AD-A217 779

NRL Report 9120

General Forms and Properties of Zero Cross-Correlation Radar Waveforms

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Target Characteristics Branch
Radar Division

January 30, 1990

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
January 30, 1990

3. REPORT TYPE AND DATES COVERED
Interim

4. TITLE AND SUBTITLE
General Forms and Properties of Zero Cross-Correlation Radar Waveforms

5. FUNDING NUMBERS
PE - 61153N
PR - 021-05-43
WU - DN480-006

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Research Laboratory
Washington, DC 20375-5000

8. PERFORMING ORGANIZATION REPORT NUMBER
NRL Report 9120

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Office of Chief of Naval Research
Arlington, VA 22217-5000

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

General forms for both the complementary and noncomplementary zero cross-correlation waveform sets are developed. Various properties of these codes and their relationship to zero sidelobe periodic codes are stated and proved. Some radar applications and practical considerations of using these codes are presented.

14. SUBJECT TERMS
Waveforms, Coding
Pulse compression, Radar

15. NUMBER OF PAGES
18

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

NSN 7540-01-280-5500

Standard Form 298 (Rev 2-89)
Prescribed by ANSI Std Z39-18
298-102

CONTENTS

GENERAL FORMS AND PROPERTIES OF ZERO CROSS-CORRELATION RADAR WAVEFORMS

1. INTRODUCTION

Modern radars generally incorporate pulse compression waveforms to obtain the desired range resolution while avoiding pulses having large peak powers. Pulse compression waveforms are exemplified by the Barker, pseudo-random shift register, chirp, and the polyphase codes [1-3]. This report describes new waveforms that have been recently investigated for use in radar systems. Of particular interest are multiple dissimilar waveforms having very low sidelobes after processing. Low sidelobes are desired to prevent the masking of weak targets in the sidelobes of strong targets or clutter returns. The multiple waveforms (whose number we set equal to M) are processed by individually match-filtering, time aligning, and summing the results.

The multiple waveforms considered in this report are derived from either complementary or noncomplementary waveforms. Complementary waveforms [4-9] are coded sequences (complex numbers in general) having autocorrelation functions (ACFs) (or equivalently the output of pulse compressors consisting of filters matched to the coded sequences) that sum to zero everywhere except at the match point when time aligned and added together. This is shown in Fig. 1 for $M = 2$.

In Refs. 2 and 3, new multiple waveforms were discussed that have zero cross-correlation response after combining the individual responses when filtered by a filter matched to a different waveform of the set. These waveforms have potential applications in cancelling stationary clutter from ambiguous ranges in a medium or high PRF radar and/or in reducing mutual interference between radars that are operating in the same frequency band in proximity to each other.

This report is an extension of that work presented in Refs. 2 and 3. Here we give general forms for both the complementary and noncomplementary zero cross-correlation waveform sets. In addition, various properties of these codes and their relationship to zero sidelobe periodic codes are stated and proved. A radar application using these codes is presented.

2. DEFINITIONS

In this section, we define our nomenclature and review the concept of periodic coded waveforms. A code word $\mathbf{a}$ is defined as a vector of length N, and

$$\mathbf{a} = (a_0, a_1, \ldots, a_{N-1}), \tag{1}$$

Manuscript approved September 1, 1989.

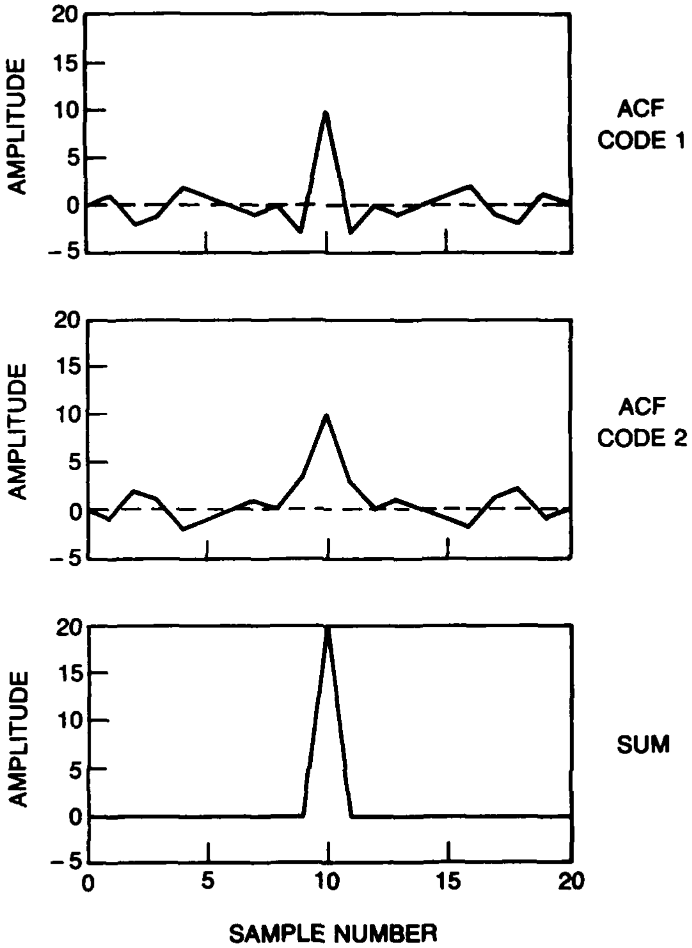


Fig. 1 — Complementary code example

where a_n, $n = 0, 1, \ldots, N-1$ are the elements of the code word. This code word modulates a carrier frequency and is match-filtered at baseband upon reception. The aperiodic autocorrelation function (ACF) of **a** is given by

$$r_a(k) = \sum_{i=0}^{N-1-k} a_i^* a_{i+k}, \quad k = 0, 1, \ldots, N-1,$$

$$r_a(-k) = \sum_{i=0}^{N-1-k} a_{i+k}^* a_i, \quad k = 1, 2, \ldots, N-1 \tag{2}$$

where * denotes complex conjugation. The $k = 0$ value of $r_a(k)$ corresponds to the match point, and the $k \neq 0$ values correspond to the right and left side sidelobes of the compressed pulse.

A periodic code is one that repeats the code word **a** indefinitely. Hence, if $\mathbf{a}_{pc}$ is the periodic code associated with **a** then

$$\mathbf{a}_{pc} = \mathbf{a} \circ \mathbf{a} \circ \mathbf{a} \ldots \tag{3}$$

where the symbol "○" denotes concatenation. On reception, a periodic code is match-filtered with its code word. The output of the correlation process is also periodic with a period, N. Hence, the matched-peak response repeats every N unit time delays as does the sidelobe response. We define the N point periodic autocorrelation function as

$$r_p(k) = \sum_{i=0}^{N-1} a_i^* a_{(i+k) mod N} , \quad k = 0, 1, \ldots, N-1. \tag{4}$$

Note that the $i + k$ subscript is taken modulo N. Thus we are computing the residue of $i + k$ with respect to the number of subpulses contained in the code word. For our development, we always compute the subscript with respect to the code order and drop the *mod* N notation from the subscript, thus $a_{N+i} = a_i$.

Define the vectors $\mathbf{h}_k$, $k = 0, \ldots, N-1$ as

$$\begin{aligned} \mathbf{h}_0 &= (a_0, a_1, \ldots, a_{N-1}), \\ \mathbf{h}_1 &= (a_1, a_2, \ldots, a_{N-1}, a_0), \\ \mathbf{h}_2 &= (a_2, a_0, \ldots, a_{N-1}, a_{0,}, a_1), \\ &\vdots \\ \mathbf{h}_{N-1} &= (a_{N-1}, a_0, a_1, \ldots, a_{N-2}), \end{aligned} \tag{5}$$

where these vectors are the circular rotations of $\mathbf{a}$. Equation (4) can be rewritten as

$$r_p(k) = \mathbf{h}_0^* \mathbf{h}_k^T , \quad k = 0, 1, \ldots, N-1 \tag{6}$$

where T denotes the vector transpose operation.

A zero sidelobe periodic code (ZSPC) has the property that

$$r_p(k) = \mathbf{h}_0^* \mathbf{h}_k^T = 0, \quad \text{for } k \neq 0. \tag{7}$$

If all the code elements of a ZSPC have unit amplitude, the code is called a perfect periodic code [1,2].

We now consider multiple waveforms. Define the code matrix C as an $M \times N$ matrix of code words:

$$C = \begin{bmatrix} c_{00} & c_{01} & \cdots & c_{0,N-1} \\ c_{10} & c_{11} & \cdots & c_{1,N-1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ c_{M-1,0} & c_{M-1,1} & \cdots & c_{M-1,N-1} \end{bmatrix} . \tag{8}$$

Let there be M code word of length N, where the mth code word ($m = 0, 1, \ldots, M - 1$) is defined by the $M + 1$th row of C or

$$\mathbf{c}_m = (c_{m0}, c_{m1}, \ldots, c_{m,N-1}) . \tag{9}$$

We define the aperiodic cross-correlation vector (CCV) between $\mathbf{c}_m$ and $\mathbf{c}_n$ as

$$\mathbf{c}_m^* \ast \tilde{\mathbf{c}}_n = (r_{-(N-1)}^{(mn)}, r_{-(N-2)}^{(mn)}, \ldots, r_0^{(mn)}, r_1^{(mn)}, \ldots, r_{N-1}^{(mn)}) , \tag{10}$$

where the bold asterisk $\ast$ denotes the linear convolution operation, $\sim$ denotes the time reversal of the sequence $\mathbf{c}_n$, and

$$r_k^{(mn)} = \sum_{i=0}^{N-1-k} c_{mi}^* c_{n,i+k} , \quad k \geq 0 , \tag{11}$$

$$r_{-k}^{(mn)} = \sum_{i=0}^{N-1-k} c_{m,i+k}^* c_{n,i} , \quad k > 0 . \tag{12}$$

Note that in general $|r_k^{(mn)}| \neq |r_{-k}^{(mn)}|$ unless $m = n$.

In addition, the summed CCV is defined as

$$\sum_{m=0}^{M-1} \mathbf{c}_m^* \ast \tilde{\mathbf{c}}_{m+l} = (q_{-(N-1)}^{(l)}, q_{-(N-2)}^{(l)}, \ldots, q_0^{(l)}, q_1^{(l)}, \ldots, q_{N-1}^{(l)}) . \tag{13}$$

We note that if

$$\sum_{m=0}^{M-1} \mathbf{c}_m^* \ast \tilde{\mathbf{c}}_m = (0, 0, \ldots, \overset{\substack{N\text{th position} \\ \downarrow}}{1}, 0, 0, \ldots, 0) , \tag{14}$$

then the code words of C form a complementary code set. If $|q_{-k}^{(l)}| = |q_k^{(l)}|$, then the summed CCV is called magnitude symmetric. Furthermore, if

$$\sum_{m=0}^{M-1} \mathbf{c}_m^* * \tilde{\mathbf{c}}_{m+l} = \mathbf{0}, \quad l \neq 0, \tag{15}$$

where $\mathbf{0}$ is a vector of $2N-1$ zeros, then we call the code words of C a zero cross-correlation code (ZCC).

In the following sections we consider periodic codes that are formed by concatenating the M rows of C. Thus a code word $\mathbf{a}$ is formed as

$$\mathbf{a} = \mathbf{c}_0 \circ \mathbf{c}_1 \circ \ldots \circ \mathbf{c}_{M-1}. \tag{16}$$

3. PROPERTIES OF ZCC COMPLEMENTARY WAVEFORMS

The relationship between ZCC complementary codes and their associated periodic code is stated in the following theorem.

Theorem 1: If the rows of C form a zero sidelobe periodic code (ZSPC), are a complementary code, and the summed CCV is magnitude symmetric, then the rows of C form a ZCC code.

Proof: Let us form the periodic code associated with C

$$\mathbf{h}_0 = (c_{00}, c_{01}, \ldots c_{0,N-1}, c_{10}, c_{12}, \ldots, c_{1,N-1}, c_{20}, \ldots, c_{N-1,N-1}). \tag{17}$$

The circular rotations of $\mathbf{h}_0$ are defined by Eq. (5).

Let $l = l_1 N + l_2$ where $l_2 \equiv l \bmod N$ and set $r_{\pm N}^{(mn)} = 0$ for all m, n. It is straightforward to show that for a ZSPC, $l \neq 0$

$$\mathbf{h}_0^* \mathbf{h}_l^T = \sum_{m=0}^{M-1} r_{l_2}^{(m,m+l_1)} + \sum_{m=0}^{M-1} r_{-(N-l_2)}^{(m,m+l_1+1)} = 0 \tag{18}$$

where $m+l_1$ and $m + l_1 + 1$ are taken modulo M. Now from Eq. (13) we know that

$$q_j^{(i)} = \sum_{m=0}^{M-1} r_j^{(m,m+i)}. \tag{19}$$

Note that $q_{\pm N}^{(i)} = 0$ because $r_{\pm N}^{(mn)} = 0$ for all m, n. It is instructive to write Eq. (18) out for successive values of l by using Eq. (19)

$$
\begin{aligned}
\mathbf{h}_0^*\mathbf{h}_1^T &= q_1^{(0)} + q_{-(N-1)}^{(1)} = 0\\
\mathbf{h}_0^*\mathbf{h}_2^T &= q_2^{(0)} + q_{-(N-2)}^{(1)} = 0\\
&\;\;\vdots\\
\mathbf{h}_0^*\mathbf{h}_{N-1}^T &= q_{N-1}^{(0)} + q_{-1}^{(1)} = 0\\
\\
\mathbf{h}_0^*\mathbf{h}_N^T &= q_N^{(0)} + q_0^{(1)} = 0\\
\mathbf{h}_0^*\mathbf{h}_{N+1}^T &= q_1^{(1)} + q_{-(N-1)}^{(2)} = 0\\
\mathbf{h}_0^*\mathbf{h}_{N+2}^T &= q_2^{(1)} + q_{-(N-2)}^{(2)} = 0\\
&\;\;\vdots\\
\mathbf{h}_0^*\mathbf{h}_{2N-1}^T &= q_{N-1}^{(1)} + q_{-1}^{(2)} = 0\\
\mathbf{h}_0^*\mathbf{h}_{2N}^T &= q_N^{(1)} + q_0^{(2)} = 0\\
\mathbf{h}_0^*\mathbf{h}_{2N+1}^T &= q_1^{(2)} + q_{-(N-1)}^{(3)} = 0\\
&\;\;\vdots
\end{aligned}
\tag{20}
$$

We note that every Nth equation of (20) is of the form $\mathbf{h}_0^*\mathbf{h}_{nN}^T = q_N^{(n-1)} + q_0^{(n)} = 0$, $n = 1,2,\ldots, N-1$. Since $q_N^{(n-1)} = 0$, it follows that $q_0^{(n)} = 0$ for $n = 1,2,\ldots, N-1$. It is seen that if the code words of C are complementary then $q_j^{(0)} = 0$ for $j \neq 0$. Thus using the first $N-1$ equations of (20) imply that $q_{-j}^{(1)} = 0$ for $j = 1, 2, \ldots, N-1$. If the summed CCV is magnitude symmetric, then $q_j^{(1)} = 0$ for $j = 1, 2, \ldots, N-1$. Hence by using the $(N+1)$th through $(2N-1)$th equation of (20), it follows that $q_{-j}^{(2)} = 0$. This argument can be repeated to show that $q_j^{(i)} = 0$ for all i, j except for when $i = j = 0$. Hence the theorem follows.

The following two theorems can be shown by using the same arguments:

Theorem 2: If C is a ZCC code and complementary, then C is also a ZSPC.

Theorem 3: If C is a ZCC code and a ZSPC, then C is complementary.

Next, consider the matrix

$$C_{aug} = \begin{array}{c} \hspace{20em} K \text{ zeros} \\ \begin{bmatrix} c_{00} & c_{01} & c_{02} & \cdots & c_{0,N-1} & 0\ 0 \ldots 0 \\ c_{10} & c_{11} & c_{12} & \cdots & c_{1,N-1} & 0\ 0 \ldots 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdots & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \\ c_{M-1,0} & c_{M-1,1} & c_{M-1,2} & \cdots & c_{M-1,N-1} & 0\ 0 \ldots 0 \end{bmatrix} \end{array} \tag{21}$$

where K is an arbitrary positive integer. This C_{aug} matrix is merely the original C matrix augmented with an $M \times K$ block of zeros. We will show that

Theorem 4: If C is a ZCC code and complementary, then C_{aug} is a ZSPC.

Proof: It is elementary to show that if C is a ZCC code and complementary, then C_{aug} is a ZCC code and complementary. Hence by using Theorem 2 the theorem follows.

4. GENERAL FORM OF ZCC COMPLEMENTARY WAVEFORMS

Consider the following $N \times N$ code matrix C where an element of C is defined by

$$c_{mi} = \lambda^m d_{i+1} W_N^{M'mi}, \quad m, i = 0, 1, \ldots, N-1, \tag{22}$$

where

$$W_N = e^{j\frac{2\pi}{N}}, \tag{23}$$

$$\lambda \epsilon \{1, W_N, W_N^2, \ldots, W_N^{N-1}\},$$

$d_1, d_2, \ldots, d_{N-1}$ are arbitrary complex numbers, and M' is an integer relatively prime to N. We show that

Theorem 5: The matrix C as defined by (22) is ZCC complementary code.

Proof: Using (11) and (12), we can show that

$$r_k^{(mn)} = \sum_{i=0}^{N-1-k} (\lambda^m d_{i+1} W_N^{M'mi})^* (\lambda^n d_{i+k+1} W_N^{M'n(i+k)})$$

$$= \lambda^{n-m} W_N^{M'nk} \sum_{i=0}^{N-1-k} d_{i+1}^* d_{i+k+1} W_N^{M'(n-m)i}, \tag{24}$$

$$r_{-k}^{(mn)} = \sum_{i=0}^{N-1-k} (\lambda^m d_{i+k+1} W_N^{M'm(i+k)})^* (\lambda^n d_{i+1} W_N^{M'ni})$$

$$= \lambda^{n-m} W_N^{-M'mk} \sum_{i=0}^{N-1-k} d_{i+k+1}^* d_{i+1} W_N^{M'(n-m)i}. \tag{25}$$

Thus if we set $n = m + l$, then

$$r_k^{(m,m+l)} = \lambda^l W_N^{M'(m+l)k} \sum_{i=0}^{N-1-k} d_{i+1}^* d_{i+k+1} W_N^{M'li}, \tag{26}$$

$$r_{-k}^{(m,m+l)} = \lambda^l W_N^{-M'mk} \sum_{i=0}^{N-1-k} d_{i+1} d_{i+k+1}^* W_N^{M'li}. \tag{27}$$

From these equations, it can be shown that

$$q_k^{(l)} = \sum_{m=0}^{N-1} r_k^{(m,m+l)} = \left[\sum_{m=0}^{N-1} W_n^{M'mk}\right] \lambda^l W_N^{M'lk} \sum_{i=0}^{N-1-k} d_{i+1}^* d_{i+k+1} W_N^{M'li}, \tag{28}$$

$$q_{-k}^{(l)} = \sum_{m=o}^{N-1} r_{-k}^{(m,m+l)} = \left[\sum_{m=0}^{N-1} W_N^{-M'mk}\right] \lambda^l \sum_{i=0}^{N-1-k} d_{i+1} d_{i+k+1}^* W_N^{M'li}. \tag{29}$$

Since

$$\sum_{m=0}^{N-1} W_N^{M'mk} = 0 \tag{30}$$

for M' relatively prime to N and $k \neq 0$, it follows that $q_k^{(l)} = q_{-k}^{(l)} = 0$ for $k, l \neq 0$. For $k = 0$ and $l \neq 0$, the second summation in both (28) and (29) is of the same form as (30). Thus $q_k^{(l)} = q_k^{(-l)} = 0$ for $k = 0$ and $l \neq 0$. Hence the theorem is proven.

We note that for $\lambda = d_1 = d_2 \ldots = d_N = M' = 1$ the general form reduces to the Frank matrix, which was shown in Ref. 2 to be a ZCC complementary waveform. In addition, if the Lewis-Kretschmer P4 code [1] has a length that is a square integer N^2, and the elements of this code are put into square matrix form where the concatenation of the rows generate the P4 code, then it is straightforward to show that this code also fits the general form given by (22) and hence is a ZCC complementary code.

5. ZCC NONCOMPLEMENTARY WAVEFORMS

In this section the following theorem is proved:

Theorem 6: If C has the form

$$C = \begin{bmatrix} a_0 b_0 & a_0 b_1 & \cdots & a_0 b_{N-1} \\ a_1 b_0 & a_1 b_1 & \cdots & a_1 b_{N-1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{M-1} b_0 & a_{M-1} b_1 & \cdots & a_{M-1} b_{N-1} \end{bmatrix} \tag{31}$$

and $\mathbf{a} = (a_0, a_1, \cdots, a_{M-1})$ *is a zero sidelobe periodic code, then the rows of C form a ZCC code.*

We call the code given by (31) an inner-outer code, because a given inner code of subpulses represented by $b_0, b_1, \ldots, b_{N-1}$ is modulated on a pulse-to-pulse basis by an outer code given by $a_0, a_1, \ldots, a_{M-1}$. We note that these waveforms have the value that M is arbitrary, whereas for the ZCC complementary waveforms, the number of code words in the matrix C always must be equal to the number of elements in a row of C.

Proof: The individual code elements are given by

$$c_{mi} = a_m b_i, \quad m, i = 0, 1, \ldots, N-1. \tag{32}$$

Using (11) and (12), it follows that

$$r_k^{(mn)} = \sum_{i=0}^{N-1-k} a_m^* b_i^* a_n b_{i+k}, \qquad k \geq 0, \tag{33}$$

$$r_{-k}^{(mn)} = \sum_{i=0}^{N-1-k} a_m^* b_{i+k}^* a_n b_i, \qquad k > 0. \tag{34}$$

Thus setting $n = m + l$

$$q_k^{(l)} = \sum_{m=0}^{M-1} r_k^{(m,m+l)} = \left(\sum_{m=0}^{M-1} a_m^* a_{m+l} \right) \left(\sum_{i=0}^{N-1-k} b_i^* b_{i+k} \right), \; k \geq 0, \tag{35}$$

$$q_{-k}^{(l)} = \sum_{m=0}^{M-1} r_{-k}^{(m,m+l)} = \left(\sum_{m=0}^{M-1} a_m^* a_{m+l} \right) \left(\sum_{i=0}^{N-1-k} b_{i+k}^* b_i \right), \; k > 0. \tag{36}$$

Since $\mathbf{a}$ is a ZSPC,

$$\sum_{m=0}^{M-1} a_m^* a_{m+l} = 0 \text{ for } l \neq 0.$$

Hence $q_k^{(l)}$ and $q_{-k}^{(l)}$ are equal to zero and the theorem follows.

6. RADAR APPLICATION EXAMPLE

In this section a radar application using the complementary or inner-outer waveforms described in the previous sections is briefly discussed. Only codes that are unit amplitude (or zero if the code element is turned off) are considered. These codes have the practical advantage that they are energy efficient on transmit. Thus for the general form of the ZCC complementary code given by (22), we stipulate that $d_1, d_2, \cdots, d_{N-1}$ must be on the unit circle.

Most radar waveforms do not have 100% duty cycles but have off-times that are used to listen for or receive the waveform. Hence the actual pulse train associated with the matrix C may look as shown in Fig. 2. Here each row of C forms a pulse (or group of subpulses). We define the code of the mth subpulse associated with the $m + 1$ row or pulse as

$$\mathbf{c}_m = (c_{m0}, c_{m1}, \cdots, c_{m,N-1}). \tag{37}$$

Each pulse is separated by a given pulse repetition interval (PRI_p) where there are "0"s transmitted between the end of one pulse and the beginning of the next. Normally this "off" time is greater than the pulse "on" time. All of the code words are transmitted in PRI_c seconds. Thereafter, they may be repeated with a period PRI_c for multiple burst processing.

One application of the ZCC complementary codes, which was first presented in Refs. 2 and 3 and is also applicable to ZCC inner-outer codes, is in removing ambiguous range radar returns for medium or high PRF radars. An example of this for a single burst is shown in Fig. 3 for $N = 4$. The waveforms are transmitted as shown in Fig. 3 according to the rows in C, but the return signals are processed only during the indicated processing interval in multiple channels having filters matched to the indicated codes in each pulse repetition interval. That is, after transmitting $\mathbf{c}_0$ in the processing interval, all received signals are processed by filters matched to $\mathbf{c}_0$, $\mathbf{c}_3$, $\mathbf{c}_2$, and $\mathbf{c}_1$ in channels 0 to 3 respectively, and so on. The result is that channel 0 is matched to the first unambiguous range interval and rejects stationary returns (those that have almost zero doppler shift) from the 2nd, 3rd and 4th time around range intervals. Likewise, channels 1, 2, and 3 are matched to the 2nd, 3rd and 4th time around returns and reject stationary clutter from the other range intervals. If the waveforms are complementary, stationary targets in the matched intervals have no sidelobes. Note that the fill pulses $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{c}_3$ are necessary for this processing scheme (as they would be for any ambiguous range radar). However, if multiple bursts are used in a particular look direction, then these fill pulses would be unnecessary, because the preceding single burst would provide the fill pulses for the current burst.

For example, the matched-filter response for a single burst of ZCC complementary waveforms is shown in Fig. 4 and for noncomplementary ZCC waveforms is shown in Fig. 5. From Fig. 4, we see that there are no sidelobes for the ZCC complementary waveforms. From Fig. 5, we observe

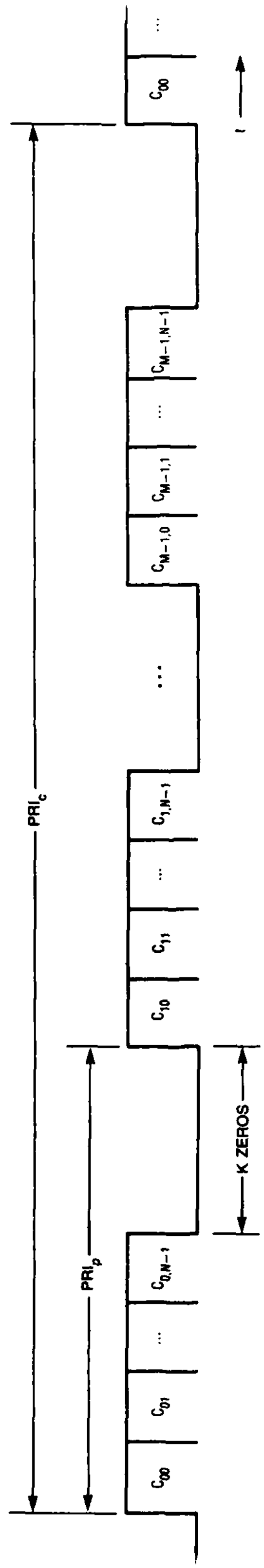
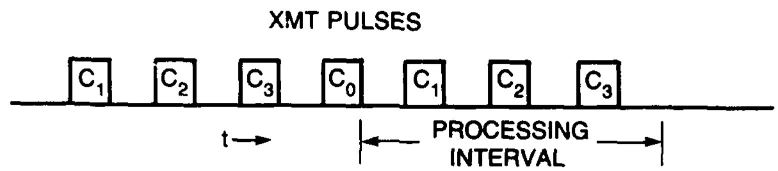


Fig. 2 — Transmitted multiple code



	MATCH TO			
CHANNEL: 0	C_0	C_1	C_2	C_3
1	C_3	C_0	C_1	C_2
2	C_2	C_3	C_0	C_1
3	C_1	C_2	C_3	C_0

Fig. 3 — Example of orthogonal waveform processing for $N = 4$

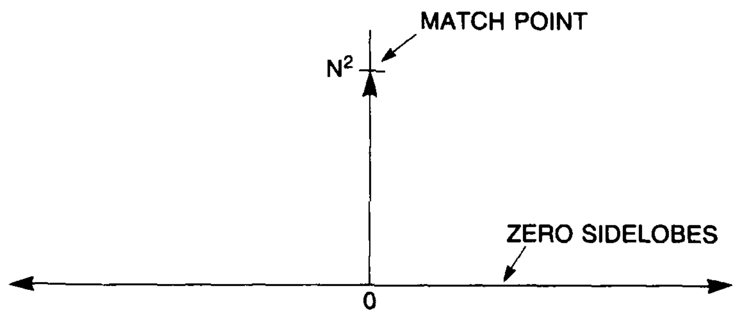


Fig. 4 — ACF for ZCC complementary waveforms

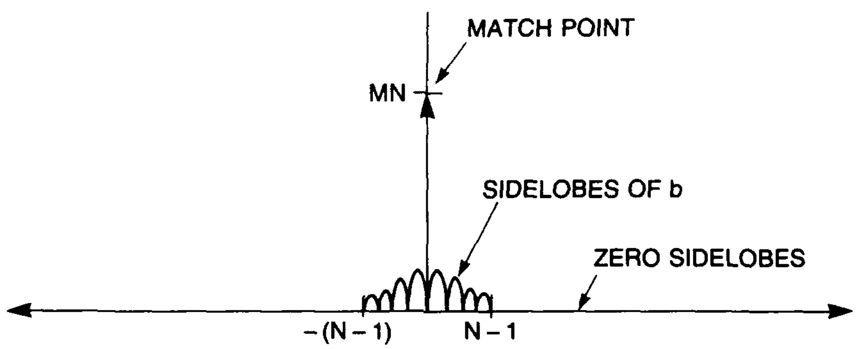


Fig. 5 — ACF for noncomplementary ZCC waveforms

that the sidelobes are nonzero only in the first $N - 1$ near-in right and left sidelobes about the match point for the noncomplementary ZCC waveforms. In fact, these sidelobes correspond to the sidelobes of the autocorrelation function (ACF) of the codeword **b** times M where the sidelobes level is measured relative to the match point gain MN. Finally, we note that for clutter having a small spectral spread about zero doppler, the nonambiguous range clutter can be reduced by using MTI processing. The PRI of the MTI canceller would equal PRI_c.

7. SUMMARY

In this report we have described the properties of zero cross-correlation waveform codes, i.e. the cross-correlation responses sum to zero everywhere. These codes, in turn, are related to periodic codes having zero sidelobe autocorrelation functions. These ideal periodic codes are important in themselves because the underlying aperiodic codes usually have useful attributes such as low sidelobes and/or good doppler tolerance. This is exemplified by the Frank, P4, and shift register codes.

Two general forms of the zero cross-correlation codes were described. The first form consists of a sequence of dissimilar waveforms that have the additional property of being complementary. The second form consists of a sequence of waveforms that are identically coded except for an outer code that results in a different phase being associated with each repetitive waveform.

A processing scheme using multiple waveforms was described that uses the zero cross-correlation codes to eliminate zero doppler ambiguous range clutter that might occur in a medium or high PRF radar. For clutter having a small spectral spread about zero doppler, the nonambiguous range clutter is reduced in a manner similar to MTI processing. A detailed assessment of the tradeoffs, and the ability to resolve the true range of a target is the subject of future work.

8. REFERENCES

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