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The goal of this project was to complete a study of Bahar's full-wave approach to rough surface scattering that was begun under ONR support. The two proposed items of technical work have been fully completed.

In the ONR-supported work, a study was begun for a simplified scattering problem: Scattering from a one-dimensional (1-D) surface with the Dirichlet (zero field) boundary condition using a Gaussian roughness spectrum. The full-wave method was implemented numerically and used in Monte Carlo calculations of the scattering cross section. Comparisons were then made with exact integral equation results, and with the Kirchhoff and perturbation approximations. It was found that the full-wave prediction did not reduce to the lowest order perturbation prediction when the perturbation result was known to be accurate, as shown by exact calculations. This is contrary to conclusions reached by Bahar and is an important finding, showing that the full-wave approach is not as

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**Final Report**  
**for**  
**An Examination of the "Full-Wave" Method for Rough Surface Scattering**

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Date: January 16, 1990

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(Dr. Walter A. Flood, Director)

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### Project Summary

The goal of this project was to complete a study of Bahar's full-wave approach to rough surface scattering<sup>1-7</sup> that was begun under ONR support.<sup>8</sup> The two proposed items of technical work have been fully completed.

In the ONR-supported work, a study was begun for a simplified scattering problem: Scattering from a one-dimensional (1-D) surface with the Dirichlet (zero field) boundary condition using a Gaussian roughness spectrum. The full-wave method was implemented numerically and used in Monte Carlo calculations of the scattering cross section. Comparisons were then made with exact integral equation results, and with the Kirchhoff and perturbation approximations. We found that the full-wave prediction did not reduce to the lowest order perturbation prediction when the perturbation result was known to be accurate, as shown by exact calculations.<sup>9</sup> This is contrary to conclusions reached by Bahar and is an important finding, showing that the full-wave approach is not as general as previously thought. The full-wave prediction did reduce to the Kirchhoff prediction when the Kirchhoff approximation is known to be accurate, as shown by exact calculations.<sup>10</sup> In fact, the full-wave result was found to agree closely with the Kirchhoff result for all cases studied, whether or not the Kirchhoff result was accurate.

In the present study, further work was done to satisfy an objection raised by Bahar and to strengthen the conclusions reached with the numerical studies. Results of this work were presented at the National Radio Science Meeting at Boulder, Colorado (January, 1989). The

Radio Science paper<sup>11</sup> summarizes the research findings and is attached. [Slide 6 in Ref. 11 shows the discrepancy between the full-wave and perturbation theory/integral equation predictions.]

The present study involved two issues, which will be briefly summarized.

1. Bahar raised one principal objection to the numerical approach used in our work. He maintained that because we used a taper function (See Ref. 11) to suppress edge effects, the true full-wave prediction was not obtained in the Monte Carlo calculations. We have shown in this study that use of the taper function does not affect the comparison with perturbation theory. This can be seen clearly in Ref. 11, Slide 8, which shows that the untapered and tapered results are essentially identical. [The need for the taper function only becomes apparent if the coherent component is included (Ref. 11, Slide 7), or if the backscattered levels are about 10 dB or more lower than in Ref. 11, Slide 8.] Thus, we find that when the full-wave method is implemented exactly as specified by Bahar, it does not reduce to the perturbation prediction when the surface heights and slopes are made small. This is contrary to Bahar's conclusions.

2. The second issue in this research concerns Bahar's method of calculating full-wave results. In past work Bahar has obtained full-wave predictions, but not by the Monte Carlo method previously mentioned. In Bahar's method (commonly used in scattering theory) the statistical properties of the surfaces are used to evaluate the needed expectations. In other words, formal averaging is used rather than Monte Carlo averaging. However, Bahar (prior to January 1989) has made two approximations to simplify the evaluation. First, the heights and slopes are taken as uncorrelated. Second, the slopes are taken as "delta" correlated. When both approximations are made, the evaluation is considerably simplified.

It appeared that the large differences between our Monte Carlo full-wave predictions and Bahar's full-wave predictions arose because of Bahar's two approximations. To investigate this issue we have formally averaged the full-wave prediction without making Bahar's approximations. The result is shown in Ref. 11, Slide 11. The formally averaged and Monte Carlo results are in essentially perfect agreement, and both disagree with perturbation theory and exact results (Ref. 11, Slide 4). When Bahar's approximations are made, the formally averaged full-wave prediction is changed significantly and agrees closely with first-order perturbation theory, as Bahar found. But "approximations" that significantly change the results are unacceptable. Thus, the true full-wave prediction must be taken as the one before Bahar's two additional approximations, and this prediction does not reduce to the perturbation result. We can now see what happened for surfaces with small heights and slopes: Bahar started with a full-wave prediction that is incorrect, made further approximations that significantly changed the result, and (by coincidence) ended up with the correct result. In the final analysis, we must conclude that the full-wave method does not reduce to first-order perturbation theory when perturbation theory is accurate.

## References

1. E. Bahar, "Full-Wave Solutions for the Scattered Radiation Fields from Rough Surfaces with Arbitrary Slope and Frequency," IEEE Trans. Antennas Propag. **AP-28**, 11-21 (1980)

2. E. Bahar, "Radio Wave Propagation over a Rough Variable Impedance Boundary: Part I - Full-Wave Analysis," IEEE Trans. Antennas Propag. **AP-20**, 354-362 (1972).
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4. E. Bahar, "Scattering cross sections for random rough surfaces: Full wave analysis," Radio Science **16**, 331-341 (1981).
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7. E. Bahar, "Review of the full wave solutions for rough surface scattering and depolarization: comparisons with geometric and physical optics, perturbation, and two-scale hybrid solutions," J. Geophys. Res. **92**, 5209-5224 (1987).
8. This work was done as part of the ONR-ARL project titled "Scattering from Very Rough Surfaces," sponsored by ONR Code 11. The investigators in this project were E.I. Thorsos, D.P. Winebrenner, and D.R. Jackson at APL-UW, and A. Ishimaru at the Department of Electrical Engineering-UW.
9. E. I. Thorsos and D. R. Jackson, "The validity of the perturbation approximation for rough surface scattering using a Gaussian roughness spectrum," J. Acoust. Soc. Am. **86**, 261-277 (1989).
10. E. I. Thorsos, "The validity of the Kirchhoff approximation for rough surface scattering using a Gaussian roughness spectrum," J. Acoust. Soc. Am. **83**, 78-92 (1988).
11. E. I. Thorsos, "An examination of the 'full-wave' method for rough surface scattering: Part II," presented at the National Radio Science Meeting, Boulder, Colorado, January 1989.

**An Examination of the "Full-Wave" Method  
for Rough Surface Scattering: Part II**

**Eric I. Thorsos**

**Applied Physics Laboratory  
College of Ocean and Fishery Sciences  
University of Washington**

**Presented at**

**National Radio Science Meeting  
University of Colorado  
Boulder, Colorado**

**January 1989**

**Work Supported by ARO**

## Abstract

Results were presented previously (National Radio Science Meeting, Boulder, Jan 5-8, 1988: Paper B/F1-1) for Bahar's full-wave approach to rough surface scattering. It was shown using Monte Carlo methods that when the surface heights and slopes become small, the full-wave prediction does not reduce to first-order perturbation theory results. Instead, the full-wave prediction agrees closely with the Kirchhoff solution. The case examined assumed the Dirichlet (zero field) boundary condition using one-dimensional surfaces with a Gaussian roughness spectrum. In this paper the full-wave method has been applied to the same case, but the average scattered intensity is found using a "formal average" rather than a Monte Carlo average. To obtain the formal average, the four-dimensional normal probability density is used to account fully for correlations between heights and slopes. It is found that the formally averaged full-wave predictions are completely consistent with the original Monte Carlo results. This work shows even more convincingly that the full-wave solution does not reduce to the first-order perturbation solution when the heights and slopes become small. [Work supported by ARO.]

## Introduction (Jan. 1988)

- Scattering problem
  - Dirichlet (zero field) boundary condition
  - Gaussian roughness spectrum, 1-D surface
  - Monte Carlo approach
- Full-wave method
  - E. Bahar, IEEE AP-28, 11-21, 1980
- Full-wave compared with
  - Exact integral equation
  - Kirchhoff
  - 1st order perturbation theory
- Integral equation method

E.I. Thorsos, JASA, Jan. 1988



## Slide 2

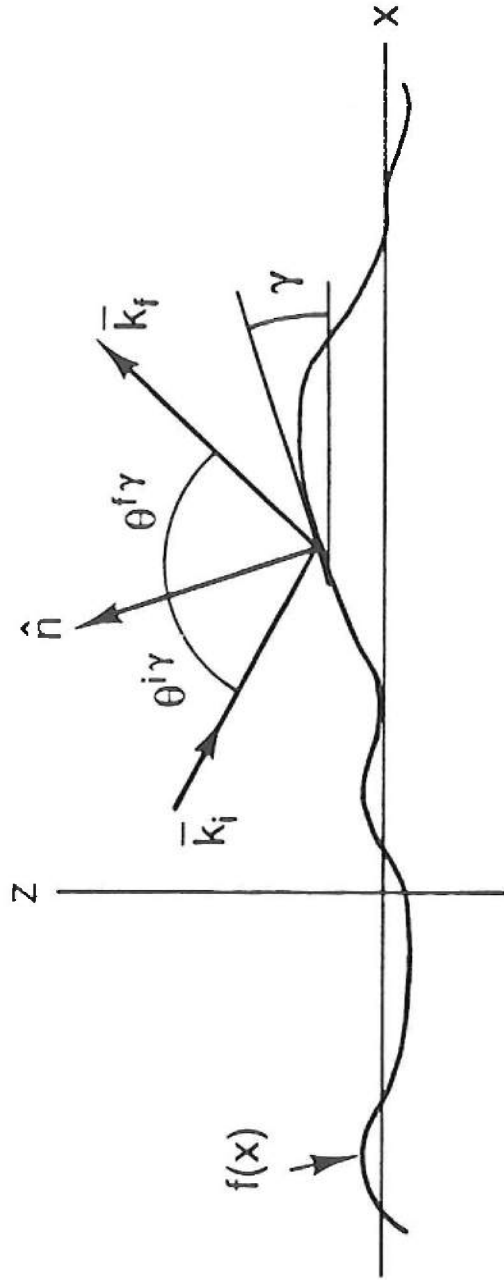
Results from an examination of Bahar's "full-wave" approach to rough surface scattering were presented at the January 1988 National Radio Science Meeting.<sup>(1)</sup> These results will be briefly reviewed. Slide 2 is reproduced from the 1988 presentation.

We are interested in the accuracy of the full-wave method applied to scattering from randomly rough surfaces. We consider a simplified scattering problem using the Dirichlet (zero field) boundary condition with surfaces which are one-dimensional (a 2-D scattering problem). This corresponds to scattering of horizontally polarized electromagnetic waves from perfectly conducting 1-D surfaces. The surfaces are assumed to have a Gaussian roughness spectrum and Gaussian height and slope distributions.

A Monte Carlo approach was used to obtain the average scattered intensity in the far field. For each example, 50 surface realizations were generated. The scattered field from each surface was computed as a function of scattered angle. The scattered intensities were then averaged over the 50-surface ensemble to obtain the bistatic radar scattering cross section.

Expressions used in computing the full-wave predictions were taken from Ref. 2, which is restricted to the 1-D surface case. The full-wave predictions were compared with those obtained with an exact integral equation method, with the Kirchhoff approximation, and with first-order perturbation theory. Details of the integral equation method are given in Ref. 3.

# Full-Wave Solution



$$E_{z_y}^r(x, z) = A \cdot \frac{1}{2L} \int_{-L}^L \frac{\cos \theta^i \gamma \cos \theta^f \gamma}{\cos \theta^i \gamma + \cos \theta^f \gamma} \frac{e^{-i\bar{v} \cdot \bar{p}'}}{\cos \gamma} dx' \quad (1)$$

$$-\frac{\pi}{2} < \theta^i \gamma, \theta^f \gamma < \frac{\pi}{2}.$$

(Otherwise contribution set to zero.)

$$-\bar{v} \cdot \bar{p} = (\cos \theta^f + \cos \theta^i) kf(x) + (\sin \theta^f - \sin \theta^i) kx$$

A is angle independent

### Slide 3

The notation and conventions are similar to those of Ref. 2. (The z-axis here corresponds to the y-axis in Ref. 2.) The full-wave solution for the scattered field is given by Eq. (4.10) of Ref. 2, which reduces to (1) for the perfectly conducting case. In (1),  $2L$  is the surface length and  $A$  is given by

$$A = \frac{i \omega I_0 L}{\pi (\rho \rho_0)^{1/2}} e^{-ik(\rho_0 + \rho)}$$

where

$\omega$  = radiation angular frequency

$k$  = radiation wave number

$I_0$  = constant proportional to incident field amplitude

$\rho_0$  = distance from source to center of surface

$\rho$  = distance from center of surface to far field point

Note that the angles  $\theta^{iy}$  and  $\theta^{fy}$  are measured relative to the local normal  $\hat{n}$  and thus vary along the rough surface. The position vector  $\vec{p}' = x' \hat{x} + f(x') \hat{z}$  denotes a point on the rough surface  $f(x')$ , and  $\vec{v} = \vec{k}_i - \vec{k}_f$ .

The condition  $-\frac{\pi}{2} < \theta^{iy}, \theta^{fy} < \frac{\pi}{2}$  will be satisfied for points on the surface that are not locally shadowed (for either incident field or scattered field shadowing). The contribution to (1) is set to zero if either  $|\theta^{iy}| > \frac{\pi}{2}$  or  $|\theta^{fy}| > \frac{\pi}{2}$ , which corresponds to the use of the simplest shadowing function. For the example discussed later, the incident angle is  $45^\circ$  and the rms surface slope angle is  $5^\circ$ , minimizing shadowing effects.

In Ref. 2 Bahar gives a separate solution referred to as the "iterative solution." The solution given by (1) is the solution of primary interest. It corresponds to the full-wave solutions given by Bahar in papers after 1980.

# Scattering Strength

- Scattering Strength =  $10 \log \left[ \frac{\sigma_{\text{rad}}(\theta^i, \theta^f)}{2\pi} \right]$

$\sigma_{\text{rad}}(\theta^i, \theta^f)$  = bistatic radar scattering cross section

$$\sigma_{\text{rad}}(\theta^i, \theta^f) = \frac{2\pi \langle I_s \rangle \rho}{I_{\text{inc}}(2L)}$$

$\langle I_s \rangle$  = ensemble-averaged scattered intensity

$\rho$  = far field range

$2L$  = surface length

- Surface field tapered with  $e^{-x^2/g^2}$  to suppress edge effects

then in  $\sigma_{\text{rad}}$ ,  $2L \rightarrow \sqrt{\frac{\pi}{2}} g$

#### Slide 4

To be consistent with the results given in Ref. 3, the scattered field is characterized by the scattering strength defined in Slide 4 in terms of the bistatic radar scattering cross section. [The scattering strength is commonly used in the underwater acoustics context.] The definition of the radar cross section is also given in Slide 4. Here  $\langle I_s \rangle$  is the 50-surface ensemble averaged scattered intensity at far field range  $\rho$  and at the scattered angle  $\theta^t$ . The surface length is  $2L$  and the incident intensity  $I_{inc} = |E_o^i|^2$  where [Eq(3.7b), Ref. 2]

$$E_o^i = \frac{I_o \omega}{2(2\pi k \rho_o)^{1/2}} e^{-i(k\rho_o - \pi/4)}$$

The definition of the radar cross section in Slide 4 is appropriate for a plane wave incident on a surface of length  $2L$ . As discussed in Ref. 3, numerical scattering computations can be affected by scattering from edges unless precautions are taken. Here, the surface field is tapered to suppress edge effects for all Monte Carlo computations, unless otherwise noted. Thus, a factor of  $e^{-x^2/g^2}$  is included inside the integral in (1). Then in the cross section the surface length  $2L$  is replaced by an "effective length"  $\sqrt{\frac{\pi}{2}} g$ . Numerical experience with this tapering procedure shows that it gives predictions in agreement with theory based on infinite surface lengths. It will be shown later that tapering can be avoided for the particular example considered if the coherent field is removed from the scattered field.

## Summary (Jan. 1988)

- **Conditions:** Dirichlet boundary condition  
1-D surface  
Gaussian spectrum
- **Full-wave solution agrees closely with Kirchhoff solution, when the Kirchhoff solution is accurate**
- **Full-wave solution does not reduce to 1st order perturbation solution as the rms surface height and slope become small**

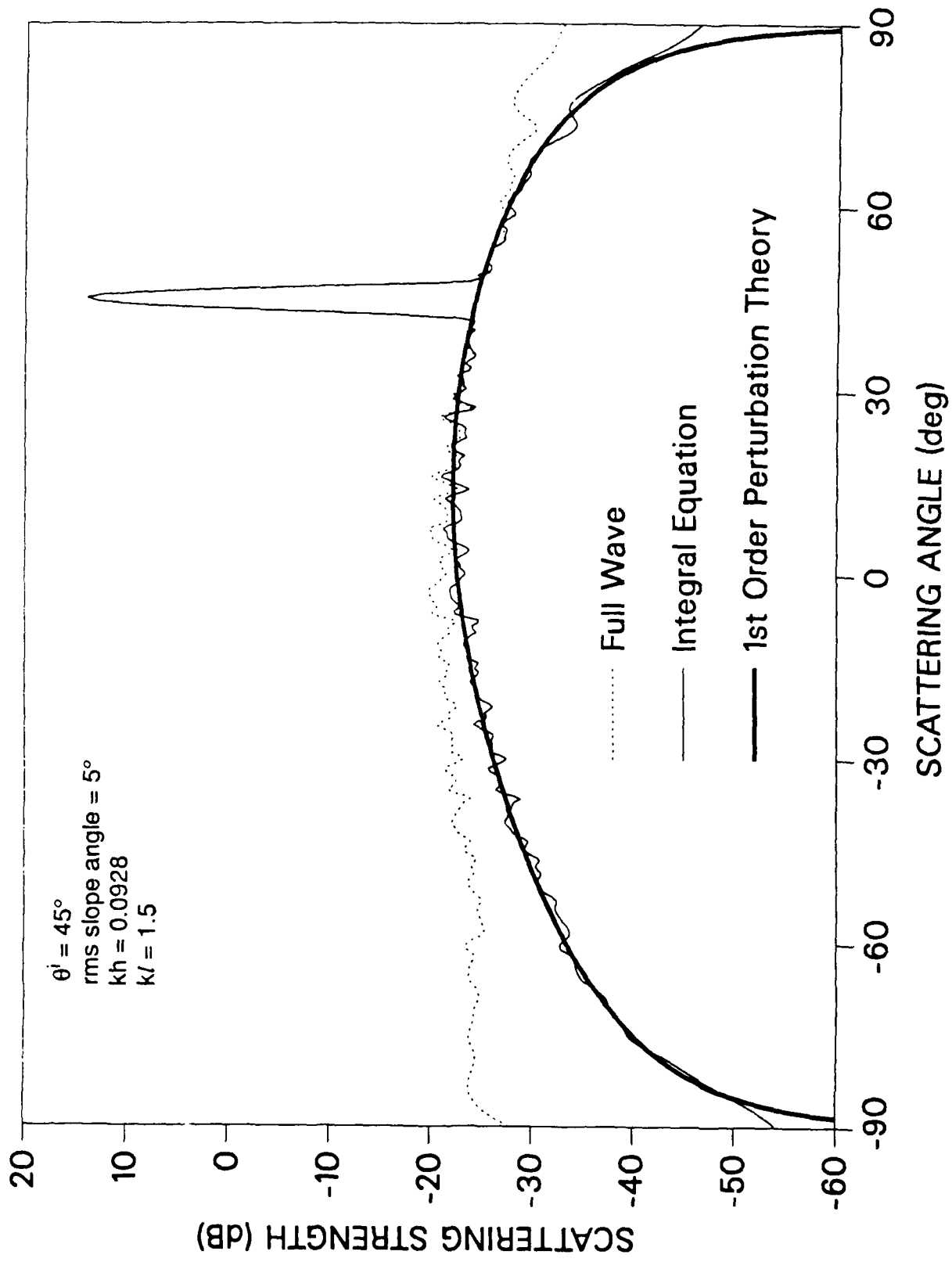
## Slide 5

Slide 5 is taken from the January 1988 Summary slide. The conditions are listed: Dirichlet boundary condition, 1-D surface, and Gaussian roughness spectrum.

First, it was found that the full-wave solution agrees closely with the Kirchhoff solution when the Kirchhoff solution is accurate. This is consistent with the conclusion reached by Bahar in Ref. 2.

Second, it was found that the full-wave solution does not reduce to the first-order perturbation solution as the rms surface heights and slopes become small. This is contrary to the conclusion reached by Bahar in Ref. 2.

In the present paper the perturbation example will be examined further.





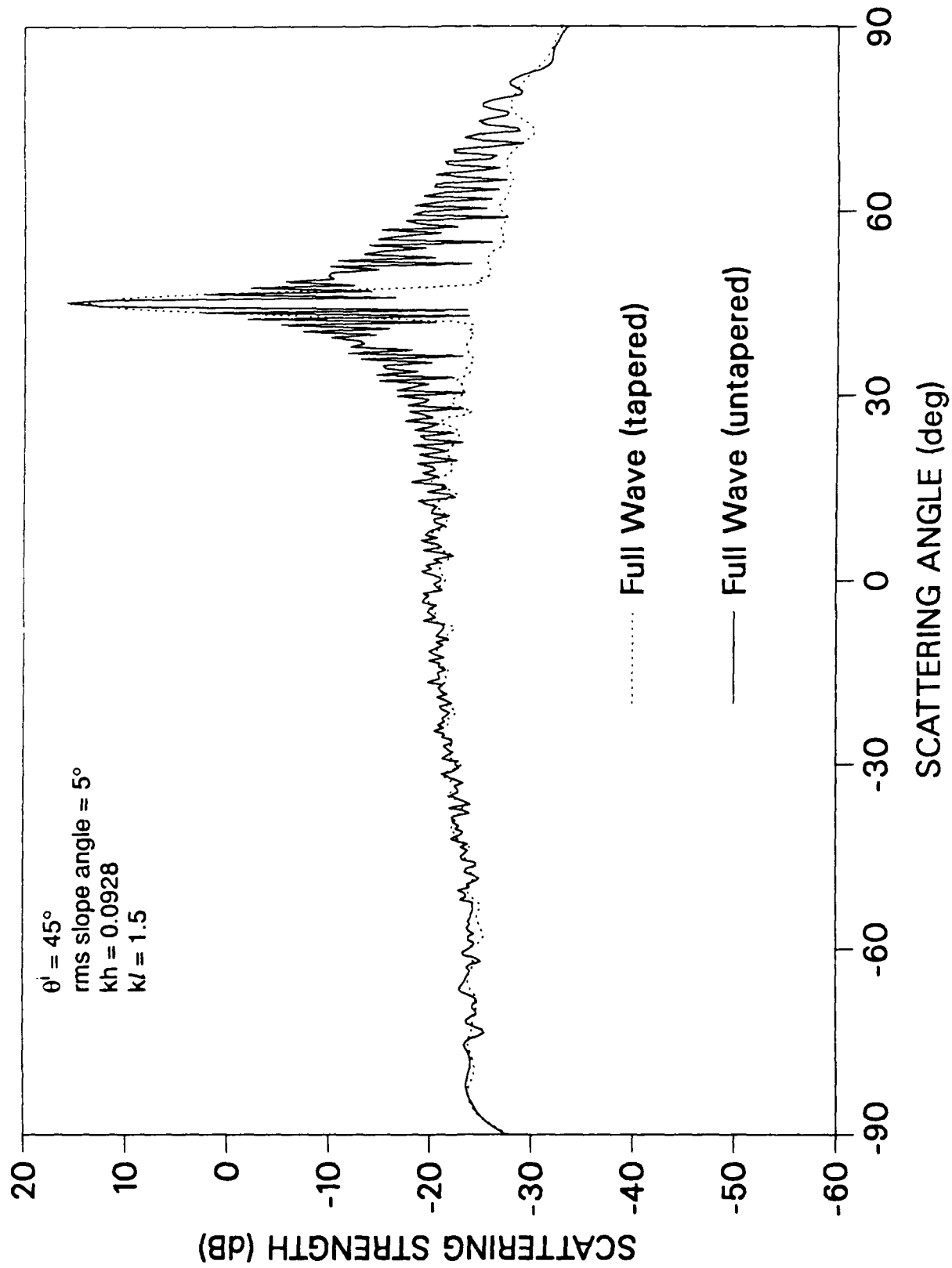
## Slide 6

Slide 6 shows a comparison between full-wave, integral equation, and first-order perturbation theory predictions for an example where first-order perturbation theory is accurate. [This was Example 3 in Ref. 1]. The incident angle is  $45^\circ$ , and the scattered angle  $\theta^f$  ranges over  $\pm 90^\circ$ . Backscattering occurs at  $\theta^f = -45^\circ$ . Here  $h$  is the rms surface height and  $l$  is the surface correlation length. The rms surface slope is  $s = \sqrt{2}h/l = \tan 5^\circ$ .

The full wave and integral equation predictions were obtained with Monte Carlo computations and include a coherent peak at  $\theta^f = 45^\circ$ ; the coherent component is large because  $kh \ll 1$ . The finite width of the coherent peak is due to the finite length ( $80\lambda$ ) of the surfaces used in the computations.

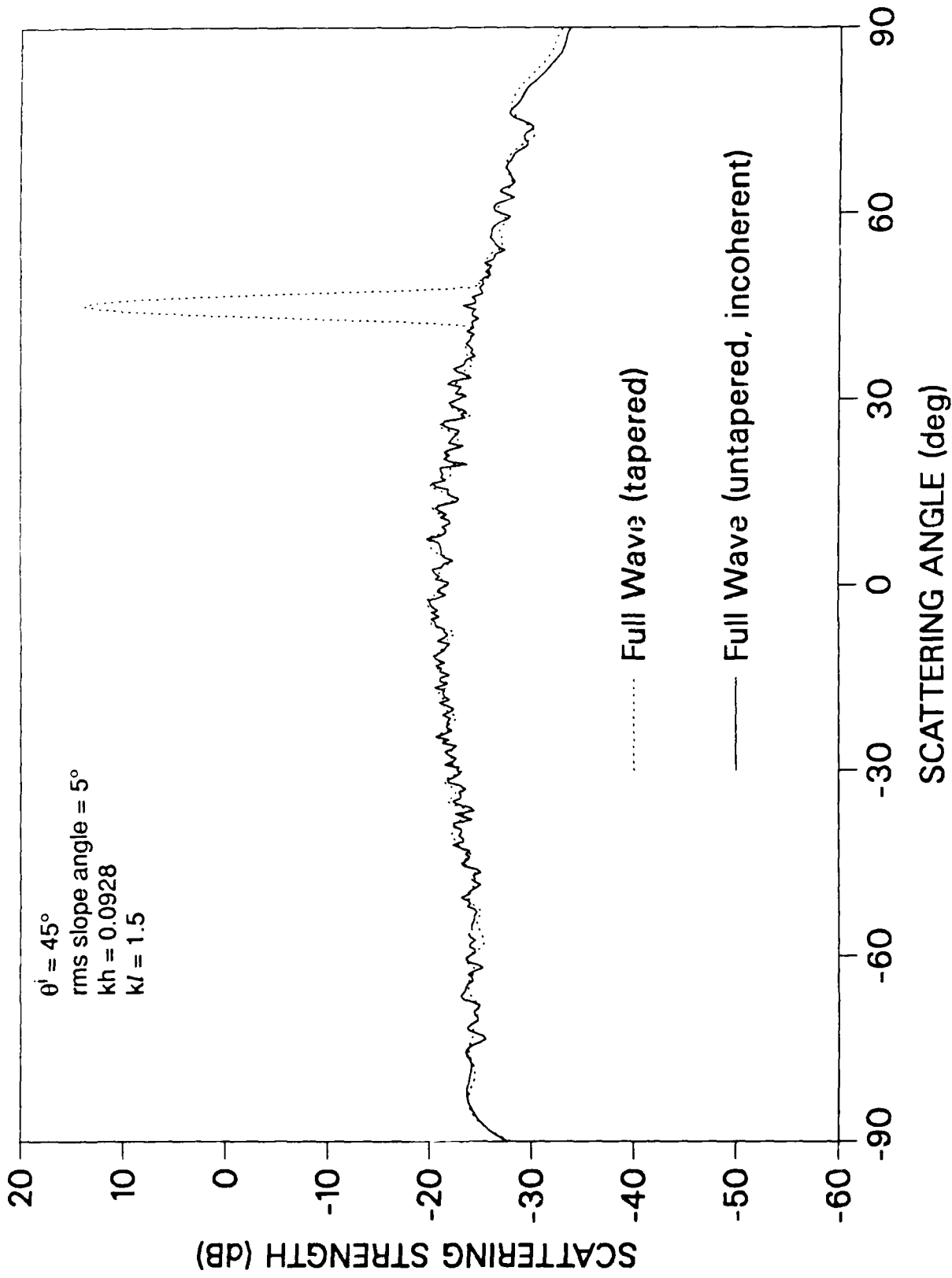
The agreement between the integral equation and perturbation theory curves shows that first-order perturbation theory is accurate for this example. The perturbation theory curve is the standard first-order theoretical result (e.g., see Ref. 3), and is not obtained by Monte Carlo methods. Only the incoherent component has been computed with perturbation theory, so the specular peak is missing. The difference between the integral equation and perturbation theory curves near  $\theta^f = \pm 90^\circ$  results from finite angular resolution effects with the integral equation prediction. Increases in surface length should lead to agreement closer to  $\theta^f = \pm 90^\circ$ .

The important result shown in slide 6 is that the full-wave prediction does not reduce to the first-order perturbation solution when the rms height and slope become small.



## Slide 7

One might argue that the use of a taper function (in order to suppress edge effects) could have significantly changed the full-wave predictions and possibly led to the discrepancy in Slide 6. Slide 7 shows a comparison of the tapered and untapered predictions for this example. The only effect of removing the taper function is the addition of side-lobes to the specular (coherent) peak. In the back direction ( $\theta' < 0^\circ$ ) the full-wave prediction is unchanged.



## Slide 8

It is possible simply to subtract out the coherent component in slide 7. This is done by first forming the coherent average, that is, the complex scattered field is ensemble averaged, then the coherent cross section is computed. Next the incoherent cross section is found by subtracting the coherent cross section from the total cross section. The result is shown in slide 8 and is compared with the original tapered full-wave prediction, which includes the coherent field.

Clearly, tapering is not responsible for the discrepancy in slide 6. The surface length has also been increased by factors of 2 and 4 with no change in the full-wave predictions.

## Full-Wave Solution: Formal Average

$$\sigma_{\text{rad}} = k \int_{-\infty}^{\infty} dx e^{-iv_x x} [M(x) - M_0 e^{-v_z^2 h^2}] \quad (2)$$

$$\bar{v} = \bar{k}_i - \bar{k}_f$$

$$M(x) = M(x_1 - x_2) = \langle F[\dot{f}(x_1)] F[\dot{f}(x_2)] e^{-iv_z[f(x_1) - f(x_2)]} \rangle \quad (3)$$

$$M_0 = \langle F[\dot{f}(x)] \rangle^2 \quad (4)$$

$$F = \frac{2 \cos \theta^i \gamma \cos \theta^f \gamma}{\cos \theta^i \gamma + \cos \theta^f \gamma} \cdot \frac{1}{\cos \gamma} = F[\dot{f}(x), \theta^i, \theta^f] \quad (5)$$

## Slide 9

Bahar also presented the results of his calculations at the Radio Science Meeting (Jan, 1988) for the example in slide 6. His results were obtained with a "formal average" method using the statistical properties of the surfaces. Bahar's method gave results in good agreement with first-order perturbation theory (and with the integral equation) and disagreed with the full-wave Monte Carlo results in slide 6. However, Bahar's approach utilized two approximations (discussed in slide 10).

We could only conjecture then that if the formal average was done without Bahar's approximations, then the results should agree with our Monte Carlo predictions. We have, therefore, computed the formal average cross section without these approximations.

We assume the surface statistics are Gaussian and stationary (independent of position). Then the incoherent radar cross section is given by (2) in terms of the moments  $M(x)$  and  $M_0$ , which are given by (3) and (4), respectively. The function  $F$ , given by (5), depends on  $f(x) = \frac{df(x)}{dx}$ ,  $\theta^i$ , and  $\theta^f$ ; in (3) and (4) this angle dependence is suppressed.

The moment  $M_0$  can be calculated easily numerically and is independent of  $x$ . The moment  $M(x)$  is much more difficult to compute, since it involves the heights and slopes at two points with separation  $x$ .

## Moment Computation

$$M(x) = \int_{-\infty}^{\infty} df_1 \int_{-\infty}^{\infty} df_2 \int_{f_{\min}}^{f_{\max}} df_1 \int_{f_{\min}}^{f_{\max}} df_2 F(\dot{f}_1) F(\dot{f}_2) e^{-lv_x(f_1 - f_2)} P(f_1, f_2, \dot{f}_1, \dot{f}_2, x) \quad (6)$$

$P(f_1, f_2, \dot{f}_1, \dot{f}_2, x)$  = joint normal probability density function

where  $f_1 = f(x_1)$ , etc.

Bahar made two approximations in previous work

(1) heights and slopes are uncorrelated

$$\rightarrow P(f_1, f_2, \dot{f}_1, \dot{f}_2, x) = P_1(f_1, f_2, x) P_2(\dot{f}_1, \dot{f}_2, x) \quad (7)$$

(2) slopes are "delta" correlated

$$\rightarrow P_2(\dot{f}_1, \dot{f}_2, x) = P(\dot{f}_1) \delta(\dot{f}_1 - \dot{f}_2) \quad (8)$$



## Slide 10

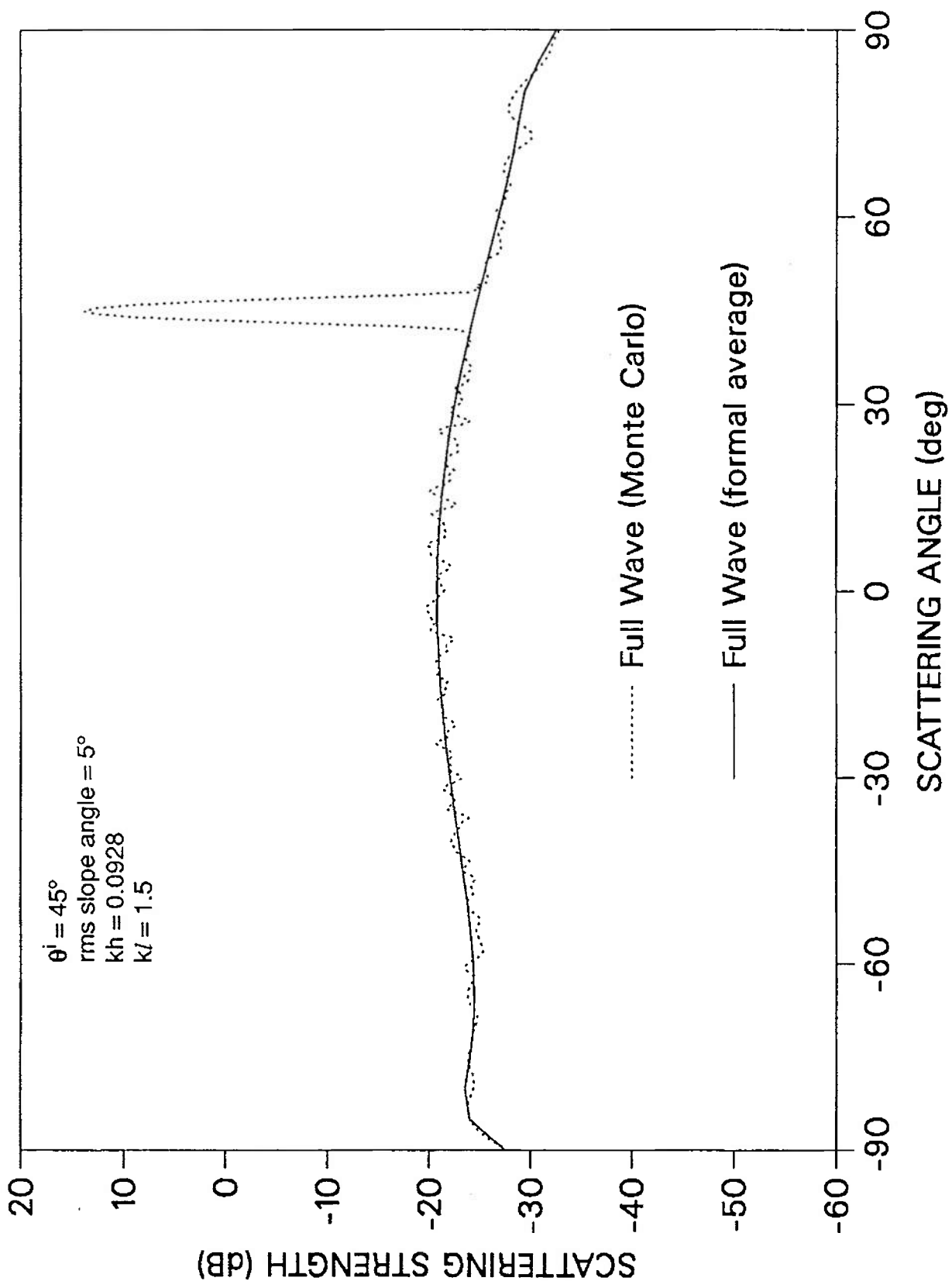
The moment  $M(x)$  is given by (6) where  $P(f_1, f_2, \dot{f}_1, \dot{f}_2, x)$  is the four-dimensional normal probability density function. (See, for example, Middleton<sup>4</sup>). In (6), the limits on the slope integrations arise because, for given  $\theta^i$  and  $\theta^f$ , the slopes are restricted so that the condition immediately below (1) is satisfied. The two height integrations (over  $f_1$  and  $f_2$ ) can be done analytically. We are then left with a three-fold integration to obtain the scattering cross section given by (2).

In previous work<sup>5</sup> Bahar has made two approximations to simplify this evaluation. First, the heights and slopes are taken as uncorrelated. This means that the probability density function factors as in (7). This approximation simplifies the algebra considerably, but one is still left with a three-fold integration to evaluate (2). Second, the slopes are taken as "delta" correlated as given by (8). Thus,  $P_2 = 0$  unless  $\dot{f}_1 = \dot{f}_2$ , and also  $P_2$  is independent of  $x$ . To be consistent with this approximation,  $M_0$  must be modified and is given by

$$M_0 = \langle F^2[f] \rangle$$

When both approximations (7) and (8) are made, only single integrations must be done.

When evaluating the complete expression (6) or when using the first approximation only, (7), special care is required in the slope integrations at small  $x$ , because the probability density functions become singular as  $x \rightarrow 0$ . Thus, separate numerical algorithms were used for small  $x$  and for general  $x$ ; detailed checks were made to ensure that the methods agreed well at the transition point.



Slide 11

In Slide 11 the full-wave formal average from (6) (i.e., accounting fully for height/slope correlations) is compared with the Monte Carlo result from slide 6. The formal average was obtained at  $5^\circ$  intervals and, as mentioned previously, does not include the coherent peak.

The agreement between the two methods is excellent.

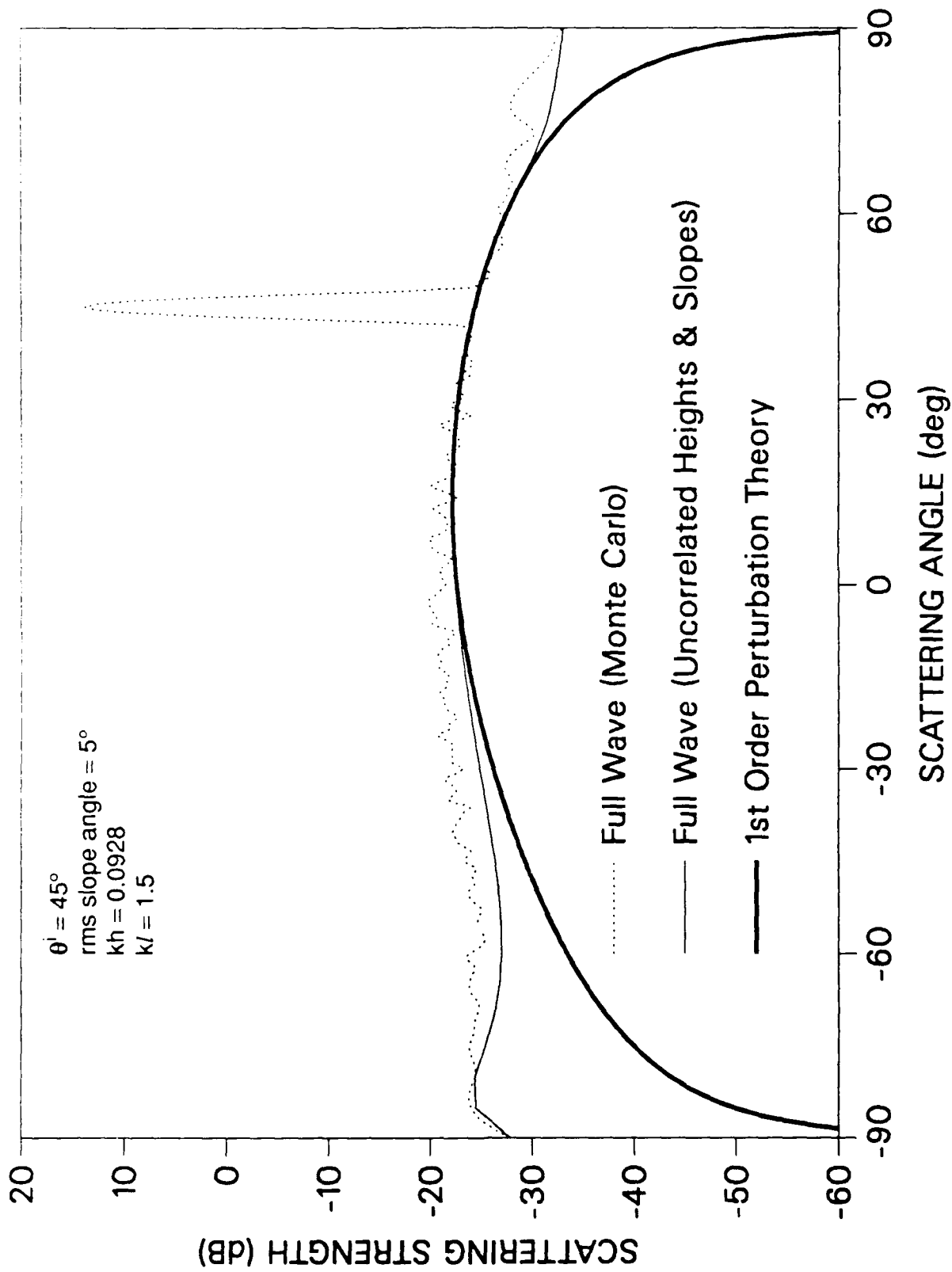
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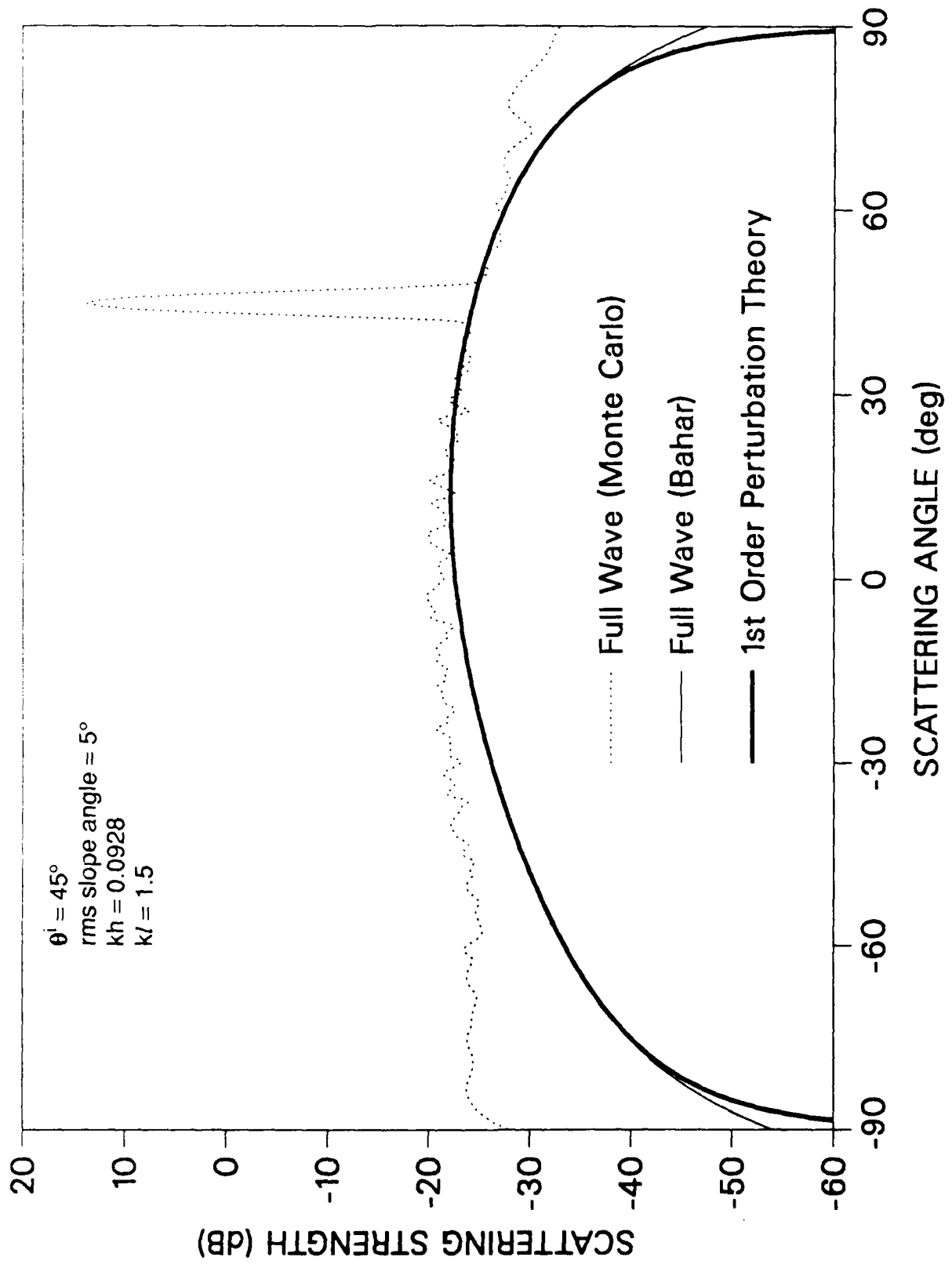
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Slide 12

Slide 12 shows the result for uncorrelated heights and slopes (7) without assuming (8). Again, the formal average curve was computed at  $5^\circ$  intervals. The "uncorrelated" prediction differs from the full-wave Monte Carlo results and is far from the first-order perturbation prediction.



Slide 13

Slide.13

Slide 13 shows the result with both approximations (7) and (8) and is labeled as "Full Wave (Bahar)". The prediction is now far from the Monte Carlo result in the back direction and in good agreement with first-order perturbation theory.



## Summary

- **Conditions:** Dirichlet boundary condition  
1-D surface  
Gaussian spectrum
- **Full-wave solution does not reduce to 1st order perturbation solution as the rms surface height and slope become small**
  - for Monte Carlo solution
  - for formally averaged solution

Slide 14

In summary, we find that for the case considered (Dirichlet boundary condition, 1-D surface, and Gaussian spectrum) the full-wave solution does not reduce to the first-order perturbation solution as the rms surface height and slope become small. This is shown by the Monte Carlo solution and by the formally averaged solution. Only when additional "approximations" are made does the formally averaged solution appear to reduce to the perturbation solution. This reduction can only be viewed as coincidental.

## References

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