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ORBITAL MECHANICS
A LEARNING TOOL ON THE MAIN FRAME

by

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Orbital Mechanics
A Learning Tool On The Main Frame

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ABSTRACT

This thesis consists of an interactive program that enables the student to study the orbital motion of satellites around the earth. The student can investigate the shape of a variety of orbits by varying the initial position and velocity of the satellite, or by supplying select orbital parameters i.e. initial orbital radius, eccentricity, and inclination. Satellite maneuvers can also be studied, like transfer orbits and inclination changes, by command velocity changes at any location in the orbit. Also the effects of the perturbing forces due to the oblateness of the earth, drag for low earth orbits, and gravitational attraction from the sun and moon can be investigated. The orbits are displayed in either the perifocal coordinate system around a model of the earth, or the ground track can be displayed on a map of the world. Orbital data is displayed below the orbital plot. The display is enabled by the use of display integrated software system and plotting language (DISSPLA) subroutines.

TABLE OF CONTENTS

I. INTRODUCTION

A visual aid for students new to orbital mechanics is required to comprehend fully the dynamics of orbital motion. This program is an interactive time step simulation program that calculates and plots either unperturbed or perturbed elliptical orbits. The program interacts with the student in developing the initial orbit. Also the program enables the student with the ability to change the velocity of the satellite at a specific location in the orbit. This feature will permit the student to investigate the effects of commanded velocity changes as in perigee kicks, apogee kicks and inclination changes. The user can also modify the initial position and velocity of the satellite at the completion of any orbit.

The student is given an opportunity to investigate the effects of perturbing forces on the satellites orbit by choosing to have the program calculate the orbit with or without perturbing forces. The variation of parameters method, as seen in [Ref. 1: pp. 396-407], is used in calculating the perturbing orbit. The perturbing forces taken into consideration are the following:

1. the oblateness of the earth
2. drag for low earth orbits
3. gravitational force of the moon
4. gravitational force of the sun

In order to review fully the operation of the program (included in appendix A) and to uncover any problems or limitations that plagued the programming, the program has been divided up as follows:

1. program design
2. unperturbed orbit
3. perturbed orbit
4. velocity changes
5. graphical plots

The programming approach and equations used in each of the above sections will be examined in there respective chapters. A review of the coordinate systems used and their transformations between them are included in appendix B. Since all the equations used in the calculation of the orbital elements are from reference 1, they will not be reviewed in each chapter but will be included in appendix C for a quick reference. Equations from other sources will be referenced in their respective chapters.

Examples of perturbed and unperturbed orbital plots for a variety of initial orbital parameters are inc d in appendix D. Included are plots of low earth orbits, transfer orbits and geosynchronous orbits.

II. PROGRAM DESIGN

In designing this program an attempt was made to make it not only as user friendly as possible, but also to make the program as simple as possible to understand. To achieve these goals, the program would have to be written in a logical manner, in a computer language that is easy to follow, the program would have to run on terminals readily available to students (at the Naval Postgraduate School (NPS)), and the program would have to be easily used by students with a minimum amount of computer or orbital mechanics knowledge.

FORTRAN was chosen as the programming language since it is a wildly used scientific language and it allows for very structured programming. By programming in a structured format, the program can be expanded in the future with a minimum amount of time required to understand the programming code. FORTRAN also allows for double precision numbers to be used in the calculation of the orbit. This is critical when round off error in single precision could be greater then the actual change that one is trying to model. The equations in the descriptions of the program might not exactly match the equations in the listings because of special programming techniques which must be included in most computer programs to handle such problems as "division by zero".

The display integrated software system and plotting language (DISSPLA) package available on the mainframe computer at NPS was used to enable a variety of graphical displays with a minimum amount of programming. DISSPLA has a set of subroutines that the programmer calls to display data contained in arrays. This requirement forces the program to load arrays with the satellites position in order for it to be plotted. The TEC618 computer terminal and associative plotter was used for ease of gaining hard copy plots of the orbits and the diversity of locations that are available here at NPS. In order to run a program in DISSPLA the user must first define storage space of 1500k and designate temporary disk space, and then call DISSPLA with the program name. This is accomplished with the following commands:

1. DEFINE STORAGE 1500K

2. ICMS
3. TDISK 4 DIS
4. DISSPLA ORBIT

To make the program user friendly, the user is prompted for inputs via the keyboard. The entry is usually a number. A yes or no response can be entered by typing "Y" or a "N". In most cases the program does a check to see if the input is appropriate. In order to make it as easy as possible for the student to get the desired orbit displayed, the program requires only the initial position and velocity of the satellite. The initial position and velocity of the satellite is supplied by the user in one of two ways. The user can input the position and velocity of the satellite, using the perifocal coordinate system (IJK), or the user can let the program place the satellite on the "I" axis of the IJK system at the radius of perigee (RP) distance supplied by the user. This latter choice gives the initial location of the satellite, but to get the velocity the program will prompt the user for one of the following:

1. the actual velocity in the IJK system.
2. the eccentricity (e) of the orbit. In which case the velocity is calculated from the following equations:

   $a = \frac{RP}{1-e}$ = semi-major axis

   $ENR = -\frac{\mu}{2a}$ = energy mass

   Where μ = MG
   M = mass of earth
   G = Universal gravitational constant

   $v = \sqrt{2(ENR + \frac{\mu}{RP})}$

3. the radius of apogee (RA) The velocity is calculated by first calculating the eccentricity (e) from the following:

   $e = \frac{RA - RP}{RA + RP}$

   With the eccentricity the same equations used above are used to calculate the velocity.

In order to give the velocity a direction the inclination (i) of the orbit is required from the user. The following equations are used to calculate the velocity vector:

$v_I = 0.0$

$v_y = v \cos\theta$

$v_x = v \sin\theta$

The program will check to ensure that the orbital eccentricity is less than 1.0, if it is not then the program will reject the inputs. After the initial input are accepted, the program will do calculations for the six orbital elements required to describe the size, shape and orientation of the orbit, and to pinpoint the position of the satellite along the orbit at a particular time. This classical set of six orbital elements are as follows:

1. a, semi-major axis.
2. e, eccentricity.
3. i, inclination.
4. Ω, longitude of the ascending node.
5. ω, argument of perigee passage.
6. T, time of perigee passage.

The program actually calculates more orbital elements than the six classical elements required to plot the orbit, this is done in an effort to make the program as robust as possible. This will add in the ability to expand the program in the future.

If the satellite is not initially at the perigee point then the satellite [illegible] first be stepped around to the perigee point. The program then enters a loop that calculates the orbit from the perigee point through one complete orbit around the earth and back to the perigee point. The orbit is calculated in steps of 2 times pi divided by an integer, i.e., 2 times pi divided by 50. This step size was used to ensure a smooth orbit for display purposes and also to get within adequate distance to the perigee point or other location for a velocity change. After the loop is completed, the program will offer the user a choice of the following plots to check the orbit:

1. perifocal
2. groundtrack

The program then goes into a loop offering the user the following choices until the user decides to end the program:

1. plot another view of the same orbit.

If the user wishes to plot another view of the same orbit then the user may use this choice to reenter the display portion of the program.

2. plot the next orbit (perturbed or unperturbed).
   To plot the next orbit the satellite is stepped around the complete orbit either with or without perturbing forces effecting the satellite.
3. change the initial conditions.
   The program goes to the beginning of the program and allows the user to change the initial position and velocity of the satellite.
4. change the velocity at a specific location
   Step the satellite around to a specific true anomaly and make a velocity change at that location.
5. clear the previous orbits from the plot.
   Clear the memory of all the previous orbits and only retain the current location and velocity as the initial position and velocity.

Before each new orbit, the orbital elements are recalculated.

There are several common assumptions and constants used throughout the program i.e. all bodies are considered to be spherically symmetric (this allows these bodies to be treated as though their masses are concentrated at their centers (point masses)). other assumptions will be covered in their respective chapters.

III. UNPERTURBED ORBIT

The subroutines that calculate the unperturbed orbit are the most widely used subroutines in the entire program. These subroutines are called to step the satellite around to the perigee point from the user supplied initial position and velocity, to calculate the next unperturbed orbit, and for any velocity change. No matter which of these sources supply the initial position and velocity the program calculates the unperturbed orbit in the same manner. The only difference is where in the orbit the satellite is initially when these subroutines are called. Before the unperturbed subroutines are called, the orbital elements are calculated.

The unperturbed subroutines are called by a single subroutine 'UNPRET' which has the following basic algorithm:

1. Increment time by the time step size (DT). The time step was chosen as the period divided by fifty to give a smooth plot, but more importantly to ensure that the satellite is within an acceptable distance from a specific location for a velocity change. The angular error caused by the step size can be as much as PI 50 from the desired point for a circular orbit and will increase for more eccentric orbits. This error becomes a factor when the user is making velocity changes, and therefore it will be covered in that chapter in further detail.

2. Calculate the new elements. The calculation of the new elements is the heart of this algorithm. The size, shape and orientation of the orbit remains unchanged. What is required is the position of the satellite along the orbit as a function of time. The problem becomes a matter to solve "the Kepler problem"-predicting the future position and velocity of an orbiting object as a function of some known initial position and velocity and the time of flight [Ref. 1: p. 181]. An algorithm using these principles will follow:

    a. A time step (DT) is added to the time of flight(TF), time of flight is the clapsed time since the satellite passed the perigee point.

       $TF = TF + DT$

    b. The new mean anomaly (MA) is calculated from the new time of flight, and the mean motion (MM).

       $MA = MM \times TF$

    c. With the new mean anomaly the new eccentric anomaly (EA) is calculated. Because the solution to the Kepler problem ($MA = EA - e \times \sin(EA)$) is transcendental, an iterative solution based on the Newton method of root finding is used. The root in question is a solution to the equation ($MA - EA + e \times \sin(EA) = 0$). This algorithm takes the form of [Ref. 1: p. 222]:

       1) $M.A. = E.A. - e \times \sin(E.A.)$

c.

$$EA_{n+1} = EA_n - \frac{(MA - MA_n)}{(1 - e \times \cos(EA_n))}$$

Where this equation is applied initially to $EA_0 = MA$ and then reapplied until the difference between MA and MA_n becomes small enough to be ignored.

d. The new true anomaly (v_0) is calculated from:

$$v_0 = \frac{\cos^{-1}(e - \cos(EA))}{e\cos(EA) - 1}$$

3. Calculate the new position and velocity. The position and velocity are calculated in the perifocal coordinate system (PQW). The PQW system uses the orbit as its fundamental plane and therefore requires only two coordinate to specify the satellite's position and velocity. The z_w coordinate is by definition always equal to zero. The position of the satellite is calculated as:

   $x_w = r \cos v$

   $y_w = r \sin v$

   $z_w = 0$

   The velocity of the satellite is calculated as:

   $v_x = \sqrt{\frac{\mu}{p}}(-\sin v_0)$

   $v_y = \sqrt{\frac{\mu}{p}}(e + \cos v_0)$

   $v_z = 0$

4. Store position and elements in arrays for plotting. In order for the program to plot the orbit the radius, true anomaly, inclination, and argument of perigee must be stored in arrays. The use of these arrays to plot the orbit will be explained in chapter 6.

5. The process is repeated until the satellite is at the perigee point and the true anomaly is two pi.

The procedure used to calculate the unperturbed orbit leave very little to be modified by a programmer. The only choices that had to be made concerned step size, how to tell the UNPRET subroutine that the perigee point had been reached, and a value of acceptable error for newtons method. For the unperturbed orbit, the step size just had to be small enough to produce a smooth plot of the orbit. Two indicators for perigee were used, one was that the true anomaly was greater than 6.21 radians (two pi equals 6.28 radians) and the time from the previous perigee point will be greater then the period. The two indicators were logically 'and' together to ensure the perigee point was reached.

The disparity between two pi and 6.21 radians is due to the error produced by the satellite not beginning the orbit at exactly the perigee point and the step size used go around the orbit. The acceptable size of error for newtons method was set at 1.0E-10, because for an unperturbed orbit this would be the major contributor to any error in the orbit and the magnitude of this error would be acceptable. However; in a perturbed orbit there are other factors contributing to determining the acceptable error, and these will be discussed in the next chapter.

IV. PERTURBED ORBIT

The perturbed orbit uses the same basic routines as the unperturbed orbit in stepping the satellite around the earth with one major difference, the perturbing forces produce a time rate of change of the orbital elements that must be applied at each time step. The variation of parameters method is used to determine this influence of the perturbing forces on the orbital elements. The analysis is simplified by using the orbital coordinate system 'RSW', as explained in appendix B. The basic algorithm is as follows [Ref. 1: p. 407]:

1. At $t = t_0$ calculate six orbital elements.
2. Compute the perturbing forces and transform it at $t = t_0$ to the 'RSW' SYSTEM.
3. Compute the time rate-of-change of the elements.
4. Calculate the change of elements for one time step, and add the changes to the old values at each step to get the new elements.
5. From the new values of the orbital elements, calculate a position and velocity.
6. Go to the step 2 and repeat until the final time is reached.

The steps in the algorithm will be explained in the following sections:

A. ORBITAL ELEMENTS

The standard orbital elements a, e, i, Ω, ω and T (or M) will be used, where

a = semi-major axis

e = eccentricity

i = inclination

Ω = longitude of ascending node

ω = argument of perigee

T = time of perigee passage

(M_e = mean anomaly at epoch = $M - n(t - t_e)$). The elements are calculated only at the beginning of the orbit from the initial position and velocity vectors. The elements are then changed continuously throughout the orbit by adding the changes due to the perturbing forces. For the perturbed orbit, the satellite will always begin at the perigee point. This is done so one complete orbit is from perigee point to perigee point.

B. COMPUTE PERTURBING FORCES

The variation of parameters method requires that the perturbing forces be calculated at each step in the orbit. In order to do this a model of each perturbing force must be developed. The following perturbing forces where used in calculating the total perturbing force effecting the satellite:

1. oblateness of the earth
2. atmospheric drag
3. gravitational attraction of the sun
4. gravitational attraction of the moon

The magnitudes of these forces have an enormous range of values and are dependent on the distance the satellite is from the perturbing body. Figure 1 on page 12 shows a graphical representation of the magnitude of the perturbing forces in a log-log plot of perturbing forces per unit mass [Ref. 2: p. IV-61]. The model of each of these forces follows:

1. NON-SPHERICAL EARTH

The earth is not perfectly spherical, but bulges around the equator. The polar and equatorial diameters are 12713.0 Km and 12756.3 Km, respectively. The oblateness results in a perturbing force per unit mass with these components in the 'RSW' coordinate system [Ref. 3: p. 81]:

$$F_r = \frac{(-3\mu J_2 r_e^2)}{2r^4}(1 - 3\sin^2(i)\sin^2(u_0))$$

$$F_s = \frac{(-3\mu J_2 r_e^2)}{r^4}(\sin^2(i)\sin(u_0)\cos(u_0))$$

$$F_w = \frac{(-3\mu J_2 r_e^2)}{r^4}(\sin(i)\cos(i)\sin(u_0))$$

The variable and constants of these equations are defined below:

1. Variables:
   a. u_0 = the argument of latitude and is equal to the true anomaly v_0 plus the argument of perigee ω.

      $u_0 = v_0 + \omega$

   b. r = the radius from the center of the earth to the satellite.

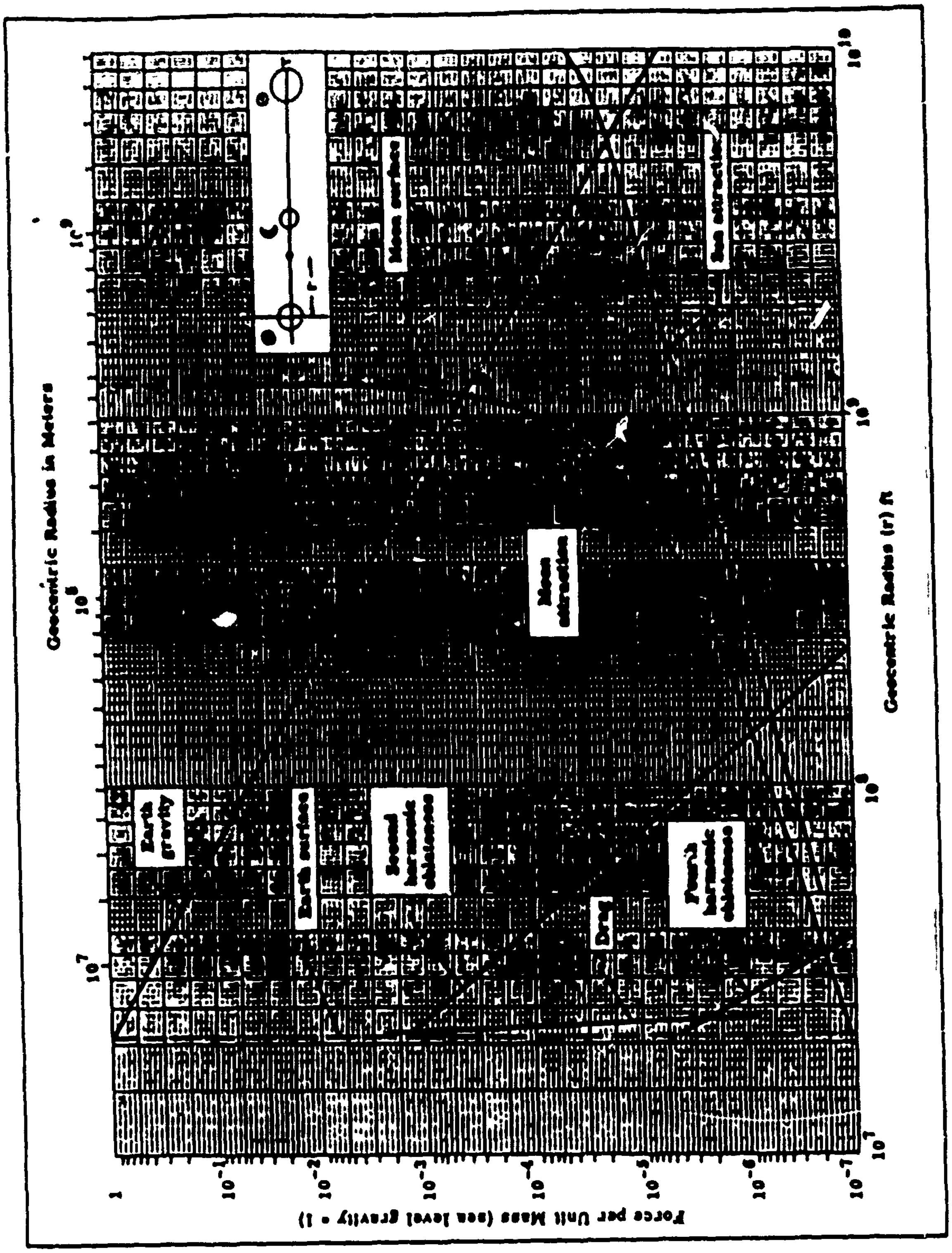

Figure 1. Comparison of perturbation magnitudes.

$r = |\bar{r}|$

2. Constants:

a. μ = the gravitational parameter of the earth.

$$\mu = 398601.2 \frac{(km^3)}{s^2}$$

b. J_2 = the second harmonic of oblateness coefficient, determined by experimental observations.

$$J_2 = 1.08263E-3$$

c. r_e = the mean radius of the earth,

$$r_e = 6.3782E3Km$$

2. ATMOSPHERIC DRAG

The formulation of atmospheric drag equations are plagued with uncertainties of atmospheric fluctuations, frontal areas of orbiting object (if not constant), the drag coefficient, and other parameters. A fairly simple formulation will be given here. Drag, by definition, will be opposite to the velocity of the vehicle relative to the atmosphere. Thus, the perturbing force is

$$\bar{F} = -\left(\frac{1}{2m}\right) \cdot CD \cdot AR \cdot DEN \cdot v \cdot \bar{v}$$

The velocity vector is in the 'IJK' system so the resulting force is also in the 'IJK' system. Therefore a transformation to the 'RSW' system is required.

The variables and constants of this equation are defined below:

1. Variables:

a. v = speed of vehicle.

b. CD = the dimensionless drag coefficient. The drag coefficient CD has a value between 1 and 2. It takes a value near 1 when the mean free path of the atmospheric molecules is small compared with the satellite size, and takes a value close to 2 when the mean free path is large compared with the size of the satellite. The drag coefficient will be modeled with CD = 2 when the satellites altitude is greater than 550km and equal to 1 otherwise. [Ref. 4: p. 295]

c. DEN = atmospheric density at the vehicle's altitude. The density is spherically symmetric, and will be modeled using exponential steps using the parameters in Table 1 on page 14 and the following formula [Ref. 1: pp. 423-424]:

$$\delta(z) = \delta_0 e^{(-\eta)} .$$

Table 1. ATMOSPHERIC PARAMETERS AND VALUES

Km	ρ	h	r	$\rho(r)$
0-150	1.225E-02	4.74E-02	0.0	1.225E-02
			150	1.0E-03
150-550	1.79840E-01	4.3614E-02	550	3.0E-8
550	1.015484E-07	2.2169SE-07	1500	3.65E-09
			2100	1.0E-12

2. Constants set to typical values:

   a. m = mass of the satellite, set equal to 100kg.

   b. AR = the sectional area of the vehicle perpendicular to the direction of motion.

3. PERT DUE TO HEAVENLY BODY

The satellite …e perturbation forces due to the gravitational effects of the sun and the moon. The perturbation force from a perturbing body is the difference between the gravitational force due to the perturbing body at the satellite and the gravitational force the satellite would experience if it were at the center of the earth. From Figure 2 on page 15, the perturbing force per unit mass of the satellite is

$$\bar{f}_p = \mu_p \frac{r_p \bar{i}_p - r\bar{i}_r}{|r_p \bar{i}_p - r\bar{i}_r|^3} - \frac{\mu_p \bar{i}_p}{r_p^2}$$

The variable and constants are defined below:

1. Variables:

   a. r_p = distance from the earth center for the perturbing body

   b. $\bar{i}_p$ = unit vector from the earth to the perturbing body

   c. r = distance from earth center to the satellite

   d. $\bar{i}_r$ = unit vector from the earth to the satellite

2. Constants:

   a. μ_p = gravitational constant of the perturbing body = $M_p G$

The subscript p is to be replaced by s if the perturbing body is the sun, and by m if the perturbing body is the moon. We will assume that $r << r_p$ then the equation above becomes

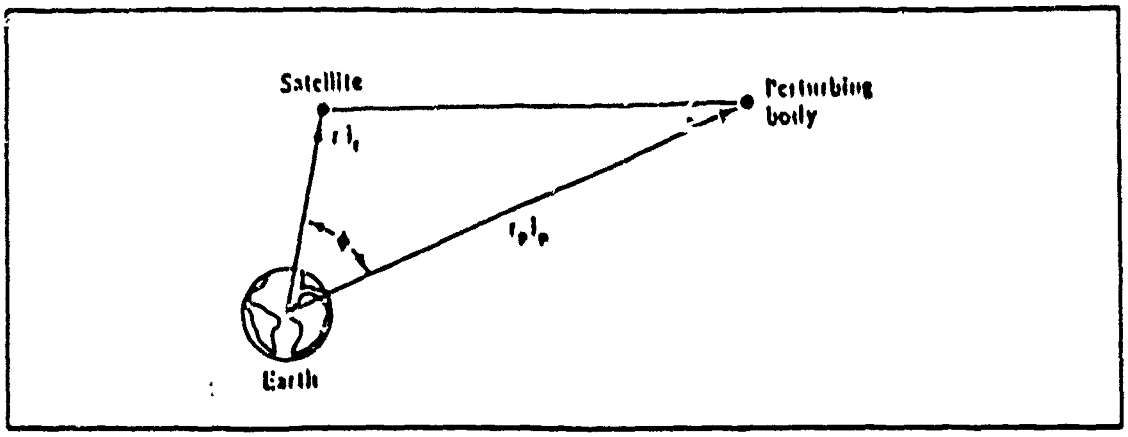

Figure 2. Perturbation forces.

$$\vec{F}_p = \left(\frac{\mu_p}{r_p^2}\right)\left(\frac{r}{r_p}\right)\left(3\overline{(\vec{i}_r \vec{i}_p)}\,\vec{i}_p - \vec{i}_r\right)$$

The unit vectors $\vec{i}_r$ and $\vec{i}_p$ can be written in terms of the 'IJK' system as:

$$\vec{i}_r = (\cos(\Omega)\cos(u_0) - \sin(\Omega)\cos(i)\sin(u_0))\vec{I} + (\cos(u_0)\sin(\Omega) + \cos(\omega)\cos(i)\sin(u_0))\vec{J} +$$

$$(\sin(i)\sin(u_0))\vec{K}$$

$$\vec{i}_p = (\cos(\Omega_p)\cos(u_{0p}) - \sin(\Omega_p)\cos(i_p)\sin(u_{0p}))\vec{I} +$$

$$(\cos(u_{0p})\sin(\Omega_p) + \cos(\omega_p)\cos(i_p)\sin(u_{0p}))\vec{J} + (\sin(i_p)\sin(u_{0p}))\vec{K}$$

where Ω, i, and u_0 are the orbital elements of the satellites and Ω_p, i_p, and u_{0p} are the orbital elements of the perturbing body. The formulas above use the 'IJK' system, and as such the resultant forces must be transformed to the 'RSW' system. Models of the sun and moon orbits are required to calculate $\vec{r}_p$ and $\vec{i}_p$. The models used in the program for the sun and moon's orbits follows: [Ref. 3: pp. 73-74]

a. SUN'S POSITION

In order to model the suns orbit, a number of simplifications had to be made in the actual parameters of the suns orbit. First the sun will be assumed to be in a circular orbit.This means that the radius (r) to the sun will be constant, and the eccentricity (e) will equal 0.0 instead of its true value of 0.017. The other assumption will be to place the sun on the 'I' axis of the 'IJK' system at the beginning of the program and have it progress through its orbit as the program runs. These changes will not effect the perturbing force in any noticeable magnitude.

The following variables and constants where used in the program to model the suns orbit after applying the simplifications: [Ref. 3: pp. 75-78]

1. Constants:
   - a. Gravitational Constant: $G = 6.67E - 11 \frac{(Nm^2)}{kg^2}$
   - b. Sun's Mass: $m_s = 1.99E30Kg$
   - c. Sun's Gravitational parameter:

     $\mu_s = 1.32733E20 \frac{Nm^2}{kg}$
   - d. Sun's eccentricity: $e_s = 0.0$
   - e. Radius of orbit, assume sun is in circular orbit: $r_s = 1.49E11m$
   - f. Sun's inclination: si = 23.45 deg. = 4.09279709d-01 radians
   - g. Longitude of ascending node: $\Omega_s = 0.0$
   - h. Argument of perigee: $\omega_s = 0.0$
2. Variables:
   - a. The true anomaly of the sun's position as a function of the time the satellite has been in orbit:

     $\nu_0(TT) = \frac{2\pi}{356 \times 24 \times 3600} TT$

     Where TT = true time, the time the satellite has been in orbit (sec)
   - b. Sun's Position vector: $\vec{r} = r \cos \nu_0 \vec{P} + r \sin \nu_0 \vec{Q}$
   - c. Unit vector from the earth to the sun: $\vec{i}_s = \frac{\vec{r}_s}{|\vec{r}_s|}$

b. MOON'S POSITION

In modeling the orbit of the moon, similar assumptions where used as with the sun. The moons orbit will be assumed to be circular, actually the eccentricity is equal to 0.055. By placing the moon initially on the 'I' axis of the 'IJK' system along with the sun, the gravitational forces of the two bodies will combine to a maximum. However; since the moons orbital period is only 27.3 days, the moon will not stay in this alignment and the magnitude of the combined forces will vary with time. The inclination of the moons orbit is not constant, but drifts between 18.3 and 28.6 degrees in ten years.

Also the longitude of the ascending node (Ω) oscillates between 13 and -13 degrees. To simplify this the inclination will be chosen as a constant 23.5 degrees and the longitude of the ascending node as 0.0 degrees. For the time period involved in calculating the perturbed orbit, these assumptions will not make any significant difference.

The following variables and constants were used in the program to model the moons orbit, after applying the simplifications:

1. Constants:
   a. Gravitational Constant: $G = 6.67E-11 \dfrac{(Nm^2)}{kg^2}$
   b. Moon's Mass: $m_{\text{☾}} = 7.35E22kg$
   c. Moon's Gravitational Parameter: $\mu_{\text{☾}} = GM_{\text{☾}} = 4.90E12 \dfrac{(Nm^2)}{kg}$
   d. Moon's eccentricity: $e_{\text{☾}} = 0.0$
   e. Radius of orbit, assume moon is in circular orbit: $r_{\text{☾}} = 3.844E8km$
   f. Moon's inclination: i = 23.5deg. = 4.10152374E-1 radians
   g. Moon's longitude of ascending node: $\Omega_{\text{☾}} = 0.0$
   h. Moon's argument of perigee: $\omega_{\text{☾}} = 0.0$
   i. Moon's period: $T = 27.3$ days [period]
2. Variables:
   a. The true anomaly of the moon's position as a function of the time the satellite has been in orbit: $v_{\text{☾}}(TT) = \dfrac{2\pi}{27.3 \times 24 \times 3600} TT$
   b. Moon's position Vector: $\bar{r} = r \cos v_{e\text{☾}} \bar{P} + r \sin v_{e\text{☾}} \bar{Q}$
   c. Unit vector from earth to moon: $\bar{i}_{\text{☾}} = \dfrac{\bar{r}_{\text{☾}}}{|\bar{r}_{\text{☾}}|}$

The models of the sun and moons orbit calculates the position vector in the 'PQW' system and therefore the position vector must be transformed to the 'IJK' system.

C. RATE-OF-CHANGE OF ORBITAL ELEMENTS

The derivations and equations of the rates-of-change of the orbital elements are contained in reference 1 pages 398 to 406. Therefore; only a summary of the actual analytic expressions for the rate-of-change of the parameters in terms of the perturbations will follow:

1. Rate-of-change of the semi-major axis:

$$\frac{da}{dt} = [\frac{2e\sin v}{n'\sqrt{1-e^2}}]F_r - [\frac{2a\sqrt{1-e^2}}{n'r}]F_t$$

Where n' is the mean motion of the satellites orbit.

$$n' = \sqrt{\frac{\mu}{a^3}}$$

2. Rate-of-change of the eccentricity:

$$\frac{de}{dt} = [\frac{\sqrt{1-e^2}\sin v_0}{n'a}]F_r + [\frac{\sqrt{1-e^2}}{n'a^2e}][\frac{a^2(1-e^2)}{r} - r]F_t$$

3. Rate-of-change of the inclination:

$$\frac{di}{dt} = [\frac{r\cos u_0}{n'a^2\sqrt{1-e^2}}]F_w$$

4. Rate-of-change of the longitude of the ascending node:

$$\frac{d\Omega}{dt} = [\frac{r\sin u_0}{n'a^2\sqrt{1-e^2}\sin i}]F_w$$

5. Rate-of-change of the argument of perigee:

$$\frac{d\omega}{dt} = (d\frac{\omega}{dt})_r + (d\frac{\omega}{dt})_t + (d\frac{\omega}{dt})_w$$

Where.

$$(\frac{d\omega}{dt})_r = [\frac{-\sqrt{1-e^2}\cos v_0}{n'ae}]F_r$$

$$(\frac{d\omega}{dt})_t = [\frac{p}{eh}][\sin v_0(1 + \frac{1}{1 + e\cos v_0})]F_t$$

$$(\frac{d\omega}{dt})_w = [\frac{-r\cot i\sin u_0}{n'a^2\sqrt{1-e^2}}]F_w$$

6. Rate-of-change of the eccentric anomaly:

$$\frac{dEA}{dt} = \frac{1}{\sin(EA)}\frac{[(\sin v_0 + \frac{de}{dt})(1 + e\cos v_0) - (\cos v_0 + e)(\frac{de}{dt}\cos v_0 + e\sin v_0)]}{[1 + e\cos v_0]^2}$$

7. Rate-of-change of the mean anomaly:

$$\frac{dMA}{dt} = \frac{dEA}{dt} - \frac{de}{dt}\sin(EA) - e \times \cos\frac{(EA)dEA}{dt} - \frac{dn'}{dt}(t - t_0)$$

This equation reduces to the following for circular and ecliptic orbits ($i < e < 1$).

$$\frac{dM_0}{dt} = \frac{-1}{na}\left[\frac{2r}{a} - \frac{(1-e^2)}{e}\cos v\right]F_r - \left[\frac{1-e^2}{nae}\right]\left[1 + \frac{r}{a(1-e^2)}\right]\sin v\, F_t - t\frac{dn'}{dt}$$

Where the Rate-of-change of the mean motion:

$$\frac{dn'}{dt} = \left[\frac{-3\mu}{2n'a^4}\right]\frac{da}{dt}$$

[ref. 1 p. 396-407]

D. NEW ORBITAL ELEMENTS

The change of each element is calculated by multiplying the rate-of-change of the element by the time step (DT). The change in the orbital elements are then added to the current values of the elements to give the new orbital elements. With the new elements calculated, the satellite is stepped forward and the new position and velocity are calculated in the same manner as the unperturbed orbit (chapter 3). Also as with the unperturbed orbit, the process is repeated until the satellite is at the perigee point, indicated by the time of flight (TF) equal to the period of the perturbed orbit.

V. VELOCITY CHANGES

The ability of the student to change the velocity of the satellite at any position in the orbit is a vital element in this program. With velocity changes the student can investigate the effects of varying the satellites velocity as in transfer orbits and inclination changes. In order to simplify the program the unperturbed orbit is used throughout this routine. The velocity change algorithm used in the program follows:

1. Rotate to velocity change location.

   The user is given the choice of changing the velocity of the satellite at the perigee, apogee or at any true anomaly. If the user chooses perigee or apogee as the change locations, the true anomaly is set equal to zero or pi radians respectfully. With the location of the velocity change, the satellite is first stepped around to the desired true anomaly. The stepping is identical with the unperturbed orbit with the exception that the stepping terminates when the true anomaly is greater or equal to the desired true anomaly. With a step size of one fiftieth of the period, the satellite is actually stepped around to a location near the desired location. This variance can be reduced by decreasing the step size but this would increase the computation time. This error will be a major factor in precise calculations of transfer orbits, or any other orbital maneuver where precise velocity changes are required. However: this program is not a tool to calculate precise orbital maneuvers, but rather a learning tool for the student to get a feel for the results of velocity changes in a satellite's orbit.

2. Change the velocity.

   With the satellite at the desired location, the program calculates and displays for the user the satellite's current velocity, escape velocity and circular velocity (the velocity required to circularize the orbit). The program will not allow velocities greater than or equal to the escape velocity. The user is given the option to enter a new velocity in the 'IJK' system or to change the magnitude of the velocity in the orbital plane. If the user chooses to change the velocity in the orbital plane, the program will prompt the user for the magnitude of the velocity change, and multiply this change by a unit vector in the direction of the satellites velocity. This velocity change vector is then added to the satellites velocity vector, to calculate the new velocity vector.

3. Calculate new elements.

   The orbital elements are calculated with the new velocity vector and the satellite's position vector.

4. Complete the orbit.

   The program will complete the orbit to the new perigee point using the satellite's position, new velocity and new elements. There are a number of problems that arise if the satellite is just stepped around to the perigee point. For example, with velocity changes in the orbital plane the apogee and perigee directions can physically swap. This is a problem when plotting with the perifocal coordinate system because the X_ω axis points toward perigee. To avoid problems like this the arrays used in plotting the orbit must be cleared and the satellite's current position and velocity be treated as initial conditions. However; to compare the old and new orbits there is a desire to retain as much of the previous orbit as possible. The velocity changes where divided into the following four cases to handle these problems:

a. Change velocity in the orbital plane at the perigee point with the new velocity greater than the circular velocity. The perigee point will remain the same so the satellite is stepped around using the unperturbed subroutines.

b. Change velocity in the orbital plane at the perigee point with the new velocity less than or equal to the circular velocity. The perigee and apogee directions will switch so the plotting arrays are first cleared and stored with the current location data. Because the satellite is now at the apogee point the satellite is stepped around to the perigee point storing the second half of the orbit. The entire next orbit is calculated and stored to get a complete orbit.

c. Change velocity in the orbital plane at the apogee point with the new velocity less than the circular velocity. The perigee and apogee directions will remain the same, so the satellite is stepped around to the perigee point completing the orbit.

d. This last case catches all the following velocity changes; velocity change in the orbital plane at the apogee point with the new velocity greater than or equal to the circular velocity, velocity changes at any other true anomaly in the orbital plane, and any velocity change out of the orbital plane. The plotting arrays are cleared and stored with the current location data. No matter where in the orbit the satellite is, the satellite is first stepped around to the perigee point, and to ensure a complete orbit is plotted the entire next orbit is also calculated and stored.

VI. GRAPHICAL PLOTS

The program provides two types of graphical displays of the orbit, a display in the perifocal coordinate system and a display of the satellite's ground track. Each display type is useful in observing different aspects of the orbit. The perifocal display will allow the user to see how certain orbital parameters change with different initial positions and velocities, and also how the parameters change with velocity changes at varying positions in the orbit. The ground track will enable the user to gain an appreciation for the physical location of the satellite above the earth, and see how the orbital parameter affects the path of the satellite. The ground track will also display the precession of a sequence of orbits. Both displays plot the position steps to give the user an understanding of how the satellite speeds up at perigee and slows down around apogee.

The DISSPLA package on the mainframe computer was used to enable the plotting of the orbits. The versatility of plotting subroutines of DISSPLA makes the actual programming of the orbit a simple matter of initializing DISSPLA for the type of monitor being used, setting up the plotting area, initializing the axis and axis scale, and then plotting the desired curve from points contained in arrays. This is a simplified explanation of DISSPLA, but for further details on DISSPLA programming refer to the DISSPLA user's manual [Ref. 5]. DISSPLA also supplies subroutines to draw a variety of projections of the world and fill the projections with coast lines, latitude lines and longitude lines. There are a couple of DISSPLA requirements that did require special handling in the program. The requirement that the data be supplied in arrays forced the program to load arrays with the required position and parameters and to keep a counter for the number in the arrays. The array format requires the size of the array be specified in the beginning of the program. The array size needs to be large enough to hold a number of orbits, but not so large as to waist storage space. The program will continue to add orbital data to the arrays until the user chooses to delete the previous orbits. If a new initial position and velocity is entered or if the arrays will overflow with the next orbit the arrays will automatically delete all previous orbits. DISSPLA also requires that all data be in single precision format. The program calculates all orbits in double precision in order to limit the effect of round-off error, but by using the single precision data for plotting will not affect the accuracy of the plot in any way.

The subroutines used to display the orbits will be covered in the following three sections:

A. PERIFOCAL PLOT

The plotting of the orbit in the perifocal coordinate system is the easier of the two types of plots. Since the perifocal coordinate system has the orbital plane as the fundamental plane, the only requirements to describe the orbit in the perifocal coordinate system are arrays with the true anomaly and the radius to the satellite. To give the user a sense of the size of the plot, the axis length varies with the eccentricity and semi-major axis length. Also a plot of the earth is plotted to the same scale, with the pole or center of the plot on the origin of the axis. The latitude of the earth at the center of the plot will vary with the inclination of the orbit. This plot will allow the user to see a relative view of the satellite's coverage in the minus 'Z' axis direction of the perifocal coordinate system.

B. GROUND TRACK

The ground track plot is a very complex subroutine compared with the perifocal plot. Because the ground track is not a continuous curve a procedure to handle the satellite ending at one end of the plot and wrapping around to the other end was developed. The wrap around problem is avoided in most orbits by plotting the orbit in segments with the following two rules. Each segment begins at the beginning of a new plot or at the edge of the plot area, and ending when the satellite would wrap around to the other side of the plot. At the beginning of a segment if the position of the satellite is within five degrees of the edge of the plot, that position and any other positions within that five degree boundary will not be plotted. The segment will end when the satellite is within ten degrees of the edge of the plot. The above restrictions imposed on the segments of the plot will not substantially affect the interpretation or usefulness of the plot. The ground track is plotted on top of a cylindrical equidistant projection of the world, with the world coast lines and a longitude-latitude grid for reference.

C. DATA

Information concerning the orbit is displayed on the lower half of the plot. The information is designed to supply the user with enough of the basic orbital elements and other parameters affecting the orbit to be able to evaluate what basic type of orbit the satellite is in, and the effects of velocity changes and perturbing forces have on the orbit. The following data are plotted: inclination(i), semi-major axis (a), eccentricity (e), period (per), apogee and perigee velocity and radius, average time rate-of-change of orbital elements, and the average magnitude of perturbing forces per unit mass.

VII. CONCLUSIONS AND RECOMMENDATIONS

The program supplies the student with an interactive tool to study the orbital motion of satellites around the earth. The student can investigate a variety of orbits by varying the orbital parameters, command velocity changes, and observe the effects of perturbing forces.

The student is provided with two options for entering the initial position and velocity of the satellite. The program could be expanded to provide the student with the additional options of entering either orbital parameters or a ground observation data and have the program calculate the initial position and velocity from this data. Also the student is limited to orbits with eccentricities less than one (elliptic orbits). The program could be also be expanded to include more eccentric orbit for Lunar, interplanetary, and missile trajectories. The perturbing orbit is calculated for orbits around the earth with relatively small perturbing forces in relation to the earths gravitational force. This fact will cause the program to produce false results if the student tries to calculate lunar trajectories. Special routines would have to be employed when the perturbing force (the moons gravitational attraction) is comparable to the earths gravitational attraction. This will not become a factor for studying current satellite orbits out to the geosynchronous radius of 42241.1km.

The velocity change subroutines move the satellite to a location close to the desired location before a velocity change is imposed. By reducing the step size in the velocity change subroutine, this error could be reduced. Precise orbital transfer maneuvers can be modeled by reducing this error caused by the positioning of the satellite prior to changing the velocity. The program will currently provide the student with useful plots for gaining experience with various transfer orbits by varying the magnitude and location of the velocity changes.

The output of the calculations of the orbit are arrays loaded with the satellite's position and select orbital parameters. The DISSPLA subroutines that plot the points are not unique. The program would become portable to personal computers with these graphics subroutines written in FORTRAN and included in the program.

A final recommendation is that the display of the ground track could be modified to show ground coverage, number of satellites in a constellation, and other elements necessary for planning a real-world artificial satellite application.

APPENDIX A. ORBIT PROGRAM

```
      PROGRAM ORBIT                                                     ORB00010
*        THIS PROGRAM IS AN INTERACTIVE TIME STEP SIMULATION OF         ORB00020
*     SATELLITES AROUND THE EARTH.  PERTURBED AND UNPERTURBED ORBITS    ORB00030
*     ARE CALCULATED AND PLOTTED.  VELOCITY CHANGES ARE ALSO PERMITTED  ORB00040
*     AT SPECIFIED TRUE ANOMALIES.                                      ORB00050
                                                                        ORB00060
*     A LIST OF VARIABLES USED BY THE MAIN PROGRAM FOLLOWS.             ORB00070
*     A    = SEMI-MAJOR AXIS                                            ORB00080
*     AL   = ARGUMENT OF LONGITUDE                                      ORB00090
*     AP   = ARGUMENT OF PERIGEE                                        ORB00100
*     CHTA = VELOCITY CHANGE LOCATION TRUE ANOMALY                      ORB00110
*     DT   = TIME STEP                                                  ORB00120
*     E    = ECCENTRICITY                                               ORB00130
*     EA   = ECCENTRIC ANOMALY                                          ORB00140
*     EI   = I VECTOR OF ECCENTRICITY                                   ORB00150
*     EJ   = J VECTOR OF ECCENTRICITY                                   ORB00160
*     EK   = K VECTOR OF ECCENTRICITY                                   ORB00170
*     FR   = R VECTOR OF TOTAL FORCE                                    ORB00180
*     FS   = S VECTOR OF TOTAL FORCE                                    ORB00190
*     FW   = W VECTOR OF TOTAL FORCE                                    ORB00200
*     H    = ANGULAR MOMENTUM                                           ORB00210
*     HI   = I VECTOR OF ANGULAR MOMENTUM                               ORB00220
*     HJ   = J VECTOR OF ANGULAR MOMENTUM                               ORB00230
*     HK   = K VECTOR OF ANGULAR MOMENTUM                               ORB00240
*     I    = INCLINATION                                                ORB00250
*     IOPT1= PERTURBED OR UNPERTURBED OPTION                            ORB00260
*     IOPT2= OPTIONS: PLOT NEXT ORBIT, CHANGE INITIAL VALUES,           ORB00270
*     CHANGE VELOCITY, PLOT ANOTHER VIEW OF ORBIT, QUIT                 ORB00280
*     LAN  = LONGITUDE OF ASCENDING NODE                                ORB00290
*     LP   = LONGITUDE OF PERIGEE                                       ORB00300
*     MA   = MEAN ANOMALY                                               ORB00310
*     MM   = MEAN MOTION                                                ORB00320
*     MU   = GRAVITATIONAL PARAMETER                                    ORB00330
*     N    = ASCENDING NODE                                             ORB00340
*     NI   = I VECTOR OF ASCENDING NODE                                 ORB00350
*     NJ   = J VECTOR OF ASCENDING NODE                                 ORB00360
*     NK   = K VECTOR OF ASCENDING NODE                                 ORB00370
*     NUM  = STEP COUNTER                                               ORB00380
*     P    = SEMI-LATUS RECTUM                                          ORB00390
*     PER  = PERIOD OF ORBIT                                            ORB00400
*     PI   = PI                                                         ORB00410
*     RA   = RADIUS OF APOGEE                                           ORB00420
*     RE   = RADIUS OF EARTH                                            ORB00430
*     R    = ORBITAL RADIUS                                             ORB00440
*     RI   = I VECTOR OF ORBITAL RADIUS                                 ORB00450
*     RJ   = J VECTOR OF ORBITAL RADIUS                                 ORB00460
*     RK   = K VECTOR OF ORBITAL RADIUS                                 ORB00470
*     T    = TIME COUNTER IN ORBIT                                      ORB00480
*     TA   = TRUE ANOMALY                                               ORB00490
*     TDA  = TOTAL CHANGE IN SEMI-MAJOR AXIS                            ORB00500
*     TDAP = TOTAL CHANGE IN ARGUMENT OF PERIGEE                        ORB00510
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*     TDE   = TOTAL CHANGE IN ECCENTRICITY                              ORB00520
*     TDH   = TOTAL CHANGE IN ANGULAR MOMENTUM                          ORB00530
*     TDI   = TOTAL CHANGE IN INCLINATION                               ORB00540
*     TDMA  = TOTAL CHANGE IN MEAN ANOMALY                              ORB00550
*     TDMM  = TOTAL CHANGE IN MEAN MOTION                               ORB00560
*     TDLAN= TOTAL CHANGE IN LONGITUDE OF ASCENDING NODE                ORB00570
*     TF    = TIME OF FLIGHT                                            ORB00580
*     TFDRA= TOTAL FORCE OF DRAG                                        ORB00590
*     TFEA  = TOTAL FORCE OF EARTH'S OBLATENESS                         ORB00600
*     TFMO  = TOTAL FORCE FROM MOON                                     ORB00610
*     TFSU  = TOTAL FORCE FROM SUN                                      ORB00620
*     TL    = TRUE Longitude AT EPOCH                                   ORB00630
*     TT    = TRUE TIME SINCE SATELLITE HAS BEEN IN ORBIT               ORB00640
*     V     = SATELLITE VELOCITY                                        ORB00650
*     VI    = I VECTOR OF SATELLITE VELOCITY                            ORB00660
*     VJ    = J VECTOR OF SATELLITE VELOCITY                            ORB00670
*     VK    = K VECTOR OF SATELLITE VELOCITY                            ORB00680
                                                                        ORB00690
*     A LIST OF THE ARRAYS USED FOLLOWS:                                ORB00700
                                                                        ORB00710
*     AINRAY  = INCLINATION                                             ORB00720
*     APRAY   = ARGUMENT OF PERIGEE                                     ORB00730
*     RARAY   = RADIUS                                                  ORB00740
*     RIRAY   = I VECTOR OF RADIUS                                      ORB00750
*     RJRAY   = J VECTOR OF RADIUS                                      ORB00760
*     RKRAY   = K VECTOR OF RADIUS                                      ORB00770
*     TARAY   = TRUE ANOMALY                                            ORB00780
*     TIMRAY  = TIME                                                    ORB00790
                                                                        ORB00800
*     A LIST OF SUBROUTINES CALLED BY THE MAIN PROGRAM WILL FOLLOW:     ORB00810
                                                                        ORB00820
*     CALCEL  = CALCULATES THE ORBITAL ELEMENTS                         ORB00830
*     CHGVEL  = ALLOW THE USER TO CHANGE THE VELOCITY OF THE SATELLITE  ORB00840
*     INPUTS  = PROMPTS USER FOR INITIAL POSITION AND VELOCITY          ORB00850
*     INTSUM  = INITIALIZES THE SUMS IN THE ARRAYS                      ORB00860
*     NEWELT  = CALCULATE NEW ORBITAL ELEMENTS FROM TIME STEP           ORB00870
*     NEWPOS  = CALCULATE NEW POSITION VECTOR                           ORB00880
*     NEWVEL  = CALCULATE NEW VELOCITY VECTOR                           ORB00890
*     OPTION  = GIVE THE USER THE OPTIONS Permitted IN THE PROGRAM      ORB00900
*     PLOTS   = PLOTS THE ORBITS                                        ORB00910
*     PRETUR  = CALCULATES THE PERTURBED ORBIT                          ORB00920
*     STORE   = STORE THE POSITION DATA IN ARRAYS                       ORB00930
*     UNPRET  = CALCULATE THE UNPERTURBED ORBIT                         ORB00940
                                                                        ORB00950
*     BEGIN MAIN PROGRAM                                                ORB00960
                                                                        ORB00970
      DOUBLE PRECISION  PI,MU,RI,RJ,RK,R,VI,VJ,VK,V,HI,HJ,HK,H,         ORB00980
     +   NI,NJ,NK,N,P,EI,EJ,EK,E,A,I,LAN,AP,TA,AL,LP,TL,PER,EA,         ORB00990
     +   MM,MA,T,DT,TF, FR,FS,FW,TT,CHTA,RA,VA,TEMPTA,RE                ORB01000
                                                                        ORB01010
      DIMENSION TARAY(500),RARAY(500),RIRAY(500),RJRAY(500),RKRAY(500), ORB01020
     +   AINRAY(500),APRAY(500),TIMRAY(500)                             ORB01030
                                                                        ORB01040
      CHARACTER*1,LOOP,YORN,ORLOOP                                      ORB01050
                                                                        ORB01060
      PI = 3.141592653589794                                            ORB01070
```

```
      MU = 3.986012D+05
      RE = 6.378145D+03

*     USER INTRO TO PROGRAM
      CALL INTRO

*     ENTERED MAIN PROGRAM LOOP
      LOOP = 'Y'
 10   IF (LOOP .EQ. 'Y') THEN

*        INITIALIZE STEP COUNTER AND TRUE TIME
 20      NUM = 1
         TT = 0.0

*        PROMPT USER FOR INITIAL POSITION AND VELOCITY
         CALL INPUTS(RI,RJ,RK,R,VI,VJ,VK,V,MU,LOOP,PI)

*        EXIT PROGRAM
         IF (LOOP .EQ. 'N') THEN
              GOTO 10
         ENDIF

*        CALCULATE AND STORE ORBITAL ELEMENTS
         CALL CALCEL(RI,RJ,RK,R,VI,VJ,VK,V,EI,EJ,EK,E,A,I,LAN,
     +               LP,TA,PER,EA,MA,AP,AL,TF,P,PI,MU,MM,N,H,HI,HJ)
         CALL STORE(RI,RJ,RK,R,TA,RIRAY,RJRAY,RKRAY,RARAY,TARAY,
     +              NUM,I,AP,AINRAY,APRAY,TT,TIMRAY)

*        PRINT DATE FOR USER TO REVIEW
         PRINT*,'VI =', VI,' KM/S'
         PRINT*,'VJ =', VJ,' KM/S'
         PRINT*,'VK =', VK,' KM/S'
         PRINT*,' V =', V,' KM/S'
         PRINT*,'RI =', RI,' KM'
         PRINT*,'RJ =', RJ,' KM'
         PRINT*,'RK =', RK,' KM'
         PRINT*,' R =', R,' KM'
         PRINT*,'ECCENTRICITY =',E
         DEGI = SNGL((180.0/PI)*I)
         PRINT*,'INCLINATION =',DEGI,' DEGREES'
         PERHRS = SNGL(PER/3600.0)
         PRINT*,'PERIOD =',PERHRS,' HOURS'
         PRINT*,'ARE THESE VALUES CORRECT?  ENTER "Y" OR "N" :'
         READ*,YORN
         CALL EXCMS('CLRSCRN')
         IF (.NOT. YORN .EQ. 'Y') THEN
              GOTO 20
         ENDIF

*        CALCULATE TIME STEP AND SET TIMER TO ONE TIME STEP
         DT = PER/50
         T = DT

*        STEP SATELLITE TO PERIGEE POINT AND RECORD
 50      IF ((TA.GT.0.063).AND.(TA.LT.6.21)) THEN
              TT = TT + DT
```

```
          CALL NEWELT(MM,MA,E,EA,TA,TF,DT,PI,PER)                     ORB01640
          CALL NPOS(RI,RJ,RK,R,LAN,AP,I, TA,A,E)                      ORB01650
          CALL NVEL(E,P,TA,LAN,AP,I,VI,VJ,VK,V,MU)                    ORB01660
          NUM = NUM + 1                                               ORB01670
          CALL STORE(RI,RJ,RK,R,TA,RIRAY,RJRAY,RKRAY,RARAY,TARAY,     ORB01680
     +         NUM,I,AP,AINRAY,APRAY,TT,TIMRAY)                       ORB01690
          T = T + DT                                                  ORB01700
          GOTO 50                                                     ORB01710
      ENDIF                                                           ORB01720
                                                                      ORB01730
*     CALCULATE ELEMENTS FROM PERIGEE POINT                           ORB01740
      CALL CALCEL(RI,RJ,RK,R,VI,VJ,VK,V,EI,EJ,EK,E,A,I,LAN,           ORB01750
     +          LP,TA,PER,EA,MA,AP,AL,TF,P,PI,MU,MM,N,H,HI,HJ)        ORB01760
                                                                      ORB01770
      DT = PER/50                                                     ORB01780
      T = DT                                                          ORB01790
                                                                      ORB01800
*     STORE FIRST Unperturbed ORBIT                                   ORB01810
      CALL UNPRET(DT,PER,AL,LAN,AP,I,RI,RJ,RK,R,VI,VJ,VK,V,           ORB01820
     +          MU,PI,H,A,E,N,TA,P,MM,MA,EA,TF,T,NUM,RIRAY,RJRAY,     ORB01830
     +          RKRAY,RARAY,TARAY,AINRAY,APRAY,TIMRAY,TT)             ORB01840
                                                                      ORB01850
*     INITIALIZE SUMS FOR FORCE AND ORBITAL ELEMENT CHANGES TO ZERO   ORB01860
      CALL INTSUM(TFEA,TFSU,TFMO,TFDRA,TDI,TDA,TDE,TDMM,TDMA,TDLAN,   ORB01870
     +          TDH,TDAP)                                             ORB01880
                                                                      ORB01890
*     PLOT FIRST UNPERTURBED ORBIT                                    ORB01900
 70   CALL PLOTS(RIRAY,RJRAY,RKRAY,RARAY,TARAY,NUM,PI,I,LP,A,E,TF,    ORB01910
     +        AINRAY,APRAY,TIMRAY,TFEA,TFSU,TFMO,TFDRA,PER,TDI,TDA,ORB01920
     +        TDE,TDMM,TDMA,TDLAN,TDH,TDAP,MM,MA,LAN,H,AP,R,V)        ORB01930
                                                                      ORB01940
*     BEGIN NEW ORBIT OPTIONS                                         ORB01950
*     IOPT1 = 1. Unperturbed ORBIT                                    ORB01960
*           = 2. Perturbed ORBIT                                      ORB01970
*           = 3. QUIT                                                 ORB01980
*     IOPT2 = 1. PLOT NEXT ORBIT                                      ORB01990
*           = 2. CHANGE INITIAL VALUES                                ORB02000
*           = 3. CHANGE VELOCITY AT A SPECIFIC TRUE Anomaly           ORB02010
*           = 4. PLOT ANOTHER VIEW OF SAME ORBIT                      ORB02020
*                                                                     ORB02030
*     ALSO ASKED IF WANT TO CLEAR ALL PREVIOUS ORBITS                 ORB02040
                                                                      ORB02050
*     CALCULATE ELEMENTS AT PERIGEE                                   ORB02060
 80   CALL CALCEL(RI,RJ,RK,R,VI,VJ,VK,V,EI,EJ,EK,E,A,I,LAN,           ORB02070
     +          LP,TA,PER,EA,MA,AP,AL,TF,P,PI,MU,MM,N,H,HI,HJ)        ORB02080
                                                                      ORB02090
*     CHECK FOR POSSIBLE ARRAY OVERFLOW                               ORB02100
      IF (NUM .GT. 425) THEN                                          ORB02110
          PRINT*,'ARRAYS ARE FULL'                                    ORB02120
          PRINT*,'PREVIOUS ORBITS WILL BE ERASED!'                    ORB02130
          NUM = 1                                                     ORB02140
          CALL STORE(RI,RJ,RK,R,TA,RIRAY,RJRAY,RKRAY,RARAY,TARAY,     ORB02150
     +              NUM,I,AP,AINRAY,APRAY,TT,TIMRAY)                  ORB02160
      ENDIF                                                           ORB02170
                                                                      ORB02180
*     PROMPT USER FOR DESIRED OPTION                                  ORB02190
```

```
      CALL OPTION(IOPT1,IOPT2,NUM,RIRAY,RJRAY,RKRAY,RARAY,              ORB02200
     +            TARAY,AINRAY,APRAY,TIMRAY)                            ORB02210
                                                                        ORB02220
*     Initialize SUMS FOR FORCE AND ORBITAL ELEMENT CHANGES TO ZERO     ORB02230
      CALL INTSUM(TFEA,TFSU,TFMO,TFDRA,TDI,TDA,TDE,TDMM,TDMA,TDLAN,     ORB02240
     +            TDH,TDAP)                                             ORB02250
                                                                        ORB02260
*     SET TIME COUNTER TO ONE TIME STEP                                 ORB02270
      T = DT                                                            ORB02280
                                                                        ORB02290
*     OPTION:  PLOT THE NEXT ORBIT                                      ORB02300
      IF (IOPT2 .EQ. 1) THEN                                            ORB02310
                                                                        ORB02320
*          CALCULATE AND PLOT UNPERTURBED ORBIT                         ORB02330
           IF(IOPT1 .EQ. 1) THEN                                        ORB02340
                CALL UNPRET(DT,PER,AL,LAN,AP,I,RI,RJ,                   ORB02350
     +               RK,R,VI,VJ,VK,V,MU,PI,H,A,                         ORB02360
     +               E,N,TA,P,MM,MA,EA,TF,T,NUM,RIRAY,RJRAY,RKRAY,      ORB02370
     +               RARAY,TARAY,AINRAY,APRAY,TIMRAY,TT)                ORB02380
                CALL PLOTS(RIRAY,RJRAY,RKRAY,RARAY,TARAY,NUM,           ORB02390
     +               PI,I,LP,A,E,TF,AINRAY,APRAY,TIMRAY,                ORB02400
     +               TFEA,TFSU,TFMO,TFDRA,PER,                          ORB02410
     +               TDI,TDA,TDE,TDMM,TDMA,TDLAN,TDH,TDAP,              ORB02420
     +               MM,MA,LAN,H,AP,R,V)                                ORB02430
                                                                        ORB02440
*          CALCULATE AND PLOT PERTURBED ORBIT                           ORB02450
           ELSEIF(IOPT1 .EQ. 2) THEN                                    ORB02460
                CALL PRETUR(DT,PER,AL,LAN,AP,I,                         ORB02470
     +                 RI,RJ,RK,R,VI,VJ,VK,V,FR,FS,FW,                  ORB02480
     +                 MU,PI,H,A,E,N,TA,P,MM,MA,EA,TF,T,NUM,            ORB02490
     +                 RIRAY,RJRAY,RKRAY,RARAY,TARAY,AINRAY,APRAY,      ORB02500
     +                 TIMRAY,TT,TFEA,TFSU,TFMO,TFDRA,                  ORB02510
     +                 TDI,TDA,TDE,TDMM,TDMA,TDLAN,TDH,TDAP)            ORB02520
                CALL PLOTS(RIRAY,RJRAY,RKRAY,RARAY,TARAY,NUM,           ORB02530
     +               PI,I,LP,A,E,TF,AINRAY,APRAY,TIMRAY,                ORB02540
     +               TFEA,TFSU,TFMO,TFDRA,PER,                          ORB02550
     +               TDI,TDA,TDE,TDMM,TDMA,TDLAN,TDH,TDAP,              ORB02560
     +               MM,MA,LAN,H,AP,R,V)                                ORB02570
           ENDIF                                                        ORB02580
                                                                        ORB02590
*     GOTO THE BEGINNING OF THE PROGRAM TO CHANGE THE INITIAL VALUES    ORB02600
      ELSEIF (IOPT2 .EQ. 2) THEN                                        ORB02610
           GOTO 20                                                      ORB02620
                                                                        ORB02630
*     CHANGE VELOCITY AT A SPECIFIC TRUE ANOMALY AND                    ORB02640
*     PLOT THE NEW ORBIT                                                ORB02650
      ELSEIF (IOPT2 .EQ. 3) THEN                                        ORB02660
           CALL CHGVEL(DT,PER,AL,LAN,AP,I,RI,RJ,RK,R,                   ORB02670
     +                      VI,VJ,VK,V,MU,PI,                           ORB02680
     +                      H,A,E,N,TA,P,MM,MA,EA,TF,T,NUM,RIRAY,       ORB02690
     +                      RJRAY,RKRAY,RARAY,TARAY,AINRAY,APRAY,       ORB02700
     +                      TIMRAY,TT,EI,EJ,EK,LP,HI,HJ,IOPT1,          ORB02710
     +                      TFEA,TFSU,TFMO,TFDRA,TDI,TDA,TDE,TDMM,      ORB02720
     +                      TDMA,TDLAN,TDH,TDAP)                        ORB02730
           CALL PLOTS(RIRAY,RJRAY,RKRAY,RARAY,TARAY,NUM,                ORB02740
     +               PI,I,LP,A,E,TF,AINRAY,APRAY,TIMRAY,                ORB02750
```

```
     +                  TFEA,TFSU,TFMO,TFDRA,PER,
     +                  TDI,TDA,TDE,TDMM,TDMA,TDLAN,TDH,TDAP,
     +                  MM,MA,LAN,H,AP,R,V)

*        PLOT ANOTHER VIEW OF THE SAME ORBIT
         ELSEIF (IOPT2 .EQ. 4) THEN
             CALL PLOTS(RIRAY,RJRAY,RKRAY,RARAY,TARAY,NUM,
     +                  PI,I,LP,A,E,TF,AINRAY,APRAY,TIMRAY,
     +                  TFEA,TFSU,TFMO,TFDRA,PER,
     +                  TDI,TDA,TDE,TDMM,TDMA,TDLAN,TDH,TDAP,
     +                  MM,MA,LAN,H,AP)

*        STOP THE PROGRAM
         ELSEIF (IOPT2 .EQ. 5) THEN
             GOTO 90
         ELSE
             PRINT*,'INVALID ENTRY!'
             GOTO 80
         ENDIF

*        CHECK IF SATELLITE Impacted THE EARTH AND GO TO THE BEGINNING
         IF (R .LE. 6450.0) THEN
             PRINT*,'SATELLITE WILL IMPACT THE EARTH!!!'
             PRINT*,'PROGRAM WILL RESET TO THE BEGINNING'
             GOTO 20
         ENDIF

*        GOTO THE TOP OF THE OPTION LOOP
         GOTO 80

*        GIVE THE USER A CHANCE TO RECOVER THE PROGRAM
 90      PRINT*,'THIS IS YOUR LAST CHANCE!'
         PRINT*,'DO YOU WANT TO CONTINUE?'
         PRINT*,'AND GOTO THE Beginning OF THE PROGRAM?'
         PRINT*,'ENTER "Y" OR "N" :'
         READ*,LOOP
         PRINT*,LOOP
         GOTO 10
      ENDIF

*     DISSPLA SUBROUTINE TO TELL GRAPHICS TERMINAL PLOTTING
*     SESSION IS DONE
      CALL DONEPL
      STOP
      END

***************************************************************

      SUBROUTINE INTRO
*     THIS SUBROUTINE WILL GIVE THE USER A Brief INTRODUCTION OF THE
*     USES OF THE PROGRAM

      PRINT*,'THIS PROGRAM IS A GRAPHICS DISPLAY OF Satellite ORBITS.'
      PRINT*,'YOU WILL BE ASKED TO INPUT THE INITIAL VELOCITY AND'
      PRINT*,'POSITION VECTORS OF THE Satellite.  THE PROGRAM WILL  '
      PRINT*,'THEN CALCULATE THE ORBITAL PARAMETERS AND THE '
```

```
      PRINT*,'Unperturbed ORBIT.  THE USER WILL THEN HAVE THE'          ORB03320
      PRINT*,'CHOICE OF DISPLAYS:'                                      ORB03330
      PRINT*,'     -PERIFOCAL (SHOWS RELATIVE SIZE OF ORBIT)'           ORB03340
      PRINT*,'     -Equatorial (SHOWS ORBIT INCLINED, USER INPUT'       ORB03350
      PRINT*,'                 LONGITUDE TO VIEW AT)'                   ORB03360
      PRINT*,'     -GROUND TRACK'                                       ORB03370
      PRINT*,' '                                                        ORB03380
      PRINT*,'THE USER IS THEN ASKED TO CHOOSE ONE OF THE FOLLOWING:'   ORB03390
      PRINT*,'     -Unperturbed ORBITS'                                 ORB03400
      PRINT*,'     -Perturbed ORBITS'                                   ORB03410
      PRINT*,'     -VELOCITY CHANGES'                                   ORB03420
      PRINT*,'THE USER"S CHOICE WILL BE USED IN DEVELOPING THE'         ORB03430
      PRINT*,'GRAPHICAL OUTPUT.'                                        ORB03440
      PRINT*,' '                                                        ORB03450
      PRINT*,'THE USER IS THEN GIVEN THE FOLLOWING CHOICES:'            ORB03460
      PRINT*,'      -CLEAR ALL THE PREVIOUS ORBITS'                     ORB03470
      PRINT*,'      -CHANGE THE INITIAL PARAMETERS'                     ORB03480
      PRINT*,'      -CHANGE VELOCITY AT A SPECIFIC TRUE Anomaly'        ORB03490
      PRINT*,'      -PLOT ANOTHER VIEW OF THE SAME ORBIT'               ORB03500
      RETURN                                                            ORB03510
      END                                                               ORB03520
                                                                        ORB03530
*****************************************************************       ORB03540
                                                                        ORB03550
      SUBROUTINE OPTION(IOPT1,IOPT2,NUM,RIRAY,RJRAY,RKRAY,RARAY,        ORB03560
     +                  TARAY,AINRAY,APRAY,TIMRAY)                      ORB03570
*     THIS SUBROUTINE GIVES THE USER A CHOICE OF OPERATIONS THAT CAN BE ORB03580
*       PERFORMED ON THE PROGRAM AND RETURNS THE USERS CHOICE WITH      ORB03590
*       VARIABLES IOPT1 AND IOPT2                                       ORB03600
                                                                        ORB03610
      DIMENSION RIRAY(500),RJRAY(500),RKRAY(500),RARAY(500),TARAY(500), ORB03620
     +   AINRAY(500),APRAY(500),TIMRAY(500)                             ORB03630
      CHARACTER*1,YORN                                                  ORB03640
      IOPT1 = 0                                                         ORB03650
                                                                        ORB03660
*     PROMPT USER FOR OPTION                                            ORB03670
 103  PRINT*,'WHICH OF THE FOLLOWING OPTIONS WOULD YOU LIKE:'           ORB03680
      PRINT*,'  1.-CALCULATE THE NEXT ORBIT USING THE SAME'             ORB03690
      PRINT*,'     PARAMETERS'                                          ORB03700
      PRINT*,'  2.-CHANGE THE INITIAL PARAMETERS OF THE ORBIT'          ORB03710
      PRINT*,'  3.-CHANGE THE VELOCITY AT A POINT IN THE ORBIT'         ORB03720
      PRINT*,'     (THE UNPERTURBED ORBIT WILL BE USED)'                ORB03730
      PRINT*,'  4.-PLOT ANOTHER VIEW OF THE ORBIT(S)'                   ORB03740
      PRINT*,'  5.-QUIT'                                                ORB03750
      PRINT*,'ENTER 1, 2, 3, 4, OR 5:'                                  ORB03760
      READ*,IOPT2                                                       ORB03770
      PRINT*,IOPT2                                                      ORB03780
      CALL EXCMS('CLRSCRN')                                             ORB03790
      IF ( IOPT2 .GT. 5) THEN                                           ORB03800
        GOTO 103                                                        ORB03810
      ENDIF                                                             ORB03820
                                                                        ORB03830
*     Prompt USER FOR TYPE OF ORBIT DESIRED                             ORB03840
 105  IF (IOPT2 .EQ. 1) THEN                                            ORB03850
        PRINT*,'WHICH TYPE OF ORBIT WOULD YOU LIKE TO SEE,'             ORB03860
        PRINT*,'   1.-Unperturbed ORBITS'                               ORB03870
```

```
         PRINT*,'   2.-Perturbed ORBITS'
         PRINT*,' ENTER 1 OR 2:'
         READ*,IOPT1
         PRINT*,IOPT1
         CALL EXCMS('CLRSCRN')
         IF ((IOPT1 .NE. 1) .AND. (IOPT1 .NE. 2)) THEN
              PRINT*,'INVALID ENTRY!'
              GOTO 105
         ENDIF
      ENDIF

*     PROMPT USER TO CLEAR PREVIOUS ORBITS
 107  IF ((IOPT2 .EQ. 1) .OR. (IOPT2 .EQ. 3)) THEN
         PRINT*,'DO YOU WANT TO CLEAR THE PREVIOUS ORBITS?'
         PRINT*,'ENTER "Y" OR "N" :'
         READ*,YORN
         PRINT*,YORN
         CALL EXCMS('Clrscrn')
         IF (YORN .EQ. 'Y') THEN
              RIRAY(1) = RIRAY(NUM)
              RJRAY(1) = RJRAY(NUM)
              RKRAY(1) = RKRAY(NUM)
              RARAY(1) = RARAY(NUM)
              TARAY(1) = TARAY(NUM)
              AINRAY(1) = AINRAY(NUM)
              APRAY(1) = APRAY(NUM)
              TIMRAY(1) = TIMRAY(NUM)
              NUM = 1
         ELSEIF (YORN .NE. 'N') THEN
              PRINT*,'INVALID ENTRY!!'
              PRINT*,'ALL INPUTS MUST BE CAPITOL LETTERS'
              GOTO 107
         ENDIF
      ENDIF

* CHECK FOR INVALID OPTION
      IF ((IOPT2 .NE. 1).AND. (IOPT2 .NE. 2).AND. (IOPT2 .NE. 3) .AND.
     +    (IOPT2 .NE. 4).AND. (IOPT2 .NE. 5)) THEN
         PRINT*,'INVALID ENTRY!'
         GOTO 103
      ENDIF
      RETURN
      END

*****************************************************************
* COORDINATE TRANSFORMATIONS
*****************************************************************

      SUBROUTINE PQWIJK(LAN,AP,INC,P,Q,W,I,J,K)
*     THIS SUBROUTINE TRANSFORMS PQW COORDINATES TO IJK COORDINATES

      DOUBLE PRECISION  INC,P,Q,W,I,J,K,R11,R12,R13,R21,R22,R23,
     +    R31,R32,R33,LAN,AP
      R11 = DCOS(LAN)*DCOS(AP) - DSIN(LAN)*DSIN(AP)*DCOS(INC)
      R12 = -DCOS(LAN)*DSIN(AP) - DSIN(LAN)*DCOS(AP)*DCOS(INC)
      R13 = DSIN(LAN)*DSIN(INC)
```

```
      R21 = DSIN(LAN)*DCOS(AP) + DCOS(LAN)*DSIN(AP)*DCOS(INC)
      R22 = -DSIN(LAN)*DSIN(AP) + DCOS(LAN)*DCOS(AP)*DCOS(INC)
      R23 = -DCOS(LAN)*DSIN(INC)
      R31 = DSIN(AP)*DSIN(INC)
      R32 = DCOS(AP)*DSIN(INC)
      R33 = DCOS(INC)
      I = R11*P + R12*Q + R13*W
      J = R21*P + R22*Q + R23*W
      K = R31*P + R32*Q + R33*W
      RETURN
      END

*****************************************************************

      SUBROUTINE IJKPQW(LAN,AP,INC,I,J,K,P,Q,W)
*     THIS SUBROUTINE TRANSFORMS IJK COORDINATES TO PQW COORDINATES

      DOUBLE PRECISION  INC,I,J,K,P,Q,W,R11,R12,R13,R21,R22,R23,
     +   R31,R32,R33,LAN,AP
      R11 = DCOS(LAN)*DCOS(AP) - DSIN(LAN)*DSIN(AP)*DCOS(INC)
      R21 = -DCOS(LAN)*DSIN(AP) - DSIN(LAN)*DCOS(AP)*DCOS(INC)
      R31 = DSIN(LAN)*DSIN(INC)
      R12 = DSIN(LAN)*DCOS(AP) + DCOS(LAN)*DSIN(AP)*DCOS(INC)
      R22 = -DSIN(LAN)*DSIN(AP) + DCOS(LAN)*DCOS(AP)*DCOS(INC)
      R32 = -DCOS(LAN)*DSIN(INC)
      R13 = DSIN(AP)*DSIN(INC)
      R23 = DCOS(AP)*DSIN(INC)
      R33 = DCOS(INC)
      P = R11*I + R12*J + R13*K
      Q = R21*I + R22*J + R23*K
      W = R31*I + R32*J + R33*K
      RETURN
      END

**************************************************************

      SUBROUTINE IJKRSW(LAN,AL,INC,I,J,K,R,S,W)
*     THIS SUBROUTINE CHANGES FROM IJK COORDINATES TO RSW COORDINATES

      DOUBLE PRECISION  INC,I,J,K,R,S,W,R11,R12,R13,R21,R22,R23,
     +   R31,R32,R33,LAN,AL
      R11 = DCOS(LAN)*DCOS(AL) - DSIN(LAN)*DCOS(INC)*DSIN(AL)
      R12 = DSIN(LAN)*DCOS(AL) + DSIN(AL)*DCOS(LAN)*DCOS(INC)
      R13 = DSIN(INC)*DSIN(AL)
      R21 = -DCOS(LAN)*DSIN(AL)-DSIN(LAN)*DCOS(INC)*DCOS(AL)
      R22 = -DSIN(LAN)*DSIN(AL) + DCOS(LAN)*DCOS(INC)*DCOS(AL)
      R23 = DSIN(INC)*DCOS(AL)
      R31 = DSIN(LAN)*DSIN(INC)
      R32 = -DCOS(LAN)*DSIN(INC)
      R33 = DCOS(INC)
      R = R11*I + R12*J + R13*K
      S = R21*I + R22*J + R23*K
      W = R31*I + R32*J + R33*K
      RETURN
      END
```

```
                                                                        ORB04990
************************************************************************ORB05000
                                                                        ORB05010
      SUBROUTINE RSWIJK(LAN,AL,INC,R,S,W,I,J,K)                         ORB05020
*     THIS SUBROUTINE CHANGES FROM RSW COORDINATES TO IJK COORDINATES   ORB05030
                                                                        ORB05040
      DOUBLE PRECISION  INC,R,S,W,I,J,K,R11,R12,R13,R21,R22,R23,        ORB05050
     +   R31,R32,R33,LAN,AL                                             ORB05060
      R11 = DCOS(LAN)*DCOS(AL) - DSIN(LAN)*DCOS(INC)*DSIN(AL)           ORB05070
      R21 = DSIN(LAN)*DCOS(AL) + DSIN(AL)*DCOS(LAN)*DCOS(INC)           ORB05080
      R31 = DSIN(INC)*DSIN(AL)                                          ORB05090
      R12 = -DCOS(LAN)*DSIN(AL)-DSIN(LAN)*DCOS(INC)*DCOS(AL)            ORB05100
      R22 = -DSIN(LAN)*DSIN(AL) + DCOS(LAN)*DCOS(INC)*DCOS(AL)          ORB05110
      R32 = DSIN(INC)*DCOS(AL)                                          ORB05120
      R13 = DSIN(LAN)*DSIN(INC)                                         ORB05130
      R23 = -DCOS(LAN)*DSIN(INC)                                        ORB05140
      R33 = DCOS(INC)                                                   ORB05150
      I = R11*R + R12*S + R13*W                                         ORB05160
      J = R21*R + R22*S + R23*W                                         ORB05170
      K = R31*R + R32*S + R33*W                                         ORB05180
      RETURN                                                            ORB05190
      END                                                               ORB05200
                                                                        ORB05210
************************************************************************ORB05220
                                                                        ORB05230
      SUBROUTINE PQWRSW(TA,P,Q,W,R,S,WN)                                ORB05240
*     THIS SUBROUTINE CHANGES FROM PQW COORDINATES TO RSW COORDINATES   ORB05250
                                                                        ORB05260
      DOUBLE PRECISION  P,Q,W,R,S,WN,R11,R12,R13,R21,R22,R23,           ORB05270
     +   R31,R32,R33,TA                                                 ORB05280
      R11 = DCOS(TA)                                                    ORB05290
      R12 = DSIN(TA)                                                    ORB05300
      R13 = 0.0                                                         ORB05310
      R21 = -DSIN(TA)                                                   ORB05320
      R22 = DCOS(TA)                                                    ORB05330
      R23 = 0.0                                                         CRB05340
      R31 = 0.0                                                         ORB05350
      R32 = 0.0                                                         ORB05360
      R33 = 1.0                                                         ORB05370
      R = R11*P + R12*Q + R13*W                                         ORB05380
      S = R21*P + R22*Q + R23*W                                         ORB05390
      WN = R31*P + R32*Q +R33*W                                         ORB05400
      RETURN                                                            ORB05410
      END                                                               ORB05420
                                                                        ORB05430
************************************************************************ORB05440
                                                                        ORB05450
      SUBROUTINE RSWPQW(TA,R,S,W,P,Q,WN)                                ORB05460
*     THIS SUBROUTINE CHANGES FROM RSW COORDINATES TO PQW COORDINATES   ORB05470
                                                                        ORB05480
      DOUBLE PRECISION  R,S,W,P,Q,WN,R11,R12,R13,R21,R22,R23,           ORB05490
     +   R31,R32,R33,TA                                                 ORB05500
      R11 = DCOS(TA)                                                    ORB05510
      R21 = DSIN(TA)                                                    ORB05520
      R31 = 0.0                                                         ORB05530
      R12 = -DSIN(TA)                                                   ORB05540
```

```
      R22 = DCOS(TA)
      R32 = 0.0
      R13 = 0.0
      R23 = 0.0
      R33 = 1.0
      P = R11*R + R12*S + R13*W
      Q = R21*R + R22*S + R23*W
      WN = R31*R + R32*S +R33*W
      RETURN
      END

************************************************************
*    STORE ELEMENTS IN ARRAYS
************************************************************

      SUBROUTINE STORE(RI,RJ,RK,R,TA,RIRAY,RJRAY,RKRAY,RARAY,TARAY,NUM,
     +   I,AP,AINRAY,APRAY,TT,TIMRAY)
*     THIS SUBROUTINE STORES THE POSITION AND ELEMENTS IN ARRAYS IN
*     SINGLE PRECISION FORM, FOR PLOTTING

      DOUBLE PRECISION RI,RJ,RK,R,TA,I,AP,TT

      DIMENSION RIRAY(500),RJRAY(500),RKRAY(500),RARAY(500),TARAY(500),
     +   AINRAY(500),APRAY(500),TIMRAY(500)

      RIRAY(NUM) = SNGL(RI)
      RJRAY(NUM) = SNGL(RJ)
      RKRAY(NUM) = SNGL(RK)
      RARAY(NUM) = SNGL(R)
      TARAY(NUM) = SNGL(TA)
      AINRAY(NUM) = SNGL(I)
      APRAY(NUM) = SNGL(AP)
      TIMRAY(NUM) = SNGL(TT)
      RETURN
      END

*******************************************************************
* INITIAL       POSITION, VELOCITY
*******************************************************************

      SUBROUTINE INPUTS(RI,RJ,RK,R,VI,VJ,VK,V,MU,QUIT,PI)
*     THIS SUBROUTINE GIVES THE USER A CHOICE TO EITHER ENTER THE
*     INITIAL POSITION AND VELOCITY VECTOR OR TO LET THE PROGRAM
*     CALCULATE THE INITIAL POSITION AND VELOCITY FROM USER PROMPTED
*     INPUTS

*     SUBROUTINES CALLED FROM THIS SUBROUTINE:
*     INELTS = Prompts USER FOR ORBITAL ELEMENTS
*     IPOS   = PROMPTS USER FOR INITIAL POSITION (IJK)
*     IVEL   = PROMPTS USER FOR INITIAL Velocity (IJK)

      DOUBLE PRECISION RI,RJ,RK,R,VI,VJ,VK,V,MU,PI
      CHARACTER*1,QUIT

*     PROMPT USER FOR METHOD TO ENTER INPUTS
 195  PRINT*,'IN WHICH MANNER WOULD YOU LIKE TO INPUT THE INITIAL'
```

```
      PRINT*,'POSITION AND VELOCITY OF THE SATELLITE?'                  ORB06110
      PRINT*,'     1: BY Inputting THE INITIAL POSITION AND VELOCITY'   ORB06120
      PRINT*,'        VECTORS IN THE PERIFOCAL COORDINATE SYSTEM (IJK)' ORB06130
      PRINT*,'     2: BY LETTING THE SATELLITE BE PLACED ON THE "I"'    ORB06140
      PRINT*,'        AXIS OF THE (IJK) SYSTEM AT A DESIRED RADIUS OF'  ORB06150
      PRINT*,'        PERIGEE(RP) AND INPUTTING EITHER A DESIRED RADIUS'ORB06160
      PRINT*,'        OF APOGEE(RA), A DESIRED ECCENTRICITY(E), OR THE' ORB06170
      PRINT*,'        DESIRED VELOCITY AT THAT RADIUS, AND A DESIRED'   ORB06180
      PRINT*,'        INCLINATION(I).'                                  ORB06190
      PRINT*,'     3: QUIT'                                             ORB06200
      PRINT*,'ENTER 1, 2 OR 3:'                                         ORB06210
      READ*,ICHC                                                        ORB06220
      PRINT*,ICHC                                                       ORB06230
      CALL EXCMS('CLRSCRN')                                             ORB06240
                                                                        ORB06250
*     USER INPUTS POSITION AND VELOCITY VECTORS                         ORB06260
      IF (ICHC .EQ. 1) THEN                                             ORB06270
         CALL IPOS(RI,RJ,RK,R)                                          ORB06280
         CALL IVEL(VI,VJ,VK,V,R,MU)                                     ORB06290
                                                                        ORB06300
*     USER INPUTS ORBITAL ELEMENTS TO GET POSITION AND VELOCITY         ORB06310
      ELSEIF (ICHC .EQ. 2) THEN                                         ORB06320
         CALL INELTS(RI,RJ,RK,R,VI,VJ,VK,V,MU,PI)                       ORB06330
                                                                        ORB06340
*     STOP PROGRAM                                                      ORB06350
      ELSEIF (ICHC .EQ. 3) THEN                                         ORB06360
         QUIT = 'N'                                                     ORB06370
      ELSE                                                              ORB06380
         PRINT*,'INVALID ENTRY!  TRY AGAIN!'                            ORB06390
         GOTO 195                                                       ORB06400
      ENDIF                                                             ORB06410
      RETURN                                                            ORB06420
      END                                                               ORB06430
                                                                        ORB06440
***********************************************************************ORB06450
                                                                        ORB06460
      SUBROUTINE IPOS(RI,RJ,RK,R)                                       ORB06470
*     THIS SUBROUTINE ASKS THE USER FOR THE INITIAL POSITION OF THE     ORB06480
*     Satellite IN GEOCENTRIC-EQUATORIAL COORDINATE SYSTEM              ORB06490
                                                                        ORB06500
      DOUBLE PRECISION RI,RJ,RK,R                                       ORB06510
                                                                        ORB06520
      CHARACTER*1, CHOICE                                               ORB06530
      LOGICAL CORREC                                                    ORB06540
      CORREC  = .FALSE.                                                 ORB06550
                                                                        ORB06560
*     PROMPT USER FOR VELOCITY VECTOR                                   ORB06570
 180  IF(.NOT.CORREC) THEN                                              ORB06580
         CALL EXCMS('CLRSCRN')                                          ORB06590
         PRINT*,'ENTER RADIUS VECTOR VALUES IN "KM"'                    ORB06600
         PRINT*,'RADIUS OF THE EARTH = 6400 KM'                         ORB06610
         CORREC  = .TRUE.                                               ORB06620
         PRINT*,'ENTER RI :'                                            ORB06630
         READ*,RI                                                       ORB06640
         PRINT*,'RI = ',RI,'KM'                                         ORB06650
         PRINT*,'ENTER RJ :'                                            ORB06660
```

```
         READ*,RJ
         PRINT*,'RJ = ',RJ,'KM'
         PRINT*,'ENTER RK :'
         READ*,RK
         PRINT*,'RK = ',RK,'KM'

*        CALCULATE TOTAL R
         R = DSQRT((RI**2) + (RJ**2) + (RK**2))
         PRINT*,'R = ',R,'KM'
         IF (R .LE. 6400.0) THEN
              PRINT*,'RADIUS TO SMALL!!  ENTER NEW VALUES!!'
              GOTO 180
         ENDIF

*        CHECK WITH USER THAT Values ARE CORRECT
         PRINT*,'ARE THESE VALUES CORRECT?'
         PRINT*,'    ENTER "Y" OR "N" :'
         READ*,CHOICE
         CHOICE = 'Y'
         PRINT*,CHOICE
         IF (CHOICE.EQ.'Y') THEN
              CORREC  = .TRUE.
         ENDIF
         GOTO 180
      ENDIF
      RETURN
      END

************************************************************************

      SUBROUTINE IVEL(VI,VJ,VK,V,R,MU)
*     THIS SUBROUTINE ASKS THE USER FOR THE INITIAL VELOCITY OF THE
*     Satellite

      DOUBLE PRECISION VI,VJ,VK,V,R,VCIR,VMAX,MU

      CHARACTER*1, CHOICE
      LOGICAL CORREC
      CORREC  = .FALSE.

*     CALCULATE ESCAPE VELOCITY AND CIRCULAR VELOCITY AND PROMPT USER
*     FOR VELOCITY VECTOR
 190  IF(.NOT.CORREC) THEN
         CALL EXCMS('CLRSCRN')
         VCIR = DSQRT(MU/R)
         VMAX = DSQRT((2.0*MU)/R)
         PRINT*,'CIRCULAR VELOCITY = ',VCIR,'KM/SEC'
         PRINT*,'MAXIMUM VELOCITY = ',VMAX,'KM/SEC'
         CORREC  = .TRUE.
         PRINT*,'ENTER VELOCITY VECTOR IN (KM/SEC)'

         PRINT*,'ENTER VI :'
         READ*,VI
         PRINT*,'VI = ',VI,'KM/SEC'
         PRINT*,'ENTER VJ :'
         READ*,VJ
```

```
         PRINT*,'VJ = ',VJ,'KM/SEC'                                     ORB07230
         PRINT*,'ENTER VK :'                                            ORB07240
         READ*,VK                                                       ORB07250
         PRINT*,'VK = ',VK,'KM/SEC'                                     ORB07260
                                                                        ORB07270
*        CALCULATE TOTAL VELOCITY (V)                                   ORB07280
         V = DSQRT((VI**2) + (VJ**2) + (VK**2))                         ORB07290
         PRINT*,'V = ',V,'KM/SEC'                                       ORB07300
                                                                        ORB07310
*        CHECK WITH USER THAT VALUES ARE CORRECTS                       ORB07320
         PRINT*,'ARE THESE VALUES CORRECT?'                             ORB07330
         PRINT*,'    ENTER "Y" OR "N" :'                                ORB07340
         READ*,CHOICE                                                   ORB07350
         CHOICE = 'Y'                                                   ORB07360
         PRINT*,CHOICE                                                  ORB07370
         IF (CHOICE.EQ.'Y') THEN                                        ORB07380
              CORREC  = .TRUE.                                          ORB07390
         ENDIF                                                          ORB07400
         IF (V .GE. VMAX) THEN                                          ORB07410
              PRINT*,'VELOCITY IS GREATER THAN THE ESCAPE VELOCITY!!'   ORB07420
              PRINT*,'RE-ENTER VELOCITY!!!'                             ORB07430
              CORREC = .FALSE.                                          ORB07440
         ENDIF                                                          ORB07450
         GOTO 190                                                       ORB07460
      ENDIF                                                             ORB07470
      RETURN                                                            ORB07480
      END                                                               ORB07490
                                                                        ORB07500
********************************************************************** ORB07510
                                                                        ORB07520
      SUBROUTINE INELTS(RI, RJ, RK, R, VI, VJ, VK, V,MU,PI)             ORB07530
*     SATELLITE PLACED ON 'I' AXIS AND USER SUPPLY ORBITAL ELEMENTS TO  ORB07540
*     GET INITIAL POSITION AND VELOCITY                                 ORB07550
                                                                        ORB07560
      DOUBLE PRECISION RI,RJ,RK,R,VI,VJ,VK,V,MU,I,ENR,A,E,RP,RA,PI,VMAX ORB07570
      CHARACTER*1,CHOICE                                                ORB07580
                                                                        ORB07590
*     PROMPT USER FOR PERIGEE RADIUS                                    ORB07600
198   PRINT*,'ENTER RADIUS OF PERIGEE(RP) IN (KM), FOR EXAMPLE:'        ORB07610
      PRINT*,'LOW EARTH ORBIT (LEO), RP = 6600.0 KM'                    ORB07620
      PRINT*,'GEOSYNCROUNOUS ORBIT, RP = 42241.1 KM'                    ORB07630
      PRINT*,'ENTER RP:'                                                ORB07640
      PRINT*,'"RP" MUST BE > 6400KM'                                    ORB07650
      READ*,RP                                                          ORB07660
      PRINT*,RP                                                         ORB07670
                                                                        ORB07680
*     CHECK FOR VALID RADIUS                                            ORB07690
      IF (RP .LT. 6400.0) THEN                                          ORB07700
         PRINT*,'YOUR "RP" IS TO SMALL!!'                               ORB07710
         GOTO 198                                                       ORB07720
      ENDIF                                                             ORB07730
                                                                        ORB07740
*     PROMPT USER FOR TYPE OF INPUT                                     ORB07750
      PRINT*,'DO YOU WANT TO ENTER THE ECCENTRICITY (E), '              ORB07760
      PRINT*,'RADIUS OF APOGEE (RA), OR VELOCITY (V)?'                  ORB07770
      PRINT*,'ENTER "E", "R", OR "V":'                                  ORB07780
```

```
      READ*,CHOICE                                                      ORB07790
      PRINT*,CHOICE                                                     ORB07800
      CALL ENCMS('CLRSCRN')                                             ORB07810
                                                                        ORB07820
*     USER ENTERS Eccentricity AND SEMI-MAJOR AXIS, ENERGY AND VELOCITY ORB07830
*     IS CALCULATED IN THAT ORDER                                       ORB07840
      IF (CHOICE .EQ. 'E') THEN                                         ORB07850
         PRINT*,'ENTER ECCENTRICITY (E):'                               ORB07860
         PRINT*,'0.0 <= E < 1.0'                                        ORB07870
         READ*,E                                                        ORB07880
         PRINT*,E                                                       ORB07890
                                                                        ORB07900
*        CHECK FOR VALID ECCENTRICITY                                   ORB07910
         IF ((E .LT. 0.0) .OR. (E .GE. 1.0)) THEN                       ORB07920
             PRINT*,'INVALID "E"'                                       ORB07930
             GOTO 198                                                   ORB07940
         ENDIF                                                          ORB07950
         A = RP/(1-E)                                                   ORB07960
         ENR = -MU/(2.0*A)                                              ORB07970
         V = DSQRT(2*(ENR+(MU/RP)))                                     ORB07980
                                                                        ORB07990
*        USER INPUTS RADIUS OF APOGEE AND ECCENTRICITY IS CALCULATED    ORB08000
*        THEN SEMI-MAJOR AXIS, ENERGY AND THEN VELOCITY.                ORB08010
      ELSEIF (CHOICE .EQ. 'R') THEN                                     ORB08020
         PRINT*,'ENTER RADIUS OF APOGEE (RA) IN KM:'                    ORB08030
         PRINT*,'"RA" MUST BE >="RP", "RP" = ',RP                       ORB08040
         READ*,RA                                                       ORB08050
         PRINT*,RA                                                      ORB08060
                                                                        ORB08070
*        CHECK FOR VALID RADIUS OF APOGEE                               ORB08080
         IF (RA .LT. RP) THEN                                           ORB08090
             PRINT*,'YOUR "RA" IS TO SMALL!!'                           ORB08100
             GOTO 198                                                   ORB08110
         ENDIF                                                          ORB08120
         E = (RA-RP)/(RA+RP)                                            ORB08130
         A = RP/(1-E)                                                   ORB08140
         ENR = -MU/(2.0*A)                                              ORB08150
         V = DSQRT(2*(ENR+(MU/RP)))                                     ORB08160
                                                                        ORB08170
*        USER INPUTS MAGNITUDE OF VELOCITY, PROGRAM PROVIDES CIRCULAR   ORB08180
*        AND ESCAPE VELOCITY FOR COMPARISON AND TO CHECK FOR VALID      ORB08190
*        INPUTS                                                         ORB08200
      ELSEIF (CHOICE .EQ. 'V') THEN                                     ORB08210
         PRINT*,'ENTER VELOCITY IN KM/SEC:'                             ORB08220
         PRINT*,'THE MINIMUM VELOCITY ALLOWED IS FOR A CIRCULAR ORBIT'  ORB08230
         VCIRC = SQRT(SNGL(MU/RP))                                      ORB08240
         PRINT*,'ORBIT. V(Circular) = ',VCIRC,' KM/S'                   ORB08250
         VMAX = DSQRT(2*(MU/RP))                                        ORB08260
         PRINT*,'THE MAXIMUM VELOCITY < ',VMAX,' KM/S'                  ORB08270
         READ*,V                                                        ORB08280
         PRINT*,V                                                       ORB08290
         IF (V .LT. VCIRC) THEN                                         ORB08300
             PRINT*,'VELOCITY TO SMALL!'                                ORB08310
             GOTO 198                                                   ORB08320
         ENDIF                                                          ORB08330
         IF (V .GE. VMAX) THEN                                          ORB08340
```

```
               PRINT*,'VELOCITY TO GREAT!!'                             ORB08350
               GOTO 198                                                 ORB08360
            ENDIF                                                       ORB08370
         ELSE                                                           ORB08380
            PRINT*,'INVALID ENTRY! TRY AGAIN'                           ORB08390
            GOTO 198                                                    ORB08400
         ENDIF                                                          ORB08410
                                                                        ORB08420
*        INCLINATION NEEDED TO GIVE Velocity A Direction                ORB08430
         PRINT*,'ENTER INCLINATION (I) IN DEGREES:'                     ORB08440
         READ*,I                                                        ORB08450
         PRINT*,I                                                       ORB08460
         I = (PI/180.0)*I                                               ORB08470
         VK = V*DSIN(I)                                                 ORB08480
         VJ = V*DCOS(I)                                                 ORB08490
         VI = 0.0                                                       ORB08500
                                                                        ORB08510
*        RADIUS VECTOR SET                                              ORB08520
         RI = RP                                                        ORB08530
         RJ = 0.0                                                       ORB08540
         RK = 0.0                                                       ORB08550
         R = RP                                                         ORB08560
         RETURN                                                         ORB08570
         END                                                            ORB08580
                                                                        ORB08590
**********************************************************************  ORB08600
* CALCULATE THE ORBITAL ELEMENTS                                        ORB08610
**********************************************************************  ORB08620
                                                                        ORB08630
         SUBROUTINE CALCEL(RI,RJ,RK,R,VI,VJ,VK,V,EI,EJ,EK,E,A,I,LAN,    ORB08640
     +               LP,TA,PER,EA,MA,AP,AL,TF,P,PI,MU,MM,N,H,HI,HJ)     ORB08650
*        THIS SUBROUTINE CALLS THE INDIVIDUAL SUBROUTINES TO CALCULATE THE ORB08660
*        ORBITAL ELEMENTS                                               ORB08670
                                                                        ORB08680
*        THIS SUBROUTINE CALLS THE FOLLOWING SUBROUTINES(RETURNED VALUES) ORB08690
*        ENERGY = ENERGY PER MASS (ENR)                                 ORB08700
*        ANGMOM = ANGULAR MOMENTUM (H,HI,HJ,HK)                         ORB08710
*        NODE   = NODE VECTOR (N,NI,NJ,NK)                              ORB08720
*        LATREC = SEMI-LATUS RECTUS (P)                                 ORB08730
*        ECC    = ECCENTRICITY (E,EI,EJ,EK)                             ORB08740
*        SMAXIS = SEMI-MAJOR AXIS (A)                                   ORB08750
*        INCL   = INCLINATION (I)                                       ORB08760
*        ASNODE = LONGITUDE OF ASCENDING NODE (LAN)                     ORB08770
*        ARP    = ARGUMENT OF PERIGEE (AP)                              ORB08780
*        IJKPQW = 'IJK' SYSTEM TO 'PQW' SYSTEM                          ORB08790
*        TANOM  = TRUE ANOMALY (TA)                                     ORB08800
*        ARLAT  = ARGUMENT OF LATITUDE (AL)                             ORB08810
*        LONPER = LONGITUDE OF Perigee (LP)                             ORB08820
*        TLON   = TRUE LONGITUDE (TL)                                   ORB08830
*        PERIOD = PERIOD (PER)                                          ORB08840
*        ECCAN  = ECCENTRIC ANOMALY (EA)                                ORB08850
*        MEANMO = MEAN MOTION (MM)                                      ORB08860
*        MEANAN = MEAN ANOMALY (MA)                                     ORB08870
*        TFLGHT = TIME OF FLIGHT (TF)                                   ORB08880
                                                                        ORB08890
         DOUBLE PRECISION RI,RJ,RK,R,VI,VJ,VK,V,EI,EJ,EK,E,A,I,LAN,AL,  ORB08900
```

```
     +   LP,TA.PER.EA.MA,AP,TF,HI,HJ,HK,H,NI,NJ,NK.N,P,PI,MU,MM,ENR,      ORB08910
     +   TL,RP,RQ,RW,NP,NQ,NW                                             ORB08920
                                                                          ORB08930
      CALL ENERGY(V,R,MU,ENR)                                             ORB08940
      CALL ANGMOM(RI,RJ,RK,VI,VJ,VK,HI,HJ,HK,H)                           ORB08950
      CALL NODE(HI,HJ,NI,NJ,NK,N)                                         ORB08960
      CALL LATREC(H,P,MU)                                                 ORB08970
      CALL ECC(RI,RJ,RK,R,VI,VJ,VK,V,EI,EJ,EK,E,MU)                       ORB08980
      CALL SMAXIS(MU,ENR,A)                                               ORB08990
      CALL INCL(HK,H,I,PI)                                                ORB09000
                                                                          ORB09010
*     SPECIAL CASE IF INCLINATION = 0.0                                   ORB09020
      IF (I.NE.0.0) THEN                                                  ORB09030
         CALL ASNODE(NI,N,LAN,NJ,PI)                                      ORB09040
         CALL ARP(NI,NJ,N,EI,EJ,EK,E,AP,PI,NP,NQ,LAN)                     ORB09050
      ELSE                                                                ORB09060
         LAN = 0.0                                                        ORB09070
         AP = 0.0                                                         ORB09080
      ENDIF                                                               ORB09090
                                                                          ORB09100
*     COORDINATE TRANSFORMATION OF 'R' AND 'V' VECTORS                    ORB09110
      CALL IJKPQW(LAN,AP,I,RI,RJ,RK,RP,RQ,RW)                             ORB09120
      CALL IJKPQW(LAN,AP,I,NI,NJ,NK,NP,NQ,NW)                             ORB09130
      CALL TANOM(EI,EJ,EK,E,RI,RJ,RK,RP,RQ,RW,R,VI,VJ,VK,TA,PI)           ORB09140
                                                                          ORB09150
*     SPECIAL CASE FOR Inclination = 0.0                                  ORB09160
      IF (I .NE. 0.0) THEN                                                ORB09170
         CALL ARLAT(NI,NJ,NK,N,RI,RJ,RK,R,AL,PI,TA,AP)                    ORB09180
      ELSE                                                                ORB09190
         AL = TA                                                          ORB09200
      ENDIF                                                               ORB09210
      CALL LONPER(LAN,AP,LP)                                              ORB09220
      CALL TLON(LAN,AP,TA,TL)                                             ORB09230
      CALL PERIOD(A,PER,PI,MU)                                            ORB09240
      CALL ECCAN(E,TA,EA,PI)                                              ORB09250
      CALL MEANMO(A,MM,MU)                                                ORB09260
      CALL MEANAN(EA,E,MA)                                                ORB09270
      CALL TFLGHT(MM,MA,TF)                                               ORB09280
      RETURN                                                              ORB09290
      END                                                                 ORB09300
                                                                          ORB09310
**********************************************************************    ORB09320
                                                                          ORB09330
      SUBROUTINE ENERGY(V,R,MU,ENR)                                       ORB09340
*     THIS SUBROUTINE CALCULATES THE ENERGY OF THE ORBIT                  ORB09350
                                                                          ORB09360
      DOUBLE PRECISION V,R,MU,ENR                                         ORB09370
                                                                          ORB09380
      ENR = ((V**2)/2) - (MU/R)                                           ORB09390
      RETURN                                                              ORB09400
      END                                                                 ORB09410
                                                                          ORB09420
**********************************************************************    ORB09430
                                                                          ORB09440
      SUBROUTINE ANGMOM(RI.RJ,RK,VI,VJ,VK,HI,HJ,HK,H)                     ORB09450
*     THIS SUBROUTINE CALCULATES THE ANGULAR MOMENTUM                     ORB09460
```

```
      DOUBLE PRECISION RI,RJ,RK,VI,VJ,VK,HI,HJ,HK,H

      HI = (RJ * VK) - (RK * VJ)
      HJ = (RK * VI) - (RI * VK)
      HK = (RI * VJ) - (RJ * VI)
      H = DSQRT((HI**2) + (HJ**2) + (HK**2))
      RETURN
      END

******************************************************************

      SUBROUTINE NODE(HI,HJ,NI,NJ,NK,N)
*     THIS SUBROUTINE CALCULATES THE NODE VECTOR

      DOUBLE PRECISION HI,HJ,NI,NJ,NK,N

      NI = -HJ
      NJ = HI
      NK = 0.0
      N = DSQRT((NI**2) + (NJ**2))
      RETURN
      END

*******************************************************************

      SUBROUTINE LATREC(H,P,MU)
*     THIS SUBROUTINE CALCULATES THE SEMI-LATUS RECTUM

      DOUBLE PRECISION H,P,MU

      P = (H**2)/MU
      RETURN
      END

********************************************************************

      SUBROUTINE ECC(RI,RJ,RK,R,VI,VJ,VK,V,EI,EJ,EK,E,MU)
*     THIS SUBROUTINE CALCULATES THE ECCENTRICITY

      DOUBLE PRECISION RI,RJ,RK,R,VI,VJ,VK,V,EI,EJ,EK,E,MU,DOT

*     CALCULATE DOT PRODUCT OF 'R' AND 'V' VECTORS
      DOT = (RI*VI) + (RJ*VJ) + (RK*VK)
      EI = (1.0D+00/MU) * (((V**2) - (MU/R)) * RI - (DOT)*VI)
      EJ = (1 0D+00/MU) * (((V**2) - (MU/R)) * RJ - (DOT)*VJ)
      EK = (1.0D+00/MU) * (((V**2) - (MU/R)) * RK - (DOT)*VK)
      E = DSQRT((EI**2) + (EJ**2) + (EK**2))
      RETURN
      END

*******************************************************************

      SUBROUTINE SMAXIS(MU,ENR,A)
*     THIS SUBROUTINE Calculates THE SEMI-MAJOR AXIS

```

```
      DOUBLE PRECISION MU,ENR,A                                         ORB10030
                                                                        ORB10040
      A = -MU/(2*ENR)                                                   ORB10050
      RETURN                                                            ORB10060
      END                                                               ORB10070
                                                                        ORB10080
*******************************************************************     ORB10090
                                                                        ORB10100
      SUBROUTINE INCL(HK,H,I,PI)                                        ORB10110
*     THIS SUBROUTINE CALCULATES THE INCLINATION                        ORB10120
*     'I' ALWAYS LESS THAN 180 DEGREES                                  ORB10130
                                                                        ORB10140
      DOUBLE PRECISION HK,H,I,PI                                        ORB10150
                                                                        ORB10160
      I = DACOS(HK/H)                                                   ORB10170
      RETURN                                                            ORB10180
      END                                                               ORB10190
                                                                        ORB10200
*******************************************************************     ORB10210
                                                                        ORB10220
      SUBROUTINE ASNODE(NI,N,LAN,NJ,PI)                                 ORB10230
*     THIS SUBROUTINE CALCULATES THE LONGITUDE OF THE ASCENDING NODE    ORB10240
*     IF 'NJ' > 0       THEN 'LAN' < 180 DEGREES                        ORB10250
                                                                        ORB10260
      DOUBLE PRECISION NI,N,LAN,NJ,PI                                   ORB10270
                                                                        ORB10280
      LAN = DATAN2(NJ,NI)                                               ORB10290
      IF (LAN .LT. 0.0) THEN                                            ORB10300
         LAN = (2*PI) + LAN                                             ORB10310
      ENDIF                                                             ORB10320
      RETURN                                                            ORB10330
      END                                                               ORB10340
                                                                        ORB10350
*******************************************************************     ORB10360
                                                                        ORB10370
      SUBROUTINE ARP(NI,NJ,N,EI,EJ,EK,E,AP,PI,NP,NQ,LAN)                ORB10380
*     THIS SUBROUTINE CALCULATES THE ARGUMENT OF Perigee                ORB10390
*     IF 'EK' GREATER THAN 0 THEN 'AP' < 180                            ORB10400
*     VARIABLE TEMP USED AS A Temporary VALUE FOR ARCTAN                ORB10410
                                                                        ORB10420
      DOUBLE PRECISION NI,NJ,N,EI,EJ,EK,E,AP,PI,NQ,NP,TEMP,LAN          ORB10430
                                                                        ORB10440
      IF ((EI .EQ. 0.0) .AND. (EJ .EQ. 0.0)) THEN                       ORB10450
         AP = 0.0                                                       ORB10460
      ELSE                                                              ORB10470
         TEMP = DATAN2(EJ,EI)                                           ORB10480
         IF (TEMP .GT. LAN) THEN                                        ORB10490
             AP = TEMP - LAN                                            ORB10500
         ELSE                                                           ORB10510
             AP = (2*PI) - (LAN - TEMP)                                 ORB10520
          ENDIF                                                         ORB10530
         IF ( AP .LT. 0.0) THEN                                         ORB10540
             AP = (2*PI) + AP                                           ORB10550
         ENDIF                                                          ORB10560
         IF (AP .GT. (2*PI)) THEN                                       ORB10570
             AP = AP - (2*PI)                                           ORB10580
```

```
         ENDIF
      ENDIF
      RETURN
      END

******************************************************************

      SUBROUTINE TANOM(EI,EJ,EK,E,RI,RJ,RK,RP,RQ,RW,R,VI,VJ,VK,
     +                 TA,PI)
*     THIS SUBROUTINE CALCULATES THE TRUE Anomaly
*     IF (R DOT V) > 0 THEN TA < 180 DEGREES

      DOUBLE PRECISION  DOT,EI,EJ,EK,E,RI,RJ,RK,R,VI,VJ,VK,TA,PI,
     +   RP,RQ,RW

      TA = DATAN2(RQ,RP)
      IF (TA .LT.0.0 ) THEN
         TA = (2 * PI) + TA
      ENDIF
      RETURN
      END

******************************************************************

      SUBROUTINE ARLAT(NI,NJ,NK,N,RI,RJ,RK,R,AL,PI,TA,AP)
*     THIS SUBROUTINE CALCULATES THE ARGUMENT OF LATITUDE
*     IF (RK > 0) THEN AL < 180 DEGREES

      DOUBLE PRECISION NI,NJ,NK,N,RI,RJ,RK,R,AL,PI,TA,AP

      AL = TA + AP
      RETURN
      END

******************************************************************

      SUBROUTINE LONPER(LAN,AP,LP)
*     THIS SUBROUTINE CALCULATES THE LONGITUDE OF PERIGEE

      DOUBLE PRECISION LAN,AP,LP

      LP = LAN + AP
      RETURN
      END

******************************************************************

      SUBROUTINE TLON(LAN,AP,TA,TL)
*     THIS SUBROUTINE CALCULATES THE TRUE LONGITUDE AT EPOCH

      DOUBLE PRECISION LAN,AP,TA,TL

      TL = AP + LAN + TA
      RETURN
      END
```

```
******************************************************************

      SUBROUTINE PERIOD(A,PER,PI,MU)
*     THIS SUBROUTINE CALCULATES THE PERIOD

      DOUBLE PRECISION A,PER,PI,MU

      PER = 2.0D+00*(PI)*DSQRT((A**3)/MU)
      RETURN
      END

******************************************************************

      SUBROUTINE ECCAN(E,TA,EA,PI)
*     THIS SUBROUTINE CALCULATES THE ECCENTRIC Anomaly

      DOUBLE PRECISION E,TA,EA,PI

      EA = DACOS((E + DCOS(TA))/(1.0D+00 + E*DCOS(TA)))
      IF (TA .GT. PI) THEN
         EA = (2*PI) - EA
      ENDIF
      RETURN
      END

******************************************************************

      SUBROUTINE MEANMO(A,MM,MU)
*     THIS SUBROUTINE CALCULATES THE MEAN MOTION

      DOUBLE PRECISION A,MM,MU

      MM = DSQRT(MU/(A**3))
      RETURN
      END

******************************************************************

      SUBROUTINE MEANAN(EA,E,MA)
*     THIS SUBROUTINE CALCULATES THE MEAN Anomaly

      DOUBLE PRECISION EA,E,MA

      MA = EA - E*DSIN(EA)
      RETURN
      END

******************************************************************

      SUBROUTINE TFLGHT(MM,MA,TF)
*     THIS SUBROUTINE CALCULATES THE TIME OF FLIGHT

      DOUBLE PRECISION MM,MA,TF

      TF = (1/MM)*MA
```

```
      RETURN                                                            ORB11700
      END                                                               ORB11710
                                                                        ORB11720
************************************************************************ORB11730
*   CALCULATE UNPERTURBED ORBIT                                         ORB11740
************************************************************************ORB11750
                                                                        ORB11760
      SUBROUTINE UNPRET(DT,PER,AL,LAN,AP,I,RI,RJ,RK,R,                  ORB11770
     +   VI,VJ,VK,V,MU,PI,H,A,E,N,TA,P,MM,MA,EA,                        ORB11780
     +   TF,T,NUM,RIRAY,RJRAY,RKRAY,RARAY,TARAY,AINRAY,APRAY,TIMRAY,    ORB11790
     +   TT)                                                            ORB11800
*     THIS SUBROUTINE CALCULATE THE UNPERTURBED ORBIT                   ORB11810
                                                                        ORB11820
*     THIS SUBROUTINE CALLS THE FOLLOWING SUBROUTINES:                  ORB11830
*     NEWELT = CALCULATE NEW ELEMENTS AFTER TIME STEP                   ORB11840
*     NEWPOS = CALCULATE NEW POSITION AFTER TIME STEP                   ORB11850
*     NEWVEL = CALCULATE NEW VELOCITY AFTER TIME STEP                   ORB11860
*     STORE  = STORES POSITION IN ARRAYS                                ORB11870
                                                                        ORB11880
      DOUBLE PRECISION T,DT,PER,AL,LAN,AP,I,RI,RJ,RK,R,VI,VJ,VK,V,      ORB11890
     +                       MU,PI,H,A,E,N,TA,P,MM,MA,EA,TF,TT          ORB11900
                                                                        ORB11910
      DIMENSION  RARAY(500),TARAY(500),RIRAY(500),RJRAY(500),           ORB11920
     +           RKRAY(500),AINRAY(500),APRAY(500),TIMRAY(500)          ORB11930
                                                                        ORB11940
*     SET TRUE ANOMALY TO NEGATIVE SO LOOP CAN BE EXECUTED              ORB11950
      IF (TA .GT. 6.21) THEN                                            ORB11960
         TA = TA - (2*PI)                                               ORB11970
      ENDIF                                                             ORB11980
                                                                        ORB11990
*     CONTINUE AROUND ORBIT TILL CLOSE TO PERIGEE                       ORB12000
 230  IF ((TA .LE. 6.21) .AND. (T .LE.PER)) THEN                        ORB12010
                                                                        ORB12020
*        Increment TRUE TIME                                            ORB12030
         TT = TT + DT                                                   ORB12040
         CALL NEWELT(MM,MA,E,EA,TA,TF,DT,PI,PER)                        ORB12050
         CALL NPOS(RI,RJ,RK,R,LAN,AP,I, TA,A,E)                         ORB12060
         CALL NVEL(E,P,TA,LAN,AP,I,VI,VJ,VK,V,MU)                       ORB12070
                                                                        ORB12080
*        INCREMENT STEP COUNTER AND STORE VALUES                        ORB12090
         NUM = NUM + 1                                                  ORB12100
         CALL STORE(RI,RJ,RK,R,TA,RIRAY,RJRAY,RKRAY,                    ORB12110
     +              RARAY,TARAY,NUM,I,AP,AINRAY,APRAY,                  ORB12120
     +              TT,TIMRAY)                                          ORB12130
                                                                        ORB12140
*        INCREMENT TIME STEP COUNTER                                    ORB12150
         T= T + DT                                                      ORB12160
         GOTO 230                                                       ORB12170
      ENDIF                                                             ORB12180
      RETURN                                                            ORB12190
      END                                                               ORB12200
                                                                        ORB12210
*******************************************************************     ORB12220
* CALCULATE THE UNPERTURBED NEW ELEMENTS                                ORB12230
*******************************************************************     ORB12240
                                                                        ORB12250
```

```
      SUBROUTINE NEWELT(MM,MA,E,EA,TA,TF,DT,PI,PER)
*     THIS SUBROUTINE CALCULATES THE Unperturbed NEW ELEMENTS

*     THIS SUBROUTINE CALLS THE FOLLOWING SUBROUTINES:
*     NEA = NEW ECCENTRIC ANOMALY
*     NTA = NEW TRUE ANOMALY

      DOUBLE PRECISION MM,MA,E,EA,TA,TF,DT,PI,PER

*     Increment TIME OF FLIGHT AND CHECK IF TF GREATER THAN PERIOD
      TF = TF + DT
      IF (TF .GT. PER) THEN
         TF = TF - PER
      ENDIF

*     CALCULATE MEAN ANOMALY AND USE TO FIND ECCENTRIC Anomaly THEN NEW
*     TRUE ANOMALY
      MA = MM*(TF)
      CALL NEA(MA,E,EA)
      CALL NTA(EA,E,TA,PI)
      RETURN
      END

*********************************************************************
*  CALCULATE PERTURBED ORBIT
*********************************************************************

      SUBROUTINE PRETUR(DT,PER,AL,LAN,AP,I,RI,RJ,RK,R,
     +   VI,VJ,VK,V,FR,FS,FW,MU,PI,H,A,E,N,TA,P,MM,MA,EA,
     +   TF,T,NUM,RIRAY,RJRAY,RKRAY,RARAY,TARAY,AINRAY,APRAY,TIMRAY,
     +   TT,TFEA,TFSU,TFMO,TFDRA,TDI,TDA,TDE,TDMM,TDMA,TDLAN,TDH,TDAP)
*     THIS SUBROUTINE CALCULATES THE PERTURBED ORBIT.

*     THIS SUBROUTINE CALLS THE FOLLOWING SUBROUTINES:
*     TFORCE = CALCULATE THE TOTAL PERTURBING FORCE ON THE SATELLITE
*     PNEWEL = CALCULATE THE Perturbed NEW ELEMENTS
*     NPOS   = NEW POSITION AFTER TIME STEP
*     NVEL   = NEW VELOCITY AFTER TIME STEP
*     PERIOD = PERIOD OF PERTURBED ORBIT
*     STORE  = STORE POSITION AND ELEMENTS IN ARRAYS FOR PLOTTING

      DOUBLE PRECISION T,DT,PER,AL,LAN,AP,I,RI,RJ,RK,R,VI,VJ,VK,V,
     +              FR,FS,FW,MU,PI,H,A,E,N,TA,P,MM,MA,EA,TF,TT,
     +   DI,DA,DE,DMM,DMA,DLAN,DH,DAP,EI,EJ,EK,HI,HJ,LP,M,
     +   DVR,DVS,DVW,DVI,DVJ,DVK

      DIMENSION  RARAY(500),TARAY(500),RIRAY(500),RJRAY(500),
     +           RKRAY(500),AINRAY(500),APRAY(500),TIMRAY(500)

*     SET MEAN RADIUS OF EARTH
      RE = 6400.0

      DT = PER/50
      T = DT
      IF (TA .GT. 6.21) THEN
         TA = TA - (2*PI)
```

```
      ENDIF
      IF (TF .GE. PER) THEN
         TF = TF - PER
      ENDIF

*     CONTINUE Around ORBIT FOR ONE PERIOD
 240  IF ((TF .LT. PER) .AND. (T .LT. PER)) THEN

*        INCREMENT TRUE TIME
         TT = TT + DT
         CALL TFORCE(AL,LAN,AP,I,RI,RJ,RK,R,VI,VJ,VK,V,
     +                 TT,FR,FS,FW,MU,PI,
     +                 FEA,FSU,FMO,FDRA,FOR,
     +               EI,EJ,EK,E,A,T,LP,TA,PER,EA,MA,TF,P,
     +               MM,N,H,HI,HJ,DT)
            CALL PNEWEL(FR,FS,FW,H,R,A,E,M,TA,DT,I,LAN,AL,
     +                             AP,P,MM,MA,EA,TF,T,MU,PI,
     +                          DI,DA,DE,DMM,DMA,DLAN,DH,DAP)
            CALL NPOS(RI,RJ,RK,R,LAN,AP,I, TA,A,E)
            CALL NVEL(E,P,TA,LAN,AP,I,VI,VJ,VK,V,MU)

*           CALCULATE NEW PERIOD AND RESET TIME STEP AND TIME COUNTER
*           IF NOT AT END OF ORBIT
            IF (T .LT. (PER-DT)) THEN
               CALL PERIOD(A,PER,PI,MU)
               DT = PER/50
               T = TF
            ENDIF

*        INCREMENT STEP COUNTER
         NUM = NUM + 1
 241     CALL STORE(RI,RJ,RK,R,TA,RIRAY,RJRAY,RKRAY,
     +                          RARAY,TARAY,NUM,I,AP,AINRAY,APRAY,
     +                          TT,TIMRAY)

*        TOTAL ELEMENT CHANGES
         TDI = TDI + SNGL(ABS(DI))
         TDA = TDA + SNGL(ABS(DA))
         TDE = TDE + SNGL(ABS(DE))
         TDMM = TDMM + SNGL(ABS(DMM))
         TDMA = TDMA + SNGL(ABS(DMA))
         TDLAN = TDLAN + SNGL(ABS(DLAN))
         TDH = TDH + SNGL(ABS(DH)'
         TDAP = TDAP + SNGL(ABS(DAP))
         TFEA = TFEA + FEA
         TFSU = TFSU + FSU
         TFMO = TFMO + FMO
         TFDRA = TFDRA + FDRA

*        CHECK FOR IMPACT
         IF (R .LE. RE) THEN
            PRINT*,'SATELLITE WILL IMPACT THE EARTH!!'
            T = PER
         ENDIF

*        INCREMENT TIME COUNTER
```

```
         T = T + DT
         GOTO 240
      ENDIF
      RETURN
      END

************************************************************************
* CALCULATE THE PERTURBING FORCES
************************************************************************

      SUBROUTINE TFORCE(AL,LAN,AP,I,RI,RJ,RK,R,VI,VJ,VK,V,TT,
     +                  FR,FS,FW,MU,PI,FEA,FSU,FMO,FDRA,FOR,
     +                  EI,EJ,EK,E,A,T,LP,TA,PER,EA,MA,TF,P,
     +                  MM,N,H,HI,HJ,DT)
*     THIS SUBROUTINE SUMS ALL THE PERTURBING FORCES FOR THE TOTAL
*     PERTURBING FORCE.

*     THE FOLLOWING SUBROUTINES WERE CALLED:
*     OBERT  = OBLATENESS OF THE EARTH
*     FSUN   = GRAVITATIONAL Attraction OF THE SUN
*     FMOON  = GRAVITATIONAL Attraction OF THE MOON
*     FDRAG  = DRAG FORCES

      DOUBLE PRECISION  FER,FES,FEW,FSR,FSS,FSW,FMR,FMS,FMW,MU,PI,
     +   FDR,FDS,FDW,FR,FS,FW,RI,RJ,RK,R,AL,I,TT,LAN,AP,VI,VJ,VK,V,
     +                  EI,EJ,EK,E,A,T,LP,TA,PER,EA,MA,TF,P,
     +                  MM,N,H,HI,HJ,DT

      CALL OBEART(RI,RJ,RK,R,AL,I,FER,FES,FEW,MU)
      CALL FSUN(TT,RI,RJ,RK,R,FSR,FSS,FSW,PI)
      CALL FMOON(TT,RI,RJ,RK,R,FMR,FMS,FMW,PI)
      CALL FDRAG(RI,RJ,RK,R,VI,VJ,VK,V,LAN,AP,I,FDR,FDS,FDW,
     +                  EI,EJ,EK,E,A,T,LP,TA,PER,EA,MA,AL,TF,P,PI,MU,
     +                  MM,N,H,HI,HJ,DT)

*     SUM VECTOR FORCES
      FR = FER + FSR + FMR + FDR
      FS = FES + FSS + FMS + FDS
      FW = FEW + FSW + FMW + FDW

*     CALCULATE TOTAL FORCE FROM EACH, AND TOTAL OF ALL
      FEA = SNGL(SQRT((FER**2)+(FES**2)+(FEW**2)))
      FSU = SNGL(SQRT((FSR**2)+(FSS**2)+(FSW**2)))
      FMO = SNGL(SQRT((FMR**2)+(FMS**2)+(FMW**2)))
      FDRA = SNGL(SQRT((FDR**2)+(FDS**2)+(FDW**2)))
      FOR = SNGL(SQRT((FR**2)+(FS**2)+(FW**2)))

      RETURN
      END

************************************************************************

      SUBROUTINE OBEART(RI,RJ,RK,R,AL,I,FER,FES,FEW,MU)
*     THIS SUBROUTINE CALCULATES THE PERTURBING FORCE DUE TO THE
*     OBLIQUENESS OF THE EARTH.

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      DOUBLE PRECISION  J2,RE,FER,FES,FEW,RI,RJ,RK,R,AL,I,MU,M          ORB13940
                                                                        ORB13950
      J2 = 1.082364D-03                                                 ORB13960
      RE = 6.3782D+03                                                   ORB13970
                                                                        ORB13980
      FER = ((-3.0D+00*MU*J2*(RE**2))/(2.0D+00*(R**4)))*                ORB13990
     +      (1.0D+00 - (3.0D+00* ((DSIN(I))**2) * ((DSIN(AL))**2)))     ORB14000
      FES = ((-3.0D+00*MU*J2*(RE**2))/(R**4))*                          ORB14010
     +      (((DSIN(I))**2)* (DSIN(AL))*(DCOS(AL)))                     ORB14020
      FEW = ((-3.0D+00*MU*J2*(RE**2))/(R**4))*                          ORB14030
     +      (DSIN(I)*DCOS(I)*DSIN(AL))                                  ORB14040
      RETURN                                                            ORB14050
      END                                                               ORB14060
                                                                        ORB14070
**********************************************************************  ORB14080
                                                                        ORB14090
      SUBROUTINE FSUN(TT,RI,RJ,RK,R,FSR,FSS,FSW,PI)                     ORB14100
*     THIS SUBROUTINE CALCULATES THE PERTURBING FORCE DUE TO THE SUN    ORB14110
                                                                        ORB14120
*     THE FOLLOWING SUBROUTINES ARE CALLED:                             ORB14130
*     SUNPOS = SUNS POSITION ORBITING AROUND EARTH                      ORB14140
*     HEVBOD = PERTURBING FORCE FROM A Heavenly BODY                    ORB14150
                                                                        ORB14160
      DOUBLE PRECISION FSR,FSS,FSW,PI,                                  ORB14170
     +    RSI,RSJ,RSK,SLAN,SI,SAL,SMU,TT,RI,RJ,RK,R,RS                  ORB14180
                                                                        ORB14190
*     SUNS GRAVITATIONAL PARAMETER                                      ORB14200
      SMU = 1.3271544D+11                                               ORB14210
      CALL SUNPOS(TT,RSI,RSJ,RSK,RS,SLAN,SI,SAL,PI)                     ORB14220
      CALL HEVBOD(RI,RJ,RK,R,RSI,RSJ,RSK,RS,SLAN,SAL,SI,SMU,FSR,FSS,FSW)ORB14230
      RETURN                                                            ORB14240
      END                                                               ORB14250
                                                                        ORB14260
**********************************************************************  ORB14270
                                                                        ORB14280
      SUBROUTINE FMOON(TT,RI,RJ,RK,R,FMR,FMS,FMW,PI)                    ORB14290
*     THIS SUBROUTINE CALCULATES THE PERTURBING FORCE DUE TO The MOON   ORB14300
                                                                        ORB14310
*     THE FOLLOWING SUBROUTINE ARE CALLED:                              ORB14320
*     MONPOS = MOONS POSITION ORBITING AROUND THE EARTH                 ORB14330
*     HEVBOD = PERTURBING FORCE FROM A HEAVENLY BODY                    ORB14340
                                                                        ORB14350
      DOUBLE PRECISION  FMR,FMS,FMW,RMI,RMJ,RMK,MLAN,MI,MAL,MMU,        ORB14360
     +            TT,RI,RJ,RK,R,RM,PI                                   ORB14370
                                                                        ORB14380
*     MOONS GRAVITATIONAL PARAMETER                                     ORB14390
      MMU = 4.90287D+03                                                 ORB14400
                                                                        ORB14410
      CALL MONPOS(TT,RMI,RMJ,RMK,RM,MLAN,MI,MAL,PI)                     ORB14420
      CALL HEVBOD(RI,RJ,RK,R,RMI,RMJ,RMK,RM,MLAN,MAL,MI,MMU,FMR,FMS,FMW)ORB14430
      RETURN                                                            ORB14440
      END                                                               ORB14450
                                                                        ORB14460
**********************************************************************  ORB14470
                                                                        ORB14480
      SUBROUTINE HEVBOD(RI,RJ,RK,R,RPI,RPJ,RPK,RP,LAN,AL,INC,MUP,       ORB14490
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     +   FHR,FHS,FHW)
*     THIS SUBROUTINE CALCULATES THE PERTURBING FORCE DUE TO A
*     HEAVENLY BODY.

*     THE FOLLOWING SUBROUTINE WAS CALLED:
*     IJKRSW = 'IJK' SYSTEM TO THE 'RSW' SYSTEM

      DOUBLE PRECISION  DOT,FHI,FHJ,FHK,RI,RJ,RK,R,RPI,RPJ,RPK,RP,
     +   LAN,AL,INC,MUP,I,J,K,IP,JP,KP,M,FHR,FHS,FHW

*     CALCULATE UNIT VECTOR FOR SATELLITE AND PERTURBING BODIES POSITION
      I = RI/R
      J = RJ/R
      K = RK/R
      IP = RPI/RP
      JP = RPJ/RP
      KP = RPK/RP

*     CALCULATE DOT PRODUCT OF UNIT VECTORS
      DOT = (( I*IP )+( J*JP )+( K*KP ))

*     CALCULATE FORCES IN THE 'IJK' SYSTEM
      FHI = (MUP/(RP**2))*(R/RP)*(3.0D+00*DOT*(IP)-(I))
      FHJ = (MUP/(RP**2))*(R/RP)*(3.0D+00*DOT*(JP)-(J))
      FHK = (MUP/(RP**2))*(R/RP)*(3.0D+00*DOT*(KP)-(K))

*     Transform FORCES TO THE RSW SYSTEM
      CALL IJKRSW(LAN,AL,INC,FHI,FHJ,FHK,FHR,FHS,FHW)
      RETURN
      END

*****************************************************************

      SUBROUTINE SUNPOS(TT,RSI,RSJ,RSK,RS,SLAN,SI,SAL,PI)
*     THIS SUBROUTINE CALCULATES THE SUNS POSITION

*     VARIABLES USED TO DESCRIBE THE SUNS ORBIT:
*     SI   = SUNS INCLINATION
*     SLAN= SUNS Longitude OF ASCENDING NODE
*     SAP  = SUNS ARGUMENT OF PERIGEE
*     RS   = SUNS ORBITAL RADIUS
*     STA  = SUNS TRUE ANOMALY
*     SAL  = SUNS ARGUMENT OF LONGITUDE

      DOUBLE PRECISION  SLAN,SI,SAL,RS,STA,SAP,TT,RSI,RSK,
     +   RSJ,RSP,RSQ,RSW,PI

      SI = 4.09279709D-01
      SLAN = 0.0D+00
      SAP = 0.0D+00
      RS = 1.4959965D+08
      STA = ((2.0*PI)/(365.0 * 86400.0) * TT)
      SAL = STA + SAP

*     CALCULATE SUNS POSITION IN 'PQW' SYSTEM
      RSP = RS*DCOS(STA)
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      RSQ = RS*DSIN(STA)                                                ORB15060
      RSW = 0.0D+00                                                     ORB15070
                                                                        ORB15080
*     TRANSFORM POSITION TO 'IJK' SYSTEM                                ORB15090
      CALL PQWIJK(SLAN,SAP,SI,RSP,RSQ,RSW,RSI,RSJ,RSK)                  ORB15100
      RETURN                                                            ORB15110
      END                                                               ORB15120
                                                                        ORB15130
*****************************************************************       ORB15140
                                                                        ORB15150
      SUBROUTINE MONPOS(TT,RMI,RMJ,RMK,RM,MLAN,MI,MAL,PI)               ORB15160
*     THIS SUBROUTINE CALCULATES THE MOONS POSITION                     ORB15170
                                                                        ORB15180
*     VARIABLES USED TO DESCRIBE THE SUNS ORBIT:                        ORB15190
*     MI   = MOONS INCLINATION                                          ORB15200
*     MLAN= MOONS Longitude OF ASCENDING NODE                           ORB15210
*     MAP = MOONS ARGUMENT OF PERIGEE                                   ORB15220
*     RM   = MOONS ORBITAL RADIUS                                       ORB15230
*     MTA = MOONS TRUE ANOMALY                                          ORB15240
*     MAL = MOONS ARGUMENT OF LONGITUDE                                 ORB15250
                                                                        ORB15260
      DOUBLE PRECISION     MLAN,MAL,RM,TM,MTA,RMP,RMQ,RMW,              ORB15270
     +    RMI,RMJ,RMK,[illegible],PI                                    ORB15280
                                                                        ORB15290
      MI = 4.99164166D-01                                               ORB15300
      RM = 3.844D+05                                                    ORB15310
      MLAN = 0.0                                                        ORB15320
      MTA = ((2.0*PI)/(27.3 * 3600) * TT)                               ORB15330
      MAP = 0.0D+00                                                     ORB15340
      MAL = MTA                                                         ORB15350
                                                                        ORB15360
*     CALCULATE MOON POSITION IN 'PQW' SYSTEM                           ORB15370
      RMP = RM*DCOS(MTA)                                                ORB15380
      RMQ = RM*DSIN(MTA)                                                ORB15390
      RMW = 0                                                           ORB15400
                                                                        ORB15410
*     TRANSFORM POSITION TO 'IJK' SYSTEM                                ORB15420
      CALL PQWIJK(MLAN,MAP,MI,RMP,RMQ,RMW,RMI,RMJ,RMK)                  ORB15430
      RETURN                                                            ORB15440
      END                                                               ORB15450
                                                                        ORB15460
*****************************************************************       ORB15470
                                                                        ORB15480
      SUBROUTINE FDRAG(RI,RJ,RK,R,VI,VJ,VK,V,LAN,AP,I,FDR,FDS,FDW,      ORB15490
     +                EI,EJ,EK,E,A,T,LP,TA,PER,EA,MA,AL,TF,P,PI,MU,     ORB15500
     +                MM,N,H,HI,HJ,DT)                                  ORB15510
*     THIS SUBROUTINE CALCULATES THE PERTURBING FORCE DUE TO DRAG       ORB15520
                                                                        ORB15530
*     THE FOLLOWING VARIABLES ARE USED TO MODEL THE ATMOSPHERE:         ORB15540
*     RE       = RADIUS OF EARTH                                        ORB15550
*     M        = MASS OF SATELLITE                                      ORB15560
*     AR       = FRONTAL SURFACE AREA OF SATELLITE                      ORB15570
*     Z        = ALTITUDE OF SATELLITE                                  ORB15580
*     K        = EXPONENTIAL DECAY FACTOR                               ORB15590
*     DENO     = NORMAL DENSITY                                         ORB15600
*     CD       COEFFICIENT OF DRAG                                      ORB15610
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                                                                        ORB15620
      DOUBLE PRECISION  MAG,M,K,FDR,FDS,FDW,RE,AR,Z,DENO,CD,DEN,        ORB15630
     +                  FDJ,FDK,FDI,RI,RJ,RK,VI,VJ,VK,V,LAN,AP,I,R,     ORB15640
     +                  EI,EJ.EK,E,A,T,LP,TA,PER,EA,MA,AL,TF,P,PI,MU,   ORB15650
     +                  MM,N,H,HI,HJ,DT,DVR,DVS,DVW,DVI,DVJ,DVK         ORB15660
                                                                        ORB15670
      RE = 6.378145D+03                                                 ORB15680
      M = 1.0D+02                                                       ORB15690
      AR = 2.0D+01                                                      ORB15700
      Z = R - RE                                                        ORB15710
                                                                        ORB15720
*     DEPENDING ON ALTITUDE SET ATMOSPHERE VARIABLES                    ORB15730
      IF (Z.LE.1.5D+02) THEN                                            ORB15740
         K = 4.74D-02                                                   ORB15750
         DENO = 1.225D+00                                               ORB15760
         CD = 1.0D+00                                                   ORB15770
      ELSEIF (Z.LE.5.5D+02) THEN                                        ORB15780
         K = 3.4614D-02                                                 ORB15790
         DENO = 1.79846D-01                                             ORB15800
         CD = 2.0D+00                                                   ORB15810
      ELSE                                                              ORB15820
         K = 2.21698D-3                                                 ORB15830
         DENO = 1.015484D-07                                            ORB15840
         CD = 2.0D+00                                                   ORB15850
      ENDIF                                                             ORB15860
                                                                        ORB15870
*     CALCULATE ATMOSPHERIC DENSITY                                     ORB15880
      DEN = DENO * DEXP(-K*Z)                                           ORB15890
                                                                        ORB15900
*     CALCULATE MAGNITUDE OF DRAG FORCE AND LIMIT IT TO 1.0E-20         ORB15910
      MAG = -(0.5D+00)*CD*AR*DEN*V*(1.0D-03)/M                          ORB15920
      IF (ABS(MAG) .LT. 1.0D-20) THEN                                   ORB15930
         MAG = -1.0D-20                                                 ORB15940
      ENDIF                                                             ORB15950
                                                                        ORB15960
*     GIVE DRAG FORCE A Direction OF MINUS THE VELOCITY                 ORB15970
      FDR = 0.0                                                         ORB15980
      FDS = MAG * V                                                     ORB15990
      FDW = 0.0                                                         ORB16000
      RETURN                                                            ORB16010
      END                                                               ORB16020
                                                                        ORB16030
*************************************************************           ORB16040
* CALCULATE PERTURBED NEW ELEMENTS                                      ORB16050
*************************************************************           ORB16060
                                                                        ORB16070
      SUBROUTINE PNEWEL(FR,FS,FW,H,R,A,E,N,TA,DT,I,LAN,AL,AP,P,         ORB16080
     +   MM,MA,EA,TF,T,MU,PI,DI,DA,DE,DMM,DMA,DLAN,DH,DAP)              ORB16090
*     THIS SUBROUTINE CALCULATES THE NEW ELEMENTS FROM THE PREVIOUS     ORB16100
*     ELEMENTS ADDED TO THE RATES OF CHANGE FOR ONE STEP                ORB16110
                                                                        ORB16120
*     THE FOLLOWING SUBROUTINES ARE CALLED:                             ORB16130
*     RATE   = CALCULATES RATES OF CHANGE OF ORBITAL ELEMENTS           ORB16140
*     NANGMO = NEW ANGULAR MOMENTUM (NEWH)                              ORB16150
*     NSMA   = NEW SEMI-MAJOR AXIS (NEWA)                               ORB16160
*     NECC   = NEW ECCENTRICITY (NEWE)                                  ORB16170
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*     NINCL  = NEW INCLINATION (NEWI)                                 ORB16180
*     NASNOD = NEW LONGITUDE OF ASCENDING NODE (NEWLAN)               ORB16190
*     NARPER = NEW ARGUMENT OF PERIGEE ( NEWAP)                       ORB16200
*     NMNMO  = NEW MEAN MOTION (NEWMM)                                ORB16210
*     MEANMO = MEAN MOTION (MM)                                       ORB16220
*     NMMAN  = NEW MEAN ANOMALY (NEWMA)                               ORB16230
*     NEA    = NEW ECCENTRIC ANOMALY (EA)                             ORB16240
*     NTA    = NEW TRUE ANOMALY (TA)                                  ORB16250
*     TFLGHT = TIME OF FLIGHT (TF)                                    ORB16260
                                                                      ORB16270
      DOUBLE PRECISION  FR,FS,FW,DMM,H,R,A,E,N,TA,DT,I,LAN,AL,AP,P,   ORB16280
     +   MM,MA,EA,TF,T,MU,PI,DA,DH,DE,DI,DLAN,DAP,DMA,                ORB16290
     +   NEWH,NEWA,NEWE,NEWI,NEWLAN,NEWAP,NEWMM                       ORB16300
                                                                      ORB16310
*     INCREMENT TIME OF FLIGHT BY ONE TIME STEP AND CALCULATE RATES   ORB16320
      TF = TF + DT                                                    ORB16330
      CALL RATES(DH,DA,DE,DI,DLAN,DAP,DMM,DMA,E,MM,R,A,FR,FS,FW,      ORB16340
     +             TA,AL,H,P,T,MU,I)                                  ORB16350
                                                                      ORB16360
*     CALCULATE NEW ELEMENTS                                          ORB16370
      CALL NANGMO(H,DT,DH,NEWH)                                       ORB16380
      CALL NSMA(A,DT,DA,NEWA)                                         ORB16390
      CALL NECC(E,DT,DE,NEWE)                                         ORB16400
      CALL NINCL(I,DT,DI,NEWI)                                        ORB16410
      CALL NASNOD(LAN,DT,DLAN,NEWLAN)                                 ORB16420
      CALL NARPER(AP,DT,DAP,NEWAP)                                    ORB16430
                                                                      ORB16440
*     SET ELEMENTS TO NEW ELEMENTS                                    ORB16450
      A = NEWA                                                        ORB16460
      E = NEWE                                                        ORB16470
      I = NEWI                                                        ORB16480
      LAN = NEWLAN                                                    ORB16490
      AP = NEWAP                                                      ORB16500
      P = A * (1 - E**2)                                              ORB16510
                                                                      ORB16520
*     MOVE THE SATELLITE ONE TIME STEP                                ORB16530
      CALL MEANMO(A,MM,MU)                                            ORB16540
      CALL NMNAN(MA,MM,DT,TF,DMA,PI)                                  ORB16550
      CALL NEA(MA,E,EA)                                               ORB16560
      CALL NTA(EA,E,TA,PI)                                            ORB16570
      CALL TFLGHT(MM,MA,TF)                                           ORB16580
      AL = TA + AP                                                    ORB16590
      RETURN                                                          ORB16600
      END                                                             ORB16610
                                                                      ORB16620
*****************************************************************     ORB16630
* CALCULATE THE RATES OF CHANGE OF THE ORBITAL Elements               ORB16640
*****************************************************************     ORB16650
                                                                      ORB16660
      SUBROUTINE RATES(DH,DA,DE,DI,DLAN,DAP,DMM,DMA,E,MM,R,A,FR,FS,FW, ORB16670
     + TA,AL,H,P,T,MU,I)                                              ORB16680
*     THIS SUBROUTINE Calls THE FOLLOWING SUBROUTINES TO CALCULATE THE ORB16690
*     TIME RATE-OF- CHANGE OF THE ORBITAL ELEMENTS:                   ORB16700
*     RSMAX  = RATE-OF-CHANGE OF THE SEMI-MAJOR AXIS (DA)             ORB16710
*     RECC   = RATE-OF-CHANGE OF THE ECCENTRICITY (DE)                ORB16720
*     RINC   = RATE-OF-CHANGE OF THE INCLINATION (DI)                 ORB16730
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*     RLAN   = RATE-OF-CHANGE OF THE Longitude OF THE ASCENDING NODE   ORB16740
*              (DLAN)                                                  ORB16750
*     RAP    = RATE-OF-CHANGE OF THE ARGUMENT OF PERIGEE (DAP)         ORB16760
*     RMM    = RATE-OF-Change OF THE MEAN MOTION (DMM)                 ORB16770
*     RMA    = RATE-OF-CHANGE OF THE MEAN ANOMALY (DMA)                ORB16780
*     RANGMO = RATE-OF-CHANGE OF THE ANGULAR MOMENTUM (DH)             ORB16790
                                                                       ORB16800
      DOUBLE PRECISION DH,DA,DE,DI,DLAN,DAP,DMM,DMA,E,MM,R,A,FR,FS,FW, ORB16810
     +  TA,AL,H,P,T,MU,I                                               ORB16820
                                                                       ORB16830
      CALL RSMAX(E,MM,R,A,FR,FS,DA,TA)                                 ORB16840
      CALL RECC(E,MM,R,A,FR,FS,TA,DE)                                  ORB16850
      CALL RINC(E,MM,R,A,FW,AL,DI)                                     ORB16860
      CALL RLAN(E,MM,R,A,I,FW,AL,DLAN)                                 ORB16870
      CALL RAP(E,MM,R,A,I,H,P,AL,TA,FR,FS,FW,DAP)                      ORB16880
      CALL RMM(MM,A,DMM,DA,MU)                                         ORB16890
      CALL RMA(E,MM,R,A,TA,DMM,FR,FS,DMA,T)                            ORB16900
      CALL RANGMO(R,FS,FW,DH)                                          ORB16910
      RETURN                                                           ORB16920
      END                                                              ORB16930
                                                                       ORB16940
*****************************************************************      ORB16950
                                                                       ORB16960
      SUBROUTINE RANGMO(R,FS,FW,DH)                                    ORB16970
*     THIS SUBROUTINE CALCULATES THE RATE OF CHANGE OF THE             ORB16980
*     ANGULAR MOMENTUM                                                 ORB16990
                                                                       ORB17000
      DOUBLE PRECISION  FS,FW,DHW,DHS,DH,R                             ORB17010
                                                                       ORB17020
      DHW = R * FS                                                     ORB17030
      DHS = R * FW                                                     ORB17040
      DH = DSQRT((DHW**2) + (DHS**2))                                  ORB17050
      RETURN                                                           ORB17060
      END                                                              ORB17070
                                                                       ORB17080
*******************************************************************    ORB17090
                                                                       ORB17100
      SUBROUTINE RSMAX(E,MM,R,A,FR,FS,DA,TA)                           ORB17110
*     THIS SUBROUTINE CALCULATES THE RATE OF CHANGE OF THE SEMI-MAJOR  ORB17120
*     AXIS                                                             ORB17130
                                                                       ORB17140
      DOUBLE PRECISION  DA,FR,FS,E,MM,R,A,TA,ET                        ORB17150
                                                                       ORB17160
*     TRAP (E) SO DENOMINATOR DOES NOT GOTO ZERO                       ORB17170
      IF (E.GT.0.9) THEN                                               ORB17180
         ET = 0.9                                                      ORB17190
      ELSE                                                             ORB17200
         ET = E                                                        ORB17210
      ENDIF                                                            ORB17220
        DA = ((2.0D+00*E *DSIN(TA))/(MM *DSQRT(1.0D+00-(ET**2))))*FR + ORB17230
     +   ((2.0D+00*A*DSQRT(1.0D+00-(E **2)))/(MM *R))*FS               ORB17240
      RETURN                                                           ORB17250
      END                                                              ORB17260
                                                                       ORB17270
*******************************************************************    ORB17280
                                                                       ORB17290
```

```
      SUBROUTINE RECC(E,MM,R,A,FR,FS,TA,DE)
*     THIS SUBROUTINE CALCULATES THE RATE OF CHANGE OF THE ECCENTRICITY

      DOUBLE PRECISION  DE,FR,FS,E,MM,R,A,TA,ET

*     TRAP (E) SO DENOMINATOR DOES NOT GOTO ZERO
      IF (E.LT.0.1) THEN
         ET = 0.1
      ELSE
         ET = E
      ENDIF
       DE = ((DSQRT(1.0D+00 - (E **2))*SIN(TA))/(MM *A))*FR +
     +   ((DSQRT(1.0D+00 - (E **2)))/(MM *ET*(A**2)))*
     +   ((A**2)*(1.0D+00 - (E **2))/(R) - (R))*FS
      RETURN
      END

*****************************************************************

      SUBROUTINE RLAN(E,MM,R,A,I,FW,AL,DLAN)
*     THIS SUBROUTINE CALCULATES THE RATE OF CHANGE OF THE LONGITUDE
*     OF THE ASCENDING NODE

      DOUBLE PRECISION  DLAN,FW,E,MM,R,A,I,AL,ET,IT

*     TRAP (E) AND (I) SO DENOMINATOR DOES NOT GOTO ZERO
      IF (E.GT.0.9) THEN
         ET = 0.9
      ELSE
         ET = E
      ENDIF
      IF (I.LT.0.01745) THEN
         IT = 0.01745
      ELSE
         IT = I
      ENDIF
      DLAN = (R*FW*DSIN(AL))/(MM *(A**2)*DSQRT(1.0D+00 - (ET**2))*
     +   DSIN(IT))
      RETURN
      END

*****************************************************************

      SUBROUTINE RAP(E,MM,R,A,I,H,P,AL,TA,FR,FS,FW,DAP)
*     THIS SUBROUTINE CALCULATES THE RATE OF CHANGE OF THE ARGUMENT
*     OF PERIGEE

      DOUBLE PRECISION  DAPR,DAPS,DAPW,DAP,FR,FS,FW,E,MM,R,I,H,P,AL,TA,
     +   ET,A,IT

*     TRAP (E) AND (I) SO DENOMINATOR DOES NOT GOTO ZERO
      IF (I.LT.0.01745) THEN
         IT = 0.01745
      ELSE
         IT = I
      ENDIF
```

```
      IF (E.GT.0.9) THEN                                                ORB17860
         ET = 0.9                                                       ORB17870
      ELSEIF (E.LT.0.1) THEN                                            ORB17880
         ET = 0.1                                                       ORB17890
      ELSE                                                              ORB17900
         ET = E                                                         ORB17910
      ENDIF                                                             ORB17920
      DAPR = (-DSQRT(1.0+00 - (E **2))*DCOS(TA))/(MM *A*ET) * FR        ORB17930
      DAPS = (P/(ET*H))*(DSIN(TA))*                                     ORB17940
     +     (1.0D+00 + 1.0D+00/(1.0D+00 + ET*DCOS(TA))) *FS              ORB17950
      DAPW = (-R*(1.0D+00/DTAN(IT))*DSIN(AL))/                          ORB17960
     +       (MM *(A**2)*DSQRT(1.0D+00 - (ET**2)))*FW                   ORB17970
      DAP = DAPR + DAPS + DAPW                                          ORB17980
      RETURN                                                            ORB17990
      END                                                               ORB18000
                                                                        ORB18010
************************************************************************ORB18020
                                                                        ORB18030
      SUBROUTINE RINC(E,MM,R,A,FW,AL,DI)                                ORB18040
*     THIS SUBROUTINE CALCULATES THE RATE OF CHANGE OF THE INCLINATION  ORB18050
                                                                        ORB18060
      DOUBLE PRECISION  DI,FW,E,MM,R,A,AL,ET                            ORB18070
                                                                        ORB18080
*     TRAP (E) SO DENOMINATOR DOES NOT GOTO ZERO                        ORB18090
      IF (E.GT.0.9) THEN                                                ORB18100
         ET = 0.9                                                       ORB18110
      ELSE                                                              ORB18120
         ET = E                                                         ORB18130
      ENDIF                                                             ORB18140
      DI = (R*FW*DCOS(AL))/(MM *(A**2)*DSQRT(1.0D+00 - (ET**2)))        ORB18150
      RETURN                                                            ORB18160
      END                                                               ORB18170
                                                                        ORB18180
************************************************************************ORB18190
                                                                        ORB18200
      SUBROUTINE RMM(MM,A,DMM,DA,MU)                                    ORB18210
*     THIS SUBROUTINE CALCULATES THE RATE OF CHANGE OF THE MEAN MOTION  ORB18220
                                                                        ORB18230
      DOUBLE PRECISION  DMM,DA,MM,A,MU                                  ORB18240
                                                                        ORB18250
      DMM =((-3.0D+00*MU)/(2.0D+00*MM *(A**4)))* DA                     ORB18260
      RETURN                                                            ORB18270
      END                                                               ORB18280
                                                                        ORB18290
************************************************************************ORB18300
                                                                        ORB18310
      SUBROUTINE RMA(E,MM,R,A,TA,DMM,FR,FS,DMA,T)                       ORB18320
*     THIS SUBROUTINE CALCULATES THE RATE OF CHANGE OF THE MEAN Anomaly ORB18330
                                                                        ORB18340
      DOUBLE PRECISION  DMAA,DMAB,DMAC,DMAD,DMM,FR,FS,DMA,E,MM,R,A,TA,  ORB18350
     +   ET,T                                                           ORB18360
                                                                        ORB18370
*     TRAP (E) SO DENOMINATOR DOES NOT GOTO ZERO                        ORB18380
      IF (E.GT.0.9) THEN                                                ORB18390
         ET = 0.9                                                       ORB18400
      ELSEIF (E.LT.0.1) THEN                                            ORB18410
```

```
         ET = 0.1
      ELSE
         ET = E
      ENDIF
      DMA  = (-1.0D+00/(MM *A))*
     +       (((2.0D+00*R)/A) - ((1 - (E **2))/ET)*DCOS(TA)) * FR -
     +    (1-(E **2))/(MM *A*ET)*(1+ R/(A*(1-(E**2))))*(SIN(TA)*FS)-
     +       (T * DMM)
      RETURN
      END

************************************************************************
* CALCULATE THE NEW ORBITAL ELEMENTS
************************************************************************

      SUBROUTINE NSMA(A,DT,DA,NEWA)
*     THIS SUBROUTINE CALCULATES THE NEW SEMI-MAJOR AXIS

      DOUBLE PRECISION DA,DT,A,NEWA

      NEWA = A + DA*DT
      RETURN
      END

************************************************************************

      SUBROUTINE NECC(E,DT,DE,NEWE)
*     THIS SUBROUTINE CALCULATES THE NEW ECCENTRICITY

      DOUBLE PRECISION DE,DT,E,NEWE

      NEWE = E + DE*DT
      RETURN
      END

************************************************************************

      SUBROUTINE NINCL(I,DT,DI,NEWI)
*     THIS SUBROUTINE CALCULATES THE NEW INCLINATION

      DOUBLE PRECISION DI,DT,I,NEWI

      NEWI = I + DI*DT
      RETURN
      END

************************************************************************

      SUBROUTINE NASNOD(LAN,DT,DLAN,NEWLAN)
*     THIS SUBROUTINE CALCULATES THE NEW LONGITUDE OF THE ASCENDING NODE

      DOUBLE PRECISION DLAN,DT,LAN,NEWLAN

      NEWLAN = LAN + DLAN*DT
      RETURN
      END
```

```
************************************************************************

      SUBROUTINE NARPER(AP,DT,DAP,NEWAP)
*     THIS SUBROUTINE CALCULATES THE NEW ARGUMENT OF PERIGEE

      DOUBLE PRECISION DAP,DT,AP,NEWAP

      NEWAP = AP + DAP*DT
      RETURN
      END

**********************************************************************

      SUBROUTINE NMNAN(MA,MM,DT,TF,DMA,PI)
*     THIS SUBROUTINE CALCULATES THE NEW MEAN Anomaly

      DOUBLE PRECISION  DMM,FR,FS,DMA,DT,MA,E,R,A,TA,MM,TF,T,PI

      MA = MM*(TF) + DMA*DT
      IF (MA .GT. (2*PI)) THEN
         MA = MA - (2*PI)
      ENDIF
      RETURN
      END

***********************************************************************

      SUBROUTINE NMNMO(MM,DMM,DT,NEWMM)
*     THIS SUBROUTINE CALCULATE THE NEW MEAN MOTION

      DOUBLE PRECISION  DMM,DT,MM,NEWMM

      NEWMM = MM + DMM*DT
      RETURN
      END

************************************************************************

      SUBROUTINE NEA(MA,E,EA)
*     THIS SUBROUTINE CALCULATES THE NEW ECCENTRIC ANOMOLY BY USING
*     NEWTONS METHOD OF ROOT FINDING

      DOUBLE PRECISION  EAN,MAN,MA,E,EA,DIFF

*     LET (EA) EQUAL (MA) FOR INITIAL GUESS AT ROOT
      EA = MA
      EAN = EA + (MA - EA +E*DSIN(EA))/(1.0D+00 - E*DCOS(EA))
      MAN = EAN - E*SIN(EAN)

*     CHECK DIFFERENCE (DIFF)
      DIFF = ABS(MA -MAN)
      EA = EAN

*     CONTINUE TO INTERATE UNTIL DIFFERENCE IS NEGLIGIBLE
 200  IF(DIFF.GT.0.0000000001) THEN
```

```
           EAN = EA + (MA - EA + E*DSIN(EA))/(1.0D+00 - E*DCOS(EA))   ORB19540
           MAN = EAN - E*DSIN(EAN)                                    ORB19550
           EA = EAN                                                   ORB19560
           DIFF = ABS(MA - MAN)                                       ORB19570
           GOTO 200                                                   ORB19580
        ENDIF                                                         ORB19590
        EA = EAN                                                      ORB19600
        RETURN                                                        ORB19610
        END                                                           ORB19620
                                                                      ORB19630
****************************************************************      ORB19640
                                                                      ORB19650
        SUBROUTINE NTA(EA,E,TA,PI)                                    ORB19660
*       THIS SUBROUTINE CALCULATES THE NEW TRUE Anomaly               ORB19670
                                                                      ORB19680
        DOUBLE PRECISION EA,E,TA,PI                                   ORB19690
                                                                      ORB19700
        TA = DACOS((E - DCOS(EA))/(E*DCOS(EA) - 1.0D+00))             ORB19710
        IF (EA.GT.PI) THEN                                            ORB19720
           TA = (2*PI) - TA                                           ORB19730
        ENDIF                                                         ORB19740
        RETURN                                                        ORB19750
        END                                                           ORB19760
                                                                      ORB19770
*******************************************************************   ORB19780
                                                                      ORB19790
        SUBROUTINE NANGMO(H,DT,DH,NEWH)                               ORB19800
*       THIS SUBROUTINE CALCULATES THE NEW ANGULAR MOMENTUM           ORB19810
                                                                      ORB19820
        DOUBLE PRECISION DH,DT,H,NEWH                                 ORB19830
                                                                      ORB19840
        NEWH = H + DH*DT                                              ORB19850
        RETURN                                                        ORB19860
        END                                                           ORB19870
                                                                      ORB19880
****************************************************************      ORB19890
                                                                      ORB19900
        SUBROUTINE INTSUM(TFEA,TFSU,TFMO,TFDRA,TDI,TDA,TDE,TDMM,TDMA, ORB19910
      +                  TDLAN,TDH,TDAP)                              ORB19920
*       THIS SUBROUTINE INITIALIZES THE SUMS OF FORCES AND ELEMENT CHANGESORB19930
                                                                      ORB19940
           TFEA = 0.0                                                 ORB19950
           TFSU = 0.0                                                 ORB19960
           TFMO = 0.0                                                 ORB19970
           TFDRA = 0.0                                                ORB19980
           TDI = 0.0                                                  ORB19990
           TDA = 0.0                                                  ORB20000
           TDE = 0.0                                                  ORB20010
           TDMM = 0.0                                                 ORB20020
           TDMA = 0.0                                                 ORB20030
           TDLAN = 0.0                                                ORB20040
           TDH = 0.0                                                  ORB20050
           TDAP = 0.0                                                 ORB20060
        RETURN                                                        ORB20070
        END                                                           ORB20080
```

```
                                                                        ORB20090
************************************************************************ORB20100
* CALCULATE THE NEW POSITION AND VELOCITY VECTORS                       ORB20110
************************************************************************ORB20120
                                                                        ORB20130
      SUBROUTINE NPOS(RI,RJ,RK,R,LAN,AP,INC, TA,A,E)                    ORB20140
*     THIS SUBROUTINE CALCULATES THE NEW POSITION VECTOR                ORB20150
                                                                        ORB20160
      DOUBLE PRECISION XW,YW,ZW,INC,RI,RJ,RK,R,LAN,AP,TA,A,E            ORB20170
                                                                        ORB20180
*     CALCULATE POSITION VECTOR IN 'PQW' SYSTEM                         ORB20190
      R = (A*(1 - (E**2)))/(1 + E*DCOS(TA))                             ORB20200
      XW = R*DCOS(TA)                                                   ORB20210
      YW = R*DSIN(TA)                                                   ORB20220
      ZW = 0                                                            ORB20230
                                                                        ORB20240
*     TRANSFORM POSITION TO 'IJK' SYSTEM                                ORB20250
      CALL PQWIJK(LAN,AP,INC,XW,YW,ZW,RI,RJ,RK)                         ORB20260
      R = DSQRT((RI**2) + (RJ**2) + (RK**2))                            ORB20270
      RETURN                                                            ORB20280
      END                                                               ORB20290
                                                                        ORB20300
************************************************************************ORB20310
                                                                        ORB20320
      SUBROUTINE NVEL(E,P,TA,LAN,AP,INC,VI,VJ,VK,V,MU)                  ORB20330
*     THIS SUBROUTINE CALCULATES THE NEW VELOCITY VECTOR                ORB20340
                                                                        ORB20350
      DOUBLE PRECISION  INC,VP,VQ,VW,MU,E,P,TA,LAN,AP,VI,VJ,VK,V        ORB20360
                                                                        ORB20370
*     CALCULATE VELOCITY IN 'PQW' SYSTEM                                ORB20380
      VP = DSQRT(MU/P)*(-DSIN(TA))                                      ORB20390
      VQ = DSQRT(MU/P)*(E + DCOS(TA))                                   ORB20400
      VW = 0.0D+00                                                      ORB20410
                                                                        ORB20420
*     TRANSFORM VELOCITY INTO 'IJK' SYSTEM                              ORB20430
      CALL PQWIJK(LAN,AP,INC,VP,VQ,VW,VI,VJ,VK)                         ORB20440
      V = DSQRT((VI**2) + (VJ**2) +(VK**2))                             ORB20450
      RETURN                                                            ORB20460
      END                                                               ORB20470
                                                                        ORB20480
************************************************************************ORB20490
*   VELOCITY CHANGE                                                     ORB20500
************************************************************************ORB20510
                                                                        ORB20520
      SUBROUTINE CHGVEL(DT,PER,AL,LAN,AP,I,RI,RJ,RK,R,                  ORB20530
     +   VI,VJ,VK,V,MU,PI,H,A,E,N,TA,P,MM,MA,EA,                        ORB20540
     +   TF,T,NUM,RIRAY,RJRAY,RKRAY,RARAY,ZARAY,AINRAY,APRAY,TIMRAY,    ORB20550
     +   TT,EI,EJ,EK,LP,HI,HJ,IOPT1,TFEA,TFSU,TFMO,TFDRA,               ORB20560
     +    TDI,TDA,TDE,TDMM,TDMA,TDLAN,TDH,TDAP)                         ORB20570
*     THIS SUBROUTINE CALCULATE VELOCITY CHANGES                        ORB20580
                                                                        ORB20590
* THE FOLLOWING SUBROUTINES ARE CALLED:                                 ORB20600
*     TACHG  = RETURNS TRUE ANOMALY FOR VELOCITY CHANGE LOCATION (CHTA) ORB20610
*              AND AN INDICATOR OF LOCATION (ITA)                       ORB20620
*     CALCEL = CALCULATE Orbital ELEMENTS                               ORB20630
*     UNPRET = CALCULATE UNPERTURBED ORBIT                              ORB20640
```

```
*     NPOS   = CALCULATE NEW POSITION
*     NVEL   = CALCULATE NEW VELOCITY
*     STORE  = STORE POSITION AND ELEMENTS IN ARRAYS
*     ENERGY = ENERGY OF SATELLITE
*     ECC    = ECCENTRICITY
*     SMAXIS = SEMI-MAJOR AXIS

      DOUBLE PRECISION T,DT,PER,AL,LAN,AP,I,RI,RJ,RK,R,VI,VJ,VK,V,
     +                   MU,PI,H,A,E,N,TA,P,MM,MA,EA,TF,TT,
     +   NEWVI,NEWVJ,NEWVK,NEWV,VMAX,CHNTA,EI,EJ,EK,LP,HI,HJ,VCIR,
     +   DI,DE,DA,DMM,DMA,DLAN,DH,DAP,NEWEI,NEWEJ,NEWEK,NEWE,NEWENR,
     +   NEWA,NEWRP,RE

      DIMENSION  RARAY(500),TARAY(500),RIRAY(500),RJRAY(500),
     +           RKRAY(500),AINRAY(500),APRAY(500),TIMRAY(500)

      CHARACTER*1,YORN,PYORN

      RE = 6.3782D+03

*     PROMPT THE USER FOR THE VELOCITY Change LOCATION
      CALL TACNG(PI,CHTA,ITA)

*     SET TIME COUNTER TO ONE TIME STEP
      T = DT

*     ROTATE TO THE VELOCITY CHANGE LOCATION
*     THIS IS IDENTICAL TO THE Unperturbed ORBIT WITH THE EXCEPTION
*     THAT A COMPLETE ORBIT IS NOT CALCULATED
      PRINT*,'ROTATE TO VELOCITY CHANGE LOCATION'
      IF ((ITA.EQ.2) .OR. (ITA.EQ.3)) THEN
         PRINT*,'BEFORE TA =',TA
         IF (TA .GT. 6.21) THEN
            TA = TA - (2*PI)
         ENDIF
 250     IF((T.LE.PER).AND.(TA.LT.CHTA)) THEN
*           PRINT*,'TA =',TA
            NUM = NUM + 1
            TT = TT + DT
                CALL NEWELT(MM,MA,E,EA,TA,TF,DT,PI,PER)
            CALL NPOS(RI,RJ,RK,R,LAN,AP,I, TA,A,E)
            CALL NVEL(E,P,TA,LAN,AP,I,VI,VJ,VK,V,MU)
            CALL STORE(RI,RJ,RK,R,TA,RIRAY,RJRAY,RKRAY,RARAY,
     +                TARAY,NUM,I,AP,AINRAY,APRAY,TT,TIMRAY)
            T = T + DT
            GOTO 250
         ENDIF
         IF (TF .GE. PER) THEN
            TF = TF - PER
         ENDIF
      ENDIF

*     PRINT ESCAPE VELOCITY AND CIRCULAR VELOCITY FOR Reference
      CALL EXCMS('CLRSCRN')
      PRINT*,'AFTER TA =',TA
      PRINT*,'THIS SHOULD BE THE DESIRED RADIUS RP OR RA'
```

```
 260  PRINT*,'RADIUS =',R
      PRINT*,'VELOCITY =',V
      VMAX = DSQRT(2.0*(MU / R))
      PRINT*,'MAX VELCCITY AT THIS RADIUS IS:',VMAX
      VCIR = DSQRT(MU/R)
      PRINT*,'CIRCULAR VELOCITY AT THIS RADIUS IS :',VCIR

*     PROMPT USER TO CHANGE VELOCITY IN ORBITAL PLANE
      PRINT*,'DO YOU WANT TO CHANGE THE VELOCITY IN THE ORBITAL PLANE?'
      PRINT*,'ENTER "Y" OR "N" :'
      READ*,PYORN
      PRINT*,PYORN
      IF (PYORN .EQ. 'Y') THEN
         PRINT*,'GIVE THE TOTAL CHANGE IN VELOCITY, I.E. 5.0 KM.'
         PRINT*,'THE PROGRAM WILL FIGURE OUT THE FINAL VELOCITY VECTOR'
         PRINT*,'  ENTER VELOCITY CHANGE:'
         READ*,CHGV
         PRINT*,CHGV

*        CALCULATE NEW VELOCITY FOR CHANGE IN THE ORBITAL PLANE
         NEWVI = VI + (CHGV * VI / V)
         NEWVJ = VJ + (CHGV * VJ / V)
         NEWVK = VK + (CHGV * VK / V)

*     Velocity CHANGE OUT OF ORBITAL PLANE
      ELSEIF (PYORN .EQ. 'N') THEN
         PRINT*,' ENTER THE NEW VELOCITY VECTOR:'
         PRINT*,' ENTER THE NEW VI'
         READ*,NEWVI
         PRINT*,NEWVI
         PRINT*,' ENTER THE NEW VJ'
         READ*,NEWVJ
         PRINT*,NEWVJ
         PRINT*,' ENTER THE NEW VK'
         READ*,NEWVK
         PRINT*,NEWVK
         NUM = 1
         ITA = 3
      ELSE
         CALL EXCMS('GLRSCRN')
         GOTO 260
      ENDIF

*     PRINT NEW VELOCITY FOR USER TO CHECK
      NEWV = DSQRT((NEWVI**2) + (NEWVJ**2) + (NEWVK**2))
      PRINT*,'NEW VI =',NEWVI
      PRINT*,'NEW VJ =',NEWVJ
      PRINT*,'NEW VK =',NEWVK
      PRINT*,'NEW V  =',NEWV
      PRINT*,' ARE THESE VALUES THE ONES YOU WANT?'
      PRINT*,'ENTER "Y" OR "N" :'
      READ*,YORN
      PRINT*,YORN
      IF (YORN  .EQ. 'N') THEN
         CALL EXCMS('CLRSCRN')
         GOTO 260
```

```
      ENDIF

*     CHECK FOR VALID VELOCITY
      IF ( NEWV .GT. VMAX) THEN
         PRINT*,'YOUR VELOCITY IS TO GREAT !!'
         GOTO 260
      ENDIF

*     Calculate PERIGEE RADIUS TO SEE IF SATELLITE WILL IMPACT EARTH
      CALL ENERGY(NEWV,R,MU,NEWENR)
      CALL ECC(RI,RJ,RK,R,NEWVI,NEWVJ,NEWVK,NEWV,NEWEI,NEWEJ,NEWEK,
     +         NEWE,MU)
      CALL SMAXIS(MU,NEWENR,NEWA)
      NEWRP = NEWA*(1.0 - NEWE)
      IF (NEWRP .LE. RE) THEN
         PRINT*,'YOUR VELOCITY AT THIS POINT IS TO SMALL!!!'
         PRINT*,'THE SATELLITE WILL IMPACT THE EARTH!!'
         PRINT*,'THE SATELLITES RADIUS OF PERIGEE WOULD BE ',NEWRP
         PRINT*,'A NEW VELOCITY WILL HAVE TO BE ENTERED!!'
         GOTO 260
      ENDIF

*     ACCEPT NEW VELOCITY
      VI = NEWVI
      VJ = NEWVJ
      VK = NEWVK
      V = NEWV

*     CALCULATE NEW ELEMENT WITH NEW VELOCITY AND SET TIME STEP
      CALL CALCEL( RI,RJ,RK,R,VI,VJ,VK,V,EI,EJ,EK,E,A,I,LAN,LP,TA,
     +                PER,EA,MA,AP,AL,TF,P,PI,MU,MM,N,H,HI,HJ)
      DT = PER/50.0
      T = DT

*     THE FOUR Different CASES OF VELOCITY CHANGES FOLLOWS:

*     VELOCITY CHANGE AT PERIGEE, AND NEWV > V Circular
      IF((ITA.EQ.1).AND.(NEWV.GT.VCIR))THEN
         CALL UNPRET(DT,PER,AL,LAN,AP,I,RI,RJ,RK,R,VI,VJ,VK,V,
     +                                  MU,PI,H,A,E,N,TA,P,MM,
     +                  MA,EA,TF,T,NUM,RIRAY,RJRAY,RKRAY,RARAY,
     +                  TARAY,AINRAY,APRAY,TIMRAY,TT)

*     Change VELOCITY AT PERIGEE, AND NEWV <= V CIRCULAR
*     APOGEE AND PERIGEE SWAP
      ELSEIF ((ITA.EQ.1).AND.(NEWV.LE.VCIR))THEN

*        CLEAR PREVIOUS PLOTS
         NUM = 1
         CALL STORE(RI,RJ,RK,R,TA,RIRAY,RJRAY,RKRAY,RARAY,TARAY,
     +              NUM,I,AP,AINRAY,APRAY,TT,TIMRAY)
         T = PER/2

*        STEP SATELLITE TO NEW PERIGEE, ONLY A HALF ORBIT
         CALL UNPRET(DT,PER,AL,LAN,AP,I,RI,RJ,RK,R,VI,VJ,VK,V,
     +                                  MU,PI,H,A,E,N,TA,P,MM,
```

```
     +                    MA,EA,TF,T,NUM,RIRAY,RJRAY,RKRAY,RARAY,
     +                    TARAY,AINRAY,APRAY,TIMRAY,TT)

*         RESET TIME COUNTER TO ONE TIME STEP
          T = DT

*         CALCULATE COMPLETE NEXT ORBIT
          CALL UNPRET(DT,PER,AL,LAN,AP,I,RI,RJ,RK,R,VI,VJ,VK,V,
     +                                   MU,PI,H,A,E,N,TA,P,MM,
     +                    MA,EA,TF,T,NUM,RIRAY,RJRAY,RKRAY,RARAY,
     +                    TARAY,AINRAY,APRAY,TIMRAY,TT)

*      CHANGE VELOCITY AT APOGEE, AND NEW V < V CIRCULAR
       ELSEIF ((ITA.EQ.2) .AND.(NEWV .LT.  VCIR)) THEN

          T = PER/2

*         FINISH ORBIT
          CALL UNPRET(DT,PER,AL,LAN,AP,I,RI,RJ,RK,R,VI,VJ,VK,V,
     +                                   MU,PI,H,A,E,N,TA,P,MM,
     +                    MA,EA,TF,T,NUM,RIRAY,RJRAY,RKRAY,RARAY,
     +                    TARAY,AINRAY,APRAY,TIMRAY,TT)

*      CHANGE VELOCITY AT Apogee, AND NEWV >= V CIRCULAR
*      OR AT ANY OTHER TRUE Anomaly
       ELSEIF (((ITA.EQ.2).AND.(NEWV.GE.VCIR)) .OR. (ITA.EQ.3)) THEN

          IF (TA .GT.  6.21) THEN
              TA = TA - (2*PI)
          ENDIF

*         CLEAR PREVIOUS ORBITS AND STEP SATELLITE TO NEW PERIGEE
          T = TF
          NUM = 1
          CALL STORE(RI,RJ,RK,R,TA,RIRAY,RJRAY,RKRAY,RARAY,TARAY,
     +               NUM,I,AP,AINRAY,APRAY,TT,TIMRAY)
          CALL UNPRET(DT,PER,AL,LAN,AP,I,RI,RJ,RK,R,VI,VJ,VK,V,
     +                                   MU,PI,H,A,E,N,TA,P,MM,
     +                    MA,EA,TF,T,NUM,RIRAY,RJRAY,RKRAY,RARAY,
     +                    TARAY,AINRAY,APRAY,TIMRAY,TT)
          IF (TF .GE.  PER) THEN
              TF = TF - PER
          ENDIF

*         CALCULATE COMPLETE NEXT ORBIT
          T = DT
          CALL UNPRET(DT,PER,AL,LAN,AP,I,RI,RJ,RK,R,VI,VJ,VK,V,
     +                                   MU,PI,H,A,E,N,TA,P,MM,
     +                    MA,EA,TF,T,NUM,RIRAY,RJRAY,RKRAY,RARAY,
     +                    TARAY,AINRAY,APRAY,TIMRAY,TT)
       ENDIF
       RETURN
       END

*************************************************************************
```

```
      SUBROUTINE TACNG(PI,CHTA,ITA)
*     THIS SUBROUTINE Asks THE USER FOR VELOCITY CHANGE LOCATION

      DOUBLE PRECISION CHTA,PI

      CALL EXCMS('CLRSCRN')
      PRINT*,'WHERE DO YOU WANT TO CHANGE THE VELOCITY?'
      PRINT*,'    1. AT CURRENT PERIGEE'
      PRINT*,'    2. AT CURRENT Apogee'
      PRINT*,'    3. AT A SPECIFIC TRUE Anomaly'
      PRINT*,'ENTER "1", "2" OR "3"'
      READ*,ITA
      PRINT*,ITA

*     SET TRUE ANOMALY CHANGE LOCATION (CHTA) TO DESIRED LOCATION
      IF (ITA .EQ. 1) THEN
         CHTA = 0.0
      ENDIF
      IF (ITA .EQ. 2) THEN
         CHTA = PI
      ENDIF
      IF (ITA .EQ. 3) THEN
         PRINT*,'AT WHAT TRUE ANOMALY DO YOU WANT TO CHANGE THE'
         PRINT*,'VELOCITY?'
         PRINT*,'ENTER TRUE ANOMALY IN DEGREES'
         READ*,CHTA
         PRINT*,CHTA
         CHTA = CHTA * PI / 180
      ENDIF
      RETURN
      END

*********************************************************************
*    OUTPUT PLOTS
*********************************************************************

      SUBROUTINE PLOTS(RIRAY,RJRAY,RKRAY,RARAY,TARAY,NUM,PI,INC,LP,A,
     +                 E,TF,AINRAY,APRAY,TIMRAY,TFEA,TFSU,TFMO,TFDRA,
     +                 PER,TDI.TDA,TDE,TDMM,TDMA,TDLAN,TDH,TDAP,
     +                 MM,MA,LAN,H,AP,R,V)

*     THIS SUBROUTINE ASKS THE USER FOR THE TYPE OF OUTPUT THAT IS
*     DESIRED PERIFOCAL, GROUND TRACK OR TO SKIP THE PLOT.

*     THE FOLLOWING SUBROUTINES ARE CALLED:
*     PERIF  = PLOT PERIFOCAL ORBIT
*     GRTRK  = PLOT GROUND TRACK
*     DATE   = DISPLAYS DATA ON PLOT
*     TEC618 = SET Disspla TO TEC 618 OUTPUT
*     ENDPL  = END THIS DISSPLA PLOT
*     REFER TO DISSPLA USER"S MANUAL FOR EXPLANATION OF DISSPLA
*     SUBROUTINES

      DOUBLE PRECISION PI,A,E,INC,LP,TF,PER,MM,MA,LAN,H,AP,R,V

      DIMENSION RIRAY(500),RJRAY(500),RKRAY(500),RARAY(500),TARAY(500),
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```
     +    AINRAY(500),APRAY(500),TIMRAY(500)                            ORB23450
      CHARACTER*1,YORN                                                  ORB23460
                                                                        ORB23470
      CALL EXCMS('CLRSCRN')                                             ORB23480
                                                                        ORB23490
* CALCULATE SINGLE PRECISION VARIABLES                                  ORB23500
      SPI = SNGL(PI)                                                    ORB23510
      SA = SNGL(A)                                                      ORB23520
      SE = SNGL(E)                                                      ORB23530
      SINC = SNGL(INC)                                                  ORB23540
      SLP = SNGL(LP)                                                    ORB23550
      STF = SNGL(TF)                                                    ORB23560
      SPER = SNGL(PER)                                                  ORB23570
      SMM = SNGL(MM)                                                    ORB23580
      SMA = SNGL(MA)                                                    ORB23590
      SLAN = SNGL(LAN)                                                  ORB23600
      SH = SNGL(H)                                                      ORB23610
      SAP = SNGL(AP)                                                    ORB23620
      SV = SNGL(V)                                                      ORB23630
      SR = SNGL(R)                                                      ORB23640
                                                                        ORB23650
*     PROMPT USER FOR DISPLAY TYPE                                      ORB23660
 340  PRINT*,'WHAT TYPE OF Display IS DESIRED:'                         ORB23670
      PRINT*,'    1. PERIFOCAL'                                         ORB23680
      PRINT*,'    2. GROUND TRACK'                                      ORB23690
      PRINT*,'    3. SKIP PLOT'                                         ORB23700
      PRINT*,'ENTER 1,2,3,4:'                                           ORB23710
      READ*,INPUT                                                       ORB23720
      PRINT350,INPUT                                                    ORB23730
 350  FORMAT(I4)                                                        ORB23740
                                                                        ORB23750
      CALL TEK618                                                       ORB23760
                                                                        ORB23770
*     CALL APPROPRIATE PLOT                                             ORB23780
      IF (INPUT .EQ. 1) THEN                                            ORB23790
         CALL PERIF(RARAY,TARAY,NUM,SPI,SINC,SLP,SA,SE)                 ORB23800
      ELSEIF (INPUT .EQ. 2) THEN                                        ORB23810
         CALL GRTRK(AINRAY,APRAY,TARAY,STF,NUM,TIMRAY)                  ORB23820
      ELSEIF (INPUT .EQ. 3) THEN                                        ORB23830
         GOTO 360                                                       ORB23840
      ELSE                                                              ORB23850
         PRINT*,'INVALID ENTRY!'                                        ORB23860
         GOTO 340                                                       ORB23870
      ENDIF                                                             ORB23880
                                                                        ORB23890
*     DISPLAY DATA                                                      ORB23900
      CALL DATA(SINC,SA,SE,TFEA,TFSU,TFMO,TFDRA,SPER,SPI,TDI,TDA,TDE,   ORB23910
     +          TDMM,TDMA,TDLAN,TDH,TDAP,SMM,SMA,SLAN,SH,SAP,SV,SR)     ORB23920
      CALL ENDPL(0)                                                     ORB23930
                                                                        ORB23940
*     PROMPT USER IF ANOTHER DISPLAY TYPE IS DESIRED                    ORB23950
      PRINT*,'WOULD YOU LIKE ANOTHER PLOT USING THE SAME ORBITAL'       ORB23960
      PRINT*,'PARAMETERS AND DATA:'                                     ORB23970
      PRINT*,'ENTER "Y" OR "N" :'                                       ORB23980
      READ*,YORN                                                        ORB23990
      PRINT*,YORN                                                       ORB24000
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      IF (YORN .EQ. 'Y') THEN                                           ORB24010
         GOTO 340                                                       ORB24020
      ENDIF                                                             ORB24030
 360  RETURN                                                            ORB24040
      END                                                               ORB24050
                                                                        ORB24060
************************************************************            ORB24070
                                                                        ORB24080
      SUBROUTINE PERIF(RARAY,TARAY,NUM,PI,INC,LP,A,E)                   ORB24090
*     THIS SUBROUTINE PLOTS OUT THE RESULTS OF THE PROGRAM USING THE    ORB24100
*     DISPLAY FEATURE ON THE MAIN FRAME.                                ORB24110
*     REFER TO DISSPLA USERS GUIDE FOR EXPLANATION OF DISSPLA           ORB24120
*     SUBROUTINES.                                                      ORB24130
                                                                        ORB24140
      REAL INC,LP                                                       ORB24150
      DIMENSION TARAY(500),RARAY(500),RIRAY(500),RJRAY(500),RKRAY(500)  ORB24160
                                                                        ORB24170
      I = 1                                                             ORB24180
                                                                        ORB24190
*     SET SCALE OF AXIS                                                 ORB24200
      RSTEP = (A*(1+E)) / 3                                             ORB24210
      CALL TEK618                                                       ORB24220
      CALL RESET(3HALL)                                                 ORB24230
      CALL SCMPLX                                                       ORB24240
      CALL PHYSOR(1.25,4.)                                              ORB24250
      CALL AREA2D(6.,6.)                                                ORB24260
      CALL MESSAG('PERIFOCAL COORDINATE SYSTEM$',100,1.0,6.5)           ORB24270
      CALL XNAME('XW',2)                                                ORB24280
      CALL YNAME('YW',2)                                                ORB24290
      CALL XAXANG(90.0)                                                 ORB24300
      CALL YAXANG(0.0)                                                  ORB24310
      CALL INTAXS                                                       ORB24320
      CALL POLAR(1.,RSTEP,3.,3.)                                        ORB24330
      CALL POLY3                                                        ORB24340
      CALL NOCHEK                                                       ORB24350
      CALL CURVE(TARAY,RARAY,NUM,1)                                     ORB24360
      CALL COMPLX                                                       ORB24370
      CALL HEIGHT(.2)                                                   ORB24380
      CALL RESET('COMPLEX')                                             ORB24390
      CALL RESET('HEIGHT')                                              ORB24400
      CALL ENDGR(0)                                                     ORB24410
                                                                        ORB24420
*     Display  EARTH PLOT                                               ORB24430
      CALL EARTH1(A,E,INC,PI,RSTEP)                                     ORB24440
      RETURN                                                            ORB24450
      END                                                               ORB24460
                                                                        ORB24470
************************************************                        ORB24480
                                                                        ORB24490
      SUBROUTINE EARTH1(A,E,INC,PI,RSTEP)                               ORB24500
*     THIS SUBROUTINE PLOTS A VIEW OF THE WORLD, LOOKING DOWN THE 'Z'   ORB24510
*     AXIS, PLACED ON THE ORIGIN.  THE Latitude IS FIXED, BUT THE       ORB24520
*     LONGITUDE VARIES WITH THE INCLINATION.                            ORB24530
*     REFER TO DISSPLA USER"S MANUAL FOR EXPLANATION OF DISSPLA         ORB24540
*     SUBROUTINES                                                       ORB24550
                                                                        ORB24560
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      REAL INC
      COMMON IWORK(3800)
      DATA IWDIM/3800/

      RE = 6378.145

*     SCALE THE EARTH PLOT AND CENTER ON THE ORIGIN
      SCFAC = RE/RSTEP
      SCFAC2 = SCFAC * 2.0
      XPHS = 1.25 + 3.0 -  SCFAC
      YPHS = 4.0 + 3.0 -  SCFAC
      YPOLE = 90 - (INC * 180 / PI)
      IF(YPOLE .GT. 90) THEN
         YPOLE = YPOLE - 90
      ENDIF
      YORIG = YPOLE - 90
      YMAX = YPOLE + 90
      CALL RESET(3HALL)
      CALL PHYSOR(XPHS,YPHS)
      CALL PROJCT('LAMBERT EQ/AREA')
      CALL MAPOLE(0.0,YPOLE)
      CALL AREA2D (SCFAC2,SCFAC2)
      CALL THKFRM (0.02)
      CALL GRAF(-90.,30.,90.,YORIG,30.,YMAX)
      CALL FRAME
      CALL MAPFIL('MAPDTA')
      CALL LBLANK('LAND',IWDIM)
      CALL GRID(1,1)
      CALL LBLANK('WATER',IWDIM)
      CALL DASH
      CALL GRID(1,1)
      CALL RESET ('DASH')
      CALL ENDGR(0)
      RETURN
      END

************************************************************************

      SUBROUTINE GRTRK(AINRAY,APRAY,TARAY,TF,NUM,TIMRAY)

      DIMENSION AINRAY(500),APRAY(500),TARAY(500),
     +   ELARAY(500),ELORAY(500),TLONG(500),TLAT(500),TIMRAY(500)

      RE = 6.3782E+03
      EROT = 7.292115856E-05
      STF = (TF)
      I = 1

*     LOAD ARRAYS WITH LATITUDE AND LONGITUDE
 410  IF (I .LE. NUM) THEN
         X = RE*COS(APRAY(I))*COS(TARAY(I))-RE*SIN(APRAY(I))*
     +        SIN(TARAY(I))
         Y = RE*COS(AINRAY(I))*SIN(APRAY(I))*COS(TARAY(I)) +
     +       RE*COS(AINRAY(I))*COS(APRAY(I))*SIN(TARAY(I))
         Z = RE*SIN(AINRAY(I))*SIN(APRAY(I))*COS(TARAY(I)) +
     +       RE*SIN(AINRAY(I))*COS(APRAY(I))*SIN(TARAY(I))
```

```
*        CALCULATE LATITUDE
         ELARAY(I) = (ASIN(Z/RE)) * (180/3.14159)

*        TRAP 'X' AND 'Y' FOR ARCTAN IN CALCULATING LONGITUDE
         IF((Y .LE. 10) .AND. (Y .GE. 0.0)) THEN
              Y = 10.
         ELSEIF ((Y .GE.-10).AND.(Y .LE. 0.0)) THEN
              Y = -10.
         ENDIF
         IF((X .LE. 10) .AND. (X .GE. 0.0)) THEN
              X = 10.
         ELSEIF ((X .GE.-10).AND.(X .LE. 0.0)) THEN
              X = -10.
         ENDIF

*        CALCULATE LONGITUDE
         ELORAY(I) = (ATAN2(Y,X) - (EROT*TIMRAY(I))) * (180/3.14159)

*        MODIFY LONGITUDES TO ( -180 TO 180)
 420     IF (ELORAY(I) .LT. -180) THEN
              ELORAY(I) = ELORAY(I) + 360
              GOTO 420
         ENDIF
         I = I + 1
         GOTO 410
      ENDIF

*     SET DISSPLA
      CALL TEK618
      CALL RESET(3HALL)
      CALL YAXANG (0.)
      CALL PHYSOR(1.0,6.0)
      CALL XNAME(' ',1)
      CALL YNAME(' ',1)
      CALL AREA2D(7.5,3.75)
      CALL HEADIN ('GROUND TRACKS',100,1.5,1)
      CALL SCMPLX
      CALL MAPGR(-180.,90.,180.,-90.,30.,90.)
      CALL GRID (1,1)
      CALL MAPFIL ('MAPDTA')
      I = 1

*     IGNORE Boundary POINTS
 430  IF ((ELORAY(I) .LT. -175) .OR.
     +    (ELORAY(I) .GT. 175) .OR.
     +    (ELARAY(I) .LT. -85) .OR.
     +    (ELARAY(I) .GT. 85)) THEN
         I = I + 1
         GOTO 430
      ENDIF

      ITEMP = 1

*     LOAD FIRST POINT OF NEW PLOT SEGMENT
      IF (I .LE. NUM) THEN
```

```
         TLONG(ITEMP) = ELORAY(I)                                       ORB25690
         TLAT(ITEMP) = ELARAY(I)                                        ORB25700
         I = I + 1                                                      ORB25710
*        IF ( I .GE. NUM) THEN                                          ORB25720
*             CALL POLY3                                                ORB25730
*             CALL CURVE(TLONG,TLAT,ITEMP,1)                            ORB25740
*        ENDIF                                                          ORB25750
      ENDIF                                                             ORB25760
                                                                        ORB25770
*     LOAD SECOND POINT IN LINE SEGMENT                                 ORB25780
      IF (I .LE. NUM) THEN                                              ORB25790
         ITEMP = ITEMP + 1                                              ORB25800
         TLONG(ITEMP) = ELORAY(I)                                       ORB25810
         TLAT(ITEMP) = ELARAY(I)                                        ORB25820
         I = I + 1                                                      ORB25830
         IF ( I .GE. NUM) THEN                                          ORB25840
              CALL POLY3                                                ORB25850
              CALL NOCHEK                                               ORB25860
              CALL CURVE(TLONG,TLAT,ITEMP,1)                            ORB25870
         ENDIF                                                          ORB25880
      ENDIF                                                             ORB25890
                                                                        ORB25900
*     LOOP UNTIL SEGMENT REACHES EDGE OR NO MORE POINTS                 ORB25910
 440  IF (I .LE. NUM) THEN                                              ORB25920
                                                                        ORB25930
*        BOTH LAT AND LONG INCREASING                                   ORB25940
         IF((ELORAY(I - 2) .LE. ELORAY(I - 1)) .AND.                    ORB25950
     +      (ELARAY(I - 2) .LE. ELARAY(I - 1))) THEN                    ORB25960
              IF((ELORAY(I) .LT. -170) .OR.                             ORB25970
     +           (ELARAY(I) .LT. -80)) THEN                             ORB25980
                   CALL POLY3                                           ORB25990
                   CALL NOCHEK                                          ORB26000
                   CALL CURVE(TLONG,TLAT,ITEMP,1)                       ORB26010
                   GOTO 430                                             ORB26020
              ELSE                                                      ORB26030
                   ITEMP = ITEMP + 1                                    ORB26040
                   TLONG(ITEMP) = ELORAY(I)                             ORB26050
                   TLAT(ITEMP) = ELARAY(I)                              ORB26060
              ENDIF                                                     ORB26070
                                                                        ORB26080
*        BOTH LAT AND LONG DECREASING                                   ORB26090
         ELSEIF((ELORAY(I - 2) .GT. ELORAY(I - 1)) .AND.                ORB26100
     +          (ELARAY(I - 2) .GT. ELARAY(I - 1))) THEN                ORB26110
              IF((ELORAY(I) .GT.  170) .OR.                             ORB26120
     +           (ELARAY(I) .GT.  80)) THEN                             ORB26130
                   CALL POLY3                                           ORB26140
                   CALL NOCHEK                                          ORB26150
                   CALL CURVE(TLONG,TLAT,ITEMP,1)                       ORB26160
                   GOTO 430                                             ORB26170
              ELSE                                                      ORB26180
                   ITEMP = ITEMP + 1                                    ORB26190
                   TLONG(ITEMP) = ELORAY(I)                             ORB26200
                   TLAT(ITEMP) = ELARAY(I)                              ORB26210
              ENDIF                                                     ORB26220
                                                                        ORB26230
*        LAT INCREASING, LONG. DECREASING                               ORB26240
```

```
         ELSEIF((ELORAY(I - 2) .GT. ELORAY(I - 1)) .AND.
     +          (ELARAY(I - 2) .LE. ELARAY(I - 1))) THEN
              IF((ELORAY(I) .GT.  170) .OR.
     +           (ELARAY(I) .LT. -80)) THEN
                   CALL POLY3
                   CALL NOCHEK
                   CALL CURVE(TLONG,TLAT,ITEMP,1)
                   GOTO 430
              ELSE
                   ITEMP = ITEMP + 1
                   TLONG(ITEMP) = ELORAY(I)
                   TLAT(ITEMP) = ELARAY(I)
              ENDIF

*        LAT. DECREASING, LONG. INCREASING
         ELSEIF((ELORAY(I - 2) .LE. ELORAY(I - 1)) .AND.
     +          (ELARAY(I - 2) .GT. ELARAY(I - 1))) THEN
              IF((ELORAY(I) .LT. -170) .OR.
     +           (ELARAY(I) .GT.  80)) THEN
                   CALL POLY3
                   CALL NOCHEK
                   CALL CURVE(TLONG,TLAT,ITEMP,1)
                   GOTO 430
              ELSE
                   ITEMP = ITEMP + 1
                   TLONG(ITEMP) = ELORAY(I)
                   TLAT(ITEMP) = ELARAY(I)
              ENDIF
         ENDIF
         IF( I .EQ. NUM) THEN
              CALL POLY3
              CALL NOCHEK
              CALL CURVE(TLONG,TLAT,ITEMP,1)
         ENDIF
         I = I + 1
         GOTO 440
      ENDIF


      CALL POLY3
      CALL NOCHEK
      CALL CURVE(TLONG,TLAT,ITEMP,1)


      CALL COMPLX
      CALL HEIGHT(.2)
      CALL THKFRM (0.03)
      CALL FRAME
      CALL RESET('COMPLX')
      CALL RESET('HEIGHT')
      CALL ENDGR (0)
      RETURN
      END

*******************************************************************

```

```
      SUBROUTINE DATA(I,A,E,TFEA,TFSU,TFMO,TFDRA,PER,PI,TDI,TDA,TDE,    ORB26810
     +                TDMM,TDMA,TDLAN,TDH,TDAP,MM,MA,LAN,H,AP,V,R)     ORB26820
*     THIS SUBROUTINE Displays THE ORBITAL DATA FOR BOTH THE PERIFOCAL  ORB26830
*     AND THE GROUND TRACK PLOTS.                                       ORB26840
*     REFER TO DISSPLA USER"S MANUAL FOR EXPLANATION OF DISSPLA         ORB26850
*     SUBROUTINES                                                       ORB26860
                                                                        ORB26870
      REAL I,MM,MA,LAN                                                  ORB26880
                                                                        ORB26890
      MU = 3.986012E+05                                                 ORB26900
                                                                        ORB26910
*     CALCULATE THE AVERAGE FORCES FROM THE TOTAL MAGNITUDE OF          ORB26920
*     FORCE CHANGES                                                     ORB26930
      AVGFE = TFEA/50.0                                                 ORB26940
      AVGFS = TFSU / 50.0                                               ORB26950
      AVGFM = TFMO / 50.0                                               ORB26960
      AVGFD = TFDRA / 50.0                                              ORB26970
                                                                        ORB26980
*     CALCULATE ORBITAL ELEMENTS IN Usable UNITS                        ORB26990
      PERH = PER/3600                                                   ORB27000
                                                                        ORB27010
      DI = I * (180.0/PI)                                               ORB27020
      DLAN = LAN * (180.0/PI)                                           ORB27030
      DAP = AP * (180.0/PI)                                             ORB27040
                                                                        ORB27050
*     CALCULATE Average CHANGE IN ELEMENTS FOR ONE PERIOD               ORB27060
      AVGDI = TDI / 50.0                                                ORB27070
      AVGDA = TDA / 50.0                                                ORB27080
      AVGDE = TDE / 50.0                                                ORB27090
      AVGDMM = TDMM / 50.0                                              ORB27100
      AVGDMA = TDMA / 50.0                                              ORB27110
      AVGLAN = TDLAN / 50.0                                             ORB27120
      AVGDH = TDH / 50.0                                                ORB27130
      AVGDAP = TDAP / 50.0                                              ORB27140
                                                                        ORB27150
*     CALCULATE RADIUS'S AND VELOCITIES                                 ORB27160
      ENR = ((V**2)/2) - (MU/R)                                         ORB27170
      RP = A*(1 - E)                                                    ORB27180
      RA = A*(1 + E)                                                    ORB27190
      VP = SQRT(2*(ENR + (MU/RP)))                                      ORB27200
      VA = SQRT(2*(ENR + (MU/RA)))                                      ORB27210
                                                                        ORB27220
                                                                        ORB27230
*     SET DISSPLA                                                       ORB27240
      CALL RESET(3HALL)                                                 ORB27250
      CALL SCMPLX                                                       ORB27260
      CALL PHYSOR(0.0,0.0)                                              ORB27270
      CALL AREA2D(8.5,4.0)                                              ORB27280
                                                                        ORB27290
*     PRINT DATA                                                        ORB27300
      CALL MESSAG('I = $',100,0.25,3.67)                                ORB27310
      CALL REALNO(DI,3,'ABUT','ABUT')                                   ORB27320
      CALL MESSAG(' DEG.$',100,'ABUT','ABUT')                           ORB27330
      CALL MESSAG('  A = $',100,'ABUT','ABUT')                          ORB27340
      CALL REALNO(A,1,'ABUT','ABUT')                                    ORB27350
      CALL MESSAG(' KM$',100,'ABUT','ABUT')                             ORB27360
```

```
      CALL MESSAG(' E = $',100,'ABUT','ABUT')
      CALL REALNO(E,3,'ABUT','ABUT')
      CALL MESSAG('   PER = $',100,'ABUT','ABUT')
      CALL REALNO(PERH,2,'ABUT','ABUT')
      CALL MESSAG(' HOURS$',100,'ABUT','ABUT')

      CALL MESSAG('AVERAGE RATE OF CHANGE OF ELEMENTS PER SECOND $',
     +          100,1.0,3.0)

      CALL MESSAG('DI/DT = $',100,0.25,2.67)
      CALL REALNO(AVGDI,-2,'ABUT','ABUT')
      CALL MESSAG('   DA/DT = $',100,'ABUT','ABUT')
      CALL REALNO(AVGDA,-2,'ABUT','ABUT')
      CALL MESSAG('   DE/DT = $',100,'ABUT','ABUT')
      CALL REALNO(AVGDE,-2,'ABUT','ABUT')

      CALL MESSAG('DMM/DT = $',100,0.25,2.33)
      CALL REALNO(AVGDMM,-2,'ABUT','ABUT')
      CALL MESSAG('   DMA/DT = $',100,'ABUT','ABUT')
      CALL REALNO(AVGDMA,-2,'ABUT','ABUT')
      CALL MESSAG('   DLAN/DT = $',100,'ABUT','ABUT')
      CALL REALNO(AVGLAN,-2,'ABUT','ABUT')

      CALL MESSAG('DH/DT = $',100,0.25,2.00)
      CALL REALNO(AVGDH,-2,'ABUT','ABUT')
      CALL MESSAG('   DAP/DT = $',100,'ABUT','ABUT')
      CALL REALNO(AVGDMA,-2,'ABUT','ABUT')

      CALL MESSAG('AVERAGE MAGNITUDE OF FORCES PER UNIT MASS (KM/S**2)
     +$',100,1.0,1.67)

      CALL MESSAG('EARTH = $',100,0.10,1.33)
      CALL REALNO(AVGFE,-1,'ABUT','ABUT')
      CALL MESSAG(' MOON = $',100,'ABUT','ABUT')
      CALL REALNO(AVGFM,-1,'ABUT','ABUT')
      CALL MESSAG(' SUN = $',100,'ABUT','ABUT')
      CALL REALNO(AVGFS,-1,'ABUT','ABUT')
      CALL MESSAG(' DRAG = $',100,'ABUT','ABUT')
      CALL REALNO(AVGFD,-1,'ABUT','ABUT')

      CALL MESSAG('PERIGEE$',100,2.75,1.0)
      CALL MESSAG('                      Apogee$',100,'ABUT','ABUT')

      CALL MESSAG('RADIUS (KM)$',100,0.25,0.67)
      CALL MESSAG('RP =$',100,2.75,0.67)
      CALL REALNO(RP,1,'ABUT','ABUT')
      CALL MESSAG('      $',100,'ABUT','ABUT')
      CALL MESSAG('   RA =$',100,'ABUT','ABUT')
      CALL REALNO(RA,1,'ABUT','ABUT')

      CALL MESSAG('VELOCITY (KM/SEC)$' 100,0.25,0.33)
      CALL MESSAG('VP =$',100,2.75,0.33)
      CALL REALNO(VP,2,'ABUT','ABUT')
      CALL MESSAG('       $',100,'ABUT','ABUT')
      CALL MESSAG('   VA =$',100,'ABUT','ABUT')
      CALL REALNO(VA,2,'ABUT','ABUT')
```

```
                                                                  ORB27930
      CALL RESET('COMPLX')                                        ORB27940
      CALL ENDGR(0)                                               ORB27950
      RETURN                                                      ORB27960
      END                                                         ORB27970
                                                                  ORB27980
```

APPENDIX B. COORDINATE SYSTEMS

A. 'IJK' : GEOCENTRIC - EQUATORIAL

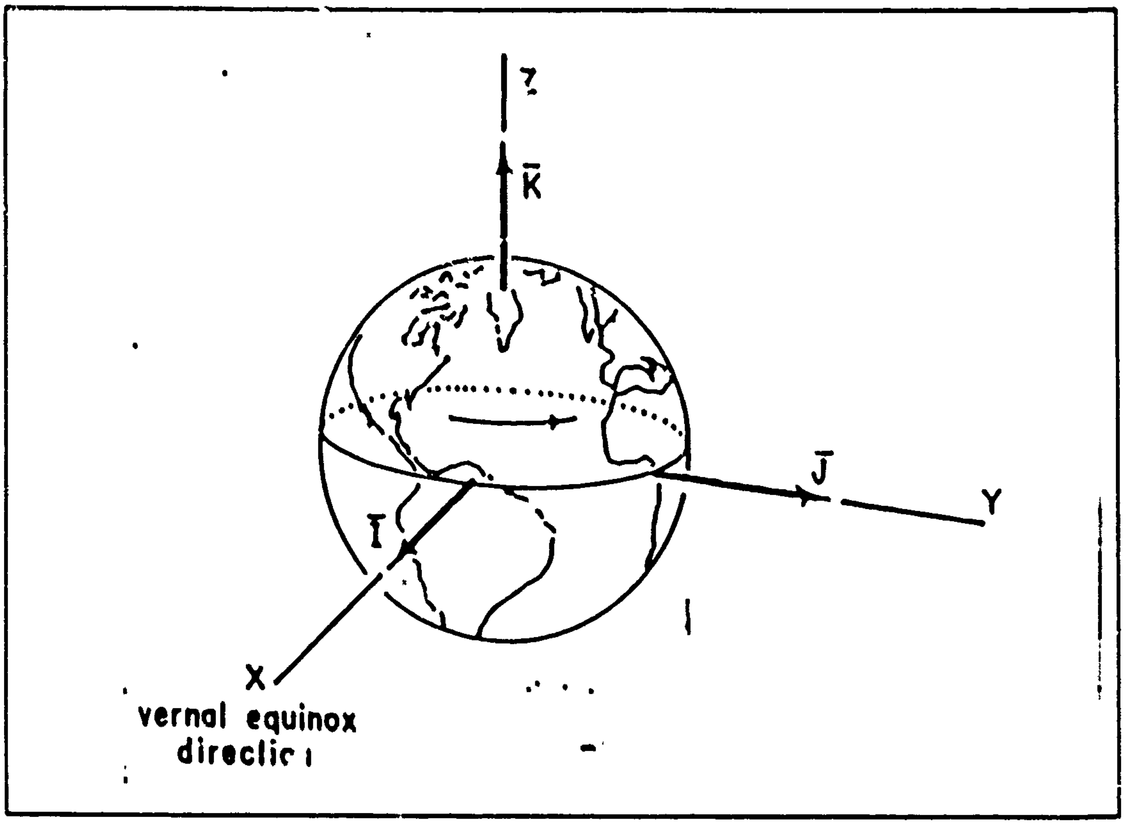


Figure 3. Geocentric-equatorial coordinate system

The geocentric-equatorial system as seen in Figure 3 has its origin at the earth's center. The fundamental plane is in the equator and the positive X-axis points in the vernal equinox direction. The Z-axis points in the direction of the north pole. This system is not fixed to the earth and turning with it; rather, the geocentric-equatorial frame is nonrotating with respect to the stars (except for precession of the equinoxes) and the earth turns relative to it. Unit vectors, $\bar{I}$, $\bar{J}$, and $\bar{K}$ shown in Figure 3, lie along the X, Y, and Z respectively. [Ref. 1: p.55]

B. 'PQW': PERIFOCAL

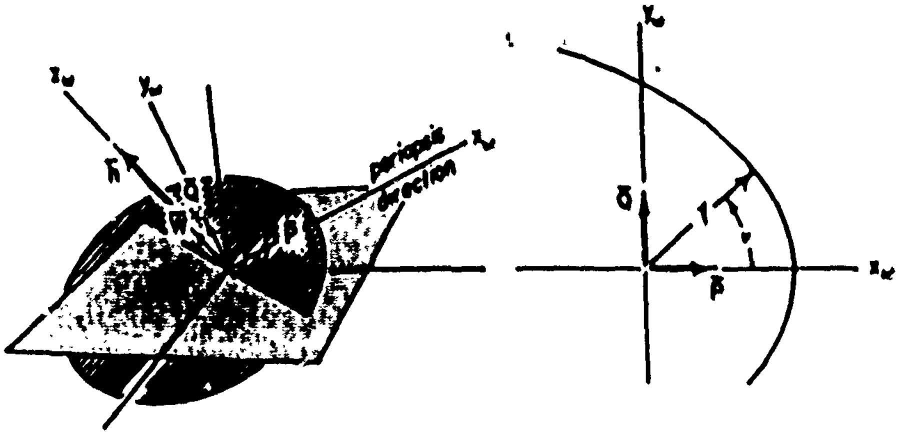


Figure 4. Perifocal coordinate system

The perifocal coordinate system has its fundamental plane in the plane of the satellite's orbit as seen in Figure 4. The coordinate axis are named, X_ω, Y_ω and Z_ω .. The X_ω axis points toward the perigee; the Y_ω axis is rotated 90 degrees in the direction of orbital motion and lies in the orbital plane; the Z_ω axis along $\bar{h}$ completes the right-handed perifocal system. Unit vectors in the direction of X_ω, Y_ω and Z_ω are called $\bar{P}$, $\bar{Q}$ and $\bar{W}$ respectively. [Ref. 1: p.57]

C. 'RSW' : ORBITAL

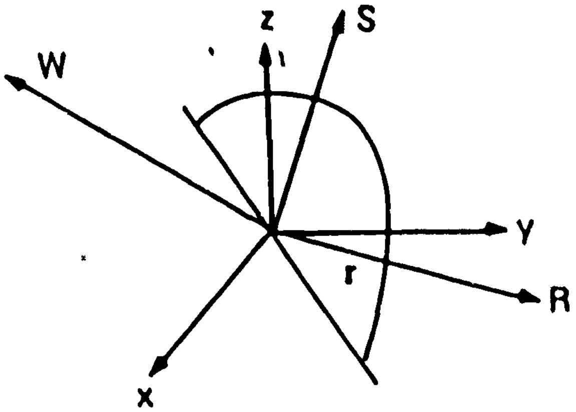


Figure 5. Orbital coordinate system

(Figure 9.4-1, Ref. 1)

The orbital coordinate system has its principle axis, R (unit vector r), along the instantaneous radius vector, r as seen in Figure 5. The axis S is rotated 90 degrees from R in the direction of increasing true anomaly. The third axis, W, is perpendicular to both R and S. Note that this coordinate system is simply rotated ν_0 from the PQW perifocal system. [Ref. 1: p.398]

D. COORDINATE TRANSFORMATIONS

The coordinate transformations, for the previous coordinate systems, use angular rotations about the axis to evaluate the transformation matrix. The matrix elements r_{ij} are calculated, then applied to the old vector to get the vector in the new coordinate system. The following orbital elements are used:

Ω = longitude of ascending node

ω = argument of perigee

i = inclination

u_0 = argument of latitude

v_0 = true anomaly

The coordinate transformations follow [Ref. 1: p.74-83]

1. PQW to IJK

$$r_{11} = \cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i$$
$$r_{12} = -\cos\Omega\sin\omega - \sin\Omega\cos\omega\cos i$$
$$r_{13} = \sin\Omega\cos\omega$$
$$r_{21} = \sin\Omega\cos\omega + \cos\Omega\sin\omega\cos i$$
$$r_{22} = -\sin\Omega\sin\omega + \cos\Omega\cos\omega\cos i$$
$$r_{23} = -\cos\Omega\sin i$$
$$r_{31} = \sin\omega\sin i$$
$$r_{32} = \cos\omega\sin i$$
$$r_{33} = \cos i$$
$$\vec{I} = r_{11}\vec{P} + r_{12}\vec{Q} + r_{13}\vec{W}$$
$$\vec{J} = r_{21}\vec{P} + r_{22}\vec{Q} + r_{23}\vec{W}$$
$$\vec{K} = r_{31}\vec{P} + r_{32}\vec{Q} + r_{33}\vec{W}$$

2. IJK to PQW (inverse of #1)

$$\vec{P} = r_{11}\vec{I} + r_{21}\vec{J} + r_{31}\vec{K}$$
$$\vec{Q} = r_{12}\vec{I} + r_{22}\vec{J} + r_{32}\vec{K}$$
$$\vec{W} = r_{13}\vec{I} + r_{23}\vec{J} + r_{33}\vec{K}$$

3. IJK to RSW

$$r_{11} = \cos\Omega\cos u_0 - \sin\Omega\sin u_0\cos i$$
$$r_{12} = \sin\Omega\cos u_0 + \sin u_0\cos\Omega\cos i$$
$$r_{13} = \sin i\sin u_0$$
$$r_{21} = -\cos\Omega\sin u_0 - \sin\Omega\cos u_0\cos i$$
$$r_{22} = -\sin\Omega\sin u_0 + \cos\Omega\cos u_0\cos i$$
$$r_{23} = \cos u_0\sin i$$
$$r_{31} = \sin\Omega\sin i$$
$$r_{32} = -\cos\Omega\sin i$$
$$r_{33} = \cos i$$
$$\vec{R} = r_{11}\vec{I} + r_{12}\vec{J} + r_{13}\vec{K}$$
$$\vec{S} = r_{21}\vec{I} + r_{22}\vec{J} + r_{23}\vec{K}$$
$$\vec{W} = r_{31}\vec{I} + r_{32}\vec{J} + r_{33}\vec{K}$$

4. RSW to IJK (inverse of #3)

$$\vec{I} = r_{11}\vec{R} + r_{21}\vec{S} + r_{31}\vec{W}$$
$$\vec{J} = r_{12}\vec{R} + r_{22}\vec{S} + r_{32}\vec{W}$$
$$\vec{K} = r_{13}\vec{R} + r_{23}\vec{S} + r_{33}\vec{W}$$

5. PQW to RSW

$r_{11} = \cos v_0$
$r_{12} = \sin v_0$
$r_{13} = 0.0$
$r_{21} = -\sin v_0$
$r_{22} = \cos v_0$
$r_{23} = 0.0$
$r_{31} = 0.0$
$r_{32} = 0.0$
$r_{33} = 1.0$

$$\vec{R} = r_{11}\vec{P} + r_{12}\vec{Q} + r_{13}\vec{W}$$
$$\vec{S} = r_{21}\vec{P} + r_{22}\vec{Q} + r_{23}\vec{W}$$
$$\vec{W} = r_{31}\vec{P} + r_{32}\vec{Q} + r_{33}\vec{W}$$

6. RSW to PQW (inverse of #5)

$$\vec{P} = r_{11}\vec{R} + r_{21}\vec{S} + r_{31}\vec{W}$$
$$\vec{Q} = r_{12}\vec{R} + r_{22}\vec{S} + r_{32}\vec{W}$$
$$\vec{W} = r_{13}\vec{R} + r_{23}\vec{S} + r_{33}\vec{W}$$

APPENDIX C. ORBITAL ELEMENTS

The user is assumed to be studying orbital mechanics and should understand the orbital elements and how to calculate them. A brief description of the elements and the equations used to calculate the elements follow. For a detailed explanation of the elements and the equations to calculate them refer to Chapters 1 and 2 of reference 1. Figure 6 on page 83 shows the orbital elements in the Geocentric-Equatorial and perifocal coordinate system.

1. Angular Momentum (h):
   The specific angular momentum is a constant of the motion of the satellite, defined as $\vec{h} = \vec{r} x \vec{v}$.

   $$\vec{h} = \vec{r} x \vec{v} = h_I \vec{I} + h_J \vec{J} + h_K \vec{K}$$

   $$h_I = r_J v_K - r_K v_J$$

   $$h_J = r_K v_I - r_I v_K$$

   $$h_K = r_I v_J - r_J v_I$$

   $$h = \sqrt{h_I^2 + h_J^2 + h_K^2}$$

2. Node Vector (n):
   The node vector is a vector pointing along the line of nodes in the direction of the ascending node.

   $$\vec{n} = \vec{K} x \vec{h} = -h_J \vec{I} + h_I \vec{J}$$

   $$n = \sqrt{h_J^2 + h_I^2}$$

3. Semi-latus rectum (p):
   The semi-latus rectum is a geometric constant of the conic section.

   $$p = \frac{h^2}{\mu}$$

4. Eccentricity (e):
   The eccentricity is a constant defining the shape of the conic orbit.

   $$\vec{e} = \frac{1}{\mu} [(v^2 - \frac{\mu}{r}) \vec{r} - (\vec{r} \vec{v}) \vec{v}]$$

   $$e = |\vec{e}|$$

5. Semi-major axis (a):

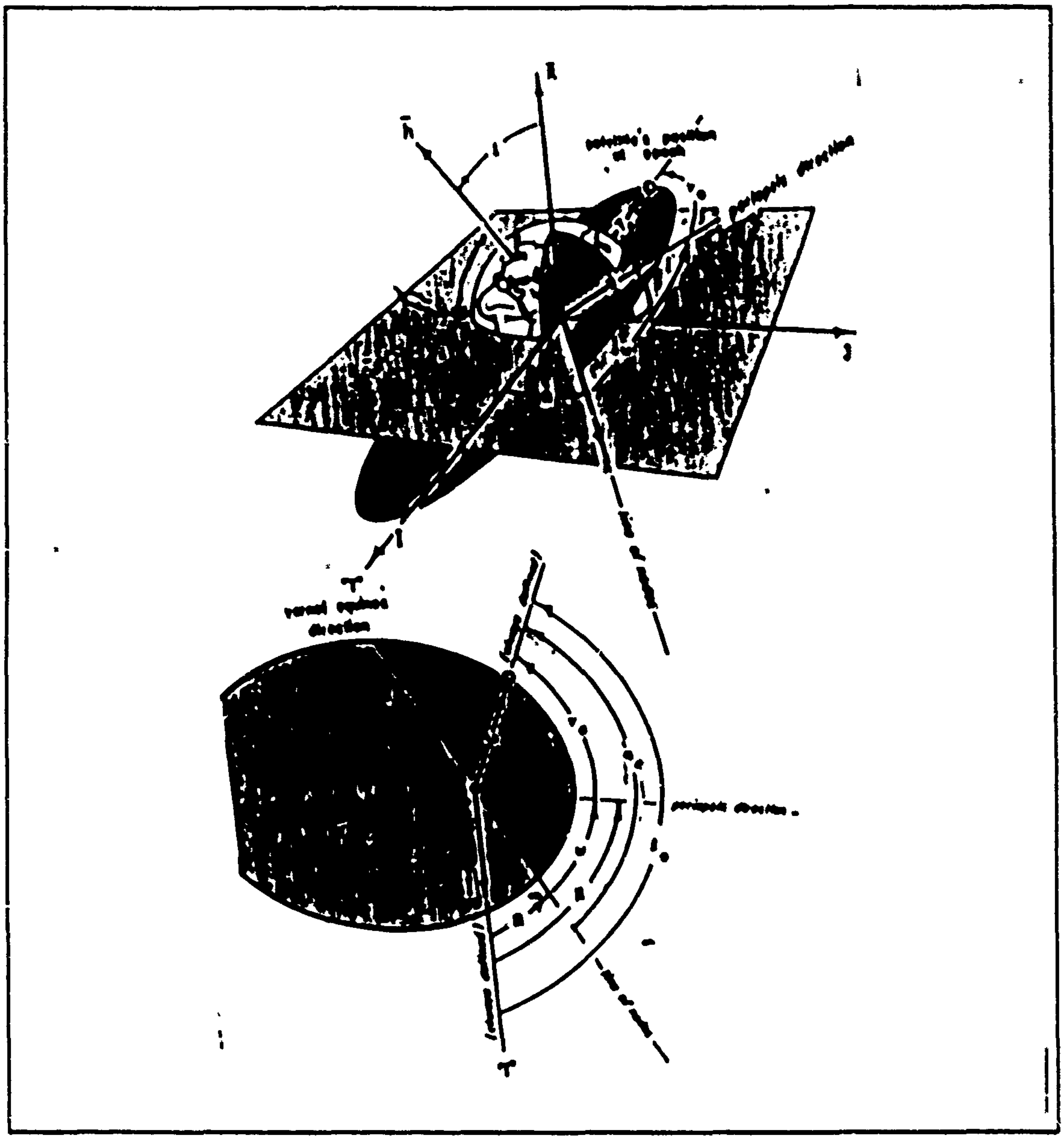


Figure 6. Orbital elements

The semi-major axis is a constant defining the size of the orbit.

$$a = \frac{(1 - e^2)}{p}$$

6. Inclination (i):

The inclination is the angle between the 'K' unit vector in the 'IJK' system and the angular momentum vector, 'h'.

$$i = \cos^{-1}\left(\frac{\bar{h}\bar{K}}{h}\right) = \cos^{-1}\left(\frac{h_z}{h}\right)$$

7. Longitude of ascending node (Ω):

The longitude of the ascending node is the angle in the fundamental plane , between the 'I' unit vector and the point where the satellite crosses through the fundamental plane in a northerly direction (ascending node) measured counterclockwise when viewed from the north side of the fundamental plane.

$$\Omega = \cos^{-1}\left(\frac{n_I}{n}\right)$$

8. Argument of perigee (ω):

The argument of perigee is the angle in the plane of the satellite's orbit, between the ascending node and the perigee point, measured in the direction of the satellite's motion.

$$\omega = \cos^{-1}\left(\frac{\bar{n}\bar{e}}{ne}\right) = \cos^{-1}\frac{(n_I e_I + n_J e_J)}{ne}$$

9. True anomaly at epoch (v_0):

The true anomaly at epoch is the angle in the plane of the satellite's orbit, between perigee and the position of the satellite at a particular time, t_0, called the "epoch".

$$v_0 = \cos^{-1}\left(\frac{\bar{e}\bar{r}}{er}\right)$$

10. Argument of latitude (u_0):

The argument of latitude is the angle in the plane of the orbit, between the ascending node and the radius vector to the satellite at time t_0.

$$u_0 = \cos^{-1}\left(\frac{\bar{n}\bar{r}}{nr}\right)$$

11. Longitude of perigee (Π):

The longitude of perigee is the angle from 'I' to perigee measured eastward to the ascending node and then in the orbital plane to perigee.

$$\Pi = \Omega + \omega$$

12. True longitude at epoch (l_0):

The true longitude at epoch is the angle between 'I' and r_0 (the radius vector to the satellite at t_0 measured eastward to the ascending node and then in the orbital plane to r_0.

$$l_0 = \omega + \Omega + v_0$$

13. Period (per):

The period is the time the for the satellite to complete one orbit.

$$Per = 2\sqrt{\frac{a^3}{\mu}}$$

14. Eccentric anomaly (EA):

The eccentric anomaly is the angle between the perigee and a position on an auxiliary circle circumscribed about the ellipse where a perpendicular line to the major axis has been extended from the epoch location of the satellite to the auxiliary circle.

$$EA = \cos^{-1} \frac{e + \cos(v)}{1 + e\cos(v)}$$

15. Mean motion (n'):

The mean motion is defined below:

$$n' = \sqrt{\frac{\mu}{a^3}}$$

16. Mean anomaly (MA):

The mean anomaly is defined below:

$$MA = n'(t - T) = EA - e\sin(EA)$$

17. Time of flight (TF):

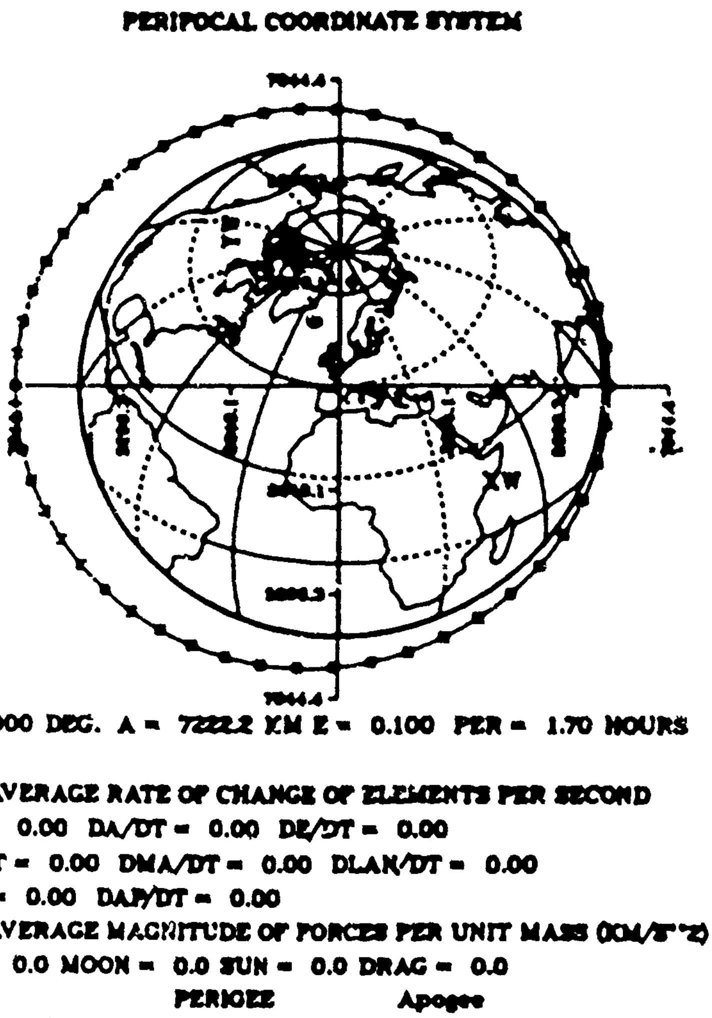
The time of flight is the elapsed time from when the satellite was at perigee to the current epoch.

$$(t - T) = \sqrt{\frac{a^3}{\mu}}\,(EA - e\sin(EA))$$

APPENDIX D. SAMPLE ORBITS

To demonstrate the capabilities of the program, a variety of orbital plots will follow:

1. Low earth orbit (LEO).



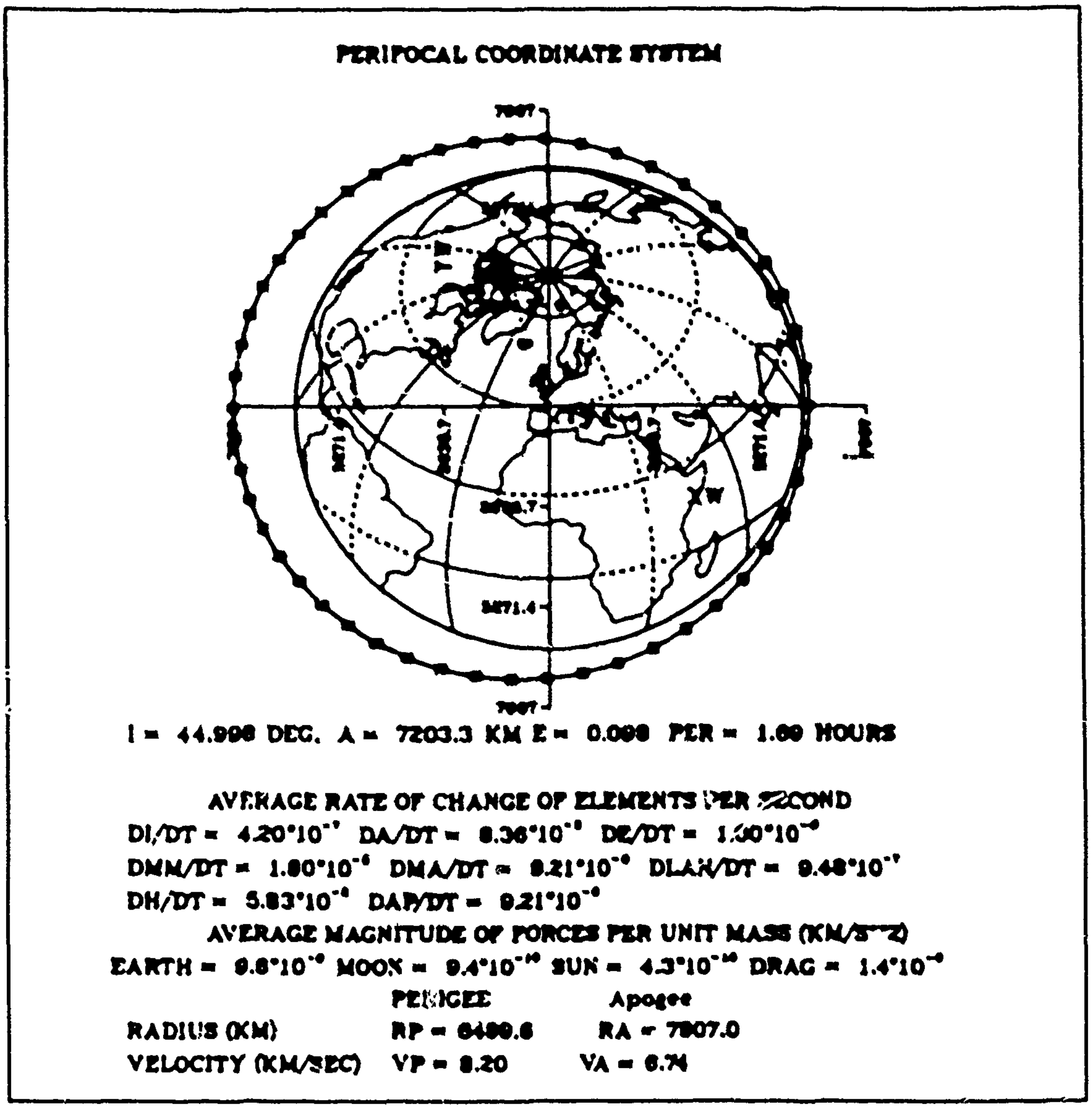
```
I =  45.000 DEG.  A =  7222.2 KM E =  0.100  PER =  1.70 HOURS

         AVERAGE RATE OF CHANGE OF ELEMENTS PER SECOND
DI/DT =  0.00  DA/DT =  0.00  DE/DT =  0.00
DMM/DT =  0.00  DMA/DT =  0.00  DLAN/DT =  0.00
DH/DT =  0.00  DAP/DT =  0.00
         AVERAGE MAGNITUDE OF FORCES PER UNIT MASS (KM/S**2)
EARTH =  0.0 MOON =  0.0 SUN =  0.0 DRAG =  0.0
                        PERIGEE          Apogee
RADIUS (KM)             RP = 6500.0      RA = 7944.4
VELOCITY (KM/SEC)       VP = 8.21        VA = 6.72
```

Figure 7. Unperturbed Low Earth Orbit (LEO)

Figure 7 shows the perifocal plot of a satellite in an unperturbed low earth orbit (LEO). The initial parameters of the orbit were entered as follows:

radius of perigee (RP) = 6500 km
eccentricity (e) = 0.1
inclination (i) = 45 degrees.



```
PERIFOCAL COORDINATE SYSTEM

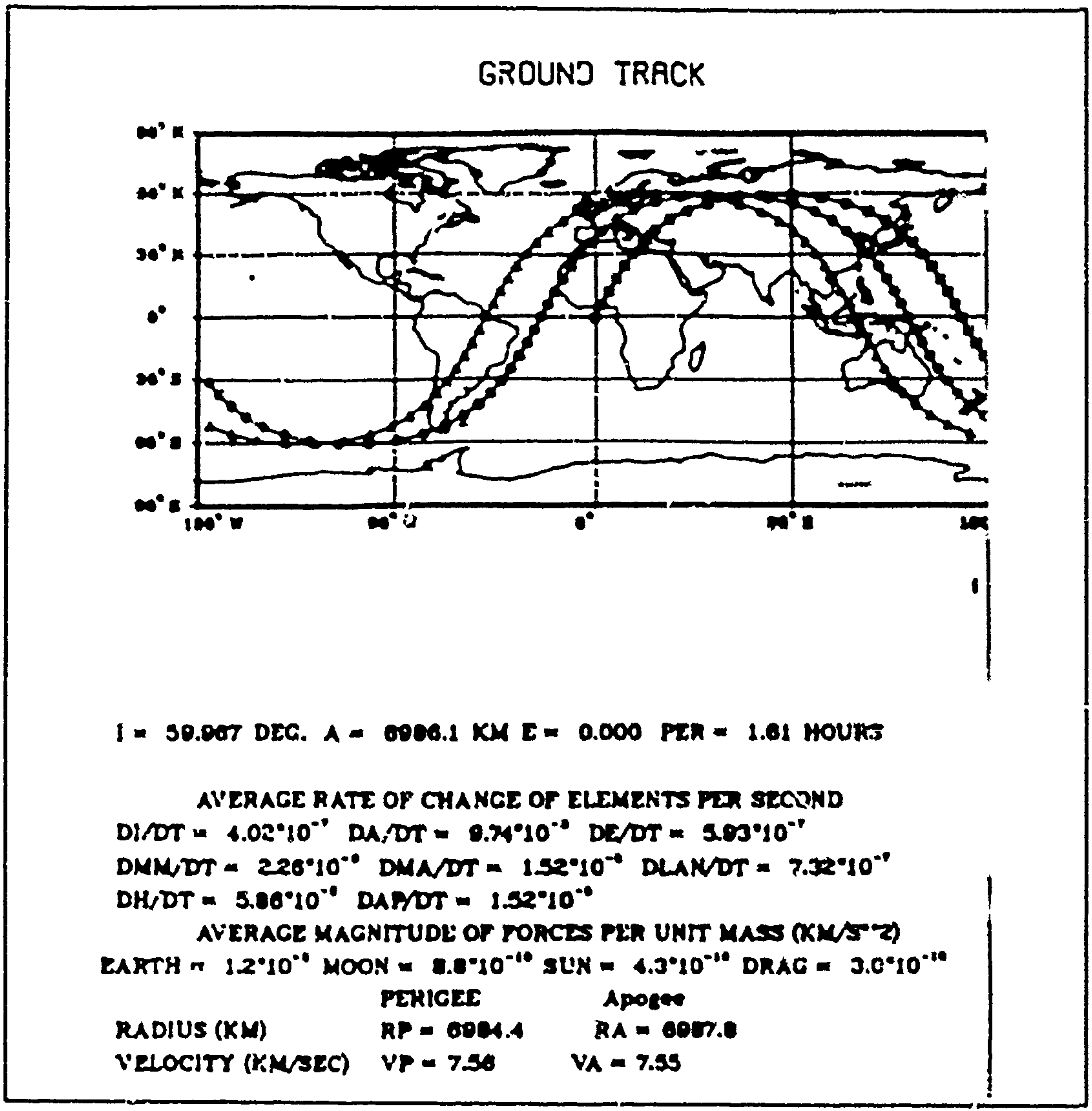
I = 44.998 DEG. A = 7203.3 KM E = 0.098 PER = 1.69 HOURS

      AVERAGE RATE OF CHANGE OF ELEMENTS PER SECOND
DI/DT = 4.20*10^-7 DA/DT = 8.36*10^-8 DE/DT = 1.30*10^-9
DMM/DT = 1.80*10^-6 DMA/DT = 9.21*10^-9 DLAN/DT = 9.48*10^-7
DH/DT = 5.83*10^-6 DAP/DT = 9.21*10^-9
      AVERAGE MAGNITUDE OF FORCES PER UNIT MASS (KM/S**2)
EARTH = 9.8*10^-9 MOON = 9.4*10^-10 SUN = 4.3*10^-10 DRAG = 1.4*10^-9
                      PERIGEE          Apogee
RADIUS (KM)           RP = 6499.6      RA = 7907.0
VELOCITY (KM/SEC)     VP = 8.20        VA = 6.74
```

Figure 8. Perturbed Low Earth Orbit (LEO)

With perturbing forces applied to the previous LEO, the drag force will be the dominate perturbing force. The drag will act as a negative velocity change applied in the area of perigee. with the result of decreasing the semi-major axis length, this in effect will decrease the eccentricity of the orbit, as can be seen by comparing the orbital data of the unperturbed LEO in Figure 7 on page 86 with the orbital data of the perturbed LEO in Figure 8.

2. Circular orbit.



```
I =  59.967 DEG.  A =  6986.1 KM E =  0.000  PER =  1.61 HOURS

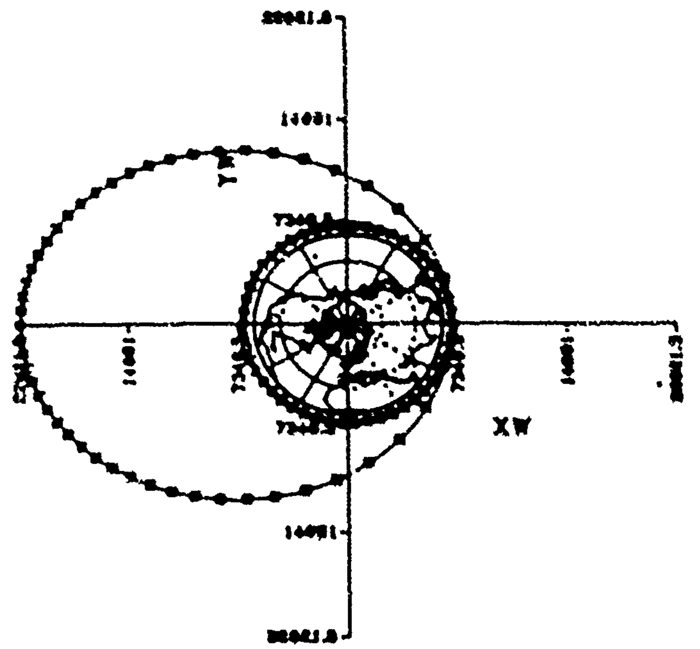
       AVERAGE RATE OF CHANGE OF ELEMENTS PER SECOND
DI/DT =  4.02*10^-7  DA/DT =  9.74*10^-3  DE/DT =  5.93*10^-7
DMM/DT =  2.26*10^-6  DMA/DT =  1.52*10^-6  DLAN/DT =  7.32*10^-7
DH/DT =  5.86*10^-6  DAP/DT =  1.52*10^-6
       AVERAGE MAGNITUDE OF FORCES PER UNIT MASS (KM/S**2)
EARTH =  1.2*10^-2 MOON =  8.8*10^-10 SUN =  4.3*10^-10 DRAG =  3.0*10^-10
                      PERIGEE          Apogee
RADIUS (KM)           RP = 6984.4      RA = 6987.8
VELOCITY (KM/SEC)     VP = 7.56        VA = 7.55
```

Figure 9.   Circular Orbit

An example of the plot of the ground track of a sequence of three 60 degree inclined perturbed circular orbits with a radius of 7000 km is shown in Figure 9. The sequence of orbits displays the precession of the orbit around the earth.

3. Transfer orbit.

PERIFOCAL COORDINATE SYSTEM



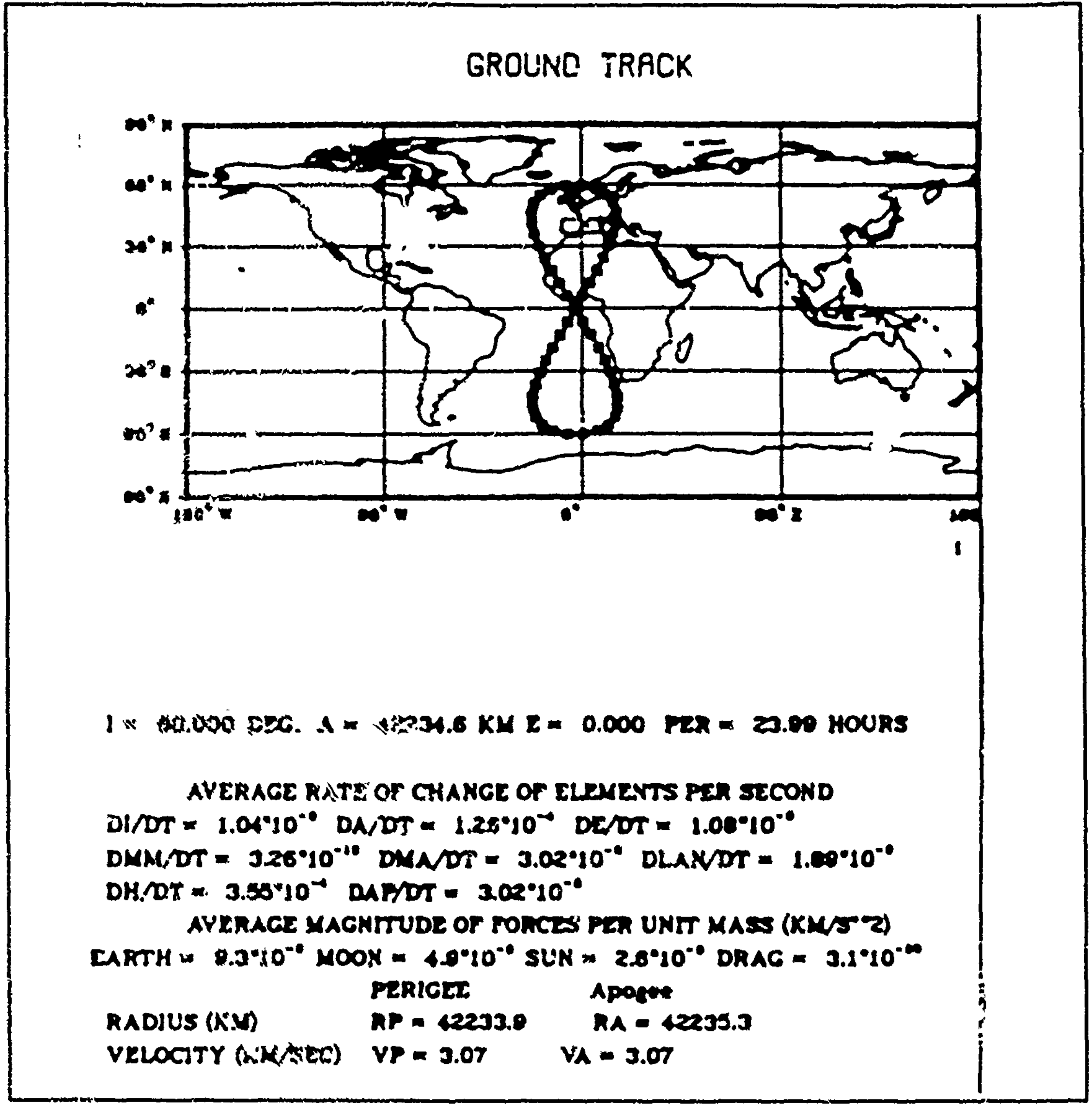
```
I = 0.000 DEG. A = 14510.8 KM E = 0.518 PER = 4.83 HOURS

      AVERAGE RATE OF CHANGE OF ELEMENTS PER SECOND
 DI/DT = 0.00 DA/DT = 0.00 DE/DT = 0.00
 DMM/DT = 0.00 DMA/DT = 0.00 DLAN/DT = 0.00
 DH/DT = 0.00 DAP/DT = 0.00
      AVERAGE MAGNITUDE OF FORCES PER UNIT MASS (KM/S**2)
EARTH = 0.0 MOON = 0.0 SUN = 0.0 DRAG = 0.0
                       PERIGEE          Apogee
 RADIUS (KM)           RP = 7000.0      RA = 22021.5
 VELOCITY (KM/SEC)     VP = 9.30        VA = 2.96
```

Figure 10. Transfer Orbit

The transfer orbit between a circular, equatorial LEO and a molniya orbit (high eccentric orbit) is shown in Figure 10. A velocity increase of 1.75 km/s was applied at the perigee to simulate a perigee kick to boost the satellite into the molniya orbit. A similar velocity change could then be applied at apogee to create a high altitude circular orbit, or a negative velocity change applied at perigee could be used to bring the satellite back to a LEO.

4. Geosynchronous orbit



I = 60.000 DEG. A = 42234.6 KM E = 0.000 PER = 23.99 HOURS

AVERAGE RATE OF CHANGE OF ELEMENTS PER SECOND

DI/DT = $1.04*10^{-9}$ DA/DT = $1.26*10^{-4}$ DE/DT = $1.08*10^{-9}$

DMM/DT = $3.26*10^{-10}$ DMA/DT = $3.02*10^{-9}$ DLAN/DT = $1.89*10^{-9}$

DH/DT = $3.55*10^{-4}$ DAP/DT = $3.02*10^{-9}$

AVERAGE MAGNITUDE OF FORCES PER UNIT MASS (KM/S**2)

EARTH = $9.3*10^{-9}$ MOON = $4.9*10^{-9}$ SUN = $2.6*10^{-9}$ DRAG = $3.1*10^{-20}$

	PERIGEE	Apogee
RADIUS (KM)	RP = 42233.9	RA = 42235.3
VELOCITY (KM/SEC)	VP = 3.07	VA = 3.07

Figure 11. Geosynchronous Orbit

The ground track of a perturbed geosynchronous orbit inclined 60 degrees is shown in Figure 11. The orbit displays the figure eight typical with inclined geosynchronous orbits.

LIST OF REFERENCES

1. Bate, R.R., Mueller, D.D., and White, J.E., *Fundamentals of Astrodynamics*, Dover Publications, Inc., 1971.

2. Martin Marrieta Corporation, Space Systems Division, *Orbital Flight Handbook*, volume 1, 1963.

3. Agrawal, B.N., *Design of Geosynchronous Spacecraft*, Prentice-Hall, Inc., 1968.

4. Roy, A.E., *Orbital Motion*, Adam Hilger Ltd., 1982.

5. Computer Associates, *DISSPLA User's Manual*, version 10.0, 1987.

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Attn: Prof. E. Milne (Code 61Mn)
Monterey, CA 93943-5000

8. Superintendent 1
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9. Commanding Oficer 2
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