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OPTIMAL ALLOCATION AND STATISTICAL CONTROL

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A Thesis in

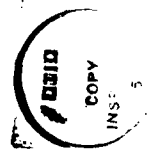Industrial Engineering and Operations Research

by

Peter J. Hofmann

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science

May 1990

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ABSTRACT

An analytic procedure was developed to predict the expected proportion of defectives produced, using a specified set of fabrication and assembly processes. This procedure was demonstrated for a General assembly process, a Peg-in-Hole assembly process and a Box-and-Cover assembly process. This procedure was compared to the formulation of a Monte Carlo simulation model and shown to require the same initial information. Two applications of the results of this procedure were developed. The first was an optimization model used to allocate a fixed amount of capital improvements to a discrete number of possible candidate assembly processes. The second was to statistically control an assembly process using a p Chart. A numerical example illustrating the procedure and its applications was also presented.

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Chapter 1

INTRODUCTION

Manufacturing processes are generally thought of as the
fabrication of component parts followed by assembly into a final
product. Traditionally, the focus of Statistical Process Control or
SPC has been to control the fabrication processes of the component
parts. The techniques used to control these processes (i.e., process
capability studies, control charts, etc.) are well developed and
widely used in industry. Assembly processes are also stochastic in
nature and can be controlled by the same statistically based
techniques that are used for fabrication processes. Since the final
product of a given manufacturing process almost always involves
assembly operations, the ability to determine if the assembly
processes are in statistical control would be very helpful in
maintaining the quality level of the final product. In addition,
knowing the expected number of defectives for a given set of
fabrication and assembly processes can be very helpful when
allocating resources for process improvements.

Several literature reviews of existing research in the areas of
assembly process variation and control yield four basic groups of
research. The first group of papers [6,7,12] studies the problem of
assembly process variation using the Monte Carlo simulation method.
While this technique is useful to get a general idea of how a

complete manufacturing process operates, it does not lend itself to the problem of optimally allocating capital improvements or real time process control. Another group of papers [8,9,10,11,14] considers the effect of assembly errors on circular arc or Novikov types of gears. These papers consider the assembly errors to be deterministic and measurable for a given gear pair. This analysis is useful for failure mode analysis and other preliminary design related tasks, but not for true manufacturing and quality control applications.

The final two groups of research are more closely related to the main thrust of this paper. The first of these groups is research into the allocation of errors in hierarchical calibration or assembly systems. Rajaraman [13] expands Crow's work [3,4] on calibration systems to assembly systems. Rajaraman develops a dynamic programming model to optimally allocate resources to each stage of assembly. The main drawback to this approach is that the assembly variations at each stage have to be independent to use a dynamic programming approach. While this is true for the linear or "telescopic" assembly process described in [13], it isn't true for a general or "tree-like" assembly process. The problem given in Chapter 3 of this paper is an example of a "tree-like" assembly process since the assembly variations in the third stage are dependent on the variations in the first stage. Rajaraman also illustrates that the manner in which errors propagate between stages is unknown and can be assumed to be a linear or nonlinear function.

The last group of research related to assembly process variation

is the analysis of basic assembly operations, used in the implementation of robotic or automated assembly stations. While there are a large number of papers to choose from, the works by Abdel-Malek [1] and Boucher [2] are used for their clear analyses of the two most basic assemblies, Peg-in-Hole and Box-and-Cover. Both authors develop probabilistic relationships for successful completion of both of these assembly operations. Their research concentrates on the probability of a robot with known repeatability being able to insert a peg into a hole or a box into a cover. They do not consider the geometries of the actual parts that are assembled and whether these parts are assembled within acceptable tolerances.

There are three basic objectives of this paper. The first objective is to analytically predict the expected number of defective units for a given set of fabrication and assembly processes. This will be accomplished using the same probabilistic analysis used for a Monte Carlo type of simulation except that the results will be determined using numerical integration and not simulation. The next objective is to develop an optimization model to allocate a limited amount of capital resources to reduce the variation in the existing assembly processes. Finally, a method for statistically controlling an assembly process is developed.

Chapter 2

ANALYTIC METHODOLOGY

The basic approach used in this method is to analyze a given assembly process from a probabilistic viewpoint. There are three basic probabilities associated with any assembly process. First, there is the probability that assembly is possible. For example, it is possible that machined components will not fit together even if there is no assembly process variation. The next event of interest is the probability that assembly will be successful in light of the existing assembly process variation. These two probabilities have been most widely researched for application in robotic assembly, as mentioned in Chapter 1. Finally, there is the probability that a successfully assembled unit meets the specified quality requirements. To have a completed assembly with the required quality, all three of these probabilities have to be satisfied.

As stated in Chapter 1, the most common methodology used to obtain an expected number of defective assemblies (i.e., an assembly that doesn't meet all three conditions above) is the Monte Carlo Simulation. In this paper, an analytic solution is found using numerical integration techniques. The method used to find this solution will be demonstrated on three basic assembly processes. The first is a General assembly process, one that doesn't contain any assembly aids (i.e., fixtures, pegs and holes, etc.). Since the

assembly variation associated with this process isn't limited by assembly aids, the assembly variation isn't restricted in any direction. Next, a Peg-in-Hole assembly will be considered. The Peg-in-Hole assembly process limits the possible variation in the X and Y directions. Finally, a Box-and-Cover assembly will be considered. The Box-and-Cover assembly process is essentially a rectangular Peg-in-Hole which limits variation in the X, Y and θ directions. It is assumed during this analysis that only one attempt is made at assembly. Also, if the assembly is successful, the part is fixed to the work, capturing the assembly errors.

Finally, a review of the basics of numerical integration including the definition of some useful notation is appropriate before beginning the analysis. The general process is to write the expression of interest as a discrete sum over each of the random variables in the expression. The limits are either the full range of the random variable (plus or minus four times the standard deviation is used as an approximation) or are defined in the expression being evaluated. Each range can then be partitioned into a discrete number (say 100) of increments and summed over all of these increments. If the increment has lower bound B1 and upper bound B2, then the point estimate (X') used in evaluation the expression is given by:

$$X' = 0.5*(B2 - B1) \tag{1}$$

The value for the expression being evaluated is then multiplied by the probability of each of the point estimators occuring, or:

$$Pr(\ B1 < X' < B2 \ ) = Pr(X') \tag{2}$$

This procedure is repeated in a nested loop fashion until all of the possible values for each random variable have been considered. The summation expressions that are obtained using this procedure may appear very complex, but they are easily programmed as a series of nested loops with a subroutine to compute the $Pr(X')$'s from equation (2). The computations are straightforward and easily computed using any personal computer.

General Assembly

An overview of the General assembly process is given in Figure 2.1. From Figure 2.1, it is clear that the General Assembly process doesn't have any assembly aids to limit variation in assembly. This means that any attempt at assembly will be successful. Therefore, if the completed assembly is within the specified tolerance, it will be of the required level of quality. Using the nomenclature given in Figure 2.1, the probability of the part being within the tolerance limits is given by the intersection of the following statements:

$$Pr\{T_X-\Delta<.5X_W-(\epsilon_X+d*\cos(\alpha-\theta))<T_X+\Delta\} \tag{3}$$

$$Pr\{T_X-\Delta<.5X_W-(\epsilon_X+d*\cos(\alpha+\theta))<T_X+\Delta\} \tag{4}$$

$$Pr\{T_Y-\Delta<.5Y_W-(\epsilon_Y+d*\cos(\beta-\theta))<T_Y+\Delta\} \tag{5}$$

$$Pr\{T_Y-\Delta<.5Y_W-(\epsilon_Y+d*\cos(\beta+\theta))<T_Y+\Delta\} \tag{6}$$

Statements (3) through (6) can be rewritten as follows:

$$Pr\{T_X-\Delta+\epsilon_X+d*\cos(\alpha-\theta)<.5X_W<T_X+\Delta+\epsilon_X+d*\cos(\alpha-\theta)\} \tag{7}$$

$$Pr\{T_X-\Delta+\epsilon_X+d*\cos(\alpha+\theta)<.5X_W<T_X+\Delta+\epsilon_X+d*\cos(\alpha+\theta)\} \tag{8}$$
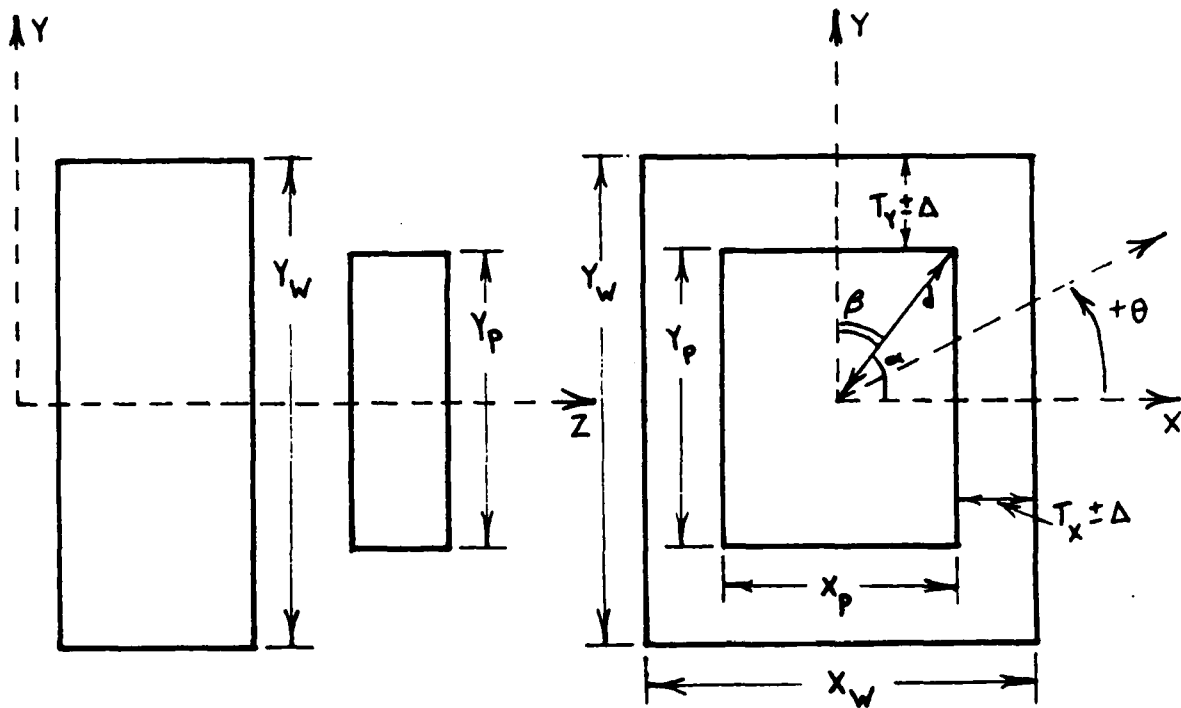
$\alpha = \cos^{-1}(.5X_P/d)$

$\beta = \cos^{-1}(.5Y_P/d)$

$d = [(.5X_P)^2 + (.5Y_P)^2]^{0.5}$

$\epsilon_X = \epsilon_Y = 0$ as shown above

T_X, T_Y and Δ are given constants

X_P, X_W, Y_P, Y_W, ϵ_X, ϵ_Y and θ are independent, normally distributed
random variables with known parameters

Figure 2.1 : General Assembly

$$Pr\{T_Y-\Delta+\epsilon_Y+d*\cos(\beta-\theta)<.5Y_W<T_Y+\Delta+\epsilon_Y+d*\cos(\beta-\theta)\} \qquad (9)$$

$$Pr\{T_Y-\Delta+\epsilon_Y+d*\cos(\beta+\theta)<.5Y_W<T_Y+\Delta+\epsilon_Y+d*\cos(\beta+\theta)\} \qquad (10)$$

Finally, statements (7) through (10) can be written as multiple sums:

$$\sum_{\epsilon_X}\ (\sum_{X_P}\ (\sum_{Y_P}\ (\sum_{\theta}(Pr\{T_X-\Delta+\epsilon_X+d*\cos(\alpha-\theta)<.5X_W<T_X+\Delta+\epsilon_X+d*\cos(\alpha-\theta)\})*Pr\{\theta\})$$
$$*Pr\{Y_P\})*Pr\{X_P\})*Pr\{\epsilon_X\} \qquad (11)$$

$$\sum_{\epsilon_X}\ (\sum_{X_P}\ (\sum_{Y_P}\ (\sum_{\theta}(Pr\{T_X-\Delta+\epsilon_X+d*\cos(\alpha+\theta)<.5X_W<T_X+\Delta+\epsilon_X+d*\cos(\alpha+\theta)\})*Pr\{\theta\})$$
$$*Pr\{Y_P\})*Pr\{X_P\})*Pr\{\epsilon_X\} \qquad (12)$$

$$\sum_{\epsilon_Y}\ (\sum_{X_P}\ (\sum_{Y_P}\ (\sum_{\theta}(Pr\{T_Y-\Delta+\epsilon_Y+d*\cos(\beta-\theta)<.5Y_W<T_Y+\Delta+\epsilon_Y+d*\cos(\beta-\theta)\})*Pr\{\theta\})$$
$$*Pr\{Y_P\})*Pr\{X_P\})*Pr\{\epsilon_Y\} \qquad (13)$$

$$\sum_{\epsilon_Y}\ (\sum_{X_P}\ (\sum_{Y_P}\ (\sum_{\theta}(Pr\{T_Y-\Delta+\epsilon_Y+d*\cos(\beta+\theta)<.5Y_W<T_Y+\Delta+\epsilon_Y+d*\cos(\beta+\theta)\})*Pr\{\theta\})$$
$$*Pr\{Y_P\})*Pr\{X_P\})*Pr\{\epsilon_Y\} \qquad (14)$$

These expressions are computed using the numerical procedure described in the previous section. Equations (3) - (6) will be used several times in the next two sections and will be abbreviated as TOL1 - TOL4, respectively. In addition, it should be noted that if the torsional errors of assembly are assumed to be insignificant, the results in this and the next two sections are <u>greatly</u> simplified.

Peg-in-Hole Assembly

An overview of a Peg-in-Hole assembly process is given in Figure 2.2. From Figure 2.2, the probability that the part can fit into the work is given by:

$$Pr\{D_p<D_h\} \qquad (15)$$

Likewise, the probability that the parts will be assembled is given
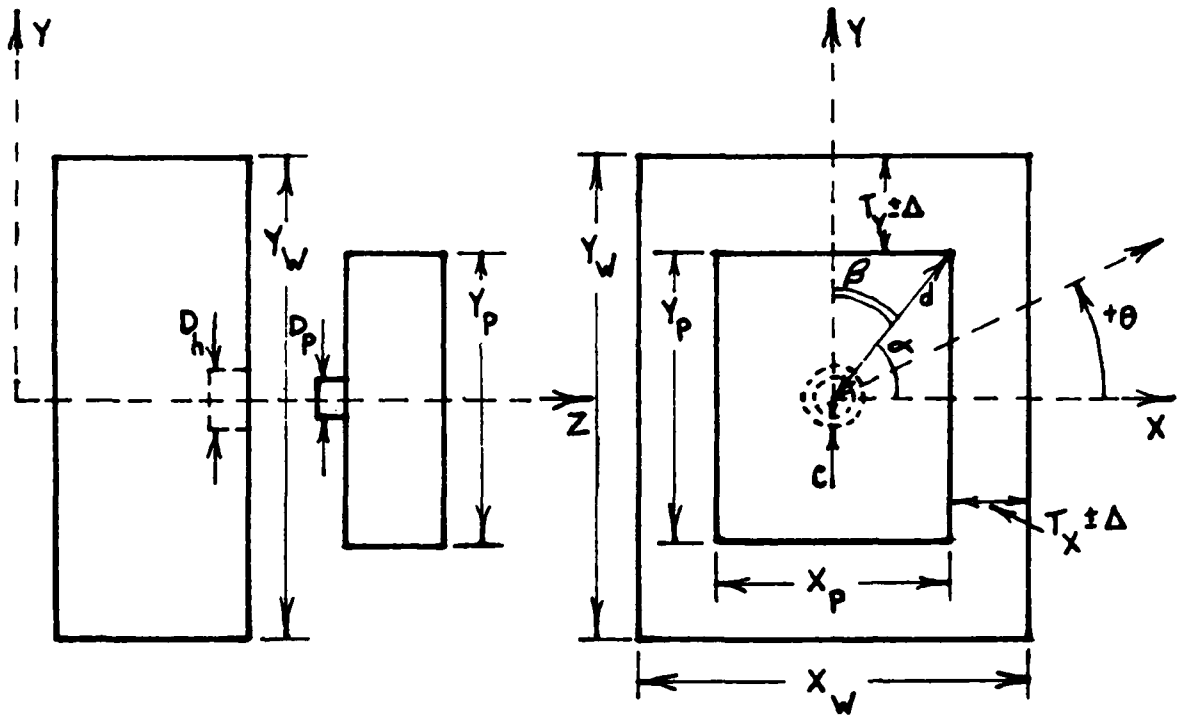
$\alpha = \cos^{-1}(.5X_p/d)$

$\beta = \cos^{-1}(.5Y_p/d)$

$d = [(.5X_p)^2 + (.5Y_p)^2]^{0.5}.$

$c = 0.5 \pm (D_h - D_p)$

$\epsilon_X = \epsilon_Y = 0$ as shown above

T_X, T_Y and Δ are given constants

X_p, X_W, Y_p, Y_W, D_h, D_p, ϵ_X, ϵ_Y and θ are independent, normally distributed random variables with known parameters

Figure 2.2 : Peg-in-Hole Assembly

by the following two probability statements:

$$\Pr\{-c<\epsilon_X<c\} \tag{16}$$

$$\Pr\{-c<\epsilon_Y<c\} \tag{17}$$

Finally, the probability that the assembled unit is within tolerance is given by the following:

$$\Pr\{TOL1 \mid -c<\epsilon_X<c\} \tag{18}$$

$$\Pr\{TOL2 \mid -c<\epsilon_X<c\} \tag{19}$$

$$\Pr\{TOL3 \mid -c<\epsilon_Y<c\} \tag{20}$$

$$\Pr\{TOL4 \mid -c<\epsilon_Y<c\} \tag{21}$$

The probability of manufacturing an acceptable unit is equal to the intersection of equations (15) through (21).

Equations (15) through (17) are easily computed using a numerical technique for normal probabilities. The last four conditional probabilities need to be massaged into a more usable form. The probability that bottom corner of the part in the assembled unit will be within tolerance in the X direction can be written as follows:

$$\frac{\Pr\{T_X-\Delta-.5X_W+d*\cos(\alpha-\theta)<-\epsilon_X<T_X+\Delta-.5X_W+d*\cos(\alpha-\theta),-c<\epsilon_X<c\}}{\Pr\{-c<\epsilon_X<c\}} \tag{22}$$

The denominator of this expression is equation (16) and is easily computed. The numerator of this expression can be rewritten as follows:

$$\Pr\{-(T_X+\Delta-.5X_W+d*\cos(\alpha-\theta))<\epsilon_X<-(T_X-\Delta-.5X_W+d*\cos(\alpha-\theta)),-c<\epsilon_X<c\}$$

$$\tag{23}$$

Finally, this probability statement can be written as the following

multiple sum:

$$\Sigma \; (\Sigma \; (\Sigma \; (\; \Sigma(Pr\{max[-c,-(T_X+\Delta-.5X_W+d*cos(\alpha-\theta))]<\epsilon_X<min[c,-(T_X-\Delta-$$
$$c \quad X_P \quad Y_P.5X_W$$
$$.5X_W+d*cos(\alpha-\theta))])\})*Pr\{.5X_W\})*Pr\{Y_P\})*Pr\{X_P\})*Pr\{c\}$$

$$(24)$$

As with the General assembly, this summation is computed using the numerical procedure outlined in the first section of this chapter. The probabilities that the top corner will be within tolerance in the X direction and both corners in the Y direction (statements (19)-(21)) are computed in the same manner outlined above. The probability of producing a defective assembly is computed in the same manner used for the General assembly process.

Box-and-Cover Assembly

An overview of a Box and Cover assembly process is presented in Figure 2.3. Using the notation given in Figure 2.3, the probability that the part can fit into the work is given by the following two probability statements:

$$Pr\{X_B<X_C\} \tag{25}$$
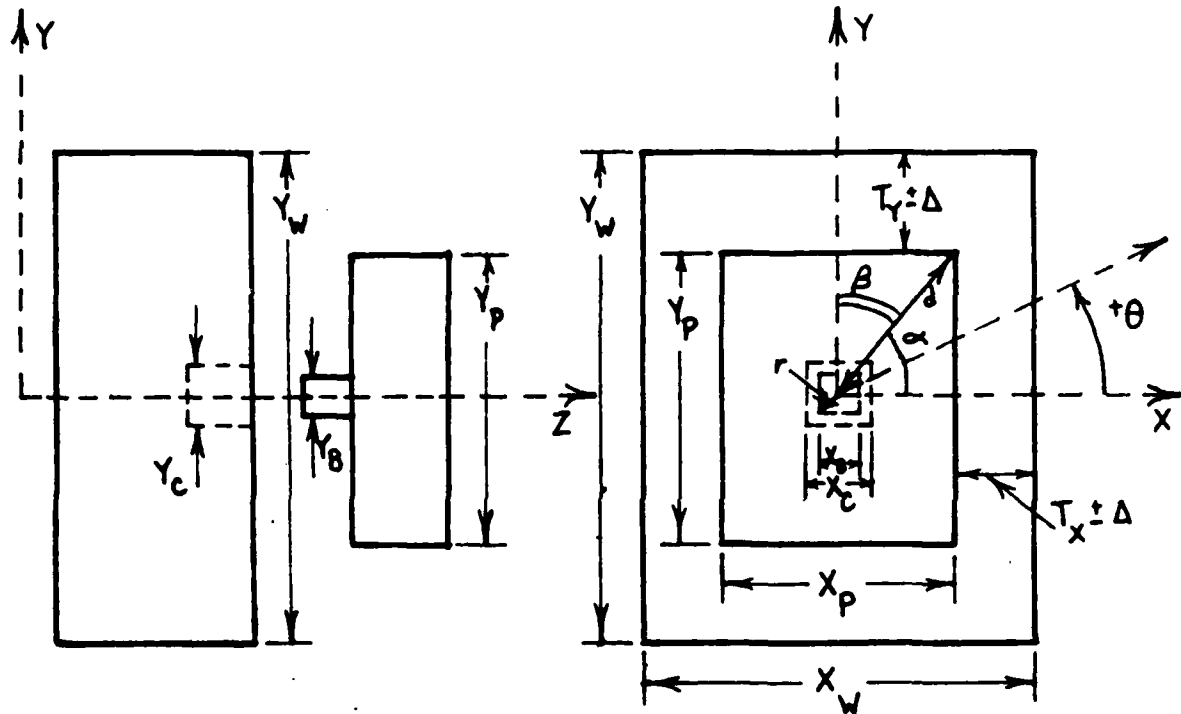
$$Pr\{Y_B<Y_C\} \tag{26}$$

Likewise, the probability that the parts will be assembled on the first attempt [2] is given by the following probabilities:

$$Pr\{\theta<cos^{-1}(.5X_B/r)-cos^{-1}((.5X_B+dX)/r) \mid dX<dY\} \tag{27}$$

$$Pr\{\theta<cos^{-1}(.5Y_B/r)-cos^{-1}((.5Y_B+dY)/r) \mid dY<dX\} \tag{28}$$

where $\quad dX = .5(X_C-X_C)-\epsilon_X$

$$dY = .5(Y_C-Y_C)-\epsilon_Y$$

$\alpha = \cos^{-1}(.5X_P/d)$

$\beta = \cos^{-1}(.5Y_P/d)$

$d = [(.5X_P)^2 + (.5Y_P)^2]^{0.5}$

$r = [(.5X_B)^2 + (.5Y_B)^2]^{0.5}$

$\epsilon_X = \epsilon_Y = 0$ as shown above

T_X, T_Y and Δ are given constants

X_P, X_W, X_B, X_C, Y_P, Y_W, Y_B, Y_C, ϵ_X, ϵ_Y and θ are independent, normally distributed random variables with known parameters

Figure 2.3 : Box-and-Cover Assembly

Finally, the probability that the assembled unit will be within tolerance is given by the following eight probability statements:

$$Pr\{TOL1 \mid \theta < \cos^{-1}(.5X_B/r) - \cos^{-1}((.5X_B+dX)/r), dX<dY\} \tag{29}$$

$$Pr\{TOL1 \mid \theta < \cos^{-1}(.5Y_B/r) - \cos^{-1}((.5Y_B+dY)/r), dY<dX\} \tag{30}$$

$$Pr\{TOL2 \mid \theta < \cos^{-1}(.5X_B/r) - \cos^{-1}((.5X_B+dX)/r), dX<dY\} \tag{31}$$

$$Pr\{TOL2 \mid \theta < \cos^{-1}(.5Y_B/r) - \cos^{-1}((.5Y_B+dY)/r), dY<dX\} \tag{32}$$

$$Pr\{TOL3 \mid \theta < \cos^{-1}(.5X_B/r) - \cos^{-1}((.5X_B+dX)/r), dX<dY\} \tag{33}$$

$$Pr\{TOL3 \mid \theta < \cos^{-1}(.5Y_B/r) - \cos^{-1}((.5Y_B+dY)/r), dY<dX\} \tag{34}$$

$$Pr\{TOL4 \mid \theta < \cos^{-1}(.5X_B/r) - \cos^{-1}((.5X_B+dX)/r), dX<dY\} \tag{35}$$

$$Pr\{TOL4 \mid \theta < \cos^{-1}(.5Y_B/r) - \cos^{-1}((.5Y_B+dY)/r), dY<dX\} \tag{36}$$

The probability of manufacturing an acceptable unit is equal to the intersection of equations (25) through (36).

The first two equations, (25) and (26), are easily computed using a numerical technique for normal probabilities. Equation 27 can be written as follows:

$$\frac{Pr\{\theta < \cos^{-1}(.5X_B/r) - \cos^{-1}((.5X_B+dX)/r), dX<dY\}}{Pr\{dX<dY\}} \tag{37}$$

The denominator of this expression is easily obtained in the same way as equations (25) and (26). The numerator can be rewritten as the following multiple sum:

$$\sum_{X_C} (\sum_{X_B} (\sum_{Y_C} (\sum_{Y_B} (\sum_{\epsilon_Y} (\sum_{\epsilon_X=\epsilon_X LB}^{\infty} (Pr\{\theta<\cos^{-1}(.5X_B/r) - \cos^{-1}((.5X_C-\epsilon_X)/r)\})$$
$$*Pr\{\epsilon_X\})*Pr\{\epsilon_Y\})*Pr\{Y_B\})*Pr\{Y_C\})*Pr\{X_B\})*Pr\{X_C\} \tag{38}$$

where $\epsilon_X LB = .5(X_C-X_B) - .5(Y_C-Y_B) + \epsilon_Y$

Equation (28) is almost identical to equation (27) and is evaluated in a similar manner.

Statements (29) through (36) are all of the same form and the evaluation of one sufficiently illustrates the procedure used on all eight. Equation (29) can be written as follows:

$$\frac{\Pr\{T_X-\Delta<.5X_W-(\epsilon_X+d*\cos(\alpha-\theta))<T_X+\Delta,\theta<\cos^{-1}(.5X_B/r)-\cos^{-1}((.5X_B+dX)/r),dX<dY\}}{\theta<\cos^{-1}(.5X_B/r)-\cos^{-1}((.5X_B+dX)/r),dX<dY\}} \quad (39)$$

The denominator of this expression is identical to the numerator of equation 37, which has already been computed. The numerator can be rewritten as the following multiple sum:

$$\sum_{X_C}(\sum_{X_B}(\sum_{X_P}(\sum_{Y_C}(\sum_{Y_B}(\sum_{Y_P}(\sum_{\epsilon_Y}(\sum_{\epsilon_X LB}^{\epsilon_X UB}(\sum_{\theta LB}^{\theta UB}(\Pr\{T_X-\Delta+\epsilon_X+d*\cos(\alpha-\theta)<.5X_W<T_X+\Delta+\epsilon_X+d*\cos(\alpha-\theta)\} \quad (40)$$

where

$$\epsilon_X LB = .5*(X_C-X_B)-.5*(Y_C-Y_B)+\epsilon_Y$$

$$\epsilon_X UB = +\infty$$

$$\theta LB = -\infty$$

$$\theta UB = \cos^{-1}(.5X_B/r)-\cos^{-1}((.5X_B+dX)/r)$$

The multiple sums given in equations (38) and (40), unlike the other multiple sums shown previously, aren't readily computable. An approximate solution can be obtained by using the relationships for the General assembly and truncating the distributions for ϵ_X, ϵ_Y and θ. For example, the distribution for θ could be truncated between $\pm\theta_{max}$ where θ_{max} is the maximum allowable torsional error. θ_{max} occurs when $\epsilon_X = \epsilon_Y = 0$.

Comparison to Simulation

The expressions derived using this analytical approach may

appear intimidating at first glance. However, they are essentially the same expressions required to accomplish a Monte Carlo simulation of the problem. In fact, that is precisely what Boucher [2] does with them. The approach used in this chapter is just a small manipulation of information required for a simulation analysis, but it produces an analytic result and not a confidence interval for the parameter of interest.

Since the numerical procedure used in this method is a discrete approximation of a continuous probability density function, this analytic result is not, strictly speaking, exact. However, there are many routinely used procedures that are also inexact algorithms. Commonly used procedures such as long division and finding the root of an expression are actually algorithms that follow a series of computations until a specified stopping rule is met. In light of this, the analytic results obtained using this method are much more useful than the random results of a simulated analysis. The analytic results for the General and Peg-in-Hole assemblies are also readily obtained using a personal computer and don't require large amounts of mainframe computer time. Since most of the work required for using this method is accomplished when setting up a simulation, why find a simulated, random answer when a good analytic result is readily available?

Optimization Model

Since an analytical result for the probability of assembling a defective unit is obtainable, several useful applications can be made. The first application is optimally allocating limited capital resources to reduce the variance of the assembly processes for the system under review. The general objective of the problem is to minimize the yearly costs due to capital depreciation and production of defective units.

For this analysis, it is assumed that a discrete number of new processes are available to choose from and each of these has an initial cost that is inversely proportional to the process variance. The new equipment is assumed to be depreciated over a known time period while the equipment currently in use is assumed to be fully depreciated. For this reason, the cost of the current equipment is assumed to be zero for the model. The optimization is summarized below as a math programming problem:

$$\min \sum_{i} \sum_{j} (c_{ij}/k)*x_{ij} + P* \sum_{j_1=1}^{n_1} x_{1j}(\sum_{j_2=2}^{n_2} x_{2j} \ldots (\sum_{j_m=1}^{n_m} x_{mj}$$
$$*(D(x_{1j}, x_{2j}, \ldots, x_{mj})) \ldots) \tag{41}$$

subject to $\sum_{i} \sum_{j} c_{ij}x_{ij} \leq$ investment amount

$\sum_{j} x_{ij}=1$ for all $i=1,2,\ldots,m$

$x_{ij}= \begin{cases} 1 & \text{if process } j \text{ is selected at stage } i \\ 0 & \text{otherwise} \end{cases}$

where $i = 1,2,\ldots,m$ assembly stages

$j = 1,2,\ldots,n_i$ processes for each stage i

k — depreciation factor

P — cost incurred due to a defective assembly

c_{ij} — cost of implementing process j at stage i

$D(x_{1j}, x_{2j}, \ldots, x_{mj})$ — number of defectives produced for a given combination of processes

It is clear that the objective of this problem is nonlinear, making solution by the usual linear techniques impossible. However, a simple manipulation that transforms this type of problem into a straight linear (0-1 integer) program is illustrated in Chapter 3. Solution is then possible on a personal computer using any commercially available software package for linear programming problems.

p Charts

Another application of the analytic results obtained using the method outlined earlier in this chapter is statistically controlling an assembly process. Since the analytically obtained result is a proportion, controlling the assembly process using a p Chart is appropriate. The expected proportion of defectives found with the analytic approach is used as the value for p_0. The statistical basis for this type of control chart is the following hypothesis:

$H_0 : p = p_0$

$H_1 : p \neq p_0$

From Grant and Leavenworth [5], the control limits for this chart are

given by:

$$UCL = p_o + Z_{\alpha/2}((p_o-(1-p_o))/n)^{0.5} \tag{42}$$

$$LCL = p_o - Z_{\alpha/2}((p_o-(1-p_o))/n)^{0.5} \tag{43}$$

For $\alpha = 0.0027$, $Z_{\alpha/2}$ is equal to three. An example of this procedure is presented in Chapter 3.

Chapter 3

NUMERICAL EXAMPLE

Problem Definition

The numerical example used in this chapter is an assembly consisting of four components that are assembled in three stages. The geometry of the problem is given in Figure 3.1. Several assumptions are used during the analysis of this problem. First, the person or machine performing the assembly operation initially determines the midpoint of the part to be joined to the base stage. The assembler then finds the alignment point on the base stage. The location of this alignment point is obtained using one-half of the nominal for the part to be joined. The assembler then attempts to join the two parts along these target lines. The assembler does this task with a repeatability error, ϵ_i for i - 1, 2, 3. The ϵ_i's are assumed to be normally distributed with mean zero and variance σ_i^2. Once the parts make contact, they are affixed to each other in some manner (glue, weld, etc.). Finally, the manufacturing processes used to manufacture the four components are also assumed to be normally distributed with known mean and variance. The mean is assumed to be equal to the nominal dimension of the part and the variance is assumed to equal one-sixth of the total tolerance.
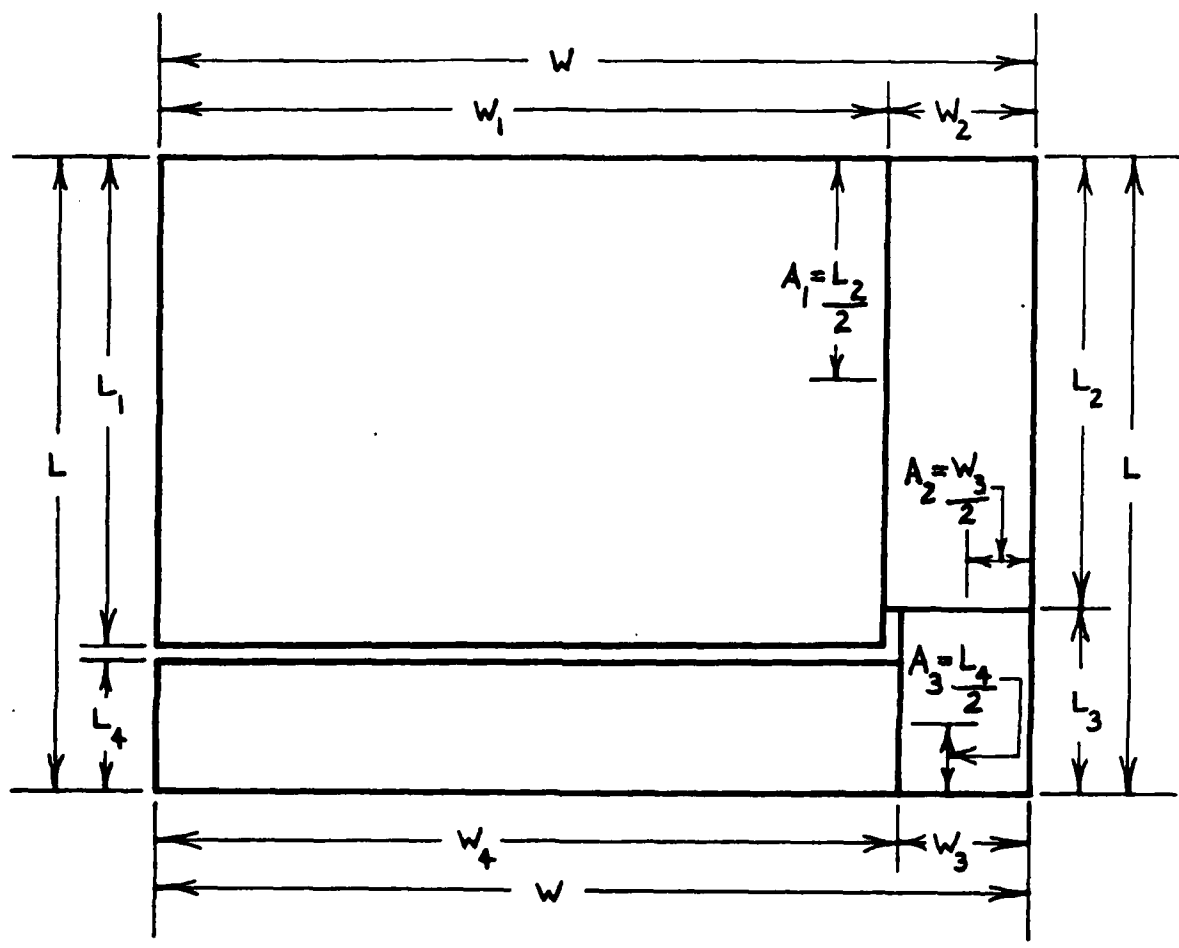
where   L = 18 ± 0.212   W = 24 ± 0.212

          L_1= 14 ± 0.15 W_1= 20 ± 0.15

          L_2= 12 ± 0.15 W_2= 4 ± 0.15

          L_3= 6 ± 0.10 W_3= 3.8 ± 0.09

          L_4= 20.2 ± 0.15 W_4= 3.8 ± 0.09

          $\epsilon_1 = \epsilon_2 = \epsilon_3 = NID(0.0, 0.03162^2)$

Figure 3.1 : Numerical Example

Determining the Expected

Number of Defectives

The first part of the analysis determine the probability of a defective assembly being produced using the current processes. Since there are many combinations that can cause an assembly to be defective, it is easiest to determine the probability of making a defective assembly using the following relationship:

Pr(defective assembly) = 1 - Pr(acceptable assembly)

The probability of having an acceptable assembly is the intersection of the following probability statements:

$$Pr\{W-\Delta<W_1+W_2<W+\Delta\} \tag{45}$$

$$Pr\{L-\Delta<A_1+L_2/2-\epsilon_1+L_3<L+\Delta \mid A_1\geq L_2/2+\epsilon_1\} \tag{46}$$

$$Pr\{L-\Delta<L_2+L_3<L+\Delta \mid A_1<L_2/2+\epsilon_1\} \tag{47}$$

$$Pr\{A_2+W_3/2-\epsilon_2\leq W_2\} \tag{48}$$

$$Pr\{W-\Delta<A_2+W_3/2-\epsilon_2+W_4<W+\Delta \mid A_2\geq W_3/2+\epsilon_2\} \tag{49}$$

$$Pr\{W-\Delta<W_3+W_4<W+\Delta \mid A_2<W_3/2+\epsilon_2\} \tag{50}$$

$$Pr\{L-\Delta<A_1+L_2/2-\epsilon_1+L_3+L_4/2-\epsilon_3-A_3<L+\Delta \mid A_3<L_4/2-\epsilon_3,A_1\geq L_2/2+\epsilon_1\} \tag{51}$$

$$Pr\{L-\Delta<L_2+L_3+L_4/2-\epsilon_3-A_3<L+\Delta \mid A_3<L_4/2-\epsilon_3,A_1<L_2/2+\epsilon_1\} \tag{52}$$

$$Pr\{A_1+L_2/2-\epsilon_1+L_3-L_1\geq L_4/2+A_3+\epsilon_3\} \tag{53}$$

The conditional probabilities are determined using numerical integration.

The probability statements that don't contain any conditional statements are easily computed using numerical techniques. To use

numerical integration, the conditional probabilities have to be rewritten in a more usable form. For example, equation (46) can be rewritten as follows:

$$\frac{Pr(L-\Delta < A_1 + L_2/2 - \epsilon_1 + L_3 < L + \Delta, A_1 \geq L_2/2 + \epsilon_1)}{Pr(A_1 \geq L_2/2 + \epsilon_1)} \qquad (54)$$

The denominator of this expression is easily computed. The numerator must be manipulated into a single probability statement on one random variable as shown below:

$$Pr(L-\Delta - A_1 - L_3 + \epsilon_1 < L_2/2 < \min[L + \Delta - A_1 - L_3 + \epsilon_1, A_1 - \epsilon_1]) \qquad (55)$$

This statement is computed by numerically integrating over all possible values for each random variable:

$$\sum_{\epsilon_1} \left(\sum_{L_3} (Pr(L-\Delta - A_1 - L_3 + \epsilon_1 < L_2/2 < \min[L + \Delta - A_1 - L_3 + \epsilon_1, A_1 - \epsilon_1]) * Pr(L_3)) * Pr(\epsilon_1) \right) \qquad (56)$$

The actual value for the original probability is given by dividing this expression by the original denominator. The listing of the BASIC program used to do the numerical integration in equation (56) can be found in the Appendix. The solution of all the remaining conditional in this example can be found using the same manner used for equation (46). It is helpful to combine random variables into joint probability distributions wherever possible to reduce the number of summations required to obtain a solution.

After determining the probability that the current process will produce a defective assembly, the following relationship is used to determine the expected number of defective assemblies produced:

E(# of defective assemblies produced/given time) =

Pr(defective assembly)*(tot # produced/time)

For this example, the probability of a defective assembly is 0.02541 and the yearly production rate is 10,000 units. This yields 254.1 expected defective assemblies per year using the current processes.

Sensitivity Analysis

A sensitivity analysis performed on the assembly process in this example is accomplished using the design of a complete 2^6 factorial experiment. The six factors studied are the mean and variance for the repeatability errors for each of the three assembly processes in this example. The two levels for each mean are 0.01 and -0.01. The two levels for each variance are 0.0004 and 0.0001. The results are summarized in Table 3.1.

Table 3.1 : Sensitivity Analysis

factor	effect = (Σhigh-Σlow)/64
μ_1	-0.00195
μ_2	0.00011
μ_3	-0.00183
σ_1^2	-0.00066
σ_2^2	-0.00009
σ_3^2	-0.00087

From Table 3.1, it is clear that the first and third assembly processes have the greatest effect on the probability of producing a defective assembly. As may be expected, a change in the means has a greater effect than a change in the variances.

Using the results of this sensitivity analysis, one important conclusion can be inferred. Priority for any reductions in variance should be given to the first and third assembly processes. This conclusion is confirmed in the next section of this chapter.

Optimization Problem

The premise of the optimization problem considered in this section is that the assembly considered is in production with known process variances. There are a discrete number of processes to choose from to improve the assembly operations. The initial costs of these processes are inversely proportional to their variances. The current and potential processes are summarized in Table 3.2. It is assumed that a finite amount of capital is available for improvements. It is also assumed that each defective assembly produced causes the company to incur a cost of P dollars.

We can further assume that a depreciating factor is applied to the capital outlays, say one-fifth, corresponding to a five year straight line depreciation of the new equipment. Finally, if we don't consider the time value of money, the general form of the optimization problem is:

$$\min \sum_{i=1}^{3} \sum_{j=1}^{4} (c_{ij}/5) * x_{ij} + P * \sum_{j_1=1}^{4} x_{1j} \left(\sum_{j_2=1}^{4} x_{2j} \left(\sum_{j_3=1}^{4} x_{3j} * \{D(x_{1j}, x_{2j}, x_{3j})\} \right) \right)$$

$$\text{subject to} \quad \sum_{j_1=1}^{4} x_{1j} = 1$$

$$\sum_{j_2=1}^{4} x_{2j} = 1$$

Table 3.2 : Current and Potential Processes

Process 1

current:	$c_{10} = \$0$	$\sigma = 0.03162$
candidate 1:	$c_{11} = \$2000$	$\sigma = 0.02236$
candidate 2:	$c_{12} = \$4000$	$\sigma = 0.01581$
candidate 3:	$c_{13} = \$6000$	$\sigma = 0.01291$

Process 2

current:	$c_{20} = \$0$	$\sigma = 0.03162$
candidate 1:	$c_{21} = \$2000$	$\sigma = 0.02236$
candidate 2:	$c_{22} = \$4000$	$\sigma = 0.01581$
candidate 3:	$c_{23} = \$6000$	$\sigma = 0.01291$

Process 3

current:	$c_{30} = \$0$	$\sigma = 0.03162$
candidate 1:	$c_{31} = \$2000$	$\sigma = 0.02236$
candidate 2:	$c_{32} = \$4000$	$\sigma = 0.01581$
candidate 3:	$c_{33} = \$6000$	$\sigma = 0.01291$

$$\sum_{j_3=1}^{4} x_{3j} - 1$$

$$\sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij}x_{ij} \leq \text{investment amount}$$

$$x_{ij} = \begin{cases} 1 \text{ if process } j \text{ is selected at stage } i \\ 0 \text{ otherwise} \end{cases}$$

This problem can be made into a straight integer programming problem by introducing some constraints to ensure that one and only one process is chosen as follows:

$$P*x_{11}*x_{21}*x_{31}*(D(x_{11}, x_{21}, x_{31})$$

is rewritten as:

$$P*x_d*(D(\text{process combination } d))$$

$$x_d \leq x_{11}$$

$$x_d \leq x_{21}$$

$$x_d \leq x_{31}$$

$$x_d \geq x_{11}+x_{21}+x_{31}-2$$

The only drawback to this problem is that it requires total enumeration of all of the possible expected number of defectives. Although this sounds monumental, it is fairly easily done since the basic probability statements are functions of one or two of the assembly processes.

A parametric analysis of this problem is performed for several levels of "defective costs" and investment capital. The results are summarized in Table 3.3. The results of this analysis confirm the sensitivity analysis performed earlier. It is optimal to reduce the

Table 3.3 : Parametric Analysis

Cost per Defective Assembly (P)

		$100	$200	$400	$800
	$5000	$19280	$37760	$74720	$148640
		$2000	$2000	$2000	$2000
		$0	$0	$0	$0
		$2000	$2000	$2000	$2000
i n v e s t m e n t	$10000	$17140	$32280	$62560	$123120
		$4000	$4000	$4000	$4000
		$2000	$2000	$2000	$2000
		$4000	$4000	$4000	$4000
a m o u n t	$15000	$17080	$31380	$59960	$117120
		$4000	$6000	$6000	$6000
		$2000	$2000	$2000	$2000
		$6000	$6000	$6000	$6000
	$20000	$17080	$31380	$59840	$116080
		$4000	$6000	$6000	$6000
		$2000	$2000	$6000	$6000
		$6000	$6000	$6000	$6000

where each cell represents: Z^*
c_{1j}^*
c_{2j}^*
c_{3j}^*

variance of the first and third assembly processes before the second process. It is also clear from this analysis that it may not be advisable to expend all of the capital available for process improvement, depending on the cost incurred per defective assembly. This analysis could also be used to justify a request for increased availability of capital improvement funds.

p Charts

A sample p Chart is presented in this section to illustrate the implementation suggested at the end of Chapter 2. The premise of this illustration is that a daily sample of 50 assemblies is inspected and the proportion of defectives is recorded on the control chart. Using the expected number of defectives found earlier in this chapter (= 0.0254) as p_0 and an α of 0.0027, the control limits are found using equations (42) and (43). These control limits are given below:

UCL = 0.0922

LCL =-0.0414 or 0.0

A simulated manufacturing process is run for 20 days. For the first ten days, all of the fabrication and assembly processes are kept in control. On the eleventh day, a 0.06324" shift in the mean of the first assembly process (i.e., ϵ_1 was now distributed N(0.06324,0.001) instead of N(0.0,0.001)) is introduced. This shift remains in effect through the 20th day. The p Chart for this time period is shown in

Figure 3.2. From Figure 3.2, this simulated process is detected as being out of control on the 15th day. Once again, this result is not unique, since each simulation run is different. This chart is presented solely as an illustration of the method developed in this paper.
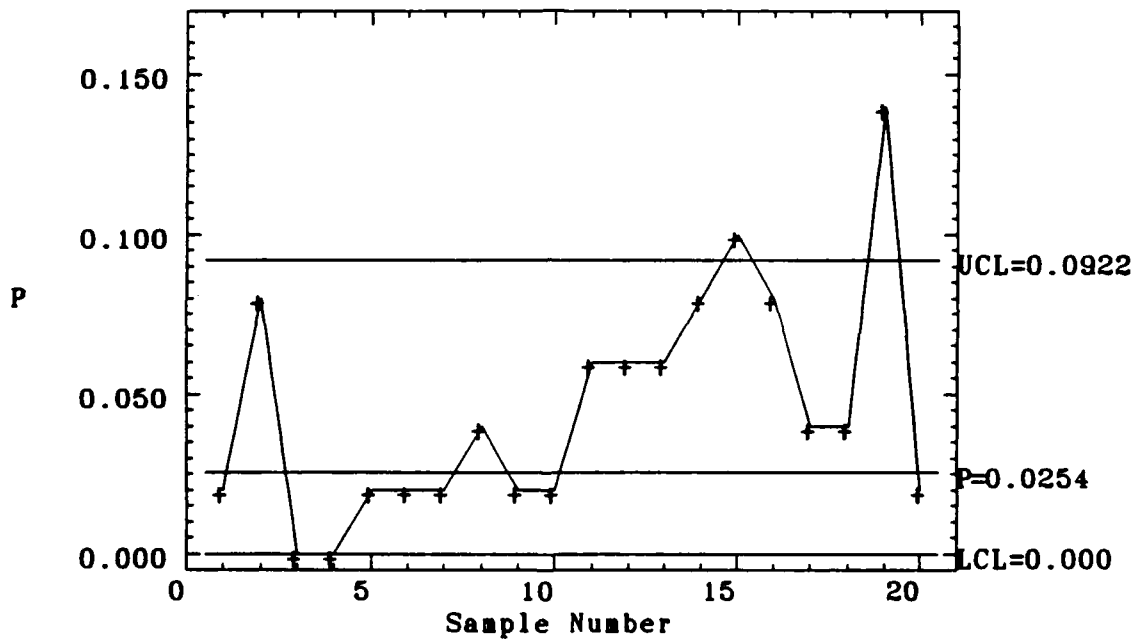
Figure 3.2 : Sample p Chart

Chapter 4

CONCLUSIONS AND RECOMMENDATIONS

A method for analytically determining the acceptance characteristics of a complex assembly process was developed. It was shown that the basic relationships needed for this analysis were also needed for the development of a simulation model, so this method did not require much more effort from the analyst than that required for a simulated result. The results of this analytic method were shown to be very useful for optimally allocating limited capital improvements and for assembly process control. An example problem was presented that illustrated this analytic methodology as well as the two applications that were developed.

There are many areas for further research in the area of assembly process allocation and control. One possible area for further research is the optimization problem. There may be some way to formulate the problem in a continuous form that is solvable by some nonlinear method. The problem can also be expanded to include the fabrication processes as well as the assembly processes, thus providing a more global optimization problem for a given system. The objective function may include many more factors. For example, if a candidate assembly process involves a piece of special tooling that not only reduces assembly variance but is also more efficient for the person doing the assembly, the reduced labor costs may be added to

the objective. Finally, the structure of the modified integer programming problem is similar to a network type of problem. If the constraint matrix can be manipulated to make the problem into a network type of problem, a faster solution may be possible.

The Box-and-Cover problem also has many possibilities for further research. If some of the random variables in this problem can be combined to form joint distributions, the numerical integrations in equations (38) and (40) may become feasible. This eliminates the need for an approximate formulation of the problem. Also, the possible forms for the approximate solution of this problem (i.e., using truncated distributions in the General assembly case) can be further researched to provide more accurate results.

The final area for possible research is to expand this method to electronic and electromechanical types of assemblies. Since purely mechanical assemblies are only one small segment of industry today, a method that can be used for different types of assembly processes can be very useful. For example, the expected number of defectives for an electronic assembly might possibly be used to predict the expected yield of the assembly through an acceptance test procedure. This expected yield can then be used to statistically control the assembly process. This eliminates the need for arbitrary target levels for acceptance yield that are commonly used, even though they have no statistical basis. Management of assembly processes is more effective when the foundation of that management is statistically developed and analytically obtained. Management by arbitrary goals

and simulated results is always less than optimal.

One area for additional study is to develop a generalized computer program that can handle a variable number of summations. Ideally, the user is able to input the distribution parameters and the limits of each sum interactively. The program should be written in a compilable language to help speed the numerous calculations. In addition, the program should minimize the number of times that the normal probability subroutine is called from within a summation. This can be accomplished through a clever use of arrays. The program in the Appendix is written in BASIC since every computer has some form of BASIC available with the initial software.

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Appendix

COMPUTER PROGRAM USED TO PERFORM

NUMERICAL ANALYSIS

```
20 REM  This part of the program is used to compute std normal probs
21 REM
25 DIM Y(6), C(6)
35  DEFDBL A-H
40  DEFDBL O-Z
44 REM  Function to implement change of variable
45 DEF FND (YT) = BO + B1 * YT
49 REM std normal function
50 DEF FNF (W) = 1! / (EXP((W ^ 2) / 2) * SQR(2 * 3.1415927#))
55 A = 0
60 REM Input of weighting factors for Gauss-Legendre Formula
65 FOR I = 1 TO 3
70 READ C(I)
75 C(7 - I) = C(I)
80 NEXT I
85 DATA .171324492,.360761573,.467913935
90 REM Input of function arguments
95 FOR I = 1 TO 3
100 READ Y(7 - I)
105 Y(I) = -Y(7 - I)
110 NEXT I
115 DATA .932469514,.661209386,.238619186
116 REM all of the above statements are used in the subroutine
117 REM now start the numerical integrations
118 REM
120 REM input parameters of given distributions
125 INPUT "mul ="; MU1
130 INPUT "sigmal ="; SIGMA1
135 INPUT "mu2 ="; MU2
140 INPUT "sigma2 ="; SIGMA2
145 INPUT "mu3 ="; MU3
150 INPUT "sigma3 ="; SIGMA3
160 REM initialise suml and sum2
165 SUM1 = 0
170 SUM2 = 0
175 L = 18
180 W = 24
181 A1 = 6
182 A2 = 1.9
183 A3 = 1.9
```

```
185 DELTA = .212
200 REM
210 REM   compute limits on first summation
211 REM
215 UPLIM1 = 4 * SIGMA1 + MU1
220 LOLIM1 = -4 * SIGMA1 + MU1
225 INCR1! = (UPLIM1 - LOLIM1) / 50
230 REM
231 REM   compute limits on second summation
232 REM
235 UPLIM2 = 4 * SIGMA2 + MU2
240 LOLIM2 = -4 * SIGMA2 + MU2
245 INCR2! = (UPLIM2 - LOLIM2) / 50
250 REM
251 REM   initiate first summation
252 REM
255 BOT1 = LOLIM1 - INCR1
260 FOR M = 1 TO 50
270 BOT1 = BOT1 + INCR1
280 TOP1 = BOT1 + INCR1
290 PT1 = (BOT1 + TOP1) / 2
300 B = (BOT1 - MU1) / SIGMA1
310 GOSUB 2000
320 PR1BOT = CUM
330 B = (TOP1 - MU1) / SIGMA1
340 GOSUB 2000
350 PR1TOP = CUM
360 PR1 = PR1TOP - PR1BOT
370 IF PR1 < 0 THEN PR1 = 0
375 REM
376 REM initiate second summation
377 REM
380     BOT2 = LOLIM2 - INCR2
390     FOR N = 1 TO 50
400     BOT2 = BOT2 + INCR2
410     TOP2 = BOT2 + INCR2
420     PT2 = (BOT2 + TOP2) / 2
430     B = (BOT2 - MU2) / SIGMA2
440     GOSUB 2000
450     PR2BOT = CUM
460     B = (TOP2 - MU2) / SIGMA2
470     GOSUB 2000
480     PR2TOP = CUM
490     PR2 = PR2TOP - PR2BOT
500     IF PR2 < 0 THEN PR2 = 0
505 REM
506 REM calculate probability for inner expression
507 REM
510 REM first calculate the lower bound
511 REM
```

```
515               BOT3A = L - DELTA - A1 - PT2 + PT1
520               BOT3B = BOT3A
524               BOT3 = BOT3A
528               IF (BOT3A - BOT3B) < 0 THEN BOT3 = BOT3B
530 REM
531 REM then calculate the upper bound
532 REM
535               TOP3A = L + DELTA - A1 - PT2 + PT1
540               TOP3B = A1 - PT1
545               TOP3 = TOP3A
547               IF (TOP3B - TOP3A) < 0 THEN TOP3 = TOP3B
550               B = (BOT3 - MU3) / SIGMA3
560               GOSUB 2000
570               PR3BOT = CUM
580               B = (TOP3 - MU3) / SIGMA3
590               GOSUB 2000
600               PR3TOP = CUM
605               PR3 = PR3TOP - PR3BOT
610               IF PR3 < 0 THEN PR3 = 0
620        SUM2 = SUM2 + PR3 * PR2
630        NEXT N
640 SUM1 = SUM1 + SUM2 * PR1
641 SUM2 = 0
645 PRINT "m ="; M
650 NEXT M
660 PRINT "sum1 ="; SUM1
670 STOP
2000 REM
2001 REM Subroutine calculates the area under a std normal curve
2002 REM
2005 IF B <= -4 THEN CUM = 0
2010 IF B >= 4 THEN CUM = 1
2015 IF B <= -4 OR B >= 4 THEN GOTO 2095
2020 A = 0
2025 SUM = 0
2030 IJ = 0
2035 IF B > 0 THEN GOTO 2055
2040 IF B = 0 THEN GOTO 2090
2045 B = -B
2050 IJ = 1
2055 B0 = (A + B) / 2
2060 B1 = (B - A) / 2
2065 FOR I = 1 TO 6
2070 SUM = SUM + C(I) * FNF(FND(Y(I)))
2075 NEXT I
2080 SUM = B1 * SUM
2085 IF IJ = 1 THEN SUM = -SUM
2090 CUM = .5 + SUM
2095 RETURN
```