AMBIVUITY BOOTSTRAPPING TO DETERMINE
GPS ORBITS AND BASELINES

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Ambiguity Bootstrapping to Determine
GPS Orbits and Baselines

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Abstract: For GPS satellite-orbit and interstation-baseline determination, the most accurate observable available is carrier phase, differenced between observing stations and between satellites to cancel both transmitter- and receiver-related errors. For maximum accuracy, the integer cycle ambiguities of the doubly differenced observations must be resolved. To perform this ambiguity resolution, Counselman (Eos, 68, 1238, 1987) proposed a bootstrapping strategy. This strategy requires the tracking stations to have a wide ranging progression of spacings. By conventional "integrated Doppler" processing of the observations from the most widely spaced stations, the orbits are determined well enough to permit resolution of the ambiguities for the most closely spaced stations. The resolution of these ambiguities reduces the uncertainty of the orbit determination enough to enable ambiguity resolution for more widely spaced stations, which further reduces the orbital uncertainty.

Abbot and Counselman (ibid., 1987) and Counselman and Abbot (JGR, 94, 7058-7064, 1989) applied this strategy to a network of six tracking stations spaced by 71 km, 245 km, ..., up to 4000 km. Resolving ambiguities for the shortest, 71-km baseline made it possible to resolve them for the next-longer, 245-km baseline, and reduced both the formal and the true errors of determining the GPS satellite orbits by a factor of 2. The precision of baseline determination was also significantly improved.

Ionospheric refraction interferes with ambiguity resolution, by systematically biasing the doubly-differenced phase observations. However, the signature of ionospheric refraction resembles that of orbital position error; either effect, although time-variable, is spatially coherent, characterized by a nearly uniform gradient across a few-hundred-kilometer-size tracking network. Thus, the same bootstrapping principle which facilitates ambiguity resolution in the presence of orbital uncertainty, can be effective in the presence of significant ionospheric refraction.

To test this prediction, Abbot, Counselman, and Gourevitch (Eos, in press, Fall 1989) analyzed GPS observations from a recent period of high solar activity, with daily observation periods spanning the morning hours during which the ionosphere varies most rapidly. The ionospheric refraction effects in these observations (5 am - noon, November 1988, in Texas) were some 20 times stronger than in the night-time, April 1985, observations originally studied by Abbot and Counselman.

Using a very simple, five-parameter, ionospheric model, Abbot et al. processed observations from 12 dual-band receivers which were arranged in a logarithmic "Nautilus" spiral with spacings from 10 to 320 km. The use of this model increased the interstation baseline length for which ambiguities could be resolved by a factor of two (to the maximum length available). Observations on successive days were processed independently; i.e., the ionospheric parameters, the position coordinates of nine receiving stations (three stations served as "fiducials"), and all the orbital elements of each satellite were determined from "single-day" arcs. The standard deviations of the horizontal station-position coordinate estimates were 2.5-4 mm, or 2-3 parts in 10^8 of the distance to the nearest fiducial.
The doubly differenced carrier phase observable:

$$\Delta \Delta \phi_{kq}^{ij} = -\left(\frac{1}{\lambda}\right) \Delta \Delta r_{kq}^{ij} + N_{kq}^{ij}$$

$\Delta \Delta r_{kq}^{ij}$ is the doubly differenced range between satellites $i, j$ and stations $k, q$;

$\lambda$ is the carrier wavelength; and

$N_{kq}^{ij}$, known as the "ambiguity parameter" or the "bias," is an integer number of cycles.
AMBIGUITY BOOTSTRAPPING:

1. The tracking stations should have a wide-ranging progression of spacings.

2. Conventional processing of observations from the most widely spaced stations (without any ambiguity resolution) determines the orbits well enough to permit resolution of ambiguities for the most closely spaced stations.

3. The resolution of these ambiguities reduces the uncertainty of the orbit determination enough to enable ambiguity resolution for more widely spaced stations, which further reduces the orbital uncertainty,...
Four widely spaced tracking stations: OV, FtD, Rich, Hays

Two additional stations, very close to OV: ML, Moj

ML–OV distance < 2% of OV–Hays distance.
OV–Moj distance = 6% .
Formal Standard Errors of Orbit Determination
With and Without Ambiguity Resolution
for ONLY THE CLOSEST (<6%) Station-Pairs
(all stations' phase obs'ns equally precise)

(m) 4

NAVSTAR Sat. Number

Without
With Ambiguity Resolution
ACTUAL Peak Errors of Orbit Determination
With and Without Ambiguity Resolution
for ONLY THE CLOSEST (<6%) Station-Pairs
(all stations used in either case)
GRADIENT OF IONOSPHERIC PHASE-SHIFT
Along E-W Axis of "Nautilus" Network
[L2-(60/77)L1 Cycles Per Unit Station-Spacing;
NAVSTAR 8 - 10 Difference]

Legend: (Station Nos.) / Spacing

- (3-1) / 1  - (1-5) / 1  - (7-3) / 2  - (5-9) / 4
- (11-7) / 8  - (11-9) / 16
IONOSPHERIC PHASE SHIFT
Along E-W Axis of “Nautilus” Network

L2-(60/77)L1 Cycles

Interstation Spacing →
(Unit = 3-1 spacing = 17 km)

Legend: Local Time on 4 Nov. 1988
-·- 10:43  -○- 10:50  -■- 10:57  -■- 11:04
-△- 11:11  -▲- 11:18  -▼- 11:25
STATION-POSITION (& ORBIT) DETERMINATIONS

Using observations from one day at a time, constraining just three fiducial-station positions, we estimated independently for each day

- all three position coordinates of each other station
- all six orbital elements of each satellite ("single day arcs")
- one tropospheric (zenith delay) parameter for each station except no. 1
- five ionospheric parameters [1 const. + 2x2 sin/cos(lat./lon.) coeff's]
- and a few receiver clock synchronization parameters,

and used bootstrapping to resolve all ambiguities from scratch each day.

Note: Antennas and receivers were replaced daily at all stations except nos. 5, 11, and 12.

STANDARD DEVIATIONS OF INDEPENDENT SINGLE-DAY DETERMINATIONS

from scatters of 17-d.o.f. samples (9 stns. on each of 3 days except for one receiver failure on one day):

LATITUDE

2.5 mm

LONGITUDE

4.0 mm

or, expressed as fractions of the distance to the nearest fiducial station (no. 10):

1.8 x 10^-8

2.9 x 10^-8
Formal Standard Errors of Orbit Determination
With the 240 x 320 km 'Nautilus' Network
(Single-Day Arc, 8 Nov. 1988)