The CECOM Center for Night Vision and Electro-Optics

OPTOELECTRONIC WORKSHOPS

V

MODERN COHERENCE THEORY

May 18-19, 1988

sponsored jointly by

ARO-URI Center for Opto-Electronic Systems Research
The Institute of Optics, University of Rochester

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This workshop on "Modern Coherence Theory" represents the fifth of a series of intensive academic/government interactions in the field of advanced electro-optics, as part of the Army sponsored University Research Initiative. By documenting the associated technology status and dialogue it is hoped that this baseline will serve all interested parties towards providing a solution to high priority Army requirements. Responsible for program and program execution are Dr. Nicholas George, University of Rochester (ARO-URI) and Dr. Rudy Buser, CCNVEO.

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OPTOELECTRONIC WORKSHOP

ON

MODERN COHERENCE THEORY

Organizer: ARO-URI-University of Rochester
and CECOM Center for Night Vision and Electro-Optics

1. INTRODUCTION

2. SUMMARY -- INCLUDING FOLLOW-UP

3. VIEWGRAPH PRESENTATIONS

   A. Center for Opto-Electronic Systems Research
   Organizer -- Emil Wolf

   Modern Coherence Theory
   Emil Wolf

   B. Center for Night Vision and Electro-Optics
   Organizer -- Ward Trussell

   Army Applications of Coherence Phenomena
   Ward Trussell

   Detection of Laser Light and Holographic Filters
   Mark Norton

4. LIST OF ATTENDEES

1. INTRODUCTION

This workshop on "Modern Coherence Theory" represents the fifth of a series of intensive academic/government interactions in the field of advanced electro-optics, as part of the Army sponsored University Research Initiative. By documenting the associated technology status and dialogue it is hoped that this baseline will serve all interested parties towards providing a solution to high priority Army requirements. Responsible for program and program execution are Dr. Nicholas George, University of Rochester (ARO-URI) and Dr. Rudy Buser, CCNVEO.

2. SUMMARY AND FOLLOW-UP ACTIONS

Dr. Rudy Buser of NVEOC made opening remarks in which he outlined the aim of the workshop and the relevance of coherence phenomena to some of the research activities that are in progress at the laboratory.

The workshop which followed consisted of two parts. In the first part Professor Wolf presented an account of the basic concepts of optical coherence theory. In particular he discussed the distinction between temporal and spatial coherence and concepts such as coherence time, coherence area, coherence volume, and the degeneracy parameter. He then introduced correlation functions that more fully characterize coherence properties of light. After this summary Professor Wolf presented a review of some of the more important recent developments. In particular he discussed coherent-mode representation of light of any state of coherence, coherence theory of laser modes, radiation from partially coherent sources, coherence properties of Lambertian sources and the effects of source coherence on the spectrum of the emitted light. Coherence effects in scattering of light from random media was also considered.

The second part was a morning session on the second day which consisted of an open discussion. It was started by Mark Norton of NVEOC who talked about practical applications of coherence. This was followed by a lively discussion regarding the possibility of making coherence filters. Suggestions were also made about future research on such devices and other applications to sensors and dicscriminators, some of which might utilize stratified media or holographic filters.

In addition to Professor Wolf the following scientists from the Unversity of Rochester took part in the workshop: Professor N. George, Dr. T. Stone, and Mr. B. Cairns. All of them participated in the discussion.

SUMMARY COMMENTS

MODERN COHERENCE THEORY - Dr. Emil Wolf
May 18-19 1988

This workshop was of great interest to CNVEO personnel and was well attended. Coherence theory has direct application to Army programs in laser protection, detection of laser radiation, laser radar, vibration sensing, and communications. In the seminars on May 18, Dr. Wolf and Dr. Brian Cairns of Univ. of Rochester presented an excellent tutorial and overview of coherence research topics. There was good interaction between CNVEO and Univ. of Rochester scientists in relating the theory to practical application. On Thursday, Mark Norton of CNVEO discussed the difficulties in detecting laser radiation remotely and optimizing a receiver for this purpose. There was further active discussion of the feasibility of "coherence filters" for broadband laser protection. Dr. Wolf gave further insight in this topic and discussed a paper which will be published soon. He said that he planned to continue research in this area.

Most participants agreed that this workshop was valuable both for general understanding and for specific applications as outlined above.

C. Ward Trussell
C, Directed Energy Team
Laser Division

AGENDA

MODERN COHERENCE THEORY

Dr. Emil Wolf
University of Rochester

Wednesday, May 18, 1988 - Main Conference Room, Bldg. 305

10:00 AM. - Introduction/Opening Remarks - CNVEO

10:05 - Modern Coherence Theory - Dr. Wolf
Univ. of Rochester

Noon - Lunch

1:30 - 4:00 - Coherence Theory, continued
Dr. Wolf, Brian Cairns
Univ. of Rochester

Thursday, May 19, 1988 - The Arena, Bldg. 309

9:00 - Practical Applications of coherence - Mark Norton
CNVEO

9:15 - Open Discussion -

1. Can coherence filters be made?
2. What theoretical research needs to be done?
3. What experiments have been done?
4. What experiments should be done?
5. What are the coherence properties of photorefractive filters.
6. Other applications of this research.

11:00 - Recommendations for continued research/study

Noon - End

CENTER FOR OPTO-ELECTRONIC SYSTEMS RESEARCH
MODERN COHERENCE THEORY

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For fuller reviews of elementary classical coherence theory, see

M. Born and E. Wolf, Principles of Optics (Pergamon Press, Oxford and New York, 6th ed., 1980), chapt. X.

E. Wolf, "Basic concepts of optical coherence theory" in Proc. Symp. on Optical Masers, ed. J. Fox, (Brooklyn Polytechnic Press and J. Wiley, 1963), pp. 29-42.

L. Mandel and E. Wolf, "Coherence properties of optical fields", Rev. Mod. Phys. 37, pp. 231-287 (1965).

J. Peřina, Coherence of Light (Reidel, Boston, second ed., 1985).

J. W. Goodman, Statistical Optics (Wiley, New York, 1985).

INTERFERENCE AND STATISTICAL SIMILARITY

$$U(t) = a(t)\cos[\varphi(t) - \bar{\omega} t] \qquad (1)$$

$$\frac{\Delta\omega}{\bar{\omega}} \ll 1 \qquad \text{(Quasi-monochromatic light)} \qquad (2)$$

At P_1: $$U_1(t) = a_1(t)\cos[\varphi_1(t) - \bar{\omega} t]$$

At P_2: $$U_2(t) = a_2(t)\cos[\varphi_2(t) - \bar{\omega} t]$$

$$\left.\begin{array}{l} U_1(t) = a_1(t)\cos[\varphi_1(t) - \bar{\omega} t] \\ U_2(t) = a_2(t)\cos[\varphi_2(t) - \bar{\omega} t] \end{array}\right\} \qquad (3)$$

$$I(t) = [U_1(t) + U_2(t)]^2$$

$$\vdots$$

$$a_1^2\cos^2[\varphi_1 - \bar{\omega} t] + a_2^2\cos^2[\varphi_2 - \bar{\omega} t]$$
$$+ a_1 a_2 \cos[\varphi_1 + \varphi_2 - 2\bar{\omega} t] + a_1 a_2 \cos(\varphi_1 - \varphi_2) \qquad (4)$$

Suppose

$$\left.\begin{array}{l} a_1(t) = a_2(t) = \text{const} (= a) \\ \varphi_1(t),\ \varphi_2(t) \ \text{fluctuate} \end{array}\right\} \quad (5)$$

$$\langle I(t) \rangle = \frac{1}{2T}\int_{-T}^{T} I(t)\,dt \qquad (T \gg 1/\bar{\omega}) \qquad (6)$$

From (4) and (6)

$$\langle I(t) \rangle = \tfrac{1}{2}a^2 + \tfrac{1}{2}a^2 + 0 + a^2 \langle \cos(\varphi_1 - \varphi_2) \rangle$$

$$= a^2 \Big[1 + \underbrace{\langle \cos(\varphi_1 - \varphi_2) \rangle}_{\text{interference term}} \Big] \qquad (7)$$

Note:

$$0 \leq \langle I(t) \rangle \leq 2a^2$$

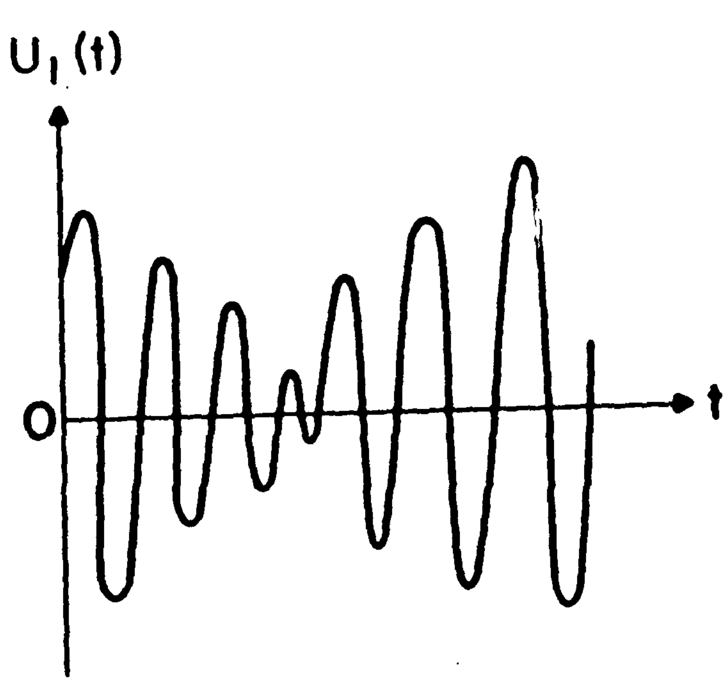
Interference term present when

$$\langle \cos(\varphi_1 - \varphi_2) \rangle \neq 0$$

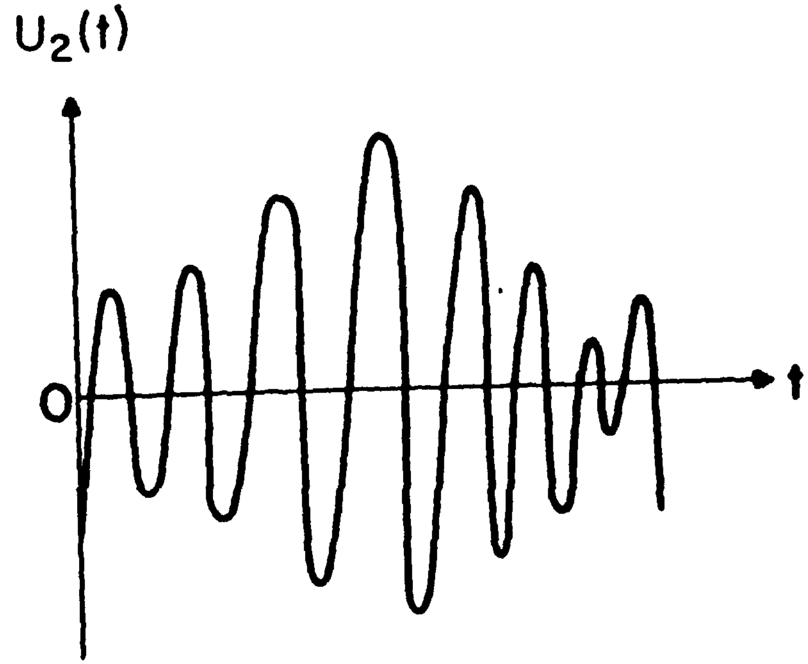
e.g. when

$$\varphi_1(t) - \varphi_2(t) = \text{Const.} \neq (2n+1)\pi/2$$

$\therefore$ <u>TO OBTAIN INTERFERENCE, LIGHT NEED NOT BE MONOCHROMATIC.</u> $\varphi_1(t)$ AND $\varphi_2(t)$ MAY EACH FLUCTUATE IN A RANDOM MANNER, PROVIDED ONLY THAT THEY HAVE SOME <u>STATISTICAL SIMILARITY</u> WITH EACH OTHER.

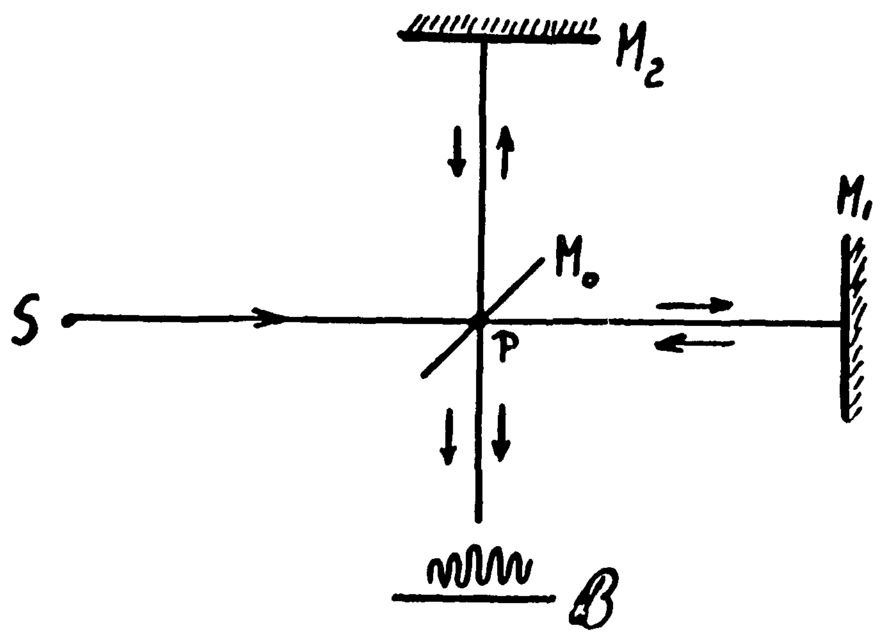


At P_1



At P_2

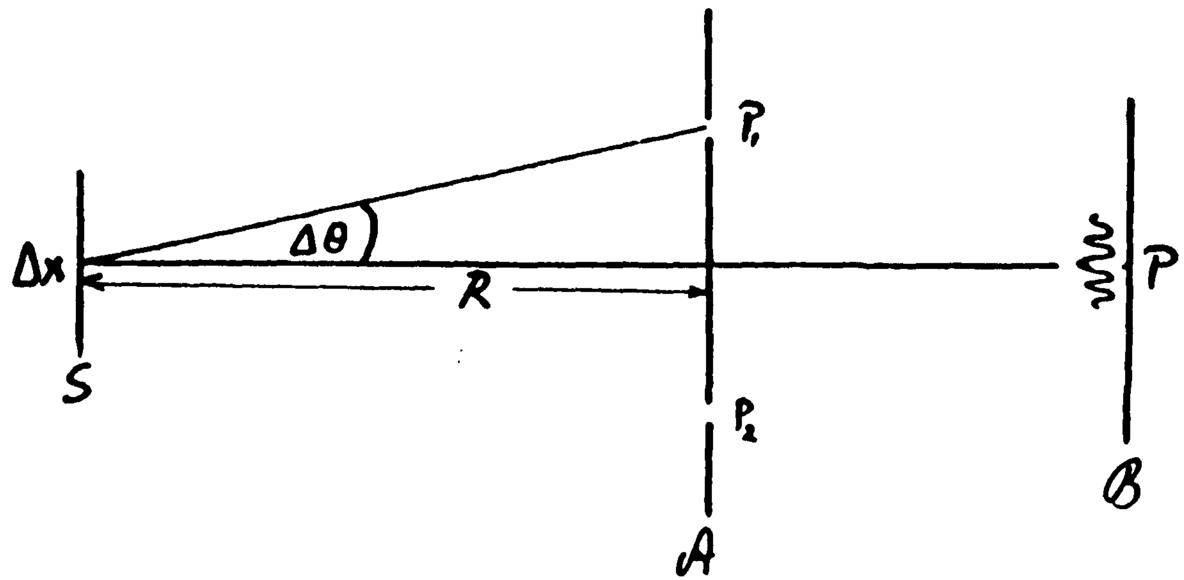
Temporal coherence and coherence time



$$\Delta t \, \Delta\omega \lesssim 1$$

Coherence time: $\Delta t \sim \dfrac{1}{\Delta\omega}$

Spatial coherence and coherence area



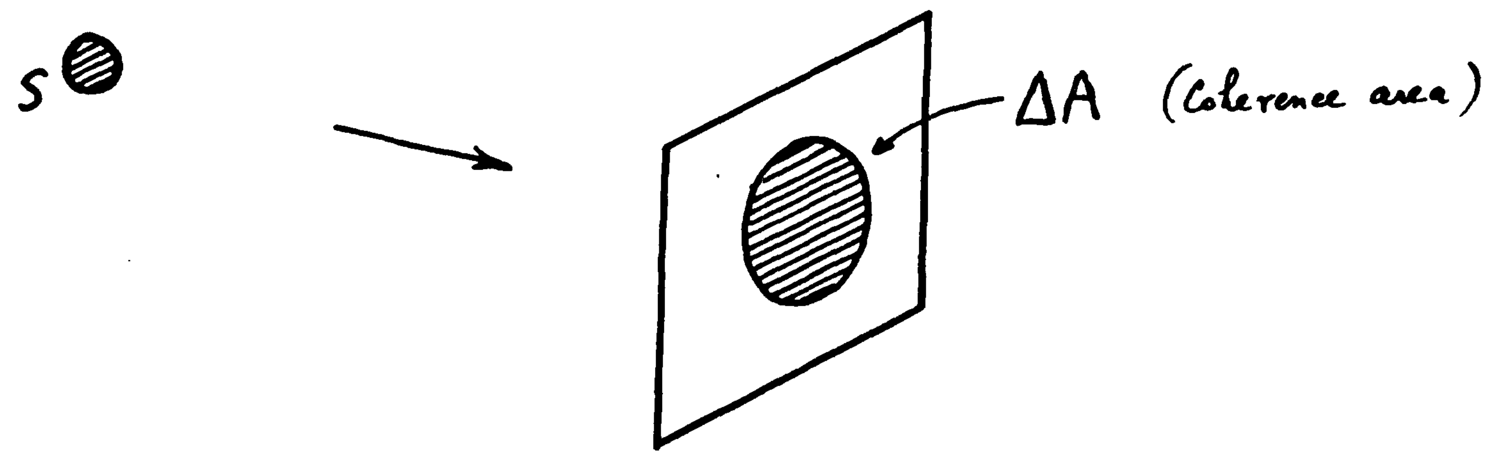
$$\left(k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \frac{2\pi\nu}{c}\right)$$

$$\Delta\theta\,\Delta x \lesssim \bar{\lambda}$$

Coherence area: $$\Delta A \sim (R\Delta\theta)^2 \sim R^2\left(\frac{\bar{\lambda}}{\Delta x}\right)^2 = \frac{c^2}{\bar{\nu}^2}\,\frac{R^2}{S} \qquad [S = (\Delta x)^2]$$

$$\Delta\Omega = \frac{\Delta A}{R^2} = \frac{c^2}{\bar{\nu}^2}\,\frac{1}{S}$$

Degeneracy parameter



Degeneracy:

$$\delta = \frac{1}{2} E_\nu \Delta\nu\, S\, \Delta\Omega\, \Delta t$$

$$\sim \frac{1}{2} \frac{c^2}{\nu^2} E_\nu \qquad (1)$$

For blackbody radiation:

$$E_\nu = \frac{2\nu^2}{c^3} \frac{1}{e^{h\nu/kT}-1} \qquad (2)$$

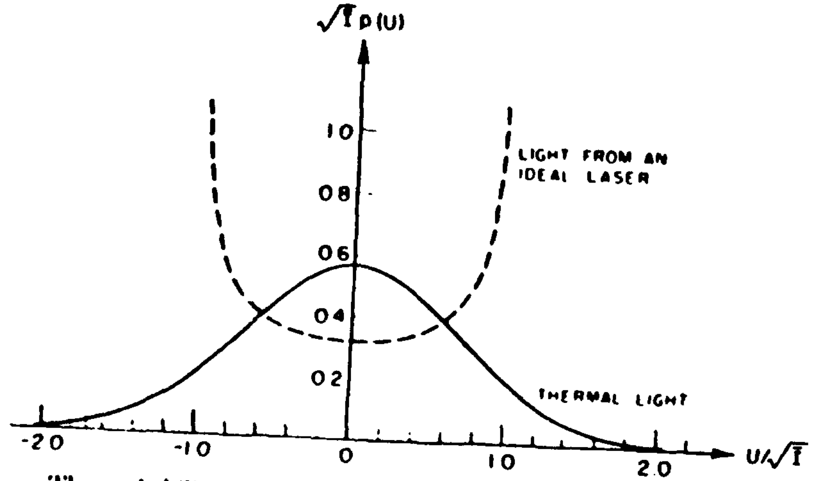
$$\delta \sim \frac{1}{e^{h\nu/kT}-1} \qquad (3)$$

L11-86

DIFFERENCES AND TYPICAL VALUES

		Thermal Source	Laser
Emission		spontaneous (for $T \lesssim 50{,}000°$ K)	stimulated
Minimum bandwidth ($\Delta\nu$)		10^9 Hz	1 Hz
Coherence time (Δt)		10^{-9} sec	1 sec
Coherence length (Δ)		1 m	10^9 m
Maximum degeneracy (δ)		10^{-3}	$10^3 - 10^{15}$
Probabilities:	$p(U)$	$\sim e^{-U^2/\sigma^2}$	$\sim \frac{1}{\sqrt{\bar{I}-U^2}}$
	$p(I)$	$\sim e^{-I/\bar{I}}$	$\sim \delta(I-\bar{I})$

22-3/77



The probability densities relating to thermal light and to light from an ideal laser. (After L. Mandel, 1964, *Quantum Electronics*, Proc. Third International Congress, edited by N. Bloembergen and P. Grivet, New York : Columbia University Press ; Paris : Dunod, p. 101.)

L11-86

THE COMPLEX ANALYTIC SIGNAL (D. GABOR, 1946)

$$U(t) = \int_{-\infty}^{\infty} v(\omega)\, e^{-i\omega t}\, d\omega \qquad (1)$$

Reality:

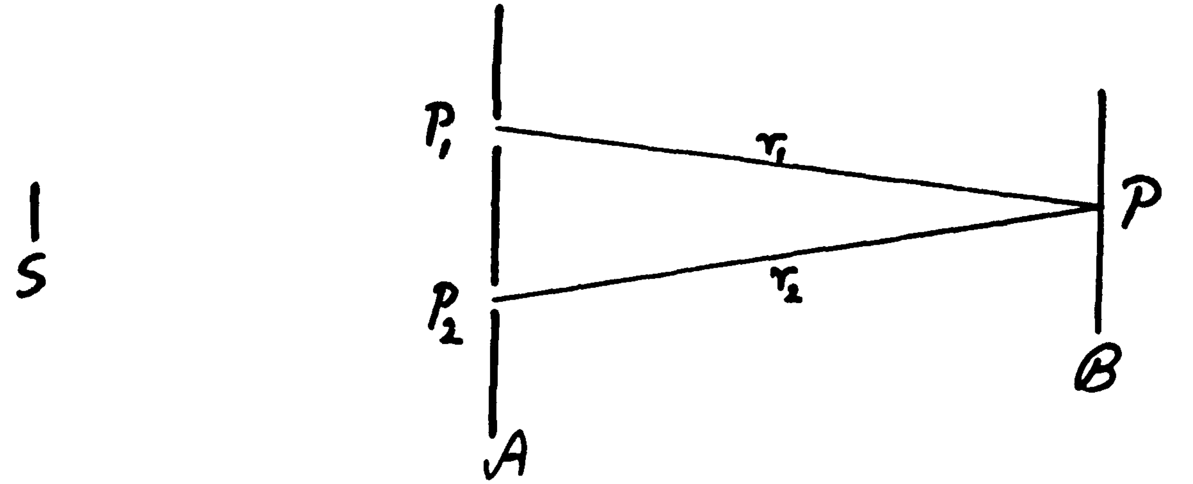
$$v(-\omega) = v^{*}(\omega) \qquad (2)$$

Analytic Signal:

$$V(t) = \int_{0}^{\infty} v(\omega)\, e^{-i\omega t}\, d\omega \qquad (3)$$

$$U(t) = 2\mathcal{R}\, V(t) \qquad (4)$$

$$V(t)\,V^{*}(t) = \frac{1}{2}\,\overline{U^{2}(t)} \qquad (5)$$



$t_1 = r_1/c$

$t_2 = r_2/c$

$$V(P,t) = K_1 V(P_1, t-t_1) + K_2 V(P_2, t-t_2)$$

$$I(P) = \langle V(P,t) V^*(P,t) \rangle$$

$$= I^{(1)}(P) + I^{(2)}(P) + \underbrace{2\mathcal{R}\,\Gamma_{12}(\tau)}_{\text{interference term}}, \quad (\tau = t_1 - t_2) \qquad (1)$$

<u>Mutual coherence function:</u>

$$\Gamma_{12}(\tau) = \langle V(P_1, t+\tau) V^*(P_2, t) \rangle \qquad (2)$$

$$\nabla_j^2 \Gamma_{12}(\tau) = \frac{1}{c^2} \frac{\partial^2 \Gamma_{12}(\tau)}{\partial \tau^2}, \quad (j = 1, 2) \qquad (3)$$

Complex degree of coherence:

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\Gamma_{11}(0)}\sqrt{\Gamma_{22}(0)}} \qquad (1)$$

$$0 \leq |\gamma_{12}(\tau)| \leq 1$$

incoherence — complete coherence

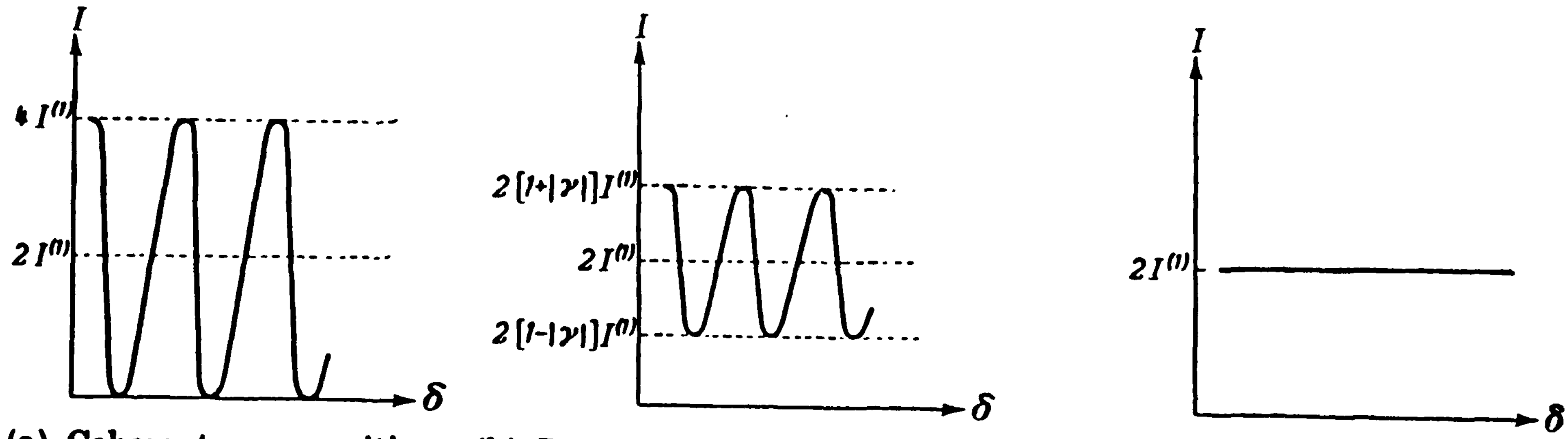
Interference law: $\left(\frac{\Delta\omega}{\bar{\omega}} \ll 1,\ |\delta| \ll \frac{\bar{\omega}}{\Delta\omega},\ I^{(2)} = I^{(1)}\right)$

$$\boxed{I(P) = 2I^{(1)}(P)\left\{1 + |\gamma_{12}(\tau)| \cos\left[\arg \alpha_{12}(\tau) - \delta\right]\right\}} \qquad (2)$$

$$\alpha_{12}(\tau) = \arg \gamma_{12}(\tau) + \bar{\omega}\tau, \quad \delta = \bar{\omega}\tau, \quad \tau = \frac{r_2 - r_1}{c} \qquad (3)$$

Visibility of fringes:

$$\mathcal{V}(P) = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = |\gamma_{12}(\tau)| \qquad (4)$$



(a) Coherent superposition ($|\gamma| = 1$)

(b) Partially coherent superposition ($0 < |\gamma| < 1$)

(c) Incoherent superposition ($\gamma = 0$)

Intensity distribution in the interference pattern produced by two quasi-monochromatic beams of equal intensity $I^{(1)}$ and with degree of coherence $|\gamma|$.

L11-86

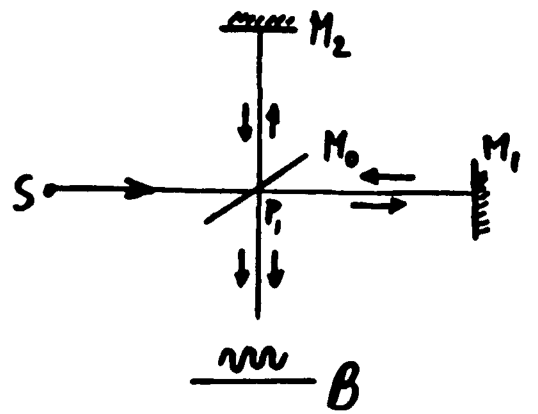
QUANTITATIVE CHARACTERIZATION OF TEMPORAL AND SPATIAL COHERENCE

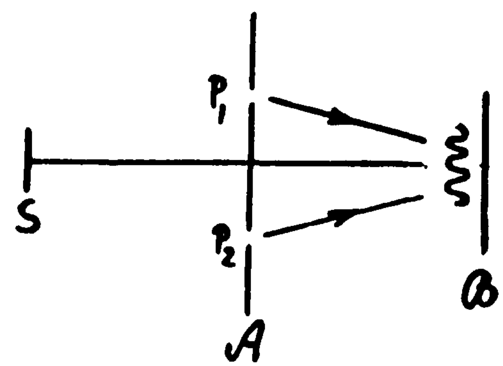
$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\Gamma_{11}(0)}\,\sqrt{\Gamma_{22}(0)}}$$

$$0 \leq |\gamma_{12}(\tau)| \leq 1$$

incoherence

complete coherence





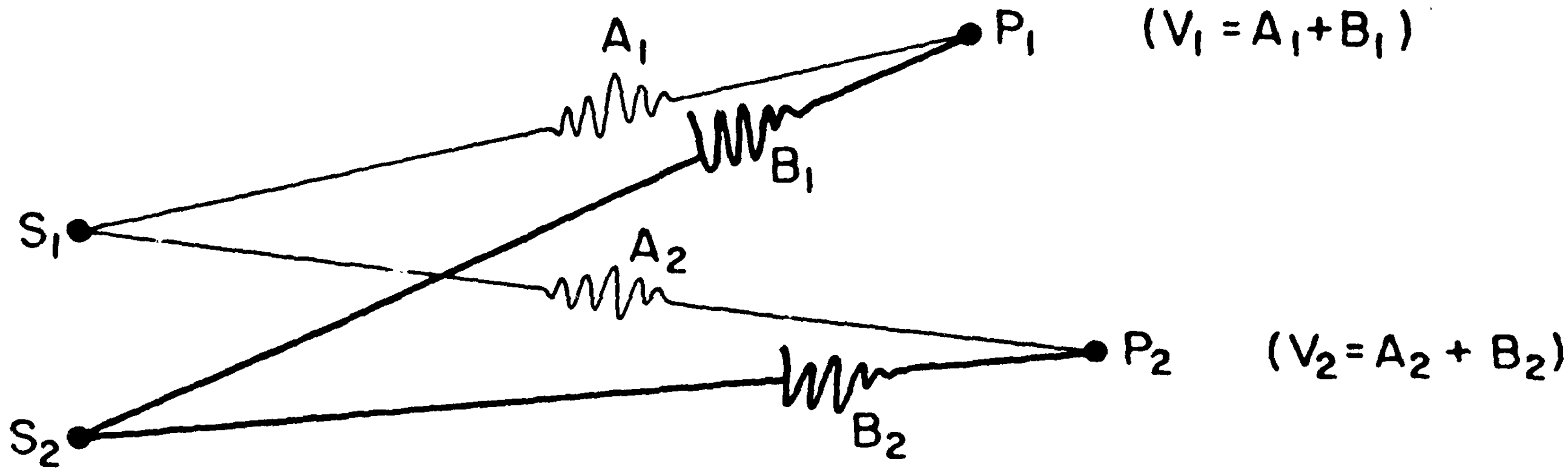
Temporal coherence: $\gamma_{11}(\tau)$

Spatial coherence: $\gamma_{12}(0)$

Visibility of fringes: $|\gamma|$

Position of minima and maxima: $\arg \gamma$

GENERATION OF SPATIAL COHERENCE FROM UNCORRELATED SOURCE



$$S_1P_1 \approx S_1P_2, \quad S_2P_1 \approx S_2P_2 \qquad (11)$$

$$\langle A_i B_j \rangle = 0, \quad (i, j = 1,2) \qquad (12)$$

$$A_2 \approx A_1 \qquad B_2 \approx B_1 \qquad (13)$$

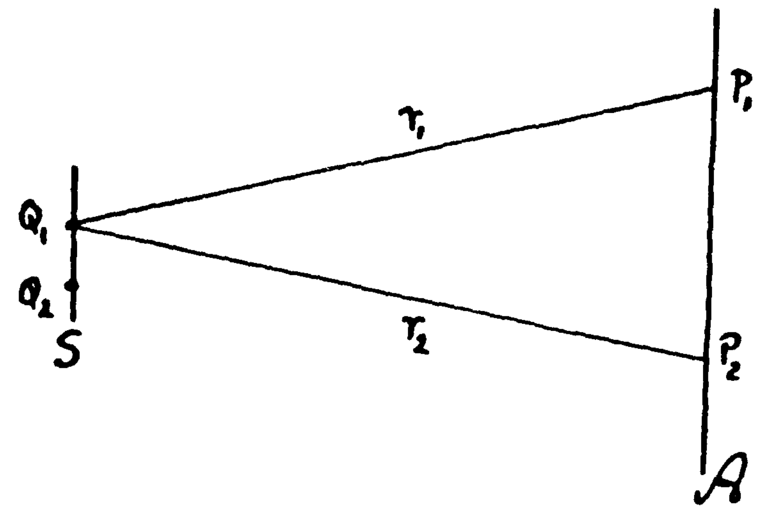
$$\text{At } P_1: \quad V_1 = A_1 + B_1 \qquad (14)$$

$$\text{At } P_2: \quad V_2 = A_2 + B_2 \qquad (15)$$

$$\boxed{V_1 \approx V_2 \qquad (16)} \quad \text{Similarity} \rightarrow \text{Coherence}$$

Generation of spatial coherence from an incoherent source

The van Cittert-Zernike theorem

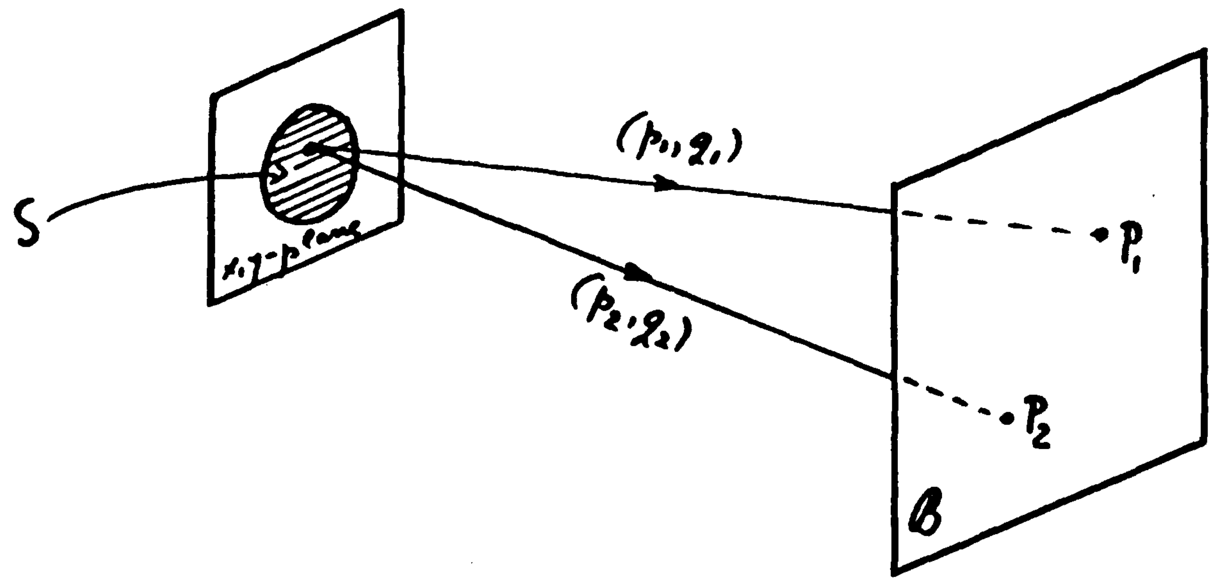


Propagation of coherence:
$$\nabla_j^2 \Gamma_{12}(\tau) = \frac{1}{c^2}\frac{\partial^2 \Gamma_{12}(\tau)}{\partial \tau^2}, \quad (j=1,2) \qquad (1)$$

Boundary conditions:
(incoherent source)
$$\Gamma(Q_1,Q_2,\tau) = j(Q_2-Q_1)\,\delta(Q_2-Q_1)\,e^{-2\pi i\bar{\nu}\tau} \qquad (2)$$
$(\tau \ll 1/\Delta\nu)$

Solution:
$$\Gamma(P_1,P_2,0) \approx \iint_S j(Q_1)\,\frac{e^{i\bar{k}(r_1-r_2)}}{r_1 r_2}\,dQ_1 \qquad (3)$$

FAR-ZONE FORM OF VAN CITTERT-ZERNIKE THEOREM



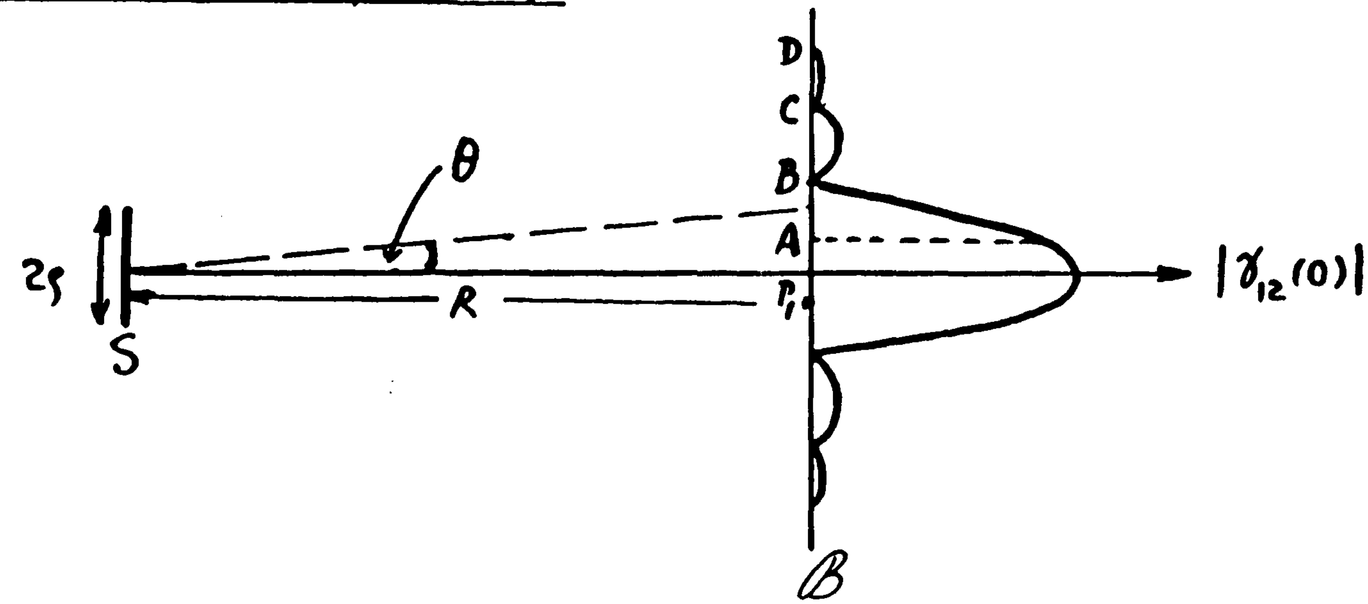
(Plane $\mathcal{B}$ in the far zone of source)

$$\gamma_{12}(0) = \frac{1}{N} \iint_S j(x,y)\, e^{i\bar{k}[(p_1-p_2)x + (q_1-q_2)y]}\, dx\, dy \quad (1)$$

$$N = \iint_S j(x,y)\, dx\, dy \quad (2)$$

Distribution of coherence from a spatially incoherent source

Uniform circular source of radius ρ:



On $\mathcal{B}$:

$$\gamma_{12}(0) = \frac{2J_1(v)}{v}, \qquad v = \bar{k}\rho\sin\theta, \qquad (\bar{k}R \gg 1)$$

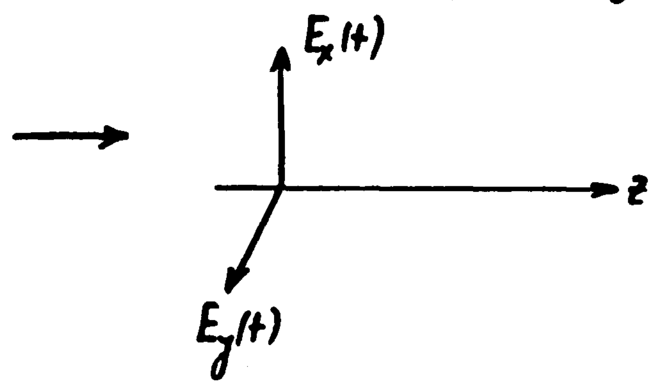
① $P_2 = A$: Radius of region of coherence: $\sin\theta \sim 0.16\bar{\lambda}/\rho$

② $P_2 = B$: First incoherence: $\sin\theta \sim 0.61\bar{\lambda}/\rho$

③ $P_2 = C$: Second incoherence: $\sin\theta \sim 1.11\bar{\lambda}/\rho$

⋮

Partial polarization — Coherency matrices



$$\mathcal{E} = \begin{bmatrix} \langle E_x E_x^* \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle E_y E_y^* \rangle \end{bmatrix} \qquad (1)$$

$$\mu_{xy} = \frac{\langle E_x E_y^* \rangle}{\sqrt{\langle E_x E_x^* \rangle}\sqrt{\langle E_y E_y^* \rangle}} \qquad (2)$$

$$0 \le |\mu_{xy}| \le 1 \qquad (3)$$

$$\underset{(\theta)}{\mathrm{Max}} \; |\mu_{xy}| = P \quad \text{(degree of polarization)} \qquad (4a)$$

$$= \sqrt{1 - \frac{4|\mathcal{E}|}{(\mathrm{Tr}\,\mathcal{E})^2}} \qquad (4b)$$

Mutual coherence function

$$\Gamma(\underline{x}_1, \underline{x}_2, \tau) = \langle V(\underline{x}_1, t+\tau) V^*(\underline{x}_2, t) \rangle$$

Coherency tensors

$$\mathcal{E}_{jk}(\underline{x}_1, \underline{x}_2, \tau) = \langle E_j(\underline{x}_1, t+\tau) E_k^*(\underline{x}_2, t) \rangle$$

$$\mathcal{H}_{jk}(\underline{x}_1, \underline{x}_2, \tau) = \langle H_j(\underline{x}_1, t+\tau) H_k^*(\underline{x}_2, t) \rangle$$

$$\mathcal{M}_{jk}(\underline{x}_1, \underline{x}_2, \tau) = \langle E_j(\underline{x}_1, t+\tau) H_k^*(\underline{x}_2, t) \rangle$$

$$\mathcal{N}_{jk}(\underline{x}_1, \underline{x}_2, \tau) = \langle H_j(\underline{x}_1, t+\tau) E_k^*(\underline{x}_2, t) \rangle$$

$(j, k = x, y, z)$

Field equations (in free space)

$$\varepsilon_{jkl}\, \partial_k^1 \mathcal{E}_{lm} + \frac{1}{c}\frac{\partial}{\partial\tau} \mathcal{N}_{jm} = 0$$

$$\varepsilon_{jkl}\, \partial_k^1 \mathcal{M}_{lm} + \frac{1}{c}\frac{\partial}{\partial\tau} \mathcal{H}_{jm} = 0$$

$$\varepsilon_{jkl}\, \partial_k^1 \mathcal{N}_{lm} - \frac{1}{c}\frac{\partial}{\partial\tau} \mathcal{E}_{jm} = 0$$

$$\varepsilon_{jkl}\, \partial_k^1 \mathcal{H}_{lm} - \frac{1}{c}\frac{\partial}{\partial\tau} \mathcal{M}_{jm} = 0$$

$$\partial_j^1 \mathcal{E}_{jk} = \partial_j^1 \mathcal{H}_{jk} = \partial_j^1 \mathcal{M}_{jk} = \partial_j^1 \mathcal{N}_{jk} = 0$$

$$\varepsilon_{jkl} = \begin{cases} +1 & \text{if } j,k,l \text{ \underline{even} permutations of } 1,2,3 \\ -1 & \text{if } j,k,l \text{ \underline{odd} permutations of } 1,2,3 \\ 0 & \text{if two indices equal} \end{cases}$$

$$\partial_k^1 = \frac{\partial}{\partial x_1^k} \quad (k = 1, 2, 3)$$

L11-86.

Higher-order coherence functions (tensors)

Mutual coherence function (2nd order): $\Gamma(\underline{x}_1, \underline{x}_2, \tau) = \langle V(\underline{x}_1, t+\tau) V^*(\underline{x}_2, t) \rangle$

Coherence function of order (m,n) – scalar field

$$\Gamma^{(m,n)}(\underline{x}_1, \underline{x}_2, \ldots \underline{x}_{m+n}; t_1, t_2, \ldots t_{m+n})$$

$$= \langle V^*(\underline{x}_1, t_1) V^*(\underline{x}_2, t_2) \ldots\ldots V^*(\underline{x}_m, t_m) V(\underline{x}_{m+1}, t_{m+1}) \ldots\ldots V(\underline{x}_{m+n}, t_{m+n}) \rangle,$$

Coherence tensors of order (m,n) – e.m. field

$$^{e}\Gamma^{(m,n)}_{j_1 j_2 \ldots j_{m+n}}(\underline{x}_1, \underline{x}_2, \ldots \underline{x}_{m+n}; t_1, t_2, \ldots t_{m+n})$$

$$= \langle E^*_{j_1}(\underline{x}_1, t_1) E^*_{j_2}(\underline{x}_2, t_2) \ldots\ldots E^*_{j_m}(\underline{x}_m, t_m) E_{j_{m+1}}(\underline{x}_{m+1}, t_{m+1}) \ldots\ldots E_{j_{m+n}}(\underline{x}_{m+n}, t_{m+n}) \rangle,$$

$^{h}\Gamma^{(m,n)}_{j_1 j_2 \ldots j_{m+n}}$, etc

CORRELATIONS IN THE SPACE-FREQUENCY DOMAIN

MUTUAL COHERENCE FUNCTION: $$\Gamma(\underline{r}_1, \underline{r}_2, \tau) = \langle V(\underline{r}_1, t+\tau) V^*(\underline{r}_2, t) \rangle \quad (1)$$

CROSS-SPECTRAL DENSITY: $$W(\underline{r}_1, \underline{r}_2, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Gamma(\underline{r}_1, \underline{r}_2, \tau) e^{i\omega\tau} d\tau \quad (2)$$

$$v(\underline{r}, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} V(\underline{r}, t) e^{i\omega t} dt \quad (3)$$

$$\langle v(\underline{r}_1, \omega) v^*(\underline{r}_2, \omega') \rangle = W(\underline{r}_1, \underline{r}_2, \omega) \delta(\omega - \omega') \quad (4)$$

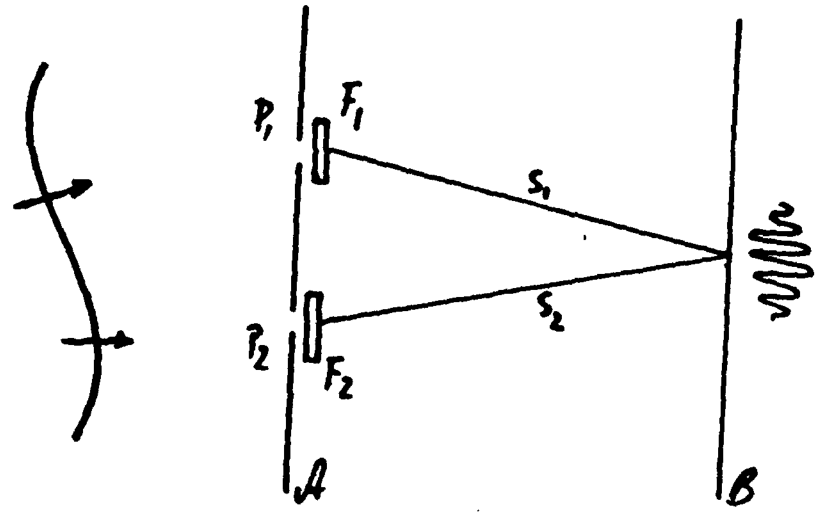
COMPLEX DEGREE OF SPECTRAL COHERENCE*:

$$\mu(\underline{r}_1, \underline{r}_2, \omega) = \frac{W(\underline{r}_1, \underline{r}_2, \omega)}{\sqrt{W(\underline{r}_1, \underline{r}_1, \omega)} \sqrt{W(\underline{r}_2, \underline{r}_2, \omega)}} \quad (5)$$

$$0 \leq |\mu(\underline{r}_1, \underline{r}_2, \omega)| \leq 1 \quad (6)$$

* L. MANDEL and E. WOLF, J. Opt. Soc. Amer., 66, 529 (1976)

YOUNG'S INTERFERENCE EXPERIMENT WITH NARROW-BAND LIGHT*



F_1, F_2 are identical narrow-band filters

$T(\omega)$ is the complex amplitude transmission function of each filter

$$\langle v(P_1,\omega)\, v^*(P_2,\omega')\rangle = W(P_1,P_2,\omega)\,\delta(\omega-\omega') \qquad (7)$$

$$v(P_j,\omega) \longrightarrow T(\omega)\, v(P_j;\omega) \qquad (j=1,2) \qquad (8)$$

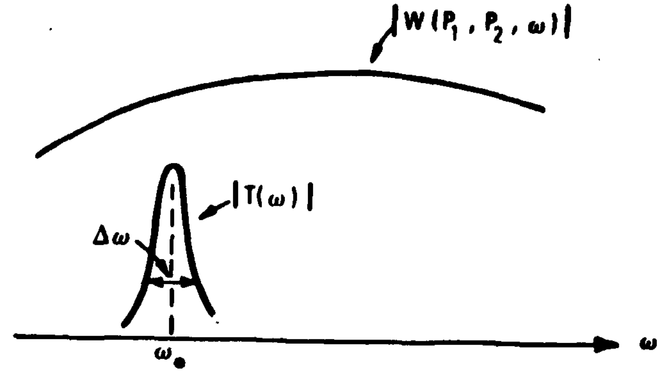
$$\langle T(\omega)\, v(P_1,\omega)\, T^*(\omega')\, v^*(P_2,\omega')\rangle = W^{(+)}(P_1,P_2,\omega)\,\delta(\omega-\omega') \qquad (9)$$

$$W^{(+)}(P_1,P_2,\omega) = |T(\omega)|^2\, W(P_1,P_2,\omega) \qquad (10)$$

$$\Gamma^{(+)}(P_1,P_2,\tau) = \int_0^\infty |T(\omega)|^2 W(P_1,P_2,\omega)\, e^{-i\omega\tau}\, d\omega \qquad (11)$$

F. WOLF, Opt. Lett., 8, 250 (1983); P. DE SANTIS, F. GORI, G. GUATTARI, C. PALMA, J.M. WEBSTER, If $\Delta\omega$ is sufficiently small (see figure), Eq.(11) gives

$$\Gamma^{(+)}(P_1, P_2, \omega) = W(P_1, P_2, \omega)\int_0^\infty |T(\omega)|^2 e^{-i\omega\tau} d\omega \qquad (12)$$



$\Downarrow$

$$\boxed{\gamma^{(+)}(P_1, P_2, \tau) = \mu(P_1, P_2, \omega_0)\,\theta(\tau) \qquad (13)}$$

$$\theta(\tau) = \frac{\int_0^\infty |T(\omega)|^2 e^{-i\omega\tau} d\omega}{\int_0^\infty |T(\omega)|^2 d\omega} \qquad (14)$$

$\theta(\tau)$ = filter function

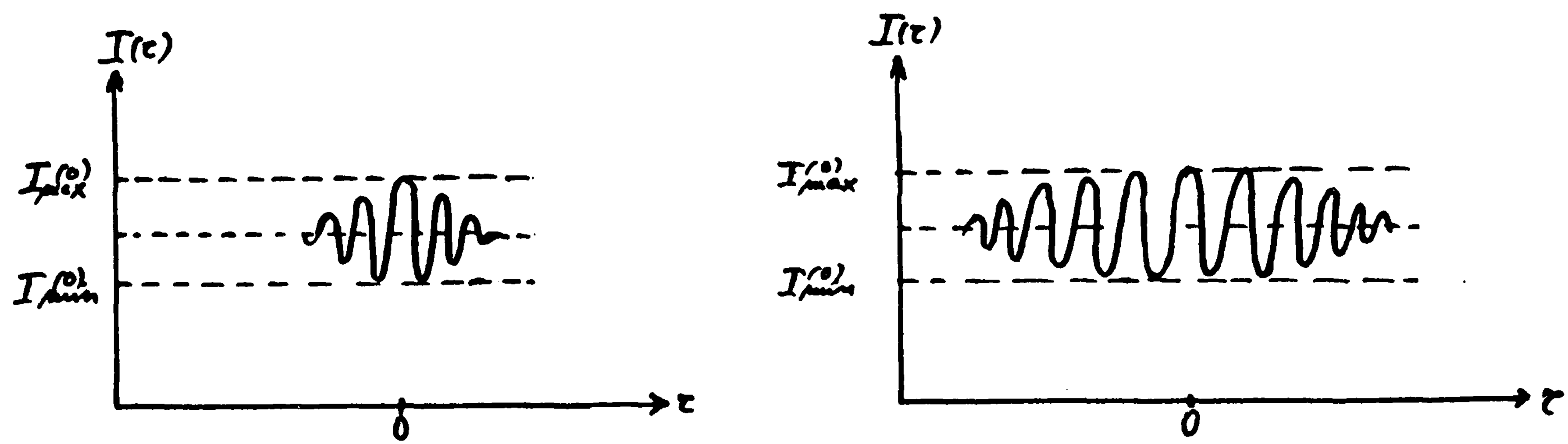
$$\max_\tau |\theta(\tau)| = \theta(0) = 1 \qquad (15)$$

$$\therefore \quad \max_\tau |\gamma^{(+)}(P_1, P_2, \tau)| = |\gamma^{(+)}(P_1, P_2, 0)| = |\mu(P_1, P_2, \omega_0)| \qquad (16)$$

NOTE: Maximum fringe visibility does <u>not</u> increase as $\Delta\omega$ is decreased! However, more fringes then become visible.

$$\underbrace{|\gamma^{(+)}(P_1,P_2,\tau)|}_{\mathcal{V}(\tau)} = \underbrace{|\mu(P_1,P_2,\overline{\omega_0})|}_{\mathcal{V}(0)}\,|\Theta(\tau)| \qquad (17)$$

$$\text{Visibility } \mathcal{V}(\tau) = \frac{I_{max}(\tau) - I_{min}(\tau)}{I_{max}(\tau) + I_{min}(\tau)} \qquad (18)$$



$(\Delta\omega)_1$ $\qquad$ $(\Delta\omega)_2 \ll (\Delta\omega)_1$

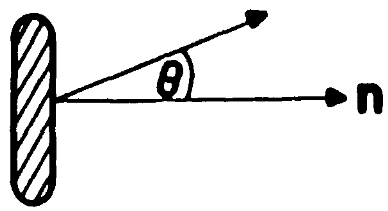
$$\gamma^{(+)}(P_1,P_2,\tau) = \mu(P_1,P_2,\omega_0)\,\Theta(\tau) \qquad [(13)]$$

Since $\Theta(0) = 1$,

$$\boxed{\gamma^{(+)}(P_1,P_2,0) = \mu(P_1,P_2,\omega_0) \qquad (20)}$$

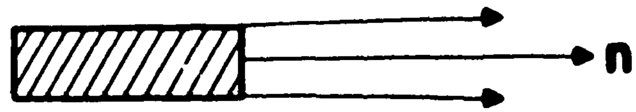
$\therefore \mu$ is a measurable quantity.

(a)



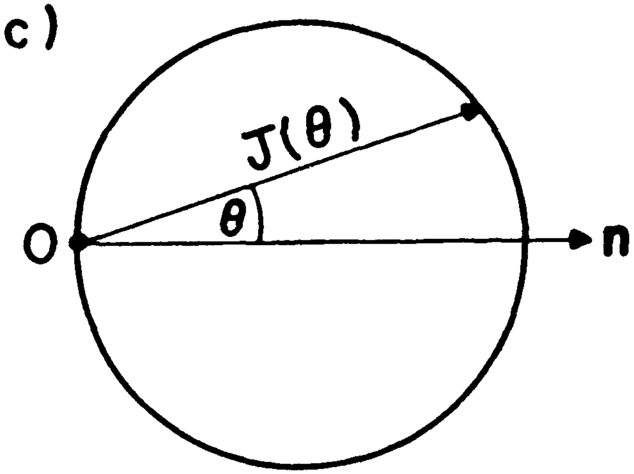
Thermal source (hot body)

(b)



Laser

(c)

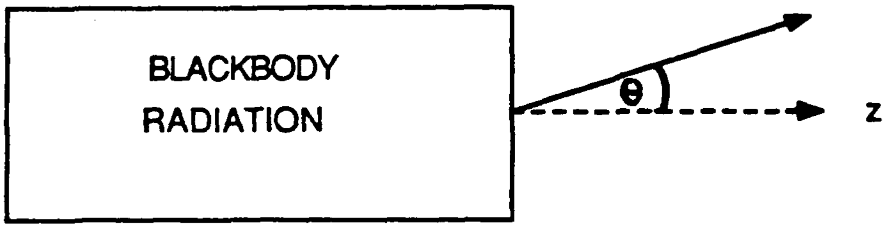


(d)

Polar diagrams of radiant intensity $J(\theta)$:

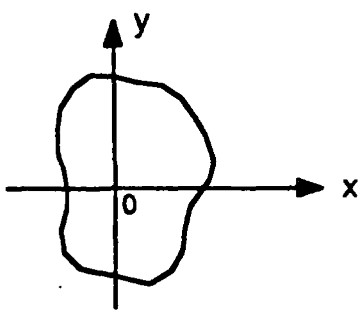
(c) From thermal source (d) From laser

RADIATION FROM THERMAL SOURCES

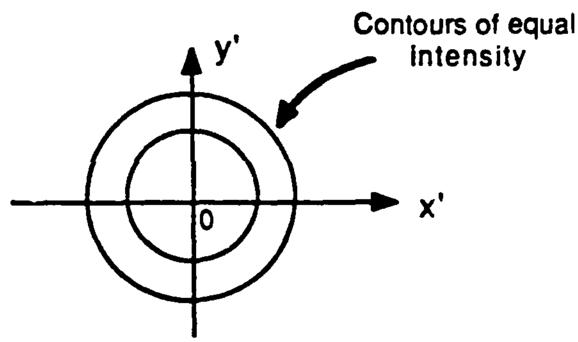


LAMBERT'S LAW

$$J(\theta) = J(0) \cos \theta$$



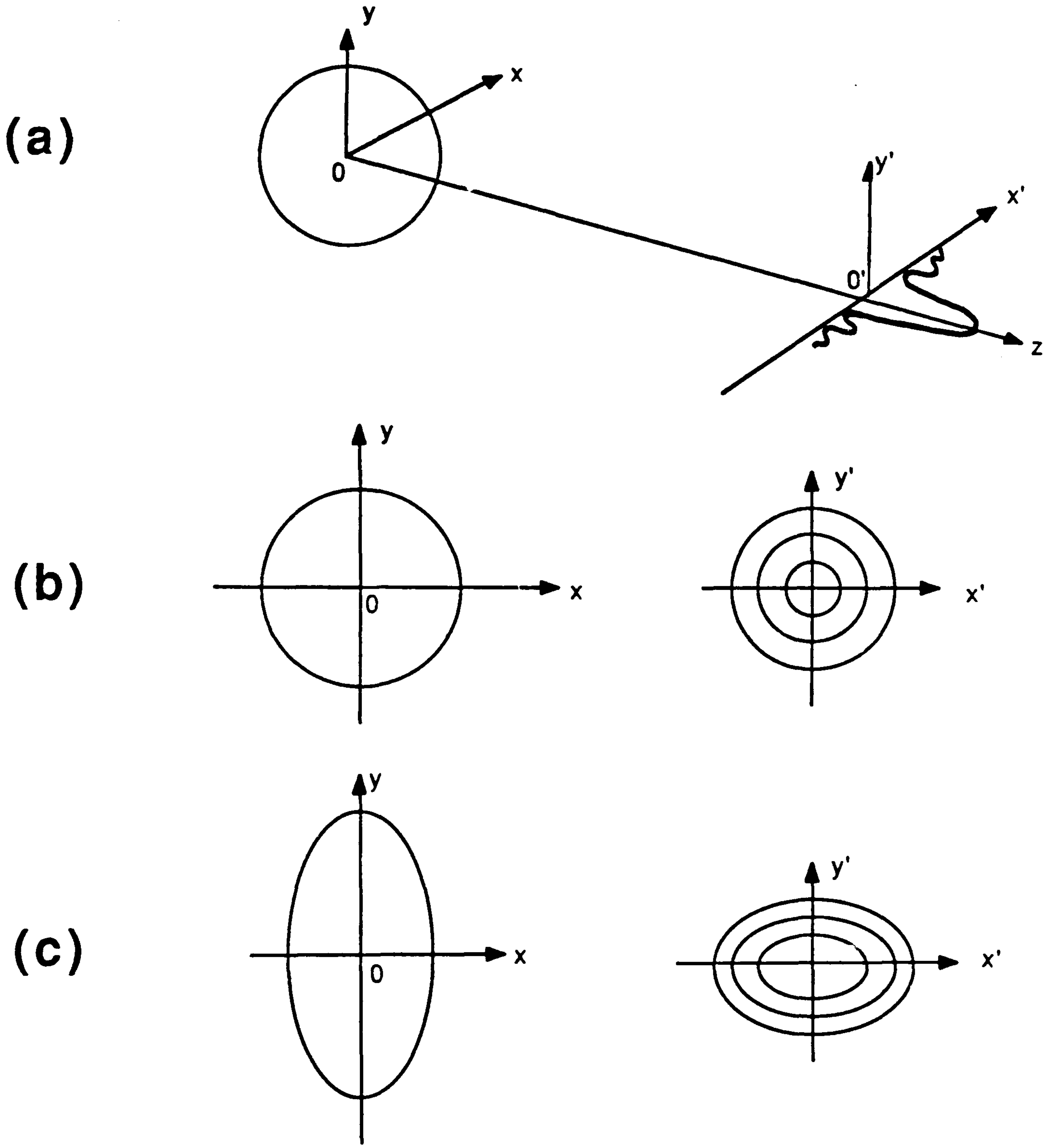
SHAPE OF EFFECTIVE SOURCE



FAR-FIELD PATTERN

FAR-FIELD <u>INTENSITY PATTERN</u> IS <u>ROTATIONALLY SYMMETRIC</u> ABOUT THE NORMAL TO THE SOURCE PLANE, <u>IRRESPECTIVE OF SHAPE OF SOURCE</u>

RADIATION FROM SPATIALLY COHERENT SOURCES



RECIPROCITY

SHAPE OF SOURCE

FAR-FIELD PATTERN

RADIANT INTENSITY

THERMAL SOURCES (SPATIALLY INCOHERENT)	LASER SOURCES (SPATIALLY COHERENT)
Broad angular distribution (Lambert's Law)	Narrow angular distribution (exponential-Gaussian)
Independent of shape of source (always rotationally symmetric)	Strongly dependent on shape of source (in general not rotationally symmetric)

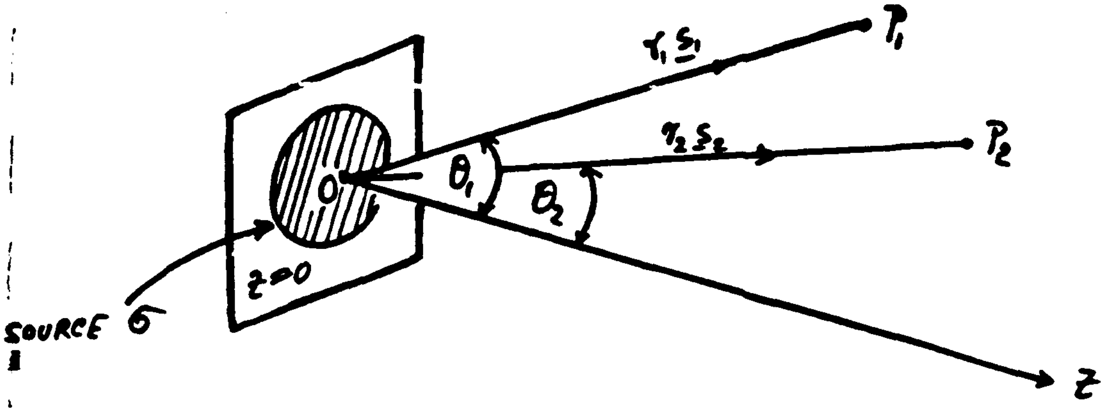
PROPAGATION OF THE CROSS-SPECTRAL DENSITY

$$\left.\begin{aligned} \nabla_1^2 W(\underline{r}_1, \underline{r}_2, \omega) + k^2 W(\underline{r}_1, \underline{r}_2, \omega) &= 0 \\ \nabla_2^2 W(\underline{r}_1, \underline{r}_2, \omega) + k^2 W(\underline{r}_1, \underline{r}_2, \omega) &= 0 \end{aligned}\right\} \qquad (1)$$

$$\nabla_j^2 \equiv \frac{\partial^2}{\partial x_j^2} + \frac{\partial^2}{\partial y_j^2} + \frac{\partial^2}{\partial z_j^2}, \qquad (j = 1, 2) \qquad (2)$$

$$k = \frac{\omega}{c} \qquad (3)$$

FAR-ZONE BEHAVIOR*



$\underline{s}_1^2 = \underline{s}_2^2 = 1$

$$\boxed{W^{(\infty)}(r_1\underline{s}_1, r_2\underline{s}_2, \omega) = (2\pi k)^2 \tilde{W}^{(0)}(k\underline{s}_{1\perp}, -k\underline{s}_{2\perp}, \omega)\, \frac{e^{ik(r_1 - r_2)}}{r_1 r_2} \cos\theta_1 \cos\theta_2 \quad (4) \qquad (kr_1 \to \infty,\ kr_2 \to \infty)}$$

$$\tilde{W}^{(0)}(\underline{f}_1, \underline{f}_2, \omega) = \frac{1}{(2\pi)^4} \iint W^{(0)}(\underline{r}_1', \underline{r}_2', \omega)\, e^{-i(\underline{f}_1\cdot\underline{r}_1' + \underline{f}_2\cdot\underline{r}_2')}\, d^2r_1'\, d^2r_2' \quad (5)$$

$$\underline{f}_1 = k\underline{s}_{1\perp}, \qquad \underline{f}_2 = -k\underline{s}_{2\perp} \quad (6)$$

$$|\underline{f}_1| \le k, \qquad |\underline{f}_2| \le k \quad (7)$$

LOW SPATIAL-FREQUENCY COMPONENTS

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* E. W. MARCHAND and E. WOLF, J. Opt. Soc. Amer., 62, 379 (1972)

L11-86

OPTICAL INTENSITY IN THE FAR FIELD:

$$I^{(\infty)}(r\underline{s}) = \frac{J(\underline{s})}{r^2} \qquad (8)$$

RADIANT INTENSITY (in direction $\underline{s}$ making angle θ with normal to source plane):

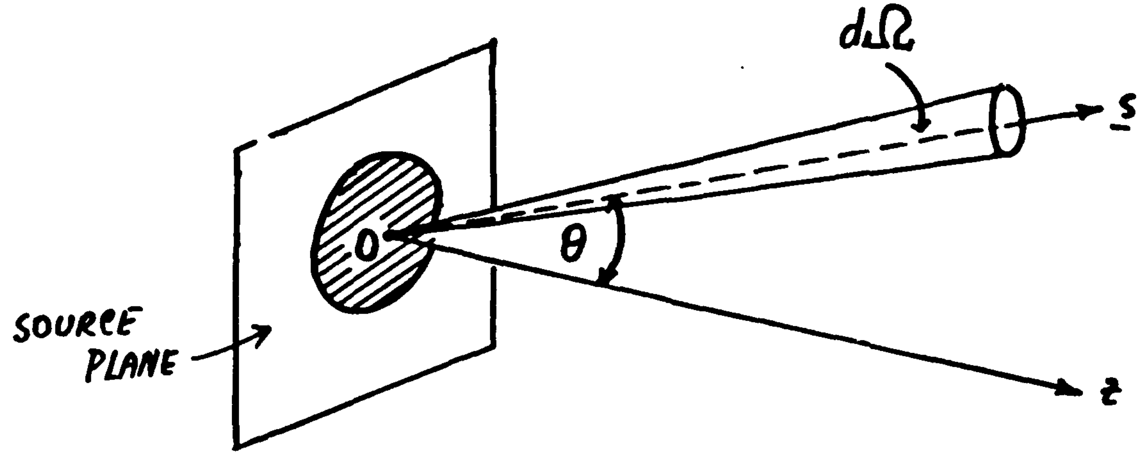
$$J(\underline{s}) = (2\pi k)^2 \, \tilde{W}^{(0)}(k\underline{s}_\perp, -k\underline{s}_\perp) \cos^2\theta \qquad (9)$$

DEGREE OF SPECTRAL COHERENCE OF FAR FIELD:

$$\mu^{(\infty)}(r_1\underline{s}_1, r_2\underline{s}_2) = \frac{\tilde{W}^{(0)}(k\underline{s}_{1\perp}, -k\underline{s}_{2\perp})}{\sqrt{\tilde{W}^{(0)}(k\underline{s}_{1\perp}, -k\underline{s}_{1\perp})}\sqrt{\tilde{W}^{(0)}(k\underline{s}_{2\perp}, -k\underline{s}_{2\perp})}} \, e^{ik(r_1 - r_2)} \qquad (10)$$

(Dependence on frequency ω not shown explicitly)

THE RADIANT INTENSITY



$$\underline{s} \equiv (s_x, s_y, s_z)$$

$$\underline{s}_\perp \equiv (s_x, s_y, 0)$$

RADIANT INTENSITY:

$$\boxed{J(\underline{s}) = (2\pi k)^2\, \tilde{W}^{(0)}(k\underline{s}_\perp, -k\underline{s}_\perp)\cos^2\theta \qquad [(9)]}$$

$$\tilde{W}^{(0)}(\underline{f}_1, \underline{f}_2) = \frac{1}{(2\pi)^4}\iint W^{(0)}(\underline{r}_1', \underline{r}_2')\, e^{-i(\underline{f}_1\cdot\underline{r}_1' + \underline{f}_2\cdot\underline{r}_2')}\, d^2r_1'\, d^2r_2' \qquad [(5)]$$

$W^{(0)}(\underline{r}_1, \underline{r}_2)$ AND CONSEQUENTLY $J(\underline{s})$ DEPEND BOTH ON THE SOURCE INTENSITY AND ON THE SOURCE COHERENCE

RADIATION FROM MODEL SOURCES

SCHELL-MODEL SOURCES*:

$$\mu^{(0)}(\underline{r}_1, \underline{r}_2, \omega) = g^{(0)}(\underline{r}_1 - \underline{r}_2, \omega) \quad (1)$$

$$\mu^{(0)}(\underline{r}_1, \underline{r}_2, \omega) = \frac{W^{(0)}(\underline{r}_1, \underline{r}_2, \omega)}{\sqrt{\underbrace{W^{(0)}(\underline{r}_1, \underline{r}_1, \omega)}_{I^{(0)}(\underline{r}_1, \omega)}} \sqrt{\underbrace{W^{(0)}(\underline{r}_2, \underline{r}_2, \omega)}_{I^{(0)}(\underline{r}_2, \omega)}}} \quad \text{[Eq. (5) on p 5]}$$

$$\therefore \quad W^{(0)}(\underline{r}_1, \underline{r}_2, \omega) = \sqrt{I^{(0)}(\underline{r}_1, \omega)} \sqrt{I^{(0)}(\underline{r}_2, \omega)} \, g^{(0)}(\underline{r}_1 - \underline{r}_2, \omega) \quad (2)$$

---

* A.C. SCHELL : (a) THE MULTIPLE PLATE ANTENNA (DOCTORAL DISSERTATION, M.I.T, 1961), § 7.5

(b) IEEE TRANS. ANTENNAS AND PROPAGATION, AP-15, 187 (1967).

QUASI - HOMOGENEOUS SOURCES*

THESE ARE SCHELL-MODEL SOURCES FOR WHICH

(1) $I^{(0)}(\underline{r},\omega)$ VARIES MUCH MORE SLOWLY WITH $\underline{r}$ THAN $g^{(0)}(\underline{r}',\omega)$ VARIES WITH $\underline{r}' = \underline{r}_1 - \underline{r}_2$.

($I^{(0)}$ = SLOW FUNCTION, $g^{(0)}$ = FAST FUNCTION)

(2) LINEAR DIMENSIONS OF SOURCE $\gg$ CORRELATION DISTANCE IN SOURCE PLANE

(3) LINEAR DIMENSIONS OF SOURCE $\gg$ WAVELENGTH

$$W^{(0)}(\underline{r}_1,\underline{r}_2,\omega) = \underset{\substack{\uparrow \\ \text{SLOW}}}{\sqrt{I^{(0)}(\underline{r}_1,\omega)}}\ \underset{\substack{\uparrow \\ \text{SLOW}}}{\sqrt{I^{(0)}(\underline{r}_2,\omega)}}\ \underset{\substack{\uparrow \\ \text{FAST}}}{g^{(0)}(\underline{r}_1-\underline{r}_2,\omega)} \qquad [(2)]$$

$$W^{(0)}(\underline{r}_1,\underline{r}_2,\omega) \approx I^{(0)}\left(\frac{\underline{r}_1+\underline{r}_2}{2},\omega\right) g^{(0)}(\underline{r}_1-\underline{r}_2,\omega) \qquad (3)$$

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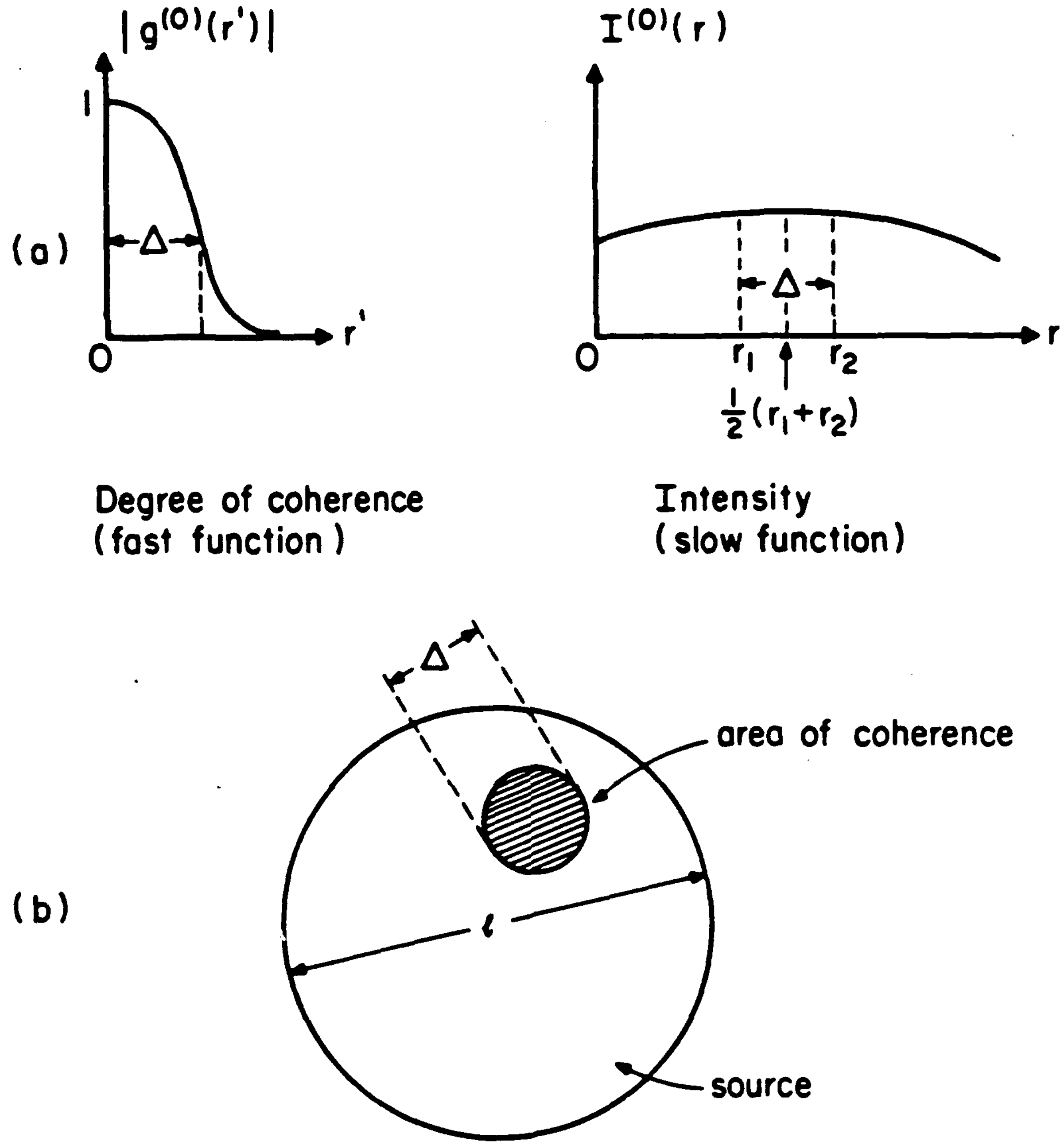
* W.H. CARTER and E. WOLF, J. Opt. Soc. Amer., 67, 785 (1977).



Illustrating the concept of a quasi-homogeneous source.

(a) The relative behavior of the degree of spatial coherence $\mu^{(0)}(\underline{r}_1, \underline{r}_2) \equiv g^{(0)}(\underline{r}_1 - \underline{r}_2)$ and of the intensity $I^{(0)}(\underline{r})$ of the light across the source.

(b) The relative linear dimensions $\ell(\gg\lambda)$ of the source and of the effective correlation length Δ of the light across the source. Such a source is always

globally incoherent: $\Delta \ll \ell$.

It is

locally incoherent if $\Delta \approx \lambda$

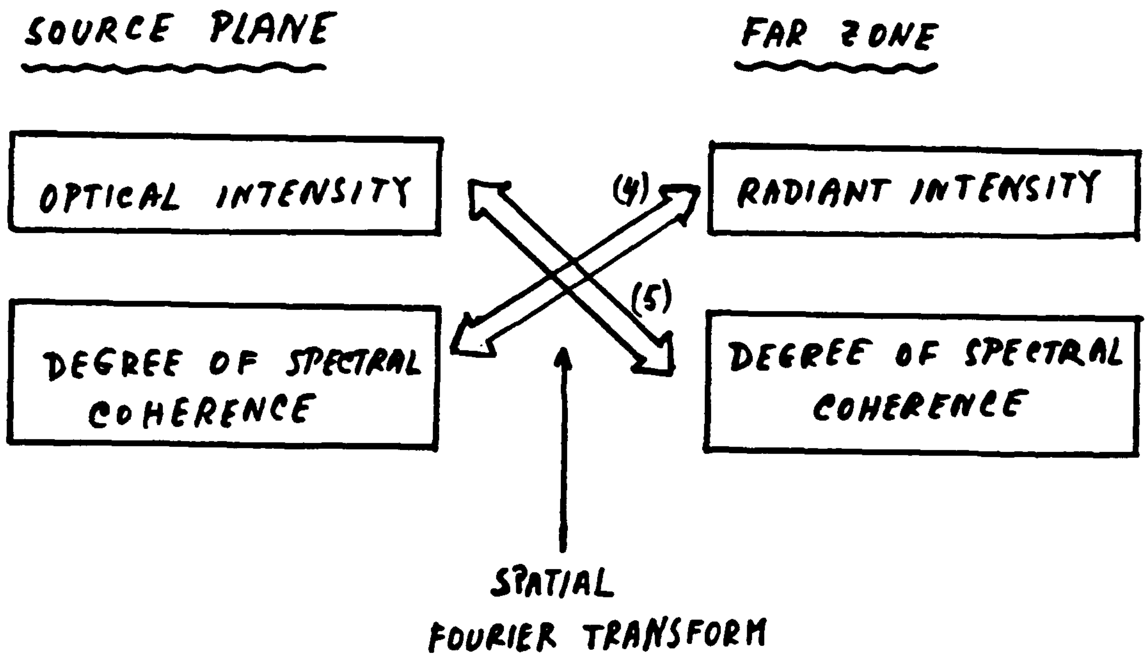
locally coherent if $\Delta \gg \lambda$

RECIPROCITY RELATIONS FOR QUASI-HOMOGENEOUS SOURCES

$$J(\underline{s}) = (2\pi k)^2 C \tilde{g}^{(0)}(k\underline{s}_\perp) \cos^2\theta \qquad (4)$$

$$\mu^{(\infty)}(r\underline{s}_1, r\underline{s}_2) = \frac{1}{C} \tilde{I}^{(0)}[k(\underline{s}_{1\perp} - \underline{s}_{2\perp})] \qquad (5)$$

$$C = \tilde{I}^{(0)}(0) = \frac{1}{(2\pi)^2} \int I^{(0)}(\underline{r})\, d^2 r \qquad (6)$$



NOTE: THE FIRST RECIPROCITY RELATION [Eq. (4)] IMPLIES THAT THE ANGULAR DISTRIBUTION OF THE RADIANT INTENSITY IS INDEPENDENT OF THE SHAPE OF THE SOURCE

GAUSSIAN CORRELATED QUASI-HOMOGENEOUS SOURCE

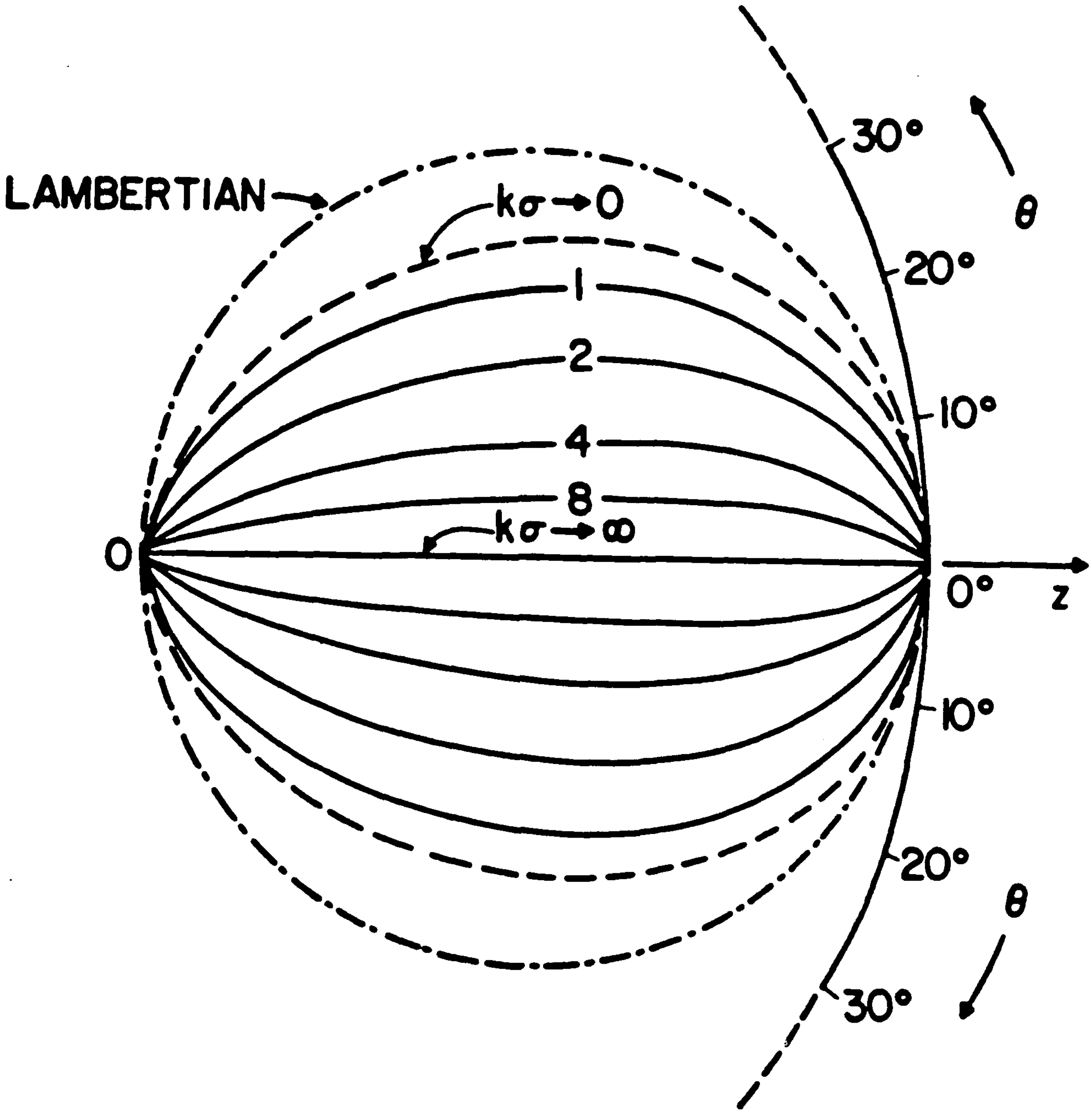
$$g^{(0)}(\underline{r}_1-\underline{r}_2) = e^{-|\underline{r}_1-\underline{r}_2|^2/2\sigma^2} \qquad (7)$$

ON SUBSTITUTING FROM EQ. (7) INTO THE FIRST RECIPROCITY RELATION [EQ. (4)] ONE FINDS THAT

$$J(\underline{s}) = J_0 \cos^2\theta \, e^{-\frac{1}{2}(k\sigma)^2 \sin^2\theta} \qquad (8)$$

WHERE

$$J_0 = \frac{(k\sigma)^2}{2\pi} \int_\sigma I^{(0)}(\underline{r})\, d^2r \qquad (9)$$



Polar diagrams of the normalized radiant intensity $J(\underline{s})/J_o$ [Eq. (8)] from a Gaussian correlated quasi-homogeneous source, for different values of the r.m.s. width σ of the degree of spatial coherence [Eq. (7)]. The length of the vector pointing from the origin to a typical point on a curve labeled by a particular value of the parameter $k\sigma$ represents the normalized radiant intensity in the direction of that vector. [After E. Wolf and W.H. Carter, Opt. Commun., 13, 205 (1975)].

LIMITING CASES

$$J(\underline{s}) = J_0 \cos^2\theta \, e^{-\frac{1}{2}(k\sigma)^2 \sin^2\theta} \qquad [(8)]$$

As $k\sigma \to 0$
(INCOHERENT LIMIT)

$$\frac{J(\underline{s})}{J_0} \longrightarrow \cos^2\theta \qquad (10)$$

As $k\sigma \to \infty$
(COHERENT LIMIT)

$$\left.\begin{aligned} \frac{J(\underline{s})}{J_0} &\longrightarrow 0 \quad \text{WHEN} \quad \theta \neq 0 \\ &= 1 \quad \text{WHEN} \quad \theta = 0 \end{aligned}\right\} \qquad (11)$$

NOTE: $k\sigma \to 0$: LOCALLY INCOHERENT GAUSSIAN CORRELATED QUASI-HOMOGENEOUS SOURCE

$k\sigma \to \infty$ LOCALLY COHERENT GAUSSIAN CORRELATED QUASI-HOMOGENEOUS SOURCE

QUASI-HOMOGENEOUS LAMBERTIAN SOURCES*

LAMBERT'S LAW: $J(\underline{s}) = j_0 \cos\theta$ (12)

FIRST RECIPROCITY RELATION [Eq.(4)]:

$$J(\underline{s}) = (2\pi k)^2 C \tilde{g}^{(0)}(k\underline{s}_\perp)\cos^2\theta \qquad [(4)]$$

FROM EQS. (12) AND (4), TAKING FOURIER INVERSE, GIVES

$$g^{(0)}_{lamb.}(\underline{r}_1 - \underline{r}_2) = \frac{\sin k|\underline{r}_1 - \underline{r}_2|}{k|\underline{r}_1 - \underline{r}_2|} + \text{H.F.C.} \qquad (13)$$

(H.F.C. = HIGH SPATIAL-FREQUENCY CONTRIBUTION –
DO NOT CONTRIBUTE TO FAR FIELD –
EVANESCENT WAVES).

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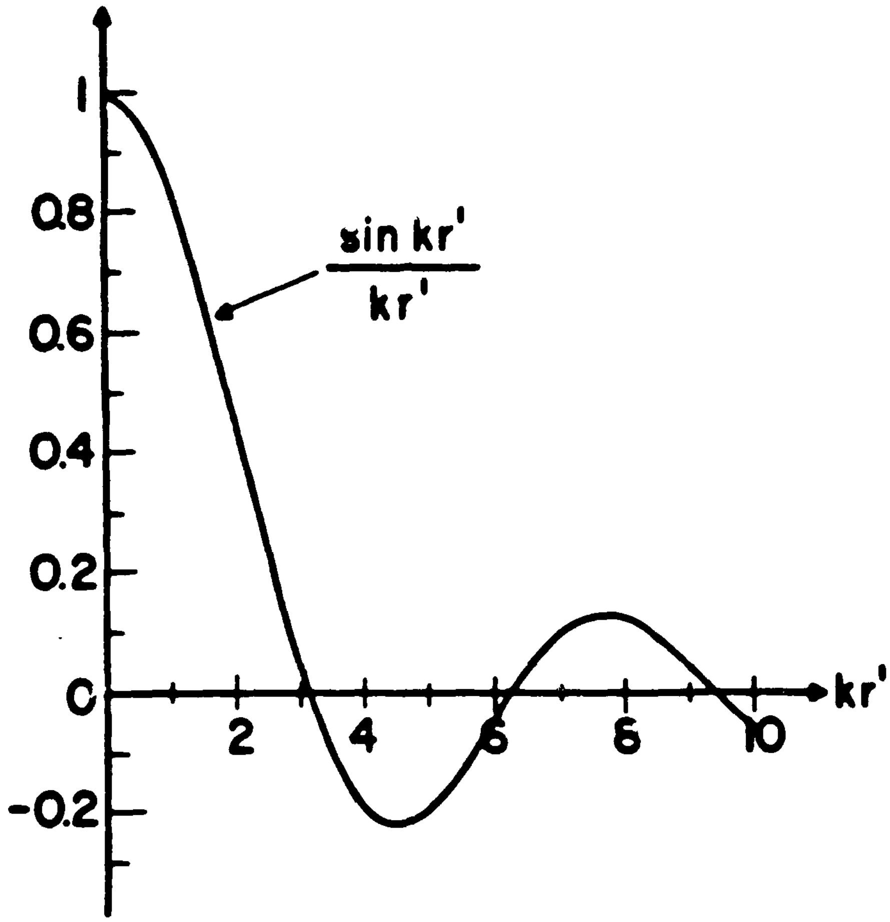
* W. H. CARTER and E. WOLF, J. Opt. Soc. Amer., 65, 1067 (1975).



The degree of spatial coherence of a quasi-homogeneous Lambertian source [Eq. (13) with high spatial-frequency contributions omitted].

CORRELATION DISTANCE ACROSS A QUASI-HOMOGENEOUS LAMBERTIAN SOURCE

$$g^{(0)}_{lamb}(\underline{r}_1 - \underline{r}_2) = \frac{\sin k|\underline{r}_1 - \underline{r}_2|}{k|\underline{r}_1 - \underline{r}_2|} + H.F.C. \qquad [(13)]$$

FIRST ZERO OCCURS WHEN

$$k|\underline{r}'| \equiv k|\underline{r}_1 - \underline{r}_2| = \pi$$

i.e. WHEN

$$\frac{2\pi}{\lambda}|\underline{r}_1 - \underline{r}_2| = \pi$$

i.e WHEN

$$\underbrace{|\underline{r}_1 - \underline{r}_2| = \frac{\lambda}{2}} \qquad (14)$$

∴ CORRELATION DISTANCE ACROSS LAMBERTIAN SOURCE IS OF THE ORDER OF A WAVELENGTH,

i.e A LAMBERTIAN SOURCE IS <u>NOT</u> STRICTLY SPATIALLY INCOHERENT.

THESE RESULTS AGREE WITH KNOWN PROPERTIES OF BLACBODY RADIATION.

SOURCES THAT GENERATE IDENTICAL DISTRIBUTIONS OF RADIANT INTENSITY

$$J(\underline{s}) = (2\pi k)^2 \tilde{W}^{(0)}(\underbrace{k\underline{s}_\perp}_{\underline{f}_1}, \underbrace{-k\underline{s}_\perp}_{\underline{f}_2}) \cos^2\theta \qquad [(1), p\, 33]$$

(ANTI-DIAGONAL COMPONENTS : $\underline{f}_2 = -\underline{f}_1$)

∴ TWO PLANAR SOURCES WHOSE CROSS-SPECTRAL DENSITIES HAVE THE SAME LOW-FREQUENCY ($f^2 \leq k^2$) ANTI-DIAGONAL ELEMENTS WILL GENERATE THE SAME DISTRIBUTIONS OF THE RADIANT INTENSITY.

—

$$W^{(0)}(\underline{r}_1, \underline{r}_2) = \sqrt{I^{(0)}(\underline{r}_1)}\sqrt{I^{(0)}(\underline{r}_2)}\ \underbrace{g^{(0)}(\underline{r}_1 - \underline{r}_2)}_{\text{More generally: } \mu^{(0)}(\underline{r}_1, \underline{r}_2)} \qquad [(2), p\, 35]$$

$$\tilde{W}^{(0)}(k\underline{s}_\perp, -k\underline{s}_\perp) = \frac{1}{(2\pi)^4}\iint \underbrace{\sqrt{I^{(0)}(\underline{r}_1)}\sqrt{I^{(0)}(\underline{r}_2)}}_{\text{INTENSITIES}}\ \underbrace{\mu^{(0)}(\underline{r}_1, \underline{r}_2)}_{\text{COHERENCE}}\, e^{-ik\underline{s}_\perp \cdot (\underline{r}_1 - \underline{r}_2)}\, d^2r_1\, d^2r_2 \qquad (1)$$

(TRADE-OFF ?)

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* E. COLLETT and E. WOLF, Opt. Lett., $\underline{2}$, 27 (1978); J. Opt. Soc. Amer., $\underline{69}$, 942 (1979).
D. McGUIRE, Opt. Commun., $\underline{29}$, 17 (1979).
B.E.A. SALEH, Opt. Commun., $\underline{30}$, 135 (1979); B.E.A. SALEH and M.I. IRSHID, Opt. Lett., $\underline{7}$, 342 (1982)

PARTIALLY COHERENT SOURCES WHICH GENERATE THE SAME DISTRIBUTION OF RADIANT INTENSITY AS A COMPLETELY COHERENT LASER SOURCE*

GAUSSIAN SCHELL-MODEL SOURCES:

$$g^{(0)}(\underline{r}') = e^{-r'^2/2\sigma_g^2}, \qquad I^{(0)}(\underline{r}) = A e^{-r^2/2\sigma_I^2}. \qquad (2)$$

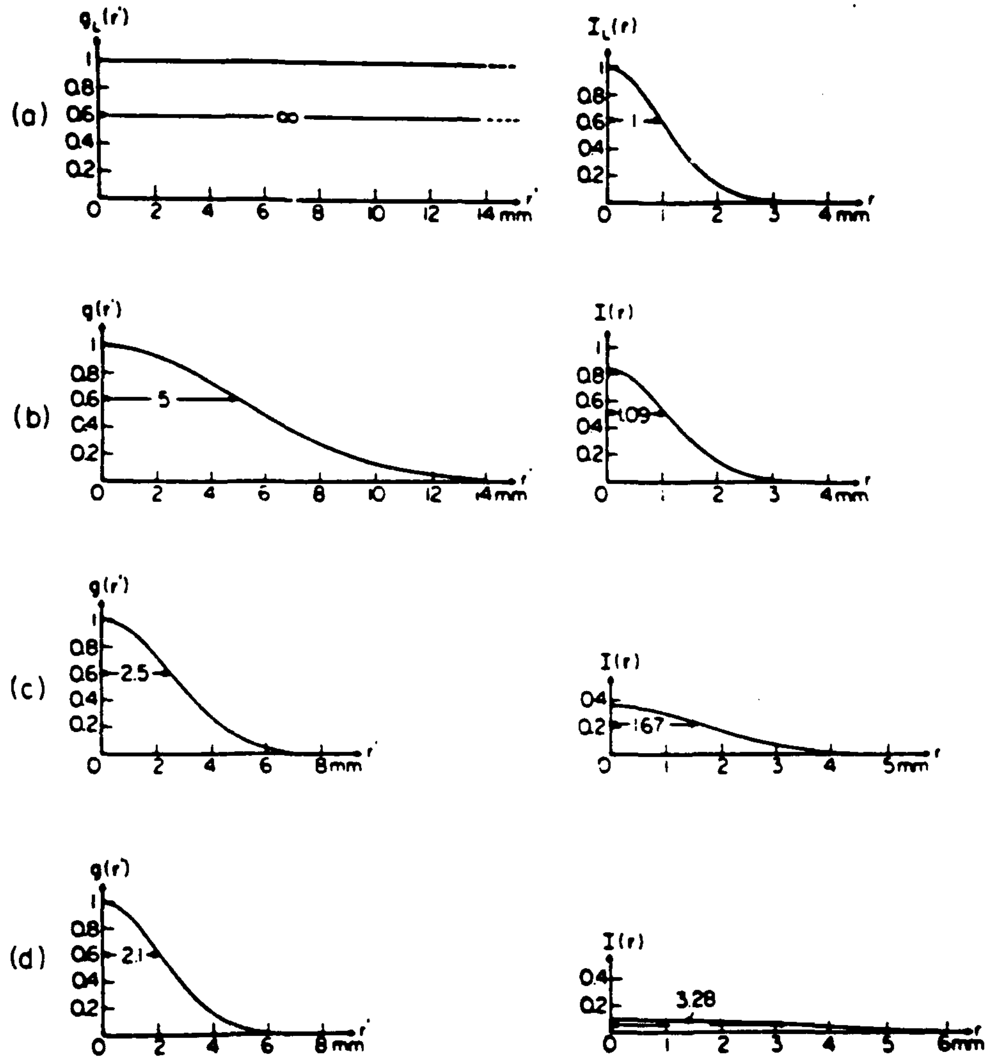
IF

$$\frac{1}{\sigma_g^2} + \frac{1}{(2\sigma_I)^2} = \frac{1}{(2\delta_L)^2}, \qquad A = \left(\frac{\delta_L}{\sigma_I}\right) A_L \qquad (3)$$

THE GAUSSIAN SCHELL-MODEL SOURCE WILL GENERATE SAME DISTRIBUTION OF RADIANT INTENSITY AS LASER, WITH

$$g_L^{(0)}(\underline{r}') \equiv 1, \qquad I_L^{(0)}(\underline{r}) = A_L e^{-r^2/2\delta_L^2} \qquad (4)$$

* E. WOLF and E. COLLETT, Opt. Commun., 25, 293 (1978)

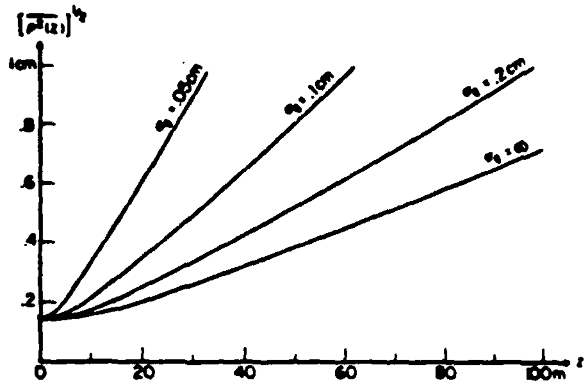


Illustrating the coherence and the intensity distributions across three partially coherent sources [(b), (c), (d)] which produce fields whose far-zone intensity distributions are the same as that generated by a coherent laser source [(a)]. The parameters characterizing the four sources are:

(a) $\sigma_g = \infty$, $\sigma_I = \delta_L = 1$ mm, $A = 1$ (arbitrary units) (b) $\sigma_g = 5$ mm, $\sigma_I = 1.09$ mm, $A = 0.84$
(c) $\sigma_g = 2.5$ mm, $\sigma_I = 1.67$ mm, $A = 0.36$ (d) $\sigma_g = 2.1$ mm, $\sigma_I = 3.28$ mm, $A = 0.09$.
The normalized radiant intensity generated by all these sources is $J(\theta)/J(0) = \cos^2\theta \exp\{-2(k\delta_L)^2 \sin^2\theta\}$. ($\delta_L = 1$ mm).

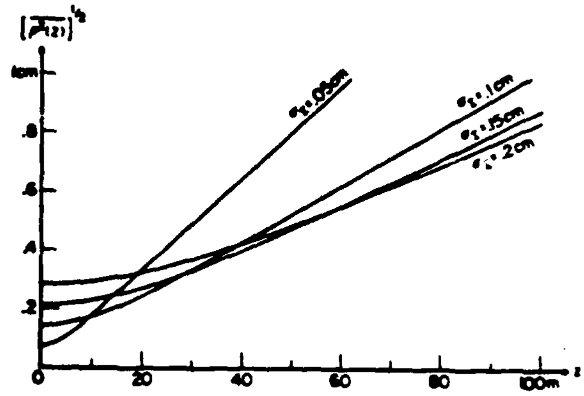
[After E. Wolf and E. Collett, Opt. Commun., 25, 293, 1978)].

(a)



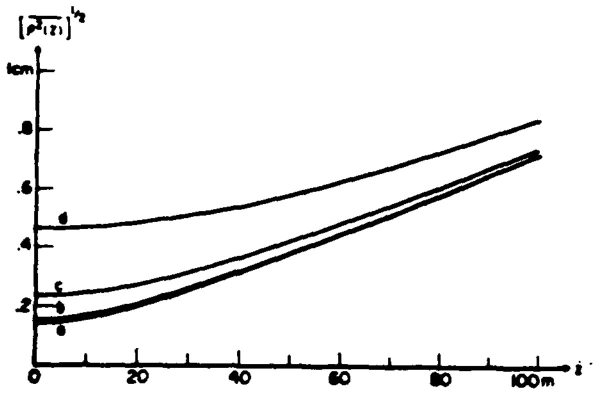
The r.m.s. beam radii for beams with the same initial r.m.s. beam radii ($\sigma_I = 0.1$ cm), but different degrees of coherence. The wavelength for each beam is 6328 A.

(b)



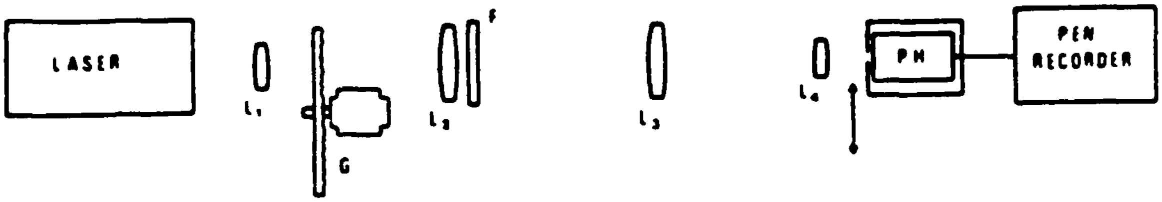
The r.m.s. beam radii for beams with the same degree of coherence ($\sigma_g = 0.2$ cm), but different initial r.m.s. radii. The wavelength for each beam is 6328 A.

(c)



The r.m.s. beam radii for beams with different initial r.m.s. beam radii and degrees of coherence, but with equal far field beam angles, θ_B. The parameters for the four beams are: (a) $\sigma_I = 0.1$ cm and $\sigma_g = \infty$, (b) $\sigma_I = 0.109$ cm and $\sigma_g = 0.5$ cm, (c) $\sigma_I = 0.167$ cm and $\sigma_g = 0.25$ cm and (d) $\sigma_I = 0.328$ cm and $\sigma_g = 0.21$ cm. The wavelength for each beam is 6328 A.

[After J. T. Foley and M.-S. Zubairy, Opt. Commun., 26, 297 (1978)].



A system used to test that sources with different coherence properties can generate identical angular distributions of the radiant intensity.

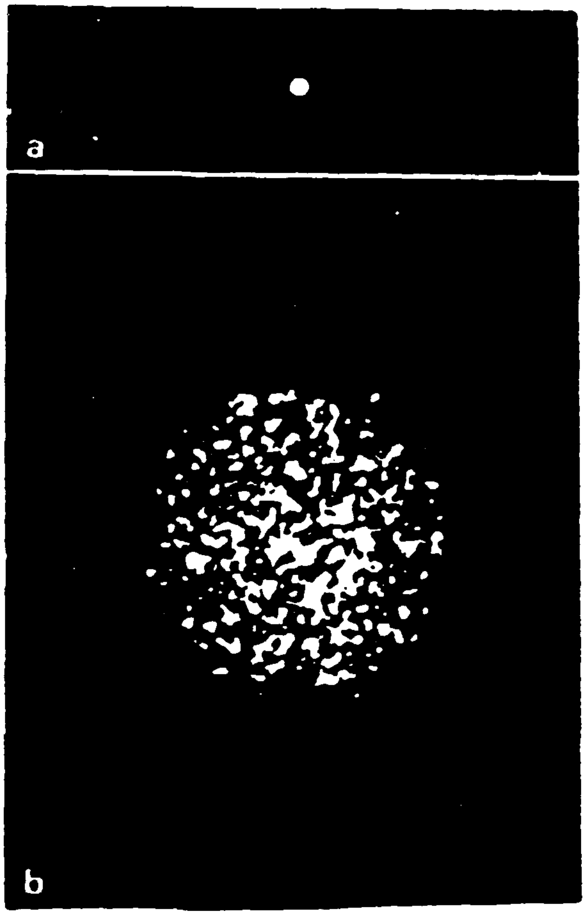
L_1, L_2, L_3, L_4: Lenses

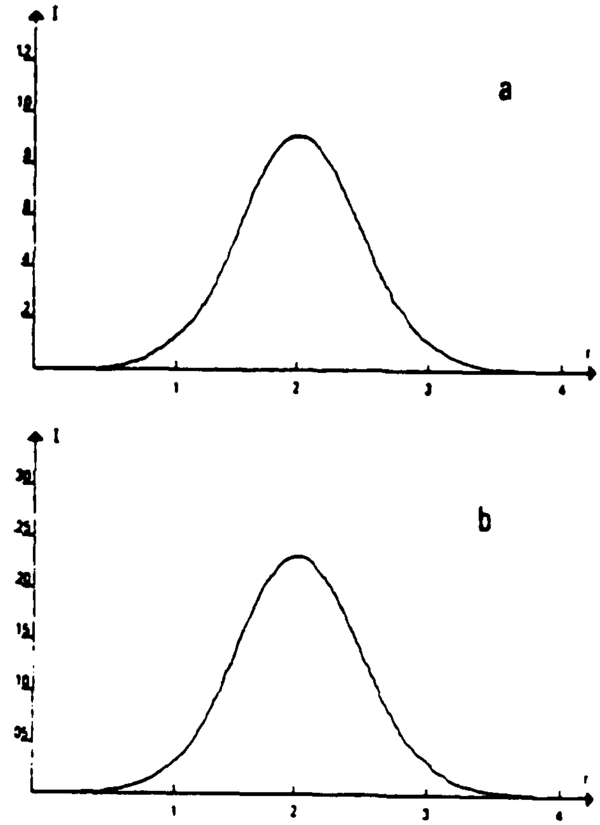
F: Amplitude filter

G: Rotating ground glass plate

PH: Photodetector

[After P. DeSantis, F. Gori, G. Guattari and C. Palma, Opt. Commun. 29, 256 (1979)]





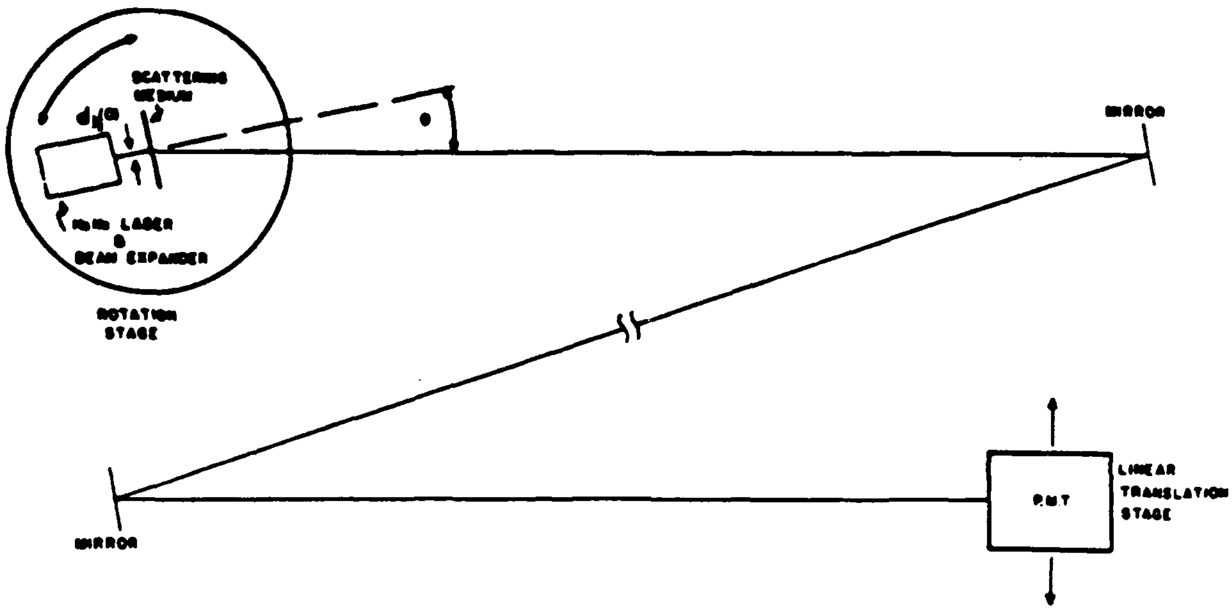
Intensity distribution across a coherent laser source (a), and across an "equivalent" partially coherent source (b).

The measured angular distribution of intensity, I (in arbitrary but same units) in the far zone of fields generated by the two sources illustrated on the left.

(For experimental arrangement see p. 49)

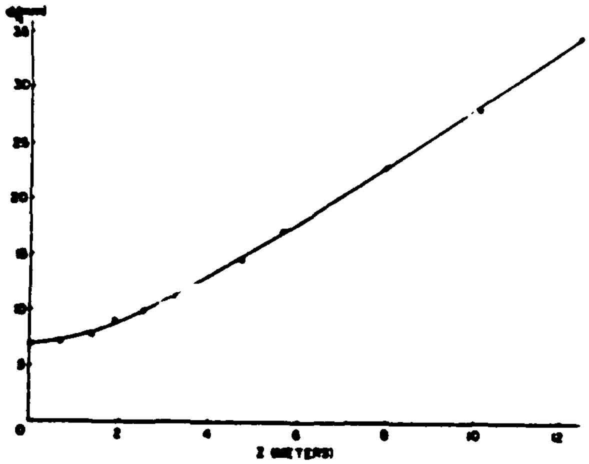
Reproduced from P. DeSantis, F. Gori, G. Guattari and C. Palma, Opt. Commun., 29, 256 (1979).

(a)



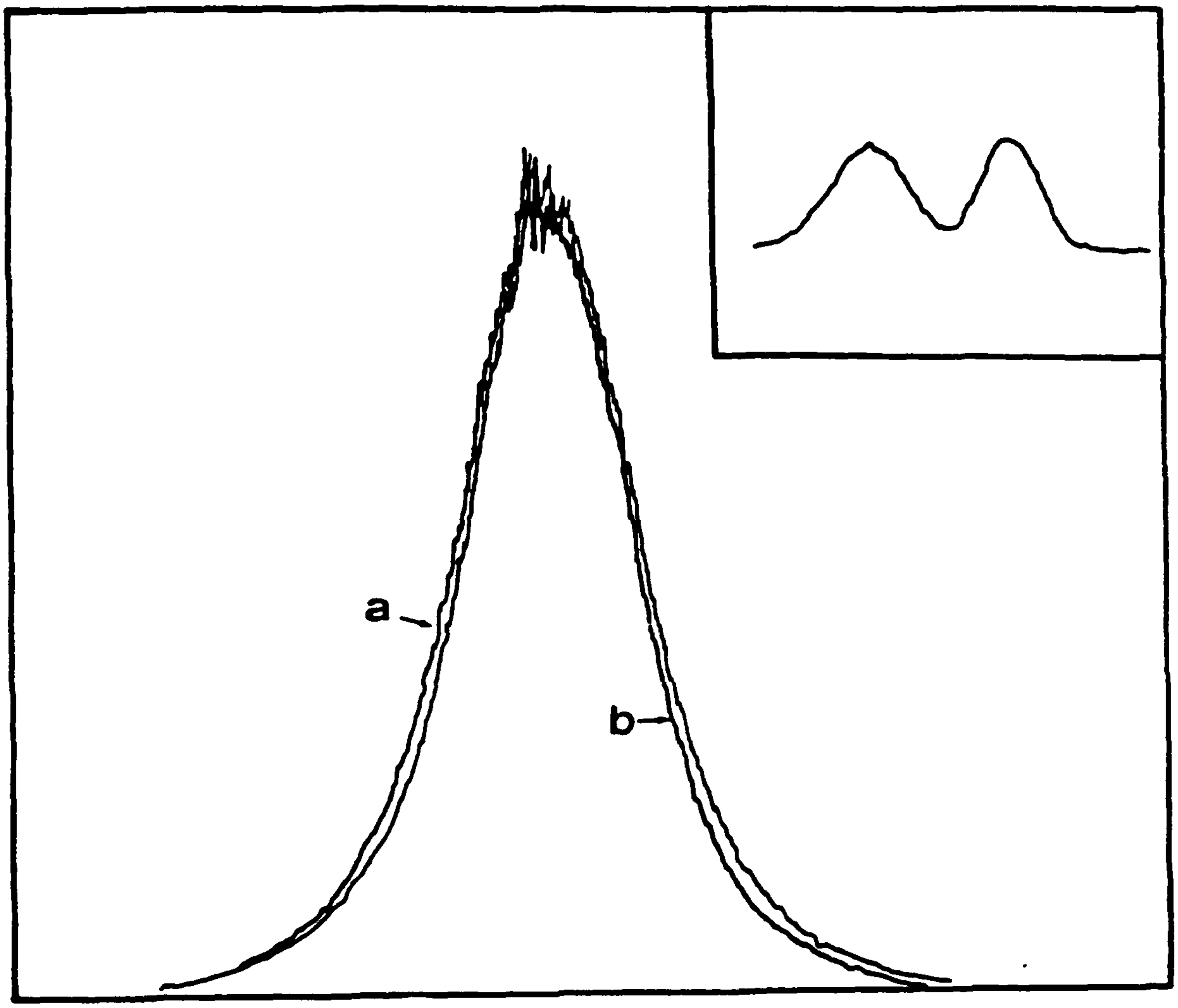
Schematic diagram of an experimental set up used to test some of the theoretical predictions relating to radiation from quasi-homogeneous sources.

(b)



Behavior of the effective beam diameter $d(z) = 2\sqrt{\overline{\rho^2(z)}}$, as a function of distance from a quasi-homogeneous source. The solid line represents theoretical predictions; the points represent results of measurements, using the arrangement shown in (a) above.

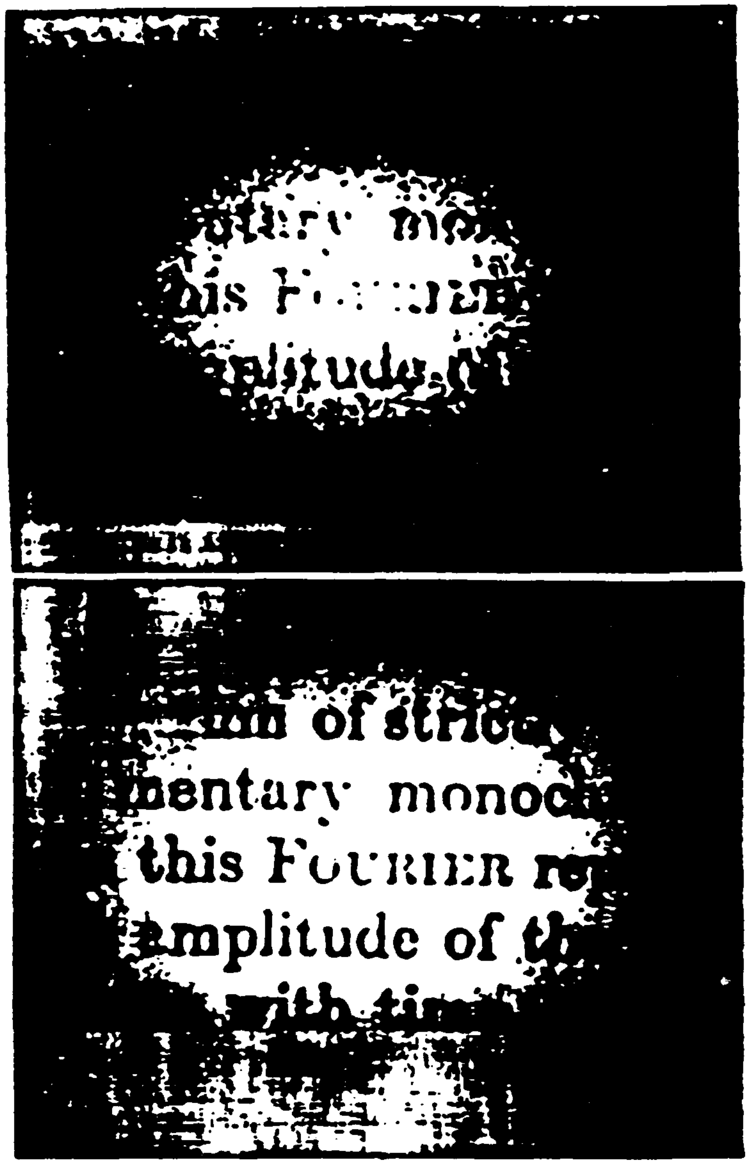
[After J. D. Farina, L. M. Narducci and E. Collett, Opt. Commun., 32, 203, (1980)].



Far field intensity profile produced by the same phase screen illuminated by (a) the Gaussian ($TEM_{0,0}$) mode of a He-Ne gas laser, and (b) the donut ($TEM_{1,0}$) mode.

The inset shows the near field intensity profile of the illuminated phase screen under conditions (b). The vertical scale in the two runs has been adjusted to match the peaks of the intensity distribution. Thus, the two curves differ only by an overall scale factor.

After L.M. Narducci and J. Farina.



Illustrating the reduction of speckle effects by changing the degree of coherence of light forming the image.

The upper figure is a photograph of a text illuminated by spatially coherent He:Ne laser light. The speckles that are produced obscure the image, making the words nearly unreadable.

The lower figure was taken under the same conditions, except that the text was illuminated by light from a quasi-homogeneous (globally incoherent) source. It is seen that the speckles have been eliminated and as a result the text has become readable.

(After L. Narducci and J. Farina)

SOME METHODS FOR PRODUCING SOURCES OF CONTROLLED COHERENCE PROPERTIES

LIQUID CRYSTALS:

F. Scudieri, M. Bertolotti and R. Bartolino
Appl. Opt. **13**, 181 (1974)

M. Bertolotti, F. Scudieri and S. Verginelli,
Appl. Opt. **15**, 1842 (1976)

ROTATING ROUGH SURFACES:

P. de Santes, F. Gori, G. Guattari and C. Palma
Opt. Commun. **29**, 256 (1979)

J.D. Farina, L.M. Narducci and E. Collett
Opt. Commun. **32**, 203 (1980)

HOLOGRAPHIC FILTERS:

D. Courjon and J. Bulabois,
Proc. S.P.I.E. **194**, 129 (1979)

ULTRASONIC WAVES:

Y. Ohtsuka and Y. Imai, *J. Opt. Soc. Amer.* **69**, 684 (1979)

Y. Imai and Y. Ohtsuka, *Appl. Opt.* **19**, 542 (1980)

Y. Ohtsuka, *J. Opt. Soc. Amer. A* **3**, 1247 (1986)

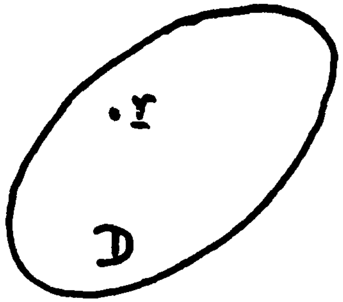
IMAGING AND LENSLESS FEEDBACK SYSTEMS

J. Deschamps, D. Courjon and J. Bulabois,
J. Opt. Soc. Amer. **73**, 256 (1983)

ACHROMATIC FOURIER TRANSFORM LENSES

G.M. Morris and D. Faklis
*Opt. Commun. **62**, 5 (1987)*

COHERENT MODE REPRENTATION OF STEADY - STATE SOURCES AND FIELDS



Source: $Q(\underline{r},t)$, localized in D

Field: $V(\underline{r},t)$

$$\nabla^2 V(\underline{r},t) - \frac{1}{c^2}\frac{\partial^2 V(\underline{r},t)}{\partial t^2} = -4\pi Q(\underline{r},t) \qquad (1)$$

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* E. WOLF, J. OPT. SOC. AMER., 72, 343 (1982);
J. OPT. SOC AMER. A, 3, 76 (1986).

Cross-correlation function of source distribution:

$$\Gamma_Q(\underline{r}_1, \underline{r}_2, \tau) = \langle Q^*(\underline{r}_1, t) Q(\underline{r}_2, t+\tau) \rangle_t \qquad (2)$$

$$\int_{-\infty}^{\infty} |\Gamma_Q(\underline{r}_1, \underline{r}_2, \tau)|^2 d\tau < \infty \qquad (3)$$

Cross-spectral density of source distribution:

$$W_Q(\underline{r}_1, \underline{r}_2, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Gamma_Q(\underline{r}_1, \underline{r}_2, \tau) e^{i\omega\tau} d\tau \qquad (4)$$

$$\int_D W_Q(\underline{r}, \underline{r}, \omega) d^3 r < \infty \qquad (5)$$

From Eq. (5) and Schwarz' inequality

$$\iint_{DD} |W_Q(\underline{r}_1, \underline{r}_2, \omega)|^2 d^3 r_1 d^3 r_2 < \infty \qquad (6)$$

$$\iint_{DD} |W_Q(\underline{r}_1, \underline{r}_2, \omega)|^2 d^3r_1\, d^3r_2 \; < \; \infty \qquad [(6)]$$

$$W_Q(\underline{r}_2, \underline{r}_1, \omega) = W_Q^*(\underline{r}_1, \underline{r}_2, \omega) \qquad (7)$$

$$\iint_{DD} W_Q(\underline{r}_1, \underline{r}_2, \omega) f^*(\underline{r}_1) f(\underline{r}_2)\, d^3r_1\, d^3r_2 \geq 0 \qquad (8)$$

Eq. (6) $\Longrightarrow$ $W_Q(\underline{r}_1, \underline{r}_2, \omega)$ is a <u>Hilbert-Schmidt kernel</u>

Eq. (7) $\Longrightarrow$ Hermitean

Eq (8) $\longrightarrow$ Non-negative definite

Mercer's theorem

$$\boxed{W_Q(\underline{r}_1, \underline{r}_2, \omega) = \sum_n \lambda_n(\omega)\, \phi_n^*(\underline{r}_1, \omega)\, \phi_n(\underline{r}_2, \omega), \qquad (9)}$$

where

$$\int_D W_Q(\underline{r}_1, \underline{r}_2, \omega)\, \phi_n(\underline{r}_1, \omega)\, d^3 r_1 = \lambda_n(\omega)\, \phi_n(\underline{r}_2, \omega), \qquad (10)$$

$$\int_D \phi_n^*(\underline{r}, \omega)\, \phi_m(\underline{r}, \omega)\, d^3 r = \delta_{nm}, \qquad (11)$$

$$\lambda_n(\omega) > 0. \qquad (12)$$

(a) <u>At least</u> one eigenvalue

(b) Expansion (9) holds whether or not the set $\{\phi_n\}$ is complete

(c) Expansion (9) is <u>absolutely</u> and <u>uniformly</u> convergent

<u>Physical significance of expansion (9):</u> Completely spatially coherent elementary sources ϕ_n. Natural oscillations

$$W_Q(\underline{r}_1,\underline{r}_2,\omega) = \sum_n \underbrace{\lambda_n(\omega)\,\phi_n^*(\underline{r}_1,\omega)\,\phi_n(\underline{r}_2,\omega)}_{W_Q^{(n)}(\underline{r}_1,\underline{r}_2,\omega)} \qquad [(9)]$$

$$\mu_Q^{(n)}(\underline{r}_1,\underline{r}_2,\omega) = \frac{W_Q^{(n)}(\underline{r}_1,\underline{r}_2,\omega)}{\sqrt{W_Q^{(n)}(\underline{r}_1,\underline{r}_1,\omega)}\sqrt{W_Q^{(n)}(\underline{r}_2,\underline{r}_2,\omega)}}$$

$$= \frac{\lambda_n(\omega)\,\phi_n^*(\underline{r}_1,\omega)\,\phi_n(\underline{r}_2,\omega)}{\sqrt{\lambda_n(\omega)\,|\phi_n(\underline{r}_1,\omega)|^2}\sqrt{\lambda_n(\omega)\,|\phi_n(\underline{r}_2,\omega)|^2}}$$

$$= \frac{\phi_n^*(\underline{r}_1,\omega)}{|\phi_n(\underline{r}_1,\omega)|}\;\frac{\phi_n(\underline{r}_2,\omega)}{|\phi_n(\underline{r}_2,\omega)|}$$

$$\therefore \quad |\mu_Q^{(n)}(\underline{r}_1,\underline{r}_2,\omega)| = 1$$

$\therefore$ EACH MODE IS SPATIALLY COMPLETELY COHERENT

(COHERENT-MODE REPRENTATION)

ANOTHER REPRESENTATION OF CROSS-SPECTRAL DENSITY OF THE SOURCE

LET

$$U_Q(\underline{r},\omega) = \sum_n a_n(\omega)\, \phi_n(\underline{r},\omega) \qquad (13)$$

$a_n(\omega)$ ARE RANDOM COEFFICIENTS WITH

$$\langle a_n^*(\omega)\, a_m(\omega)\rangle_\omega = \lambda_n(\omega)\, \delta_{nm} \qquad (14)$$

THEN

$$\vdots$$

$$\boxed{W_Q(\underline{r}_1,\underline{r}_2,\omega) = \langle U_Q^*(\underline{r}_1,\omega)\, U_Q(\underline{r}_2,\omega)\rangle_\omega \qquad (19)}$$

CROSS - SPECTRAL DENSITY OF THE FIELD DISTRIBUTION

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) V(\underline{r},t) = -4\pi Q(\underline{r},t) \qquad (20)$$

$$W_Q(\underline{r}_1,\underline{r}_2,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \langle Q^*(\underline{r}_1,t)\,Q(\underline{r}_2,t+\tau)\rangle_t \, e^{i\omega\tau}\, d\tau \qquad (21a)$$

$$W_V(\underline{r}_1,\underline{r}_2,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \langle V^*(\underline{r}_1,t)\,V(\underline{r}_2,t+\tau)\rangle_t \, e^{i\omega\tau}\, d\tau \qquad (21b)$$

$$(\nabla_1^2 + k^2)(\nabla_2^2 + k^2)\, W_V(\underline{r}_1,\underline{r}_2,\omega) = (4\pi)^2\, W_Q(\underline{r}_1,\underline{r}_2,\omega) \qquad (22)$$

$$k = \frac{\omega}{c}.$$

L11-86

COHERENT-MODE REPRESENTATION OF RADIATED FIELD

SOURCE:

$$W_Q(\underline{r}_1, \underline{r}_2, \omega) = \sum_m \lambda_m \phi_m^*(\underline{r}_1, \omega)\, \phi_m(\underline{r}_2, \omega) \qquad [(9)]$$

$$= \left\langle U_Q^*(\underline{r}_1, \omega)\, U_Q(\underline{r}_2, \omega) \right\rangle_\omega \qquad [(19)]$$

$$U_Q(\underline{r}, \omega) = \sum_m a_m \phi_m(\underline{r}, \omega) \qquad [(13)]$$

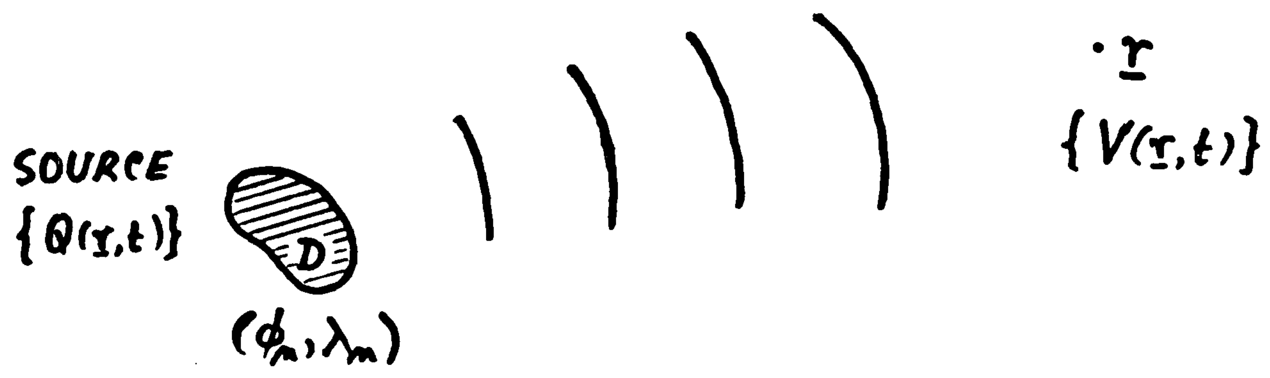
USING EQ. (22):

FIELD:

$$\boxed{\begin{aligned} W_V(\underline{r}_1, \underline{r}_2, \omega) &= \sum_m \lambda_m \psi_m^*(\underline{r}_1, \omega)\, \psi_m(\underline{r}_2, \omega) \qquad (23) \\ &= \left\langle U_V^*(\underline{r}_1, \omega)\, U_V(\underline{r}_2, \omega) \right\rangle_\omega \qquad (24) \end{aligned}}$$

$$U_V(\underline{r}, \omega) = \sum_m a_m \psi_m(\underline{r}, \omega) \qquad (25)$$

$$\psi_m(\underline{r}, \omega) = \int \phi_m(\underline{r}', \omega) \frac{e^{ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|}\, d^3 r' \qquad (26)$$



SOURCE:

$$W_Q(\underline{r}_1,\underline{r}_2,\omega) = \sum_n \lambda_n(\omega)\,\phi_n^*(\underline{r}_1,\omega)\,\phi_n(\underline{r}_2,\omega) \qquad [(9)]$$

$$= \left\langle U_Q^*(\underline{r}_1,\omega)\,U_Q(\underline{r}_2,\omega)\right\rangle_\omega \qquad [(19)]$$

$$U_Q(\underline{r},\omega) = \sum_n a_n(\omega)\,\phi_n(\underline{r},\omega) \qquad [(13)]$$

FIELD:

$$\phi_n(\underline{r},\omega) \longrightarrow \psi_n(\underline{r},\omega) = \int_D \phi_n(\underline{r}',\omega)\,\frac{e^{ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|}\,d^3r' \qquad [(26)]$$

FIELD:

$$W_V(\underline{r}_1,\underline{r}_2,\omega) = \sum_n \lambda_n(\omega)\, \psi_n^*(\underline{r}_1,\omega)\, \psi_n(\underline{r}_2,\omega) \qquad (24)$$

$$= \langle U_V^*(\underline{r}_1,\omega)\, U_V(\underline{r}_2,\omega)\rangle_\omega \qquad (25)$$

$$U_V(\underline{r},\omega) = \sum_n a_n(\omega)\, \psi_n(\underline{r},\omega) \qquad (26)$$

COMPARE WITH

SOURCE:

$$W_Q(\underline{r}_1,\underline{r}_2,\omega) = \sum_n \lambda_n(\omega)\, \phi_n^*(\underline{r}_1,\omega)\, \phi_n(\underline{r}_2,\omega) \qquad [(9)]$$

$$= \langle U_Q^*(\underline{r}_1,\omega)\, U_Q(\underline{r}_2,\omega)\rangle_\omega \qquad [(19)]$$

$$U_Q(\underline{r},\omega) = \sum_n a_n(\omega)\, \phi_n(\underline{r},\omega) \qquad [(13)]$$

$$\boxed{W_V(\underline{r}_1,\underline{r}_2,\omega) = \sum_m \lambda_m(\omega)\,\psi_m^*(\underline{r}_1,\omega)\,\psi_m(\underline{r}_2,\omega) \qquad (28)}$$

THIS EXPANSION IS A <u>MODE REPRENTATION OF PARTIALLY COHERENT FIELDS.</u> EACH MODE IS A COMPLETELY SPATIALLY COHERENT WAVE, OF FREQUENCY ω, GENERATED BY ELEMENTARY COHERENT SOURCE OSCILLATIONS $\phi_m(\underline{r},\omega)\,e^{-i\omega t}$.

APPLICATIONS

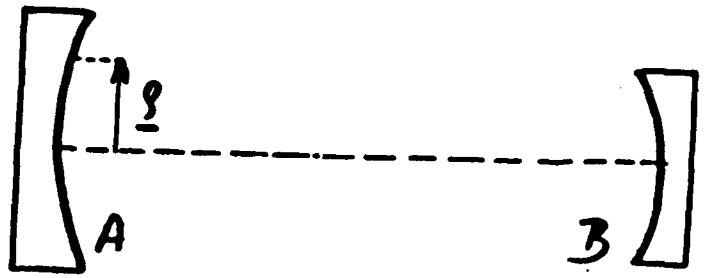
Coherence properties of laser modes

Statistical properties of speckles

Light propagation through the atmosphere

Scattering from fluctuating media

COHERENCE THEORY OF LASER MODES*



CYCLES	CROSS-SPECTRAL DENSITY ON MIRROR A
0 (INITIAL DISTR.)	$W_0(\underline{\rho}_1, \underline{\rho}_2, \omega)$
1 $A \to B \to A$	$W_1(\underline{\rho}_1, \underline{\rho}_2, \omega)$
2 $\underbrace{A \to B \to A}\underbrace{\to B \to A}$	$W_2(\underline{\rho}_1, \underline{\rho}_2, \omega)$
⋮	⋮

* E. WOLF and G.S. AGARWAL, J. Opt. Soc. Amer., A, **1**, 541 (1984)

$$W_j(\underline{\rho}_1, \underline{\rho}_2, \omega) = \langle U_j^*(\underline{\rho}_1, \omega)\, U_j(\underline{\rho}_2, \omega) \rangle_\omega \qquad (1)$$

$$U_{j+1}(\underline{\rho}, \omega) = \int_A L(\underline{\rho}, \underline{\rho}', \omega)\, U_j(\underline{\rho}', \omega)\, d^2\rho', \qquad (j = 0, 1, 2, \ldots), \qquad (2)$$

(2) $\rightarrow$ (1)

$$W_{j+1}(\underline{\rho}_1, \underline{\rho}_2, \omega) = \iint_{A\,A} L^*(\underline{\rho}_1, \underline{\rho}_1', \omega)\, L(\underline{\rho}_2, \underline{\rho}_2', \omega)\, W_j(\underline{\rho}_1', \underline{\rho}_2', \omega)\, d^2\rho_1'\, d^2\rho_2'. \qquad (3)$$

FOR STEADY STATE:

$$W_{j+1}(\underline{\rho}_1, \underline{\rho}_2, \omega) = \sigma(\omega)\, W_j(\underline{\rho}_1, \underline{\rho}_2, \omega) \qquad (4)$$

SINCE $W(\underline{\rho}, \underline{\rho}, \omega) \geq 0$,

$$\sigma(\omega) \geq 0 \qquad (5)$$

$$\boxed{\iint\limits_{A\,A} W(\underline{\rho}_1', \underline{\rho}_2', \omega)\, L^*(\underline{\rho}_1, \underline{\rho}_1', \omega)\, L(\underline{\rho}_2, \underline{\rho}_2', \omega)\, d^2\rho_1'\, d^2\rho_2' = \sigma(\omega)\, W(\underline{\rho}_1, \underline{\rho}_2, \omega) \qquad (6)}$$

(BASIC INTEGRAL EQUATION OF PRESENT THEORY)

U11-86

NATURE OF SOLUTIONS OF INTEGRAL EQUATION (6):

$$\int_A L(\underline{\rho}_1,\underline{\rho}_2,\omega)\,\phi_m(\underline{\rho}_2,\omega)\,d^2\rho_2 = \alpha_m\,\phi_m(\underline{\rho}_1,\omega) \qquad (7)$$

$$\int_A L^*(\underline{\rho}_2,\underline{\rho}_1,\omega)\,\chi_m(\underline{\rho}_2,\omega)\,d^2\rho_2 = \beta_m\,\chi_m(\underline{\rho}_1,\omega) \qquad (8)$$

$$\beta_m = \alpha_m^* \qquad (9)$$

$$\int \phi_n^*(\underline{\rho},\omega)\,\chi_m(\underline{\rho},\omega)\,d^2\rho = \delta_{nm} \qquad (10)$$

BI-ORTHOGONAL EXPANSION* OF $L(\underline{\rho}_1,\underline{\rho}_2,\omega)$:

$$\boxed{L(\underline{\rho}_1,\underline{\rho}_2,\omega) = \sum_m \alpha_m(\omega)\,\phi_m(\underline{\rho}_1,\omega)\,\chi_m^*(\underline{\rho}_2,\omega) \qquad (11)}$$

---

* P.M. MORSE AND H. FESHBACH, METHODS OF MATHEMATICAL PHYSICS (1953), Vol II, pp 919-920.

IF THERE IS NO DEGENERACY, SOLUTIONS OF THE INTEGRAL EQUATION (6) CAN BE SHOWN TO BE NECESSARILY OF THE FORM

$$\boxed{W_K(\underline{\rho}_1, \underline{\rho}_2, \omega) = \lambda_K(\omega)\, \phi_K^*(\underline{\rho}_1, \omega)\, \phi_K(\underline{\rho}_2, \omega), \quad (12)}$$

WITH EIGENVALUES

$$\boxed{\sigma_K(\omega) = \alpha_K^*(\omega)\, \alpha_K(\omega), \quad (13)}$$

WHERE*

$$\int_A L(\underline{\rho}_1, \underline{\rho}_2, \omega)\, \phi_K(\underline{\rho}_2, \omega)\, d^2\rho_2 = \alpha_K(\omega)\, \phi_K(\underline{\rho}_1, \omega) \qquad [(7)]$$

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* EQUATION (7) IS THE FOX-LI EQUATION FOR (MONOCHROMATIC) LASER MODES [A.G. FOX and T. LI, Bell Syst. Tech. J, 40, 453 (1961)].

L11-86

NATURE OF THE SOLUTIONS

$$W_K(\underline{\rho}_1, \underline{\rho}_2, \omega) = \lambda_K(\omega)\, \phi_K^*(\underline{\rho}_1, \omega)\, \phi_K(\underline{\rho}_2, \omega). \qquad [(12)]$$

DEGREE OF SPECTRAL COHERENCE AT FREQUENCY ω OF EACH MODE:

$$|\mu_K(\underline{\rho}_1, \underline{\rho}_2, \omega)| = \left| \frac{W_K(\underline{\rho}_1, \underline{\rho}_2, \omega)}{[W_K(\underline{\rho}_1, \underline{\rho}_1, \omega)]^{1/2} [W_K(\underline{\rho}_2, \underline{\rho}_2, \omega)]^{1/2}} \right| = 1 \qquad (14)$$

$\therefore$ COMPLETE SPATIAL COHERENCE AT FREQUENCY* ω.

SPECTRAL DENSITY:

$$S_K(\underline{\rho}, \omega) \equiv W_K(\underline{\rho}, \underline{\rho}, \omega) = \lambda_K(\omega) |\phi_K(\underline{\rho}, \omega)|^2 \qquad (15)$$

$$\lambda_K(\omega) = \int_A S_K(\underline{\rho}, \omega)\, d^2\rho \qquad (16)$$

---

* M. BERTOLOTTI et al. Nuovo Cimento, 38, 1505 (1965).

611-86

IN THE SPACE - TIME DOMAIN:

$$W_K(\underline{\rho}_1,\underline{\rho}_2,\omega) = \lambda_K(\omega)\,\phi_K^*(\underline{\rho}_1,\omega)\,\phi_K(\underline{\rho}_2,\omega) \qquad [(12)]$$

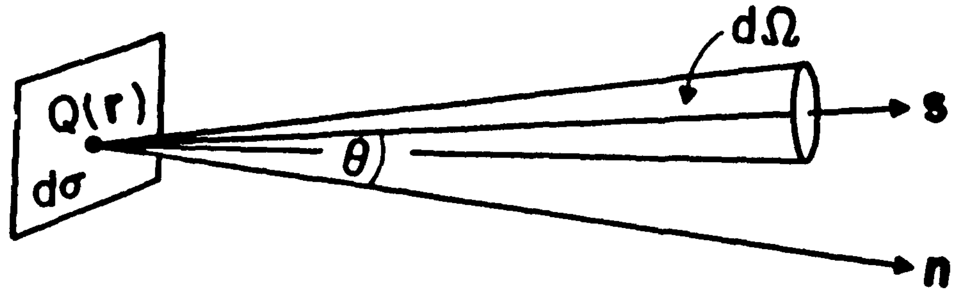
$$\Rightarrow \boxed{\Gamma_K(\underline{\rho}_1,\underline{\rho}_2,\tau) = \int_0^\infty \lambda_K(\omega)\,\phi_K^*(\underline{\rho}_1,\omega)\,\phi_K(\underline{\rho}_2,\omega)\,e^{-i\omega\tau}\,d\omega \qquad (17)}$$

FOX-LI MODES

$$\mu_K(\underline{\rho}_1,\underline{\rho}_2,\omega) = \frac{W_K(\underline{\rho}_1,\underline{\rho}_2,\omega)}{[W_K(\underline{\rho}_1,\underline{\rho}_1,\omega)]^{1/2}[W_K(\underline{\rho}_2,\underline{\rho}_2,\omega)]^{1/2}} \Longrightarrow |\mu_K(\underline{\rho}_1,\underline{\rho}_2,\omega)| = 1 \qquad (18)$$

$$\gamma_K(\underline{\rho}_1,\underline{\rho}_2,\tau) = \frac{\Gamma_K(\underline{\rho}_1,\underline{\rho}_2,\tau)}{[\Gamma_K(\underline{\rho}_1,\underline{\rho}_1,0)]^{1/2}[\Gamma_K(\underline{\rho}_2,\underline{\rho}_2,0)]^{1/2}} \Longrightarrow |\gamma_K(\underline{\rho}_1,\underline{\rho}_2,\tau)| \neq 1 \qquad (19)$$

FOUNDATIONS OF RADIOMETRY



$$d\mathcal{E}_\nu = B_\nu(\mathbf{r},\mathbf{s})\cos\theta \, d\sigma \, d\Omega \, dt \qquad (1)$$

Radiated power: $$P_\nu = \int_\sigma d\sigma \int_{(2\pi)} d\Omega \; B_\nu(\mathbf{r},\mathbf{s})\cos\theta \qquad (2)$$

$$= \int_\sigma E_\nu(\mathbf{r})\, d\sigma = \int_{(2\pi)} J_\nu(\mathbf{s})\, d\Omega \qquad (3)$$

$$E_\nu(\mathbf{r}) = \int_{(2\pi)} B_\nu(\mathbf{r},\mathbf{s})\cos\theta \, d\Omega = \text{Radiant emittance} \qquad (4)$$

$$J_\nu(\mathbf{s}) = \cos\theta \int_\sigma B_\nu(\mathbf{r},\mathbf{s})\, d\sigma = \text{Radiant intensity} \qquad (5)$$

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EQUATION OF RADIATIVE ENERGY TRANSFER

$$\boxed{\underline{s} \cdot \nabla B_\gamma(\underline{r},\underline{s}) = -\alpha_\gamma(\underline{r},\underline{s})\, B_\gamma(\underline{r},\underline{s}) + \int_{(4\pi)} \beta_\gamma(\underline{r},\underline{s},\underline{s}')\, B_\gamma(\underline{r},\underline{s}')\, d\Omega' + D_\gamma(\underline{r},\underline{s}) \qquad (6)}$$

$\alpha_\gamma(\underline{r},\underline{s})$ = EXTINCTION COEFFICIENT

$\beta_\gamma(\underline{r},\underline{s},\underline{s}')$ = DIFFERENTIAL SCATTERING COEFFICIENT

$D_\gamma(\underline{r},\underline{s})$ = SOURCE FUNCTION

SPACE DENSITY OF RADIATION

$$u_\nu(\underline{r}) = \frac{1}{c}\int_{(4\pi)} B_\nu(\underline{r},\underline{s})\, d\Omega \qquad \text{(SCALAR)} \qquad (7)$$

NET FLUX

$$F_\nu(\underline{r}) = \int_{(4\pi)} B_\nu(\underline{r},\underline{s})\,\underline{s}\, d\Omega \qquad \text{(VECTOR)} \qquad (8)$$

(MARGINAL RELATIONS)

RADIATIVE TRANSFER IN FREE SPACE

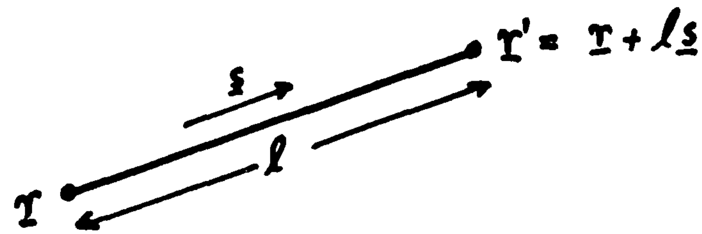
$$\underline{s} \cdot \nabla B_\nu(\underline{r},\underline{s}) = -\alpha_\nu(\underline{r},\underline{s})\, B_\nu(\underline{r},\underline{s}) + \int_{(4\pi)} \beta_\nu(\underline{r},\underline{s},\underline{s}')\, B_\nu(\underline{r},\underline{s}')\, d\Omega' + D_\nu(\underline{r},\underline{s}) \qquad [(6)]$$

IN FREE SPACE: $\alpha_\nu = \beta_\nu = D_\nu = 0$ (9)

EQ. (6) REDUCES TO $\underline{s} \cdot \nabla B_\nu(\underline{r},\underline{s}) = 0$ (10a)

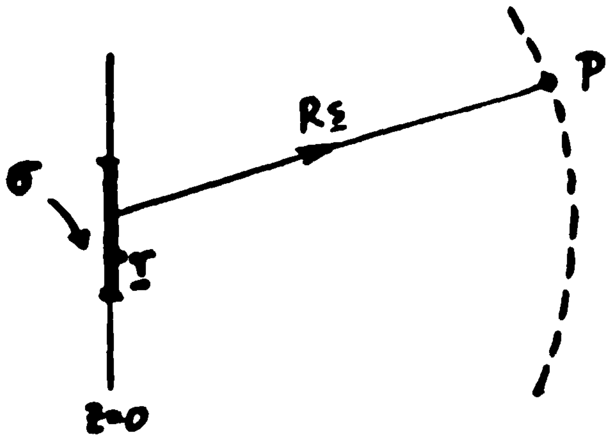
i.e. $\dfrac{\partial B_\nu(\underline{r},\underline{s})}{\partial \ell} = 0$ (10b)

$\therefore$ WITH $\underline{s}$ FIXED, $\underline{B_\nu(\underline{r},\underline{s}) = \text{CONSTANT}}$ (10c)



$B_\nu(\underline{r},\underline{s}) = B_\nu(\underline{r}',\underline{s})$ (10d)

GENERALIZED RADIANCE



$$F_\nu = \int_{(2\pi)} \underbrace{W(R\underline{s}, R\underline{s}, \nu)}_{\text{"Optical" intensity}} R^2 d\Omega_s \qquad (11)$$

$$(kR \to \infty)$$

$$\boxed{F_\nu = \int_{(2\pi)} d\Omega_s \int_{z=0} d^2\underline{r}\; B_\nu(\underline{r}, \underline{s}) \cos\theta \qquad (12)}$$

$$\boxed{B_\nu(\underline{r}, \underline{s}) = \left(\frac{k}{2\pi}\right)^2 \cos\theta \int_{z=0} W(\underline{r} + \tfrac{1}{2}\underline{r}', \underline{r} - \tfrac{1}{2}\underline{r}', \nu)\, e^{-ik\underline{s}\cdot\underline{r}'}\, d^2\underline{r}' \qquad (13)}$$

A. WALTHER, J. OPT. SOC. AMER., 58, 1256 (1968).

E. W. MARCHAND AND E. WOLF, OPT. COMMUN., 6, 305 (1972); J. OPT. SOC. AMER., 64, 1219 (1974).

$$B_\nu^{(1)}(\underline{r},\underline{s}) = \left(\frac{k}{2\pi}\right)^2 \cos\theta \int_{(z=0)} W(\underline{r}+\tfrac{1}{2}\underline{r}', \underline{r}-\tfrac{1}{2}\underline{r}', \nu)\, e^{-ik\underline{s}\cdot\underline{r}'}\, d^2r' \qquad [(13)]$$

$$B_\nu^{(2)}(\underline{r},\underline{s}) = \left(\frac{k}{2\pi}\right)^2 \cos\theta \int_{(z=0)} W(\underline{r}, \underline{r}', \nu)\, e^{-ik\underline{s}\cdot(\underline{r}-\underline{r}')}\, d^2r' \qquad (14)$$

$$B_\nu^{(2)}(\underline{r},\underline{s}) \neq B_\nu^{(1)}(\underline{r},\underline{s}) \qquad (15)$$

But $$F_\nu^{(2)} = F_\nu^{(1)} \qquad (16)$$

A. WALTHER, J. OPT. SOC. AMER., 63, 1622 (1973); 64, 1275 (1974)

E. W. MARCHAND AND E. WOLF, J. OPT. SOC. AMER., 64, 1273 (1974)

L. S. DOLIN (1964)

V. I. TATARSKII (1971); G. I. OVCHINNIKOV AND V. I. TATARSKII (1972)

RECENT RESEARCH ON

FOUNDATION OF RADIOMETRY*

1.) Restriction to globally incoherent sources

(Quasi-homogeneous sources)

2.) Short wavelength limit ($\lambda \rightarrow 0$)

(Asymptotic limit: $k = 2\pi/\lambda \rightarrow \infty$)

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* J. FOLEY and E.WOLF, *Opt. Commun.* 55, 236 (1985).
K. KIM and E. WOLF, *J. Opt. Soc. Amer. A* 4, 1233 (1987).
G.S. AGARWAL, J.T. FOLEY and E. WOLF, *Opt. Commun.* 62, 67 (1987).
Review of earlier researches: E. WOLF, *J. Opt. Soc. Amer.* 68, 6 (1978); A.T. FRIBERG, *Opt. Eng.* 21, 927 (1982).

FOR A QUASI-HOMOGENEOUS SOURCE

$$W^{(0)}(\underline{r}_1, \underline{r}_2, \nu) = \underbrace{I^{(0)}\left(\frac{\underline{r}_1 + \underline{r}_2}{2}, \nu\right)}_{\substack{\text{INTENSITY} \\ \text{(SLOW)}}} \underbrace{g^{(0)}(\underline{r}_1 - \underline{r}_2, \nu)}_{\substack{\text{COHERENCE} \\ \text{(FAST)}}}$$

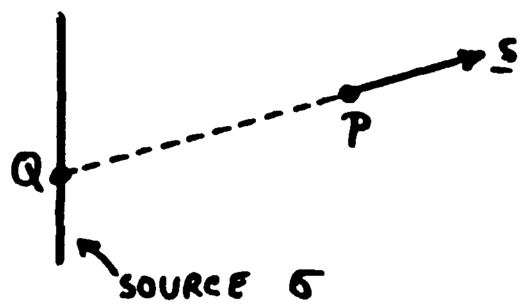
[Eq. (3), transp. 36]

ASYMPTOTIC LIMIT AS $k \to \infty$

$$\boxed{B_\nu(\underline{r}, \underline{s}) \sim k^2 s_z I^{(0)}\left(\underline{r}_\perp - \frac{z}{s_z} \underline{s}_\perp, \nu\right) \tilde{g}^{(0)}(k \underline{s}_\perp, \nu) \qquad (17)}$$

GEOMETRICAL INTERPRETATION :

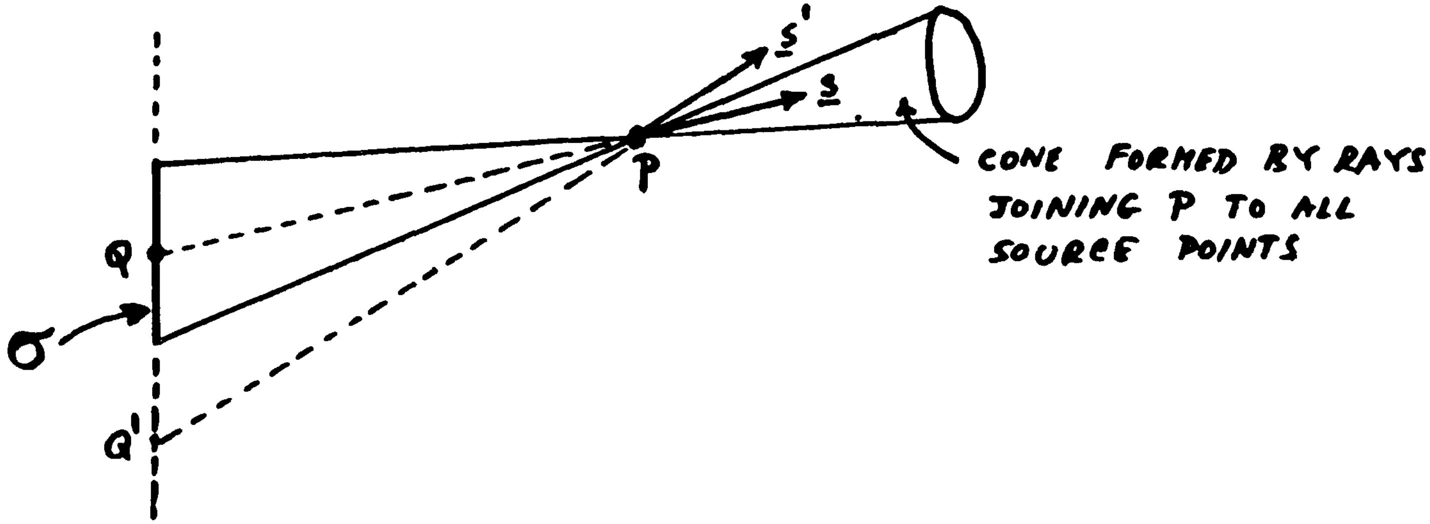
$$\boxed{B_\nu(P, \underline{s}) \sim k^2 s_z I^{(0)}(Q, \nu) \tilde{g}^{(0)}(k \underline{s}_\perp, \nu) \qquad (17a)}$$



L11-86

CONSISTENCY WITH RADIOMETRY

$$B_\nu(P,\underline{s}) \sim k^2 s_z\, I^{(0)}(Q,\nu)\, \tilde{g}^{(0)}(k\underline{s}_\perp,\nu) \qquad [(17a)]$$



(1) $B_\nu(P,\underline{s}) \geq 0$ IF $Q \in \sigma$ ($\underline{s}$ WITHIN CONE)

(2) $B_\nu(P,\underline{s}) = 0$ IF $Q \notin \sigma$ ($\underline{s}$ OUTSIDE CONE)

(3) $B_\nu(P,\underline{s})$ CONSTANT ALONG EVERY $\underline{s}$-DIRECTION THROUGH P; i.e. SATISFIES EQUATION OF RADIATIVE TRANSFER IN FREE SPACE

SPECIAL CASE : LAMBERTIAN SOURCE

$$B_\nu(P, \underline{s}) \sim k^2 s_z \, I^{(0)}(Q,\nu)\, \tilde{g}^{(0)}(k\underline{s}_\perp,\nu) \qquad [(17a)]$$

FOR A LAMBERTIAN SOURCE

$$g^{(0)}(\underline{r}_1-\underline{r}_2,\nu) = \frac{\sin(k|\underline{r}_1-\underline{r}_2|)}{k|\underline{r}_1-\underline{r}_2|} \qquad [\text{Eq. (13), transp. 42}]$$

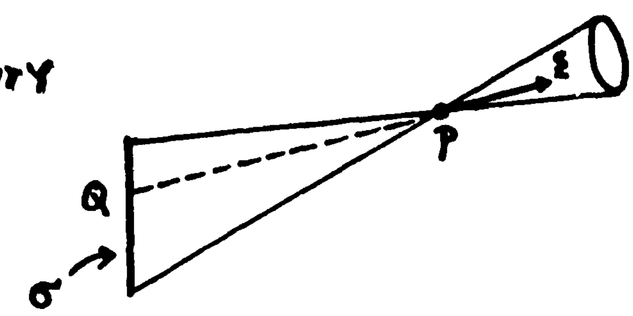
$$\downarrow$$

$$\tilde{g}^{(0)}(k\underline{s}_\perp,\nu) = \frac{1}{2\pi k^2 s_z} \qquad (s_z = \cos\theta) \qquad (18)$$

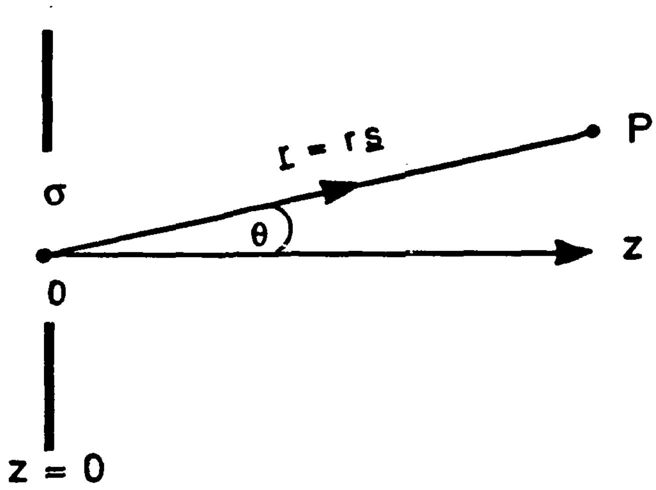
FROM EQS (17a) AND (18)

$$\boxed{\left[B_\nu(P,\underline{s})\right]_{\text{LAMBERTIAN}} \begin{array}{ll} \sim \dfrac{1}{2\pi} I^{(0)}(Q,\nu) & \text{if } Q \in \sigma \\ \sim 0 & \text{if } Q \notin \sigma \end{array} \Bigg\} \quad (19)}$$

NOTE: THE RADIANCE NOW DEPENDS ONLY ON THE SOURCE INTENSITY



EFFECTS OF SOURCE CORRELATIONS ON THE SPECTRUM OF EMITTED LIGHT



Planar secondary quasi-homogeneous source σ

Same source spectrum, $S^{(0)}(\omega)$, at every source point

Reciprocity formula (4) on transparency 38 implies that spectrum in the far zone is

$$\boxed{S^{(\infty)}(\underline{r},\omega) \equiv \frac{J_\omega(\underline{s})}{r^2} = \frac{k^2 A}{r^2}\, S^{(0)}(\omega)\,\tilde{\mu}^{(0)}(k\underline{s},\omega)\cos^2\theta \qquad (1)}$$

A = area of source

$\tilde{\mu}^{(0)}$= Fourier transform of degree of spectral coherence of light in source plane $[g \rightarrow \mu]$

<u>The spectrum of the far field depends not only on the source spectrum</u> $S^{(0)}(\omega)$ <u>but also on the coherence properties of the source</u>.

In general, <u>the normalized spectrum of light changes on propagation</u>.

RADIATION FROM A PRIMARY SOURCE

$$(\nabla_1^2 + k^2)(\nabla_2^2 + k^2)\, W_V(\underline{r}_1, \underline{r}_2, \omega) = (4\pi)^2\, W_Q(\underline{r}_1, \underline{r}_2, \omega) \qquad [(7)]$$

RADIATION FROM A PLANAR, HOMOGENEOUS SECONDARY SOURCE

$$(\nabla_1^2 + k^2)(\nabla_2^2 + k^2)\, W_V(\underline{r}_1, \underline{r}_2, \omega) = 0 \qquad (z \geq 0) \qquad (7a)$$

B.C.:

$$\left.\begin{aligned} W_V(\underline{r}_1', \underline{r}_2', \omega)\Big|_{z=0} &= W_V^{(0)}(\underline{r}_2' - \underline{r}_1', \omega) && \underline{r}_1', \underline{r}_2' \in \sigma \\ &= 0 && \underline{r}_1', \underline{r}_2' \notin \sigma \end{aligned}\right\} \qquad (14)$$

THE DEGREE OF SPECTRAL COHERENCE (A = V or Q)

$$\mu_A(\underline{r}_1, \underline{r}_2, \omega) = \frac{W_A(\underline{r}_1, \underline{r}_2, \omega)}{\sqrt{W_A(\underline{r}_1, \underline{r}_1, \omega)}\sqrt{W_A(\underline{r}_2, \underline{r}_2, \omega)}}, \quad (0 \leq |\mu_A| \leq 1) \qquad (15)$$

NORMALIZED SPECTRUM

$$s_A(\underline{r}, \omega) = \frac{S_A(\underline{r}, \omega)}{\int S_A(\underline{r}, \omega)\, d\omega} \qquad \left(\int s_A(\underline{r}, \omega)\, d\omega = 1\right) \qquad (16)$$

SCALING LAW [E Wolf, *Phys. Rev. Lett.* 56 1370 (1986)]

A SUFFICIENCY CONDITION FOR THE NORMALIZED SPECTRUM OF LIGHT PRODUCED BY A PLANAR, HOMOGENEOUS SECONDARY SOURCE TO BE THE SAME THROUGHOUT THE FAR ZONE AND ACROSS THE SOURCE ITSELF IS THAT THE DEGREE OF SPECTRAL COHERENCE OF THE LIGHT DISTRIBUTION ACROSS THE SOURCE HAS THE FORM

$$\boxed{\mu_V^{(0)}(\underline{r}_2' - \underline{r}_1', \omega) = f[k(\underline{r}_2' - \underline{r}_1')], \qquad \left(k = \frac{\omega}{c} = \frac{2\pi}{\lambda}\right) \quad (17)}$$

I.E. THAT IT IS A FUNCTION OF THE VARIABLE

$$\xi = k(\underline{r}_2' - \underline{r}_1') = 2\pi \frac{\underline{r}_2' - \underline{r}_1'}{\lambda} \qquad (18)$$

ONLY.

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SCALING LAW

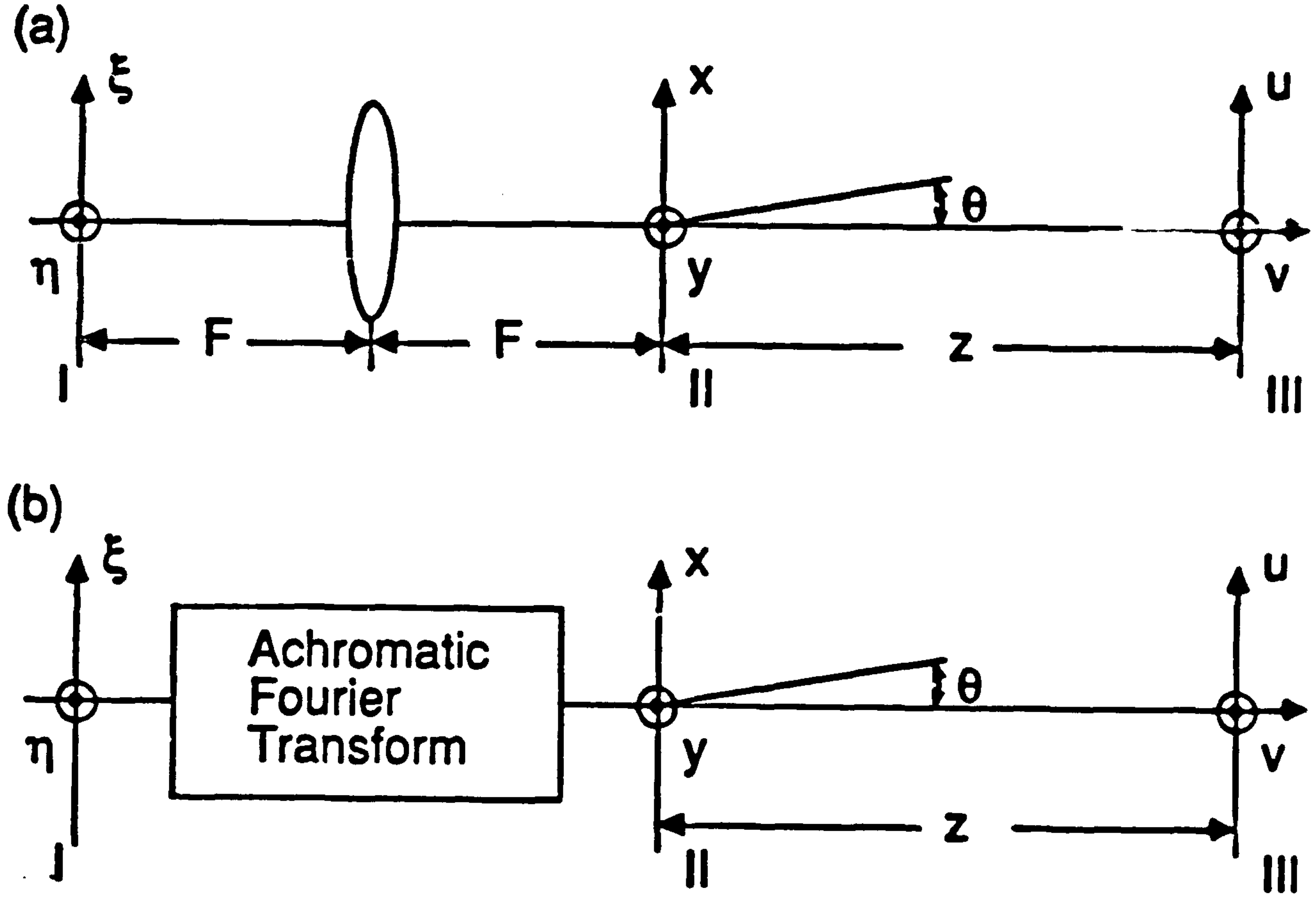
$$\boxed{\mu_V^{(0)}(\mathbf{r}_2'-\mathbf{r}_1',\omega) = f\left[k(\mathbf{r}_2'-\mathbf{r}_1')\right], \quad \left(k=\frac{\omega}{c}=\frac{2\pi}{\lambda}\right) \qquad [(17)]}$$

EXAMPLES:

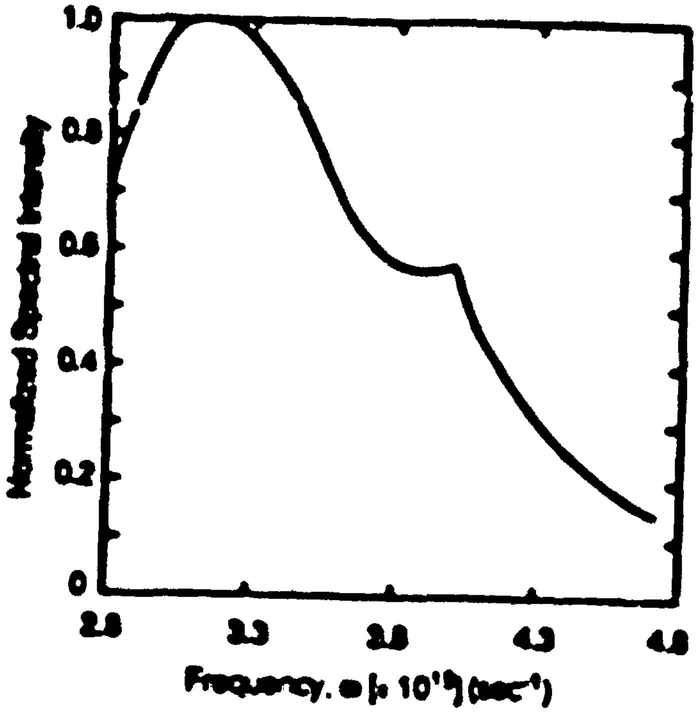
BLACKBODY SOURCES

LAMBERTIAN SOURCES

$$\mu_V^{(0)}(\mathbf{r}_2'-\mathbf{r}_1',\omega) = \frac{\sin(k|\mathbf{r}_2'-\mathbf{r}_1'|)}{k|\mathbf{r}_2'-\mathbf{r}_1'|} \qquad (19)$$



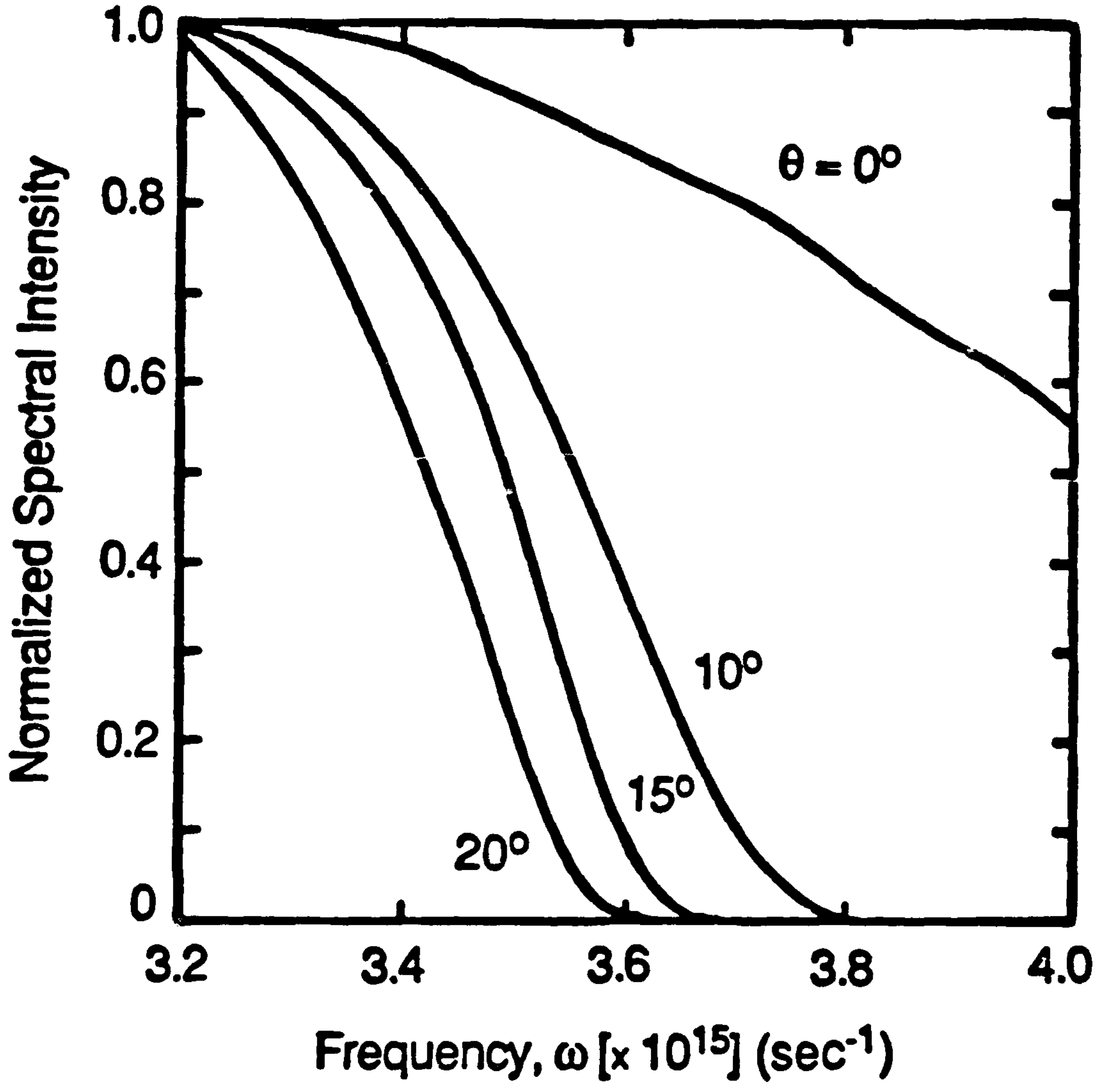
AFTER G. M. MORRIS AND D. FAKLIS, OPTICS COMMUNICATIONS 62, 5 (1987).



(a) SCALING LAW SATISFIED

AFTER G M MORRIS AND D. FAKLIS.

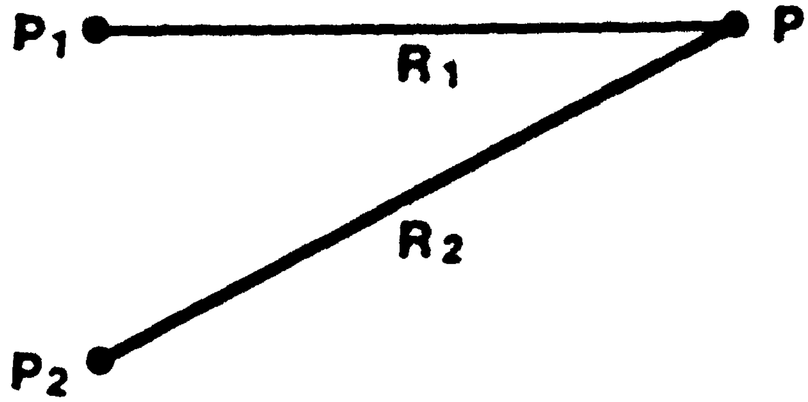
OPTICS COMMUNICATIONS 62, 5 (1987)



(b) SCALING LAW NOT SATISFIED

AFTER G. M. MORRIS AND D. FAKLIS, OPTICS COMMUNICATIONS 62, 5 (1987).

EXAMPLE: TWO SMALL CORRELATED SOURCES*



SOURCE ENSEMBLES: $\{Q(P_1,\omega)\}$, $\{Q(P_2,\omega)\}$

FIELD ENSEMBLE: $\{U(P,\omega)\}$

$$U(P,\omega) = Q(P_1,\omega)\frac{e^{ikR_1}}{R_1} + Q(P_2,\omega)\frac{e^{ikR_2}}{R_2} \tag{9}$$

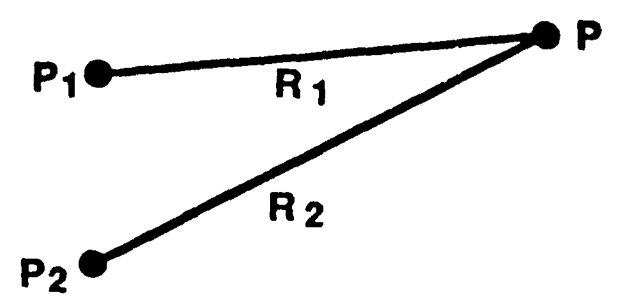
$$k = \frac{\omega}{c}$$

SPECTRUM OF THE FIELD AT P:

$$S_V(P,\omega) = \langle U^*(P,\omega)U(P,\omega)\rangle \tag{10}$$

(9) → (10)

* E. Wolf, Phys. Rev. Lett. 58, 2646(1987).



$$S_V(P,\omega) = \underbrace{\langle Q^*(P_1,\omega)Q(P_1,\omega)\rangle}_{S_Q(\omega)}\frac{1}{R_1^2} + \underbrace{\langle Q^*(P_2,\omega)Q(P_2,\omega)\rangle}_{S_Q(\omega)}\frac{1}{R_2^2} + \left[\underbrace{\langle Q^*(P_1,\omega)Q(P_2,\omega)\rangle}_{W_Q(P_1,P_2,\omega)}\frac{e^{ik(R_2-R_1)}}{R_1R_2} + cc.\right]$$

$$\boxed{S_V(P,\omega) = S_Q(\omega)\left[\frac{1}{R_1^2}+\frac{1}{R_2^2}\right] + \left[W_Q(P_1,P_2,\omega)\frac{e^{ik(R_2-R_1)}}{R_1R_2} + cc.\right] \quad (11)}$$

$\therefore$ $S_V(P,\omega)$ IS **NOT** PROPORTIONAL TO $S_Q(\omega)$, IN GENERAL

EXEPTIONAL CASES:

(1) $W_Q(P_1, P_2, \omega) \equiv 0$: Uncorrelated **(INCOHERENCE)**

(2) $W_Q(P_1, P_2, \omega) \propto S_Q(\omega)$, $R_2 = R_1$: Completely correlated **(COHERENCE)**

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WITH THE CHOICE $R_2 = R_1 = R$, Eq.(11) REDUCES TO

$$\boxed{S_V(\omega) = 2\, S_Q(\omega)\left[\, 1 + \mathrm{Re}\,\mu_Q(\omega)\right] \qquad (12)}$$

$$S_V(\omega) = \frac{2}{R^2} S_V(P, \omega) = \text{REDUCED FIELD SPECTRUM} \qquad (12a)$$

$\mu_Q(\omega)$ = Degree of correlation between the sources

$$\mu_Q(\omega) = \frac{W_Q(P_1, P_2, \omega)}{S_Q(\omega)}$$

$$= \frac{\langle Q^*(P_1, \omega) Q(P_2, \omega)\rangle}{S_Q(\omega)} \qquad (13)$$

$$0 \leq |\, \mu_Q(\omega) \,| \leq 1$$

UNCORRELATED   FULLY CORRELATED

EXAMPLE:

SOURCE:

$$\boxed{S_Q(\omega) = A\, e^{-(\omega-\omega_0)^2/2\delta_0^2} \qquad (\delta_0/\omega_0 \ll 1)}$$

$$\mu_Q(\omega) = a\, e^{-(\omega-\omega_1)^2/2\delta_1^2} - 1 \qquad (\delta_1/\omega_1 \ll 1,\ a \leq 2)$$

$$\downarrow$$

FIELD:

$$\boxed{S_V(\omega) = A' e^{-(\omega-\omega_0')^2/2\delta_0'^2}}$$

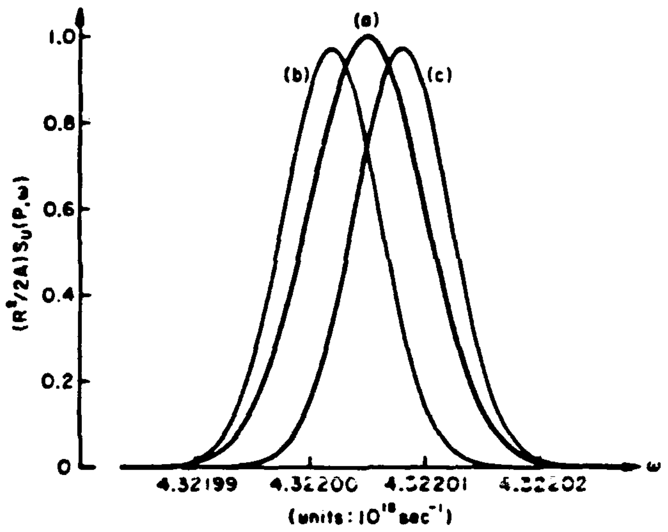
$$A' = \frac{2Aa}{R^2}\, e^{-(\omega-\omega_0')^2/2(\delta_0^2+\delta_1^2)}$$

$$\omega_0' = \frac{\delta_1^2\omega_0 + \delta_0^2\omega_1}{\delta_0^2 + \delta_1^2}$$

$$\frac{1}{\delta_0'^2} = \frac{1}{\delta_0^2} + \frac{1}{\delta_1^2}$$

<u>REDSHIFT</u> IF $\omega_0' < \omega_0 \quad \Rightarrow \quad \omega_1 < \omega_0$

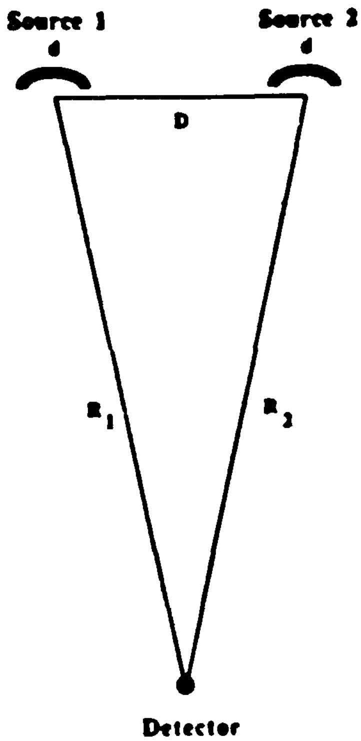
<u>BLUESHIFT</u> IF $\omega_0' > \omega_0 \quad \Rightarrow \quad \omega_1 > \omega_0$

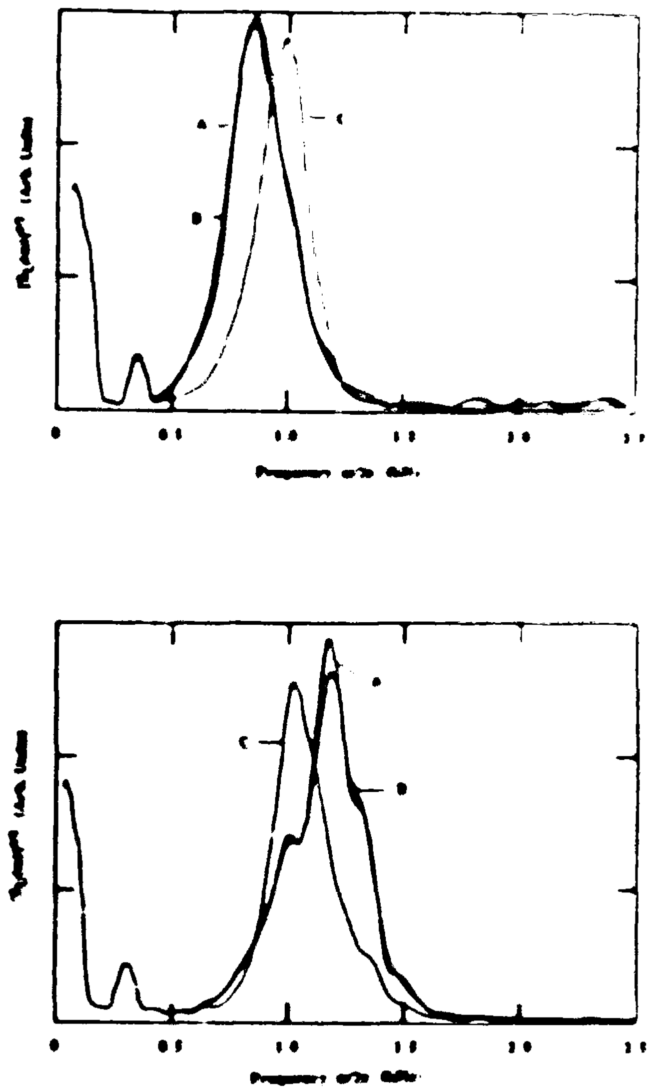


From E. Wolf, *Phys. Rev. Lett.* **58**, 2646 (1987)

EXPERIMENTAL TEST WITH ACOUSTICAL SOURCES

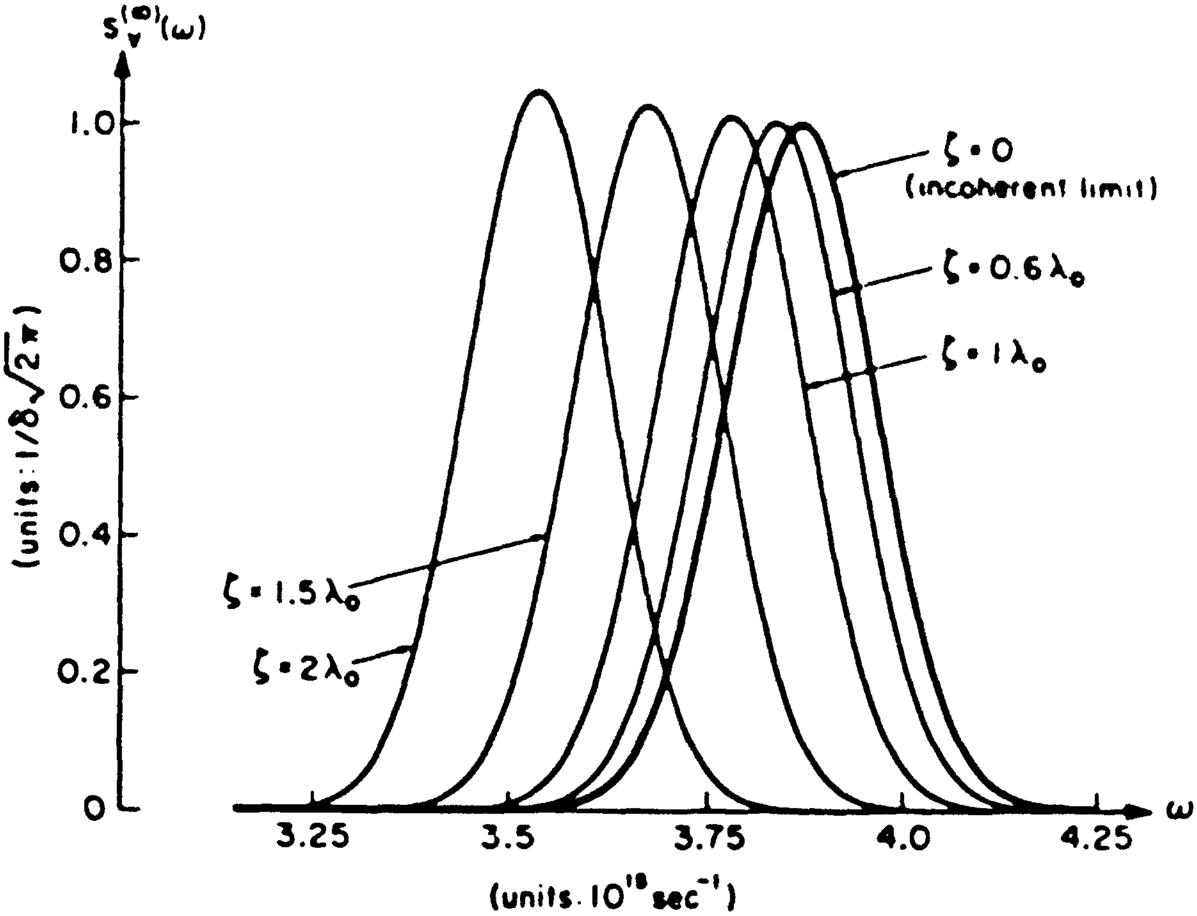
M.F. Bocko, D.H. Douglass and R.S. Knox
Phys. Rev. Lett. **58**, 2649 (1987)





REDSHIFTS WITH THREE-DIMENSIONAL SOURCES

Similar results hold for spectra of fields radiated by three-dimensional sources*

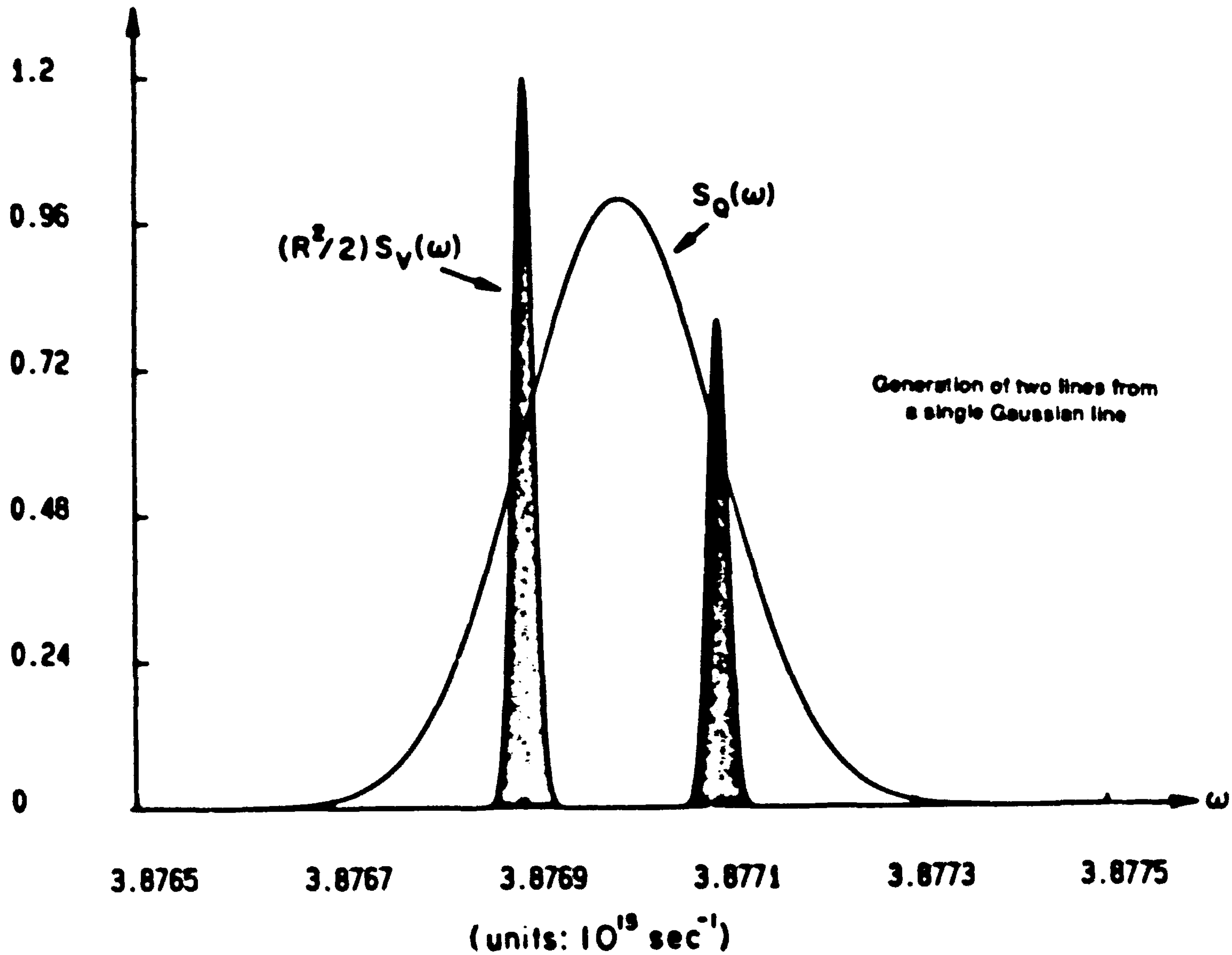


[$\omega_0 = 3.887 \times 10^{15}$ sec^{-1} ($\lambda_0 = 4841$ Å), $\delta = 9.57 \times 10^{13}$ sec^{-1}]

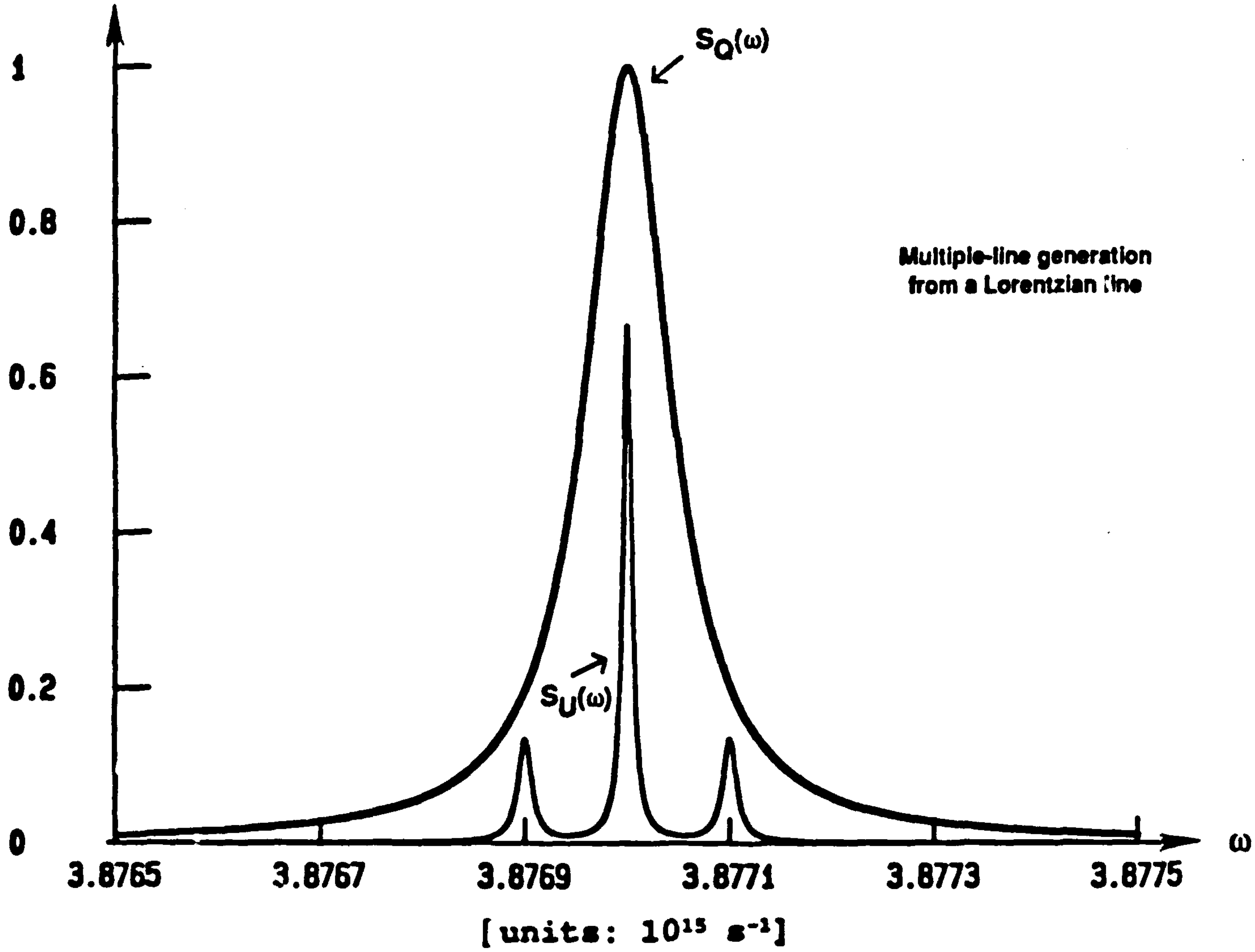
* E. Wolf, *Nature* **326**, 363 (1987); *Opt. Commun.* **62**, 12 (1987).

OTHER KINDS OF SPECTRAL MODULATION

BY CONTROL OF SOURCE CORRELATIONS



After A. Gamliel and E. Wolf, *Opt. Commun.*, in press



After A. Gamliel and E. Wolf, *Opt. Commun.*, in press.

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(a) *Theoretical*

E. Wolf, "Invariance of spectrum of light on propagation", Phys. Rev. Lett. **56**, 1370-1372 (1986).

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A. Gamliel and E. Wolf, "Spectral modulation by control of source correlations", Opt. Commun. **65**, 91-96 (1988).

Z. Dacic and E. Wolf, "Changes in the spectrum of partially coherent light beam propagating in free space", J. Opt. Soc. Amer., A, in press.

J. T. Foley and E. Wolf, "Partially coherent sources which generate the same far field spectra as completely incoherent sources", J. Opt. Soc. Amer., A, in press.

(b) *Experimental*

G. M. Morris and D. Faklis, "Effects of source correlation on the spectrum of light", Opt. Commun. **62**, 5-11 (1987).

M. F. Bocko, D. H. Douglass and R. S. Knox, "Observation of frequency shifts of spectral lines due to source correlations", Phys. Rev. Lett. **58**, 2649-2651 (1987).

continued . . .

D Faklis and G M Morris, 'Spectral shifts produced by source correlations', Opt Lett 13, 4-6 (1988)

F Gori, G Guattari, C Palma and G Padovani, 'Observation of optical redshifts and blueshifts produced by source correlations', Opt Commun, in press

W Knox and R S Knox, "Direct observation of the optical Wolf shift using white-light interferometry", Opt Lett (submitted); see also abstract of postdeadline paper PD21, Annual Meeting of the Optical Society of America (Rochester, NY), October, 1987 J Opt Soc Amer A 4, No 13, P131 (1987)

CECOM CENTER FOR NIGHT VISION AND ELECTRO-OPTICS
ARMY APPLICATIONS OF COHERENCE PHENOMENA

SOME APPLICATIONS - COHERENCE

1. LASER PROTECTION

2. LASER DETECTION

3. MOTION/VIBRATION SENSING

4. COHERENT IMAGING - ACTIVE/PASSIVE

5. COMMUNICATIONS

Related Programs

Coherence Filters - Physical Optics Corp.
U. S. Army Natick R & D Center

Acoustic - Optic coherence deflection filters - MTL

In-house Research in coherent filters - WPAFB

Photorefractive material research - CNVEO

LIST OF ATTENDEES

Modern Coherence Theory

Attendees List

18 May 1988

Center for Night Vision & Electro-Optics:

Rudy Buser	Al Pinto
Robert Rohde	Gary Wood
Mark Norton	Bill Clark
Ed Sharp	Gerri Daunt
Mark Savan	Martin Lenhart
Richard Utano	Gertrude Kernfield
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