reload only occurs in conjunction with the repositioning (move) event. Repositioning however does not imply a reload will occur.
6. The attrition cycle begins with the target search activity and ends with a target kill.

The resulting FAADS attrition model is shown in Figure 2.

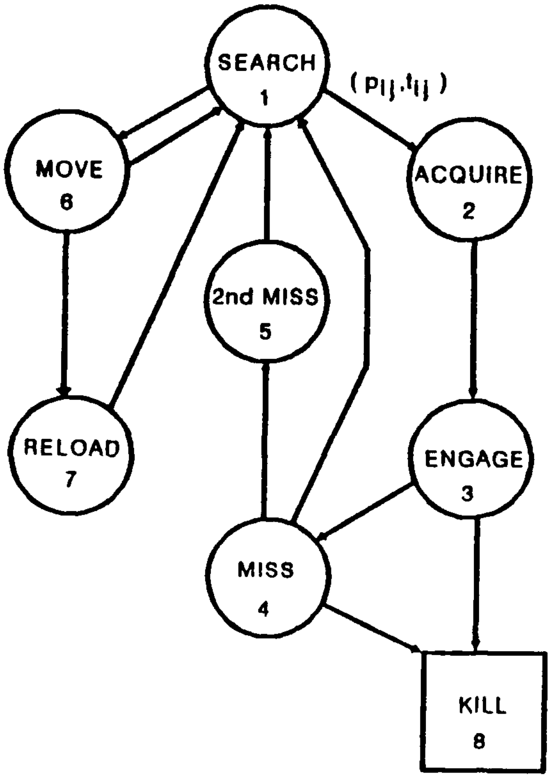

Figure 2. FAADS Attrition Model

Individual states contained within the FAAD attrition process are defined as follows:

State 1:	Search: The initial state. The active process of searching for a target using the primary search technique organic to the weapon system to be modeled. Techniques include: visual; forward looking infrared (FLIR); television guidance; active radar systems; etc.
State 2:	Acquire: This state is achieved only upon acquisition of a potential target. The acquisition state includes such activities as locking on the target, tracking the target, the target identification.

State 3: Engage: The fire order has been given, the round is fired. Reacquisition of the target is conducted concurrent with missile flight to target in the event a second shot is required.

State 4: Miss: The round misses the target. A decision is made to either reengage or break off engagement and return to search.

State 5: 2nd Miss: The second round fired at a particular target misses.

State 6: Move: A FAAD weapon system in support of offensive operation repositions to maintain coverage of the maneuver force. A system protecting static assets in the rear area or in support of a defensive operation moves to alternate firing positions. Weapon system setup and breakdown activities are considered a subset of the repositioning state.

State 7: Reload: The ammunition reload drill is conducted.

State 8: Kill: The absorbing state. Achieved when a missile hit on target results in a tactical kill.

The discrete transition probabilities and times assigned to the connecting arcs between states of the FAAD attrition model are a function of one or more of four independent variables and are fixed for each attrition cycle. The four independent variables are described below.

1. Weapons System Type: Two types of FAAD weapon systems are considered:

   Weapon System Type 1 - LOS-F-H
   Weapon System Type 2 - LOS-R

   Note: The NLOS FAAD weapon system requires a modified version of the proposed FAAD attrition model. Recommendations for the development of an NLOS attrition model is addressed in Chapter VI.

2. Target System Type: The air threat consists of two major system types:

   Target System Type 1-Rotary Wing (attack helicopters)
   Target System Type 2-Fixed Wing (tactical air)

   The analyst should consider target system tactics, infrared signature, anti-missile defense systems, and any other performance characteristics which might enable it to evade detection or destruction.

3. Slant Range to Target: The length of the range vector between the FAAD system in the XY plane and the target in the XYZ dimension. The actual slant range is rounded to the nearest integer and is currently restricted to values between 1 and 8 km.

4. Target Exposure: The degree to which a target is exposed impacts on its ability to be seen and successfully engaged. The FAAD attrition model considers two levels of exposure:

   Target Exposure Level 1 - High Exposure
   Target Exposure Level 2 - Low Exposure

   When determining the level of target exposure, the analyst should consider the following criteria:

   - Attack profiles for fixed wing aircraft
   - Attack helicopter tactics
   - Fixed and rotary wing air routes in and out of the area of operations and FAAD line of sight with those routes.
   - The terrain and vegetation in the area of operations and its ability to conceal aviation stand off systems or support "pop up" tactics.

Transition probabilities for the FAAD attrition model are derived from a set of discrete probabilities, each of which are a function of the independent variables previously discussed. These discrete probabilities are defined as follows:

P_{ijkl} = Single Shot Probability of Kill: The single shot probability of FAAD weapon system type i hitting and destroying target system type j at slant range k and target exposure level l.

PR_i = Probability of Reposition: The probability of FAAD weapon system type i repositioning at any point in the battle.

PRL_i = Probability of Reload: The probability that FAAD weapon system type i will need to reload at some point during the battle.

PSS_{ijkl} = Probability of a Second Shot: The probability that FAAD weapon system type i will take a second shot at target system j at slant range k and target exposure level l, given a first round miss.

Transition times between states of the FAAD attrition model are derived from the combination of expected times to complete specific activities required to enter that state. The times to complete these specific activities are a function of one or more of the four independent variables previously discussed. Activity times are defined as follows (all times are in seconds):

TA_{ijkl} = Time to Acquisition: The expected time required for FAAD weapon system type i to acquire target system type j at slant range k and target exposure level l.

TT_{ijkl} = Time to Tone: The expected time required for FAADS weapon system type i to "lock on" to target system type j at slant range k and target exposure level l.

TF_{ik} = Time of Flight: The expected time required for a missile fired from FAAD weapon system type i to cover slant range k.

TR_i = Time of Reposition: Given its current mission, the expected time required for FAAD weapon system type i to complete reposition (recall that this time includes system setup and breakdown time.

TRL_i = Time to Reload: The expected time required for FAAD weapon system type i to conduct a complete reload drill.

Objective data can be obtained for **PK**, **TA**, **TT**, **TF**, and **TRL** from various sources such as AMSAA, BRL, Test and Evaluation results, etc.. However, the values assigned **PR**, **PRL**, **PSS** and **TR**, are subjective and as such should be developed using the following guidelines:

With respect to **PR** and **TR**:

a. Conduct a detailed mission analysis followed by consultation with the "experts" on FAAD tactics and doctrine. Determine a recommended FAAD employment scheme and expected frequency of movement to support the mission.

b. Conduct a detailed terrain analysis with emphasis on potential FAAD positioning and mobility constraints.

c. Review FAAD weapon system capabilities/limitations with respect to mobility, emplacement time, and tear down time.

d. Answer the questions:

(1) What proportion of the time would an individual FAAD weapon system not be in a "ready-to-fire" state?

(2) What is the expected time to completion of those activities that distract from ready-to-fire status?

With respect to **PRL**:

a. Consider the total number of air defense systems in the immediate area and the expected number of threat sorties into that area.

b. Consider the total number of FAAD weapon systems in the immediate area and number of rounds ready to fire on a fully uploaded system.

c. Review ammunition expenditures for similar scenarios generated by high resolution simulations.

d. An approximation for **PRL** might be:

$$\mathbf{PRL} = (a/b - 1)t \qquad (10)$$

where

a = E(# sorties during battle)x(#A/C per sortie)

b = (PK)x(# of FAAD sys.)x(# missiles ready to fire per sys.)

t = (time required to conduct reload)/(battle duration)

1. Example 3. Approximation for **PRL**

A FAAD LOS-F-H platoon has four ready-to-fire units, with four ready to fire rounds each, and is deployed in support of a battalion task force. Four sorties of four attack helicopters each are expected in the area over a thirty minute period. The median LOS-F-H singleshot probability of kill for the given scenario is 0.45. System reload time averages three minutes.

$$\mathbf{PRL} = ((4\text{x}4)/(.45\text{x}4\text{x}4) - 1)\text{x}(3/30) = .12$$

By equation (10), a LOS-F-H employed in this scenario would have a twelve percent change of needing to conduct a reload.

The probability of engaging a target with a second missile, **PSS**, is best represented as a binary variable. Depending on the current conditions, i.e., range to target, target exposure, target and weapon system types, **PSS** is either 1 or 0. Sample values for a LOS-F-H versus an attack helicopter are given in Table 1.

TABLE 1. **PSS** FOR LOS-F-H VERSUS ATTACK HELICOPTER

RANGE TO TARGET	TARGET EXPOSURE	**PSS**
1 - 6 km	high	1
7 - 8 km	high	0
1 - 3 km	low	1
4 - 8 km	low	0

Because of the subjective nature of the values of **PR**, **PRL**, **PSS** and **TR** it is necessary to fix the values with respect to i-j pairings for the duration of the battle. This will eliminate attrition rate variation due to subjective inputs.

Once the values for respective transition probabilities and times have been identified, they are placed in a "look-up" table with the following column format:

Type FAAD Sys	Type Red Air	Tgt Exp Lvl	Rng to Tgt	**PK**	**PR**	**PRL**	**PSS**	**TA**	**TT**	**TF**	**TR**	**TRL**
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)

The resulting table is a 64 x 13 matrix in which column entries (5) through (13) of any given row are a function of the combination of independent variables found in columns (1) through (4) of that same row. A completed table is provided at Appendix C. Figure 3 presents the attrition rate curves attained when applying input data from the table in Appendix C to the FAADS attrition model. The curves represent the rate at which a LOS-F-H will attrite an attack helicopter over various ranges at full or partial levels of exposure.

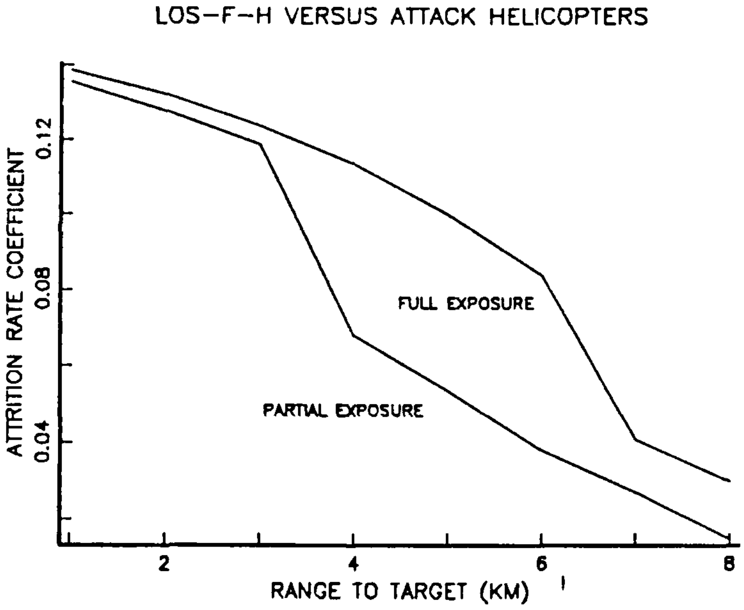

Figure 3. FAADS Attrition Rate Curves

The resulting attrition rate curves exhibit the downward trend expected when looking at attrition with respect to range. The significant drops on both the partial exposure and the full exposure curves after three and six kilometers respectively, reflect assignment of the PSS values given in Table 1. As expected, the attrition rate of fully exposed targets is greater than that of partially exposed targets at every range. Also, as expected, the rate of decrease in the attrition rate is greater for the partially exposed target as range increases. The results of inverting the attrition rates to get the expected time between kills are seen in Figure 4.

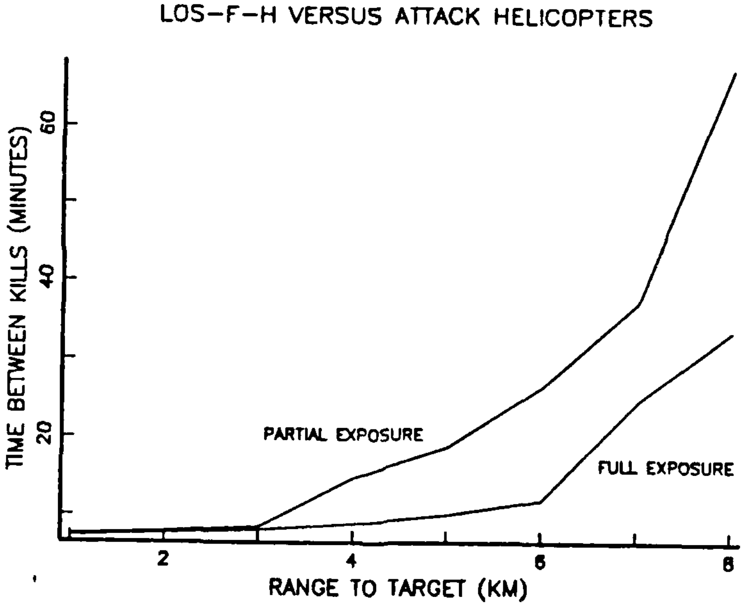

Figure 4. Expected Time Between Kills

The resulting time between kills ranges from approximately six to seventy minutes, implying that a single LOS-F-H could kill up to five attack helicopters in a thirty minute period. This is not unreasonable given the fact that at least one reload/ reposition would be required during that 30 minute period to achieve such a number of kills. The attrition rate values for the FAADS attrition model were derived using the algebraic method discussed in Chapter II. Actual calculations were accomplished using lines [38] through [71] of the APL function MODEL at Appendix B.

C. SUMMARY

By representing a FAAD weapon system's attrition cycle as a Semi-Markov process, a more realistic attrition rate is attained. The resulting FAAD attrition model is scenario dependent, producing attrition rate estimates which are a function of several independent variables. As shown, transition probabilities and times associated with the FAADS attrition model are a function of combinations of the independent variables. The problem then becomes how to determine what values the independent variables take on for each time-step of the battle simulation. This problem is addressed in Chapter IV, along with the development of the FAADS mix analysis model.

IV. FAADS FORCE MIX ANALYSIS MODEL (FFMAM)

A. GENERAL

An attack helicopter's lethality and survivability are enhanced through its ability to use the terrain. Its superior mobility allows the attack helicopter to move between firing positions using the terrain to mask its movement. Similarly, tactical aircraft rely on their ability to approach at high speeds and low altitude to strike targets before air defense systems can react. Therefore, when modeling air defense versus combat aviation scenarios, it is important to capture as many of aviation's tactical advantages as possible. Accordingly, consideration of those tactical advantages was paramount in the development of the FAADS force mix model introduced in this chapter.

B. BASIC MODEL

1. Model description

The FAADS force mix analysis model (FFMAM) is a low resolution, heterogeneous, Lanchester time-stepped simulation of air defense versus combat aviation engagements in the division forward area. Specifically, FFMAM simulates a maneuver brigade's area of operation. The model is written in A Programming Language (APL) and will run on a personal computer with 640K in RAM. The model APL code is presented at Appendix B with definitions of model variables at Appendix A.

2. Model resolution

The task organization of air defense assets to support a tactical operation would be outlined in the tactical operations order. The lowest level of task organization is the air defense platoon (or section), consisting of four or five fire units of the same system type. Historically, individual platoons are assigned the mission of supporting a battalion sized task force in the brigade forward area, or a critical asset(s) in the brigade rear area. Air defense coverage of designated priorities in the brigade area of operation is accomplished through the integration of individual system coverages, resulting in an air defense "umbrella" over protected assets. Because the platoon is the smallest independent grouping of air defense fire units, aggregation of air defense assets in the model is at the platoon level. Accordingly, combat aviation assets engaging targets in the battalion task force area are aggregated by system type.

A maneuver task force consists of numerous direct and indirect fire weapon systems, each of which is capable of inflicting damage on opposing air defense and aviation assets. In its current configuration, the model does not account for attrition of air defense or aviation assets due to those direct and indirect fire systems. Although such attrition might prove significant when properly modeled, it is beyond the scope of this research and left to future embellishment of FFMAM.

3. User interface

Although the attrition methodologies used in FFMAM are range dependent, the model does not calculate the distance between attacking and defending forces at each time step. To determine the current distance between opposing systems, the FFMAM user must plot the center of mass of each aggregated system for each simulation time-step. Although these projected locations are not actually input to FFMAM, they are used to determine the input values of key independent variables such as target exposure or target range. By plotting the projected maneuver force location for each time-step of a simulated attack, the user can determine the likely positions from which enemy aviation assets will engage that force. This can be achieved by conducting a detailed analysis of the surrounding terrain for each time-step. Such a technique allows the modeler to account for the impact of mixed terrain on the employment of combat assets throughout the conduct of a battle. Specific techniques used to determine the deployment of both fixed and rotary wing aircraft for each simulation time-step are discussed in the ensuing paragraphs.

4. Attrition

a. Independent Variables

The FAADS attrition model developed in Chapter III is used to determine the rate at which combat aviation assets are killed. Given a scenario in which a FAAD system of type A must defend against an aviation system of type B, the resulting attrition rate coefficient becomes a function of the two remaining undefined independent variables; target exposure and range to target. Both target exposure and range to target are a function of the distinct attack profiles (or tactics) employed by attack helicopters and fixed wing tactical aircraft.

b. Attack helicopter employment

When modeling attack helicopter tactics it was necessary to make the following assumptions:

1. The primary use of attack helicopters in support of defensive operations is to destroy hard targets in the battle area, i.e., tanks and other mechanized force weapon systems. Accordingly, attack helicopter penetration into the brigade rear area is not modeled.

2. Attack helicopters in the forward area will select firing positions that maximize their ability to kill mechanized forces while minimizing their exposure to enemy fires.

3. Movement to and from these positions will be via routes concealed from enemy observation due to terrain masking.

4. Flank shots are preferred over head-on shots.

Given these assumptions, it is then necessary to determine where the maneuver force will be at any given point in the battle.

The scheme of maneuver presented in the brigade tactical operations order, coupled with associated map overlays, provide battalion task force commanders with their specific avenues of attack. A FAADS platoon, assigned the mission of supporting a battalion task force in the attack, would be integrated into the battalion's scheme of maneuver to provide air defense coverage that keeps pace with the battalion advance. The resulting air defense "umbrella" would move down the attack axis generally centered on the supported battalion. Representation of a FAAD platoon's location at any given time-step during an attack can be estimated by the progress of the attacking force along the attack avenue. Given a battalion task force's position for each one minute time step of a hypothetical battle, a graphic representation of FAAD platoon positioning by time-step might be described as shown in Figure 5.

Having determined the approximate task force/air defense center of mass for each time-step of the attack, it is then possible to determine the "optimal" positioning of attack helicopters for each respective time-step. This is

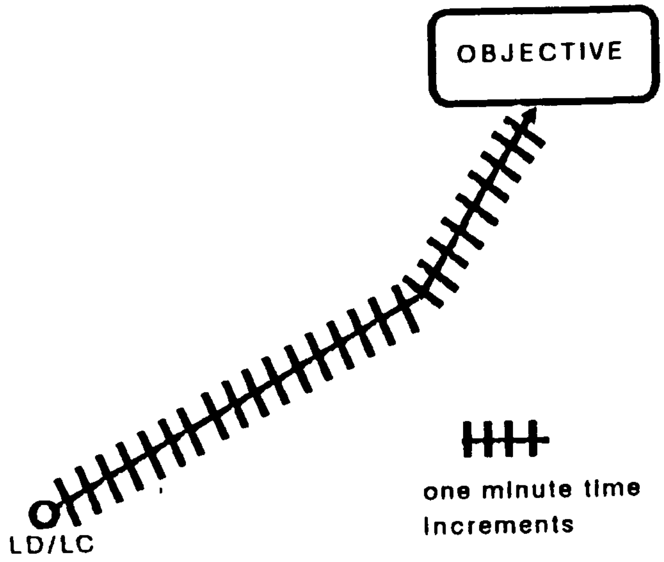

Figure 5. Time-Stepped Representation of Attack Axis

accomplished through a detailed terrain analysis and thorough understanding of actual or projected attack helicopter capabilities and tactics. For example, at simulation time-step one, the "optimal' positioning of an attack helicopter might be at a range of three kilometers, masked by trees and hilly terrain, ready to "pop-up" and take a flank shot at advancing forces. In contrast, at time-step ten, the "optimal" firing position might be at six kilometers directly in front of the approaching force in a relatively exposed stand-off position. Needless to say, attack helicopter positioning is a strong function of the "usable" terrain surrounding the axis of advance of an attacking force.

Consequently, when simulating a brigade in the offense, it is necessary to conduct a detailed terrain analysis for each battalion task force axis of advance.

c. Fixed wing tactical aircraft attack profile

When modeling fixed wing tactical aircraft it was necessary to make the following assumptions:

1. The priority targets for fixed wing tactical aircraft will be soft targets such as command and control centers, logistical assets, forward aviation resupply points, air defense sites and supply routes. Hard target selection is limited to reserves and field artillery assets. Accordingly, fixed wing engagements of maneuver forces in the forward area are not modeled.
2. Fixed wing aircraft will fly nap of the earth (NOE) using available high speed corridors until initiating attack profiles at target destination.
3. Fixed wing aircraft are not attrited by FAAD systems in the brigade forward area while enroute to rear area targets.
4. Fixed wing aircraft spend a maximum of fifty seconds within range of rear area air defense systems during attack execution.
5. Fixed wing aircraft have good target location information.
6. Fixed wing aircraft do not engage targeted assets until they are within bomb release range (approximately two kilometers).

Given these assumptions, the next step is to model air defense coverage of critical assets in the brigade rear area. The brigade tactical operations order, along with attached overlays, prescribes the positioning of key assets in the brigade rear area. Assets are prioritized for air defense coverage with the most critical receiving a FAADS section. When modeling air defense coverage of those priority assets in the brigade rear area, the following assumptions are made:

1. The overall air defense design is a series of individual critical asset defenses (point defenses), as opposed to a totally integrated area coverage of all assets.
2. The rear area air defense battles consist of a series of independent engagements in which each critical asset is attacked only once.
3. Air defense coverage of an asset is centered on the asset.
4. A maximum of three assets are defended due to the limited number of FAAD LOS-R systems organic to the air defense battery supporting the brigade area.

Because fixed wing aircraft approach, attack, and depart at such high speeds, it is necessary to simulate the fixed wing versus FAADS engagement cycle using five second time-steps. Figure 5 shows a standard profile for fixed ordnance delivery. When flying NOE prior to initiating its targeting and ordnance delivery run, the attacking aircraft is at a relatively low exposure level to air defense acquisition and fires. Upon initiation of its attack profile, the aircraft becomes fully exposed as seen in Figure 6.

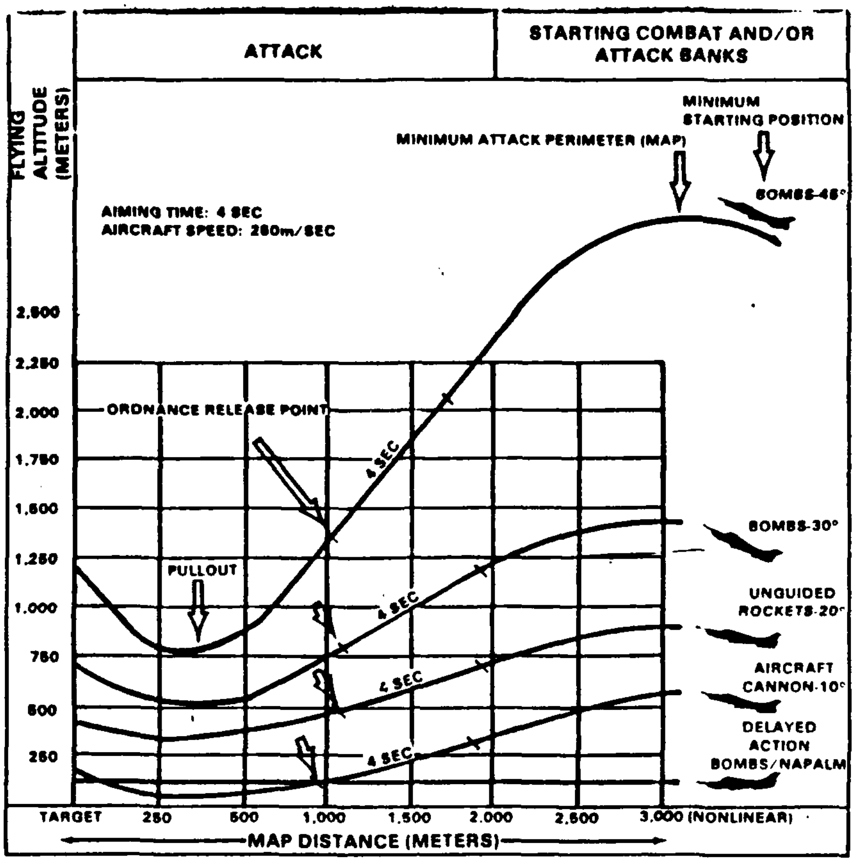

Figure 6. Fixed Wing Aircraft Attack Profile

Given the location of a critical asset, it then possible to conduct a terrain analysis to determine the potential high speed avenues of approach into and out of the asset area. Once the best routes in and out have been selected, the aircraft attack profile should be applied starting at eight kilometers from the target. Assuming an aircraft will cover two kilometers with each five second time-step, the appropriate values for target exposure and range can be determined. As in the case of the various battalion task force areas, the terrain surrounding critical assets in the brigade rear area varies from asset to asset. Therefore, aircraft exposure levels may differ based on the degree of terrain masking.

d. Attrition of FAAD weapon systems

As currently configured, FFMAM does not adequately account for the attrition of FAAD weapon systems. As previously discussed, the effects of indirect and direct fire weapon systems are not modeled. FFMAM does, however, allow for attrition from combat aviation systems. Attrition rates are determined using the key parameters of equation (5) presented in Chapter II. Although use of equation (5) is not the preferred attrition methodology of this research, it does produce acceptable attrition coefficients for the purpose of model demonstration given the following assumptions:

1) The rate of fire of both fixed and rotary wing systems is one round per minute.
2) Fifteen percent of all rotary wing fires and ten percent of all fixed wing fires are allocated to air defense targets.
3) The probability of a combat aviation system acquiring an air defense target is a constant value independent of range.

Specific input values developed to support the scenario presented in Chapter V are given in Appendix C. The associated FFMAM input variables are defined in Appendix A.

5. FFMAM input

Input for FFMAM consists of several matrices, each of which is described in sufficient detail in Appendix A. It is important to note that the column dimensions of most of the input matrices are a function of the tactical scenario modeled. Although most of the input matrices are self explanatory, TSTEP requires additional discussion and is addressed below.

Once a scenario for a brigade in the offense is selected, a first step in structuring model input is to determine the number of forward area battalion task force sized battles to be simulated. The next step is to draw a map overlay of the attack axis for each attacking force as described in Figure 5. Each axis is then partitioned into thirty, one minute time-steps to reflect expected battle progression over a thirty minute time period. Next attack helicopter firing positions are selected for each time-step on each attack avenue. The associated target exposure and target range data are estimated and recorded for each time-step. The next step is to identify those rear area assets receiving dedicated air defense coverage and to overlay the optimal ingress and egress routes of attacking fixed wing aircraft. Partition those routes into ten, five second time-steps and record the associated range to target and target exposure data. The recorded data for both the forward and rear areas are then stored in the input matrix TSTEP. Assuming the scenario called for two battalion task forces attacking along avenues A and B, respectively, and three critical assets with FAAD coverage in the brigade rear area, TSTEP would be structured as follows:

Time step	Target exposure level			Range to target		
	A	B	REAR	A	B	REAR
(1)	(2)	(3)	(4)	(5)	(6)	(7)

For the prescribed thirty minute battle, the resulting matrix would be a 30x7 table. It is important to note that input data for the three rear area battles are contained in columns (4) and (7). Columns (4) and (7) are subdivided into three groups each: rows (1)-(10), (11)-(20), and (21)-(30). The ten time-step data for each of the three rear area battles are placed into columns (4) and (7) accordingly. The completed TSTEP matrix provides input for the FAADS attrition model discussed in Chapter III. A sample matrix is presented at Appendix C.

6. Measures of Effectiveness

A measure of an air defense platoon's mission effectiveness is its ability to prevent combat aviation from attriting protected assets. The longer aviation is permitted to fire on friendly forces, the greater the attrition of those assets. The following measures of effectiveness (MOE) will be used to assess the effectiveness of the various FAAD force mix alternatives:

MOE-1: Number of enemy fixed or rotary wing aircraft destroyed.

MOE-2: Total number of surviving rotary wing firing minutes (on-station time).

MOE-3: Number of FAADS weapon systems killed.

A larger value is always better when comparing force mix alternatives using MOE-1 or MOE-3. A smaller value is desirable when evaluating MOE-2.

C. SUMMARY

The FAADS force mix analysis model emphasizes the impact of terrain on the three dimensional battle. As currently configured, FFMAM focuses on the attrition of combat aviation systems. To provide added realism, FFMAM exercises the FAADS attrition model introduced in Chapter III. Detailed mission and terrain analysis is required to generate input for the model. Emphasis is placed on the impact of terrain on attack helicopter employment and fixed wing ingress and egress routes. Chapter V demonstrates the model and provides discussion of model output. Several FAADS force mix alternatives are compared along with variations in other selected inputs.

V. FFMAM OUTPUT AND ANALYSIS

A. GENERAL

The purpose of this chapter is to demonstrate the capabilities of FFMAM as currently configured. Although some comparisons will be made between selected FAADS force mix alternatives, it is not the intention of this research to conduct a force mix analysis. The focus of discussion will be on attrition related trend analysis. Several input parameters will be varied to demonstrate model sensitivity. An hypothetical scenario is provided that supports the current model structure. Model output is presented in MOE format and discussed accordingly.

B. SCENARIO

A mechanized infantry brigade(-) is attacking along two avenues. Task Force Sam attacks along Avenue A to sieze objective Pipe. Task Force Bill attacks along Avenue B to sieze objective Smoke. Both task forces have a platoon of FAADS LOS-F-H in direct support. The brigade combat trains, brigade tactical operation center, and direct support field artillery battalion are designated as priorities for air defense. A section of FAADS LOS-R is in general support of each critical asset. The enemy air situation supports twelve to eighteen fixed wing aircraft sorties and fourteen to twenty-one attack helicopter sorties during the operation.

C. DEMONSTRATION OF MODEL TRENDS

Model input was developed in accordance with the guidelines presented in preceeding chapters. The input matrices constructed in support of the scenario are at Appendix C. Values assigned the input variable TSTEP are designed to reflect terrain representative of a "European" scenario. Values assigned the input matrix TABLE were derived through discussion with various air defense officers having heavy division experience. The number of fire units assigned FAADS LOS-F-H platoons and LOS-R sections were extracted from the force mix alternatives listed in the FAADS Update Study Plan [Ref. 1:encl.4].

Four input parameters were varied to demonstrate model sensitivity and trends:

1. Number of fire units per FAADS platoon or section.
2. Number of threat aircraft.
3. Repositioning time for LOS-F-H systems in the forward area.
4. Single shot kill probability of LOS-R systems.

Repositioning time for LOS-F-H fire units in the forward area was decreased from 600 to 480 seconds. The LOS-R "basic" missile was given a performance upgrade (PIP) that uniformly increased its lethality by five percent over its effective range. Table 2 presents FFMAM output by MOE for each of the input variations addressed above. Cases including changes in LOS-F-H repositioning time or LOS-R single shot probability of kill are listed as such in the DELTA VALUE column of Table 2. Cases 19 through 36 reflect a fifty percent increase in combat aviation assets in each of the brigade's three mission areas.

The "end game" data presented in Table 2 is consolidated by MOE and FAADS weapon system type and presented graphically in figures 7 through 11. The reader is reminded that battles 7 through 12 (cases 19 through 36) represent a fifty percent increase in combat aviation force composition over battles 1 through 6 (cases 1 through 18).

TABLE 2. MOE COMPARISONS OF FFMAM OUTPUT FOR VARIOUS INPUTS

BATTLE NUMBER	CASE NUMBER	FAADS SYSTEMS	MISSION AREA	DELTA VALUE	NUMBER A/C	MOE-1	MOE-2	MOE-3
1	1	5	A	600	8	8	89.5	.42
	2	5	B	600	6	6	37.8	.22
	3	12	REAR	BASIC	12	6.06	N/A	1.45
2	4	5	A	480	8	8	78.0	.34
	5	5	B	480	6	6	30.7	.17
	6	12	REAR	PIP	12	6.87	N/A	1.39
3	7	4	A	600	8	8	113.6	.51
	8	4	B	600	6	6	45.1	.27
	9	9	REAR	BASIC	12	4.44	N/A	1.56
4	10	4	A	480	8	8	91.0	.43
	11	4	B	480	6	6	37.5	.22
	12	9	REAR	PIP	12	5.04	N/A	1.52
5	13	3	A	600	8	5.80	150.4	.62
	14	3	B	600	6	6	61.7	.36
	15	6	REAR	BASIC	12	2.82	N/A	1.67
6	16	3	A	480	8	7.21	127.2	.55
	17	3	B	480	6	6	49.0	.30
	18	6	REAR	PIP	12	3.20	N/A	1.64
7	19	5	A	600	12	9.8	207.2	.88
	20	5	B	600	9	9	80.7	.50
	21	12	REAR	BASIC	18	5.84	N/A	2.40
8	22	5	A	480	12	12	168.2	.75
	23	5	B	480	9	9	66.5	.41
	24	12	REAR	PIP	18	6.63	N/A	2.33
9	25	4	A	600	12	7.56	244.1	.99
	26	4	B	600	9	9	109.3	.62
	27	9	REAR	BASIC	18	4.23	N/A	2.51
10	28	4	A	480	12	9.39	213.6	.89
	29	4	B	480	9	9	83.4	.51
	30	9	REAR	PIP	18	4.80	N/A	2.46
11	31	3	A	600	12	5.26	280.9	1.11
	32	3	B	600	9	6.75	147.6	.81
	33	6	REAR	BASIC	18	2.61	N/A	2.62
12	34	3	A	480	12	6.54	259.2	1.04
	35	3	B	480	9	8.20	127.1	.70
	36	6	REAR	PIP	18	2.96	N/A	2.59

The total number of attack helicopters killed in the brigade area of operations for each of the twelve battle simulations is shown in Figure 7. An effect of increasing the threat by fifty percent is seen when comparing the number of attack helicopter kills in battles 3 and 9. The increased threat resulted in the reduction of LOS-F-H killing efficiency from 100 percent in battle 3 to 79 percent in battle 9.

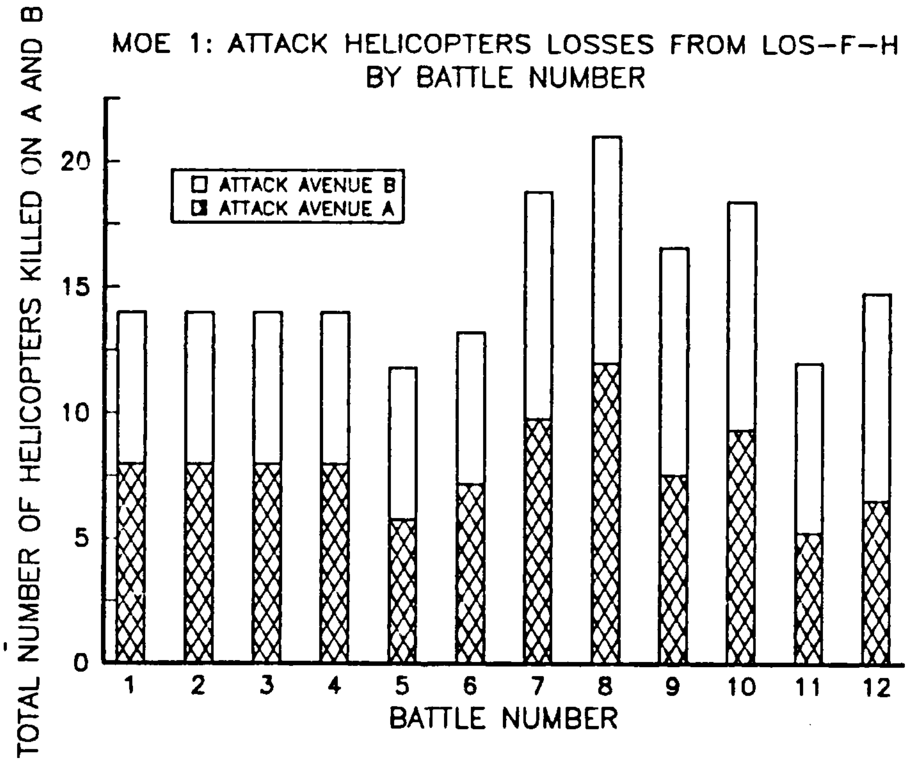

Figure 7. Number of Attack Helicopters Killed

The total number of fixed wing aircraft destroyed for each of the twelve battles is shown in Figure 8. Note that the fifty percent increase in fixed wing sorties resulted in a slightly smaller number of fixed wings killed. The decreasing trend is realistic given the increase in aircraft sorties over an already saturated area. The fixed number of LOS-R systems cannot attrite the attacking aircraft any faster, while the increased number of aircraft can bring additional fires to bear on air defense systems.

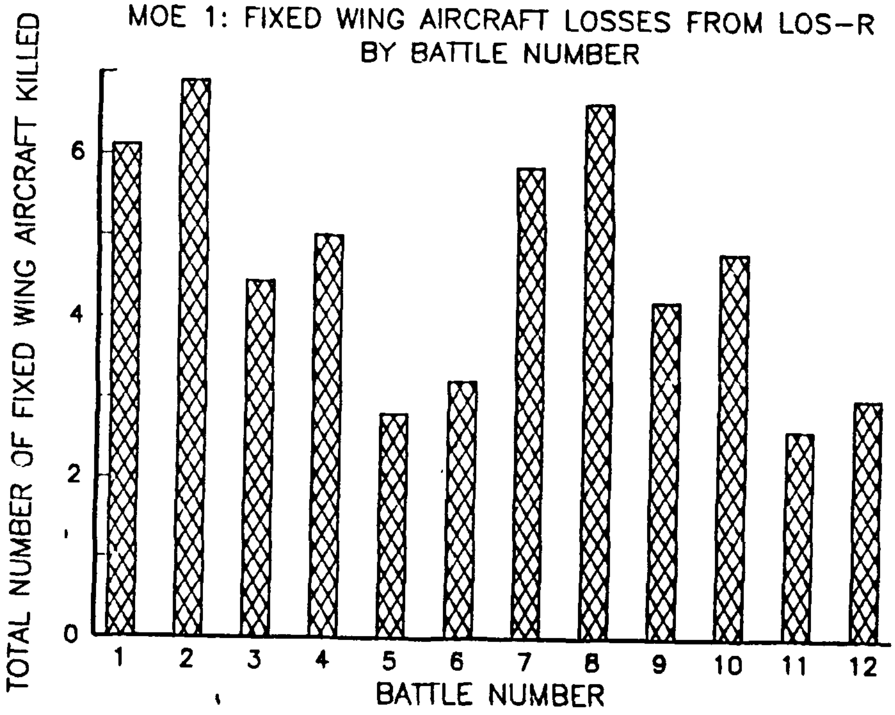

Figure 8. Number of Fixed Wing Aircraft Killed

The combined totals of attack helicopter firing minutes on avenues A and B for each battle are shown in Figure 9. Again, an expected trend is observed. The increase in attack helicopters results in a much greater increase in the number of minutes in which they are firing on potential targets. For example, battle 7 shows a 126 percent increase in firing minutes over battle 1 while battle 9 shows a 123 percent increase over battle over battle 3.

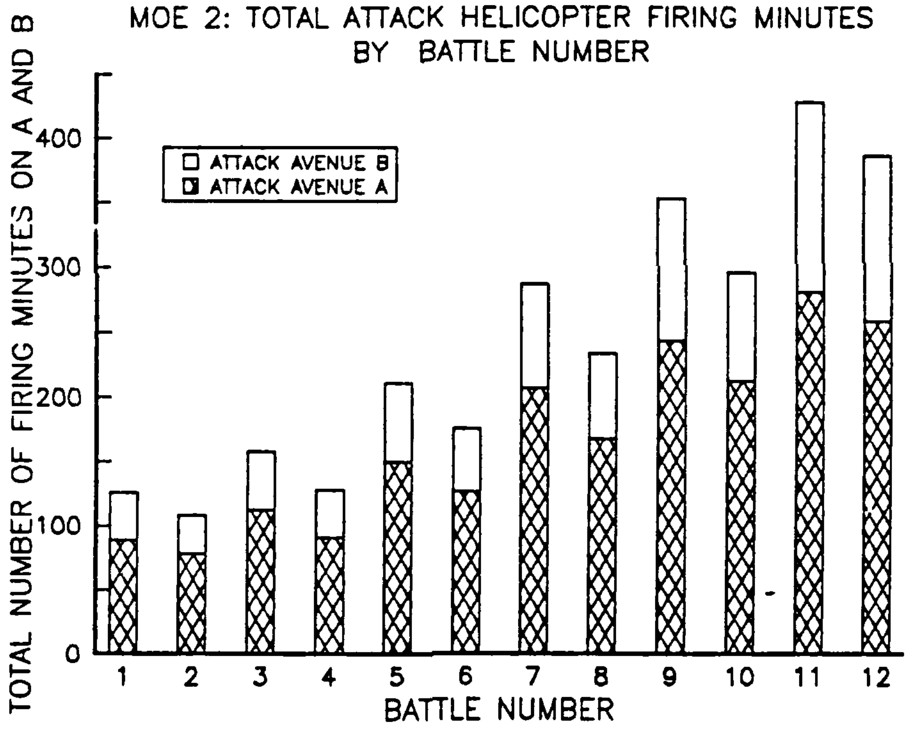

Figure 9. Attack Helicopter Firing Minutes

The combined totals for attrition of FAADS LOS-F-H systems on attack avenues A and B are shown in Figure 10. As expected, an increase in the aircraft to air defense system ratio results in an increase in the percentage of air defense systems killed.

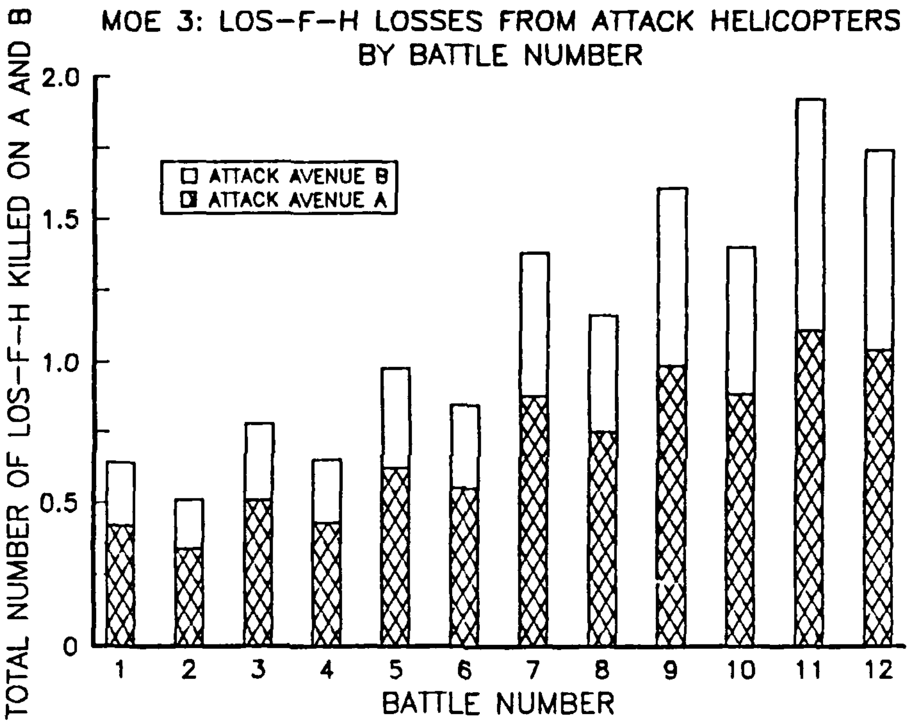

Figure 10. Number of LOS-F-H Killed

The attrition of FAADS LOS-R systems in the brigade rear area is shown in figure 11. Again, an increase in fixed wing to LOS-R system ratio results in greater LOS-R losses.

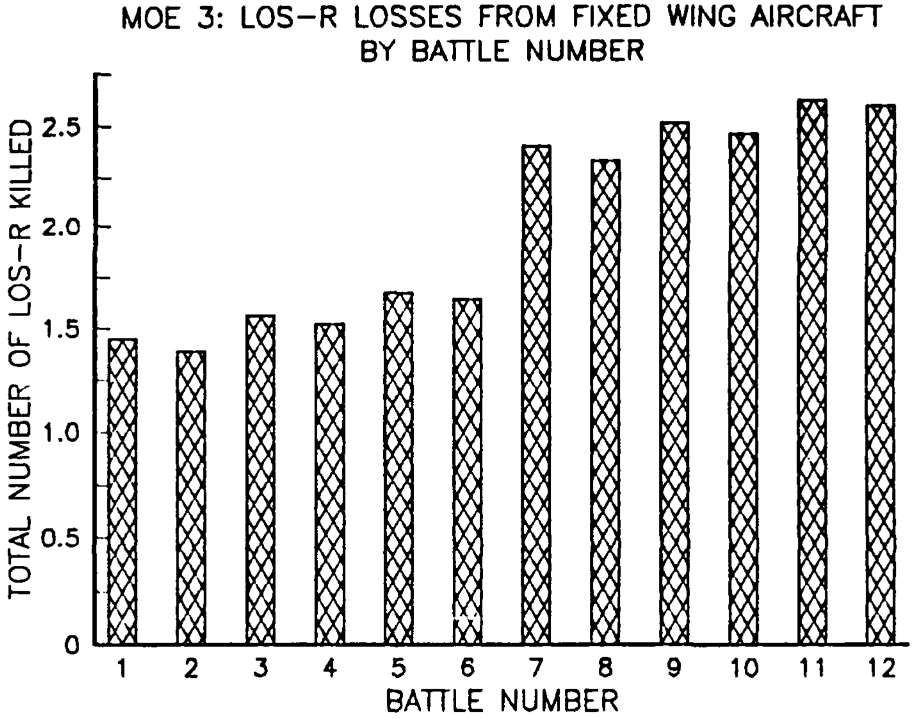

Figure 11. Number of LOS-R Killed

The graphic comparisons of FFMAM end game results presented in figures 7 through 11 highlight the model's sensitivity to input variation. Additionally, figures 7 through 11 demonstrate that resulting MOE values follow expected attrition related trends.

The most revealing MOE for FAADS LOS-F-H systems in the foward area is attack helicopter firing minutes (MOE-2). When assessing force mix performance based on attack helicopter attrition per MOE-1, LOS-F-H cases for battles one through four would appear equally effective. Comparison of the cases using MOE-2 reveals a difference in effectiveness. A more detailed picture is provided considering the accumulation of attrition over time. Figures 12 through 15 present time-stepped results of the accummulation of MOE values for various case comparisons.

Figures 12a and 12b compare the lethality of a LOS-F-H platoon requiring 600 seconds repositioning time per system (case 7) versus a platoon that requires 480 seconds (case 10). In both cases, all attacking aircraft were destroyed, but case 10 required only 20 minutes to do so while case 7 required the full thirty minutes. As expected, the less time a system is available to engage potential targets, the longer it is going to take that system to destroy them.

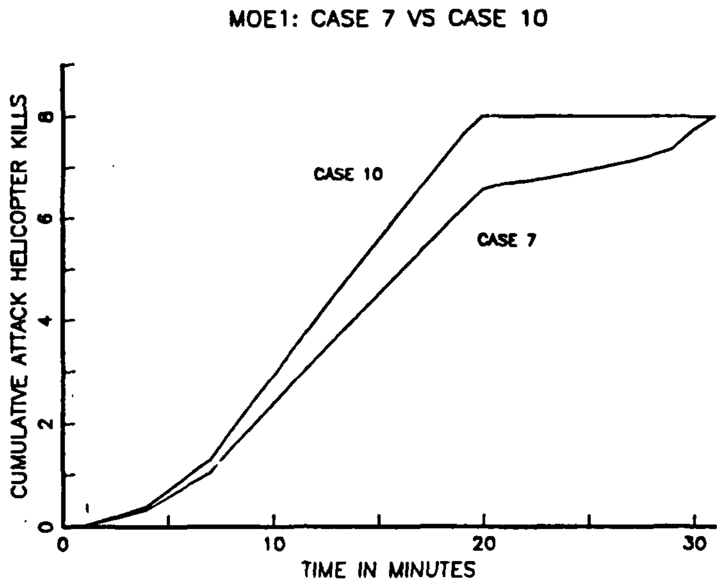

Figure 12a. Cumulative Attack Helicopter Kills

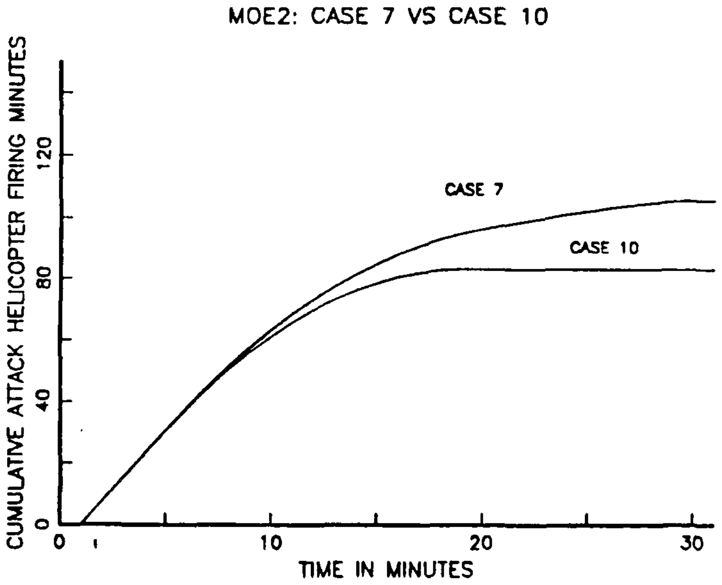

Figure 12b. Cumulative Attack Helicopter Firing Minutes

Figure 13 demonstrates the increased number of kills obtained by LOS-R systems employing the improved missile over the the basic model. The periodic flattening of the attrition curve is a result of fixed wing aircraft being beyond the effective range of the LOS-R system. As expected, the increased lethality of the improved round (case 6) results in a greater number of fixed wing kills over the basic round (case 3).

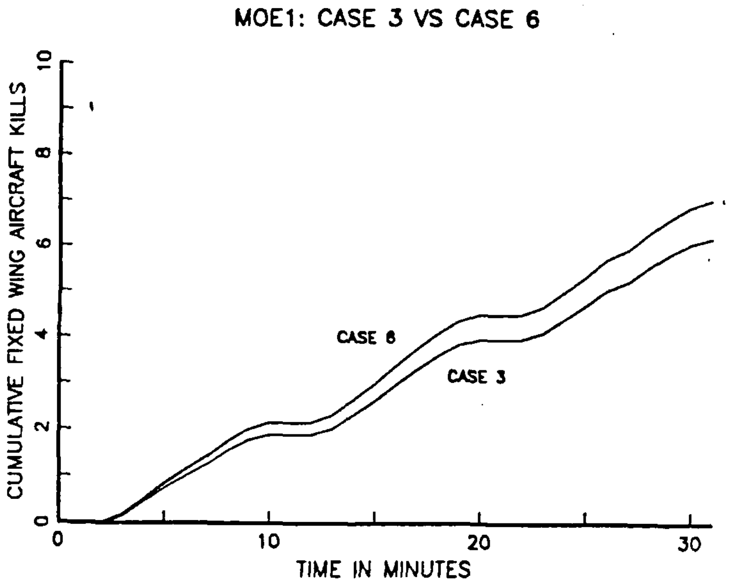

Figure 13. Cumulative Fixed Wing Aircraft Kills

Figures 14a and 14b present a comparison of the three possible LOS-F-H platoon configurations against an increased threat. Although the four system configuration destroyed all threat systems, it required approximately twenty-eight minutes to do so, where the five system configuration required only seventeen minutes.

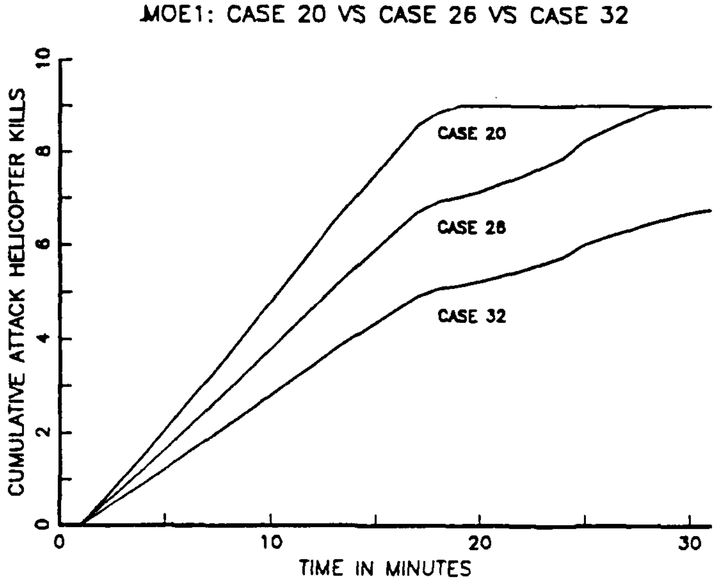

Figure 14a. Cumulative Attack Helicopter Kills (increased threat)

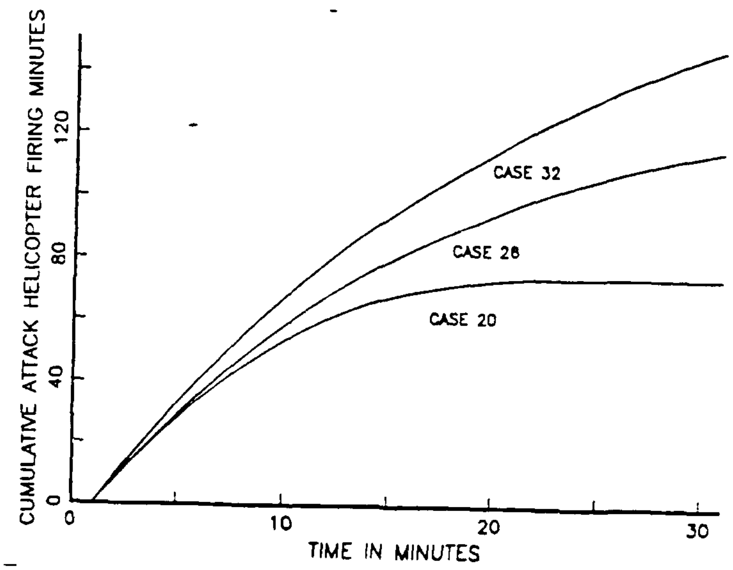

Figure 14b. Cumulative Attack Helicopter Kills (increased threat)

A similar comparison is made in Figure 15 for the potential configurations of a LOS-R section with the improved missile.

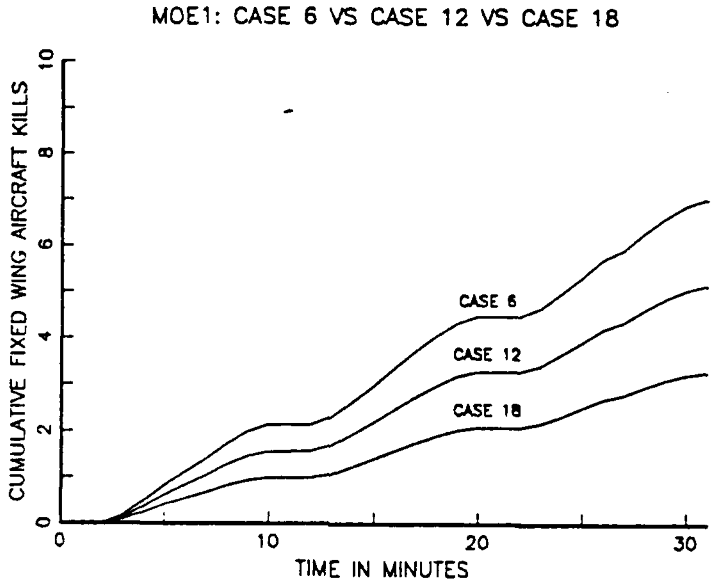

Figure 15. Cumulative Fixed Wing Aircraft Kills

D. SUMMARY

As demonstrated, FFMAM generates the attrition related trends expected from air defense versus combat aviation scenarios. Model sensitivity to changes in selected input variables is of the magnitude anticipated for such changes.

Finally, although MOE-1 provides a bottom line for the number of aircraft killed, MOE-2 provides a better assesment of how efficiently that was accomplished. The longer an enemy is permitted to engage friendly forces, the more potential damage he can inflict.

VI. FUTURE ENHANCEMENTS

A. SUMMARY

The FAADS force mix analysis model (FFMAM) was developed as an analytic filter model for use in the ongoing FAADS force mix analysis study at the US Army Air Defense Center. The current analytic filter model, CARMO-FAAD, does not adequately represent the air defense battle. FFMAM was developed as a functional area model which focuses on the air defense versus combat aviation battle. Representation of combat aviation tactics, and the impact of varying terrain on both air defense and aviation system employment, were key considerations in FFMAM model design. Additional emphasis is placed on detailed mission and terrain analysis during the generation of FFMAM input. Finally, because the model is written in APL, it is easy to embellish given a rudimentary understanding of the language.

B. RECOMMENDATIONS

Having demonstrated FFMAM capabilities with respect to FAAD LOS-F-H and LOS-R systems, the next step is to embellish FFMAM to model effects of NLOS systems in the brigade area of operations. A second improvement would be to account for the attrition of air defense and aviation systems due to other direct or indirect fire systems. A final improvement would be the development of attrition models for tactical fixed wing aircraft and attack helicopters. FFMAM would be an excellent analytical tool specifically tailored for studies of FAADS in the forward areas.

APPENDIX A. MODEL VARIABLES

LOCAL VARIABLES

A	Matrix of transition probabilities between transient states.
ATTRITA	Vector of FAAD weapon system attrition coefficients for current time-step.
ATTRITB	Vector of combat aviation attrition coefficients for current time-step.
BADLOSS	Matrix of blue air defense losses for each time-step by mission area.
BLUEAD	Matrix of blue air defense force strength at each time-step.
D	Index for the number of force mix alternatives input to current model run.
I	Index identifying battalion task force or brigade rear areas.
IDEN	Identity matrix.
J	Index identifying aviation system type.
L	Index identifying air defense system type.
M	Matrix of transition times from transient states to all states.
N	Counter to keep track of current FAADS force mix.
P	Matrix of transition probabilities between all states.
P1	Matrix of transition probabilities from transient states to all states.
PK	Probability of a kill given a hit.

PR	Probability of repositioning.
PRL	Probability of conducting a reload.
PSS	Probability of taking a second shot.
RAIRLOSS	Matrix of red aviation losses for each time step by mission area.
T	Time-step counter.
TA	Time to target acquisition.
TF	Missile flight time to target impact.
TR	Time required to complete repositioning.
TRL	Time required to complete reload.
TT	Time to tone.

OUTPUT VARIABLES

BCUMLOSS	Matrix of blue air defense losses accumulated up to and including the current time-step.
RCUMMINS	Matrix of red attack helicopter firing minutes accumulated up to and including the current time step.
RCUMLOSS	Matrix of red aviation losses accumulated up to and including the current time-step.
TLOSSBAD	Matrix of end game total losses of blue air defense systems.
TLOSSRA	Matrix of end game total losses of red aviation systems.
TOTMIN	Matrix of end game total attack helicopter firing minutes.

INPUT VARIABLES

ACQ	Matrix of acquisition probabilities for combat aviation system type i acquiring FAADS target type j. Rows 1 and 2 are attack helicopter and fixed wing respectively. Columns 1 and 2 are LOS-F-H and LOS-R respectively.
ALLOCATE	Matrix of the percent allocation of aviation system type i fires to FAADS system type j.
MAXR	Vector of combat aviation rates of fire. Columns 1 and 2 are attack helicopter and fixed wing respectively.
PKILL	Matrix of the single shot kill probabilities for aviation system type i engaging FAADS system type j.
ROF	Vector of combat aviation rates of fire per minute. Columns 1 and 2 are attack helicopter and fixed wing respectively.
SHAPE	Matrix of the shaping parameters for the probability of an aviation system hitting a target over its effective range. Columns 1 and 2 are attack helicopter and fixed wing respectively.
STARTAD	Matrix of the number of FAAD systems assigned each area of the battlefield for each force mix alternative.
STARTAIR	Matrix of the number and type of combat aviation systems. Row 1 is the number of systems in area j. Row 2 is the aviation system type in area j. System type 1 is attack helicopters. System type 2 is fixed wing aircraft.

TABLE	Matrix of transition probabilities and times as described in Chapter III.
TSTEP	Matrix of target exposure levels and ranges to target for each time-step as described in Chapter IV.
TYPEAD	Vector of the type FAAD system assigned area j.

APPENDIX B. THE FAADS FORCE MIX ANALYSIS MODEL

```
      ∇MODEL[⎕]
      ∇ MODEL
[1]  ⍝ THE FUNCTION  MODEL IS THE FAADS FORCE MIX ANALYSIS
[2]  ⍝ MODEL (FFMAM).  THE MODEL IS CURRENTLY STRUCTURED TO
[3]  ⍝ SIMULATE A BRIGADE SCENARIO CONSISTING OF TWO FORWARD
[4]  ⍝ BATTALION TASK FORCE SIZED BATTLES AND THREE REAR AREA
[5]  ⍝ BATTLES.  THE MODEL RUNS FOR A MAXIMUM OF 30 TIME-STEPS
[6]  ⍝ PER BRIGADE SCENARIO.
[7]  ⍝ INITIALIZE VARIABLES
[8]   D←3
[9]   BLUEAD←(D, 31 3)⍴0
[10]  REDAIR←(D, 31 3)⍴0
[11]  BADLOSS←(D, 30 3)⍴0
[12]  RAIRLOSS←(D, 30 3)⍴0
[13]  ATTRITA← 1 3 ⍴0
[14]  ATTRITB← 1 3 ⍴0
[15]  TOTMINS←(D,2)⍴0
[16]  TLOSSBAD←(D,3)⍴0
[17]  TLOSSRA←(D,3)⍴0
[18]  BCUMLOSS←(D, 31 3)⍴0
[19]  RCUMLOSS←(D, 31 3)⍴0
[20]  RCUMMINS←(D, 31 3)⍴0
[21]  M← 7 8 ⍴0
[22]  P← 7 8 ⍴0
[23]  N←1
[24] ⍝ START CLOCK
[25] L6:T←1
[26] ⍝ INITIALIZE FORCE LEVELS
[27]  BLUEAD[N;T;]←STARTAD[N;]
[28]  REDAIR[N;T;]←STARTAIR[1;]
[29] ⍝ UPDATE PROBABILITY TRANSITION MATRIX
[30]  I←1
[31] L1:R←((TYPEAD[;I]-1)×32)+((STARTAIR[2;I]-1)×16)+((TSTEP[T;I+1]-1)×8)
[32]  R←R+TSTEP[T;I+4]
[33]  PK←TABLE[R;5]
[34]  PR←TABLE[R;6]
[35]  PRL←TABLE[R;7]
[36]  PSS←TABLE[R;8]
[37]  P[1;2]←1-PR
[38]  P[1;6]←PR
[39]  P[3;4]←1-PK
[40]  P[3;8]←PK
[41]  P[4;1]←1-PSS
[42]  P[4;5]←(1-PK)×PSS
```

```
 P[4;8]←PK×PSS
 P[6;1]←1-PRL
 P[6;7]←PRL
 P[2;3]←P[5;1]←P[7;1]←1
⍝ UPDATE TRANSITION TIMES MATRIX
 TA←TABLE[R;9]
 TT←TABLE[R;10]
 TF←TABLE[R;11]
 TR←TABLE[R;12]
 TRL←TABLE[R;13]
 M[1;2]←TA
 M[1;6]←TR
 M[2;3]←TT
 M[3;4]←M[3;8]←TF
 M[4;5]←M[4;8]←TT+TF
 M[4;1]←M[5;1]←M[6;1]←M[7;1]←0
 M[6;7]←TRL
⍝ CALCULATE ATTRITION MATRIX FOR FIRER BLUEAD USING SEMI-MARKOV PROCESS
 P1← 7 8 ↑P
 PI← 7 1 ⍴+/(P1×M)
 A← 7 7 ↑P1
 IDEN← 7 7 ⍴1,7⍴0
 TIME←PI⌹(IDEN-A)
 →(STARTAIR[2;I]≠2)⍴L8
⍝ CALCULATE ATTRITION COEFFICIENT FOR REAR AREA
 ATTRITA[;I]←5÷TIME[1;]
 →L9
⍝ CALCULATE ATTRITION RATE COEFFICIENT FOR FORWARD AREA
L8:ATTRITA[;I]←60÷TIME[1;]
⍝ CALCULATE ATTRITION MATRIX FOR FIRER REDAIR USING RDFPS
L9:J←STARTAIR[2;I]
 L←TYPEAD[;I]
 →(TSTEP[T;I+4]≤MAXR[;J])⍴L10
 ATTRITB[;I]←0
 →L11
L10:ATTRITB[;I]←ALLOCATE[J;L]×ACQ[J;L]×PKILL[J;L]×ROF[1;J]
 ATTRITB[;I]←ATTRITB[;I]×(1-(TSTEP[T;I+4]÷MAXR[1;J]))*SHAPE[1;J]
 →(STARTAIR[2;I]≠2)⍴L11
 ATTRITB[;I]←ATTRITB[;I]÷3
⍝ EXECUTE BATTLE FOR TIME INCREMENT USING LANCHESTER EQUATIONS
L11:→((T≠11)∧(T≠21))⍴L7
 →(I≠3)⍴L7
 BLUEAD[N;T;3]←STARTAD[N;3]
 REDAIR[N;T;3]←STARTAIR[1;3]
L7:BADLOSS[N;T;I]←ATTRITB[;I]×REDAIR[N;T;I]
 RAIRLOSS[N;T;I]←ATTRITA[;I]×BLUEAD[N;T;I]
⍝ UPDATE FORCE LEVELS
 BLUEAD[N;T+1;I]←BLUEAD[N;T;I]-BADLOSS[N;T;I]
```

```
[91]  REDAIR[N;T+1;I]←REDAIR[N;T;I]-RAIRLOSS[N;T;I]
[92]  BCUMLOSS[N;T+1;I]←BCUMLOSS[N;T;I]+BADLOSS[N;T;I]
[93]  RCUMLOSS[N;T+1;I]←RCUMLOSS[N;T;I]+RAIRLOSS[N;T;I]
[94]  RCUMMINS[N;T+1;I]←RCUMMINS[N;T;I]+REDAIR[N;T+1;I]
[95]  →(BLUEAD[N;T+1;I]>0)ρL2
[96]  BLUEAD[N;T+1;I]←0
[97]  BCUMLOSS[N;T+1;I]←STARTAD[N;I]
[98] L2:→(REDAIR[N;T+1;I]>0)ρL3
[99]  REDAIR[N;T+1;I]←0
[100]  RCUMLOSS[N;T+1;I]←STARTAIR[1;I]
[101]  RCUMMINS[N;T+1;I]←(+/REDAIR[N;;I])-STARTAIR[1;I]
[102] L3:I←I+1
[103]  →(I≤3)ρL1
[104] ⍝ CHECK STOPPING CRITERIA
[105]  →(((+/BLUEAD[N;T+1;])=0)∨((+/REDAIR[N;T+1;])=0)∨(T=30))ρL5
[106]  T←T+1
[107]  I←1
[108]  →L1
[109] ⍝ CALCULATE TOTALS FOR BRIGADE BATTLE NUMBER N
[110] L5:TOTMINS[N;1]←+/REDAIR[N;;1]
[111]  TOTMINS[N;2]←+/REDAIR[N;;2]
[112]  TLOSSBAD[N;1,2]← 1 2 ρ(BLUEAD[N;1;1,2]-BLUEAD[N;T+1;1,2])
[113]  TLOSSBAD[N;3]←(3×BLUEAD[N;1;3])-(+/BLUEAD[N;10,20,30;3])
[114]  TLOSSRA[N;1,2]← 1 2 ρ(REDAIR[N;1;1,2]-REDAIR[N;T+1;1,2])
[115]  TLOSSRA[N;3]←(3×REDAIR[N;1;3])-(+/REDAIR[N;10,20,30;3])
[116]  N←N+1
[117]  →(N≤D)ρL6
[118] ⍝ MODEL OUTPUT
[119]  'TOTAL REDAIR LOSSES'
[120]  ' '
[121]  TLOSSRA
[122]  ' '
[123]  ' '
[124]  'TOTAL REDAIR MINUTES ENGAGING BLUE FORCES'
[125]  ' '
[126]  TOTMINS
[127]  ' '
[128]  ' '
[129]  'TOTAL BLUEAD LOSSES '
[130]  ' '
[131]  TLOSSBAD
[137] ∇
```

APPENDIX C. SCENARIO INPUTS

TSTEP

1	1	1	2	8	4	7
2	1	1	2	8	4	5
3	1	1	1	8	4	3
4	2	1	1	4	3	1
5	2	1	2	4	3	3
6	2	1	2	4	3	3
7	2	1	1	2	3	1
8	2	1	2	2	2	3
9	2	1	2	2	2	5
10	2	1	2	2	2	7
11	2	1	2	2	2	7
12	2	1	2	2	1	5
13	2	2	1	2	2	3
14	2	2	1	2	2	1
15	2	2	1	2	2	2
16	2	2	1	2	2	2
17	2	2	1	2	4	1
18	2	2	1	2	6	3
19	2	2	2	2	6	5
20	2	2	2	6	5	7
21	2	2	2	8	5	7
22	2	2	2	8	4	5
23	2	2	1	7	4	3
24	2	2	1	7	3	1
25	2	2	1	7	4	2
26	2	2	2	7	4	4
27	2	2	1	6	4	2
28	2	2	1	6	5	1
29	2	2	2	3	5	3
30	2	2	2	3	6	5

STARTAIR

8	6	4
1	1	2

STARTAD

5	5	4
4	4	3
3	3	2

TYPEAD

1 1 2

ACQ

0.1	0.1
0.6	0.5

ALLOCATE

0.15	0.1
0.85	0.9

PKILL

0.5	0.6
0.5	0.6

ROF

1 1

MAXR

5 2

SHAPE

0.2 0.2

PTABLE

1	1	1	1	0.64	0.4	0.06	1	1	4	1.5	600	210
1	1	1	2	0.6	0.4	0.06	1	3	4	3	600	210
1	1	1	3	0.55	0.4	0.06	1	5	5	4.5	600	210
1	1	1	4	0.49	0.4	0.06	1	7	5	6	600	210
1	1	1	5	0.42	0.4	0.06	1	9	6	7.5	600	210
1	1	1	6	0.34	0.4	0.06	1	11	6	9	600	210
1	1	1	7	0.27	0.4	0.06	0	13	7	10.5	600	210
1	1	1	8	0.2	0.4	0.06	0	15	7	12	600	210
1	1	2	1	0.62	0.4	0.06	1	3	4	1.5	600	210
1	1	2	2	0.57	0.4	0.06	1	5	4	3	600	210
1	1	2	3	0.52	0.4	0.06	1	7	5	4.5	600	210
1	1	2	4	0.44	0.4	0.06	0	9	6	6	600	210
1	1	2	5	0.35	0.4	0.06	0	11	6	7.5	600	210
1	1	2	6	0.25	0.4	0.06	0	13	7	9	600	210
1	1	2	7	0.18	0.4	0.06	0	15	7	10.5	600	210
1	1	2	8	0.1	0.4	0.06	0	17	8	12	600	210
1	2	1	1	0.52	0.4	0.1	0	1	5	1.5	600	210
1	2	1	2	0.5	0.4	0.1	1	3	5	3	600	210
1	2	1	3	0.45	0.4	0.1	1	5	5	4.5	600	210
1	2	1	4	0.39	0.4	0.1	1	7	6	6	600	210
1	2	1	5	0.33	0.4	0.1	1	9	6	7.5	600	210
1	2	1	6	0.26	0.4	0.1	1	11	6	9	600	210
1	2	1	7	0.19	0.4	0.1	1	13	7	10.5	600	210
1	2	1	8	0.09	0.4	0.1	0	15	7	12	600	210
1	2	2	1	0.49	0.4	0.1	0	3	7	1.5	600	210
1	2	2	2	0.47	0.4	0.1	1	5	7	3	600	210
1	2	2	3	0.41	0.4	0.1	1	7	7	4.5	600	210
1	2	2	4	0.34	0.4	0.1	1	9	8	6	600	210
1	2	2	5	0.27	0.4	0.1	1	11	8	7.5	600	210
1	2	2	6	0.19	0.4	0.1	1	13	10	9	600	210
1	2	2	7	0.1	0.4	0.1	0	15	10	10.5	600	210
1	2	2	8	0.02	0.4	0.1	0	17	11	12	600	210
2	1	1	1	0.54	0.2	0.05	0	3	5	1.5	300	180
2	1	1	2	0.5	0.2	0.05	1	3	5	3	300	180
2	1	1	3	0.45	0.2	0.05	1	4	6	4.5	300	180
2	1	1	4	0.39	0.2	0.05	1	5	6	6	300	180
2	1	1	5	0.32	0.2	0.05	1	7	7	7.5	300	180
2	1	1	6	0.21	0.2	0.05	0	9	7	9	300	180
2	1	1	7	0.001	0.2	0.05	0	0	0	0	300	180
2	1	1	8	0.001	0.2	0.05	0	0	0	0	300	180
2	1	2	1	0.52	0.2	0.05	0	3	5	1.5	300	180
2	1	2	2	0.48	0.2	0.05	1	3	6	3	300	180
2	1	2	3	0.42	0.2	0.05	1	5	6	4.5	300	180
2	1	2	4	0.35	0.2	0.05	1	6	7	6	300	180
2	1	2	5	0.26	0.2	0.05	1	7	7	7.5	300	180

2	1	2	6	0.15	0.2	0.05	0	10	8	9	300	180
2	1	2	7	0.001	0.2	0.05	0	0	0	0	300	180
2	1	2	8	0.001	0.2	0.05	0	0	0	0	300	180
2	2	1	1	0.47	0.05	0.12	0	3	6	1.5	300	180
2	2	1	2	0.44	0.05	0.12	1	3	6	3	300	180
2	2	1	3	0.4	0.05	0.12	1	4	7	4.5	300	180
2	2	1	4	0.35	0.05	0.12	1	5	7	6	300	180
2	2	1	5	0.29	0.05	0.12	0	7	8	7.5	300	180
2	2	1	6	0.18	0.05	0.12	0	9	8	9	300	180
2	2	1	7	0.001	0.05	0.12	0	0	0	0	300	180
2	2	1	8	0.001	0.05	0.12	0	0	0	0	300	180
2	2	2	1	0.47	0.05	0.12	0	3	6	1.5	300	180
2	2	2	2	0.43	0.05	0.12	1	3	6	3	300	180
2	2	2	3	0.38	0.05	0.12	1	5	7	4.5	300	180
2	2	2	4	0.32	0.05	0.12	1	6	8	6	300	180
2	2	2	5	0.25	0.05	0.12	1	7	9	7.5	300	180
2	2	2	6	0.15	0.05	0.12	1	10	9	9	300	180
2	2	2	7	0.001	0.05	0.12	1	0	0	0	300	180
2	2	2	8	0.001	0.05	0.12	0	0	0	0	300	180

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