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A HETEROGENEOUS SHOOT-LOOK-SHOOT ATTRITION PROCESS

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Shoot-look-shoot attrition generally refers to cases in which the shooting side has (or can be adequately modeled as having) sufficient coordination among its shooters that (1) it can assign any particular shooter to engage any particular target, (2) engagements occur in succession, and the shooting side can assess the results of each engagement before being required to make succeeding assignments, and (3) the shooting side can assign shooters who have not yet made an attack (or who are capable of making another attack) to engage only those targets that either have not yet been engaged or have survived all prior engagements against them. This paper describes formulas that can be used to simulate shoot-look-shoot attrition processes in deterministic combat models.

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PREFACE

This paper was prepared under IDA contract MDA 903 89 C 0003, Task Order T-I6-682, Net Assessment Methodologies and Critical Data Elements for Strategic and Theater Force Comparisons, for the Capabilities Assessment Division of the Force Structure, Resource, and Assessment Directorate (J-8) of the Joint Chiefs of Staff, and has been written in partial fulfillment of that Task Order.

This paper describes formulas that can be used to simulate shoot-look-shoot attrition processes in deterministic combat models.

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ABSTRACT

Shoot-look-shoot attrition generally refers to cases in which the shooting side has (or can be adequately modeled as having) sufficient coordination among its shooters that (1) it can assign any particular shooter to engage any particular target, (2) engagements occur in succession, and the shooting side can assess the results of each engagement before being required to make succeeding assignments, and (3) the shooting side can assign shooters who have not yet made an attack (or who are capable of making another attack) to engage only those targets that either have not yet been engaged or have survived all prior engagements against them. This paper describes formulas that can be used to simulate shoot-look-shoot attrition processes in deterministic combat models.

CONTENTS

TABLE

A. INTRODUCTION

1. Purpose

Shoot-look-shoot attrition generally refers to cases in which the shooting side has (or can be adequately modeled as having) sufficient coordination among its shooters that (1) it can assign any particular shooter to engage any particular target, (2) engagements occur in succession, and the shooting side can assess the results of each engagement before being required to make succeeding assignments, and (3) the shooting side can assign shooters who have not yet made an attack (or who are capable of making another attack) to engage only those targets that either have not yet been engaged or have survived all prior engagements against them.

Shoot-look-shoot fire clearly requires a very high level of coordination among the shooters. If such coordination exists, there are several cases in which the shooting side would want to take advantage of this capability. For example, and perhaps most importantly, shoot-look-shoot fire generally results in killing more targets than other types of fire because fire is not wasted against previously destroyed targets.[1] Additionally, shoot-look-shoot fire might be able to save munitions (even if all of the targets are eventually killed) by not using munitions against targets that have already been destroyed. Finally, if the targets are of different value and if the shooting side can choose the order in which targets are attacked, it could engage only the most valuable targets until such targets are destroyed, and then switch to the second most valuable targets, and so on. This paper directly addresses the first rationale above for using shoot-look-shoot fire. Future work, perhaps using the results of this paper, could address other advantages of using shoot-look-shoot fire.

The goal of this paper is to describe a set of formulas that can be used to simulate shoot-look-shoot attrition processes in deterministic combat models. Such models generally represent time in terms of steps through time intervals, and they assess attrition for each time interval at the end of that interval. Further, these time intervals are generally sufficiently long that multiple engagements and multiple kills can occur within any one

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[1] In fully unconstrained shoot-look-shoot fire, no fire is ever wasted against previously destroyed targets. There are some variations of shoot-look-shoot fire that allow some fire to be so wasted; but, in general, the amount of such wasted fire will be less than that which occurs for other types of fire. One such variation in which some fire can occasionally be wasted is preallocated shoot-look-shoot fire against heterogeneous targets, which is the type of fire described in this paper.

interval. That is, these models take the numbers and capabilities of the various weapons on each side at the beginning of a relatively long time interval and compute the numbers of kills that occur during that interval directly as a function of these inputs, not by dividing the time interval into small subintervals and assessing attrition separately for each subinterval. See Section A of Chapter V of Reference [1] for a more thorough discussion of this structure for representing attrition over time.

Two versions of shoot-look-shoot attrition processes are discussed here. The first explicitly considers multiple types of shooters and multiple types of targets, but not multiple types of munitions. In particular, it uses probabilities of kill that depend on the type of shooter and type of target involved, but must be averaged over the types of munitions that could be used in a given shooter-target combination. The second version explicitly considers the use of multiple types of munitions.

2. Background

In addition to discussing general approaches for modeling attrition over time, Chapter V of Reference [1] defines and describes several specific attrition processes. (Computer code implementing those attrition processes is also given in Reference [1].) One of those processes involves shoot-look-shoot attrition with heterogeneous shooters and homogeneous targets. (Heterogeneity here means that multiple types of weapons on the side in question can be distinctly simulated; homogeneity means that only one, perhaps notional, type of weapon can be simulated on the given side.) That chapter also presents a somewhat simplistic extension of that process to handle heterogeneous targets, and it suggests (but does not describe in detail) a more organic extension to that process. In Section C below, the present paper describes in detail that suggested method for considering both heterogeneous shooters and heterogeneous targets in a shoot-look-shoot attrition process. Thus, this paper can be viewed as a natural follow-on to Reference [1].

Reference [2] presents a taxonomy for attrition processes that considers whether these processes address point fire or area fire, whether or not they address multiple types of munitions, and which of several levels of coordination among different shooters is being modeled. In addition, Reference [2] describes in detail the equations for all but two of the processes in that taxonomy--the two not described are the two versions of shoot-look-shoot fire considered here (i.e., shoot-look-shoot fire without explicit consideration of munitions, which is denoted by P5 in [2], and shoot-look-shoot fire with explicit consideration of munitions, which is denoted by PM5 in [2]). Thus, this paper can also be viewed as completing the structure proposed in Reference [2].

Clearly, the interested reader here may want to consult Chapter V of Reference [1] and all of Reference [2]. However, with one exception, the discussion below stands by itself in that it does not require that the reader be familiar with these references. The one exception is as follows. The attrition process below is unilateral in that invulnerable shooters on one side are firing at impotent targets on the other side. Reference [1] (in Section C of Chapter V) describes a method for converting such unilateral attrition assessments into bilateral attrition involving both lethal and vulnerable weapons on each side. This description is repeated in Section G of Reference [2]. This method is not described again here; the interested reader should consult one of these references for details.

3. Some Generic Types of Shoot-Look-Shoot Fire

There are many types of shoot-look-shoot fire. To put this paper in context, this section briefly describes some of the types not discussed here.

One type of shoot-look-shoot fire is to place an independent upper bound on the number of times that any particular target can be engaged per time period, no matter how many shooters are involved in the interaction. The versions of shoot-look-shoot fire discussed here place no such upper bound on engagements.

Another type of shoot-look-shoot fire involves assessing the results of firing at a target only after several engagements of that target, instead of after each engagement of that target. For example, the shooting side could assign two shooters to engage a target, then assess attrition after both engagements to determine if a third shooter should be assigned to that target. (In a sense, this is an example of shoot-shoot-look-shoot fire.) This form can be useful when targets are vulnerable only for a limited time, and the act of making an attrition assessment consumes part of this time. In the fire described below, attrition is assessed after each engagement.

If there are multiple types of shooters and there is an upper bound on the number of times any particular target can be engaged, then (in general) the attrition will depend on the order in which the shooters by type engage the targets (even if there is only one type of target). However, this is not the case when the targets are homogeneous and there is no upper bound on the number of engagements against any individual target.

If there are multiple types of targets, then the attrition can depend on the order in which the targets are engaged. In general, it is not computationally practical to compute an average attrition over all possible permutations of the order in which targets can be engaged. Also, it is not desirable to have the results of the attrition process depend on some particular but arbitrarily selected order for engaging targets. Reference [1] suggests (but does not implement) a method for addressing this problem that involves preallocating shooters to targets by type of target, and this approach is taken here.

Basically, this preallocation assigns a calculated number of shooters of each type to engage each type of target; those shooters can engage any target of their assigned type, but they can only engage targets of that type. Thus, this preallocation turns an attrition process with heterogeneous shooters and heterogeneous targets into J independent attrition processes, where J is the number of types of targets. Further, each of these J attrition processes is heterogeneous in shooter types but considers only one type of target. Accordingly, since the targets are homogeneous and there is no set upper bound on the number of shooters that can engage any particular target, the expected attrition is independent of the order in which the shooters engage the targets in each of these J attrition processes (this assertion is formally stated and proved in Section C.2 below).

It should be noted that preallocated shoot-look-shoot fire will generally kill somewhat fewer targets than unconstrained shoot-look-shoot fire. This happens because there is a possibility that one type of target is annihilated while another is not, thereby wasting the fire of those shooters assigned to attack the annihilated type of target but whose turn to fire comes after that annihilation has occurred. In unconstrained fire, those shooters could attack targets of the types that were not annihilated, but here the assumption of preallocation of fire precludes them from doing so. However, the impact of this characteristic of preallocated shoot-look-shoot fire may be very minor relative to the computational and order-independence advantages that this preallocation offers.

B. NOTATION

Reference [2] classifies attrition processes according to its Table 1 (which is reproduced as Table 1 here), and that reference gives attrition equations for all of the processes indicated on that table except for P5 and PM5. As noted above, this paper describes the calculation of attrition for P5 and PM5.

To facilitate using shoot-look-shoot fire in conjunction with the types of fire discussed in [2], the notation discussed here will generally follow the notation described in

Table 1. A Taxonomy for Attrition Equations

Coordination Assumptions	Point-Fire Equations Are Munitions Considered? No	 Yes	Area-Fire Equations Are Munitions Considered? No	 Yes
1) Uncoordinated Fire	P1	PM1	A1	AM1
2) Preallocated Fire	P2	PM2	n/a	n/a
3) Coordinated Fire within Shooter Types	P3		A3	
1) But only within Munition Types		PM3.1		AM3.1
2) And Across all Munition Types		PM3.2		AM3.2
4) Coordinated Fire Across all Shooter (and Munition) Types				
1) Uniform Fire by Numbers of Engagements or Salvos	P4.1	PM4.1	A4	AM4
2) Proportional Fire by Potential Kills	P4.2	PM4.2	n/a	n/a
5) Shoot-Look-Shoot Fire	P5	PM5	n/a	n/a

[2]. In particular, the definitions in Section 1, below, are identical to the definitions given in [2]--the notation defined here corresponds to the subset of the notation defined in [2] that is relevant to shoot-look-shoot fire. Section 2 then presents some extensions to that notation that are needed for the consideration of shoot-look-shoot fire.

1. Some General Notation Concerning Point Fire

Since shoot-look-shoot fire is a special case of point fire, the following general notation concerning point fire is relevant here.

a. Point-Fire Notation With or Without Munitions

The following notation applies whether or not multiple types of munitions are being considered.

I = the (input) number of types of shooters being considered; $I \in \{1,2,\ldots\}$.

s_i = the (input) number of shooters of type i for $i = 1,\ldots,I$; $s_i \in [0,\infty)$.

J = the (input) number of types of targets being considered; $J \in \{1,2,\ldots\}$.

t_j = the (input) number of targets of type j for $j = 1,\ldots,J$; $t_j \in [0,\infty)$.

v_j = the (input) fraction of targets of type j that are vulnerable to both point fire and area fire for $j = 1,\ldots,J$; $v_j \in [0,1]$.

z = the (input) number of point-fire combat zones where $1/z$ of the shooters are assumed to be attacking $1/z$ of the targets in each of these z zones; $z \in (0,\infty)$.

u_j = the (input) fraction of targets of type j that are vulnerable to point fire but not to area fire for $j = 1,\ldots,J$; $u_j \in [0,1-v_j]$.

$\tilde{t}_j$ = $(u_j + v_j)\, t_j/z$ = the (calculated) number of targets of type j per combat zone that are vulnerable to point fire in the attrition process being considered for $j = 1,\ldots,J$.

e_i = the average number of point-fire engagements that a shooter of type i makes per time period for $i = 1,\ldots,I$; $e_i \in [0,\infty)$.

$\tilde{s}_i$ = $e_i s_i/z$ = the (calculated) average number of point-fire engagements per combat zone that are made by all shooters of type i during the time period in question for $i = 1,\ldots,I$.

If multiple types of munitions are not being explicitly considered, then e_i is an input to the attrition calculation. If multiple types of munitions are being addressed, then e_i either can be an input or can be calculated from other inputs to the attrition calculations as described in Section C.2.c of Reference [2]. Either way, e_i here should incorporate relevant factors that affect average engagement rates, such as shooter readiness and target acquisition. In particular, note that e_i does not depend on t_j. Thus, in the attrition processes described below, the average number of engagements that a shooter of type i makes is assumed to be adequately approximated by a term, e_i, that is independent of the number of targets present (provided, of course, that there are some targets present). See Section A.2 of Reference [2] for further discussion of this assumption.

b. Point-Fire Notation Without Munitions

The following notation is used in point-fire attrition equations when multiple types of munitions are not being addressed.

p_{ij} = the (input) probability of kill per engagement by a shooter of type i when that shooter is making a point-fire engagement against a target of type j for i = 1,...,I and j = 1,...,J; $p_{ij} \in [0,1]$.

a_{ij} = $a_{ij}(\tilde{t})$ = the average fraction of engagements that shooters of type i make against targets of type j (out of all of the point-fire engagements made by those type-i shooters) when the target force, $\tilde{t}$, is $\{\tilde{t}_1,\ldots,\tilde{t}_J\}$, where i = 1,...,I and j = 1,...,J.

Allocations of fire can be computed in many ways. See Chapters III and IV of [1] for a discussion of a relatively wide variety of methods to compute such allocations. For the purpose of this paper, assume that these allocations are computed by the method described in Section B of Chapter III of [1]. (This method is used to determine allocations of fire in IDAGAM, INBATIM, TACWAR, JCS FPM, and IDAPLAN, all of which are dynamic combat models. Discussions of various aspects of this method can be found in Chapter II of [3], on pages 98 through 100 of [4], on pages 31 and 32 of [5], on pages 53 and 54 of [6] (see also pages 42 and 43 of [6]), and on pages 4 through 8 of [7].) This method uses the following inputs.

t^*_j = the (input) number of targets of type j in a typical target force, where this target force must contain a strictly positive number of targets of each type for j = 1,...,J; $t^*_j \in (0,\infty)$.

a^*_{ij} = the (input) fraction of point-fire engagements that shooters of type i would make, on average, against targets of type j (out of all of the point-fire engagements made by shooters of that type) when the target force consists of $t^*_{j'}$ weapons of type j', where i = 1,...,J, j = 1,...,J, and j' = 1,...,J; $a^*_{ij} \in [0,1]$.

The allocations of fire a_{ij} are then calculated by the formula

$$a_{ij} = a_{ij}(\bar{t}) = \begin{cases} \dfrac{a^*_{ij}\,\bar{t}_j/t^*_j}{\sum\limits_{j'=1}^{J} a^*_{ij'}\,\bar{t}_{j'}/t^*_{j'}} & a^*_{ij}\,\bar{t}_j/t^*_j > 0 \\ 0 & \text{otherwise} \end{cases}$$

for $i = 1,\ldots,I$ and $j = 1,\ldots,J$. See the aforementioned references for discussions concerning this method for allocating fire in combat models.

c. Point-Fire Notation With Munitions

The following notation is used in point-fire attrition equations when multiple types of munitions are being addressed.

M = the (input) number of types of munitions being considered; $M \in \{1,2,\ldots\}$.

p_{imj} = the (input) probability of kill per engagement by a shooter of type i when that shooter is making a point-fire engagement using munitions of type m against a target of type j for $i = 1,\ldots,I$, $m = 1,\ldots,M$, and $j = 1,\ldots,J$; $p_{imj} \in [0,1]$.

c_{imj} = $c_{imj}(\bar{t})$ = the average fraction of point-fire engagements by shooters of type i that are made using munitions of type m against targets of type j (out of all of the point-fire engagements made by the type-i shooters) when the target force, $\bar{t}$, is $\{\bar{t}_1,\ldots,\bar{t}_J\}$, where $i = 1,\ldots,I$, $m = 1,\ldots,M$, and $j = 1,\ldots,J$; $c_{imj} \in [0,1]$.

In general, c_{imj} will depend both on the composition of the target force (i.e., on $\bar{t}$) and on the number of munitions of the various types that are available for use by shooters of type i during the time period in question. This paper assumes the c_{imj} have been calculated in some reasonable manner based, in part, on these quantities

2. Notation Concerning Shoot-Look-Shoot Fire

a. Shooters

Section F of Reference [2] defines f_{i5} as being the (input) average fraction of the point-fire engagements by shooters of type i that are made using shoot-look-shoot fire. Since this paper is only concerned with this type of fire, it is useful to define s'_i here as

$$s'_i = f_{i5}e_is_i/z = f_{i5}\bar{s}_i \qquad \text{for } i = 1,\ldots,I ,$$

so that s'_i is the average number of shoot-look-shoot engagements per combat zone that can be made by all shooters of type i during the time period in question.

b. Targets

Reference [2] suggests assessing the attrition (if any) due to all other types of fire before computing and assessing attrition due to shoot-look-shoot fire. This suggestion is adopted here. Section F of Reference [2] defines $\Delta\bar{t}_j$ as the number of targets of type j that are killed by all area fire and all point fire except for shoot-look-shoot fire. Assessing this attrition first (and, as in [2], assuming that vulnerability to shoot-look-shoot fire is independent of vulnerability to other types of fire) means that the potential number of targets of type j facing shoot-look-shoot fire is given by

$$t_j - \Delta\bar{t}_j$$

for j = 1,...,J. Accordingly, for j = 1,...,J, let

$$t'_j = (u_j + v_j)\,(t_j - \Delta\bar{t}_j)\,/\,z ,$$

so that t'_j is the (calculated) number of targets of type j per combat zone that are vulnerable to shoot-look-shoot fire in the attrition process being considered.

3. Notation Concerning Functions

For any non-negative number x, let $\lfloor x \rfloor$ denote the largest integer that is less than or equal to x, and let $\langle x \rangle$ denote the fractional part of x so that

$$x = \lfloor x \rfloor + \langle x \rangle .$$

C. A HETEROGENEOUS SHOOT-LOOK-SHOOT ATTRITION PROCESS THAT DOES NOT CONSIDER MULTIPLE TYPES OF MUNITIONS

1. Assumptions

1) There can be multiple types of shooters and multiple types of targets, but each shooter of any particular type when engaging a target of any particular type uses the same munitions, on average, as any other shooter of that shooter type uses when engaging any target of that target type. Accordingly, the notation introduced in Sections B.1.a, B.1.b, B.2, and B.3 above applies.

2) At a fixed time in each combat zone, t'_j targets of type j become vulnerable to $s'_i a_{ij}$ engagements by shooters of type i for all $i = 1,\ldots,I$; and these targets are vulnerable only to these engagements. (That is, a_{ij} of the s'_i engagements that can be made by shooters of type i are allocated against targets of type j; these engagements can be made against any vulnerable target of type j in the combat zone, but they cannot be made against any other target.)

3) For all relevant i and j, t'_j and the products $s'_i a_{ij}$ are integers. (This assumption will be discussed further in Section 3, below.)

4) The shooters do not all fire at the same time. Instead, the shooters make engagements one-engagement-at-a-time according to the following rules. For $j = 1,\ldots,J$, let

$$\bar{s}_j = \sum_{i=1}^{I} s'_i a_{ij} ,$$

so $\bar{s}_j$ is the total number of engagements that can be made (per combat zone) against targets of type j. Label these engagements from 1 through $\bar{s}_j$ so that each engagement against a target of type j in a given zone has its own numeric label. Let σ_j be a permutation of $\{1,\ldots,\bar{s}_j\}$. That is, for each $v \in \{1,\ldots,\bar{s}_j\}$, $\sigma_j(v) \in \{1,\ldots,\bar{s}_j\}$, and if $v \neq v'$ then $\sigma_j(v) \neq \sigma_j(v')$. Engagement $\sigma_j(1)$ occurs first, followed by engagement $\sigma_j(2)$, and so on through engagement $\sigma_j(\bar{s}_j)$. When an engagement occurs, the shooter involved selects one target to fire upon from among the targets of type j remaining alive. That is, each shooter in each of its engagements knows the outcome of all previous engagements before it selects a target to attack, and it never attacks a target that was killed in a previous engagement. Since all targets of type j are identical, the choice of target (from among those of type j remaining alive) is irrelevant. If all of the targets of type j are killed before all of the possible engagements against them have occurred, the remaining engagements do not occur (and so the shooters involved lose these "turns" to fire).

5) Given that a shooter of type i engages a target of type j, it kills that target in that engagement with probability p_{ij}, otherwise the target is unaffected.

6) The firing processes are independent of the target selection process and are mutually independent of each other.

2. Independence of the Order of Fire

Theorem: Let $T_j(\sigma_j)$ denote the random number of targets of type j that are killed according to the assumptions above. Then $T_j(\sigma_j)$ is independent of σ_j. That is, if σ_j and σ''_j are two different permutations of $\{1,\ldots,\bar{s}_j\}$, then

$$\text{Prob}\ \{T_j(\sigma_j) = x\} = \text{Prob}\ \{T_j(\sigma''_j) = x\}$$

for all x.

Corollary: The expected number of targets of type j killed is independent of σ_j.

Proof of Theorem: Let $L_j(\bar{s}_j)$ denote the random number of lethal shots that would be fired if the number of engageable targets of type j were equal to the number of engagements, $\bar{s}_j$, that the shooters involved can make against type-j targets. By the independence assumptions above, $L_j(\bar{s}_j)$ is independent of σ_j. Since, given any value for t'_j,

$$T_j = \min\{t'_j,\ L_j(\bar{s}_j)\}\ ,$$

T_j is also independent of σ_j.

3. Results

This section is divided into two subsections. The first subsection gives equations that correctly calculate the expected number of targets killed when all of the assumptions stated above hold. However, in deterministic combat models, assumption 3 is not likely to hold. Accordingly, the second subsection below gives a reasonable procedure for calculating the number of targets killed when some (or all) of the $s'_i a_{ij}$ and t'_j are not integers but the other five assumptions above hold.

a. The Integral Case

Let $\Delta t'_j$ denote the expected number of targets of type j that would be killed in a given combat zone if all of the assumptions stated above hold, and let Δt_j denote the expected number of targets of type j killed in all of the combat zones according to these assumptions. Since the z combat zones are replicas of each other,

$$\Delta t_j = z\Delta t'_j \ .$$

The following equations calculate the exact (not estimated) value of $\Delta t'_j$ (and hence of Δt_j) under these assumptions.

If $t'_j \geq \bar{s}_j$, then each shooter against targets of type j is guaranteed a (live) target for each of its possible engagements, and so

$$\Delta t'_j = \sum_{i=1}^{I} s'_i a_{ij} p_{ij} \ .$$

To address the case where $t'_j < \bar{s}_j$, consider the following structure. Since, by the theorem above, the expected attrition is independent of the order of fire, assume for simplicity that the shooters fire in order by type (with all type-1 shooters firing first, followed by all type-2 shooters, and so forth). For $i = 1,\ldots,I$ and $\iota = 0,\ldots,t'_j$, let $r_{ij}(\iota)$ denote the probability that exactly ι targets of type j remain alive after all of the shooters of type i have fired but (for $i = 1,\ldots,I-1$) before any shooter of type $i + 1$ has fired. Set

$$r_{0j}(\iota) = \begin{cases} 1 & \iota = t'_j \\ 0 & \text{otherwise} \ . \end{cases}$$

Then, starting with $i = 1$, $r_{ij}(\iota)$ can be calculated recursively as follows.

Assume that $r_{i-1,j}(\iota)$ has already been calculated for all relevant ι. Set

$$S_{ij} = s'_i a_{ij}$$

and calculate values for $r_{ij}(\iota)$ using the formulas:

$$r_{ij}(\iota) = \sum_{\iota'=1}^{w} r_{i-1,j}(\iota')\, b(\iota'-\iota, S_{ij}, p_{ij}) \qquad \iota = 1,\ldots,t'_j$$

and

$$r_{ij}(0) = \sum_{\iota'=0}^{w} r_{i-1,j}(\iota')\, \bar{b}(\iota', S_{ij}, p_{ij}) \ ,$$

where, for $\iota = 0,1,\ldots,t'_j$,

$$w = w(t'_j, S_{ij}, \iota) = \min\{t'_j, S_{ij}+\iota\} \ ,$$

and, for all relevant values of their arguments,

$$b(\iota, s, p) = \binom{s}{\iota} p^{s}(1-p)^{1-s}$$

$$= \frac{s!\,p^{s}(1-p)^{1-s}}{\iota!(s-\iota)!}$$

and

$$\bar{b}(\iota', s, p) = \sum_{\iota=\iota'}^{s} b(\iota, s, p)\ .$$

Once values for $r_{\iota j}(\iota)$ have been determined for all relevant ι, $\Delta t'_j$ can be calculated by the formula:

$$\Delta t'_j = \begin{cases} t'_j - \sum_{\iota=1}^{t'_j} \iota\, r_{\iota j}(\iota) & t'_j < \bar{s}_j \\ \sum_{i=1}^{I} s'_i a_{ij} p_{ij} & t'_j \geq \bar{s}_j\ . \end{cases}$$

b. The Non-Integral Case

The formulas above can be used in one-time-only calculations in which $s'_i a_{ij}$ and t'_j are integers for all relevant i and j. However, if $s'_i a_{ij}$ or t'_j is not an integer for any relevant i or j, the formulas above for the case where $t'_j < \bar{s}_j$ cannot be evaluated, and so are useless as written. Further, a primary use for formulas that calculate expected attrition is in models that use these formulas to make deterministic estimates of attrition over multiple periods of combat. (See Section A of Chapter V of Reference [1] for further discussion of this structure.) In such models, the values of $s'_i a_{ij}$ and t'_j are unlikely to be integers anytime after the first assessment of attrition (and may not be integers even for the first attrition assessment). Accordingly, the formulas for the case in which $t'_j < \bar{s}_j$ as presented above need to be extended to cover non-integral shooters and targets in order to be useful for representing shoot-look-shoot fire in deterministic models of combat.

In addition to being able to address non-integral shooters and targets, the extended formulas here should have the followng three properties. First, they should reduce to the formulas given above when the numbers of shooters and targets are all integers. Second, the number of targets of type j killed according to these formulas should be a continuous function of the numbers of shooters and targets involved (else, a very small change in the numbers of weapons involved could yield a large change in results, which is generally not desirable for relevant combat models). Third, these extended formulas should be relatively as tractable as the formulas given for the all-integer case above. These criteria rule out many rounding and interpolation schemes for considering non-integral shooters and targets. However, the structure below appears to satisfy all three criteria.

Assume that assumptions 1, 2, 4, 5, and 6 stated above hold, but that assumption 3 may not hold. Let Δt_j and $\Delta t'_j$ be as defined above (except that assumption 3 may not hold).

If $t'_j \geq \bar{s}_j$, then the formulas presented above extend directly to the non-integral case, and so, if $t'_j \geq \bar{s}_j$, the formula

$$\Delta t'_j = \sum_{i=1}^{I} s'_i a_{ij} p_{ij}$$

can be used whether or not $s'_i a_{ij}$ and t'_j are integers.

To address the case where $t'_j < \bar{s}_j$ and the numbers of shooters and targets involved are not necessarily integers, consider the following extensions to the structure presented above. Let

$$T_j = \begin{cases} t'_j & \text{if } t'_j \text{ is an integer} \\ \lfloor t'_j \rfloor + 1 & \text{otherwise ,} \end{cases}$$

let

$$S_{ij} = \lfloor s'_i a_{ij} \rfloor ,$$

and let

$$r_{0j}(\iota) = \begin{cases} 1 & \iota = T_j \\ 0 & \text{otherwise .} \end{cases}$$

Then, starting with i = 1, the terms $r_{ij}(\iota)$ can be calculated recursively as follows.

Assume that $r_{i-1\,j}(\iota)$ has already been calculated for all relevant ι. To calculate $r_{ij}(\iota)$, first calculate $\bar{r}_{ij}(\iota)$ for all relevant ι as described next ($\bar{r}_{ij}$ will then be used to calculate r_{ij}). Let

$$\bar{r}_{ij}(\iota) = \sum_{\iota'=\iota}^{w} r_{i-1\,j}(\iota')b(\iota'-\iota,S_{ij},p_{ij}) \qquad \iota = 1,\ldots,T_j$$

and

$$\bar{r}_{ij}(0) = \sum_{\iota'=0}^{w} r_{i-1\,j}(\iota')\bar{b}(\iota',S_{ij},p_{ij}) \ ,$$

where, for $\iota = 0,1,\ldots,T_j$,

$$w = w(T_j,S_{ij},\iota) = \min\{T_j,S_{ij}+\iota\} \ ,$$

and $b(\bullet)$ and $\bar{b}(\bullet)$ are as defined above. Let

$$x_{ij} = \langle s'_i a_{ij} \rangle p_{ij} \ .$$

Then, once $\bar{r}_{ij}(\iota)$ has been calculated for all relevant ι, $r_{ij}(\iota)$ is calculated by setting

$$r_{ij}(\iota) = \begin{cases} \bar{r}_{ij}(0) + x\bar{r}_{ij}(1) & \iota = 0 \\ (1-x)\bar{r}_{ij}(\iota) + x\bar{r}_{ij}(\iota+1) & \iota = 1,\ldots,T_j-1 \\ (1-x)\bar{r}_{ij}(T_j) & \iota = T_j \end{cases}$$

Note that if $s'_i a_{ij}$ is an integer then $r_{ij}(\iota) = \bar{r}_{ij}(\iota)$ here, and if t'_j is also an integer then these terms equal $r_{ij}(\iota)$ as defined in the integral case above.

Once values for $r_{ij}(\iota)$ have been determined by these formulas, the resulting expected number of targets that would be killed if T_j targets were initially present, say k_j, is given by

$$k_j = T_j - \sum_{\iota=1}^{T_j} \iota r_{Ij}(\iota) .$$

Since (only) t'_j targets were initially present, a reasonable estimate for $\Delta t'_j$ (when $t'_j < \bar{s}_j$) is given by

$$\Delta t_j = k_j(t'_j / T_j) .$$

In particular, if the numbers of targets and engagements by shooters are all integers, this estimate is exact.

Combining the two cases ($t'_j < \bar{s}_j$ and $t'_j \geq \bar{s}_j$) gives

$$\Delta t'_j = \begin{cases} k_j(t'_j / T_j) & t'_j < \bar{s}_j \\ \sum_{i=1}^{I} s'_i a_{ij} p_{ij} & t'_j \geq \bar{s}_j . \end{cases}$$

4. Bounds

As discussed in Reference [1], estimating shoot-look-shoot attrition by using uniform fire attrition would (in general) understate the proper number of kills, while estimating shoot-look-shoot attrition by using Lanchester fire would (in general) overstate the proper number of kills. Accordingly, if when using the same force strengths and effectiveness parameters, uniform attrition is about equal to Lanchester attrition, then either of these can be used as a good estimate of shoot-look-shoot attrition, and the time-consuming recursive technique proposed here need not be used. However, if when using the same data, uniform fire attrition is significantly lower than Lanchester attrition, and if an estimate of shoot-look-shoot attrition is desired, then (at least) a few cases could be examined using this recursive technique to help make a (judgmental) estimate as to whether uniform attrition, Lanchester attrition, or some weighted average of attrition computations appear to most reasonably approximate the results of shoot-look-shoot fire.

D. AN EXTENSION THAT EXPLICITLY CONSIDERS MULTIPLE TYPES OF MUNITIONS

1. Assumptions

1) There can be multiple types of shooters which can use multiple types of munitions against multiple types of targets. Accordingly, the notation introduced in Sections B.1.a, B.1.c, B.2, and B.3 above applies.

2) At a fixed time in each combat zone, t'_j targets of type j become vulnerable to $s'_i c_{imj}$ engagements by shooters of type i using munitions of type m for all $i = 1,\ldots,I$ and all $m = 1,\ldots,M$; and these targets are vulnerable only to these engagements. (That is, c_{imj} of the s'_i engagements that can be made by shooters of type i using munitions of type m are allocated against targets of type j; these engagements can be made against any vulnerable target of type j in the combat zone, but they cannot be made against any other target.)

3) For all relevant i, m, and j, t'_j and the products $s'_i c_{imj}$ are integers.

4) The shooters do not all fire at the same time. Instead, the shooters make engagements one-engagement-at-a-time according to the following rules. For $j = 1,\ldots,J$, let

$$\bar{s}_j = \sum_{i=1}^{I} \sum_{m=1}^{M} s'_i c_{imj},$$

so $\bar{s}_j$ is the total number of engagements that can be made (per combat zone) against targets of type j. Label these engagements from 1 through $\bar{s}_j$ so that each engagement against a target of type j in a given combat zone has its own numeric label. Let σ_j be a permutation of $\{1,\ldots,\bar{s}_j\}$. That is, for each $v \in \{1,\ldots,\bar{s}_j\}$, $\sigma_j(v) \in \{1,\ldots,\bar{s}_j\}$, and if $v \neq v'$ then $\sigma_j(v) \neq \sigma_j(v')$. Engagement $\sigma_j(1)$ occurs first, followed by engagement $\sigma_j(2)$, and so on through engagement $\sigma_j(\bar{s}_j)$. When an engagement occurs, the shooter involved selects one target to fire upon from among the targets of type j remaining alive. That is, each shooter in each of its engagements knows the outcome of all previous engagements before it selects a target to attack, and it never attacks a target that was killed in a previous engagement. Since all targets of type j are identical, the choice of target (from among those of type j remaining alive) is irrelevant. If all of the targets of type j are killed before all of the possible engagements against them have occurred, the remaining engagements do not occur (and so the shooters involved lose these "turns" to fire).

5) Given that a shooter of type i engages a target of type j using munitions of type m, it kills that target with probability p_{imj}, otherwise the target is unaffected.

6) The firing processes are independent of the target selection process and are mutually independent of each other.

2. Independence of the Order of Fire

Let $T_j(\sigma_j)$ denote the random number of targets of type j that are killed according to the assumptions above. Then, as in Section C.2 above, $T_j(\sigma_j)$ is independent of σ_j. Accordingly, the expected number of targets of type j killed here is also independent of σ_j.

3. Results

As in Section C, if $t'_j \geq \bar{s}_j$ then each shooter on each of its possible engagements against targets of type j can attack a "live" target. Accordingly, the expected number of targets of type j that would be killed, Δt_j, is given by

$$\Delta t_j = \sum_{i=1}^{I} \sum_{m=1}^{M} s'_i c_{imj} p_{imj}$$

when $t'_j \geq \bar{s}_j$.

Due to the independence of the order of fire, the recursive technique proposed in Section C extends directly here to handle the case in which $t'_j < \bar{s}_j$. In particular, that scheme stepped through the shooters, one shooter-type at a time, calculating the probability (denoted by $r_{ij}(\iota)$) that ι targets of type j remain alive just after each particular type of shooter has been considered. The extension here is, within each shooter type, to also step through the munitions one munition-type at a time. That is, let $\bar{r}_{imj}(\iota)$ denote the probability that exactly ι targets of type j remain alive just after all of the shooters of type i using munitions of type m have fired. Then, when shooters of type i are being considered, first the probability that ι targets remain alive after the first type of munition for those shooters has been considered, say $\bar{r}_{i1j}(\iota)$, would be calculated using $s'_i c_{i1j}$ and p_{i1j}, then

$\tilde{r}_{i2j}(\iota)$ would be calculated using $s_i'c_{i2j}$ and p_{i2j}, and so on, until $\tilde{r}_{iMj}(\iota) = r_{ij}(\iota)$ is found. Given $r_{ij}(\iota)$, the relevant probabilities concerning shooters of type i+1 can be calculated by stepping through their munitions one type at a time.

Clearly, if the numbers of shooters and targets are large, then this technique is computationally intractable. A beneficial use of this technique might be to determine, for a small number of relatively small cases, the (uniform, Lanchester, or other) approximations that appear to be most suitable, and then to use these approximations for larger cases.

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